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Pages 174, 176, and 178, Tables 13, 15, and 17, Points in right-hand diagrams should be numbered from 11 to 1, left to right.

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Page 1002, Equation (4), for

$$''\alpha = \frac{1}{\frac{E I n^2}{l^2} \frac{1}{E \sin\theta \cos^2\theta A}}'', \text{ read}$$

$$''\alpha = \frac{1}{1 + \frac{E I n^2}{l^2} \frac{1}{E \sin\theta \cos^2\theta A}}''.$$

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Paper No. 1827

AN ANALYSIS OF MULTIPLE-SKEW ARCHES ON ELASTIC PIERS

BY J. CHARLES RATHBUN,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. B. L. WEINER, G. D. HOUTMAN, CHARLES L. NORD, F. W. KEYSER, A. A. EREMIN, RICHARD M. HODGES, AND J. CHARLES RATHBUN.

SYNOPSIS

The purpose of this paper is to extend the analysis of the ring of a skew arch that was given in a previous paper[2] to include skew arches of more than one span and, by placing the skew equal to zero, to show how this method may be applied to the multiple right arch on elastic piers. In this latter case, the effect of eccentric loads is brought out by the formulas.

With the idea of being consistent with the method of analysis used in the first paper, the simple method of deflections was retained instead of using one of the more elegant methods, such as virtual work, or a procedure of a similar nature. In order that this paper may be more complete in itself an outline of the method of analysis of the single-span skew arch will be reviewed briefly. For the derivation of the formula for the deflection of a skewed cantilever under the various loads used, reference should be made to the previous paper.

As the use of the integral signs has had a tendency to confuse some of the designing engineers who might otherwise use this theory, the deflection formula is also given as a summation of a series of elastic blocks, the designer dividing the ring in somewhat the same manner as is customary in designing right arches. In this latter case, the terms due to ring-shortening, deformations due to shear, both radial and horizontal, and the terms containing the moment of inertia about the normal to the neutral surface may be omitted. Except possibly in rings of very unusual shapes, these terms are small, and

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NOTE.—Published in April, 1931, *Proceedings*.

[1] Associate Prof. of Civ. Eng., Coll. of the City of New York, New York, N. Y.

[2] "Analysis of the Stresses in the Ring of a Concrete Skew Arch," by J. Charles Rathbun, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

the effect of omitting them will not be as great as the approximation of dividing the ring into blocks of finite lengths. In the case of ribbed arches the latter term, however, may reach an appreciable value.

The paper is divided into four parts:

Part I.—Outline of the Theory of the Skew Arch and a description of the method of extending it to two or more spans. By placing the angle of skew equal to zero the effect of eccentrically placed loads on a multiple right arch is found, and the results are summarized briefly.

Part II.—Method of Computing the Unit Stresses that occur in the concrete, and the reinforcing steel required.

Part III.—Application of the Theory to the Analysis of a Skew Arch with approach spans for concentrated loads.

Part IV.—An Experiment to Verify the Computations of Part III.

The notation used in the paper is given in Appendix I; the formulas for the skew cantilever are given in Appendix II, and those for the two-hinged single-span skew arch ring, in Appendix III.

It is believed that the examples of Part III should make clear the method of dividing the multiple-skew arch on elastic piers into a series of skew cantilevers that are determinate structures and can be designed by the ordinary methods. The unusual features of these computations are given in Part III, while the method of reducing them to the forces, and of reducing moments to unit stresses, is given in Part II.

INTRODUCTION

The multiple-span skew arch bridge will be found to be a solution for the case shown in Figs. 1 and 2, in which the roadways are parallel on both sides of the crossing. By using three skew spans with the piers between consisting of posts, the curves at the crossing can be made comparatively easy and the line of sight correspondingly long.

In adopting this type of structure the wing-walls that are generally used to retain the fill are replaced by approach spans (see Fig. 2). These spans not only present a better appearance, but they also increase the line of sight in many cases. The extra cost of the approach span will be offset approximately by the saving in the cost of the wing-walls.

The ring of a skew arch is a statically indeterminate structure containing forces and moments in such positions that, in general, all cannot be made parallel to a single plane. It must be analyzed, therefore, in three dimensions instead of the two that are used in the right arch. However, for the preliminary analysis for dead load, or any other loading in which the funicular polygon may be made to follow the arch ring approximately (as viewed in the direction parallel to the barrel of the arch), and in which the loads are placed symmetrically with respect to a plane midway between spandrel walls, a section parallel to the spandrels can be used. The dead load and total uniform load usually approximate these requirements. This same situation prevails with respect to the multiple skew with elastic piers. The skew introduces certain forces and moments not present in right arches for loads on the

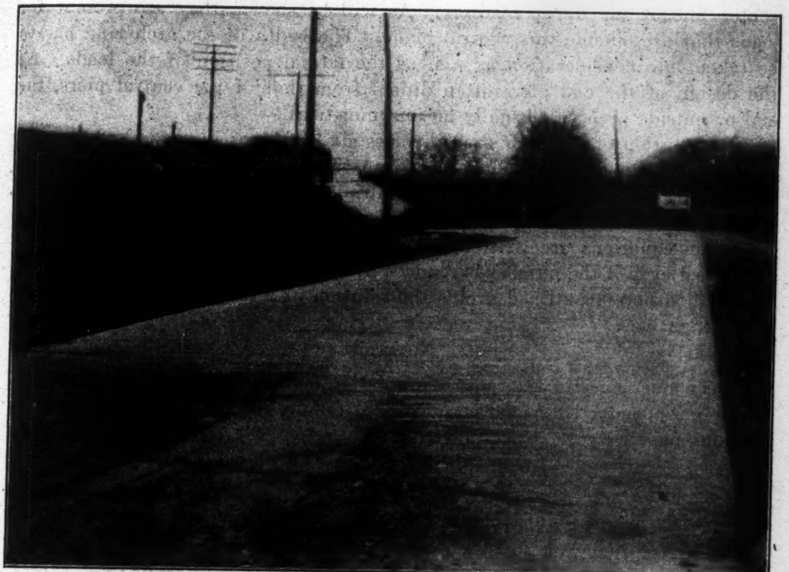


FIG. 1.—AN IDEAL SITE FOR A SKEW UNDER-CROSSING

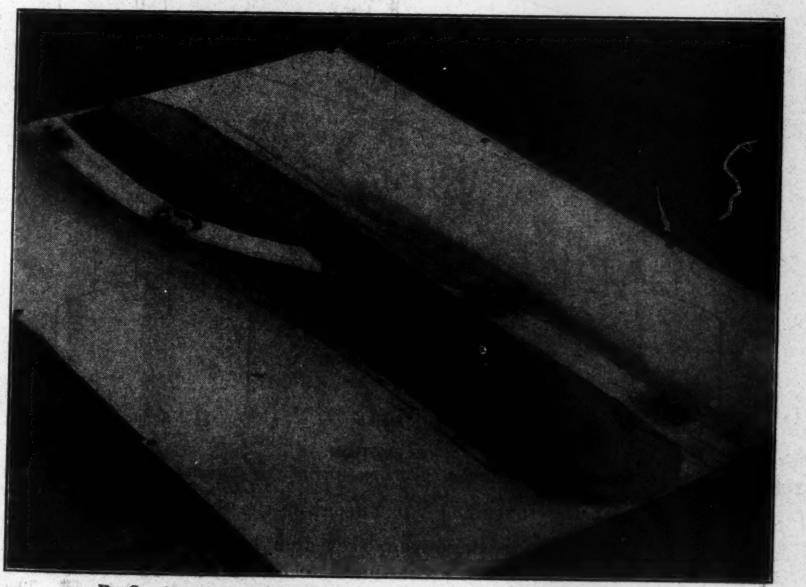


FIG 2.—MODEL OF MULTIPLE-SPAN SKEW UNDER-CROSSING, SHOWING INCREASED LINE OF VISION

center line of the upper roadway. As will be shown, they are present for loads that are outside this central plane. The width of the arch ring has to be taken into consideration as well as the lateral position of the loads. As the design of the end piers often differs from that of the central piers, the end or outside arch will usually be unsymmetrical.

The solution of the skew arch involves six equations in six unknowns. In the symmetrical arch these break up into two groups, one of four equations and one of two. In the case of the multiple-skew arch the addition of each pier will involve the addition of six more unknowns. If the corresponding deflection equations are written, and are solved simultaneously, the labor is very great even if the number is reduced by symmetry. Although it is lengthy, the method herein outlined avoids the solution of a large number of simultaneous equations.

NOTATION

For convenience of reference the notation adopted for use in this paper, is given in Appendix I. As far as possible it follows the notation of the writer's previous paper.

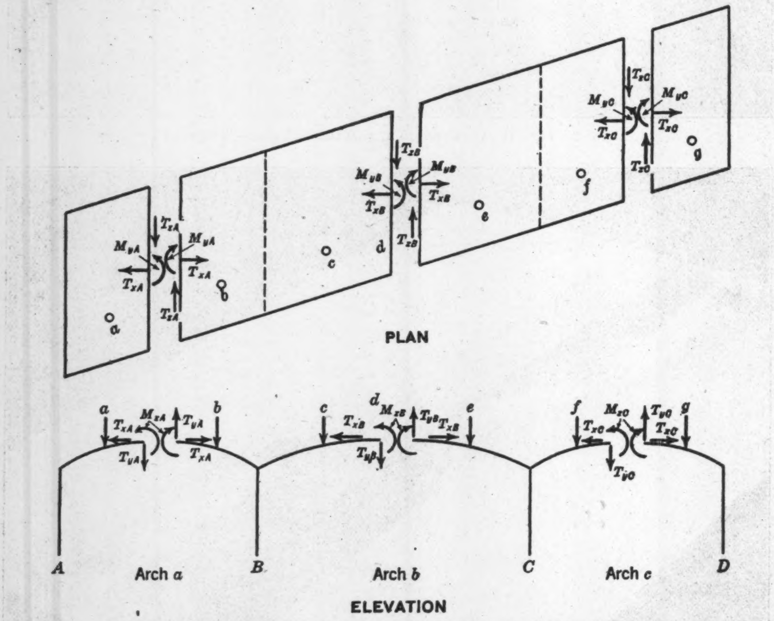
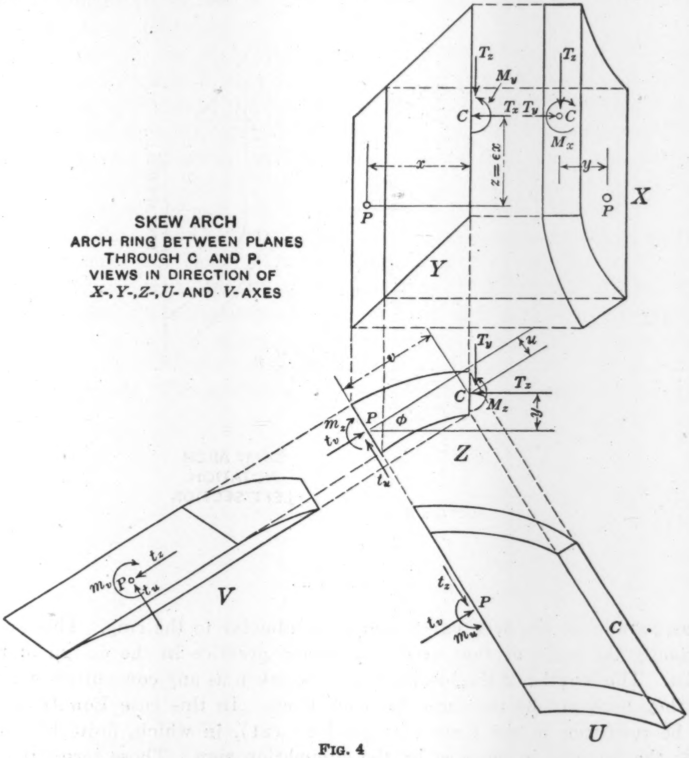

T_{zA}
M_{yA}
M_{yA}
T_{xA}
T_{xA}
T_{zA}
a
b
c
d
T_{zB}
M_{yB}
M_{yB}
T_{xB}
T_{xB}
T_{zB}
e
f
T_{zC}
M_{yC}
M_{yC}
T_{xC}
T_{xC}
T_{zC}
g
PLAN
a
T_{xA}
M_{xA}
T_{yA}
b
T_{yA}
T_{xA}
c
T_{xB}
d
M_{xB}
T_{yB}
T_{xB}
T_{yB}
e
f
T_{xC}
M_{xC}
T_{yC}
T_{xC}
g
T_{yC}
A
Arch a
B
Arch b
C
Arch c
D
ELEVATION

FIG. 3.—MULTIPLE-SKEW ARCH BRIDGE DIVIDED BY CUTTING PLANES

PART I.—OUTLINE OF THE THEORY OF THE SKEW ARCH

Let Fig. 3 represent a three-span, skew-arch bridge on vertical elastic piers (*A*, *B*, *C*, and *D*) which are assumed to be rigidly supported at their bases. The arch can be considered as loaded with concentrated loads. The ring can be divided into sections, as shown, by cutting planes, making a series of skewed cantilevers. If the arch ring has symmetry about any plane, as is usually the case, some labor can be saved by choosing that plane as the cutting plane.

First, consider the cantilever at the left, the one containing Pier *A*. This part is acted upon by the external loads and by three unknown forces and three unknown moments at the cutting plane. After the values of these six unknown quantities are obtained the stresses at any plane section of the cantilever can be determined by the usual processes of static mechanics from a diagram similar to Fig. 4.



SKEW ARCH
ARCH RING BETWEEN PLANES THROUGH C AND P.
VIEWS IN DIRECTION OF *X*-, *Y*-, *Z*-, *U*- AND *V*- AXES

FIG. 4

The deflection at the cutting plane of this cantilever can be computed from Equations (20) (see Appendix II), the unknown reactions being carried as algebraic quantities, which are to be solved subsequently. The derivation of this formula is not given because it was developed in full in the paper on the single-span skew arch.[3] The origin of co-ordinates has been taken at the center of the cut section and the usual rectangular co-ordinate system used. The method of choosing the co-ordinate system and the meaning of a part of the notation are given in Fig. 5. In order to shorten the formulas the terms, v and u, are introduced to replace the expressions:

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[3] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 640.

$$v = x \cos\phi + y \sin\phi \text{......................} (1)$$

and,

$$u = x \sin\phi - y \cos\phi \text{......................} (2)$$

If the arch is of a mathematical shape, Equations (20) may be used as written if the integrations present no difficulties. However, in most designs, the arch ring is of a shape that does not lend itself to this form of treatment, and the analyst must divide the ring into a finite number of blocks by means of planes parallel to the arch barrel and perpendicular to the ring. This process is exactly the same as that used in common practice in the design of right arches. The lengths of the blocks, s, may be taken as any convenient quantity, but they need not be the same for each block. In this case Equations (20) can be rewritten in the form of Equations (21), in which, finite blocks are used; the integral is replaced by the summation sign. Those terms that are normally small may be neglected, giving Equations (22). It is suggested that the designer check the writer's judgment in the matter of neglecting terms for any arch that he may be called upon to design. These neglected terms represent deformations due to shear, rib-shortening, and rotation about the radial axis. In rib arches the rotation, particularly, may be quite large. For the various types of loading terms it is suggested that the previous paper be consulted. The terms for vertical, dead load, uniform load, temperature,

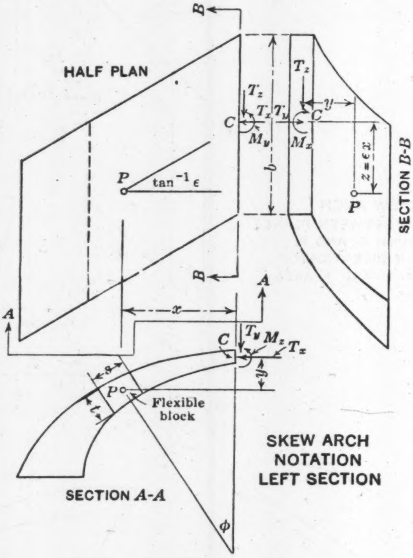

FIG. 5

and concentrated load at a point, $x' z'$, are given. The terms due to grade and horizontal earth pressure are not repeated.

If Equations (21) are used the length, thickness, co-ordinate of the center, as well as the slope of the arch ring, can be obtained by scaling. The values of u and v can also be obtained from the layout, or they may be computed.

The deflection of this cantilever is expressed in terms of its components along each of the three axes and its rotation about each of these axes. Thus, six quantities, or six equations of deflection, are set up, which involve the six unknown quantities at the cutting plane; namely, the three components of the force and the three components of the moment. These quantities of force and moment are independent of each other. This is also true of the six quantities of deflection.

Next, consider the cantilever containing Pier *B* in Fig. 3. Except for the effect of the arch, *b*, this cantilever can be treated in exactly the same way, and the deflection of the cut section can be obtained. The forces at this section are the same as those of the cantilever, *A*, but opposite in direction, being action and reaction. As the abutments of each cantilever are fixed, the movement of the crown of one is necessarily equal to the movement of the crown of the other in each of the directions or rotations, because they must coincide both before and after deflecting. These six conditions can be expressed as six equations, care being taken as to the selection of the sign of the movement. Thus, six equations are set up in terms of the known external loads, certain physical characteristics of the arch ring, and the six unknown literal reactions at the cutting plane. These are taken so that they are mutually independent. As all these equations are linear, they can be solved by simple algebraic methods. The mathematical treatment may be lengthy, but it is not involved.

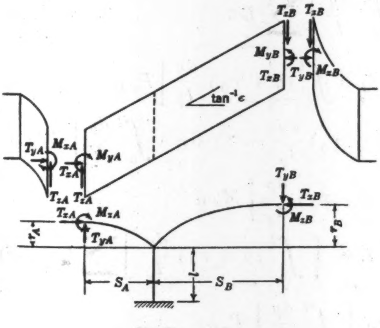

FIG. 6.—DIRECTIONS OF POSITIVE FORCES AND MOMENTS

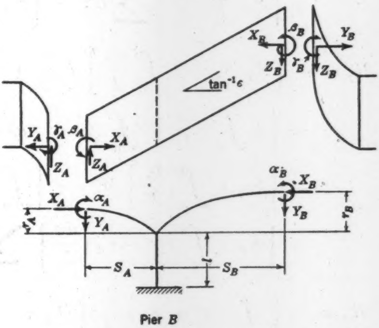

FIG. 7.—DIRECTIONS OF POSITIVE MOVEMENTS

If the skew structure consists of one span only, the indeterminate feature of the problem is complete. If another arch, such as Arch *b* in Fig. 3, is present, in computing the deflection of the right cantilever of Arch *a*, recognition must be taken of the deflection caused by the loads from this second arch. These loads consist of the six unknown components of the reaction at the cutting plane and the external loads between the cutting plane of Arch *b*, and the pier, *B*. By examining the forces acting on this cantilever in Fig. 6 and the direction of the movement of the crown in Fig. 7, one can write the following equations of deflection:

For the right side:

$$\begin{aligned} R_x = &-\left[\int_0^l \frac{\lambda}{A}\,dy + \int_0^l (y + r_A)(y + r_B)\frac{dy}{I_z} - S_A S_B \varepsilon^2 \lambda \int_0^l \frac{dy}{F}\right] T_{xB} \\ &+ \left[S_B \int_0^l (y + r_A)\frac{dy}{I_z}\right] T_{yB} - \left[S_A S_B \varepsilon \lambda \int_0^l \frac{dy}{F}\right] T_{zB} \\ &- \left[\int_0^l (y + r_A)\frac{dy}{I_z}\right] M_{zB} + \left[S_A \varepsilon \lambda \int_0^l \frac{dy}{F}\right] M_{yB} \\ &+ \left[S_B \int_0^l (y + r_A)\frac{dy}{I_z}\right] \sum L - \left[\int_0^l (y + r_A)\frac{dy}{I_z}\right] \sum L x' \;..(3a) \end{aligned}$$

$$\begin{aligned} R_y = &\left[S_A \int_0^l (y + r_B)\frac{dy}{I_z}\right] T_{xB} \\ &+ \left[\int_0^l \frac{dy}{A} - S_A S_B \int_0^l \frac{dy}{I_z} - S_A S_B \varepsilon^2 \int_0^l \frac{dy}{I_y}\right] T_{yB} \\ &+ \left[S_A \varepsilon \int_0^l (y + r_B)\frac{dy}{I_y}\right] T_{zB} + \left[S_A \int_0^l \frac{dy}{I_z}\right] M_{zB} \\ &- \left[S_A \varepsilon \int_0^l \frac{dy}{I_y}\right] M_{xB} \\ &+ \left[\int_0^l \frac{dy}{A} - S_A S_B \int_0^l \frac{dy}{I_z} - S_A S_B \varepsilon^2 \int_0^l \frac{dy}{I_y}\right] \sum L \\ &+ \left[S_A \int_0^l \frac{dy}{I_y}\right] \sum L z' + \left[S_A \int_0^l \frac{dy}{I_z}\right] \sum L x' \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3b) \end{aligned}$$

$$\begin{aligned} R_z = &-\left[S_A S_B \varepsilon \lambda \int_0^l \frac{dy}{F}\right] T_{xB} + \left[S_B \varepsilon \int_0^l (y + r_A)\frac{dy}{I_y}\right] T_{yB} \\ &- \left[\lambda \int_0^l \frac{dy}{A} + \int_0^l (y + r_B)(y + r_A)\frac{dy}{I_y} - S_A S_B \lambda \int_0^l \frac{dy}{F}\right] T_{zB} \\ &- \left[S_A \lambda \int_0^l \frac{dy}{F}\right] M_{yB} + \left[\int_0^l (y + r_A)\frac{dy}{I_y}\right] M_{xB} \\ &+ \left[S_B \varepsilon \int_0^l (y + r_A)\frac{dy}{I_y}\right] \sum L - \left[\int_0^l (y + r_A)\frac{dy}{I_y}\right] \sum L z' . (3c) \end{aligned}$$

$$\begin{aligned} R_\alpha = &-\left[\int_0^l (y + r_B)\frac{dy}{I_z}\right] T_{xB} + S_B \left[\int_0^l \frac{dy}{I_z}\right] T_{yB} \\ &- \left[\int_0^l \frac{dy}{I_z}\right] M_{zB} + S_B \left[\int_0^l \frac{dy}{I_z}\right] \sum L - \left[\int_0^l \frac{dy}{I_z}\right] \sum L x' \;\ldots (3d) \end{aligned}$$

$$R_\beta = \left[S_B \varepsilon \lambda \int_0^l \frac{dy}{F}\right] T_{xB} - \left[S_B \lambda \int_0^l \frac{dy}{F}\right] T_{zB} + \left[\lambda \int_0^l \frac{dy}{F}\right] M_{yB} \;\ldots (3e)$$

and,

$$\begin{aligned} R_\gamma = {} & \left[-S_B\,\varepsilon\int_0^l\frac{dy}{I_y}\right]T_{yB} + \left[\int_0^l(y+r_B)\frac{dy}{I_y}\right]T_z - \left[\int_0^l\frac{dy}{I_y}\right]M_x \\ & - \left[S_B\,\varepsilon\int_0^l\frac{dy}{I_y}\right]\sum L + \left[\int_0^l\frac{dy}{I_y}\right]\sum L z' \ldots\ldots\ldots\ldots (3f) \end{aligned}$$

For the left side :

$$\begin{aligned} L_x = {} & -\left[\int_0^l\frac{\lambda}{A}\,dy + \int_0^l(y+r_A)(y+r_B)\frac{dy}{I_z} - S_A S_B\,\varepsilon^2\lambda\int_0^l\frac{dy}{F}\right]T_{xA} \\ & - \left[S_A\int_0^l(y+r_B)\frac{dy}{I_z}\right]T_{yA} - \left[S_A S_B\,\varepsilon\lambda\int_0^l\frac{dy}{F}\right]T_{zA} \\ & - \left[\int_0^l(y+r_B)\frac{dy}{I_z}\right]M_{zA} - \left[S_B\,\varepsilon\lambda\int_0^l\frac{dy}{F}\right]M_{yA} \\ & + \left[S_A\int_0^l(y+r_B)\frac{dy}{I_z}\right]\sum L - \left[\int_0^l(y+r_B)\frac{dy}{I_z}\right]\sum L x' \,.\,.\, (4a) \end{aligned}$$

$$\begin{aligned} L_y = {} & \left[S_B\int_0^l(y+r_A)\frac{dy}{I_z}\right]T_{xA} \\ & + \left[S_A S_B\int_0^l\frac{dy}{I_z} - \int_0^l\frac{dy}{A} + S_A S_B\,\varepsilon^2\int_0^l\frac{dy}{I_y}\right]T_{yA} \\ & + \left[S_B\,\varepsilon\int_0^l(y+r_A)\frac{dy}{I_y}\right]T_{zA} + \left[S_B\int_0^l\frac{dy}{I_z}\right]M_{zA} \\ & - \left[S_B\,\varepsilon\int_0^l\frac{dy}{I_y}\right]M_{xA} \\ & - \left[S_A S_B\int_0^l\frac{dy}{I_z} - \int_0^l\frac{dy}{A} + S_A S_B\int_0^l\frac{dy}{I_y}\right]\sum L \\ & + \left[S_B\int_0^l\frac{dy}{I_z}\right]\sum L x' + \left[S_B\,\varepsilon\int_0^l\frac{dy}{I_y}\right]\sum L z' \ldots\ldots\ldots\ldots (4b) \end{aligned}$$

$$\begin{aligned} L_z = {} & -\left[S_A S_B\,\varepsilon\lambda\int_0^l\frac{dy}{F}\right]T_{xA} - \left[S_A\,\varepsilon\int_0^l(y+r_B)\frac{dy}{I_y}\right]T_{yA} \\ & + \left[S_A S_B\lambda\int_0^l\frac{dy}{F} - \lambda\int_0^l\frac{dy}{A} - \int_0^l(y+r_A)(y+r_B)\frac{dy}{I_y}\right]T_{zA} \\ & + \left[S_B\lambda\int_0^l\frac{dy}{F}\right]M_{yA} + \left[\int_0^l(y+r_B)\frac{dy}{I_y}\right]M_{xA} \\ & + \left[S_A\,\varepsilon\int_0^l(y+r_B)\frac{dy}{I_y}\right]\sum L - \left[\int_0^l(y+r_B)\frac{dy}{I_y}\right]\sum L z' \,.\, (4c) \end{aligned}$$

$$\begin{aligned} L_\alpha = {} & -\left[\int_0^l(y+r_A)\frac{dy}{I_z}\right]T_{xA} - \left[S_A\int_0^l\frac{dy}{I_z}\right]T_{yA} - \left[\int_0^l\frac{dy}{I_z}\right]M_{zA} \\ & + \left[S_A\int_0^l\frac{dy}{I_z}\right]\sum L - \left[\int_0^l\frac{dy}{I_z}\right]\sum L x' \ldots\ldots\ldots\ldots\ldots\ldots (4d) \end{aligned}$$

$$L_\beta = \left[S_A\,\varepsilon\lambda\int_0^l\frac{dy}{F}\right]T_{xA} - \left[S_A\lambda\int_0^l\frac{dy}{F}\right]T_{zA} - \left[\lambda\int_0^l\frac{dy}{F}\right]M_{yA} \,.\,.\, (4e)$$

and,

$$L_\gamma = \left[S_A\,\varepsilon \int_0^l \frac{dy}{I_y}\right] T_{y\,A} + \left[\int_0^l (y + r_A)\,\frac{dy}{I_y}\right] T_{z\,A} - \left[\int_0^l \frac{dy}{I_y}\right] M_{x\,A}$$
$$- \left[S_A\,\varepsilon \int_0^l \frac{dy}{I_y}\right] \sum L - \left[\int_0^l \frac{dy}{I_y}\right] \sum L\,z' \quad\ldots\ldots\ldots\ldots\ldots(4f)$$

It is to be noted in this cantilever that, except for forces and moments at the cut sections, the section from the crown of Arch *a* to the top of the pier has no force from Arch *b* acting on it, and also that the deformation of the section from the crown of Arch *b* to the top of the pier does not influence the deflection of the crown of Arch *a*. Therefore, the summation of the terms of deflection need be taken over the pier only. As this pier is usually vertical and straight, the formulas are written with this condition in mind. The deflections thus obtained must be added to those already computed for the right half of Arch *a*, before combining it with those of the left half to obtain the six equations. Incidentally, the deflection of the cut section of Arch *b*, due to the forces at the cut section of Arch *a*, can be written at once from Equations (3) producing Equations (4).

Solving the six equations obtained for Arch *a*, the values of the reactions at the crown of this arch are obtained in terms of all loads between the section of Arch *b* and the abutment of Pier *A*, and of the six reactions at the section of Arch *b*.

The cantilever consisting of Pier *B* and the left portion of Arch *b* can now be taken up in the same way as the cantilever of Arch *a*. The deflection of this cantilever is caused by the loads acting directly upon it and the reactions from the section of Arch *a*, as well as the six unknowns from the section of Arch *b*. For the deflection due to the loads on the ring of Arch *b*, Equations (20) or Equations (21) in Appendix II should be used, while for the deflection due to the reactions from Arch *a*, Equations (4) are to be taken. This deflection can thus be written in terms of the external loads and their reactions from the cut sections of Arch *a* and Arch *b*. The reactions from Arch *a* involve the unknown reactions from Arch *b*. Writing the deflections, equations similar to those for the left cantilever of Arch *a* are obtained after collecting the terms. No operations of a higher degree than the first are involved (six first-degree equations in six unknowns), in obtaining the reactions at the crown of Arch *b*. These represent the deflections of the system consisting of all of Arch *a* and the portion of Arch *b* to the left of the section.

If the system is symmetrical about the crown of the center span, as is true in the example given, the deflections of the arch on the right and the right side of Arch *b* will involve the same equations, except possibly for loading terms. Equating these six deflections against their corresponding six on the left half of the system gives four equations in four unknowns and two equations in two unknowns, which when solved fully determine the reactions in the ring of Arch *b*. If, however, the ring is not symmetrical, Arch *a*, together with the left portion of Arch *b*, can be treated in the same manner that was used on the left part of Arch *a*. The right part of Arch *b* is treated in the same way that was used on the right part of Arch *a*. In this case the reactions of the section of Arch *b* will be found in terms of the loads and the reactions from Arch *c*. This process must be continued until the arch on the extreme right is reached, when the right cantilever will have no unknown loads acting on it.

Having obtained the reaction of the right-hand arch, or the center arch in the case of symmetry, the reaction of the arch to the left can be found by substituting these values in the reaction at the cut section of this second arch. The process is continued until the left arch, or Arch *a*, is reached.

The entire system is now broken up into a system of cantilevers acted upon by known loads. By means of Fig. 4 and Equations (5), which are presented under "Method of Computing Unit Stresses", the stresses on any other section can be obtained.

It is interesting to note that if the skew is made equal to zero (a right multiple arch), the crown forces and moments, T_x, T_y, and M_z, are independent of the three remaining forces and moments. The six equations break up into two groups of three equations each. This independence remains throughout the entire set of computations. (Note Equations (30), (31), (32), and (33) which develop from Equations (20), (24), (3), and (4), by placing ϵ equal to zero.) As a result the equations and method herein given can be used to study the effect of loads that are not in the center of the upper roadway on a multiple arch with elastic piers, or, in fact, the effect on a single span right arch.

This procedure also gives a method of analysis for a right multiple arch on elastic piers which at no time involves the solution of more than three equations in three unknowns in one group. A discussion on the relative merits of this solution of the multiple arch and the others known to the profession would be valuable.

PART II.—METHOD OF COMPUTING THE UNIT STRESSES

As the writer showed[4] in the previous paper, the polygon of forces follows the central line of the skew arch as closely as possible. In an arch of the ideal shape, therefore, it follows this line exactly, and the distribution of forces and moments along the cutting plane, such as shown in Fig. 8, or the crown section, is uniform; that is, the thrust is uniformly distributed along this section while the shear is non-existent. If the adjustment is not so perfect the thrust can still be taken as uniformly distributed. Consider, therefore, any section at a point, *P*, Fig. 4. The computed components of the force on this section are t_v, t_u, and t_z, and the moments are m_z, m_u, and m_v. The force can also be resolved into three components: One parallel to the spandrel wall and perpendicular to the radius, t; another parallel to the radius, t_u; and the third, perpendicular to these two forces, s. The moment can be resolved into components around these axes. The main reinforcement steel is parallel to the force, t. This force can be considered as uniformly distributed over a section the area of which is the projected area of the section in the direction of this force, t, proper allowance being made for steel area. The shear, s, can be distributed over this same projected section using the parabolic distribution. This gives zero shear at the edges. At this place the force, of necessity, must be parallel to the spandrel walls. Three halves of the average shear value is the maximum at the center. These values will usually not be great enough to require reinforcement.

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[4] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

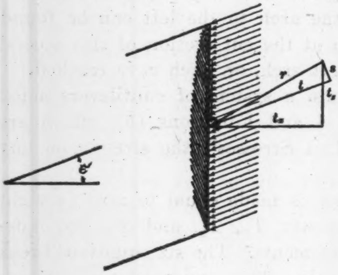

FIG. 8

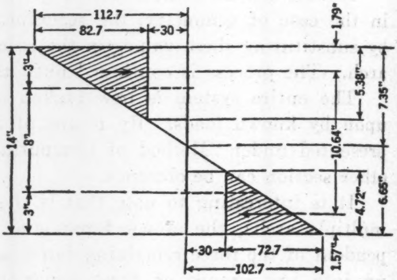

FIG. 9

The moment, *m*, about the axis parallel to *s* is treated in the same way as the force, *t*; it acts directly on the steel, and the projected area of the concrete is used in computing width. The torque, *f*, about the axis parallel to *t* will usually be small in value, but it can be computed on the projected section. The distribution of this stress is parabolic along each side of the section, with a maximum value in the center of the long side, and a zero value at the corners.

As the main reinforcing steel is parallel to the spandrel walls, the thrust and moment acting on this steel are next computed, as are the shear and thrust at the maximum point of the section; that is, the center of the top and bottom sides.

In making these computations after having obtained the crown reactions, the following formulas are used. Equations (5) are set up directly from Fig. 4, by taking forces and moments in the direction of and about the various axes.

$$t_v = T_x \cos \phi + T_y \sin \phi + V \sin \phi - H \cos \phi \quad \text{(5a)}$$

$$t_u = - T_x \sin \phi + T_y \cos \phi + V \cos \phi + H \sin \phi \quad \text{(5b)}$$

$$t_z = T_z \quad \text{(5c)}$$

$$m_z = T_x y - T_y x + M_z - M_{L\,z} \quad \text{(5d)}$$

$$m_u = T_x z \cos \phi + T_y z \sin \phi - T_z v + M_y \cos \phi + M_x \sin \phi + \varepsilon M_{L\,z} \sin \phi \quad \text{(5e)}$$

and,

$$m_v = - T_x z \sin \phi + T_y z \cos \phi + T_z u - M_y \sin \phi + M_x \cos \phi + \varepsilon M_{L\,z} \cos \phi \quad \text{(5f)}$$

For brevity substitute ϵ' for $\tan^{-1} \epsilon'$; then, by resolving forces about a set of axes making an angle of ϕ with the original set of *u*, *v*, *z*:

$$\varepsilon' = \varepsilon \cos \phi \quad \text{(6)}$$

$$t = t_v \cos \varepsilon' + t_z \sin \varepsilon' \qquad (7a)$$

$$s = t_v \sin \varepsilon' - t_z \cos \varepsilon' \qquad (7b)$$

$$m = m_z \cos \varepsilon' - m_v \sin \varepsilon' \qquad (7c)$$

and,

$$f = m_z \sin \varepsilon' + m_v \cos \varepsilon' \qquad (7d)$$

Example 1.—To illustrate the method here outlined a numerical example will be given. For this purpose an arch is chosen other than the one used to illustrate the method of computing the crown reactions in the multiple-skew arch, which is tested with the model. The following quantities have been measured or computed: $\epsilon = \tan 47° 11' = 1.079$; $b = 45.6$ ft.; crown thickness = 14 in.; and $b \cos 47° 11' = 31$ ft. = 372 in.

By methods described in Appendix II of this paper, the crown stresses are as listed in Table 1.

TABLE 1.—THRUST AND MOMENTS AT THE CROWN; EXAMPLE 1

Load	T_x	T_z	M_z	M_x
Dead	259.33	273.43	262.94	—234.37
Temperature	—37.92	—55.71	145.93	— 10.28
Uniform	6.16	6.04	20.50	— 16.21
Concentrated	13.80	13.44	104.20	—101.08

At the crown, $\phi = 0$; $\cos \phi = 1$; $\sin \phi = 0$; $\varepsilon = \varepsilon' = 1.079$; $\cos \varepsilon' = 0.6796$; and $\sin \epsilon' = 0.7335$. Substituting these values in Equations (7),

$$t = 259.33 \times 0.6796 + 273.43 \times 0.7335 = 176.35 + 200.57 = 376.92$$

$$s = 259.33 \times 0.7335 - 273.43 \times 0.6796 = 190.22 - 185.33 = 4.89$$

$$m = 262.94 \times 0.6796 + 234.37 \times 0.7335 = 178.71 + 171.92 = 350.63$$

$$f = 262.94 \times 0.7335 - 234.37 \times 0.6796 = 192.87 - 159.29 = 35.58$$

In a similar manner the stresses for the other loads may be computed. The values are as shown in Table 2.

TABLE 2.—STRESSES DETERMINED BY EQUATIONS (7); EXAMPLE 1

Load	t	s	m	f
Dead	376.92	4.89	350.63	33.58
Temperature, 50°	68.64	— 10.04	106.73	100.06
Uniform	8.62	0.42	25.80	4.00
Concentrated	19.28	1.04	140.04	7.76
Total with 50° drop	336.18	16.39	628.19	145.40
Total with 30° rise	446.00	0.32	457.47	— 14.66

It is now necessary to design a section, 14 in. thick and 372 in. wide, to carry the thrusts and moments in Table 2. The design for t and m presents no unusual problem, being the simple case of an eccentric load on a column.

The horizontal shear at the center equals $\frac{16.39 \times 1.5}{14 \times 372} = 5.0$ lb. per sq. in.

Shear due to torque, or f, using the formula for torque,

$$v = \frac{9\,f}{2\,b\,d^2} \qquad \text{(8)}$$

is equal to $\frac{9 \times 145.40 \times 12}{2 \times 372 \times 14^2} = 107.7$ lb. per sq. in., and the shearing stress on the section equals $\pm\, 107.7 + 5.0 = -\,102.7$, or $+\,112.7$ lb. per sq. in.

Thus, at the upper surface of the crown there is a shear stress of 112.7 lb. per sq. in., while on the lower surface the shear is 102.7 lb. per sq. in. Assuming a straight-line variation of shear between surfaces, Fig. 9 represents this distribution. If 30 lb. per sq. in. is allowed for concrete, a force represented by the volume of a wedge (base, 82.7 lb. per sq. in., height, 5.38 in., and depth, 12 in.) must be taken up by reinforcement on the upper surface. On the lower surface the force is likewise represented by the volume of a wedge, $\frac{72.7 \times 4.72 \times 12}{2}$. This reinforcement must be adequate to take the torque of these two forces when acting through a lever arm equal to the distance apart of the two rows of steel, in this case, 8 in.

Noting that the center of gravity of the triangle is one-third the distance from the base to the altitude and using a steel stress of 22 000 lb. per sq. in.,

$$\frac{82.7 \times 5.38 \times 12 \times 10.64}{2 \times 8 \times 22\,000} = 0.1614 \text{ sq. in.}$$

and,

$$\frac{72.7 \times 4.72 \times 12 \times 10.64}{2 \times 8 \times 22\,000} = 0.1245 \text{ sq. in.}$$

This steel is laid at right angles to the main reinforcement and if considered advisable, may be reduced toward the spandrel walls, since the shear is distributed according to the ordinates of a parabola.

The stresses in any other section than the crown present no unusual difficulties that have not been clarified in the foregoing computations.

PART III.—APPLICATION OF THE THEORY TO THE ANALYSIS OF A SKEW ARCH

In order to illustrate the application of this theory to the analysis of a structure, Fig. 10 will be taken as an example. The entire set of computations will not be given, but it is hoped that the outline will prove sufficiently complete to be followed readily by those familiar with the method as applied to the single-span skew arch.

Example 2.—The arch rings, including the outside piers (Fig. 10), have been divided into blocks of finite length and, by scaling, the values listed in Table 3 have been obtained. Furthermore, from Fig. 10, $b = 42$ ft.; $\varepsilon = \tan 34° 55' = 0.698$; and $\lambda = 2.5$.

In computing temperature the loading terms for Arch *a* are taken as:

$$L_x + R_x = 42 \times 23.5 \times 0.0000065 \times 2\,000 \times 144 = 1\,848$$

and,

$$L_z + R_z = \varepsilon \times 1\,848 = 1\,290$$

and for Arch *b* these terms are:

$$L_x = 42 \times 26.5 \times 0.0000065 \times 2\,000 \times 144 = 2\,083.5$$

and,

$$L_z = \varepsilon \times 2\,083.5 = 1\,454.31$$

TABLE 3.—SCALES, VALUES, AND COMPUTATIONS ACCOMPANYING EXAMPLE 2

Point	x	y	u	t	v	s	ϕ	x	y	t	u	s	v	ϕ
	LEFT HALF OF ARCH *a*							LEFT HALF OF ARCH *b*						
1	0.7	0.00	0.00	1.02	0.70	1.43	0.031	1.30	0.00	1.50	0.00	2.58	1.30	0.000
2	2.14	0.07	0.06	1.14	2.14	1.43	0.069	3.86	0.07	1.54	0.12	2.58	3.90	0.047
3	3.55	0.19	0.16	1.33	3.55	1.43	0.099	6.46	0.22	1.63	0.22	2.58	6.47	0.067
4	4.97	0.33	0.25	1.63	5.00	1.43	0.119	9.02	0.43	1.76	0.43	2.58	9.04	0.095
5	6.42	0.52	0.45	2.05	6.45	1.43	0.151	11.59	0.71	1.96	0.72	2.58	11.62	0.124
6	7.82	0.77	0.73	2.55	7.85	1.43	0.191	14.16	1.10	2.16	1.15	2.58	14.16	0.160
7	9.25	1.07	1.13	3.17	9.26	1.43	0.238	16.70	1.50	2.42	1.45	2.58	16.73	0.177
8	10.70	1.82	5.33	3.00	9.45	1.84	0.810	19.25	2.00	2.74	1.93	2.58	19.26	0.206
9	11.96	2.80	9.34	2.80	7.95	1.35	1.915	21.75	2.55	3.08	2.53	2.58	21.75	0.238
10	11.88	4.60	12.05	2.80	4.11	2.20	−22.000	24.20	3.06	3.67	1.30	2.50	24.36	0.186
11	11.82	6.80	12.05	2.62	6.32	2.20	−22.000	25.95	3.40	4.50	0.42	1.05	26.20	0.151
12	11.72	9.00	12.05	2.43	8.53	2.20	22.000	(26.50)	(3.50)	....	....	....	......	......
13	11.63	11.20	12.05	2.24	10.75	2.20	−22.000	......	......	....	....	....	......	......
14	11.55	13.40	12.05	2.05	13.96	2.20	−22.000	......	......	....	....	....	......	......
	RIGHT HALF OF ARCH *a* (POINTS 1 TO 7 ARE THE SAME AS THE LEFT SIDE)													
8	10.95	1.50	1.35	4.10	10.95	2.10	0.265	......	......	....	....	....	......	......

In both cases this is *b E*, or 42 *E* times the expansion in the direction of *X* or *Z*. The multiple, 42, is used because each term has the common factor, 42, cancelled from the denominator (as is also *E*), or 2 000 × 144 kips per sq. ft. The coefficient of expansion was taken as 0.0000065.

By computation from the scaled quantities in Table 3 the summations or approximations of the integrals used in the formulas are found to be as shown in Table 4. The summations are taken from the crown to the abutment, or from the load to the abutment in the case of partial summations for the left side of Arch *a*. In the case of the right side of Arch *a* and the left side of Arch *b*, the posts of the intermediate pier presented a problem somewhat different from the blocks. This section of the arch was omitted, therefore, and the effect of the posts was treated separately.

In this example the writer carried the computations to a larger number of significant figures than a practical designer would have done. This was done in order to study the results better. Furthermore, it required little more effort, because a multiplying machine was used. The values are published as computed, because it is felt that an experienced designer can balance time against accuracy as he sees fit, and would not take these computations as an example in this particular. Others may be interested in the numbers as computed.

The piers consist of five 3 by 3-ft. posts, 9 ft. 6 in. long and 9 ft. 8 in. on centers. They were assumed to be held rigidly, top and bottom. The

TABLE 4.—SUMMATION OF BLOCKS IN EXAMPLE 2

Quantity	SUMMATIONS, ABUTMENT TO CROWN			SUMMATIONS, ABUTMENT TO LOAD		
	Arch *a*		Arch *b*	Arch *a*		Arch *b*
	Left	Right	Left	Left	Right	Left
$\frac{s}{t}$	12.000	7.164	13.042	....	....	....
$\frac{s}{t^3}$	4.504	3.615	3.633	1.254	0.365	0.751
$\frac{s\,v}{t}$	635.846	270.922	2 322.118	....	....	....
$\frac{s\,u^2}{t}$	726.480	50.036	249.020	....	....	....
$\frac{s \sin \phi}{t}$	6.050	1.118	1.729	....	....	....
$\frac{s \sin^2 \phi}{t}$	5.292	0.456	0.548	5.262	0.426	0.504
$\frac{s \cos \phi}{t}$	6.757	6.764	12.606	....	....	....
$\frac{s \cos^2 \phi}{t}$	6.703	6.703	12.500	2.128	2.128	4.783
$\frac{s \sin \phi \cos \phi}{t}$	0.932	0.772	1.377	0.595	0.435	0.894
$\frac{s\,x \sin \phi \cos \phi}{t}$	6.617	4.895	21.216	5.519	3.797	17.123
$\frac{s\,x^2 \sin \phi \cos \phi}{t}$	52.298	38.729	377.995	53.022	34.453	339.598
$\frac{s\,u \cos \phi}{t}$	3.750	2.339	9.806	3.287	1.876	7.689
$\frac{s\,u\,x \cos \phi}{t}$	22.300	18.883	166.927	20.454	17.038	147.347
$\frac{s\,x \cos^2 \phi}{t}$	29.524	29.506	137.303	17.813	17.795	90.280
$\frac{s\,x^2 \cos^2 \phi}{t}$	196.646	196.157	2 157.617	155.555	155.065	1 771.467
$\frac{s\,u \sin \phi}{t}$	61.566	4.467	....	....	....	....
$\frac{s\,u\,x \sin \phi}{t}$	718.482	52.134	....	....	....	....
$\frac{s\,x \sin^2 \phi}{t}$	61.348	4.957	12.565	61.234	4.846	12.156
$\frac{s\,x^2 \sin^2 \phi}{t}$	714.270	56.408	300.789	713.808	55.946	300.695
$\frac{s\,v \cos \phi}{t}$	29.005	30.030	139.483	....	....	....
$\frac{s\,v\,x \cos \phi}{t}$	188.702	200.607	2 198.265	....	....	....
$\frac{s\,v \sin \phi}{t}$	51.322	9.678	26.822	50.212	8.568	22.718
$\frac{s\,v\,x \sin \phi}{t}$	578.481	95.777	524.447	574.153	91.448	485.723
$\frac{s\,x \sin^2 \phi}{t^3}$	10.637	0.591	1.036	....	....	....
$\frac{s\,x^2 \sin^2 \phi}{t^3}$	123.436	6.333	35.175	123.220	6.117	34.000
$\frac{s\,y}{t^3}$	9.075	1.056	2.433	8.783	0.764	1.806
$\frac{s\,y^2}{t^3}$	91.519	7.537	11.824	....	....	....
$\frac{s\,x}{t^3}$	20.085	9.740	29.313	13.278	2.932	13.504
$\frac{s\,x^2}{t^3}$	166.272	45.830	374.946	145.375	24.933	253.737
$\frac{s\,x\,y}{t^3}$	102.496	9.330	44.138	101.399	8.234	38.445

TABLE 4.—(*Continued.*)

Quantity	SUMMATIONS, ABUTMENT TO CROWN			SUMMATIONS, ABUTMENT TO LOAD		
	Arch *a*		Arch *b*	Arch *a*		Arch *b*
	Left	Right	Left	Left	Right	Left
$\frac{s\,u^2}{t^3}$	127.544	5.565	28.155	....	....	....
$\frac{s \sin \phi}{t^3}$	1.182	0.303	0.325	....	....	....
$\frac{s \sin^2 \phi}{t^3}$	0.927	0.064	0.074	....	....	....
$\frac{s \cos \phi}{t^3}$	3.578	3.564	3.578	....	....	....
$\frac{s \cos^2 \phi}{t^3}$	3.593	3.551	3.559	0.357	0.317	0.691
$\frac{s \sin \phi \cos \phi}{t^3}$	0.278	0.264	0.285	0.071	0.058	0.125
$\frac{s\,x \sin \phi \cos \phi}{t^3}$	1.166	1.033	3.523	0.590	0.458	2.252
$\frac{s\,x^2 \sin \phi \cos \phi}{t^3}$	7.143	5.782	52 870	5.110	3.749	41.459
$\frac{s\,u \cos \phi}{t^3}$	0.513	0.461	1.736	0.276	0.225	1.075
$\frac{s\,x \cos^2 \phi}{t^3}$	9.449	9.149	27 800	2.696	2.396	12.098
$\frac{s\,u\,x \cos \phi}{t^3}$	3.150	2.723	25.278	2.276	1.848	19.442
$\frac{x^2 \cos^2 \phi}{t^3}$	42.845	39.502	339.906	22.160	18.817	219.740
$\frac{s\,u \sin \phi}{t^3}$	10 753	0.513	1.248	....	....	....
$\frac{s\,u\,x \sin \phi}{t^3}$	125.199	5.812	30.379	....	....	....

evaluations of the deflections are given in a few cases in the following computations. As the other computations for deflection of the post are of a similar nature, they are here omitted.

The deflection of five posts due to the moment about the Y-axis equals:

Shaft action:

$$T = \frac{E_s\,F}{l}\,\theta = \frac{180 \times 3^4 \times 5 \times \theta}{2.5 \times 9.5 \times 2.05 \times \pi \times (3^2 + 3^2)} = 2.383\,\theta$$

Cantilever action:

$$T = 2\,(19\tfrac{1}{3}^{\,2} + 9\tfrac{2}{3}^{\,2})\,\frac{3 \times 8 \times 3^4}{12 \times 9.5^3 \times 2}\,\theta = 88.281\,\theta$$

Total:

$$T = (2.383 + 88.281)\,\theta = 90.664\,\theta$$

$$b\,\frac{\lambda\ s}{F} = \frac{42}{90.664} = 0.4632$$

$$\int y\,\frac{ds}{I_z} = \frac{12}{5 \times 3^4}\int_4^{13.5} y\,dy = \frac{12}{2 \times 5 \times 3^5}\,(13.5^2 - 4^2) = 2.463$$

$$\int y^2\,\frac{ds}{I_z} = \frac{12}{5 \times 3^4 \times 3}\,(13.5^3 - 4^3) = 23.668$$

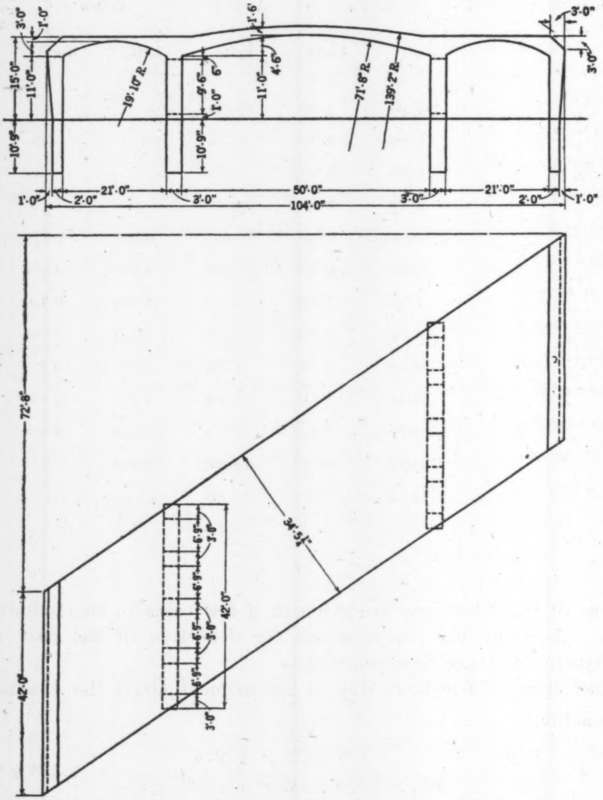
3'-0"
1'-0"
15'-0"
11'-0"
10'-9"
19'-10" R
9'-6"
6"
1'-0"
10'-9"
1'-6'
11'-0"
4'-6"
71'-8" R
139'-2" R
3'-0"
3'-0"
1'-0"
2'-0"
21'-0"
3'-0"
50'-0"
3'-0"
21'-0"
2'-0"
1'-0"
104'-0"
72'-8"
42'-0"
34'-5¼"
6'-9"
3'-0"
6'-9"
6'-5"
3'-0"
6'-9"
42'-0"
3'-0"

FIG. 10.—DIMENSIONAL DRAWING OF ARCH ANALYZED IN EXAMPLE 2

The following values are obtained for the various terms for the pier of five posts, in which allowance is made for the elasticity of the base:

$$\frac{b\,\lambda\,s}{F} = 0.3330 + 0.4632 = 0.7962$$

$$\frac{b\,s}{I_y} = 0.00227 + 42 \times 0.001125 = 0.00227 + 0.04725 = 0.04952$$

$$\frac{b\,s}{I_z} = 0.4444 + \frac{12 \times 42 \times 9.5}{3^4 \times 5} = 0.4444 + 11.8222 = 12.2666$$

$$\int (y + 4)\,\frac{b\,dy}{I_y} = 14 \times 0.00227 + \frac{b}{I_y}\int y\,dy + \frac{4\,b}{I_y}\int dy = 0.03175 + 0.41347 = 0.44522$$

and,

$$\int (y + 5.75)\,\frac{b\,dy}{I_y} = 15.75 \times 0.00227 + 0.49616 = 0.53188$$

Terms containing I_z are similarly computed.

For simplicity of arrangement, the equations for deflection of the skewed cantilevers may be written in the following form:

$$A\,T_x + B\,T_y + C\,T_z + D\,M_z + E\,M_y + F\,M_x + L_x \;\ldots\ldots (9a)$$
$$B\,T_x + G\,T_y + H\,T_z + I\,M_z + F\,M_y + K\,M_x + L_y \;\ldots\ldots (9b)$$
$$C\,T_x + H\,T_y + L\,T_z + N\,M_y + O\,M_x + L_z \;\ldots\ldots (9c)$$
$$D\,T_x + I\,T_y + P\,M_z + L_\alpha \;\ldots\ldots (9d)$$
$$E\,T_x + F\,T_y + N\,T_z + Q\,M_y + R\,M_x + L_\beta \;\ldots\ldots (9e)$$

and,

$$F\,T_x + K\,T_y + O\,T_z + R\,M_y + S\,M_x + L_\gamma \;\ldots\ldots (9f)$$

The coefficients are evaluated from the summations as given in Equations (20) and are made up of the following terms:

$A = 6.703 + 13.230 + 1\,098.224 + 0.652 + 1.765 + 537.922 = +\ 1\,658.496$
$B = -\ 1.399 - 1\,229.946 + 0.189 - 0.141 - 31.277 = -\ 1\,262.574$
$C = -\ 0.896 - 2.543 - 781.677 = -\ 785.118$
$D = \qquad = 108.900$
$E = +\ 0.140 + 0.217 + 66.411 = 66.768$
$F = +\ 0.031 - 0.023 - 7.278 = -\ 7.270$
$G = +\ 5.292 + 16.759 + 1\,995.259 + 2.367 + 0.486 + 186.717 = 2\,206.880$
$H = -\ 2.742 + 0.079 + 19.669 = 17.006$
$I = \qquad = -\ 241.024$
$K = 0.291 + 0.104 + 58.993 = 59.388$
$L = 30.000 + 4.325 + 3.684 + 1\,140.855 = 1\,178.864$
$N = -\ 0.197 - 0.312 - 96.181 = -\ 96.690$
$O = -\ 0.350 + 0.019 + 4.586 = 4.255$
$P = \qquad = 54.052$
$Q = +\ 0.032 + 0.067 + 20.721 = -\ 20.820$
$R = +\ 0.006 - 0.005 - 2.484 = -\ 2.483$
$S = 0.036 + 0.034 + 32.137 = 32.207$

The values of these coefficients for the right side of Arch *a* are computed as follows:

$A = 6.703 + 1.141 + 90.444 + 0.650 + 0.139 + 27.597 + (22.116) + (994.056) + (32.500) = 1\,175.396$
$B = -\ 1.158 - 111.959 + 0.128 - 0.096 - 25.196 - (1\,241.333) = -\ 1\,379.614$
$C = -\ 0.952 - 0.184 - 36.287 - (46.562) = -\ 83.985$
$D = 12.673 + (103.444) = 116.117$

$$\begin{aligned}
E &= 0.140 + 0.018 + 3.690 + (3.880) &&= 7.728\\
F &= 0.023 - 0.017 - 6.450 &&= -6.444\\
G &= 0.456 + 16.758 + 549.958 + 0.187 + 0.485\\
&\quad + 172.148 + (8.866) + (1\,720.400) + (3.316) &&= 2\,472.574\\
H &= -0.454 + 0.067 + 17.000 - (3.463) &&= 13.150\\
I &= -116.879 - 141.867 &&= -258.746\\
K &= 0.024 + 0.104 + 57.121 + 0.396 &&= 57.645\\
L &= 17.910 + 1.843 + 0.254 + 49.779 + 22.166\\
&\quad + 3.973 + 66.708 &&= 162.633\\
N &= -0.204 - 0.023 - 4.590 - 5.559 &&= -10.376\\
O &= -0.066 + 0.012 + 4.125 - (0.413) &&= 3.658\\
P &= 43.382 + 11.822 &&= 55.204\\
Q &= 0.032 + 0.006 + 1.442 + (.463) &&= 1.943\\
R &= 0.005 - 0.004 - 2.365 &&= -2.364\\
S &= 0.003 + 0.034 + 31.759 + 0.047 &&= 31.843
\end{aligned}$$

Deflections due to the post are enclosed in parentheses. To compute the deflections due to the loads, the partial summations are substituted in Equations (24), in which formula, the load is not at the crown.

For load at Point *a* (Fig. 3):

$$\begin{aligned}
L_x &= -0.893 - 1\,216.792 + 0.176 - 0.131 - 22.270\\
&\quad + 632.376 - 0.385 + 0.287 + 54.142 &&= -553.490\\
L_y &= 5.262 + 5.321 + 1\,744.501 + 2.366 + 0.384\\
&\quad + 96.572 - 956.002 - 4.271 - 0.926 - 247.271 &&= 645.936\\
L_z &= -2.726 + 0.072 + 14.210 + 5.017\\
&\quad - 0.245 - 36.274 &&= -19.946\\
L_\alpha &= -159.334 + 90.259 &&= -69.075\\
L_\beta &= 0.026 - 0.020 - 3.405 - 0.059 + 0.044 + 9.367 &&= 5.935\\
L_\gamma &= 0.291 + 0.063 + 16.835 - 0.526\\
&\quad - 0.158 - 46.864 &&= -30.349
\end{aligned}$$

For load at Point (*b*):

$$\begin{aligned}
R_x &= -0.652 - 98.804 + 0.114 - 0.085 - 16.339\\
&\quad + 55.008 + 0.114 - 0.085 - 18.307\\
&\quad - 1\,241.331 + 620.666 &&= -699.431\\
R_y &= 0.427 + 5.322 + 299.200 - 0.185 + 0.383\\
&\quad + 82.004 - 211.133 + 0.144 + 0.398\\
&\quad + 94.439 + 8.866 + 1\,702.399 + 3.315\\
&\quad - 851.200 + 7.498 &&= 1\,137.247\\
R_z &= -0.432 + 0.060 + 11.540 - 0.368 + 0.105\\
&\quad + 12.681 - 3.463 - 2.609 &&= 17.514\\
R_\alpha &= -35.189 + 26.244 - 141.867 + 70.933 &&= -79.879\\
R_\beta &= 0.018 - 0.013 - 2.858 + 0.019 - 0.014 - 3.275 &&= -6.123\\
R_\gamma &= 0.023 + 0.063 + 14.962 + 0.018 + 0.068\\
&\quad + 17.869 + 0.396 + 0.314 &&= 33.713
\end{aligned}$$

In the case of the crown loads, the value of the load for T_y and the load times z' for M_x may be substituted in Equations (9). With the co-ordinates, x', of Point (a) equal to 6.00, $z' = 14.688$, and with the co-ordinates, x', of Point (b) equal to 6.00, $z' = -6.312$.

From Equations (3) the deflections of the right cantilever of Arch a, due to the crown reactions and loads on Arch b are computed, as follows:

$$\begin{aligned} R_x = & -(23.0 + 1\,274.184 - 123.356)\,T_{x\,B} + 2\,906.152\,T_{y\,B} \\ & - 176.728\,T_{z\,B} + 109.666\,M_{z\,B} - 6.669\,M_{y\,B} + 1\,590.167\,c \\ & + 2\,906.152\,d \ldots\ldots\ldots\ldots \end{aligned} \tag{10a}$$

$$\begin{aligned} R_y = & \ 1\,573.600\,T_{x\,B} + (9.2 - 3\,900.800 - 7.672)\,T_{y\,B} + 4.455\,T_{z\,B} \\ & + 147.200\,M_{z\,B} + 0.4148\,M_{x\,B} - 2\,125.030\,c - 3\,894.910\,d \ldots \end{aligned} \tag{10b}$$

$$\begin{aligned} R_z = & -176.728\,T_{x\,B} + 8.235\,T_{y\,B} - (23.0 + 5.340 - 253.192)\,T_{z\,B} \\ & - 9.554\,M_{y\,B} + 0.4452\,M_{x\,B} + 0.169\,c + 3.561\,d \ldots\ldots\ldots \end{aligned} \tag{10c}$$

$$\begin{aligned} R_\alpha = & -131.133\,T_{x\,B} + 325.068\,T_{y\,B} - 12.267\,M_{z\,B} + 177.267\,c \\ & + 325.068\,d \ldots\ldots\ldots\ldots \end{aligned} \tag{10d}$$

$$R_\beta = 14.727\,T_{x\,B} - 21.099\,T_{z\,B} + 0.7962\,M_{y\,B} \ldots\ldots\ldots\ldots \tag{10e}$$

and,

$$\begin{aligned} R_\gamma = & -0.916\,T_{y\,B} + 0.532\,T_{z\,B} - 0.0495\,M_{x\,B} + 365.088\,c \\ & + 669.490\,d \ldots\ldots\ldots\ldots \end{aligned} \tag{10f}$$

Equations (10) may be summed up, as follows:

$$\begin{aligned} R_x = & -1\,171.927\,T_{x\,B} + 2\,906.152\,T_{y\,B} - 176.728\,T_{z\,B} \\ & + 109.666\,M_{z\,B} - 6.669\,M_{y\,B} + 1\,590.167\,c + 2\,906.152\,d \ldots \end{aligned} \tag{11a}$$

$$\begin{aligned} R_y = & \ 1\,573.600\,T_{x\,B} - 3\,899.265\,T_{y\,B} + 4.455\,T_{z\,B} + 147.200\,M_{z\,B} \\ & - 0.4148\,M_{x\,B} - 2\,125.030\,c - 3\,894.910\,d \ldots\ldots\ldots\ldots \end{aligned} \tag{11b}$$

$$\begin{aligned} R_z = & -176.728\,T_{x\,B} + 8.235\,T_{y\,B} - 224.995\,T_{z\,B} + 9.5544\,M_{y\,B} \\ & - 0.4452\,M_{x\,B} + 0.169\,c + 3.561\,d \ldots\ldots\ldots\ldots \end{aligned} \tag{11c}$$

$$\begin{aligned} R_\alpha = & -131.133\,T_{x\,B} + 325.068\,T_{y\,B} - 12.267\,M_{z\,B} + 177.267\,c \\ & + 325.068\,d \ldots\ldots\ldots\ldots \end{aligned} \tag{11d}$$

$$R_\beta = 14.727\,T_{x\,B} - 21.099\,T_{y\,B} + 0.7962\,M_{y\,B} \ldots\ldots\ldots\ldots \tag{11e}$$

and,

$$\begin{aligned} R_\gamma = & -0.916\,T_{y\,B} + 0.532\,T_{z\,B} - 0.0495\,M_{x\,B} + 365.088\,c \\ & + 669.490\,d \ldots\ldots\ldots\ldots \end{aligned} \tag{11f}$$

The deflections of the left cantilever of Arch b, due to the crown reactions of Arch a, can be written at once without further computation; thus:

$$\begin{aligned} L_x = & -1\,171.927\,T_{x\,A} - 1\,573.600\,T_{y\,A} - 176.728\,T_{z\,A} \\ & - 131.133\,M_{z\,A} - 14.727\,M_{y\,A} + 786.800\,b \ldots\ldots\ldots\ldots \end{aligned} \tag{12a}$$

$$\begin{aligned} L_y = & \ 2\,906.152\,T_{x\,A} + 3\,899.265\,T_{y\,A} + 8.235\,T_{z\,A} + 325.068\,M_{z\,A} \\ & - 0.916\,M_{x\,A} - 1\,931.582\,b \ldots\ldots\ldots\ldots \end{aligned} \tag{12b}$$

$$\begin{aligned} L_z = & \ 176.728\,T_{x\,A} + 4.455\,T_{y\,A} - 224.995\,T_{z\,A} - 21.099\,M_{y\,A} \\ & - 0.532\,M_{x\,A} - 5.585\,b \ldots\ldots\ldots\ldots \end{aligned} \tag{12c}$$

$$L_\alpha = -109.666\,T_{x\,A} - 147.200\,T_{y\,A} - 12.267\,M_{z\,A} + 73.600\,b \ldots \tag{12d}$$

$$L_\beta = 6.669\, T_{x\,A} - 9.554\, T_{z\,A} - 0.7962\, M_{y\,A} \ldots\ldots\ldots\ldots (12e)$$

$$L_\gamma = 0.4148\, T_{x\,A} + 0.4452\, T_{z\,A} - 0.0495\, M_{x\,A} + 0.5292\, b \ldots\ldots (12f)$$

The equations for obtaining the crown reactions of Arch *a* may now be set up by placing the deflections of the left side equal to those of the right side, thus producing the following six equations:

$$\begin{gathered} 2\,833.892\, T_{x\,A} + 117.162\, T_{y\,A} - 869.103\, T_{z\,A} + 225.017\, M_{z\,A} \\ + 59.040\, M_{y\,A} - 13.714\, M_{x\,A} = 1\,171.927\, T_{x\,B} - 2\,906.152\, T_{y\,B} \\ + 176.728\, T_{z\,B} + 109.666\, M_{z\,B} - 6.669\, M_{y\,B} + 1\,848.00\, t^\circ \\ + 553.490\, a + 699.431\, b - 1\,590.167\, c - 2\,906.152\, d \ldots\ldots (13a) \end{gathered}$$

$$\begin{gathered} 117.162\, T_{x\,A} + 4\,679.454\, T_{y\,A} + 3.856\, T_{z\,A} + 17.722\, M_{z\,A} \\ - 13.644\, M_{y\,A} + 1.743\, M_{x\,A} = 1\,573.600\, T_{x\,B} - 3\,899.265\, T_{y\,B} \\ + 4.455\, T_{z\,B} + 147.200\, M_{z\,B} - 0.4148\, M_{x\,B} - 645.939\, a \\ + 1\,137.247\, b - 2\,125.030\, c - 3\,894.910\, d \ldots\ldots (13b) \end{gathered}$$

$$\begin{gathered} - 869.103\, T_{x\,A} + 3.856\, T_{y\,A} + 1\,341.497\, T_{z\,A} - 86.314\, M_{y\,A} \\ + 0.7913\, M_{x\,A} = 176.728\, T_{x\,B} - 8.235\, T_{y\,B} - 224.995\, T_{z\,B} \\ + 9.5544\, M_{y\,B} - 0.4492\, M_{x\,B} + 1\,290.000\, t^\circ - 19.946\, a \\ - 17.514\, b + 0.169\, c - 3.561\, d \ldots\ldots (13c) \end{gathered}$$

$$\begin{gathered} 225.017\, T_{x\,A} + 17.722\, T_{y\,A} + 109.256\, M_{z\,A} = 131.133\, T_{x\,B} \\ - 325.068\, T_{y\,B} + 12.267\, M_{z\,B} + 69.075\, a + 79.879\, b - 177.267\, c \\ - 325.068\, d \ldots\ldots (13d) \end{gathered}$$

$$\begin{gathered} 59.040\, T_{x\,A} - 13.644\, T_{y\,A} - 86.314\, T_{z\,A} + 22.763\, M_{y\,A} - 0.119\, M_{x\,A} \\ = 14.727\, T_{x\,B} - 21.099\, T_{z\,B} + 0.7962\, M_{y\,B} - 5.953\, a - 6.123\, b \ldots (13e) \end{gathered}$$

$$\begin{gathered} - 13.714\, T_{x\,A} + 1.743\, T_{y\,A} + 7.913\, T_{z\,A} - 0.119\, M_{y\,A} \\ + 64.050\, M_{x\,A} = 0.916\, T_{y\,B} - 0.532\, T_{z\,B} + 0.0495\, M_{x\,B} + 30.349\, a \\ - 33.713\, b - 0.019\, c + 0.396\, d \ldots\ldots (13f) \end{gathered}$$

The solution of these six equations is as follows:

$$\begin{gathered} 1\,804.086\, T_{x\,A} = 982.464\, T_{x\,B} - 2\,170.911\, T_{y\,B} + 38.049\, T_{z\,B} \\ + 81.730\, M_{z\,B} - 0.7666\, M_{y\,B} - 0.2681\, M_{x\,B} + 2\,666\,278\, t^\circ + 415.954\, a \\ + 499.621\, b - 1\,186.400\, c - 2\,168.103\, d \ldots\ldots (14a) \end{gathered}$$

$$\begin{gathered} 4\,661.994\, T_{y\,A} = 1\,524.094\, T_{x\,B} - 3\,743.919\, T_{y\,B} - 24.430\, T_{z\,B} \\ + 141.197\, M_{z\,B} + 1.1091\, M_{y\,B} - 0.4243\, M_{x\,B} - 65.639\, t^\circ \\ - 682.886\, a + 1\,102.424\, b - 2\,038.019\, c - 3\,735.900\, d \ldots\ldots (14b) \end{gathered}$$

$$\begin{gathered} 1\,013.343\, T_{z\,A} = 598.806\, T_{x\,B} - 821.460\, T_{y\,B} - 291.600\, T_{z\,B} \\ + 30.616\, M_{z\,B} + 12.311\, M_{y\,B} - 0.5510\, M_{x\,B} + 2\,240.613\, t^\circ + 95.312\, a \\ + 152.825\, b - 444.135\, c - 815.579\, d \ldots\ldots (14c) \end{gathered}$$

$$\begin{gathered} 109.256\, M_{z\,A} = 2.800\, T_{x\,B} - 40.066\, T_{y\,B} - 4.653\, T_{z\,B} + 1.536\, M_{z\,B} \\ + 0.0914\, M_{y\,B} + 0.0350\, M_{x\,B} - 332.306\, t^\circ + 19.791\, a \\ + 13.372\, b - 21.545\, c - 40.447\, d \ldots\ldots (14d) \end{gathered}$$

$$\begin{gathered} 22.763\, M_{y\,A} = 38.045\, T_{x\,B} - 9.897\, T_{y\,B} - 47.249\, T_{z\,B} + 0.346\, M_{z\,B} \\ + 1.873\, M_{y\,B} - 0.0392\, M_{x\,B} + 103.406\, t^\circ - 13.385\, a - 6.289\, b \\ - 4.978\, c - 9.464\, d \ldots\ldots (14e) \end{gathered}$$

$$\begin{gathered} 64.050\, M_{x\,A} = 2.422\, T_{x\,B} - 7.823\, T_{y\,B} + 1.796\, T_{z\,B} + 0.331\, M_{z\,B} \\ - 0.0925\, M_{y\,B} + 0.0518\, M_{x\,B} + 3.337\, t^\circ + 32.952\, a - 31.553\, b \\ - 4.834\, c - 8.369\, d \ldots\ldots (14f) \end{gathered}$$

The labor of dividing by the coefficients of the six unknowns need not be performed if the computations are made with the aid of a machine. Substituting the values from Equations (14) in Equations (12), the loading terms on Arch *b* are found in terms of the crown reaction of this arch and the external loads. The substitutions are, as follows:

$$\begin{aligned} L_x = &- 1\,285.052\, T_{x\,B} + 2\,871.685\, T_{y\,B} + 70.539\, T_{z\,B} - 108.157\, M_{z\,B} \\ &- 3.345\, M_{y\,B} + 0.3968\, M_{x\,B} - 1\,738.666\, t^\circ - 71.419\, a \\ &- 735.296\, b + 1\,565.128\, c + 2\,866.306\, d \ldots\ldots\ldots\ldots\ldots\ldots (15a) \end{aligned}$$

$$\begin{aligned} L_y = &\ 2\,871.685\, T_{x\,B} - 6\,754.194\, T_{y\,B} + 24.577\, T_{z\,B} + 254.566\, M_{z\,B} \\ &+ 0.0659\, M_{y\,B} - 0.6878\, M_{x\,B} + 3\,269.587\, t^\circ + 158.075\, a \\ &+ 1\,768.366\, b - 3\,683.369\, c - 6\,744.073\, d \ldots\ldots\ldots\ldots\ldots\ldots (15b) \end{aligned}$$

$$\begin{aligned} L_z = &\ 70.540\, T_{x\,B} + 24.577\, T_{y\,B} - 112.229\, T_{z\,B} - 1.019\, M_{z\,B} \\ &+ 4.542\, M_{y\,B} - 1.315\, M_{x\,B} + 332.239\, t^\circ - 31.064\, a - 22.155\, b \\ &+ 14.902\, c + 26.030\, d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15c) \end{aligned}$$

$$\begin{aligned} L_a = &- 108.157\, T_{x\,B} + 254.675\, T_{y\,B} - 1.020\, T_{z\,B} - 9.598\, M_{z\,B} \\ &+ 0.0013\, M_{y\,B} + 0.0258\, M_{x\,B} - 122.694\, t^\circ - 5.902\, a \\ &- 66.680\, b + 138.886\, c + 254.294\, d \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15d) \end{aligned}$$

$$\begin{aligned} L_\beta = &- 3.345\, T_{z\,B} + 0.066\, T_{y\,B} + 4.543\, T_{z\,B} + 0.001\, M_{z\,B} \\ &- 0.1844\, M_{y\,B} + 0.0056\, M_{x\,B} - 14.887\, t^\circ + 1.107\, a + 0.626\, b \\ &- 0.024\, c + 0.006\, d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15e) \end{aligned}$$

$$\begin{aligned} L_\gamma = &\ 0.397\, T_{x\,B} - 0.687\, T_{y\,B} - 0.131\, T_{z\,B} + 0.026\, M_{z\,B} \\ &+ 0.0056\, M_{y\,B} - 0.0002\, M_{x\,B} + 0.975\, t^\circ + 0.044\, a + 0.189\, b \\ &- 0.372\, c - 0.683\, d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15f) \end{aligned}$$

The coefficients in Equations (9) for Point (*b*) are determined in the same way as for Point (*a*). The computed values are as follows:

$$\begin{aligned}
A &= 12.500 + 1.370 + 141.887 + 7.150 + 0.753 \\
&\quad + 153.289 + (22.167) + (1\,392.313) \\
&\quad + (158.494) &&= 1\,889.923 \\
B &= -2.065 - 529.654 + 1.253 - 0.934 - 229.953 \\
&\quad - (3\,289.533) &&= -4\,050.886 \\
C &= -10.437 - 0.940 - 189.669 - (227.070) &&= -428.116 \\
D &= 29.201 + (124.133) &&= 153.334 \\
F &= 0.100 - 0.075 - 21.996 &&= -21.971 \\
G &= 0.548 + 31.250 + 4\,499.351 + 1.010 + 5.330 \\
&\quad + 1\,481.291 + (8.867) + (8\,302.157) \\
&\quad + (16.185) &&= 14\,345.989 \\
H &= -2.490 + 0.591 + 157.823 - (9.178) &&= 146.746 \\
I &= -351.760 - (313.289) &&= -665.049 \\
K &= 0.060 + 0.486 + 173.571 + (0.874) &&= 174.991 \\
L &= 32.620 + 15.797 + 1.263 + 251.842 + (22.167) \\
&\quad + (5.553) + (325.316) &&= 654.558 \\
O &= -0.182 + 0.050 + 15.529 - (0.496) &&= 14.901 \\
P &= 43.598 + (11.822) &&= 55.420
\end{aligned}$$

$$\begin{aligned}
Q &= 0.059 + 0.007 + 1.655 + (0.463) &&= 2.184\\
R &= 0.009 - 0.007 - (2.547) &&= -\,2.545\\
S &= 0.004 + 0.063 + 31.838 + (0.047) &&= 31.952
\end{aligned}$$

As before, deflections due to the post are enclosed in parentheses.

For load at Point (c), Fig. 31 ($x' = 12.00$ and $z' = 18.876$):

$$\begin{aligned}
L_x = (&-1.338 - 461.340 + 1.125 - 0.839\\
&- 180.674 + 260.078 - 1.535 + 1.144\\
&+ 265.355 - 3\,289.533 + 1\,489.600) &&= -\,1\,917.960\\
L_y = (&0.504 + 11.959 + 3\,044.843 + 0.997 + 4.376\\
&+ 957.615 - 1\,944.633 - 1.089 - 6.031\\
&- 1\,425.817 + 8.866 + 8\,302.157 + 16.167\\
&- 3\,759.467 - 16.496) &&= +\,5\,193.951\\
L_z = (&-2.306 + 0.521 + 121.384 + 2.917 - 0.736\\
&- 181.455 + 9.178 + 9.365) &&= -\,41.132\\
L_\alpha = & &&= -\,225.363\\
L_\beta = (&0.081 - 0.060 - 14.058 - 0.115 + 0.086\\
&+ 21.190) &&= +\,7.124\\
L_\gamma = (&+0.058 + 0.320 + 75.536 - 0.065 - 0.458\\
&- 116.602 + 0.874 - 0.892) &&= -\,41.229
\end{aligned}$$

Finally, the six equations in this case are as follows:

$$\begin{array}{rrrrr}
1\,889.923\,T_{x\,B} & -\,428.116\,T_{z\,B} & +\,153.334\,M_{z\,B} & -\,21.971\,M_{x\,B} & \\
-\,1\,285.052\,T_{x\,B} & +\,70.539\,T_{z\,B} & -\,108.157\,M_{z\,B} & +\,0.397\,M_{x\,B} & -\,71.491a\\
\hline
604.871\,T_{x\,B} & -\,357.577\,T_{z\,B} & +\,45.177\,M_{z\,B} & -\,21.574\,M_{x\,B} & -\,71.491a
\end{array}$$

$$\begin{array}{rrrrl}
+\,786.800\,b & -\,1\,917.960\,c & -\,3\,820.201\,d & -\,2\,083.536\,t^\circ & \\
-\,735.296\,b & +\,1\,565.128\,c & +\,2\,866.306\,d & -\,1\,738.666\,t^\circ & \\
\hline
+\,51.504\,b & -\,352.832\,c & -\,953.895\,d & -\,3\,822.202\,t^\circ & = 0\,..\,(16a)
\end{array}$$

$$\begin{array}{rrrrr}
-\,428.116\,T_{x\,B} & +\,654.558\,T_{z\,B} & & +\,14.901\,M_{x\,B} & \\
70.539\,T_{x\,B} & -\,112.229\,T_{z\,B} & -\,1.020\,M_{z\,B} & -\,0.131\,M_{x\,B} & -\,31.063\,a\\
\hline
357.577\,T_{x\,B} & +\,542.329\,T_{z\,B} & -\,1.020\,M_{z\,B} & +\,14.770\,M_{x\,B} & -\,31.063a
\end{array}$$

$$\begin{array}{rrrrl}
+\,7.812\,b & -\,41.132\,c & -\,9.714\,d & -\,1\,454.308\,t^\circ & \\
-\,22.155\,b & +\,14.902\,c & +\,26.030\,d & +\,332.239\,t^\circ & \\
\hline
-\,14.343\,b & -\,26.230\,c & +\,16.316\,d & -\,1\,122.069\,t^\circ & = 0\,\ldots\ldots(16c)
\end{array}$$

$$\begin{array}{rrrrr}
153.334\,T_{x\,B} & & +\,55.420\,M_{z\,B} & & \\
-\,108.157\,T_{x\,B} & -\,1.020\,T_{z\,B} & -\,9.598\,M_{z\,B} & +\,0.026\,M_{x\,B} & -\,5.902a\\
\hline
45.177\,T_{x\,B} & -\,1.020\,T_{z\,B} & +\,45.822\,M_{z\,B} & +\,0.026\,M_{x\,B} & -\,5.902a
\end{array}$$

$$\begin{array}{rrrrl}
+\,73.600\,b & -\,225.363\,c & -\,665.049\,d & & \\
-\,66.680\,b & +\,138.886\,c & +\,254.294\,d & -\,122.694\,t^\circ & \\
\hline
+\,6.920\,b & -\,86.477\,c & -\,410.755\,d & -\,122.694\,t^\circ & = 0\,\ldots\ldots(16d)
\end{array}$$

$$\begin{array}{l}
-21.971\,T_{x\,B} + 14.901\,T_{z\,B} \qquad\qquad + 31.952\,M_{x\,B}\\
\underline{\quad 0.397\,T_{x\,B} - \quad 0.131\,T_{z\,B} + 0.026\,M_{z\,B} \qquad\qquad\qquad + 0.044a}\\
-21.574\,T_{x\,B} + 14.770\,T_{z\,B} + 0.026\,M_{z\,B} + 31.952\,M_{x\,B} + 0.044a\\
\qquad -0.728\,b - 41.229\,c - 160.505\,d\\
\qquad \underline{+0.189\,b - \quad 0.372\,c - \quad 0.683\,d + 0.975\,t^\circ}\\
\qquad -0.559\,b - 41.601\,c - 161.188\,d + 0.975\,t^\circ = 0 \ldots\ldots.(16f)
\end{array}$$

$$\begin{array}{l}
\quad 14\,345.989\,T_{y\,B} - 21.971\,M_{y\,B} \qquad\qquad\qquad\quad -1\,954.654b\\
\underline{-\quad 6\,754.194\,T_{y\,B} + \quad 0.066\,M_{y\,B} + 158.075\,a + 1\,768.366b}\\
+\quad 7\,591.795\,T_{y\,B} - 21.906\,M_{y\,B} + 158.075\,a - \quad 186.288b\\
\qquad +5\,193.951\,c + 12\,508.596\,d\\
\qquad \underline{-3\,683.369\,c - \quad 6\,744.073\,d + 3\,269.587\,t^\circ}\\
\qquad +1\,508.657\,c + \quad 5\,764.523\,d + 3\,269.587\,t^\circ = 0 \ldots\ldots.(16b)
\end{array}$$

$$\begin{array}{l}
-21.971\,T_{y\,B} + 2.184\,M_{y\,B}\\
\underline{+\quad 0.066\,T_{y\,B} - 0.184\,M_{y\,B} + 1.107\,a + 0.626b}\\
-21.905\,T_{y\,B} + 2.000\,M_{y\,B} + 1.107\,a + 0.626b\\
\qquad +7.124\,c + 4.753\,d\\
\qquad \underline{-0.024\,c + 0.006\,d - 14.887\,t^\circ}\\
\qquad +7.100\,c + 4.759\,d - 14.887\,t^\circ = 0. \ldots\ldots\ldots\ldots.(16e)
\end{array}$$

In each of Equations (16), the first line of values represents the loads on Arch *b* itself; the second line represents the reaction from the crown of Arch *a*; and the final result is given in the third line. The result of solving Equations (16) is, as follows:

$$T_{x\,B} = +0.134\,a - 0.055\,b + 0.469\,c + 0.941\,d + 13.780\,t^\circ \ldots (17a)$$
$$T_{y\,B} = -0.012\,a + 0.012\,b - 0.108\,c - 0.396\,d \qquad \ldots (17b)$$
$$T_{z\,B} = +0.117\,a - 0.023\,b + 0.312\,c + 0.533\,d + 11.020\,t^\circ \ldots (17c)$$
$$M_{z\,B} = -0.065\,a - 0.022\,b + 0.488\,c + 3.565\,d + 10.665\,t^\circ \ldots (17d)$$
$$M_{y\,B} = -0.404\,a - 0.023\,b - 2.958\,c - 5.522\,d \qquad \ldots (17e)$$
$$M_{x\,B} = +0.036\,a - 0.018\,b + 0.823\,c + 2.908\,d + 4.188\,t^\circ \ldots (17f)$$

Substituting these values back into Equations (14) and collecting terms, the values of the crown reactions of Arch *a*, in terms of loads only, are:

$$T_{x\,A} = +0.317\,a + 0.231\,b - 0.242\,c - 0.039\,d + 9.697\,t^\circ \ldots (18a)$$
$$T_{y\,A} = -0.096\,a + 0.208\,b - 0.185\,c - 0.072\,d + 4.768\,t^\circ \ldots (18b)$$
$$T_{z\,A} = +0.142\,a + 0.114\,b - 0.185\,c - 0.042\,d + 4.974\,t^\circ \ldots (18c)$$
$$M_{z\,A} = +0.183\,a + 0.117\,b - 0.154\,c - 0.177\,d - 3.006\,t^\circ \ldots (18d)$$
$$M_{y\,A} = -0.635\,a - 0.329\,b - 0.272\,c - 0.178\,d + 4.860\,t^\circ \ldots (18e)$$
$$M_{x\,A} = +0.525\,a - 0.497\,b - 0.028\,c - 0.003\,d + 0.941\,t^\circ \ldots (18f)$$

The entire system is now broken up into a system of cantilevers which are determinate in every way, and from Equations (5) the stresses at any section can be obtained and the arch properly reinforced.

The reactions in the Y and X directions at the base of the second pier (Pier B), have been computed and found to be, respectively:

$$T_x = 0.183\,a + 0.286\,b - 0.711\,c - 0.980\,d - 0.515\,e + 0.136\,f - 0.007\,g \qquad (19a)$$

and,

$$T_y = 0.085\,a + 0.816\,b + 1.077\,c + 0.677\,d + 0.139\,e + 0.012\,f - 0.018\,g \qquad (19b)$$

PART IV.—An Experiment to Verify the Computations of Part III

In order to obtain an experimental verification of the theory and to check the computations given in Part III, a celluloid model was made of the structure to a scale of $\frac{3}{16}$ in. to the foot (Figs. 10 to 13), which was tested by the deflection method devised by George E. Beggs, M. Am. Soc. C. E.,[5] and contours of deflection were obtained for the several movements of the piers. In tests applied to solid figures, Professor Beggs used very rigid apparatus.[6] The writer was not able to secure this rigidity and, therefore, used the method herein described to annul the influence of the deflections of the apparatus.

The celluloid sheet used was $1\frac{1}{8}$ in. thick, and was built up by cementing three thinner sheets together. The arch rings were cut from this sheet and the four piers were fastened to the rings with cement furnished for the purpose. The ring was cut on a milling machine. During this operation a stream of water was kept flowing on the tool to prevent fire and to keep the surface of the material cool and hard.

Although the work on the model was carefully and accurately done, it did not prove as satisfactory for the purpose as was anticipated. During the milling of the ring the sheet showed some tendency to split into the three original sheets. After several trials a ring was obtained in which this tendency did not appear to be very serious, but doubtless it had some influence on the results. The joints at which the legs were cemented to the horizontal part could not be relied upon to be 100% perfect for deflection, although they appeared to be rigid as far as could be observed. The model showed a strong tendency to warp out of shape when left out of the clamps, and doubtless there were stresses induced in it by the testing frame when the legs were in a neutral position. A line across the crown that should have been level and was made true in the milling machine, showed a crown in the center of more than $\frac{1}{20}$ in. However, even with these imperfections the writer questions whether a better model could have been made of this material.

Steel collars were secured on the model at the positions that represented the plane that was considered immovable in the bridge; that is, the contact of the ground and the foundation. As shown in Figs. 11, 12, and 13, each collar consisted of two sheets of $\frac{1}{8}$-in. steel, one on each side of a leg. These sheets were drawn toward each other by screws, thus clamping them securely together with the leg between them. One collar was placed on each leg, so that it responded accurately to any movement of the leg at the place where it was fastened.

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[5] This method is described in *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1208, by Professor Beggs as applied to plane figures.

[6] *Engineering News-Record*, July 21, 1927, as applied to solid figures.

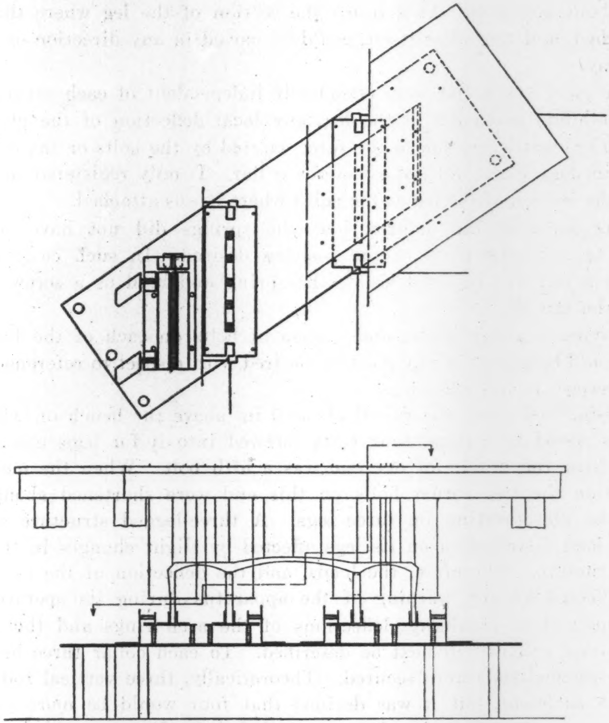


FIG. 11.—SCHEMATIC DRAWING OF TESTING APPARATUS

A horizontal plate was clamped to the bottom of each leg. Two angles, one on either side of the leg, were fastened together with bolts running through holes in the leg, thus clamping them securely to it and to each other. These angles, in turn, were bolted to the plate. This plate was approximately 2 in. below the collar. Figs. 11, 12, and 13 should serve to make clear the position and method of fastening both the collar and plate.

Each horizontal plate was fastened to a $\frac{3}{8}$-in. steel base plate by seven bolts set in slotted holes. Four of these bolts were vertical and three, horizontal. Each horizontal bolt was passed through holes in angles that, in turn, were bolted one to the base-plate and one to the horizontal plate. Around each bolt was a strong spring, so that the horizontal plate could be moved one way by tightening the nut on a bolt, and in the opposite direction by the spring when the nut was loosened. A vertical motion could be induced by tightening or loosening all vertical bolts; a horizontal motion could be produced by means of the horizontal bolts. Rotation about the vertical axis could be obtained by loosening one of the bolts and tightening another. By this system the base of the leg could be moved in any direction, or could be rotated about any axis. As a result the section of the leg where the collar was attached, and the collar itself, could be moved in any direction or rotated in any way.

As the plate and collar were completely independent of each other, except for the celluloid extension of the leg, any local deflection of the plate as a beam slab or cantilever, due to the force exerted by the bolts or the deflection of the $\frac{3}{8}$-in. base-plate, did not affect the collar. It only registered the movement of the section of the leg at the point where it was attached.

During some of the deformations the springs did not have sufficient strength to force the plate to the position desired. In such cases a short bolt with a nut was inserted beside the spring and used as a screw-jack by backing the nut off.

By having a collar, plate, and system of bolts on each of the four legs, any leg could be placed in any position desired with respect to reference planes or with respect to the other legs.

The $\frac{3}{8}$-in. base-plate was raised about 6 in. above the bench on which the apparatus rested by having four bolts screwed into it for legs, one in each corner. Near the middle of one end was a fifth bolt. When the model was in operation the two corner bolts on this end were shortened about $\frac{1}{4}$ in., leaving the plate resting on three legs. A three-legged structure will not have the load distribution on its legs affected by slight changes in the shape of the structure. Therefore, the loads, and the deflection of the bench, will not be affected by any warping of the apparatus during its operation.

The method of obtaining deflections of the arch rings and the motions of the several collars will next be described. To each collar three horizontal and four vertical rods were secured. Theoretically, three vertical rods would have been sufficient, but it was decided that four would be more practical. The same was true of the four vertical bolts. Three could have been used. The ends of the horizontal rods were passed through holes in heavy steel stands. The vertical rods were passed through holes in the top of a small table which was especially made for the purpose. At the position on the arch where the reading of the vertical movement was desired, a vertical rod was passed through this table top.

The amount that these rods projected above the table or through the stands was read by means of an Ames dial (reading to 0.001 in.) mounted on a tripod. This was carried from point to point by the operator and was read before and after moving one of the legs of the model. It was found that the readings were consistent by this method and no error within the limits of accuracy of the dial was found when the legs of the tripod were shifted about while the dial was on one definite rod.

With this arrangement it is to be noted that the movement of any or all of the four collars can be obtained with reference to (1) the bench on which the apparatus rests; (2) the table under which it is placed; and (3) the steel

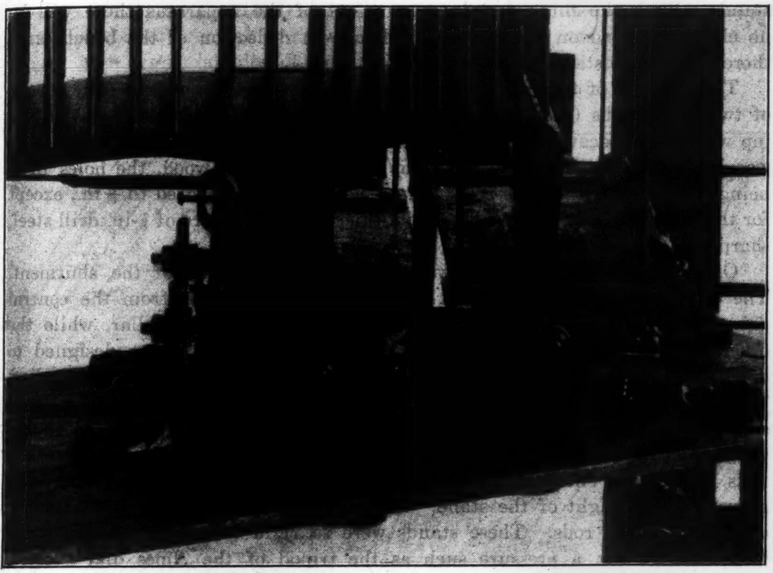


FIG. 12.—CELLULOID MODEL BEING TESTED, SHOWING COMPLETE SET-UP

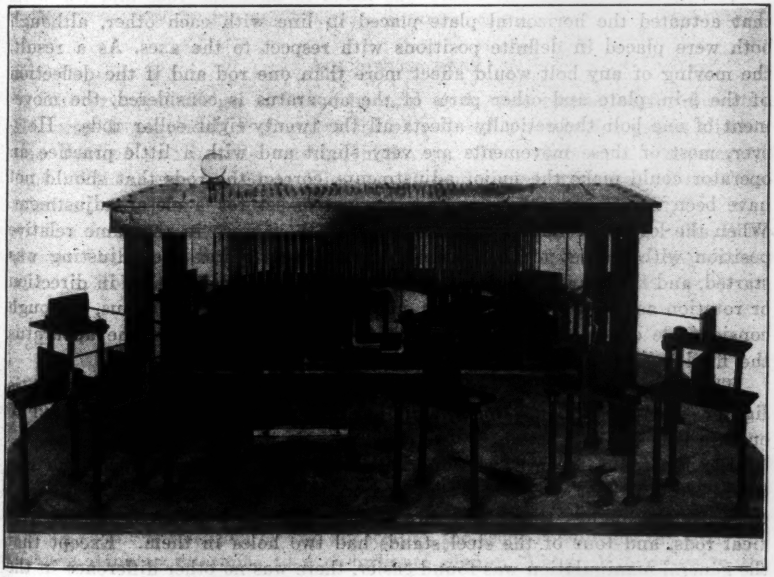


FIG. 13.—DISTANT VIEW OF END PIER OF MODEL BEING TESTED

stands. With the three-legged arrangement of the apparatus there will be no change in load on the bench, no change in deflection of the bench, and, therefore, of the stands or small table.

The position of the table with the holes in the top was adjusted by means of two plumb-bobs (see Fig. 12), the string passing through two holes in the top which were located over two punch-marks in the base-plate when the table was in proper position. The table top was of three-ply wood, the holes in it being 1/32 in. larger than the rods. These holes were back-bored to 1/4 in., except for the upper ply, which was about 1/8 in. thick. All rods were of 1/8-in. drill steel, sharpened at both ends.

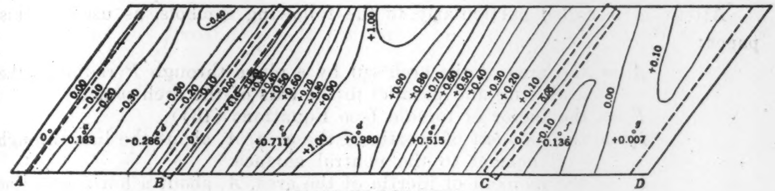
One of the horizontal rods was located on the **Z**-axis of the abutment. The other two were at right angles to it and equi-distant from the central point. One end of the rod rested in a punch-mark on the collar, while the other extended about 1/8 in. through the steel stand which was designed to hold it. A rubber band was used to hold the rod firmly against the collar. The stands were made by sawing a 4-in. length from an 8-in. **H**-beam, and sawing this section in two through the web. Three holes were bored in the web, and bolts were inserted in them to form legs. On these bolts were two nuts so that the entire stand was heavy and rigid. This design allowed for adjusting the height of the stand. Holes through the web were bored 1/32 in. over-size for the rods. These stands were so rigid that no deflection could be observed from a pressure such as the tripod of the Ames dial exerted. The vertical rods rested in punch-marks on the collar or on the model.

It will be noted that no effort was made to have the rods and the bolts that actuated the horizontal plate placed in line with each other, although both were placed in definite positions with respect to the axes. As a result, the moving of any bolt would affect more than one rod and if the deflection of the 3/8-in. plate and other parts of the apparatus is considered, the movement of one bolt theoretically affects all the twenty-eight collar rods. However, most of these movements are very slight and with a little practice an operator could make the major adjustments, correct the rods that should not have been moved, and then go over the entire set for a closer adjustment. When the legs were finally adjusted three of them were in the same relative position with respect to the bench that they were before the adjusting was started, and the fourth had been given a definite movement either in direction or rotation as indicated by the rods on the collar of that leg. Thus, although considerable deflection had been given to the several parts of the apparatus the final position was obtained independent of these deflections.

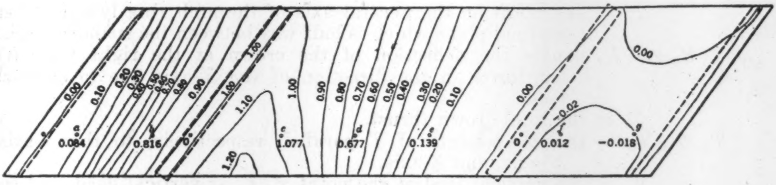
The advantage of this apparatus over the more rigid type is that it is lighter, and takes up less room, and that only one Ames dial is required to operate it.

For the square arch or the one with zero skew, the same apparatus was used. Another set of holes was bored in the base-plate for the bolts and to secure the angles; the table had a second top with holes in it for the vertical rods, and four of the steel stands had two holes in them. Except that the general manipulation was found easier, there was no other difference in the operation between the two arches.

Fig. 14(*a*) and Fig. 14(*b*) show the contour surfaces obtained for the movement of the left central pier along the *X* and the *Y*-axes (Pier *B*, Fig. 3). These give a general idea of the work and a comparison with the computed quantities.


0.00
-0.10
-0.20
-0.30
-0.30
-0.20
-0.10
-0.40
+1.00
+0.90
+0.80
+0.70
+0.60
+0.50
+0.40
+0.30
+0.20
+0.10
0.00
+0.10
-0.183
-0.286
+0.711
+0.980
+0.515
0.136
+0.007
A
B
C
D

(a) THRUST *X* ON PIER *B*


0.00
0.10
0.20
0.30
0.90
1.00
1.10
1.20
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.02
0.00
0.084
0.816
1.077
0.677
0.139
0.012
-0.018

(b) THRUST *Y* ON PIER *B*

FIG. 14.—CONTOURS OF INFLUENCE SURFACE

ACKNOWLEDGMENT

The practical design and experimental work were conducted for the State Highway Department of Connecticut as part of a study for grade elimination between two heavily traveled highways. It is through the courtesy of E. C. Weldon, Deputy State Highway Commissioner, and that of the Commission itself, that this analysis and description are being published.

The formulas in Appendix III were derived by the writer for the Westchester County Park Commission and are reproduced through the courtesy of L. G. Holleran and Arthur G. Hayden, Members, Am. Soc. C. E.

APPENDIX I

NOTATION

As far as possible, the notation used follows the precedent established by the first paper on the single-span skew arch.[7] However, the interaction of adjacent arch spans requires the introduction of some new symbols. For example, when a factor is modified by two subscripts, the second identifies the span and the first denotes reference to a co-ordinate plane: Thus, $M_{x\,A}$ refers to the moment in Span A, taken about the X-axis, etc. Dimensional notation may be identified most readily by reference to the various diagrams. The capital letter coefficients introduced in Equations (9) duplicate some of the symbols used elsewhere in the paper, with other meanings. It is felt, however, that no confusion will result.

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[7] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), pp. 615–616.

Attention is called particularly to the following symbols, as used in this paper:

A = the area of the arch cut by a plane through P radial to the arch ring and parallel to the axis of the arch panel.
F = the factor of torsion (see Equations (16)).
I_y = the moment of inertia of the area, A, about the line through P normal to the neutral surface.
I_z = the moment of inertia of the area, A, about a horizontal line through P parallel to the Z-axis.
M = the total crown moment.
M_x, M_y, M_z = the components of M about the X-axis, Y-axis, and Z-axis, respectively.
P = any point on the neutral axis of the arch ring lying in that vertical plane which is half way between the spandrel walls.
R_x and L_x denote the deflection of the crown at the right (or left) cantilever in the direction of the X-axis due to external loads.
T = the total crown thrust.
T_x, T_y, T_z = the components of T parallel, respectively, to the X-axis, Y-axis, and Z-axis.
W = the vertical load at the point x', z', or vertical dead load between the crown and P.
b = the width of the arch ring measured parallel to the Z-axis.
f = the torsional force on a block corresponding to the moment about the V-axis.
h = rise from hinge to crown on a hinged skew arch.
l = the half span of hinged skew arch.
r_A, r_B = rise of Arch a or Arch b above vertical pier.
S_A, S_B = the half span of Arch a, or Arch b, measured in X-direction.
t = the thickness of the arch measured radially at the point, P.
u, v = the co-ordinates of the crown center referred to the tangent and radial lines through P and parallel to the plane, $Z = 0$ (see Equations (1) and (2)).
x, y = the co-ordinates of P referred to vertical and horizontal axes through the crown.
x', z' = co-ordinates with respect to the crown of a concentrated load.
α, β, and γ = subscripts denoting angular rotation or moment about the Z, Y, or X-axes, respectively.
θ = angular deflection at the top of a pier due to a torsion, T, about the vertical or Y-axis.
λ = the ratio, $\dfrac{\text{modulus of elasticity in compression}}{\text{modulus of elasticity in shear}}$.
ϵ = the tangent of the angle of skew of the arch.
ϵ' = the complement of the angle between the cut section at P and the plane of the spandrels.
ϕ = the angle of slope of a plane tangent to the neutral surface of the arch at P.
w = the uniform live load per square unit of surface.

APPENDIX II

FORMULAS FOR THE SKEW CANTILEVER

The deflections of a skewed cantilever in the directions of the several axes and the rotations of the end of this cantilever about these axes when acted upon by loads, are given by the following equations:

$$\begin{aligned}
E\,\Delta x = &\Big\{ \int \cos^2\phi \frac{ds}{A} + \lambda \int \sin^2\phi \frac{ds}{A} + \int y^2 \frac{ds}{I_z} + \varepsilon^2 \int x^2 \cos^2\phi \frac{ds}{I_y} \\
&+ \varepsilon^2 \lambda \int x^2 \sin^2\phi \frac{ds}{F} \Big\} T_x + \Big\{ (1-\lambda) \int \sin\phi \cos\phi \frac{ds}{A} - \int x\,y \frac{ds}{I_z} \\
&+ \varepsilon^2 \int \left(\frac{1}{I_y} - \frac{\lambda}{F}\right) x^2 \sin\phi \cos\phi \, ds \Big\} T_y - \Big\{ \varepsilon \int v\,x \cos\phi \frac{ds}{I_y} \\
&+ \varepsilon \lambda \int u\,x \sin\phi \frac{ds}{F} \Big\} T_z + \Big\{ \int y \frac{ds}{I_z} \Big\} M_z + \Big\{ \varepsilon \int x \cos^2\phi \frac{ds}{I_y} \\
&+ \varepsilon \lambda \int x \sin^2\phi \frac{ds}{F} \Big\} M_y + \Big\{ \varepsilon \int \left(\frac{1}{I_y} - \frac{\lambda}{F}\right) x \sin\phi \cos\phi \, ds \Big\} M_x + L_x \text{..(20}a\text{)}
\end{aligned}$$

$$\begin{aligned}
E\,\Delta y = &\Big\{ (1-\lambda) \int \sin\phi \cos\phi \frac{ds}{A} - \int x\,y \frac{ds}{I_z} \\
&+ \varepsilon^2 \int \left(\frac{1}{I_y} - \frac{\lambda}{F}\right) x^2 \sin\phi \cos\phi \, ds \Big\} T_x + \Big\{ \int \sin^2\phi \frac{ds}{A} + \lambda \int \cos^2\phi \frac{ds}{A} \\
&+ \int x^2 \frac{ds}{I_z} + \varepsilon^2 \int x^2 \sin^2\phi \frac{ds}{I_y} + \varepsilon^2 \lambda \int x^2 \cos^2\phi \frac{ds}{F} \Big\} T_y \\
&- \Big\{ \varepsilon \int v\,x \sin\phi \frac{ds}{I_y} - \varepsilon \lambda \int u\,x \cos\phi \frac{ds}{F} \Big\} T_z - \Big\{ \int x \frac{ds}{I_z} \Big\} M_z \\
&+ \Big\{ \varepsilon \int \left(\frac{1}{I_y} - \frac{\lambda}{F}\right) x \sin\phi \cos\phi \, ds \Big\} M_y + \Big\{ \varepsilon \int x \sin^2\phi \frac{ds}{I_y} \\
&+ \varepsilon \lambda \int x \cos^2\phi \frac{ds}{F} \Big\} M_x + L_y \ldots\ldots\ldots\ldots\ldots\text{(20}b\text{)}
\end{aligned}$$

$$\begin{aligned}
E\,\Delta z = &-\Big\{ \varepsilon \int v\,x \cos\phi \frac{ds}{I_y} + \varepsilon \lambda \int u\,x \sin\phi \frac{ds}{F} \Big\} T_x - \Big\{ \varepsilon \int v\,x \sin\phi \frac{ds}{I_y} \\
&- \varepsilon \lambda \int u\,x \cos\phi \frac{ds}{F} \Big\} T_y + \Big\{ \lambda \int \frac{ds}{A} + \int v^2 \frac{ds}{I_y} + \lambda \int u^2 \frac{ds}{F} \Big\} T_z \\
&- \Big\{ \int v \cos\phi \frac{ds}{I_y} + \lambda \int u \sin\phi \frac{ds}{F} \Big\} M_y - \Big\{ \int v \sin\phi \frac{ds}{I_y} \\
&- \lambda \int u \cos\phi \frac{ds}{F} \Big\} M_x + L_z \ldots\ldots\ldots\ldots\ldots\text{(20}c\text{)}
\end{aligned}$$

$$E\,\Delta \alpha = \Big\{ \int y \frac{ds}{I_z} \Big\} T_x - \Big\{ \int x \frac{ds}{I_z} \Big\} T_y + \Big\{ \int \frac{ds}{I_z} \Big\} M_z + L_\alpha \ldots\text{(20}d\text{)}$$

$$\begin{aligned}
E\,\Delta \beta = &\Big\{ \varepsilon \int x \cos^2\phi \frac{ds}{I_y} + \varepsilon \lambda \int x \sin^2\phi \frac{ds}{F} \Big\} T_x \\
&+ \Big\{ \varepsilon \int \left(\frac{1}{I_y} - \frac{\lambda}{F}\right) x \sin\phi \cos\phi \, ds \Big\} T_y - \Big\{ \int v \cos\phi \frac{ds}{I_y} + \lambda \int u \sin\phi \frac{ds}{F} \Big\} T_z \\
&+ \Big\{ \int \cos^2\phi \frac{ds}{I_y} + \lambda \int \sin^2\phi \frac{ds}{F} \Big\} M_y \\
&+ \Big\{ \left(\frac{1}{I_y} - \frac{\lambda}{F}\right) \sin\phi \cos\phi \, ds \Big\} M_x + L_\beta \ldots\ldots\ldots\ldots\text{(20}e\text{)}
\end{aligned}$$

and,

$$\begin{aligned}E\,\Delta\,\gamma = {} & \Big\{ \varepsilon \int \Big(\frac{1}{I_y} - \frac{\lambda}{F}\Big) x \sin\phi \cos\phi \, ds \Big\} T_x + \Big\{ \varepsilon \int x \sin^2\phi \frac{ds}{I_y} \\ & + \varepsilon\,\lambda \int x \cos^2\phi \frac{ds}{F} \Big\} T_y - \Big\{ \int v \sin\phi \frac{ds}{I_y} - \lambda \int u \cos\phi \frac{ds}{F} \Big\} T_z \\ & + \Big\{ \int \Big(\frac{1}{I_y} - \frac{\lambda}{F}\Big) \sin\phi \cos\phi \, ds \Big\} M_y + \Big\{ \int \sin^2\phi \frac{ds}{I_y} \\ & + \lambda \int \cos^2\phi \frac{ds}{F} \Big\} M_x + L_\gamma \ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \tag{20f}$$

This set of equations may be written as follows, if finite blocks are used:

$$\begin{aligned}E\,\Delta\,x = {} & \Big\{ \sum \cos^2\phi \frac{S}{A} + \lambda \sum \sin^2\phi \frac{S}{A} + \sum y^2 \frac{S}{I_z} + \varepsilon^2 \sum x^2 \cos^2\phi \frac{S}{I_y} \\ & + \varepsilon^2\,\lambda \sum x^2 \sin^2\phi \frac{S}{F} \Big\} T_x + \Big\{ (1-\lambda) \sum \sin\phi \cos\phi \frac{S}{A} - \sum x\,y \frac{S}{I_z} \\ & + \varepsilon^2 \sum x^2 \sin\phi \cos\phi \frac{S}{I_y} - \varepsilon^2 \sum x^2 \sin\phi \cos\phi \frac{S}{F} \Big\} T_y - \Big\{ \varepsilon \sum v\,x \cos\phi \frac{S}{I_y} \\ & + \varepsilon\,\lambda \sum u\,x \sin\phi \frac{S}{F} \Big\} T_z + \Big\{ \sum y \frac{S}{I_z} \Big\} M_z + \Big\{ \varepsilon \sum x \cos^2\phi \frac{S}{I_y} \\ & + \varepsilon\,\lambda \sum x \sin^2\phi \frac{S}{F} \Big\} M_y + \Big\{ \epsilon \sum x \sin\phi \cos\phi \frac{S}{I_y} \\ & - \varepsilon\,\lambda \sum x \sin\phi \cos\phi \frac{S}{F} \Big\} M_x + L_x \ldots\vdots\ldots\ldots\ldots\ldots \end{aligned} \tag{21a}$$

$$\begin{aligned}E\,\Delta\,y = {} & \Big\{ (1-\lambda) \sum \sin\phi \cos\phi \frac{S}{A} - \sum x\,y \frac{S}{I_z} + \varepsilon^2 \sum x^2 \sin\phi \cos\phi \frac{S}{I_y} \\ & - \varepsilon^2\,\lambda \sum x^2 \sin\phi \cos\phi \frac{S}{F} \Big\} T_x + \Big\{ \sum \sin^2\phi \frac{S}{A} + \lambda \sum \cos^2\phi \frac{S}{A} \\ & + \sum x^2 \frac{S}{I_z} + \varepsilon^2 \sum x^2 \sin^2\phi \frac{S}{I_y} + \varepsilon^2\,\lambda \sum x^2 \cos^2\phi \frac{S}{F} \Big\} T_y \\ & - \Big\{ \varepsilon \sum v\,x \sin\phi \frac{S}{I_y} - \varepsilon\,\lambda \sum u\,x \cos\phi \frac{S}{F} \Big\} T_z - \Big\{ \sum x \frac{S}{I_z} \Big\} M_z \\ & + \Big\{ \varepsilon \sum x \sin\phi \cos\phi \frac{S}{I_y} - \varepsilon\,\lambda \sum x \sin\phi \cos\phi \frac{S}{F} \Big\} M_v + \Big\{ \varepsilon \sum x \sin^2\phi \frac{S}{I_y} \\ & + \varepsilon\,\lambda \sum x \cos^2\phi \frac{S}{F} \Big\} M_x + L_y \ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \tag{21b}$$

$$\begin{aligned}E\,\Delta\,z = {} & - \Big\{ \varepsilon \sum v\,x \cos\phi \frac{S}{I_y} + \varepsilon\,\lambda \sum u\,x \sin\phi \frac{S}{F} \Big\} T_x \\ & - \Big\{ \varepsilon \sum v\,x \sin\phi \frac{S}{I_y} - \varepsilon\,\lambda \sum u\,x \cos\phi \frac{S}{F} \Big\} T_y + \Big\{ \lambda \sum \frac{S}{A} \\ & + \sum v^2 \frac{S}{I_y} + \lambda \sum u^2 \frac{S}{F} \Big\} T_z - \Big\{ \sum v \cos\phi \frac{S}{I_y} + \lambda \sum u \sin\phi \frac{S}{F} \Big\} M_y \\ & - \Big\{ \sum v \sin\phi \frac{S}{I_y} - \lambda \sum u \cos\phi \frac{S}{F} \Big\} M_x + L_z \ldots\ldots\ldots \end{aligned} \tag{21c}$$

$$E\,\Delta\,\alpha = \Big\{ \sum y \frac{S}{I_z} \Big\} T_x - \Big\{ \sum x \frac{S}{I_z} \Big\} T_y + \Big\{ \sum \frac{S}{I_z} \Big\} M_z + L_\alpha \ldots \tag{21d}$$

$$\begin{aligned} E\,\Delta\,\beta = {} & \Big\{ \varepsilon \sum x \cos^2 \phi \frac{S}{I_y} + \varepsilon\,\lambda \sum x \sin^2 \phi \frac{S}{F} \Big\} T_x \\ & + \Big\{ \varepsilon \sum x \sin \phi \cos \phi \frac{S}{I_y} - \varepsilon\,\lambda \sum x \sin \phi \cos \phi \frac{S}{F} \Big\} T_y \\ & - \Big\{ \sum v \cos \phi \frac{S}{I_y} + \lambda \sum u \sin \phi \frac{S}{F} \Big\} T_z + \Big\{ \sum \cos^2 \phi \frac{S}{I_y} \\ & + \lambda \sum \sin^2 \phi \frac{S}{F} \Big\} M_y + \Big\{ \sum \sin \phi \cos \phi \frac{S}{I_y} \\ & - \lambda \sum \sin \phi \cos \phi \frac{S}{F} \Big\} M_x + L_\beta \;\ldots\ldots\ldots\ldots\ldots\; (21e) \end{aligned}$$

and,

$$\begin{aligned} E\,\Delta\,\gamma = {} & \Big\{ \varepsilon \sum x \sin \phi \cos \phi \frac{S}{I_y} - \varepsilon\,\lambda \sum x \sin \phi \cos \phi \frac{S}{F} \Big\} T_x \\ & + \Big\{ \varepsilon \sum x \sin^2 \phi \frac{S}{I_y} + \varepsilon\,\lambda \sum x \cos^2 \phi \frac{S}{F} \Big\} T_y - \Big\{ \sum v \sin \phi \frac{S}{I_y} \\ & - \lambda \sum u \cos \phi \frac{S}{F} \Big\} T_z + \Big\{ \sum \sin \phi \cos \phi \frac{S}{I_y} - \lambda \sum \sin \phi \cos \phi \frac{S}{F} \Big\} M_y \\ & + \Big\{ \sum \sin^2 \phi \frac{S}{I_y} + \lambda \sum \cos^2 \phi \frac{S}{F} \Big\} M_z + L_\gamma \;\ldots\ldots\ldots\; (21f) \end{aligned}$$

In the usual type of arch the terms due to shear and direct thrust may be omitted; this is also true for those due to rotation about the radial axis if the arch is of the barrel type:

$$\begin{aligned} E\,\Delta\,x = {} & \Big\{ \sum y^2 \frac{S}{I_z} + \varepsilon^2 \lambda \sum x^2 \sin^2 \phi \frac{S}{F} \Big\} T_x + \Big\{ -\sum x\,y \frac{S}{I_z} \\ & - \varepsilon^2 \lambda \sum x^2 \sin \phi \cos \phi \frac{S}{F} \Big\} T_y - \Big\{ \varepsilon\,\lambda \sum u\,x \sin \phi \frac{S}{F} \Big\} T_z \\ & + \Big\{ \sum y \frac{S}{I_z} \Big\} M_z + \Big\{ \varepsilon\,\lambda \sum x \sin^2 \phi \frac{S}{F} \Big\} M_y \\ & - \Big\{ \varepsilon\,\lambda \sum x \sin \phi \cos \phi\, S \Big\} M_x + L_x \;\ldots\ldots\ldots\ldots\; (22a) \end{aligned}$$

$$\begin{aligned} E\,\Delta\,y = {} & \Big\{ -\sum x\,y \frac{S}{I_z} - \varepsilon^2 \lambda \sum x^2 \sin \phi \cos \phi \frac{S}{F} \Big\} T_x + \Big\{ \sum x^2 \frac{S}{I_z} \\ & + \varepsilon^2 \lambda \sum x^2 \cos^2 \phi \frac{S}{F} \Big\} T_y + \Big\{ \varepsilon\,\lambda \sum u\,x \cos \phi \frac{S}{F} \Big\} T_z - \Big\{ \sum x \frac{S}{I_z} \Big\} M_z \\ & - \Big\{ \varepsilon\,\lambda \sum x \sin \phi \cos \phi \frac{S}{F} \Big\} M_y + \Big\{ \varepsilon\,\lambda \sum x \cos^2 \phi \frac{S}{F} \Big\} M_x + L_y \ldots (22b) \end{aligned}$$

$$\begin{aligned} E\,\Delta\,z = {} & -\Big\{ \varepsilon\,\lambda \sum u\,x \sin \phi \frac{S}{F} \Big\} T_x + \Big\{ \varepsilon\,\lambda \sum u\,x \cos \phi \frac{S}{F} \Big\} T_y \\ & + \Big\{ \lambda \sum u^2 \frac{S}{F} \Big\} T_z - \Big\{ \lambda \sum u \sin \phi \frac{S}{F} \Big\} M_y \\ & + \Big\{ \lambda \sum u \cos \phi \frac{S}{F} \Big\} M_x + L_z \;\ldots\ldots\ldots\ldots\ldots\; (22c) \end{aligned}$$

$$E\,\Delta\,\alpha = \Big\{ \sum y \frac{S}{I_z} \Big\} T_x - \Big\{ \sum x \frac{S}{I_z} \Big\} T_y + \Big\{ \sum \frac{S}{I_z} \Big\} + L_\alpha \ldots\ldots (22d)$$

$$\begin{aligned} E\,\Delta\,\beta = & \left\{ \varepsilon\,\lambda \sum x \sin^2 \phi \frac{S}{F} \right\} T_x - \left\{ \varepsilon\,\lambda \sum x \sin \phi \cos \phi \frac{S}{F} \right\} T_y \\ & + \left\{ \lambda \sum u \sin \phi \frac{S}{F} \right\} T_z + \left\{ \lambda \sum \sin^2 \phi \frac{S}{F} \right\} M_y \\ & - \left\{ \lambda \sum \sin \phi \cos \phi \frac{S}{F} \right\} M_x + L_\beta \ldots\ldots\ldots\ldots\ldots \quad (22e) \end{aligned}$$

and,

$$\begin{aligned} E\,\Delta\,\gamma = & - \left\{ \varepsilon\,\lambda \sum x \sin \phi \cos \phi \frac{S}{F} \right\} T_x + \left\{ \varepsilon\,\lambda \sum x \cos^2 \phi \frac{S}{F} \right\} T_y \\ & + \left\{ \lambda \sum u \cos \phi \frac{S}{F} \right\} T_z - \left\{ \lambda \sum \sin \phi \cos \phi \frac{S}{F} \right\} M_y \\ & + \left\{ \lambda \sum \cos^2 \phi \frac{S}{F} \right\} M_x + L_\gamma \ldots\ldots\ldots\ldots\ldots \quad (22f) \end{aligned}$$

Terms for various types of load have been developed, as follows:

Temperature Loading Terms:

$$L_x = S_A\, t^\circ\, e\, E \ldots\ldots\ldots\ldots\ldots \quad (23a)$$

$$L_z = \varepsilon\, S_A\, t^\circ\, e\, E \ldots\ldots\ldots\ldots\ldots \quad (23b)$$

Concentrated Loading Terms (the integrations or summations of Equations (24) and (25) are taken from the load to the abutment only):

$$\begin{aligned} L_x = & \left\{ (1 - \lambda) \int \sin \phi \cos \phi \frac{ds}{A} - \int x\, y \frac{ds}{I_z} \right. \\ & \left. + \varepsilon^2 \int x^2 \sin \phi \cos \phi \left(\frac{1}{I_y} - \frac{\lambda}{F} \right) ds \right\} W + \left\{ \int y \frac{ds}{I_z} \right\} W\, x' \\ & - \left\{ \varepsilon \int x \sin \phi \cos \phi \left(\frac{1}{I_y} - \frac{\lambda}{F} \right) ds \right\} W\, z' \ldots\ldots\ldots \quad (24a) \end{aligned}$$

$$\begin{aligned} L_y = & \left\{ \int \sin^2 \phi \frac{ds}{A} + \lambda \int \cos^2 \phi \frac{ds}{A} + \int x^2 \frac{ds}{I_z} + \varepsilon^2 \int x^2 \sin^2 \phi \frac{ds}{I_y} \right. \\ & \left. + \varepsilon^2 \lambda \int x^2 \cos^2 \phi \frac{ds}{F} \right\} W - \left\{ \int x \frac{ds}{I_z} \right\} W\, x' - \left\{ \varepsilon \int x \sin^2 \phi \frac{ds}{I_y} \right. \\ & \left. + \varepsilon\, \lambda \int x \cos^2 \phi \frac{ds}{F} \right\} W\, z' \ldots\ldots\ldots\ldots\ldots \quad (24b) \end{aligned}$$

$$\begin{aligned} L_z = & - \left\{ \varepsilon \int v\, x \sin \phi \frac{ds}{I_y} - \varepsilon\, \lambda \int u\, x \cos \phi \frac{ds}{F} \right\} W + \left\{ \int v \sin \phi \frac{ds}{I_y} \right. \\ & \left. - \lambda \int u \cos \phi \frac{ds}{F} \right\} W\, z' \ldots\ldots\ldots\ldots\ldots \quad (24c) \end{aligned}$$

$$L_\alpha = - \left\{ \int x \frac{ds}{I_z} \right\} W + \left\{ \int \frac{ds}{I_z} \right\} W\, x' \ldots\ldots\ldots\ldots\ldots \quad (24d)$$

$$\begin{aligned} L_\beta = & \left\{ \varepsilon \int \left(\frac{1}{I_y} - \frac{\lambda}{F} \right) x \sin \phi \cos \phi \, ds \right\} W \\ & - \left\{ \int \left(\frac{1}{I_y} - \frac{\lambda}{F} \right) \sin \phi \cos \phi \, ds \right\} W\, z' \ldots\ldots\ldots \quad (24e) \end{aligned}$$

and,

$$L_\gamma = \left\{ \varepsilon \int x \sin^2 \phi \frac{ds}{I_y} + \varepsilon \lambda \int x \cos^2 \phi \frac{ds}{F} \right\} W$$
$$- \left\{ \int \sin^2 \phi \frac{ds}{I_y} + \lambda \int \cos^2 \phi \frac{ds}{F} \right\} W z' \dots\dots\dots\dots (24f)$$

As in the case of Equations (21) the terms of Equations (24) may be written for finite blocks, as follows:

Concentrated Loading Terms:

$$L_x = - \left\{ \sum x y \frac{S}{I_z} + \varepsilon^2 \sum x^2 \sin \phi \cos \phi \frac{\lambda}{F} S \right\} W + \sum y \frac{S}{I_z} W x'$$
$$+ \varepsilon \sum x \sin \phi \cos \phi \frac{\lambda s}{F} W z' \dots\dots\dots\dots (25a)$$

$$L_y = \left\{ \sum x^2 \frac{S}{I_z} + \varepsilon^2 \lambda \sum x^2 \cos^2 \phi \frac{S}{F} \right\} W - \left\{ \sum x \frac{S}{I_z} W x' \right.$$
$$\left. - \varepsilon \lambda \sum x \cos^2 \phi \frac{S}{F} \right\} W z' \dots\dots\dots\dots (25b)$$

$$L_z = \left\{ \varepsilon \lambda \sum u x \cos \phi \frac{S}{F} \right\} W - \left\{ \lambda \sum u \cos \phi \frac{S}{F} \right\} W z' \dots\dots\dots (25c)$$

$$L_\alpha = - \left\{ \sum x \frac{S}{I_z} \right\} W + \left\{ \sum \frac{S}{I_z} \right\} W x' \dots\dots\dots\dots (25d)$$

$$L_\beta = - \left\{ \varepsilon \sum x \sin \phi \cos \phi \frac{\lambda}{F} S \right\} W + \left\{ \sum \sin \phi \cos \phi \frac{\lambda}{F} S \right\} W z' . (25e)$$

and,

$$L_\gamma = \left\{ \varepsilon \lambda \sum x \cos^2 \phi \frac{S}{F} \right\} W - \left\{ \lambda \sum \cos^2 \phi \frac{S}{F} \right\} W z' \dots\dots\dots\dots (25f)$$

In the case of dead load and uniform load the terms due to horizontal earth thrust have been omitted for the sake of brevity. The remaining terms, have been reproduced for the finite blocks only:

Dead Vertical Loading Terms:

$$L_x = \sum (1 - \lambda) \sum W \sin \phi \cos \phi \frac{S}{A} - \sum W \bar{x} y \frac{S}{I_z}$$
$$+ \varepsilon^2 \sum W \bar{x} x \sin \phi \cos \phi S - \varepsilon^2 \lambda \sum W \bar{x} x \sin \phi \cos \phi \frac{S}{F} .. (26a)$$

$$L_y = \sum W \sin^2 \phi \frac{S}{A} + \lambda \sum W \cos^2 \phi \frac{S}{A} + \sum W \bar{x} x \frac{S}{I_z}$$
$$+ \varepsilon^2 \sum W \bar{x} x \sin^2 \phi \frac{S}{I_y} + \varepsilon^2 \lambda \sum W \bar{x} x \cos^2 \phi \frac{S}{F} \dots\dots (26b)$$

$$L_z = - \varepsilon \sum W \bar{x} v \sin \phi \frac{S}{I_y} + \varepsilon \lambda \sum W \bar{x} u \cos \phi \frac{S}{F} \dots\dots\dots\dots (26c)$$

$$L_\alpha = - \sum W \bar{x} \frac{S}{I_z} \dots\dots\dots\dots\dots\dots\dots\dots (26d)$$

$$L_\beta = \varepsilon \sum W \overline{x} \sin\phi \cos\phi \frac{S}{I_y} - \varepsilon \lambda \sum W \overline{x} \sin\phi \cos\phi \frac{S}{F} \ldots\ldots (26e)$$

and,

$$L_\gamma = \varepsilon \sum W \overline{x} \sin^2\phi \frac{S}{I_y} + \lambda \varepsilon \sum W \overline{x} \cos^2\phi \frac{S}{F} \ldots\ldots (26f)$$

Uniform Vertical Loading Terms:

$$L_x = b\, w \left\{ (1-\lambda) \sum x \sin\phi \cos\phi \frac{S}{A} - \frac{1}{2} \sum x^2 y \frac{S}{I_z} \right.$$
$$\left. + \frac{1}{2} \varepsilon^2 \sum x^3 \sin\phi \cos\phi \frac{S}{I_y} - \frac{1}{2} \varepsilon^2 \lambda \sum x^3 \sin\phi \cos\phi \frac{S}{F} \right\} \ldots (27a)$$

$$L_y = b\, w \left\{ \sum x \sin^2\phi \frac{S}{A} + \lambda \sum x \cos^2\phi \frac{S}{A} + \frac{1}{2} \sum x^3 \frac{S}{I_z} \right.$$
$$\left. + \frac{1}{2} \varepsilon^2 \sum x^3 \sin^2\phi \frac{S}{I_y} + \frac{1}{2} \lambda \varepsilon^2 \sum x^3 \cos^2\phi \frac{S}{F} \right\} \ldots\ldots (27b)$$

$$L_z = b\, w \left\{ -\frac{1}{2} \varepsilon \sum x^2 v \sin\phi \frac{S}{I_y} + \frac{1}{2} \lambda \varepsilon \sum x^2 u \cos\phi \frac{S}{F} \right\} \ldots (27c)$$

$$L_\alpha = b\, w \left\{ -\frac{1}{2} \sum x^2 \frac{S}{I_z} \right\} \ldots\ldots\ldots\ldots (27d)$$

$$L_\beta = b\, w \left\{ \frac{1}{2} \varepsilon \sum x^2 \sin\phi \cos\phi \frac{S}{I_y} - \frac{1}{2} \varepsilon \lambda \sum x^2 \sin\phi \cos\phi \frac{S}{F} \right\} \ldots (27e)$$

and,

$$L_\gamma = b\, w \left\{ \frac{1}{2} \varepsilon \sum x^2 \sin^2\phi \frac{S}{I_y} + \frac{1}{2} \lambda \varepsilon \sum x^2 \cos^2\phi \frac{S}{F} \right\} \ldots\ldots (27f)$$

The value of F is given by the expression:

$$\frac{I}{F} = \frac{205\, \pi\, (t^2 + b^2)}{180\, b^3\, t^3} \ldots\ldots\ldots\ldots (28)$$

When λ is 2.5 this becomes $\frac{\lambda}{F} = 8.95 \frac{1}{b} \left(\frac{1}{b^2 t} + \frac{1}{t^3} \right)$, which may be written:

$$\frac{\lambda}{F} = 9 \times \frac{1}{b} \left(\frac{1}{b^2 t} + \frac{1}{t^3} \right) \ldots\ldots\ldots\ldots (29)$$

If the angle of skew is made equal to zero, these equations reduce as follows:

Equations (20) become,

$$E\, \varDelta x = \left\{ \int \cos^2\phi \frac{ds}{A} + \lambda \int \sin^2\phi \frac{ds}{A} + \int y^2 \frac{ds}{I_z} \right\} T_x$$
$$+ \left\{ (1-\lambda) \int \sin\phi \cos\phi \frac{ds}{A} - \int x\, y \frac{ds}{I_z} \right\} T_y + \left\{ \int y \frac{ds}{I_z} \right\} M_z + L_x \ldots (30a)$$

$$E\,\Delta y = \left\{(1-\lambda)\int \sin\phi\cos\phi\,\frac{ds}{A} - \int x\,y\,\frac{ds}{I_z}\right\} T_x + \left\{\int \sin^2\phi\,\frac{ds}{A} + \lambda\int \cos^2\phi\,\frac{ds}{A} + \int x^2\,\frac{ds}{I_z}\right\} T_y + \left\{\int x\,\frac{ds}{I_z}\right\} M_z + L_y \ldots\ldots (30b)$$

$$E\,\Delta\alpha = \left\{\int \gamma\,\frac{ds}{I_z}\right\} T_x - \left\{\int x\,\frac{ds}{I_z}\right\} T_y + \left\{\int \frac{ds}{I_z}\right\} M_z + L_\alpha \ldots\ldots (30d)$$

$$E\,\Delta z = \left\{\lambda\int \frac{ds}{A} + \int v^2\,\frac{ds}{I_y} + \lambda\int u^2\,\frac{ds}{F}\right\} T_z - \left\{\int v\cos\phi\,\frac{ds}{I_y} + \lambda\int u\sin\phi\,\frac{ds}{F}\right\} M_y - \left\{\int v\sin\phi\,\frac{ds}{I_y} - \lambda\int u\cos\phi\,\frac{ds}{F}\right\} M_x + L_z \ldots\ldots (30c)$$

$$E\,\Delta\beta = -\left\{\int v\cos\phi\,\frac{d\,s}{I_y} + \lambda\int u\sin\phi\,\frac{ds}{F}\right\} T_z + \left\{\int \cos^2\phi\,\frac{ds}{I_y} + \lambda\int \sin^2\phi\,\frac{ds}{F}\right\} M_y + \left\{\int\left(\frac{1}{I_y} - \frac{\lambda}{F}\right)\sin\phi\cos\phi\,ds\right\} M_x + L_\beta \ldots\ldots (30e)$$

and,

$$E\,\Delta\gamma = -\left\{\int v\sin\phi\,\frac{ds}{I_y} - \lambda\int u\cos\phi\,\frac{ds}{F}\right\} T_z + \left\{\int\left(\frac{1}{I_y} - \frac{\lambda}{F}\right)\sin\phi\cos\phi\,ds\right\} M_y + \left\{\int \sin^2\phi\,\frac{ds}{I_y} + \lambda\int \cos^2\phi\,\frac{ds}{F}\right\} M_x + L_\gamma \ldots\ldots (30f)$$

Equations (24) become

$$L_x = \left\{(1-\lambda)\int \sin\phi\cos\phi\,\frac{ds}{A} - \int x\,y\,\frac{ds}{I_z}\right\} W + \left\{\int y\,\frac{ds}{I_z}\right\} W x' \,.\, (31a)$$

$$L_y = \left\{\int \sin^2\phi\,\frac{ds}{A} + \lambda\int \cos^2\phi\,\frac{ds}{A} + \int x^2\,\frac{ds}{I_z}\right\} W - \left\{\int x\,\frac{ds}{I_z}\right\} W x' \ldots\ldots (31b)$$

$$L_\alpha = -\left\{\int x\,\frac{ds}{I_z}\right\} W + \left\{\int \frac{ds}{I_z}\right\} W x' \ldots\ldots (31d)$$

$$L_z = -\left\{\int v\sin\phi\,\frac{ds}{I_y} - \lambda\int u\cos\phi\,\frac{ds}{F}\right\} W z' \ldots\ldots (31c)$$

$$L_\beta = -\left\{\int\left(\frac{1}{I_y} - \frac{\lambda}{F}\right)\sin\phi\cos\phi\,ds\right\} W z' \ldots\ldots (31e)$$

and,

$$L_\gamma = -\left\{ \int \sin^2 \phi \, \frac{ds}{I_y} + \lambda \int \cos^2 \phi \, \frac{ds}{F} \right\} W z' \ldots\ldots (31f)$$

Equations (3) become,

$$\begin{aligned} R_x = & \left\{ \int \frac{\lambda}{A} \, dy + \int (y + r_A)(y + r_B) \frac{dy}{I_y} \right\} T_{x\,B} \\ & + \left\{ s_B \int (y + r_A) \frac{dy}{I_z} \right\} T_{y\,B} - \left\{ (y + r_A) \frac{dy}{I_z} \right\} M_{z\,B} \\ & + \left\{ s_B \int (y + r_A) \frac{dy}{I_z} \right\} \sum L - \left\{ (y + r_A) \frac{dy}{I_z} \right\} \sum L \, x' \ldots (32a) \end{aligned}$$

$$\begin{aligned} R_y = & \left\{ s_A \int (y + r_A) \frac{dy}{I_z} \right\} T_{x\,B} + \left\{ \int \frac{dy}{A} - s_A \, s_B \int \frac{dy}{I_z} \right\} T_{y\,B} \\ & + \left\{ s_A \int \frac{dy}{I_z} \right\} M_{z\,B} + \left\{ \int \frac{dy}{A} - s_A \, s_B \int \frac{dy}{I_z} \right\} \sum L \\ & + \left\{ s_A \int \frac{dy}{I_y} \right\} \sum L \, z' + \left\{ s_A \int \frac{dy}{I_z} \right\} \sum L \, x' \ldots\ldots (32b) \end{aligned}$$

$$\begin{aligned} R_\alpha = & -\left\{ \int (y + r_B) \frac{dy}{I_z} \right\} T_{x\,B} + s_B \left\{ \int \frac{dy}{I_z} \right\} T_{y\,B} - \left\{ \int \frac{dy}{I_z} \right\} M_{z\,B} \\ & + \left\{ s_B \int \frac{dy}{I_z} \right\} \sum L - \left\{ \int \frac{dy}{I_z} \right\} \sum L \, x' \ldots\ldots (32d) \end{aligned}$$

$$\begin{aligned} R_z = & -\left\{ \lambda \int \frac{dy}{A} + \int (y + r_A)(y + r_B) \frac{dy}{I_y} - s_A \, s_B \, \lambda \int \frac{dy}{F} \right\} T_z \\ & - \left\{ s_A \, \lambda \int \frac{dy}{F} \right\} M_{y\,B} + \left\{ \int (y + r_A) \frac{dy}{I_y} \right\} M_{x\,B} \\ & - \left\{ \int (y + r_A) \frac{dy}{I_y} \right\} \sum L \, z' \ldots\ldots (32c) \end{aligned}$$

$$R_\beta = -\left\{ s_B \, \lambda \int \frac{dy}{F} \right\} T_{z\,B} + \left\{ \lambda \int \frac{dy}{F} \right\} M_{y\,B} \ldots\ldots (32e)$$

and,

$$R_\gamma = \left\{ (y + r_B) \int \frac{dy}{I_y} \right\} T_z - \left\{ \int \frac{dy}{I_y} \right\} M_x + \left\{ \int \frac{dy}{I_y} \right\} \sum L \, z' \ldots (32f)$$

and Equations (4) become,

$$\begin{aligned} L_x = & -\left\{ \int \frac{\lambda}{A} \, dy + \int (y + r_A)(y + r_B) \frac{dy}{I_z} \right\} T_{x\,A} \\ & - \left\{ s_A \int (y + r_B) \frac{dy}{I_z} \right\} T_{y\,A} - \left\{ \int (y + r_B) \frac{dy}{I_z} \right\} M_{z\,A} \\ & + \left\{ s_A \int (y + r_B) \frac{dy}{I_z} \right\} \sum L - \left\{ \int (y + r_B) \frac{dy}{I_z} \right\} \sum L \, x' \ldots (33a) \end{aligned}$$

$$L_y = \left\{ s_B \int (y + r_A) \frac{dy}{I_z} \right\} T_{x\,A} + \left\{ s_A \; s_B \int \frac{dy}{I_z} - \int \frac{dy}{A} \right\} T_{y\,A}$$

$$+ \left\{ s_B \int \frac{dy}{I_z} \right\} M_{z\,A}$$

$$- \left\{ s_A \; s_B \int \frac{dy}{I_z} - \int \frac{dy}{A} + s_A \; s_B \int \frac{dy}{I_y} \right\} \sum L$$

$$+ \left\{ s_B \int \frac{dy}{I_z} \right\} \sum L\,x' \ldots\ldots\ldots\ldots\ldots\ldots \text{(33}b\text{)}$$

$$L_\alpha = - \left\{ \int (y + r_A) \frac{dy}{I_z} \right\} T_{x\,A} - \left\{ s_A \int \frac{dy}{I_z} \right\} T_{y\,A} - \left\{ \int \frac{dy}{I_z} \right\} M_{z\,A}$$

$$+ \left\{ s_A \int \frac{dy}{I_z} \right\} \sum L - \left\{ \int \frac{dy}{I_z} \right\} \sum L\,x' \ldots\ldots\ldots \text{(33}d\text{)}$$

$$L_z = \left\{ s_A \; s_B \; \lambda \int \frac{dy}{F} - \lambda \int \frac{dy}{A} - \int (y + r_A)\,(y + r_B) \frac{dy}{I_y} \right\} T_{z\,A}$$

$$+ \left\{ s_B \; \lambda \int \frac{dy}{F} \right\} M_{y\,A} + \left\{ \int (y + r_B) \frac{dy}{I_y} \right\} M_{x\,A}$$

$$- \left\{ \int (y + r_B) \frac{dy}{I_y} \right\} \sum L\,z' \ldots\ldots\ldots\ldots\ldots \text{(33}c\text{)}$$

$$L_\beta = - \left\{ s_A \; \lambda \int \frac{dy}{F} \right\} T_{z\,A} - \left\{ \lambda \int \frac{dy}{F} \right\} M_{y\,A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(33}e\text{)}$$

and,

$$L_\gamma = \left\{ \int (y + r_B) \frac{dy}{I_y} \right\} T_{z\,A} - \left\{ \int \frac{dy}{I_y} \right\} M_{x\,A} - \left\{ \int \frac{dy}{I_y} \right\} \sum L\,z' . \text{(33}f\text{)}$$

All groups of equations for zero skew are arranged in two parts of three equations each, so that two sets of computations can be carried through entirely independent of each other.

APPENDIX III

Equations for Skew Arch, When Abutments Are Free to Rotate

As the formulas for the skew arch, when the abutments are considered free to rotate, have been requested on numerous occasions they are given in Appendix III with a brief outline of their derivation.

From Fig. 3 and Equations (5),

$$dv = t_v \frac{ds}{A\;E} = (T_x \cos \phi \pm T_y \sin \phi + V \sin \phi - H \cos \phi) \frac{ds}{A\;E} \ldots\ldots \text{(34}a\text{)}$$

$$du = t_u \frac{\lambda\;ds}{A\;E} = (- T_x \sin \phi \pm T_y \cos \phi + V \cos \phi + H \sin \phi) \frac{\lambda\;ds}{A\;E} \ldots \text{(34}b\text{)}$$

$$dz = t_z \frac{\lambda\;ds}{A\;E} = T_z \frac{\lambda\;ds}{A\;E} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(34}c\text{)}$$

$$d\alpha = m_z \frac{ds}{E\,I_z} = [T_x\,y \mp T_y\,x + M_z - V\,(x - x') - H\,(y - y')]\frac{ds}{E\,I_z} \,\ldots (34d)$$

$$d\beta = m_u \frac{ds}{E\,I_y} = [T_x\,\varepsilon\,x \cos\phi \pm T_y\,x \sin\phi - T_z\,v \pm M_y \cos\phi$$
$$+ M_x \sin\phi + V\,(\varepsilon\,x - z') \sin\phi - H\,(\varepsilon\,x - z') \cos\phi]\,v \ldots\ldots (34e)$$

$$d\gamma = m_v \frac{\lambda\,ds}{E\,F} = [- T_x\,\varepsilon\,x \sin\phi \pm T_y\,x \cos\phi + T_z\,u \mp M_y \sin\phi$$
$$+ M_x \cos\phi + V\,(\varepsilon\,x - z') \cos\phi + H\,(\varepsilon\,x - z') \sin\phi]\,v \ldots\ldots (34f)$$

If a unit force, p_x, is placed at the right abutment, the thrust on any block length, *ds*, making an angle, ϕ, with the horizontal is,

$$T = p_x \cos\phi\,d\,v \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

Table 5, which gives the forces and moments at each section due to forces and moments at the abutments, may be derived with the guidance of Fig. 15.

TABLE 5.—FORCES AND MOMENTS AT VARIOUS SECTIONS OF AN ARCH DUE TO ABUTMENT REACTIONS

(1)	x (2)	y (3)	z (4)	α (5)	β (6)	γ (7)
p_v	$p_x \cos\phi$	$\pm p_y \sin\phi$	…………	……	………	………
p_u	$- p_x \sin\phi$	$\pm p_y \cos\phi$	…………	……	………	………
p_z	…………	…………	p_z	……	………	………
n_z	$p_x (y - h)$	$\mp p_y (l + x)$	…………	n_z	………	………
n_u	$p_x (\epsilon\,l + \epsilon\,x) \cos\phi$	$\pm p_y (\epsilon\,l + \epsilon\,x) \sin\phi$	$- p_z (v + l \cos\phi - h \sin\phi)$	……	$\pm n_y \cos\phi$	$n_x \sin\phi$
n_v	$- p_x (\epsilon\,l + \epsilon\,x) \sin\phi$	$\pm p_y (\epsilon\,l + \epsilon\,x) \cos\phi$	$+ p_z (u \pm l \sin\phi + h \cos\phi)$	……	$\pm n_y \sin\phi$	$n_x \cos\phi$

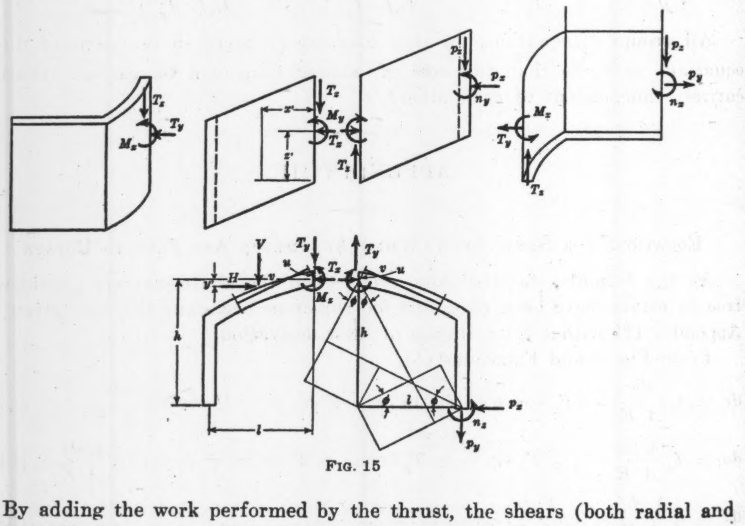

FIG. 15

By adding the work performed by the thrust, the shears (both radial and horizontal), the moments about the horizontal and radial axes, and the torque, the total work performed in deforming this block may be determined. This is equal to the work performed by the force, p_x, moving through the distance that the abutment is deflected in the direction in which the force is acting.

If p_x equals unity, the expression for the deflection of the abutment in the direction, X, due to the deformation of one block of length, ds, is:

$$d\,\Delta = p_v\,dv + p_u\,du + p_z\,dz + n_z\,d\alpha + n_u\,d\beta + n_v\,d\gamma \ldots\ldots (36)$$

The total deflection is the integral of the several terms producing Δ.

In obtaining $d\,\Delta$ to find the expressions for p_v, p_u, n_z, etc., for a deflection in the direction, X, only those terms in Column (2) Table 5 are used; for the y-deflection, Column (3) is used, etc.

Collecting terms and using the values of Δ_x, Δ_y, etc., as defined in Equations (20), the several deflections of the right abutment of the arch when considered as fixed at the left abutment, are:

$\Delta x - h\,\Delta\,\alpha + \varepsilon\,l\,\Delta\,\beta$ = deflection of right abutment in x-direction.....(37a)

$\Delta y + l\,\Delta\,\alpha - \varepsilon\,l\,\Delta\,\gamma$ = deflection of right abutment in y-direction.....(37b)

$\Delta z + h\,\Delta\,\gamma - l\,\Delta\,\beta$ = deflection of right abutment in z-direction.....(37c)

$\Delta\,\alpha$ = rotation of right abutment about z-axis........(37d)

$\Delta\,\beta$ = rotation of right abutment about y-axis........(37e)

$\Delta\,\gamma$ = rotation of right abutment about x-axis........(37f)

If the right sides of Equations (37) are all placed equal to zero, they represent the fixed-ended skew arch and the equations reduce at once to those already obtained for this condition, as they should. However, with the right abutment on a hinge, the axis of which is parallel to the z-axis, Equation (37d) cannot be placed equal to zero although Equations (37a), (37c), (37e), and (37f) may; and these four equations reduce to:

$$\Delta\,x - h\,\Delta\,\alpha = 0 \ldots\ldots (38a)$$

$$\Delta\,z = 0 \ldots\ldots (38c)$$

$$\Delta\,\beta = 0 \ldots\ldots (38e)$$

$$\Delta\,\gamma = 0 \ldots\ldots (38f)$$

With the left abutment hinged Equations (37b) and (37d) can be produced from purely static considerations, and a little algebraic transforming produces the following equations:

$$T_y = -\frac{1}{2\,l}(l - x')\,V - \frac{1}{2\,l}(h - y')\,H \ldots\ldots (39a)$$

$$Q\,M_y = -F\,T_y + e_v + e_h \ldots\ldots (39b)$$

$$A'\,T_x + C\,T_z + F\,M_x = a_v + a_h \ldots\ldots (39c)$$

$$C\,T_x + L\,T_z + O\,M_x = c_v + c_h \ldots\ldots (39d)$$

$$F\,T_x + O\,T_z + S\,M_x = f_v + f_h \ldots\ldots (39e)$$

$$M_z = \frac{1}{2}(l - x')\,V - h\,T_x + \frac{1}{2}(h - y')\,H \ldots\ldots (39f)$$

The formulas for the coefficients in Equations (39), are:

$$\begin{aligned} A' = A - 2\,h\,D + P\,h^2 = {} & \sum \cos^2\phi\,\frac{S}{A} + \lambda \sum \sin^2\phi\,\frac{S}{A} + \sum y^2\,\frac{S}{I_z} \\ & + \varepsilon^2 \sum x^2 \cos^2\phi\,\frac{S}{I_y} + \varepsilon^2\,\lambda \sum x^2 \sin^2 \frac{S}{F} - 2\,h \sum y\,\frac{S}{I_z} + h\,\varepsilon^2 \sum \frac{S}{I_z} \end{aligned} \quad \text{(40}a\text{)}$$

$$C = -\left\{ \varepsilon \sum v\,x \cos\phi\,\frac{S}{I_y} + \varepsilon\,\lambda \sum u\,x \sin\phi\,\frac{S}{F} \right\} \ldots\ldots\ldots\ldots \text{(40}b\text{)}$$

$$F = \varepsilon \sum x \sin\phi \cos\phi\,\frac{S}{I_y} - \varepsilon\,\lambda \sum x \sin\phi \cos\phi\,\frac{\lambda}{F} \ldots\ldots\ldots\ldots \text{(40}c\text{)}$$

$$L = \sum v^2\,\frac{S}{I_y} + \lambda \sum u^2\,\frac{S}{F} + \lambda \sum \frac{S}{A} \ldots\ldots\ldots\ldots \text{(40}d\text{)}$$

$$O = -\left\{ \sum v \sin\phi\,\frac{S}{I_y} - \lambda \sum u \cos\phi\,\frac{S}{F} \right\} \ldots\ldots\ldots\ldots \text{(40}e\text{)}$$

$$Q = \sum \cos^2\phi\,\frac{S}{I_y} + \lambda \sum \sin^2\phi\,\frac{S}{F} \ldots\ldots\ldots\ldots \text{(40}f\text{)}$$

and,

$$S = \sum \sin^2\phi\,\frac{S}{I_y} + \lambda \sum \cos^2\phi\,\frac{S}{F} \ldots\ldots\ldots\ldots \text{(40}g\text{)}$$

In Equations (39) the terms due to vertical loads are:

$$\begin{aligned} a_v = -\frac{1}{2} \Big\{ & (l - x') \left[\sum_A^B y\,\frac{S}{I_z} - h \sum_A^P \frac{S}{I_z} \right] \\ & + (1 - \lambda) \sum_A^P \sin\phi \cos\phi\,\frac{S}{A} - \sum_A^P x\,y\,\frac{S}{I_z} + h \sum_A^P x\,\frac{S}{I_z} \\ & - h\,x' \sum_A^P \frac{S}{I_z} + x' \sum_A^P y\,\frac{S}{I_z} + \varepsilon^2 \sum_A^P x^2 \sin\phi \cos\phi\,\frac{S}{I_y} \\ & - \varepsilon^2\,\lambda \sum_A^P x^2 \sin\phi \cos\phi\,\frac{S}{F} - \varepsilon^2\,x' \sum_A^P x \sin\phi \cos\phi\,\frac{S}{I_y} \\ & + \varepsilon^2\,x'\,\lambda \sum_A^P x \sin\phi \cos\phi\,\frac{S}{F} \Big\} V \ldots\ldots\ldots\ldots \text{(41}a\text{)} \end{aligned}$$

$$\begin{aligned} c_v = -\frac{1}{2} \Big\{ & -\varepsilon \sum_A^P v\,x \sin\phi\,\frac{S}{I_y} + \varepsilon\,\lambda \sum_A^P u\,x \cos\phi\,\frac{S}{F} \\ & + \varepsilon\,x' \sum_A^P v \sin\phi - \varepsilon\,x'\,\lambda \sum_A^P u \cos\phi\,\frac{S}{F} \Big\} V \ldots\ldots\ldots\ldots \text{(41}b\text{)} \end{aligned}$$

$$\begin{aligned} f_v = -\frac{1}{2} \Big\{ & \varepsilon \sum_A^P x \sin^2\phi\,\frac{S}{I_y} + \varepsilon\,\lambda \sum_A^P x \cos^2\phi\,\frac{S}{F} \\ & - \varepsilon\,x' \sum_A^P \sin^2\phi\,\frac{S}{I_y} - \varepsilon\,x'\,\lambda \sum_A^P \cos^2\phi\,\frac{S}{F} \Big\} V \ldots\ldots\ldots \text{(41}c\text{)} \end{aligned}$$

and,

$$\begin{aligned} e_v = -\frac{1}{2} \Big\{ & \varepsilon \sum_A^P x \sin\phi \cos\phi \left(\frac{S}{I_y} - \frac{\lambda\,S}{F} \right) \\ & - \varepsilon\,x' \sum_A^P \sin\phi \cos\phi \left(\frac{S}{I_y} - \frac{\lambda\,S}{F} \right) V \ldots\ldots\ldots\ldots \text{(41}d\text{)} \end{aligned}$$

In Equations (39) the terms due to horizontal loads are:

$$\begin{aligned} a_h = -\frac{1}{2}\Big\{ & (h-y')\left[\sum_A^P y\frac{S}{I_z} - h\sum_A^P \frac{S}{I_z}\right] - \sum_A^P \cos^2\phi\,\frac{S}{A} \\ & -\lambda\sum_A^P \sin^2\phi\,\frac{S}{A} - \sum_A^P y^2\frac{S}{I_z} + y'\sum_A^P y\frac{S}{I_z} + h\sum_A^P y\frac{S}{I_z} \\ & -h\,y'\sum_A^P \frac{S}{I_z} - \varepsilon^2\sum_A^P x^2\cos^2\phi\,\frac{S}{I_y} + \varepsilon^2 x'\sum_A^P x\cos^2\phi\,\frac{S}{I_y} \\ & -\varepsilon^2\sum_A^P x^2\sin^2\phi\,\frac{\lambda S}{F} + \varepsilon^2 x'\sum_A^P x\sin^2\phi\,\frac{\lambda S}{F}\Big\} H \quad\ldots\ldots\ldots(42a) \end{aligned}$$

$$\begin{aligned} c_h = -\frac{1}{2}\Big\{ & \varepsilon\sum_A^P v\,x\cos\phi\,\frac{S}{I_y} - \varepsilon\,x'\sum_A^P v\cos\phi\,\frac{S}{I_y} \\ & +\varepsilon\sum_A^P x\,u\sin\phi\,\frac{\lambda S}{F} - \varepsilon\,x'\sum_A^P u\sin\phi\,\frac{\lambda S}{F}\Big\} H \quad\ldots\ldots\ldots(42b) \end{aligned}$$

$$\begin{aligned} f_h = -\frac{1}{2}\Big\{ & -\varepsilon\sum_A^P x\cos^2\phi\,\frac{S}{I_y} - \varepsilon\sum_A^P x\sin^2\phi\,\frac{\lambda S}{F} \\ & +\varepsilon\,x'\sum_A^P \cos^2\phi\,\frac{S}{I_y} + \varepsilon\,x'\sum_A^P \sin^2\phi\,\frac{\lambda S}{F}\Big\} H \quad\ldots\ldots\ldots\ldots(42c) \end{aligned}$$

and,

$$\begin{aligned} e_h = -\frac{1}{2}\Big\{ & -\varepsilon\sum_A^P x\cos^2\phi\,\frac{S}{I_y} - \varepsilon\sum_A^P x\sin^2\phi\,\frac{\lambda S}{F} \\ & +\varepsilon\,x'\sum_A^P \cos^2\phi\,\frac{S}{I_y} + \varepsilon\,x'\sum_A^P \sin^2\phi\,\frac{\lambda S}{F}\Big\} H \quad\ldots\ldots\ldots(42d) \end{aligned}$$

For symmetrical loading, $T_y = 0$ and $M_y = 0$. Therefore e_v and e_h need not be determined.

DISCUSSION

B. L. WEINER,[8] ASSOC. M. AM. SOC. C. E. (by letter).—Parts I, III, and IV of Professor Rathbun's paper are of as much interest to designers of right arches and other statically indeterminate frames as they are to the designers of skew arches. The author has presented a general method for analyzing indeterminate structures, which has a certain marked advantage over other methods known to the profession.

Generally speaking, a right arch has three unknown reactions for each span, which must be obtained by the solution of simultaneous equations. In a three-span arch, there would be nine unknowns, necessitating nine simultaneous equations, each containing all the unknown quantities. This, in general, would be the case, except for certain simplifications which are possible because of symmetry, or because such conditions as hinged bases make certain unknowns zero, or because of other conditions which assign known values to some of the reactions. For the skew arch, the various standard multiple-span analyses could be applied and Professor Rathbun's theory of single-span arches[9] could be extended to multiple spans. In the skew arch, however, there are six unknowns for each arch, and, in general, eighteen equations with eighteen unknowns in each would be required for the analysis of a three-span skew arch. While this is possible, the mere mechanical difficulty of the process would be great even though the solution of simultaneous equations is no particular hardship if determinates are not used.[10] The difficulty increases with the number of spans, and for n spans, there would be $6n$ equations, each containing $6n$ unknowns.

Professor Rathbun has shown a way out, since the method he proposes restricts the number of equations to be solved at any one time to that of one span, that is, in a right arch only three equations with three unknowns, and in a skew arch no more than six equations with six unknowns need be solved at any one time. This is an advantage even though the saving in the numerical work involved may not be very much, since a set of the six equations must be solved for each span and, except for the last span, the equations of one span contain constants which are the unknown reactions of an adjacent span. Furthermore, after the equations for one span are solved, giving the reactions of this span in terms of the reactions of an adjacent span, substitutions must be made to eliminate these reactions which have been obtained from the equations of the adjacent span. Finally, after the equations of the last (end) span have been solved, resulting in numerical values for the reactions of this span, a series of numerical substitutions must be made, working backword from the end span to obtain the reactions of the other spans. Furthermore, since the underlying principle of statically indeterminate analysis is that of deflections, the same deflections which appear as constants must be computed no matter what method is used.

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[8] New York, N. Y.

[9] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

[10] *Loc. cit.*, Vol. 96 (1932), Table 6, p. 1243 *et seq.*

However, even if there is no saving in direct labor, in Professor Rathbun's solution, the process is comparatively easy to follow. In this method, the effect of one span is apparent only in an adjacent span and is not directly carried over into other spans, thus making it possible to break the work up into a series of comparatively short steps. Besides reducing the number of simultaneous equations which have to be solved at any one time, this process has another important advantage.

The writer has found by experience in the drafting-room that even when the analysis of an arch is given in complete algebraic formulas, together with a numerical example, an enormous amount of explanation is required by the computers. Without complete algebraic formulas, the task of getting computers to follow through an analysis is made more difficult. While the standard methods of analysis lend themselves readily to expression in algebraic formulas, it is not generally possible to write one set of equations for an arch of any number of spans. Professor Rathbun's solution makes the process the same for any number of spans. The same procedure is repeated from the left-hand to the right-hand span regardless of the number of intervening spans.

It is unfortunate that the author has not given a set of deflection formulas for the case of abutments hinged at the base. Curiously enough, except for the single span with the footings at the same elevation, the assumption that the bases are hinged makes the theory more difficult, even though there are fewer unknowns. It seems to the writer that, for this case, the multiple-span frame will have to be broken up into fundamental types of structures other than cantilevers, and that a more "elegant" method of obtaining deflections will have to be used instead of that of the single cantilever. The cantilever method of obtaining deflections requires certain corrections when the bases are hinged and at different elevations. These corrections involve considerable laborious work and are difficult to apply.

The case of hinged bases is a very important one. The true state of the footings is that of partial fixity. When the first "Rigid Frames" of one span were designed by the Westchester County Park Commission, the analysis was made for both conditions, that is, with the bases fixed, and with the bases hinged. The sections were proportioned for whichever condition was the more severe. After several designs, it was found possible to design for hinged bases only, as the principal difference was found to be at the bottom of the posts where the dowels provided ample reinforcement for either condition. A similar procedure should be followed in the present case and both conditions taken into account.

In the derivation of the equations for the loading terms, the author has taken into account the effect of forces which are located unsymmetrically with respect to the spandrel walls. The only effect seems to be that the forces, T_z (or t_z), M_x (or m_v), and M_y (or m_u), are changed in the skew arch or introduced in the right arch. Taking the latter case, for simplicity, M_z remains unchanged if the load is shifted from a symmetrical, to an unsymmetrical, position. Since Professor Rathbun distributes the moment uniformly over the width of the section, it follows that as far as bending is con-

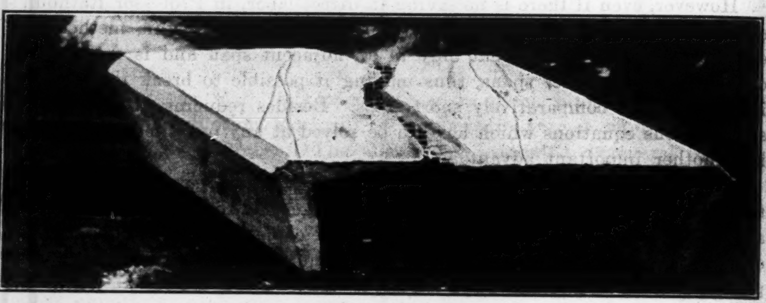

FIG. 16.—BASES FIXED, LOAD PLACED SYMMETRICAL, AND AT CENTER OF SPAN.

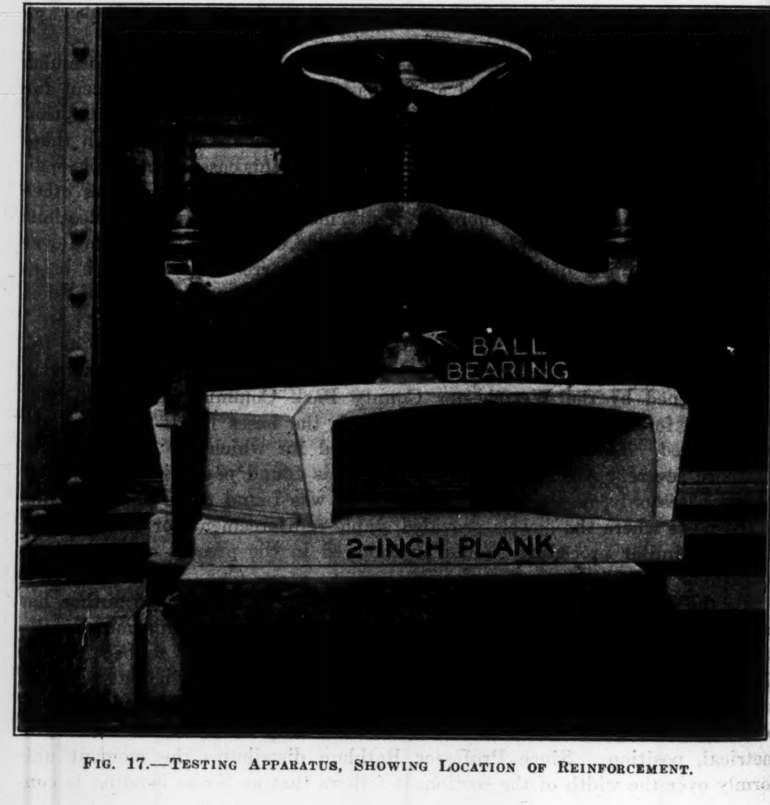
BALL BEARING
2-INCH PLANK

FIG. 17.—TESTING APPARATUS, SHOWING LOCATION OF REINFORCEMENT.

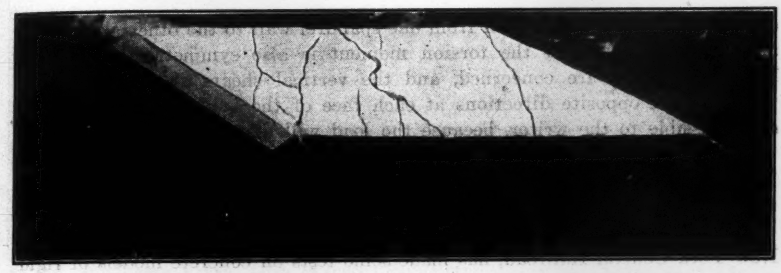

FIG. 18.—BASES HINGED, LOAD PLACED SYMMETRICAL AND AT CENTER OF SPAN.

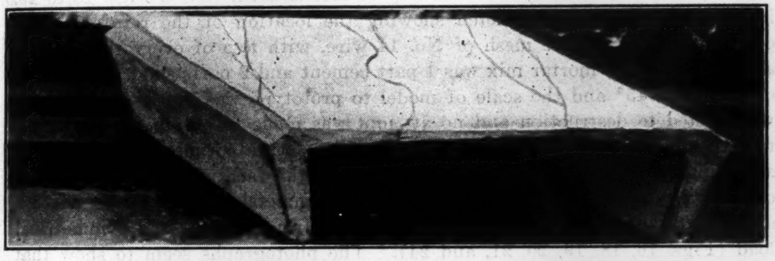

FIG. 19.—BASES HINGED, LOAD PLACED SYMMETRICAL AND AT CENTER OF SPAN.

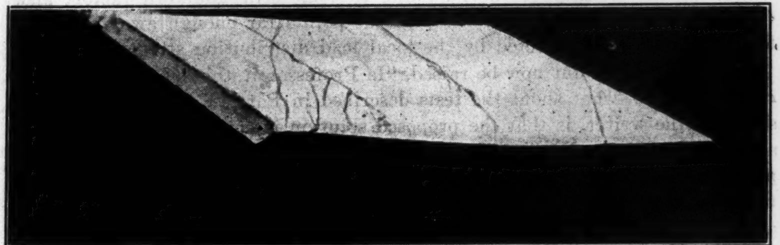

FIG. 20.—BASES HINGED, LOAD PLACED SYMMETRICAL AND AT QUARTER POINT OF SPAN.

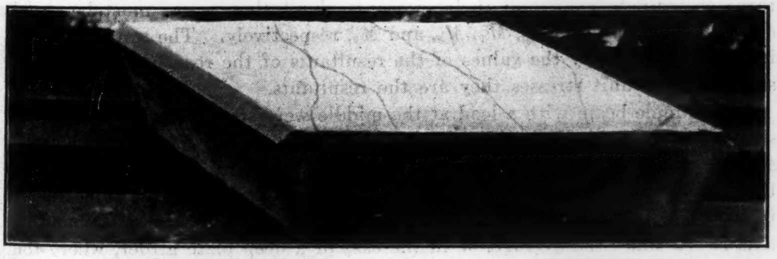

FIG. 21.—BASES HINGED, LOAD PLACED SYMMETRICAL AND AT QUARTER POINT OF SPAN.

cerned, all parts of the section from one spandrel wall to the other are equally affected. The effect of the torsion moment is also symmetrical as far as horizontal shears are concerned, and the vertical shears are also of equal value, but of opposite directions at each face of the bridge. This does not seem possible to the writer, because the load will distribute itself in such a way that the area immediately adjacent to it will be stressed most severely. The question of load distribution on slabs and arches, which has not yet been solved, enters into this problem.

E. F. Gifford, Assistant Engineer in the Engineering Department of the New York Central Railroad, has made some tests on concrete models of rigid frames. The photographs showing the results and method of testing are reproduced herein through the courtesy of Mr. Gifford. (See Figs. 16 to 25, inclusive.) Fig. 26 is a sketch showing the location of the reinforcement, comprising $\frac{1}{2}$-in. square mesh of No. 14 wire, with two of every three cross-strands cut. The mortar mix was 1 part cement and 2 parts sand. The angle of skew was 45° and the scale of model to prototype was 1 : 64. The models were tested to destruction and no attempt was made to obtain any quantitative results. The first cracks appeared at the obtuse corners of the top slab and continued approximately perpendicular to the face of the bridge. It is obvious from the photographs, that the unsymmetrical load (Figs. 22, 23, and 25) produces a type of failure different from that of the symmetrical load (Figs. 16, 18, 19, 20, 21, and 24). The photographs seem to show that for the unsymmetrical load there are two types of failures: (1) A local failure in the vicinity of the load due to some still unknown factor; and, (2) a general failure which takes place at points far enough away from the load so as not to be affected by the local load distribution stresses.

The question might now be raised, "Is Professor Rathbun's solution incorrect, and, if so, what about the tests described in Part IV?." The answer, it seems to the writer, is that the proposed solution is correct as far as it goes, and that Professor Rathbun's tests prove only what they were intended to prove and no more. If the crown section is taken, for example, it is shown that three forces and three moments act on it, that is, T_x, T_y, T_z, M_x, M_y, and M_z. These stresses are resisted by the unit stresses developed in the material of the arch. No matter what distribution of these unit stresses is adopted, their summation over the section, or their resultants parallel to the three co-ordinate axes, and their moments about these axes, must be equal and opposite to T_x, T_y, T_z, M_x, M_y, and M_z, respectively. The tests described in Part IV give only the values of the resultants of the reactions and do not show of what unit stresses they are the resultants.

If a simple beam with a load at the middle were similarly tested, it would show that one-half the load is carried to each support. The vertical shear acting on any section would then be equal to either reaction, but the distribution of the shear is another matter. The test would produce the same result in the case of a rectangular beam in which the maximum shear at the neutral axis is 3/2 times the average, or in the case of a deep plate girder, where the shear at any point in the web is practically equal to the average shear.

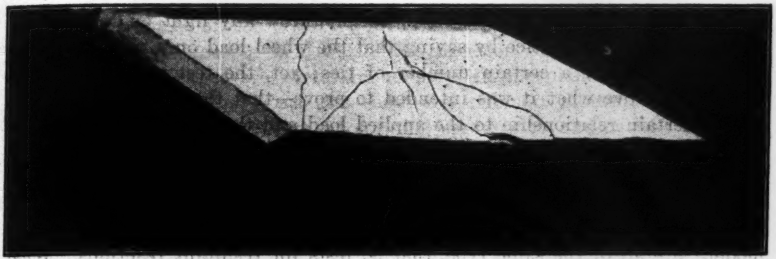

FIG. 22.—BASES FIXED, LOAD PLACED UNSYMMETRICAL AND AT CENTER OF SPAN.

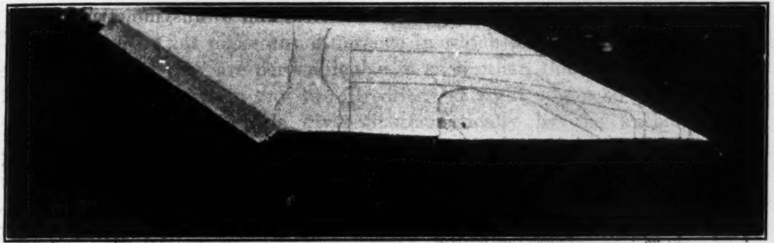

FIG. 23.—BASES FIXED, LOAD PLACED UNSYMMETRICAL AND AT CENTER OF SPAN.

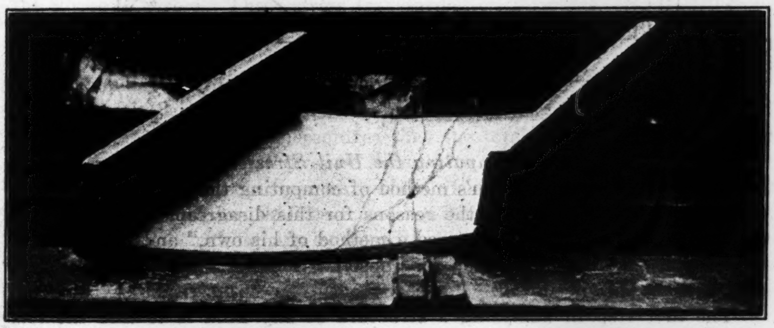

FIG. 24.—BASES FIXED, LOAD PLACED SYMMETRICAL AND AT QUARTER POINT OF SPAN.

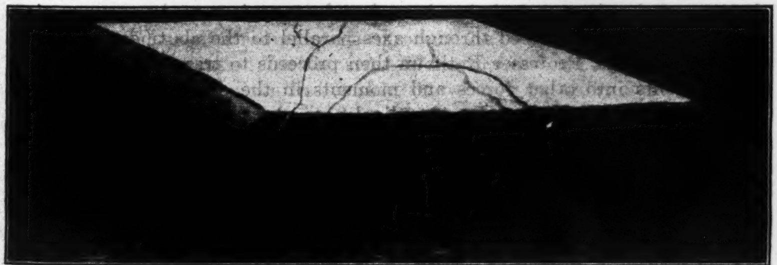

FIG. 25.—BASES HINGED, LOAD PLACED UNSYMMETRICAL AND AT CENTER OF SPAN.

Neither would the test of this simple beam throw any light on the problem which is solved in practice by saying that the wheel load on a railroad bridge is distributed over a certain number of ties; yet, the test would be correct and would prove what it was intended to prove—that the resultant reactions have a certain relationship to the applied load and that they satisfy the laws of equilibrium. The present case, it seems to the writer, is analogous. The tests described by Professor Rathbun in the present paper, those made on a plaster model and reported in a previous paper entitled, "Crown Stresses in a Skew Arch,"[11] and those made by him and Professor Beggs on a rubber model,[12] are all of the same type, that is, tests for resultant reactions. What is required now are tests to determine internal unit stresses and their distribution, and this applies equally to symmetrical and to unsymmetrical loads.

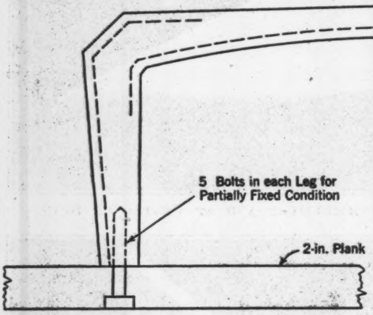

FIG. 26.

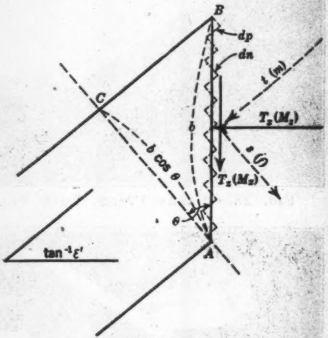

FIG. 27.

Part II.—Method of Computing the Unit Stresses.—The writer does not agree with Professor Rathbun's method of computing the unit stresses. It is not necessary to go into all the reasons for this disagreement in any great detail since the writer has proposed a method of his own,[13] and the interested reader can make the comparison for himself. Some of these differences, however, will be discussed.

The first step in analyzing a structure of any kind is to find the total or resultant forces and moments acting on the various critical sections. In the case of the skew arch, it has been found convenient to do so on sections cut by radial planes passed through axes parallel to the abutments (Z-axes). Having done this, Professor Rathbun then proceeds to transform these forces and moments into other forces and moments in the direction of and about axes, which are parallel and perpendicular to the faces of the bridge. In Fig. 27, the forces, T_x and T_z, are shown transformed into t and s, and the moments, M_x and M_z, into m and f. The author then assumes that the force and moment in the plane parallel to the face of bridge, t and m, act as a

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[11] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 135.

[12] *Loc. cit.*, Vol. 88 (1925), p. 1208.

[13] "Design of a Reinforced Concrete Skew Arch," *Transactions*, Am. Soc. C. E., Vol. 96 (1932), Part II, p. 1249 *et seq.*

thrust and moment on the projected area of the section on a radial plane perpendicular to the face of bridge (in the direction of the plane, AC). He similarly assumes that the force and moment in the plane perpendicular to the face of bridge, s and f, act as a shear and a torsion moment on this same projected area. No proof is given for this assumption and as will be shown later, it leads to incorrect results. If the zig-zag line, AB (composed of segments parallel and perpendicular to the face of bridge), and the dimensions, dp, dn, are considered, it might be argued that t and m produce tension and compression on the segments, dn, and shear on the segments, dp. Similarly, it might be argued, that s and f produce tension and compression on the segments, dp, and shear on the segments, dn. This is what actually happens, and the exact proportion of each kind of stress in each direction can be found. It does not make any difference in which direction the segments are taken, provided they are perpendicular to each other, thus giving a means of obtaining the stresses in any two directions at right angles to each other.

The operation which is given mathematically by the author's Equations (7), is merely a transformation of axes, but what is required is something entirely different. Textbooks on mechanics[14] derive formulas for finding the unit stresses on any plane when the unit stresses on any other two planes at right angles to each other are known. These formulas, when referenced to the principal planes, result in the equation of what is known as the ellipse of stress. These stress transformation formulas, or the ellipse of stress, apply equally well to moments and forces per unit width as to unit stresses.

A complete set of these equations in terms of the skew arch notation adopted by the writer is given in the paper previously mentioned[15] and need not be repeated. Their use requires that the total or resultant forces and moments on a section shall first be reduced to forces and moments per unit width, and that the stresses per unit width, if any, acting on a plane at right angles to the cut section shall be found. With these data, the stress per unit width on a plane making any angle with the cut section, and on the plane on which the maximum stresses occur, can be computed. This procedure gives a different result from that obtained by the method outlined by the author.

The writer's method based on the ellipse of stress requires that the total forces and moments first be distributed on the plane on which they are computed, that is, on the radial plane through the Z-axis. This distribution, as well as the derivation for the stresses on the plane at right angles to it, through the V-axis, is given in detail in the writer's paper.[16] Professor Rathbun has made a suggestion regarding the distribution of T_z on the plane on which it is computed, which shows that it can be done. He has stated that a portion of T_z equal to ϵT_x can be distributed uniformly and the remainder, parabolically.[17] This is proven by the writer to be correct and similar derivations are worked out for the moments.

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[14] "Strength of Materials," by Timoshenko, Pt. I, Chapter II; "Applied Mechanics," by Fuller and Johnson, Chapter II; "Theory of Elasticity," by Prescott, Chapter I, etc.

[15] "Design of a Reinforced Concrete Skew Arch," *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1297, Equation (53).

[16] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), Equations (35), (36), (43), (44), (48), (49), (51), and (52), p. 1297.

As an example of the differences between the two methods, the numerical illustration given by the author under the heading, "Example 1,"[18] will be used. The value of m for which the main or longitudinal steel is designed, is 628 kip-ft. acting over a width of $b \cos 47° 41' = 45.6 \times 0.680 = 31$ ft. The moment per unit width is $\frac{628}{31} = 20.2$ kip-ft. The writer obtains two values for this moment per unit width: One at the face of bridge and another for the central portion. The first value equals $\frac{M_z}{b} \sec^2 47° 11' = \frac{534}{45.6} \times 2.16 = 25.3$ kip-ft. The moment at the face of bridge is chosen because the proof of its correctness[19] is simple and is not subject to the uncertainty surrounding the correct formula to use for the torsion moment. The torsion moment does not enter into this part of the problem in the solution proposed by the writer.

Incidentally, Professor Rathbun uses Merriman's torsion formula whereas the writer uses St. Venant's formula. This is a matter of minor importance as neither is correct, as explained in the writer's paper previously mentioned. As far as the writer has been able to discover from the available literature on this subject, there is no derivation which applies exactly. St. Venant's formula is the most accurate, and for large ratios of $\frac{b}{t}$, that of Merriman is the least accurate.[20] Unless a correct formula, when developed, reveals something quite unexpected, its results need only be substituted for those used herein, and the discussion remains the same.

The author has attempted to transform the computed moments and thrusts, T_x, T_z, M_x, and M_z, into the stresses, t, s, f, and m, respectively, because they are in the directions in which the steel is to be placed. This transformation is convenient, but the writer questions its correctness. Sections should be designed for the maximum moments and thrusts. The writer has referred previously to the stress transformation formulas which are a form of the equation for the ellipse of stress. These show that the maximum moments occur on one of the principal planes. The directions of the principal planes are determined by the relative values of the various moments, and, similarly, for the forces. It therefore happens, except at the face of bridge, that the steel will not be perpendicular to the principal planes. This fact requires a new set of reinforced concrete design formulas for the condition in which the steel is not normal to the plane on which the moments are computed.[21]

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[17] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 678.

[18] See p. 13.

[19] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1254, Equation (34), and p. 1297, Equation 53(*d*).

[20] *Bulletin No. 3*, Faculty of Applied Science and Eng., School of Eng. Research, Univ. of Toronto, Toronto, Ont., Canada.

[21] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1261, and pp. 1298 to 1320, inclusive.

The ellipse of stress shows that the maximum shearing stresses occur on a pair of planes at 45° to the principal planes. Here, again, the planes form an angle with the steel. It is shown by the writer[21] that where the steel is not normal to a section it resists shearing stresses and direct stresses (bending and thrust) simultaneously and the two types of stresses can be separated only where the steel is normal to the section. Leaving out the question of the correctness of the transformation formulas given by the author, the writer still does not agree with the design for torsion in "Example 1."[22] Besides the points already enumerated, no account is taken of the relative deformation of the concrete and the steel. Incidentally, in designing for the maximum or principal flexural stresses, the fact that the steel is not normal to the section and that, therefore, the concrete and the reinforcement do not deform in the same direction must be taken into account. The same applies to the shearing stresses.

The writer has found that if the longitudinal steel in a skew "Rigid Frame" is placed parallel to the faces of bridge, no transverse steel is required for skew arch action. The photographs (Figs. 16 to 25, inclusive) seem to prove this in a general way, since the failure cracks are perpendicular to the face of bridge. At the center, some of the cracks make a small angle with the perpendicular to the face of bridge, and this also is to be expected. In a design of a similar bridge (skew angle, 50°),[23] this angle is found to be 11° 50′ for loads symmetrical with respect to the faces of bridge. In a slab, it can be shown that for symmetrical loads, as understood herein, the principal stresses will be parallel to the face of bridge. Apparently, a rigid frame differs very little from a slab as far as a certain type of skew arch action is concerned. For other reasons, transverse steel, of course, should be used even in a right arch, but that is another matter entirely. The transverse steel can be placed parallel to the abutments (and, if found necessary, it can be designed that way) for simplicity in detailing. By the writer's method, the steel can be placed and designed in any direction desired, and the stress computed at any point and in any direction.

Conclusion.—Professor Rathbun's mathematics and the experimental work done by him and by Professor Beggs, demonstrate the relation between the load and the resultant reactions.

The mathematical demonstration of the relation between the reactions and the internal stresses is covered in Part II of the writer's paper.[24] The proportioning of the sections and design of steel, as suggested in this paper, are sufficiently accurate for design purposes, although certain factors which have not been experimentally demonstrated enter into the derivations. This experimental verification should be made so that no doubt will remain as to the exactness of the theory developed, even though a design based upon it, may be adequate for all practical purposes.

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[22] See p. 14.

[23] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1275.

[24] *Loc. cit.*, p. 1249.

G. D. HOUTMAN,[25] ASSOC. M. AM. SOC. C. E. (by letter).—The writer has observed one bridge of several arches built on a pronounced skew, in apparent disregard of skew-arch theory, which shows no extraordinary crack or blemish. On the other hand, he has seen a lesser arch, built along lighter lines, showing a peculiar crack near the acute corner, as noted in Fig. 28. The direction of this crack is almost certain proof that the arch, near that crack, is not in compression, which confirms some of the current beliefs regarding skew-arch action.

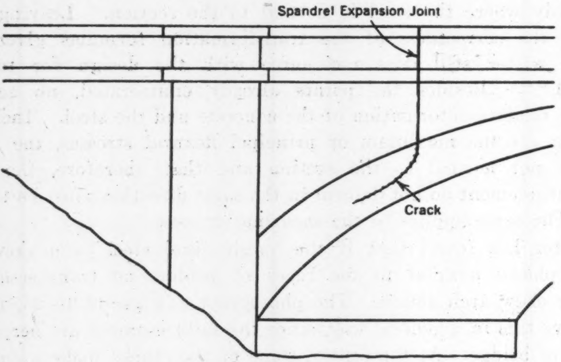

FIG. 28

CHARLES L. NORD,[26] M. AM. SOC. C. E. (by letter).—To any one familiar with the first paper[27] written by Professor Rathbun, it will be perfectly apparent that the method presented in this paper is, in the main, the same as that used in multiple right arches, in which the reaction of one is considered in the computation of the other.

The arch rings are divided into cantilevers, loaded with uniform and concentrated loads and with the reaction of the cutting plane or planes as in the case of the intermediate spans. The entire mathematical analysis of the single skew arch as well as the multiple arches is based on the flexure theory and can readily be understood and applied by engineers familiar with the elastic theory.

Other methods, such as "virtual work," or "least work," could be used. These, however, are too tedious and complicated for the average designer.

Professor Rathbun's mathematical analysis of the single skew arch is of primary importance to the profession since it has been proved by tests that, for vertical loads, the reactions are transferred to the supports along the lines parallel to the skew axis and not along those normal to the support. This, apparently, is contrary to the fundamental law of Nature that work is done by forces along the shortest lines and in the most economical way; although the writer fails to see where there can be an error in Professor Rathbun's method.

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[25] Designing Engr., County Engr.'s Office, Delaware County, Media, Pa.

[26] Bridge Engr., State Highway Dept., New Haven, Conn.

[27] "Analysis of Stresses in the Ring of a Skew Arch," *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

That other authorities do not agree with the author may be seen from a paper[28] by Professor Oskar Muy, of the Technische Hochschule, Stuttgart, Germany, describing the design and construction of a reinforced skew arch bridge over the Saar River, near Voelklingen. In this paper it is stated that in a skew arch the stresses cannot follow the arch as far as the sharp angles at the haunches, but will run more in the direction of the plane normal to the abutments, and that, therefore, the tendency is to overstress the obtuse corners greatly, with the likelihood of breaking them. To meet this situation, these points were heavily reinforced in a transverse direction. Professor Muy advanced no theory to support his view, but based his assumption solely on what he called "common sense." This would seem to indicate that a theory or method of designing a skew arch was a universal need of the Engineering Profession.

TABLE 6.—COMPUTATIONS FOR AN 80-FOOT SPAN OF A SKEW ARCH.

Term	Value	Term	Value
$\Sigma \cos^2 \phi \frac{s}{A}$	0.47*	$\epsilon \lambda \Sigma x \sin \phi \cos \phi \frac{s}{F}$	0.12
$\lambda \Sigma \sin^2 \phi \frac{s}{A}$	0.32*	$\Sigma v^2 \frac{s}{I_y}$	1.20*
$\Sigma y^2 \frac{s}{I_z}$	32.37	$\lambda \Sigma u^2 \frac{s}{F}$	187.40
$\epsilon^2 \Sigma x^2 \cos^2 \phi \frac{s}{I_y}$	0.47*	$\lambda \Sigma \frac{s}{A_1}$	1.51*
$\epsilon^2 \lambda \Sigma x^2 \sin^2 \phi \frac{s}{F}$	58.14	$\Sigma v \sin \phi \frac{s}{I_y}$	0.017*
$2h \Sigma y \frac{s}{I_z}$	95.77	$\lambda \Sigma u \cos \phi \frac{s}{F}$	0.13
$h^2 \Sigma \frac{s}{I_z}$	569.38	$\Sigma \sin^2 \phi \frac{s}{I_y}$	0.0001*
$\epsilon \Sigma VX \cos \phi \frac{s}{I_y}$	0.79*	$\lambda \Sigma \cos^2 \phi \frac{s}{F}$	1.295
$\epsilon \lambda \Sigma u x \sin \phi \frac{s}{F}$	105.00	$\Sigma \cos^2 \phi \frac{s}{I_y}$	0.004*
$\epsilon \Sigma x \sin \phi \cos \phi \frac{s}{I_y}$	0.008*	$\lambda \Sigma \sin^2 \phi \frac{s}{F}$	0.137

* Terms in Equation (11) that may be eliminated.

The writer unintentionally checked Professor Rathbun's statement that certain terms may be eliminated and found it correct. Table 6 was prepared in the computation of a skew arch of 80 ft. span, at an angle of 30°, which, later, was checked by Professor Rathbun personally.

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[28] *Deutsche Bauzeitung*, Nos. 14–18, 1915.

F. W. KEYSER,[29] ESQ. (by letter).—The importance of Professor Rathbun's mathematical achievement is obvious when one realizes the many grade separations made necessary by highways crossing other highways, railroads, and rivers. The constantly expanding public roads program and its accompanying economy has made, for the skew arch, a definite place in the world of structures.

The mathematical analysis advanced by Professor Rathbun in this paper and in his previous paper on skew arches, namely, "Analysis of the Stresses in the Ring of a Concrete Skew Arch,"[30] ends the long period of obscurity in which the skew-arch problem has rested.

The slow increase in bridge width has been greatly accelerated in recent years, and, in a very short time, the effect of eccentric loads on the right arch will be a primary and not a secondary factor in the design of right arches. This eccentricity of loads on right arches can no longer be neglected. Professor Rathbun's analysis with its accompanying fundamental assumptions definitely shows the effects of eccentric loads. Not only that, but it points the way to the solution of eccentric stresses in the design of arches in the future.

The fact that Professor Rathbun's analysis is an exact one and is based on simple fundamental assumptions does not eliminate the case in which the stresses, computed as such, would be only approximate at best. For example, he has assumed rigid supports which means absolutely unyielding foundations, since the reactions do no work. This condition is never quite encountered in practical cases because even rock has a certain degree of elasticity and will deform under the action of thrusts and moments (although the deformation is insignificant).

For most other soils the deformation will be greater and more permanent, and, therefore, the writer believes that it remains for the designer to decide how far to go in accepting soil conditions as found and, at the same time, consider the work of the reactions as zero. In other words, Professor Rathbun's analysis is based on an assumption that never actually occurs in practical cases, although it may be approached.

The deformations as calculated are comparatively small; they will have to be so, in order to apply the elastic theory. If a settlement of the foundation occurs, it can at best only be estimated; it certainly cannot be calculated. The calculated deformations and, therefore, the crown reactions which are dependent upon the rigidity of the support, will necessarily be approximate only. In regard to the author's assumption of rigid supports the writer would like to issue a solemn warning that the skew arch, as well as any other indeterminate structure, should not be considered unless absolutely reliable data on the subsoil condition are at hand, to be studied carefully and deliberately, so as to determine the exact degree of rigidity that can be relied upon. In other words, engineers are all too apt to equate the work of the abutment reactions to zero. In many instances this is not so, due to subsoil conditions that are not apparent until the excavation for the structure has started.

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[29] New Haven, Conn.

[30] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

Then, spread footings, tie-beams, pile foundations, and other methods are used to make up for the lack of assumed rigidity. In such cases the indeterminate structure would not be economical and might even prove dangerous.

The modulus of elasticity is another factor to be considered. The writer is fully aware of the fact that common practice of present-day design is to consider this factor a constant when dealing with materials that follow the constant ratio of stress to deformation. Within the limits of stress commonly used, this assumption is correct; that is, if concrete is one of these materials. Tests made by Professor Emil Probst, in the testing laboratory in Zurich, published in 1906, give a different picture. Dr. Ing. Rudolph Saliger[31] has called attention to several tests made by Professors Bach, Wayss, and Freytag, of Berlin, which indicate that concrete is not one of these materials. The results of these tests show that the modulus of elasticity for concrete is a function of the stress, and, therefore, must be treated as a variable and not as a constant. However, the striking of an average stress deformation constant might be justifiable for an approximation except for another complication that enters in concrete structures, namely, the initial stresses induced in the steel and concrete during the setting process of the concrete. This subject has received little attention and much less investigation and, therefore, must be considered unsolved. These stresses may be of such intensity as to cause considerable variation in the stress computed by the elastic theory.

Another subject that has received very little attention and investigation is the behavior of large sections under torsional moments. Their behavior is most uncertain and this problem must be considered as unsolved.

These points have been cited to show why structures of the skew-arch type should be designed with conservative stresses, the judgment of experience as to conditions of site, and an absolute understanding of the analysis and its assumptions. The writer's views are those of the practicing engineer and not of the scientific investigator and mathematical expert. They are given in the spirit of co-operation to the end that a practical solution of the problem may be accepted by the profession.

With these points in mind, and under the assumption that they may be discounted, the writer has made a study of Professor Rathbun's analysis of skew arches by the Method of Virtual Work, or as it is more commonly known, the Principle of Least Work. He has checked his formulas for the single-span skew arch as outlined in Part I of the paper, but in the development of Equations (3) for the right side, sign differences were encountered because the directions of forces and moments were assumed to act as in Professor Rathbun's first paper,[32] where they are opposite in direction to those in the present paper. These sign differences occur in the writer's analysis only when based on his own interpretation of the method of virtual work and the sign system for the right half of Arch *A* (Fig. 3) as developed in Professor Rathbun's first paper.[32] By assuming that the directions of moments, forces, and crown deformations, as shown in Figs. 6 and 7, are correct, the writer has checked Professor Rathbun's analysis in sign and terms for both the right side and the left side.

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[31] "Ueber die Festigkeit Veränderlich Elastischer Konstructionen," Leipzig, 1904.

[32] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 613.

The method of virtual work is based upon Castigliano's first theorem:[33] "The derivative of the total internal work, W, with respect to any one load or external force, X, is equal to the displacement, Δ, in the direction of this load."

In other words,

$$\frac{dW}{dX} = \Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(43)$$

The general formula for the elastic work is,

$$W = \tfrac{1}{2}\int \frac{t^2_v\,ds}{EA} + \tfrac{1}{2}\int \frac{m^2_z\,ds}{EI} + \tfrac{1}{2}\int \frac{t^2_u\,ds}{GA} + \tfrac{1}{2}\int \frac{m^2_u\,ds}{GI_p} \quad \ldots(44)$$

in which, $G =$ modulus of shear $\left(= \frac{E}{\lambda}\right.$ in Professor Rathbun's analysis); and,

$GI_p =$ factor of torsion $\left(= \frac{EF}{\lambda}\right.$ in Professor Rathbun's analysis).

Differentiation of Equation (44) by partial differentiation for the unknown elastic values, which is denoted by X, determines the virtual work; thus,

$$\frac{dW}{dX} = \Delta = \int \frac{t_v}{EA}\,\frac{\partial t_v}{dX}\,ds + \int \frac{m_z}{EI}\,\frac{\partial m_z}{\partial X}\,ds + \lambda \int \frac{t_u}{EA}\,\frac{\partial t_u}{\partial X}\,ds + \lambda \int \frac{m_v}{EF}\,\frac{\partial m_v}{\partial X}\,ds. \quad (45)$$

When the abutment deformation is neglected Equation (45) represents 1 × Deformation. The modulus of elasticity, E, considered constant, is next removed from the integrals and multiplied by the deformation.

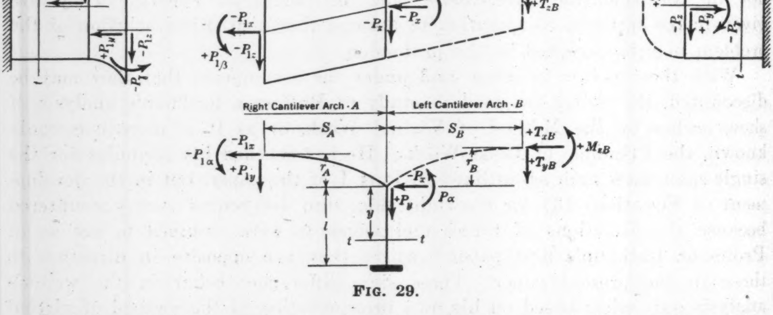

FIG. 29.

In his analysis the writer treated the problem as a single cantilever, the right cantilever of Arch A, acted upon by the forces of Arch B, as shown in Fig. 29. The P forces acting on the cantilever are due to the crown reactions of Arch B and their relationship may be demonstrated as follows:

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[33] "Die Neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen," and, "Die Graphische Statik der Baukonstruktionen," by Prof. H. Müller-Breslau. (Lectures by the same Professor as recorded by the writer during his student days.)

$$P_x = T_{xB} \qquad \text{(46a)}$$

$$P_y = T_{yB} \qquad \text{(46b)}$$

$$P_z = T_{zB} \qquad \text{(46c)}$$

$$P_\alpha = T_{xB}\, r_B - T_{yB}\, S_B + M_{zB} \qquad \text{(46d)}$$

$$P_\beta = T_{xB}\, \epsilon\, S_B - T_{zB}\, S_B + M_{yB} \qquad \text{(46e)}$$

$$P_\gamma = T_{yB}\, \epsilon\, S_B - T_{zB}\, r_B + M_{xB} \qquad \text{(46f)}$$

The P_1 forces are the unit forces applied in the direction of the deformations caused by the P forces. The relationship of the forces acting on the Section (t-t) of the pier is shown by the following formulas:

$$t_x = - P_x - P_{1x} \qquad \text{(47a)}$$

$$t_y = + P_y + P_{1y} \qquad \text{(47b)}$$

$$t_z = - P_z - P_{1z} \qquad \text{(47c)}$$

$$t_\alpha = - P_\alpha - P_{1\alpha} - P_x y - P_{1x} (r_A + y) - P_{1y}\, S_A \qquad \text{(47d)}$$

$$t_\beta = + P_\beta + P_{1\beta} - P_{1x}\, \epsilon\, S_A + P_{1z}\, S_A \qquad \text{(47e)}$$

$$t_\gamma = - P_\gamma - P_{1\gamma} + P_{1y}\, \epsilon\, S_A + P_{1z} (r_A + y) + P_z y \qquad \text{(47f)}$$

The partial differentiation of Equations (45) with respect to the P_1 forces are given in Table 7.

TABLE 7.—PARTIAL DERIVATIVES FOR USE IN EQUATIONS (45)

Terms	∂P_{1x}	∂P_{1y}	∂P_{1z}	$\partial P_{1\alpha}$	$\partial P_{1\beta}$	$\partial P_{1\gamma}$
∂t_x	-1	0	0	0	0	0
∂t_y	0	$+1$	0	0	0	0
∂t_z	0	0	-1	0	0	0
∂t_α	$-(r_A+y)$	$-S_A$	0	-1	0	0
∂t_β	$-\epsilon S_A$	0	$+S_A$	0	$+1$	0
∂t_γ	0	$+\epsilon S_A$	$+(r_A+y)$	0	0	-1

The P_1 forces are equated to zero, after differentiation, because they do no work. Then, the forces acting on the cantilever are:

Shear:

$$t_x = - P_x \qquad \text{(48a)}$$

Normal:

$$t_y = + P_y \qquad \text{(48b)}$$

Shear:

$$t_z = - P_z \qquad \text{(48c)}$$

Moment:

$$t_\alpha = - P_\alpha - P_x y \qquad \text{(48d)}$$

Torsion:

$$t_\beta = + P_\beta \qquad \text{(48e)}$$

Moment:

$$t_\gamma = - P_\gamma + P_z y \qquad \text{(48f)}$$

By inserting the values of the shears, normal force, moments, and torsion, as obtained from Equations (48), with their respective differential values, in the general formula for virtual work (Equation (45)), the equations of deformations in terms of the P forces are determined. By the proper substitution of the crown forces of Arch B (Equations (46), in the equations of deformation, the author's Equations (3) are derived except that, as noted previously, the signs are reversed completely in Equations (3*a*), (3*c*), (3*d*), and (3*f*).

It will be noted that the load terms have been neglected. The writer considered the cantilever unloaded. The load terms are independent of the elastic values and should be treated separately. Their omission will not affect, in any way, the equations as developed.

In closing, the writer wishes to acknowledge the assistance of Joseph D. Drury, a co-worker in the Engineering Division of the Connecticut State Highway Department, in checking the mathematical analysis and in the preparation of this discussion.

A. A. EREMIN,[34] ASSOC. M. AM. SOC. C. E. (by letter).—The author has extended the application of the deflection equations of a cantilever skew arch to the solution of a continuous multiple-span rigid frame.

The equations in the terms of the loading and their reactions at the cut planes developed by the author may be solved either simultaneously for all spans of the frame, or in the groups of six equations for each span, by substituting the unknown reactions in the terms of the reactions at the cut planes in the adjacent spans, as described by Professor Rathbun. In both cases, however, the arithmetic error in solving the equations accumulates with the number of equations connected by continuous properties of a structure.

If a frame is cut by the planes at the crown, as shown by Professor Rathbun in Fig. 3, the procedure of solution of the frame is the same, regardless of the variation in the relative elastic properties of the arches and piers. The solution of a frame may be simplified by placing the cutting planes at the skewbacks, as shown in Fig. 30.

The deflections, Δ_y, Δ_z, and $\Delta\gamma$, at the top of the piers are usually small and, therefore, it may be assumed that the reactions T_y, T_z, and M_x, at the cutting plane of one span (Fig. 30) will not vary with the changes in the loading and their reactions at the adjacent spans. According to this assumption fewer substitutions will be required to solve the equations.

If the frame and loading are symmetrical then one cutting plane in Fig. 30 may be placed symmetrically to the frame and the other sections at the skewbacks in the adjacent spans.

It is interesting to compare Equation (8), used by Professor Rathbun for computing the unit shear stress due to torsion moment, to Equation (49) which was experimentally developed by Professor Bach[35]:

$$v = \left(3 + \frac{2.6}{0.45 + \dfrac{b}{d}} \right) \frac{f}{b d^2} \qquad \text{.....................(49)}$$

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[34] Asst. Designing Engr., Bridge Dept., State Highway Comm., Sacramento, Calif.

[35] "Deutscher Ausschuss für Eisenbeton," Heft 16, 1912, W. Ernst & Sohn, Berlin.

The numerical coefficients in both Equations (8) and (49) will be the same for the cross-section with the ratio, $\frac{b}{d} = 1.29$.

In his test, Professor Bach used beams of a square cross-section and the rectangular section with the ratio, $\frac{b}{d} = 2$. Therefore, his equation and, apparently, Equation (8) cited by Professor Rathbun, are true only in this range of the sections.

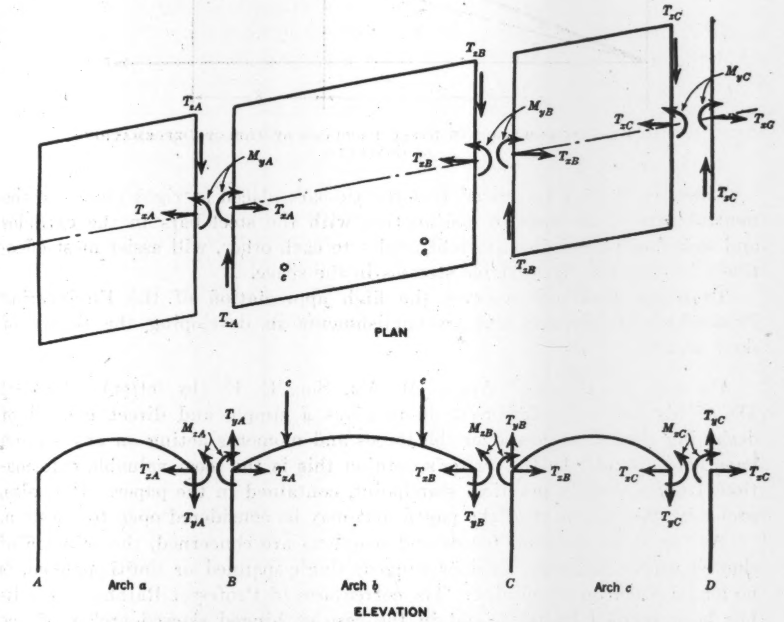
T_{zC}
T_{zB}
M_{yC}
T_{zA}
M_{yB}
T_{xC}
T_{xG}
M_{yA}
T_{xB}
T_{xB}
T_{zC}
T_{xA}
T_{xA}
e
e
T_{zB}
T_{zA}
PLAN
e
e
T_{yA}
M_{zA}
M_{zB}
T_{yB}
T_{yC}
M_{zC}
T_{xA}
T_{xA}
T_{xB}
T_{xB}
T_{xC}
T_{xC}
T_{yA}
T_{yB}
T_{yC}
A
Arch a
B
Arch b
C
Arch c
D
ELEVATION

FIG. 30.—MULTIPLE-SPAN RIGID FRAME DIVIDED BY CUTTING PLANES.

The writer agrees with Professor Rathbun that the shear reinforcing steel should be placed at right angles to the main reinforcement. A very noticeable loss in the efficiency of steel reinforcement occurs if it is placed inclined to a force. This may be proved by simple geometrical relations.

In Fig. 31 is shown a steel bar, *AB,* inclined to the force, *P.* at an angle, α. If the length of a strand, *l,* changes to $l + \delta$, the strain in the bar will be:

$$\frac{\delta}{l} \cos^2 \alpha$$

Whence, the stress in the bar in the direction of the force, *P,* is:

$$E \frac{\delta}{l} a_s \cos^3 \alpha$$

Therefore, the effective area of the cross-section of the inclined bar is:

$$a_s \cos^3 \alpha$$

This area will be maximum if the angle, α, is equal to zero, or if the bar is placed in the direction of the force.

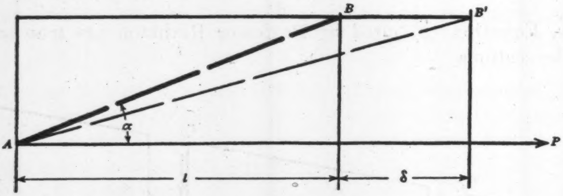

FIG. 31.—DEFORMATION IN STEEL PRODUCED BY DIRECT DEFORMATION IN CONCRETE.

Likewise, it may be proved that the tie-bars placed at right angles to the neutral axis of an arch in conjunction with the steel bars in the extrados and intrados, placed also at right angles to each other, will assist most effectively in carrying the variable stresses in the space.

Professor Rathbun deserves the high appreciation of the Engineering Profession for his excellent accomplishments in developing the theory of skew arches.

RICHARD M. HODGES,[36] ASSOC. M. AM. SOC. C. E. (by letter).—In Part IV of his paper, Professor Rathbun gives a simple and direct method of designing skewed arches after the forces and moments acting on any section have been found. In the writer's opinion this is the most valuable information, from a strictly practical standpoint, contained in the paper. It is also, probably, the only part of the paper that may be considered open to question.

As far as all external forces and reactions are concerned, the solution of skewed arches, whether fixed or hinged, single-spanned or multi-spanned, is no longer a matter of opinion. The correctness of Professor Rathbun's results has been verified by tests, and in the case of hinged skewed arches of one span, at least, has been checked by an independent analytical procedure,[37] as well as by the investigations of B. L. Weiner,[38] Assoc. M. Am. Soc. C. E.

However, when it comes to the matter of finding the distribution of stresses on any section, and of designing the arch and its reinforcement to withstand these stresses safely and economically, no such certainty exists. It seems evident that, whatever method is used, an exact design will be impossible until tests have been made to verify the fundamental assumptions of stress distribution which must be made at the outset, the correct value of the constant, K, in the torsion formula, $v = \frac{K f}{b d^2}$ (and the applicability of the formula itself to skewed arches), and the practical effectiveness of whatever method of torsion reinforcement may be proposed. Pending such tests, it will remain a matter of opinion as to what degree of analytical precision is required in deriving design formulas, and as to what degree of reliance can be safely placed on such formulas in practical design. The writer is of the opinion that the author's method, with the modification hereinafter suggested, is the best practical method yet proposed for designing skew arches, on account of its directness and simplicity, and because it leads to results which seem safe and reasonable for all ordinary skews.

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[36] Asst. Engr., Westchester County Park Comm., White Plains, N. Y.

[37] "The Rigid Frame Bridge," by A. G. Hayden, M. Am. Soc. C. E., N. Y., John Wiley & Sons, 1931.

[38] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 1212–1229.

Luckily, after all, indications point to the probability that for moderate skews, the old method of designing a skewed arch as a rectangular arch for a span equal to the skewed span is fairly safe. In the writer's opinion, the author's method is applicable to skews up to 45°, more or less, which comprises the great majority of grade-crossing eliminations. Skews of more than 50° should be avoided, pending the results of tests.

In applying the author's method of design, the following modification is suggested. It is often found advisable from the standpoint of practical simplicity to place the transverse reinforcement parallel to the abutments. In such a case, the forces and moments, being resolved into directions parallel to spandrels and parallel to abutments, become:

$$t = t_v \sec \epsilon' \tag{50}$$

$$s = t_v \tan \epsilon' - t_z \tag{51}$$

$$m = m_z \sec \epsilon' \tag{52}$$

and,

$$f = m_z \tan \epsilon' + m_v \tag{53}$$

The remaining procedure then follows, as shown in the paper. This change seems to result usually in an increase in the longitudinal reinforcement, but sometimes a decrease, depending on the relative magnitudes of t_v and t_z, or m_z and m_v. Wherever the resultant, r (see Fig. 8), is in the exact direction of the arch axis, Equations (50), (51), (52), and (53) will lead to the same results as the author's formulas. It is suggested that, in practice, the design should be made in every case according to the formulas leading to the larger amount of longitudinal reinforcement regardless of the selected direction of the transverse steel. The increase in cost will be inconsiderable in comparison with the increased margin of safety.

The design of the transverse reinforcement becomes a matter of academic interest only, since the resulting steel is nearly always less than the transverse spacer rods which would be inserted in any case to tie the longitudinal steel. Inadvertently the author seems to have equated an uubalanced shear torque to an unbalanced resisting steel stress couple in order to find the required steel areas. A more correct procedure would seem to be to take moments first on one side of the neutral axis and then on the other. However, this correction leads to very small variations, in application, from the results obtained by doing it the author's way.

J. Charles Rathburn,[39] **M. Am. Soc. C. E. (by letter).**—Some of the points that have been brought out by those who have contributed to this paper are pertinent to the subject of arches in general rather than to the effect of the skew. As they add to the knowledge of arches and arch design the writer will give his contribution as well as discuss those points that arise in the design of skew arches only.

Mr. Weiner indicates that a set of deflection formulas should be worked out for the case of a multiple-skew arch hinged at the base. The paper contains, in Appendix III, the formulas for the skew arch when the abutments are free to rotate, as he suggests. The body of the paper gives the general method of extending the analysis from one to two or more spans. The writer feels that this constitutes a solution of the problem even if an example of application is not given. The extension of these formulas to include the multiple arch involves no new ideas that have not been brought out in this paper, in the writer's previous paper,[40] and in Appendix III. Working out and publishing these formulas would have involved considerable labor and would have lengthened the paper materially. The writer suggests to any one who may have use for these formulas and may wish to derive them that he divide the structure by cutting planes as shown in Fig. 32.

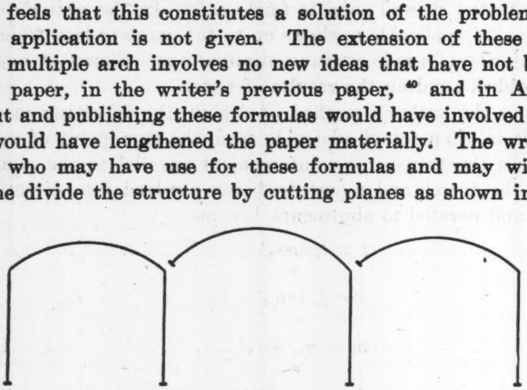

Fig. 32

The local load distribution due to a concentrated load, as demonstrated by tests made by Mr. E. F. Gifford (Figs. 16 to 25, inclusive),[41] shows that dangerous local stresses surround the point of application of this load. This very important problem arises in the design of arch rings where the load covers a comparatively restricted area and will arise in right, as well as in skew, arches.

The writer conducted a simple experiment which may add to the findings of Mr. Gifford. It also gives some idea of the lateral distribution over the ring of a concentrated load. Fig. 33 gives the general set-up of the experiment. A sheet of galvanized iron about 20 in. wide was bent to form an arch with about 5 in. rise and a span of 13½ in. The ring was a segment of a circular cylinder, quite wide and very thin. The ends may be considered fixed, because the sheet extended about 4 in. beyond each abutment. This 4-in. extension was placed under a 1 by 4-in. plank which, in turn, was nailed to the table. A concentrated load was applied by weighting a 60d nail which had been driven through a plank, as shown in Fig. 33. Weights were placed on this plank, but no attempt was made to evaluate the load.

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[39] Associate Prof. of Civ. Eng., Coll. of the City of New York, New York, N. Y.

[40] "Analysis of the Stresses in the Ring of a Concrete Skew Arch," *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

[41] See p. 50.

A central load was placed on the arch and the deflections along the crown from face to face were noted by means of an Ames dial placed on a carrier. There was some permanent set, as shown, when the load was removed. The load was then placed near one edge of the crown and another set of deflections taken. The deflection and permanent set due to the second load were taken from the final position after the removal of the first load.

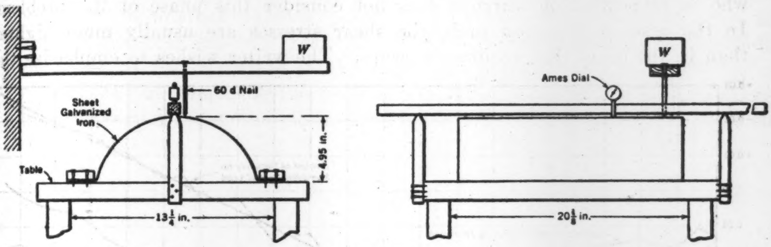

FIG. 33

Fig. 34 shows three curves for each load, first the deflection along the crown of the arch while the load is in position; second, the position after the load is removed (the permanent set); and, third, the difference between the two, which gives the distance that the crown rose after the load was removed. These curves show that there is considerable deviation from a straight line near the load. This simple experiment gives some emphasis to Mr. Gifford's work. A practical as well as a theoretical solution of this problem would be very valuable, but until it is made it is evident that designers should reinforce structures so as to avoid failures of the type indicated by Figs. 16 to 25.

Mr. Weiner takes issue with the writer because he does not use the ellipse of stress, and states that his Equations (7) are "merely a transformation of axes." This is evidently a misunderstanding because the writer has not transformed axes, as is done in the common method of deriving the ellipse of stress. The writer computes the components of a force and from them he finds that force, which is then broken up into other components that are more convenient to use in obtaining unit stress. The ellipse of stress that is found by considering these forces may have one axis equal to zero; that is, it may be a straight line. The properties of such an ellipse do not contribute to the understanding of the problem, as the writer sees it, and, therefore, he has not used it in his solution.

To make this point clear the stresses on a point near the edge of the ring will be considered. The line of force, or direction of force, must be parallel to the surface when it is a very short distance from this surface. This means that there is no shear across the plane perpendicular to the force and it is, therefore, one axis of the ellipse. The force perpendicular to it (if it is of any value) will be perpendicular to a surface. It is therefore zero, and the ellipse is a straight line. This is the point used by Mr. Weiner in the illustration in his discussion.

The writer feels that the problem of the ellipse of stress in the skew arch is somewhat the same as that in a reinforced concrete beam. It is interesting to study these ellipses at various points in a beam, but a designer who is proportioning stirrups does not consider this phase of the problem. In the case of the skew arch, the shear stresses are usually much lighter than in the beam that requires stirrups. The writer wishes to emphasize the fact that although T_x, T_z, and T_y are evaluated separately, they are only a part of the force, T. Combining components of a force to find the force is only one step in what Mr. Weiner refers to as the first part of the skew-arch problem.

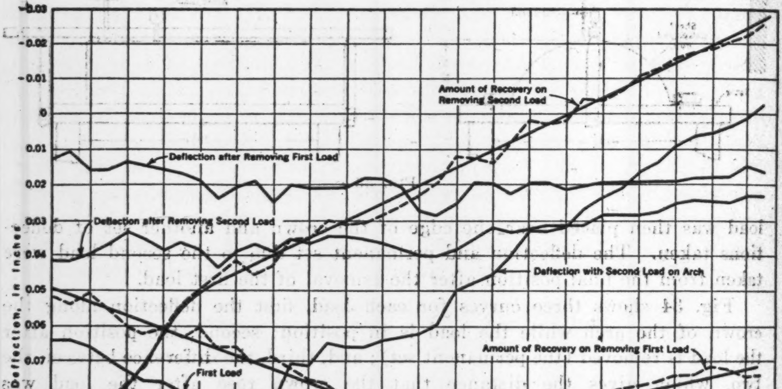

FIG. 34.—DEFLECTION ALONG THE CROWN OF A WROUGHT-IRON SEGMENTAL ARCH UNDER CONCENTRATED CROWN LOADS

Mr. Weiner takes issue with the writer on the method of computing unit stresses and shows by an example the difference between the results obtained by his method and that presented in this paper. He states that the writer is in error in claiming that the two methods are the same, which is admitted. As Mr. Weiner states, the one given by the writer has been used for several years and until carefully conducted laboratory tests or other demonstration proves the superiority of another method, he will consider it as close an approximation as has been found. The assumptions have been stated and are quite clear, the entire method is easy to understand and simple to apply. Mr. Weiner has presented a method which is based on assumptions of the same general character, slightly different from those given by the writer, but no more logical, and without proof. As his method has no better foundation than that of the writer and leads to a result with a shear on one plane and a thrust on another at each point, it is difficult to understand why the change has been made. Like Mr. Weiner, the writer does not consider the effect of the curvature of the arch ring.

One way of comparing the two methods is as follows: The writer assumes uniform thrust across the plane, *hi* (Fig. 35), in the direction parallel to the

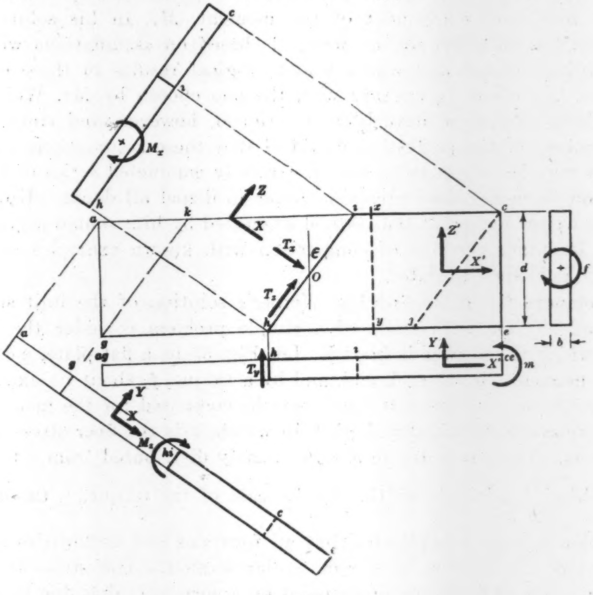

FIG. 35

X'-axis (in the direction of the major line of reinforcement), and a shear parabolically distributed at right angles to it or parallel to the *Z'*-axis. Mr. Weiner assumes uniform thrust across the plane, *hi*, in the direction of the *X*-axis. From this assumption he deduces the writer's assumption as to thrust. He also assumes a shear parabolically distributed in the direction of the *Z*-axis, on the *hi* plane. He then has at each point a thrust in the direction of the *X'*-axis across a plane parallel to the *Z'*-axis and a shear (along a different plane), parallel to the *Z*-axis. This shear has a component parallel to the line of thrust, the *X'* direction, which makes the thrust non-uniformly distributed although the thrust plus the component of the shear are so distributed. The writer considers the thrust and shear perpendicular to each other and thus avoids this phase of the problem.

Comparing the internal work of the shear and thrust the writer has uniform thrust against non-uniform thrust by Mr. Weiner, the former giving the less work. Both have parabolic shear at right angles to this thrust. Therefore, the writer's method assumes less internal work then that of Mr. Weiner. This is not a proof of the correctness of the assumptions, but does show they are more nearly correct than those submitted as a substitute.

In the problem chosen in the discussion by Mr. Weiner his method gives a result 25% in error and, therefore, the writer considers that the statement that "his method is sufficiently accurate for design purposes" is questionable. The writer feels that the assumptions made in his paper do not lead to such large errors. It is to be noted that Mr. Weiner did not take into account the very important component of the moment, M_x, in his solution. The stress distribution given by the writer is based on assumptions which have been definitely stated and which lead to logical results in those cases that the writer can check in another way, the one chosen by Mr. Weiner being one of them. As these have been questioned, however, and since, perhaps, some members of the profession may feel that these assumptions are unwarranted, it may be necessary to have a carefully conducted series of laboratory tests made on model skew arches in order to dispel all doubt. Until this is done, the writer feels that the method suggested by him should serve for arch design. It stands the test of comparison with known examples better than any other published to date.

To compare the results of Mr. Weiner's solution of the unit stresses on a diagonal surface with those of a simple problem consider the following case, in which the answer is known. Let Fig. 35 be a flat plate, *a c e g*, acted upon by moments, m, at each end, and by a torque, f, about its axis. At any right section (as xy) the unit stress may be computed for the moment, m, by the well-known formula, $m = \frac{1}{6} pbd^2$, in which, p is the fiber stress at the top and bottom. The stress due to m is uniformly distributed from x to y, and is expressed by $\frac{m}{b}$ per unit width. In the case of the torque, f, the unit stress distribution is more complicated, but all formulas and authorities agree that it is zero at the corners of a rectangular section. The stress then at the points, x, y, i, and h, or any other point on a corner, is that due to a moment of $\frac{m}{b}$ and is in a direction parallel to the corner line. This is true for all values of f. The force across a plane parallel to and near the vertical face, *ac*, is zero. The ellipse of stress has degenerated into a straight line. If this flat plate is cut by an imaginary plane, *hi*, making an angle, ϵ, with the right section, and if the portion to the right of the section is removed, it must be replaced by the moment and force produced by the removed portion. This moment may be broken up into three components, M_z, M_x, and M_y, while the force is broken up into T_z, T_x, and T_y. Taking the forces and moments about each axis equal to zero, six equations of equilibrium are evolved; $M_y = 0$; $T_x = T_y = T_z = 0$; while $M_x \cos \epsilon + M_z \sin \epsilon = f$; and, $M_z \cos \epsilon - M_x \sin \epsilon = m$.

If M_x and M_z are known, m and f may be found and the unit stresses computed. As m may be taken as uniformly distributed at the ends of the plate the stress at i must be $\frac{m}{b}$ and in the direction of ac no matter what the value of f. Taking the case chosen by Mr. Weiner in which $M_x = -361.9$; $M_z = 533.57$; $\tan \epsilon = 1.079$, and $d = 31$ ft. By computation, $m = 628.1$, and the unit stress at i and h is that produced by a moment of $\frac{628.1}{31}$, or 20.2 kip-ft. per lin. ft., and not 25.3, as reported by Mr. Weiner. The writer, therefore, does not wish to endorse Mr. Weiner's method.

Mr. Eremin and Mr. Hodges bring up the question of the reinforcement in the steel which is placed at an angle with the force. It is interesting to compare Mr. Eremin's equations with those given by Mr. Weiner[43] in Equation (60) of his paper "Design of a Reinforced Concrete Skew Arch," noting that his δ_z is zero when the ellipse of stress is only a straight line. The shear induced in the concrete by the steel placed at an angle tends further to reduce the load on the steel. In most cases the direction of the force follows the direction of the steel very closely, and the writer questions whether there is great error in the customary method of reinforcing for the parallel component of the force and then reinforcing for shear with bars acting somewhat as stirrups do in a beam. Of course, in extreme cases this point should be investigated.

Mr. Nord states that the direction of stresses in a skew arch are "apparently contrary to the fundamental law of Nature that work is done by forces along the shortest lines and in the most economical way." The latter part of the statement is most important, the former being assumed, at times incorrectly, from it. It is just this assumption that has made it difficult for engineers to believe the results of tests and the theory in the case of the skew arch. The equations used in this theory have been checked by the "method of least work" by the writer and by others, including Mr. Keyser. Such solutions as the one suggested by Professor Oskar Muy involve more internal work than that given by the writer. Although they under-estimate or neglect the force, T_z, and so make the force at the crown assume a direction nearer the obtuse corner, the resulting lateral bending of the arch produces more internal work than is gained by reducing the distance traveled by this force.

Mr. Houtman's discussion brings out quite clearly the fallacy of this line of thought. If an arch is divided into parts by a cutting plane all the forces across this plane should be considered. This includes thrust. Those who divide the arch by planes perpendicular to the abutment into two triangles and a rectangle (as viewed in plan) do not evaluate these forces and moments; nor do they even estimate this thrust. At least, the writer has known of no case in which the analysis has been carried through.

As Mr. Nord suggests, certain terms may be eliminated in the practical design of an arch. His Table 6 should be interesting and valuable as a reference to one who has an arch to analyze of the type studied by Mr. Nord. He can there see the relative value of the various terms.

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[43] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1263.

Mr. Keyser calls attention to the fact that the stresses due to eccentric loads on a right arch can be worked out by the equations given by the writer. In this connection the local stresses caused by a concentrated load, as shown by Mr. Gifford's experiments, must be considered in the final design. Mr. Keyser's check of the writer's equations by an entirely independent method—the method of virtual work—has been very valuable and encouraging. His suggestions about foundation movement can well be enlarged upon. If the movement of a foundation is anticipated it may be assumed that it is proportional to the load, or that it is a definite amount. In the latter case the deflection at the crown section can be computed. Since the load terms are computed as deflections of this same section this added deflection can be used as a load term and the stresses can be found in the usual way. If the deflection of the abutment is assumed to be a function of the abutment load the pier may be assumed to be extended, the extension being of such dimensions that it will have the same elastic properties as those assumed for the soil. The wisdom of building an indeterminate structure on yielding foundations is questionable, unless due consideration is taken of the effect of the yielding; so Mr. Keyser's warning is a wise one.

Mr. Eremin suggests several simplifications which will doubtless shorten the work in the analysis of those designs when they do not create serious error. It has been the writer's policy to develop the equations without neglecting small terms, in order that these equations may be used in types of design where they may be of considerable influence. It is a comparatively easy task to drop small terms, but difficult to supply them if dropped in the original development and afterward found to be of importance. Mr. Eremin suggests that arithmetical errors may accumulate, but this can be avoided if one uses a mechanical computation machine. It involves little more labor to carry the work out to enough decimal points to keep the error well within the range of the accuracy required.

Mr. Hodges is correct in his statement that the theory of the two-hinged arch has been checked as to the algebra of the analytical solution. This check was made by Mr. Weiner and H. G. Babcock, M. Am. Soc. C. E., under the direction of A. G. Hayden, M. Am. Soc. C. E. Mr. Hayden has given this theory in the book referred to by Mr. Hodge, but not as a check on the theory. Mr. Weiner states quite clearly in the "Introduction" to his paper,[43] that he is simply presenting this theory as the writer developed it, but has "revised the form of the equations." No changes or additions affecting the theory were claimed (except for unsymmetrical skews), but the changes were made in order to simplify or clarify the work. As brought out in the discussion of that paper the writer does not think any of the changes an improvement and some were unwise because they were not consistent with accepted usage in mathematics, analytics, and the beam theory, and because they tend to confuse.

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[43] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1213.

The writer questions the limitation of 45° placed by Mr. Hodges on the skew-arch theory. There may be some question as to the assumptions of the writer in calculating the unit stress, and Mr. Weiner has shown that other assumptions can be made that lead to different results. However, the general theory has been definitely corroborated. As Mr. Hodges states "the transverse reinforcement becomes a matter of academic interest only" and so this matter of the difference in the two methods of computing unit stress is not as great as might be supposed at first as far as transverse reinforcement is concerned.

In the case of unbalanced torque the writer has simply attempted to place steel in the form of stirrups that is an equivalent for the unbalanced concrete, as shown in the shaded triangle of Fig. 9. This steel must be an equivalent in moment as well as in thrust. As the force has a lever arm of 10.64 in. and the steel has only 8 in., the area must be changed accordingly. The direct thrust must also be checked if the change affects it, as it might where the lever arm of the steel is greater than that of the concrete.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1828

FINANCING STREET AND HIGHWAY IMPROVEMENTS[1]

BY R. W. CRUM,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. A. P. GREENSFELDER, C. R. THOMAS, J. C. CARPENTER, SAMUEL ECKELS, HORACE H. SEARS, AND R. W. CRUM.

SYNOPSIS

The Symposium on "Equitable Distribution for Highway Purposes of Motor Vehicle Licenses and Gasoline Taxes," conducted by the Highway Division of the Society during 1929, brought out the status of contemporary practice and opinion representative of the various governmental jurisdictions interested. The papers and discussions presented in this Symposium disclosed the fact that engineers in general concur in three basic principles.

In this paper the writer discusses from an engineering point of view, and without prejudice, some of the factors that appear to have a controlling effect in the equitable assessment of highway costs, and calls attention to some research projects that could aid materially in ascertaining the facts that are needed. These facts themselves will vary from place to place and from State to State, but the same methods of attack upon the problem should result in some uniformity in procedure.

IMPORTANT BASIC PRINCIPLES

In the separate papers presented by O. E. Carr, George B. Sowers, George H. Henderson, and Frederic E. Everett, Members, Am. Soc. C. E., and in the discussions elicited from all parts of the United States on this subject, there is evidence of concurrence in the following principles:

> 1.—Motor vehicle users should pay a fee to cover what is called, in public utility parlance, "readiness to serve." This may be considered to represent the motorist's contribution to the capital investment, and is generally collected through license fees.

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[1] Presented at the meeting of the Highway Division, New York, N. Y., January 22, 1931.

[2] Director, Highway Research Board, National Research Council, Washington, D. C.

2.—Motor vehicle users should also pay some amount based upon their use of the roads. This amount may represent their share of the operating costs of the highways, and is universally collected at the present time through imposts on gasoline.

3.—No part of the funds raised by means of motor vehicle license fees or gasoline taxes should be diverted to any other use than the construction, maintenance, and control of highways.

These three principles, and especially the last, are of great importance. It will be noted that they do not touch upon the controversial question of the allocation of funds among the various classes of roads and streets, or governmental units. With respect to this question many well-considered opinions were presented by authors and discussers. In general, the engineers reasoned from their own experience with one or more phases of road or street work, although they recognized that the equities of the situation have not yet been defined.

Some emphasized the point that engineers should consider all the facts and should not favor any particular interest—the small town, the city, or the State highway department. This is obvious, and there is no doubt that if engineers could study the facts underlying the problem they could recommend an equitable procedure. The difficulty at present is that all the facts are not available for study.

Determining the Road Users' Share of Costs

The real problem is not how the funds being raised at the present time should be distributed, but rather what share of the annual expenditures on the various classes of roads and streets should be borne by motor vehicle users.

In order to determine correct amounts, sources, and distribution of funds for the construction, maintenance, and administration of any group of roads or streets, certain questions must be answered. Among such questions are:

1.—What are the roadway costs?
  (*a*) Cost of building; and,
  (*b*) Annual cost for administration, maintenance, and periodic replacement or renewal.
2.—What part of the costs should be borne by each of the various interests benefited?
  (*a*) Federal Government;
  (*b*) State government;
  (*c*) Local community;
  (*d*) Adjacent property; and,
  (*e*) The user.

Having the answers to these questions for all the various systems of roads and streets in a State, the grand total of what the users should pay, could be added up and assessed if it is within the "ability to pay." If the entire sum could not be assessed, consideration would have to be given to the allocation of the funds raised so that they would yield the greatest good to the greatest number.

The answers will vary widely, depending upon such factors as class, type, location of roads, and traffic. The classification of roads adopted by the American Association of State Highway Officials is adapted to discussion of these questions. This classification is: (1) Primary roads, comprising the Federal and State systems; (2) secondary roads, the principal county trunk highways, or State-aid highways; and (3) third-class roads, the purely local township roads. To these should be added city and town streets.

PRIMARY ROADS

Primary roads are the highways that provide for interstate, inter-city, and inter-county transportation which, it is generally agreed, should be built and maintained by the State with such aid as is contributed by the Federal Government. This system, as thus far defined, stops at city limits. However, the discussions in the Symposium indicate that a considerable number believe that the extensions of these roads into and through the cities should be considered a part of the State primary system. In so far as engineers are concerned, there can be no doubt that, in planning for the movement of traffic over primary roads, the city links cannot be neglected, therefore, these streets will be considered in connection with the primary roads.

In any business undertaking, public or private, there must first be an outlay of capital. Then, if the enterprise is to be on a sound basis, it must earn enough to pay interest on the capital, to pay the operating and fixed charges, and to provide for anticipated renewals and replacements. In the case of a public project, such as a system of highways, the soundness of the enterprise can be assured if the governmental unit concerned levies enough taxes upon the benefited parties to provide the necessary sum each year to cover the financing of the capital outlay and the annual cost.

The rate at which a system can be built will depend upon what the benefited interests can afford, and upon whether the payments for construction are to be made directly from the tax money or from borrowed money. The reasons why the issuance of bonds for road improvements under certain conditions is sound public policy have been ably presented[3] by T. R. Agg, M. Am. Soc. C. E. and John E. Brindley, Professor of Economics, Iowa State College, Ames, Iowa. The writer is principally interested in the determination of the total annual cost so that the necessary taxes can be levied.

NEED FOR CO-OPERATION

The building and operating of a highway system is a huge co-operative enterprise of the people of a State, for which they must first furnish the capital, and then must pay individually for the use of the facilities. Their profit comes in lowered vehicle operating costs and in other more or less intangible benefits.

At any given date, the money and time necessary to bring the entire system to the desired standard can be estimated. If the pay-as-you-go plan is followed, the tax levy for construction will naturally be whatever the benefited interests can afford each year; and if bonds are to be issued, the construction fund should be sufficient to pay them as they mature, as well as the accrued interest. By issuing serial bonds, these annual imposts can be made uniform over any number of years.

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[3] Highway Administration and Finance," by T. R. Agg and John E. Brindley, McGraw Hill Book Co., First Edition, 1927, p. 160.

The determination of the amounts needed per year for operating expenses and for periodic renewals is more laborious, but there is no reason why they cannot be calculated on a sound basis for any highway system. This involves computation of the annual highway cost for those parts of the system that are already completed, and estimates of the probable annual cost of the remainder during and after the construction period.

Annual Roadway Costs

The Committee on Transportation of the Highway Research Board, National Research Council, has proposed a method for computing the annual roadway costs as of the date when the data are assembled, for any specific section of highway.[4]

The annual cost of a section of highway consists of the interest on the cost of construction, or value at the time of the analysis, plus the annual maintenance cost, including administration and engineering, plus the sum that must be set aside each year at compound interest to provide money for periodic renewal or replacement. This is expressed in a formula, as follows:

$$C = r\left(A + \frac{B}{r} + \frac{E}{(1+r)^{n}-1} + \frac{E'}{(1+r)^{n'}-1} + \ldots\right) \ldots\ldots (1)$$

in which, C is average annual cost; A, the cost to construct; B, the yearly maintenance cost; E, the expenditure for periodic maintenance every n years; E' the expenditure for reconstruction every n' years; and r, the rate of interest prevailing in current financing.

An estimate of the total annual cost of a highway system would have to be built up piecemeal since the factors would vary on different roads. On those sections of the system included in the construction program, the annual cost would be calculated for the date on which the analysis was made, and then could be estimated for each year in advance, until the end of the construction period. The result of such calculations would be a tabulation showing the amounts that the system would actually cost the people of the State for capital investment and for annual cost.

If this annual cost should be collected through some form of taxes, the revenue would be in excess of the expenditures, at least to the extent of the interest factor. This excess would no doubt be needed for those changes and betterments in the system, that would appear from time to time.

Knowledge of Costs Helpful

Although it might not be feasible, for administrative or legislative reasons, to assess the interest and carry a sinking fund as provided in these calculations of costs, a knowledge of the actual annual cost of providing the roads would be helpful in planning both revenue measures and expenditures. These costs, after deductions for Federal aid, would be the amount that might be fairly assessed against the vehicle operators, if a State highway system were to be built and operated on money raised by license fees on motor vehicles and taxes on gasoline.

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[4] Ninth Annual Proceedings, Highway Research Board, National Research Council, p. 360, and Tenth Annual Proceedings, Highway Research Board, p. 329.

Anticipating the acquisition of fundamental data for use in this motor-vehicle tax-distribution problem, it would be a worth-while research project to have the annual highway costs computed for all or for part of a State highway system. This would be a laborious job, it is true, but how else can such basic data be secured?

SOURCES OF FUNDS

The parties that would be benefited by the completion of a State highway system are: (1) The United States; (2) the State; (3) the local communities; (4) real estate; and (5) the road users.

The Federal Government recognizes the value to the Nation of roads that are needed for interstate transportation and National defense. Hence, Federal aid on the construction of these highways is well established and will probably continue for some time.

Since it is generally realized that the State is the principal beneficiary of the primary system of rural highways, it is a well-established principle that these roads should be built and maintained by the State organization, on money raised by the State. As the entire population is affected to some degree by the use of the main highways, it seems equitable to spread the cost over the population by means of taxes borne by motor vehicles, consisting in part of a service charge and in part of a payment proportionate to the amount of use. As far as can be determined, this arrangement appears to be acceptable to the users.

It is probable that the benefits to the local community and private property are relatively greater on extensions of primary roads into and through towns and cities than in the rural districts. Little information has been assembled to show the true relationship existing between community interests, private property, and road users. As a matter of fact, this relationship undoubtedly varies from place to place and from State to State, but a thorough study of even one case would be of immense value in pointing the way to a profitable solution in other places, and in establishing some principles of general application.

Factors needing study are: (*a*) The volume of through traffic, local traffic, and public carrier traffic; (*b*) the effect of through and local traffic upon the design of pavements; (*c*) the use of streets for parking, this being an extensive street use that cannot be measured by a tax upon fuel; (*d*) the relation of the highway to local business; and (*e*) the effect of street improvement upon property values. Many data and painstaking analyses are required. If the interests of the various parties could be evaluated, it should not be difficult to secure a fair share of the cost from each.

SECONDARY ROADS

Secondary roads are the principal county trunk highways, mainly important to the immediate agricultural regions and market centers which they serve. An analysis of annual highway costs would also be valuable on roads of this class. Some of the States, recognizing that these roads are necessary to general prosperity, grant aid on important secondary projects; others take them over into the primary system when funds for extensions become available. As State money is ordinarily derived from user taxes, these methods can be adapted to take care of that portion of the total cost of secondary roads that should be borne by the road user.

As the communities served are the chief beneficiaries of the secondary roads, it is usual to finance them by general property taxes. Benefits to adjacent property by secondary road improvements depend upon such factors as the extent to which the entire region is served, the character of the traffic, and the character of the improvements. Although improvement is undoubtedly a benefit to farm land, it may not be of value beyond a certain degree of all-year usableness. When the residents along a particular stretch of road desire to expedite the improvement of it, financing by means of an assessment district may be advantageous. As many secondary roads have considerable transient traffic that is of no value to the local district, the only way to get a contribution from this traffic is through State taxes affecting the user.

Studies of land values, land income, and the amount, character, origin, and destination of traffic are needed to arrive at the proportion of secondary road costs that should be borne by the various interested parties.

An investigation of secondary road conditions has been conducted by the United States Bureau of Public Roads, the North Carolina State Highway Commission, and the Tax Commission of North Carolina. Their report[8] is entitled "North Carolina County Road and Finance Survey." According to the finding of the Committee on Highway Finance of the Highway Research Board:

> "The report of North Carolina goes far to point out the reasons why no faster progress is being made in the improvement of secondary roads, and the principles that must be made effective if, within any possible expenditure, adequate secondary roads are to be secured."

THIRD-CLASS ROADS

Improvement of third-class roads, which exist primarily to give access to the farms of the country, constitutes one of the most pressing and difficult of highway problems. The principal beneficiary of these roads must be the farmer, because an outlet is essential to his success. However, the interest of the community in them can scarcely be less than his, because successful farms are necessary to its prosperity, and the general property tax touching both is probably fair.

In particular cases the improvement of certain roads by means of special assessment districts may be profitable to the land owners. Traffic is light and the relative share of the users, as such, in the costs is slight.

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[8] *Public Roads*, Vol. II, No. 12, February, 1931, p. 237.

CITY STREETS

In the cities, besides the extensions of rural through routes, there are residential streets, business streets, and intra-city through routes. The residents are the greatest beneficiaries of improvements on strictly residential streets, and the practice of assessing the major part of the cost against the real estate benefited has become established as equitable.

The city at large and the users, especially public carriers, have considerable interest in business streets, which are also vitally necessary to the business houses using the adjacent property, so that their share of the cost is probably relatively large. The street railways pay for their share of the pavement and upkeep, while buses, taxicabs, and public carrier trucks have an interest that can be assessed by the city in various ways. It is true that many of these vehicles do not operate outside the city limits although they contribute, through license fees and gasoline taxes, to the primary road system. It can be claimed in justification that the prosperity of the city is enhanced by the existence of a good system of tributary roads.

On business streets, the problem of who should pay for that portion used for parking space is perplexing. It is probable that the subject is of greatest interest to the vehicle users, although the city at large, or the adjacent property, may have to bear the burden. The value of street parking to business houses needs to be evaluated, and there is much evidence relative to this problem in the traffic data that have been collected in many of the larger cities.

The intra-city through highways may be a detriment to the abutting property although benefiting a zone of some width. The city at large and the public carriers have considerable interest in these streets, and it may be that the interests of the private users and the city coincide. Study of the available data, to evaluate these factors in terms of benefits to the various parties concerned, is needed as well as a study of property values and the earning capacity of real estate in the zone benefited by these streets.

NEEDED RESEARCH

This Symposium has brought out the need for factual studies before sound conclusions can be reached as to the relative parts of the cost of each road or street system that should be paid by the State, the community, the adjacent property, and the users. Although it does not appear probable that the relationships can be reduced to formulas, an engineer finds such studies essential to the exercise of judgment.

Four needed research projects may be mentioned:

1.—Calculation of the annual cost of all or part of a State highway system. It is expected that the Committee on Transportation of the Highway Research Board will continue its work in this field.

2.—A study of the amount, character, origin, destination, and purpose of traffic on city streets that are extensions of the State primary road system.

3.—A study of real estate values and earning capacity in the vicinity of through streets in cities as affected by street improvements.

4.—A study of the effect of road improvements upon rural property values and earning power.

CONCLUSIONS

As a result of this study, the following conclusions have been drawn:

1.—Motor vehicles should pay a "readiness-to-serve" charge.

2.—Motor vehicle users should pay for roads in proportion to the extent of their use.

3.—Funds raised through special taxes on motor vehicles or motor-vehicle users should be used only for highway improvement.

4.—It is not possible, from the information at hand, to determine what proportion of road costs should be paid from user taxes. General practice concedes that the major part of the primary road costs shall be paid from motor-vehicle license fees and taxes on gasoline.

5.—The State should be the sole agency levying special taxes upon the motor vehicle or highway user.

DISCUSSION

A. P. Greensfelder,[8] M. Am. Soc. C. E. (by letter).—An analysis of the broad principles involved in financing highway improvements has been ably presented in this paper.

There would seem to be no worthy reason for diverting for other than highway use the funds levied on motor vehicles and fuel. In spite of the equity in the case, a number of States are using gasoline tax money for a variety of other purposes. To date, however, motorists have been found willing to support these taxes because of their desire to secure ample highway facilities. If American legislators break faith and use such gasoline funds derived from motor vehicles for other than motor highways, there is likely to be "back fire" which they will not enjoy. America has needed, and still needs, thousands of miles of major highways. The user of these highways should be given every consideration because he is paying the major part of the bill.

In determining the road user's share of costs, it would seem essential to consider the separation of highway bond funds into road-bed bonds and paving bonds. The road-bed bonds should include costs of right of way, drainage, and grading. These costs might be allocated to the various political subdivisions of the Government, and might be issued for a term of thirty to fifty years. Serial bonds should be issued only for the estimated life of the particular pavement and then might be allocated logically to the users, including amortization, interest, and maintenance of the highway.

Just as it is logical to tax motor vehicles for highways, so it would seem only equitable that fuel tax from inland vessels be allocated to the cost of providing navigable inland waterways, and the fuel tax from aircraft should be allocated to airport and airway lighting.

Relative to primary roads, it would seem that the community benefits derivable from private trucking and buses should be weighed against the very probable damage and heavier pavement costs which these vehicles will presently cause. Railways are required to have private rights of way and yet State and interstate truck and bus traffic is permitted to block the highways without regard to other vehicles. Perhaps long hauls by rail upon private rights of way and local distribution from pivotal centers by trucks on public highways will be found to be the economic answer. The writer agrees with Mr. Crum that there is need for co-operation between all transportation groups. There is danger, however, that highways may be damaged by fast heavy truckage of low-class freight more than the gain in freight charges or the value of time saved in delivery.

There is little doubt but that automobile transportation of individuals is extremely expensive per capita per mile. That the saturation point per capita has been temporarily reached is evidenced from the 1931 automobile sales. Whether individual passenger automobiles per mile per capita will increase or decrease is debatable.

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[8] Pres., Fruin-Colnon Contr. Co., St. Louis, Mo.

In considering sources of funds, it is interesting to recall that in 1880 when local highways were desired, the benefited parties were frequently required to pay direct toll. While toll highways have been practically eliminated, toll bridges are still being constructed for the same obvious reason, namely, that the toll charge is more than justified in the time and mileage saved. Tolls are merely direct taxes paid by the benefited users, whereas the gasoline fund is an indirect tax.

Federal aid may have been more justifiable in former times than at present. It is easier to educate from the top down than from the bottom up. Federal aid served as a bonus to stimulate State and county highway construction. Just how long and how much the National Government should continue to contribute would seemingly be a function of interstate vehicular traffic.

It seems only equitable that both primary and secondary roads should be paved through a city or village as a continuous highway. That this was not done originally was due to the fact that county legislators overwhelmingly predominated the representatives of the urban centers in any plan. In justice to the city taxpayers who collectively contribute a very large sum for highway construction, the through highway should be continuous. It seems equally vital that the expenditure of funds for through highways in cities should be under the control of the State Highway Department, the same as it is throughout the counties. If the State Highway Department can design, contract, maintain, and pay for the highways in the counties of the State, it would seem perfectly reasonable that they should do the same thing through the cities of such counties, since both are political subdivisions of the same State.

There is another type of highway which Mr. Crum does not mention, namely, those that go to and through various State Parks. American citizens to-day demand outdoor recreational facilities and are increasing the number of National, State, and County Parks, which, in turn, increasingly require highways to reach them. In a bond issue of $75 000 000, in Missouri, several million dollars were thus allocated in 1928.

In connection with third-class roads the author questions just how much the land owner should contribute. It seems fundamental that the least the farmer can furnish is a fair amount of right of way for the highway which will undoubtedly benefit his farm. Perhaps a tax for advertising signs per square foot per 1 000 passers-by per annum would furnish a considerable sum for highway improvements, since without the highway the signs are worthless. Signs are distasteful to most motorists and to tax them thus would minimize their area.

In discussing city streets, the author very appropriately presents the problem of parking space. It would be interesting if the cities would consider providing parking space on unpaved city blocks rather than on the paved street itself. Perhaps it would decrease traffic congestion and, at the same time, be considerably less expensive per car parked. In the down-town section the city might make inducements to private capital for the construction of multi-story garages.

The writer concurs with the author that continuous research along these lines is imperative. The traffic surveys made in the City of Cleveland, Ohio, and in the State of Connecticut during the years 1928, 1929, and 1930, are splendid examples of what can be done. City, county, and regional planning must be based on adequate information.

C. R. THOMAS,[7] M. AM. SOC. C. E. (by letter).—The largest single factor in the stabilization of highway programs is the relative continuity of the income from motor vehicle taxes used for highway purposes. The marked diversion of such funds to purposes other than roads and streets would be a great blow to stabilization of income that permits budgeting of funds several years in advance, and to the amount of road building now (1931) in progress. As Mr. Crum states, the problem of allocation of the motor vehicle taxes among various classes of roads and streets or governmental units, which is controversial, is overshadowed by the greater problem of stopping diversion to purposes other than highways.

The principal reasons advanced for the diversion of the motor vehicle taxes to other purposes than roads are: (1) Other State activities need money more than roads; (2) the road system is well advanced toward completion, and the funds raised by motor vehicle taxes should go to other purposes; (3) motor vehicle taxes belong to the general funds of the State and can be used as the State Legislatures desire. These reasons are advanced with the object of influencing Legislatures so that appropriations may be obtained for many purposes foreign to roads and streets. Back of these reasons may be the desire of property owners to use motor vehicle taxes as a substitute for levies on real estate.

The original purpose of motor vehicle taxes, which include both gasoline and license taxes, was to provide roads for motor vehicles. Taxes are assessed for schools and other special purposes and diversion of such taxes is not generally considered. The justice of the motor vehicle taxes (which are really "use" charges in lieu of tolls), like telephone charges, has been proved by the apparent popularity of the tax and the lack of opposition to repeated increases. However, some opposition to the motor vehicle taxes has developed since diversions have taken place in seventeen States, so that the motor vehicle drivers no longer receive full benefits. The growth of the idea for diversion has followed the increase in returns which amounted to $850 388 000 in 1930, and the demands of other State interests for a part of this large sum of money. Diversion to purposes other than roads amounted in 1930 to 3% of the gasoline tax income. Practically all the diversion is from the gasoline tax.

The reason advanced that other State activities need the money more than roads opens up the whole field of discussion of the value of highways, both tangible and intangible, as compared with the public value of other State activities. The subject is too broad for discussion here. It is sufficient to state that the savings in transportation costs alone to the public more than pays the cost and sinking fund for replacement of heavy traffic highways.

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[7] Alexandria, Va.

No other State activity appears to be on such a sound financial basis. The danger of changing from an investment that is returning such tangible dividends is at once apparent.

While more roads are being paved in 1931 than a few years previously, the highway system is by no means approaching completion for several reasons. There are 3 000 000 000 miles of public roads and only 700 000 miles have been surfaced. New developments in vehicles, such as the change in the few years preceding 1931 from low to high road speeds, bring new burdens on the roads that call for complete revision of highway-design practice. The rapid increase in the number of motor vehicles has led to congestion on the highways with resulting costly delays and accidents. Highway facilities are far behind the motor vehicle facilities both as to rate of production and mileage of improved roads. It has been stated that in Cook County, Illinois, if all the vehicles in the county were on the improved roads at one time, they would be two deep.

The idea that motor vehicle taxes belong to the general fund of a State is perhaps sound in some States, due to constitutional provisions, but the use of such funds for purposes other than roads and streets is undoubtedly a violation of the intent of the motor vehicle taxes as originally imposed, and is contrary to the public understanding of the purpose of the taxes. It is certain that opposition to motor vehicle taxes will develop in almost direct ratio to the amount of the diversion to illegitimate purposes.

J. C. CARPENTER,[8] M. AM. SOC. C. E. (by letter).—The problem involved in this paper is the determination of the most equitable method of obtaining highway funds in such a manner that the burden may be adjusted according to ability to pay and the funds distributed according to the needs of traffic. This determination has been made customarily by legislative bodies with very little advice from engineers. Several years ago a legislator made a public statement that it is the function of the Legislature to decide on the amount of funds necessary as well as to determine the method of collection, and all that will be expected of the engineer is proper expenditure.

It is becoming more and more evident that the engineering and financing are so intimately connected that they cannot be considered separately, and engineers are being called upon for advice as to methods of financing. When times are good the average citizen is too busy to inform himself on routine governmental action. He grumbles about payment of an excessive tax, but does not bother to look into the use that is made of his funds. When times are not so good, however, the taxpayer takes an active interest in knowing just what use is made of his hard-earned dollars and also inquires as to the functions performed by those entrusted with his money. Equitable collection of funds and proper distribution of the finances so as to furnish satisfactory service for those who provide these funds are necessary if the public is to remain satisfied with results.

The classification of highways into the three groups outlined by Mr. Crum is commonly accepted. The primary roads, as defined, cover the State highway systems as now laid out. The Federal Aid Act provides for a system not to exceed 7% of the public highway mileage of the State, at the time of the passage of the Act, and divides the highways thus designated into primary (not to exceed three-sevenths of the system) and secondary. Since this law was passed the American Association of State Highway Officials has mapped out a system of numbered highways extending across the country in all directions. The established primary highways in all cases, do not follow the numbered highways, and there are a few isolated instances in which the numbered highway may not be a part of the Federal Aid System. The Association, in establishing this system of marked highways, performed a service of inestimable value to the traveling public. In the writer's opinion, the highways on this system should be considered the primary highways of the country.

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[8] Fort Worth, Tex.

Secondary highways should include those of enough importance, outside of the primary system, to be included in the State highway system. Any highway within the description given by Mr. Crum for secondary highways should be a part of the State highway system.

Tertiary highways, or third-class roads, should include all other necessary public highways, except city and village streets. If a highway is open to public travel and serves one or more individuals, it should be included in the classification.

Changing conditions make reclassification necessary as is obvious from the many changes in the Federal Aid System since the first system was adopted after the passage of the Federal Road Act in 1916. A highway that is in the secondary class to-day may be a primary highway five years from now, or *vice versa,* by change of traffic trend, a primary highway may become a secondary or even a tertiary highway to-morrow. Nevertheless, with careful planning and far-sighted determination of the location of routes, the systems will finally become established and the changes which could not be foreseen in the early days of highway planning will become much less extensive and of minor importance. This condition is eminently desirable from the standpoint of economical expenditure of highway funds.

The problem of determining just what portion of the cost should be paid by each individual on a complete highway system for the entire country is by no means simple. Consider the case of the farmer who lives at the end of the third-class road. When he leaves his garage he travels to the farm gate over a right of way which is reserved for his special use to the exclusion of all others. Obviously, he should pay the entire cost of any improvement on this location; but when he passes his gate and enters the third-class highway he may mix with trucks hauling farm products to the community center, tax assessors from the county seat, State officials on business for the Commonwealth, and the United States mail carrier on his regular round of duty. He may legitimately expect this traffic to help him finance the improvement and maintenance of the roadway in proportion to the service it receives. After he emerges into the secondary system he finds a well-paved road which may be provided by the State, and this service may be of more value to him than the service his tertiary road affords to the occasional State officials who may use it. If he visits his cousin in the next State he uses the primary highway system. With Federal Aid on this highway this farmer is receiving some return for the use of his tertiary road by the United States in carrying the mail. This simple example illustrates the complications that surround a solution of the problem of financing and service.

During 1930 almost $2 000 000 000 were provided for State highways and local roads. A study of available statistics for 1931 indicates that there is much variation in the amounts collected and the methods of distribution in the different States. There are almost as many different schedules of license fees as there are States. Gasoline tax rates run from 2 cents to 6 cents, with an average of 3.35 cents for the entire country. One of the States with the lowest rate of 2 cents has the largest total mileage of a high type of surfaced highway. The income for State highways was $1 136 673 437 and for local roads, $818 379 508.

The following figures are for the entire United States. There are many variations for the individual States, but these divisions are representative of the entire situation as far as collections and distribution are concerned. For State highways the total funds were derived in the following manner:

	Percentage
State tax levied	1
Appropriation by States	2.9
Motor vehicle fees, etc	25.5
Gasoline tax receipts, etc	36.2
Miscellaneous income	1.5
State bonds and notes	19.5
Federal Aid post roads fund, allotment used	8.1
Funds transferred from local authorities	5.3

State highway disbursements total $1 139 676 601, and disbursements by local authorities, $851 686 625. State highway disbursements were distributed as follows:

	Percentage
For construction and right of way	62.5
For maintenance	16.9
For miscellaneous expenses	0.2
For equipment and machinery	2.0
For interest payments on bonds and notes outstanding	4.4
For principal payment on bonds and notes	6.1
For transfer to county or town for local roads	5.9
For other obligations assumed	2.0

These figures show that construction and right of way absorbed the equivalent of all receipts from Federal Aid, State bonds and notes, motor-vehicle fees, and 9.4% of other receipts. After deducting this 9.4% from the gasoline tax receipts there remains 26.8%, which is almost enough to pay the total cost of maintenance and the principal and interest payments on bonds and notes.

With the increase in constructed mileage of the State systems there will be, of course, a reduction in the percentage spent for construction and an increase in the percentage paid for maintenance.

The grand total of collections from gasoline taxes for 1930 was $494 683 410. This was distributed, as follows:

	Percentage
Collection cost	0.2
Construction and maintenance of State highways	68.5
Construction and maintenance of local roads	19.5
State and county road bond payment	6.3
For miscellaneous purposes	5.5

The total for miscellaneous purposes is $27 378 986. It is encouraging to note that $13 509 516 is transferred to cities for expenditure on streets, aviation, free bridges, highway administration, and refund reserve. These amounts are expended for purposes indirectly related to rural highway traffic. Some States recognize, in this way, that city streets carrying highway traffic are entitled to a portion of the revenues from gasoline taxes. For school purposes Texas, Florida, and Georgia divert $13 423 880, or 2.7%, of the entire fund collected; and an additional 0.1% is used for purposes that have no connection with highways. While it is generally admitted that the diversion for schools of $7 381 774 from the Texas gasoline tax receipts is not equitable, there is some consolation in the fact that the State Highway Department receives in transfers from local authorities $10 037 600. The $355 704 860 received for license fees, registrations, etc., is distributed as follows:

	Percentage
For collection and administration	5.4
For State highways	62.4
For local roads	19.3
For State and county bonds	10.2
For other purposes	2.7

The larger part of the 2.7% for other purposes is expended for motor patrols. This tabulation indicates practically the same major distribution as for gasoline taxes.

There is need for a thorough study of all legislation providing funds for highway work, such as is now being undertaken by a committee of the American Road Builders Association. This will be valuable in promoting uniform laws throughout the country. The results of such a study when presented to a legislative body would be convincing and no doubt would be welcome to those who formulate the laws. The Tax Commission of Wisconsin has been authorized under an Act of the Legislature of 1911, to collect information on tax assessment, receipts, and expenditures, and the reports have been found exceedingly valuable by legislatures and economists.

The 1930 joint survey of the North Carolina State authorities and the United States Bureau of Public Roads is a valuable contribution toward the solution of problems of financing. This indicates that all *ad valorem* taxation is not a just method of collecting funds for highway purposes. It also makes clear that the county unit is unsatisfactory as an administrative division. Among the 3 000 or more counties of the United States, there are numerous instances of competent engineering and administrative organization, but almost one-quarter of all the counties have no definite, organized, engineering control over their highway work. The centralization of highway authority for several counties would undoubtedly result in more efficient and economic expenditure of funds.

Before the annual cost of all or parts of any State highway system can be calculated there must be careful study of the adequacy of the system as at present adopted. The systems now in use (1932) were laid out about 1922 and were generally well suited for the needs of the various States; but in ten years there has been a marked change in traffic conditions, industrial and agricultural communities, etc. In addition, it was necessary, in the early days of highway construction, to locate the highways so as to meet the needs of individual communities of small size. With the "nation on wheels" the public is now interested in, and demands, direct routings, good alignment and grades, smooth surfaces, and freedom from congestion. The highway engineer can now, with public support, use correct engineering principles in location.

A revised system should be laid out in such a manner as better to serve local and through traffic for the entire country. It is important that such a system be developed at an early date. Based on present and probable traffic trends, on the development of the industrial and agricultural sections of the country, the growth of cities, and the controlling topographic features, a careful study should be made in each State and a major system laid out and filed for use on new work. It will be necessary to consider, carefully, entirely new locations, extending across the State, in some instances. The location through and adjacent to cities, the distance between the highways and other utilities, such as railways, etc., should be carefully determined in considerable detail.

While it would be impossible to change at once from the present system to an entirely new one, the data should be collected and the system developed and placed on file with the expectation of utilizing the ideal locations for construction of parts of the system as funds become available. Under the present policy, large outlays of money are required to reconstruct old projects on narrow rights of way with undesirable alignment, and traffic congestion is increased, rather than diminished. With these same funds a new highway can often be built on good alignment, with adequate width of right of way for future development, and with much better planned surfacing, leaving the old road to serve local traffic, with a much smaller maintenance cost for the two routes.

The new location serves and develops an additional local community, and also provides ample room, at low ground rental, for commercial activities necessary for highway traffic (such as garages and hotels) without interfering with established stores and other services essential to the community, on the old road.

Most of the State highway departments now have a large volume of information on costs of all the elements of highway construction and maintenance. After having laid out the system, it should be possible for a department to work out the cost of construction and with it the annual cost of systems to the States.

There are in excess of 2 400 000 miles of local roads in the United States. Not more than 300 000 miles have been graded, drained, and surfaced. With the increased demand for "farm to market roads" and consequent increase in the amount of funds being expended for local roads, many more of them will be improved and surfaced. Logical co-ordinated planning is essential for economical and efficient expenditure of these funds. In addition to combining and re-routing local highways so as to furnish the best service for the fewest miles, the well planned system will be comprehensive and will be based on proper consideration of such factors as correct utilization of farm lands (such as prospective abandonment of some farm lands which are being worked without profit and probable change in population density after a soil survey to establish correct utilization of the land), re-adjustment of so-called marginal or sub-marginal lands, etc.

The information and plans established by regional planning boards and commissions should be correlated with the plans for the system of primary, secondary, and tertiary highways. The establishment of regional planning areas in rapidly growing communities is desirable. Any form of governmental organization which tends to ascertain regional requirements officially is better than no organization at all. Such an organization will create an informed body of public opinion on the subject. The success of regional planning depends on the extent to which those who furnish the funds are convinced of its practical utility. Enabling acts are desirable in those States where regional planning has not been sanctioned by law.

Traffic studies are essential in the planning of any system of highways. Many States have traffic records extending over a period of years and find them valuable in prophesying future construction. A study of city streets traversed by rural traffic, compared to that on other streets of the city, should furnish a basis of determining the portion of the cost of improvement to be paid by rural traffic.

A study of real estate values as affected by road improvement would prove valuable, but there are so many other elements which change values that it is difficult to apply any factor of assessment derived by this means. The existing governmental agencies organized for the purpose of assessing property should be utilized to furnish these data and any adjustments in values should be determined by the regularly constituted assessors. There is no doubt that improved highways change property values and those benefited should pay a part of the original cost, but as previously stated, the North Carolina survey indicates that the *ad valorem* tax is not an equitable method of financing all improvement costs.

The necessity for careful and extensive planning of locations for highways is much more obvious than ever before. In issuing bonds for highways it has been considered sound to assume that the location, grading, and drainage structures are permanent features, but numerous instances are found in which modern traffic demands locations which mean the abandonment of considerable sections of these old roads. The planning cannot stop at county lines, but must be extended to develop locations which will not be subject to change.

The magnitude of the problem is difficult to grasp, because there are many elements to be considered in proper planning, and failure to include all essentials will mean that the entire scheme will be temporary rather than permanent. This is a certainty: Planning must precede any scheme of financing and only such financing programs as are based on a carefully worked out plan will be successful and result in satisfaction for the individuals paying for the work.

SAMUEL ECKELS,[9] M. AM. SOC. C. E. (by letter).—This presentation of contemporary practice and opinion, representative of various governmental jurisdictions connected with the financing of street and highway improvements, is interesting and is particularly meritorious in pointing out quite clearly the need for factual studies before sound conclusions of great importance can be reached.

Until recently financing methods have been predicated upon the theory of localized use. Improvement of streets and highways has been considered to benefit adjacent properties, and, generally, within limits of ability to pay, such improvement has been considered largely chargeable to adjacent properties. At present (1932), there are some modifications of method recognizing changed traffic conditions, but traffic interest and acceptance of public responsibility for costs are naturally to be measured by the extent of public use, other than local, and, in this regard (particularly with respect to improvements other than on State highways), there is deficiency of reliable information.

The most conservative theories of financing public improvements may be stated as: (1) Financing responsibility coincident with property rights and authority; (2) public contribution from public general funds; and (3) public general funds from general tax.

(1) Financing Responsibility Coincident with Property Rights and Authority.—With careful location, extensive grading, construction of artificial surfaces, erection of structures, and intensive public use, private ownership of public right of way becomes a legal myth. Facility of motor-vehicle movement nullifies boundaries. In many cases, with respect to rural roads and busy streets, there is overburdening of local responsibility, and adjustment by public contribution is under way.

(2) Public Contribution from Public General Funds.—Historically, public improvements have lagged behind demand; transportation facilities have been incommensurate with traffic requirements; and demands for public contribution, especially with reference to State highways, have been so great that an extraordinary increase of State revenues has been required.

(3) Public General Funds from General Tax.—The urgent demand for public contribution, together with opposition to an increase of a general tax levy, and the recognition of special benefits to the operation of motor vehicles, has led, throughout the country, to special levies on motor fuel and motor

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[9] Chf. Engr., State Dept. of Highways, Harrisburg, Pa.

vehicles that are remarkable for lack of uniformity. Table 1 was arranged from data given in a *Bulletin* of the Motor Vehicle Conference Committee, issued in January, 1931.

TABLE 1.—SUMMARY OF MOTOR-VEHICLE TAX RATES

Item	MOTOR-FUEL TAX		MOTOR-VEHICLE LEVIES ON PRIVATE PASSENGER CARS	
	Rate, in dollars per gallon	Number of States	Basis of tax	Number of States
(1)	(2)	(3)	(4)	(5)
1	0.020	7*	Flat rate	2*
2	0.030	10	Value	3
3	0.035	1	Weight	20
4	0.040	17	Horse-power †	14
5	0.060	3	Flat rate and weight	2
6	.....	....	Value and weight	1
7	.....	....	Weight and horse-power.	5
8	.....	....	Value, weight, and horse-power	1

* Also, the District of Columbia.

† Including also cubic-inch displacement in one State.

Thirty-four States and the District of Columbia tax motor vehicles as personal property, in addition to charging annual fees, computed variously as indicated in Table 1, Columns (4) and (5).

Important Basic Principles.—Variations in the rate of gas taxes and registration fees are unfortunate, of course, particularly with reference to adjoining States, but the extent of the need of revenue and the degree of ability or willingness to pay are not uniform throughout the States. It would seem, however, that there might be some elimination of differences in the bases of computing license fees.

From the Symposium on "Equitable Distribution for Highway Purposes of Motor Vehicle Licenses and Gasoline Taxes," conducted by the Highway Division of the Society during 1929, Mr. Crum draws three principal deductions, as stated in the paper under "Important Basic Principles." The following comments apply to these deductions in the order given by the author.

Principle 1.—License fees are readily defensible in so far as they are designed to cover costs of necessary supervision, protection, and exclusive or special benefit. In excess of these indicated limits, license fees are discriminatory, and are explainable only as the product of necessity or policy. The statement that motor vehicle users should pay a fee to cover "readiness to serve," is open to argument inasmuch as road construction constitutes a public improvement, as logically chargeable to public general funds as education, for instance. Considered in the light of expediency rather than equity, argument is avoided. There is apparent difference of opinion as to whether the license fee should be in lieu of other direct tax, and the answer to this question is a prerogative of the State Legislatures. Where the motor vehicle is exempted from personal property assessment it would appear that the license fee to some extent should be based upon valuation. Other bases of computation of the license fee logically are horse-power and weight.

The collection of license fees computed on valuation, weight, and horsepower might appear difficult, but this idea has been used in North Dakota for some years and appears to be practicable. The component part of the registration fee based on valuation—when the motor vehicle is exempted from personal property assessment—is reasonably equivalent to the State average of personal property tax exemption. The horse-power and weight components of the fee are computed supposedly with reference to the highway surface requirements of motor vehicles. The influence of weight is of particular importance with reference to heavy wheel loads, but unfortunately there is a general lack of sufficient information in this regard. The graduated fee in accordance with horse-power is quite reasonable with reference to incurring maintenance costs and to ability of car owners to pay; but it might be injudicious in favoring under-powered freight carriers.

Traffic conditions as well as physical characteristics of surface should be studied in connection with the proportioning of license fees and the theory of the "readiness to serve" charge. Large or slow vehicles might not exceed the width capacity or the carrying capacity of a road surface, but still might interfere seriously with the usefulness of the road to the preponderant class of motor vehicles of moderate dimensions and moderate weight. The speeding up of large heavy vehicles would not make them entirely unobjectionable, especially in mountainous districts where they might get out of control.

Principle 2.—The fact that all the States and the District of Columbia collect gas tax is evidence of the popularity of this form for the production of revenue. Since gasoline consumption increases with weight of load and with speed of travel, it would seem that this tax might be depended upon for total revenue for road construction and maintenance in States where motor vehicles are not exempt from personal property assessment. From a practical standpoint the revenues from gas tax should not be set aside exclusively for maintenance because separate funds for separate purposes frequently hamper administration.

The argument that no part of the funds raised by motor-vehicle license fees or gasoline taxes should be diverted to any use other than the construction, maintenance, and control of highways, is an effort toward consistency in the application of the theory of special fund for special benefit development. This argument is more reasonable in States where the motor vehicle is taxed as personal property than in other States. Such theories, however, should be considered with respect to necessity and expediency and with the recognition that there is less objection to the determination of levies or imposts on ability to pay than in consideration of ability to resist payment.

Determining the Road Users' Share of Costs.—The problem as to what share of the annual expenditures on the various classes of roads and streets should be borne by motor-vehicle users is unavoidably a problem of compromise. The determination of roadway costs, including cost of building and annual cost for administration, maintenance, and periodic replacement or renewal, simply shows a total that has been fixed by practical limits. What part of the costs should be borne by each of the various interests benefited, namely, the Federal Government, State Government, local community, adjacent property, and the user, is a problem of voluntary contribution and adjustment.

Under present conditions this country has a Federal Aid System, other State highways, county roads, township roads, and streets of incorporated municipalities. The designation of Federal Aid and State highways has very probably been judicious, but it has not been based on qualifications that furnish factors for accurate apportionment of costs. Briefly, the Federal Government has encouraged the development of correlated State highway systems by Federal aid apportioned among the States, one-third by area, one-third by population, and one-third in proportion to the total mileage of all highways at the time of the passage of the Act. The various States have relieved counties or townships as the case might be of responsibility for construction and maintenance of certain principal roads, and have set aside motor funds for State highway purposes. To some extent the Federal Aid highways enter the jurisdiction of municipalities, and to some extent motor funds have been distributed among counties, townships, and municipalities for highway purposes. The conditions involving the designation of Federal Aid highways and State highways and of the apportionment of Federal, and the distribution of State, motor funds are scarcely capable of analysis with respect to interests benefited, or to proper allocation.

Primary Roads.—It is established practice that the State (with Federal aid, as it becomes available) construct and maintain the principal rural roads and the State highway system, including the Federal Aid System. The discontinuance of State responsibility and Federal aid at municipal corporation lines has been considered impractical. The Federal Aid Act has recognized the importance of assisting in the development of streets in municipalities of small population and in the scantily developed sections of the more populous municipalities, and, in some cases, the burden of carrying large volumes of State highway traffic into and through boroughs and cities has been accepted in part as State responsibility. Future development of by-passes will not entirely eliminate the State highway significance of connecting streets.

There is force to Mr. Crum's statement that "the rate at which a system can be built will depend upon what the benefited interests can afford, * * *." This is especially true with reference to streets carrying through traffic with limited assistance of Federal Aid and State contribution.

Need for Co-Operation.—Viewing a highway system as a co-operative enterprise of the people of the State and considering the relative advantages of the "pay as you go" plan and the issuance of bonds, it may be stated that the use of bond funds has appeared desirable where the expected value of contemplated improvements has been greater than the anticipated total of annual costs and when rate of increase of annual revenue indicated future sufficiency for amortization of bonds, without impairing working-fund requirements. The present approach of motor vehicle registration to stabilized totals lessens regard for bond issues as a means of financing highway improvements. When there is recourse to the sale of bonds for financing highway improvement, however, the advantages of serial issues are important. A serial issue can be designed to develop equal annual costs, the loss by difference between interest rate on sinking fund and interest rate on bonds is overcome, and the responsibility for custody of the sinking fund is avoided.

Annual Roadway Costs.—On the basis of expenditure from current funds, actual road costs are chargeable to depreciation and maintenance. Actual annual road charges, then, may be stated as average annual depreciation plus average annual maintenance cost.

When the proceeds of bond issues are used for financing improvements, and in studies of costs where various rates of initial investment and depreciation are to be taken into account, annual interest on the investment is to be considered as well as depreciation charge and maintenance cost.

The formula mentioned by Mr. Crum as having been developed by a committee for the Highway Research Board differs slightly from the method that has been used in the Pennsylvania State Department of Highways for a number of years. Equation (1) charges simple interest on construction cost; it indicates perpetuity of initial investment, and capitalizes cost of periodic maintenance.

The Pennsylvania method charges off all investments in an estimated period of economic life and enters periodic maintenance cost as a simple annual average. The Pennsylvania formula uses as a factor the expression, $\frac{i}{(1+i)^n-1} + i$, values of which are directly obtainable from tabulations in handbooks of accounting and financing, known as "the annuity 1 will buy," or "the annual sinking fund which will accumulate to 1 at the end of n years" plus interest.

The Pennsylvania formula is as follows:

$$F = Gg + C'c + Dd + M + N \dots\dots\dots\dots\dots\dots (2)$$

in which, F is the average annual total charge; G is the grading and drainage cost; g is the annual sinking fund rate for the period of years in which the grading and drainage cost is to be charged off, plus interest; C' is the cost of a durable type of surface, or the salvage value anticipated in temporary or semi-durable type surface; c is the annual sinking fund rate for the years of economic life of the surface, plus interest; D is the re-surfacing cost, including initial surface treatment; d is the annual sinking fund rate for the years of period preceding re-surfacing, plus interest; M is the average annual cost of general maintenance; and N is the average annual cost of periodic maintenance.

Knowledge of Costs Helpful.—A method tried for a short time by the Pennsylvania State Department of Highways to show annual road-mile and car-mile costs, made use of records of costs of construction and maintenance and estimates of depreciation or salvage value and anticipated further economic life of improvements. For each year the cost was computed as the depreciation for the year, plus interest on the depreciated value, plus maintenance costs. This method required two charts for each section of road: One for recording traffic, itemized maintenance costs, depreciation and interest, total annual cost, average annual cost per mile, and average car-mile cost, with expectation of adjustment for relative weighting of truck-mile costs; and the other for special use to show annual costs of pavement only, including a column for reporting traffic, current value of pavement, depreciation and interest, maintenance costs, annual pavement-mile costs, and average pavement costs per car-mile.

Source of Funds.—Explanation of absorption of costs of highway improvements by the Federal Government, the States, local communities, real estate, and road users, in terms of respective benefits, suggests the need of extensive studies of classification and of source and destination of traffic. It is to be remembered, however, that ratios of allotment of cost are not to be fixed definitely for long terms. The public serviceability and use of the principal roads of the nation at the present time (1932), for instance, are very different from what they were in 1916 when the first Federal Aid Road Act was approved, and the mapping of State highway systems is carried out with such flexibility of purpose and standard that classification of roads as to jurisdiction can only be considered a matter of expediency.

Secondary and Third-Class Roads.—The distinction between primary, secondary, and third-class roads by definition is difficult. In some cases, there might be classification of highways in a State such that the Federal Aid System was identical with the State highway system. This might be described as the primary system; a recognized system of other roads under county supervision, eligible to certain State-aid benefit, might be termed secondary; while the remaining roads might be identified as third class. There might be still other cases of more extensive State highway systems, including secondary roads as well as primary roads, in the Federal Aid System. If there is any effort to classify highways on a basis of traffic counts, it is evident that there must be a sliding scale, because the distinction between primary and secondary roads in one State might be an absurdity in another. From a practical point of view classification of highways must be more elaborate than "primary, secondary, and third-class" roads; that is, in accordance with present conceptions, more than three traffic ratings are required. The principal consideration is in the question of roadside surface, width, and type, although there are questions of relocation and grading for reduction of curvature and gradient that cannot be settled without consideration of traffic classification.

A tentative road classification based on traffic is suggested, as follows:

Class	Traffic, in vehicles per day	Class	Traffic, in vehicles per day
1	More than 3 000	4	250 to 500
2	1 000 to 3 000	5	100 to 250
3	500 to 1 000	6	Less than 100

In addition to traffic, each class would be defined as to: Width, grades, alignment, sight distance, and depth of pavement.

City Streets.—The greatest difficulties in the problems of planning and financing street improvements are traffic congestion and relief. After giving some recognition to corporate limits and corporation rights, and assuming that by-pass routes are furnished for so-called through traffic, the street problem is largely localized, and the consideration of special benefits and users' costs by special license fees appears to be possible. The task is difficult, however, and requires careful investigation and consideration.

Needed Research.—The city street problem is acute in many cases and it is probably the most important field for investigation at the present time. In addition to other lines of investigation suggested by Mr. Crum, another of great importance is to determine the responsibility for carrying all classes of traffic at all times on all public roads. At present, it is generally recognized to be impracticable to build and maintain all the public roads to sustain maximum, generally permissible, loads in all seasons. Adverse conditions are snow, grades, curves, and surface bearing capacity. Where local use does not justify high type construction and all-year maintenance, and where local ability to pay does not permit it, the determination of facts of other than local use offers a wide field of study.

HORACE H. SEARS,[10] M. AM. SOC. C. E. (by letter).—A serious phase of engineering design and construction is discussed in this paper. Future financing of these improvements is dependent upon the sale to private investors of millions of dollars in new and refunding securities. These securities are issued by highway districts and special assessment districts in the form of bonds, notes, and warrants. Each of the securities is a promise to pay by the district, as a corporation, for these improvements. The improvements become the property of the district when installed; yet they cannot be attached or sold to satisfy the costs for labor and materials as is the case in constructing improvements for private corporations.

The intent of this discussion is to call the attention of engineers to the need of recognizing the financial limitations of these highway and special assessment districts under the limited powers which the Legislatures of various States have conferred upon them. The design of roads, the appropriation budget, and the prospect of securing funds are radically different for the city special assessment district as compared with the county road district or the State highway district. It is of no avail to design highways unless funds are permissible under highway district powers and unless authorized securities can be sold to investors. The payment for construction and for engineering services comes from the sale of corporate securities. Repayment of these loans must be assured or no funds will be forthcoming.

The sale of securities for city street improvements is best assured when bonds are voted and payable from *ad valorem* taxes. Preliminary engineering designs and costs must be determined in advance of the actual authorization of these voted bonds. To-day, more than previously, the city special assessment district is a corporate form of State agency which must give the engineer cause for thought. The sale of such securities for many cities meets with a great deal of sales resistance from the investing public. The city credit may be stated on the bonds, but experience shows that the repayment to the investor is dependent upon collection of assessments against the property in the district benefited.

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[10] Cons. Engr. and Attorney-at-Law, Hastings-upon-Hudson, N. Y.

The sale of securities for county road districts and for State highway district improvements is direct to the purchaser, without the intermediary of a city to supervise collections. This form of security is dependent for repayment, in the event of delinquent tax collections, upon foreclosure proceedings against delinquent property in the district. These district corporations are agencies of the State, created for a special purpose by the Legislature. They have limited powers in the matter of issuing securities, which are, therefore, not as readily sold to investors as the *ad valorem* tax bond voted by cities. In view of the present financial stringency, the design of renewals, repairs, and new road construction requires a knowledge of the probable financing of construction in the present securities market, as well as a knowledge of the corporate powers of the district. The legislative powers may be examined by attorneys for the engineer, but if funds are not forthcoming in the desired amount, the design and budget should be shaped accordingly. The resourceful engineer will prove his value to the community in maintaining traffic under these limited sources of funds. Highway districts must economize as many such corporations have reached the statutory debt limit.

District financing and special assessment financing for cities are very different so far as securing funds from the investor at the present time (1932). The engineer should investigate the terms, "highway district bond," and "special assessment bonds," and note the exact promise of the debtor corporation. Funds are secured at less cost when the credit of a city is back of the special assessment bond, than in the case of the district bond. The district road warrant, note, or tax anticipation certificate, should be studied with care. These securities are restricted, as a general rule, to payment for an existing indebtedness. When issued for loans and based on the future credit of the district, the enabling act of the Legislature should receive special attention from attorneys who specialize in municipal securities.

The investor has learned, and the engineer should know, that the district corporation is termed in law a quasi-municipal corporation, which is of limited character when compared with the municipal corporation, which is usually designated as the city. An approving opinion by a bond attorney, on procedure for a district security proposed to be issued, is in the nature of a preliminary statement. The final approving opinion of the attorney may not be given. Engineering designs, both preliminary and final, may be tied up with insufficient funds because of the failure to secure a final approving opinion, or because litigation is pending over the proposed issue of bonds. Highway construction may be imperatively needed, but if funds are not forthcoming the traveling public may detour for months because of incomplete paving. In such cases alternate highway plans must be provided until securities are sold.

The need for further assurance in this matter of financing by sales to the investor of special assessment securities and of district bonds has resulted in the recent publication by The Investment Bankers Association of

America of proposed Uniform Assessment Laws which will be submitted to the various State Legislatures for consideration and adoption. The State, through the Legislature, subject to constitutional limitations, has authorized these quasi-municipal corporations of limited powers to act as its agents. The customary legal liability of the State as principal for its agent's authorized acts in issuing securities does not result in the State becoming liable for the debts of these corporations. The State Constitutions limit the powers of municipal corporations in the matter of incurring debt, and, to a greater degree, the indebtedness of district corporations. The issue of securities to pay for a present debt, such as a warrant represents, differs greatly from the future pledging of district credit. A warrant promises to pay when funds are available, while a bond is a future promise to pay for funds presently loaned by the purchaser.[11]

The cost of designed roads must be financed from funds secured by the sale of district securities to the investor. The engineer must consult with his attorney and his district's banker before he can be assured of financing his projects. He must have an understanding of the corporate financial powers and the sale of its securities to the same extent that he has materials tested and traffic conditions analyzed.

In conclusion, it may be stated that the estimated annual revenues from motor vehicle users in the nature of automobile licenses and gasoline tax will prove most valuable as collateral assurance that the original sources of financing construction through municipal corporate securities will be performed. These license taxes are operating revenues and should be distinguished from highway district bonds, which are in the nature of first mortgage liens on benefited real estate. The highway engineer must not ignore the funds from bond sales, which make the original cost of construction possible. It is the finished structure that creates the "readiness to serve" the public (mentioned in Mr. Crum's paper), which is productive of annual revenues from motor vehicles. The owners of the latter, in turn, pay the State license and gasoline tax.

R. W. CRUM,[12] M. AM. SOC. C. E. (by letter).—By common consent some of the factors of the problems of highway financing are becoming settled. For instance, the principle that the users should pay directly the entire cost of the main roads (except that part borne by the Federal Government) has been widely adopted, and the States generally finance their primary road improvements and maintenance by means of license fees on vehicles and taxes on gasoline.

Of course, a part of the total sum raised by these imposts is derived from travel on roads and streets not included in the self-supporting system. However, the determination of that part of the total sum raised that is derived from travel on the main highways does not appear to be impossible. In a given case the data needed are the car-miles of traffic on the system in question and the total traffic on all roads and streets in the jurisdiction. Since the primary system is definitely located and limited in extent, traffic surveys will give that part of the needed data.

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[11] Dillon, "Municipal Corporations," Chapters XIX and XX.

[12] Director, Highway Research Board, National Research Council, Washington, D. C.

The total traffic can be estimated from the gasoline consumption, as revealed by the tax returns, or from the total number of trucks and passenger cars registered, together with the average annual mileage per vehicle, and valuable data for use in making such estimates have been compiled.[13] The ratio of primary road traffic to total traffic will also be the ratio of the tax derived from primary road use to the total tax, and can be utilized in making an equitable allocation of the funds.

This ratio, in connection with the annual cost of the primary road system, should indicate also the total money that can be equitably raised by taxes on users of vehicles. As an illustration, consider some data from a certain State: The average daily traffic during 1929, at 360 stations, covering the 3 490 miles of the State primary highway system, was found to be 732 vehicles. The total average traffic per day on these roads was, therefore, 3 490 × 732, or 2 554 860 car-miles.

An estimate of the total traffic on all the roads and streets of the State was made from the returns of the gasoline tax. In 1929, this tax produced $6 794 000, which means that at the tax rate of 5 cents per gal., 135 880 000 gal. of gasoline were consumed during the year, or an average of 372 274 gal. per day. Assuming that an average vehicle in this State will go 14 miles on 1 gal., the total average car-miles per day were 5 211 836. Of these daily car-miles, the 2 554 860 on the State Highway System amounted to 49 per cent.

The total revenue from motor vehicle fees and the 5-cent gasoline tax in this year was $9 334 000, of which 49%, or $4 573 660, was derived from travel on the State Highway System. Allowing for reasonable increase in traffic over a 30-year period, the amount derivable from the traffic on this system, without change in tax rate, was estimated to be about $5 300 000 per year.

An estimate of the average annual cost over a 30-year period of improving and maintaining this road system was $5 239 000, exclusive of the Federal Aid to be expected during construction. The amount to be realized, therefore, was just about equal to the demand.

If the amount derivable from the traffic on the primary road system was insufficient to cover the annual road costs, the State would be faced with the necessity of deciding between the alternate policies of increasing the tax rates, or of using some money on the primary system that was generated by traffic on other roads and streets. The justice of the latter procedure would depend largely upon the need of the State as a whole for improved highway transportation facilities.

This discussion is offered merely as an illustration of an attempt at a rational analysis of one phase of the problem. In addition to the list of needed research projects in the paper, it is suggested that a detailed study of the various comprehensive traffic surveys that have been carried on in numerous States would probably shed considerable light on the fixing of rates of taxation on motor vehicle users and the equitable allocation of the funds raised by these taxes.

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[13] "Automobile Operating Costs and Mileage Studies", by Robley Winfrey, *Bulletin 106*, Eng. Experiment Station, Iowa State Coll., Ames, Iowa.

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DESIGN OF LARGE PIPE LINES

BY HERMAN SCHORER,[1] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. L. J. MENSCH, W. P. ROOP, JOHANNES SKYTTE, DONALD E. LARSON, RAYMOND J. ROARK, F. W. HANNA, PAUL BAUMAN, L. J. LARSON, C. M. ORR, EDWARD J. BEDNARSKI, H. C. BOARDMAN, R. L. TEMPLIN AND R. G. STURM, F. KNAPP, C. P. VETTER, L. E. GRINTER, OREN REED, ROBERT E. GLOVER, AND HERMAN SCHORER.

SYNOPSIS

The object of this paper is to present an exact theory for the design of large pipe lines and their supports, which may also be used for the design of horizontal tanks, under-water tubes supported on piers, etc. Instead of the saddles commonly used, the pipe is supported and kept in shape by full ring girders which were first introduced in Sweden.

The ring girders act as stiff, disk-shaped members, preventing large distortions over the supports and their transmission into the pipe shell proper. Under these conditions stresses can be analyzed by the modern elastic theory of thin shells. It is found that the shell then is mainly subject to direct beam and ring stresses, the loads being transmitted to the supporting rings by shearing stresses. A narrow zone of secondary bending stresses occurs in the shell portions adjacent to the supporting rings, induced by the restraining action.

Theory and experience show that a pipe line so designed does not require any intermediate stiffening angles, even in case of comparatively thin shells. Much larger spans than those permitted by the customary saddle supports are entirely feasible, and considerable savings in steel and excavation may be realized with a larger factor of safety. The governing design stresses in the shell are derived and, for completeness, the stress analysis of a circular supporting ring is also given.

NOTE.—Published in September, 1931, *Proceedings*.

[1] Chf. Engr., Thebo, Starr & Anderton, Inc., San Francisco, Calif.

INTRODUCTION

Large steel pipe lines of usual design, supported on saddles and subject to beam action, are typified by short spans and a large number of circumferential stiffeners. Although the cost of such conduits is by no means small, present design practice is still largely based on crude, empirical rules, derived from observations on the unsuccessful behavior of some of the earlier structures, especially during filling and when operating under low pressure.

A radical departure from the old type saddle support, first introduced in Sweden about 1911, has been described by K. I. Karlsson[2] in an interesting paper. The pipe shell is riveted to and supported by circular girders, which transmit the load to the ground through a two-point support near the ends of the horizontal diameter. Tests and observations made on pipe lines thus supported have demonstrated the remarkable stability of the thin pipe shell built without any intermediate stiffening rings, and combined with unusual spans. For instance, the Kungfors line, built in 1921, has a shell thickness of $\frac{1}{4}$ to $\frac{5}{16}$ in., a diameter of almost 11 ft., with 41-ft. spans, and no stiffening angles between the ring-girder supports.

During the development stage of the Swedish type of support the elastic theory of thin shells had not advanced sufficiently to furnish a theoretical explanation for the small shell deformations under low heads. Although Karlsson has made an attempt to analyze the bending stresses in the thin shell, for the pipe full and half full, his assumptions were necessarily of a rather arbitrary nature, so that the uncertainty concerning the actual stress condition in the pipe shell was probably one of the reasons why the stiff-ring girder support has not replaced the saddle support in other countries.

Since the publication of Karlsson's paper, the general theory of shells has progressed to such an extent that the exact stress condition can be derived in a fairly simple manner. In the following analysis the pipe is considered as a thin shell, unsymmetrically loaded.

NOTATION

The following notation is used throughout this paper. Units must be consistent, for instance, inches, pounds, pounds per square inch, pounds per cubic inch, etc.

a = eccentricity of reaction, $\frac{Q}{2}$ (see Fig. 6), from a tangent to the center of the pipe shell.

b = moment arm of the tangential shear, s, from Point D in Fig. 6.

c = width of circular girder ring (see Fig. 4 (*a*)).

d = radial deflection:

- d_H = radial deflection accompanying the radial thrust, H, on the free cylinder rim.
- d_M = radial deflection caused by a radial moment, M, at a free rim.

e = 2.7183 = base of natural logarithms.

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[2] "Ueber Schwerkraftspannungen in Rohrleitungen von grossen Durchmessern und deren rationelle Konstruktion," by Karl I. Karlsson, *Schweizerische Bauzeitung*, Vol. 80, September 2, 1922, p. 105.

f = unit stress:

f'_r = maximum ring stress in shell, pipe precisely full.

f_r = maximum combined ring stress in shell, pipe under pressure.

f_{bx} = unit longitudinal bending stress in the rim at a distance, x, from the rim (see Fig. 4(*b*)).

f_L = combined maximum longitudinal beam stress.

f_{bo} = maximum longitudinal rim bending stress in the shell.

f_{bm} = absolute maximum value of the longitudinal rim bending stress.

h = head.

l = elementary length; $d\,l$ = segment lengths of a ring element (see Fig. 2).

m = Poisson's ratio.

p = variable pressure on the inside of the pipe circumference.

q = unit weight of the fluid flowing in a pipe.

r = mean radius of a pipe.

r'_p = increase in radius, r, at unrestricted sections of pipe.

r'_d = increase in radius, r, of the ring disk.

s = tangential shearing force per unit width of shell element.

t = thickness of shell.

u = angular location of a cylinder element, in the cross-section of a pipe, measured from the vertical (Fig. 2); du = angle subtended by the arc element, $d\,l$.

v = angular location of the point of tangency of s, measured from the vertical.

w = weight of pipe shell per unit of area.

x = distance from shell rim (see Fig. 4(*b*)).

x_1 = abscissas in the cross-section of a pipe, to any point (D, Fig. 5) in the supporting ring.

x' = distance along the center line of the pipe to any point measured from the origin, O (see Fig. 1).

dx' = length of an arc element, $d\,l$ wide, measured along the axis of the pipe.

y = ordinate of any point (D, Fig. 5) in a supporting ring.

z = a constant for the cylindrical shell $= \dfrac{\sqrt{r\,t}}{\sqrt[4]{3\,(1 - m^2)}}$.

A_r = area of supporting ring in a plane along the axis of the pipe.

D = diameter of pipe = $2\,r$ (see Fig. 3).

E = modulus of elasticity.

H = uniformly distributed radial thrust (see Fig. 4), measured per unit width of rim.

L = length of span, from center to center of ring-girder supports.

M = bending moments in ring girder:

M_e = moment of external forces.

M_s = moment due to tangential shearing forces.

M_A = moment due to left reaction (Fig. 6); M_B = moment due to right reaction.

M_0 = moment at the top of the ring when u = zero.

M_a = ring moment at $u = \dfrac{\pi}{2}$.

M_1 = ring moment in the first quadrant; M_2, second quadrant; etc.

M_i = maximum intermediate moment.

M_s = moment at the elastic center of the supporting ring.

N = normal forces in ring girder:
  N_0 = radial force at top of ring, when u = zero.
  N_a = maximum ring force due to the shearing forces, s.
  N_t = total tension in the ring caused by the combined loads, p and H.
P = external forces acting on a pipe.
Q = total load of the pipe shell, transmitted by shear to one supporting ring.
R = radius of curvature of a cylinder element (see Fig. 2).
S = section modulus of pipe, acting as a beam.
S' = section modulus of shell element.
T = stress components on shell element, measured per unit width:
  T_1 = acting in a longitudinal direction.
  T_2 = acting in a tangential direction.
X = longitudinal load component on shell element.
X_1 = horizontal force applied at the elastic center of supporting ring.
Y = tangential load component on shell element.
Y_1 = vertical force applied at the elastic center of supporting ring.
Z = radial load component on shell element.
δ = angular change:
  δ_H = change that accompanies the deflection, d_H.
  δ_M = change that accompanies the deflection, d_M.

DIRECT STRESSES IN PIPE SHELL

Fig. 1 shows a horizontal pipe section connected to, and supported by, two stiff, disk-shaped members, which prevent large deformations of the rim and their transmission into the shell proper. Under such rim conditions the theory of thin shells shows that continuous surface loads, applied over the shell area, will cause mainly direct stresses in the pipe walls. The cylindrical shell acts as a membrane, free from bending stresses normal to its surface, except for a narrow zone of secondary transverse stresses at the restrained rims. This behavior is quite in accordance with Castigliano's theorem of least work. Since a direct stress system is possible, much less internal work is required than with a system of large bending stresses in the pipe walls.

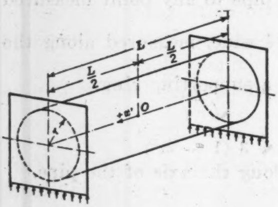

FIG. 1.

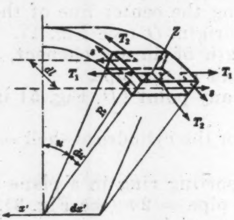

FIG. 2.

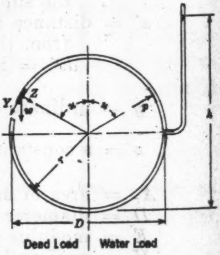

FIG. 3.

The following derivation of the direct stresses is based on the general theory of cylindrical shells, developed by Bauersfeld, and, later, independently by Finsterwalder, for the purpose of analyzing the stresses in the Zeiss-Dywidag, barrel-type roof, as described by Dischinger.[3]

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[3] Handbuch für Eisenbetonbau, Vol. 6, Fourth Edition: "Dachbauten Kragdächer, Schalen, und Rippenkuppeln," by H. J. Kraus and Franz Dischinger, p. 269, Wilhelm Ernst & Sohn, Berlin, 1928.

Fig. 2 shows a small element of the pipe shell, loaded by the external force components, X, Y, and Z. The element is held in equilibrium by the stress components, T_1, T_2, and the shearing forces, s, which act in pairs of equal magnitude on all four faces. Designating tension by the positive sign, and the radius of curvature of the cylinder element by R, Bauersfeld (as mentioned by Dischinger[3]) has derived the following general differential equations,

$$T_2 = -RZ \quad \text{.........................} (1)$$

$$s = -\int \frac{\partial T_2}{R\,\partial u}\,dx' - \int Y\,dx' + f_1(u) \quad \text{..............} (2)$$

and,

$$T_1 = -\int \frac{\partial s}{R\,\partial u}\,dx' - \int X\,dx' + f_2(u) \quad \text{..............} (3)$$

The functions, $f_1(u)$ and $f_2(u)$, depend on the origin of the co-ordinate system and the conditions at the supports. With the origin at the center of the span, as shown in Fig. 1, $f_1(u) = 0$. Assuming that any inside pressure on the end disks is resisted by corresponding forces in the opposite direction, then, in the case of Fig. 1, the stress component, T_1, must become zero at the supports, which condition determines $f_2(u)$.

Equations (1), (2), and (3), can be integrated if the radius of curvature and the load components can be expressed as trigonometric functions of the angle, u. For a circular pipe shell, $R = r$ is constant.

The most important loadings are: First, the dead load of the shell; and, second, the pipe filled with a liquid, so that the free level just reaches the top of the shell.

Case 1.—Dead Load Stresses.—Designating the weight of the pipe shell per unit area by w, Fig. 3 gives the following values for the load components: $Z = w \cos u$; $Y = w \sin u$; and $X = 0$. Substituting Z in Equation (1), the ring stress component becomes,

$$T_2 = -r\,w\cos u \quad \text{.....................} (4)$$

Substituting the partial derivative, $\frac{\partial T_2}{\partial u} = r\,w\sin u$, and the value of Y in Equation (2):

$$s = -2\,w\,x'\sin u \quad \text{.........................} (5)$$

The partial derivative of s, $\frac{\partial s}{\partial u} = -2\,w\,x'\cos u$, is then substituted in Equation (3). By integration, $T_1 = \frac{w\,(x')^2}{r}\cos u + f_2(u)$. Since $T_1 = 0$, when $x' = \frac{L}{2}$, $f_2(u) = -\frac{w\,L^2}{4\,r}\cos u$. Therefore,

$$T_1 = \frac{w}{r}\left[(x')^2 - \frac{L^2}{4}\right]\cos u \quad \text{.................} (6)$$

The unit stresses are obtained by dividing the expressions for the stress components by the shell thickness, t.

Equation (4) shows that the ring stress is independent of the span. The maximum values occur at the top (compression) and at the bottom (tension). The stress at the horizontal diameter is zero.

The shearing stresses, Equation (5), reach their maximum values at the end disks and are distributed over the cross-section as in a hollow, cylindrical beam, subject to bending. Equation (6) shows that the longitudinal shell stresses are also identical with those obtained by the ordinary theory of flexure, because cos u defines a straight-line stress distribution. Considering the shell as a simply supported beam, then, with the load assumptions previously given, the maximum moment is, $M_{\text{max.}} = \frac{2\,\pi\,r\,w\,L^2}{8}$.

The section modulus of a thin circular ring is, $S = \pi\ t\ r^2$, therefore, the maximum beam stress,

$$f_{\text{max.}} = \frac{M_{\text{max.}}}{S} = \frac{w\ L^2}{4\,r\,t} \qquad (7)$$

By comparing Equation (7) with Equation (6), it may be noted that the maximum stresses are identical.

The direct longitudinal stresses, therefore, are the same as those in a hollow, cylindrical beam, but the addition of the ring stress, T_2, changes the principal stress system.

Case 2.—Stresses for Pipe Precisely Full.—Assuming that a liquid of the unit weight, q, precisely fills the pipe to the top, with the free level at $u = 0$, the load components are given by Fig. 3, as, $Z = -\ q\ r\ (1 - \cos u)$; $Y = 0$; and $X = 0$.

From Equation (1),

$$T_2 = q\ r^2\ (1 - \cos u) \qquad (8)$$

Proceeding as shown for Case 1, from Equation (2),

$$s = q\ r\ x'\ \sin u \qquad (9)$$

Equation (3) becomes,

$$T_1 = -\ \frac{q}{8}\left[L^2 - 4\ (x')^2\right]\cos u \qquad (10)$$

Equation (8) shows that the ring stress is tension, and again is independent of the span. It increases from zero at the top to a maximum value at the bottom.

The shearing stress and the longitudinal stress are also identical with those obtained by the ordinary beam theory. It is interesting to note that the longitudinal stress due to liquid load is independent of the diameter.

Case 3.—Pipe Half Full.—A detailed analysis of the direct and secondary bending stresses in the pipe shell for this case of loading is given in a paper by A. F. Samsioe,[4] which came to the writer's attention after the present paper was submitted. The example given by Samsioe indicates that the maximum combined shell stresses for a pipe half full are of the same order as those obtained by the present method, when the direct stresses of Case 2 are combined with the rim bending stresses, as shown subsequently. For practical purposes, a design based on Case 2, including the rim stresses, will, therefore, also be safe for a pipe half full.

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[4] "Die Spannungen in einem auf mehreren Stützen in gleicher gegenseitiger Entfernung aufgelegten und zur Hälfte mit Wasser gefüllten Rohr," by A. Frey Samsioe, pub. by Svenska Bokhandelscentralen, Stockholm, 1926.

Case 4.—Combined Direct Stresses.—Combining the two cases of loading and designating the maximum unit ring stress by f'_r, then from Equations (4) and (8), with $D = 2r$,

$$f'_r = \frac{D}{2t}(w + qD) \qquad (11)$$

This stress occurs at the bottom of the shell, where the tension due to the dead load is increased because of the weight of the liquid.

If the pipe is under pressure, corresponding to a head, h, above the bottom of the pipe, then the maximum combined ring stress, designated by f_r, becomes,

$$f_r = \frac{D}{2t}(w + qh) \qquad (12)$$

The combined maximum longitudinal stress (f_L) is obtained from Equations (6) and (10); with $x' = 0$, and $u = 0$, or $u = \pi$,

$$f_L = \mp \frac{L^2}{4t}\left(\frac{2w}{D} + \frac{q}{2}\right) \qquad (13)$$

It can be shown that, in case of continuous pipe lines, the direct longitudinal stresses can also be derived from the theory of continuous beams.

If the pipe is prevented from expansion and contraction due to temperature changes, the longitudinal stresses due to this cause must be superimposed to the stresses, f_L. However, by providing expansion joints and bearings of low frictional resistance the temperature stresses can practically be eliminated.

Ring buckling of the pipe shell must be investigated only in case of possible outside pressure, for instance, that due to partial vacuum. It will generally be more economical to avoid vacuum pressure by the installation of stand-pipes and relief valves. Buckling due to longitudinal compressive stresses in thin, hollow cylinders has been analyzed by Timoshenko.[5] It may be expected that no reduction of the usual design stresses will be required until the ratio, $\frac{r}{t} > 500$, which is a remote possibility in practical cases.

Longitudinal and circumferential joints of riveted and welded construction must be designed to transmit the full shell stress.

RIM STRESSES

Assume that the pipe section shown in Fig. 1 is under uniform inside pressure. Due to the elasticity of the shell the pipe diameter will increase by a small amount in direct proportion to the ring stress. The shell rims, on account of their rigid connection with the end disks, cannot widen in the same manner as the more distant shell portions, so that bending deformations occur in the restrained rim zone.

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[5] "Einige Stabilitätsprobleme der Elastizitätstheorie," by Timoshenko, *Zeitschrift für Mathematik und Physik*, 1910, p. 337.

In practical cases the pipe is built as a continuous structure over more than two supports. In order to provide an unrestricted flow area the full disks of Fig. 1 are replaced by ring disks members. Fig. 4 (*a*) shows the state of deformation of a pipe shell, which is under a uniform inside pressure, p, and is rigidly connected to a circular ring of constant cross-sectional area, A_r. The radius of the unrestricted shell portions has increased by the elastic amount, r'_p, and the radius of the ring disk, by the amount, r'_d. Fig. 4 (*b*) shows one-half the pipe shell cut at its connection with the ring. The original deformation of the free rim is restored by applying a uniformly distributed radial thrust, H, acting inward, and a uniformly distributed radial moment, M, which has the tendency to turn the rim outward. H and M are measured per unit length of circumference.

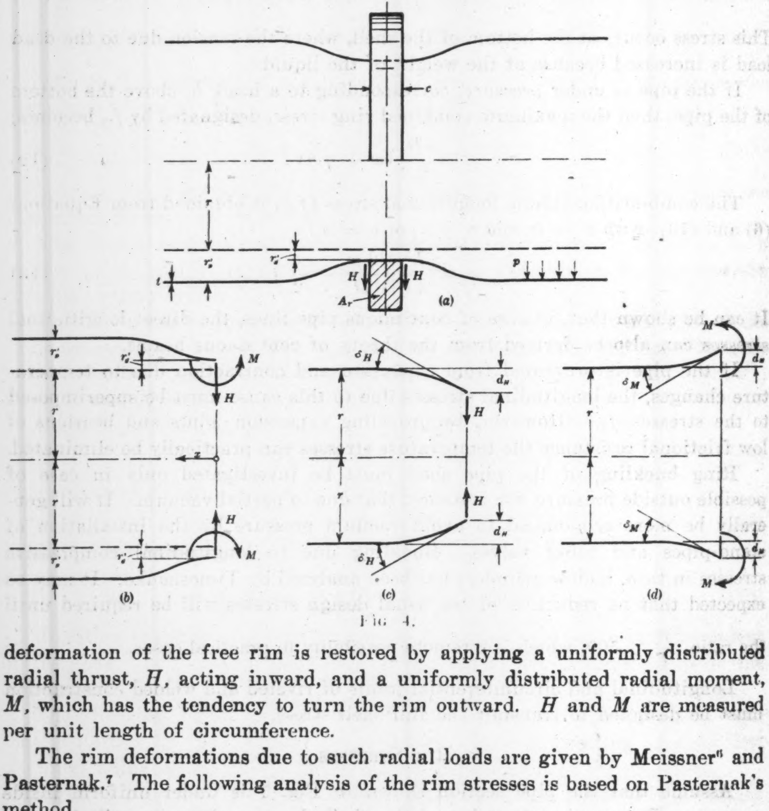

FIG. 4.

The rim deformations due to such radial loads are given by Meissner[6] and Pasternak.[7] The following analysis of the rim stresses is based on Pasternak's method.

If the free cylinder rim is loaded by the radial thrust, H, alone, as shown in Fig. 4 (*c*), the rim deformation is expressed by the radial deflection,

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[6] "Zur Festigkeitsberechnung von Hochdruck-Kesseltrommeln," by E. Meissner, *Schweizerische Bauzeitung*, Vol. 86, July 4, 1925, p. 1.

[7] "Die praktische Berechnung biegefester Kugelschalen, kreisrunder Fundamentplatten auf elastischer Bettung und kreiszylindrischer Wandungen in gegenseitiger monolither Verbindung," by P. Pasternak, *Zeitschrift für angewandte Mathematik und Mechanik,* Vol. 6, 1926, p. 1.

d_H, and the angular change of the generatrix, designated by δ_H. If the free rim is loaded by the radial moment, M, alone, as shown in Fig. 4 (d), the corresponding deformations are designated by d_M and δ_M, respectively.

Pasternak has derived the following expressions for the deformations of the rim, due to the radial loads, H and M:

$$d_H = \frac{2\, r^2\, H}{t\, z\, E} \quad \text{(14)}$$

$$\delta_H = \frac{2\, r^2\, H}{t\, z^2\, E} \quad \text{(15)}$$

$$d_M = \frac{2\, r^2\, M}{t\, z^2\, E} \quad \text{(16)}$$

and,

$$\delta_M = \frac{4\, r^2\, M}{t\, z^3\, E} \quad \text{(17)}$$

in which z is a constant for the shell, equal to $\dfrac{\sqrt{r\, t}}{\sqrt[4]{3\,(1 - m^2)}}$.

Due to the inside pressure, p, per unit area, the radius of the unrestricted shell portions increases by the amount,

$$r'_p = \frac{p\, r^2}{t\, E} \quad \text{(18)}$$

The disk ring is loaded by the direct pressure, p, acting over the width, c, and the radial thrust, H, transmitted from both shell rims. Designating by N_t the total tension in the ring caused by the combined loads, then,

$$N_t = (p\, c + 2\, H)\, r \quad \text{(19)}$$

Furthermore, if A_r designates the cross-sectional area of the ring,

$$r'_a = (p\, c + 2\, H)\, \frac{r^2}{A_r\, E} \quad \text{(20)}$$

With reference to Fig. 4 (b), the following relation can next be written,

$$r'_p - d_H + d_M = r'_a \quad \text{(21)}$$

Since the connection between the ring and the shell is assumed as fixed,

$$\delta_H = \delta_M \quad \text{(21a)}$$

Substituting the corresponding values for the deformations, Equations (14), (15) (16), (17), (18), and (20), in Equations (21) and (21a), respectively,

$$H = p\, z\, \frac{(A_r - c\, t)}{(A_r + 2\, t\, z)} \quad \text{(22)}$$

and,

$$M = \frac{p\, z^2}{2}\, \frac{(A_r - c\, t)}{(A_r + 2\, t\, z)} \quad \text{(23)}$$

For steel, with $m^2 = 0.10$, the shell constant equals,

$$z = \frac{\sqrt{r\, t}}{\sqrt[4]{3\,(1 - m^2)}} = 0.78\, \sqrt{r\, t} \quad \text{(24)}$$

Therefore, Equation (22) becomes,

$$H = 0.78\, p \sqrt{r\, t}\, \frac{(A_r - c\, t)}{(A_r + 1.56\, t \sqrt{r\, t})} \quad \ldots\ldots\ldots\ldots (25)$$

and Equation (23),

$$M = 0.304\, p\, r\, t\, \frac{(A_r - c\, t)}{(A_r + 1.56\, t \sqrt{r\, t})} \quad \ldots\ldots\ldots\ldots (26)$$

The section modulus, S', of the shell plate, per unit width of circumference, is $\frac{t^2}{6}$. Designating by f_{bo} the maximum rim bending stress of the shell, from Equation (26),

$$f_{bo} = \frac{M}{S'} = 1.82\, \frac{(A_r - c\, t)}{(A_r + 1.56\, t \sqrt{r\, t})} \times \frac{p\, r}{t} \quad \ldots\ldots\ldots\ldots (27)$$

In Equation (27), the factor, $\frac{p\, r}{t}$, is identical with the ring stress in the unrestricted shell, also called the cylinder stress. The rim bending stress is, therefore, a function of the cylinder stress in the more distant shell portions. If A_r becomes very large, comparable to an absolutely unyielding disk, the rim bending stress approaches an absolute maximum value, designated by f_{bm}, for which Equation (27) gives,

$$f_{bm} = 1.82\, \frac{p\, r}{t} \quad \ldots\ldots\ldots\ldots (28)$$

In this extreme case the rim bending stress exceeds the cylinder stress by fully 82 per cent. The smaller the area of the ring disk, the smaller the rim bending stress will be. The bending stresses in the rim zone cannot be neglected in the design, because in continuously supported pipe lines the longitudinal stresses due to beam action are combined with the longitudinal rim bending stresses.

Equation (27) also shows the influence of the width, c, on the magnitude of the rim bending stress. The wider the ring disk, the smaller will be the bending stress for a constant area, A_r.

In all thin shells of revolution, subject to symmetrical rim loads, the bending stresses decrease rapidly with increasing distances from the rim in the shape of a damped wave, as shown by Geckeler.[8] Designating by f_{bx} the rim bending stress at the distance, x, from the rim,

$$f_{bx} = f_{bo} \sqrt{2}\; e^{\frac{-x}{z}} \cos\left(\frac{x}{z} + \frac{\pi}{4}\right) \quad \ldots\ldots\ldots\ldots (29)$$

Equation (29) shows that the thinner the shell compared to its diameter, the narrower the rim zone of secondary bending stresses will be.

The total tension, N_t, in the supporting ring can now be determined by substituting H, from Equation (25), in Equation (19), which gives,

$$N_t = p\, r \left[c + 1.56 \sqrt{r\, t}\, \frac{(A_r - c\, t)}{(A_r + 1.56\, t \sqrt{r\, t})} \right] \quad \ldots\ldots\ldots\ldots (30)$$

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[8] "Ueber die Festigkeit achsensymmetrischer Schalen," by J. Geckeler, *Forschungsarbeiten* auf dem Gebiete des Ingenieurwesens, Heft 276, VDI-Verlag, Berlin, 1926.

This determination of the rim stresses applies only to symmetrical rim loads. Dischinger[9] has shown that in case of unsymmetrical loads the maximum rim bending stress is obtained with good approximation by substituting the maximum ring stress in place of the cylinder stress, $\frac{p\,r}{t}$, in Equation (27). In the present case, the maximum ring stress is given by Equation (12).

STRESSES IN SUPPORTING RING

Fig. 5 shows a circular ring of uniform cross-section, loaded and held in equilibrium by the external forces, *P*. For the purpose of analysis the ring is cut at the top and the right half is fixed at *C′*. The left half is provided with a rigidly connected lever arm, *C O*, in which, *O* represents the elastic center of the ring (identical in this case with the geometric center). At *O*, the unknown forces, X_1, Y_1, and the unknown moment, M_z, are applied so that the ring deformations at *C*, with respect to *C′*, disappear.

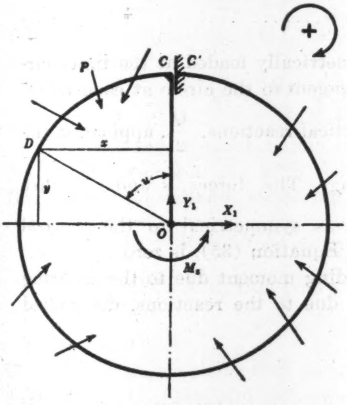

FIG. 5.

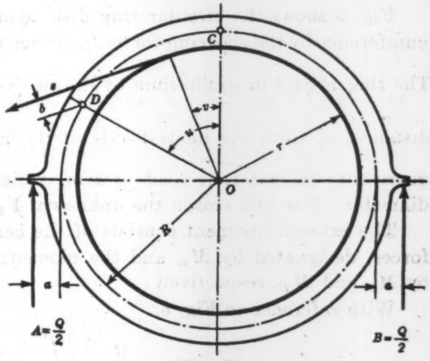

FIG. 6.

If *dl* designates the length of a small ring element and M_e defines the moment of the external forces between *C* and *D* with respect to *D*, then the unknowns are found from the following equations, as given by Mueller-Breslau:[10]

$$X_1 = \frac{\int M_e\, y\, dl}{\int y^2\, dl} \qquad \text{(31)}$$

$$Y_1 = \frac{\int M_e\, x\, dl}{\int x_1^{\,2}\, dl} \qquad \text{(32)}$$

and,

$$M_z = \frac{\int M_e\, dl}{\int dl} \qquad \text{(33)}$$

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[9] Handbuch für Eisenbetonbau, Vol. 6, Fourth Edition, p. 247.

[10] "Die neueren Methoden der Festigkeitslehre," by H. Mueller-Breslau, pp. 125, 134, pub. by Alfred Kroener, Leipzig, 1913.

The forces, X_1 and Y_1, are assumed positive when acting in the direction shown in Fig. 5, and all moments are taken as positive when acting clockwise.

Substituting $dl = R\,du$; $x = R \sin u$; and $y = R \cos u$; and integrating between the limits, $u = 0$ and $u = 2\pi$, Equations (31), (32), and (33) become,

$$X_1 = \frac{1}{\pi R} \int_0^{2\pi} M_e \cos u \, du \dots\dots\dots\dots\dots (34)$$

$$Y_1 = \frac{1}{\pi R} \int_0^{2\pi} M_e \sin u \, du \dots\dots\dots\dots\dots (35)$$

and,

$$M_z = \frac{-1}{2\pi} \int_0^{2\pi} M_e \, du \dots\dots\dots\dots\dots (36)$$

Designating by Q the total load of the pipe shell, transmitted by shear to one ring disk, the distribution of the shear, s, per unit width of circumference is given by Equations (5) or (9); therefore, with reference to Fig. 6,

$$s = \frac{Q}{\pi r} \sin v \dots\dots\dots\dots\dots (37)$$

Fig. 6 shows the circular ring disk, symmetrically loaded at the inner circumference by the shearing forces, s, acting tangent to the circle at each point. The ring is held in equilibrium by the two vertical reactions, $\frac{Q}{2}$, applied at the distance, a, from the neutral axis of the ring. The forces, s and $\frac{Q}{2}$, then represent the external load system, which is symmetrical to the vertical diameter. For this reason the unknown, Y_1, Equation (35), is zero.

The external moment consists of the bending moment due to the shearing forces, designated by M_s, and the moments due to the reactions, designated by M_A and M_B, respectively.

With reference to Fig. 6,

$$M_s = \int_0^u b \, s \, r \, dv \dots\dots\dots\dots\dots (38)$$

From analytic geometry,

$$b = R\left(\sin u \sin v + \cos u \cos v - \frac{r}{R}\right) \dots\dots\dots\dots\dots (39)$$

and, therefore, with s from Equation (37),

$$M_s = \frac{Q\,R}{\pi} \int_0^u \sin v \left(\sin u \sin v + \cos u \cos v - \frac{r}{R}\right) dv \dots\dots (40)$$

By integration,

$$M = \frac{Q\,R}{\pi}\left[\frac{u}{2} \sin u - \frac{r}{R}\,(1 - \cos u)\right] \dots\dots\dots\dots\dots (41)$$

The left reaction causes a moment between $u = \frac{\pi}{2}$ and $u = 2\,\pi$, equal to,

$$M_A = \frac{Q\,R}{2}\left(\frac{a}{R} - \sin u + 1\right) \dots\dots\dots\dots\dots (42)$$

and the right reaction between $u = \frac{3\pi}{2}$ and $u = 2\pi$, causes a moment,

$$M_B = -\frac{Q\,R}{2}\left(\frac{a}{R} + \sin u + 1\right) \ldots\ldots\ldots\ldots\ldots (43)$$

Equation (34) then becomes,

$$X_1 = \frac{1}{\pi\,R}\left[\int_0^{2\pi} M_s \cos u\, du + \int_{\frac{\pi}{2}}^{2\pi} M_A \cos u\, du + \int_{\frac{3\pi}{2}}^{2\pi} M_B \cos u\, du\right]. (44)$$

Substituting Equations (41), (42), and (43) in Equation (44), and integrating between the given limits,

$$X_1 = \frac{Q}{\pi}\left(\frac{r}{R} - \frac{a}{R} - \frac{3}{4}\right) \ldots\ldots\ldots\ldots\ldots (45)$$

For the moment, M_z, Equation (36) gives,

$$M_z = -\frac{1}{2\pi}\left[\int_0^{2\pi} M_s\, du + \int_{\frac{\pi}{2}}^{2\pi} M_A\, du + \int_{\frac{3\pi}{2}}^{2\pi} M_B\, du\right] \ldots (46)$$

Substituting Equations (41), (42), and (43), and integrating between the limits shown,

$$M_z = \frac{Q\,R}{\pi}\left[\frac{r}{R} - \frac{\pi}{4}\left(1 + \frac{a}{R}\right)\right] \ldots\ldots\ldots\ldots\ldots (47)$$

The resulting moment at any point of the ring is obtained by combining the external moment, M_e, with the moment, M_z, and the moment due to X_1. Designating the moment at the top of the ring, $u = 0$, by M_0;

$$M_0 = M_z - R\,X_1 = \frac{Q\,R}{\pi}\left[\frac{3}{4} - \frac{\pi}{4} + \frac{a}{R}\left(1 - \frac{\pi}{4}\right)\right] \ldots\ldots (48)$$

The normal force at this point, designated by N_0, is equal to X_1,

$$N_0 = \frac{Q}{\pi}\left(\frac{r}{R} - \frac{a}{R} - \frac{3}{4}\right) \ldots\ldots\ldots\ldots\ldots (49)$$

Designating the resulting ring moment in the first quadrant by M_1, then, from Equations (41), (48), and (49),

$$M_1 = M_0 + N_0\,R\,(1 - \cos u) + M_s \ldots\ldots\ldots\ldots\ldots (50)$$

or,

$$M_1 = \frac{Q\,R}{2\pi}\left[u \sin u + \frac{3}{2}\cos u - \frac{\pi}{2} + \frac{2\,a}{R}\left(\cos u - \frac{\pi}{4}\right)\right] \ldots (51)$$

Designating the resulting ring moment in the second quadrant by M_2, then

$$M_2 = M_0 + N_0\,R\,(1 - \cos u) + M_s + M_A \ldots\ldots\ldots\ldots (52)$$

and, with reference to Equation (42),

$$M_2 = -\frac{Q\,R}{2\pi}\left[(\pi - u)\sin u - \frac{3}{2}\cos u - \frac{\pi}{2} - \frac{2\,a}{R}\left(\cos u + \frac{\pi}{4}\right)\right]. (53)$$

A comparison of Equations (51) and (53) shows that the absolute moment values in both quadrants are the same. Except for the opposite sign, the moment curves are symmetrical about the horizontal diameter. On account of symmetry in loading the moments in the third and fourth quadrant are identical with those in the second and first quadrant.

In order to determine the maximum moments it will be sufficient, therefore, to consider the first quadrant only. Extreme values occur at $u = 0$, at $u = \frac{\pi}{2}$, and at some intermediate angle, which is found by differentiating Equation (51) with respect to u, thus,

$$u \cot u = \frac{1}{2} + \frac{2\,a}{R} \ldots\ldots\ldots\ldots\ldots\ldots (54)$$

The maximum intermediate moment, designated by M_i, is obtained by substituting in Equation (51) the value of u given by Equation (54). Designating the moment for $u = \frac{\pi}{2}$ by M_a, Equation (51) gives,

$$M_a = -\frac{Q\,R}{4} \times \frac{a}{R} \ldots\ldots\ldots\ldots\ldots\ldots (55)$$

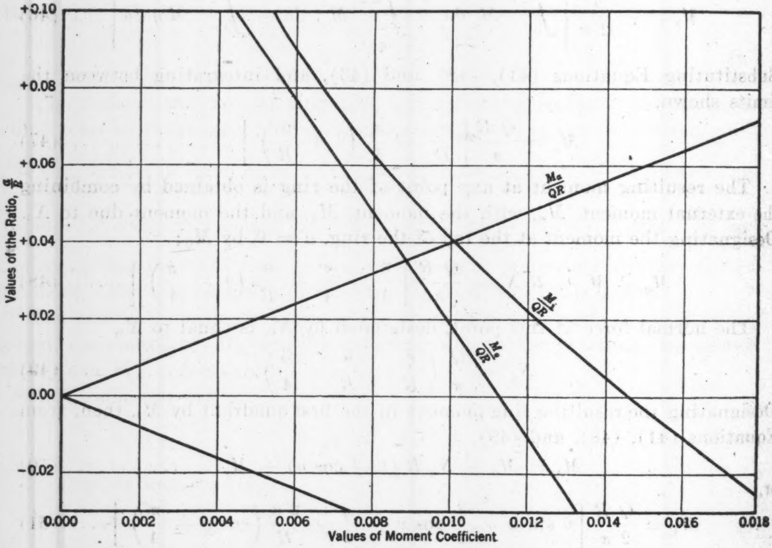

FIG. 7.—BENDING MOMENTS IN SUPPORTING RING.

Fig. 7 shows the absolute values of M_o, M_i, and M_a plotted in the function of the ratio, $\frac{a}{R}$. The eccentricity, a, is taken as positive when the reactions are applied outside the neutral axis of the supporting ring. The minimum possible ring moment is obtained for $\frac{a}{R} = +\,0.04$, in which case,

$$M_i = M_a = 0.010\;Q\,R \ldots\ldots\ldots\ldots\ldots\ldots (56)$$

Fig. 8 illustrates the moment distribution for this case.

Karlsson,[11] by a different method, has derived the maximum ring moment for the special case, $a = 0$. He also quotes the absolute minimum moment derived by Hoeckerberg.[12] Fig. 7 checks both these values.

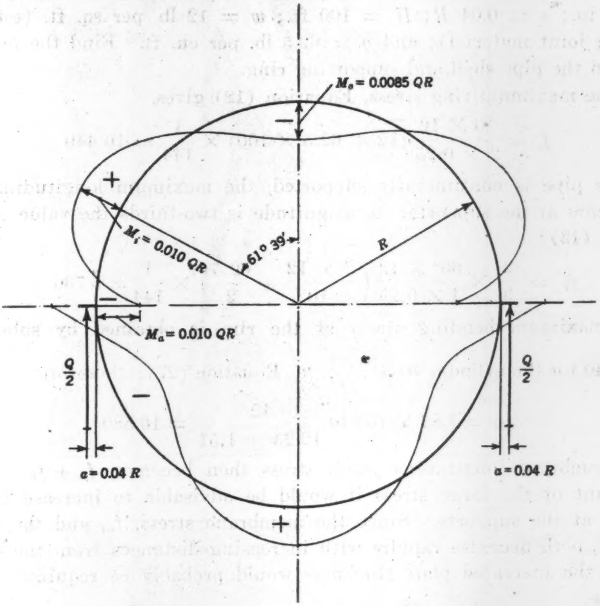

FIG. 8.—BENDING MOMENTS IN SUPPORTING RING FOR $\frac{a}{R} = 0.04$.

The maximum normal ring force, due to the shearing forces, s, occurs at $u = \frac{\pi}{2}$ and is designated by N_a, expressed by the relation,

$$N_a = \frac{Q}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

This force exerts compression just above, and tension just below, the horizontal diameter. Since, for $\frac{a}{R} \geqq +0.04$, the maximum bending stress is also found at this point, a somewhat smaller combined stress will be obtained with $+0.04 > \frac{a}{R} > 0.00$.

In addition to the normal force, N_a, the ring is stressed by the tensile force, N_t, Equation (30), which is independent of $\frac{a}{R}$, but can assume large values for higher heads in combination with thicker shells.

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[11] "Ueber Schwerkraftspannungen in Rohrleitungen von grossen Durchmessern und deren rationelle Konstruktion," by Karl I. Karlsson, *Schweizersche Bauzeitung*, Vol. 80, September 2, 1922.

[12] *Teknisk Tidskrift*, 1919.

EXAMPLE AND APPLICATIONS

Assume a pipe line, continuously supported by circular rings, and having the following characteristics: $D = 10$ ft.; $t = \frac{1}{4}$ in.; $L = 60$ ft.; $A_r = 1$ in. by 12.25 in.; $a = 0.04\ R$; $H = 100$ ft.; $w = 12$ lb. per sq. ft. (estimated, including joint material); and $q = 62.5$ lb. per cu. ft. Find the maximum stresses in the pipe shell and supporting ring.

For the maximum ring stress, Equation (12) gives,

$$f_r = \frac{10 \times 12}{2 \times 0.25}(12 + 62.5 \times 100) \times \frac{1}{144} = 10\ 440$$

Since the pipe is continuously supported, the maximum longitudinal beam stress occurs at the supports; its magnitude is two-thirds the value given by Equation (13):

$$f_L = \frac{2}{3} \times \frac{60^2 \times 12}{4 \times 0.25}\left(\frac{2 \times 12}{10} + \frac{62.5}{2}\right) \times \frac{1}{144} = 6\ 730$$

The maximum bending stress at the rim is obtained by substituting $f_r = 10\ 440$ for the cylinder stress, $\frac{p\ r}{t}$, in Equation (27); therefore,

$$f_{bo} = 1.82 \times 10\ 440\ \frac{12}{12.25 + 1.51} = 16\ 580$$

The combined longitudinal shell stress then becomes, $f_L + f_{bo} = 23\ 310$. On account of the large stress it would be advisable to increase the shell thickness at the supports. Since the membrane stress, f_L, and the bending stress, f_{bo}, both decrease rapidly with increasing distances from the supporting disk, the increased plate thickness would probably be required for only one course.

The total reaction, in pounds, transmitted to one supporting ring is, $Q = 60\ (\pi \times 10 \times 12 + \pi \times 5^2 \times 62.5) = 317\ 000$. Fig. 7, with $\frac{a}{R} = 0.04$, gives for the maximum ring moment, in inch-pounds, $M = 0.010 \times 317\ 000 \times 66.25 = 210\ 000$. The maximum bending stress in the supporting ring then is, $f_1 = \frac{210\ 000 \times 6}{12^2} = 8\ 750$.

From Equation (57) the maximum normal ring stress due to the shearing forces, s, equals $f_2 = \frac{317\ 000}{4 \times 12} = 6\ 610$. The normal ring stress due to radial forces, from Equation (30), equals $f_3 = \frac{43.4 \times 60}{16}\left(1 + 6.04 \times \frac{12}{12.25 + 1.51}\right) = 1\ 020$. Since all these stresses occur combined at the horizontal diameter, the maximum fiber stress, (tension), in the supporting ring becomes, $f = f_1 + f_2 + f_3 = 16\ 380$.

Although principally developed for large pipe lines, the preceding analysis, with due modifications, may also be used for the design of horizontal storage tanks, tank cars, flumes, and under-water tubes supported on piers, etc.

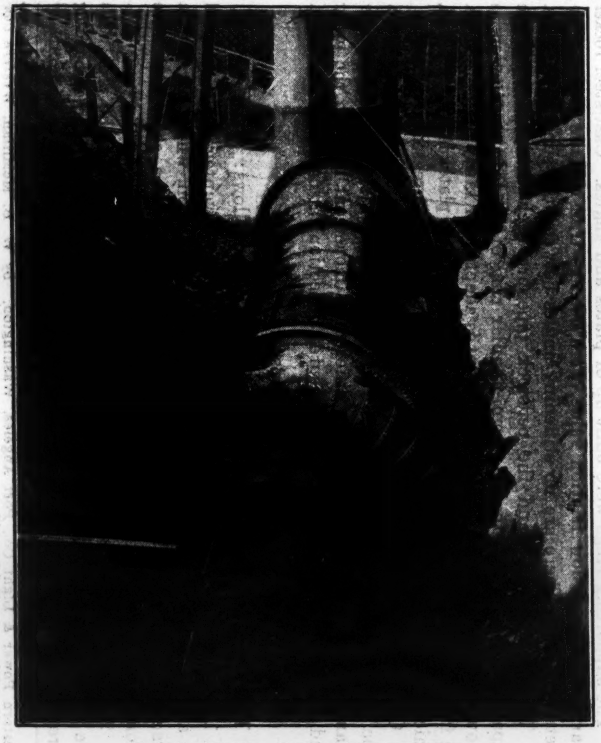

FIG. 9.—GLINES CANYON PRESSURE PIPE.

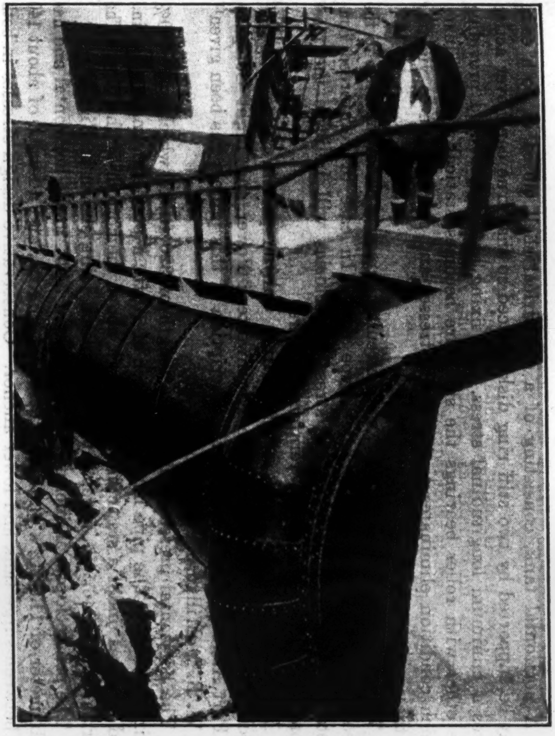

FIG. 10.—GLINES CANYON PENSTOCK PIPE.

A horizontal tank, consisting of a cylindrical shell closed at both ends, may be supported by two stiff ring disks spaced so that true cantilever action gives a minimum longitudinal stress. By fixing one support and providing the other with roller bearings the structure becomes statically determinate, which condition eliminates temperature stresses and stresses due to yielding of the supports.

The design of self-supporting, semi-circular flumes is closely related to the theory of the Zeiss-Dywidag barrel-type roof. In case of reinforced concrete construction, the principal stress trajectories will indicate the most economical direction of the reinforcing bars.

Figs. 9 and 10 are views of the Glines Canyon pressure pipe and penstock, designed and built for the Northwestern Power and Light Company at Port Angeles, Wash. A general description of this project has been given[13] by W. B. McMillan, M. Am. Soc. C. E.

The horizontal pressure pipe has a length of 320 ft., of which about 70% is in a tunnel. The pipe has a diameter of 10 ft., a plate thickness of $\frac{7}{16}$ in., and operates under a static head of 120 ft. above the bottom of the pipe. The normal span is 46.5 ft., except at the expansion joint, shown in Fig. 9.

The penstock pipe is inclined at an angle of 34.5° to the horizontal and has a diameter of 11.5 ft., a plate thickness of $\frac{1}{2}$ to $\frac{3}{4}$ in., and a length of about 140 ft. between surge tank and lower anchor. Conservative design stresses in both pipes were adopted on account of water-hammer action.

The supporting rings were built up of plates and angles. Cast-steel rocker bearings, giving a low friction value, were used to transmit the load to concrete footings. There are no stiffening angles between the supporting rings.

The pressure pipe and the penstock pipe are each equipped with one expansion joint, consisting of a semi-annular ring of riveted construction, as shown in Figs. 9 and 10. This type of joint is adapted for fairly high pressures. In future installations, electric butt-welding of the joint plates is contemplated, which will give a more economical joint combined with increased flexibility.

Observations made on the empty pipe by M. Juul Hvorslev, Assoc. M. Am. Soc. C. E., show that the temperature variation at the top is considerably larger than at the bottom, which causes a differential movement between the upper and lower part, as confirmed by the measured deflections at the expansion joint.

CONCLUSIONS

The writer has reached six major conclusions as follows:

1.—Cylindrical shells, subject to water loads, are advantageously designed in accordance with the theory of thin shells of revolution, which permits an exact knowledge of the stress distribution and leads to a safer and more economical design than the customary empirical methods.

2.—If the pipe is supported by stiff, disk-shaped members, the pipe walls are mainly subject to direct stresses, except for a very narrow zone of bending stresses at the supports. For this reason the shell is entirely stable without any

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[13] "The Glines Canyon Hydro-Electric Development on the Elwha River of the Northwestern Power & Light Co., Port Angeles, Washington," by W. B. McMillan, *Western Construction News*, April 10, 1927.

intermediate stiffening rings, thus permitting the use of large spans in combination with thin plates.

3.—On account of the reduced number of supports the use of bearings with small friction values becomes feasible. In connection with expansion joints it is possible practically to eliminate temperature stresses.

4.—The larger spans permit easy access for painting, and, in many cases, the cost of excavation can be considerably reduced.

5.—Due to the small deformations under all conditions of loading the joints remain tight.

6.—Pipe lines thus designed present a pleasant appearance, because the finished structure expresses the typical properties of a thin shell.

DISCUSSION

L. J. MENSCH,[14] M. AM. SOC. C. E. (by letter).—This interesting paper is too abbreviated for American practice. It is based on a number of German articles on the strength of membranes, the authors of which claim to have obtained simple working formulas by making simplifying assumptions in the treatment of the subject as found in Professor Love's "Treatise of the Mathematical Theory of Elasticity." The author accepts their differential equations on trust and then calls his product an exact theory. This latter expression is constantly found in all prominent works on elasticity from Professor St. Venant on, but the writer considers it very misleading and unjustified by tests and experience. Geometry is considered an exact science for the purpose of the engineer (but not for that of the astronomer as Riemann and Einstein have showed), but the new sciences of mechanics and strength of materials are not advanced far enough to treat of exact theories in these branches. Engineers have no exact column theory, nor an exact beam theory, but simply a belief that their hypotheses lead to formulas with a more or less approximation of the truth.

Equations (2) and (3) were obtained by omitting a great many factors from what Professor Love calls the equations of equilibrium of an infinitesimal part of the surface of a membrane. It seems that for the case of a full-circle pipe, simply supported around its periphery, these equations lead, as the author shows, to useful results which agree with the common theory; it should be noted, however, that his assumption of stresses for a continuous pipe on many supports cannot be derived from Equations (2) and (3), but are based on the common theory of bending only. Dischinger used these equations for the design of elliptically shaped roofs, but in such cases the writer found that his stresses deviate as much as 50% from those obtained by the common theory, and although the firm with which he is connected, made large-sized tests to prove to the public the safety of its concepts, the writer finds that the test objects were largely over-designed, although to the ordinary engineer they appear extremely bold and novel. Let it be stated that similar roofs were constructed in the United States ten to twenty years before they were tried in Germany.

The practicing engineer will not have the time nor the opportunity to look up all the authorities cited by the author and this might lead him to disregard, or look with suspicion upon this valuable paper. The writer is pleased to state that more than twenty years ago he obtained the same results as the author by a much simpler method, and can attest the correctness of Equation (28). He knows that Equation (29) was obtained by a difficult chain of reasonings, with many approximations; it may be correct. He can give, however, some data on the condition of restraint which will simplify the understanding and work of the designer. The rim stresses of $1.82\,\frac{pr}{t}$

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[14] Gen. Contr., Chicago, Ill.

become zero at the distance, $x = 0.588\sqrt{rt}$. A reversed maximum moment occurs at nearly twice this distance from the rim, namely, $1.15\sqrt{rt}$, and these reversed stresses amount to $0.4\frac{pr}{t}$. In the author's example, $r = 60$ in. and $t = \frac{1}{4}$ in.; hence, the two critical distances are 2.75 in. and 4.40 in., respectively. This shows that there are great changes of stress in a very short distance, involving considerable shear, and this part of the pipe should be carefully designed, with low stresses, which practice was recommended to his students by Professor Grashof more than fifty years ago. In the design of the circular ring disk the consideration of the Lamé stresses might be advisable in most cases.

W. P. ROOP,[15] M. AM. SOC. C. E. (by letter).—This interesting and important paper gives a theoretical analysis of a novel type of support for horizontally disposed, thin-walled tubes. In the section headed, "Direct Stresses in Pipe Shell," solutions are offered for the stresses due to weight of tube and contents, carried by the tube itself, considered as a girder, and also as modified by the tensile stresses in the shell (rim stresses) due to hydrostatic pressure. In a second section these rim stresses receive more detailed study, especially as affected by the supports, consisting of very rigid peripheral stiffeners. The stresses in these rings themselves are considered in a third section, and the paper closes with examples showing the practical advantages of this type of construction.

The mathematical procedure is somewhat formal, but the details could doubtless be readily reconstructed by one who wished to follow the analysis through closely. The summary is excellent. The present comment is confined to references to related work not quoted, and to emphasizing some features which may possibly not otherwise come to the notice of the casual reader.

A structure of circular form draws enormous benefit with respect to its rigidity from its symmetry, and the application studied by Mr. Schorer, while extending to various cases mentioned under "Example and Applications," by no means exhausts the usefulness of such a symmetrical form. As examples, the writer wishes to cite the cantilevers upon which he reported in 1929[16] and the tubes under external pressure discussed by Messrs. H. E. Saunders and D. F. Windenburg in a paper[17] entitled "Strength of Thin Cylindrical Shells Under External Pressure."

It is necessary, of course, that the circular form be adequately maintained, and the peripheral stiffener has been found to be an effective means of doing this. In the cases thus far mentioned, the interior of the shell is left open and the stiffeners are rings, inside or outside. In other cases, however, it is possible to use diaphragms extending entirely across the section of the cylinder. Such, for example, are the transverse bulkheads in a submarine. Although surface ships depart widely from circular sections, the form is approximately cylindrical on a section partly symmetrical. The typical structure is reproduced naturally in the stem of the bamboo plant.

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[15] Lieut. (C. C.), U. S. N.; New York Shipbuilding Co., Camden, N. J.

[16] *Transactions*, Am. Soc. Mech. Engrs., Vol. 51 (1), 1929 (A. P. M.-51-7), p. 69.

[17] *Loc. cit.*, Sept.-Dec., 1931, Applied Mechanics, (A. P. M.-53-17A).

Turning again to Mr. Schorer's paper, it is noted that his numerical example shows stresses of an acceptable intensity in pipe the diameter of which is 480 thicknesses supported only by peripheral stiffeners at intervals of 6 diameters. Contrast such a structure with one of cast pipe, supported by trusses, such as may be seen in many large industrial plants. The writer believes that there is no doubt that savings in weight of the order of one-half or more of that used in continuous structures of traditional type can be made in many cases by close analysis of the requirements of accurately specified loads, and that a structure of continuous character may be built up of plates shockingly thin if due attention is given to their favorable disposition and to their adequate support even at rather widely spaced boundaries. Such a structure will generally be characterized by high rigidity and by relative immunity to complete failure due to moderate local damage. In general, the more specific the load, however, the greater the liability to injury through occurrence of unforeseen loads.

The use of thin plating perhaps has extended further in ship building (and aircraft construction) than in other branches of engineering. Structures of high weight efficiency and, at the same time, high adaptability to undetermined loads have been developed, very largely through empirical and evolutional methods. It has more than once occurred that analysis like that used by Mr. Schorer has led to the conclusion that the type of structure evolved by experience has merits which the analysis reveals clearly without being able to carry them much further. Experience has thus rather blindly found the way in advance of analysis. Such cases, however, are exceptional; analysis can proceed systematically and, at least, greatly accelerate progress toward the structure perfectly designed for its function.

JOHANNES SKYTTE,[18] **ASSOC. M. AM. SOC. C. E.** (by letter).—The author is to be commended for producing this paper. It is the writer's belief that this method can be applied with advantage in many cases to the design of pipe lines and other cylindrical structures.

Primary Stresses in the Pipe Shell.—The stress condition will be general only when it involves the radial shears, S_r and S_l, and the moments, M_r and M_l (see Fig. 11). However, S_l and M_l are very small quantities for any partial filling of pipe and will be herein considered equal to zero.

For the three cases treated in the paper, namely, (1) dead weight of pipe; (2) pipe just full; and (3) pipe under pressure,

$$S = -C \sin u \quad \text{.......................} (58)$$

in which, C is a constant. Consequently, T_1 has a straight-line variation as for ordinary beams, and S_r and M_r become equal to zero.

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[18] Asst. Hydr. Engr., Hetch Hetchy Project, San Francisco, Calif.

The general formulas, Equations (59), (60), and (61), are derived by projection on the three axes. Equation (62) is derived by taking the moment around the X-axis.

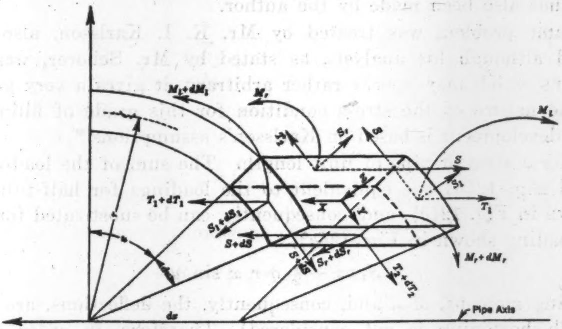

FIG. 11.

Projection on the Z-axis gives:

$$Z\ r\ du\ dx + (T_2 + dT_2)\ dx\ \tfrac{1}{2}\ du + T_2\ dx\ \tfrac{1}{2}\ du + dS_r\ dx = 0$$

or,

$$T_2 = -r\left(Z + \frac{dS_r}{r\ du}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (59)$$

Projection on the Y-axis gives:

$$Y\ r\ du\ dx + (T_2 + dT_2)\ dx - T_2\ dx - 2S_r\frac{du}{2}dx + (S + dS)\ r\ du - S\ r\ du = 0$$

or,

$$dS = -\frac{d\,T_2\ dx}{r\ du} + \frac{S_r}{r}\ dx - Y\ dx$$

and, consequently,

$$S = -\int\frac{d\ T_2}{r\ du}\ dx + \int\frac{S_r}{r}\ dx - \int Y\ dx + C_1 \ \ldots\ldots\ldots (60)$$

Projection on the X-axis gives, in a similar way:

$$T_1 = -\int\frac{dS}{r\ du}\ dx - \int X\ dx + C_2 \ \ldots\ldots\ldots\ldots\ldots (61)$$

Moment around the X-axis gives:

$$dM_r + r\ du\ S_r = 0$$

or,

$$M_r = -r\int S_r\ du + C_3 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (62)$$

Pipe Partly Filled.—For the case of a pipe partly filled the stress condition becomes very complicated, and, to the writer's knowledge, the general solution for this case has not yet been made. For half-full condition, however, a very thorough and complete analysis has been made[19] by Mr. A. Frey Samsioe, of Stockholm, Sweden, in a volume of about 100 pages, to which reference has also been made by the author.

The same problem was treated by Mr. K. I. Karlsson, also of Stockholm, and although his analysis, as stated by Mr. Schorer, was based on assumptions which may appear rather arbitrary, it gives a very good indication of the nature of the stress condition for this grade of filling, and the following development is based on Karlsson's assumptions.[20]

Consider a slice of pipe of unit length. The sum of the loadings in Fig. 12(*a*) and Fig. 12(*b*) are equivalent to the loadings for half-full condition, or as shown in Fig. 12(*c*), and, consequently, can be substituted for the latter. For the loading shown in Fig. 12(*a*),

$$S_1 = -\tfrac{1}{2}\, q\, r\, x \sin u$$

and the ring moment, M_{r1}, and, consequently, the deflections, are zero at all points (rib-shortening is not considered). Therefore, in order to find the resulting ring moment, M_{r1}, one needs only to consider the loading as in Fig. 12(*b*). In this case the deflection will cause a shortening of the horizontal diameter and a lengthening of the vertical diameter.

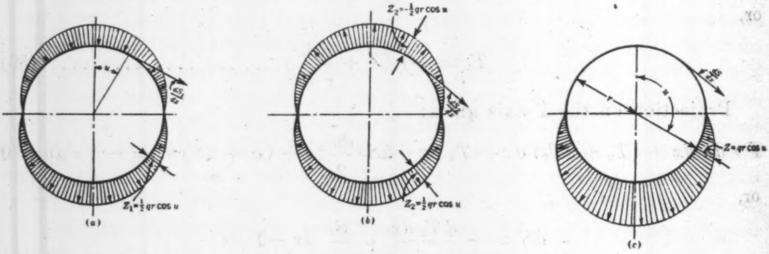
u
$\frac{1}{2}\frac{dS_1}{dx}$
$Z_1 = \frac{1}{2} qr \cos u$
(a)
$Z_2 = -\frac{1}{2} qr \cos u$
$\frac{1}{2}\frac{dS_2}{dx}$
$Z_2 = \frac{1}{2} qr \cos u$
(b)
$\frac{dS}{dx}$
r
u
$Z = qr \cos u$
(c)

FIG. 12.

This deformation will be resisted greatly by the rigid ring support and will cause a tangential shear, gradually decreasing from the ring support to the center of the span, where it becomes equal to zero. Since the shear must be symmetrical about the two axes and zero at the ends of the axes it is reasonable to assume that:

$$S_2 = f \times \tfrac{1}{2}\, q\, r\, x \sin 2\, u$$

or,

$$\frac{dS_2}{dx} = f \times \tfrac{1}{2}\, q\, r \sin 2\, u$$

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[19] "Die Spannungen in einem auf mehreren Stützen in gleicher gegenseitiger Entfernung aufgelegten and zur hälfte mit Wasser gefüllten Rohr," by A. Frey Samsioe, Ingeniörs Vetenskaps Akademiens Handlingar Nr. 50, Stockholm, 1926; pub. by Svenska Bokhandelscentralen, Stockholm, 1926.

[20] "Ueber Schwerkraftspannungen in Rohrleitungen von grossen Durchmessern und deren rationelle Kronstruktion," by Karl I. Karlsson, *Schweizerische Bauzeitung*, Vol. 80, September 2, 1922, p. 105.

Differentiation of Equation (60), with respect to x gives,

$$\frac{dS_2}{dx} = -\frac{dT_2}{r\,du} + \frac{S_r}{r} - Y$$

since $Y = 0$,

$$\frac{dS_2}{dx} = -\frac{dT_2}{r\,du} + \frac{S_r}{r} = f \times \tfrac{1}{2}\, q\, r \sin 2u \ldots\ldots\ldots\ldots\ldots (63)$$

Next, substitute $Z_2 = -\frac{1}{2}\, q\, r \cos u$, in Equation (59), and differentiate with respect to u, thus:

$$\frac{dT_2}{du} = -r\left(\tfrac{1}{2}\, q\, r \sin u + \frac{d^2S_r}{r\,du^2}\right) \ldots\ldots\ldots\ldots\ldots (64)$$

Eliminating T_2 between Equations (63) and (64):

$$\frac{d^2S_r}{du^2} + S_r = \tfrac{1}{2}\, q\, r^2\, (f \sin 2\,u - \sin u)$$

The complete integral is:

$$S_r = \tfrac{1}{2}\, q\, r^2 \left[\frac{u \cos u}{2} - f\,\frac{\sin 2\,u}{3}\right] + C_4\, e^{+u\sqrt{-1}} + C_5\, e^{-u\sqrt{-1}}$$

For $u = 0$ and $u = \frac{\pi}{2}$, $S_r = 0$, or, $C_4 = C_5 = 0$, and, consequently,

$$S_r = \tfrac{1}{2}\, q\, r^2 \left[\frac{u \cos u}{2} - f\,\frac{\sin 2\,u}{3}\right] \ldots\ldots\ldots\ldots\ldots (65)$$

Substituting for S_r in Equation (62) and integrating:

$$M_r = -\tfrac{1}{2}\, q\, r^3 \left(\frac{\cos u}{2} + \frac{u \sin u}{2} + f\,\frac{\cos 2\,u}{6} + C_3\right) \ldots\ldots (66)$$

Since the angle between the tangents at $u = 0$ and $u = \frac{\pi}{2}$ remain unchanged:

$$\int_0^{0.5\pi} M_r\, r\, du = 0$$

or,

$$-\tfrac{1}{2}\, q\, r^3 \left[\frac{\sin u}{2} + \frac{\sin u}{2} - \frac{u \cos u}{2} + f\,\frac{\sin 2\,u}{12} + C_3\, u\right]_0^{0.5\pi} = 0$$

or, $C_3 = \frac{1}{2}\, q\, r^3 \frac{2}{\pi}$, and, consequently Equation (66) becomes:

$$M_r = -\tfrac{1}{2}\, q\, r^3 \left(\frac{\cos u}{2} + \frac{u \sin u}{2} + f\,\frac{\cos 2\,u}{6} - \frac{2}{\pi}\right) \ldots\ldots (67)$$

For a minimum of internal work, f is found equal to 0.85. Substituting this value for f:

$$M_{r0} = -\,0.0050 \times \tfrac{1}{2}\, q\, r^3$$

and,

$$M_{0.5\pi r} = -\,0.0072 \times \tfrac{1}{2}\, q\, r^3$$

For a pipe 12 ft. in diameter:

$$M_{0.5\pi r} = 0.0072 \times \tfrac{1}{2} \times 62.5 \times 6^2 \times 12 = 585 \text{ in-lb.}$$

For a $\frac{5}{16}$-in. shell:

$$f_s = \frac{585 \times 6}{\left(\frac{5}{16}\right)^2 \times 12} = 3\,000 \text{ lb. per sq. in.}$$

The value of T_1 is found next by substituting for $\dfrac{dS}{du}$ in Equation (61):

$$S = S_1 + S_2 = -\tfrac{1}{2}\, q\, r\, x\, [\sin u - 0.85 \sin 2\, u] \;\ldots\ldots\ldots (68)$$

and,

$$\frac{dS}{du} = -\tfrac{1}{2}\, q\, r\, x\, [\cos u - 1.70 \cos 2\, u] \;\ldots\ldots\ldots\ldots (69)$$

Then, by substitution in Equation (61), since $X = 0$,

$$T_1 = -\frac{1}{r}\int -\tfrac{1}{2}\, q\, r\, x\, (\cos u - 1.70 \cos 2\, u)\, dx + C_2$$

$$= \tfrac{1}{2}\, q\, \frac{x^2}{2} (\cos u - 1.70 \cos 2\, u) + C_2 \ldots\ldots\ldots\ldots\ldots (70)$$

For a simply supported pipe, with $x = \dfrac{L}{2}$, $T_1 = 0$, and Equation (70) becomes,

$$T_1 = \tfrac{1}{2}\, q\, (\cos u - 1.70 \cos 2\, u) \left(\frac{x^2}{2} - \frac{L^2}{8}\right) \ldots\ldots\ldots\ldots (71)$$

For $u = \pi$,

$$T_1 = -1.35\, q \left(\frac{x^2}{2} - \frac{L^2}{8}\right)$$

or 1.35 times the value derived for a pipe exactly full.

For continuously supported pipe:

$$T_1 = \tfrac{1}{2}\, q\, (\cos u - 1.70 \cos 2\, u) \left(\frac{x^2}{2} - \frac{L^2}{24}\right) \ldots\ldots\ldots\ldots (72)$$

Mr. Karlsson has also given some measurements for the deformations due to ring stresses for the old Kungfors pipe line in Sweden.[22] If the pipe when empty is considered to have zero deformation, then the maximum deformation for the pipe half full at the center line of the span amounts to $\frac{1}{6}$ in. The pipe is built of $\frac{5}{16}$-in plate with no stiffeners between supporting rings and has a diameter of about 9 ft. and is 38 ft. between the supports. This condition indicates clearly that a direct stress condition exists, practically, for this grade of filling since a possible ring moment of any consequence would cause considerably more deflection.

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[22] *Schweizerische Bauzeitung*, September 2, 1922.

According to Mr. Samsioe,[22] pipe lines of $\frac{1}{4}$-inch. plate, 12.5 ft. in diameter, and 43 ft. span, and with no stiffeners between the supporting rings, are common practice in Sweden.

***Rim Stresses.*—An approximate value of f_{bm} for unyielding ring support can be found in the following manner. The author's notations are used**

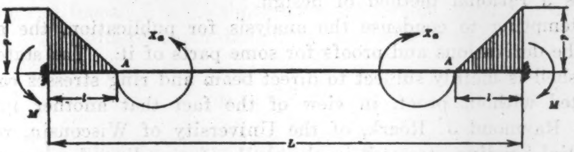

FIG. 13.

and l (Fig. 13) is the distance along the pipe axis from the center line of the ring support to the point, A, where the pipe shell practically has expanded an amount, r'_p, and, consequently, all the load is taken by the ring stresses. It is obvious that the moment at this point, A, must have a value between zero and the value, X_a, which makes the tangent to the elastic line of the pipe shell in A parallel to the pipe axis. It is also evident that the load causing the rim stresses is equal to p at the ring support; and from this point it is decreasing and becomes practically equal to zero at Point A and remains very close to zero between the two points, A.

Assume a straight-line variation of the load between the center line of the ring support and Points A. First, consider the case in which the moment at A is assumed equal to zero. Equating r'_p and the cantilever deflection:

$$\frac{\frac{1}{2}\, p\, l^4}{15\, E\, I} = \frac{p\, r^2}{E\, t}$$

or,

$$l = 1.26 \sqrt{r\, t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (73)$$

$$M = 0.264\, p\, r\, t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

and,

$$f_{bm} = 1.58\, \frac{p\, r}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

In a similar way, for the case in which the moment at A is assumed equal to $X_a = \frac{1}{24}\, p\, l^2$,

$$f_{bm} = \frac{2.58\, p r}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76)$$

Taking the average of the two values in Equations (75) and (76):

$$f_{bm} = 2.08\, \frac{p\, r}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (77)$$

or 14% more than the value found by the author.

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[22] **By letter to the writer.**

Donald E. Larson,[23] **Jun. Am. Soc. C. E.** (by letter).—There has been a noticeable scarcity of data for use in the design of large pipe lines. Something can be learned from records of past failures, but such information is not generally available to engineers because failures are not advertised. Mr. Schorer's analysis is a valuable contribution to the subject because it introduces a rational method of design.

In attempting to condense the analysis for publication, the author has omitted the derivations and proofs for some parts of it. The statement that the pipe shell is mainly subject to direct beam and ring stresses can scarcely be accepted without proof, in view of the fact that another investigator, Professor Raymond J. Roark, of the University of Wisconsin, reports circumferential bending stresses[24] in the shells of pipe lines having supports of the same type as those discussed by the author. Furthermore, the results of the two studies should be in agreement because both investigators used the same distribution of longitudinal horizontal shearing stress.

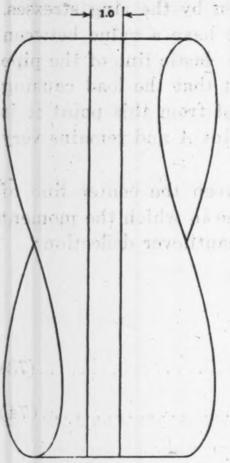

Fig. 14.

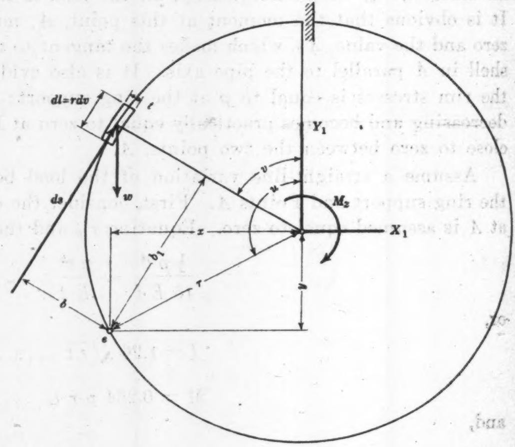

Fig. 15.

The writer has made a study to determine the reason for the disagreement in regard to circumferential bending stresses in the shells of pipes subject to dead load and full water load. An analysis is submitted as proof for the following statements:

(1) There are no circumferential bending stresses in the pipe shell under dead load if the shearing stresses act tangentially as indicated by the author.

(2) There are no circumferential bending stresses in the pipe shell under full water load if the shearing stresses act tangentially.

(3) The shearing stresses do act tangentially as indicated in the author's analysis.

The notation is that introduced by Mr. Schorer, except as otherwise given.

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[23] With Chicago Bridge & Iron Works, Chicago, Ill.

[24] "A Study of Circumferential Bending of Pipes and Cylindrical Containers", *Bulletin No. 69*, Eng. Experiment Station Series, Univ. of Wisconsin, Madison, Wis.

Example (1).—Circumferential Bending Due to Dead Load.—A portion of a pipe having a unit length is shown in Fig. 14. The forces acting on this ring are the shears on the end faces and the weight of the ring itself. The forces tending to produce circumferential bending in the ring are: (1) The weight of the ring; and (2), the difference of the tangential shearing forces on the end faces. Fig. 15 shows these forces acting on a small element of the ring. The difference in tangential shearing forces is given by the following equation:

$$ds = \frac{A\bar{y}\,dQ}{2I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (78)$$

in which, ds is the tangential shear per unit of circumference, due to change in total shear, dQ. In this case,

$$dQ = 2\pi\, r\, w \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (79)$$

$$A\bar{y} = 2\, r^2\, t \sin v \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (80)$$

and,

$$I = \pi\, r^3\, t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (81)$$

Substituting,

$$ds = -\, 2\, w \sin v \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (82)$$

Following the Müller-Breslau equations for forces at the neutral point,

$$M_z = \frac{\int M_e\, dl}{\int dl} = \frac{1}{2\,\pi}\int_0^{2\pi} M_e\, du \quad \ldots\ldots\ldots\ldots (83)$$

$$X_1 = \frac{\int M_e\, y\, dl}{\int y^2\, dl} = \frac{1}{\pi\, r}\int_0^{2\pi} M_e \cos u\, du \quad \ldots\ldots\ldots\ldots (84)$$

$$Y_1 = \frac{\int M_e\, x\, dl}{\int x^2\, dl} = \frac{1}{\pi\, r}\int_0^{2\pi} M_e \sin u\, du \quad \ldots\ldots\ldots\ldots (85)$$

in which, M_e is the moment of all forces between the top of the ring and the point, e, about the point, e.

Referring to Fig. 15,

$$M_e = -\int_0^u ds\; b\; r\; dv + \int_0^u w \sin v\; b\; r\; dv - \int_0^u w \cos v\; b_1\; r\; dv \;.. (86)$$

and,

$$b_1 = r \sin (u - v) = r (\sin u \cos v - \cos u \sin v) \quad \ldots\ldots (87)$$

$$b = r\, [1 - \cos (u - v)] = r\, (1 - \cos u \cos v - \sin u \sin v) \quad \ldots (88)$$

Substituting these values of b_1 and b in Equation (86) and performing the indicated integration,

$$M_e = -\, w\, r^2\, (1 - \cos u) \quad \ldots\ldots\ldots\ldots\ldots\ldots (89)$$

Substituting this value of M_e in Equations (83), (84), and (85), and integrating between the limits, 0 and 2π,

$$M_z = -\, w\, r^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (90)$$

$$X_1 = +\, w\, r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (91)$$

$$Y_1 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (92)$$

The moment in the ring at any point is,

$$M = -M_z - X_1 r \cos u + M_e \ldots\ldots\ldots\ldots\ldots (93)$$

Substituting the values of M_z, X_1, and M_e from Equations (90), (91), and (89), respectively,

$$M = w r^2 - w r^2 \cos u - w r^2 (1 - \cos u) = 0 \ldots\ldots\ldots (94)$$

This indicates that there is no circumferential bending in the shell of a truly circular pipe due to dead load, providing the shearing forces act tangentially as shown in Fig. 15.

Example (2).—Circumferential Bending Due to Water Load.—Fig. 16 shows the water-load forces tending to produce circumferential bending in the ring. These are: (1) The outward forces due to water pressure; and (2) the difference between tangential shearing forces on the end faces. In this case,

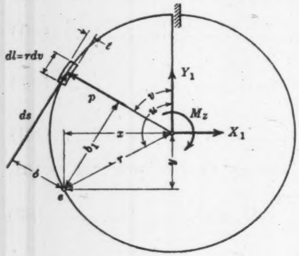

FIG. 16.

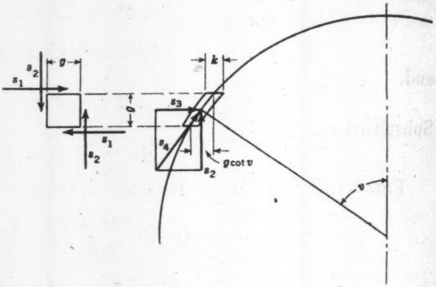

FIG. 17.

$$dQ = q \pi r^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (95)$$

Substituting this value in Equation (78),

$$ds = -q r \sin v \ldots\ldots\ldots\ldots\ldots\ldots\ldots (96)$$

The outward force at any point is given by the relation,

$$p = qr (1 - \cos v) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (97)$$

The moment at the point, *e*, of all the forces between the top of the ring and *e* is,

$$M_e = -\int_0^u ds\ b\ r\ dv + \int_0^u p\ b_1\ r\ dv \ldots\ldots\ldots\ldots (98)$$

Substituting the values of *s*, *b*, *p*, and b_1 from Equations (96), (88), (97), and (87), respectively, and integrating between the limits, 0 and *u*,

$$M_e = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (99)$$

Therefore, $M_z = 0$, $X_1 = 0$, and $Y_1 = 0$, and the moment in the ring at any point is equal to zero. This proves that there is no circumferential bending in the pipe shell due to full water load, providing the shearing stresses act tangentially as shown in Fig. 16.

Example (3).—Tangential Shearing Stresses.—It now remains to demonstrate that the shearing forces do act tangentially. It is in this respect that the author's analysis differs from that of Professor Roark. Fig. 17 shows an element of the pipe shell having a length, g, and a vertical height, g, such that the area of each of the four faces is equal to unity. The shearing stress on the horizontal faces is given by the relation,

$$s_1 = \frac{Q A \bar{y}}{2 I k} \quad \ldots\ldots (100)$$

in which, s_1 is the unit horizontal shearing stress and,

$$k = \frac{t}{\sin v} \quad \ldots\ldots (101)$$

Substituting this value for k and the values of $A\bar{y}$ and I from Equations (80) and (81), Equation (100) becomes,

$$s_1 = \frac{Q \sin^3 v}{\pi r t} \quad \ldots\ldots (102)$$

These shearing stresses on the top and bottom faces form a couple, $s_1\, g$, tending to rotate the element in a vertical plane and a couple, $s_1\, g \cot v$, tending to rotate the element in a horizontal plane. In order to balance these couples, the shearing stresses, s_2 and s_3, must be applied to the end faces. Equating moments,

$$s_2 = \frac{s_1\, g}{g} = s_1 \quad \ldots\ldots (103)$$

$$s_3 = \frac{s_1\, g \cot v}{g} = s_1 \cot v \quad \ldots\ldots (104)$$

The resultant of s_2 and s_3 acts tangentially and has a magnitude,

$$s_4 = \sqrt{s_2^2 + s_3^2} \quad \ldots\ldots (105)$$

in which, s_4 is the tangential unit shear on the end faces.

Substituting the values of s_2 and s_3 given by Equation (103) and Equation (104),

$$s_4 = s_1 \sqrt{1 + \cot^2 v} = \frac{s_1}{\sin v} \quad \ldots\ldots (106)$$

Substituting the value of s_1 given by Equation (102),

$$s_4 = \frac{Q \sin v}{\pi r t} \quad \ldots\ldots (107)$$

This indicates that the shearing forces shown by the author are correct in both magnitude and direction.

The foregoing analysis has been based on the assumption that the horizontal longitudinal shearing stresses in the pipe vary as they would in a cylindrical beam. The author has not given the derivation of the differential equation from which this distribution of shearing stresses was obtained, but it has been shown that such a distribution results in a system of forces which hold each element of the shell in static equilibrium without the application of circumferential bending moments. Such a system of forces does a minimum amount of work and it may be argued from the principle of least not yet

work that the assumed distribution of horizontal longitudinal shearing stresses is the correct one. This being the case, there can be no doubt about the correctness of the author's statement that the pipe shell is mainly subject to direct beam and ring stresses.

Raymond J. Roark,[25] Esq. (by letter).—Because of work the writer had done some time ago on stresses in pipes and cylindrical containers, he is especially interested in Mr. Schorer's valuable paper. The conclusions to which the analysis of such stresses by the theory of elasticity leads, are surprising, and at variance with those reached from cursory consideration. The writer confesses with some chagrin that in his own analysis of these stresses, undertaken some years ago, an error with respect to the shearing stresses led to erroneous expressions for the circumferential bending moments in thin shells of the type here under consideration. The calculations then made indicated the presence of such moments in cylinders either partly or quite filled with liquid, whereas, as Mr. Schorer shows, the uneven pressure in a completely filled pipe can be resisted by a system of shearing and normal stresses which are at every point in the plane of the wall. As a consequence, there is no primary bending under these circumstances, and even a perfectly flexible diaphragm, if rigidly constrained at the ends, will retain its circular form. This fact first came to the writer's attention through an article by Thoma,[26] who shows, in essentially the same manner as Mr. Schorer, that not only in the case of a circular cylinder such as a pipe, but in the case of any developable surface, a distributed pressure can be resisted without primary bending stress.

The demonstration for the full vessel is clear and convincing, but it would appear that for a partly filled vessel some circumferential bending must occur. The writer has had no opportunity to read the paper by Mr. A. F. Samisoe to which Mr. Schorer refers, and Mr. Schorer does not state whether this analysis of the half-filled cylinder indicates the existence of bending or not; but that the pressure in any partly filled vessel cannot be resisted by stresses which are at every point in the plane of the shell would seem to be sufficiently proved by the following.

Let f_u, f_x, and τ denote, respectively, the unit stress in circumferential tension, longitudinal tension, and tangential shear; let u and x, respectively, denote distance measured circumferentially and longitudinally from any chosen origin; let p, t, and r, respectively, denote the unit pressure, thickness of the shell, and radius of the cylinder. Then, if the assumption of plane stress is correct, the following equations must apply:

$$f_u = \frac{p\,r}{t} \qquad (108)$$

$$\frac{\partial f_u}{\partial u} + \frac{\partial \tau}{\partial x} = 0 \qquad (109)$$

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[25] Associate Prof. of Mechanics, Univ. of Wisconsin, Madison, Wis.

[26] "Die Beanspruchung freitragender gefüllten, Rohre durch das Gewicht der Flussigkeit," *Zeitschrift für das gesamte Turbinenwesen,* February 20, 1920. The discussion is extended by Schwerin in a paper "Uber die Spannungen in freitragender gefüllten Rohren," *Zeitschrift für angewanete Mathematik und Mechanik,* Vol. II, October, 1922.

$$\frac{\partial f_x}{\partial x} + \frac{\partial \tau}{\partial u} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (110)$$

Equation (108) shows that for any point above the water line, $f_u = 0$; hence, $\frac{\partial f_u}{\partial u} = 0$ in that region, and it follows from Equation (109) that $\frac{\partial \tau}{\partial x} = 0$ and that, since $\tau = 0$ at, say, the middle of a cylinder supported at the ends, it must be zero everywhere above the water-line; but this is obviously inconsistent with the assumption of plane sections and ordinary beam action of the cylinder as a whole.

It seems, therefore, that in the partly filled cylinder there must be some circumferential bending. An analysis of this circumferential bending may be made, as follows:[27]

Let a segment of the pipe of unit length be divided by a vertical diameter; the forces acting on the resulting half-ring are as shown in Fig. 18. They comprise: (1) The unknown couples, C_1 and C_2; (2) the unknown tensions, F_1 and F_2; (3) the water pressure which, at any point at an angular distance, α, from the bottom of the ring, amounts to $p = q\, r\, (\cos \alpha - \cos \theta)$ lb. per in. of arc (q being the specific weight of the contained liquid, in pounds per cubic inch); and (4) the tangential shearing stress which, at any point at an angular distance, α, from the bottom of the ring, amounts to $\tau_s\, t = \lambda\, q\, r \sin \alpha$ lb. per in. of arc (λ being the fractional degree of fullness of the cylinder). It is easy to show that the bending moment due to these forces at any point, A, at an angular distance, α', from the bottom of the ring is, for points below the water level ($\alpha' < \theta$):

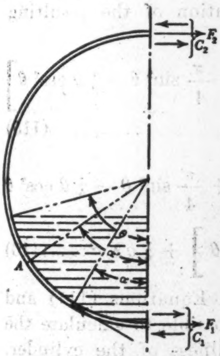

FIG. 18.—FORCES ON FREE RING (WEIGHT NEGLECTED).

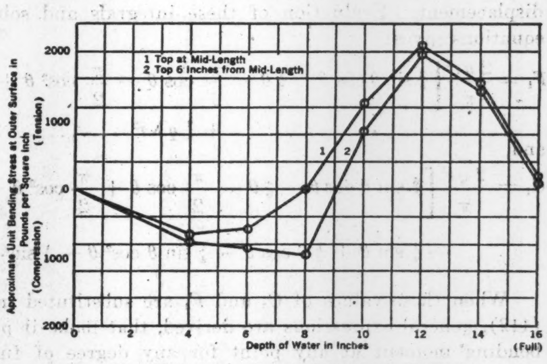

FIG. 19.—MEASURED CIRCUMFERENTIAL BENDING STRESSES ON HORIZONTAL CYLINDER.

$$M = C_1 - F_1 r\, (1 - \cos \alpha') - \lambda\, q\, r^3 \left(\tfrac{1}{2}\, \alpha' \sin \alpha' + \cos \alpha' - 1\right)$$
$$+\, q\, r^3 \left(\tfrac{1}{2}\, \alpha' \sin \alpha' - \cos \theta + \cos \theta \cos \alpha'\right) \ldots\ldots\ldots\ldots (111)$$

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[27] For complete analysis, see *Bulletin No. 69*, Univ. of Wisconsin, "A Study of Circumferential Bending of Pipes and Cylindrical Containers." Some of the results therein presented are invalidated by the error, corrected in the present discussion, in the expression for the shear stress.

and for points above the water level ($\alpha' > \theta$), the moment is:

$$M = C_1 - F_1 r (1 - \cos \alpha') - \lambda q r^3 (\tfrac{1}{2} \alpha' \sin \alpha' + \cos \alpha' - 1)$$
$$+ q r^3 [\sin \alpha' (\tfrac{1}{2} \theta - \tfrac{1}{2} \sin \theta \cos \theta)$$
$$+ \cos \alpha' (\cos \theta - \cos^2 \theta - \tfrac{1}{2} \sin^2 \theta] \ldots\ldots\ldots\ldots\ldots (112)$$

If the shell is assumed to be perfectly flexible, $C_1 = 0$ and

$$F_1 = q r^2 (1 - \cos \theta)$$

If these values are submitted in Equations (111) and (112), they vanish for $\theta = 180°$; that is, for the case of the completely filled vessel, but do not vanish for any smaller value of θ, showing that for the partly filled vessel some bending must exist.

The existence of the couples, C_1 and C_2, is thus shown to be necessary for equilibrium, and C_1, C_2, F_1, and F_2, all become indeterminate. They may be found from the conditions of statics and the additional relations:

$$\int_0^{2\pi} M r \, d \alpha' = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (113)$$

$$\int_0^{2\pi} M r^2 (1 - \cos \alpha') \, d\alpha = 0 \ldots\ldots\ldots\ldots\ldots\ldots (114)$$

which equations simply express the facts, obvious from symmetry, that as between the bottom and the top of the ring there is neither lateral nor angular displacement. Evaluation of these integrals and solution of the resulting equations gives:

$$F_1 = \frac{2 q r^2}{\pi} \left[\tfrac{3}{8} \sin \theta \cos \theta - \tfrac{1}{8} \theta - \frac{\pi}{2} \cos \theta - \frac{\pi}{2} \cos^2 \theta + \frac{\pi}{4} \sin^2 \theta - \tfrac{1}{4} \theta \cos^2 \theta \right]$$
$$+ \tfrac{3}{4} q \lambda r^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (115)$$

and,

$$C_1 = \frac{2 q r^3}{\pi} \left[\tfrac{5}{8} \sin \theta \cos \theta - \tfrac{3}{8} \theta - \frac{\pi}{2} \cos \theta + \frac{\pi}{2} \cos^2 \theta + \frac{\pi}{4} \sin^2 \theta - \tfrac{1}{4} \theta \cos^2 \theta \right.$$
$$\left. - \tfrac{1}{4} \sin \theta + \tfrac{1}{2} \theta \cos \theta - \tfrac{1}{4} \sin \theta \cos^2 \theta - \tfrac{1}{4} \sin^3 \theta \right] + \tfrac{1}{4} q \lambda r^3 \ldots (116)$$

When these values of C_1 and F_1 are substituted in Equations (111) and (112), general expressions are derived, that make it possible to calculate the bending moment at any point for any degree of fullness of the cylinder. Obviously, such calculations must be made with the greatest precision if the results are to have any value, because the expression for M represents the algebraic sum of a large number of nearly equal terms, and even a small error in these individual terms will produce a large relative error in the result.

In order to secure a verification of the main points of the foregoing discussion, namely, that there is no circumferential bending in the full cylinder, and that there is such bending in the partly filled cylinder, the writer made some tests which, although of a rather "rough and ready" nature, may be of interest. The results are shown in Fig. 19.

A specimen was constructed which consisted essentially of a circular pipe with closed ends; the diameter was 16 in.; the length, 8 ft.; the thickness

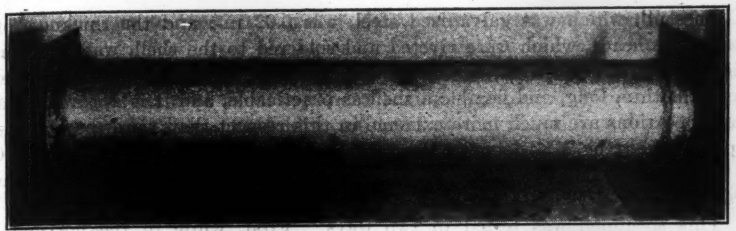

Fig. 20.—Circular Test Pipe, with Square Heads.

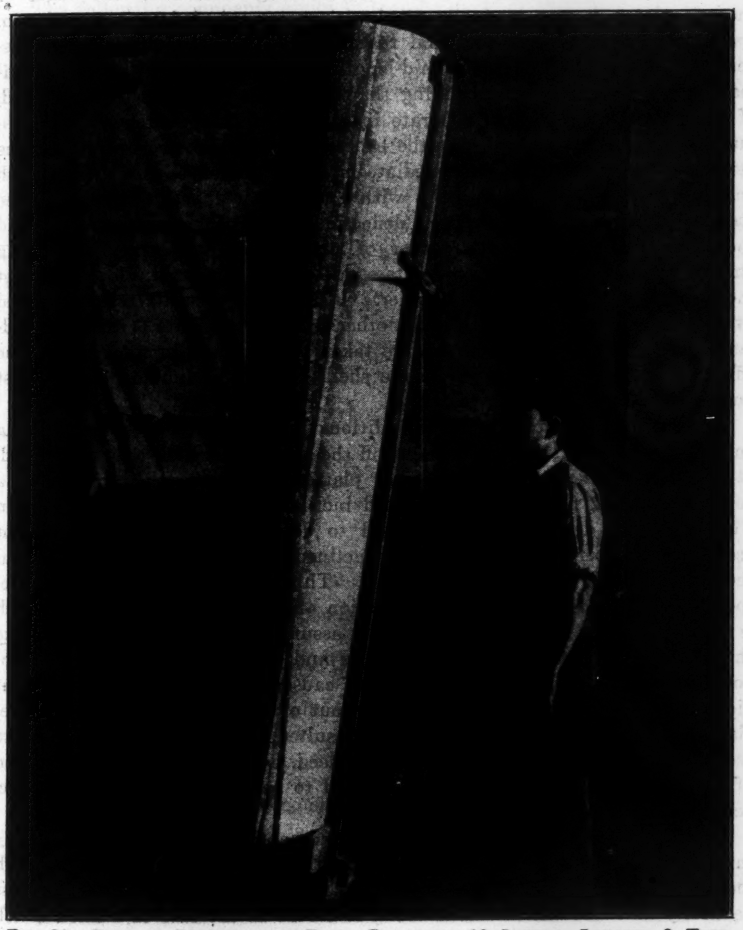

Fig. 21.—Pipe in Position for Test; Diameter, 16 Inches, Length, 8 Feet, Thickness of Shell, 0.02 Inches, and Thickness of Square Heads, 0.125 Inches.

of the shell, which was galvanized steel, was 0.02 in.; and the thickness of the square heads, which were riveted and soldered to the shell, was 0.125 in. Figs. 20 and 21 represent photographs of this specimen. It was intended to secure as thin, long, and flexible a shell as practicable, and it is apparent that the proportions are much more extreme in this respect than were those of the pipe described by Mr. Schorer, the ratio of diameter to thickness being 800 as against 44, and the ratio of unsupported length to diameter being 6 as against 3.7.

This specimen was tested in two ways. First, while in the horizontal position as shown in Fig. 20, the weight being carried by the edges of the end plates, it was gradually filled with water. A deflectometer of 4-in. span, provided with an Ames dial reading to 0.0001 in., was placed transversely on top of the vessel near mid-length, to detect and to measure approximately any circumferential bending that took place. The readings secured, of course, do not afford an accurate indication of the bending stresses produced, but if it is assumed that the bending of the 4-in. arc over which the deflectometer reached, was intermediate in form between that of a beam with end couples and that of a beam with end supports and center load, the stresses corresponding to a given deflection could be easily computed, and should represent roughly the order of magnitude of the actual bending stresses produced. Fig. 19 represents stresses thus computed, plotted against the degree of fullness of the cylinder. One curve represents results secured by measurements at mid-length, the other by measurements 6 in. from mid-length; the two sets of readings were taken to eliminate in some degree the effect of irregularities in form of the shell—flat spots, dimples, etc.—almost unavoidable in such a thin plate.

Second, in order to simulate conditions in a stand-pipe or tank during an earthquake shock, one end was cut off the specimen, which was then bolted, at the other end, to a heavy timber platform. It was then placed in a vertical position, filled with water, and inclined to the vertical as shown in Fig. 21. The deflectometer was used to detect any bending that might occur at the upper end, but even when inclined as steeply as 9° to the vertical the bending was practically negligible. This degree of inclination corresponds roughly to a horizontal acceleration of 5.1 ft. per sec. per sec., which is in excess of the acceleration ordinarily assumed in design when earthquake shock is taken into account. By raising and lowering one end of the supporting platform rapidly, the tank was caused to oscillate with sufficient violence to result in water being thrown out over the top, and yet no appreciable distortion of the cross-section resulted. No effort was made to ascertain accurately the acceleration produced, but from a rough observation of the amplitude and period it was judged to be at least 4 or 5 ft. per sec. per sec.

Although these tests were manifestly crude and the results unquestionably influenced by geometrical irregularities in the specimen and the very rough method used for measuring the stresses, they seem to show none the less definitely that circumferential bending is absent in the case of a filled vessel, and is present, but ordinarily negligible, in the case of a partly filled vessel. It is quite possible, however, that circumstances might occur in which the stresses in this latter case were large enough to be serious in the event of frequent reversal, as in the case of a paper digester or other rotating vessel. For pipes and tanks of usual proportions, however, these stresses would almost certainly be well below the endurance limit of the materials ordinarily used.

F. W. Hanna,[28] **M. Am. Soc. C. E.** (by letter).—The author has presented a useful technical analysis of the design of large pipe lines. The writer has checked this analysis with much interest, and agrees in general with the methods and results obtained.

The following comments apply to specific parts of the paper:

(1) In Equations (38) and (40) the moment, M_s, is negative rather than positive, as given by the author, because he has elsewhere assumed counter-clockwise moments to be negative and because the signs of moments are not determinable from the signs of either the arms or the forces involved.

(2) Equation (39) seems to be incorrectly stated and Equation (37) seems to be incorrectly used in the derivation of Equation (41). The writer suggests the following development.

Using the author's diagrams and nomenclature, the load on the pipe carried to one disk due to the pipe's weight and the water's weight for the condition when the pipe is filled just to the top,

$$Q = [\pi (r + t)^2 - \pi r^2] \frac{w L}{2} + q \pi r^2 \frac{L}{2} \text{ .... } (117)$$

or,

$$Q = \frac{\pi L}{2} [(2 r t + t^2) (w + q r^2)] \text{ .... } (118)$$

Since t^2 is small compared with $2 r t$, it may be dropped without serious error; then, multiplying by $\frac{\sin v}{\pi r}$,

$$\frac{Q \sin v}{\pi r} = \frac{L}{2} (2 t w + q r) \sin v \text{ .... } (119)$$

Applying the author's Equations (5) and (9) to Fig. 6 for the angle, v,

$$s = \frac{L}{2} (2 t w + q r) \sin v \text{ .... } (120)$$

Equating the values of the right members of Equations (119) and (120),

$$s = \frac{Q}{\pi r} \sin v \text{ .... } (121)$$

Since counter-clockwise moments are assumed to be negative, the moment, M_s, of the shearing force, s, is,

$$M_s = - \int_0^u b s r \, dv \text{ .... } (122)$$

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[28] Chf. Engr. and Gen. Mgr., East Bay Municipal Utility Dist., Oakland, Calif.

Also, from Fig. 6, it is clear that,

$$\cos (u - v) = \frac{r - b}{R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (123)$$

Transposing, solving for b, and expanding $\cos (u - v)$ in Equation (123),

$$b = r - R (\sin u \sin v + \cos u \cos v) \ldots\ldots\ \ldots\ldots (124)$$

Substituting the value of b from Equation (124) and the value of s from Equation (121) in Equation (122), and re-arranging,

$$M_s = \frac{Q\,R}{\pi} \int_0^u (\sin u \sin^2 v + \cos u \cos v \sin v - \frac{r}{R} \sin v)\, dv \ldots (125)$$

Intergrating Equation (125) between the limits of u and 0, the following final value of M_s is obtained,

$$M_s = \frac{Q\,R}{\pi} \left[\frac{u \sin u}{2} - \frac{r}{R} (1 - \cos u) \right] \ldots\ldots\ldots\ldots (126)$$

Thus, it is apparent that the error in the author's value for b in Equation (39) and his omission of the minus sign before the right-hand member of Equation (38) have cancelled one another and given the correct value of M_s in his Equation (41). The remaining results based on Equation (41) are apparently correct.

The author has developed equations for finding the total normal and bending stresses in the circular supporting disk to moment only. There are stresses due also to thrust and shear, and while those due to shear are generally very small, those due to thrust may be quite considerable. The total normal bending and shearing stresses involving the effect of rib-shortening, moment, and shear, may be found by the writer's method of arch dam analysis.[29]

In short, this method involves the principle of deflections through the application of a rigid bracket between the free end of the cut section and the elastic center. Redundant forces at this point replace the released forces, and also the dummy unit loads at the elastic center. The deflection formula when complete contains terms that provide: (1) For deflection due to rib-shortening; (2) for that due to bending moment; (3) for that due to shear; and (4) for that due to volume changes. For the problem under discussion the third and fourth terms of the formula may be omitted, but the first two terms should be used for accuracy. The author has used only the second term, that providing for deflection due to bending moments.

Paul Bauman,[30] M. Am. Soc. C. E. (by letter).—The ring support of large pipe lines, as outlined by the author, is a decided advance from a standpoint of statical clearness, as compared to the conventional saddle support. The reactions are concentrated and their points of application exactly defined. As the actions too are well established, the actual stress due to moments and shear in the continuous system necessarily must tally closely with the theoretical ones.

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[29] "The Design of Dams," by Messrs. Hanna and Kennedy, p. 227.

[30] Chf. Designer, Quinton, Code & Hill-Leeds & Barnard, Engrs. Consolidated, Los Angeles, Calif.

The maximum stresses in the pipe shell are likely to be higher with the ring supports than with the saddle supports of equal spacing, as in the latter case the distortion at the supports is less accentuated. The stress distribution in the latter case is too complex for an exact solution, however, and the types compare statically somewhat as a beam-and-girder slab and a flat-slab system of buildings.

Longitudinal shear is a maximum at the support where it combines with the longitudinal stress and the ring stress of the shell to a diagonal or principal stress. Referring to the author's numerical example, the shear, in pounds, on one side of the supporting ring is, $\frac{Q}{2} = \frac{317\,000}{2} = 158\,500$.

The maximum unit shear stress, s, due to $\frac{Q}{2}$, occurs in the plane of the neutral axis,

$$s = \frac{\frac{Q}{2} M_{sT}}{2\,t\,I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(127)$$

in which, M_{sT} is the statical moment of the upper half of the shell about the horizontal axis of symmetry, and I, the moment of inertia of the shell about the horizontal axis of symmetry; but $M_{sT} = \pi\, r^2\, t \frac{\sin u}{u} = 2\, r^2\, t$; and $I = \pi\, t\, r^3$. Substituting in Equation (21), the maximum unit shear stress, in pounds per square inch, is, $s = \frac{Q\, r^2\, t}{2 t^2 \pi r^3} = \frac{Q}{2\, r\, \pi\, t} = 3\,365$. This value is checked by the author's Equations (5) and (9).

The maximum rim stress for $x \neq 0$, and a rigid ring is given by Equation (29) for,

$$\frac{d(f_{bx})}{dx} = 0 = f_{bo} \sqrt{2} \left[-\frac{e^{-\frac{x}{z}}}{z} \cos\left(\frac{x}{z} + \frac{\pi}{4}\right) - \frac{e^{-\frac{x}{z}}}{z} \sin\left(\frac{x}{z} + \frac{\pi}{4}\right) \right]$$

or,

$$\sin\left(\frac{x}{z} + \frac{\pi}{2}\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots(128)$$

which is the case for $\frac{x}{z} = \frac{\pi}{2}$; $\frac{3\pi}{2}$; .. etc.; and, since, $z = 0.78 \sqrt{r\,t} = 3.025$, x for $f_{bx} \Big]_{\text{min.}}^{\text{max.}} = 4\tfrac{3}{4}$ in.; $14\tfrac{1}{4}$ in.; etc. For $x = 4\tfrac{3}{4}$ in.,

$$f_{bx} \text{ min.} = 1.82 \frac{pr}{} \sqrt{2} \left[\frac{1}{4.81} \times \frac{-\sqrt{2}}{2} = -0.379 \frac{pr}{t} \right] \ldots.(129)$$

and for $x = 14\tfrac{1}{4}$ in.,

$$f_{bx} \text{ max.} = \frac{4.81}{112} \times 0.379 \frac{pr}{t} = +0.0163 \frac{pr}{t} \ldots\ldots\ldots\ldots(130)$$

Introducing numerical values, namely, $p = 100 \frac{62.50}{144} = 43.30$ lb. per sq. in.; $r = 60$ in.; and $t = \frac{1}{4}$ in.; f_{bx} min. $= -3\,950$ lb. per sq. in.; and f_{bx} max. $= +170$ lb. per sq. in.

Neglecting the weight of the pipe, the ring stress at $x = 4\frac{3}{4}$ in. is not due to the full pressure, p, because that is partly carried longitudinally by beam action. To determine this beam load:

$$M_x = Ke^{-\frac{x}{z}} \cos\left(\frac{x}{z} + \frac{\pi}{4}\right) \ldots\ldots\ldots\ldots\ldots\ldots (131)$$

in which, $K = f_{bm} \frac{t^2}{6} \sqrt{2}$. Then,

$$\frac{dM_x}{dx} = Q_x = -\frac{Ke^{-\frac{x}{z}}}{z}\left[\sqrt{2} \sin\left(\frac{x}{z} + \frac{\pi}{2}\right)\right] = -\frac{Ke^{-\frac{x}{z}}}{z} \sqrt{2} \cos\frac{x}{z} \, .(132)$$

and,

$$\frac{d^2M_x}{dx^2} = q_x = -\frac{K\sqrt{2}}{z}\left[-\frac{e^{-\frac{x}{z}}}{z} \sin\frac{x}{z} - \frac{e^{-\frac{x}{z}}}{z} \cos\frac{x}{z}\right]$$

$$= +\frac{2K}{z^2} e^{-\frac{x}{z}} \sin\left(\frac{x}{z} + \frac{\pi}{4}\right) \ldots\ldots\ldots\ldots\ldots\ldots (133)$$

for $x = 0$, $q_x = p$, which checks; and for $x = 4\frac{3}{4}$ in.,

$$q_x = \frac{2 \times 1.82 \frac{pr}{t} \times \frac{t^2}{6} \sqrt{2}}{0.607\, r\, t} \left[\frac{1}{4.81} \frac{+\sqrt{2}}{2}\right] \ldots\ldots (134)$$

or, $q_x = 0.208\ p$. This gives a ring stress, $f_r = \frac{rp\,(1 - 0.208)}{t} = +8\,250$ lb. per sq. in. The unit shear stress, s, at $x = 4\frac{3}{4}$ in., based on $s = 0$ for $x = \frac{L}{2}$, is $s = \frac{12 \times 30 - 4.75}{360} \times 3\,365 = 3\,320$ lb. per sq. in.

In the plane of the neutral axis the longitudinal stress, $f_L = 0$. At $x = 4\frac{3}{4}$ in., therefore, $s = \pm 3\,320$ lb. per sq. in. $= \tau$; $f_{bx} = -3\,950$ lb. per sq. in. $= \sigma_x$; and $f_r = +8\,250$ lb. per sq. in. $= \sigma_y$.

The principal stresses are: $\sigma_{\text{max.}} = \frac{-3\,950 + 8\,250}{2} + \frac{1}{2}\sqrt{4\,(3\,320)^2 + 12\,200^2}$ $= +2\,150 + 6\,940 = +9\,090$ lb. per sq. in.; $\sigma_{\text{min.}} = -4\,790$ lb. per sq. in.; and $\tau_{\text{max.}} = \pm 6\,940$ lb. per sq. in.

If the unit shear stress, s ($= \pm 3\,365$ lb. per sq. in.), is combined with the maximum rim bending stress, f_{bo} ($= \pm 16\,580$ lb. per sq. in.), and if $\sigma_y = 0$: $\sigma_{\text{max.}} = 8\,290 + \frac{1}{2}\sqrt{4\,(3\,365)^2 + 16\,580^2} = +17\,240$ lb. per sq. in., which is still only 74% of $f_L + f_{bo}$, which occurs on top of the shell at the supports. The latter is the critical tensile stress which is economically met by thickening the pipe locally as suggested by the author.

Equation (29), properly modified, permits determining the elastic line of the shell, for example,

$$\frac{EI}{1-m^2}\frac{d^2y}{dx^2}=M=Ke^{-\frac{x}{z}}\cos\left(\frac{x}{z}+\frac{\pi}{4}\right)=K'\left[e^{-\frac{x}{z}}\cos\frac{x}{z}-e^{-\frac{x}{z}}\sin\frac{x}{z}\right]. \quad (135)$$

in which, $K'=1.82\,\frac{prt}{6}$. Furthermore,

$$\frac{EI}{1-m^2}\frac{dy}{dx}=K'\left[\int e^{-\frac{x}{z}}\cos\frac{x}{z}dx-\int e^{-\frac{x}{z}}\sin\frac{x}{z}dx\right]+C_1\ \ldots(136)$$

or,

$$\frac{EI}{1-m^2}\frac{dy}{dx}=K'z\left[e^{-\frac{x}{z}}\sin\frac{x}{z}\right]+C_1\ldots\ldots\ldots\ldots\ldots(137)$$

for $x=0$; $\frac{dy}{dx}=0$; and $C_1=0$. Also,

$$\frac{EI}{1-m^2}y=-\frac{K'z^2}{2}\left[e^{-\frac{x}{z}}\left(\sin\frac{x}{z}+\cos\frac{x}{z}\right)\right]+C_2$$

$$=-\frac{K'z^2}{2}\left[e^{-\frac{x}{z}}\sqrt{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right]+C_2\ldots\ldots\ldots\ldots\ldots(138)$$

for $x=0$; $y=0$; and $C_2=+\frac{K'z^2}{2}$. Thus,

$$\frac{EI}{1-m^2}y=\frac{K'z^2}{2}\left[1-\sqrt{2}\,e^{-\frac{x}{z}}\sin\left(\frac{x}{z}+\frac{\pi}{4}\right)\right]\ldots\ldots\ldots(139)$$

in which, $\frac{K'z^2}{2}=\frac{1.11\,pr^2t^3}{12}$; and $I=\frac{t^3}{12}$.

Making the proper substitutions in Equation (139),

$$Ey=\frac{pr}{t}\left[1-\sqrt{2}\,e^{-\frac{x}{z}}\sin\left(\frac{x}{z}+\frac{\pi}{4}\right)\right]\ldots\ldots\ldots\ldots(140)$$

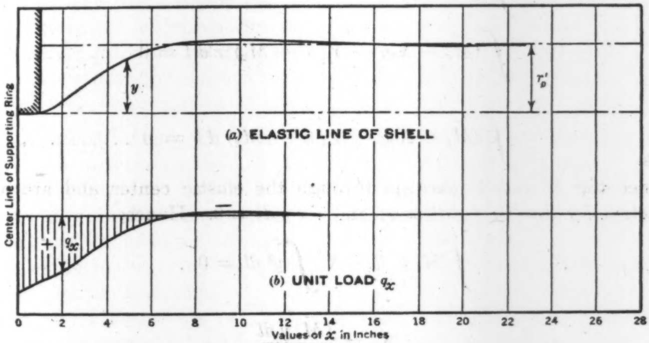

FIG. 22.

In using the numerical values as given in the author's example, a line as shown in Fig. 22, results. As x continues to grow, the term with $e^{-\frac{x}{z}}$ approaches zero and $E\,y$ approaches the ring distortion, r'_p. The loading, q_x, per unit area is shown in Fig. 22(b).

The line of pressure of the statically indeterminate supporting ring could readily be found graphically or analytically by means of the ellipse of elasticity, as previously shown by the writer.[31]

The problem is particularly simple, because the ellipse of elasticity in this case is a circle. Due to symmetry, it is sufficient to consider one-half only, either left or right. As a boundary condition, Reaction *A* or Reaction *B* must be tangent to the line of pressure.

A Check of Equations (31), (32), and (33).—The work of deformation is expressed by,

$$A = \int \frac{M^2}{2\,EI}\,dl \quad\text{.....................}(141)$$

and the condition of least work is,

$$\frac{\partial A}{\partial X_1} = \int \frac{M\left(\dfrac{\partial M}{\partial X_1}\right)}{EI}\,d\,l = 0\text{..............}(142a)$$

$$\frac{\partial A}{\partial Y_1} = \int \frac{M\left(\dfrac{\partial M}{\partial Y_1}\right)}{EI}\,d\,l = 0\text{..............}(142b)$$

and,

$$\frac{\partial A}{\partial M_z} = \int \frac{M\left(\dfrac{\partial M}{\partial M_z}\right)}{EI}\,d\,l = 0\text{..............}(142c)$$

Since $\dfrac{\partial M}{\partial X_1} = -\,y$; $\dfrac{\partial M}{\partial Y_1} = -\,x$; and $\dfrac{\partial M}{\partial M_z} = -\,1$:

$$\int (M_e - X_1 y - Y_1\,x - M_z)\,y\,d\,l = 0 \quad\text{............}(143a)$$

$$\int (M_e - X_1 y - Y_1\,x - M_z)\,x\,d\,l = 0 \quad\text{...........}(143b)$$

and,

$$\int (M_e - X_1 y - Y_1\,x - M_z)\,d\,l = 0 \quad\text{............}(143c)$$

Since the *X* and *Y* axes go through the elastic center and are axes of symmetry, $\int y\,dl = 0$; $\int x\,dl = 0$; and $\int x\,y\,dl = 0$. Hence,

$$\int M_e\,y\,dl - X_1 \int y^2\,dl = 0 \quad\text{...................}(144)$$

or,

$$X_1 = \frac{\int M_e\,y\,dl}{\int y^2\,dl} \quad\text{........................}(145a)$$

$$Y_1 = \frac{\int M_e x\,dl}{\int x^2\,dl} = 0\text{....................}(145b)$$

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[31] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), pp. 1672–1674.

and,

$$M_s = \frac{\int M_s\, dl}{\int dl} = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (145c)$$

which check Equations (31), (32), and (33).

The total tension in the supporting ring is given by Equation (30), or, in general terms, by Equation (19). This is only approximately correct as the ratio between the outside and inside diameters of these rings may vary quite considerably from unity. The tension is then no longer uniformly distributed over the cross-section, but has a maximum intensity on the inside, and a minimum on the outside, of the ring.

If r_i is the inside radius; r_o, the outside radius; and $p' = pc + 2\,H$; then,

$$f_i = p'\,\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} = p'\,\frac{r_o^2 + r_i^2}{(r_o + r_i)A_r} \;\ldots\ldots\ldots\ldots\ldots (146)$$

instead of, $f_s = \frac{p'\,r_1}{A_r} = 1\,020$ lb. per sq. in.

Let $pc + 2H = p' = \frac{A_r\,f_s}{r} = 204$ lb. per sq. in.; $r_i = 60$ in.; and $r_o = 72$ in. Then, the maximum tensile stress, using the same numerical values as before, is, $f_s = 204\,\frac{5\,200 + 3\,600}{1\,600} = 1\,125$ lb. per sq. in., or an increase of 10 per cent.

The author's formulas are also applicable to flanged pipes of small shell thickness.

For penstocks on steep inclines the weight component in the direction of the axis must be introduced. It produces a tensile stress if the pipe is anchored on top of the incline, which is a desirable arrangement. The expansion joint is then located either at the bottom of each run or immediately below the anchor point, depending on its type.

The elimination of uncertainties in the outer or inner stability of structures should be made an outstanding task by all designing engineers.

The author has again contributed to the clarification of statical systems and deserves the appreciation of his colleagues.

L. J. LARSON,[82] ASSOC. M. AM. SOC. C. E. (by letter).—This interesting paper gives a concise analysis of the action of a thin-walled large pipe supported by ring girders. It also indicates, to a striking degree, the advantage that may be gained by supporting a structure so as to carry the loads by direct, instead of bending, stresses. The dead-load stresses in a thin pipe improperly supported, may be greater than those due to the pressure for which the pipe is designed.

Referring to Equations (4), (5), and (6), it is interesting to note that all the unit stresses obtained by dividing each by the shell thickness, t, are independent of the thickness, since $\frac{hw}{t}$ is the weight per unit volume of the material in the pipe.

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[82] Research Engr., A. O. Smith Corporation, Milwaukee, Wis.

Taking the weight of steel as 0.283 lb. per cu. in., the maximum values for the unit stresses are: $\frac{T_2}{t} = 0.283\,r$; $\frac{S}{t} = 0.283\,L$; and $\frac{T_1}{t} = \frac{0.283}{r} \times \frac{L^2}{4}$.

The unit ring stress obtained from Equation (4) is particularly striking when it is compared to the bending stresses in a similar pipe without ring girders, supported continuously along the bottom. Using the values assumed by the author under "Example and Applications," the ring stress due to dead load is equal to: $\frac{T_2}{t} = 0.283 \times 60 = 16.98$ lb. per sq. in. If a pipe of the same dimensions were supported on a horizontal plane the maximum bending stress would be, approximately, $2.5\,\frac{r^2}{t}$, or 36 000 lb. per sq. in. While it is true that this bending stress can be reduced materially by supporting the pipe on a larger area, or along two lines at the ends of the horizontal diameter, still the stress will be proportional to $\frac{r^2}{t}$, and large, thin pipes will be subject to high stresses and considerable deformation.

In the derivation of stresses in the pipe and the supporting ring, the author assumes that the shearing stresses in the pipe are applied to the ring. It would be of interest to know to what extent the results are affected if the ring is attached to the pipe only at intervals and, also, if the ring is not attached but fits the pipe snugly. The amount of slip or relative movement between the pipe and the ring, necessary to relieve all the shearing stresses may be very small, and if such movement occurs the ring will act as if it were unattached. The action of a loose ring may not be serious unless it changes the entire state of stress in the pipe and invalidates the equations developed. If the author has made any analysis based on a loose ring support, the results will be helpful.

In the application of the author's equations to tanks it would also be of interest to know to what extent the usual type of convex, "dished" heads act as ring girders. Such heads are not as stiff as flat plates in the radial direction, but they will withstand a considerable radial load without collapsing. As far as the head itself is concerned it could probably be used as a disk or ring girder in supporting a tank, but, it is a question whether the small movements or deformations permitted in the shell by such a support are sufficient to change the action of the entire shell in resisting the dead and live loads. If the author can show that the theory holds for tanks with various types of heads, his analysis will find many additional applications.

C. M. ORR,[33] ESQ. (by letter).—Reference has frequently been made to the bending stresses in the rim zone of a pipe line due to restraint of the pipe at the support. It has been customary to say that these stresses "shall be considered in the design," or "cannot be neglected in the design," although cases of similar restraint may be pointed out in which the effects of the restraint are neglected. While the existence of these stresses, approaching 1.82 times the $\frac{pr}{t}$ cylinder stresses, is not questioned, it may be argued that they may be neglected as logically or safely in the case of a pipe shell as in the case of the following:

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[33] Asst. Research Engr., Chicago Bridge & Iron Works, Chicago, Ill.

(1) In the shells of vertical, flat-bottom storage tanks, near the connection to the bottom, the bending stresses approach the theoretical 1.82 $\frac{pr}{t}$ value; yet it is in accordance with standard specifications to consider only the $\frac{pr}{t}$ tension stresses in the design.

(2) In the shells of elevated, suspended bottom tanks, near the connection to the bottom, bending stresses are introduced, due to the restraint of a horizontal circular girder. This girder is necessary to resist the inward thrust from inclined tower posts, but its effect on shell stresses is commonly neglected, it being usual to consider only $\frac{pr}{t}$ tension stresses in the shell design.

(3) In the case of every pressure container composed of a cylindrical shell with closed ends, the shell is restrained to some extent by the ends, yet the design is based on stresses due to pressure.

While it is recognized that, in some respects, the foregoing examples are not identical with the pipe line, they are presented with the idea of suggesting that bending stresses due to restraint are not necessarily of prime importance in pipe-line design.

The combination of rim bending stresses with longitudinal stresses due to beam moments would affect only a small section of the pipe. The effect of overstressing the pipe by this combination would be some local change of shape affecting a small area, rather than failure of the pipe as a whole. If this change of shape were sufficient to cause a permanent distortion over this small area, it would be of such small extent and relatively small magnitude as to be negligible. Common precice permits not only the use of material previously stressed beyond its yield point, but it also permits working stresses in this material comparable with stresses in material which has not been previously distorted.

Complete elimination of all bending stresses in the rim zone due to restraint may be imagined. If it were practical to make the diameter of the ring equal to that of the pipe when expanded by full pressure, and then to stretch the shell plate at the ring to this increased diameter, leaving the remainder of the shell unchanged, no rim bending would occur under full load.

If it were possible to support the shell at the ring by radial members projecting from the shell through holes in the ring, in such a manner that expansion of the pipe under pressure were permitted, no rim bending stresses would occur.

Either of these suggestions would probably be impractical, but the writer mentions them with the idea of emphasizing the thought that they are also unnecessary. Because similar restraint stresses are safely ignored in other structures, and because of the limited area of pipe affected by such stresses, and the small possibility of serious distortion over this area, may not these stresses be neglected in large pipe lines?

Edward J. Bednarski,[34] Assoc. M. Am. Soc. C. E. (by letter).—The author has presented an interesting study of the reaction exerted by a thin-walled pipe ($\frac{r}{t}$ = 200 and more) on a support in the form of a rigid ring or a diaphragm when the pipe is horizontal and precisely full. In Conclusion 2 he states that,

> "If the pipe is supported by stiff, disk-shaped members * * *, the shell is entirely stable without any intermediate stiffening rings, thus permitting the use of large spans in combination with thin plates."

The "Complete Report on Construction of the Los Angeles Aqueduct" contains the statement that $\frac{1}{4}$ in. is the minimum thickness of steel plate that can be used for pipes as large as 10 ft. in diameter, and still maintain a satisfactory rigidity. This limit, it is claimed, gives satisfactory tensile strengths to sustain a head of as much as 144 ft.

Before it was filled, the Nine-Mile Siphon (designed diameter, 9.5 ft.), of the Los Angeles Aqueduct[35] measured 9 ft. vertically and 9.81 ft. horizontally. Under a pressure of 50 lb. per sq. in., these inside dimensions changed to 9.54 ft. and 9.58 ft., respectively.

The pipe was placed on concrete piers, 2 ft. wide, spaced 24 ft., center to center, and cast to fit the pipe. When two adjacent piers on a side hill, settled away from the pipe, the span was 72 ft. This pipe when filled remained suspended without injury until the defective supports could be replaced.

In another instance[36] a break occurred in a section of the aqueduct composed of a 10-ft. pipe, with $\frac{1}{4}$-in. and $\frac{5}{16}$-in. shells, crossing the Antelope Valley in the Mojave Desert region. The water escaped rapidly, under a head of 200 ft., and the pipe collapsed like an emptied fire hose for a distance of nearly 2 miles. At some points the top of the pipe was forced down to within a few inches of the bottom. After the break was mended the water was turned on and the pipe restored to its original form by pressure.

In the example given by the author the pipe has all the characteristics of the Los Angeles Aqueduct line, with a pressure of almost 50 lb. per sq. in. Therefore, it might possibly stand up for some time on similar piers, with a span of 60 ft.

The property of this kind of a pipe, when empty, to change its shape to another cross-section when under pressure, proved to be an asset. The thin-walled pipe cannot stand the bending stresses any more than the cables of a suspension bridge. It seems that under a liquid load of comparatively light specific gravity a thin-walled circular pipe with a comparatively high modulus of elasticity, just full, cannot assume any shape except that of a circle (regardless of the method by which the dead weight of the pipe and the liquid is supported), because a circle is a contour containing a maximum possible area for a given perimeter or a contour of a maximum expansion. As any other shape contains less area, a pipe in order to take this other shape would have to expel a part of the liquid it contains. If it is assumed that the water cannot escape from the pipe, the pipe may change the form of its cross-section to another form containing the same area only by elongation of its perimeter. For a very small elongation a distortion consequently should be insignificant. In the case of a thin-walled rubber pipe filled with mercury the shape of the pipe will change quite considerably with every particular method of support. It is probable, that for the ring-like support at the ends, the rubber pipe will retain a circular cross-section throughout its length.

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[34] La Verne, Calif.

[35] "Complete Report on Construction of the Los Angeles Aqueduct," Dept. of Public Service, Los Angeles, Calif., 1916, p. 195.

[36] *Loc. cit.*, p. 29.

Due to the flexibility of the pipe walls the bending stresses should be insignificant. Any additional internal pressure will add to the stability of the circular shape of the pipe and make it more nearly a perfect circle. In the case of a 10-ft. pipe, a head of 100 ft. makes a difference of only 10% between the pressures at the top and at the bottom, or 4.33 lb. per sq. in., as compared with the total of 43.3 lb. per sq. in. This is not enough to produce any considerable distortion. In the foregoing case the distortion is $\frac{1}{2}$ in. for 50-lb. pressure on a pipe 9 ft. 6 in. in diameter. It should be noted that the pipe in this case was supported by concrete piers "2 ft. wide, and only 3 or 4 ft. of circumference of the pipe was bearing on the pier."[37]

The supports of the Los Angeles Aqueduct type seem to have the following advantages:

1.—They do not require special reinforcement for the pipe at the pier due to the difference between the rigidity of the pipe and the supporting ring, causing the additional bending stresses near the ring.[38] Thus, the pipe may be made of plates with uniform thicknesses for its entire horizontal length.

2.—The location of the piers may be changed if necessary in the field without causing any inconvenience.

3.—Uniform thickness of the pipe wall insures the pipe from being damaged beyond recovery in case a sudden vacuum occurs in the line and the relief valves fail to act promptly and efficiently.

The disadvantage of the Los Angeles Aqueduct type of pier seems to be the necessity of using shorter spans of 24 ft., as against 60 ft. for the ring-girder supports. It would be interesting to know what would happen to the Antelope Pipe Line in the Mojave Desert region if it were supported by ring girders and piers.

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[37] "Complete Report on Construction of the Los Angeles Aqueduct," Dept. of Public Service, Los Angeles, Calif., 1916, p. 196.

[38] See "Applied Elasticity," by Timoshenko and Lessels, p. 146, Westinghouse Technical Night School Press, East Pittsburgh, Pa., 1925.

H. C. Boardman,[39] **M. Am. Soc. C. E.** (by letter).—As used by Mr. Schorer, the terms "exact theory," "exact stress condition," and "exact knowledge," are repugnant to the writer. "Exact theory" teases the mind for definition; "exact knowledge" of "exact stress conditions" is a goal constantly striven for, but seldom attained.

The author states that the governing design stresses in the shell of a pipe line are "derived," but his derivation consists essentially of a statement of the Bauersfeld differential Equations (1), (2), and (3), which are unfamiliar to American engineers, and their application to a simple beam with cylindrical section. The resulting equations for transverse shear and direct longitudinal stresses are shown to agree with those obtained by the ordinary theory of flexure applied to a simple, hollow, cylindrical beam. Is this, in the mind of the average reader, an indication of the validity of the Bauersfeld equations, or of the validity of the beam formulas? The transition to the case of multiple supports is made by the statement, "It can be shown that, in case of continuous pipe lines, the direct longitudinal stresses can also be derived from the theory of continuous beams." The implication is that the longitudinal stresses in continuous pipe lines have been derived by the use of the Bauersfeld equations alone, without the use of the theory of continuous beams. The writer is unable to make this derivation. Partial liquid loadings, to which the Bauersfeld equations are presumably not applicable, because the surface loads are discontinuous, are treated by a rather cursory reference to a paper[40] written by Mr. A. Frey Samsioe covering half-full liquid loading.

The writer feels that the author would have accomplished his purpose more directly, and to the greater satisfaction of American engineers, if he had omitted Equations (2) and (3) and simply stated, as in effect he has, that:

(*a*) The shell of a cylindrical pipe line full of liquid, and kept essentially round at two or more supports, acts as a memorane free from bending stresses, except for a narrow zone near the supports.

(*b*) Under the conditions of Statement (*a*), the shearing stresses on a transverse section of the shell are necessarily tangential, and are distributed in accordance with the ordinary theory of flexure as applied to a hollow, cylindrical beam.

(*c*) Under the conditions of Statement (*a*), the longitudinal shell stresses may be obtained by an application of the ordinary beam formulas.

With Statement (*a*) granted, Equations (1), (4), (8), (11), and (12) follow directly from the equations of statics expressing the equilibrium of inward and outward radial forces on a differential element of the shell.

Still more convincing, in the writer's opinion, would have been an application of the well-known Mueller-Breslau Equations (31), (32), and (33), to an analysis of the dead load and the full liquid load stresses in a ring cut out of the pipe some distance from the supports by two transverse planes, 1 in. apart—the ring being held in equilibrium by tangential shearing forces distributed as in a hollow, cylindrical beam, by vertical dead weight forces, and by radial pressure forces. Such an analysis leads very simply to Equations (4) and (8), and to zero bending stresses.

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[39] Research Engr., Chicago Bridge & Iron Works, Chicago, Ill.

[40] "Die Spannungen in Einem auf Mehreren Stützen in gleicher, gegeseitiger Entfernung Aufgelegten und zur Hälfte mit Wasser Gefüllten Rohr," by A. Frey Samsioe, Stockholm, 1926.

Mr. Schorer has done a great service in calling attention to the fact that the shearing stresses on a ring between two transverse planes are tangential. Other investigators, the writer among them, have fallen into the error of considering only the vertical components of the shearing forces on the edges of the ring. A little consideration will disclose the fact that the shearing forces must be tangential to satisfy the laws of statics as applied to a differential arc of the ring.

It is unfortunate that the author treats so lightly the conditions of partial liquid loading. It is feared that many engineers who, like the writer, do not read German, will fail to obtain a translation of Mr. Samsioe's paper, and, as a result, will harbor a suspicion that the full liquid loading emphasized by the author is not critical. For this reason the following analysis of ring stresses in a pipe half full of liquid is offered in the hope that, in combination with a test to be discussed later, it will help others, as it has helped the writer, to gain a clearer idea of the true stress conditions. The notation is essentially that of the author; departures therefrom are self-explanatory.

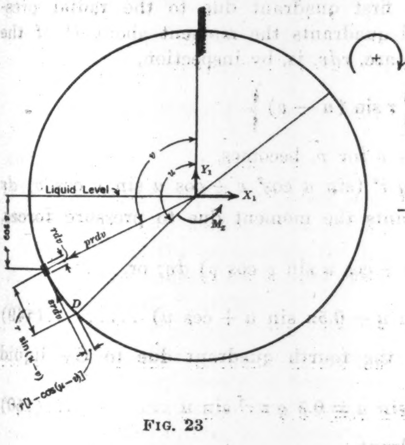

FIG. 23

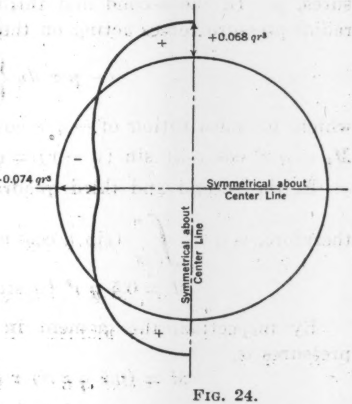

FIG. 24.

The ring chosen is 1 in. wide longitudinally, and the transverse plane of one edge is midway between two pipe supports. The shearing stresses are first assumed to be distributed in accordance with the ordinary theory of flexure as applied to hollow, cylindrical beams, their intensities being one-half those for full liquid loading. Later, it is shown that a different shear distribution would result in practically zero moments throughout the ring, a condition which seems to correspond with the observed behavior of an experimental pipe section subsequently described.

In Fig. 23, $p = 0$ above the liquid, and below the liquid surface, $p = -q\,r \cos v$. Furthermore, $s = 0.5\,q\,r \sin v$, which is one-half the value of s for full liquid load. The forces, s and p, represent the external load system, which is symmetrical about the vertical diameter. For this reason, Y_1 is zero. The moment at the point, D, caused by the tangential shearing force on the arc, rdv, is, by inspection of Fig. 23:

$$M_D = s\,r\,dv\left\{r\,[1-\cos(u-v)]\right\}$$

which, by substitution of $0.5\,q\,r\sin v$ for s, becomes,

$$M_D = 0.5\,q\,r^2\sin v\,dv\left\{r\,[1-\cos(u-v)]\right\}$$

or,

$$M_D = 0.5\,q\,r^3\,(\sin v - \cos u\sin v\cos v - \sin u\sin^2 v)\,dv$$

In all quadrants the moment due to shearing forces, therefore, is,

$$M = 0.5\,q\,r^3\int_0^u(\sin v - \cos u\sin v\cos v - \sin u\sin^2 v)\,dv \text{ . (147)}$$

which by integration becomes,

$$M = 0.5\,q\,r^3\,(1-\cos u - \tfrac{1}{2}\,u\sin u)\ldots\ldots\ldots\ldots(148)$$

There are no moments in the first quadrant due to the radial pressures, p. In the second and third quadrants the moment about D of the radial pressure forces acting on the arc, rdv, is, by inspection,

$$-\,p\,r\,dv\left\{r\sin(u-v)\right\}$$

which, by substitution of $-\,q\,r\cos v$ for p, becomes,

$$M_D = q\,r^2\cos v\,dv\sin(u-v) = q\,r^3\,(\sin u\cos^2 v - \cos u\sin v\cos v)\,dv$$

In the second and third quadrants the moment due to pressure forces, therefore, is $q\,r^3\int_{0.5\pi}^{u}(\sin u\cos^2 v - \cos u\sin v\cos v)\,dv$; or,

$$M = 0.5\,q\,r^3\,(u\sin u - 0.5\pi\sin u + \cos u)\ldots\ldots\ldots(149)$$

By inspection, the moment in the fourth quadrant due to the liquid pressures is,

$$M = (0.5\,q\,\pi\,r^2)\,r\sin u = 0.5\,q\,\pi\,r^3\sin u\ldots\ldots\ldots\ldots(150)$$

Summarizing: In the first quadrant,

$$M_{e1} = 0.5\,q\,r^3\,(1-\cos u - 0.5\,u\sin u)\ldots\ldots\ldots\ldots(151)$$

in the second and third quadrants,

$$M_{e23} = 0.5\,q\,r^3\,(1-\cos u - 0.5\,u\sin u)$$
$$+\,0.5\,q\,r^3\,(u\sin u - 0.5\,\pi\sin u + \cos u)\ldots\ldots\ldots\ldots(152)$$

and, in the fourth quadrant,

$$M_{e4} = 0.5\,q\,r^3\,(1-\cos u - 0.5\,u\sin u) + 0.5\,q\,r^3\,(\pi\sin u)\ldots(153)$$

Equation (34) then becomes,

$$X_1 = \frac{1}{\pi\, r}\left[\int_0^{\frac{\pi}{2}} M_{e1} \cos u\, du + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} M_{e23} \cos u\, du \right.$$

$$\left. + \int_{\frac{3\pi}{2}}^{2\pi} M_{e4} \cos u\, du \right] \ldots\ldots\ldots\ldots\ldots\ldots (154)$$

from which,

$$X_1 = -\tfrac{1}{4}\, q\, r^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (154)$$

Equation (36) becomes,

$$M_Z = -\frac{1}{2\pi}\left[\int_0^{\frac{\pi}{2}} M_{e1}\, du + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} M_{e23}\, du + \int_{\frac{3\pi}{2}}^{2\pi} M_{e4}\, du\right] \ldots (155)$$

from which,

$$M_Z = q\, r^3 \left(\frac{1}{\pi} - \frac{1}{2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (156)$$

By combining M_z and the moments due to X_1, with the M_e moments expressed by Equations (151), (152), and (153), the following ring moments are obtained,

$$M_1 = q\, r^3 \left(\frac{1}{\pi} - \frac{1}{4} \cos u - \frac{1}{4}\, u \sin u\right) \ldots\ldots\ldots\ldots (157)$$

$$M_2 = M_3 = q\, r^3 \left(\frac{1}{\pi} + \frac{1}{4} \cos u - \frac{\pi}{4} \sin u + \frac{1}{4}\, u \sin u\right) \ldots (158)$$

and,

$$M_4 = q\, r^3 \left(\frac{1}{\pi} - \frac{1}{4} \cos u + \frac{\pi}{2} \sin u - \frac{1}{4}\, u \sin u\right) \ldots\ldots (159)$$

A study of the expressions for Equations (157), (158), and (159) will show that the moment curves are symmetrical about both the horizontal and vertical axes (see Fig. 24).

Attention is called to the fact that the moments in Fig. 24 are those which would exist if the transverse shears were distributed in accordance with the ordinary theory of flexure as applied to hollow, cylindrical beams. They are of considerable magnitude; for example, if applied to the pipe line chosen by the author to illustrate the application of his formulas, the moment of 0.074 $q\, r^3$ for a half-full water load would result in a calculated bending stress of $\dfrac{0.074 \times 0.362 \times 60^3}{\frac{1}{6} \times \left(\frac{1}{4}\right)^2}$ = 55 000 lb. per sq. in., indicating large deformations and over-stress while the pipe is being filled.

The writer believes no such over-stress or deformation would actually occur. He has recently directed the testing of a copper cylinder, 22½ in. in diameter and 68 in. long, made of No. 24 gauge material. A wooden disk was closely fitted into each end and the cylinder was supported on four vertical wooden legs, two nailed to each disk, as shown in Fig. 25.

The cylinder was slowly filled with water, and measurements were made at the mid-section to determine the changes in shape. The recorded variations were very small, and not perceptible to the eye, although the calculated

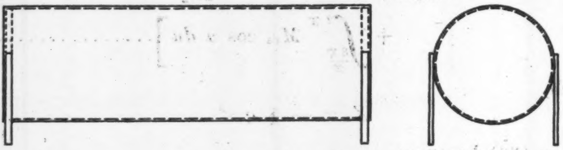

FIG. 25.

bending stress due to a moment of $0.074\ q\ r^3$ is $\frac{0.074 \times 0.0362 \times 12.25^3}{\frac{1}{6} \times (\frac{1}{40})^2}$ = 36 000 lb. per sq. in., probably more than the ultimate strength of the copper. As a check the filling and measuring were done three times, always with essentially the same result. The ratio of diameter to thickness was 900. The shells of large oil-storage tanks show a similar remarkable resistance to deformation under wind loads, if they are kept round at the top by suitable circular girders.

Only one explanation seems possible. The transverse shears must be distributed so as to result in practically no moments in rings cut out by transverse sections. A clue to the approximate magnitude and direction of the shearing forces which must be applied to a ring acted upon by shearing stresses distributed in accordance with the ordinary theory of flexure, in order to result in zero moments, may be gained by an inspection of Fig. 24. The moment graph is the general shape the ring would take if the indicated moments actually existed. It is apparent that four tangential forces acting as shown in Fig. 26 would tend to pull the ring into its original circular shape.

Assuming each of these forces to be unity, the resulting moments through the ring can be found as follows, using the Mueller-Breslau equations. Since the load system is symmetrical about the vertical diameter, Y_1 is zero. The moment of the force at A about any point, P, if $u > \frac{\pi}{4}$, is, by inspection, $-\left[r\left\{1-\cos\left(u-\frac{\pi}{4}\right)\right\}\right]$, or,

$$M = r\,(-1 + 0.707 \cos u + 0.707 \sin u) \ldots\ldots\ldots (160)$$

Similarly, the moment of the force at B about any point P, if $u > \frac{3\pi}{4}$, is,

$$M = r\,(1 + 0.707 \cos u - 0.707 \sin u) \ldots\ldots\ldots (161)$$

the moment of the force at C about any point, P, if $u > \frac{5\pi}{4}$, is,

$$M = r\,(-1 - 0.707 \cos u - 0.707 \sin u) \ldots\ldots\ldots (162)$$

and the moment of the force at D about any point, P, if $u > \frac{7\pi}{4}$, is,

$$M = r\,(1 - 0.707 \cos u + 0.707 \sin u) \qquad (163)$$

Combining: For values of u (Fig. 26), between 0° and 45° and 315° and 360°,

$$M_e = 0 \qquad (164)$$

for values of u between 45° and 135°,

$$M_{ea} = r\,(-1 + 0.707 \cos u + 0.707 \sin u) \qquad (165)$$

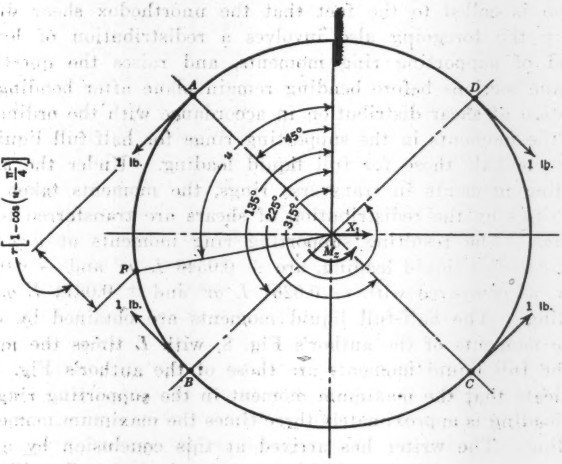

FIG. 26.

for values of u between 135° and 225°,

$$M_{eb} = r\,(1.414 \cos u) \qquad (166)$$

and, for values of u between 225° and 315°,

$$M_{ec} = r\,(-1 + 0.707 \cos u - 0.707 \sin u) \qquad (167)$$

By inspection, $X_1 = 0.707$ and Equation (36) becomes,

$$M_Z = -\frac{1}{2\pi}\left[\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} M_{ea}\,du + \int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} M_{eb}\,du + \int_{\frac{5\pi}{4}}^{\frac{7\pi}{4}} M_{ec}\,du\right] \qquad (168)$$

from which, $M_Z = 0.5\,r$.

By combining M_z in Equation (168), and the moments due to X_1 with the moments given by Equations (165), (166), and (167), ring moments of $+ 0.207\,r$, $+ 0.207\,r$, $- 0.207\,r$, and $+ 0.207\,r$, are obtained for points, $u = 0°$, $u = 90°$, $u = 180°$, and $u = 270°$, respectively. The corresponding moments of Fig. 24 are, $+ 0.068\,qr^3$, $- 0.074\,qr^3$, $+ 0.068\,qr^3$, and $- 0.074\,qr^3$, respectively, for which the arithmetical mean is $0.071\,qr^3$. It follows that a

shearing force at A, B, C, and D, of $\frac{0.071}{0.207} qr^2$, or 0.343 qr^2, in addition to the shearing forces of Fig. 23, will result in practically zero moments throughout the ring.

It is not claimed that concentrated shearing forces of 0.343 qr^2 exist at the points, A, B, C, and D of Fig. 26. The actual forces are doubtless distributed over the entire circle according to a law for which the writer has not obtained a mathematical expression. However, the foregoing crude demonstration serves to reveal the nature of the shear distribution necessary to result in practically zero ring moments under half-full liquid loading.

Attention is called to the fact that the unorthodox shear distribution suggested in the foregoing also involves a redistribution of longitudinal stresses and of supporting ring moments, and raises the question as to whether plane sections before bending remain plane after bending. Under the assumption of shear distribution in accordance with the ordinary theory of flexure, the moments in the supporting rings for half-full liquid loading are exactly one-half those for full liquid loading. Under the assumption of no bending moments in transverse rings, the moments taken from the transverse rings by the redistribution of shears are transferred to the supporting rings. The resulting supporting ring moments at $u = 0°$ and $u = 90°$, for half-full liquid loading, are $+ 0.0546\ L\ qr^3$ and $- 0.0897\ L\ qr^3$, respectively, as compared with $- 0.0267\ L\ qr^3$ and $\pm\ 0.0314\ L\ qr^3$, for full liquid loading. The half-full liquid moments are obtained by combining 50% of the moments of the author's Fig. 8, with L times the moments of Fig. 24; the full liquid moments are those of the author's Fig. 8. These results indicate that the maximum moment in the supporting ring for half-full liquid loading is approximately three times the maximum moment for full liquid loading. The writer has arrived at this conclusion by a combination of observation and logic. If his reasoning is faulty, he will appreciate correction.

Mr. Schorer is to be congratulated on his effort to rationalize pipe design. His paper is instructive and stimulating. However, he has thrown himself open to the charge of concentrating on the condition of loading for which the stress analysis is simplest, namely, full liquid loading. For this reason his paper should be supplemented by further investigation, both analytical and experimental, to establish, conclusively, the stress conditions in supporting rings and cylinder for all partial liquid loadings.

R. L. TEMPLIN,[41] M. AM. SOC. C. E., AND R. G. STURM,[42] ASSOC. M. AM. SOC. C. E. (by letter).—The author is to be commended for bringing before the Engineering Profession so vital a subject. His paper presents one method of supporting a thin-walled cylinder so that the stresses set up in the shell are not serious even when spans greater than those in common practice are used. This, however, should not be construed as the only method of supporting a pipe to obtain small distortions and low stresses.

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The writers agree with the fundamental principles and basic assumptions set forth, but feel that several questions which have a direct bearing on the usefulness of the theory might well be clarified. The first question is in regard to the eccentricity, *a*, of the reaction which is the first term defined in the "Notation." Referring to Fig. 6 and the subsequent analysis, it seems that the distance, *a*, should be defined as measured from the neutral axis of the ring. The location of the neutral axis of the ring and shell requires some knowledge or assumptions as to how much of the shell acts with the stiffener, because the pipe shell itself will tend to act integrally with the ring to a certain degree. In the illustrative example given by the author, the variation in the location of the neutral axis may be more than 100% of the computed eccentricity of reaction. As no measured values are given of stresses that actually exist in a pipe so stiffened and so supported, deviations of the theoretical stresses from the actual stresses are not known.

It is interesting to note that the two radii, r and R, both occur in the first set-up of the equations, but the final formulas contain only R, thereby eliminating the effect of the relative size of stiffener and pipe. The writers believe that neglecting this effect is a justifiable approximation in the practical cases, although it does not give an exact theory.

Another question which may be raised is: "How does the example of a pipe, supported as in the case of the Glines Canyon pipe, prove or disprove the theory when it does not appear to comply with the assumptions made in the analysis?" In the first place, the supports of the Glines Canyon pipe are riveted securely to the stiffener ring over a considerable distance extending from the horizontal diameter to a point well below it. Therefore, the point of support is not definitely at the ends of the horizontal diameter nor at any fixed point with reference to the center line of the girder; nor is it supported on knife-edges as assumed in the analysis of the paper. The writers would like to ask, therefore, "Does the fact that this pipe stands without appreciable distortion or without developing serious stresses, necessarily indicate the validity of the analysis presented in the paper?" Apparently, it does show that a pipe reinforced by stiffeners and supported on strut supports has appreciable advantages over pipe supported by other means.

Early in 1928, one of the writers was called upon to make extensive field tests of the Santeetlah Pipe Line which was part of one of the hydro-electric power developments of the Tallassee Power Company in Western North Carolina. This line, including tunnels, is about 5 miles long, and was fabricated from steel plates varying in thickness from $\frac{3}{8}$ in. to $\frac{1}{2}$ in. so as to form a circular conduit having an inside diameter of 11 ft. The pipe is supported on saddles at intervals of about 30 ft. These saddles were fabricated from steel plates and angles so as to extend over an arc of 90° of the pipe shell, and, in turn, are supported on concrete piers or steel trestle bents, depending on the topography. The pipe was stiffened at the supports with 4 by 6 by $\frac{3}{8}$-in. angles riveted to the shell circumferentially, with the longer leg outstanding. The saddles were spaced so as to be adjacent to or framed into one of the stiffening rings.

Tests were made to determine the change in the stresses at critical points in the pipe shell when it was subjected to various water loads up to the maximum load. The places at which the stresses were measured included various points on the stiffening rings, stresses in different directions in the pipe shell around the ends of the saddles and adjacent to the stiffeners, and at various points around the entire circumference of the pipe at mid-span and near the supports. The deformation of the shell from the initial circular section was also determined.

The results of these first field tests showed that when the pipe was almost full of water high stresses occurred in both the saddles and the stiffening rings at certain critical points, namely, in the shell near the ends of the saddles, in the saddle stiffener rings at the top of the vertical diameter, and at both ends of the horizontal diameter. The amount of stiffening ring steel was increased, therefore, by arc welding additional angles to those already on the pipe so as to form a Z-section. These added angles were 4 by 4 in. by $\frac{3}{8}$ in., and were attached to the others so that the final depth of the stiffening ring was about 7 in.

After the completion of this reinforcing work, the pipe was again tested with more extensive measurements than previously. These later tests showed that the high stresses found in the first tests, were decreased substantially to acceptable safe working values. Strengthening the stiffener rings automatically relieved the high stresses in the saddles and in the pipe shell.

Although this pipe line had been designed according to a theory that gave satisfactory results on smaller sizes yet deficiencies in the design theory became apparent when it was applied to lines of the size of that just described. Accordingly, an extensive research was carried out on the design of pipe lines and their supports. This research included tests of models, together with a development of a theory of design which differs from that given by Mr. Schorer.

The models were built to approximately one-tenth the scale of the Santeetlah Pipe Line, and were made of aluminum alloy sheet, so as to magnify the strains and distortions. The models were supported on saddle supports which were in contact with the pipe shell over distances of 66°, 90°, and 180° of arc. These saddles were arranged so as to give a series of spans varying from 3 to 15 diameters, the total length of the model pipe being 15 diameters.

Since models of such size preclude any extensive measurement of localized strains it was decided to depend for the most part on the deformation or distortion of the pipe shell as an index of such strains and hence stresses resulting from various water loads. Strains were actually determined, however, with $\frac{1}{2}$-in. "tensometers," at some points in the models.

Fig. 27 shows a sketch of one of the models in position for test with supports at intervals of 3 diameters. Fig. 28 shows typical distortion curves for an unstiffened pipe on three kinds of saddles. The maximum distortion was found to occur when the pipe was just full of water, but under no pressure.

It was found that the effects of the saddles were quite independent of the length of span and that the effects of the span were nearly independent of the type of saddle.

In view of these results more models were built and tested, using different types of stiffeners. One type was a series of bands of heavier gauge than the material in the shell, varying in width from one-fourth to three-fourths the diameter of the pipe. The main shell was cut for the space occupied by the bands, except for the metal required to make the lap-joints. The bands were bolted to the pipe with small machine screws (No. 2), spaced approximately 1 in. on centers.

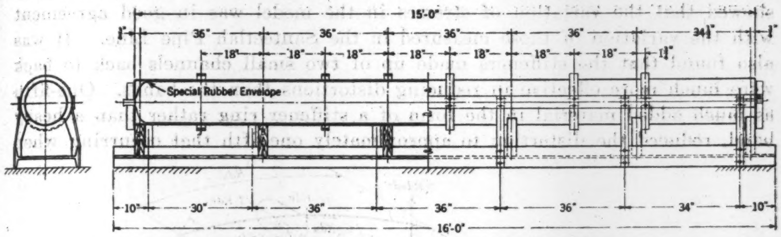

FIG. 27.—GENERAL ARRANGEMENT OF MODEL PIPE LINE AND SUPPORTS.

Another type of stiffener was made up of two channels, which were placed back to back at the joints of the pipe. The pipe was arranged so that the saddles came adjacent to one side of the stiffener rings thus formed. The types of saddles and spans were the same as before. Stresses were measured with "tensometers" attached to the stiffeners as shown subsequently in Fig. 33.

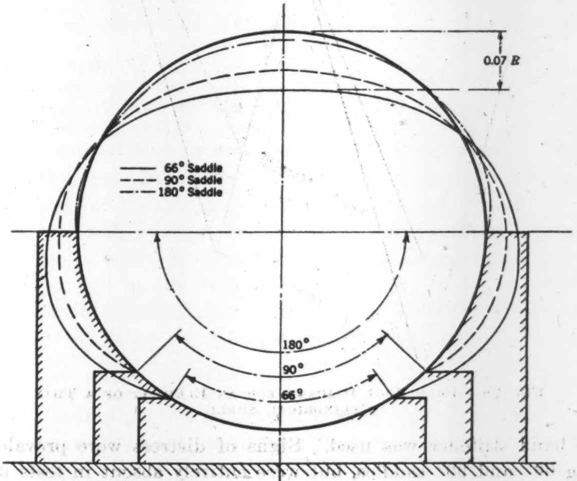

FIG. 28.—DISTORTION OF UNSTIFFENED PIPE SHELLS USING SADDLE TYPE SUPPORTS AND A SPAN OF THREE DIAMETERS.

It was found that the distortion of the pipes, stiffened with the bands at the supports, was very little less than the distortion of the unstiffened pipe, and was found to vary over wide limits because of the indefinite connection between the bands and the shell by means of the bolts used. However, in cases where the bolts apparently held, the distortion varied inversely with the moment of inertia of the combined shell and band. The points of distress and points of contraflexure were found to be the same as in the prototype, and the points of distress as determined by stress measurements on the stiffeners showed that the variation of stresses in the model was in good agreement with the variation of those measured in the Santeetlah Pipe Line. It was also found that the stiffeners made up of two small channels back to back were much more effective in reducing distortions than the bands. One-fifth as much added material in the form of a stiffener ring rather than a heavy band, reduced the distortion to approximately one-fifth that occurring when

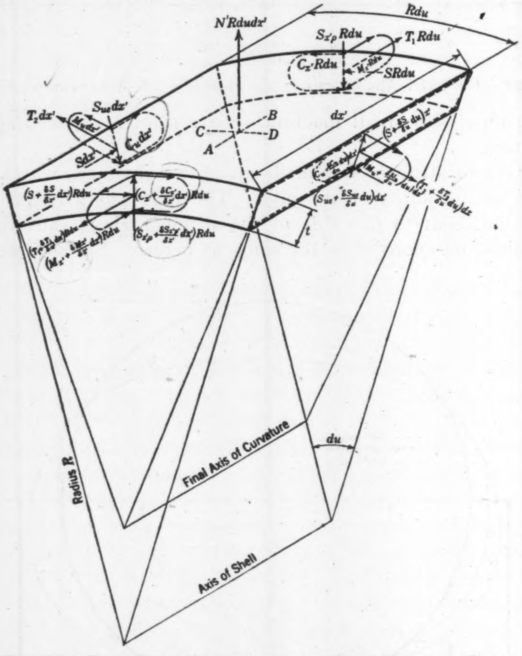
Rdu
N'Rdudx'
$S_{x'p}Rdu$
T_1Rdu
$C_{x'}Rdu$
$M_{x'}Rdu$
SRdu
$S_{ue}dx'$
T_2dx'
M_udx'
C_udx'
Sdx'
B
C
D
A
dx'
$(S+\frac{\delta S}{\delta u}du)x'$
$(S+\frac{\delta S}{\delta x'}dx')Rdu$
$(C_{x'}+\frac{\delta C_{x'}}{\delta x'}dx')Rdu$
$(C_u+\frac{\delta C_u}{\delta u}du)dx'$
$(M_u+\frac{\delta M_u}{\delta u}du)dx'$
$(T_2+\frac{\delta T_2}{\delta u}du)dx'$
$(S_{ue}+\frac{\delta S_{ue}}{\delta u}du)dx'$
$(T_1+\frac{\delta T_1}{\delta x'}dx')Rdu$
$(M_{x'}+\frac{\delta M_{x'}}{\delta x'}dx')Rdu$
$(S_{x'p}+\frac{\delta S_{x'p}}{\delta x'}dx')Rdu$
t
Final Axis of Curvature
du
Radius R
Axis of Shell

FIG. 29.—FREE BODY DIAGRAM FOR AN ELEMENT OF A THIN CYLINDRICAL SHELL

that wide band stiffener was used. Signs of distress were prevalent in the tests using 66° and 90° saddles, but were entirely absent in tests using 180° saddles. The general conclusions might be drawn that the use of 180° saddles would be entirely safe, although prohibitively expensive for general use.

With these data as a background, it was possible to develop a theory of the action of a horizontal pipe line, just filled with liquid, supported on saddles which would give reliable indications of stresses and distortions of stiffened or unstiffened pipe shells supported on various types of saddles. The general differential equation expressing the equilibrium of any elemental section of the pipe shell was derived from a consideration of a free body diagram of such a small portion. Fig. 29 is a sketch of a small element of a pipe shell showing all the forces acting on that small part. From a consideration of equilibrium of such a small particle, a general differential equation involving the forces acting on such an element is obtained. Then, assuming that, since the shell is thin, ordinary flexural formulas hold, the equations of continuity may be combined with the differential equation of equilibrium so as to give a general differential equation in terms of the distortion, ρ, of the shell. This equation was found to be:

$$\frac{EI}{R^4\,(1-m^2)}\left[\frac{\partial^4 \rho}{\partial u^4} + R^2\,\frac{\partial^2 \rho}{\partial u^2} + 2\,R^2\,\frac{\partial^4 \rho}{\partial u^2\,\partial\,(x')^2} + m\,R^4\,\frac{\partial^2 \rho}{\partial\,(x')^2} + R^4\,\frac{\partial^4 \rho}{\partial\,(x')^4}\right]$$

$$= -N' - \frac{T_2}{R_2}\left(R - \frac{\partial^2 \rho}{\partial u^2}\right) - T_1\,\frac{\partial^2 \rho}{\partial\,(x')^2}\;\ldots\ldots\ldots\ldots(169)$$

in which,

x' = distance along the pipe from the center line to the point considered;
ρ = change in radius at any point;
R = mean radius of the pipe shell;
N' = intensity of the hydrostatic pressure at any point;
E = modulus of elasticity of the material in the shell;
$I = \frac{t^3}{12}$ = moment of inertia of a cross-section of the shell of unit length;
t = thickness of the shell;
m = Poisson's ratio for the material in the shell;
T_2 = tangential pull, in pounds per linear inch, circumferentially;
T_1 = tangential pull, in pounds per linear inch, longitudinally.

In connection with Fig. 29 it should be noted also that ρ is positive outward; x' is positive toward the reader; u is positive clockwise; and, M is positive when it tends to cause tension in the outer surface.

The general solution of Equation (169) involves work very similar to that carried through in laborious detail by Mr. A. Frey Samsioe.[43] In view of the experimental data, certain simplifying assumptions were made with the result that the following equation was derived:

$$S = -\,qRx'\sin u\ldots\ldots\ldots\ldots\ldots\ldots(170)$$

in which, q = weight per cubic foot of the liquid in the pipe and u = angular position at any point. The meaning of the other terms are as defined for Equation (169).

Equation (170) has been derived by a different line of reasoning by D. Thoma.[44] In passing, it would be well to point out that in the mathematical work involved in deriving Equation (170) it was found that, of all the possible types of solutions for the general differential equation, a series of the form, $\rho = A_0 + A_2 \cos 2\theta + A_4 \cos 4\theta$ * * *, fulfills the conditions if A_0, A_2, and A_4 are considered as functions of x'. Considering only the first two terms of the series, one would conclude that the general shape of the distorted pipe with the ends fixed would tend to be that of a cosine curve. This would mean that the distortions of a shell with perfectly fixed ends would be of the same general shape as the measured distortions of the shell tested (see Fig. 28).

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[43] "Die Spannungen in einem auf mehreren Stützen in gleicher gegenseitiger Entfernung aufgelegten und zur Hälfte mit Wasser gefüllten Rohr," by A. Frey Samsioe, Ingeniörs Vetenskaps Akademien, Handlingar Nr. 50, 1926.

[44] *Zeitschrift* für das Gesamte Turbinenwesen, February 20, 1920.

In the models and in the practical case of the Santeetlah Pipe Line the distortions at the support are very large compared with those of a "fixed ended" shell. Hence, the distortions at the supports were considered as the controlling factor. Using Equation (170) as the distribution of shearing forces on the cross-section of the shell, it seems reasonable that at the point of support a stiffening ring or some other rigid member should be attached to the pipe shell to transmit these forces to the support. The actual conditions may be quite closely approximated by considering that these forces are transmitted to a stiffening ring by the shell at the junction between the ring and the shell.

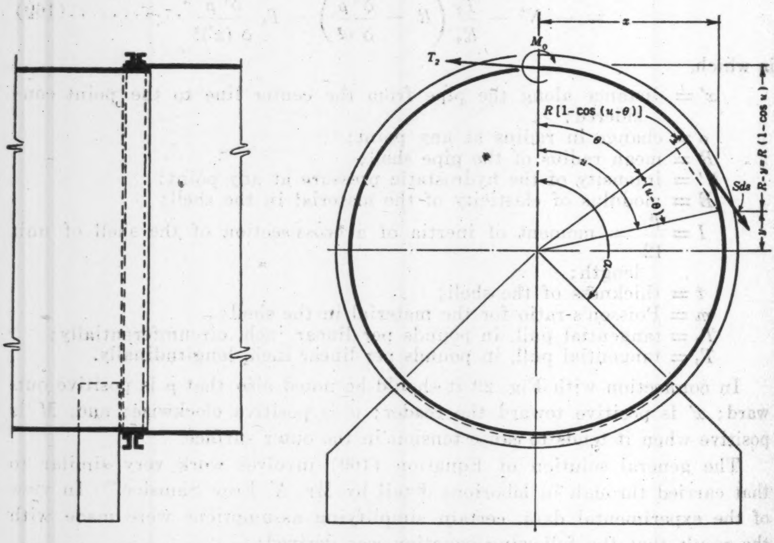

FIG. 30.—SCHEMATIC SKETCH OF PIPE AT SUPPORT, SHOWING FORCES ACTING ON IT.

Fig. 30 is a schematic sketch of a portion of the shell near the support showing the forces acting on it. Treating this part of the structure as an isolated continuous ring with these forces acting on it, equations for the bending moments in the ring resulting from water load are found to be as follows:

For the 66° saddle:

$$M_u = qR^3L\left[0.6327 - 0.5046 \cos u - \frac{u}{2} \sin u\right] \ldots\ldots (171)$$

For the 90° saddle:

$$M_u = qR^3L\left[0.6791 - 0.5847 \cos u - \frac{u}{2} \sin u\right] \ldots\ldots\ldots (172)$$

For the 180° saddle:

$$M_u = qR^3L\left[0.8411 - 0.8211 \cos u - \frac{u}{2} \sin u\right] \ldots\ldots\ldots (173)$$

From the theory of elasticity of curved members involving the relationship,

$$M_u = \frac{EI}{R^2}\left(\frac{\partial^2 \rho}{\partial u^2} + \rho\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (174)$$

the radial distortions may be found to be:

For the 66° saddle ($\alpha = 147°$):

$$\rho = \frac{q R^5 L}{EI}\left[0.6327 - 0.6976 \cos u - 0.3773\, u \sin u + 0.125\, u^2 \cos u\right] . (175)$$

For the 90° saddle ($\alpha = 135°$):

$$\rho = \frac{q R^5 L}{EI}\left[0.6791 - 0.7173 \cos u - 0.4173\, u \sin u + 0.125\, u^2 \cos u\right] \,.. (176)$$

For the 180° saddle ($\alpha = 90°$):

$$\rho = \frac{q R^5 L}{EI}\left[0.8411 - 0.8441 \cos u - 0.5355\, u \sin u + 0.125\, u^2 \cos u\right] \,.. (177)$$

A comparison between the theoretical distortions and the distortions measured on the model involves a consideration of several important factors: First, since the stiffeners were bolted to the pipe shells there is a possibility of as much as 10% variation in the combined moment of inertia of the stiffener and that part of the plate acting with it. A difference between the measured stresses at the same water levels in the prototype during filling and emptying indicates that there was a tendency for re-adjustment to take place there as well as in the models. By checking the bolts very carefully and keeping them tight, it was possible to hold this variation well within 10 per cent. In connection with the combined moment of inertia of the stiffener and plate, supplementary tests were run to determine what proportion of the plate would act monolithically with the stiffener. From tests made on a flat sheet stiffened with two channels back to back, bolted together and to the plate, it was found that on an average about eight times the thickness of the plate outside of each row of bolts could be considered as acting monolithically with the stiffener so as to give computed load-deflection and load-stress values that agreed with experimental results. This value was used in the computations mentioned herein.

A second factor which enters into a comparison between the theoretical and measured deflections or stresses is that the model pipes did not fit the saddles exactly and thereby permitted the pipe to deform until this discrepancy was rectified before the type of action theoretically assumed would begin. The maximum discrepancies were in the order of 0.01 or 0.02 in., which would not seriously affect the distortion of the shell with 66° and 90° saddles; but it would make a decided difference for the 180° saddles since it was assumed that the horizontal diameter of the 180° saddles would not change, which would be the case for the pipe shell if rigidly attached to the saddle or a perfect fit was obtained.

Another factor which entered into the measurements was that the pipe was tested in a room of ordinary temperature, but was filled with water at a considerably lower temperature. The temperature difference between the water and room was occasionally as great as 15° cent. Such variations made stress determination rather difficult because of the large correction necessary for temperature variation. In spite of these factors the curves given in Fig. 31 show that the agreement between the actual measured and computed distortions is quite close.

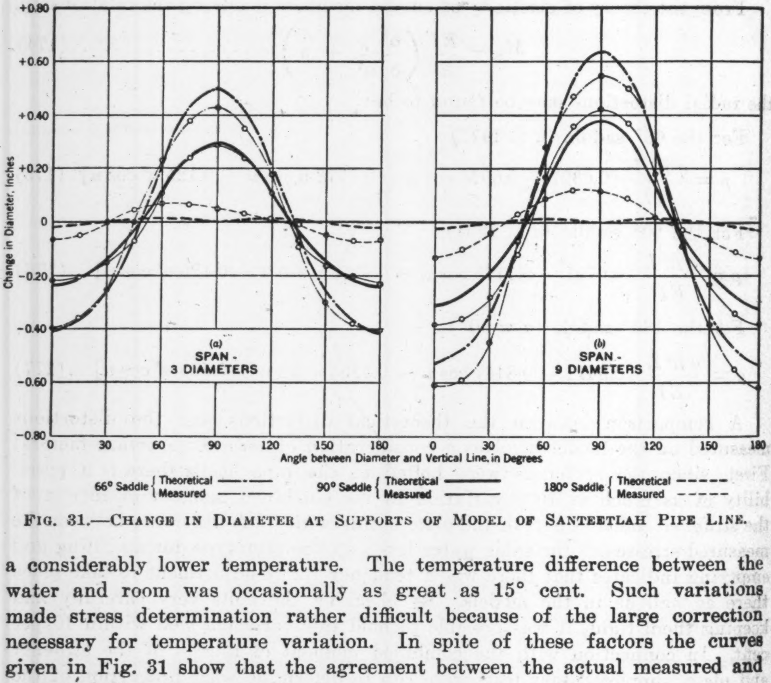

FIG. 31.—CHANGE IN DIAMETER AT SUPPORTS OF MODEL OF SANTEETLAH PIPE LINE.

Table 1 gives the measured and computed stresses in the stiffeners for two span lengths on the three types of saddles. In Item 4, Columns (4) and (6), a highly localized yielding was noted beside the gauge line. This caused the stiffener to pull away from the pipe shell so as to deflect noticeably past the edge of the support, thereby relieving the stress in the stiffener. The 180° saddles showed a clearance of 0.01 to 0.02 in. at the sides. This would tend to make the stresses at the top and sides of the pipe with the 180° saddles greater than the computed stress.

A consideration of the bending moments in the shell of the pipe, supported by saddles, reveals the fact that relatively high stresses must always occur at the ends of the saddles. This phenomenon was also quite apparent in the prototype and in the models. The nature of the theoretical distortion curve shows that if the edge of the saddle were free to move and, at the same time, the center portion of the saddle were also relieved, the change in curvature would be greatly reduced. This observation suggested that the pipe line may be supported by struts attached to stiffener rings at some point below the horizontal diameter and in so doing minimize the moments throughout the ring.

TABLE 1.—MEASURED AND COMPUTED STRESSES IN A STIFFENED MODEL PIPE LINE 12 INCHES IN DIAMETER

(In kips per square inch.)

Item	Position	Span length, in inches	66° SADDLES		90° SADDLES		180° SADDLES	
			Measured	Computed	Measured	Computed	Measured	Computed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1......	At top of pipe............	36	8.0	6.2	6.6	4.5	2.6	1.0
2......		108	16.8	12.9	10.1	9.4	......	2.0
3......	Near edge of saddles......	36	16.3	17.1	8.4	12.0	3.2	2.6
4......		108	18.6	36.0	12.0	26.0	......	5.6

In order to eliminate excessive stresses in the top of the strut caused by the twisting resulting from the distortions of the shell, the horizontal component of the resultant reaction of the strut was chosen so that there would be no rotation of the top of the strut under a full water load. Since the strut would probably be joined to the stiffener by a gusset-plate, a concentration of bending moments at the point of support would be eliminated because of the distribution of the forces through the gusset-plate. For the purposes of analysis, however, the resultant reaction of the strut was considered as acting at a point on the neutral axis of the stiffening ring and pipe shell combined. Again applying the theory of elasticity, involving the consideration that the member is curved, three equations involving the moment at the top, the tangential force at the top, the horizontal component of the strut reaction, and the angular position of the point of application of the strut reaction are obtained.

For various values of the angular position, α, corresponding values for bending moment and tangential force at the top of the pipe and horizontal component of strut reaction were found. By plotting these values against the corresponding values of α it was found that the moment at the top of the shell will pass through zero for some particular α and for the same α the other values would be nearly a minimum. From this it may be concluded that if the resultant reaction of a strut intersects the neutral axis of the combined stiffener and shell at a point 117.3° below the top center, the bending moment at the top will be practically zero and that at the point of support will be nearly a minimum. The reasons involved in choosing this point for the application of the strut reaction may be enumerated, as follows:

1.—Since the point of application of the struts does not rotate (a condition imposed by the choice of the horizontal component of thrust), the slope at this point does not change and, therefore, the distortion must be a maximum.

2.—The distortion of the shell is caused by changes in curvature which, in turn, are caused by bending moments.

3.—The bending moment at the top of the shell is indicative then of the change in curvature of the top and, consequently, to a degree an indication of the distortion.

4.—The magnitude of the distortion at the top is indicative of the magnitude of the distortions throughout the shell.

5.—Consequently, by making M_1 as small as possible, the distortions and stresses are kept small.

Using the value of $\alpha = 117.3°$, equations for the bending moment throughout the ring have been determined. The bending moment at any point in the stiffening ring may be expressed by the following equations:

For values of u between 0 and α,

$$M_u = qR^3L\,[0.825 - 0.825 \cos u - 0.500\, u \sin u] \;\ldots\ldots\ldots (178)$$

and for values of u between α and π,

$$M_u = qR^3L\,[0.564 - 0.452 \cos u + 1.571 \sin u - 0.500\, u \sin u] \;\ldots (179)$$

Again using the fundamental differential equation, (174), the equations for the distortion of the shell are found to be:

For values of u between 0 and α,

$$\rho = \frac{q\,R^5\,L}{E\,I}\,[0.825 - 0.809 \cos u - 0.537\, u \sin u + 0.125\, u^2 \cos u]\,.(180)$$

and for values of u between α and π,

$$\rho = \frac{q\,R^5\,L}{EI}\,[- 0.568 + 0.641 \cos u + 1.103 \sin u - 0.785\, u \cos u$$
$$- 0.351\, u \sin u + 0.125\, u^2 \cos u] \;\ldots\ldots\ldots\ldots (181)$$

Because the distortions of the pipe on this type of support are so small it would be desirable to make corrections in the computed distortions for the temperature change. A very close approximation to the actual distortion caused by temperature change may be expressed by the equation,

$$\rho = \Delta\,(4.29 - 3.37 \cos u - 3.37\, u \sin u] \;\ldots\ldots\ldots (182)$$

in which, Δ equals the change in diameter caused by the temperature change.

In order to check the validity of this theoretical investigation, another model was made using the same scale as for the previous models. The stiffeners were again bolted to the shell and the struts joined to the stiffeners by gusset-plates. Fig. 32 shows an end view of the model built to check the

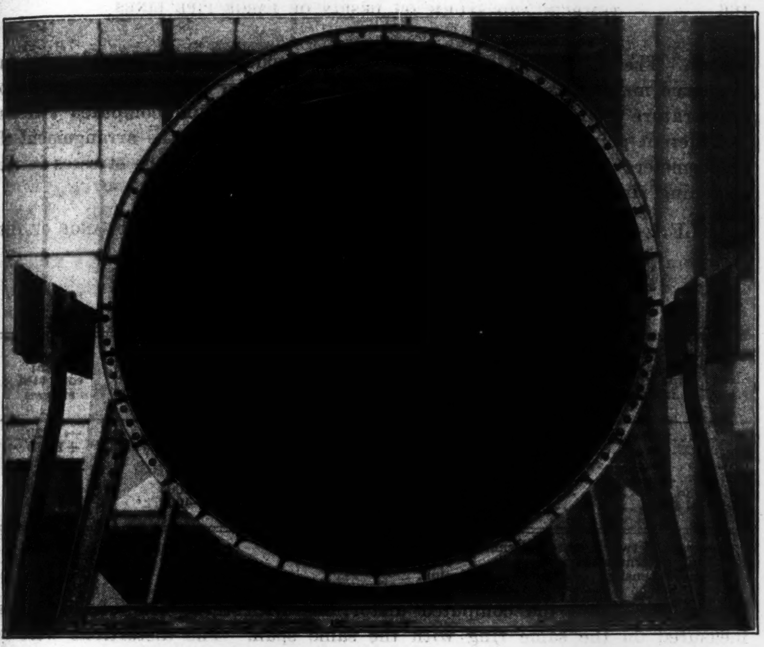

FIG. 32.—END VIEW OF PIPE SUPPORTED BY SPECIAL STRUTS.

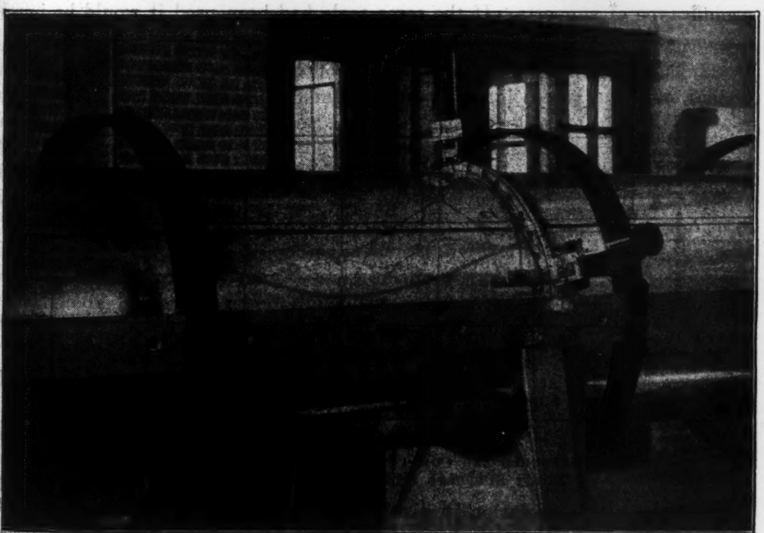

FIG. 33.—ARRANGEMENT OF TENSOMETERS FOR MEASURING STRESSES IN THE STIFFENING RINGS ON A MODEL PIPE LINE.

theory. The distortions and stresses of the model were measured again in the same manner as in former tests, with particular consideration given to temperature changes. Table 2 shows the measured and computed stresses at different points around the pipe shell. Fig. 33 shows the arrangement of "tensometers" previously mentioned herein, for measuring the stresses in the stiffening rings on a model pipe line.

TABLE 2.—MEASURED AND COMPUTED STRESSES IN THE OUTER FLANGE OF THE STIFFENERS OF THE MODEL PIPE LINE ON STRUT SUPPORTS WITH 108-INCH SPAN

(In pounds per square inch.)

Angle around pipe from top, in degrees	Measured stresses	COMPUTED STRESSES		Total computed stresses
		From water load only	From temperature change only	
0	— 500*	0	—230	— 230
30	+ 160†	+1 400	—150	+1 250
70	+1 800†	+1 900	+180	+2 080
80	+ 500*	0	+300	+ 300
Outside of strut	— 500*	— 750	+400	— 350
Inside of strut	....	— 750	—400	—1 150

* Average of two or more determinations.
† Single determination.
+ Denotes tension; — denotes compression.

Fig. 34 shows the computed distortions with three sets of distortions measured on the same ring with the same span. The distortions were so small that the scale necessary to show them is ten times that used for the saddle type of supports. If the same scale had been used it would be impossible to detect the variation in distortion resulting from this type of support because they would all lie so near the zero axis. It was concluded, therefore, that for all practical purposes the distortions are negligible. In view

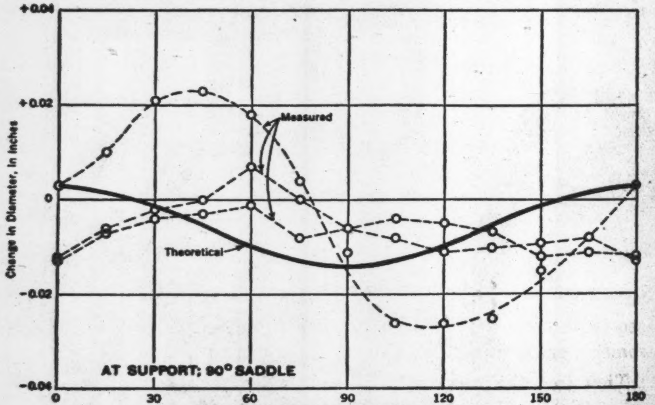

FIG. 34.—MEASURED AND COMPUTED DISTORTIONS OF A MODEL SUPPORTED ON STRUT SUPPORTS SHOWN AS CHANGES IN DIAMETER.

of the agreement between observations on the model and computed values, the writers believe that the theory may be considered a reliable guide in the design of supports and stiffeners for large, thin-shelled cylindrical containers.

From the nature of the moment equation it may be seen that the stresses in the stiffening ring and struts are directly proportional to the span length for any given diameter and that they vary as the cube of the radius of the pipe for spans of equal length. The critical bending moment for design purposes will be that at the bottom of the shell because the moment at the point of support is not only absorbed by the gusset-plate, but also diminished by the distribution of the strut reaction by the gusset-plate. The maximum bending moment at the bottom of the shell reduces to,

$$M_{\pi} = 0.112\ qR^3L \ \ldots\ldots\ldots\ldots\ldots\ldots (183)$$

This equation is on the side of safety because of the stiffening effect of the gusset-plates and the fact that the strut reaction is distributed over a considerable portion of the ring by the gusset-plate. The maximum shear stresses in the side of the shell may be found from the equation,

$$S_{ys} = \frac{qR\,x'\,\sin u}{t} \ldots\ldots\ldots\ldots\ldots\ldots (184)$$

which reduces to,

$$S = \frac{qRL}{2t} \ \ldots\ldots\ldots\ldots\ldots\ldots (185)$$

for the maximum shear stress in the side of the shell.

F. Knapp,[45] Esq. (by letter).—The subject of this interesting paper has been treated[46] previously by Professor Dr. Dieter Thoma, of Munich, Germany, in a paper entitled "Die Beanspruchung freitragender gefüllter Rohre durch das Gewicht der Flüssigkeit." Using the fundamental equations of the theory of elasticity as a basis, Professor Thoma considers the case of a completely filled pipe, either "encastrée" at the ends, or having the end sections connected to rigid rings which may move normally and in the direction of, but not inclined to, the axis. However, he does not consider the design of these rings.

Through the special courtesy of Professor Dr. L. Föppl, of the Technische Hochschule of Munich, the writer was fortunate enough to study, in the original, the work of Dr. R. Abdank, entitled "Die Berechnung freiaufliegender Rohre bei beliebiger Füllhöhe," regarding the stresses in large pipe lines caused by partial filling of the pipe.[47]

Based on the extensive experiments made on a model pipe for the Shannon Development, in Ireland, Dr. Abdank studies the following two cases:

(1) That in which the inclination of the pipe is such that partial filling is possible without the water level cutting the supporting rings. (In this

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[45] Asst. Hydr. Engr., The Saõ Paulo Tramway, Light & Power Co., Ltd., Saõ Paulo, Brazil.

[46] *Zeitschrift* für das gesamte Turbinenwesen, 1920, p. 49.

[47] Submitted as a thesis in partial fulfillment of requirement for the degree of Doctor of Engineering at the Technische Hochschule, of Münich. An abstract of this thesis was published by Dr. Abdank in *Die Bautechnik*, for June 19, 1931.

case, Professor Thoma's formulas for a completely filled pipe may be applied with safety); and,

(2) That in which the pipe is horizontal. In Case (2) the maximum normal stress in the membrane in the direction of the axis of the pipe and for a freely supported pipe, may be calculated by the approximate formula:

$$f_{(\max.)} = \frac{3}{32} L \times \frac{L}{t} \sqrt{\frac{R}{t}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (186)$$

In the case of a continuous pipe, this stress is reduced to $\frac{2}{3}$ $f_{(\max.)}$, tension at the connection of the ring, and to $\frac{1}{3}$ $f_{(\max.)}$, compression midway between the rings. These stresses are always greater than in the case of a completely filled pipe.

C. P. Vetter,[48] Assoc. M. Am. Soc. C. E. (by letter).—The author has made a valuable contribution to the art of designing large pipe lines subject to low pressures. The manner of support advocated by him is not entirely new, and several attempts have been made by others to produce a satisfactory stress analysis of which at least one is entirely rational, although, unfortunately, far too cumbersome for practical use. This is the analysis by Mr. A. Frey Samsioe mentioned by the author under "Case 3." Mr. Schorer deserves credit, therefore, for the suggestion of applying the membrane theory (developed largely for the design of domes and vaulted roofs) to the problem, and thereby establishing surprisingly simple formulas.

Some time may elapse before designers gain confidence in the author's membrane equations, as has been the case with every innovation in engineering mathematics. Mr. Schorer would undoubtedly help his cause by giving details of some of the tests that have been conducted under his auspices. There is no doubt that his solution is an approximation, good only for thin pipe shells. Its value, however, may best be determined by tests since the exact mathematical solution is so complicated that it is practically unavailable.

Unfortunately, no details are given of the derivation of the primary equations, (1), (2), and (3), and Fig. 2 is of little help because the all-important increments of stress which form the basis for the stress equations, have been omitted. Therefore, an attempt at a rational stress analysis along the lines suggested by the author may be of interest.

Fig. 35 shows a section of pipe shell subjected to a radial load, *p*, in pounds per square inch, and a mass load (weight), *w*, in pounds per cubic inch. The pipe shell is assumed to possess that special quality by which it is capable of sustaining normal forces and shear, but not bending moments. The section is part of a circular cylinder with its axis along the *X*-axis (in the diagram). In so far as its incapacity to resist bending moments is concerned the shell represents, in space, something similar to a string or a cable in a plane. The principal difference is that a string cannot sustain internal shear while the pipe shell can. If it is assumed that the shape of the string under load were known by intuition, and the corresponding internal stresses were found by applying the principles that shear and bending moments must not occur, then the assumed shape is a true one if each and every point of the string is found to be in equilibrium.

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[48] San Francisco, Calif.

Fig. 36 will illustrate this reasoning: Let A and B be two forces acting on a string of the assumed shape, *c-a-b-d*. Under the assumptions that shear and bending cannot exist, the internal stresses, p and q, are found. If p is equal and opposite to q, then every point of the string is in equilibrium and the assumed shape of the string is stable.

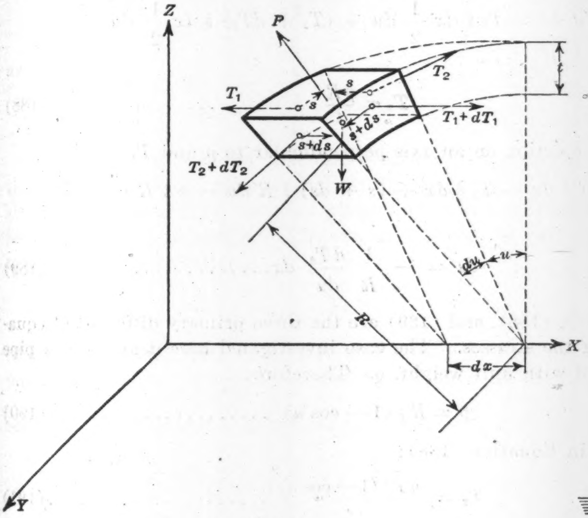

FIG. 35.

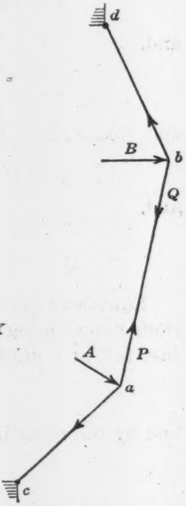

FIG. 36.

Applying this principle to the pipe shell of which a section is shown in Fig. 35, the outside forces are p and w; the assumed shape is a circular cylinder; and the problem is to find the internal stresses under the assumption that bending moments cannot occur. If, after the internal stresses have been determined, it can be proved that every point of the shell is in equilibrium, then the assumed circular cylinder shape is correct and the structure is stable.

The author starts out, as do the originators (Messrs. Kraus and Dischinger) of the membrane formulas, by assuming an outside force with components, X, Y, and Z. If the outside force is to be taken as a surface load this is not possible, because any component in a tangential plane would necessarily produce both radial shearing stresses and bending moments in the shell. Surface loads, therefore, must be confined to radial loads, that is, loads perpendicular to the surface. Mass loads, such as gravity, are not surface loads, and are not affected by this rule. They may have any direction and, in Fig 35, are assumed parallel to the Z-axis (or the Z-axis is assumed vertical).

Returning to Fig. 35, the influence of p only is considered. Assuming $w = 0$ and by projection on the X-axis:

$$(T_1 + dT_1)\ R\ t\ du - T_1\ R\ t\ du + (s + ds)\ t\ dx - s\ t\ dx = 0$$

and,

$$dT_1 = -\frac{1}{R}\frac{ds}{du}\ dx \qquad (187)$$

and by projection on p:

$$p\ R\ du\ dx = T_2\ t\ dx\ \frac{1}{2}\ du + (T_2 + dT_2)\ t\ dx\ \frac{1}{2}\ du$$

and,

$$T_2 = \frac{pR}{t} \qquad (188)$$

and, finally, by projection on an axis perpendicular to p and T_1:

$$(T_2 + dT_2)\ t\ dx - T_2\ t\ dx + (s + ds)\ t\ R\ du - s\ t\ R\ du = 0$$

and,

$$ds = -\frac{1}{R}\ \frac{dT_2}{du}\ dx \qquad (189)$$

Equations (187), (188), and (189) are the three primary differential equations determining the stresses. The case investigated here deals with a pipe just full of a fluid with unit weight, q. Therefore:

$$p = Rq\ (1 - \cos u) \qquad (190)$$

and by inserting in Equation (188):

$$T_2 = \frac{q\ R^2\ (1 - \cos u)}{t} \qquad (191)$$

and by differentiation: $\frac{dT_2}{du} = \frac{qR^2}{t}$ sin u; which inserted in Equation (189) gives,

$$ds = -\ \frac{qR}{t}\ \sin u\ dx \qquad (192)$$

By integration with respect to x: $s = -\frac{qR}{t} x \sin u + f_1\ (u)$ and, again, by differentiation with respect to u: $\frac{ds}{du} = \frac{qR}{t}\ x \cos u + \frac{d\,[f_1(u)]}{du}$.

Inserting in Equation (187):

$$dT_1 = \frac{q}{t}\ x \cos u\ dx - \frac{1}{R}\ \frac{d\,[f_1(u)]}{du}\ dx$$

and by integration with respect to x:

$$T_1 = \frac{1}{2}\ x^2\ \frac{q}{t}\ \cos u - \frac{1}{R}\ x\ \frac{d\,[f_1(u)]}{du} + f_2\ (u) \qquad (193)$$

in which, f_1 (u) and f_2 (u) are the integration constants. They represent functions that may contain the variable, u, but not x, and their determination is not the least part of the problem.

Let it first be assumed that the pipe considered is the overhanging portion of a pipe line, such as would occur at an expansion joint. Then, by choosing the origin of the co-ordinate system at the end of the pipe, it is evident that for $x = 0$, $T_1 = 0$, and $s = 0$. Therefore, $f_1\ (u) = 0$ and $f_2\ (u) = 0$, and, consequently:

$$s = -\frac{qR}{t}\, x \sin u = -\frac{q}{t}\, x\, y \dots\dots\dots\dots\dots\dots (194)$$

and,

$$T_1 = \frac{1}{2}\, \frac{q}{t}\, x^2 \cos u = \frac{1}{2}\, \frac{q}{tR}\, x^2\, z \dots\dots\dots\dots\dots\dots (195)$$

Equation (195) shows that for constant x, the direct stress is proportionate to the Z-co-ordinate, indicating that except for deflections due to shear, the cross-section remains plane under load, assuming proportionality between stress and strain. The stress, T_1, may be expressed, therefore, by the well-known equation:

$$T_1 = \frac{M}{I}\, z \dots\dots\dots\dots\dots\dots\dots\dots\dots (196)$$

which is based on this same assumption only and where M is the bending moment of the exterior forces and I, the moment of inertia of the cross-section. The moment is expressed by $M = \frac{1}{2}\, \pi\, R^2\, x^2\, q$, and,

$$I = 4 \int_0^{\frac{\pi}{2}} R^3\, t \cos^2 u\, du = \pi\, R^3\, t$$

Therefore, $T_1 = \dfrac{\frac{1}{2}\pi R^2 x^2 q}{\pi R^3 t}\, z = \dfrac{1}{2}\, \dfrac{q\, x^2}{tR}\, z$, which is identical to Equation (195). However, the analogy goes still further. If the cross-sections remain plane, the shearing stresses may be expressed by another well-known equation: $s_z = \dfrac{M_z}{I}\, s_v$, in which, s_z is the total horizontal shear per unit length in two radial sections having a distance, z, from the neutral axis; M_z is the static moment with regard to the neutral axis of that part of the cross-section which lies outside the radial sections; and s_v is the total vertical shear. The moment is expressed by:

$$M_z = 2 \int_0^u t\, R^2 \cos u\, du = 2\, t\, R^2 \sin u$$

Furthermore, $s_v = \pi\, R^2\, x\, q$, and, therefore, $s_z = 2\, q\, x\, R \sin u$. The unit shear equals $s = \dfrac{\frac{1}{2} s_z}{t} = \dfrac{qR}{t}\, x \sin u$, which numerically is identical to the value given by Equation (194).

In a pipe supported at either end the origin of the co-ordinate system may be chosen at the section where the total vertical shear, s_v, is zero. For a pipe with uniform load, such as investigated herein, the origin would be in the middle between the supports, as suggested by the author.

From Equation (192) for $x = 0$,

$$s = f_1(u) \qquad (197)$$

and,

$$s_v = 2\int_0^{\pi} s\,R\,t \sin u\,du = 0 \qquad (198)$$

By inserting Equation (197) in Equation (198) and eliminating the constant values:

$$\int_0^{\pi} f'(u) \sin u\,du = 0 \qquad (199)$$

This equation is satisfied by:

$$f'(u) = 0 \qquad (200)$$

and also by numerous other trigonometrical functions. It is evident, however, that these other functions would not agree with the law of continuity, at least not for uniformly distributed load. It must be assumed, therefore, that Equation (200) gives the only possible solution of Equation (199).

Equation (192) then becomes:

$$s = -\frac{qR}{t}\,x \sin u \qquad (201)$$

as for cantilevers, Equation (193) becomes:

$$T_1 = \frac{1}{2}\,x^2\,\frac{q}{t}\cos u + f_2(u) \qquad (202)$$

for $x = \frac{L}{2}$, $T_1 = 0$, in which, L is the length between supports, therefore:

$$f_2(u) = -\frac{1}{8}\,L^2\,\frac{q}{t}\cos u \qquad (203)$$

and,

$$T_1 = -\frac{q}{8t}\,[L^2 - 4x^2]\cos u$$

or,

$$T_1 = -\frac{q}{8tR}\,[L^2 - 4x^2]\,z \qquad (204)$$

By comparing Equation (204) with Equation (195) it is seen that in the case of a pipe supported at either end, the normal stresses are also proportionate to the Z-co-ordinate. It is evident, therefore, that the stresses are the same as those found by the common beam formulas.

This point is of particular interest because in the ordinary beam theory it is necessary to assume that a normal section remains plane under stress in order to establish the usual formulas for the stresses. In the case of thin shells this assumption is not necessary. Neither is it necessary to assume proportionality between stress and strain nor a constant elastic modulus. If, on the other hand, this proportionality exists, the cross-sections will necessarily remain plane.

Thus, it has been shown that the ordinary bending formulas may be applied to this type of cylindrical shells in the case of cantilevers and simple beams. The reasoning may easily be extended to show that they apply also in the case of continuous beams. In addition to the bending stresses, hoop stresses occur, which are determined by Equation (191). They are not affected by the manner in which the pipe is supported.

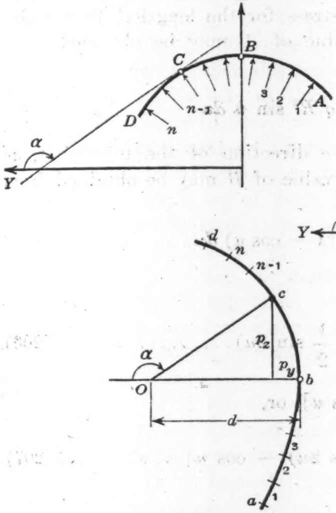

FIG. 37.

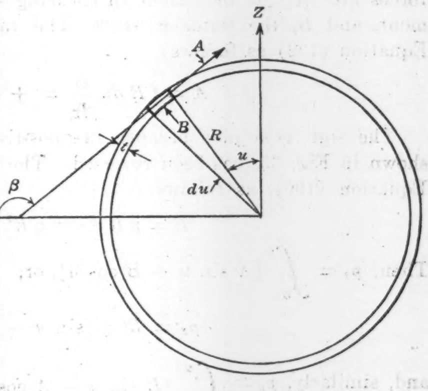

FIG. 38.

It remains to be proved that an arbitrarily chosen element of the shell is in equilibrim when subject to the forces and stresses developed in the preceding equations. A circular ring, cut from the pipe, and of unit length, may be chosen for the proof. It is well known that a circular ring is in equilibrium if the string polygon to all the forces acting upon it is a circle. The equation for the string polygon may be found as follows: Let 1 to *n* (Fig. 37(*a*)) be the external forces, which are infinitely small, infinitely close together, and which form a continuous load. Let *A B C D* be a string polygon to these forces, *B* being the point at which the string polygon is parallel to the *Y*-axis, and *C* being an arbitrary point with co-ordinates (*y*, *z*). The force polygon is shown in Fig. 37(*b*). The points, *a*, *b*, *c*, and *d*, correspond to *A*, *B*, *C*, and *D*, and *O* is the pole. Then, the differential equation for the string polygon may be expressed by $\frac{dz}{dy} = \tan \alpha = -\frac{p_z}{d - p_y}$, in which, *d* is the pole distance. The equation is obviously correct irrespective of the magnitude of d. If $d = 0$, the string polygon becomes the pressure line to the forces, 1 to n, the equation for which becomes:

$$\frac{dz}{dy} = \frac{p_z}{p_y} \quad \text{.........................(205)}$$

in which, p_z and p_y are, respectively, the sum of the Z-components and the sum of the Y-components of all the forces between B and C.

Returning again to the circular element of the pipe (Fig. 38), the external forces are: A, the increment in shearing stress for the length l, of the element, and B, the water pressure. The value of A may be obtained from Equation (192) as follows:

$$A = t\,R\,du\,\frac{ds}{dx} = +\,q\,R^2 \sin u\,du$$

The sign is a plus because the positive direction of the increment, as shown in Fig. 35, has been reversed. The value of B may be obtained from Equation (190), as follows:

$$B = p\,R\,du = q\,R^2\,(1 - \cos u)\,du$$

Then, $p_z = \int_0^u [A \sin u + B \cos u]$, or,

$$p_z = R^2\,q\,(\sin u - \frac{1}{2} \sin 2u) \quad \text{................(206)}$$

and, similarly, $p_y = \int_0^u [B \sin u - A \cos u]$, or,

$$p_y = R^2\,q\,[\frac{1}{2}\,(1 + \cos 2u) - \cos u] \quad \text{...........(207)}$$

and,

$$\frac{p_z}{p_y} = \frac{\sin u - \frac{1}{2} \sin 2\,u}{\frac{1}{2}\,(1 + \cos 2u) - \cos u} = -\tan u = \tan \beta \quad \text{........(208)}$$

and, $\frac{dz}{dy} = \frac{p_z}{p_y} = \tan \beta$.

This is readily seen to be the differential equation for a circle with the center at the origin of the co-ordinate system.

It has thus been proved that the pressure line for the forces acting on a circular ring cut from the pipe is itself a circle. Therefore, the circular shape is stable, and the assumption that no bending moments would occur is correct.

If the same reasoning were applied to the case of dead load without water pressure, it would be found that the author's equations throughout rest on a solid foundation of logic.

L. E. GRINTER,[40] ASSOC. M. AM. SOC. C. E. (by letter).—In spite of its brevity, this paper has real value. If it succeeds in calling the attention of designers to the serious stresses that exist in ring stiffeners and in the shell near the supports, the author will have performed a valuable service. Observation shows that the use of inadequate ring stiffeners has been a common mistake.

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[40] Prof. of Structural Eng., Agri. and Mech. Coll. of Texas, College Station, Tex.

At the beginning of this paper Mr. Schorer states that the theory developed is applicable to horizontal tanks as well as to pipe lines. He then advocates the use of ring stiffeners at supports only. It is important to realize that horizontal tanks exceeding 12 ft. in diameter are used in industrial plants where the shell may not be exposed to internal pressure. Since the most serious condition exists with the shell full or partly full of liquid, it is clear that the thickness required to resist direct ring stresses is exceedingly small.

Storage tanks of this type might easily be damaged during erection or while in service so that the cross-section would be far from circular. The writer considers the use of thin plates, stiffened at reasonable intervals by intermediate ring stiffeners, to be the proper design in such cases. Wherever resistance to corrosion or internal pressure requires reasonably thick plates, the intermediate stiffeners may be omitted. The important consideration is to produce a structure of such stiffness that any possible change in shape will not produce leakage at the joints.

The safety of a large pipe line or storage tank in which intermediate stiffeners are not used, is dependent largely upon the proper design of ring stiffeners at the supports. The computation of bending stress in the ring is discussed by the author for a special case. The general problem may be solved by any one of several methods, but the "column analogy" method[50] introduced by Hardy Cross, M. Am. Soc. C. E., seems to have the advantage of simplicity. With the aid of Mr. Schorer's equations for shear, which control the load on the stiffener ring, the column analogy may be used to solve directly for the moments irrespective of the type of support.

The writer is fundamentally opposed to the common procedure of analyzing special cases and recording the results as formulas for use in practice. For instance, Mr. Schorer's equations for moments in the ring stiffener are developed for a special type of support, namely, a lug support at each end of the horizontal diameter. On the other hand, the structures which the author lists in his "Synopsis" are quite as likely to be supported by a cradle or by some other special device. The designer would find considerable difficulty in revising these formulas to fit other types of support.

Another criticism inherent with formularization is that the simplified appearance of the final results leads poorly trained engineers to attempt their use. The outcome is likely to be unsatisfactory to all concerned since such a designer may have almost no conception of the important change in stresses which frequently accompanies an apparently slight change in the type of support. The use of a general method of analysis where numerical computations are carried through the entire process tends to eliminate such careless design work. The column analogy method, therefore, may be considered as a tool for extending the usefulness of the author's work.

Many of Mr. Schorer's equations are taken from the mathematical theory of elasticity. Unfortunately, it is not possible to suggest a general method of analysis to take the place of such formulas. Most technical designers do not have the mathematical training which is required to make use of the mathematical theory of elasticity, and, therefore, they must depend upon the results of the work of specialists in this field. The writer believes that a large amount of poor design work is inevitable unless the analysis of all such problems is simplified until it can be understood by the designer.

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[50] "The Column Analogy," by Hardy Cross, M. Am. Soc. C. E., *Bulletin No. 215*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill.

Mr. Schorer's equations for stresses in the shell near stiffener rings are of great importance. Such flexural stresses would be overlooked by many designers, although they could scarcely miss the stresses caused by longitudinal beam action. The author's numerical example is well chosen since it points out the importance of the rim stress by showing it to be of much greater magnitude than the longitudinal beam stress despite the fact that the unsupported length is 60 ft.

Oren Reed,[61] Assoc. M. Am. Soc. C. E. (by letter).—Ring supports for large pipe lines have several advantages over the usual broad concrete saddles. Generally, no attempt is made to determine the stresses in the pipe shell near the supports with the old type saddle. This is not necessary with a pipe having a heavy shell, but a knowledge of the stress distribution and the possibility of distortion of the pipe over the supports are essential, when a pipe of large diameter and thin shell is to be used. Computation of the stresses in the pipe shell over a saddle support is complicated and uncertain, while with ring supports the stress distribution can be computed more readily.

Ring supports probably should not be used on a pipe line where there is a possibility of collapse due to vacuum. In the event of even a partial collapse, joints might be pulled apart in such a way that a reconditioning of the line would be difficult. However, air-valves and vents should always be provided at summits in the line.

In 1923, the writer inspected the Svaelgfos No. 2 penstock, in Norway. This pipe line is about 7 ft. in diameter and has heavy stiffener angles at about 6-ft. intervals. As pointed out by the author, such a pipe line does not have a pleasing appearance due to the use of the stiffeners.

If the pipe sections are to be transported a long distance, especially by boat, or handled several times before they reach the point of installation, it may be necessary to use stiffening angles for the thin plate sections, unless a definite minimum thickness is adopted. Such a minimum thickness is often much heavier than would be required to withstand the static and dynamic pressures at the location in the line.

At most Swedish hydro-electric plants, heads are not great and the fall is concentrated so that an open penstock is seldom used. However, there are two recent examples of the use of ring supports for pipe lines, at Atrafors and at Tanger. The Atrafors pipe has a diameter of 13.1 ft., a plate thickness of $\frac{7}{32}$ to $\frac{9}{32}$ in., and the span between supports is 44.2 ft. The Tanger power plant was completed in 1931. The penstock has a diameter of 9.2 ft. and operates under a static head of 98.5 ft. above the bottom of the pipe. The line is 2 625 ft. long, and the normal span is 40 ft. between supports. The supporting rings are built up of plates and angles, and rocker-bearings transmit the load to small concrete piers on each side of the pipe. The use of such rocker-bearings makes the design of anchors less difficult. The connection point of the bearings makes an angle of about 30° with the horizontal center line. Ring supports were adopted in order to keep the project construction cost as low as possible.

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[61] St. Paul, Ind.

Ring supports were also used on the Rempen penstock, of the Waggital Project, in Switzerland, which was completed in 1924. The two penstocks have a diameter of 7.2 ft., a plate thickness of $\frac{3}{4}$ in., and the span between supports is 73.5 ft.

The author will have done the Engineering Profession a worthwhile service, if by his design of the Glines Canyon penstock, and the detailed description, a more general use of ring supports for large pipe lines is adopted in America,

Robert E. Glover,[52] Esq. (by letter).—A rigid connection between stiffener ring and pipe shell will enforce equality of displacements along their common surface of contact. Mr. Schorer has shown how the enforcement of equality of radial displacements results in relieving the circumferential stresses in the pipe shell in the vicinity of the stiffener ring at the expense of increased tension in the ring and local bending stresses in the pipe shell.

It is of interest also to consider the results that follow from the enforcement of equality of circumferential displacements. In this case the bending stresses in the ring are relieved at the expense of circumferential stresses induced locally in the pipe shell, and the net effect is much the same as if a certain part of the pipe shell acted as a flange of the stiffener ring.

In order to ascertain approximately how much of the pipe shell might be considered to act with the stiffener ring to resist bending, the writer calculated, by the theory of elasticity,[53] the strains produced in a flat plate of indefinite extent subjected to forces in its plane such that the intensity, measured in units of force per unit of length, varied as the ordinate of a sine curve. These strains were compared with those calculated for a plate of the same thickness and of finite width under the usual assumption of uniform stress distribution, and the following formula was obtained:

$$\beta = \frac{\lambda}{\pi\left[2 - \frac{(1-m)^2}{2}\right]} \qquad \text{(209)}$$

in which, β represents the width of a uniformly stressed plate that is equivalent to the infinite plate in its ability to resist deformation, and λ represents the wave length of the applied forces.

In order to apply this result to the present problem it was necessary to represent the moment diagram shown on the author's Fig. 8 as nearly as possible by a single sinusoidal variation, and a curve was chosen for the purpose such that its ordinates were proportional to cosine $(3u)$. If it is recalled that forces are transmitted from web to flange in a beam by means of shear stresses the intensity of which is proportional to the rate of change of bending moment with respect to length, it may be seen that for the idealized curve the circumferential forces between stiffener ring and pipe shell will be proportional to sine $(3u)$. Under these conditions:

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[52] Engr., U. S. Bureau of Reclamation, Denver, Colo.

[53] The stress function used for this purpose may be found in Paragraph 238 of "Applied Elasticity," by J. Prescott.

$$\lambda = \frac{2\,\pi\,r}{3} \quad \text{.........................} (210)$$

and,

$$\beta = \frac{2r}{3\left[2 - \dfrac{(1-m)^2}{2}\right]} \quad \text{.....................} (211)$$

If Poisson's ratio, m, is given the value, 0.28, this relation reduces to,

$$\beta = 0.383\,r \quad \text{.........................} (212)$$

The effect of circumferential equalization on the bending stresses in the stiffener ring may be shown by the following computation based on the data of the author's example, thus: $\beta = 0.383\,r = 0.383 \times 60 = 22.98$ in.; the area of the section is $1 \times 12 + 22.98 \times 0.25 = 17.75$ sq. in.; the center of gravity of the section is 4.27 in. from the inner surface of the pipe shell; $R = 64.27$ in.; the moment of inertia is 293.5 in.^4; and the section modulus $= \dfrac{293.5}{12.25 - 4.27}$ $= 36.75$ in.^3

Hence, $M = 0.010\,Q\,R = 0.010 \times 317\,000 \times 64.27 = 203\,700$ lb-in., and the bending stress in the outer fiber of the stiffener ring is, $f_1 = \dfrac{203\,700}{36.75}$ $= 5\,540\ \dfrac{\text{lb.}}{\text{in.}^2}$. A comparison of these figures with those of the author shows a smaller value of R, a smaller moment, and a reduced bending stress as results accruing from the equalization of the circumferential displacements.

Herman Schorer,[54] Assoc. M. Am. Soc. C. E. (by letter).—The discussion has brought out many important points which the writer, admittedly, had not treated with sufficient detail. It will not be possible to answer all the questions raised, because the entire subject deserves further theoretical investigation, although the practically important points are now fairly well established.

Mr. Mensch refers to the fundamental work by Professor Love[55] who established the general theory of shells forty years ago. Few engineers would be in a position to apply Professor Love's complete theory directly for practical purposes. Messrs. Templin and Sturm, in Fig. 29, present a complete picture of the normal and shearing stress components, the bending and torsional moments, and the load components, that act on a small element in the most general case.

Mr. Mensch doubts the justification of simplifying assumptions made by several investigators. Most engineers will welcome a comparatively simple method, giving results within a few per cent. of the elastic theory. The mathematical physicist will necessarily take a different point of view. Engineering progress, to a large extent, depends on the creation of reliable working tools consistent with the accuracy of the other assumptions made in structural design.

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[54] Chf. Engr., Thebo, Starr & Anderton, Inc., San Francisco, Calif.

[55] "The Mathematical Theory of Elasticity," by A. E. H. Love, Third Edition, Cambridge Univ. Press, 1920.

Regarding the bending stresses in symmetrical shells of rotation, symmetrically loaded, Geckeler[56] has presented a detailed discussion of his simplified method and that derived by Pasternak,[57] as compared with the fundamental solution by Meissner,[58] the latter strictly conforming to the requirements originally stated by Love. For design purposes, the small deviations can be safely neglected.

The reader is also referred to the publications in the English language by Prescott,[59] Greene,[60] and Coates.[61] The bending stresses in unsymmetrically loaded, cylindrical shells have been treated by Miesel[62] for the case of continuous surface load components, which can be expressed by trigonometric series. Miesel also treats the problem of a full pipe line, inclined at any angle.

All the investigations on bending and torsional shell stresses in closed cylindrical shells, for both symmetrical and unsymmetrical loading, bring out the fact that these stresses are limited to a narrow rim zone, provided the surface loads are distributed over the total shell area in such a manner that a direct stress system is possible. This system of so-called membrane stresses is comparable to the condition of a statically determinate structure.

Mr. Bauman's Fig. 22 clearly illustrates the rapid decrease of the bending deformations with increasing distances from the rim. It is mainly due to the recognition of this point, so typical for structures of the shell type, that their analysis and design can be simplified greatly without any appreciable sacrifice in accuracy.

The development of the Zeiss-Dywidag type of roof, as described by Dischinger,[3] represents a very remarkable step in the application of these principles to open shells for the purpose of wide-span construction over large areas, rectangular or polygonal in plan. Mr. Mensch finds that the stresses given by Dischinger deviate as much as 50% from those obtained by the common theory and claims that the test structures built by the originators were largely over-designed. The writer rather suspects that the common theory used by Mr. Mensch may not represent the actual conditions.

The theory of the Zeiss-Dywidag structures has been verified by numerous model tests made by the designers for the purpose of ascertaining the validity of theoretical simplifications and also at the request of the official building departments entrusted with the authority of guarding the interests of the public and the approval of plans. In addition, independent investigations were made by disinterested experts, among them Professor Kleinlogel. The theory of these structures can now be considered as sufficiently advanced for general application, with due consideration of limiting conditions in the case of discontinuous and concentrated surface loadings. Dischinger[63] has further developed the theory of polygonal shells and their relation to inscribed shells of rotation.

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[56] "Zur Theorie der Elastizität flacher rotations-symmetrischer Schalen," by J. W. Geckeler, *Ingenieur-Archiv,* Vol. 1, 1930, p. 255.

[57] "Vereinfachte Berechnung der Biegebeanspruchung in dünnwandigen, kreisrunden Behältern," by P. Pasternak, Verhandlungen des zweiten internationalen Kongresses für Technische Mechanik, Zürich, 1926, p. 427.

[58] "Über Elastizität und Festigkeit dünner Schalen," by E. Meissner, Vierteljahrsschrift der Naturforsch, Gesellschaft in Zürich, Vol. 60 (1915), p. 23.

[59] "Applied Elasticity," by John Prescott, p. 530, Longmans, Green & Co., Lond., 1924.

[60] "Design of Ellipsoidal Heads for Pressure Vessels," by T. W. Greene, *Transactions,* Am. Soc. M. E., Vol. 51 (1), 1929, p. APM-51-13-137.

[61] "The State of Stress in Full Heads of Pressure Vessels," by W. M. Coates, *Transactions,* Am. Soc. M. E., Vol. 52 (1), 1930, p. APM-52-12-117.

[62] "Ueber die Festigkeit von Kreiszylinderschalen mit nichtachsensymmetrischer Belastung," by Kurt Miesel, *Ingenieur-Archiv,* Vol. 1 (1930), p. 22.

Mr. Roop's references to related work give an excellent idea of possible applications of shell design and diaphragm action in other branches of engineering. He mentions, for instance, the bulkheads in a submarine, which correspond to the ring girder of the pipe line. The importance of an accurate knowledge of the stress conditions in this case is evident, not only in order to obtain minimum weights, but also to prevent certain loss of life in case of structural weakness. The same considerations apply to aircraft construction.

Mr. Skytte, Professor Roark, and Mr. Boardman have presented extremely valuable contributions in connection with the condition of stress in a pipe partly filled. As pointed out in the paper,[3] Equations (1), (2), and (3), derived by Bauersfeld, apply only in case the load components can be expressed as continuous trigonometric functions of the angle, u.

When a pipe is partly filled a discontinuous surface load is created, which, in addition to the direct stresses, must also induce bending and torsional stresses in the pipe shell. The fact that equilibrium cannot be established without bending in this case, is clearly brought out by Professor Roark in his Equation (112). Theoretically, it would be necessary to consider all the forces and moments on a small element, as pointed out by Messrs. Templin and Sturm in Fig. 29. Miesel[64] has shown that nineteen differential equations are required for the complete solution of this problem. Their integration, however, presents a mathematical problem of considerable magnitude.

In order to illustrate the general character of the stress condition in a pipe half full, assume the shell to be cut open over its entire length, along the horizontal diameter. Neglecting the dead load, the upper half shell then is free from external loads. With the designations of Fig. 2, the lower half, now comparable to a semi-circular flume, is loaded by the external load components, $X = 0$, $Y = 0$, and $Z = -q\,r\cos(\pi - u)$. The Bauersfeld Equations (1), (2), and (3), applied to the flume section, give the following stress components between the limits, $u = \frac{\pi}{2}$ and $u = \pi$:

$$T_2 = q\,r^2 \cos(\pi - u) \quad \text{.................(213)}$$

$$s = q\,r\,x \sin(\pi - u) \quad \text{.................(214)}$$

and,

$$T_1 = \frac{q}{8}(L^2 - 4x^2)\cos(\pi - u) \quad \text{.................(215)}$$

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[63] "Die Theorie der Vieleckkuppoln," by Franz Dischinger, Wilhelm Ernst & Sohn, Berlin, 1929.

[64] "Ueber die Festigkeit von Kreiszylinderschalen mit nichtachsensymmetrischer Belastung," by Kurt Miesel, *Ingenieur-Archiv*, Vol. 1 (1930), p. 32.

The shearing stress component, s, for $u = \frac{\pi}{2}$, has a finite value, $s = q\,r\,x$. Since this stress is acting on all four sides of a small edge element, it is necessary to apply an external shearing force of equal magnitude, acting along the edge in the opposite direction, in order to establish equilibrium. This force requires a longitudinal compression member, as shown in Fig. 39(*a*), monolithically connected to the edge of the shell, similar to the tension member in the Zeiss-Dywidag barrel type of roofs. The force in the compression member, designated by P, is equal to the total longitudinal tension in one lower quadrant and is of opposite sign. From Equation (215), by integration between the limits, $u = \frac{\pi}{2}$ and $u = \pi$:

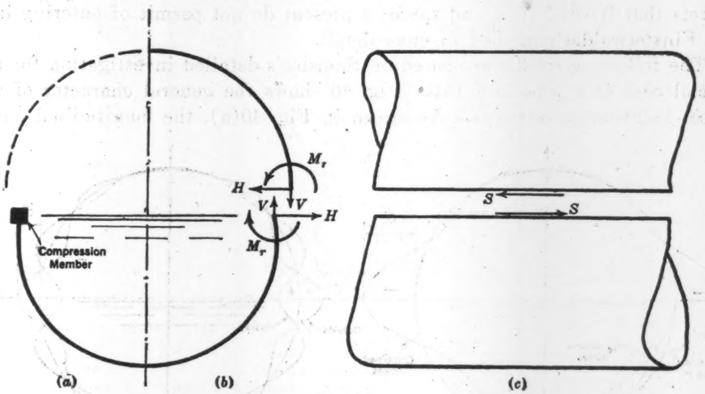

FIG. 39.

$$P = -\frac{qr}{8}\,(L^2 - 4\,x^2) \qquad (216)$$

The semi-circular flume then is subject to direct stresses only, except for the secondary stresses caused by the weight and rigidity of the compression member.

This edge member may now be assumed to be replaced by the full shell. Due to the deformation of the loaded lower half shell, a system of edge forces, H, V, and S, and a radial bending moment, M_r, as shown in Fig. 39(*b*), will be required to close the gap and to restore the continuous curvature of the shell. The deformation of a small element also induces longitudinal bending moments, M_l, and corresponding shears, S_l, as shown in Mr. Skytte's Fig. 11. In addition, there are torsional moments acting in radial and longitudinal planes. The values of M_l and S_l, as pointed out by Mr. Skytte, are very small; the same may be said of the torsional moments. This fact is well supported by the careful tests and analysis of the Zeiss-Dywidag roofs. Samsioe's analysis also indicates comparatively small values of M_l. The longitudinal bending stresses in the narrow rim zone of the supporting ring must be considered as a separate stress system, which is of no importance for the condition of partly filled pipes.

Dr. Finsterwalder[65] has shown that the five stress components, T_1, T_2, S, S_r, and M_r (Fig. 11), can then be derived from a stress function, F, similar to Airy's stress function in case of a plane plate. The resulting differential equation is of the eighth order and the integral consists of two damped waves, originating at the loaded edges. Although mainly developed for the purpose of analyzing the barrel type of roofs, Dr. Finsterwalder's method undoubtedly can be adapted for the analysis of any partial filling of a pipe. The writer regrets that limited time and space at present do not permit of entering into Dr. Finsterwalder's method in more detail.

The following results are based on Samsioe's detailed investigation for the special case of a pipe half full. Fig. 40 shows the general character of the direct longitudinal stresses. As shown in Fig. 40(a), the longitudinal membrane stress (Equation (215)), acts in the lower half of the shell only, and is superimposed by the stress due to the longitudinal force, P (Equation (216)), which is symmetrically distributed about the horizontal diameter in the shape of a damped wave. The relative stress concentration of this force near the horizontal diameter increases with comparatively short spans and thin shells, in which case only a narrow longitudinal zone of the pipe shell acts as a compression member. The combined stress system is shown in Fig. 40(b), indicating that a straight-line stress diagram could not represent the actual conditions.

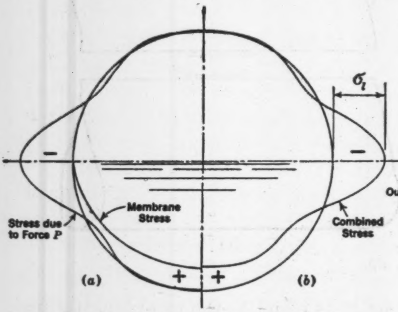

FIG. 40.—PIPE HALF FULL, LONGITUDINAL STRESS.

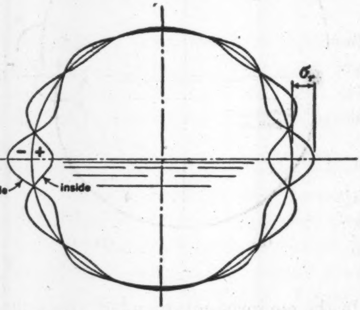

FIG. 41.—PIPE HALF FULL, RADIAL BENDING STRESS.

Fig. 41 shows the character of the radial bending stresses, corresponding to the moment, M_r. They are also distributed symmetrically about the horizontal diameter in the form of a damped wave, with a maximum value at the center of the span, decreasing to zero at the rigid supports.

Fig. 42 shows the distribution of the transverse and longitudinal shearing stress, S. This stress (Fig. 42(a)), is composed of the shear in the membrane system (Equation (214)), acting in the lower half of the shell only, and the secondary shear due to the edge conditions. The combined value is shown in Fig. 42(*b*). For comparatively short spans and thin shells the combined shear distribution approaches the membrane condition. For large spans and thick shells the distribution becomes identical with that of a full pipe.

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[65] "Die Theorie der zylindrischen Schalengewolbe, System Zeiss-Dywidag: Anwendung auf die Grossmarkthalle in Budapest," by Dr. Ing. Ulrich Finsterwalder, International Assoc. for Bridge and Structural Eng., Zurich, 1932.

For design purposes it is convenient to compare the absolute maximum longitudinal stress, σ_l (Fig. 40), and the absolute maximum radial bending stress, σ_r (Fig. 41), with the maximum beam stress f_L, of a simply supported, full pipe (Equation (13); $w = 0$). The corresponding stress ratios, designated by n_l and n_r, can be computed by Samsioe's equations which, after a few transformations, become a function of the pure number, k, defined as follows:

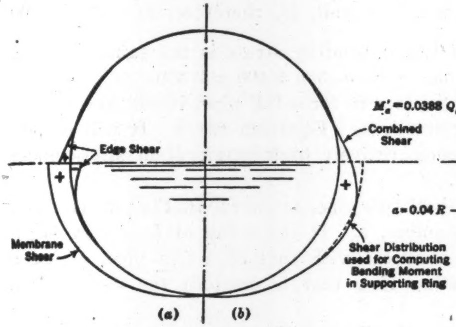
Edge Shear
+
Membrane Shear
Combined Shear
+
Shear Distribution used for Computing Bending Moment in Supporting Ring
(a)
(b)

FIG. 42.—PIPE HALF FULL, TRANSVERSE AND LONGITUDINAL SHEARING STRESS.

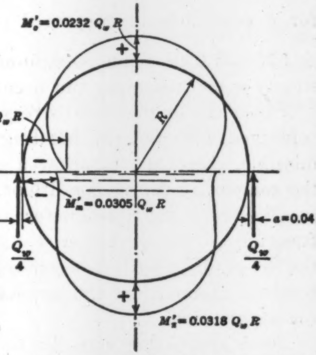

FIG. 43.—BENDING MOMENT IN SUPPORTING RING, PIPE HALF FULL.

$$k = \frac{L}{r}\sqrt{\frac{t}{r}} \quad \text{.........................} (217)$$

In practical cases this characteristic number varies approximately from $k = 0.20$ to $k = 1.20$. A series of values for n_l and n_r has been computed within this range, whereby it was possible to establish the following relations:

$$n_l = \frac{\sigma_l}{f_L} = \frac{1}{\sqrt{k}} \quad \text{....................} (218)$$

and,

$$n_r = \frac{\sigma_r}{f_L} = \frac{0.31}{\sqrt{k}} \quad \text{....................} (219)$$

Equation (218) shows that for values of $k \leqq 1$, the half-full condition will give a larger longitudinal stress than in case of a full pipe. The ratio, n_l, remains the same in case of a continuously supported pipe, because both stresses are reduced in the same proportion. The absolute value of the radial bending stress, σ_r, is independent of the type of support.

A comparison of Equations (218) and (219) shows that the maximum radial bending stress is less than one-third the maximum longitudinal stress at the horizontal diameter. In case $k \leqq 1$, the stress, σ_l, will therefore govern the design, but for values of $k \geqq 1$, it will be sufficient to investigate the maximum beam stress of a full pipe.

Applied to the design example, from Equation (217), $k = \frac{720}{60}\sqrt{\frac{0.25}{60}} = 0.775$. From Equation (218), $n_l = \frac{1}{\sqrt{0.775}} = 1.13$, and $n_r = \frac{0.31}{\sqrt{0.775}} = 0.35$.

The maximum beam stress, in inch-pounds, for a full pipe, simply supported, is: $f_L = \frac{62.5 \times 60^2 \times 12}{8 \times 0.25 \times 144} = 9\,400$. The maximum longitudinal stress for a continuously supported pipe, half full, is, therefore, $\sigma_l = \frac{2}{3} \times 9\,400 \times 1.13 = 7\,050$. The maximum radial bending stress is the same as for a simply supported pipe, and becomes, $\sigma_r = 0.35 \times 9\,400 = 3\,300$.

Since the longitudinal stress, σ_l, is zero for a full pipe, it will never occur combined with the rim bending stress, f_{bo} (Equation (27)). It will be permissible, therefore, to allow a comparatively high longitudinal stress under the temporary filling conditions.

The shear distribution for a half-full pipe, as shown in Fig. 42, is also a function of the characteristic number, k. If the value of k is very large, the shear distribution becomes identical with that of a full pipe, giving a bending moment in the supporting ring that is one-half the value found for a full pipe.

As k approaches zero the shear distribution becomes more and more concentrated in the lower half of the shell, as shown in Fig. 42(*b*), drawn for $k = 0.20$. In the extreme case, $k = 0$, the shear follows the law defined by Equation (214), acting between the limits, $u = \frac{\pi}{2}$ and $u = \pi$. By means of Samsioe's equations it would be possible to find the shear distribution for any given value of k, whereby the corresponding bending moment in the supporting ring could then be evaluated. However, the amount of work is considerable and scarcely justified, because the maximum moment is not greatly affected by a variation of k within the practical range.

For design purposes it will be advisable to assume that $k = 0$. This assumption is on the safe side and can be treated similarly to the method shown in the paper under the heading, "Stresses in Supporting Ring." With the designations of Fig. 6, except for Q, the following equations for the bending moments, M'_1 and M'_2, in the first and second quadrants, respectively, have been obtained:

$$M'_1 = \frac{Q_w R}{2\pi}\left[\frac{2r}{\pi R} - \frac{\pi}{4}\left(1 + \frac{a}{R}\right) + \left(\frac{5}{4} - \frac{r}{R} + \frac{a}{R}\right)\cos u\right] \quad \ldots(220)$$

and,

$$M'_2 = \frac{Q_w r}{2\pi}\left[\frac{2r}{\pi R} + \frac{\pi}{4}\left(1 + \frac{a}{R}\right) + \left(\frac{1}{4} + \frac{r}{R} + \frac{a}{R}\right)\cos u - (\pi - u)\sin u\right] \quad \ldots(221)$$

In Equations (220) and (221), Q_w designates the total water load over one support for a full pipe. The moment coefficients for the half-full condition are, therefore, directly comparable with those obtained by Equations (51)

and (53), when the weight of the pipe is neglected in the latter equations. The dead load moment, as found by Equations (51) and (53), must then be added in both cases.

Fig. 43 shows the moment diagram in the supporting ring, for a pipe half full, with $\frac{r}{R} = 0.90$ and $\frac{a}{R} = +0.04$. Fig. 44 gives the moment coefficients for the maximum moments in the function of $\frac{a}{R}$. It will be noted that, for $\frac{a}{R} = +0.04$, the maximum moment in the supporting ring for the half-full condition is 3.88 times the moment value of a full pipe. In case $\frac{a}{R} = 0$, the corresponding ratio is 2.4.

By observation and logic, Mr. Boardman finally arrives at an approximate ratio of three. A similar value has also been estimated by Karlsson. It is evident that the radial bending moment in the pipe shell, due to partial fillings, is greatly relieved at the expense of the increased bending moment in the supporting ring.

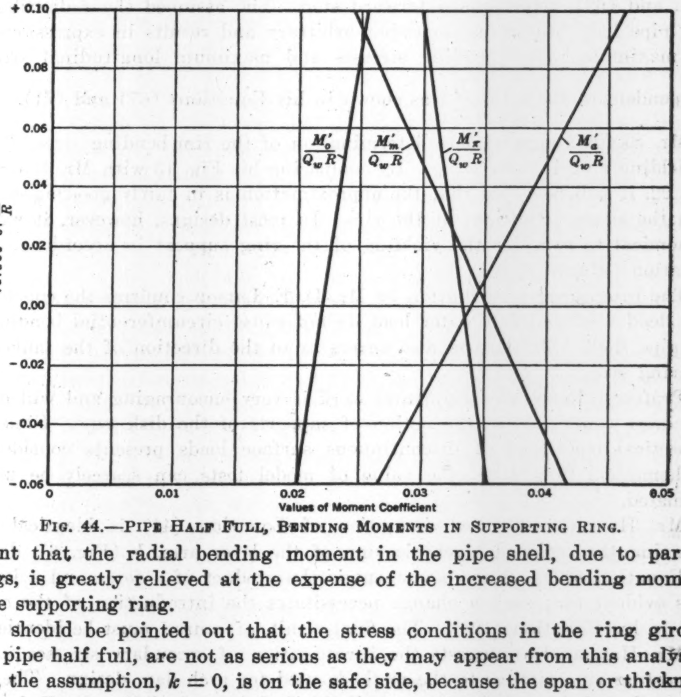

FIG. 44.—PIPE HALF FULL, BENDING MOMENTS IN SUPPORTING RING.

It should be pointed out that the stress conditions in the ring girder, for a pipe half full, are not as serious as they may appear from this analysis: First, the assumption, $k = 0$, is on the safe side, because the span or thickness of the shell never is zero; second, the direct stress in the supporting ring, due to the shearing forces, is only one-half the value given by Equation (57) for a full pipe; third, the direct stress due to the radial pressure (Equation (30)), is practically zero; and fourth, the rigid connection between pipe shell and supporting ring actually increases the effective section modulus of the ring girder; also, the large gusset-plate is placed at the point of maximum

moment. Water-hammer stresses at such a filling are excluded and any inclination of the pipe line will also relieve the bending stresses.

Nevertheless, the stresses in the ring girder for half-full condition should be investigated and compared with the stresses obtained when the pipe is under pressure. In view of the temporary filling conditions and the safe assumptions made in arriving at the moment values, it will be entirely permissible to use stresses close to the elastic limit for this case, although additional resistance to bending can be obtained at small expense. A value, $\frac{a}{R} = 0$, will give more favorable conditions under half-full loading, but higher stresses for full load, than in case $\frac{a}{R} = +0.04$.

In estimating the possible magnitude of the bending stresses in the pipe walls for half-full condition, Mr. Skytte presents Karlsson's reasoning, as illustrated in Fig. 12. His derivation of the general Equations (59), (60), (61), and (62), represents a forward step. The assumed shear distribution of a pipe half full seems somewhat arbitrary and results in expressions for the maximum radial bending stresses and maximum longitudinal stresses independent of the ratio, $\frac{L}{r}$, as shown in his Equations (67) and (71).

Mr. Skytte's approximate determination of the rim bending stress for an unyielding ring is interesting. By comparing his Fig. 13 with Mr. Bauman's Fig. 22, it will be noted that the approximation is in fairly good agreement with the stress conditions at the rim. In most designs, however, it will be economical to consider the yielding of the ring support in accordance with Equation (27).

The interesting contribution by Mr. D. E. Larson confirms the conclusion that dead load and full water load do not cause circumferential bending in the pipe shell. Mr. Larson also agrees as to the direction of the tangential shearing stresses.

Professor Roark's experimental work is very encouraging and will establish more confidence in the action of supports of the disk type. Since the theoretical treatment of discontinuous surface loads presents considerable mathematical difficulties, the value of model tests can scarcely be underestimated.

Mr. Hanna's suggested derivation of Equation (41) is identical with reversing the assumed algebraic sign of the lever arm, b (Fig. 6), defined by Equation (39). Since a moment is the product of a force and a length, it is evident that such a change necessitates the introduction of the minus sign in his Equation (122). The final result, of course, must be identical.

Mr. Hanna also suggests the investigation of secondary stresses in the supporting ring similar to the methods used for arch dam design. The supports of the ring girder are usually designed as flexible columns or are provided with rocker bearings, so as to allow reasonable movement. In addition, it is well known that the secondary stresses in a semi-circular, fixed arch are generally small, except in comparatively thick arches, in which case, the refinements proposed by Mr. Hanna are justified.

The writer is indebted to Mr. Bauman for his able discussion, especially for his presentation of the elastic deformation of the shell and the load distribution at the rim zone, as shown by Fig. 22, which clearly illustrates the damped wave character of the rim conditions.

Mr. Bauman's derivation of Equations (31), (32), and (33) is of convincing simplicity. The more accurate determination of the tensile stresses in the ring also represents a desirable contribution.

Mr. L. J. Larson presents an interesting comparison of a pipe supported on a horizontal plane, in which a direct stress system can not come into play until large deformations have taken place.

Regarding Mr. Larson's question as to the action of a pipe shell attached to the ring at intervals only, it may be stated that such a design is comparable to the action of a riveted connection. Provided the connections are spaced comparatively close and designed to transmit the corresponding forces in a continuous structure, it is reasonable to assume that the general state of stress would not be greatly affected.

The action of a pipe fitted to the ring in a more or less loose manner would change the assumptions decidedly, because the friction forces can not be depended upon to transmit the shearing stresses, except in case of a shrink fit. If the ring fits rather loosely, then the load will be transmitted mostly by direct bearing on the lower half of the ring. In the extreme case the ring would act as a saddle support, which condition is reached more quickly with a comparatively thick shell, than in case of a thin shell. Fig. 28, presented by Messrs. Templin and Sturm, is well adapted to illustrate this point.

Mr. Larson's suggestion as to the use of the dished heads of horizontal tanks as supporting disks is very timely and of great practical importance. The writer has not made any analysis of this possibility, but it is evident that a direct stress system is possible with tangential column supports attached at the center of the dished heads.

As a first approximation, the two end reactions could then be considered as a concentrated force, applied to a point in the direction of a plane of an infinite flat plate, which problem is treated by Prescott.[66] The reaction must be applied over a sufficiently large area, so as to keep the local stresses within safe limits. For this purpose the central area of the dished head may require a gradual increase in thickness. In many cases the flange of a manhole or outlet, located at this point, could be used to transmit the forces.

Mr. Orr finds that the bending stresses in the rim zone can be safely neglected, although he does not question their existence and considerable magnitude. The writer is unable to agree with Mr. Orr on this point, in spite of the fact that standard specifications may not require their consideration. Working stresses close to the elastic limit, especially when accompanied with frequent stress reversal, sooner or later may lead to failures.

Mr. Orr's statement regarding the design of pressure containers, is decidedly in error. The proposed changes in the A. S. M. E. Code for Pressure Vessels, especially the specifications for ellipsoidal heads, are a direct attempt to eliminate or minimize the high stress concentration in the rim zone.

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[66] "Applied Elasticity," by John Prescott, Longmans Green & Co., Lond., 1924, p. 375.

Mr. Bednarski gives an interesting account of the experience on the pipe line of the Los Angeles Aqueduct, supported on piers of the conventional type. In most practical cases the advantages mentioned by Mr. Bednarski are more than offset by other factors. Although it is quite true that the pipe line, when continuously kept under pressure, would not require any rigid ring supports to maintain its circular shape, it should be pointed out that such ideal operating conditions are seldom realized.

The large deformations at partial filling must invite failure or, at least, cause serious leakage, requiring frequent caulking and replacement of protective paint coatings. A pipe line supported on piers, with comparatively short spans, must necessarily follow the natural contours of the ground very closely, whereas long-span construction permits the choice of a shorter route, which may result in considerable savings in material, together with smaller friction losses.

Mr. Boardman's excellent contribution is of decided value, because it raises important questions in connection with partial filling, already answered in connection with Mr. Skytte's discussion and that of Professor Roark. Mr. Boardman's explanation of the small deformations for discontinuous loadings, as observed on his model, and his reasoning regarding the re-distribution of transverse shearing stresses, closely coincide with the approximation already advanced by Karlsson.[2] Instead of the concentrated shearing forces as indicated by Mr. Boardman in Fig. 26, Karlsson assumed a continuous distribution proportional to sin 2 *u*, also used by Mr. Skytte.

By letter dated June 6, 1932, Mr. Boardman has informed the writer of an interesting test on the tank illustrated in Fig. 45, which is a convincing application of theory to practice. The tank is 12 ft. 0 in. in diameter, 35 ft. 0 in. long, and both shell and heads are made of $\frac{1}{4}$-in. plates butt-welded together, each being stiffened by two horizontal channels, 9 in. by 13.4 lb., welded on the inside. According to Mr. Boardman, this tank, supported entirely on the four vertical angles, two on each head (see Fig. 45), was filled with water and then emptied; no deformations were noticeable to the eye.

Equation (217) gives a characteristic number, $k = 0.343$, for this tank. The maximum longitudinal stress for half-full condition, from Equation (218), is $\sigma_l = 3\,180$ lb. per sq. in., and the maximum bending stress in the pipe walls, from Equation (219), is $\sigma_r = 1\,670$ lb. per sq. in. The magnitude of these stresses sufficiently explains the small deformations observed by Mr. Boardman.

Messrs. Templin and Sturm are to be congratulated for presenting their extremely valuable investigation on the pipe-line problem. All their deductions are based on extensive experimental data, combined with theoretical considerations. It is gratifying to note that, beginning with the tests made on the Santeetlah Pipe Line, Messrs. Templin and Sturm finally arrive at the type of support illustrated in Fig. 32.

Regarding the relative location of the neutral axis of the ring, the writer quite agrees that the integral action of pipe shell, stiffener, and column support requires certain assumptions as to the proper value of *a* (Fig. 6). In

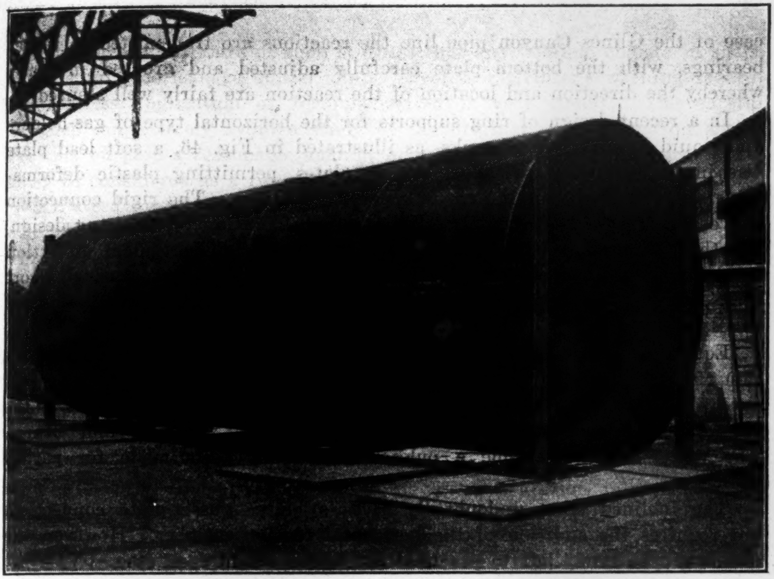

FIG. 45.—TANK FILLED WITH WATER AND EMPTIED WITHOUT CAUSING DEFORMATIONS NOTICEABLE TO THE EYE.

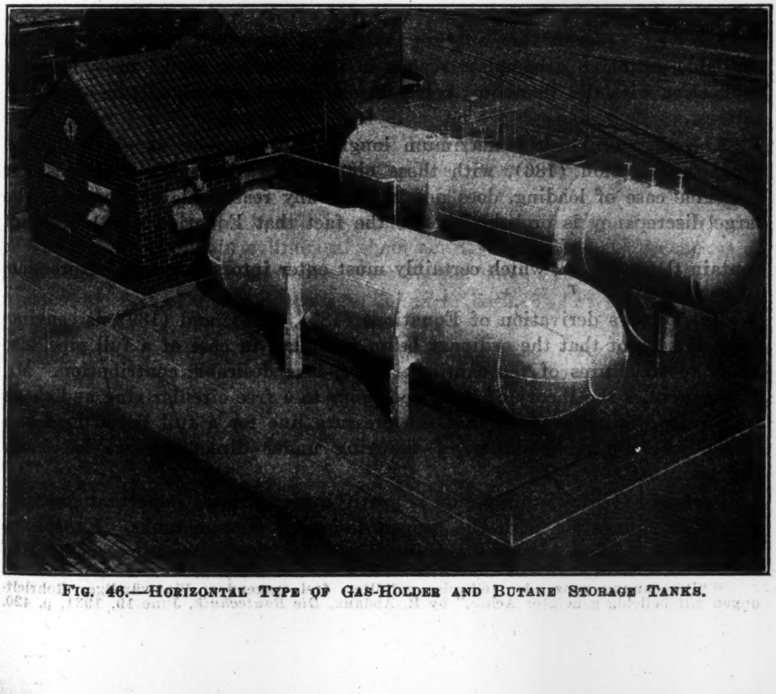

FIG. 46.—HORIZONTAL TYPE OF GAS-HOLDER AND BUTANE STORAGE TANKS.

case of the Glines Canyon pipe line the reactions are transmitted to rocker bearings, with the bottom plate carefully adjusted and grouted in place, whereby the direction and location of the reaction are fairly well assured.

In a recent design of ring supports for the horizontal type of gas-holders and liquid butane storage tanks, as illustrated in Fig. 46, a soft lead plate was inserted between two steel bearing-plates, permitting plastic deformations and re-adjustment in case of eccentric loadings. The rigid connection between ring and shell was not taken into consideration for the ring design, but this assumption is evidently on the safe side. The typical distortion curves of unstiffened pipe shells on saddle supports, with comparatively short spans, as shown in Fig. 28, will serve as an instructive guide for designs of this type.

Equation (170), based on simplifying assumptions, is identical with Equation (9). The same qualifying considerations for partial fillings, as brought out by Mr. Boardman and already answered, also hold for the analysis of the ring supports of the type investigated by Messrs. Templin and Sturm. The writer would welcome a publication of any tests made for partial fillings.

The inclined strut undoubtedly gives a better gusset-plate connection, but requires the addition of the horizontal tension member. However, the choice can be safely left to practical considerations in each case.

Mr. Knapp's reference to the previous paper by Professor Thoma, also referred to by Professor Roark and Messrs. Templin and Sturm, is appreciated.

The writer regrets that only an abstract of the thesis by Dr. Abdank[67] is available, although a complete report on the extensive model tests, made in connection with the Shannon River penstock, would undoubtedly be of general interest.

A comparison of the maximum longitudinal stresses, as given by Mr. Knapp's Equation (186), with those obtained by Samsioe's equations, for the same case of loading, does not indicate any reasonable agreement. The large discrepancy is probably due to the fact that Equation (186) does not contain the ratio, $\frac{L}{r}$, which certainly must enter into any general expression.

Mr. Vetter's derivation of Equations (187), (188), and (189), is convincing. His proof that the ordinary beam formulas, in case of a full pipe, also apply to structures of the cantilever type, is a desirable contribution. Mr. Vetter applies the theory of string polygons to a free circular ring and shows in a very original manner that the pressure line for a full liquid load is a circle, whereby the stability of the ring under direct stresses is clearly demonstrated.

Professor Grinter's discussion is gratifying and raises important practical points in connection with the proper design of storage tanks. Experience will always confirm his excellent statement that the important consideration is to produce a structure of such stiffness that any possible change in shape will not produce leakage at the joints. Professor Grinter's warning regarding careless application of formularization is very timely. The careful designer can scarcely avoid a study of the original publications on shells. For this purpose a reasonable amount of mathematical training must necessarily be considered as a fundamental requirement.

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[67] "Berechnung ganz oder teilweise gefüllter, frei tragender, dünnwändiger Rohrleitungen mit beliebig geneigter Achse," by R. Abdank, *Die Bautechnik*, June 19, 1931, p. 420.

Mr. Reed's description of some of the more recent Swedish pipe lines is well adapted to establish further confidence in supports of the ring type, which evidently has become common practice in Sweden for economical reasons.

Mr. Glover's discussion is of decided value, because it raises the question as to what part of the pipe shell can be considered as a flange of the ring girder, when monolithic action is assured. The same problem has also been discussed by Messrs. Templin and Sturm, who suggest that, based on tests of a flat plate, stiffened with two channels back to back, about eight times the thickness of the plate outside each row of bolts can be considered as acting monolithically with the stiffener.

Mr. Glover's analysis is based on the stress condition in an infinitely large flat plate, loaded by forces in its plane along a straight line, the magnitude of these forces varying as the ordinate of a sine curve. Assuming that the closed shell acts in a manner similar to a plate, Mr. Glover then arrives at a total effective width of the shell, $\beta = 0.383\, r$ (Equation (212)), whereas Messrs. Templin and Sturm suggest $\beta = 16\, t$, when both sides of the ring are considered.

The writer is unable to agree with either suggestion. Evidently, the effective width in the case of a shell is a function of both r and t. This fact may be observed roughly by compressing rubber tubes of different dimensions at the two opposite points of a diameter. From the theory of shells it may be concluded that a wave-shaped load around the circumference will not change the effective width greatly, unless the number of waves becomes fairly large.

As an approximation it will be sufficient to compute the effective width of a shrink fit on a thin cylindrical shell, similar to Fig. 4(a). By means of Geckeler's[8] equation for the ring stress in the shell, the width of the corresponding zone of uniform stress is found to be $0.78\sqrt{r\, t}$ on each side of the ring, giving a total effective width, $\beta = 1.56\sqrt{r\, t}$. Applied to the design example, $\beta = 1.56\sqrt{60 \times 0.25} = 6.0$ in., Mr. Glover's formula gives about 23 in. and that of Messrs. Templin and Sturm gives 4 in. for the same case.

As shown by Mr. Glover, the consideration of monolithic action between shell and supporting ring results in reduced bending stresses in the ring girder. In order to take full advantage of the imaginary shell flange, it would be advisable to provide a corresponding flange of equal width and thickness on the outside of the stiffener web, so as to obtain true I-beam action.

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HIGHWAY CONSTRUCTION MANAGEMENT[1]

By T. Warren Allen[2], M. Am. Soc. C. E.

With Discussion by Messrs. A. E. Holcomb, and Chester Mueller

Synopsis

The aim of construction management is to conduct operations properly in the least expensive way, whether the task be to construct a bridge, lay a pavement, or move cubic yards of earth.

In its broadest sense such management should begin with the consideration of the economic necessity and priority of procedure. Thus: Should the highway, A-B, be improved? What valid reasons are there for its improvement? What adequate return will there be for the expenditure that may be made? Should this road be improved prior to the improvement of the highway, C-D? What valid reasons are there for its improvement in advance of other highways that should also be improved? To avoid waste, an expenditure of public funds should await suitable answers to these questions. This paper describes how some of these problems have been approached by the Division of Management of the United States Bureau of Public Roads.

The Influence of Administration

Consciously or unconsciously, every administrator having charge of highway improvements, every locator, and every designer is frequently confronted with problems of construction management. During the 1929 construction season the writer encountered two outstanding illustrative cases. One was the construction of a part of a route out of turn. The location of this section was such that it could neither be used to local advantage nor to forward the construction of the remainder of the route. Thus, a sum of several hundred thousands of dollars was tied up, yielding no returns toward paying its interest cost for a period of nearly two years. Furthermore, had an adjoining section been constructed first it would have provided an improved road instead of an unimproved one for transporting the materials used in the first section.

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[1] Presented at the meeting of the Highway Division, New York, N. Y., January 16, 1930.

[2] Chf., Div. of Management, U. S. Bureau of Public Roads, Washington, D. C.

The other case was that of a highway under contract to be paved. The route covered was mostly new location and several new bridges were required. These bridges were not included in the paving contract; they were let as a separate contract, but not until the paving work was nearly completed. As a result, the paving contractor had to move his plant around each stream, crossing by various detours and often at considerable expense. Incidentally, the paving contract also included the placing of the pavement over these bridges, which could not be done until long after the regular paving had been completed. The loss due to the failure to complete the bridges at an earlier date extends much farther than the mere cost of moving the plant around the detours. For example, while the contractor, in the course of seven days, had to move his equipment to six new locations to dodge unbridged streams, the length of haul from move to move varied as much as 4 miles. Thus, one day it was 3.2 miles; the next it was 7 miles; and the following day, 3 miles. It was impossible to keep a properly balanced hauling equipment on hand, and the rate of production was naturally low even for the actual working time.

These illustrations show the influence of administration on construction management. Another example concerns the mixing time of concrete for the pavement slab. State highway specifications for the time the concrete materials must be held in the drum during mixing vary from 60 to 105 sec. Since modern paving machines require about 15 sec. additional for charging and discharging, it follows that a 60-sec. mixing requirement permits of turning out a maximum of 48 batches per hour, but a 105-sec. specification reduces the maximum possible output to 30 batches per hour. Bearing in mind that the daily operating cost is practically the same in both cases, it is evident that in the latter case the cost of production is increased tremendously over what it would be for the former. The mixing action is not changed by merely moving the paver across State lines. If 60 sec. of mixing produces concrete of the required strength and quality in one State, why is this not also true in a neighboring State? From another viewpoint, if quality is improved by a longer mixing time, is it, for example, improved sufficiently by 45 sec. of additional mixing to pay the cost of 37½% reduction in the rate of production?

HIGHWAY MANAGEMENT RESEARCH

In this connection Table 1 shows typical averages of results of more than three years' work by the Division of Management of the Bureau of Public Roads, which seem conclusive. It gives the average breaking strength of standard cylinders made from samples taken, respectively, from each corner and from the center of the batch as dumped on the sub-grade when the mixing time was varied from 45 to 75 sec. Each entry represents an average of from five to ten cylinders. Cylinders *A* and *B* were taken, respectively, from the left and right-hand corners farthest from the mixer, Cylinders *C* and *D,* from the right and left-hand corners nearest the mixer, and Cylinder *E,* from the center of the batch as dumped on the sub-grade. The locations of the jobs given by numbers are, as follows:

Job No.	Location
1	Mahoning County, Ohio
2	Miami County, Ohio
3	Coshocton County, Ohio
4	Crawford County, Arkansas
5	Gadsden County, Florida
6	Leon County, Florida
7	Harnett County, North Carolina

TABLE 1.—EFFECT OF MIXING TIME ON STRENGTH OF CONCRETE

Job No.	Mixing time, in seconds	PART OF BATCH FROM WHICH SAMPLES WERE TAKEN					Job averages
		A	*B*	*C*	*D*	*E*	
1	45	4496	4710	3960	4568	4428	4432
2		4968	4715	4753	4780	4724	4788
3		4727	4900	5108	4994	4961	4938
4		4670	4562	5205	4744	4941	4829
5		3950	4028	3985	3519	3988	3894
6		3664	3845	3977	3924	4120	3905
7		3123	2803	3212	3381	3370	3177
Averages		4228	4223	4314	4272	4362	4280
1	60	4210	4314	4578	4406	4038	4309
2		4826	5092	5078	4955	5084	5007
3		4967	5015	5081	4942	5028	5007
4		4174	3854	4000	4240	4174	4085
5		4327	4243	4124	4141	4123	4191
6		3404	3183	3099	3168	3221	3215
7		3002	2500	2946	3084	3169	2948
Averages		4130	4028	4130	4134	4120	4109
1	75	4420	4596	4608	4254	4072	4390
2		4700	4683	4644	4636	4738	4680
3		4982	5065	5026	4843	4927	4969
4*		4425	4140	4310	4604	4822	4260
5		4188	4180	4236	3941	4007	4110
6		3972	4002	3888	4188	4137	4037
7		3456	3278	3152	3210	3314	3282
Averages		4306	4277	4266	4242	4145	4247
General averages		4221	4176	4237	4216	4209	4212

* This series was mixed in 90 sec.

Fig. 1 also shows graphically the result of more than 3 000 individual tests made on about forty different highway construction projects under a great variety of actual field conditions. These show definitely that the modern road paver in good condition will produce a high quality of concrete in 45 sec. and that certainly nothing of material value is to be gained in strength by extending the mixing time beyond 60 sec.

The field tests also show that with modern equipment and alert management, operation can be maintained on the shorter mixing time at as high a degree of efficiency as on longer mixing times. Furthermore, this additional production can be secured with the employment of little, if any, additional labor except the necessary additional trucks and drivers to deliver the additional batches.

The influence of design on construction management and costs is far-reaching. In the case of a bridge, for example, the design largely controls the methods which can be used in both the fabrication and the erection. In the case of the highway itself the design usually includes many items which vary widely, depending on whether the designer gives real thought to his work and is familiar with the construction problems involved or merely follows the lines of least resistance. The average designer in dealing with the item of grading, for instance, is quite careful to balance his cuts and fills, but he usually fails to give sufficient consideration to the lengths of haul he imposes or how widely or frequently these lengths may fluctuate. Inattention to these details makes it very difficult, if not impossible, for the contractor to adjust the size of his hauling outfit properly so as to fit the work to be done, and increases the cost materially.

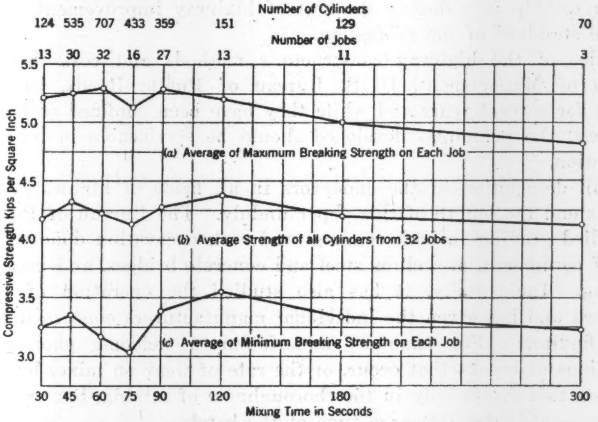

FIG. 1.—EFFECT OF MIXING TIME ON STRENGTH OF CONCRETE.

This may be better appreciated by reference to the actual variations in haul from day to day encountered on a power-shovel grading project. A few consecutive days selected at random from this project show the following average daily haul lengths, in feet; 900, 400, 200, 1 000, 1 000, 550, 800, 1 000, 350, 1 100, and 650.

In this case assume that the shovel was excavating at the rate of 750 cu. yd. of common excavation per 10-hour day. Then, for example, if 1.5-yd. wagons, traveling at a speed of 240 ft. per min., were used for hauling, the number of wagons required for each day would be 9, 5, 4, 10, 10, 6, 9, 10, 5, 11, and 7, respectively.[8] Under such conditions it is practically impossible for the contractor properly to adjust the size of his hauling equipment to the output of the shovel. The condition is really worse than shown, because quite wide variations in hauling lengths also occurred during the progress of each day's work.

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[8] A full discussion of the method of determining the number of wagons may be found in *Public Roads*, March, 1928, and for the number of trucks in paving work, in *Public*

As a matter of fact in a case like this, the contractor being unable to vary his hauling equipment in such a zigzag fashion, usually decides upon a fixed number of hauling units, say eight, for illustration. He then finds that for the first day's work he is one wagon short, and as the shovel can only dig when there is hauling equipment available the output for the day is 83⅓ cu. yd. short, and at, say, 60 cents per cu. yd., this means a loss of $50 minus $10 (estimated cost of one wagon, including driver), or $40. The following day there is a surplus of hauling equipment of three wagons which at $10 per day means a loss of $30. In like manner, for the succeeding days, losses occur amounting to $40, $70, $20, $40, $70, $30, $92.73, and $10, respectively, or an average loss of about $46.63 per day.

The foregoing discussion is introduced solely to bring to the attention of highway engineers matters which if given due consideration should operate not only to help in reducing the cost of highway improvement, but also to raise the standard of the profession.

Studies of the highway constructor's methods and costs made by the Division of Management, U. S. Bureau of Public Roads, have been in progress for several years and while they have been confined to highways, it is believed the principles developed should be serviceable in other lines of construction.

A full description of the endeavors in all fields of highway contracting would extend the length of this paper unduly. The Bureau of Public Roads has studied grading in both common and rock excavation done with various kinds of equipment, as well as steel and concrete bridges, and many types of surfacing. Incidentally, it has also studied the operation of the equipment used and has given the individual manufacturers concerned the benefit of its findings. For example, manufacturers welcome photographs and descriptions of breaks that occur, or the rate of wear on mixer buckets. The latter is a factor, not only in the thoroughness of mixing, but excessive wear seriously retards the discharge time of the batch.

Analysis of Time Losses

In general, the method used has been to make stop-watch analyses of the various activities on many going projects, covering practically all classes of road work. By this means average performance data have been secured for construction work both by districts and for the entire country. Of course, data showing excellent performance have also been secured. From this experience, and with these averages as a measure, the performance of the individual constructor as determined by stop-watch readings can be analyzed so as to show him not only wherein he fails to attain the fruits of average or excellent management, as the case may be, but also to indicate to him how his future performance can be improved.

The studies invariably disclose time losses. These are divided into major losses (which are of 15 min., or more, in duration), and minor losses (which are of less than 15 min. duration). Major and minor losses are then usually each subdivided into Classes A and B. Class A, in general, includes all those arising from the action of the elements and from contingencies which the exercise of good managerial ability would not have been expected to anticipate and prevent; and Class B includes all time losses which good management, under the conditions of the particular job in question, should have found possible to eliminate. The degree to which a contractor profitably eliminates the Class B losses indicates his grade of management. The results of these studies are recorded on a weekly report known as "Form C. S. 13" (see Table 2). A copy of this form properly filled out is given to the contractor whose job is being studied.

Its purpose is to bring together the data collected in regular production studies into a concise and compact form so as to emphasize as clearly and forcibly as possible the extent or amount of the various classes of time losses which are being incurred, and the effect that these time losses (which seem possible of elimination) are having on the rate of production.

The major time losses consisting of definite stops involving several minutes or hours in duration are, of course, fully apparent and every contractor makes efforts to prevent their occurrence or recurrence. The minor time losses, which are largely repetitive and involve only seconds, are not so apparent to the management; and yet on the average road job, as determined from studies on many hundreds of going projects, the minor time losses consume from 20 to 50% of the total time that the crew is at work. Furthermore, definite stops of from 15 min. each to several days in length consume from 25 to 50% of the otherwise available calendar days. In other words, on the average job about one-half the theoretically available calendar working time is a total loss in so far as any useful production is concerned.

On the basis of studies of nearly a hundred concrete paving jobs with modern equipment, fairly well distributed over the entire United States, the Bureau of Public Roads has found that, on what might be called the average concrete paving job, after the contract is secured and the plant and crew are assembled, the mixer is only used for pouring concrete about 60% of the available calendar working hours. The remainder of the time is lost from the following causes:

MAJOR TIME LOSSES

	Percentage
Rain	9.5
Wet sub-grade	8
Lack of prepared sub-grade	3
Lack of materials	3.5
Moving plant set-up	3.5
Inadequate supply and faulty operation of hauling equipment	3
Mixer trouble	2
Lack of water at mixer	2
Loading plant trouble	1.5
Miscellaneous causes	4
Total	40

TABLE 2.—Summary of Time Losses and Their Effect on Production (Form C. S. 13).

State__________ County__________ F. A. P. No._____ Project No._____

Type of project__________ Dates, including or week ending__________

Major equipment__________ Auxiliary equipment__________

Total available working time, hours__________ Total estimated production__________

Total major delays occurring during available working time *Character*	Class A		Class B		Total all major delays	
	Hours	Per cent	Hours	Per cent	Hours	%
Totals						
Time major equipment actually operated—						

Total minor delays occurring during time of actual operation, as extended from stop-watch studies *Character*	Class A		Class B		Total all minor delays	
	Hours	Per cent	Hours	Per cent	Hours	%
Totals						
Time major equipment operated at 100% efficiency....................						

Grand total all Class B time losses, hours....................

Possible operating time, all Class B losses eliminated, hrs....................

With all Class B losses eliminated, production would have been....................

Over-all efficiency of major equipment operation, %....................

Notes.—"Major delays" include all stops of 15 min. or more in duration, while "minor delays" include all stops less than 15 min. each in duration as found by stop-watch studies and extended to time of actual operation. Use back of this form for Remarks. Give total extent of stop-watch studies, in hours, for week.

These definite stops, each of 15 min., or more, in duration, do not comprise the entire time loss or time during which the key equipment is idle. Stop-watch studies on these same jobs show that an average of nearly 16% of the total available time, or 26% of the time the crew is at work, is lost in minor stops and interruptions, each less than 15 min. in duration. These are due to the following causes:

MINOR TIME LOSSES

	Percentage
Supply and operation of hauling equipment	7.3
Lack of, or trouble with, water at mixer	1.9
Lack of materials and supplies in place	0.7
Mixer trouble	1.1
Mixer operator	1.3
Finishing	0.4
Sub-grade delays	1.6
Miscellaneous causes	1.3
Total	15.6

In other words, the average concrete road paving contractor is not only out on the road with his crew only about 60% of the available calendar hours, but during this time he is actually operating at only about 74% of the rate at which his key equipment is capable of maintaining. The details from which these averages are derived were assembled in forms, of which samples are illustrated by Tables 3 and 4.

Table 3 is arranged to show the time available, time lost, production, and stop-watch data observed on about eighty concrete paving jobs during 1926, 1927, 1928, and 1929. The code numbers in Columns (5) and (21) denote: 1, new; 2, very good, or excellent; 3, good; 4, fair; 5, poor; and 6, very poor. The distribution of time losses for various causes is recorded by the Bureau of Public Roads as illustrated in Table 4. The major losses, involving stops of 15 min. or more, each, are segregated in a record form identical with Table 4, except for some modifications in corresponding column headings. The totals and the column headings for this table, on the jobs listed in Tables 3 and 4 are, as follows:

Total time that should have been available, and the percentage of this time that was lost for various causes, in definite stops of 15 min., or more:

Column (3), total available time, in hours	33 309.72

Total time that the mixer was idle:

Column (4), hours	13 342.46
Column (5), percentage of total available time	39.95

Causes from which time losses (in percentage of total time) were incurred:

Column (6), miscellaneous	2.63
Column (7), operation	0.47
Column (8), shortage	2.15
Column (9), moving mixer or plant set-up	3.62
Column (10), mixer trouble	2.04
Column (11), lack of materials at yard	3.55
Column (12), trouble at batcher plant	1.47

Column (13), lack of, or trouble with, water	2.15
Column (14), finishing	0.50
Column (15), wet sub-grade	7.29
Columns (16) and (17), joints, or steel	0.10
Column (18), cold	1.05
Column (19), sub-grade not prepared	3.37
Column (20), rain	9.56

In road grading with the power shovel, much the same condition is found as in paving, except that the weather losses are much smaller and the minor or short-period interruptions or delays consume a much larger proportion of the working time. Thus, on the basis of more than 100 power-shovel, road-grading jobs, the Bureau of Public Roads has found that on the average, after the contract is secured and the equipment is on the ground, the major losses are about 30% of the total available calendar hours, these being in definite stops each of 15 min., or more, in duration. Such major delays are due to the following causes (in percentage of total losses):

Rain and wet ground	9.4
Repairs to shovel and other equipment	10.5
Moving from cut to cut	2.8
Drilling and blasting	2.1
Hauling equipment trouble, or shortage	1.4
Taking on, or lack of, fuel or water	0.9
Miscellaneous causes of various kinds	3.0
Total	30.1

Stop-watch studies also show that about 27% of the total available calendar hours, or about 39% of the time the shovel is supposedly at work, is unproductive. These minor delays or interruptions to continuous operation are due to the following causes (in percentage of total available calendar hours):

Lack of sufficient hauling equipment	8.2
Improper operation of hauling equipment	5.4
Moving shovel within cut	3.3
Checking grade	0.3
Shovel repairs, minor	2.3
Handling large rocks, boulders, stumps, etc.	2.5
Blasting boulders, rocks, etc.	0.4
Taking on fuel	0.7
Operator	1.1
Miscellaneous causes of various kinds	2.9
Total	27.1

These figures are averages of a great many jobs and must not be confused with the performance on individual jobs. Weather and other uncontrollable causes often affect one job seriously, but may leave another practically untouched. Thus, the Bureau of Public Roads has found concrete paving jobs which have operated for a month or more at a time loss not exceeding 10% of the available calendar hours and which utilized more than 90% of the time at work in useful production. This, however, is exceptional and prob-

TABLE 3.—JOB RECORDS SUMMARY OF CONCRETE PAVING WORK. TIME AVAILABLE, TIME LOST, PRODUCTION, AND STOP-WATCH DATA

		PAVER					STOP-WATCH STUDIES					PRODUCTION RECORD FOR PERIOD JOB WAS UNDER OBSERVATION						HAULING EQUIPMENT		
Job location (1)	Job No. (2)	Make, code (3)	Size, E (4)	Condition, code (5)	Mixing time required, 1929 standards, in seconds (6)	Year study made (7)	Total time of study, in minutes (8)	Total time lost, in minutes (9)	Percentage of time mixer idle (10)	Total batches during studies (11)	Average rate of production batches per hour (12)	Total hours expected to be available (13)	Total time mixer idle, stops of 15 min. or more, (14)	Percentage of time mixer idle (15)	Total batches poured (16)	Average rate of production, batches per hour (17)	Average length of haul (18)	Kind (19)	Batches per load or train (20)	Condition average, code (21)
14-09-01	1	06	27	3	90	1928	3 878.84	744.7	19.2	1 941	30.0	403.00	108.00	26.8	8 635	29.3	4.2	Trucks...	4	2
21-11-01	2	10	27	1	75	1927	3 720.00	1 380.0	37.1	1 542	24.9	531.50	251.30	47.3	7 299	26.0	4.1	Trucks...	2	3
21-11-02	3	04	27	4	75	1927	2 400.00	918.5	38.3	921	23.0	434.50	236.30	54.4	4 015	20.2	0.5	Trucks...	3	2
23-06-02	6	04	27	2	75	1927	1 120.00	246.3	21.9	452	24.2	156.90	44.50	28.4	2 822	25.2	4.2	Trucks...	2	3
23-06-06	7	06	21	3	75	1927	1 260.00	192.9	15.3	640	30.5	191.75	62.23	32.5	3 851	29.8	2.1	Trucks...	2	2
23-06-09	8	09	21	5	75	1927	1 140.00	131.9	11.6	505	26.6	137.75	84.75	61.5	1 235	23.1	1.2	Trucks...	3	3

TABLE 4.—JOB RECORDS OF STOP-WATCH STUDIES ON CONCRETE PAVING WORK SHOWING TOTAL TIME OF STUDIES AND PERCENTAGE OF THIS TIME WHICH WAS LOST DUE TO VARIOUS CAUSES

			TOTAL TIME LOST		CAUSES FROM WHICH TIME LOSSES (IN PERCENTAGE OF TOTAL TIME) WERE INCURRED													
						HAULING EQUIPMENT												
Job location (1)	Job No. (2)	Total study time, in minutes (3)	Seconds (4)	Percentage of study time (5)	Miscellaneous (6)	Operation (7)	Supply (8)	Dumping batches (9)	Mixer trouble (10)	Inspection (11)	Mixer operator (12)	Water supply at mixer (13)	Finishing (14)	Handling cement (15)	Joints (16)	Steel (17)	Setting parting strip (18)	Subgrade not prepared (19)
14-09-01.....	1	3 878.84	44 684	19.1	0.7	1.4	1.8	5.5	1.0	0.1	4.2	2.6	0.3	....	0.2	1.1	0.1	....
21-11-01.....	2	3 720.00	82 800	37.1	0.6	1.8	15.1	6.1	1.1	0.0	2.7	8.5	1.1	....	0.0	0.1	0.0	0.0
21-11-02.....	3	2 400.00	55 109	38.3	2.0	1.5	3.1	2.7	2.3	....	15.9	4.0	0.9	1.7	0.1	1.5	1.1	1.3
23-06-02.....	6	1 120.00	14 777	21.9	....	0.8	5.4	0.8	1.0	2.8	2.4	0.5	....	0.2	1.6	....	....	6.4
23-06-06.....	7	1 260.00	11 573	15.3	0.4	1.5	2.2	1.9	0.0	....	0.7	2.0	....	0.8	0.2	0.9	0.3	4.4
23-06-09.....	8	1 140.00	7 914	11.6	....	3.1	0.4	1.3	0.1	....	3.3	1.9	....	0.3	....	0.1	0.4	0.7

ably could be expected to be attained only during the most favorable part of the construction season. The studies also indicate that as contractors become better aware of the losses incurred, the percentage of such losses decreases.

METHOD OF DETERMINING REQUIRED NUMBER OF HAULING UNITS

When hauling units are used in different types of road construction an immediate problem confronts the contractor as to the number of units required in hauling material. Usually, a contractor does not predetermine the proper number of trucks, wagons, or other typical hauling equipment that is necessary to handle the particular material which is involved. If it appears that he started with an insufficient supply he orders more, but not until an unnecessary amount of time and money has been lost.

In concrete road construction or similar work, trucks are usually hired and paid for at so much for each batch or each batch-mile. This price is influenced by the length of the haul. If too many trucks are on the job they eliminate one another by the income they receive, or they are discharged by the contractor's superintendent. If not enough trucks are present, more are hired. It is only by careful planning and management on the part of the superintendent that a sufficient number of trucks are present to supply the mixer. This is true in other lines of construction where hauling units are employed.

In power-shovel grading where the haul is relatively long, large-sized hauling equipment is frequently used. In shorter hauls, teams and wagons, light trucks, and iron mules are often used. With the larger units extreme care must be taken when deciding on the proper number of hauling units. It may appear that an insufficient number of units is employed, yet, if another large-sized unit is added, it may not be economical; but whether large or small units are employed it is necessary to keep all of them busy. If they are not sub-contracted, it is necessary, if possible, to shift part of them to longer hauls when an over-supply of units is available. The foregoing statements concerning power-shovel grading may be applied also to the elevating grader.

In any construction it is absolutely essential to maintain a good haul road. After a rain the road is soft and becomes rutted and full of "chuck" holes. One or more men with the proper equipment should be assigned as rapidly as possible to the constant maintenance of the hauling road. Failure to observe these rather elementary requirements probably explains why the average contractor loses more money through shortage of hauling equipment than from any other controllable cause.

EFFICIENCY IN PRODUCTION

These time losses would not be so serious if the cost of operation varied with the output. Unfortunately, however, the daily or hourly cost of operating almost any kind or type of modern well-equipped road-building outfits is nearly the same whether production is at only a fraction of its capacity or at its maximum rate. In concrete road construction, for instance, the operating cost of a modern outfit, exclusive of materials, but including all labor, supplies, overhead, and rental value of the equipment, is seldom less than $40 per actual working hour and may readily exceed $50 per working hour. Frequently, however, a plant with an operating cost of, say, $45 per working hour and an actual capacity of 45 to 48 batches per hour is, at least at times, producing only 20 batches per hour. The operating cost is then $2.25 per batch, whereas capacity production would cut the cost to about $1.00 per batch.

It is thus quite plain that the contractor should do everything in his power to increase production. Frequently, the observation is made by engineers that it is no concern of the engineer whether the contractor makes or loses money. With but few exceptions, the cheapest method of construction is by contract, and a prosperous contracting fraternity is essential if work is to be well done. Increased efficiency in contract work will ultimately be reflected in lower bid prices. Probably without realizing it, highway contractors are bidding on a basis of doing work at a little more than 50% efficiency. Some of the contractors are doing a profitable business and some are not. The proper procedure for all of them, in order that they may do a more profitable business, should be more efficient management rather than increased prices.

The Bureau of Public Roads, therefore, believes it is working in a worthy cause and in a fertile field when it undertakes to assist highway contractors in increasing production. To this end the Bureau has met with gratifying success. In some few cases it has been possible nearly to double the rate of production with little or no addition to the cost of operation. These cases are exceptional, of course, but still fewer are the cases in which it has not been possible to show some improvement in the rate of production without entailing a corresponding increase in the cost of operation.

Having in mind the causes of low production, attention then logically turns to their elimination or reduction. Frequently, it is an easy matter to remedy some of the causes which have been most influential in keeping down the rate of production. For example, it is not uncommon to find that in power-shovel grading improper placing of the hauling units imposes an unnecessarily long boom-swing which increases the bucket cycle by as much as 5 sec. In good common excavation the elimination of this one delay alone may mean an increase of from 20 to 25% in the rate of production, an item surely worth while.

As a specific example of what was accomplished in the summer of 1929 by ascertaining losses and bringing their meaning to the attention of the contractor the following is given:

On a Portland cement concrete paving job 12.41 miles long and 18 ft. wide, with a thickness of 8 in., 6 in., and 8 in., the pavement was being laid during the summer months. Standard, modern equipment was used throughout, and production studies were conducted between August 19 and October 5. The daily record of production and time losses before these studies were

TABLE 5.—Example of Summary Sheet of Weekly Reports of Time Losses

State, Tennessee. County, Haywood. Project No., S. A. P. 358-A.

Type of project, Portland cement concrete. Duration of study, from August 19, 1929, to October 5, 1929. Weeks, 7.

Major equipment, 27-E paver. Auxiliary equipment, two-batch trucks.

Week ending, 1929	Production, in square yards	Total available working time, in hours	Major Delays			Minor Delays			Total of all delays, Column (6) + Column (9)	Time major equipment operated at 100% efficiency. Column (3) to Column (10)	Total of all Class B delays, Column (5) + Column (8)	Possible operating time eliminating Class B delays, Column (11) + Column (12)	Possible production eliminating Class B delays	Percentage of over-all efficiency, $\frac{\text{Column (11)}}{\text{Column (13)}}$
			Class A, in hours	Class B, in hours	Total, in hours	Class A, in hours	Class B, in hours	Total, in hours						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Aug. 24....	10 981	68.0	1.75	3.15*	4.90	0.55	10.75*	11.30	16.20	51.80	13.90	65.70	13 910	78.8
Aug. 31....	12 052	68.0	2.00	2.43	4.43	0.00	11.00*	11.00	15.43	52.57	13.43	66.00	15 100	79.8
Sept. 7....	9 982	67.5	7.5‡	4.25	11.75	1.53	11.50*	13.03	24.78	42.72	15.75	58.47	13 700	72.8
Sept. 14....	8 422	68.0	23.08§	1.00	24.08	1.65	4.06*†	5.71	29.79	38.21	5.06	43.27	9 540	88.4
Sept. 21....	12 812	68.0	1.58	0.00	1.58	0.00	(12.53*)	12.53	14.11	53.89	12.53	66.42	15 800	81.2
Sept. 28....	10 426	68.0	17.50†‖	0.00	17.50	1.08	4.33*	5.41	22.91	45.09	4.33	49.42	11 426	91.2
Oct. 5....	6 920	67.5	(33.58‡)	0.00	33.58	3.40	0.72	4.12	37.70	29.8	0.72	30.52	7 058	98.2
Total														

* Truck supply and operation delays comprise greater part of this total: September 21, 6.84 hours; September 28, 2.43 hours.

† Water supply delays comprise greater part of this total: September 21, 5.09 hours.

‡ Rain delays comprise greater part of this total.

§ Wet sub-grade delays comprise greater part of this total.

‖ Move pipe line delays comprise greater part of this total.

started, is not given. Paving operations had been in progress for one month prior to August 19 and during this time only 1.53 miles, or 16 157 sq. yd. of pavement were laid. The biggest day's run during this period was 330 batches, or 991 lin. ft. of pavement. On only one or two days did the outfit lay more than 800 lin. ft. of pavement. During the seven weeks that the study was in progress, 6.76 miles of pavement were laid. During the week ending September 21, 2 168 batches, or 6 406 lin. ft. of pavement were laid. The biggest day's run was 411 batches, or 1 226 lin. ft. of pavement in 11.5 hours. The summary in Table 5 shows that the efficiency of the job increased from 78.8% for the week ending August 24, to 88.4% for the week ending September 14, and to 98.2% for the week ending October 5 (see Column (15)).

Another example of increased production with unit costs decreased is shown in the results of studies made on Wisconsin Federal-Aid Project No. 460-A. This was a Portland cement concrete paving job, 14.7 miles long, in Dane and Columbia Counties. Studies were conducted between June 3 and August 24, 1929.

The contractor's major operations were analyzed and the cause and extent of all time losses determined. This information was furnished to the contractor who used it as a guide in his efforts to increase production. The trend of operations during the studies is shown in Table 6.

TABLE 6.—PRODUCTION STUDIES, FEDERAL-AID PROJECT NO. 460-A IN 1929

Item No.	Description	June	July	August
	DISTRIBUTION OF TIME LOSSES, IN PERCENTAGE OF TOTAL TIME SPENT			
1	Mixer (operator)	2.3	2.8	1.7
2	Mixer (mechanical)	1.4	1.2	1.2
3	Truck (operation)	4.6	4.0	3.7
4	Truck (supply)	13.0	7.6	1.6
5	Installing joints	0.9	4.0	2.1
6	Sub grade	2.1	2.0	1.9
7	Water supply	2.4	1.4	2.2
8	Moving plates (track for trucks)	1.0	0.7	0.3
9	Materials	1.3	0.0	1.3
10	Miscellaneous	2.3	1.7	2.4
11	Dry batches	0.0	0.7	1.0
12	Total delays	31.3	26.1	19.4
	COSTS			
13	Cost per square yard	$1.44	$1.30	$1.26
14	Reduction, in percentage, of June costs	....	9.7	12.5
	MISCELLANEOUS DATA			
15	Number of days in which concrete was poured	24	23	15
16	Average length of working days, in hours	11	11.5	11.5
17	Total production, in square yards	36 636	40 285	30 381
18	Average production per day in which concrete was poured, in square yards	1 526	1 749	2 025

The greatest cause of lost time, as shown by the studies, was due to a shortage of hauling equipment. This was pointed out to the contractor early in the studies and, as a result, he realized the importance of supplying a sufficient number of hauling units for full mixer production. This delay was materially reduced. The reduction in unit cost is due to the increase in production resulting from the elimination of delays as a result of the studies.

The foregoing are only a few of many studies in which production was materially increased without a proportionate increase in unit costs. It is not always possible to increase production at the rates shown in the examples given, but on the other hand failures to record some improvement are rare.

CONCLUSIONS

A general observation of the following rather elementary suggestions should do much to increase the efficiency with which the average road construction outfit is operating:

1.—The rate of production is so dependent on the performance of the key equipment as to demand that it always be maintained in good mechanical condition, and to forbid the employment of any other than a high-grade operator.

2.—Auxiliary or dependent equipment which is of insufficient capacity or which cannot maintain the rate demanded by the operations of the key equipment, should not be tolerated. This applies also to the operators of such equipment.

3.—The use of equipment or methods of operation which cannot readily be adapted to operating efficiently in connection with the assembly or outfit as a whole, or to sustain economically the production rate of the key equipment, should be avoided.

An assembly of men and equipment, no matter how fine, is practically worthless as a producing unit until supplied with a directing head, because on the managerial ability which it exercises depends not only the rate of production obtainable from any construction outfit, but also the unit cost at which such production is secured. A man with real managerial ability, free to run the job as a whole, is essential to high rates of production, low unit costs, and best possible profits; but if the construction industry is to reap the full advantages which management offers in translating designs and plans into realities, it must also develop a spirit of mutually helpful co-operation and understanding between those who manage and those who plan and supervise. The relation of costs and quality are so interwoven that the ideal adjustment can never be attained except through mutual co-operation and assistance.

DISCUSSION

A. E. HOLCOMB,[4] M. AM. SOC. C. E. (by letter).—This paper is both interesting and instructive. The cost of work is frequently affected materially by its location and its timeliness with reference to other work. Unfortunately, economy of construction is seldom the controlling factor in the passing of a bond issue; the volume of noise made by the respective interested parties is usually much more potent. In fact, the psychology of selfish interest is quite often as important a study as the engineering problems involved.

The politician makes the welkin ring with his pre-election grief over the hardships of his constituents and promises to raise them "out of the mud" if he is elected. After election, that locality containing the most votes gets the improvements regardless of whether the work in that section forms a link in a system that will serve the greatest number or, due to location, is more expensive to build than it would be later after another section has been constructed.

Insufficiency of funds and the necessity of giving some relief frequently compel the construction, for instance, of an inadequate drainage system or other project that later must be reconstructed and enlarged at an increased unit cost. Many other reasons of expediency often affect ultimate costs. These are practical considerations that have to be borne in mind. Mr. Allen, however, refers to controllable factors subject to job management.

One of the most frequent decisions that a contractor is called upon to make, is whether to use the equipment he already owns, which is not properly adapted, economically, to the particular job in question, or to purchase new and proper equipment of a size and type to handle the work to the best advantage. In other words, will the increased earnings of new equipment exceed the depreciation and interest charges of the idle equipment? Also, will he be justified financially in depleting his liquid assets at that time by purchasing the needed new equipment? It may be more expedient, or even more profitable, in the end to use his old and less efficient equipment.

On the other hand, because of an inherent, natural reluctance on the part of every one to scrap anything of value, contractors will frequently keep and use obsolescent equipment that is still mechanically in good condition, but "out of step" with newer methods and faster operation. A study of relative output and cost of operation will often show that it would be more profitable to run the obsolescent equipment over the bank—use it as a part of the fill—and buy more modern equipment.

There is often a wide range between the cost to own and the cost to operate. One can well afford to pay considerably more than the average price for equipment, provided the increased earnings within a reasonable length of time will exceed the sum of the increase in first cost, depreciation, and interest charges.

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[4] Sales Mgr., Koehring Shovel and Crane Div., National Equipment Corporation, Milwaukee, Wis.

Among the major delays, Mr. Allen mentions 10.5% lost time due to repairs to the shovel and other equipment. However, he does not state the nature of the other equipment. The writer's records covering eleven shovels and dragline excavators, over a period of six years, show an average of only 7.8% delay for repairs of sufficient duration to cause a record on the daily report blanks. Such delays are affected by so many things that may or may not be controllable by management that it is impossible, in estimating, to depend on any given range of minimum and maximum repair time figures that would not be excessive. For ordinary and usual conditions, a range of 8 to 10% would be adequate where equipment is maintained in reasonably proper condition. The closer the inspection and the better the upkeep management, the less will be the amount of upkeep delays.

Speaking generally, the more common causes of major delays are unbalanced equipment and lack of foresight in planning operations. For example, in rock work the drilling and blasting are frequently allowed to tie up expensive equipment through the use of the wrong size or kind of drills, lack of sufficient drilling equipment, or improper placing and character of charges. On other work, as Mr. Allen very clearly indicates, one of the greatest causes of loss is the difficulty in balancing and dispatching the hauling equipment.

The real savings, however, are usually to be effected through close personal supervision to keep down the minor, or Class B, delays, since they are largely subject to management control. Mr. Allen refers to improper placing of hauling equipment with relation to the shovel on grading work as an influential factor in the production rate. The writer would suggest in connection with this, the study of the proper width of bench and height of cut to give the most economical results in bank work through deep cuts.

Pre-analysis of jobs, with determination of general plan and layout of work before bidding, supplemented afterward with time studies and distributed costs, is the great need, instead of so much blind and hungry bidding with ever-springing hope and faith in Providence to show the bidder how to cut his costs in two after the award has been made.

CHESTER MUELLER,[5] ASSOC. M. AM. SOC. C. E. (by letter).—This paper forcefully calls attention to the avoidable waste prevalent in the highway construction industry and the efforts of the U. S. Bureau of Public Roads to reduce it. As shown by Mr. Allen, this branch of engineering is in need of, and is entitled to, the painstaking form of research heretofore confined to the realms of pure science and only recently applied to manufacturing processes by the Mechanical Engineering Profession through the media of time studies.

A few comments on some of the features of the construction industry that were treated by the paper might not be amiss, particularly if they are examined in the light of municipal paving projects.

Policies.—Wisely, Mr. Allen foregoes a lengthy discussion of policy. While "economic necessity and priority" are proper topics for engineering consideration, nevertheless, a pavement is frequently constructed at the insistence of an organized group of citizens—when the cost is not to be locally assessed—

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[5] Prin. Asst. Engr., Dept. of Public Affairs, Newark, N. J.

in spite of the lack of necessity for it. Likewise, a much needed pavement may be opposed by abutting property owners when such improvement is to be paid for by assessment against such owners. Preparation of data clearly will be expected of an engineer, but where local sentiment or the pocketbook is involved, the exercise of well-founded engineering judgment will be denied him.

Contracts.—The paper, dealing essentially with the execution of work, implies that an eingineer must do his share to foster efficient methods. He can do this in the very beginning by making the time of letting a paving contract seasonable. Receiving bids during the period of uncertain markets, for work under adverse weather conditions, and with unreasonable time clauses, will certainly affect bid prices and will have considerable bearing on subsequent construction administration.

By making the "Information to Bidders" clear and complete, the contractor may better formulate his plan of operation. Adequate plans of the work will also aid better planning.

It is possible that, even though they are poorly executed, the foregoing items may not prevent efficient administration; but if they are properly constituted, they will certainly be conducive to it.

Subsurface Work for Utility Companies.—In cities, probably the largest single problem handicapping successful administration, is the subsurface work that public utility companies perform in advance of paving. Such work is essential, and in order to reduce the disruption of traffic to a minimum, it often becomes necessary for a paving contractor to begin his operations while underground work is also in process. In co-ordinating this work and in expediting the subsurface work the engineer can do much to obtain for the public and the contractor a paving job efficient in time and in cost. Subsurface work by utility companies in the public streets and highways certainly tends to resist efficient highway construction administration, and in the extensive field of endeavor which it covers, much is yet to be found to aid the paving industry.

Specifications.—Engineers are generally prone to let existing specifications do service from year to year without any changes other than the addition of clauses occasionally, as local experience dictates.

Specifications should be kept modern by elimination and alteration as well as by addition. In other words, complete revision may be desirable after existing specifications are scrutinized in the light of recent researches and practices. As Mr. Allen states clauses that require a mixing time for concrete in excess of the time actually required with modern equipment, result in wasteful administration. In some communities and localities, clauses of that nature may persist in specifications for years after they have been proved obsolete by competent investigators.

Design.—Failure to consider the utility of roads and approaches, and the sequence of contracts, as illustrated in the paper, is indicative of poor design and thwarts efficient administration. The writer does not fully subscribe, however, to the author's suggestion that paving design should attempt to provide for the 100% economic use of the contractor's equipment. In establishing lines and grades it should be only a minor influence since in most present-day designs the cost to the public of operating vehicles over a road after the improvement is completed, is as important as the initial construction cost. In the illustration given by Mr. Allen, the average loss of about $46.63 per day, due to irregular requirements for hauling units, may add about 4 cents per sq. yd. to the cost of the pavement, whereas changes in grade may cost far more than that, if not in construction, possibly in the operating cost of the vehicular traffic using the road. A design permitting a practically uniform daily number of hauling units and the efficient use of a contractor's equipment, is, of course, highly desirable provided other essential elements are not sacrificed. To determine to what extent the design of a road can be influenced, without adding to its cost, requires considerable additional research, and the efforts of the Bureau of Public Roads in this direction will be most helpful.

If the provision of suitably located disposal areas is considered as a function of design and as the task of the engineer rather than that of the contractor, then the design of the improvement, in so far as these areas are concerned, is most vitally associated with efficient administration. Most contracts leave the question of disposal of excess earth to the contractors, which is one reason why an alert contractor who plans his work can under-bid his competitors.

Equipment.—Equipment is given an important place by Mr. Allen, a place that it well deserves. Serviceable equipment with readily available spare parts, used and maintained in a proper fashion, determines more than any other factor, the speed and smoothness with which a paving job can be completed. The forceful and convincing sales data of leading equipment manufacturers prove to be a boon to contractors in compelling them to keep abreast of the times.

Personnel.—Many contractors are short-sighted with respect to the employment of labor on the maintenance of approach and utility roads. Use of men in what is sometimes referred to as "non-productive" work frequently has a direct bearing on the efficiency with which a contractor can operate his equipment and almost always indirectly contributes to it.

Time Losses.—In this part of Mr. Allen's paper, the most important and most convincing results of the studies conducted are brought to the reader's attention. It sets an example that many engineers could emulate, in some measure, in the study of highway jobs under his supervision. Any improvement that the engineer can introduce to the contractor will certainly benefit the contractor and ultimately the public that the engineer serves.

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Paper No. 1831

CONSTRUCTION MANAGEMENT[1]

BY RICHARD HOPKINS,[2] ESQ.

WITH DISCUSSION BY MESSRS. EDWARD W. BUSH, AND A. P. GREENSFELDER

SYNOPSIS

Management of construction work may well be classified into two departments: (1) The objective art of bidding for the contract, planning the work, and bringing it to completion; and (2) the subjective art of combining the elements in Part (1) to discover leaks in profit due to avoidable time losses in the various steps involved. This paper is confined to the first of these departments.

Standards of success in conservative bidding are discussed, as are also the ideal qualifications of the most important men on any job, namely, the superintendent, master mechanic, and the foreman. The paper deals, specifically, with the best attitude to hold concerning the maintenance of a construction schedule.

Intelligent bidding is the first requisite of good management. Anything that is well bought is half-way along the road to being well sold. A "lemon" on the day of the letting scarcely ever becomes a "ripe plum" when the final ledger accounts of the job are closed.

Each organization should appreciate what class of work it is best equipped to handle. Doing work in one locality only is generally not a good policy. Because two jobs lie close to each other is no reason why the same organization should attempt to build them, if they are dissimilar. Furthermore, in some years, the volume of work let may be small and the competition severe in the region where the contractor has been doing business. Prices, however, may be much better in another section. The contractor must meet the market price, but he can always choose the market in which to sell.

As a general rule, a man who obtains more than 20% of the work on which he bids is not a good bidder. He is bidding below the market. Good bidding is more often found among those who are awarded contracts for fewer than 10% of their bids. It is much better to let machinery depreciate, and parts of an organization stay idle, than it is to lose cash from a bank account. From two to six months before a job is planned to be finished the contractor should be looking over prospective work and putting in occasional bids on desirable jobs. There is no more pitiful figure in the construction world than the man who has a smoothly running and efficient organization on a particular job and waits until a week or two before it is finished to start bidding on new work. The result is likely to be bad from three angles: The new work may not be the right kind, it may not be obtained in time, and the price may be much too low.

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[1] Presented at the meeting of the Highway Division, New York, N. Y., January 16, 1930.

[2] Richard Hopkins Co., Albany, N. Y.

The head of an organization should be introspective and critical of the quantity of work that he can have under way at one time. Most contractors do too much work rather than too little. The financial strain and lack of proper attention, both to general planning and small details, are generally harmful to profits under such conditions.

It is assumed that the bidder makes up an estimated field cost which is based on his own records. It is further assumed that to this field cost he adds his estimated over-head which includes such items as interest, depreciation, insurance, repairs, workmen's compensation, camps, office, and freight and loading charges, in moving equipment to the job. This estimated total should be close to his real total cost if labor, the elements, and the supervising engineers all act in a perfectly natural way. At this point in making up the bid, the question of contingencies enters. What are the soil conditions? How much harm will a week of rain do to progress? Will some other contractor or some public service company be working on the job at the same time, and, if so, what are their records for co-operation? Are there bridges or other structures on the job that will slow down the entire program? The bidder should judge what these contingencies will amount to in dollars and add them to the previous total.

The bidder is then ready to decide what percentage of profit or what lump-sum profit he will add to his total estimated cost. The sum added for profit will depend on whether or not the bidder is "hungry." It will depend on whether or not the job is in a market that has more than its share of labor troubles. It may vary through a wide range, depending on whether it is largely a labor job or largely a material job, and on the definiteness or indefiniteness of the engineering inspection that can be expected.

Planning the Work

It is assumed that the contractor has a head office that directs all field operations on the different jobs and handles all administrative work. A superintendent may be assigned to a particular job because he is the only one available, or because the job is suited to him and his organization. If possible, the latter reason should be the one for giving him the job. The superintendent is the all-important man on the job. His personality is of vital importance, very much more so even than that of the contractor himself. It is not always possible to have a halo around the head of the superintendent, but all construction men are learning to appreciate that nothing pays better dividends than sentiment. The superintendent should be a real leader, a friendly czar, a man who knows all the letters of the construction alphabet. Above all, he should be a man who knows how to plan and how to get performance and who knows how to dovetail the different operations into a completed whole.

Every foreman should have such ability as to command the respect of his men. They may find fault with his temperament and with some of his methods, but they must respect his ability if he is to be successful. They must feel that he knows his job. There should be a foreman on each particular operation; there should be definite responsibility for each task. If only two men are working by themselves, one of them should have the responsibility for getting the task done in the right way. Co-operation among the foremen is a prime necessity. Personal feeling and jealousy between two foremen should never be allowed to interfere with the progress of the work.

A large number of contractors are coming to realize that it is wise to combine their engineering and accounting work on a job, and place both under the control of a trained engineer. There is a close inter-relation between accounting and engineering, and nobody should be able to interpret cost figures better than an engineer. His costs will be up to date, and, with his help, it will be possible to "lock the barn door before the horse is backed out of the stall." On account of his training, the engineer is able to suggest changes in methods and personnel and even in equipment when the proper costs are not being obtained. The items of extra work, special measurements, necessary departures from plans and specifications—all have engineering and accounting woven so closely together that it is best to have them handled by one man.

The selection, purchase, and care of equipment were never so important as at present. In 1910, or thereabouts, 85 to 90% of the cost of a grading job was the payroll. About 1915, when hard-surfaced roads came to be the National vogue, the payroll was still more than 50% of the total cost. Now (1931) all the major operations and most of the minor operations on a job are done by machinery. Steam machinery has been greatly improved, the automotive engine has taken a preponderant hold on all construction equipment, and the Diesel engine is making steady progress. No construction organization can long face the present competition without being machine-minded.

Next to the superintendent the master mechanic is the most important man on the job. He should be an all-round mechanic, of course, but with the present-day machinery he should be particularly competent on automotive work. He should have enough help to make quick repairs in the event of the inevitable break-down. Most of the minor repairs and some of the major ones should be made nights, Sundays, and rainy days. If the master mechanic is not given authority to employ and discharge machine operators, he should at least be consulted by the superintendent on both employment and discharge. When the purchase of any piece of equipment is contemplated, his opinion should be asked on three questions: (1) Is the machine staunchly built? (2) Is it well designed? (3) Is a large repair bill or a small repair bill being bought with the machine?

He should see and approve all repair bills before they are paid. He should have authority to keep on hand whatever repair parts his experience teaches him are most often needed. He should be "construction conscious"; and should appreciate thoroughly the earning power of each machine and the consequent loss when the machine is not in operation.

The importance of the master mechanic's duties has been particularly stressed, but no good master mechanic can show results unless he has the help of experienced and competent operators on the different machines. Good foremen can show only indifferent results when key machines have poor operators. The machine cannot function without a hand to guide it. Often the major portions of big jobs are run with old equipment that has long since been charged off, but in the hands of good operators it is earning dividends every working day. On the other hand, a visit to a similar job with new machinery, manned by incompetent operators, often reveals numerous breakdowns which cripple any effective schedule, and, at the finish of the job, there remains a pile of junk and a depreciation charge double or even triple what a reliable estimator had a right to expect. Physicians and hospitals exercise a big part in every-day life to keep the public in good health. The construction physician is the master mechanic and when he and his internes—the machine operators—all know their job, the piece of construction under their hands is in good financial health.

Saving one shovel, one roller, or any other piece of equipment by speeding up the production of those units working on the job, cuts down the machinery charge against a job. Loading a job up with unnecessary machinery increases the machinery charge. Moving, loading, and freight charges on heavy machinery constitute a cost that is not sufficiently appreciated. Frequently, if a heavy machine is to be transported a long way and the work required of it will take only a few weeks, it is cheaper to rent a machine close at hand.

A point about which there is a sharp difference of opinion among contractors is that of maintaining a schedule. All will agree that a job that drags is seldom profitable. One school of thought upholds the ideal of 110 cu. yd. of earth per hour for the small shovel and 50 cu. yd. of concrete per hour for the paver, and there is the continual drive to equal these figures on a few big days. A few trucks and a few teams as well as a number of men are added to the payroll. The other school of thought insists on an orderly progress of the work along the line of a well-planned schedule. There are no big days, but there are some good weeks and some wonderful months. All work proceeds at an orderly consistent pace. The advocates of this method argue that the job that is built the quickest is not always built the cheapest. Final unit cost is the yardstick, to decide on the relative merits of the arguments put forth by these two widely divergent points of view.

There is an old axiom that no well-conceived military operation can be a success without good infantry. No construction operation can hope to be successful without good laborers, who are satisfied with their tasks. Their length of service is measured in years—not in days or weeks—and turn-over is small. Their camps are clean, their food is good, they know that the superintendent is a fine boss and always treats them square. They boast a little about how much better they are than the outfit on the other side of the ridge. Underlying it all is the firm conviction that they know their job and that the boss knows his.

DISCUSSION

Edward W. Bush,[3] M. Am. Soc. C. E. (by letter).—No doubt members of the Society feel greatly indebted to the author for this instructive paper. It deals with questions about which little has been written, but concerning which much can be said.

The author points out that "it is much better to let machinery depreciate, and parts of an organization stay idle, than it is to lose cash from a bank account"; also, that "most contractors do too much work rather than too little. The financial strain and lack of proper attention, both to general planning and small details, are generally harmful to profits under such conditions." The files of any large surety company will disclose cases without number which affirm the correctness of these statements. Bankers as well as surety underwriters recognize over-extension as one of the leading causes retarding the financial advancement of contractors. Analytical studies of the technical records and financial information of contractors, such as the surety underwriters are constantly making in their daily work, will disclose that advancement in financial strength is more likely to occur when the contractor makes a gradual increase in activities, rather than when he is trying to become a big contractor within a short period of time. The author has stressed the importance of a good superintendent and organization, and these are best developed by the contractor through a moderate expansion.

The contractor must be an optimist to be successful, as the business demands that he have the utmost faith in his own ability to perform, and in many cases it is this optimism which leads him to over-extend. Especially is this likely to occur after getting the "breaks" on a few contracts and thus "cleaning up" some nice profits. Under such conditions the business of contracting looks very attractive indeed and it is only a strong-minded person who can resist the temptation to load up with about everything that can be taken. Many contractors have traded net quick assets or working capital for new construction plant to find to their sorrow that there was nothing left with which to operate. A contractor may be "plant poor" and not fully realize it until he tries to keep all of it at work.

A. P. Greensfelder,[4] M. Am. Soc. C. E. (by letter).—There is a fair difference of opinion, particularly in this time of depression, as to how much a construction organization should specialize or diversify its bidding. Specialization is seemingly the order of the day. An organization with specialized machinery and crew, theoretically, should have the advantage over one that does not. On the other hand, unless there is a great deal of that particular class of construction work continuously at hand in the community, it means that the organization must enlarge its territory and follow such class of work wherever it may be found. Diversification, on the contrary, permits a smaller radius of travel and guards against having all one's organization and capital tied up in one particular type of work.

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[3] Engr., Aetna Casualty & Surety Co., Hartford, Conn.

[4] Pres., Fruin-Colnon Contr. Co., St. Louis, Mo.

Mr. Hopkins' comments are sound relative to percentage of work secured from the volume bid. Post-bidding comments always indicate that the marginal loss between the low and the next low bid is the hardest money to regain actually. The statement is correct that one should not undertake too great a volume of work for the capital of the company. Over-eagerness in this respect has too often proved fatal.

It would seem vital that contractors, together with engineers, should evolve practical measuring rods for determining the relation between volume of work, quick and deferred assets, and construction equipment for various classes of work. The Bureau of Contract Information, Incorporated, set up jointly in Washington, D. C., by surety companies, is determining the performance record of contractors throughout the country. It should presently have sufficient experience rating to serve as a guide in making these determinations.

Unbalanced bidding is another feature which should be guarded against. One is likely to feel that his practical experience will warrant certain manœuvering of unit bid prices. There is usually enough gambling in construction without this and it is only warranted through the viewpoint of equity of the owner when the engineer's quantities are themselves unbalanced (which is, unfortunately, quite often the case). Engineers too frequently guess the quantities of excavation classification instead of predetermining them by making borings.

All estimates should include equipment rental or there will soon be no equipment to rent to oneself. Depreciation of construction equipment necessarily goes on at a very rapid rate. Obsolescence also plays its part. American inventors are daily producing new equipment so much more efficient than previous machinery that construction equipment five years old is frequently obsolete.

In planning the work, contractors should realize the value of time studies. The current studies of the U. S. Bureau of Public Roads indicate that these are indeed worth while. Lost time can be minimized when one realizes the extent.

The increasing prevalence of including engineers in contractors' organizations is noteworthy. Such engineers, on the other hand, must realize the need of being practical as well as theoretical. Regardless of cost brains are the cheapest purchase an organization can make. The analytical mind of the engineer is essential to proper cost-keeping, provided such cost-keeping furnishes, daily, the current trends of costs.

Mr. Hopkins very truly stresses the need of having on hand repair parts of machinery. Time lost awaiting repairs is expensive when a large organization is delayed. On the other hand, contractors are too apt to think in terms of labor-saving devices. They should analyze their jobs and consider machine-saving costs by not over-equipping them. Too frequently they use theoretical costs and realize to their sorrow that they have not found the operator whose hands fit the levers, or whose mind is sufficiently quick thinking or his body sufficiently active to make the operation of that machine profitable.

The preparation of advance schedules of work is essential. Such schedules serve as guides to the superintendent in the field as well as to the "boss" at his desk. They also furnish the necessary check as to the average performance. One hears of the wonderful day's over-run when records are broken, but not of the many days of under-run when machinery or organization is broken.

Construction management is rapidly becoming more and more scientific. It takes mind and matter jointly to produce smooth running construction jobs.

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Paper No. 1832

SOIL MECHANICS RESEARCH[1]

BY GLENNON GILBOY,[2] JUN. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. J. C. MEEM, H. DE B. PARSONS, JACOB FELD, JOHN R. JAHN, C. H. EIFFERT, A. A. EREMIN, F. N. MENEFEE, E. G. WALKER, D. P. KRYNINE, J. STUART CRANDALL, FRANK ALWYN MARSTON, GEORGE PAASWELL, JOHN H. GRIFFITH, H. E. GRUNER, F. KÖGLER, AND GLENNON GILBOY

SYNOPSIS

In the fall of 1925, Charles Terzaghi, M. Am. Soc. C. E., joined the staff of Massachusetts Institute of Technology for the purpose of inaugurating advanced courses and establishing a program of extensive research in the field of Soil Mechanics. Under his direction a great amount of pioneer work was carried on, which served firmly to establish the importance of research in foundation engineering and allied subjects, and to define clearly the most promising lines of endeavor.

It is the purpose of this paper to summarize briefly the salient elements of this work. The problems described fall into two main groups: (1) Soil Physics, comprising studies of the composition and grain distribution of soils, permeability, compressibility, consolidation, compressive strength, internal friction, and cohesion; and (2) Soil Engineering, including investigations on the bearing capacity and settlement of foundations, on hydraulic-fill dams, on the frost heaving of highway sub-grades, and on the lateral pressure of earth against retaining walls.

The first group includes studies of the physical properties of soils and of the interrelation between these properties, with the ultimate object of understanding the nature of the effects observed in soils under various conditions. The second group comprises investigations of the behavior of soil as an engineering material forming part of a structure. The two groups are closely interrelated, inasmuch as a knowledge of the behavior of a soil mass connotes a knowledge of the physical properties of its component parts.

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[1] Presented at the meeting of the Structural Division, Boston, Mass., October 11, 1929.

[2] Asst. Prof. of Soil Mechanics, Mass. Inst. Tech., Cambridge, Mass.

In the last analysis, engineering problems control the trend of development of soil mechanics. A thorough knowledge of soil physics in itself is interesting and valuable from a scientific standpoint; but it is economically worthless unless it provides a means for answering the questions that puzzle the practicing engineer. It has been the aim of Dr. Terzaghi and his associates to keep this principle constantly in mind, and to plan the research work accordingly.

RESEARCH IN SOIL PHYSICS

Mechanical Analysis.—The oldest and most obvious method of soil analysis is the separation of a soil sample into fractions on the basis of grain size. The results of such an analysis are usually represented by a curve showing the weight of all grains smaller than any given diameter, expressed as a percentage of the total weight of the sample. A typical curve is shown in Fig. 1. Point *A*, for example, indicates that 30% of the sample is made up of grains smaller than 0.1 mm. in diameter.

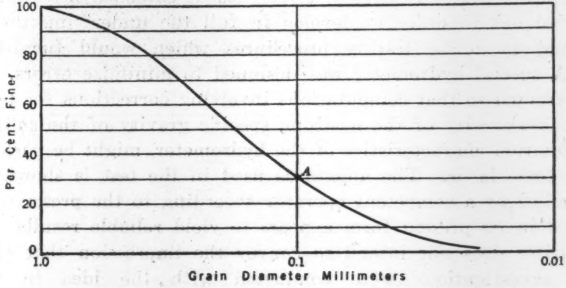

FIG. 1.—TYPICAL MECHANICAL ANALYSIS DIAGRAM

For coarser materials, such as sand and gravel, the analysis is made in a simple manner by passing the sample through a series of screens. In separating soils composed of grains which are finer than the openings in the finest screen available, one of the many sedimentation methods may be used. Such methods are based on the principle that the rate at which a sphere sinks in a liquid is directly proportional to the square of the diameter of the sphere.

Considerable research on sedimentation has been done at the Massachusetts Institute of Technology with a view to standardizing equipment and procedure. Simple sedimentation in jars was first used, but it was found tedious and inaccurate. Attention was next directed to Wiegner's method.[8] In this system the soil suspension is contained in a long vertical cylinder, which communicates near its lower end with a small stand-pipe. The latter contains distilled water. Since the specific gravity of the suspension is greater than that of water, the level in the stand-pipe is higher than that in the cylinder. As sedimentation proceeds, the difference in level decreases, approaching zero as the suspension clears. A plot of the difference in level against time affords a means of constructing graphically a mechanical analysis curve for the sample.

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[8] "Über eine neue Methode der Schlämmanalyse," von G. Wiegner, *Zentralblatt* für die gesamte Landwirtschaft, Bd. I (1920), No. 1.

The principal source of error in the simplest form of the apparatus is the difficulty of making an accurate visual measurement of the level difference, which is initially about 1 cm. Gessner has devised a photographic apparatus which is reported to give good results.[4] It was felt, however, that something simpler and less expensive would be advisable. The scheme finally adopted was based on magnification of the level difference by means of an inclined gauge glass connected to the stand-pipe. After the apparatus had been carefully calibrated and charts had been worked out to simplify the computations, the method was used successfully for a considerable length of time.

An even simpler method was later developed from ideas suggested by the investigations of Professor G. J. Bouyoucos.[5] The basic principle resembles Wiegner's, in that it involves time observations on the variation in density of a soil suspension as the particles settle. Instead of a piezometer, however, an immersion hydrometer is used to measure the density. In co-operation with the United States Bureau of Public Roads extended investigations have been carried out in order to develop in full the underlying theory of the method and to devise testing procedures which would furnish accurate results. A special hydrometer was designed to minimize errors, and charts were constructed so that computations involving corrections for temperature, density, and viscosity of the medium, specific gravity of the solid particles, and the flotation characteristics of the hydrometer, might be performed with relatively little labor. The apparatus used in the test is shown in Fig. 2. When handled by a competent operator according to the prescribed schedule the method in its present form appears to yield reliable results.

The writer does not intend to convey the impression that these rather elaborate investigations were conducted with the idea in mind that mechanical analyses could be expected to furnish conclusive information as to the character of a given soil. It was fully recognized that other factors, such as grain shape, chemical and mineralogical composition, structure, density, and previous geological history, are of such importance that any analysis based on size alone will constitute at best only a rough indication of the properties of the material. Nevertheless, a grain-distribution curve is of considerable assistance to an experienced soil analyst, and the writer feels that the time and effort expended in the investigations have been fully justified.

Limits of Consistency.—Among the many routine tests proposed for cohesive materials, the limit tests made by A. Atterberg[6] were considered of sufficient importance to warrant investigation. A sample of clay containing

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[4] "Der Wiegnerische Schlämmapparat und seine praktische Anwendung," by H. Gessner, *Kolloid Zeitschrift*, Vol. 38, 1926.

[5] "The Hydrometer Method for Making a Very Detailed Mechanical Analysis of Soils," by G. J. Bouyoucos, *Soil Science*, September, 1928. References to previous publications will be found under this article.

[6] "Die Plastizität der Tone," von A. Atterberg, *Internationale Mitteilungen für Bodenkunde*, 1911, Heft 1.

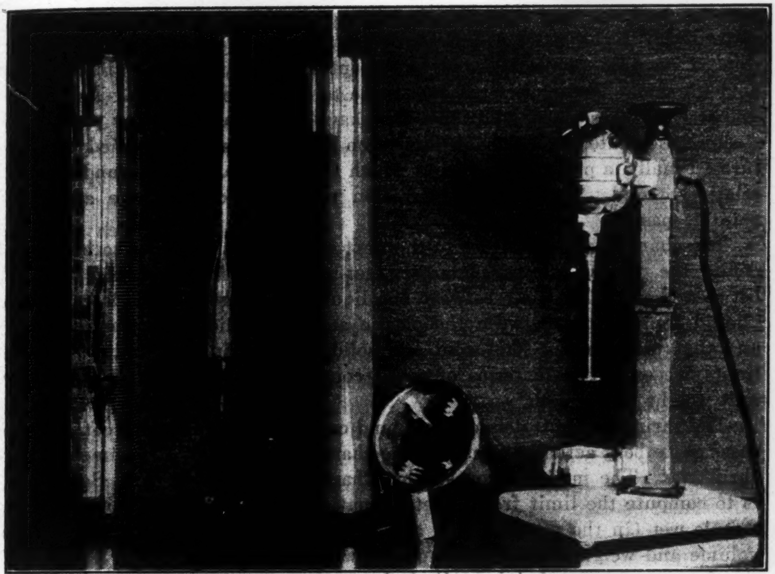

FIG. 2.—APPARATUS FOR HYDROMETER TEST

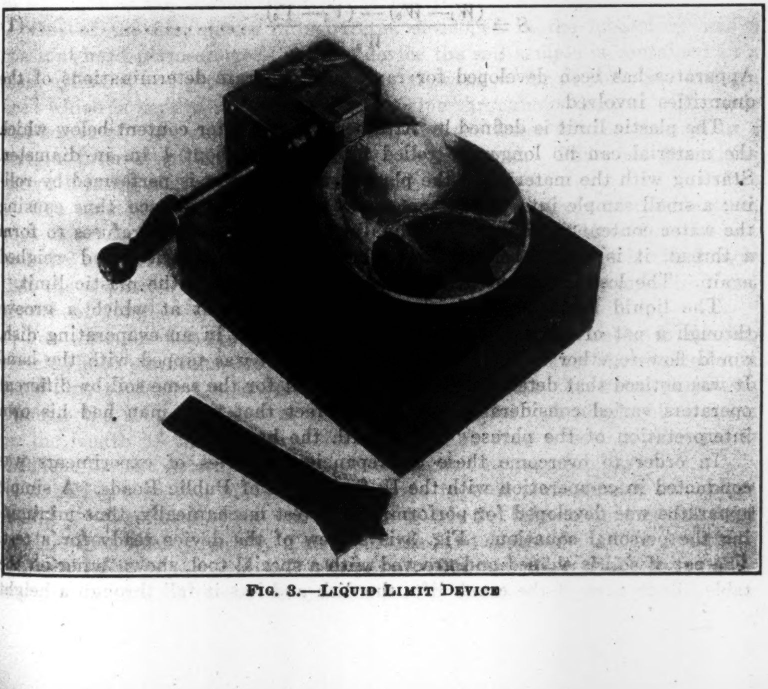

FIG. 3.—LIQUID LIMIT DEVICE

excess water acts like a liquid; it will not hold any definite shape unless restrained. If the water escapes by evaporation, the clay shrinks, loses certain of its fluid properties, and passes into the plastic state. It will now hold its shape, but offers little resistance to a change in shape. As further evaporation takes place, additional shrinkage occurs, and the clay passes into a semi-solid state, in which an alteration of shape is accompanied by fracture. Finally, a point is reached at which the clay changes color and becomes relatively solid. Continued evaporation produces no shrinkage after this point is passed.

The change from one state to the next is gradual. In order to be able to define the various states, Atterberg set up and defined certain "limits" between them. The limit between the liquid and plastic states is called the "liquid limit"; that between the plastic and semi-solid, the "plastic limit"; and that between the semi-solid and solid, the "shrinkage limit." All the limits are expressed in terms of water content, that is, weight of water per unit weight of dry soil.

The shrinkage limit defines itself, and can be determined with comparative ease. Atterberg actually measured the change in length of a specimen during shrinkage. The method proposed by Charles Terzaghi, M. Am. Soc. C. E., is to compute the limit from a determination of the weight and volume of a sample wet (in the plastic state) and bone dry. If V_1 and W_1 represent the volume and weight, respectively, of the wet sample, and V_0 and W_0 those of the dry sample, the shrinkage limit is,

$$S = \frac{(W_1 - W_0) - (V_1 - V_0)}{W_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Apparatus has been developed for rapid and accurate determinations of the quantities involved.

The plastic limit is defined by Atterberg as the water content below which the material can no longer be rolled into threads about $\frac{1}{8}$ in. in diameter. Starting with the material in the plastic state, the test is performed by rolling a small sample into threads on an absorbent paper surface, thus causing the water content to decrease continually. When the sample refuses to form a thread, it is placed between watch glasses, weighed, dried, and weighed again. The loss in weight divided by the dry weight is the plastic limit.

The liquid limit was defined as the water content at which a groove through a pat of the soil about 1 cm. high, formed in an evaporating dish, would flow together at the bottom when the dish was tapped with the hand It was noticed that determinations of this limit for the same soil by different operators varied considerably, due to the fact that each man had his own interpretation of the phrase "tapped with the hand."

In order to overcome these discrepancies, a series of experiments was conducted in co-operation with the U. S. Bureau of Public Roads. A simple apparatus was developed for performing the test mechanically, thus minimizing the personal equation. Fig. 3 is a view of the device ready for a test. The pat of soil is shaped and grooved with a special tool, shown lying on the table. Each turn of the crank lifts the dish and lets it fall through a height of 1 cm. Instead of trying to work the soil exactly to the liquid limit, it has been found advisable to determine, for several water contents, the number of blows required to close the bottom of the groove over a distance of $\frac{1}{2}$ in. The higher the water content, the fewer the number of blows will be. A curve of water content against number of blows is plotted, and the water content corresponding to twenty-five blows, as read from the curve, is taken as the liquid limit.

Limit tests are in somewhat the same category as mechanical analyses, inasmuch as their results have only an indirect bearing on engineering problems. They are valuable, on account of their simplicity, in a rapid preliminary grouping of a set of soil samples prior to performing more elaborate tests.

Permeability.—One of the most important problems in soil mechanics is the study of the percolation of water through soils. In the normal case, with laminary flow throughout, percolation takes place in substantial accordance with Darcy's law, which states that the rate of percolation per unit area is directly proportional to the hydraulic gradient. The constant of proportionality is known as the "coefficient of permeability" of the soil. Symbolically, if Q represents the quantity flowing through an area of soil, A, in time, t, under a hydraulic gradient, i, Darcy's law may be represented by the equation,

$$Q = k\ i A\ t \qquad (2)$$

in which, k is the coefficient of permeability.

One of the first pieces of apparatus developed in the laboratory was a constant-head permeameter. In this device the soil sample is contained in a glass cylinder, and provision is made for allowing water to percolate under a head which is kept at a known constant value throughout the test. The area and length of the sample are determined, and the water flowing through in a certain interval of time is collected and measured. Thus, all quantities except k in Equation (2) are determined.

The constant-head apparatus is suited only to relatively permeable soils, such as sands. In finer soils, the rate of percolation is so slow that the test occupies an extremely long period of time, during which there is the danger that the character of the soil may be altered by organic decomposition and growth of fungi.

For tests on fine-grained materials, it was found advisable to develop variable-head permeameters. In these devices the feed-water is contained in a stand-pipe of known area just above the sample. As the water percolates through the soil, the level in the stand-pipe drops. A single observation on the length of time required for the head to drop a certain distance is sufficient for the determination of the coefficient of permeability.

A half-section and half-elevation of one type of permeameter are shown in Fig. 4. The level of the tail-water in which the base is immersed is kept constant by a waste overflow. The apparatus is so designed that the resistance to flow offered by the stop-cock, the porous disk, and the various channels is negligible compared to that of the soil itself. Hence the height of the water surface in the stand-pipe, measured from the tail-water surface, represents the energy available to drive the water through the soil.

Let *a* represent the area of the stand-pipe; *A*, the area of the sample; *L*, the length of the sample; and *h*, the head at any time, *t*. Then, in the next succeeding interval of time, *dt*, during which the head drops a distance, *dh*, conditions may be represented, according to Equation (2), by the relation,

$$-a\,dh = k\frac{h}{L}A\,dt \qquad (3)$$

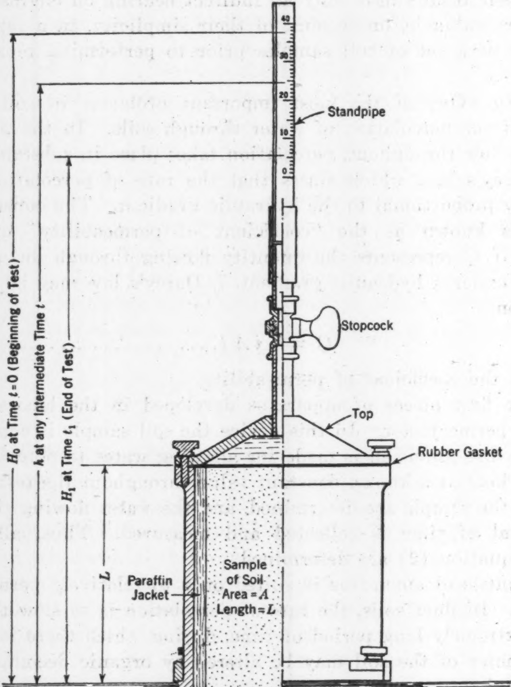
40
30
20
10
0
Standpipe
Stopcock
Top
Rubber Gasket
H_0 at Time $t=0$ (Beginning of Test)
h at any Intermediate Time t
H_1 at Time t_1 (End of Test)
L
Paraffin Jacket
Sample of Soil Area = A Length = L
Porous Disc
Base
To Waste

FIG. 4.—VARIABLE HEAD PERMEAMETER

If the measured head at any time, t_1, is H_1, and at some later time, t_2 is H_2, Equation (3) may be integrated between these limits, giving the following general value for the coefficient of permeability,

$$k = \frac{aL}{A}\,\frac{1}{t_2 - t_1}\,\log_e \frac{H_1}{H_2} \qquad (4)$$

Thus, the coefficient may be evaluated from the observed data by a single computation.

By proper selection of the various component parts, apparatus of this type may be applied to a wide range of soils. The porous disks are commercially available in three grades, fine, medium, and coarse. If the soil is too porous to permit the use of the coarse-grade disk, a perforated rubber disk covered with wire mesh may be used. The stand-pipe may be of any convenient size, ranging from a small pipette, for fine-grained soils, to graduated tubes $\frac{3}{4}$ in. in diameter for coarser materials. Very pervious sands may be tested by an extension of this method, in which the entire apparatus consists of a long glass tube with a screen across the bottom. The sample is placed in the lower part of the tube, and the upper part serves as a stand-pipe.

Another important application is found in the direct testing of the permeability of soils in their natural state. Methods have been developed whereby undisturbed samples may be removed from the earth and encased in paraffin at once, thus being well protected until the tests can be made. A portion of the sample may be placed in the permeameter in its original paraffin jacket, as indicated in Fig. 4, so that it is subjected to a minimum of handling in preparation for the test. The method is at present applicable only to soils which possess at least some degree of cohesion.

Measurements of permeability by observation of the rate of capillary saturation have received considerable attention. Extended studies were made on the relation between the permeability of a soil and the rate at which water is drawn up into a dry sample of the material contained in a vertical glass tube with its lower end immersed. As a result of these studies, the method was successfully applied to field control of the embankment material of an earth dam.[7]

Certain fundamental defects associated with the vertical capillarity test, which necessitate careful and extensive preliminary calibration, seem to disappear in the horizontal capillarity test, first proposed by Mr. Arthur Casagrande. The dry pulverized sample of soil is contained in a glass tube with a screen at one end and a vented stopper at the other, as indicated in Fig. 5. The tube is immersed in a horizontal position in a shallow depth of water, and observations are made upon the rate at which the line of capillary saturation proceeds through the soil. If x represents the length of saturation in time, t, it may be shown theoretically that,

$$x^2 = t \sqrt{k F \frac{1+e}{e}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

A plot of observed values of x^2 against t is, therefore, a straight line, of which, $\sqrt{k F \frac{1+e}{e}}$ is the slope. The void-ratio, e (ratio of volume of voids to volume of solid) can readily be determined by measuring the volume and the dry weight of the sample. Thus, if the factor, F, is known, the coefficient of permeability, k, is readily found.

Experiments made on a wide range of soil types give a mean value of F of 100 000, when t is in minutes and k is in centimeters per second.

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[7] "Soil Studies for the Granville Dam at Westfield, Mass.," by Charles Terzaghi, M. Am. Soc. C. E., *Journal*, New England Water Works Assoc., Vol. XLIII, No. 2.

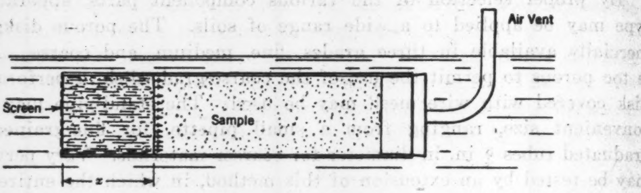

FIG. 5.—HORIZONTAL CAPILLARITY TEST

Deviations from this mean are ± 50% for ordinary soils. The value of F is smallest for very mixed-grained soils and largest for very uniform soils.

For a rapid approximate determination of the permeability, the result of a capillarity test, using a value, $F = 100\,000$, will in many cases be sufficiently accurate. For closer determinations, a series of calibrating tests should be made in order to furnish a value of F applicable to a particular range of soil types. The method appears at present to be reliable, and by far the simplest ever devised, especially for rapid field determinations.

***Compressibility and Consolidation.*— The compressibility of a soil is studied by measuring its change in volume produced by a change in load. Under the pressures usually encountered, the compression of the solid material is negligible, so that a decrease in volume of a soil mass under load is almost entirely due to decrease in the volume of voids. Results of compressibility tests are usually represented as curves of void-ratio (ordinates) against unit pressure (abscissas). The curves have a characteristic shape as shown in Fig. 6.**

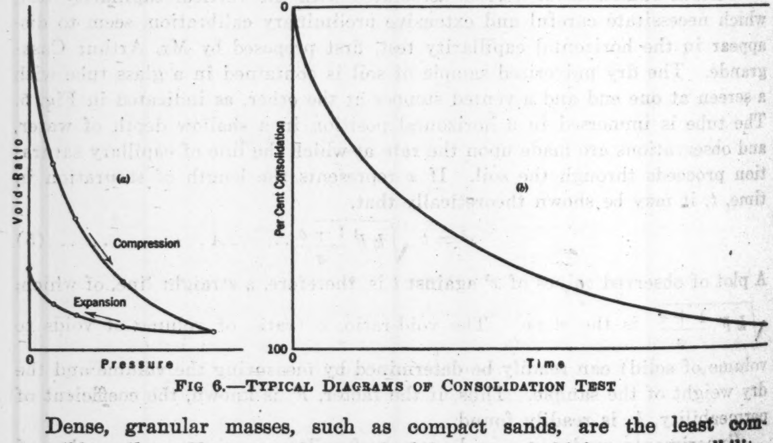

FIG 6.—TYPICAL DIAGRAMS OF CONSOLIDATION TEST

Dense, granular masses, such as compact sands, are the least compressible soils. Clays are characterized by a much higher compressibility. Dr. Terzaghi[8] has shown that the compression diagrams of sands and clays

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[8] "Erdbaumechanik," pp. 86 and 90.

could be represented by the same logarithmic equations, the only difference being in the scale of void-ratio.

A rational explanation for the high compressibility of clays was furnished by a study of the compressibility of mixtures of sand and mica.[9] Both were granular, non-cohesive materials, and exhibited none of the characteristic properties of clay; yet it was found possible to select a mixture of sand and mica such that the compressibility characteristics of typical clays could be reproduced in detail. The researches showed definitely that the shape of the grains has an extraordinary influence on this important property of soils.

If a coarse, dry, granular material is placed under load in such a manner as to prevent lateral displacement, a decrease in void-ratio takes place almost instantaneously. A quantity of air corresponding to the change in volume is forced out, but there is practically no resistance to its escape. If the air is replaced by water, that is, if the mass is saturated, the change in volume is necessarily accompanied by a forcing out of some of the water. Since the resistance offered by coarse-grained soil to the passage of water is relatively small, compression is again almost instantaneous. If the soil is fine-grained, however, it may offer considerable resistance to the escape of the excess water. The compression in this case does not take place instantaneously, but occupies an appreciable period of time. The gradual compression of fine-grained, saturated soils under load is known as consolidation.

Consolidation tests are made with an apparatus which has been gradually developed from the primitive device first used by Dr. Terzaghi.[10] A half cross-section and half elevation of this instrument is shown in Fig. 7.

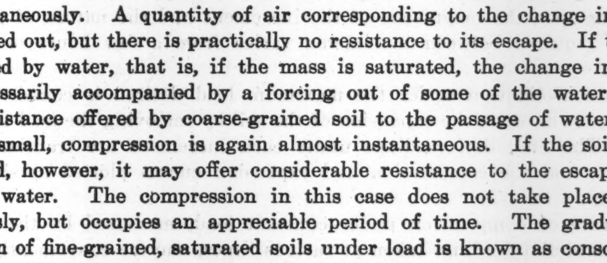

FIG. 7.—CONSOLIDATION APPARATUS

The soil sample is retained within a cylinder (2) and is compressed by means of load applied to a piston (5). The aligning pin (8) holds the piston immovable within the cylinder while the test is being prepared, and is removed when the first load is applied. The amount of compression produced by a given increase of load is measured by two Ames dial gauges reading to 0.0001 in. Free passage of water at the two faces of the sample is provided by the porous disks in the base and cylinder. By filling the base stand-pipes and the interior of the piston with water, the test may be performed with the sample entirely submerged, so that a state of complete saturation exists at all times.

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[9] "The Compressibility of Sand-Mica Mixtures," by Glennon Gilboy, Jun. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, p. 555.

[10] "Erdbaumechanik," p. 83.

In performing the test, loads are applied in definite increments. While any given increment is acting, the dial readings are noted at convenient intervals of time. The dials are allowed to come to rest—that is, consolidation is allowed to proceed to completion—before the next increment is added. Depending upon the magnitude of the increment and the nature of the soil, the time required between increments varies from a few hours to one or two days.

After consolidation under the maximum load desired is complete, it is customary to measure the expansion of the soil by removing the load in increments. The final dial readings under each increment are translated into void-ratio, and the compression and expansion curves plotted as shown in Fig. 6(*a*).

The rate of compression under constant load, measured by the various intermediate dial readings taken at various times, may be conveniently represented by a curve such as that of Fig. 6 (*b*). The total compression under the given increment is taken as 100%, and the intermediate dial readings, expressed in percentage of total movement, are plotted against the corresponding values of time. Although only one curve is illustrated, actually as many curves may be obtained as there are increments of load applied. The curves are all of the same shape, but do not, in general, coincide.

The consolidation diagrams offer a means of determining the permeability of very fine-grained soils. The quantity of water which is forced out of the sample by a given increment of load depends upon the compressibility of the soil, and the rate at which the water can escape depends on the permeability. Therefore, the rate of consolidation, as indicated by the time-consolidation curve, depends upon both these factors. The compressibility is determined from the pressure-void-ratio diagram, thus leaving the permeability as the only unknown to be evaluated mathematically.[11]

True clays are relatively compressible and impermeable. Their time curves are flat and well defined, permitting accurate computation of the constants. Border-line materials, such as mixtures of very fine sand and silt, are much more permeable and much less compressible than clays. Consequently, the time curves are rather steep in the first stages, and the dials move so rapidly that accurate readings are difficult to obtain. To overcome this difficulty an arrangement has been devised whereby direct permeability tests can be performed on the sample while the compression test is going on. One of the stand-pipes in the base of the apparatus is closed, and the other is connected to a vertical pipette tube hung above the device in such a manner that water from the pipette flows into the base, up through the sample, into the piston, and out of the holes previously occupied by the aligning pin. The action is analogous to that of the variable-head permeameter previously described.

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[11] "Principles of Final Soil Classification," by Charles Terzaghi, M. Am. Soc. C. E., *Public Roads*, Vol. 8, No. 3, May, 1927.

The combined consolidation and permeability test furnishes a means of checking the accuracy of the permeability coefficients computed from the consolidation curves. The close agreement obtained in two tests selected at random is shown in Fig. 8.

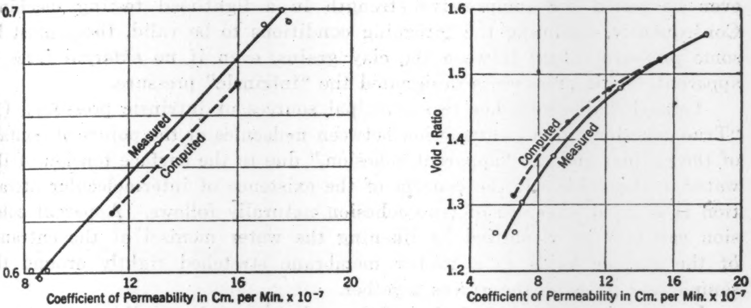

FIG. 8.—COMPARISONS OF DIRECT AND INDIRECT PERMEABILITY DETERMINATIONS

Inasmuch as the consolidation test yields information upon the amount of compression which a soil sample will undergo under load and upon the rate at which this compression proceeds, it ranks as one of the most important soil tests ever devised. In addition to its purely scientific value, it has many practical applications, among which may be mentioned the analysis of settlements of buildings underlaid by clay strata, and the consolidation characteristics of hydraulic-fill cores.

Internal Friction and Cohesion.—One of the most important physical properties of soil, especially with regard to stability and lateral pressure, is the shearing resistance. Attempts have been made by numerous investigators to develop methods for evaluating this property, but the problems involved are still far from a final solution.

The underlying conceptions of the nature of shearing resistance are not difficult to visualize. If two soil grains are pressed together, the resistance against sliding of one grain on another is equal to the pressure between the grains multiplied by the coefficient of friction of grain on grain. The shearing strength of an accumulation of grains is the sum of a great number of such individual sliding resistances. Therefore, in order that a mass of soil shall exhibit shearing strength, two conditions must be fulfilled: (1) There must be pressure between the grains; and (2) there must be a definite coefficient of friction between the grains. If one of these quantities is zero, the mass behaves like a liquid, no matter what the value of the other factor may be.

A familiar illustration of this statement is furnished by the so-called "quicksand" phenomenon. The dynamic energy of a current of water flowing upward through a mass of sand tends to counterbalance the downward force of gravity. If the energy of the water is sufficiently great to counteract entirely the action of gravity, the effective weight of each individual sand particle is zero. Hence, the intergranular pressures are reduced to zero, and the mass behaves essentially like a liquid.

On the other hand, a stiff clay exhibits the properties of a solid. A cylindrical specimen cut out of the mass retains its shape unaided, and may even be tested for compressive strength in a light-load testing machine. Consequently, assuming the foregoing conditions to be valid, there must be some pressure acting between the clay grains, even if no external force is apparent. This pressure is designated the "intrinsic" pressure.

Terzaghi[12] distinguishes two principal sources of intrinsic pressure: (1) "True cohesion," due to attraction between molecules at the points of contact of the grains; and (2) "apparent cohesion," due to the surface tension of the water in the voids. If the concept of the existence of intermolecular attraction is granted, the idea of true cohesion naturally follows. Apparent cohesion can best be visualized by likening the water menisci at the entrance of the surface voids to a rubber membrane stretched tightly around the sample and pressing the grains together.

It should be noted that neither of these phenomena is peculiar to clay alone. They are usually associated with clay, because their effects are most pronounced in very fine-grained material; but they can occur to a certain degree in any type of soil. For example, a moistened sand can be moulded into a cylinder, because the surface tension of the water produces some apparent cohesion. If the cylinder is immersed in water, the surface tension disappears, and the cylinder disintegrates. True cohesion also exists in a mass of sand, but its magnitude is vanishingly small on account of the relatively small number of points of contact per unit area.

True cohesion depends upon a great number of factors, such as grain size, grain shape, structure, density, and previous geological history of the deposit. It also appears to be influenced by chemical factors and electrolytic effects about which relatively little is known. Apparent cohesion is a function of many such variables as those mentioned, and, in addition, is subject to wide variations produced by changing moisture conditions. Thus, it is fairly evident that considerable time and study are required before the complications involved in the shearing resistance of soils can be properly understood.

The first method used at Massachusetts Institute of Technology in the study of these effects was based upon tests of the compressive strength of clay cylinders. When the sample fails in compression, shear cracks become visible on the surface, as indicated in Fig. 9(*a*). According to Mohr's theory,[13] the angle, α, between two such shearing planes is $90° - \phi$, wherein, ϕ is the angle of internal friction of the material. With the compressive strength, p_c, also determined from the test, it is possible to construct a rupture diagram for the specimen, as shown in Fig. 9(*b*). The intrinsic pressure is represented by the distance marked p_i. The vertical intercept marked *c* is the cohesion as defined by Coulomb. Thus, it is theoretically possible from this test to determine the important strength constants of the material.

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[12] "Erdbaumechanik," p. 94.

[13] "Technische Mechanik," by Otto Mohr, Ernst, Berlin, 1906.

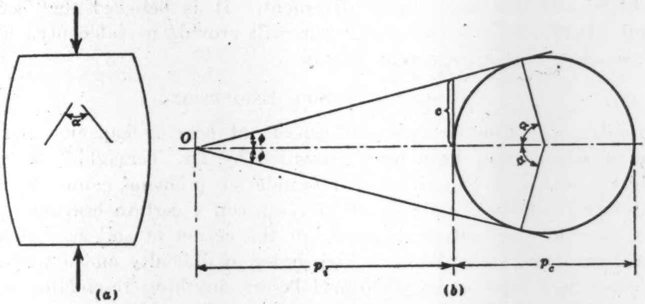

FIG. 9—RUPTURE OF A CYLINDER ACCORDING TO MOHR'S THEORY

Many investigators have endeavored to measure the shearing resistance of soils by subjecting samples to direct shearing stress. An important contribution in this line was made under Dr. Terzaghi's direction in 1929. Under a co-operative agreement with the New England Power Construction Company, a large and elaborate shearing device was constructed, based on the principle of applying a known vertical pressure to the soil sample, and measuring the amount of horizontal force required to produce failure in shear. A brief description of the apparatus and an outline of the results of the first series of tests are available in published form.[14]

Subsequently, a second machine was constructed, embodying certain of the principles of the first device, but designed to be simpler, more accurate, and more flexible. A close-up of one of the shearing units and a general view of the apparatus are shown in Figs. 10 and 11. A distinct advantage of this machine is that six units may occupy the table simultaneously, whereas only one rig for producing horizontal shear is required.

The apparatus has been in constant use since its construction, and has yielded new and valuable information upon various types of soil. It is impossible to formulate any general conclusions from the data at present available. Certain interesting indications, however, may be enumerated tentatively. A sand in a dense state exhibits a definitely higher angle of friction than the same sand loose. The difference appears to decrease with decreasing grain size. Shear failure in sand is accompanied by marked deformations of the mass normal to the failure plane. Saturation does not decrease the angle of friction of sand. The clays thus far tested exhibit considerably higher angles of friction than were expected. In stiff clays possessing high true cohesion it is difficult to develop the full shearing strength of the material;

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[14] *Engineering*, June 13, 1930.

the soil appears to fail progressively, due to local concentrations of high stress, which in turn are due to the inability of the material to undergo sufficient plastic yield to equalize the stress distribution. If this hypothesis proves to be correct, it may have an important bearing upon the question of the stability of slopes cut into soil of this character.

Opinions as to the physical nature and effect of shearing resistance in soils have been, and still are, widely divergent. It is believed that continued research along the lines outlined herein will provide useful contributions to the knowledge of this important subject.

RESEARCH IN SOIL ENGINEERING

Foundations.—The fundamental aspects of bearing capacity and settlements of foundations have been presented by Dr. Terzaghi.[15] It may be readily seen from his exposition that foundation problems cannot be resolved into the simple task of assigning to a given soil a certain bearing value per unit area. On the contrary, a study of the action of soil as a supporting material involves mechanics of a high order of difficulty and a vast amount of research will have to be performed before anything resembling a simple solution can come into being.

Studies of the load-settlement characteristics of footings resting on sand have shown that even in this material wide variations are to be expected. The same sand exhibits quite different properties according to whether it is loose or dense. Under loads applied to the surface of a loose sand, a settlement of ½ in. was produced by a load of about ½ ton per sq. ft.; when the same material was tested in a compact state, the same settlement occurred under a load of about 5 tons per sq. ft. Results of this type indicate the need of a more accurate classification of sand, especially with regard to density, than is usually found in foundation codes. Corroborating previous evidence, the tests showed that the settlement under a given unit load, not too near the ultimate, is practically independent of the size of the loaded area.

A promising method for field exploration of sand deposits was successfully tried on the Houston-Essex Subway, in New York City, in co-operation with the contractors, the Corson Construction Corporation. The variations in the force required to press a steel cone into the soil were measured, and the results were correlated with a series of loading tests performed at various levels. The penetration device appeared to be very sensitive to changes in the density of the deposit, and these changes were also reflected in the loading tests. Similar investigations made later at the Massachusetts Institute of Technology, under controlled conditions, agreed very well with the field explorations. A distinct advantage of the method is that it permits of testing the soil in place, thereby avoiding the errors which are associated with sampling operations. Descriptions of the apparatus and procedure and outlines of the results thus far obtained have been published.[16]

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[15] "The Science of Foundations—Its Present and Future," by Charles Terzaghi, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 270.

[16] "Die Tragfähigkeit von Pfahlgründungen," by Charles Terzaghi, M. Am. Soc. C. E., *Die Bautechnik*, 1930, Heft 31 u. 34; and "Penetration Tests Give Bearing Power of Deep Subsurface Soils," by George Paaswell, M. Am. Soc. C. E., *Engineering News-Record*, April 2, 1931.

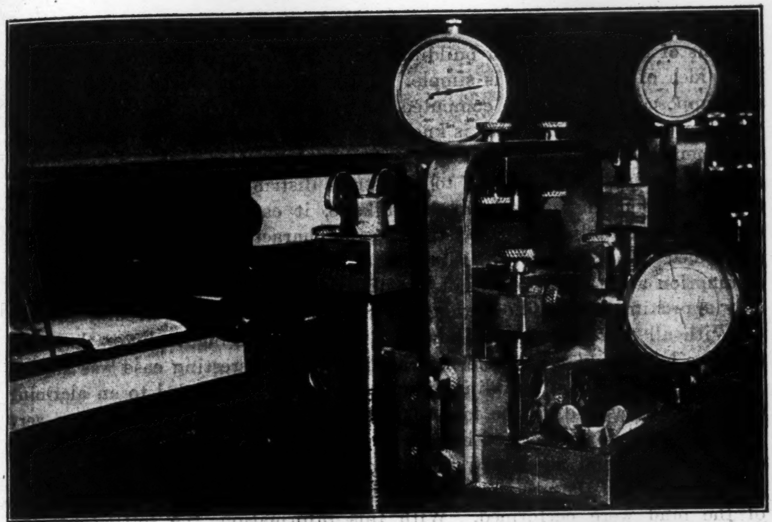

FIG. 10.—VIEW OF SHEARING UNIT

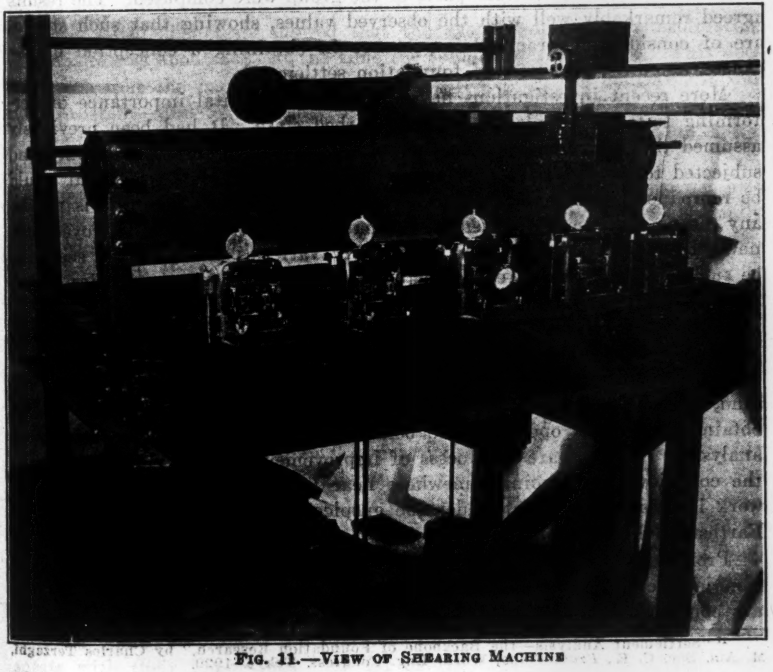

FIG. 11.—VIEW OF SHEARING MACHINE

A practical application of the theory of consolidation is found in the analysis of the settlement of buildings underlaid by deep strata of clay. The basic idea of the analysis is simple. If the stresses produced in the soil by superimposed loads can be computed, and the deformation that the soil will undergo under a given stress is known, it should be possible to determine the settlement of the structure, both in magnitude and in rate. Actually, the problem is very complex. As to the stress distribution, while the classical theory of Boussinesq is the best available, it can yield only approximate results. The analysis of the stress-strain characteristics of soils thus far performed includes only the simplest cases. Most important of all, some assumption as to homogeneity of the underground must be made, and this is only approximately fulfilled, even under the most favorable circumstances.

With all these drawbacks, however, it is encouraging to know that such analyses have yielded valuable information. An interesting case was studied in 1929.[17] A group of buildings founded on piles had settled to an alarming extent, notwithstanding the fact that the load applied to the piles was very low, compared with the rated bearing capacity. Borings disclosed the presence of a layer of mud about 120 ft. below the surface. A stress analysis was made, neglecting the presence of the piles, and consolidation tests on samples of the mud were performed. With this information, the settlements and rates of settlement of various parts of the group were computed. The results agreed remarkably well with the observed values, showing that such studies are of considerable practical importance in obtaining a conception of the causes, amounts, and rates of foundation settlements.

More recent investigations have established the vital importance of performing tests on samples in an undisturbed state. It had been previously assumed that if a sample were thoroughly remoulded to a liquid state and subjected to pressure, the previous geological history of the material would be reproduced on a laboratory scale. It seems, however, that this is not by any means the case. Apparently, the natural structure, once destroyed, can never be duplicated, and even in ordinary varieties of clay there seems to be sufficient structural effect so that the consolidation characteristics in the natural and the remoulded states are quite different.

In order to provide means for obtaining large undisturbed samples of cohesive material, a new type of rotary drill, operating without water, was designed. It has been used successfully in sinking a 6-in. hole through fill, sand, and clay to a depth of nearly 100 ft. The undisturbed samples obtained from this operation are being utilized in an elaborate settlement analysis. The tools are in process of improvement, with a view to bringing the cost down to a point somewhere near to that of wash borings. This work is being carried out under the auspices of the Special Committee on Earths and Foundations of the Society.

Progress along these lines would be greatly facilitated if every construction job were looked upon as a large-scale experiment. The data required are comparatively easy to collect. A schedule of loads, a series of bench-mark observations taken at various times, a geological profile of the underground, and a few soil tests on the different strata will furnish the material required for a settlement analysis. The additional cost of such observations would be a negligible item in the total cost of construction; the information obtained would be of highest value.

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[17] "Settlement Analysis—the Backbone of Foundation Research," by Charles Terzaghi, M. Am. Soc. C. E., *Proceedings*, World Eng. Congress, Tokyo, 1929.

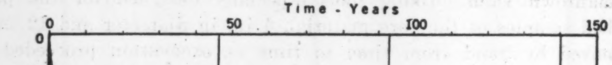
Hydraulic-Fill Dams.—In co-operation with the Miami Conservancy District, a thorough investigation of the properties and behavior of hydraulic-fill dams was started in the summer of 1927. A vertical shaft, 5 ft. in diameter, lined with concrete, was sunk to a depth of 80 ft. into the core of the Germantown Dam. Using tools especially designed for the purpose, undisturbed samples of the core material, 4 in. in diameter and 12 in. long, were removed by hand from time to time as excavation proceeded. The samples were sealed into iron containers, carefully packed, and shipped to the laboratory. Undisturbed samples, in 2-in. pipes, 2 ft. long, sealed at both ends, were removed from a drill hole near-by. Many unforeseen difficulties impeded the progress of the field work, and the District is to be commended for its persistence in mastering these difficulties and for the thoroughgoing manner in which it fulfilled its part of the program.

The experimental work included extensive tests on the physical properties of the soil, with emphasis on permeability, consolidation, and compressive strength tests. The tests showed that the average degree of consolidation of the core was about 25% at the time the samples were removed. This does not mean that the material was soft or yielding. On the contrary, it was very resistant—due to its relatively high angle of internal friction (about 27°)—and exhibited a compressive strength varying from 1 kg. to 2 kg. per sq. cm.

A mathematical analysis of the rate of consolidation of the core was also undertaken. The theory is extremely complicated, and the equations finally obtained present a forbidding appearance. It was found possible, however, to obtain from these equations a comparatively simple method of solution. Using this method, an investigator can construct a curve showing the progress of consolidation with time. The curve for the Germantown Dam is reproduced in Fig. 12. It is interesting to note that the dam was about seven years old when the samples were removed, and that the theoretical percentage consolidation corresponding to seven years is 25%, which checks exactly the results of the experimental investigations. It will be still more interesting to take samples from the core at the end of another ten or twenty years, to see whether the theory is still corroborated.

As studies on the general problem of the stability of hydraulic-fill dams progressed, it became increasingly apparent that a major rôle is played by the cohesionless, permeable outer sections of the dam, which the writer suggests might be designated by the term "shell." The impression was obtained that the emphasis usually placed upon the composition and behavior of the core had tended to obscure the functions of the shell to a degree incommensurate with their importance. As far as the core is concerned, the worst that can happen is that it does not consolidate at all. If the shell is sufficiently strong to hold the core in place under these circumstances, the structure should be stable.

In accordance with this idea, an attempt was made to evaluate the stress conditions in the shell with the core assumed to be in a liquid state. Mathematical analysis leads to rather complicated equations, but it was found possible to develop a graphical solution whereby the position of the weakest plane through the shell and the angle of friction which the shell material must develop in order to insure stability, can readily be determined. From the standpoint of safety, therefore, it appears that studies of the physical properties of the shell material, with a view to selecting proper constituents and designing an adequate section, are even more important than the tests usually made on the material of the core.



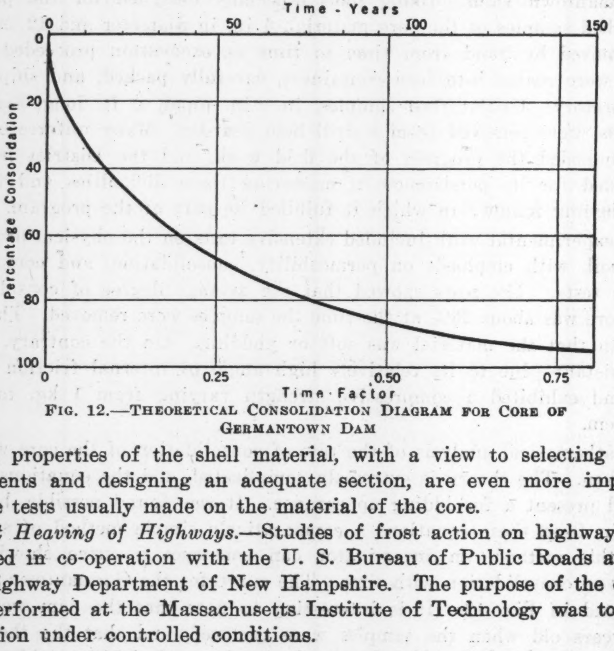


FIG. 12.—THEORETICAL CONSOLIDATION DIAGRAM FOR CORE OF GERMANTOWN DAM

Frost Heaving of Highways.—Studies of frost action on highways were conducted in co-operation with the U. S. Bureau of Public Roads and the State Highway Department of New Hampshire. The purpose of the experiments performed at the Massachusetts Institute of Technology was to study frost action under controlled conditions.

The experimental unit consisted of a wooden box, 3 ft. square and 3 ft. deep. On one side of the box was an observation well, 1 ft. square, communicating with the interior of the box at the bottom. A gravel filter was placed in the bottom of the box and well, thick enough to cover the opening between them. In this manner, the water level within the box could be controlled from the well, because the gravel prevented any soil from entering. The sides of the box were coated on the inside with a thin layer of grease, over which were laid small overlapping sheets of Manila wax paper. The object of the grease and paper was to minimize the retarding effect of friction between the soil and the sides of the box. The soil to be investigated was carefully tamped into the box, filling it level full. On top was placed a slab, 2 ft. 8 in. square, of concrete or macadam, representing the road surface.

Eight such units were buried to full depth in the ground, and observations on water level, depth of frost, heaving of the slabs, etc., were continued throughout the winter of 1928-29. Difficulties were experienced in holding the water levels constant; the boxes appeared to leak continuously, in spite of all precautions. Nevertheless, the experiment was successful in general, and paved the way for more detailed investigation of the phenomenon.

The researches showed that frost heaving cannot occur to any appreciable extent unless there is a continuous supply of water from below the frost level. The pressures associated with freezing water are due to the growth of ice crystals in layers which, in turn, is due to the travel of the large trihydrol molecules $(H_2 O)_3$, toward the focus of crystallization. As long as trihydrols are supplied, crystal formation will continue, and pressure will be exerted to lift the soil above the ice layer. If the supply of trihydrols is cut off, equilibrium will be established and the heaving will cease.

It has been observed that ice layers will grow in mixed fine-grained soils, while the water in the voids of clean sands will freeze without causing growth of the ice crystals beyond the usual increase in volume associated with the change in state. This behavior is due to the fact that water in very small spaces exhibits the property of remaining in a liquid state at temperatures considerably below the normal freezing point. Since the freezing temperature of water in very fine voids decreases with decreasing void diameter, it can be seen that the water will crystallize in the larger voids, while in the surrounding smaller voids it will remain in a liquid state, thus keeping supply channels open for the growth of the ice crystals.

As the frost effect penetrates deeper into the soil mass, another ice layer will start to form lower down. As the lower layer grows, it gradually cuts off the water supply, so that further growth of the upper layer is prevented.

The frost effect is thus shown to be controlled by three factors: First, the position of the ground-water level, or of veins and pockets of water which may serve as a source of supply for excess ice accumulation; second, the void-size distribution of the soil in which freezing occurs; and, third, the capillary properties of the soil between the source of water supply and the frost level.

Earth Pressure on Retaining Walls.—The most elaborate research project undertaken at the Soil Mechanics Laboratory was a large-scale experimental study of the lateral pressure exerted by earth behind a retaining wall, conducted in co-operation with the New England Power Construction Company. The project was laid out in detail by Dr. Terzaghi in December, 1928, and construction was started the following month. The apparatus was ready for operation in the latter part of April.

An isometric sketch of the device is shown in Fig. 13. The soil to be tested is contained in a concrete bin, 14 ft. square and 10 ft. high. The retaining wall, forming one side of the bin, consists of a structural steel frame, *F*, faced with a concrete slab, *K*. By means of counterweights, *Q*, at each end of the wall, the center of gravity of the entire wall assembly is made to lie in the face of the slab. The wall is suspended from hangers,

V_1 and V_2, the points of suspension being also in the face of the slab. Thus, when it is free, the wall hangs in a strictly vertical position; and any vertical component exerted by the earth on the face produces a direct pull on the hangers. The hangers are supported by scales above, which, in turn, rest on Columns C_1 and C_2 and on structural members supported by heavy columns

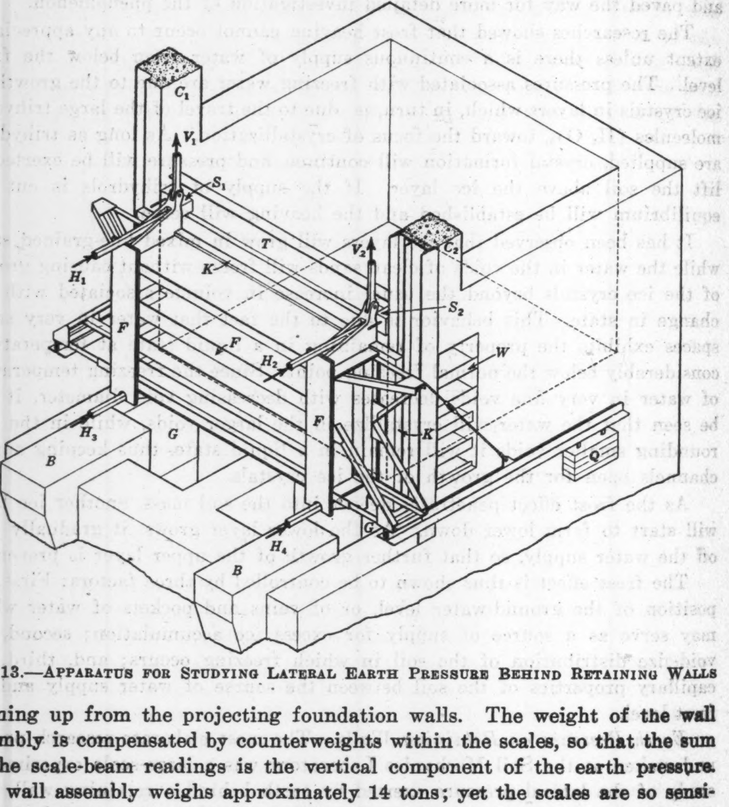


FIG. 13.—APPARATUS FOR STUDYING LATERAL EARTH PRESSURE BEHIND RETAINING WALLS

running up from the projecting foundation walls. The weight of the wall assembly is compensated by counterweights within the scales, so that the sum of the scale-beam readings is the vertical component of the earth pressure. The wall assembly weighs approximately 14 tons; yet the scales are so sensitive that they will record the weight of a man standing on the wall, with an error of ± 2 lb.

The horizontal pressures on the wall are balanced by thrust rods at the four corners, H_1, H_2, H_3, and H_4. The magnitudes of the thrusts are recorded by four scales. The two lower scales rest on concrete bases, B, in the substructure; the upper scales are supported by a rigid structural frame tied to the bin, the frame being supported by the same columns which carry the rear ends of the vertical scales. Thus, the bin, wall, and scales are included in a rigid unit which is independent of the building enclosing the apparatus. The unit is supported by a heavily reinforced concrete raft, 3 ft thick, resting on fifty piles, 20 ft. long. The space above the raft and below the floor-slab of the bin forms an inspection gallery, *G*.

A window, *W*, of 1-in. plate-glass in a structural steel frame, is inserted in one wall of the bin to allow inspection of the earth within. The tie-beam, *T*, is introduced to prevent excessive deflections of the bin walls under the lateral earth pressure. Four studs, arranged to bear on the inside faces of the thrust castings directly opposite the horizontal thrust bars, are operated by hand-wheels through gear-boxes, two of which, S_1 and S_2, are visible. The four horizontal scales are geared to move forward and backward. The combination of movable scales on one side and movable studs on the other gives a very accurate control of the horizontal movements of the wall. The two vertical scales can likewise be moved backward and forward to keep the hangers always vertical. The vertical scales are further provided with gearing to move the wall up or down. All wall movements can be controlled to within 0.0001 in.

The method of determining the magnitude, direction, and point of application of the earth pressure is practically self-evident. The vertical component is $V_1 + V_2$. The horizontal component is $H_1 + H_2 + H_3 + H_4$. The point of application is determined by the relation between $H_1 + H_2$ and $H_3 + H_4$.

In the design of the apparatus, an attempt was made to eliminate the shortcomings of previous installations of similar character, particular attention being given to the rigidity of the retaining wall and the possibility of moving the wall at will through an appreciable distance from the bin. The distinguishing features of the apparatus are as follows: The model wall is practically rigid, deflections under maximum pressure being less than 0.001 in.; the wall can be tipped or moved parallel to itself through a distance of more than 1 in.; and the joint between the bin and the wall is equipped with a water-tight rubber bellows or diaphragm so that the back-fill can be completely saturated without any leakage of water.

The apparatus has been designed with a view to making all manner of retaining wall tests. Surcharges up to 50 tons can be applied to the surface of the back-fill. The effects of different slopes of the surface of the back-fill can be measured. The action of filters and drains, as well as the effect of partial and complete saturation, can be studied. A brief summary of the results of the first series of tests has appeared in published form.[18]

ACKNOWLEDGMENTS

In addition to the acknowledgments and references embodied in the text of this paper, the writer wishes especially to mention the work of Mr. Arthur Casagrande, of the U. S. Bureau of Public Roads. Together with other important contributions, Mr. Casagrande has been chiefly responsible for the development of the liquid limit device, the hydrometer analysis, the horizontal capillarity test, the latest type of shearing device, and the frost-heaving experiments. Much valuable assistance has been obtained from the work of students, whose researches have been presented as theses to the Faculty of the Massachusetts Institute of Technology.

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[18] *Engineering*, May 30 and June 13, 1930.

DISCUSSION

J. C. Meem,[19] M. Am. Soc. C. E. (by letter).—This paper is a contribution of unquestioned value to the profession, particularly if or when the further research in connection with foundations and lateral earth pressures shall have been carried out.

The writer believes that it is practicable to establish definitely, through practical tests:

(*a*) The actual subsidence of various soils under varying loads at various depths; and

(*b*) The actual lateral pressure of soils at varying depths.

In order to conduct such tests so that the results will be of definite practical value, it will be necessary first, as to Test (*a*), to re-arrange tables for the so-called bearing values of soils so that they will show the expected settlement or subsidence under loading rather than the load which such soils may be expected to carry; that is, the writer advocates making a simple classification of soils, such as loam, clay, sand, gravel, and rock, with combinations of each, and testing them:

(*c*) Superficially to loads of 1, 2, 4, and 8 tons;

(*d*) At depths of 10 to 20 ft., unconfined to loads of 4, 8, 10, and 20 tons; and,

(*e*) At depths of 20 ft., or more, confined (as at the base of piles or small shafts) to loads of 20, 40, 60, and 80 tons per sq. ft.

The subsidence under the various loads should be noted in each case. The tests should be made:

(*f*) On normally dry soils; and,

(*g*) On saturated soils.

With proper co-ordination and co-operation the results of tests made in accordance with an established procedure could be collected from widely varying sources and tabulated; and, eventually, tables of real practical value, showing the subsidence of soils under loadings, could be permanently established.

In connection with Test (*b*), the apparatus for testing lateral pressure, the writer is convinced that the use of the apparatus described in the paper will mark an important step in the advancement of soil testing, but that it does not go far enough.

He is convinced that the most effective apparatus for making such tests would be a box, rigidly braced externally and having four or more sliding bulkheads confining one side; that is, in a box of a 20-ft. cube, there should be four superimposed bulkheads, 5 by 20 ft., to confine the soil on one face. These bulkheads or louvers should be of beam or box-girder construction, having arms extended and designed to slide independently of each other on cross-arms or tracks extended beyond the box. Four adjustable weights to each louver operating through pulleys and cords attached to the extending arms of the louvers, should be arranged to offset or balance the sliding friction, so that light pressure applied at any point in the louver face will cause it to move. Then, by filling the box with kiln-dried sand, and increasing the inside weights to balance the pressure of the soil, the exact pressure on each independent louver can be determined.

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[19] Cons. Engr., Brooklyn, N. Y.

The tests should be made with dry sand or dry disintegrated materials only, such as kiln-dried sand, pea gravel, ordinary gravel, broken stone, and, if desired, wheat, and buckshot. In the writer's judgment, any attempt to make these tests on soils or materials which are cohesive, or are made so by moisture, will be abortive and misleading. It will be a simple matter to co-relate the results of the tests on the dry materials with those having cohesive properties, by observing or measuring and comparing the "break" lines of each.

The writer believes that if exhaustive tests are made on the dry soils, as noted, they will establish conclusively the fact that arching properties and lateral pressures in soils are definitely measurable; and that they will further confirm the fact that the pressure of soil due to its lateral transmission is non-accumulative vertically, as in tunnels, although it increases laterally in direct ratio to the depth, being always at a maximum at a point slightly above the center of the vertical face.

In suggesting further procedure, which the writer feels may be of value, he does not wish to imply any lack of his appreciation of the paper itself, which is of great interest and value to the profession.

H. de B. Parsons,[30] M. Am. Soc. C. E. (by letter).—The writer, having read this interesting paper, would like to ask Professor Gilboy why temperature does not enter Equation (2), for finding permeability, or the quantity of water flowing through a soil? Perhaps the author had in mind experiments made in a laboratory where variations in temperature were small.

In a study, made by the writer, of water in a pervious soil in its natural state, the effect of temperature was very marked. In this study, the temperature range was about 30° Fahr. Furthermore, the formula used for determining the flow of water through filter sands shows that the velocity varies with the temperature.

Jacob Feld,[31] Assoc. M. Am. Soc. C. E. (by letter).—It is an ambitious program which Professor Gilboy outlines, and his reports of progress will be watched with great interest. It is a hopeful sign that he permits engineering problems to control the trend of research in soil mechanics, for only in that way can the practical observations collected in the field of construction be tied into the same network of facts.

The Wiegner method for mechanical analysis of finely pulverized materials brings out an important fact. The pressure of water containing suspended soil or any other material is greater than that of pure water. This fact applies even to such suspensions as freshly mixed concrete. The pressures are in direct proportion to the specific gravities of the suspensions. In other words, as far as engineering data are concerned, suspensions are liquids and follow the laws of liquids. Large-scale experiments by Professor E. B. Smith and by Shunk on concrete pressure, and work by Résal on clay fills, and by La Habra Dam Commission on mud, give results on various materials which check the theoretical values.

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[30] Prof. Emeritus, Rensselaer Polytechnic Inst.; Cons. Engr., New York, N. Y.

[31] Cons. Engr., New York, N. Y.

The tests developed for the various consistency limits may result in methods for distinguishing and classifying various soils. However, the writer cannot see any value in the definitions of the limits, as given in this paper. For example, under "Limits of Consistency," the author states: "All the limits are expressed in terms of water content, that is, weight of water per unit weight of dry soil." How is the unit weight of dry soils to be determined? What packing or loose condition can be taken as a standard, so that all experimenters will agree on a unit weight for any given sample?

A more serious objection, in addition to this one, applies to the definition of the shrinkage limit as given in Equation (1). It is a fundamental check in mathematical analysis of physical phenomenon that all equations must balance in dimensions. The definition requires the shrinkage limit value to be without dimension, that is, a pure number or ratio of weight to weight of equal volumes. The equation is weight minus volume, both divided by weight, and cannot truly represent the factor defined.

In studying the permeability of soils, it must be pointed out that Equation (3) is true only if: (1) The soil is entirely saturated before any reading is taken; (2) the flow through the soil sample is constant under constant pressure, that it is not clogging up or washing out; (3) the value of the constant, k, is independent of the pressure head, h; and (4) the temperatures of soil and water are the same. A very fine discussion of the variables controlling the flow of water through earth has been given[22] by John H. Griffith, M. Am. Soc. C. E.

The further data on the compression, tension, and shear values of soils add to the store of knowledge started by the very earliest experimenters in soil physics. It is these functions of a soil which distinguish it from the fluid and solid states of matter.

The author's statement under "Research in Soil Engineering," to the effect that, "corroborating previous evidence, the tests showed that the settlement under a given unit load, not too near the ultimate, is practically independent of the size of the loaded area," is interesting if it can be proved, even for sand. It is the writer's impression that, from recent observations of existing structures as well as from experiments, exactly the opposite has been the general trend. It is hoped that a detailed description of facts from which this conclusion is drawn, will be presented.

The work on lateral pressure on walls has been of special interest to the writer. The report of the experimental work performed for the New England Power Construction Company to which the author refers, is not complete enough to be conclusive. Apparently, from the results reported,[23] a great number of tests have been performed.

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[22] "Physical Properties of Earths," *Bulletin 101*, Iowa Eng. Experiment Station.

[23] *Engineering*, May 30 and June 13, 1930.

Professor Gilboy is to be complimented on his work in continuing Dr. Terzaghi's research, and for his clear exposition of the various lines of research in soils which he is covering.

JOHN R. JAHN,[24] **ASSOC. M. AM. SOC. C. E.** (by letter).—This interesting paper by Professor Gilboy shows the amount of careful study and the number of mechanical devices needed to produce complete information concerning the physical properties of soils. The writer was particularly interested in the short description of the hydrometer method of size analysis and feels that it was unfortunate that more was not written of the details of laboratory work and mathematical analysis. The author has wisely pointed out that accurate knowledge of size distribution is not vital to soil mechanics, since other factors are of such importance that size analyses give only a rough indication of the properties of the material. If he has found a method of making accurate analyses, however, it would be of interest to many laboratories that are attempting to make determinations. It would also be of assistance to laboratories in other industries which depend on fine comminution, such as the sliming of ores, fine grinding of paints and pigments, cements, inks, etc.

The writer feels that the Bouyoucos method, with some of its improvements, is a useful tool for engineering information, but that it has inherent faults which will eliminate its use if accurate information is sought. These faults are fundamental, as will be brought out.

The hydrometer is shown at the left in the author's Fig. 2. It consists of an enlarged bulb portion with a weighted point and a thin stem with graduations, and at the level of submersion it registers the grammes of suspended matter per liter of liquid. A known weight of sample is thoroughly deflocculated and diluted to a known volume in the test cylinder. At various time intervals after the start of the analysis the hydrometer is lowered into the cylinder and the decreasing densities of the system give the data from which the size analysis of the sample is computed. Stokes' law[25] is used to determine the size of particle which will fall from the surface of the liquid to a critical depth point on the hydrometer in the observed time, t, and some quantitative method is used for the mathematical analysis.[26]

The following faults in the hydrometer method are apparent:

(*a*) With its irregular shape and conical point, the hydrometer does not have a practicable bottom level, to which to refer the amount of fall of particles.

(*b*) Due to its bulbous shape, too much emphasis is given to the density of the suspensoid opposite the region of the enlargement.

(*c*) The hydrometer must be withdrawn after each observation to avoid deposits on its upper shoulder. This causes turbulence, with an indeterminate translation of particles.

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[24] Civ. Engr., San Francisco, Calif.

[25] *Transactions*, Cambridge Philosophical Soc., Vol. 8, p. 287, and Vol. 9, p. 8.

[26] "The Hydrometer Method for Making a Very Detailed Mechanical Analysis of Soils," by G. J. Bouyoucos, *Soil Science*, September, 1928.

(*d*) The meniscus on both the cylinder and the hydrometer at the level of the liquid prevents close reading of the scale unless other means, such as external gauges, are used.

(*e*) Unless due correction is made, an error of considerable magnitude will remain in the analysis because of the settling out of particles smaller than the size determined by Stokes' formula, which have fallen beyond the hydrometer, from their initial subsurface positions, that is, from less than the full depth. If the semi-log graph of "the hydrometer reading" plotted against logarithm of "the elapsed time of settlement" is straight, the error will be constant throughout, but if the graph is curved, as is usual, the error will be variable in the different size classifications.

Corrections for all these faults, except (*c*), are being made in the laboratory of Charles H. Lee, M. Am. Soc. C. E., to adapt the hydrometer method to a more accurate size analysis of fine-grained engineering material, such as muds, soils, etc. Fault (*e*) is being corrected by a modification of Rubey's method[27] adapted to semi-log graphs; Fault (*d*) is avoided by arranging a mirror against which an index point is reflected; Fault (*c*) cannot be eliminated, but is minimized by very careful manipulation; and Faults (*a*) and (*b*) are corrected by a calibration method in which the errors are grouped and the correction factor ascertained by control tests, using a similar cylinder, but having a parting plane of separation at the lower limit of the hydrometer's subsidence. Its purpose is to establish the actual average density of the suspension at the instant of reading the hydrometer. The depth of the parting plane being known, it is possible to obtain a correction curve to convert hydrometer readings into average densities of a standard height of column at the same instants. These calibration corrections will have to be established for each class of material or until its character can be anticipated. The corrections are not constant over the range of the analysis, but have varied, according to texture, from 0 to 12%, within a period of a few minutes in the "run."

During the fall of 1924, the writer was seeking a method of making an accurate size analysis, using an adaptation of the Oden method[28] as developed at the University of Upsala, at Upsala, Sweden. This method weighs the accumulations of sediment on a scale pan, suspended from a balance and situated near the bottom of a test cylinder. At the Soil Technology Laboratory, of the University of California, Professor E. V. Winterer had assembled a well-designed apparatus for weighing the sediment and recording the time intervals for equal weight accumulations. It had close thermostatic control and was automatic in every particular after commencement of the test. The method was found to be grossly inaccurate in a very unusual way, in that the particles refused to fall vertically, but migrated away from the center line of the cylinder, that is, horizontal layers did not have uniform distribution of suspensoids. Errors of from 10 to 35%,[29] all of under-registry, were found, and experiments with cylinders of glass, metal, rubber, and wood and having electrical charges of from 1.5 to 30 000 volts impressed thereon in various combinations, were found to be ineffectual. The cause of the trouble was finally decided to be the electrostatic repulsion of particles away from the center line of mass of the test cylinder with concentration at the side walls. The method was abandoned as unreliable, and rather definitely disposed of sedimentation methods depending on a collection of particles in fractional areas of containers.

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[27] *Professional Paper 165A*, U. S. Geological Survey, p. 25.

[28] *Soil Science*, v. 19.

[29] *Proceedings and Papers*, First International Congress of Soil Science, by Charles F. Shaw and E. V. Winterer, June 13–22, 1927, Vol. 1.

The writer feels that a more elaborate method of utilizing the changing suspension densities offers a solution of the problem. While the hydrometer method is unsuitable for exact laboratory work, its objections or faults can be met in an apparatus in which a rod of uniform cross-section is suspended to a fixed level in a sedimentation cylinder and instantaneous records of its apparent weight are found by its being hung from one arm of a sensitive balance. As the suspended matter in the liquid settles out, the apparent weight of the rod will increase. The time rate of change in weight may be obtained by noting the times at which a given weight is equalled, electrical contacts on the balance recording the instant. A magazine can then release a known weight increment upon the opposing balance pan, the balance will swing back, and a new time interval will be recorded following sufficient change in density. The vertical travel may be kept less than a millimeter, by the use of stops under the pan or arms.

The proposed apparatus would avoid Fault (*a*), by having a constant subsiding range; Fault (*b*), by having a truly cylindrical body; and Fault (*c*), by not disturbing the rod after the test had started. Fault (*d*) would not exist, and Fault (*e*) could be avoided by Rubey's graphical, or Oden's mathematical, methods.

The falling range for subsiding particles will be the distance from the surface of the liquid to the lower end of the rod. This fact was established by the writer during the 1924 work. It was found that a cup submerged in clear water, carrying suspended material, registered its computed weight when the bottom was impervious, but registered only the weight of the matter resting on the bottom of the cup, when an aperture was opened therein. The change in weight was immediate and indicated that that weight was transmitted as a pressure due to frictional effects on the falling particles and not as an increased suspension density which would have affected the weight of the cup until slow diffusion through the aperture had taken place. Hence, the suspended rod will be acted upon vertically upward by all the displaced suspended matter above its lower end and by none of the matter that has passed this horizon.

An automatic data-recording device is desirable to avoid the need of close personal attention during 24-hour runs. The weight increments should be no larger than is necessary to bring the balance back promptly. In the 1924 work, small steel ball-bearings weighing 0.05 gramme were found to be uniform in weight. They can be discharged upon the balance pan from a magazine actuated by the same electric current that marks the swing of the balance.

The writer regrets that he has not had an opportunity to experiment with this apparatus. Inasmuch as he believes that other devices have failed to give much more than approximate results, this is suggested as a possibility, with the hope that some dependable analytic procedure can be developed. If such procedure were found, it would then be possible to compare less elaborate analytical devices, such as Bouyoucos' hydrometer, to determine the extent of their reliability and establish calibration corrections thereto.

C. H. EIFFERT,[30] M. AM. SOC. C. E. (by letter).—Since the completion of the dam of the Miami Conservancy District the plan of exploring the cores of one or more or these dams had been under consideration. The desire to make such an exploration was prompted by the lack of knowledge of core conditions in the completed structure and by the difference of opinion among prominent engineers as to the exact nature of the core material some time after the completion of a dam. It was maintained by some that the core remained in a semi-liquid condition for a long period, while others were of the opinion that there might be a possibility of the core drying out and cracking to such an extent as to destroy its imperviousness. That the cores of the Miami Conservancy Dams did not long remain in a fluid or semi-liquid condition was indicated by the Goldbeck pressure cell readings and by some of the exposures made during construction.[31] These readings showed that the core material possessed considerable inherent stability at that time. Further knowledge as to the nature of the cores was not available until the recent experiments were made.

One reason for not making explorations prior to those under consideration was that facilities for testing samples and experimenting with the materials to the best advantage were not available. When it became possible to enter into a co-operative agreement with the Massachusetts Institute of Technology, it was arranged for the District to do the field work and the Institute to make the laboratory tests and experiments.

Two general methods of obtaining samples were available. One was to sink a shaft large enough for men to work in and obtain samples, and in which the materials could be inspected in their undisturbed condition. The other method was to secure the samples from some type of drill hole or boring. In order to obtain the relative value of the two methods, it was determined to use both. The shaft, of course, would be much more expensive than the boring, but it was fairly certain that good samples could be obtained by this method, whereas some uncertainty existed as to the value of those from a drill hole or boring. The primary consideration in either case was to obtain the samples as nearly as possible in an undisturbed condition.

During the construction of these dams Goldbeck pressure cells[32] had been installed at Germantown and Taylorsville. The most satisfactory results from these cells were obtained at Taylorsville, and it was at first thought desirable to obtain the samples from this dam, but because of the paved roadway and heavy traffic conditions a vertical shaft appeared impracticable there, while a sloping shaft or drift would have been more expensive and a lesser depth of core would have been penetrated by it. On account of its greater height and the accessibility of the core, the Germantown Dam was finally selected.

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[30] Chf. Engr. and Gen. Mgr., Miami Conservancy Dist., Dayton, Ohio.

[31] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), pp. 1195–1197.

[32] *Loc. cit.*, p. 1188.

The shaft was located about 12 ft. west of the center line of the dam at a point where its height was approximately 110 ft. A shaft on the center line would have been more desirable, but was impracticable on account of its obstruction to traffic over the dam which is traversed by a public road.

It was decided to sink a vertical shaft with a circular concrete casing, with walls 5 in. thick and 5 ft. in internal diameter. A sheet-metal form somewhat similar to those used for monolithic concrete silos, was provided. The casing was cast at the ground level in sections about 3 ft. high. As the excavation proceeded the concrete sank under its own weight and new sections were cast in place on top of the old ones as the sinking proceeded. As there is no true core in the upper 15 or 20 ft. of the dam and the shaft was located to one side of the center line, no core material was encountered until a depth of 33 ft. was reached. The first set of samples was secured at this elevation, Professor Gilboy being present when they were taken.

The excavating for the shaft was done by hand by one man, and the material elevated with a hand-hoist and derrick (see Fig. 14), the boom being placed so that in a lowered position the bucket could be dumped into a wagon beyond the shaft. A small shed to house materials, tools, and samples was constructed to act as a counterweight for the boom. Being placed to one side of the center line more layers of fine gravel and sand were encountered than would have been the case had the shaft been directly on the center line of the dam. This led to the unexpected difficulty of handling a considerable quantity of water contained in these layers, which entered the shaft beginning at a point within 30 ft. of the top of the dam and far above the maximum water level ever recorded in the reservoir. A hand-pump was installed but as the excavation proceeded pumping by hand became too tedious and a small gas engine and pump-jack were provided for this work.

The water encountered had undoubtedly remained from the construction operations, with some possible addition due to the rainfall on the dam. It is retained by layers of core material which project outward from the main core between the layers of gravel, all the layers sloping downward toward the center of the dam. The retention of this water is evidence of the imperviousness of the core material.

The tools for obtaining samples are shown in Fig. 15. They were designed by Professor Gilboy under the direction of Charles Terzaghi, M. Am. Soc. C. E. At (*a*) is shown a seamless steel sampling tube, 4 in. in diameter, and 12 in. long, with one end turned down to a knife-edge, and this edge burnished in 0.001 in. to give clearance to the sample. The tube, which had been wiped with an oiled rag before using in order to reduce friction on the inside, was forced into the core material by hand, the material around it being removed with a trowel as fast as the tube penetrated. As soon as the tube was filled it was carefully removed and the surplus material at the ends was cut off by means of a fine wire in a hack-saw frame. Sheet metal caps (*b*) were then placed over the ends and the sample was brought to the surface. Samples were taken in pairs at each location, one vertically and the other horizontally, at right angles to the axis of the dam.

On the work table were two circular iron bases, $\frac{3}{4}$ in. by $4\frac{1}{2}$ in. (*c*). When a sample was brought to the surface to be prepared for shipment, the iron base was covered with a thin layer of melted paraffin and the sample set vertically on it after the caps had been removed. The steel tubing was then slipped from the sample. If it did not come off easily, a wooden disk slightly smaller in diameter than the tube, was placed on top of the sample and held down while the tubing was pulled up off the sample. A cylinder (*d*) of 24-gauge galvanized iron, $4\frac{1}{2}$ in. in diameter, and $13\frac{1}{2}$ in. long, was then placed over the sample, the bottom sealed with modeling clay, and the space around and above the sample filled with melted paraffin. When the paraffin had hardened the cylinder was reversed, heat being applied to the iron disk if necessary to loosen it, and the other end poured full of the paraffin. The entire sample was thus enclosed in paraffin, the structure of the sample and its water content being well preserved until it arrived at the laboratory. The tube (*e*) was used for obtaining samples from the drill hole.

When the work began it was the intention to obtain a pair of samples every 10 ft. This was found to be impracticable and only four good sets of samples were secured. The first 30 ft. was not in core material, and the shaft was sunk to a depth of only 80 ft. At a depth of 40 or 50 ft., the casing separated at one of the joints and side-slip occurred as it continued to sink. It was necessary to install some timber braces to prevent this. At this time the sinking process was very slow, and it became necessary to weight the casing and to force it down with jacks. At a depth of about 60 ft. it appeared impracticable to sink it any farther because it moved out of alignment and some cracks occurred. At this point a new casing was started inside the first one, the concrete being taken down in buckets. This was sunk to an additional depth of 20 ft. After a depth of 40 or 50 ft. had been reached it was necessary to supply air to the workman at the bottom of the shaft. This was done by a small compressor attached to the engine which ran the pump. At the depth of 80 ft. progress had become rather slow. The shaft also appeared to be somewhat dangerous. The four good sets of samples were thought to be sufficient and, therefore, the work was stopped.

The writer believes that if another shaft were to be sunk in material similar to that at Germantown, it could be done very satisfactorily by the methods herein described, but that the walls should be 8 in. thick to give them additional weight for sinking and that they should be reinforced to prevent any separation at the construction joints.

In order to determine whether or not any of the porous strata encountered, extended across the center line of the dam, 4-in., horizontal holes were bored at right angles to the center line in the two heaviest layers of this kind. In both cases, solid core material was encountered within a few feet of the shaft.

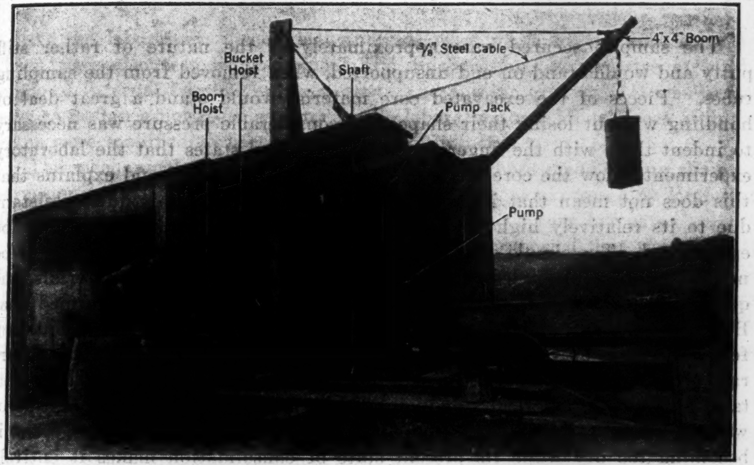
⅞" Steel Cable
4"x4" Boom
Bucket Hoist
Shaft
Boom Hoist
Pump Jack
Pump

FIG. 14.—ARRANGEMENT AT SHAFT HEAD, GERMANTOWN DAM, FOR SINKING TEST PIT.

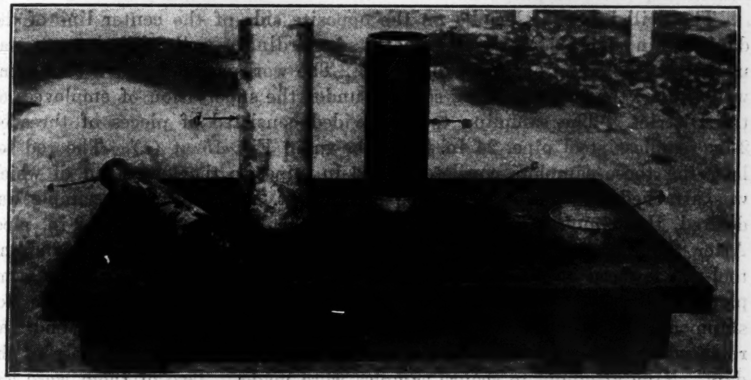

FIG. 15.—APPARATUS FOR TAKING SAMPLES OF CORES, GERMANTOWN DAM.

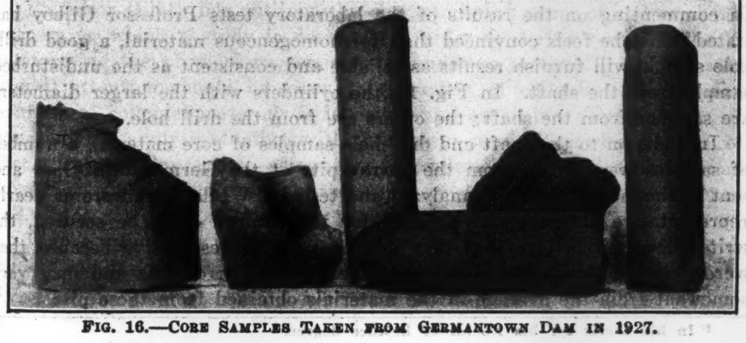

FIG. 16.—CORE SAMPLES TAKEN FROM GERMANTOWN DAM IN 1927.

The samples secured were approximately of the nature of rather stiff putty and would stand on end unsupported, when removed from the sampling tubes. Pieces of the excavated core material would stand a great deal of handling without losing their shape, and considerable pressure was necessary to indent them with the fingers. Professor Gilboy states that the laboratory experiments show the core material to be 25% consolidated and explains that this does not mean that it is soft or yielding, but that it is quite resistant due to its relatively high angle of internal friction. This point should be emphasized. The simple statement that the core is 25% consolidated may be misleading to some unless a comparison is made with other materials, the qualities of which are better known. The writer has been informed that Boston blue clay, which is considered a satisfactory material for building foundations, is only 10% consolidated by the same standards. The future rate of consolidation of the Germantown core is necessarily somewhat uncertain, but from a practical standpoint this is unimportant. The quantity of water encountered in sinking the shaft shows that there is no danger of it becoming too dry while its present state of consolidation makes it entirely satisfactory for core purposes.

The drill hole was located on the opposite side of the center line of the dam within about 25 ft. of the shaft. An ordinary well-drilling outfit was used, the pipe being 5⅝ in. in diameter. The work was done by a competent well driller, the samples being secured under the supervision of employees of the District. The sampling tools provided consisted of pieces of threaded 2-in. seamless steel pipe, 24 in. long, shown in Fig. 15 at (*e*). The rod for lowering these samplers consisted of 1¼-in. pipe, to the lower end of which was attached a piece of 2-in. pipe about 2 ft. long. When a sample was desired, the sample tube was screwed into the lower end of the 2-in. pipe. After the well had been carefully cleaned out with a dipper the sampling tool was lowered and driven into the core about 3 ft. This insured the complete filling of the sampling tube. The samples remained in these tubes for shipment, about ¾ in. of the sample being removed from each end and replaced with melted paraffin. The drill hole was driven to the base of the dam. Nine attempts to secure samples were made. Two of them were lost by slipping from the tube, but the others appeared to be very satisfactory. In commenting on the results of the laboratory tests Professor Gilboy has stated,[33] that he feels convinced that, for homogeneous material, a good drill-hole sample will furnish results as reliable and consistent as the undisturbed sample from the shaft. In Fig. 16, the cylinders with the larger diameters are samples from the shaft; the others are from the drill hole.

In addition to the shaft and drill-hole samples of core material, a number of samples were taken from the borrow-pits at the Germantown Dam and sent to the laboratory to be analyzed and tested. While these were as nearly representative of the materials used for the dam as could be secured, the writer does not believe that they can be truly representative because they were necessarily obtained from the edges of the borrow-pits and may vary somewhat from the average of the materials obtained from these pits.

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[33] In his thesis for his Doctor of Science degree.

At the laboratory, mechanical analyses were made of the core samples. They were also tested for permeability, compression, compressive strength, angle of internal friction, water content, and specific gravity. It was necessary to design a new permeameter at the laboratory and other pieces of apparatus were modified to better meet the requirements.

The mechanical analyses were consistent with those made during the construction period. The average rate of permeability, as expressed by Professor Gilboy, was approximately 100×10^{-7} cm. per min., a very low rate.

Tests for compression and compressive strength showed that the material possessed a high degree of stability. The friction angle averaged about 27 degrees. The water content for a number of samples, expressed in percentage of dry or solid matter, averaged approximately 25 per cent. The specific gravity was found to be 2.67 for the solids and 2 for the mass. This gives a weight of 125 lb. per cu. ft. for the undisturbed core.

As a result of this work the following conclusions may be drawn:

1.—The samples from the drill hole were fully as satisfactory as those from the shaft, and were obtained with much less expense.

2.—The mechanical analyses showed the core material to be well graded from fine sand to clay in such proportions as to make a combination ideally adapted for a hydraulic-fill core.

3.—The water content does not differ materially from that found at Englewood and Lockington at the end of the construction period. While the Germantown Dam was not tested for water at that time, the mechanical analyses show the material to be quite similar. This would seem to indicate that there has been little change in the cores during the seven years between 1920 and 1927 and that they will change but little in the future.

It is hoped that the tests may be repeated at intervals of five or ten years, and that similar tests will be made on other dams.

A. A. EREMIN,[34] ASSOC. M. AM. SOC. C. E. (by letter).—Present interpretation of soil behavior under sustained loads is erroneous and unsafe. The soil problem is complicated, due to the facts that no two foundation materials can be found alike and that it is difficult to obtain test samples under natural conditions.

Professor Gilboy has outlined the characteristic factors of soil mechanics concerning which a study is desirable in order to derive reliable rules for interpreting the physical properties of foundation materials. In the study of soil attention should be given to the effect of colloids. The chemical analysis of colloidal materials is laborious and expensive. Furthermore, the results of the analysis obtained with the present state of knowledge often conflict with actual practice. However, to neglect the colloidal effect would introduce considerable error in the study of permeability and cohesion of soil, which are the most important factors in soil mechanics.

Tensile strength of loams and clays increases with the increased colloidal content, as has been shown[35] by John H. Griffith, M. Am. Soc. C. E. Therefore, the angle of internal friction of the test cylinder, Fig. 9, may also vary due to the binding effect of the colloidal particles between soil grains.

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[34] Asst. Designing Engr., Bridge Dept., State Highway Comm., Sacramento, Calif.

[35] "Physical Properties of Earth," by John H. Griffith, M. Am. Soc. C. E.

Likewise, the permeability of soil varies, due to expansion and contraction of the colloidal materials and to its reactions with the chemicals in the ground. From research on soil mechanics for hydraulic dam cores, Mr. F. E. Hance has stated that soil chemically treated may become from 200 to 1 000% more impermeable than the same material when neutral.[36]

In the study of foundation materials an investigation of the bearing on piles in compressible soils should be included. In the future, estimation of the bearing capacity of piles will probably be based on the physical properties of the foundation material. Moreover, the penetration curve of piles may give useful information about the character of the soil.

F. N. MENEFEE,[37] M. AM. SOC. C. E. (by letter).—The subdivision of this study into (1), soil physics, and (2), soil engineering, is good. It is analogous to the division of other studies of an engineering nature. The very refined, detailed, and sometimes little used, although valuable, properties of a material correctly belong in a classification by themselves, for a larger part of the engineer's professional knowledge is based upon scientific studies which at the time they were made had no direct application. In the case at hand Charles Terzaghi, M. Am. Soc. C. E., and the author have aptly called that part of their study soil physics, and that which follows, soil engineeering.

Due to studies by Duff A. Abrams,[38] M. Am. Soc. C. E., and the late William B. Fuller,[39], M. Am. Soc. C. E., on concrete aggregates, American engineers in general have adopted the practice of plotting their mechanical analysis curves with the smaller diameter at the origin, thus giving a curve of positive rather than of negative slope, as does the author. There is no question of right or wrong to either method, but standardization of method in presenting data does assist interpretation and facilitates the use of an idea.

Since Mr. Fuller's ideal curve gave the densest mixture of sand and gravel it probably would assist in determining the permeability as applied to soils because, aside from the possible influence of colloids and swelling tendencies of, or chemical changes with addition of, moisture, the hydrostatic head lost in water flowing through soil is inversely proportional to the diameter of the voids. To state it in another way, the velocity of water through soil increases or decreases with increase or decrease in diameter of the inter-granular space. An ideal gradation from fine to coarse fills up the interstices and reduces the permeability.

Atterberg's choice of terms—"liquid limit," "plastic limit," and "shrinkage limit"—seemed very good to the writer, but the methods of determining them were subject to at least some personal equation. The methods described by the author seem to be an improvement.

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[36] *Engineering News-Record*, October 3, 1929, p. 542.

[37] Prof., Eng. Mechanics, Univ. of Michigan, Ann Arbor, Mich.

[38] *Bulletins*, Structural Materials Research Laboratory, Lewis Inst., Chicago, Ill.

[39] "The Laws of Proportioning Concrete," *Transactions*, Am. Soc. C. E., Vol. LIX (1907), p. 67.

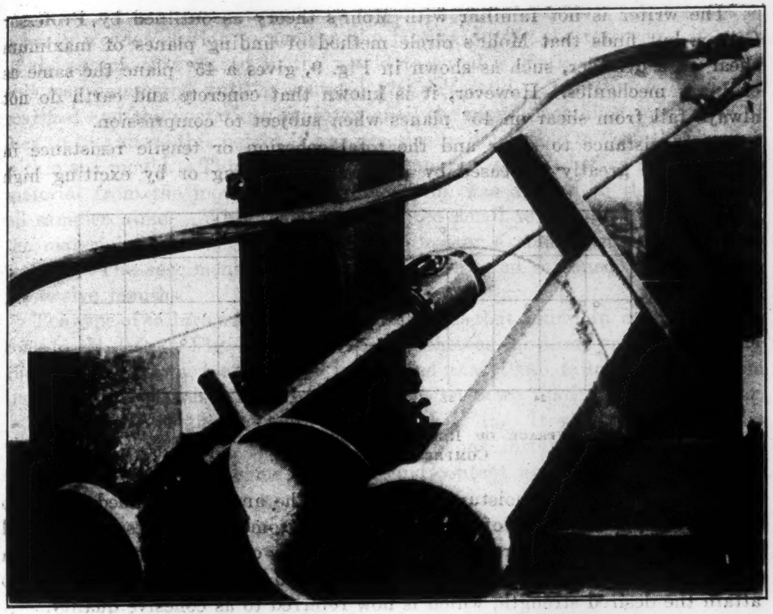

FIG. 17.—APPARATUS USED FOR MAKING SPECIMENS.

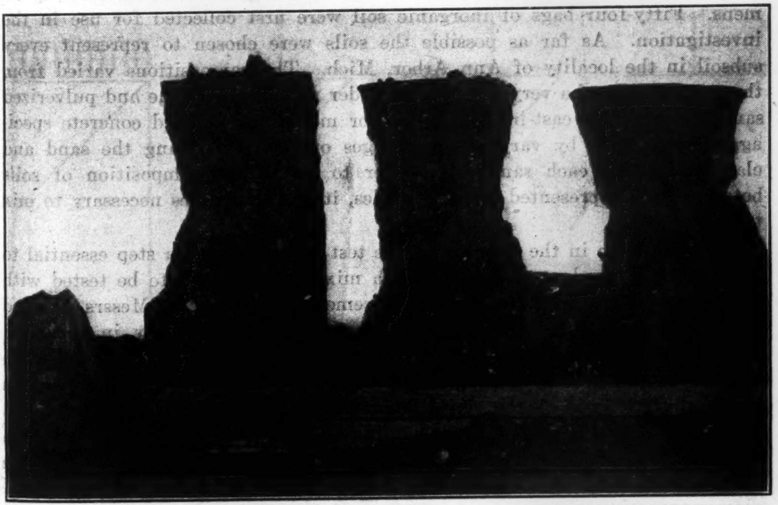

FIG. 18.—CHARACTERISTIC FAILURES.

The writer is not familiar with Mohr's theory as outlined by Professor Gilboy, but finds that Mohr's circle method of finding planes of maximum shear on a cylinder, such as shown in Fig. 9, gives a 45° plane the same as ordinary mechanics. However, it is known that concrete and earth do not always fail from shear on 45° planes when subject to compression.

The resistance to shear and the total cohesion or tensile resistance in soil may be greatly increased by mechanical working or by exciting high pressure under proper moisture conditions. The ancients rammed soils into forms for building walls of dwellings and, in some cases, these walls stood for centuries. Such literature as can be found on the subject describes a method of ramming or tamping the earth into wooden forms in order to attain the desired strength, which is now referred to as cohesive quality.

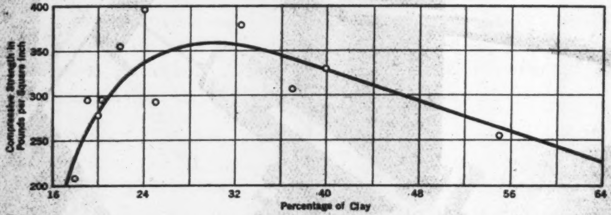

FIG. 19.—AVERAGE OF RELATIONS BETWEEN CLAY CONTENTS AND COMPRESSIVE STRENGTHS.

The writer has performed a series of experiments on mixtures of clay and sand rammed into cast-iron cylinders for moulding standard concrete specimens. Fifty-four bags of inorganic soil were first collected for use in the investigation. As far as possible the soils were chosen to represent every subsoil in the locality of Ann Arbor, Mich. The compositions varied from the one extreme of a very heavy clay binder mixed with a fine and pulverized sand rammed into cast-iron cylinders for moulding standard concrete speci-aggregate bonded by varying percentages of clay. Knowing the sand and clay analysis of each sample, in order to obtain the composition of soils between those represented by the samples, it was sometimes necessary to mix two or more.

The first step in the forming of the test cylinders, and a step essential to uniformity of product, was a thorough mixing of the soil to be tested with sufficient water to fulfill the rough requirements described[40] by Messrs. Edward W. Coffin and H. B. Humphrey. A sample squeezed tightly in the hand should adhere in the form of a ball, but will break into small pieces when dropped to the ground from the height of the waist line. This property is essentially governed by the clay content of the sample, but the writer found that over a large range of soils, varying in clay content, a tempering with water to fulfill this test gave a mixture that could be best handled with the air-hammer.

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[40] Handbook on "Building Walls with Rammed Earth."

These moulds were clamped to a metal base which, in turn, was screwed to a heavy wooden section to dampen the jarring during the tamping. Enough of the sample was placed in the mould to equal a depth of about 5 in. and this was hammered until it sounded and felt solid. The surface was then scarified and the operations repeated until the mould was filled to a depth of 9 or 10 in.; above this height it was practically impossible to keep the rammer within the mould. This left the surface uneven, so that before removing the material from the mould a load of 4 000 lb. was applied to the surface of all samples alike. While this load was too small to change the density of the material appreciably it did tend to present a smooth surface for later testing. The specimens were then stored away and seasoned, most of them for twelve months.

The type of failure was very much the same as that found in concrete. The cone to the left in Fig. 18 is of concrete, whereas the three specimens to the right are of rammed earth. It is believed that these failures depart from the 45° plane because they are due to tension more than to diagonal shear.

The results of the tests are shown in Fig. 19. It will be noted that a value of 400 lb. per sq. in. was attained in one instance. At a later date a cylinder of about the same clay and sand content was attained which withstood nearly 500 lb. per sq. in. of compression. The apparatus used for making specimens is shown in Fig. 17.

The author's Fig. 9(*a*) shows a barreling effect of the clay cylinder which in itself indicates that the diameter increases between the top and bottom, introducing tension.

Fig. 20 represents a clay cylinder subject to compression; *P* is the total load, and *f* is the frictional resistance tending to keep the ends from spreading or increasing in diameter. This friction is a maximum at the periphery and diminishes to zero at the center or axis of the cylinder. If *f* could be eliminated the maximum shear would be,

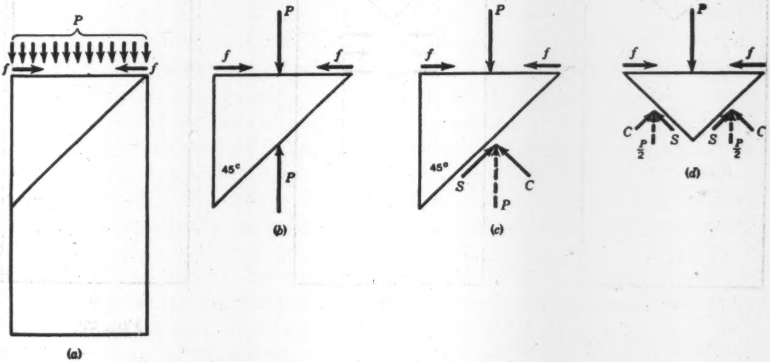


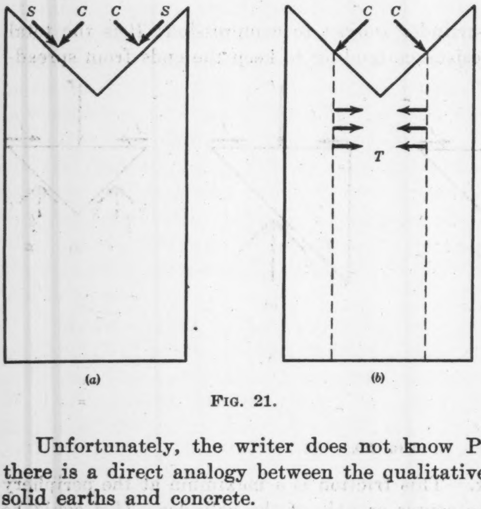
FIG. 20.

$$S = \frac{1}{2}\,\frac{P}{A} \qquad \text{......................} (6)$$

in which, A is the cross-sectional area, and would be on planes 45° with the upper and lower faces. (Fig. 20(a), Fig. 20(b), and Fig. 20(c)). With f always existing to a certain extent the plane of maximum shear will be less than 45° with the upper and lower faces, or the cone will be flatter than shown; but actual tests show that the failures in earth cylinders and concrete as well, are along planes of greater slope than 45°, as indicated in Fig. 18.

From a purely static consideration, the shearing and comprehensive forces on the slant faces of the cone neutralize each other and the resultant splitting tendency is nil; but when the load is applied the forces actually produce deformation, and the cone settles into the cup (Fig. 21(a)), through shearing deformation. Thus, the values of S are reduced, the values of C increase, and spreading action takes place introducing tension in concentric cylinders from top to bottom (Fig. 21(b)).

The illustration is made clearer by assuming a highly polished cone placed into the correspondingly polished cup (Fig. 21(b)). The values of S disappear, and the values of C increase until the sum of their vertical components is equal to P, their horizontal components pushing outward, increasing the diameter of the cylinder. Although there are no external forces pulling the sides of the cylinder outward the internal horizontal forces pushing outward radially from the axis produce diametral strain and corresponding stress.



(a) (b)

FIG. 21.

(c)

FIG. 22.

Unfortunately, the writer does not know Poisson's ratio for earth, but there is a direct analogy between the qualitative analyses of the behavior of solid earths and concrete.

Knowing Poisson's ratio for concrete to be about 0.12 and taking $E = 3\,000\,000$, a compression of 4 000 lb. per sq. in. will produce $\dfrac{4\,000}{3\,000\,000}$

= 0.00133 in. per in. of strain or shortening (Fig. 22). This will be accompanied by a diametral elongational strain of 0.12 × 0.00133 = 0.00016 in. per in. The corresponding tensile stress would be 3 000 000 × 0.00016, or 480 lb. per sq. in., which is enough to cause a tension failure that is becoming known as "longitudinal split." The failures of earth cylinders shown in Fig. 18 indicate a similar behavoir.

E. G. Walker,[41] M. Am. Soc. C. E. (by letter).—This paper is of value to engineers interested in the practical application of the mechanics of soils because it is a concise account of the lines along which research in this comparatively new branch of engineering science is being pursued. It would appear from the paper that the laboratory investigations into the physical properties of soils are being developed with a view to obtaining data on four general properties, namely: (1) Their mechanical analysis; (2) their "limits of consistency"; (3) their permeability; and, (4) their internal friction and cohesion.

As far as these properties are real physical values, a knowledge of their variations relative to one another and to external phenomena may be expected to be of considerable use to the engineer in the future in assisting him to deal with the many practical problems which arise from day to day in civil engineering practice. At present, most of these problems can only be solved, in the light of previous experience.

The subject is undoubtedly one of the greatest complexity and—no matter how successful laboratory research ultimately may be in connecting the various physical properties of individual soils with their behavior in Nature—the practical application of the results is almost certain to be fraught with uncertainties. At the same time, this is no argument against the pursuit of laboratory research, because this should enable engineers to predict the behavior of soils under the ideal conditions of experimental work, and in this way guidance may be given to the application of practical experience.

The methods described for measuring consistency certainly seem very empirical and it would be of interest if the author would elaborate this section of the paper by explaining what is really obtained from the measurements therein described. The change from the state of wet consistency in which the clay acts practically as a liquid down to that in which, all the water having been removed, it may act as an imperfectly elastic solid, is a gradual one and is not marked by any complete changes of properties, such as occur, for example, when an increase of temperature converts a solid, successively, into a liquid and a gas. It seems undesirable, therefore, to attempt to define limits in the manner suggested in the paper, since this straightway leads to the deduction that there are points at which definite changes of properties exist.

The writer suggests that the line of research should rather be to attempt to correlate, with the variation of moisture content, some representative property of the clay, such as the force necessary to produce some agreed distortion of the sample. If a detailed study of the phenomena should show points to exist at which changes of properties become apparent, a more scientifically defined limit could be designated.

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[41] (Maxted & Knott), London, England.

The writer would like some further explanation of Equation (1). The quantity, $\frac{W_0 - W_0}{W_0}$, is the change in weight due to drying divided by the weight of the dry sample and is, therefore, the ratio of the weight of water to clay in the wet sample. The quantity, $\frac{V_1 - V_0}{W_0}$, is the ratio of the shrinkage to the weight of the dry sample and is, therefore, the reciprocal of the apparent change of density of the clay. Hence, this equation requires that a measure of bulkiness, which cannot be dissociated from units of weight and volume, be subtracted from an abstract ratio in order to obtain a value which is called a "shrinkage limit." The task of defining limits becomes doubly difficult when it is proposed to measure them in so obscure and empirical a manner.

The portable capillary permeameter shown in Fig. 5 would appear to be a good type of apparatus for approximate field determinations when the more exact apparatus shown in Fig. 4 would not be available. The author states that the coefficient, F, in the formula used for reducing the measurements made with the horizontal capillary permeameter, may be taken as 100 000, with a range of deviation of ± 50 per cent. Equation (5) reduces to $k = \frac{x^4}{t^2} \times \frac{e}{1 + e} \times \frac{1}{F}$; that is, for any given set of observations the permeability varies as $\frac{1}{F}$. Hence, if F may vary within the stated limits the actual permeability may vary from two-thirds to twice that calculated by assuming a mean value, $F = 100\,000$. Therefore, unless calibration of the instrument against the standard type of permeameter can reduce the range of possible variation of F, the capillary type does not appear to be of much practical value.

Research with the consolidation apparatus, in which the sample is constrained laterally, together with mechanical tests in compression and shear on the lines described under "Internal Friction and Cohesion," should lead to results of considerable practical value. It is hoped that this part of the work of the laboratory will be developed to the fullest extent in combination with the field testing of foundation soils described under "Foundations."

The four classes of engineering researches described in the second part of the paper foreshadow great possibilities of adding to the knowledge facts that will enable the design of civil engineering works to be placed on a more satisfactory basis.

As far as the writer is aware, no organized research on the pressures on retaining walls has ever been attempted before with so large an apparatus as that shown in Fig. 13. Many experiments have been made on earth pressures during the last hundred years and more, but the great majority have been useless as far as the direct practical application of the results to engineering design is concerned, solely because the scale upon which the experiments have been conducted, has been too small. It has not been recognized sufficiently in the past that there must be a distinct scale effect in experiments of this nature. Whereas a model wall quite conceivably may be constructed to a small scale to represent the original in dimension, weight, distribution of material, etc., to a close degree of exactness, it is impossible, proportionately, to scale down the soil which presses against it, or to represent adequately the effects of consolidation or loosening of the soil resultant from actual practical works of construction. The only hope of obtaining better design data than are now available is to carry out experiments on a large scale. The writer has more sanguine hopes of the success of the large apparatus described in the paper, therefore, than he has had of previous researches of this nature.

There is one detail, however, which the writer ventures to criticize. The bin in which the earth is to be retained is 14 ft. square and 10 ft. high. That is, the ratio of width to height of the measuring panel is only 1 to 4. With so low a ratio, unless special precautions are taken, the measurements are likely to be distorted considerably by end effects. The introduction of watertight joints between the movable panel and the side walls, when experiments on saturated soils are to be undertaken, will only accentuate this difficulty. As no mention of this point is made in the paper, it would be of interest to learn how it is proposed to deal with it.

D. P. KRYNINE,[42] M. AM. SOC. C. E. (by letter).—Prior to 1925, when Charles Terzaghi, M. Am. Soc. C. E., published his findings in the field of soil mechanics[43] that science was a rather weak sub-division of engineering mechanics. It dealt with imaginary dry substances affected by friction, and sometimes cohesion. The development of the science was accelerated when it was emphasized that the backbone of engineering soil research is the study of the physical properties of the material.

Rôle of Soil Physics.—In the present epoch a new science of soil mechanics is being created and no necessary tool should be disregarded. Soil physics has proved to be a mighty "base" on which engineering theories may be erected; therefore, a thorough knowledge of it should not be underestimated. Obviously, such knowledge is economically worthless, as Professor Gilboy rightly states, unless it provides a means for answering engineering questions. Unfortunately, the branch of soil physics that may happen to be necessary in the future is not known at present; and a modern worker in soil engineering research should be able to study any new branch of soil physics, at any time, and even to develop it independently if necessary. For this purpose a certain training and familiarity with soil physics and related sciences are required.

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[42] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

[43] "Erdbaumechanik auf bodenphysikalischer Grundlage," by Charles Terzaghi, Leipzig and Vienna, 1925.

Some time ago soil colloids were considered to be the cause of many phenomena in soils. Professor Terzaghi did not hesitate to plunge into a detailed study of the mechanics of adsorption, and he published a paper on the subject that would have done credit to any specialist in physical chemistry.[44] The attitude of any modern soil engineer should be similar; otherwise, the development of the science will be retarded. However, his writings, addressed to the general technical public, should be different from those addressed to specialists. They should be concise and should contain as little as possible of complicated formulas and physical speculations not directly applicable in engineering practice.

Mechanical Analysis.—Civil engineers received their first knowledge of soil research procedure from the agriculturalists, and from that source came a tendency to over-estimate the value of mechanical analysis. Those who work in soil mechanics know how much time has been spent in studying and improving methods of making mechanical analyses.

Considerable research on sedimentation has been done by most investigators. Sedimentation methods depend on the fact that in a viscous medium, such as water, particles sink at different speeds depending on their weight, specific gravity, and configuration. The relation between their diameter and their velocity of sinking is given by the well-known formula introduced by Stokes.[45] This equation is valid for spherical particles. However, clay particles are scale-like and, therefore, Stokes' formula can give only the "effective diameter" of the particle, which is defined as the diameter of an imaginary sphere of the same material which would sink in water with the same velocity as the given particle. The general idea of the test consists in stirring the soil solution well and allowing it to stand for a given time, *t*, when the turbid liquid is poured or siphoned into a receptacle; it does not contain particles coarser than a certain diameter, *d*, computed from Stokes' formula.

The quantity of suspended particles in the turbid liquid may also be measured directly, without pouring or siphoning (hydrometer method). Results obtained by even the most refined sedimentation methods, however, are only approximate. The Russian physicist, G. J. Pokrowski, studied[46] some optical properties of soils and found that variations of such properties correspond only to variations in the size of particles more than 0.3 microns in magnitude. In other words, it seems to be inadequate to apply Stokes' formula to particles less than 0.3 microns in magnitude, that is, to small colloidal particles and to those somewhat coarser. The walls of the container exert a certain influence on the particles according to their electric charge; and the path of sinking particles is not at all straight, as Stokes' formula assumes.

The conclusion to which engineering soil investigators have come lately, may be formulated as follows. Since complicated methods give only approximate results, it is preferable to save time and energy by using simpler methods provided the degree of accuracy remains practically the same. In this way, the methods of Wiegner, of Gessner, of Sabanine (used in Russia),"[47] of Robinson, and others, which may be adequate for agricultural purposes, have become merely of historical interest to a civil engineer. For engineering routine research, sieve analysis for coarser particles and the Bouyoucos hydrometer test for minute particles are to be recommended.

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[44] "The Mechanics of Adsorption and of Swelling of Gels," by Charles Terzaghi, Colloid Symposium Monograph, 1926.

[45] "The Physical Properties of Soil," by Bernard A. Keen, Lond., 1931, p. 39 *et seq.*

[46] "Optical Investigation of Soil Suspensions," by G. J. Pokrowski, Soil Investigations in 1927 (in Russian), Moscow, 1928, p. 57 *et seq.*

A full description of methods of procedure for mechanical analysis for the combined sieve and hydrometer method has been published by the U. S. Bureau of Public Roads.[48] In applying the Bouyoucos test, proper use of deflocculating agents should be emphasized (sodium silicate solution for soils having a plasticity index of less than 20 and hydrogen peroxide for heavier clays). Professor Bouyoucos himself describes[49] this procedure in a manner somewhat different from that of the Bureau of Public Roads.

The accumulated percentages of particles of different effective diameters may be plotted to a logarithmic scale as shown in Fig. 1 of Professor Gilboy's paper, and thus a particle—size accumulation curve—may be obtained. In his routine research for highway purposes the writer seldom makes the complete hydrometer test. It has been felt that in the majority of practical cases, the 15-min. Bouyoucos test is all that is needed. Admittedly this is rather empirical; but it gives a fair idea of the amount of colloidal matter in the soil. This is especially true of low percentages when obtained in the 15-min. test.

Moisture Equivalent.— The writer understands that no research on moisture equivalent has been made at the Massachusetts Institute of Technology. Possibly this fact may be explained by the lack of interest in this item on the part of foundation engineers. On the contrary, highway engineers consider the study of the moisture equivalent as an important feature. For reasons which follow, the writer wishes to suggest the importance of a certain familiarity with the moisture equivalent in foundation engineering as well.

The moisture equivalent represents the percentage of water, based on the dry weight of the soil sample (usually 5 grammes), retained by the soil after first being soaked in water for about 6 hours, then drained in a humidifier for 12 hours, and, finally, subjected to the action of a centrifugal force with an acceleration one thousand times that of gravity, for 1 hour. This is the procedure of the U. S. Bureau of Public Roads; the duration of soaking and draining, as recommended by the U. S. Bureau of Soils, is different. The centrifuge method was introduced by Messrs. Lyman J. Briggs and John McLane[50] about 1907; a few years ago a field method was proposed[51] by A. C. Rose, Assoc. M. Am. Soc. C. E. However, on applying the centrifuge method difficulties arise. In the case of heavy clays, especially of the sticky type, the water is not thrown out through the holes in the bottom of the container (Gooch crucible), but collects on the top of the layer. This phenomenon, called "water-logging," should be of interest to foundation engineers. According to computations by Professor Terzaghi the process of centrifuging is equivalent to the application of a load equal to 2 kg. per sq. cm.

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[47] "Textbook on Highway Engineering," by D. P. Krynine, Third Edition (in Russian), Moscow, 1930, p. 190.

[48] "Procedures for Testing Soils, etc.," by A. M. Wintermeyer, E. A. Willis, and R. C. Thoreen, *Public Roads*, Vol. 12, No. 8 (1931), p. 198 *et seq.*

[49] "A Comparison of the Hydrometer Method and the Pipette Method, etc.," by George John Bouyoucos, *Journal*, Am. Soc. of Agronomy, Vol. 22, No. 8 (1930), p. 747 *et seq.*

[50] "The Moisture Equivalent of Soils," by Lyman J. Briggs and John McLane, U. S. Dept. of Agriculture, *Bulletin 45*, Washington, D. C., 1907. A great number of papers along these lines are to be found in *Soil Science* and other magazines.

[51] "Foundations and Drainage of Highways," by Albert C. Rose, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 64 *et seq.*

The theory of hydrodynamic phenomena in soils requires the pressing out of water under load. Possibly in a loaded soil layer the tendency is for water to be squeezed out at both top and bottom. Owing to the sticky nature of some gels the clay may become impermeable at the bottom of the layer, or close to it. This is analogous to the phenomenon of water-logging in the process of centrifuging. The same may happen close to the top, or in the middle, of a layer. Thus, instead of being thrown out, water remains in the interior of the layer, which may conserve its voids-ratio close to the liquid limit. Such cases have been disclosed in some borings and, as far as the writer knows, have not been thoroughly understood.

Further study will possibly establish closer relationship between the values of both the moisture equivalent and the coefficient of permeability. Furthermore, there is interdependence between the contrifuge moisture equivalent and the colloidal content of the soil as determined by the 15-min. Bouyoucos test. This fact has been stated by Professor Bouyoucos;[52] an analogous interdependence has been found by the writer in testing more than one hundred samples of soils in New Haven County, Connecticut.[53]

Limits of Consistency.—The procedure of determining the limits of consistency has been fully described in *Public Roads.*[48] The method of determining the liquid limit is stated as follows: "The dish is held firmly in one hand, with the groove parallel to the line of sight, and tapped lightly with a horizontal motion against the palm of the other hand ten times." Obviously, the apparatus constructed by Mr. A. Casagrande[54] minimizes the personal equation and is therefore superior to that adopted by the Bureau of Public Roads. The Bodman and Tamaschi method[55] involves dropping a flat-bottomed, straight-sided, metal container (50 mm. in diameter, and 13 mm. high), holding the plastic mass of soil (weight, 20 grammes when in water-free state), from a height of 30 cm. A ditch of triangular cross-section is excavated along the dish with two razor blades held at an angle of 45° to the earth's surface. The number of impacts is plotted against the moisture content. The equation of the curve is:

$$N = aw^n \qquad (7)$$

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[52] "A New, Simple, and Rapid Method for Determining the Moisture Equivalent," by George John Bouyoucos, *Soil Science,* Vol. 27 (1929), p. 234 *et seq.*

[53] "Some Principles of Soil Surveying and Soil Mapping for Road Purposes," by D. P. Krynine. Paper presented to the Highway Research Board, National Research Council, in December, 1931.

[54] See p. 222.

[55] "Studies of Soils in the Plastic State," by G. B. Bodman and M. Tamaschi, *Soil Science,* Vol. 30 (1930), p. 175 *et seq.* Another example is the method of Mr. Roy C. Roberts, as described in the Report of the Am. Soil Survey Assoc., Ames, Iowa, 1930, p. 56 (mimeographed copy).

in which, N is the number of impacts needed to fill the ditch; w, the percentage of water as referred to the dry weight of the sample; and a and n are constants. Obviously, the Bodman and Tamaschi method has much in common with the idea developed by Mr. Casagrande.

There have been many other attempts to measure plasticity, advantage of which may be taken by a soil engineer. Dr. J. W. Mellor takes[56] a clay ball of standard size, applies a sufficient stress on a standard area to cause steady compression, and measures the total motion required to split the ball open at the edges. Thus, two factors may be obtained—stress and strain—both depending on the water content. Plasticity is expressed by the product of these two factors, when this product reaches a maximum. In Bischoff's test,[57] mixtures of soil powder, sand, and water are prepared. After drying, a moist brush is passed twenty-five times back and forth on the surface of each cylinder. Some of the cylinders (the bulging ones) will still remain convex, while others will become concave. Suppose that after the test there are n convex cylinders; then, according to Bischoff, n is the plasticity coefficient. In the method introduced by Messrs. T. W. Talwalkar and C. W. Parmelee[58] a plastic clay bar is subjected to torsional stresses, and a curve showing the stress-strain relations is drawn. There are still other methods, as, for instance, that of Professor Zemiatchensky.

Attempts have been made to explain soil plasticity by colloidal content. Mr. M. L. Nichols[59] has found experimentally that the lower plastic limit and the plasticity index depend on the colloid content of the soil. According to Mr. Nichols, the lower plastic limit decreases with the increase of the "activity" of the colloids acting in a given soil; and the plasticity index (or "plasticity number") may be expressed by an exponential function of the colloid content. According to Mr. Nichols' computations, a soil with 15 to 17% colloid content would have a plasticity index of 1; and this is quite closely in agreement with practical data. The writer does not mean that the attempt to explain so many soil properties by its colloidal behavior should be revived in soil mechanics; but he wishes to point out that colloid content determined by even a rudimentary method represents a valuable datum which should never be lost sight of by the engineering soil investigator.

Influence of Scale-Like Particles.—Atterberg has found[60] that only soils having disk or plate-shaped (scale-like) particles showed signs of plasticity. Mr. L. D. Baver has attributed[61] plasticity to the orientation of plate-shaped colloidal particles. The influence of scale-like particles, such as mica, on the compressibility of soils has been evidenced by the research of Professor

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[56] "On the Plasticity of Clays," by J. W. Mellor, *Transactions*, Ceramic Soc. (British), Vol. 21 (1921–22), p. 93–94.

[57] "Die feuerfeften Tone," by Bischoff, 1904, p. 84.

[58] "Measurement of Plasticity," by T. W. Talwalkar and C. W. Parmelee, *Journal*, Am. Ceramic Soc., Vol. 10 (1927), p. 670.

[59] "An Explanation of Dynamical Properties of Soils," by M. L. Nichols, *Agricultural Engineering*, Vol. 12, No. 7 (1931), p. 259 *et seq.*

[60] "Die Konsistenz und die Bindigkeit der Böden," by A. Atterberg, *International Mitteilungen Bodenkunde*, Vol. 2 (1912), p. 149 *et seq.*

[61] "The Atterberg Consistency Constants," by L. D. Baver, *Journal*, Am. Soc. of Agronomy, Vol. 22 (1930), pp. 935–948.

Terzaghi and Professor Gilboy.[62] Scale-like colloids belong to the same category as mica. Possibly, Baver's opinion[61] relative to plasticity should be generalized and applied to compressibility as well. In other words, compressibility of soils should be explained, not by the "scale-likeness" of particles as such, but by their orientation, or by the possibility of such orientation in applying the load.

In order to clarify the idea of orientation, suppose a small paper slip is dropped on the floor. Its path of falling is more or less vertical, and the slip will be deposited with its face touching the floor; that is, it will be oriented horizontally. Let a great number of small paper slips be dropped simultaneously. Although most of them will be oriented on the floor, some will not be so oriented. Such is the case of sedimentary clays consisting of small scale-like particles; and the degree of orientation can be different in different clays. Therefore, in a disturbed sample all the particles are mixed and the degree of orientation has little to do with the natural state. Obviously, in using disturbed samples in the form of powder, not only the degree of orientation, but the voids-ratio and the character of the particle itself are not the same as in Nature. It has already been stated that according to Stokes' formula clay particles lying higher should be finer, as a rule, than those from lower layers. Therefore, if particles possessing different properties are mixed so as to form a "disturbed" sample, the testing would scarcely give very desirable data. The initiative of Professor Gilboy in studying the question of how to obtain large undisturbed samples is welcomed.

An attempt of Dr. Pokrowski to express the degree of orientation numerically should be mentioned.[63] The intensity of polarized light reflected by the surface of the soil was proved to depend on the degree of orientation. The latter may be measured by the ratio of two light intensities reflected in predetermined directions. The maximum value of this ratio was proved to be equal to 1.13 (talc), and it varies between 1.03 and 1.07 for clays.

Permeability.—Darcy's law (Equation (2) of Professor Gilboy's paper) may be written in the form:

$$v = \frac{Q}{At} = k\,N \qquad (8)$$

in which, v is the velocity of flow. Forchheimer showed[64] that generally:

$$v^m = k\,N \qquad (9)$$

in which, the limits of m are 1 and 2.

Smreker[65] proposed an equation in the form of Equation (9):

$$N = a\,v^2 + b\,v^{\frac{3}{2}} \qquad (10)$$

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[62] "The Compressibility of Sand Mica Mixtures," by Glennon Gilboy, *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, p. 555 *et seq.*

[63] On Orientation of Particles in Soils," by G. J. Pokrowski, Soil Investigations in 1928–1929 (in Russian), Moscow, 1930, p. 88.

[64] "Wasserbewegung durch Böden," by Ph. Forchheimer, *Zeitschrift* des Vereins Deutscher Ingenieur, Vol. 45 (1901), p. 1781 *et seq.* This formula is also mentioned by Forchheimer in "Hydraulik," Berlin, 1914, p. 420.

[65] "Das Grundwasser, seine Erscheinungsformen, Bewegungsgesetze, und Mengenbestimmung," by O. Smreker, Leipzig und Berlin, 1914, pp. 29–33.

in which, *a* and *b* are constants. Afterward, he omitted the first member in the right-hand side of Equation (10) and obtained:

$$N = b\ v^{\frac{3}{2}} \quad \text{.........................(11)}$$

This formula provoked considerable discussion in Europe.

On proposing his formula Darcy had a limited application[66] in mind; therefore, its application in other cases may serve only as an approximation. Actual research, however, is generally based on the assumption that Darcy's law is an exact one. In this connection, it should also be emphasized that the coefficient of permeability, *k*, is not a constant, but fluctuates during the percolation. In the beginning of the percolation, the initial value of *k* decreases. This fact has been established by many investigators, beginning with Forchheimer[64] and others before 1901, to the modern research workers of the U. S. Bureau of Chemistry and Soils in 1931.[67] After a certain time, the value of *k* may increase again considerably. Strictly speaking, such fluctuations should be taken into account in designing permeameters.

Another inconvenience of the apparatus in Fig. 4 is the possibility of "colloid erosion" in using high values for hydraulic gradients. Under the action of a water column, colloids should migrate downward, rapidly, if the column is high, and slowly, otherwise. A proof of this statement is furnished by Nature herself; as known, colloidal matter from the upper soil horizons has been partly removed to form lower illuvial horizons that are richer in colloids.

The following simple device for determining the coefficient of permeability has been used by the writer for loams and sandy soils in Connecticut. In Fig. 23 there are two metallic containers, *A* and *B*. The bottom of the inner cylinder, *A*, is perforated and covered with a filter paper. The soil sample is placed on this paper so that its upper surface corresponds to the edge of the outer cylinder. The edge serves as an overflow device. First, the outer cylinder is filled full of water, and the sample is allowed to absorb as much as it can. Additional water should be added gradually in order to maintain a constant level in the outer cylinder. Afterward the inner cylinder is carefully filled without disturbing the surface of the sample; percolation begins immediately. Suppose that at a certain time, *t*, from the beginning of percolation, the water level in the inner cylinder has dropped to a distance, *a*, from the top. Let, $H =$ the vertical distance from the top of the inner cylinder to the top of the outer cylinder; $d =$ the thickness of the sample; and $k =$ the coefficient of permeability. Then, applying the Smreker formula:

$$k = \frac{3\ d^{\frac{2}{3}}}{t}\left(\sqrt[3]{H} - \sqrt[3]{H - a}\right) \text{...............(12)}$$

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[66] "Les fontaines publiques de la ville de Dijon," by H. Darcy, Paris, 1856, p. 590.

[67] "A Laboratory Study of the Field Percolation Rates of Soils," by C. S. Slater and H. G. Byers, *Technical Bulletin No. 232*, U. S. Dept. of Agriculture, January, 1931, p. 9.

The height, H, and the thickness of the sample, d, are supposed to be known. For a given value of a,

$$k = \frac{c}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

in which, c is a constant, corresponding to the given value of a.

The apparatus is unusually easy to handle and adequate for rough work. The inner cylinder may be taken out and driven into the ground; and thus an undisturbed sample may be obtained and tested. In order to avoid mistakes in the beginning of the percolation, the writer generally observes the time, t, necessary for the water to drop from a level, $a = a_1$, to another level, $a = a_2$. Both levels are marked with white lines on the inner surface of the cylinder. In this case, the form of Equation (13) is the same as before; and the constant, c, has been computed once and for all. It should be added that the high level of the overflow permits one to keep the sample entirely saturated from the beginning of percolation. It is obvious that the results of permeability tests should be reduced to a standard temperature (usually + 10° cent.).

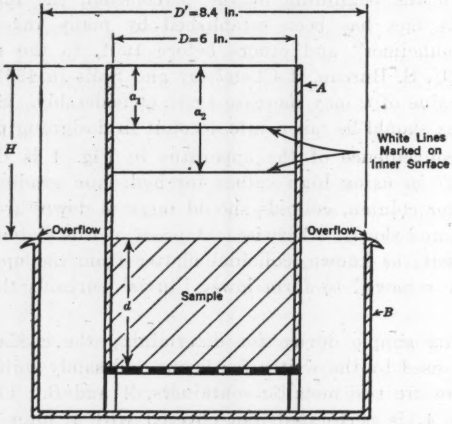

FIG. 23.—A SIMPLE DEVICE FOR MEASURING PERMEABILITY OF SOILS.

The horizontal capillarity test proposed by Mr. Casagrande should receive special attention because of its unusual interest (see Fig. 5). From Equation (5) of the paper, the following may be written.

$$\frac{x^2}{t} = C_1 \sqrt{k} \text{ (a constant)} \ldots\ldots\ldots\ldots\ldots\ldots(14)$$

in which, C_1 (for a given soil in a given condition) is a constant. The value of k is assumed to be constant.

As far as the writer knows, formulas for the speed of capillary movement, both vertical and horizontal, were first developed[68] by Diro Kitao, in 1897.

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[68] "Handbuch der Bodenkunde," Edited by E. Blanck, Vol. 6, p. 104 *et seq.*; also, p. 172.

He was also the first to draw the attention of investigators to the meaning of the ratio, $\frac{x^2}{t}$.

Later investigators, for instance, J. S. Kozeny[69] also paid due attention to the study of the ratio, $\frac{x^2}{t}$. However, the idea of applying observations on capillary movement to the determination of the coefficient of permeability, k, was clearly expressed only in 1925, by Professor Terzaghi.[43] In 1930, Zunker[68] proposed the determination of the value of k from data referring to the downward capillary movement. His formula is:

$$\frac{x^2}{t} = 2k\ (H - Z + L) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which, Z and L are values of excess atmospheric pressure at the bottom and the top of the capillary column, respectively, and H, the pulling force of the meniscus. Approximately:

$$\frac{x^2}{t} = 2k\ H \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

or,

$$\frac{x^2}{t} = C_2\, k \text{ (a constant)} \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

in which, C_2 is a constant for a given soil. The form of the right side of both Equations (14) and (17) being somewhat different, it would be interesting to see a development of the formula. It should also be emphasized, that the capillary saturation, mentioned by Professor Gilboy in introducing Fig. 5, does not accompany the moving meniscus at all. Saturation takes place gradually from the entrance of the experimental tube to its outlet; and sometimes many hours are needed for complete saturation. Observations were made on the gradual saturation of soil in the experimental tube without extracting samples therefrom for the determination of moisture content.[70] For this purpose the "reflectometer" of Dr. Pokrowski[71] was used. This is an apparatus which permits the determination of the moisture content of a given soil by observing certain optical phenomena on the visible surface of the moist soil through the walls of the experimental tube. Progressive saturation curves for tubes, 17 and 23 mm. in diameter, are shown in Fig. 24. In these curves, h is the time, in hours, from the beginning of capillary movement. The visible moisture line is not shown.

Compressibility and Consolidation.—The theories of soil mechanics referring to compressibility and consolidation, and the consolidation apparatus shown in Fig. 7, have been developed almost exclusively by Professor Terzaghi.[43] Fig. 25 shows, diagrammatically, the general installation of the same apparatus at Yale University: *A* is a wooden plank fixed by the clamps, *B,* to a solid laboratory table, *T.* A metallic column, *C,* carries the axis of rotation of a lever, *L,* on the other side of which a variable load, *P,* may be applied (bags containing different quantities of shot). Thus, loads may be applied to the experimental cylinder, *E,* in definite increments. The installation is very simple and inexpensive.

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[69] "Ueber den kapillaren Aufstieg des Wassers in Boden," by J. S. Kozeny, *Sitzungsberichte,* Akademie Wissenschaft, Wien, Vol. 136 (1927), p. 271 *et seq.*

[70] "Penetration of Water into Clay," by D. P. Krynine, Soil Investigations in 1928–1929 (in Russian), Moscow, 1930, p. 41 *et seq.*

[71] "The Reflectometer," by G. J. Pokrowski, Soil Investigations in 1928–1929 (in Russian), Moscow, 1930, p. 80 *et seq.*

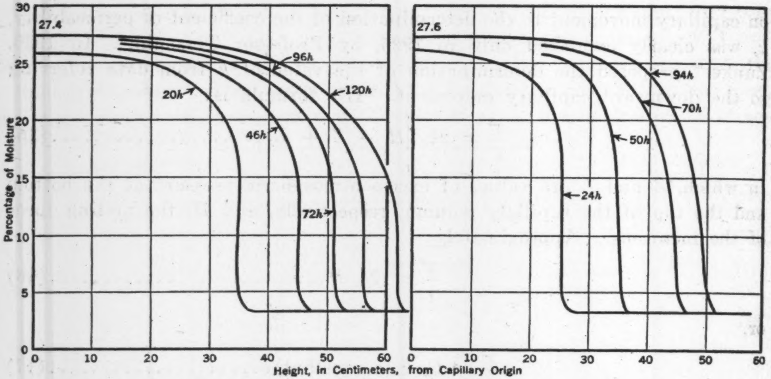

FIG. 24.—PROGRESSIVE CURVES OF CAPILLARY SATURATION IN A GLACIAL LOAMY CLAY.

In the writer's opinion the shape of the curves obtained for samples ½ in. thick, as shown in Fig. 6, may be considered only an indication (not conclusive) of the consolidation of a natural sedimentary clay layer. Obviously, it is understood that the difference in thickness should be taken into account. A simple consideration of Stokes' formula[45] leads one to the conclusion that, as a rule, a sedimentary clay layer cannot be homogeneous. The water in the basin where such a clay is being formed, is practically a col-

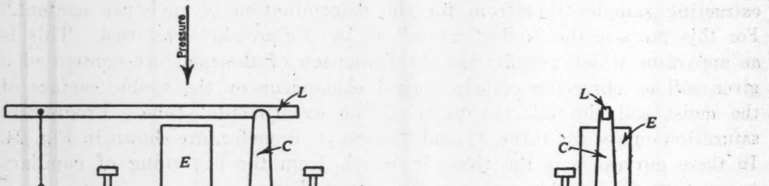

FIG. 25.—SKETCH OF A SIMPLE INSTALLATION OF THE CONSOLIDATION DEVICE.

loidal suspension; colloids settle gradually to the bottom, flocculating or simply mixing with fresh material reaching the basin from eroded localities. The influx of the eroded material is of intermittent intensity, and possibly it stops periodically. The topography of the bottom of the basin and the water level of the basin may change. In this connection superficial crusts may sometimes be formed. The mechanics of the formation of such crusts was developed by Professor Terzaghi.[48] Furthermore, clay may be sticky, and traps for percolating water may be formed under the action of loads as stated in the foregoing remarks under "Moisture Equivalent."

As a result, some faulty permeable or completely impermeable colloid "sheets" may exist in the interior of a clay layer. This sometimes should disturb the whole picture of consolidation. The writer believes, therefore, that special attention should be paid to the study of the geological profile in the particular case. Especially,.places at which the natural water content is close to the liquid limit should be examined carefully. These places in addition to disturbing the consolidation, may reveal the existence of lateral flow and thus cause the unexpected settlement of a building.

Internal Friction and Cohesion.—Both internal friction and cohesion are responsible for the shearing resistance of the soil as stated by Professor Gilboy (see, "Internal Friction and Cohesion"). Numerous investigations of friction have been made; and its nature is somewhat clearer than that of cohesion. An adequate definition of cohesion was given by Newton in his "Opticks," before 1721; and about 1773 the laws of friction and cohesion, as affecting a mass of soil, were formulated by Coulomb.[87] To-day, cohesion is being studied to a considerable extent by metallographists and physicists working in that field. Although some progress has been made, the question of cohesion in metals seems still to be open. Further details may be found in the publications of the Faraday Society.[72] The problem of cohesion appears to be even more complicated in soil mechanics than in metallography. In this case the investigator has to deal not with a "continuum" as in metals, but with millions and millions of small bodies, often surrounded by water films stuck to the surface of the particle, and separated one from another by cushions of water and air of variable thickness. Professor Terzaghi, as stated by Professor Gilboy, has distinguished between true and apparent cohesion. For practical purposes the apparent cohesion only is interesting in the majority of cases.

A short statement as to the nature of apparent cohesion may be formulated as follows:

(*a*) A mass of soil generally consists of air, water, and hard soil particles. The molecular forces of hard particles may cause water to stick to their surfaces; or particles may adsorb water in the form of films. In their turn, there are molecular forces between particles of water; and these forces are responsible for the tensile and shearing resistance of water in films. It should be emphasized that water in capillary films is rather hard, similar to hardened glue.

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[72] "Cohesion and Related Problems": A General Discussion held by the Faraday Society, November, 1927. Attention should be drawn to the paper of N. K. Adam, entitled "Cohesion in Surface Films," p. 149.

(*b*) The molecular forces act within a very short distance from the surface of a particle (range of molecular forces). They are strong close to the surface and rapidly decrease toward the boundary of their range. When a surface of separation (or surface of rupture) passes between two particles surrounded by water films, a phenomenon takes place analogous, to a certain extent, to the shearing of a beam, relatively weak in the middle of the span and very strong on the ends (Fig. 26). In cohesive soils this span is microscopically small. The smaller the span (that is, the distance between the particles), the stronger is the beam, principally because of the dropping of the weak part from the middle.

The following conclusions may be drawn from Fig. 26, which may easily be checked in practice: (1) Ramming or packing of soil increases its cohesion, because of the decrease of the span of the imaginary beam (Fig. 26); (2) settlement of soil produces the same effect, for the same reason; and (3) saturation of soil with water makes the span of the imaginary beam increase; as a result, cohesion is reduced. Thus, the principal cause of cohesion in soils is water in proper proportion, capillary or even hygroscopic. There is also an opinion that a mineral binder may contribute to the increase of cohesion of smaller particles.[73]

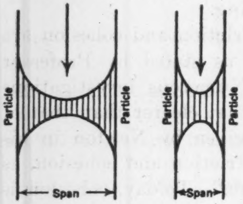

Fig. 26. — Graphic Representation of Increase of Shearing Resistance With a Decrease of Distance Between Particles.

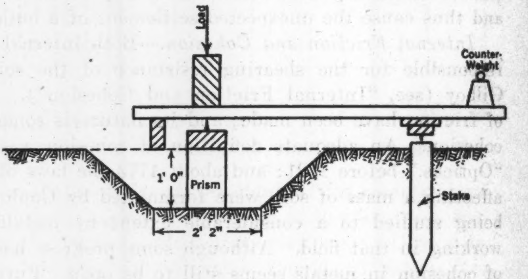

Fig. 27.—Determination of Shearing Resistance of the Soil After Buisman.

Although Professor Gilboy does not formulate conclusions drawn from working with the machine of Figs. 10 and 11, the few results he cites are very interesting, and that referring to the angle of friction of clays, should be especially emphasized. Contrary to a widespread opinion, the angles of friction of clay, ϕ, appear to be considerable. Perhaps this is the eve of important events in this branch of research.

The existing methods of determining cohesion are principally those of the laboratory. The well-known idea of pulling one box of soil along the surface of another was developed by the late William Cain, M. Am. Soc. C. E., and has appeared many times in literature. The method based on the rupture of a cylinder (Fig. 9), was elaborated by Professor Janicsek, of the Technical University of Hungary, when working under the supervision of Professor Terzaghi, at Cambridge, Mass. The procedure in question is simple and efficient, but inconvenient in some respects. In addition to a certain difficulty in measuring the angle, α, accurately, there remains unconsidered the influence of the friction between the top and bottom of the experimental cylinder, and the compression machine, as Professor Janicsek himself points out in his writings.[74]

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[73] "Interrelationship of Load, Road, and Subgrade," by C. A. Hogentogler, Assoc. M. Am. Soc. C. E., and Charles Terzaghi, M. Am. Soc. C. E., *Public Roads*, Vol. 10, No. 3 (1929), p. 37 *et seq.*

Professor A. S. Buisman, of Holland, finds the coefficients of friction and cohesion by a similar procedure.[75] In addition to his laboratory work, Professor Buisman made experiments in the field destroying soil prisms as shown in Fig. 27. It is noteworthy that for a clay and a loam, tested in the laboratory, Professor Buisman found the values of ϕ to be between $27\frac{1}{2}$ and 29 degrees.

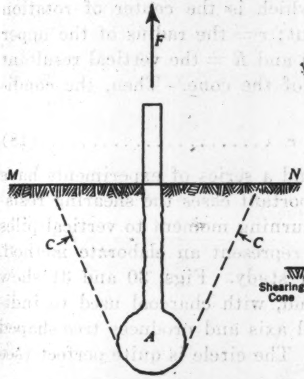

FIG. 28.—POSSIBILITY OF DETERMINING SHEARING RESISTANCE OF SOIL BY PULLING OUT A MALONE ANCHOR.

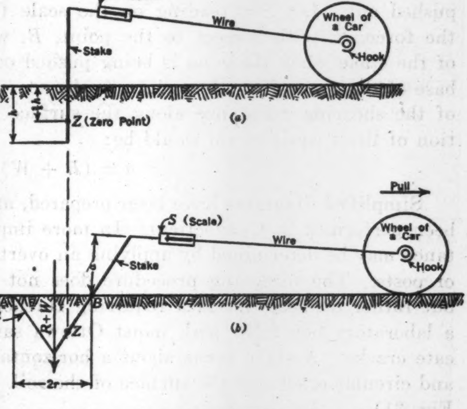

FIG. 29. — SIMPLE METHOD OF DETERMINING SHEARING RESISTANCE OF SOIL BY PULLING A STAKE WITH AN AUTOMOBILE.

Among his other experiments, the strength of earth in shear has been studied[76] by John H. Griffith, M. Am. Soc. C. E. He used a method somewhat similar to that proposed by Professor Cain, but dealt with blocks of natural earth about 12 in. wide and 24 in. long. The ratio of stress to strain, or the shearing modulus, was computed, and corresponding curves were drawn.

In 1923, dynamometer tests on a special type of anchors for transmission towers were conducted for the Blaw-Knox Company in Pittsburgh, Pa., and, in 1931, similar tests made in Java, were described[77] by O. J. Herz. The Java experiments consisted in pulling some buried concrete masses from the ground by applying a vertical force. Both the Pittsburgh and the Java experiments disclose the possibility of determining shearing resistance of soil. In Fig. 28, *MN* is the surface of the ground; *A* represents the anchor used in the Pittsburgh experiments; *F* is a vertical pulling force measured with the dynamometer; and *C* is the surface of the cone of destruction (shearing surface).

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[74] "Alkalmas—e a Koska a Töröszilárdság Megallapitásápará?" by Jozsef Janicsek, *Technika*, Vol. 8, Nos. 3 and 4 (1927), pp. 73 and 101.

[75] "Eenige beproevingsmethoden ter bepaling van den inwendigen wrijvingeweers tand van grondsorten," by A. S. Buisman, *De Ingenieur*, Vol. 26 (1928), p. 135 *et seq.*

[76] "Physical Properties of Earths," by John H. Griffith, *Bulletin 101*, Iowa Eng. Experiment Station, Ames, Iowa, 1931.

In 1930 the writer proposed a field-stake method for determining shearing resistance of loamy soil. The method consists of driving a rigid metallic stake into the soil and pulling it with an automobile. In the beginning of the experiment the stake turns about a zero point, *Z* (Fig. 29). Cracks are then formed within a circle on the ground; and, finally, a conic body is pushed out. Let $S =$ reading on the scale (Fig. 29); $a =$ moment arm of the force, *S,* with respect to the point, *B,* which is the center of rotation of the stake when the cone is being pushed out; $r =$ the radius of the upper base of the cone; $W =$ the weight of the cone; and $R =$ the vertical resultant of the shearing resistance along the surface of the cone. Then, the condition of limit equilibrium would be:

$$S\ a = (R + W)\ r \quad \text{......................} (18)$$

Simplified diagrams have been prepared, and a series of experiments have been performed in Connecticut. In more important cases the shearing resistance may be determined by applying an overturning moment to vertical piles or posts. The foregoing procedure does not represent an elaborate method, but rather discloses an idea requiring further study. Figs. 30 and 31 show a laboratory box filled with moist Ottawa sand, with charcoal used to indicate cracks. A stake turns about a horizontal axis and produces tree-shaped and circular cracks on the surface of the soil. The circle is quite perfect (see Fig. 31).

Foundations.—The question of foundations has been thoroughly investigated by Professor Terzaghi.[78] The statics of foundations should not be studied further in the laboratory unless a completely new line of research is evolved. A great number of experimental data are available, and a large percentage of these data are known not only to investigators, but to practical engineers as well. For instance, transmission of pressure through soils has been the subject of many investigations since 1879, and a more or less complete bibliography thereof may be found in a German publication.[79] Its authors, Professor Koegler and Dr. Scheidig, conducted comprehensive experiments and made attempts to develop a theory of pressure distribution. In some directions their results are similar to those obtained[80] by A. T. Gold-

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[77] "Zugwiderstand eines Mastfundamentes und Scherfestigkeit des Lehmbodens," by O. J. Herz, *Zeitschrift* des Oesterreichischer Ingenieur und Architekten Vereins, Vol. 83 (1931), Nos. 9–14, pp. 59, 73, 93 *et seq.*

[78] "The Science of Foundations," by Charles Terzaghi, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 93 (1929), p. 270.

[79] "Druckverteilung im Baugrunde," by F. Koegler and Dr. Scheidig. A series of articles published in *Bautechnik,* 1927–1929.

[80] *Proceedings,* Am. Soc. for Testing Materials, 1916, Pt. 2, p. 309, and 1917, pp. 640–641; *Proceedings,* Am. Soc. C. E., May, 1925, Papers and Discussions, p. 893; and *Public Roads,* Vol. 5 (1925), p. 1.

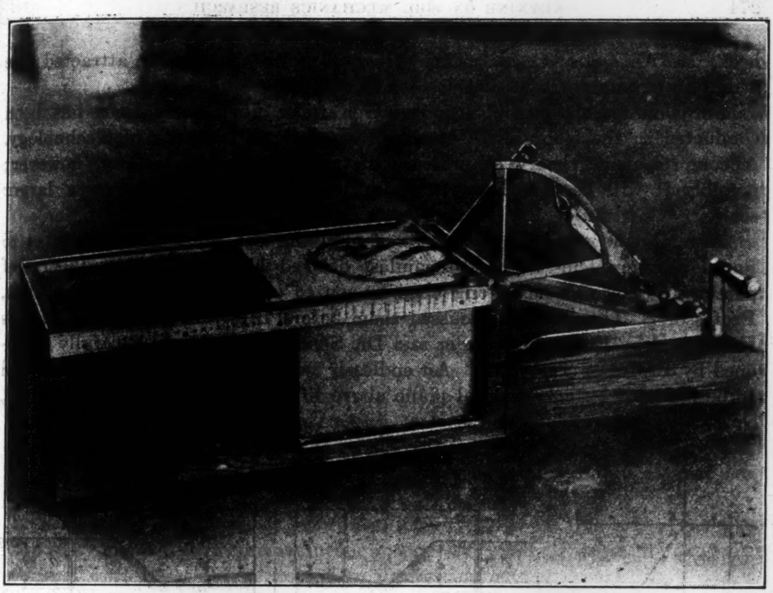

FIG. 30.—CONE OF MOIST OTTAWA SAND CREATED BY TURNING A METALLIC STAKE ABOUT A HORIZONTAL AXIS.

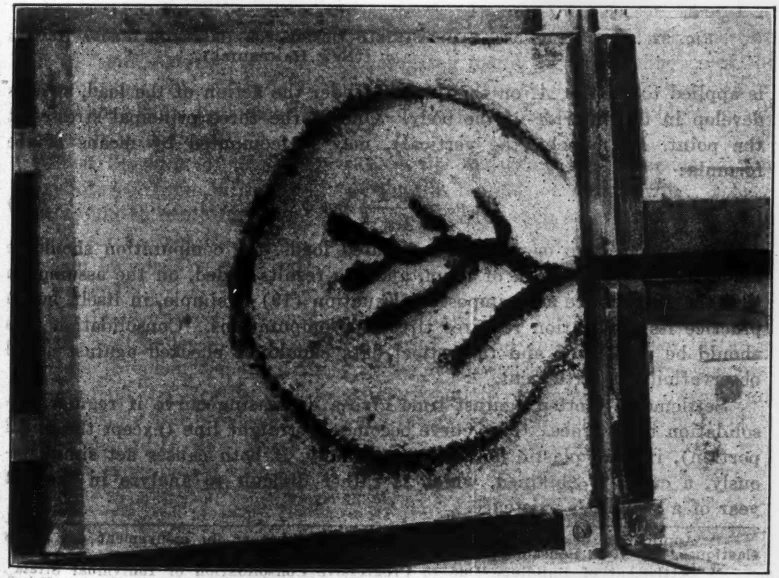

FIG. 31.—PLAN VIEW IN WHICH CHARCOAL INDICATES HOW NEARLY A TRUE CIRCLE IS FORMED.

beck, M. Am. Soc. C. E. The "pressure bulbs" have also attracted the attention of engineers.

For this reason, possibly, no research work along these lines has been conducted in the laboratories of the Massachusetts Institute of Technology. A comprehensive theory of pressure distribution is still lacking, however; possibly it will be elaborated in connection with observations on a larger scale.

A few years ago, the idea arose of applying the Boussinesq formulas to foundation problems. These formulas, derived by applying the theory of potential function,[81] have long been known to civil engineers; for instance, Professor S. Timoshenko discussed them before 1918. In 1928 the idea was formulated by Professor Koegler and Dr. Scheidig[79], and, in 1929, fully developed by Professor Terzaghi.[82] An endlessly wide and elastic body is assumed to be bounded by a horizontal plane above it (Fig. 32). A vertical load, *P*, is applied to a point *A*, on this plane. Under the action of the load, stresses develop in the interior of the body. One of the three principal stresses at the point, *B* (namely, the vertical), may be computed by means of the formula:

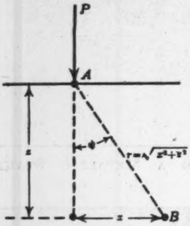

FIG. 32.

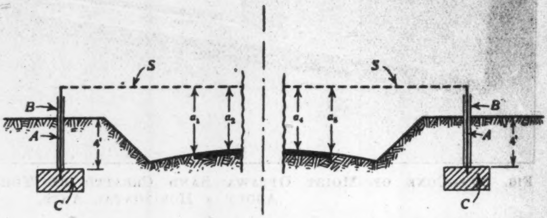

FIG. 33.—SAMPLE FIELD DEVICE FOR OBSERVING FROST HEAVING (NEW HAMPSHIRE).

$$p_z = \frac{3\,P}{2\,\pi\,z^2}\cos^5\psi \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

If there are two or more concentrated loads, the computation should be made separately for each of them, and the results added, on the assumption that the stresses are superimposed. Equation (19) is simple, in itself, but in practice its application requires tiresome computations. Consolidation tests should be performed and theoretical data should be checked against actual observations of settlement.

Settlements plotted against time give a decreasing curve if regular consolidation takes place. This curve becomes a straight line (except the initial portion), if pure plastic flow exists. Finally, if both causes act simultaneously, a curve is obtained, which is rather difficult to analyze in the first year of a building's existence.

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[81] "Application des potentiels à l'étude de l'equilibre et du mouvement des solides élastiques," by V. J. Boussinesq, Paris, 1885, p. 104 *et seq.*

[82] "Settlement of Buildings Due to Progressive Consolidation of Individual Strata", by Charles Terzaghi, M. Am. Soc. C. E., *Journal of Mathematics and Physics*, Vol. 8, No. 4 (1929), p. 266 *et seq.*

In this connection the necessity of obtaining good statistical material on settlements (as Professor Gilboy observes under the heading, "Foundations"), should be emphasized. At present, many superintendents and contractors consider even the possibility of settlement of their building as shameful; it should be borne in mind, however, that, as a rule, any building settles. The data on settlement should be surveyed and represented uniformly throughout the country. It would be better obviously, if the study of such settlements could be started on an international scale. Professor Terzaghi has collected numerous data along these lines in Europe, and the possibility of such an international study in close connection with him, deserves further consideration.

The subject of pile foundations has been well studied; especially valuable information has been presented[78] by Professor Terzaghi.[83] However, there is a branch of pile technique, somewhat different from foundations proper, that has scarcely been touched by research workers, namely, the stability of isolated piles and sheet-piles under the action of horizontal forces. Engineers require rules concerning the design of such features, but no adequate theory exists.

Hydraulic-Fill Dams.—It is rather difficult to discuss this important question so briefly outlined by the author in one and one-half pages of text. The research work described is more than interesting.

The writer believes that the path of hydrodynamical phenomena in the core should depend on the physical properties of the material of the shell; and this fact should be taken into account. In other words, the theoretical consolidation diagram (Fig. 12), which possibly has been traced for a certain homogeneous material, should vary according to the physical influence of the surrounding shell. This is because the shell forms a peculiar medium in which those hydrodynamical processes develop. If so, a new problem in soil physics is introduced.

Frost Heaving of Highways.—Generally speaking, the influence of low temperatures is not an important problem in foundation engineering. Exception should be made for permanently frozen soils, such as are met in Siberia and in some other localities. A vast literature on the subject exists in the Russian language; a short account in English may be found in a paper by the writer.[84]

The frost-heaving phenomena on highways have been thoroughly studied and are well understood by American scholars and engineers. For instance, Professor Stephen Taber, of the University of South Carolina, was the first to formulate the idea that the continuous supply of water from lower layers is responsible for heavings. The old theory assumed that frost heaving is due to change in the volume of water frozen in the voids; this theory was based on experiments with closed systems[85] In applied research the work of the late Mr. V. R. Burton and of Mr. A. C. Benkelman should be mentioned.[86] Progressive research in Michigan tends to introduce some corrections in the foregoing theories. Tests have disclosed some soil textures that are known to be productive of the majority of severe frost disturbances in Michigan and in other States in the frost area.

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[83] "Die Tragfestigkeit von Pfahlgründungen," by Charles Terzaghi, M. Am. Soc. C. E., *Die Bautechnik*, Vol. 8 (1930), No. 31, p. 475 *et seq.*; and No. 34, p. 517 *et seq.*

[84] "Soil Investigations in Russia," by D. P. Krynine, M. Am. Soc. C. E., *Proceedings*, Ninth Annual Meeting, Highway Research Board, 1929, p. 68 *et seq.*

[85] "Frost Heaving," by Stephen Taber, *The Journal of Geology*, Vol. 37 (1929), No. 5, p. 428 *et seq.*; and "The Mechanics of Frost Heaving," by Stephen Taber, *The Journal of Geology*, Vol. 38 (1930), No. 4, p. 303 *et seq.*; see. also, other publications by the same author.

A simple field device used in the central part of New Hampshire for observing frost heaving, is shown in Fig. 33. In this diagram, *A* is a metallic pipe, $\frac{7}{8}$ in. in diameter, embedded in a concrete block, *C*. There is also an outer pipe, *B*, 2 in. in diameter. The space between the two pipes is filled with grease. The block, *C*, is located 4 ft. below the earth's surface and is thus protected from the action of the frost, since frost penetration is known to be less than 4 ft. The purpose of the outer pipe, *B*, is to insure the inner pipe, *A*, from any disturbance which may be caused by the frozen soil. A similar device is constructed on the other side of the road, and a horizontal string, *S*, may be drawn between them during the observation. The level of the string, *S*, is constant and may be considered as the datum plane. Vertical distances, a_1, a_2, a_3, etc., are recorded at predetermined points on the string every ten days during the winter. Thus, the rate of heaving may be determined. These experiments were begun under the supervision of Mr. Casagrande.

Retaining Walls.—Only a few years ago the theory of retaining walls constituted practically all the science now known as "soil mechanics."[87] Those who are attracted by the modern theory of a perfect fluid as acting on a retaining wall would possibly find new material for reflection in the descriptions of the tests cited by Professor Gilboy.[88] These tests demonstrate that, besides being characteristic of a given soil, internal friction is also indicative of its state of compactness, and that this varies within a wide range in the same structure.

Soil Mechanics Research Abroad.—As a rule, good scientific results may be reached when investigators are familiar with results obtained by others. Neither the branches of soil mechanics research in the United States, nor the names of American investigators have been specified completely in the preceding discussion. America is especially famous for its road and highway soil research. If, however, this country is far ahead of others in road soil research, its laboratories in some other branches are not as well equipped as some in Europe.

In the understanding of the writer, foreign countries in which soil mechanics research is being conducted may be divided into two categories: (*a*) Those with highly developed research, such as Germany, Austria, and Sweden; and (*b*) those with less developed research, such as England and its Dominions, Russia, Poland, Holland, Hungary, Japan, and others. France has given to the world many original theories of "classical" soil mechanics but is not participating in modern soil research as based on soil physics.

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[86] "The Relation of Certain Frost Phenomena to the Subgrade," by V. R. Burton and A. C. Benkelman, *Proceedings*, Tenth Annual Meeting, Highway Research Board, National Research Bureau, 1930, p. 259 *et seq.*; see, also, many other publications by the same authors.

[87] "History of the Development of Lateral Earth Pressure Theories," by Jacob Feld, Assoc. M. Am. Soc. C. E., *Proceedings*, Brooklyn Engr.'s Club, January, 1928.

[88] *Engineering*, May 30 and June 13, 1930.

Great Britain is especially noted for excellent research work on the physical properties of soil, including soil dynamics, carried on in the Rothamsted Agricultural Experiment Station.[45] Isolated papers from other English-speaking countries appear from time to time in the technical press. Russia has followed principally the American path of research; but has developed several interesting ideas of its own. The influence of the highly developed and unique Russian agricultural soil science was evident in the initial soil mechanics research in that country. Roads and highways form the principal province of activity for the Russian investigators, who would carry their work much further with better laboratories. Poland has also conducted its research along road lines. Holland works very actively in the field of engineering geology, which is closely allied to soil mechanics, and in soil mechanics itself, as may be seen from Dutch technical publications. Hungary has such an efficient investigator as Professor Janicsek. Japan has studied earthquake problems as applied to civil engineering, and some research on soil mechanics proper has been done as, for instance, the study of shearing resistance.[89] A special Geotechnical Committee has been organized in Japan. Some soil research work seems to have been done in South America, in the Argentine Republic, in connection with the problem of dirt roads.[90]

As far as the writer knows, some of the well-equipped laboratories in Austria, Germany, and Sweden are: (*a*) Technische Hochschule, Vienna, Austria; (*b*) Preussische Versuchsanstalt für Wasserbau und Schiffsbau, Berlin, Germany; (*c*) Deutsche Gesellschaft für Bodenbaumechanik, Berlin, Germany; (*d*) Foundations and Hydraulics Institute, Hanover, Germany; and, (*e*) in Sweden, at the Technical University of Stockholm, and the Swedish Bureau of Public Roads.

In establishing the Laboratory of Soil Mechanics at the Technische Hochschule, in Vienna, Professor Terzaghi perfected the actual research equipment and developed new methods. The Austrian Government made a considerable appropriation for its installation.

At the "Preussische Versuchsanstalt für Wasserbau und Schiffsbau," in Berlin, special emphasis has been laid on soil mechanics research in the Department of Foundation Engineering.

The laboratory of the German Society for the Development of Soil Mechanics ("Deutsche Gesellschaft für Bodenbaumechanik") was formed by governmental engineers, with headquarters at the Technische Hochschule, in Charlottenburg, Berlin. Its principal aim is the study of emergencies in connection with earth structures (water-works failure, landslides, embankment failures, etc.).

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[89] "'Shearing Resistance of Soils with Various Constituents," by N. Yamaguti, *Bulletin*, Japanese Govt. Railways, Vol. 19 (1931), Nos. 1 and 25, in Japanese.

[90] "Construcción de Caminos en Paises Nuevos," by Roberto Kurtz, Assoc. M. Am. Soc. C. E., *Proceedings*, Sixth International Road Congress, Washington, D. C., 1930.

The Soil Mechanics Laboratory of the Foundations and Hydraulics Institute, in Hanover, has a separate new building provided with good equipment; for example, a huge retaining wall is a part of it. The dimensions of the wall are about the same as those of the apparatus in the Massachusetts Institute of Technology, as described by Professor Gilboy.

Among the Swedish establishments there are two small separate laboratories of soil mechanics at the Technical University of Stockholm, and a small laboratory of the Goetechnical Commission of the Swedish Railroads. The soils laboratory of the Swedish Bureau of Public Roads ("Svenska Väginstitutet"), maintains an active contact with geologists, and there are members of the Swedish Geological Survey who have specialized in the problems of both soil physics and road engineering, approaching the latter, of course, from the geological point of view.[91]

J. STUART CRANDALL,[92] M. AM. SOC. C. E. (by letter).—This paper gives an excellent résumé of some of the progress made during the last few years in determining the nature and physical characteristics of soils. The importance to the engineer of these developments is apparent, because all structures must be supported by the soil through some type of foundation. The rate of settlement of these foundations over a period of years is important, and occasionally vital. Soil mechanics is a connecting link between geology and civil engineering. Further and more rapid advances may confidently be expected in the future. It must be admitted that, on the whole, the technique of foundation design has been rather empirical, with little actual rational basis. There is need for change. It is to be hoped that the profession may sincerely seek logically to apply the results of these studies to actual foundation design.

Most building codes are very definite as to the allowable load on different soils, but are rather vague as to the classification of these soils. The author might have pointed out the objectionableness of the present common practice of leaving to the foreman of a boring crew the identification of the type of soil under a foundation. He might have pointed out also the common lack of attention and care to preserve boring samples in the natural state. Frequently, soils are classified as "clay," when the material is actually silt. This occurred recently at Rochester, N. Y., where borings were taken at the site of the Charlotte Terminal at the mouth of the Genesee River. The boring foreman identified the underlying soil as "soft blue clay" and "medium soft gray clay." The proposed quay wall was of the L-type of reinforced concrete supported by wood piles, with a steel, sheet-pile bulkhead along the face, tied back by rods attached to a back-wall of steel piling. Analysis of the entire structure indicated a small factor of safety against rotation, on account of the low angle of internal friction and low shearing resistance of clay. Samples from the borings were then analyzed, which indicated that the material was not clay, but silt with a small admixture of clay. The coefficient of internal friction and shearing resistance of the silt being much higher than clay, there was no longer any question of stability. Such occurrences show the necessity for the adoption of a reliable and accurate method of classifying the type and condition of soils.

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[91] Svenska Väginstitutet, Meddelanden 13; 15 (1929); 21; 24; 25; 26 (1930); 30 (1931), av Dr. G. Bescow, Stockholm.

[92] Treas. and Engr., The Crandall Eng. Co., Cambridge, Mass.

Of great importance is the very evident fact that for cohesive, impervious soils, such as clay, the usual load-settlement tests required by various building codes are of little value, for within the usual short duration of such tests there is not sufficient time to determine the actual rate of settlement. Furthermore, the zone and intensity of stress in the soil directly under the small area loaded are very different from the zone and intensity of stress under the entire foundation.

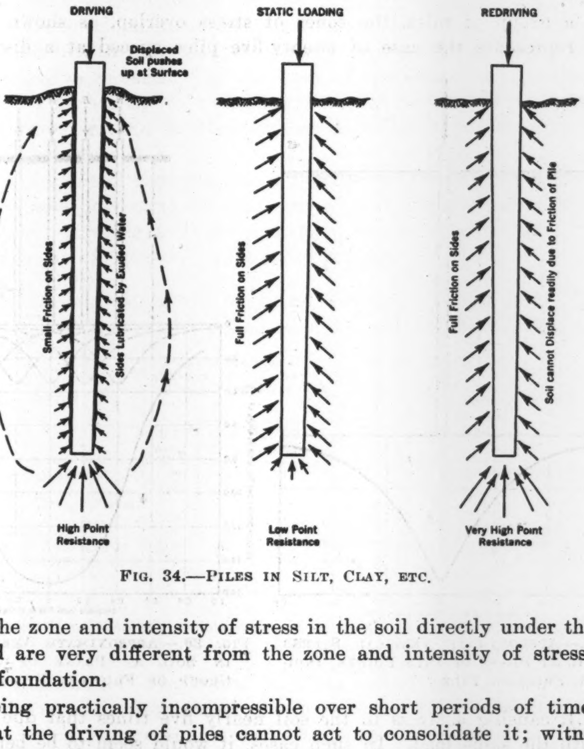
DRIVING
Displaced Soil pushes up at Surface
Soil Displaced Upward
Small Friction on Sides
Sides Lubricated by Exuded Water
High Point Resistance
STATIC LOADING
Full Friction on Sides
Low Point Resistance
REDRIVING
Full Friction on Sides
Soil cannot Displace readily due to Friction of Pile
Very High Point Resistance

FIG. 34.—PILES IN SILT, CLAY, ETC.

Clay being practically incompressible over short periods of time, it is evident that the driving of piles cannot act to consolidate it; witness the swelling of the surface when piles are driven in clay. This being the case, the very work of driving must disturb and knead the clay, and thus increase the rate of settlement. It is logical to assume, therefore, that the driving of piles into clay should be avoided, and that it is better practice in such soils to sink caissons or caisson piles so that there may be little disturbance and so that the foundation will bear on the material in its natural state.

Charles Terzaghi, M. Am. Soc. C. E., has demonstrated[93] that there is no known relationship between the dynamic resistance of piles driven in clay and the resistance to static load. In Fig. 34 is shown another way of illustrating the causes of this phenomenon.

This being true, the attempt to determine the bearing capacity of piles driven in clay by the use of some pile-driving formula has no logical or scientific basis and cannot give results of any value. Load tests of individual piles give no indication of the settlement of a group of piles with the same load per pile. The approximate distribution of the vertical component of stress in the soil at the plane of the point of a single pile is shown in Fig. 35.

For a group of piles, the zones of stress overlap, as shown in Fig. 36 (which represents the case of twenty-five piles spaced at a distance equal to 0.2L), causing a stress in the soil nearly five times that due to a single pile with the same load. In such cases, it would seem to be better practice to determine analytically the stress in the soil along the plane of the points, compare this with the bearing resistance of the soil, due allowance being made for probable settlement based on an analysis of the underlying material, and then, by trial and error, calculate the most economical length, spacing, and load per pile.

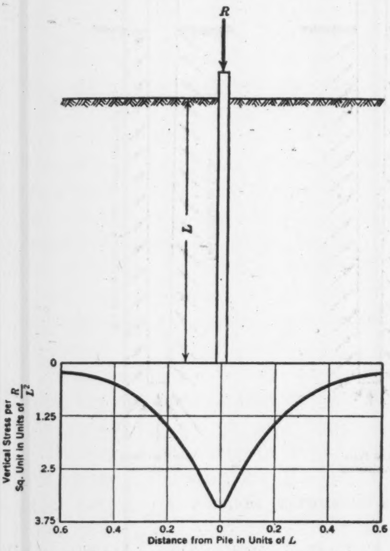

FIG. 35.—APPROXIMATE VERTICAL STRESS IN SOIL AT PLANE OF PILE POINTS, INDIVIDUAL FRICTION PILE.

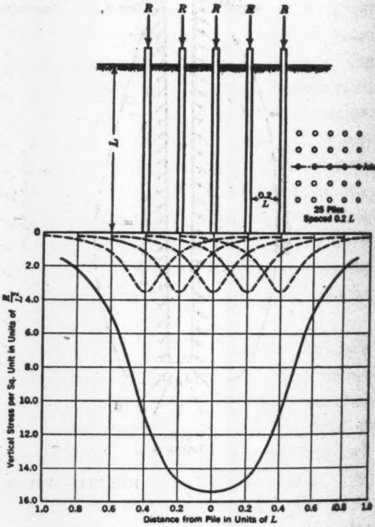

FIG. 36.—APPROXIMATE VERTICAL STRESS IN SOIL AT PLANE OF PILE POINTS, GROUP OF FRICTION PILES.

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[93] The Science of Foundations—Its Present and Future," *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 270.

Frank Alwyn Marston,[94] M. Am. Soc. C. E. (by letter)—Substantial progress was made during the five years, 1927 to 1932, in devising methods of determining the physical properties of soils. The practical application, however, of data concerning these physical properties to studies of the behavior of soil in engineering structures is somewhat less advanced. This paper is, therefore, a valuable contribution, because it gives in brief form a description of certain tests which have been found to be of distinct aid to judgment in passing on the suitability of soils for the building of dams and embankments and the solving of foundation problems.

The opportunities for research are great. Programs of scientific study such as are being carried on in various institutions of learning are most necessary for the development of methods of analyzing soils. Few laboratories outside of such institutions have the opportunity or the personnel that will permit of extensive research. To the work of the scientific laboratory must be added the opportunity to make practical application of the results of research by investigations in the field on full-sized structures, if tangible results are to be obtained.

Soil physics and soil engineering are destined to become of greater importance as time goes on because it will be more generally recognized that judgment aided by scientific soil analyses may be more dependable than the unaided judgment of even the experienced engineer.

TABLE 1.—A Comparison of Some Typical Fine-Grained Soils

Source of sample	Percentage by Weight of Grains of Size Less Than			Liquid limit, percentage*	Plasticity index†	Angle of internal friction‡	Sample number	Percentage of organic matter
	0.1 mm.	0.01 mm.	0.002 mm.					
Sewer trench, Pawtucket, R. I....	97	24	0	20.7	0	35°–10′	2 145	....
China clay....................	100	78	40	52.2	11.8	......	A-28	....
Core, Alexander Dam, Kauai, Hawaii§....................	94	53	9	75.2	11.8	37°–57′	A-23	10.6
Marine mud, Boston, Mass.......	98	55	18	78.9	42.0	19°–16′	A-2	7.3
Blue gray clay, Port Arthur, Tex..	100	87	61	93.7	59.1	9°–39′	A-11	....
Fossiliferous earth, Springfield, Ohio	89	28	9	118.8	71.7	......	A-32	20.1

* Ratio of weight of water to weight of dry solids expressed in percentage.
† Difference between liquid and plastic limits.
‡ Determined from shearing resistance.
§ Contained 64.8% of iron as Fe_2O_3.

Separation of the soil sample into fractions according to grain size is helpful, but if the analysis stops there, conclusions may be misleading. Table 1 gives data relating to several samples of soil in each of which nearly all the grains were less than 0.1 mm. in size, that is, of a size approximately, that would pass the 150-mesh sieve. On the basis of the sieve analysis, the samples are more or less comparable. The fine fractions, determined by sedimentation, vary widely. The china clay, containing 40% by weight of grains less than 0.002 mm. in size, has a low index of plasticity, whereas the fossiliferous earth with only 9% finer than 0.002 mm. has a plasticity index of 71.7. Generally, the larger the plasticity index the more plastic and slippery is the material. The index may be considered as a rough measure of the cohesion existing between the soil particles. A high index may indicate high compressibility and slow rate of consolidation. Non-plastic soils have an index of zero. It is evident from the examples given, that the percentage content of fine particles may not be a controlling factor in the relative behavior of soils from various sources. On the other hand, with a given soil, the proportion of fine particles may have a strong influence on the behavior of the composite sample.[95]

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[94] Cons. Engr. (Metcalf & Eddy), Boston, Mass.

The mechanical analysis is useful, but should be supplemented by tests in order to determine other characteristics of the soil, as pointed out by Professor Gilboy.

After some studies of the results of analyses by the Wiegner and Bouyoucos sedimentation methods, the writer is of the opinion that the Bouyoucos method is the more convenient, and when conducted according to the procedure developed by the U. S. Bureau of Public Roads, is more likely to give reliable results.

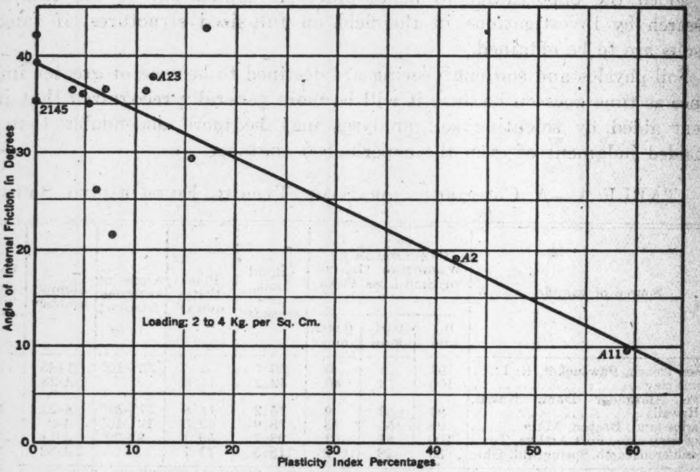

FIG 37.—TYPICAL RELATION BETWEEN ANGLE OF INTERNAL FRICTION AND PLASTICITY INDEX.

The Atterberg limits of consistency—liquid and plastic limits—are easily determined, and they furnish a valuable index to the plasticity of the soil. The liquid limit device has been used for some time and has been found to be superior to the hand method because more consistent results can be obtained and there is less variation due to the personal equation.

The interrelation of soil characteristics is such that it is difficult to select two characteristics and show that one is a function of the other. With a

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[95] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 354, Table 9.

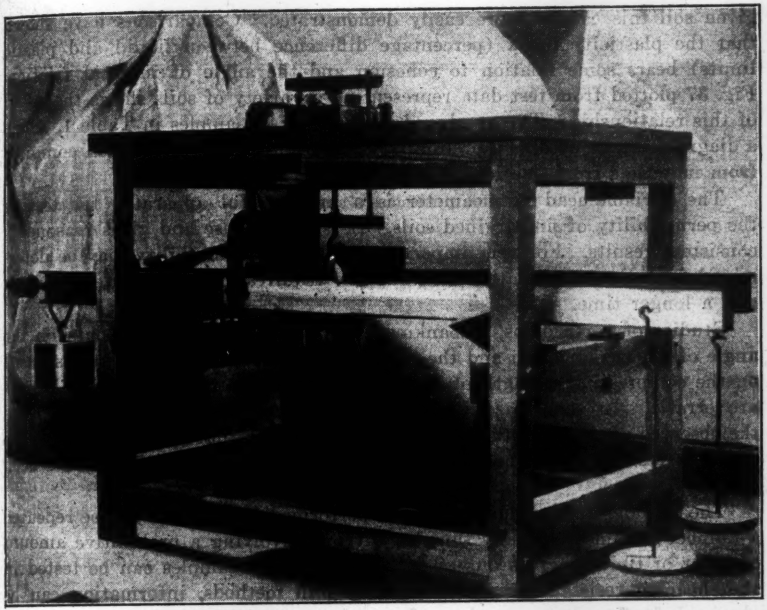

FIG. 38.—VIEW OF SHEARING MACHINE.

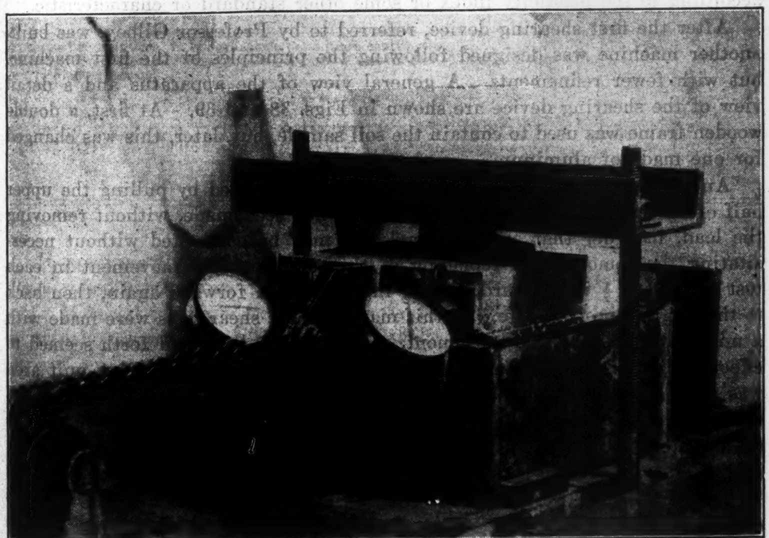

FIG. 39.—VIEW OF SHEARING UNIT.

given soil this can be more easily demonstrated. Comparisons have shown that the plasticity index (percentage difference between liquid and plastic limits) bears some relation to cohesion and the angle of internal friction. Fig. 37 plotted from test data representing a variety of soils shows the trend of this relationship. The numbered points refer to samples in Table 1. Such a diagram can be used as a preliminary rough guide especially when compiled from materials from one locality.

The variable-head permeameter is a most useful apparatus for testing the permeability of fine-grained soils. It is easy to use and gives reasonably consistent results. For very impervious soils, the permeability may be determined from the consolidation test, but this requires more elaborate apparatus and a longer time.

Studies of earth dams, embankments, and earth cuts require data on the angle of internal friction and the cohesion of the soil. Determinations based on the compressive strength of clay cylinders are interesting, but the results are erratic. In some cases the angle of internal friction computed from the observed angle of rupture agrees fairly well with that determined from the shearing resistance test and, in other cases, considerable variation has been found.

The cylinder rupture method can be performed quickly and can be repeated a number of times on many samples without involving a prohibitive amount of labor or time, whereas only a few representative samples can be tested in the shearing resistance machine. Using both methods, information can be obtained from a large number of samples, especially if they are grouped according to the plasticity index or some other standard or characteristic.

After the first shearing device, referred to by Professor Gilboy, was built, another machine was designed following the principles of the first machine, but with fewer refinements. A general view of the apparatus and a detail view of the shearing device are shown in Figs. 38 and 39. At first, a double wooden frame was used to contain the soil sample, but, later, this was changed for one made of aluminum.

An interesting series of experiments have been tried by pulling the upper half of the frame back, after one shear test has been made, without removing the load, in order that an additional test may be conducted without necessitating the removal and replacing of the sample. The movement in each test was about $\frac{1}{2}$ in. forward, then $\frac{1}{2}$ in. back, then forward again, then back to the original position, etc. In this manner three shear tests were made with a minimum of horizontal movement. The moving back and forth seemed to effect a slight consolidation of the material, because the shear per unit area was generally higher on the second and third tests than on the first, but not always so. Table 2 is typical of the results of such a procedure. The data are of interest as showing the prompt re-adjustment of the soil particles after a slip.

It is important that sufficient time be allowed after loading the soil for consolidation to take place. With a cohesive soil the time required may be from one to two days. After that, the change is comparatively small. What

TABLE 2.—TYPICAL TEST OF SHEARING RESISTANCE—SAMPLE PULLED BACK AFTER EACH SLIP

Test characteristic	YELLOW SANDY CLAY:† TESTS		
	A	*B*	*C*
Pressure per unit area*, in kilogrammes per square centimeter	0.93	3.03	5.04
Shear per Unit Area, in Kilogrammes per Square Centimeter:			
First slip	0.46	1.76	3.56
Second slip	0.55	1.89	3.71
Third slip	0.59	2.03	3.60
Moisture content at start, percentage	23.9	....	....
Moisture content at end, percentage	15.7	....	....

* Time for consolidation allowed after each change in loading.
† Sample saturated with water during test.

happens over a period of months or years under constant load is not known, but it seems probable that some further consolidation or increase in cohesion takes place, resulting in increased shearing resistance. Table 3 shows the results of a shearing resistance test covering 26 days. There was very little change in resistance after 42 hours, and these differences may be within the limits of error of the test.

TABLE 3.—EFFECT OF TIME OF CONSOLIDATION UNDER LOAD UPON SHEARING RESISTANCE

Test data	Brownish sandy clay*
Pressure per unit area, in kilogrammes per square centimeter	3.01
Shear per Unit Area, in Kilogrammes per Square Centimeter (Time, in Hours, after Placing Load):	
22	1.97 2.30
42	2.55 2.55
119	2.55
284	2.63
454	2.63
625	2.46

* Sample saturated with water during test.

In applying the methods described by Professor Gilboy to studies of soils for a rolled earth dam, it is not so necessary to use samples of undisturbed soil. Material in the borrow-pit is excavated and then deposited in the dam in thin layers which are thoroughly rolled. Much the same process can be carried out on a small scale in the laboratory. Undisturbed samples should be taken from the dam from time to time to check the degree of consolidation, moisture content, etc.

A helpful basis of study can be had by comparing the void-ratio of undisturbed samples of the embankment with the void-ratio of laboratory compacted samples. In general, the field-compacted samples are likely to have a little larger void-ratio than samples thoroughly rammed, kneaded into compact form, and similarly loaded in the laboratory.

GEORGE PAASWELL,[96] M. AM. SOC. C. E. (by letter).—This paper gives a sober account of the work done under the direction of Professor Gilboy, and it is primarily educational.

There can no longer be any doubt that the rule-of-thumb era in foundation work has passed. Foundation failures can no longer be ascribed to the perverse workings of a balky Nature. The proper design of a foundation is not simply a surface study of the loaded area. Bearing in mind that the loaded surface of the soil creates a certain stress distribution through a depth, comparable in extent with the area of surface loaded, it can easily be seen that an extensive soil exploration is required before a proper anticipation may be made of the distortion of the soil under load. Surface tests form merely one link, then, in a complete study of the foundation.

The vast amount of sub-surface construction in large cities has created a field for an extensive study of foundation problems. Not only must the foundation selected be of such character as to sustain safely the load to be placed upon it, but also the method of constructing this foundation must be such as not to endanger existing neighboring foundations. This has required a type of research that finds its conclusions from the applications of soil mechanics as given in the paper.

The writer was privileged to collaborate with both Professors Terzaghi and Gilboy on an extensive series of experiments in connection with the construction of the Houston Street Subway in New York City, mentioned by the author. On this work the sand extended in a deep layer down to rock which varied from 60 ft. to more than 100 ft. below the ground surface. The sand varied in texture from extremely coarse to extremely fine, distributed in irregular layers in so far as its texture was concerned. Its density bore no relation to the depth at which the density was ascertained so that the rule-of-thumb generally applied, which gives the bearing capacity as increasing with the depth, was entirely in error. The angle of internal friction as found by experiments on small samples of the sand was fairly constant, a fact which the writer believes has been justified by some later experiments conducted by Professor Gilboy, which show only small variations in the angle of internal friction in varying densities of sand. It was also found that for the small loads applied (none exceeded 5 tons per sq. ft.) the settlement was not a function of the area loaded.

This is an important conclusion, although it is one that has been frequently checked by experiment in sands, and it must be emphasized that it probably applies only to moderate loads. For very heavy loads, such as are carried by pile clusters with concentrated point loads, experiments seem to show that the settlement is a function of the area loaded. The writer hopes that this important topic will be a subject of future study and experiment, since it has been found that in clays the settlement is intimately related to the shape and area of the surface loaded.

The effect of surcharge was studied carefully in these experiments, and the settlement was noticeably decreased by the presence of a surcharge of sand around the loaded area; that is, the surface loading settles more rapidly under a given load than under the same load applied on the same area at a distance below the surface of the ground. When the load is applied at a depth below the surface equal approximately to the maximum diameter of the loaded area, the maximum effect of surcharge is obtained. This fact should be carefully noted in connection with the underpinning of buildings adjacent to a future excavation. The piers sustaining the existing building should be carried to a depth equal at least to the thickness of the pier below the depth of the future excavation.

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[96] Engr., Corson Constr. Corporation, Brooklyn, N. Y.

The sum total result of the experiments gave quite a comprehensive picture of the behavior of sand of this character within the range of loads to be placed upon it, and was sufficiently substantiated by other observations and by the later behavior of the finished structure to vindicate the time and money spent on these experiments and to give striking proof that soil mechanics was able to answer decisively the questions of practical and immediate importance.

The design of a system of sheeting and bracing of unique character was governed primarily by the observations taken on the retaining-wall experiments quoted by the author. The various phases of movement in a retained bank of sand behind a yielding support (such as sheeting), were checked by observations during the construction of the subway. In some cases, the deflections of the sheeting were quite considerable, but no fracture occurred, showing the diminution in pressure as the retaining wall moved away from the applied load. It need hardly be mentioned here that a system of sheeting and bracing to sustain safely a deep bank of sand must be most carefully designed if the results of the quoted experiments are to be applied in a safe manner. Vast economies in subway construction have resulted to the City of New York because of the successful application of this type of sheeting and bracing, and it would not be "far fetched" to trace back this vast saving of money to the city to the series of experiments, starting in Constantinople, Turkey, and ending in Boston, Mass., as conducted in that brilliant series of studies made by Professor Terzaghi.

JOHN H. GRIFFITH,[97] M. AM. SOC. C. E. (by letter).—In regard to the question of stress distribution, Professor Gilboy states (see "Foundations"):

> "* * * while the classical theory of Boussinesq is the best available, it can yield only approximate results. The analysis of the stress-strain characteristics of soils thus far performed includes only the simplest cases. Most important of all, some assumption as to homogeneity of the underground must be made, and this is only approximately fulfilled, even under the most favorable circumstances."

Following this is the apparently contradictory statement that such analyses yield valuable information. He cites as proof the instance of a highly heterogeneous matter consisting of piles overlying an area of mud 120 ft. below the surface, and comments, as follows: "The results agreed remarkably well with the observed values, showing that such studies are of considerable practical importance, * * *."

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[97] Prof. of Experimental Eng., Iowa State Coll., Ames, Iowa.

To understand the disparities of opinions which have originated with respect to earth analyses, or to use Professor Gilboy's term, "soils," it is well to consider that the theory of earth resistance has developed two schools; those who, like Professor Gilboy, take the physicist's, or molecular, point of view involving grain parameters, shapes of particles, true and apparent cohesions, intrinsic stresses, and structures of atoms; and those who adhere to the viewpoints of mathematicians, that such conceptions have no place in the systematic working formulations of science. It is important to consider the facts in this case.

Strictly speaking there are no systematic atomic or molecular theories formulated as such. They are unmanageable in science and much more so in applied physics, such as engineering and geology. A true molecular analysis is a problem in *n* body dynamics, with *n* very large, and involves a complex theory of numbers or a calculus of finite differences which the proponents of such methods do not seem to have at their command.

On the contrary the systematic theories of science involve molar matters the properties of which are defined by continuous functions except for discontinuities at particular points and boundaries. One may consider as examples among others the theories of electro-magnetism, thermodynamics, elasticity, hydrodynamics, meteorology, seismology, and relativity. They are fundamentally field theories. The field is the measurable entity, the only thing that can be measured. If an oscillator vibrates in mercury it immediately diffuses its effect into waves as in the case of a tuning fork. The tuning fork is a singularity or peculiarity of the wave field, the point or region where the lines of force converge, and the function may increase without limit.

Having the equation defining the field it is necessary to find the peculiarities the function may assume in limiting cases and at points and special boundaries.[98] In seismology the engineer seeks for the focus of disturbance, in electro-magnetism for the pole or electrode, in acoustics for the source, and their functional aspects, together with the types of existence theorems. The same type of problem arises in all branches of dynamics, for the different modes of energy.

The potential theory formulated for matter by George Green,[99] while strictly a molar theory, is really applicable to molecular systems composed of an immense number of mutually re-acting particles, as stated by its author, and without the necessity of multitudinous borings, and chemical and colloidal analyses, as the writer will proceed to show.

Several investigators have made tests[100] showing that the vertical distribution of pressure under a load appears to be in agreement with the mathematical distributions formulated by Boussinesq[101] and Michel.[102] The writer has suggested the availability of Boussinesq's formula,

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[98] "Physical Properties of Earths," *Bulletin No. 101,* Iowa Eng. Experiment Station, 1930.

[99] "Mathematical Papers," p. 243.

[100] *Bulletin,* Pennsylvania State Coll., Eng. Experiment Station, Vol. 17, p. 650; "The Distribution of Pressure by Granular Material", by M. L. Enger, M. Am. Soc. C. E., *Engineering,* Vol. 101, 1916, p. 170; *Proceedings,* Am. Soc. for Testing Materials, Vol. 17, 1917, p. 650; *Proceedings,* Am. Soc. C. E. August, 1920, Papers and Discussions, pp. 931–937; *Transactions,* Am. Soc. C. E., Vol. LXXXIII (1919–1920), pp. 1527–1580.

[101] "Theory of Elasticity", by E. A. H. Love, Formula, p. 189.

[102] *Proceedings,* London Mathematical Soc., Vol. 32, 1900, p. 23.

$$Z_z = \frac{3}{2\pi}\frac{Pz^3}{r^5} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

treating the coefficient as an empirical constant in the usual manner illustrated in hydraulic formulas for orifices, which is entirely independent of physical constants, involving only the load, *P,* and the location of stress at the point distant *r* from *P.*

Anson Marston,[103] Past-President, Am. Soc. C. E., applied Boussinesq's theory to the problem of bridge rollers in 1894, and supplemented this by researches on large wheels in later years. In the Progress Report on Culvert Pipe Investigations covering 1915 to 1921, submitted to the U. S. Bureau of Public Roads and the various State Highway Departments, he showed by tests that Boussinesq's formula for a concentration (Equation (20)) applies to all levels of fill to within 2 ft. of the load "source" where the full concentration applies; the analytic function becomes infinite with the radius vector measured from the load reducing to zero. A little later, Messrs. Spangler, Mason, and Winfrey[104] demonstrated that the formula gave the upper limit of load and generally showed a close parallelism with Boussinesq's law. In 1920, at a lecture before a group of railroad officials, Dean Marston stated that the lines of force crowd into the culvert or pipe tending to break it, the weight sustained being more than the vertical surcharge over the extrados.

In the most recent researches of Dean Marston[105] the theory of potential based on molar matter analyses, where the integrations have been carried out for "concentrated super-loads" taken over the "conduit top horizontal plane projection," show excellent correlations according to a recent communication to the writer by Professor Robley Winfrey, of the Iowa Experiment Station. He has made a detailed study of the problem by comparing the results of summing Boussinesq's simpler formula under a concentration with the results obtained by integration and by dividing the stressed area into relatively small squares. The recent researches not only show that the potential formulas give the upper limits of the distributions, but actually apply when the load has close proximity with the extrados. As in the case of the simpler formula the results are independent of the moduli of the material applying to either gravel, sand, or clay, as long as the lattice structure of the medium is not too much deformed as would be the case in straining an ordinary bridge truss. If it experienced finite deflections, the panel shapes would change involving a suitable correction.

Researches of the kind mentioned are valuable epistemologically in showing inductively the sufficiency of the potential algorithm, as a complete physical method. One instance only will suffice to show the possibilities in the distribution of forces by railroad tracks.

The researches of Harold Medway Martin[106] show that the distribution over the track from the train load is a "witch-shaped" curve of the type originally deduced by Agnesi. Suppose a culvert is under the track, the stress may be obtained as given by the general formula (Equation (20)), with a load equal in amount to the ordinate of the track distribution of Martin, and summed or integrated for continuous distributions. In the case of the attractions of body upon body according to the Newtonian potential theory[107] the general potential of mass, m', with respect to the mass, m, is:

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[103] ***Transactions*, Am. Soc. C. E., Vol. XXXII (1894), p. 99.**

[104] ***Bulletin No.* 79, Iowa Eng. Experiment Station, pp. 14–15.**

[105] ***Bulletin No.* 96, Iowa Eng. Experiment Station, 1930, p. 17, Formula 13.**

[106] **"Statically Indeterminate Structures and the Principle of Least Work", rev. and reprinted from *Engineering* (Lond.), 1895, pp. 63–71.**

$$V = \int\int\int\int\int\int \frac{\rho\,\rho'\,dm\;dm'}{r} \quad \ldots\ldots\ldots\ldots(21)$$

in which, r is the distance between the mass elements of the bodies. Where there is an intervening medium, such as the earth matrix, it is necessary to introduce the direct and logarithmic potentials of Boussinesq in the manner shown in a previous paper.[108] In general, it is not possible to dispense with the elastic constants. This is the case in the instance of a retaining wall and for other elastic or semi-elastic distribution in which lateral pressures occur. The potential distributions hold very closely in the case of impact.[109]

The retaining wall problem is one of the oldest historically and is most difficult to formulate. The author's apparatus, designed with meticulous care, will doubtless throw new light on the problem. A solution by molecular dynamics will be of the greatest interest to the profession. In addition to his rational solution for the weir problem of hydraulics,[110] Boussinesq's theory of the retaining wall is perhaps the most important of his purely engineering analyses. He devotes more than 170 pages of his treatise on the subject[111] to this important case treating a molecular problem throughout by a strictly molar theory. He shows that it is a more general case than the theory of elasticity, and he arranges his parameters so that as a limiting aspect the theory reduces to the elastic case as a limit.[112]

The hangers, V_1 and V_2, of the retaining wall apparatus (Fig. 13) will balance Boussinesq's wall friction which he takes into account, except for infinitely smooth walls. It will be impossible to measure the maximum overturning moment because this is exerted when no energy has been expended in frictions and small displacements, and the author's apparatus will give some indeterminate equilibria and settlings in manipulations of the "wall" part as a result of the movements. The three-faced holder for the earth will act as a clamp on the specimen; under the wall friction there will be considerable restraint, making it impossible for the "retaining wall" to experience the maximum thrust. It was suggested to the writer by Dr. O. H. Basquin, of Northwestern University, that dental rubber placed on a wall would eliminate the friction since this material is nearly incompressible and has only a small modulus of rigidity. His theoretical reasons are sound, but the writer has never tried the material in this manner. It may eliminate the frictions of the side walls in the present case.

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[107] "Peirce's Newtonian Potential Function", Third Edition, rev. and enl., p. 42.

[108] "Theory of Elasticity", by E. A. H. Love, Second Edition, p. 189; and *Bulletin No. 101*, Iowa Eng. Experiment Station, pp. 31–35.

[109] *Bulletin No. 79*, Iowa Eng. Experiment Station, pp. 22, 34 *et seq.*

[110] "Hydraulics", by F. C. Lea, 1908, Edwin Arnold, Lond., pp. 104–108.

[111] "Essai théorique sur l'Equilibre des Massifs Pulverulents", Bruxelles, 1876; "Application des Potentials", Paris, 1885.

[112] *Loc. cit.*, p. 61.

In conclusion, the writer does not wish to be construed as asserting that atomic, molecular, colloidal, and crystalline researches are not valuable in themselves. On the contrary, they are of the highest scientific importance in developing correct philosophies of the nature of energy, ether, matter, and atomic and crystalline structures. The point emphasized in the discussion is that the systematic working theories of science are not molecular; they are and always have been essentially molar theories involving apparent continuous entities and transmission coefficients, and not discrete entities and grain parameters, except in isolated instances ultimately to be displaced as a wave theory displaces a discrete theory.

H. E. GRUNER,[113] M. AM. SOC. C. E. (by letter).—The interesting information contained in Professor Gilboy's paper prompts the writer to contribute a description of certain simplified methods of conducting similar experiments, so designed that the most important properties of soil can be measured right at the construction site.

After the general line of attack had been established by the pioneer work of Terzaghi and Krey, and testing apparatus had been developed at the Staatliche Versuchsanstalt für Wasserbau und Schiffbau, in Berlin, the writer undertook the construction of simplified devices for testing the soil used in the construction of the dam for the Albbruck-Dogern Power Station on the Rhine. The purpose of this research was twofold: First, to determine those soil constants required for calculations of the stability of the dam; and, second, to obtain continuous data on the properties of the material during the construction process.

On account of the wide variations in the properties of soils, it is necessary to have a definite basis for the selection of material suitable to a given purpose, and for the rejection of unsuitable material, such as soil containing a large proportion of colloids. It is the writer's opinion, therefore, that in the case of water-power projects involving large dams and canals a continuous series of soil tests is just as important as the continuous control tests customarily performed on cement and concrete.

Apparatus and Procedure.—A general view of the testing layout is shown in Fig. 41. Fig. 42 is a demonstration of the shear test, described subsequently. The devices are not intended to be as accurate as those used in research laboratories. They are well adapted, however, to the purpose for which they were designed, namely, to the rapid testing of a large number of samples, with a degree of accuracy sufficient for the analysis of dam construction materials. These instruments undoubtedly can be improved; in fact, the experiments to date have indicated the advisability of certain changes. The present description is intended merely to suggest one method of attacking the problem.

The tests thus far undertaken may be divided into five groups:

(1) Measurements of specific gravity, unit weight, and percentage of voids;

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[113] Cons. Engr., Basel, Switzerland.

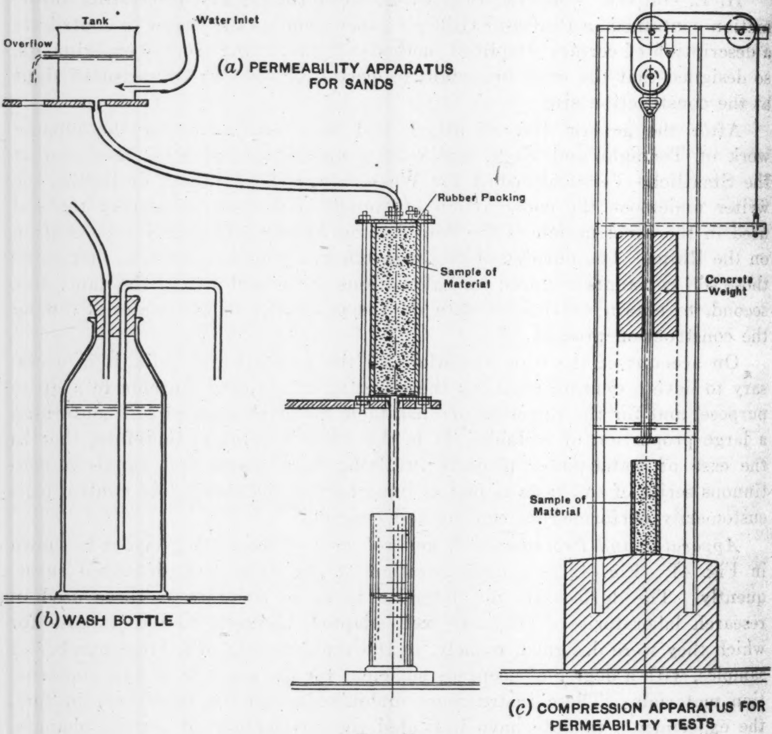
(2) Mechanical analyses, with the object of classifying the soil with regard to fineness;

(3) Permeability tests, to obtain data with which to determine the position of the line of saturation and the velocity of the seepage water;

(4) Measurements of shearing resistance, to determine those constants required in an analysis of the stability of the slopes; and,

(5) Tests of the compressibility, the variations in water content, and the lateral pressure of fine-grained materials.

The information in Group (5) is required for studies on the settlement of the dam, on the loss of water associated with consolidation, and on the possibility of compacting the earth by means of pressure.



(*a*) PERMEABILITY APPARATUS FOR SANDS

(*b*) WASH BOTTLE

(*c*) COMPRESSION APPARATUS FOR PERMEABILITY TESTS

FIG. 40.

Group (1).—These quantities enter into the determination of loading conditions in the dam. The specific gravity is determined by Schumann's volumenometer. The unit weight is obtained by the use of gauged capacity meters, different methods of filling being compared. In order to determine the change in unit weight produced by compression, the results of the consolidation test described under Group (5) are utilized.

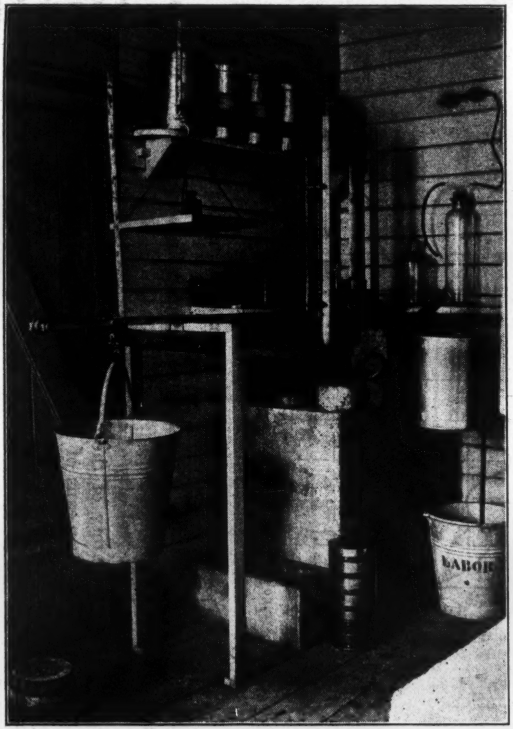


FIG. 41.—ARRANGEMENT OF APPARATUS.

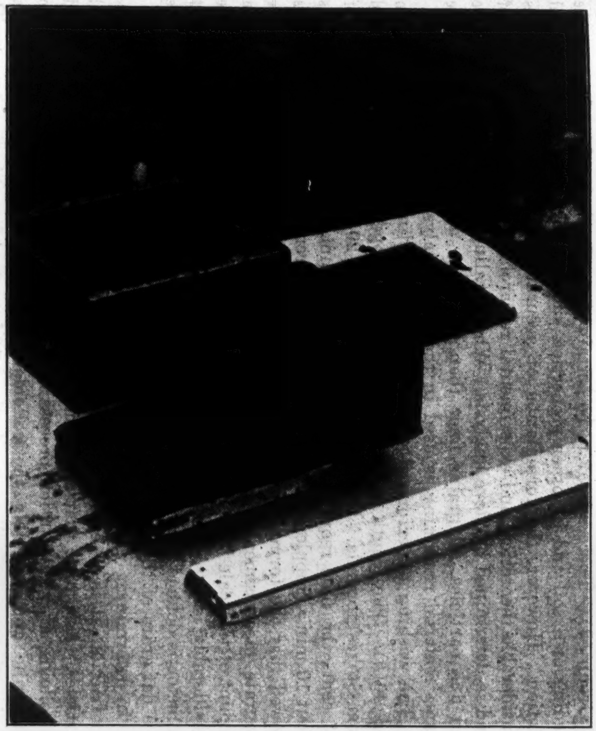


FIG. 42.—SAMPLE AFTER SHEAR TEST.

Group (2).—There appear to be certain general relationships between the grading of a soil and its other properties, such as permeability and internal friction. However, the formulation of accurate relationships still requires considerable theoretical and experimental research. The grain distribution of sand is determined by sieve analysis. When dealing with mixtures of coarse and fine particles, all grains finer than 0.02 mm. are washed out, and the coarse material remaining is used in the sieve analysis.

The washing is performed with the aid of a cylindrical bottle, shown in Fig. 40(*b*). The length of time required for the operation can be computed from Stokes' formula. Using a depth of water of 20 cm. at a temperature of 15° cent., the time required for a particle, 0.02 mm. in diameter, to settle is about 10 min. Supplementary microscopic measurement of the largest grains showed that the formula gives very satisfactory results. However, the more accurate measurements made by Atterberg and Terzaghi showed that on account of the scale-like character of the particles the actual grain diameters are considerably larger, possibly twice the equivalent diameters. The equivalent diameter is defined as the diameter of a sphere which settles at the same rate as a flat grain of the same specific gravity.[114]

In determining the shearing resistance as a measure of the angle of internal friction, it is of especial importance to know the percentage of colloidal particles. Assuming that all particles smaller than 0.002 mm. may be classed as colloids, a sedimentation process similar to the foregoing may be used to determine approximately the colloidal content. The settling time for a particle, 0.002 mm. in diameter, is about 16 hours. After this time has elapsed, the percentage of colloids is determined by optical measurements of the transparency of the turbid water.

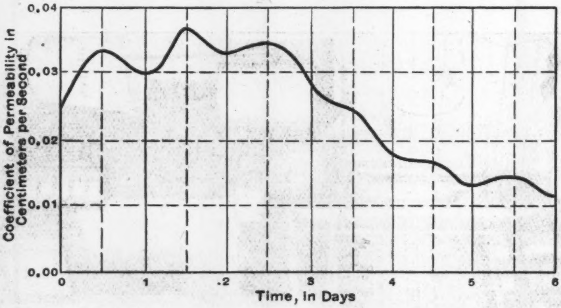


FIG. 43.—VARIATION OF PERMEABILITY WITH TIME.

Group (3).—The velocity of the seepage water is of particular importance in analyzing the safety of the dam against washouts. The permeability of sand is measured by means of a cylinder, obtained from the Versuchsanstalt für Wasserbau und Schiffbau in Berlin, and illustrated in Fig. 40(*a*). To ensure that the voids in the sample will be free of air, water is allowed to flow slowly into the cylinder from the bottom during the filling process, keeping the water level always slightly above the level of the sand. Thus, the sample is always completely immersed, and at the same time the washing out of very fine particles is avoided.

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[114] "Erdbaumechanik," by Charles Terzaghi, M. Am. Soc. C. E., p. 40.

The influence of temperature on the coefficient of permeability may be determined from Justin's data.[115] In the test, measurements are first made on the value of the coefficient of permeability, k_0, which is defined as the apparent velocity of the water through the uncompressed soil under unit hydraulic gradient. Then, the sample is compressed under a pressure of 1 kg. per sq. cm. in the apparatus shown in Fig. 40(*c*), the change in volume is measured, and the permeability test is repeated on the compressed material, in order to determine the influence of the compression on the permeability.

The phenomenon of a partial sealing of the sample within itself during a period of continuous flow was investigated on several occasions. The permeability appeared to increase slightly during the first few days; but afterward it decreased rapidly and continuously, finally attaining a certain minimum value. A graph of this relation is shown in Fig. 43. In addition to the general tendency, it was observed that the permeability was subject to daily fluctuations of variable amplitude. The causes of these fluctuations could be studied only by performing accurate experiments in a scientific laboratory.

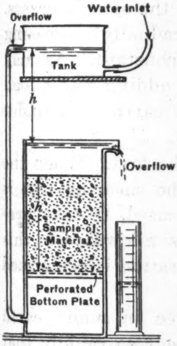

FIG. 44.—PROPOSED APPARATUS FOR MEASURING PERMEABILITY OF SAND-GRAVEL MIXTURES.

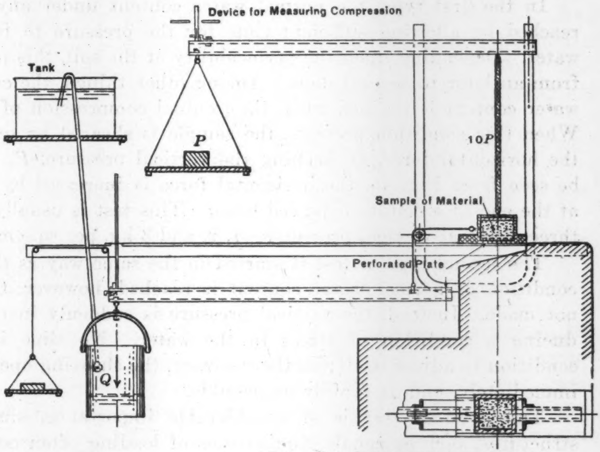

FIG. 45.—SHEARING APPARATUS.

In order to determine the permeability of sand-gravel mixtures, the sand fraction, consisting of all grains smaller than 7 mm. in diameter, was separated from the sample by sifting; this fraction was then subjected to permeability tests as outlined herein. By making certain theoretical assumptions it is possible to determine the permeability of the mixture from the permeability of the sand fraction and the proportion of sand to gravel, provided the latter is such that all the voids in the gravel are completely filled with sand. However, the assumptions that must be made are open to question. In order to eliminate them, and to arrive at a correct result without the necessity of determining the proportions of the constituents of the mixture, it seems preferable to obtain the permeability of the mixture by direct measurement in an apparatus such as that illustrated in Fig. 44. Parallel experiments on the mixture and on the sand fraction alone provide means for determining empirically the relations outlined herein.

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[115] "The Design of Earth Dams," by Joel D. Justin, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1.

Group (4).—The shearing resistance of fine-grained soils, as a measure of the angle of internal friction, is determined by means of the apparatus shown in Fig. 45. Fig. 42 shows the appearance of a sample after it has been tested to failure in shear.

Two different types of tests are made: (*a*) Normal tests, in which the water in the voids of the sample is under no excess pressure; and (*b*) rapid tests, in which the water carries part or all of the superimposed load.

In the first type, the normal water content under any given pressure is reached by allowing sufficient time for the pressure to force out all excess water. Depending upon the permeability of the soil, this process may require from an hour to several days. Among other things, the condition of normal water content is reached when the gradual compression of the sample ceases. When this condition prevails, the sample is sheared by gradually increasing the horizontal force, Q, keeping the vertical pressure, P, constant. As may be seen from Fig. 45, the horizontal force is increased by adding water load at the end of a counterbalanced beam. This test is usually carried out under three different vertical pressures—1, 2, and 3 kg. per sq. cm.

The second type of test is started in the same way as the first. When the condition of normal water content is reached, however, the shearing test is not made. Instead, the vertical pressure is suddenly increased, thereby producing a condition of stress in the water. No time is allowed for this condition to adjust itself; on the contrary, the shearing operation is performed immediately, and as rapidly as possible.

This type of test is of considerable importance, since in many earth structures, such as canals, fluctuations of loading often occur so rapidly that the stresses produced in the water within the voids of the soil have no time to adjust themselves. The results of rapid shear tests, therefore, serve as a basis for analyzing the most unfavorable loading conditions.

Comparisons between the friction angles determined from the two types of test offer interesting indications concerning the character of the soil. To provide a numerical basis for such comparisons, a coefficient, λ_2, is defined by the relation,

$$\lambda_2 = \frac{\tan \mu_{1+1}}{\tan \mu_2}$$

in which, μ_2 is the angle of internal friction determined from a normal test under a pressure of 2 kg. per sq. cm., and μ_{1+1} is the angle determined from a rapid test in which a pressure of 1 kg. per sq. cm. was applied; time was allowed for the stress in the water to adjust itself; then an additional load of 1 kg. per sq. cm. was applied, and the sample was sheared off immediately afterward.

The finer the grains of a soil, the smaller are the angles of internal friction, μ_{1+1} and μ_2, and the smaller is the coefficient, λ_2. For pure clays, λ_2 may attain very low values—perhaps in the neighborhood of 0.3—whereas for pure sands, or soils containing a large percentage of sand, it approaches unity.

Group (5).—The compressibility of the soil is measured by means of the apparatus shown in Fig. 46. The device is arranged so that the lateral pressure may be determined at the same time.

After the water content of the soil has been measured independently, the material is filled into the brass cylinder in a plastic condition, and the total weight is determined. Then a pressure of 1 kg. per sq. cm. is applied to the soil through a precisely machined piston. The compression, magnified ten times, is measured by the movement of the end of the load beam. Motion ceases when the water content is reduced to the normal value corresponding to the applied pressure. When this condition is reached a measurement of the lateral pressure is made. The horizontal force, H, is gradually diminished by siphoning water from a tank (not shown) until the index, Z, shows a small movement. The index is arranged to magnify the motion fifty times, because only a very small amount of movement is permissible if arching action and other disturbing effects are to be avoided.

Let σ_I and σ_{II} denote the vertical and horizontal pressures, respectively. Let β be their ratio, defined by $\beta = \frac{\sigma}{\sigma_I}$. From Mohr's diagram it is possible to determine the angle of internal friction, ϕ, as a function of β. The relation is $\tan \phi = \frac{1 - \beta}{2\sqrt{\beta}}$.

After several measurements of the lateral pressure have been made, the weight of the cylinder and sample is again determined. The difference between this weight and the original weight represents the quantity of water forced out by the applied pressure. Next, the load is suddenly increased from 1 kg. per sq. cm. to 2 kg. per sq. cm. A lateral pressure measurement is made immediately, that is, while the water in the voids is still in a state of stress. Then follows a period of gradual compression under the increased load. When the compression is complete, the lateral pressure is determined once more. This continuous process is especially advantageous in that a whole series of measurements is made on the same sample without disturbing it in any way. Thus, a clear understanding of the relations between the various properties of the material is readily obtained.

Both the shearing apparatus (Fig. 45) and the consolidation apparatus (Fig. 46) can be improved. The more favorable characteristics of both might be combined advantageously in a single device, such as that illustrated in Fig. 47.

Summary.—The foregoing description illustrates the possibility of making continuous tests, using relatively cheap and simple instruments right at the job, to measure the most important characteristics of the soils encountered in construction work. Without claiming that the results are highly accurate, the writer believes that field laboratories, designed to fulfill the needs of a single construction project, serve as a practical supplement to the investigations undertaken in scientific institutions. Such investigations should make it possible to improve upon the economy and safety of earth structures.

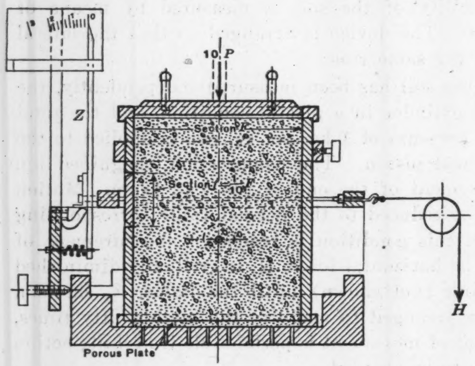

10 P
Z
Section F
Section $\frac{1}{10}F$
H
Porous Plate

FIG. 46.—APPARATUS FOR CONSOLIDATION AND LATERAL PRESSURE TESTS.

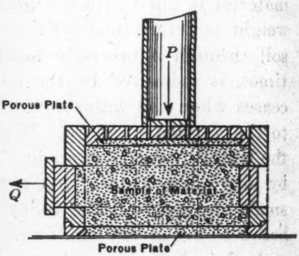
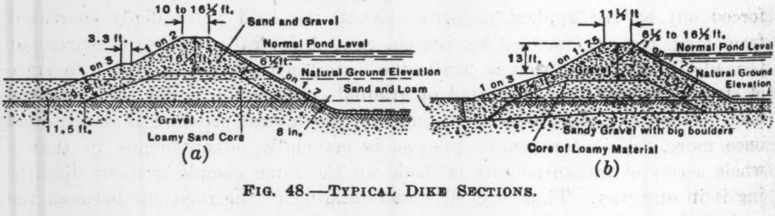

P
Porous Plate
Q
Sample of Material
Porous Plate

FIG. 47.—PROPOSED APPARATUS FOR CONSOLIDATION AND SHEARING TESTS.

To illustrate the practical application of such studies, experience in connection with the power plant at Albbruck-Dogern, on the Rhine, and the Kockovce Ladce project, on the Waag, may be cited. In both cases the canals were lined with concrete to insure water-tightness (see Fig. 48). In both cases it was necessary to economize on gravel. Hence, the dikes had to be composed partly of silt. The question as to the best use of the available material was decided from the results of tests on the soil, and from statical computations based on the methods of Ehrenberg.[116]


10 to 16½ ft.
Sand and Gravel
3.3 ft.
1 on 2
Normal Pond Level
1 on 3
16½ ft.
6½ ft.
9.8 ft.
1 on 1.7
Natural Ground Elevation
Sand and Loam
Gravel
11.5 ft.
Loamy Sand Core
8 in.
11½ ft
6½ to 16½ ft.
13 ft.
1 on 1.75
Normal Pond Level
1 on 1.75
Gravel
Natural Ground Elevation
1 on 3
6½ ft
Sandy Gravel with big boulders
Core of Loamy Material

(*a*) (*b*)

FIG. 48.—TYPICAL DIKE SECTIONS.

Although the same factor of safety was applied to the slopes of both canals, the resulting cross-sections differed materially, as a comparison of Fig. 48(*a*) and Fig. 48(*b*) will show. In the case of the Waag project, the silt core had to be much smaller than that in the Albbruck-Dogern dikes, because the silt found in the Waag had a much smaller shearing resistance than the Rhine material.

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[116] "Grundlagen der Berechnung von Staudämmen," *Wasserkraft und Wasserwirtschaft*, 1929, Heft 23.

The writer was assisted in all the tests by Mr. Robert Haefeli, who is in charge of the Albbruck-Dogern Laboratory and of similar laboratories on other projects with which the writer is associated.

PROF. DR.-ING. F. KÖGLER,[117] (by letter).—Under the heading, "Research in Soil Engineering (Foundations)," Professor Gilboy states that his studies have shown that the settlement under a given unit load not too near the ultimate is practically independent of the size of the loaded area.

An extended series of tests performed under the writer's direction in the Soils Laboratory of the Bergakademie, in Freiberg, Germany, yielded valuable information on the subject as early as 1924. Figs. 49 and 50 show the relations between settlement and diameter of loaded area as derived from these studies. The laboratory tests agreed, in the main, with tests performed in the field.[118]

The first part of this research program was devoted to careful measurements of the distribution of vertical stress under loaded areas.[119] Later, the settlement characteristics of areas of similar shapes but of different sizes were studied. The curves of Fig. 49 were obtained from the results of a series of such tests on seven different circular areas with diameters ranging from 8 cm. to 100 cm.[120] The researches were next extended to include studies of the settlement characteristics and ultimate bearing capacities of areas of equal size but of different shapes.[118]

The shape of the curves in Figs. 49 and 50 may at first appear rather peculiar; but it can easily be explained. The settlement of a loaded area is due to two factors: First, to the compression (elastic and non-elastic), of the soil; and, second, to the displacement of the soil sidewise and upward. The latter factor is especially marked in the case of loads applied to the surface of the soil over small areas; the smaller the area, the more marked is the influence of lateral displacement.

This relation is reflected in the appearance of the left branches of the curves in Fig. 49. An understanding of this action is essential to the proper interpretation of the results of loading tests on small areas. The larger the area and the greater its depth, the smaller becomes the influence of this factor. Even so, however, it may still be appreciable if the soil is of a character offering relatively low resistance to lateral displacement.

With regard to the direct compression of the soil, theoretical considerations lead to the conclusion that the settlement due to this factor is proportional to the diameter of the loaded area. The writer's tests confirm this relation, as indicated by the right branches of the curves in Fig. 49. Tests performed in the field by Dr.-Ing. Heinrich Press[121] and the tests by W. S. Housel, Assoc. M. Am. Soc. C. E., gave similar results. The relation appears to hold for sand as well as for clay. Therefore, it is not correct to state that the settlement is independent of the size of the loaded area; it increases with increasing size, and, in certain ranges, is approximately in direct proportion.

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[117] Director, Foundations Laboratory, Bergakademie, Freiberg, Saxony, Germany.

[118] Kögler, "Ueber Baugrund-Probebelastungen," *Die Bautechnik,* 1931, H. 24.

[119] Kögler-Scheidig, "Druckverteilung im Baugrunde," *Die Bautechnik,* 1927, H. 29 und 31; 1928, H. 15 und 17; 1929, H. 18 und 52.

[120] E. W. Goerner, Assoc. M. Am. Soc. C. E., "Der Einfluss der Flächengrösse auf die Eisenkung von Gründungskörpern," Dissertation, Freiberg, 1928, *Geologie und Bauwesen,* Heft 3, 1932.

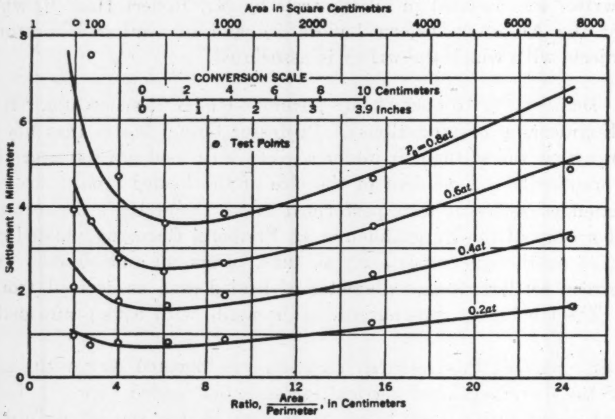

FIG. 49.—RELATION BETWEEN SETTLEMENT AND LOADED AREA, OF SAND.

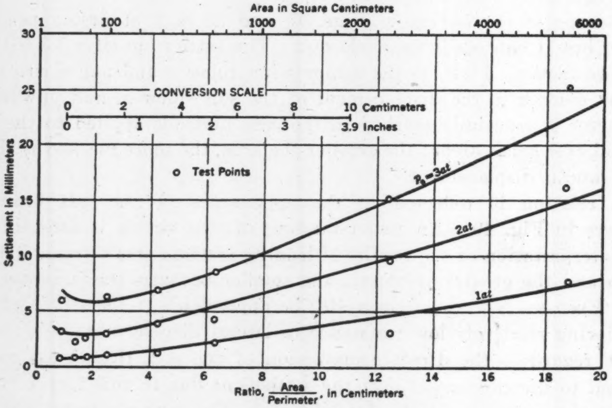

FIG. 50.—RELATION BETWEEN SETTLEMENT AND LOADED AREA, IN CLAY.

However, it is not justifiable to assume that this proportionality extends to very large areas. For such areas the settlement becomes correspondingly smaller. The reason is that the stresses produced in the soil by a large area extend to much greater depths than those under a small area, and at such depths the soil is denser and the lateral resistance is greater.[122] If it is assumed that the coefficient of compressibility (modulus of elasticity) of the soil increases in direct proportion with depth, theoretical analysis of the problem gives a curve of the type, shown at *B*,[123] Fig. 51. Even this assumption, however, leads to the results that under large areas the settlement will be large. It is unfortunate that there are not sufficient data on the behavior of large areas to permit a definite answer to the question.

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[121] *Die Bautechnik*, September 26, 1930.

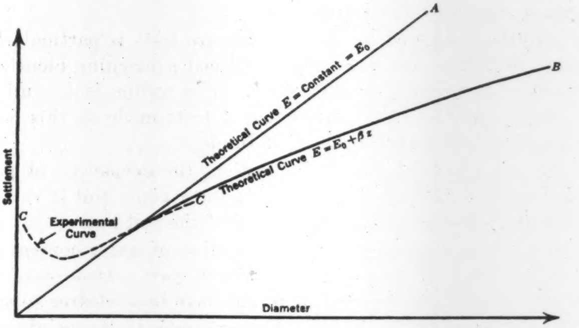

FIG. 51.

The fact that Professor Gilboy's studies appear to have yielded different results may possibly be explained by the consideration that the areas which he tested, if plotted on Fig. 49, would lie near the minimum points of the curves. An exact interpretation of his results would, under these circumstances, conform to the curves of Fig. 49. At all events, it is highly desirable to obtain settlement data on very large areas.

GLENNON GILBOY,[124] JUN. AM. SOC. C. E. (by letter).—The number of discussions contributed to this paper is indicative of the rather widespread interest now being taken in soil mechanics. In spite of the difficult nature of the problems involved, there appears to be a growing sentiment in favor of continuing the attack from all angles, thereby adding slowly but steadily to the somewhat meager fund of knowledge available. The information thus far accumulated affords a clearer understanding of the behavior of soil as a construction material than has hitherto been possible; and persevering study is certain to produce comparable results in the future.

Mr. Meem's idea that bearing values should be stated in terms of settlement is quite logical. However, the writer is not optimistic with regard to the possibility of reducing the problem of settlement to a relatively simple classification as outlined by Mr. Meem. Furthermore, there is not much hope that loading tests, even on the broad scale advised by Mr. Meem, will furnish adequate information. To mention only one difficulty, the time element involved in the settlement of foundations under-laid by clay makes it difficult, perhaps impossible, to predict the behavior of a large building over a long period of years from no other data than the results of a test on a limited area lasting a relatively short time.

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[122] Scheidig. "Versuche über die Formänderung von Sand und ihre Anwendung auf die Setzungsanalyse von Bauwerken," Wien, 1931 (unpublished).

[123] Aichhorn, "Ueber Zusammendrückung des Bodens infolge örtlicher Belastung," Dissertation, Freiberg, 1931, *Geologie und Bauwesen,* 1932, H. 1.

[124] Asst. Prof. of Soil Mechanics, Mass. Inst. Tech., Cambridge, Mass.

The apparatus suggested for lateral pressure tests is particularly interesting because, some time ago, the writer outlined a machine identical in principle, although on a much smaller scale. The writer is in full agreement with Mr. Meem regarding the importance of tests made on this basis. Their performance is impeded only by lack of funds.

In answer to Professor Parsons' question, the coefficient of permeability as used in Equation (2) is not a constant; for a given soil it varies not only with temperature, but also with the density of the soil mass.

Mr. Feld's misunderstanding of the definition of water content is probably due to a poor choice of terms on the writer's part. Instead of "weight of water per unit weight of dry soil," it might have been clearer to say, "weight of water per unit weight of solid." For example, if the original weight of a sample of soil is W, and the weight of the same sample "bone dry" is W_0, the water content of the original sample, w, is, $w = \frac{W - W_0}{W_0}$.

The objection by Mr. Feld to the lack of homogeneity of Equation (1) is sound. Here, again, a looseness in statement is apparent. To be strictly correct, the quantity, $V_1 - V_0$, should be multiplied by the weight of water per unit volume. Since, however, this is unity in the centimeter-gramme-seconds system, one easily falls into the habit of forgetting its existence in routine laboratory work.

There is still considerable controversy concerning the effect of the size of the loaded area on the magnitude of settlement. Although the tests with which the writer has had to deal justify no other conclusion than the one stated, it is possible that the range of areas involved was not sufficiently great to show the true situation. Extended series of tests performed in Freiberg under the direction of Professor Kögler indicate that the settlement on sand increases with the size of the area. On the other hand, this increase can scarcely be unlimited. If it were, large and heavy buildings founded on deep sand deposits should undergo considerable settlement; but this does not seem to be the case. A number of fairly heavy buildings on Lower Manhattan Island are founded on deposits of sand, frequently rather loose in structure, extending to depths of 100 ft. and more; yet if the writer's information is correct, settlements in this locality are relatively small.

To quote some figures, one series of the Freiberg tests on different areas under a pressure of 0.8 ton per sq. ft., gives a curve of which the asymptote may be represented approximately by the equation, $S = 0.026\,R$, in which, S is the settlement and R, the ratio of area to perimeter. This extension leads to the result that a building founded on a mat of 100 by 200 ft., with the very light load of less than 1 ton per sq. ft., should settle about 10 in.; or that a subway 50 ft. wide, with the same light load, should settle about 8 in. Such conclusions are manifestly absurd.

Just where the discrepancy lies, the writer is not at present prepared to state. It is evident, at any rate, that considerable work remains to be done in order to bring out the true relations between the results of loading tests and the behavior of actual structures, even when operating with a relatively simple material, such as cohesionless sand.

Mr. Jahn's pertinent comments on the hydrometer method illustrate the difficulties involved in working out a procedure for wet mechanical analysis. Certain of the faults enumerated can be minimized by the use of proper corrections. It is true, as Mr. Jahn intimates, that even with all possible corrections taken into account the method still remains approximate; but the margin of error is small enough so that the method fulfills the usual requirements of practice. Its many advantages, including the low cost of equipment, the ease of operation, and the short time consumed, make it a useful adjunct to routine laboratory work.

The writer was favorably impressed by Mr. Jahn's scheme of using a balanced immersion cylinder for this work, although it seems probable that the apparatus might be expensive. The scale-beam would have to be extremely sensitive, and the weights accurate, on account of the small changes in the uplift of the soil suspension. In addition, the temperature of the outfit would have to be kept constant, in order to eliminate changes in uplift due to changes in density of the water.

Mr. Eiffert's clear description of the field work at the Germantown Dam requires no comment. It may be useful, however, to clarify the meaning of the term, "percentage consolidation." To say that a soil is 25% consolidated means nothing, of itself, with regard to solidity. It merely means that of the total quantity of water that must eventually be forced out of the soil mass by the pressure to which it is subjected, one-fourth has already been forced out. In order to determine the solidity of the material, its angle of internal friction and the magnitude of the pressure acting upon it must also be known. For example, if a soil having the characteristics of the Germantown core material was subjected to a pressure of 1 ton per sq. ft., it would develop, if 100% consolidated, a compressive strength of about 1.7 tons per sq. ft. On the other hand, if the same soil were under a pressure of 4 tons per sq. ft., but only 25% consolidated, its strength would still be 1.7 tons per sq. ft. Thus, it is evident that the degree of consolidation alone is not sufficient to define the characteristics of the material.

With regard to the slow rate of consolidation of the Miami cores, it is the writer's opinion that this is a favorable feature, much to be preferred to the situation arising in a core which has been made coarser purposely to facilitate rapid consolidation. If the core is stable at a low degree of consolidation, additional hardening merely adds strength where it is not needed. Rapid and complete consolidation is, therefore, neither necessary nor desirable. Conversely, a core that consolidates slowly has, in addition to its high degree of water-tightness, the advantages that the re-adjustments of the entire dam accompanying consolidation are very gradual, and that there is no danger that drying and cracking of the core, in the manner described by Mr. Eiffert, will take place.

Mr. Eremin's remarks on colloids touch on one of the many effects that complicate the physical and chemical properties of soils. In fact, it is on account of these complications that direct methods of testing appear much more promising than attempts to infer the physical and mechanical properties from detailed technical analyses.

Professor Menefee has brought out some interesting points concerning the rupture of materials under compression. It is true that specimens of concrete and of soil frequently fail due to lateral tension, the failure being accompanied by vertical cracks; but this action is not the reason why true shear failures do not occur on planes making an angle of 45° with the principal planes. Rupture in shear takes place along a given plane when the obliquity of the resultant stress on that plane equals the angle of internal friction of the material. Therefore, if a specimen fails in shear, it will fail along planes upon which the obliquity of stress is a maximum. These planes are not necessarily planes of maximum shear intensity. In a material, such as steel, having a low angle of internal friction, the planes of maximum obliquity and the planes of maximum shear practically coincide; but this condition no longer prevails when the angle of internal friction has a value appreciably greater than zero. The greater the angle of internal friction, the greater is the angular displacement between these two sets of planes.

Referring again to Mohr's diagram in the original paper (Fig. 9), the positions of the planes of maximum obliquity are represented by the two radii drawn to the points of tangency. Suppose that the angle of internal friction of the soil specimen is 26°; then the angle, α, is 64 degrees. This is twice the angle between each plane of maximum obliquity and the plane of minor principal stress. Therefore, shear failure of the specimen will take place along planes making an angle of 32° with the vertical, in spite of the fact that the maximum intensity of shearing stress occurs on 45° planes. It is thus evident that Professor Menefee's observation in connection with Fig. 18 that "actual tests show that the failures in earth cylinders and concrete as well, are along planes of greater slope than 45°," substantiates theoretical concept.

Mr. Walker is correct in stating that the changes in consistency of clay as the water content decreases are not all marked by well-defined points of division. The shrinkage limit is an easily recognized boundary, but the plastic and liquid limits—especially the latter— are really transition states. It is precisely for this reason that standardized methods of determination of these limits are essential.

Mr. Walker suggests the correlation of moisture content with the force required to produce some agreed distortion of the sample. This is, in effect, the basis of the liquid limit determination. The agreed distortion is represented by the flowing together of the two halves of the sample over a specified distance. The force is measured by the number of impacts required to produce this distortion. When the number of impacts required equals an agreed number—say, 25—the water content of the sample is said to correspond to the liquid limit. To this extent the definition is arbitrary; but, as long as the adopted standard is adhered to, the results will be consistent, and the values will afford a definite measure of the character of the material.

It should also be noted that any elaborate method of determining the limits, or of setting up comparable standards, would defeat its own purpose. As the writer has pointed out, the chief advantage of the limit determinations is their simplicity. As soon as they become difficult to handle, their utility ceases.

Mr. Walker's unfavorable opinion of the shrinkage limit equation undoubtedly arises from the same difficulty that misled Mr. Feld. If Equation (1) is written:

$$S = \frac{(W_1 - W_0) - \rho\,(V_1 - V_0)}{W_0}$$

in which, ρ is the weight of water per unit volume, the obscurity disappears. The shrinkage limit is, by definition, the weight of water in the sample at the limit, divided by the dry weight of the sample. The quantity, $W_1 - W_0$, is the total weight of water in the saturated sample at the outset. The quantity, $\rho\,(V_1 - V_0)$, is the weight of water that disappeared from the sample as it dried from its original state to the shrinkage limit. The difference must be the weight of water that the sample contained at the shrinkage limit. Dividing this by the dry weight gives the desired value.

It is advisable, as Mr. Walker suggests, to calibrate the capillary permeameter before using it on an extended program of tests. However, the writer adheres to his original opinion that the apparently large variation in the coefficient, F, does not prevent the use of the uncalibrated apparatus for approximate measurements. If one could be absolutely certain of seepage calculations within 50% or even 100%, it would represent a fair degree of accuracy. For example, the permeability of the apparently homogeneous core of the Germantown Dam, determined from direct tests on undisturbed samples, varied from 20×10^{-7} to 400×10^{-7} cm. per min. In heterogeneous natural deposits the variations are many times greater. Hence, an approximate measurement that is within 50% of the truth is by no means worthless. In scientific research, a high degree of accuracy is essential; but in the practical field, the most elaborate computations frequently can do no more than define the general region within which the true solution lies.

The remark by Mr. Walker concerning the end effect in the retaining-wall apparatus brings out one of the main difficulties associated with devices of this type. The effect is not quite as marked as Mr. Walker states. Although the bin is 10 ft. deep, the model wall is only 7 ft. high, so that the ratio of depth to width of restrained earth is 1 to 2. In addition, the depths actually used in the tests were somewhat less than 7 ft., thus giving an even more favorable ratio. A further attempt was made to minimize the side friction by greasing the wall before placing the fill. Therefore, while certain end effects were always apparent, it is believed that they were not of sufficient magnitude to invalidate the results.

Professor Krynine's bibliography and synopsis of the investigations pursued by many scientists should be useful to those unfamiliar with the field of soil mechanics as an indication of the time and effort expended on the study of various important problems.

Mr. Crandall's remarks on the present unsatisfactory methods of soil classification are most pertinent. The writer did not touch on this subject because of the lack of anything concrete to offer. The problems involved in an attempt to arrive at a rational basis of classification are rather numerous. In the first place, the scheme of classification would have to be such as to meet the requirements of the particular purpose for which the standards are intended. A system satisfactory to an agronomist might be misleading to a foundation engineer. The sub-divisions should be based on variations in those physical properties of the soil that have a direct bearing on the behavior of the material from an engineering standpoint. There should be a sufficient number of sub-divisions so that any known soil would find a definite place, but not so many as to make the system cumbersome to use. The criteria of allocating soils to their proper sub-divisions should be definite, simple, and readily reproducible. Any tests required for applying these criteria should be of such an elementary nature as to be readily performed in the field, at least for a rough preliminary classification. The system as a whole should be of such a sensible and straightforward nature as to appeal immediately to the practical man. Finally would come the not inconsiderable task of educating engineers, architects, contractors, and drillmen in the essentials of the system, in order to make its use sufficiently widespread as to constitute a generally recognized and acceptable standard. The entire affair, therefore, represents a difficult and lengthy undertaking.

The sound ideas on the subject of pile foundations, mentioned briefly in Mr. Crandall's discussion, were presented in 1931 in a separate paper of greater length.[125] A valuable analysis of the effect of kneading on the properties of clay may be found in a paper published early in 1932 by Mr. Arthur Casagrande.[126]

Mr. Marston's brief discussion gives an insight into the careful studies which have been conducted under his supervision. The establishment of the Metcalf and Eddy laboratory represents a praiseworthy endeavor to apply the results of scientific analysis to practical problems, and to correlate laboratory findings with field observations.

The writer is indebted to Mr. Paaswell for the definite statement that the soil studies performed in connection with the subway construction in New York brought about a substantial saving of money. This is direct evidence of the fact that funds invested in such studies may be expected to pay good dividends. It is hoped that this idea will soon be grasped by the construction world in general, so that soil research will receive financial backing comparable to that accorded investigations on other engineering materials.

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[125] "Piles and Pile Foundations," by J. Stuart Crandall, M. Am. Soc. C. E., *Journal*, Boston Soc. of Civ. Engrs., April, 1932.

[126] "The Structure of Clay and Its Importance in Foundation Engineering," *Journal*, Boston Soc. of Civ. Engrs., April, 1932.

Professor Griffith apparently wishes to classify the writer as a member of a definite school of thought. To this the writer objects strenuously. He refuses allegiance to any particular school, believing that such an attitude is certain to result in a narrow field of view without adequate compensation in depth of perception.

It is quite true that the writer is interested in the physicist's concept of matter, and has been guilty of discoursing at some length on grain size and shape, intrinsic stresses, and soil structure, since he does not agree with Professor Griffith's idea that "such conceptions have no place in the systematic working formulations of science." On the other hand, the writer is equally interested in the methods of theoretical and applied mechanics, involving field equations subject to singularities and boundary conditions. Perhaps there was insufficient evidence in the original paper to make this clear to Professor Griffith. As an illustration, it may be interesting to elaborate a trifle upon the theory of consolidation. If a mass of saturated clay under pressure is taken as the field, it can be shown that the hydrodynamic conditions in the field are governed by the equation, $K \nabla^2 u = \frac{\partial u}{\partial t}$, in which, K is a constant of the material, and u is the hydrostatic excess at any given point in the mass at any given time, t. The solution of this equation in conformity with a given set of boundary conditions results in a theoretical expression determining the extent of consolidation as a function of space and time. A solution for the simplest case—a layer of clay infinite in horizontal extent, drained at the upper and lower faces, and subjected to uniform stress—was published by Professor Terzaghi in 1923.[127] The writer has since generalized the theory and has investigated a number of more complicated cases. This is, without doubt, exactly the kind of attack that Professor Griffith recommends.

In brief, the writer feels that it is not only possible but highly desirable to adopt a middle course, not relying exclusively on the methods of one or the other school, but investigating the favorable features of both.

Concerning the application of Boussinesq's theory to stress analysis in soils, the writer feels that his statements have been quite consistent. He merely pointed out that at best only an approximate solution is possible, but that in many cases an approximate solution comes close enough to actual conditions so that information of real value can be obtained therefrom.

Mr. Gruner's description of the field tests made under his supervision furnishes an interesting example of the manner in which properly conducted research can be used to aid in the solution of practical problems. Much still remains to be accomplished in this line; but it is encouraging to realize that the zone of uncertainty invariably associated with earthwork engineering can even now be narrowed appreciably by the use of physical measurements.

The proposed permeability apparatus shown in Fig. 44 would probably work better if it were arranged so that the water flowing through the sample were to travel downward instead of upward. An upward current tends to loosen the soil, and the results may be somewhat erratic for that reason.

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[127] "Die Berechnung der Durchlässigkeitsziffer der Tone aus dem Verlauf der hydrodynamischen Spannungserscheinungen," *Sitzungsberichte* der Akademie der Wissenschaften in Wien, Mathem. Naturw. Klasse, Abt. 11-*a*, 1923.

The depth of the consolidation device illustrated in Fig. 46 appears to be rather large compared to the diameter. There is every reason to believe that a considerable part of the applied pressure is transferred to the walls, so that the pressure intensity at the bottom of the sample is only a fraction of that at the top. This means, of course, that the measurements will indicate a smaller compressibility than actually exists.

The proportions of the proposed device, Fig. 47, are much better. However, the writer is somewhat doubtful of the advisability of measuring the shearing resistance in the manner suggested. At first glance, it looks as if the withdrawal of the central section will produce pure shear along two horizontal surfaces, as indicated by the broken lines; but the actual situation is not so simple. The application of the horizontal force, *Q*, reduces the lateral stress on one side of the sample (on the left in Fig. 47) to zero, and produces an intense compression on the other side. This compression must be resisted by an equal and opposite comparison on the left side of the upper and lower elements of the sample. Consequently, the main compressive stresses within the sample will be transferred along curved paths from the right of the center element to the left of the two outer elements. This represents a rather complicated stress distribution which will not result in the desired condition of simple shear. For the present, at least, the writer believes that it is more satisfactory to use separate devices for consolidation and shearing tests.

Professor Kögler's remarks on the settlement of footings bring up the same question as that raised by Mr. Feld, and the writer's comments on Mr. Feld's discussion apply in both cases. As Professor Kögler suggests, it will be advisable, when circumstances permit, to execute a series of tests on a somewhat larger scale than usual, in order to bridge the gap that seems to exist between the behavior of model footings and the action of full-sized structures.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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ENGINEERING FEATURES OF THE ILLINOIS WATERWAY[1]

By WALTER M. SMITH,[2] M. AM. SOC. C. E.

WITH DISCUSSIONS BY MESSRS. A. W. SARGENT, E. G. WALKER, DAVID A. MOLITOR, CHARLES A. D. YOUNG, AND WALTER M. SMITH

SYNOPSIS

Locks, dams, and power houses along the Illinois Waterway in the State of Illinois constitute the primary subjects of this paper. A general description of the structures at Lockport, Brandon Road, Dresden Island, Marseilles, and Starved Rock, is followed by a theoretical analysis of culvert valve chambers, operating machinery, etc., as designed for this project.

The steps in the design of miter-gates are reviewed briefly, including a description of a metal sealing-spring plate which permitted a more accurate and a lighter design. The gate anchorages were planned so as to make possible a fine adjustment of the rotation axis, similar to the arrangement for the locks at Sault Ste. Marie, Mich.

Interference with existing structures at Lockport, Ill., made necessary the installation of vertical lift-gates at this point. Furthermore, an emergency gate was necessary to take the place of movable gates that regulated water levels at the other dam sites in case of accidents. The physical features of all these units are treated herein.

An important part of the paper is an analysis of stresses in the side walls of the locks where the entry of the culverts complicates the design. The problem is solved by the Principle of Least Work, and one complete numerical example is given.

INTRODUCTION

The canal officially known as the Illinois Waterway, extends from the down-stream end of the Sanitary District Canal at Lockport, Ill., to Starved

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[1] Presented at the meeting of the Waterways Division, Milwaukee, Wis., July 11, 1929.

[2] Asst. Chf. Engr., Div. of Waterways, State of Illinois, Chicago, Ill.

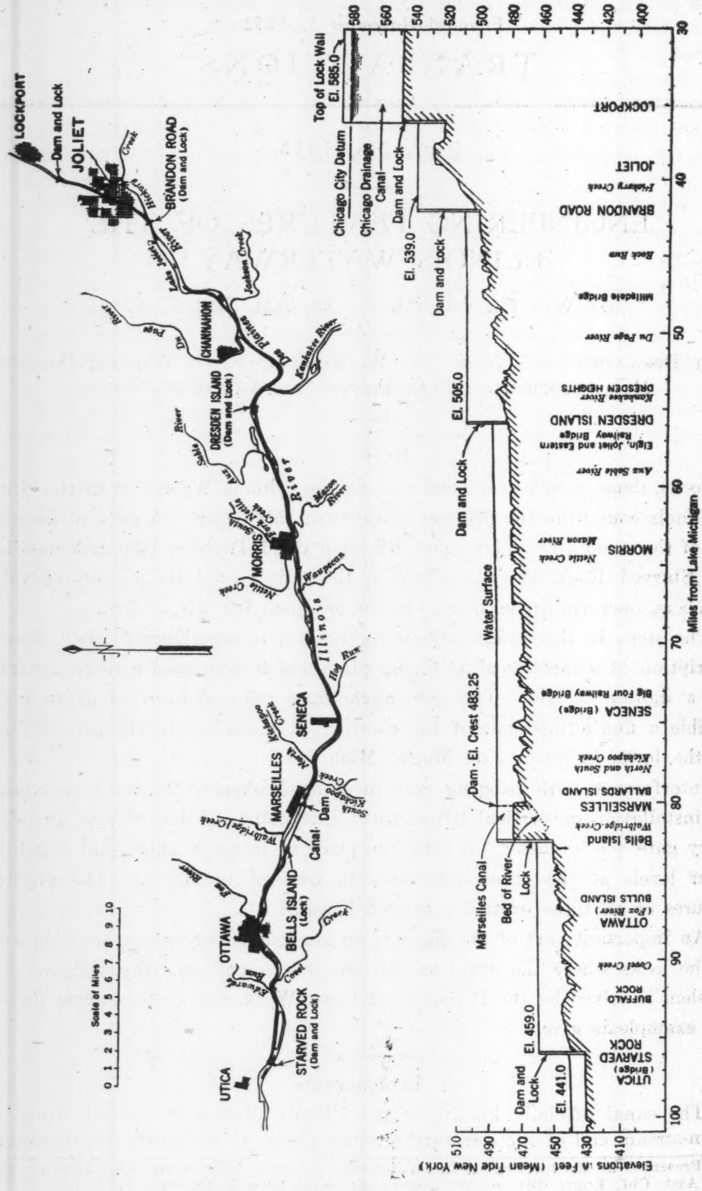
LOCKPORT
Dam and Lock
JOLIET
BRANDON ROAD
(Dam and Lock)
Hickory Creek
Lake Joliet
Jackson Creek
Des Plaines River
Du Page River
CHANNAHON
DRESDEN ISLAND
(Dam and Lock)
Kankakee River
Aux Sable River
South Fork Nettie Creek
Mazon River
MORRIS
Nettie Creek
Waupecan
Illinois River
Hog Run
SENECA
North Kickapoo Creek
South Kickapoo Creek
MARSEILLES
Dam
Canal
Walbridge Creek
Fox River
BELLS ISLAND
(Lock)
OTTAWA
Covel Creek
Edwards Run
STARVED ROCK
(Dam and Lock)
UTICA
Scale of Miles
0 1 2 3 4 5 6 7 8 9 10
Top of Lock Wall El. 585.0
Chicago City Datum
Chicago Drainage Canal
Dam and Lock
El. 539.0
Dam and Lock
El. 505.0
Dam and Lock
Water Surface
Dam - El. Crest 483.25
Marseilles Canal
Bed of River
Lock
El. 459.0
Dam and Lock
El. 441.0
Elevations in Feet (Mean Tide New York)
580
560
540
520
500
480
460
440
420
400
510
490
470
450
430
410
LOCKPORT
JOLIET
Hickory Creek
BRANDON ROAD
Rock Run
Milldale Bridge
Du Page River
DRESDEN HEIGHTS
Kankakee River
DRESDEN ISLAND
Elgin, Joliet and Eastern Railway Bridge
Aux Sable River
MORRIS
Nettie Creek
Mazon River
SENECA (Bridge)
Big Four Railway Bridge
North and South Kickapoo Creek
BALLARDS ISLAND
MARSEILLES
Walbridge Creek
Bells Island
BULLS ISLAND
OTTAWA
(Fox River)
Covel Creek
BUFFALO ROCK
STARVED ROCK
UTICA
(Bridge)
30
40
50
60
70
80
90
100
Miles from Lake Michigan

FIG. 1.—GENERAL PLAN AND PROFILE ILLINOIS WATERWAY

Rock, a distance of about sixty miles. The plan and profile of the canal are shown in Fig. 1.

Locks have been located at Lockport, Brandon Road, Dresden Island, Bell's Island (about 2½ miles below Marseilles and known as the Marseilles Lock), and at Starved Rock. Dams and head-gate structures for power houses have been constructed at Brandon Road, Dresden Island, Marseilles, and Starved Rock. The Sanitary District of Chicago had a dam and power house in operation at Lockport before the beginning of work on the Illinois Waterway.

The head-gate structures of the power houses, except at Marseilles, were built first as a part of the dam, so that the power-house construction could be left for a later date. For a distance of 2½ miles up stream from Marseilles Lock, the Waterway consists of a canal entirely separate from the river. This is the case also for a distance of 2 miles down stream from Lockport Lock. Otherwise, the Waterway consists of a series of pools, dredged to a proper depth and width wherever necessary. The dimensions established were a minimum width of 200 ft. and a minimum depth of 9 ft.

DESCRIPTION OF STRUCTURES

Lockport Lock.—The structure a short distance southwest of Lockport consists of a lock with a maximum lift of 40 ft., which has miter-gates at the lower, and vertical lift-gates at the upper, end. An emergency dam has been constructed at the up-stream entrance to the lock.

Brandon Road Pool.—Des Plaines River enters the canal between Lockport and Brandon Road. Hence, there will be much greater flood flow at this site than at Lockport, and provision has been made to take care of a flood of 35 000 sec-ft. It is desired to hold this water at a very uniform level (Elevation 539.0) because this is the pool elevation through Joliet, Ill. A concrete retaining wall was built along the east side of the river through Joliet from the upper end of the city, southward 13 500 ft. to the dam, because the water level is much higher than the lower parts of the surrounding land. On the west side of the river a concrete wall was built from Jackson Street to the dam, a distance of 11 500 ft. As the retaining wall along the east side had to be built in any case, about 1 200 ft. of the lower end of it was constructed as a spillway weir. The crest of this weir is 2 ft. 3 in. below the water level as maintained by a series of twenty-one Taintor gates, 2 ft. 3 in. high by 50 ft. long. A gate has been constructed for passing ice, similar to the one at the Lockport Power Plant, of the Chicago Sanitary District.[3]

The crest of the long spillway weir is at Elevation 536.75, New York mean tide datum; the crest of the Taintor gates is at Elevation 539; and the crest of the ice-chute weir, at Elevation 534. A study was made to see if it would be better and more economical to construct several "Stickney" siphon spillways[4] rather than the long ogee weir. It was found that, in order to secure a considerable flow with very slight rise, the siphon spillways were slightly more economical; but there was a serious disadvantage in the fact that there is almost no increase in flow in these spillways as the water rises in the reservoir, whereas in the ogee type the capacity increases as the three-halves power of the head. For a 3-ft. head on the ogee weir, for example, the flow would be about 5.2 times as great as for a head of 1 ft. and, for a 4-ft. head, it would be 8 times as great. Because this great increase in discharge for a small increase in head was very desirable, it offset decidedly the slight economy of the siphon spillway. In addition, six sluice-gates with clear openings of 6 by 8 ft. were provided in the base of the dam. The normal lower-pool elevation is about 506.2, making the normal lift of the lock, 32.8 ft. The section of canal from Lockport to Brandon Road (see Fig. 1) is about 6 miles long.

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[3] *Engineering News*, Vol. 60, 1908, p. 512.

[4] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1098.

Intercepting sewers are constructed in the base of the retaining walls at Joliet, the one on the east intercepting all the sewage and storm-water run-off from that part of the city and discharging it below the dam. The west side of Joliet is much higher than the east side, except for a narrow strip along the back of the retaining wall. Therefore, most of the storm-water run-off is conveyed into the canal through submerged cast-iron pipes that pass through the wall. The sewage from the west side is conveyed into the intercepting sewer in the base of the wall. A 36-in. pipe has been placed across the river bed to convey this sewage to the larger intercepting sewer in the wall on the east side near its down-stream end.

Dresden Island Pool.—From Brandon Road to Dresden Island Dam is about 14 miles and two rivers of consequence enter this stretch, the Du Page from the north and the Kankakee, with a very large drainage area, from the south. According to the best data available the maximum flood flow that will occur at this site is 75 000 sec-ft. The dam consists of a concrete weir with nine Taintor gates, 60 ft. long—each sustaining a head of 14 ft. of water normally—and an ice chute, 30 ft. long, with a gate sustaining a head of only 5 ft. when closed. This gate is similar to the ice chute for the dam at Brandon Road.

The elevation of the normal upper pool and the Taintor gates is 505; the crest of the Taintor-gate weirs is at Elevation 491; the crest of the ice-chute weir is at Elevation 500; and the normal lower-pool elevation is 486.3.

Marseilles Pool.—From Dresden Island to Marseilles Dam is about 24.5 miles; but no rivers of consequence enter the canal between these two points so that the maximum flood flow for this site is 77 000 sec-ft. The Marseilles Lock is about 2½ miles below the dam and a canal has been excavated beside the river from the dam to the lock instead of using the river. A short distance down stream from the old dam, the new one, 970 ft. long, is planned to extend to the side of the canal entrance. It includes seven Taintor gates, each 60 ft. long and 14 ft. high. The tops of the Taintor gates were set at Elevation 483.25.

Marseilles Canal and Lock.—A canal separate from the river extends about 2½ miles from the Marseilles Dam to the Marseilles Lock. Since this canal is intended both for navigation and power purposes the power house is near the lock. The general plan and sections of the lock are shown in Fig. 2, while Fig. 3 shows the details at the upper gates.

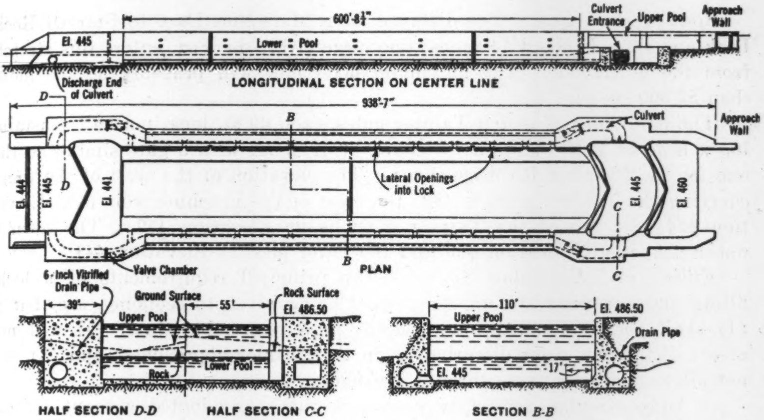
600'-8¼"
Culvert Entrance
Upper Pool
Approach Wall
El. 445
Lower Pool
Discharge End of Culvert
LONGITUDINAL SECTION ON CENTER LINE
938'-7"
D
B
Culvert
Approach Wall
El. 444
El. 445
El. 441
Lateral Openings into Lock
El. 445
El. 460
C
Valve Chamber
PLAN
6-Inch Vitrified Drain Pipe
Ground Surface
Rock Surface
39'
55'
El. 486.50
Upper Pool
Lower Pool
Rock
110'
El. 486.50
Upper Pool
Drain Pipe
El. 445
17'
HALF SECTION D-D
HALF SECTION C-C
SECTION B-B

FIG. 2.—GENERAL PLAN AND SECTIONS, MARSEILLES CANAL AND LOCK

All the locks on the Illinois Waterway have a usable length of 600 ft. and a width of 110 ft., with lifts varying from 15 to 40 ft. Miter-gates will be used at both up-stream and down-stream ends of locks, except at Lockport, where a different type of gate had to be used at the up-stream end for reasons to be given subsequently.

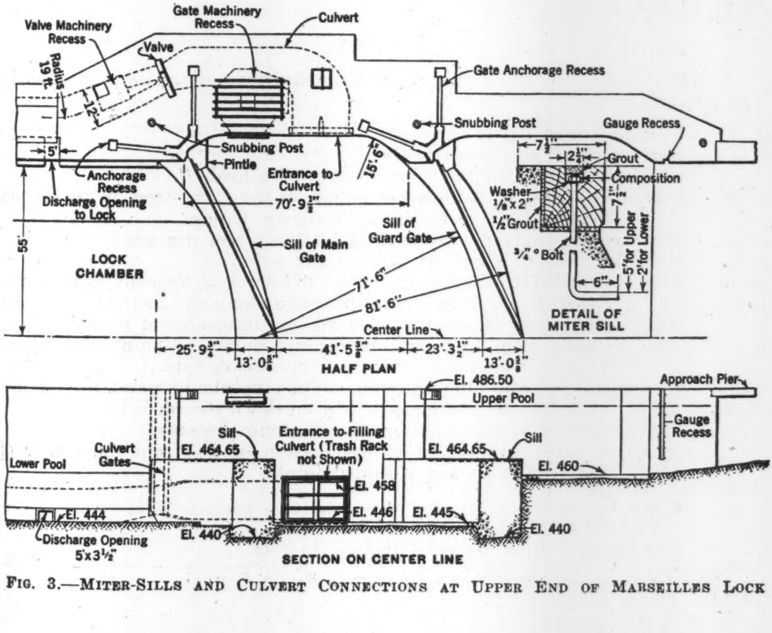
Valve Machinery Recess
Gate Machinery Recess
Culvert
Valve
Radius 19 ft.
12'
Gate Anchorage Recess
Snubbing Post
Gauge Recess
5'
Snubbing Post
Pintle
15'-6"
Anchorage Recess
Discharge Opening to Lock
Entrance to Culvert
70'-9½"
7½"
2¼"
Grout
Composition
Washer ⅛x2"
½ Grout
7"
¾" Bolt
6"
5' for Upper
2' for Lower
55'
LOCK CHAMBER
Sill of Main Gate
Sill of Guard Gate
71'-6"
81'-6"
DETAIL OF MITER SILL
Center Line
25'-9¾"
13'-0⅝"
41'-5⅜"
23'-3½"
13'-0⅝"
HALF PLAN
El. 486.50
Approach Pier
Upper Pool
Gauge Recess
Sill
El. 464.65
Entrance to Filling Culvert (Trash Rack not Shown)
El. 464.65
Sill
El. 460
Lower Pool
Culvert Gates
El. 458
El. 444
El. 446
El. 445
El. 440
El. 440
Discharge Opening 5'x3½'
SECTION ON CENTER LINE

FIG. 3.—MITER-SILLS AND CULVERT CONNECTIONS AT UPPER END OF MARSEILLES LOCK

Starved Rock Pool.—The distance from Marseilles Dam to Starved Rock Lock and Dam is about 14 miles; one large river, the Fox, enters this stretch from the north side. The maximum flood flow will probably not be more than 85 000 sec-ft.

The dam consists of ten Taintor gates, each 60 ft. long, normally sustaining a head of 17 ft., and an ice chute, 30 ft. long, with a gate similar to the one for the Brandon Road ice chute. The elevation of the crest of the weirs carrying the Taintor gates is 442; the crest of the ice-chute weir is at Elevation 454; the top of the Taintor gates is at Elevation 459. The normal upper pool is at Elevation 459 and the lower pool at Elevation 441.

Filling and Emptying System.—The principal requirements of a lock-filling and emptying system, due regard being given to economy, are that it (1) shall permit equalization of levels in a reasonable time; (2) will not create dangerous water disturbances in the locks or approaches; and (3) will not allow excessive waste through leakage.

A large circular culvert in each lock-wall was adopted without further study because the economy of such design has been demonstrated by past experience. Studies showed that two culverts, 12 ft. in diameter, would ensure equalization of levels in from 7 to 12 min., depending on the location of the lock and subject to the limitation in equalization of 4 ft. vertically per min. A complete description of the filling and emptying system and lock valves has been written by L. D. Cornish, M. Am. Soc. C. E.,[5] and the operating machinery for the valves has been described by Mr. A. S. Grossberg,[6] Designing Engineer.

NOTATION

In the analytical parts of this paper the definitions of the various symbols are as follows:

b = length of culvert arch.

c = strength coefficient = Hs.

e = eccentricity, or the moment arm of a load, P.

f = unit stress, in pounds per square inch; f_u = stress in up-stream flange; f_d = stress in down-stream flange.

g = acceleration of gravity = 32.2 ft. per sec. per sec.

h = thickness of a culvert arch at any point.

i = a dimension, in feet (see Fig. 6(b)); i_1 = moment arm of the force, P_3; i_2 = horizontal distance along the neutral axis from the center of rotation of a gate to its center of gravity.

j = distance from line through contact points to point of application (air chambers), skin on up-stream side.

j' = distance from line through contact points to point of application (no air chambers), skin on down-stream side.

l = length of arch culvert measured along the axis.

e_n = distance of the point of application of P_n from the center of the section at any assumed joint.

$r = \dfrac{P_1}{2 \sin \alpha}$.

s = spacing of miter-gate girders, in feet.

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[5] Manuals of Engineering Practice, Am. Soc. C. E., No. 3, p. 17.

[6] *Loc. cit.*, p. 21.

t = time required to fill a lock chamber, in seconds.

w = weight, in pounds; w_g = weight of miter-gate; w_w = weight of 1 cu. ft. of water.

x = transverse horizontal distance from the center of rotation of a gate to its center of gravity.

y = horizontal distance from the down-stream face of a gate to its center of gravity.

A = area, in square feet; A_c = combined area of the pair of culverts leading into a lock chamber; A_l = area of lock chamber; A_n = area of a culvert section at any assumed joint, acted upon by the force, P_n.

A_1 and A_2 = pull in the anchorages with gates closed.

C = a constant, varying from 0.75 to 1.15.

E = modulus of elasticity.

F = total stress, in pounds; F_u = stress in the up-stream flange; F_d = stress in the down-stream flange.

H = head, or lift, in feet.

I = moment of inertia of a section of any assumed joint that is acted upon by P_n.

K = resisting force, is equal and opposite to $\frac{P_1}{2}$.

L = distance from a culvert support, measured along the axis; dL = an infinitely small distance along the axis.

M = bending moment in a culvert section at any assumed joint.

N = resisting force, is equal and opposite to P_1.

P = direct pressure, in pounds; P_1 = total water pressure perpendicular to flange on one girder; P_2 = total water pressure against ends of one girder; P_3 = total pull exerted on anchors, A_1 and A_2, at the top of a miter-gate $= \frac{W_g i^2}{i_1}$; P_n = total pressure normal to a section at any assumed joint of a culvert.

r = radius, in feet.

$T = P_1 + P_2$.

V = velocity, in feet per second, in up-stream approach; and V_t = velocity of water at the throat section of a Venturi meter.

W = total work of resistance in a culvert wall.

α = angle of miter-gate with a line perpendicular to the lock-walls.

Culvert Valve Chambers and Operating Machinery

The culvert valve chambers were designed according to the principles of the Venturi meter as developed[7] by the late Clemens Herschel, Past-President, and Hon. M. Am. Soc. C. E., the culverts being circular (12 ft. in diameter), and the valve openings, 9 ft. square.

For some reason it has been the practice of canal engineers to design culvert valves with a considerably larger opening than the area of the culvert. On the other hand it has been the practice of water supply engineers to put gates in a conduit or pipe line of considerably less area than that of the conduit or pipe, knowing that if the reduction is not very great the loss of head is negligible and the smaller gates are much more economical.

Herschel took advantage of this fact in his development of the Venturi meter. In 1887, he performed many experiments with two meters of different sizes. The first was with a tube 1 ft. in diameter and a throat diameter of $\frac{1}{3}$ ft., so that the throat area was one-ninth the area of the tube. In the second set the tube was 9 ft. in diameter, with a throat diameter of 3 ft., making the throat area one-ninth the area of the tube, the same as in the other case.

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[7] *Transactions*, Am. Soc. C. E., Vol. XVII (1887), p. 228, and Vol. XVIII (1888), p. 133.

The loss of head of the water passing from the inlet to the throat may be expressed in two ways: First, in terms of velocity at the throat, V_t; and, second, in terms of increase in velocity at the throat, $V_t - V$.

In the experiments with the small meter, with velocities at the throat varying from 10 to 35 ft. per sec., the loss of head in terms of the velocity at the throat was $0.00172\ V^2_t$ and in terms of the increased velocity at the throat, $0.00218\ (V_t - V)^2$.

In the case of the large meter the loss of head was slightly higher, probably due to the fact that it was not so well made; also, there was a greater variation in values, the loss of head being about $0.00230 V^2_t$, or $0.00290\ (V_t - V)^2$. In the case of the small meter a very uniform coefficient of 3.0 was obtained, but in the larger size the coefficient was about 2.5 and varied considerably more than in the small one. In these experiments the velocity at the throat of the meter was nine times as great as that in the tube.

In designing the inlet gates in the main gate-house at the Ashokan Dam, of the Catskill Water Supply System for New York City, advantage was taken of the Venturi principle by the Engineers of the Board of Water Supply. The valve, which is the throat of the meter, is 5 ft. in diameter, with an area of 19.6 sq. ft.

The sides flare on the down-stream side at an angle of 2° 30′, the same as in the Herschel meters, to a diameter of 7 ft. 4¼ in. and an area of 42.6 ft. at a distance of about 29 ft. from the valve. This gives an increase of 117% in area over the throat area. In testing these valves a coefficient of 1.18 was obtained before the Venturi action was broken.

In the filling and emptying system of the locks of the Illinois Waterway the culverts are circular, 12 ft. in diameter, with an area of 113 sq. ft. The valves are 9 ft. square. A transition section down stream becomes a circle 12 ft. in diameter in a distance of 19 ft. from the valve, or an increase in area of 40 per cent. For the same distance, the increase in area of the tail-tube in the Ashokan valve is 72 per cent. It is hoped that as high a coefficient will be obtained in the valves as that obtained in the Ashokan tests.

The formula for finding the time required for filling a lock chamber is,

$$t = \frac{2\ A_l\ \sqrt{H}}{C\ A_c\ \sqrt{2g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and it has been the practice to take $C = 75$. It was found in testing the Panama Canal locks after completion that when the valve area was much greater than the culvert area, $C = 0.83$. Using this value in Equation (1) the time required to fill the Marseilles Lock, if the rise is unlimited and $H = 21.6$ ft., is: $t = \frac{2 \times 74\,910\ \sqrt{21.6}}{0.83 \times 226\ \sqrt{64.4}} = 464$ sec., or 7 min. 44 sec.

Assuming that a coefficient of $C = 1.15$ will be obtained in the valve constructed in this manner and substituting this coefficient and the area of the valves, $A_c = 162$, for the area of the culverts in Equation (1), the time required will be 467 sec., or an increase of 3 sec. in the time of filling the lock due to using the smaller size of valves. This shows that with this size and type of valve the time of filling and the time required for a vessel to pass through the locks will not be materially increased by using the smaller valves unless the coefficient falls below 1.15 which is not at all likely.

If a coefficient of 1.10 only should be obtained in the valves, the increase required in filling one lock is only 25 sec.; the increase in time of the boat going through the five locks is less than 3 min.

The gate-valves are raised 10 ft. in about 1 min. The machinery is driven by a 10-h.p. electric motor, at 900 rev. per min., through a worm-wheel drive, and a train of gears revolving a drum, to lock the gate in any position of opening or closing. A close-link chain, attached to the gate, is deflected from a vertical direction by an idler sheave near the top of the coping. It runs a short distance horizontally through a hawse pipe and winds around the scored drum. The machinery can be operated by hand from the top of the masonry by a windlass with a sliding bevel-gear pinion that can be engaged very quickly by means of a detachable collar on the shaft. For a more complete description of the culvert valves see Lock Valve Manual No. 3, previously mentioned.[8]

MITER LOCK-GATES

With the exception of the upper gates for the structure at Lockport all lock-gates are of the mitering type. There will be thirteen miter-gates of two leaves each, the total weight of which will be about 3 000 tons. The leaves are 65 ft. long, 7 ft. wide, and vary in height from 20 to 59 ft., and in weight from 90 to 320 tons.

Structurally, each leaf consists of several horizontal girders rigidly connected to a miter post on the free end and a heel post on the hinge end. The leaf is stiffened vertically by five sets of diaphragms between all girders, and has diagonal tension rods with turnbuckles on the up-stream side to provide means of adjusting the miter-posts to a vertical position. The miter and quoin-posts are built of structural steel shapes as shown in Fig. 4, the web-plate being in sections which extend well in between the girder flanges as vertical diaphragms which transfer the thrust of the leaves along the quoin and miter-posts to the web-plates in the horizontals. The sheathing or skin-plates are on the down-stream side. Buckle-plates were utilized where possible and they will be in tension except where a double skin was used to form air chambers. Intercostals are not necessary with buckle-plates; hence, this type of sheathing costs less than flat plates.

No other type of gate was considered because mitering gates have long been the standard form for canals and have given excellent satisfaction in operation and maintenance. Estimates based on actual designs have shown that usually, for the lock as a whole, the most economical rise of mitered gates is approximately one-fourth the lock width and that, for a leaf 65 ft. long, the economical width of the horizontal girder is about 7 ft. The flanges should be straight except for a short curved length near each end of the up-stream flange.

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[8] Manuals of Engineering Practice, Am. Soc. C. E. No. 3, p. 18.

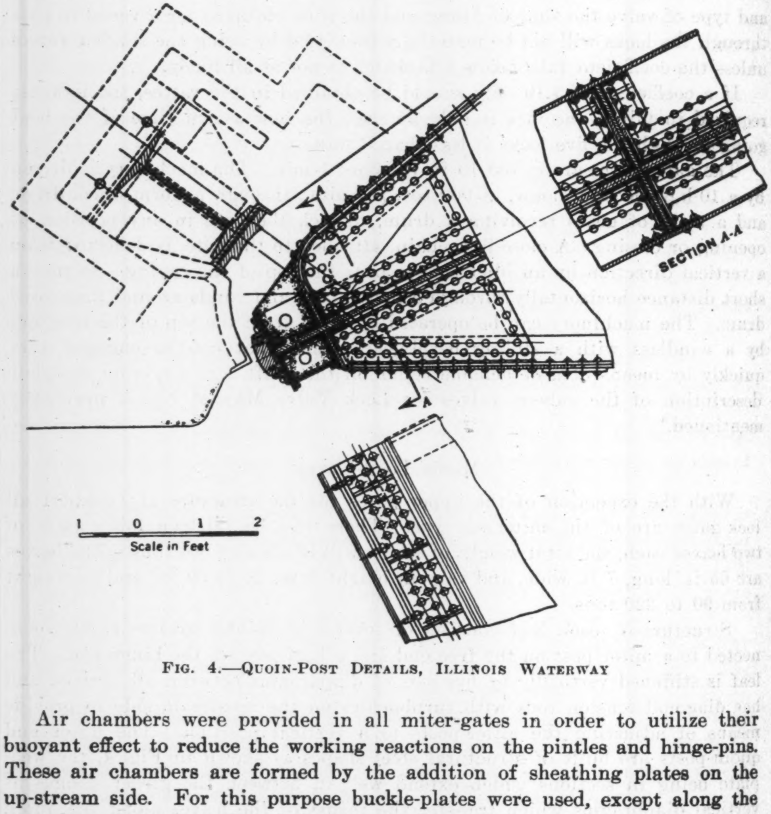


Fig. 4.—Quoin-Post Details, Illinois Waterway

Air chambers were provided in all miter-gates in order to utilize their buoyant effect to reduce the working reactions on the pintles and hinge-pins. These air chambers are formed by the addition of sheathing plates on the up-stream side. For this purpose buckle-plates were used, except along the curved portion, where flat plates supported by intercostals were necessary.

The bearing plates along the quoin and miter-posts are of steel with convex curved surfaces. The bearing plates in the hollow quoin or reaction post are also of steel, but with a concave surface, the radius of which is slightly greater than the convexity radius of the quoin-post. The bearing plates are accurately adjusted with set screws after which melted lead-antimony alloy is poured into the spaces left for that purpose behind the plates.

The determination of the stresses in the gate and gate anchorage is not described at length in this paper because it would involve merely a repetition of what has been written before on this subject.

It was originally thought that air chambers would not be needed in any of the miter-gates except those previously mentioned, but after both the upper and lower gates at Marseilles Lock had been completed according to the original designs, and after making operating tests, it was found desirable to construct air chambers in all the gates to relieve the heavy load on the pintles. Fig. 5 shows the various details.

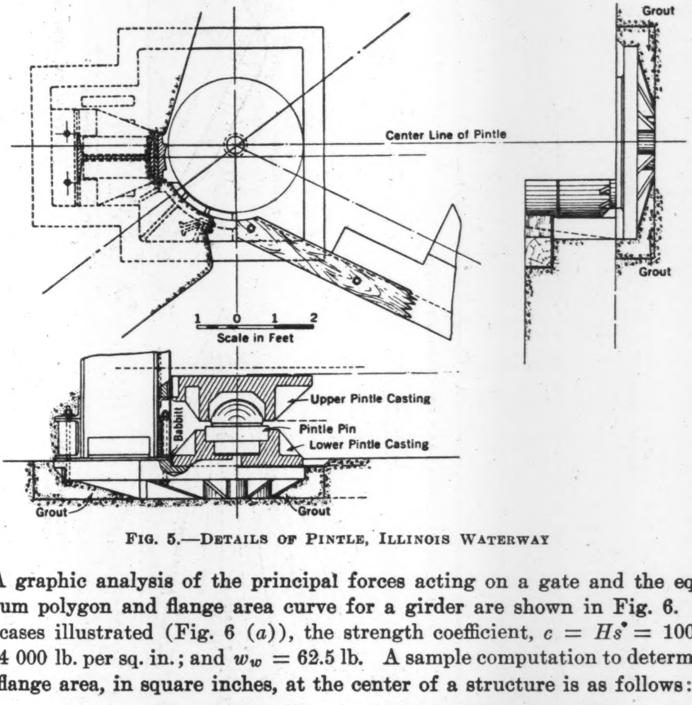


FIG. 5.—DETAILS OF PINTLE, ILLINOIS WATERWAY

A graphic analysis of the principal forces acting on a gate and the equilibrium polygon and flange area curve for a girder are shown in Fig. 6. In the cases illustrated (Fig. 6 (*a*)), the strength coefficient, $c = Hs^* = 100$; $f = 14\,000$ lb. per sq. in.; and $w_w = 62.5$ lb. A sample computation to determine the flange area, in square inches, at the center of a structure is as follows:

$$\text{Flange area} = \frac{Tj}{7f} = \frac{71.8\,w_w H\,s \times j}{7f} = \frac{71.8 \times 100 \times 62.5 \times 7.78}{7 \times 14\,000} = 35.6, \text{ in}$$

which, 71.8 was measured from Fig. 6(*c*); and $w_w H = c = 100$. To determine the value of $j = 7.78$ ft., refer to Fig. 6(*d*). Taking moments about O:

$$j - x = \frac{0.112\,N + 3.55\,P_2 + 32.456\,K - 16.228 \times 0.5\,P_1}{P_1 + P_2} = 7.78$$

and, similarly,

$$j' = \frac{0.112\,N + 32.456\,K - 16.228 \times 0.5\,P_1}{P_1} = 8.226$$

The total stresses, in pounds, in the flanges then are equal to:

$$F_u = \frac{Tj}{7} \text{ and } F_d = \frac{T(j-7)}{7}$$

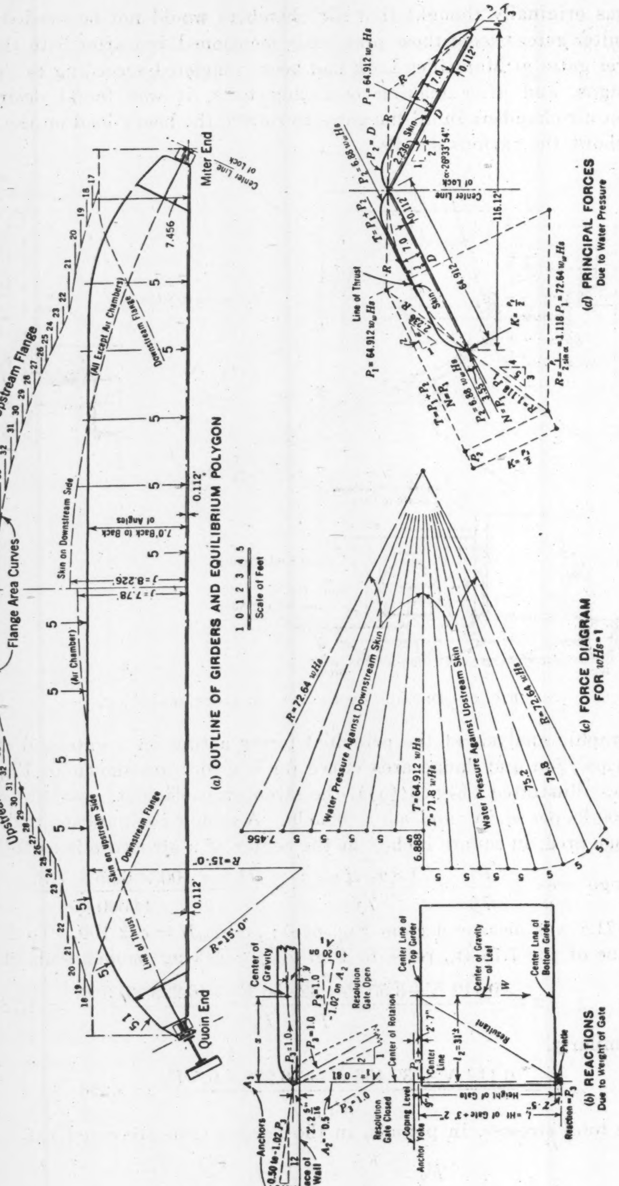
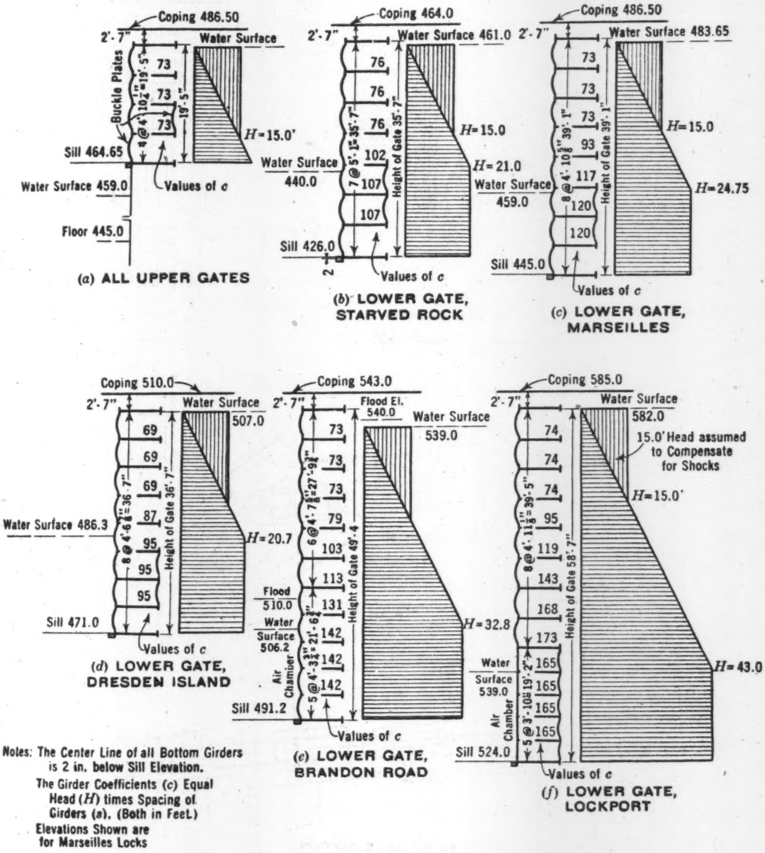

(a) OUTLINE OF GIRDERS AND EQUILIBRIUM POLYGON
Upstream Flange
Flange Area Curves
Skin on Downstream Side
Skin on Upstream Side
Downstream Flange
Line of Thrust
(Air Chamber)
(All Except Air Chambers)
7.0' Back to Back of Angles
j = 8.226'
j = 7.78'
0.112'
7.456
R = 15'-0"
Quoin End
Miter End
Center Line of Lock
Scale of Feet
(b) REACTIONS
Due to Weight of Gate
Anchors
Face of Wall
Anchor Yoke
Coping Level
Center of Rotation
Center of Gravity
Resolution Gate Closed
Resolution Gate Open
Center Line of Top Girder
Center of Gravity of Leaf
Center Line of Bottom Girder
Resultant
Height of Gate
Pintle
Reaction = P_3
W
(c) FORCE DIAGRAM
FOR $wHs = 1$
R = 72.64 wHs
T = 64.912 wHs
T = 71.8 wHs
Water Pressure Against Downstream Skin
Water Pressure Against Upstream Skin
(d) PRINCIPAL FORCES
Due to Water Pressure
$P_1 = 64.912\,w_0 Hs$
$P_2 = 6.88\,w_0 Hs$
$P_1 = 72.64\,w_0 Hs$
$T = P_1 + P_2$
$N = P_1$
Line of Thrust
Skin
116.12'
α = 26°33'54"

FIG. 6.—DESIGN DATA FOR MITER-GATES, ILLINOIS WATERWAY

Fig. 6(*b*) shows that the maximum pull on one anchor is, for gate closed, $-0.84\,P_s$ and, for gate open, $+0.12\,P_s$, and for the other anchor, $1.03\,P_s$ maximum with gate nearly open.

On the diagrammatic cross-sections of the various gates for the waterway are shown the girder spacing and the coefficient, *c*, of each girder. The water pressure diagrams in Fig. 7 show the actual or assumed head acting on each leaf and the coefficients, $c = Hs$, for each girder. The load per linear foot on any girder equals $w_w\,c$. As the required cross-sections of girders are directly proportional to their coefficient (Fig. 6(*c*)), a correct determination of flange areas for a girder the coefficient of which is, say, 100, solves the flange-area requirements of all girders. It should be noted that the total flange stresses

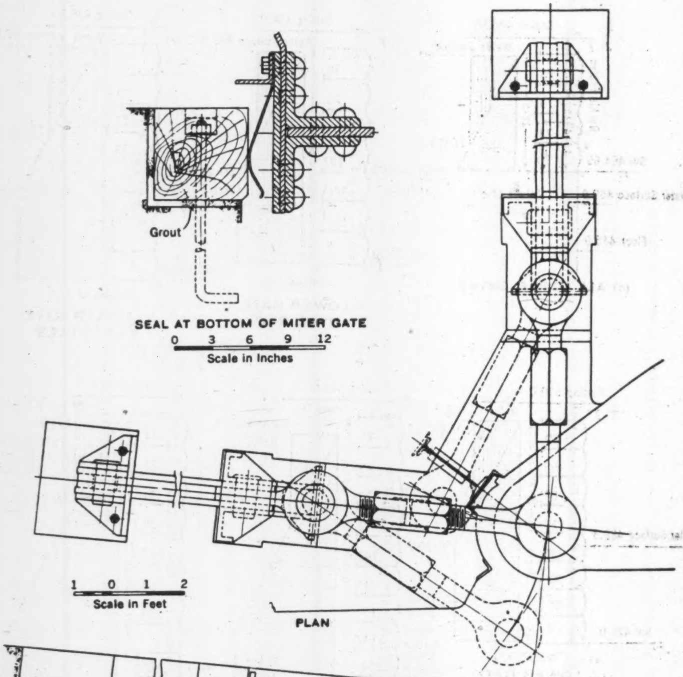

Coping 486.50
2'-7"
Water Surface
Buckle Plates
4 @ 4'-10 7/8" = 19'-5"
19'-5"
73
73
73
H=15.0'
Sill 464.65
Water Surface 459.0
Values of c
Floor 445.0
(a) ALL UPPER GATES
Coping 464.0
2'-7"
Water Surface 461.0
76
76
76
102
107
107
7 @ 5'-1" = 35'-7"
Height of Gate 35'-7"
H=15.0
H=21.0
Water Surface 440.0
Sill 426.0
2
Values of c
(b) LOWER GATE, STARVED ROCK
Coping 486.50
2'-7"
Water Surface 483.65
73
73
73
93
117
120
120
8 @ 4'-10 5/8" = 39'-1"
Height of Gate 39'-1"
H=15.0
H=24.75
Water Surface 459.0
Sill 445.0
Values of c
(c) LOWER GATE, MARSEILLES
Coping 510.0
2'-7"
Water Surface 507.0
69
69
69
87
95
95
95
8 @ 4'-6 7/8" = 36'-7"
Height of Gate 36'-7"
Water Surface 486.3
H=20.7
Sill 471.0
Values of c
(d) LOWER GATE, DRESDEN ISLAND
Coping 543.0
2'-7"
Flood El. 540.0
Water Surface 539.0
73
73
73
79
103
113
131
142
142
142
6 @ 4'-7 3/8" = 27'-9 3/4"
5 @ 4'-3 1/2" = 21'-6 3/4"
Height of Gate 49'-4
Flood 510.0
Water Surface 506.2
Air Chamber
H=32.8
Sill 491.2
Values of c
(e) LOWER GATE, BRANDON ROAD
Coping 585.0
2'-7"
Water Surface 582.0
15.0' Head assumed to Compensate for Shocks
H=15.0'
74
74
74
95
119
143
168
173
165
165
165
165
8 @ 4'-11 3/8" = 39'-5"
5 @ 3'-10 3/8" = 19'-2"
Height of Gate 58'-7"
Water Surface 539.0
Air Chamber
H=43.0
Sill 524.0
Values of c
(f) LOWER GATE, LOCKPORT

Notes: The Center Line of all Bottom Girders is 2 in. below Sill Elevation.
The Girder Coefficients (*c*) Equal Head (*H*) times Spacing of Girders (*s*). (Both in Feet.)
Elevations Shown are for Marseilles Locks

FIG. 7.—WATER PRESSURE DIAGRAMS FOR MITER-GATES, ILLINOIS WATERWAY

are less when the skin is on the down-stream side than when it is on the up-stream side. The difference is due to the elimination of the force, P_2, shown for the left-hand leaf in the analysis of "Principal Forces (Fig. (*6d*))."

Figs. 6 and 7 were devised by L. D. Cornish, M. Am. Soc. C. E., Chief Engineer of the Division of Waterways, State of Illinois, and were of great value in the designing of all miter-gates.



SEAL AT BOTTOM OF MITER GATE

PLAN

SECTIONAL ELEVATION

FIG. 8.—ANCHORAGE OF MITER-GATE, ILLINOIS WATERWAY

Contact at Miter-Sill.—In the design of mitering gates it has been the practice heretofore to permit close contact and heavy bearing between the lower gate girders and the miter-sill. The result does not permit an accurate and scientific design of either the lock-gate or the miter-wall. Both must be designed unnecessarily strong to withstand the unknown and indeterminate pressures, which are assumed to be heavy. This indeterminate condition has been remedied by placing a metal sealing-spring plate on the gate, which alone will be in contact with the miter-sill when the gate is closed, and which cannot transmit to the sill more than a part of the pressure due to the water in contact with the spring itself. This permits a more accurate and lighter design of both gate and miter-wall and results in decidedly lower costs (see Fig. 8).

The gate anchorage designs for these locks differ in many respects from any other designs of which the writer has knowledge. (See Fig. 8.) Large gate anchorages usually are extended deep into the massive concrete against which the gate abuts and are made of eye-bars or built-up structural members. Such anchorages are difficult to set and hold accurately while the concrete is poured. To secure the same results, that is, to carry the pull of the gate when hanging free, well down into the concrete mass, the gate-reaction monoliths of these locks are reinforced by a network of reinforcing bars and only straight eye-bar anchors are used, the anchor ends of which are about 4 ft. below the coping level of the concrete. Troughs are left in the concrete so that the bars may be accurately set at leisure and subsequently embedded in concrete. A rather fine adjustment of the rotation axis of a gate is necessary and this is provided for by adjustable double eye-bars which connect the hinge pin of the gate and the fixed anchor. The adjustment is accomplished by means of sleeve nuts with threads of slightly different pitch and size at each end, but in the same direction. This design is the same as that used in the new locks at Sault Ste. Marie, Mich. The advantage of this design over the ordinary turnbuckle is a much finer adjustment per turn and, therefore, less force is required, which is an important item as adjustments must be made when the anchor is under maximum stress.

The quoin-post which is embedded in the concrete to distribute the thrust of the gate is made up of I-beams with pressure-distributing angles riveted on the back and a channel section on the front to which the hollow quoin plates are fixed. A cross-section of the quoin-post is shown in Fig. 5. The posts are in one piece and the bottom end is bolted to a projection of the pintle bed-plate casting so as to facilitate holding the quoin-post in an exact position during concreting operations.

MITER-GATE OPERATING MACHINERY

The so-called bull-wheel type of operating machinery devised for the Panama Canal is used to open and close the miter-gates (see Fig. 9). It has been described in detail in a paper by Edward Schildhauer, M. Am. Soc. C. E.[*]

This machine, under constant motor speed, imparts an ideal motion to the gate, that is, slow speed at the ends of travel and high speed at the mid-point. The force required mainly depends upon the vertical area which is submerged during operation as the movement causes a difference in water level on the two sides of the gate, which is very marked near the recess and mitered position and during acceleration. As the depth of submergence of the Illinois Waterway gates in no case exceeds 16 ft. when operated, or less than one-half the submergence of the Panama gates, the forces are much less and the machinery is lightened accordingly. Several other changes have been made in the design, among which may be mentioned the omission of the spring-center spur gear to reduce the starting torque of the motor. Experience has shown that the value of the spring center is not commensurate with its cost.

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[*] *Transactions*, Inter. Eng. Cong., 1915, Vol. II, Paper No. 20, p. 256.

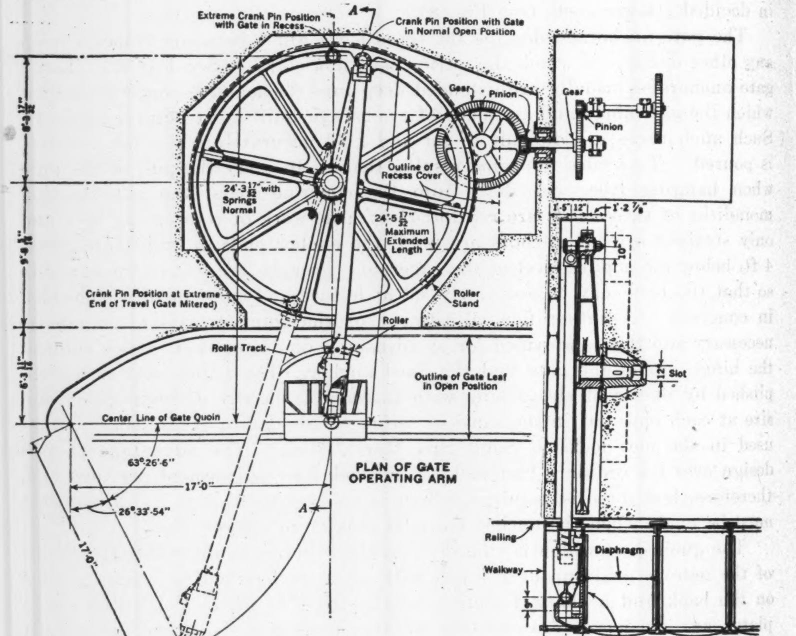
Extreme Crank Pin Position with Gate in Recess
A
Crank Pin Position with Gate in Normal Open Position
Gear
Pinion
Gear
Pinion
6'-3 17/32"
9'-8 1/16"
6'-3 11/16"
24'-3 17/32" with Springs Normal
Outline of Recess Cover
24'-5 17/32" Maximum Extended Length
Crank Pin Position at Extreme End of Travel (Gate Mitered)
Roller Stand
Roller
Roller Track
Outline of Gate Leaf in Open Position
Center Line of Gate Quoin
63°26'-6"
17'-0"
26°33'-54"
17'-0"
A
PLAN OF GATE OPERATING ARM
1'-5"
12"
1'-2 7/8"
10"
12
Slot
Railing
Walkway
Diaphragm
9"
1'-5"
1'-2"
2'-7"
Top Girder of Leaf
Up Stream Skin Plate
CROSS SECTION A-A

FIG. 9.—PLAN OF STRUT ARM AND BULL WHEEL FOR OPERATING MITER-GATES, ILLINOIS WATERWAY

LIFT-GATES AND OPERATING MACHINERY

At Lockport there exists a small modern lock built by the Sanitary District of Chicago to connect the Drainage Canal with the old Illinois and Michigan Canal. The necessity of retaining this lock so restricted the remaining available lock site in width that, adjoining the small lock, there was not sufficient area for the mass of concrete required for the thrust of a mitering gate. Hence, for these upper gates, a vertical-lift, horizontal, truss-type of gate was adopted (see Fig. 10).

Two gates similar in construction and operation have been installed. One acts as an emergency gate and will be used only if it becomes necessary to remove the other for repairs. The structure consists of a movable gate, counterweighted, with a tower at each end, which is connected at the top by a service bridge placed at a clear elevation of 50 ft. above the high-water level. These gates have been described previously in a paper by the writer.[10]

The lock chamber is 110 ft. wide, and is provided at the upper end with recesses in the side walls about 5 ft. wide and 14 ft. long, that extend the full depth of the lock, and the gate operates within these recesses. The vertical travel is 23 ft. 8½ in. and is accomplished in 2 min. by a 50-h.p. electric motor, placed at the center of the service bridge, and acting through the necessary reductions on internal gears integral with the sprocket drums. Two link chains at each end of the gate pass over these sprocket drums at the top of the towers and connect to a steel-skeleton counterweight box, filled with sufficient concrete to balance the weight of the gate when immersed.

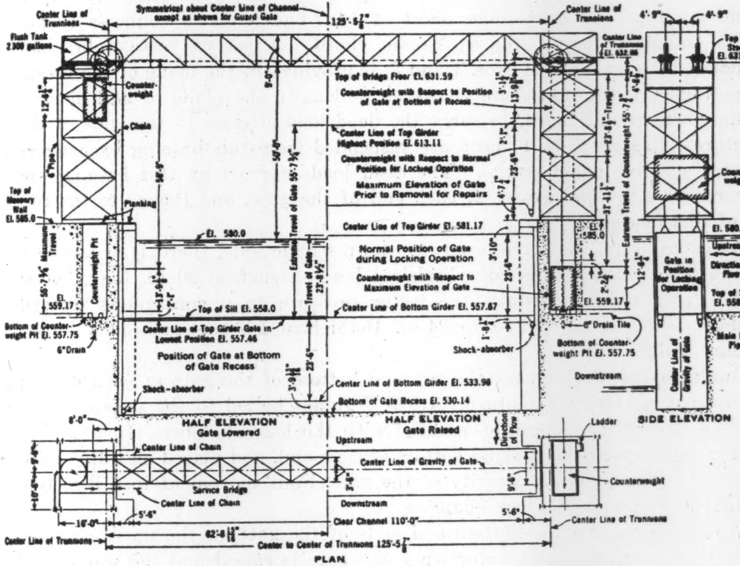
Center Line of Trunnions
Symmetrical about Center Line of Channel except as shown for Guard Gate
125'-5 7/8"
Flush Tank 2 300 gallons
Counterweight
Chain
6" Pipe
17'-4 1/4"
Top of Masonry Wall El. 585.0
Planking
9'-0"
58'-5"
50'-0"
Extreme Travel of Gate 55'-7 3/4"
El. 580.0
Maximum Travel
55'-7 3/4"
Counterweight Pit
El. 559.17
13'-0 1/2"
2'-1"
Top of Sill El. 558.0
Travel of Gate 23'-8 1/2"
Bottom of Counterweight Pit El. 557.75
6" Drain
Center Line of Top Girder Gate in Lowest Position El. 557.46
Position of Gate at Bottom of Gate Recess
23'-6"
3'-9 13/16"
Shock-absorber
HALF ELEVATION
Gate Lowered
Top of Bridge Floor El. 631.59
Counterweight with Respect to Position of Gate at Bottom of Recess
Center Line of Top Girder Highest Position El. 613.11
Counterweight with Respect to Normal Position for Locking Operation
Maximum Elevation of Gate Prior to Removal for Repairs
Center Line of Top Girder El. 581.17
Normal Position of Gate during Locking Operation
Counterweight with Respect to Maximum Elevation of Gate
Center Line of Bottom Girder El. 557.67
Center Line of Bottom Girder El. 533.98
Bottom of Gate Recess El. 530.14
HALF ELEVATION
Gate Raised
3'-1 1/4"
13'-9"
2'-1"
23'-6"
4'-7 1/4"
3'-10"
23'-6"
1'-8 1/4"
Center Line of Trunnions El. 632.98
23'-8 1/2" Travel
37'-11 1/4"
Extreme Travel of Counterweight 55'-7 3/4"
4'-8 1/4"
El. 585.0
2'-2 1/2"
12'-4 1/4"
El. 559.17
6" Drain Tile
Bottom of Counterweight Pit El. 557.75
Downstream
4'-9"
4'-9"
Top of Steel El. 631.24
Counterweight
El. 580.0
Upstream
Direction of Flow
Gate in Position for Locking Operation
Top of Sill El. 558.0
Main Flush Pipe
Center Line of Gravity of Gate
SIDE ELEVATION
8'-0"
Center Line of Chain
Service Bridge
Center Line of Chain
5'-6"
16'-0"
10'-6"
Upstream
Center Line of Gravity of Gate
3'-8"
9'-6"
Downstream
Direction of Flow
Ladder
Counterweight
5'-6"
Clear Channel 110'-0"
Center Line of Trunnions
62'-8 15/16"
Center Line of Trunnions
Center to Center of Trunnions 125'-5 7/8"
PLAN

FIG. 10.—LIFT-GATE OPERATING DIAGRAM, LOCKPORT LOCK

The lift-gate consists of five horizontal trusses, hereinafter designated as Trusses Nos. 1, 2, 3, 4, and 5, and placed one below the other, Truss No. 1 being uppermost. Trusses Nos. 1, 2, and 3 are spaced 6 ft. 8 in., center to center; Trusses Nos. 3, 4, and 5 are spaced 5 ft. 1 in., center to center. Each truss is of the ordinary Pratt type, with a single web system. The web members and posts are composed of angles, and the down-stream or tension chord of 15-in. channels is reinforced with 12-in. side-plates where necessary. The up-stream or compression chord is built up of four angles spaced 20 in. back to back, two 19 by $\frac{11}{16}$-in. web-plates, a 13-in. cover-plate outside the skin-plates, and reinforcing side plates where required. In addition to the direct compression, this chord was designed to carry the bending moments resulting from the uniformly applied water-pressure load transferred to it from the skin-plates.

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[10] *Engineering News-Record*, Vol. I, 1926, p. 722.

Each truss is 11 ft. 9 in., center to center of chords, and consists of ten panels of 11 ft. 9 in. each, making a distance of 117 ft. 6 in., center to center of end bearings. A continuous sheet of $\frac{1}{4}$-in. buckle-plates connects to the outside flange angles of the up-stream chord. A set of vertical bracing, extending between Trusses Nos. 1 and 3, is provided in the plane of the inside flange angles of the up-stream chord, as well as in the plane of both flanges of the down-stream chord, to carry the dead load stresses to the ends of the structure. Trusses Nos. 4 and 5 are suspended from this bracing by hangers at the respective panel points. The dead loads carried by this bracing are delivered to a vertical girder at each end of the gate, and thence to the two sets of link chains.

At each end of each truss, on the down-stream side, there is a cast-steel bearing block with a finished cylindrical bearing surface 20 in. long, bolted to the gate, which transmits the water pressure to a continuous vertical finished steel casting, bolted to 24-in. Bethlehem I-beams embedded in the masonry walls.

Guide rollers, bolted near the top and bottom of the gate at each end on the up-stream side, act against vertical T-guides bolted to the masonry, and limit movement in a direction parallel with the lock chamber. In a similar manner rollers near the top and bottom at each end, and placed on a line coincident with the center of gravity of the gate, limit movement in a direction at right angles with the lock chamber.

Water seals along the bottom and ends of the gate on the up-stream side prevent serious leakage of water when the gate is raised and the water level in the lock chamber is lowered. The bottom seal consists of a continuous line of $4\frac{1}{2}$-in. standard wrought-iron pipe, connected together by screw plugs and resting on a horizontal curb angle just below the top of the breast wall. This pipe terminates at each end in the side of a casting, resting on, but not bolted to the masonry, and the top of this casting serves as a support for another $4\frac{1}{2}$-in. pipe forming the vertical seal. This vertical pipe also terminates in a casting bolted to the masonry at the upper end. Screw plugs are provided at each end of the vertical pipe and enter recesses in the castings. These recesses allow the necessary movement for the pipe to come in contact with the gate. The horizontal pipe forms a contact between the curb angle on the masonry and the cover-plate on Truss No. 5. Rivets coming in contact with all pipes are countersunk. A series of T-headed bolts, spaced 3 ft. 6 in., center to center, throughout the length of the horizontal pipe and anchored to the masonry (the heads being inserted through slotted holes in the pipe and then turned 90°) prevent this pipe from falling out of position when the gate is raised for repairs. At the same time this arrangement allows freedom of movement for the pipe to assume a curve corresponding to the deflection of the gate under maximum pressure. The vertical seal acts in a similar manner to the horizontal, forming a contact between a vertical curb angle embedded in the masonry and vertical plates riveted to the ends of the gate.

Owing to the great quantity of sludge in the drainage-canal water a flushing system for the bottom seal and recess is necessary. A flushing pipe is placed in the breast-wall masonry just back of the recess with smaller pipes leading from it into the recess at frequent intervals. This pipe is connected with a tank placed on one of the towers of the operating bridge so that water can be discharged into the recess under a 60-ft. head.

The operating machinery of the lift-gate is at the center of a tower bridge, 50 ft. above the upper pool water level. Sprocket drums are mounted on the top of the towers and by specially constructed link chains suspend the lift-gates and the counterweights at each corner, equilibrium being established when the gate is in its middle position (see Fig. 10). A 50-h.p. electric motor, at 600 rev. per min., was agreed upon as the basis of calculations for the moving machinery, the time of operation to be 2 min. for a lift of 25 ft. Internal gears cast *en bloc* with the sprocket drums are meshed with operating pinions mounted on a shaft between them and are revolved by a large gear at the center through a gear train ending in a set of miter gears. A line shaft running the entire length of the tower bridge rigidly connects the two systems of gears and drums at each end, a small gear being coupled at the center. A short counter-shaft is driven by the motor pinion at one end, while at the other a sprocket-wheel and roller chain facilitate hand operation.

Lockport Emergency Dam

A lock-gate may be damaged or wrecked by boats going in either direction. The most common accident occurs from a misunderstanding of signals on an upbound boat approaching the lock when it is filled. In case of accidents to the lock-gates at Starved Rock, Marseilles, Dresden Island, or Brandon Road, the water can be drawn down almost to the level of the upper miter-sill by opening the movable dams.

At the summit level at Lockport it is not possible to draw the pool down in this manner. An emergency gate has been designed therefore to be installed up-stream from the upper lock-gate. In case any of the gates at this lock are carried away the emergency dam can be raised and the flow of water reduced.

An outline of this emergency dam is shown in Fig. 11. It consists of an immense steel shutter swiveling on a horizontal axis operated by valves in manholes in the side walls. Ordinarily, this shutter rests in a horizontal position in a recess in the floor. Beneath the up-stream half of this shutter there is a recess in which this part of the gate swings. This recess can be connected with either the upper or lower pool by manipulating the proper valves in the side walls. When the recess is connected with the upper pool the pressure on both sides of this half of the shutter is equalized and, as the other leg of the shutter is longer, it will maintain a horizontal position. When it is desired to raise the shutter to an upright position the valves connecting the up-stream recess to the upper pool are closed and those connecting it to the lower pool are opened. The pressure from the upper pool on the up-stream leg of the shutter, being then unbalanced, will force this part of the shutter down.

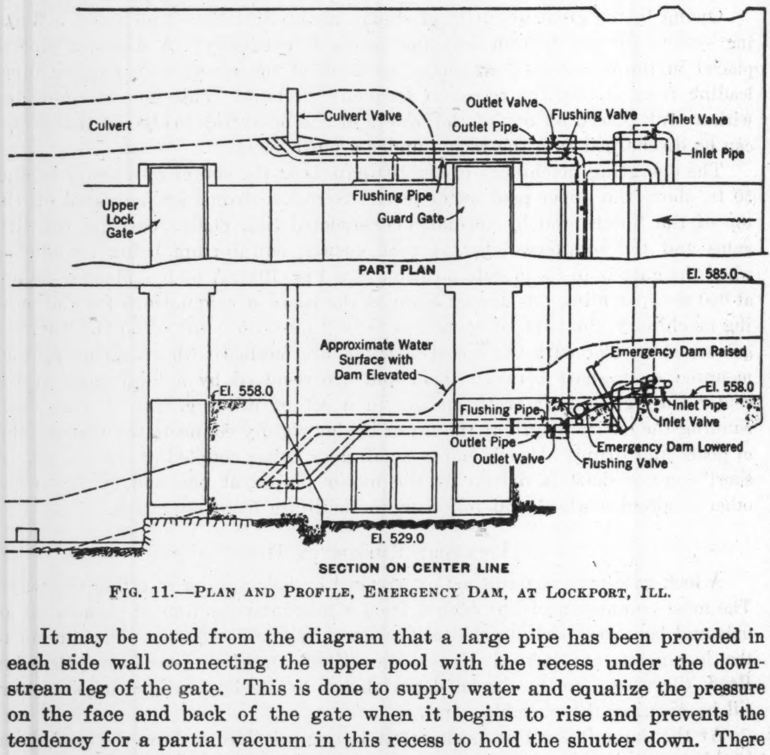

Culvert
Culvert Valve
Outlet Valve
Flushing Valve
Inlet Valve
Outlet Pipe
Inlet Pipe
Upper Lock Gate
Flushing Pipe
Guard Gate
PART PLAN
El. 585.0
Approximate Water Surface with Dam Elevated
Emergency Dam Raised
El. 558.0
El. 558.0
Flushing Pipe
Inlet Pipe
Inlet Valve
Outlet Pipe
Outlet Valve
Emergency Dam Lowered
Flushing Valve
El. 529.0
SECTION ON CENTER LINE

FIG. 11.—PLAN AND PROFILE, EMERGENCY DAM, AT LOCKPORT, ILL.

It may be noted from the diagram that a large pipe has been provided in each side wall connecting the upper pool with the recess under the down-stream leg of the gate. This is done to supply water and equalize the pressure on the face and back of the gate when it begins to rise and prevents the tendency for a partial vacuum in this recess to hold the shutter down. There is a pipe in the concrete beneath the recess, below the down-stream half of the shutter. At intervals there are smaller pipes, connecting the recess with this pipe, which blow out sludge and other deposits that may form in the recess from time to time. This can be done by connecting the pipe with the lower pool. It is very necessary on account of the large quantity of sludge carried by the Sanitary Canal.

When the dam has been lifted, the flow is cut down sufficiently for the water surface to take the elevations shown in Fig. 11. As all the water will then be going through the culverts the upper lock-gate, or the emergency gate, whichever is uninjured, can be raised. After this has been done and before the valves are closed, the emergency dam can be lowered by closing the valve connecting the space below the up-stream leg of the shutter to the lower pool and opening the one connecting it to the upper pool. The dam will then revolve into the recess in the floor.

SIDE-WALL ANALYSIS

As the culverts in the locks are circular, the stress in the wall at the culvert cannot be found by the method used when they are rectangular. It is believed in this case that the most accurate method of determining the stresses in the masonry around the culvert is that of the "Principle of Least Work."

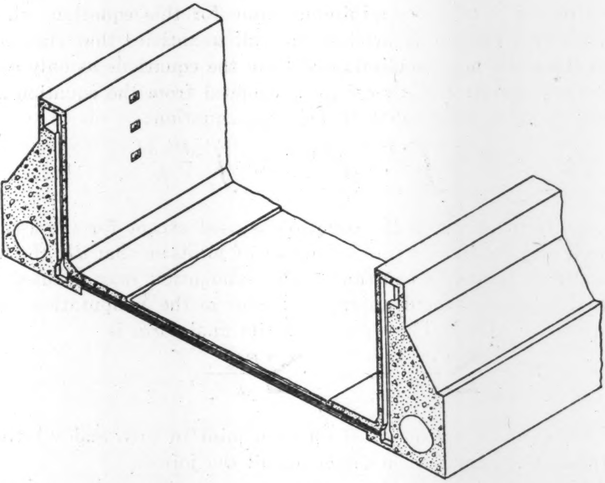


FIG. 12.—ISOMETRIC VIEW OF SECTION OF LOCK, ILLINOIS WATERWAY

In this analysis the assumption is made that the arch will do the least amount of work necessary to support the load upon it. Hence, if an expression can be found for the work done by each pressure line, the pressure line giving the least amount of work is the true one, and the stresses in the joints can then be computed.

The formula for the work of resistance in an arch (see Fig. 12) is:

$$W = \int_0^l \frac{M^2\,dL}{2\,EI} + \int_0^l \frac{P^2_n\,dL}{2\,EA_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

in which,

W = total work of resistance in the structure.
M = bending moment at any assumed joint.
P_n = total pressure normal to section at any assumed joint.
L = distance from support measured along axis.
dL = an infinitesimally small distance along the axis of the arch.
l = length of arch measured along axis.
E = modulus of elasticity of material of arch.
A_n = area of section at any assumed joint where P_n acts.
I = moment of inertia of section of any assumed joint where P_n acts.
e_n = distance of point of application of P_n from center of section at any assumed joint.

Substituting h for the thickness of the arch at any joint and making the length of the arch considered equal to $b = 1$ ft., $A_n = hb = h$. The equation then becomes:

$$W = \int_0^1 \frac{M^2 dL}{2E\frac{h^3}{12}} + \int_0^1 \frac{P^2_n dL}{2Eh} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

The next object is to get a minimum value for this equation, which can only be done by trial. It is evident that all quantities that stay constant throughout the trials may be eliminated from the equations as only comparative values are wanted; 2 E, therefore, is dropped from the equation and for M its value, $P_n e_n$, is substituted, making the equation:

$$W = \int_0^1 \frac{P^2_n e^2_n dL}{\frac{h^3}{12}} + \int_0^1 \frac{P^2_n dL}{h} \quad \ldots\ldots\ldots\ldots\ldots(4)$$

This is the form in which the equation is used except for the assumption that the arch may be divided into a number of joints so that dL, the distance between assumed joints, is constant. The summation may be used instead of the actual integral without appreciable error in the computations to determine comparative value. The equation in its final form is:

$$W = \sum \frac{P^2_n e^2_n}{\frac{h^3}{12}} + \sum \frac{P^2_n}{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

in which, h, P_n, and e are obtained for each joint in turn and substituted in the equation. The sum is then taken for all the joints.

After the arch has been divided and the forces acting on the various blocks have been found, a force polygon can be drawn and the resultants obtained from it in direction and magnitude. The point of application of each resultant can then be found by taking the sum of the moments of all the forces acting on one-half the arch about any point and dividing this moment by the resultant. This will give the lever arm of the resultant about the center of moments, and the resultant can then be drawn in position on the arch.

There are three unknown quantities to be obtained, the horizontal and vertical components and the point of application of the crown thrust, and all are determined very easily by this process. Any point on the springing line and crown joint may be taken and the horizontal crown thrust is obtained that will pass a pressure line through the point on the crown joint and the springing line. This is accomplished by taking moments of all the forces acting on that half of the arch about the point on the springing line and dividing the sum by the distance to the line of action of the crown thrust. This thrust can then be applied at various points on the crown joint, the pressure line can be drawn, and the work computed for each.

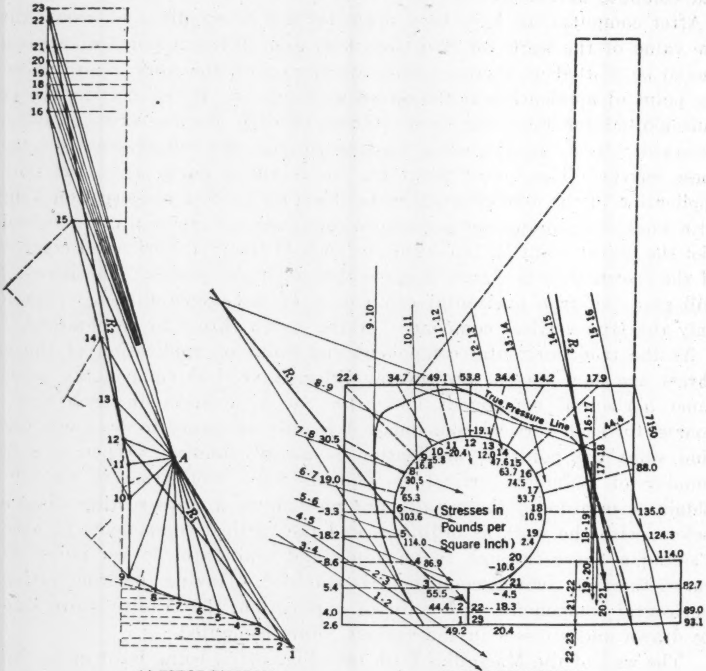
It is generally desirable to apply the thrust at about five points on the crown joint at equal distances apart. Then it is only necessary to draw the two outer pressure lines because the distance between these lines can be measured at each joint and divided by four in getting the values of e_n for each line. After the work has been obtained for the set of pressure lines for this crown thrust, assume a thrust a little larger or smaller, pass pressure lines through the same points on the crown joint and through a minimum, and compute as before.

After computations have been made for five or six different crown thrusts the value of the work for each thrust at each different point of application should be plotted on a cross-section diagram with the work as ordinates and the point of application at the crown as abscissas. By connecting the points thus plotted for the same crown thrust in each case, a series of curves is obtained. Next, draw a curve passing through the lowest point on each of these curves. The lowest point on the resulting curve gives the point of application of the true crown thrust. Then on another cross-section diagram, with work as ordinates and horizontal component of crown thrust as abscissas, plot the lowest point on the curve for each of the trial horizontal components of the crown thrust. Draw a curve through these points. The lowest point will give the true horizontal component of the crown thrust. This leaves only the true vertical component of the crown thrust to be obtained.

As the true horizontal component and point of application of the crown thrust are now known, try several different vertical components using the same horizontal component and point of application in each case. Of course, for each vertical component it is only necessary to draw one pressure line since the point of application is already known. When a sufficient number of different vertical components have been tried in an effort to obtain a minimum, the results are plotted on a cross-section diagram as before, with the work as ordinates and the vertical components as abscissas. Passing a curve through these points and taking the lowest point on that curve, the true vertical component is obtained. Having the true vertical and horizontal components and point of application, the true pressure line may be drawn and stresses in the various joints computed.

The wall of the Marseilles Lock (see Fig. 13) is being built up to the level of the shoulder after which a wait occurs until the concrete sets sufficiently to erect the forms for the back of the wall upon it. This results in a well-defined joint on this line. The wall above this line is designed so that for the most adverse conditions possible there will be no tension at the heel. The stress diagram for this joint is then drawn as shown in Fig. 13(*a*) and the upper wall is assumed to be replaced by this pressure on the top of the arch. The arch is then divided into sections by imaginary joints and by the forces found acting on each of these blocks. A force diagram is then drawn, and the resultants acting on the two halves are obtained in direction, magnitude, and point of application, as previously explained. With the moments of all the forces on each half of the arch determined, it is very easy to obtain the horizontal component of crown thrust to pass a pressure line through any point on the springing line. It will be noticed from Fig. 13 that the trial crown thrusts used are not horizontal, but very much inclined. It is not necessary to use a horizontal crown thrust, but in the first set of computations to find the true horizontal component and point of application, the vertical component must be the same for each trial.

A trial computation for a horizontal component of 12 000 lb. is shown by Fig. 13 and the computations are given in Table 1. In drawing the pressure lines on the arch it is well first to prolong the crown thrusts at the extremes out to the resultants and then draw lines from their intersections on the resultants, parallel with the line in the force diagram representing the resultant of the crown thrust and all the forces on one-half the arch. These lines are $A1$ and $A23$ in the force diagram, Fig. 13(a). This will give points on the springing line to which the pressure lines drawn from joint to joint must come if correct. Only the two outside pressure lines are drawn although there are five used in the computation. The intermediate values of e_n can be obtained very easily, as they are all an equal distance apart. The values of P_n are the same on each joint for all the lines. It will be noticed in Table 1 that they are divided by 1 000, so that the numbers in the computations will not be too large.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3
2
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A
R2
R1
9·10
10·11
11·12
12·13
13·14
14·15
15·16
16·17
17·18
18·19
19·20
20·21
21·22
22·23
8·9
7·8
6·7
5·6
4·5
3·4
2·3
1·2
22.4
34.7
49.1
53.8
34.4
14.2
17.9
True Pressure Line
7150
44.4
-19.1
-20.4
12.0
-5.8
16.8
47.6
63.7
74.5
30.5
65.3
53.7
10.9
103.6
114.0
2.4
86.9
-10.6
55.5
-4.5
44.4
18.3
49.2
20.4
19.0
-3.3
-18.2
-8.6
5.4
4.0
2.6
88.0
135.0
124.3
114.0
82.7
89.0
93.1
(Stresses in Pounds per Square Inch)

(a) FORCE DIAGRAM (b) SECTION OF WALL

FIG. 13.—ANALYSIS OF STRESSES IN CULVERT WALL, MARSEILLES LOCK

After completing this computation another value of crown thrust is taken and a similar table is computed for it. Four different values of crown thrust were used in this case, the horizontal components being 13 000, 12 000, 11 000, and 9 000 lb. The values of work for each of these crown thrusts for each

TABLE 1.—COMPUTATION OF INTERNAL WORK FOR MARSEILLES LOCK FOR A HORIZONTAL COMPONENT OF 12 000 POUNDS

Joint	h, in feet	$\frac{h^3}{12}$	$\frac{P_n}{1000}$ (kips)	$\left(\frac{P_n}{1000}\right)^2$	$\frac{\text{Column (5)}}{\text{Column (3)}}$	$\frac{\text{Column (5)}}{\text{Column (2)}}$ $\frac{\left(\frac{P_n}{1000}\right)^2}{h}$	e_{-2} (feet)	e^2_{-2} (square feet)	Column (6) × Column (9)	e_{-1} (feet)	e^2_{-1} (square feet)	Column (6) × Column (12)	e_0 (feet)	e^2_0 (square feet)	Column (6) × Column (15)	e_1 (feet)	e^2_1 (square feet)	Column (6) × Column (18)	e_2 (feet)	e^2_2 (square feet)	Column (6) × Column (21)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
1......	10.0	83.3	38.1	1452	17.5	145.2	1.70	2.89	50.5	1.38	1.90	33.2	1.07	1.14	11.9	0.75	0.56	9.8	0.43	0.19	3.2
2......	10.0	83.3	36.6	1340	16.1	134.0	1.22	1.49	24.0	0.89	0.70	11.3	0.55	0.30	4.9	0.22	0.05	0.8	0.12	0.10	0.2
3......	7.5	35.2	33.6	1129	32.1	150.6	1.52	2.31	74.3	1.15	1.32	42.4	0.78	0.61	19.5	0.41	0.17	5.4	0.04	0.00	0.0
4......	5.6	14.6	32.3	1043	71.3	186.5	1.67	2.79	199.0	1.30	1.69	120.4	0.93	0.87	61.7	0.56	0.31	22.4	0.19	0.04	2.6
5......	4.4	6.9	30.7	942	137.5	216.5	1.77	3.13	430.0	1.37	1.88	258.0	0.97	0.94	129.4	0.58	0.34	46.2	0.18	0.03	4.4
6......	4.0	5.3	29.6	876	164.5	219.0	1.63	2.66	438.0	1.23	1.51	249.0	0.83	0.69	113.3	0.43	0.19	30.5	0.03	0.00	0.2
7......	4.4	6.9	27.7	767	112.0	176.3	1.50	2.25	270.0	1.07	1.14	137.0	0.64	0.41	49.2	0.21	0.04	5.3	0.22	0.05	5.8
8......	5.5	13.9	25.8	666	48.1	121.0	1.41	1.99	95.7	0.94	0.88	42.3	0.47	0.22	10.6	0.00	0.00	0.0	0.47	0.22	10.6
9......	8.1	44.3	24.6	605	13.6	74.8	1.49	2.22	31.0	0.99	0.98	13.3	0.49	0.24	3.3	0.00	0.00	0.0	0.50	0.25	3.4
10......	5.6	14.2	13.6	185	13.0	33.7	1.84	3.39	44.1	0.96	0.92	12.0	0.08	0.01	0.1	0.80	0.64	8.3	1.68	2.82	36.7
11......	4.4	6.9	11.1	123	18.0	28.3	2.10	4.41	79.5	1.02	1.04	18.8	0.06	0.00	0.1	1.14	1.30	23.4	2.22	4.93	88.8
12......	4.0	5.3	12.0	144	27.1	36.0	2.00	4.00	108.4	1.00	1.00	27.1	0.00	0.00	0.0	1.00	1.00	27.1	2.00	4.00	108.4
13......	4.4	6.9	16.2	262	38.3	60.3	1.92	3.69	141.5	1.18	1.39	53.3	0.44	0.19	7.4	0.30	0.09	3.5	1.04	1.08	41.4
14......	5.6	14.2	26.1	681	47.9	122.6	1.75	3.06	146.5	1.30	1.69	81.0	0.85	0.72	34.6	0.40	0.16	7.7	0.85	0.72	34.6
15......	8.2	45.9	48.7	2372	51.8	289.0	1.41	1.99	103.0	1.17	1.37	71.0	0.93	0.87	44.8	0.69	0.48	24.6	0.45	0.20	10.6
16......	9.0	60.7	72.6	5271	86.8	585.0	0.68	0.46	40.1	1.52	0.27	23.4	0.36	0.13	11.3	0.20	0.04	3.5	0.04	0.00	0.2
17......	7.5	34.4	75.8	5746	167.0	772.0	0.05	0.00	0.5	1.23	0.05	8.9	0.41	0.17	28.1	0.59	0.39	58.1	0.77	0.59	97.4
18......	7.1	28.6	73.3	5373	187.6	762.5	0.90	0.81	152.0	1.07	1.14	214.0	1.24	1.54	289.0	1.41	1.99	374.0	1.58	2.50	469.0
19......	8.4	33.8	75.5	5700	168.6	683.0	1.28	1.64	276.5	1.43	2.04	344.0	1.58	2.50	421.0	1.73	2.99	504.0	1.88	3.53	595.0
20......	10.6	72.6	77.9	6068	83.6	575.0	2.06	4.24	354.0	1.22	4.93	412.0	2.38	5.66	473.0	2.54	6.45	539.0	2.70	7.23	609.0
21......	14.5	254.0	80.6	6496	25.5	448.0	2.66	7.07	180.0	2.81	7.90	202.0	2.96	8.76	223.5	3.12	9.73	248.2	3.28	10.76	274.0
22......	17.0	410.0	85.7	7344	17.9	432.0	4.28	18.32	328.0	4.43	19.62	352.0	4.58	20.98	376.0	4.73	22.37	400.0	4.88	23.81	427.0
23......	17.0	410.0	88.3	7797	19.0	459.0	4.49	20.16	384.0	4.63	21.44	480.0	4.77	22.75	433.0	4.91	24.11	459.0	5.05	25.5	486.0
Summation..	.....	.....	.....	.....	.....	6710.8	.....	.....	3 950.6	.....	.....	3 134.4	.....	.....	2 745.7	.....	.....	2 800.8	.....	.....	3 308.5
									6 710.8	.....	.....	6 710.8	.....	.....	6 710.8	.....	.....	6 710.8	.....	.....	6 710.8
	.....	.....	.....	.....	.....	.....	.....	.....	10 661.4	.....	.....	9 845.2	.....	.....	9 456.5	.....	.....	9 511.6	.....	.....	10 019.3

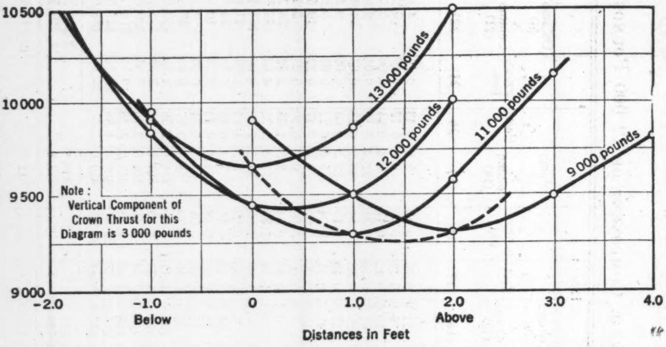
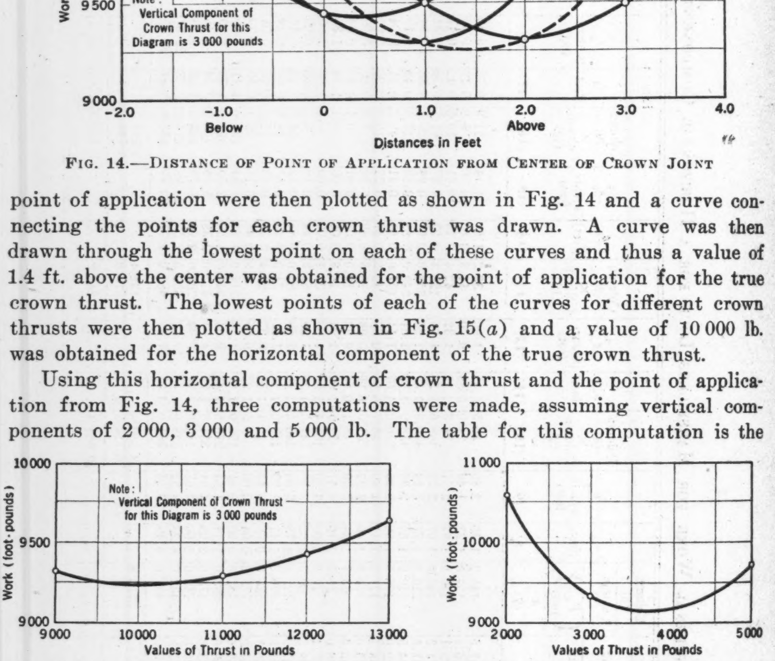


FIG. 14.—DISTANCE OF POINT OF APPLICATION FROM CENTER OF CROWN JOINT

point of application were then plotted as shown in Fig. 14 and a curve connecting the points for each crown thrust was drawn. A curve was then drawn through the lowest point on each of these curves and thus a value of 1.4 ft. above the center was obtained for the point of application for the true crown thrust. The lowest points of each of the curves for different crown thrusts were then plotted as shown in Fig. 15(*a*) and a value of 10 000 lb. was obtained for the horizontal component of the true crown thrust.

Using this horizontal component of crown thrust and the point of application from Fig. 14, three computations were made, assuming vertical components of 2 000, 3 000 and 5 000 lb. The table for this computation is the



(*a*) HORIZONTAL COMPONENT

(*b*) VERTICAL COMPONENT

FIG. 15.—COMPONENTS OF THRUST AT THE CROWN

same as that used for obtaining the horizontal component, except that for each vertical component assumed only one pressure line is drawn, as the point of application is known. The values obtained for the different vertical components are then plotted as illustrated in Fig. 15(*b*), and a curve is drawn through these points. The lowest point on this curve gives a value of 3 700 lb. as the true vertical component.

Having now obtained the true vertical and horizontal components and point of application of the crown thrust it is a simple matter to draw the true pressure line and compute the stresses in the various joints. The pressure line and stresses obtained for the section in question are shown in Fig. 13.

It is not claimed that this method will give exact results, but it is believed to give results nearer the truth in a case of this kind than any other method. The computation would be rigidly true if the arch were divided so that $\frac{dL}{l}$ is constant for the first term of the formula. Perform this part of the computation, then divide the arch so that $\frac{dL}{A_c}$ is constant and use this division for the second term of Equation (2).

In designing the arched conduits through the dams for the Miami Conservancy District, at Dayton, Ohio, this was done as a check for one of the arches after the true pressure line had been obtained by the method described herein. It was found that the pressure lines obtained by the two methods were practically identical; the crown thrust differed only 0.17% and the point of application was practically the same.

TAINTOR GATE DAMS AND OPERATING MACHINERY

The dams at all the sites except Brandon Road consist of a concrete weir of moderate height, with piers at certain intervals. A movable gate that may be opened during flood periods is placed between each two piers. At one end of each of the dams the height of the gate is different from the others, similar to the sector gate at Lockport. This is for the purpose of taking care of the ice (see Section *B-B*, Fig. 16).

Three types of gates were studied for the movable parts of these dams. The first consisted of a shutter, hinged along the up-stream edge, which is at the level of the crest of the permanent masonry portion of the dam. At the two down-stream corners chains from heavy counterweights are attached, the counterweights sliding along the backs of the piers.

The second consisted of the well-known "Stoney gate" type, consisting of a large, counterweighted shutter traveling vertically in grooves. Instead of having a train of rollers for each end bearing traveling with the gate at half speed, as is the custom in this type, fixed wheels with roller bearings were substituted.

The third type is a "Taintor gate" and consists of a part of a cylindrical surface braced to and revolving about a horizontal axis. It is counterweighted so that the operating machinery will have only friction to overcome. This gate can be counterweighted accurately much more cheaply and simply than either of the others. The operating machinery is carried by a light steel footbridge spanning from pier to pier.

The first type was quickly discarded because of the great difficulty of counterweighting due to the extreme variation of the load for different positions of the gate. This would make the cost of operating machinery and power very heavy.

In studying the second and third types the cost was found to be about the same, but there were several disadvantages of the second, or "Stoney" type. A heavy operating car with powerful machinery would have to be provided for installing the gates and removing them when necessary, and a heavy

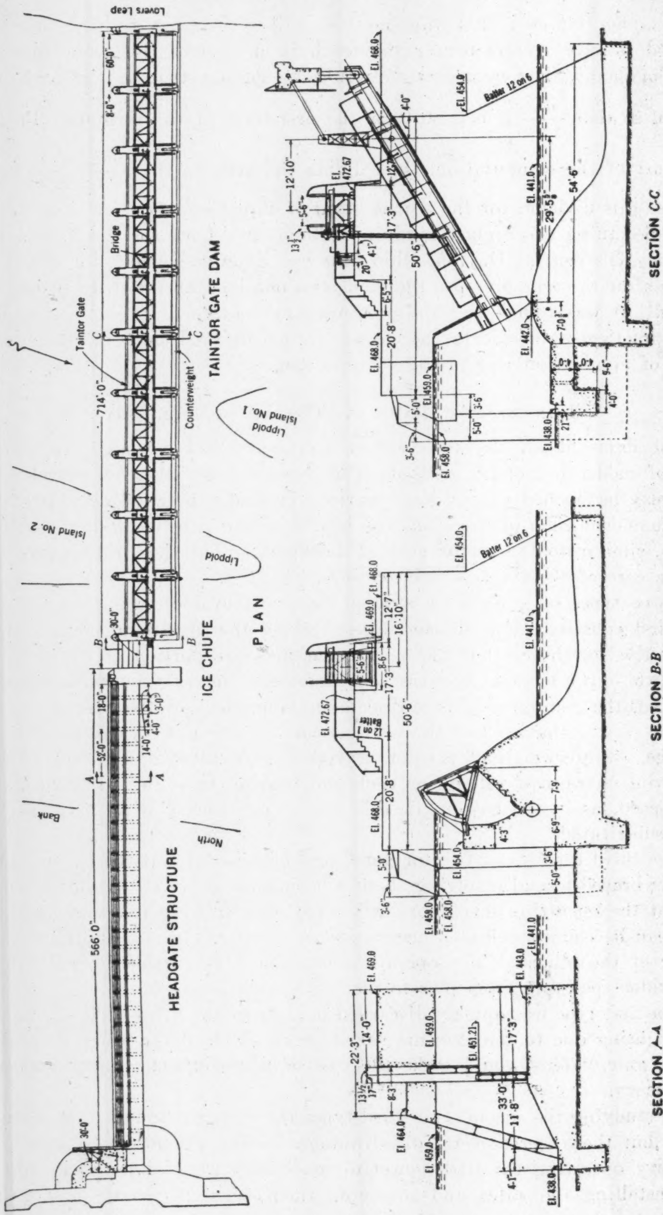

Lovers Leap
60'-0"
8'-0"
Bridge
Taintor Gate
C
C
TAINTOR GATE DAM
Counterweight
714'-0"
Lippold Island No. 1
Island No. 2
Lippold
30'-0"
B
B
ICE CHUTE
PLAN
18'-0"
52'-0"
A
A
14'-0"
4'-0"
3'-0"
Bank
North
HEADGATE STRUCTURE
566'-0"
36'-0"
El. 468.0
El. 454.0
Batter 12 on 6
4'-0"
El. 472.67
12'-10"
5'-6"
13½"
4'-1"
20'
17'-3"
50'-6"
6'-5"
20'-8"
El. 468.0
El. 459.0
5'-0"
3'-6"
5'-0"
3'-6"
El. 458.0
El. 441.0
29'-5½"
54'-6"
El. 442.0
7'-0"
4'-0"
4'-0"
9'-6"
4'-0"
El. 438.0
21'
SECTION C-C
El. 454.0
Batter 12 on 6
El. 468.0
El. 469.0
12'-7"
16'-10"
El. 472.67
12 on 1 Batter
5'-6"
8'-6"
17'-3"
50'-6"
20'-8"
El. 468.0
El. 441.0
7'-9"
6'-9"
4'-3"
3'-6"
5'-0"
El. 454.0
5'-0"
3'-6"
El. 459.0
El. 458.0
SECTION B-B
El. 441.0
El. 469.0
El. 443.0
22'-3"
14'-0"
El. 459.0
El. 451.21
17'-3"
13½"
17"
8'-3"
33'-0"
11'-8"
El. 464.0
El. 459.0
4'-1"
El. 438.0
SECTION A-A

FIG. 16.—PART PLAN AND SECTIONS OF METAL WORK AT STARVED ROCK DAM

overhead bridge would be necessary to carry this car and the suspended gate. This bridge would have to be on high piers with its floor far above the tops of the lock walls, thus necessitating a stairway at each end and making it difficult to install or take out either the operating car or the gates. To offset this cost, the piers in the second type were much lighter than those in the third type, the weir being the same in each. This saving in concrete practically balanced the extra cost of the operating car and heavy bridge, but on account of the difficulty in installing and operating the Stoney type the "Taintor" gate type was adopted. Fig. 16 shows the layout of metal work at Starved Rock Dam.

A method has been provided to close any one Taintor gate-opening so that a gate may be inspected, repaired, and, if necessary, taken out and replaced without loss of water. Notches are provided on each side of each pier at the water surface into which a steel coffer (the bottom of which is a very strong horizontal beam) can be floated. Steel needles are inserted with the lower ends resting in a groove in the concrete apron and the upper ends against the bottom girder of the coffer. After the needles are placed, cocks in the bottom of the coffer are opened, and the water is drained out. These cocks are provided so that the coffer can be made to float at the proper level to fit into the notches in the piers.

All Taintor gates are operated by machinery placed on the piers. An alternative hand and motor operation is provided as indicated in Section *C-C*, Fig. 16. A shaft operated by a worm-wheel at the pier extends to the center of the operating bridge. A pinion on the end of this shaft, at the center of the bridge, engages a spur gear on a shaft that extends the entire length of the bridge. On the ends of this shaft are pinions that engage in the operating struts attached to the end struts of the gates, which extend beyond the center of rotation and carry the counterweight. The purpose of this method of operation is to insure the same amount of torsion at both ends of the operating shaft and thus prevent twisting in the gate.

Adjustable Crest-Sector Dam at Each Site

This dam (Section *C-C*, Fig. 16) is built of structural steel and is a part of a cylinder rotating about a horizontal axis. The curved surface is faced up stream so that the pressure of the water is exerted through the center of rotation. The weir is 30 ft. long, with an adjustable crest of 7 ft. maximum variation. It is constructed to facilitate the movement of ice. The curved sheet of the sector has a radius of 10 ft. A weir tube controlling the water level under the dam can be manipulated to regulate the amount of pressure head within the movable dam. The hinge consists of steel castings bolted together in sections. The upper, being part of the gate, is held by four pins to the lower castings embedded in the masonry and anchored. The castings run the entire length of the dam. A seal is obtained by arranging a curved overlapping apron of the upper casting so that it is in rubbing contact with a movable strip of metal secured to the lower stationary casting by set screws. Steam pipes prevent ice from forming about the hinges. The sides of the movable structure are sealed by pipes pressed against steel plates attached to the walls of the flanking masonry piers. The gate can be locked in the elevated position by bolts keyed to a shaft on the breast-wall which holds the gate in position, while the water pressure inside can be released. A suitable manhole gives access to the interior of the chamber for cleaning or repairs. This dam is like that of the Sanitary District of Chicago at Lockport.[11]

BRIDGES

Twelve highway and street bridges that crossed the river channel either had to be rebuilt entirely or a draw span had to be constructed. Five of these bridges are in the City of Joliet and the others at various points along the Waterway. The five in Joliet and one at Brandon Road are rolling-lift bascule structures of the Scherzer type. The others are ordinary fixed bridges built at a high level.

In addition, there are draw spans in five railroad bridges that cross the river. One of these located in Joliet was entirely rebuilt.

WATER-PIPE TUNNEL

The Illinois Waterway interfered with the water pipes that crossed on existing bridges. The law authorizing construction required the State to provide a means of conveying the water across the canal before the existing mains could be disturbed. As the bridges at Joliet are of the rolling-lift bascule type, it was impossible to place any water mains on them and, therefore, a tunnel in which water mains could be placed had to be constructed under the river. The Electric Light and Power Company, the Telephone Company, and the Gas Company, also desired to use the tunnel, so they each paid their proportional part of the cost of the structure.

The section of the tunnel is oval, the longer diameter being horizontal. It has a maximum height of 9 ft. and a maximum width of 10 ft. The shafts are the same size as the tunnel. Conduits for electric light and power wires are provided in the side wall of the tunnel and in shafts on one side, with a manhole at the top of each shaft. Similar conduits are provided on the other side for telephone wires, also with manholes at the top of each shaft. Two 16-in. water mains and three gas pipes occupy the open space of the tunnel and shafts.

The center line of the tunnel is about 38.5 ft. below the bed of the stream and is in first-class limestone. There is a working chamber at the head of each shaft. Ordinarily, the tunnel and shafts will be kept flooded to within a few feet of the working chamber floors as this makes it safe to place gas mains, and electric light, power, and telephone wires in the same chamber. A sump is provided at the bottom of one of the shafts, in which a rotary pump is placed with a shaft running up to a motor in the working chamber and a discharge pipe running up to and entering an intercepting sewer in the wall along the side of the canal. When it is desired to inspect the tunnel this pump will clear the tunnel of water in about seven hours. A blower is provided in the same working chamber to force air down the shaft, through the tunnel and up the shaft on the far side of the canal, before any one enters.

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[11] *Engineering News*, Vol. II, 1908, p. 512.

DISCUSSION

A. W. Sargent,[12] M. Am. Soc. C. E. (by letter).—Some new and interesting features in lock design have been brought out in this paper. The use of the Venturi principle in the design of lock culverts has resulted in a saving in the cost of valves and valve-operating machinery with little loss in efficiency. The valves, which apparently are designed to be lowered by their dead weight, without strut or stem to force them to the closed position, would operate satisfactorily in quiet water and under a small head, but it is the writer's opinion that some mechanism should have been provided to force them to their seats under a head equal to the maximum lift of the lock. It will be interesting to know whether in actual practice the valve-operating mechanism, which is quite simple and inexpensive compared with many other types now in use, has proved entirely satisfactory; and, also, it would be interesting to know the maximum head under which a valve can be lowered to its seat.

The design of the gates, with air chambers in the lower part, single skin on the upper part on the tension or down-stream side, and buckle-plates for sheathing, has no doubt resulted in economy, but the additional protection afforded by a complete double skin would be beneficial. It would prevent the accumulation of driftage on the girders and the painted surfaces would be better protected. The interior surfaces of the double-skin gates of the Lake Washington Ship Canal, which is between salt water and fresh water, require little attention. A coating of bituminous solution and enamel applied about 1916 is still (1931) in excellent condition, while the exterior surfaces have required extensive repairs annually when the lock is unwatered for general overhauling of all under-water equipment.

The design of the part in contact with the miter-sill is new, and the writer believes it will be uniformly satisfactory when the strut-arm method of operating the gates is used (Fig. 9). This method does not allow the leaf to swing beyond the proper mitering position. The motion is eccentric at the heel and does not contact with the quoin-post except when mitered (Fig. 4). Where gates are operated by wire cables a stop should be provided to prevent a leaf from swinging too far and injuring the sealing strip as well as stressing the hinge-pins and anchorage. The writer had this in mind when he designed the water-seals for the lower guard-gates of the Lake Washington Ship Canal.[13]

To insure the successful operation of an emergency dam when an emergency arises, all parts should be easily accessible for inspection and repair. A structure of this kind which is continually submerged is not considered the best type. This condition could possibly have been avoided by placing the guard-gate up stream from the emergency dam. All parts of the dam would then be accessible after unwatering the lock. The spar bull-wheel type of gate-operating machinery which is now generally used for mitering lock-gates, has proved very satisfactory.

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[12] Civ. Engr., U. S. Engr. Office, Govt. Locks, Seattle, Wash.

[13] *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 1008.

The Lockport Lock, which has a lift of 40 ft., is one of the highest in the United States. The Wilson Dam Lock has a lift of 45.5 ft., and the Keokuk Lock, one of 41.0 ft. The locks of the Welland Ship Canal, in Canada, have a maximum lift of 46.5 ft. The difficulty of handling vessels in high-lift locks, while the water is being raised and lowered, is quite apparent. At the Welland Ship Canal mooring passages are provided in the lock-walls to facilitate the mooring of vessels at lower pool levels.[14] This method possibly has some drawbacks as the passages no doubt become slippery making it dangerous for the men handling the lines.

The writer believes that it would be practicable to equip high-lift locks with movable mooring devices which could be held in vertical recesses in the lock-walls, and raised or lowered with the water, either mechanically or by floats. A mechanism equipped with roller-bearings would facilitate the raising or lowering when a ship's lines are taut. A ship entering a lock which has been equipped with these devices, would have the eye on the mooring lines attached to two or more as desired and this should be secured to the ship's cleats or windlass. Lengthening and shortening of lines, as is usual when lowering or raising a vessel, would not be necessary. The ship's crew would do the principal work now required of the linesmen on the lock-wall, and the safety of the vessel in the lock would be dependent upon the vessel's crew and its own lines. With workable mooring devices which would be held a certain distance above the water in a lock while the water is being raised or lowered, the operation of locks of extreme lift would be feasible and much of the difficulty of providing lockage facilities at high dams would be overcome, and the time of passage between pools would be reduced.

E. G. Walker,[15] M. Am. Soc. C. E. (by letter).—The arrangement of the sluices on the culverts of the locks along the Illinois Waterway is certainly unusual, and it would be interesting to learn whether the advantages claimed for the reduction of the size of the valve, relative to the cross-section of the culvert, are fully justified in practice. In order to obtain efficiency, the principle of the convergent and divergent mouthpiece which was used by the late Clemens Herschel, Past-President and Hon. M. Am. Soc. C. E., in the development of the Venturi meter, presumes a gradual convergence to the smallest cross-section and subsequent divergence of the stream line without varying the general shape of the stream, except possibly by such gradual changes of curvature as will prevent the formation of eddies and turbulence.

In the design which the author describes, a circular culvert, 12 ft. in diameter, is reduced at the valve face to an opening 9 ft. square. In practice, any transition from the circular to the square must introduce a certain amount of eddying, however gradual it may be, and, therefore, the full benefit of the contraction of sections cannot be obtained. The type of sluice-valve described by L. D. Cornish, M. Am. Soc. C. E., as being used on the Illinois Waterway,[16] could not readily be adapted to suit a circular opening, and this accounts for the change in cross-section of the water passage. The principal benefit that is obtained by using a smaller sluice-valve is that the cost of the valve is thereby reduced. It does not seem likely that there can be much practical value in the theoretical hydrodynamical advantage of increased rate of flow, when the practical circumstances of the problem are considered. The author calculates a theoretical advantage of only 25 sec., and it seems doubtful whether even this could be fully justified if a practical comparison could be made between the larger and smaller valves.

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[14] *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, p. 2066.

[15] (Maxted & Knott), London, England.

[16] Manuals of Engineering Practice, Am. Soc. C. E., No. 3, on Lock Valves, Appendix 3, p. 17.

The proportion, 1 to 4, of rise to span for the lock-gates seems high. It is stated that it was adopted as the result of estimates based on actual designs. Is it to be assumed that these were designs prepared in connection with the job, or is it the author's generalization of an examination of previous practice? If the latter it would seem that he has examined only the more abnormal cases. The writer knows of many instances of mitered gates in which the rise is as little as one-sixth of the width between the centers of the quoin-posts, and he believes that although cases can be found, especially on small locks, of rises of one-fourth and even one-third of the width, an average would probably be considerably less than one-fourth.

It would be of interest if the author would enlarge a little upon his reasons for stating, as he does under "Miter Lock-Gates," that mitering lock-gates should be of the parallel type. It is common practice in America for this to be so, but in many other places the cylindical-backed gate has been found to be suitable, and there are definite features in favor of it. The parallel gate has the advantage of taking up considerably less space in the lock-wall when open, and may enable a reduction to be made in the over-all width of the lock between the backs of the walls of the gate recesses (usually the widest part of the structure). In many instances, particularly when two locks or entrances have to be constructed side by side in a limited width, this may be a great advantage in enabling the works to be arranged so as to give the most economical utilization of the available space. On the other hand, the cylindrical-backed gate does not present any difficulties of construction; the plates have only to be rolled to a relatively simple curvature in one plane and this applies to all the structural details involved. Therefore, it does not present any construction difficulties as compared with the parallel gate. At the same time, it has the advantage that the line of thrust can usually be kept well inside the thickness of the gate at all points, and thus enable better distribution of metal to be arranged with smaller variation of stress from point to point of the structure. It is noted that in Fig. 6 (*a*), the value of *j* (the distance of the line of thrust from the chord of the gate) reaches a maximum of 8.226 ft., as compared with a gate width of 7 ft.

The writer was interested to learn of the use of buckle-plates on the gates, a feature which would not be feasible with cylindrical-backed gates. This appears to be somewhat of a new departure in lock-gate construction and it is of value to learn that a saving of cost was effected thereby. If such saving can be effected in the general case it is a strong feature in favor of the use of parallel gates.

The statement in the paragraph introducing Fig. 5 is peculiar. It seems to the writer that in all lock-gates there is a self-evident benefit in endeavoring to balance as much of the weight of the gate by buoyancy as is suitable to the average circumstances under which the gate is to operate. This applies whether the gate is small or large. If the weight of the gate can be balanced by flotation there must be a self-evident advantage in equalizing the distribution of pressure on quoins, miters, and sills, and, therefore, of enabling more water-tight joints to be made and maintained. All the gates on the Illinois Waterway are certainly large enough to have made the provision of buoyancy chambers worth while and it seems remarkable, therefore, that any attempt to design gates without buoyancy chambers should have been made at all.

The writer would suggest the desirability of adding to the paper the detail of the miter-post, similar to the clear detail of the quoin-post shown in Fig. 4. He suggests that this, in company with the other details of the gates given in the other diagrams in the paper, would complete the author's clear description of the gates as a whole.

The design of the pintle, as shown in Fig. 5, while it is in accordance with American practice, seems to be rather more complicated than is necessary. With properly supported gates in which, under most of the conditions of operation, the weight is balanced by the buoyancy of the air chambers, there should not be a large load upon the pintle and, therefore, a plain steel casting with a corresponding plain cast-steel cup on the heel of the gate, should be sufficient and would appear to be a cheaper solution.

The design of the sill on the miter-gate is an interesting attempt to overcome the disadvantages inherent upon the older methods of attempting to make a tight joint by means of timber or metal clapping sills. Provided no obstruction gets between the lock-sill and the spring-plate, there should be every possibility of making a tight joint, if the spring-plate has been properly adjusted in the first place. There are, however, one or two points upon which the writer would like information. It would be interesting to learn the amount of deflection allowed in the design. It is possible that small obstructions might easily cause inordinate leakage. With a solid clapping sill the introduction of such obstruction would be sufficient to prevent the gate from closing properly and so most likely the existence of the obstruction would be made known. With the spring-plate, however, it seems quite conceivable that the obstruction might force the plate back, but yet not be sufficient to prevent the gate from mitering, and in this way a permanent opening would be formed along the sill of the gate after the lock had ostensibly been closed, and considerable leakage would thus result. There is also the possibility of damage to the plate by obstructions, such that the damage could only be discovered by under-water inspection and yet not be of such a nature as would prevent the gates closing properly.

The design of anchorage shown in Fig. 8 is good. The writer agrees strongly with the author's complaint as to the common design of anchorages, which frequently consist of huge tie-bars carried back into enormous masses of concrete out of all proportion to the load which the anchorage is called upon to carry. The writer has known of cases in which it was simple to demonstrate that—entirely apart from the resistances of anchorage of old rails and large washer-plates at the ends of the ties—if the adhesion of the concrete to the tie-bar could be relied upon, it would be possible to break the anchor-bars by tension before reaching an admissible working adhesion between the steel and the concrete. The gate anchorages shown in Figs. 3 and 8 prove that, by rational design, it is possible to construct anchorages of reasonable dimensions which will stand up fully to the requirements of the job. The differentially threaded turn-buckle is ingenious and should enable closer adjustments of the quoin-post to be made relative to the quoin.

The gate-operating machinery shown in Fig. 9 seems well adapted for its purpose, in that it enables a maximum of force to be exerted in the operating arm at the beginning and the end of the stroke, the times when extra effort is most likely to be required. It has the disadvantage, however, that the effort is applied to the gate at a short radius relative to the radius of the center of water pressure on the gate, thereby inducing greater dynamic stresses in the structure of the gate than would otherwise be the case. At the same time, except by the use of the old method of attaching chains at the center of pressure, it is not possible to eliminate entirely the dynamic stress in gates during opening and closing, and the only designs that the writer has seen for substituting an arm for the chain for this purpose had a number of features which were obviously detrimental. It appears necessary to accept the fact, therefore, that compact and convenient gate-operating mechanism can only be obtained at the expense of a certain amount of dynamical stressing of the gate structure.

The writer was interested to note the relatively high degree of elaboration which it has been thought necessary to adopt in designing the structure of the lock-wall where it is intersected by culverts. He questions its value on the generally accepted principle that no structure can be safer than the foundations upon which it stands. The author of this comparatively elaborate graphical and analytical method of designing the wall gives no indication of why he assumes that all of the regular cross-section of the lock-wall acts as an arch. As far as can be followed from the paper, the lock-walls are simply plain mass concrete structures, and it is impossible, without making assumptions of the most arbitrary kind, to know how the pressures on the back and front of the wall are transmitted through the monolithic mass. It is surely going very far to assume that a rectangular wall section with a toe and a circular opening through it, as shown in Fig. 13(*b*), acts as a pure arch composed of a number of independent voussoirs each of special and arbitrary shape. The writer has strong belief in the application of scientific principles to design whenever benefit can be obtained thereby, but he does not think that this particular instance is one suited to extremes of mathematical analysis.

DAVID A. MOLITOR,[17] M. AM. SOC. C. E. (by letter).—In this paper, the author has attempted to describe briefly a work comprising a variety of

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[17] U. S. Constr. Engr. for Superv. Archt., Treasury Dept., Lansing, Mich.

structures entering into the construction of the Illinois Waterway, and he gives special consideration to the stress analysis of the lock-walls with large culvert openings, which he claims to have solved by applying the principle of least work. Several of the subjects mentioned, since they are not adequately treated in books, might properly cover more space than was allotted to the entire paper.

***Mitering Lock-Gates.*—In designing horizontal girders of mitering lock-gates, the chief difficulty is in determining appropriate flange areas for the case of direct stress and bending, where the direct stress, the bending moment, and the moment of inertia, all change at each girder section, as shown on the author's Fig. 6(*a*).**

The writer has found the following formulas very useful in avoiding the customary trial-and-error method of proportioning the required girder flanges.

In addition to the notation used by the author, the following are introduced (see Fig. 17):

$F =$ area of flanges in Fig. 17; F_1 refers to Flange No. 1 and F_2 refers to Flange No. 2.
$f =$ unit stress; f_1 refers to extreme fiber of Flange No. 1 and f_2 refers to extreme fiber of Flange No. 2.
$d =$ depth of web-plate.
$t =$ thickness of web-plate.
$j =$ lever arm of resultant, T.
$T =$ resultant force acting on the girder section, A-A.

***Case I.*—To find the theoretical areas, F_1 and F_2, required for each flange, when the maximum allowable unit stresses are assigned, positive for tension and negative for compression. When T, j, d, t, f_1, and f_2, are given:**

$$F_1 = -\frac{1}{df_1}\left[Tj + \frac{d^2 t}{6}(2f_1 + f_2) \right] \ldots\ldots\ldots\ldots (6)$$

and,

$$F_2 = -\frac{1}{df_2}\left[T(d - j) + \frac{d^2 t}{6}(f_1 + 2f_2) \right] \ldots\ldots\ldots (7)$$

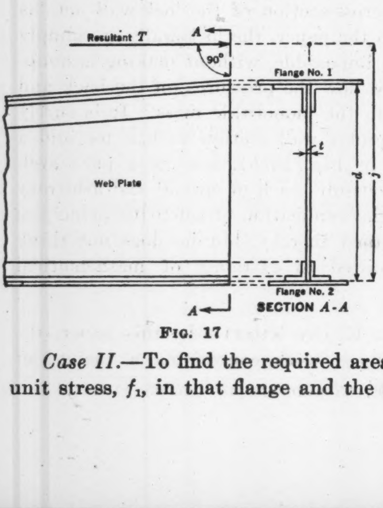
Resultant T
A
90°
Flange No. 1
t
Web Plate
d
j
Flange No. 2
A
SECTION A-A

FIG. 17

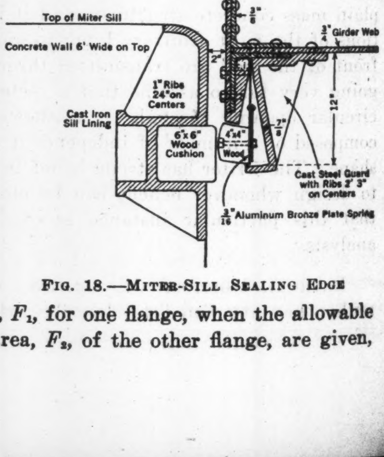
1/2"
3/8"
Top of Miter Sill
3/4" Girder Web
Concrete Wall 6' Wide on Top
2"
1" Ribs 24" on Centers
12"
Cast Iron Sill Lining
7/8"
6"x6" Wood Cushion
4"x4" Wood
Cast Steel Guard with Ribs 2' 3" on Centers
3/16" Aluminum Bronze Plate Spring

FIG. 18.—MITER-SILL SEALING EDGE

***Case II.*—To find the required area, F_1, for one flange, when the allowable unit stress, f_1, in that flange and the area, F_2, of the other flange, are given,**

but the unit stress in the given flange is not assigned. When T, j, d, t, f_1, and F_2 are given:

$$F_1 = -\frac{T\left(\frac{6\,j\,F_2}{d^2\,t} + \frac{3j}{d} - 1\right) + \frac{f_1}{2}\left(d\,t + 4\,F_2\right)}{2\,f_1\left(\frac{3\,F_2}{d\,t} + 1\right)} \quad \ldots\ldots\ldots(8)$$

and,

$$f_2 = -\left[\frac{6\,f_1\,F_1}{d\,t} + \frac{6\;T\,j}{d^2\,t} + 2\,f_1\right] \quad \ldots\ldots\ldots\ldots\ldots(9)$$

Case III.—The areas, F_1 and F_2, for the flanges, when the eccentricity of the axial thrust, T, varies between equal limits on both sides of the gravity axis of the section, are given by the formula:

$$F_1 = F_2 = -\frac{1}{6\,f_1\,d}\left[3\,Tj + 2\,f_1 d^2 t + \sqrt{(3\,Tj)^2 + f_1\,d^2 t\,[f_1 d^2 t - 6\,T(j-d)]}\right]. (10)$$

Equations (6), (7), (8), (9), and (10) look rather lengthy, but will be found very serviceable in designing columns and girders subject to direct stress and bending. The important subject of deflection of lock-gates is not mentioned in this discussion because it has been comprehensively treated elsewhere.[18]

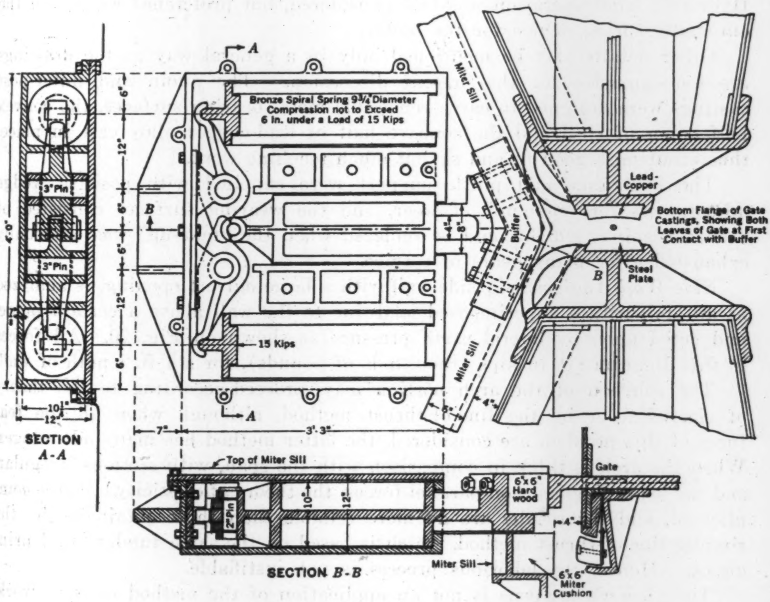
A
A
Bronze Spiral Spring 9¾" Diameter
Compression not to Exceed
6 In. under a Load of 15 Kips
15 Kips
Miter Sill
Miter Sill
Buffer
Lead-Copper
Bottom Flange of Gate Castings, Showing Both Leaves of Gate at First Contact with Buffer
Steel Plate
B
B
3" Pin
3" Pin
4'-0"
6"
12"
6"
6"
12"
6"
4"
8"
10½"
12"
7"
3'-3"
5"
4"
SECTION A-A
Top of Miter Sill
Gate
6"x6" Hard wood
2" Pin
10"
12"
4"
Miter Sill
6"x6" Miter Cushion
SECTION B-B

FIG. 19.—MITER-SILL BUFFER

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[18] "Kinetic Theory of Engineering Structures," by David A. Molitor, Chapter 14.

Contact at Miter-Sill.—The indeterminate conditions introduced by sill contact can be entirely eliminated in the manner described by the author, but when this is done, there should be provision for forcing centric mitering of the two gate-leaves at the instant contact is made on the miter-sill. With only a spring bearing at the sill, the chances for eccentric mitering are very great, and would lead to excessive stresses in the horizontal girders, besides causing delays in operation.

In January, 1912, the writer prepared a design for the lower lock-gate for the Mississippi River Power Company, at Keokuk, Iowa, which embodied several novel features not previously used in mitering gates. Among these was a spring-sill bearing having wooden contact surfaces with guards, and a miter-sill buffer, which forces simultaneous centric bearing of the miter-posts, prior to making spring contacts along the sill. The water pressure, as it gradually increases after closing the gate, forces all contact surfaces along the sill and miter-posts into tight bearing.

The Keokuk gate consisted of two leaves, 49 ft. high and 64.9 ft. wide, for a lock, 110 ft. wide, and a maximum lift of 34 ft. Figs. 18 and 19 show the miter-sill sealing edge and the miter-sill buffer designed for this gate. Hydraulic hand operation was first considered, but preference was given the automatic spring device for the buffer.

Other details may be mentioned only in a general way as the drawings are too extensive for the present discussion. The quoin and miter-post castings were designed to carry renewable metal bearing surfaces, the convex half being of steel and the concave half of lead-copper alloy cast in place, thus ensuring a good fit and saving much machine work.

The anchorage and pintle support were supplied with positive wedge adjustments for "take-up" of wear, and the wearing surfaces consisted of bronze bushings which could be replaced when the "take-up" was eventually exhausted after many years of service.

Side-Wall Analysis.—A side-wall with a large culvert opening, is analyzed as an arch with a superimposed load due to the wall above a certain plane, and acted upon by lateral earth pressure as shown on Fig. 20. All forces in this diagram are in kips (thousands of pounds), for a 1-ft. length of wall.

The solution of the arch portion may proceed according to the theory of elasticity, or by the line-of-thrust method, although when all the features of this problem are considered, the latter method has many advantages. When the arch is thick in comparison with the span, with sections irregular, and the loads diagonal non-parallel forces, the theory of elasticity becomes complicated, and the results are no more reliable than those obtainable by the simpler line-of-thrust method, which is based on the same fundamental principles. Hence, the laborious process is not justifiable.

The author's analysis is not an application of the method of least work, but represents an attempt to show that the most probable line of thrust actually corresponds to a condition of minimum internal work, a fact already well established. The following outline for a solution according to the theory of elasticity will clarify these matters and will establish the criteria for locating the most probable line-of-thrust.

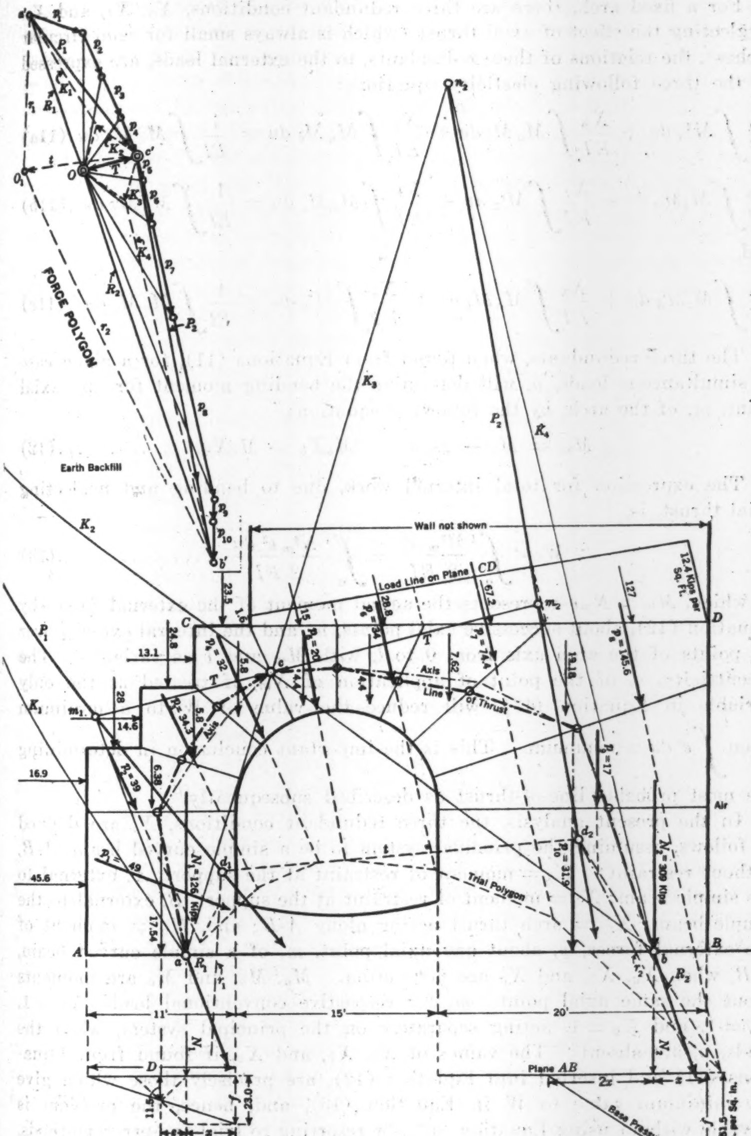
FORCE POLYGON
a'
O
O₁
c'
b'
K₁
K₂
K₃
K₄
R₁
R₂
r₁
r₂
t
T
P₁
P₂
m₁
m₂
Earth Backfill
Wall not shown
Load Line on Plane CD
12.4 Kips per Sq. Ft.
C
D
A
B
c
a
b
d₁
d₂
23.5
1.6
24.8
13.1
28
14.6
16.9
45.6
15
28.8
67.2
127
p₁ = 49
p₂ = 39
p₃ = 34.3
p₄ = 32
p₅ = 20
p₆ = 34
p₇ = 74.4
p₈ = 145.6
p₉ = 17
p₁₀ = 31.9
5.8
5.8
4.4
4.4
7.62
19.18
6.38
17.54
N₁ = 126 Kips
N₂ = 309 Kips
N₁
N₂
Axis
Axis Line of Thrust
Trial Polygon
Air
11'
15'
20'
Plane AB
2x
x
f
81.5
357 Lb. per Sq. In.
Base Pressure
11.5
23.0

FIG. 20

For a fixed arch, there are three redundant conditions, X_a, X_b, and X_c. Neglecting the effect of axial thrust (which is always small for semi-circular arches), the relations of these redundants, to the external loads, are expressed by the three following elasticity equations:

$$\frac{X_a}{EI}\int M^2_a\,du + \frac{X_b}{EI}\int M_a M_b\,du + \frac{X_c}{EI}\int M_a M_c\,du = \frac{1}{EI}\int M_o M_a\,du. \quad (11a)$$

$$\frac{X_a}{EI}\int M_b M_a\,du + \frac{X_b}{EI}\int M^2_b\,du + \frac{X_c}{EI}\int M_b M_c\,du = \frac{1}{EI}\int M_o M_b\,du \quad .(11b)$$

and,

$$\frac{X_a}{EI}\int M_c M_a\,du + \frac{X_b}{EI}\int M_c M_b\,du + \frac{X_c}{EI}\int M^2_c\,du = \frac{1}{EI}\int M_o M_c\,du \quad .(11c)$$

The three redundants, when found from Equations (11), for a given case of simultaneous loads, p, will determine the bending moment for any axial point, m, of the arch, by the following equation:

$$M_m = M_o - M_a X_a - M_b X_b - M_c X_c \ldots\ldots\ldots\ldots\ldots (12)$$

The expression for total internal work, due to bending and neglecting axial thrust, is,

$$W = \int_0^l \frac{M^2_m\,du}{2\,EI} = \int_0^l \frac{N^2_m e^2\,du}{2\,EI} \ldots\ldots\ldots\ldots\ldots (13)$$

in which, $M_m = N_m e$ represents the actual moment of the external forces by Equation (12), about successive axial points, m, and the integral extends over all points of the arch axis from 0 to l, with M_m and I as variables. The eccentricity, e, of the point of application of N_m, if treated as the only variable in Equation (13), will reduce the value of W to a minimum when $\int e^2\,du$ is minimum. This is the important conclusion in determining the most probable line-of-thrust as described subsequently.

In the present analysis, the three redundant conditions, X, are defined as follows, assuming the principal system to be a simple curved beam, A-B, without restraint: X_a = moment of restraint at the support, A, external to the simple beam; X_b = moment of restraint at the support, B, external to the simple beam; X_c = arch thrust acting along A-B; and, M_o = moment of the external forces, p, about any axial point, m, of a simple curved beam, A-B, when X_a, X_b, and X_c are not acting. M_a, M_b, and M_c are moments about the same axial points, m, for respective conventional loads, $X_a = 1$, $X_b = 1$, and $X_c = 1$, acting separately on the principal system, when the loads, p, are absent. The values of X_a, X_b, and X_c, if found from Equations (11) and inserted into Equation (12), are precisely those which give the minimum value to W in Equation (13), and, hence, the problem is solvable without using Equation (13), or resorting to trial-and-error methods.

However, when the external loads, p, are not parallel and are otherwise unrelated, then all functions involving M_o must be evaluated by some process of graphic summation which complicates matters to a point where the method becomes impracticable as in the present case; but even if a solution were carried through in this manner, the results would be no more accurate or reliable than those obtainable by the "line-of-thrust method" which is the method devised by the writer in 1907, for analyzing the walls of the Gatun Locks of the Panama Canal, as shown on Fig. 20.

The Line-of-Thrust Method.—The portion of wall, *ABCD* (Fig. 20), is treated as an arch, with the wall above acting as a superimposed load as shown. The external earth pressures acting between *A* and *C*, and the weights of the several voussoir stones (separated by assumed voussoir joints), constitute the system of external loads on this arch. These loads are combined graphically into Loads p_1 to p_{10}, acting on the respective voussoir stones. Owing to the unsymmetric shape and loading of this arch, the joint at *c* is chosen as the crown joint, while the springing joints are on the line, *A-B*.

First, the resultants, P_1 and P_2, of all external forces each side of the crown section, *c*, are found. This is done with the aid of a force polygon, by adding graphically the several loads, p_1 to p_{10}, in their proper order, true directions, and magnitudes. Then, the resultant, P_1, of the loads, p_1 to p_4, and the resultant, P_2, of the loads, p_5 to p_{10}, are found in direction and magnitude in the force polygon. A point on the line of action of each is found by drawing an equilibrium polygon, using any convenient pole, O_1, with pole distance, *t*. The intersections, d_1 and d_2, of the end rays, r_1 and r_2, with the pole ray, *t*, of the trial polygon, are the desired points, so that P_1 and P_2 are now fully determined. This much of the line drawing, including the dotted arch axis, should be drawn in ink so that the next steps, which are first drawn in pencil, may be erased after the final line of thrust is located by trial.

The foregoing really constitutes the preparatory work, and drawing the most probable line-of-thrust completes the analysis, giving the data for stress calculations for all voussoir joints.

The line-of-thrust in an arch ring, is a polygon found by joining, successively, the points of application of the resultant thrusts of the several assumed voussoir joints, as determined by an equilibrium polygon drawn for the given forces. Since any number of equilibrium polygons can be drawn through an arch ring, it becomes necessary to select the most probable one, for which the line-of-thrust satisfies the criterion: That the internal work of deformation, due to the actual loads, *p*, shall be a minimum. The terms, "equilibrium polygon," and, "resultant polygon," are synonymous, but distinct from the line-of-thrust.

The most probable line-of-thrust, according to the theory of elasticity, is that which reduces to a minimum, the residual departures from the arch center line according to the method of least squares. This is equivalent to making Σe^2 minimum, which is in agreement with Equation (13) for the condition producing the minimum internal work, *W*.

Hence, if a certain line-of-thrust can be made to coincide with the arch center line, then this particular thrust line will be the most probable one of all the possible lines for the given case of loads, making $\Sigma e^2 = 0$. For any other line-of-thrust, $\Sigma e^2 > 0$. Since values of e may be plus or minus, the sum of the squares can never become zero unless all values of e are zero.

As a first trial, draw a resultant polygon through three points *a, b,* and *c,* chosen on the axis of each section. This line, when obtained, will indicate how to shift the three points first assumed, so as to bring the resultant polygon into the closest possible coincidence with the arch axis. After two or three trials the line-of-thrust, *a c b,* was located as best satisfying the requirement, $\Sigma e^2 = 0_{\min}$.

There is a further condition which must be satisfied as follows: The line of thrust must intersect the arch axis in at least three points, and when the arch and its loading are both symmetrical, there must be at least four such intersections. Fig. 20 shows three intersections as required for unsymmetric cases.

The method of drawing a resultant polygon through three points, *a, c,* and *b,* is described as follows: Draw the two lines, K_2 and K_3, through the three given points and continue to the intersections, n_1 and n_2, with the resultants, P_1 and P_2. Then, draw the lines, K_1 and K_4. Now draw four lines, K_1, K_2, K_3, and K_4, in the force polygon, respectively, parallel to the lines just drawn and complete the parallelogram with the diagonal, $Oc' = T$, thus locating the pole, O, and determining the crown thrust, T, in direction and magnitude. This thrust acts at c, and the line, $m_1 m_2$ parallel to T, determines the final reactions, $m_1a = R_1$, and $m_2b = R_2$. The vertical component of R_1 is N_1, and the vertical component of R_2 is N_2, from which the stresses on the sections, *AB,* are found.

The stresses on any other voussoir joint are obtained from the line of thrust, by completing the resultant polygon for the correct pole, O. The rays, drawn from O to the several corners of the load line of the force polygon, represent the resultants on the several voussoir joints, measured to the scale of forces. The normal components, N, with the eccentricities, e, all scaled from the diagram, determine the stresses on the sections.

The stresses on any voussoir joint are now found from Navier's law for direct stress and bending, as follows:

When $e < \frac{D}{6}$,

$$f = -\frac{N}{D}\left(1 \pm \frac{6e}{D}\right) \dots\dots\dots\dots\dots\dots (14)$$

and, when $e > \frac{D}{6}$, allowing no tension,

$$f = -\frac{2N}{3\left(\frac{D}{2} - e\right)} \dots\dots\dots\dots\dots\dots (15)$$

In Equations (14) and (15), N represents the normal component of the resultant acting on the joint of width, D, and unit thickness, and e is the eccentricity of the point of application of N, as shown for the base sections on *A-B* (Fig. 20).

It is easily seen from Equation (14) that when the line-of-thrust remains everywhere within the middle-third, there will be no tensile stresses.

This completes the analysis, on a theoretically sound basis and with sufficient accuracy for all practical purposes. Had the author applied the criteria as here presented, for identifying the most probable one of all possible lines-of-thrust, all his involved calculations would have been avoided.

In Table 1 the sum of Column (7), representing the internal work due to axial thrust, and treated as a constant, contributes nothing of value, for which reason the author should have neglected axial thrust. If the sums are taken for Columns (9), (12), (15), (18), and (21), it will be found that Column (18) gives $\Sigma e^2 =$ minimum, which is the only conclusion of any value. These sums are not a true measure for minimum Σe^2, because the voussoirs were not taken at constant distances apart on the axis; hence, $d\,l$ should have been taken in as a variable factor.

The author further states, following Equation (5):

> "There are three unknown quantities to be obtained, the horizontal and vertical components, and the point of application of the crown thrust, and all are determined very easily by this process."

If he means by this that these three unknowns constitute the three redundant conditions involved in the arch problem, then he is seriously in error, because the three unknowns of which he speaks, merely determine one of the three redundants. He has thus solved a threefold statically indeterminate problem by considering only one.

All methods for the evaluation of the three redundant conditions in arches of the kind here treated, are more or less speculative; but they yield results which are within knowable limits, especially when the heterogeneous character of building materials is considered. Whether the resultant polygon and line-of-thrust are located correctly by the method of least work, by the elastic theory, or by a few trials observing the criteria stated herein, makes very little difference except in the labor involved, and certainly the least laborious solution is the most acceptable. If the problem dealt with large flat arch ribs, having thin rings and carrying vertical loads, then the theory of elasticity would be applicable and would afford the best and most accurate method of arch analysis.[19]

Charles A. D. Young,[20] M. Am. Soc. C. E. (by letter).—The author has presented a thorough description of the many interesting problems which arise in the design of navigable waterways and the accompanying locks and dams. The adaptation of the principles of the Venturi meter to the design of the culvert valve chambers is a valuable contribution to the art, and the results will be watched with interest by all concerned with the design of locks. If the results obtained in operation are as anticipated, this design, no doubt, will be adopted wherever applicable because it will afford a material saving in the cost of the valves and their operating machinery.

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[19] "Kinetic Theory of Engineering Structures," by David A. Molitor, Chapter 15.

[20] Senior Engr., U. S. Reclamation Bureau, Denver, Colo.

It is only within comparatively recent years that the scientific design of lock-gates has been given much consideration, and as yet no method has been devised for determining the stress transferred to the upper girders by the vertical framing when the gate is in contact with the sill. It has been the custom to compensate for these indeterminate stresses by adding to the water load a certain amount of static head at the upper part of the gate; but by eliminating the cause of the indeterminate stresses as the author does, the design of a lock-gate becomes a comparatively simple problem and this excess head may be omitted.

In the design of the gates, as described by Mr. Smith, there is no contact at the sill and hence no indeterminate stress; but it is noted that an allowance of a 15-ft. head at the upper part of the gate was made to compensate for shock.

Standard specifications for structural steel construction provide that no steel less than $\frac{5}{16}$ in. thick shall be used. Due to the fact that lock-gates are particularly susceptible to corrosion, it is considered good practice to limit this thickness to $\frac{3}{8}$ in. When the girders are equally spaced and of equal depth, it is found that the upper ones will be much stronger than required to resist the static head.

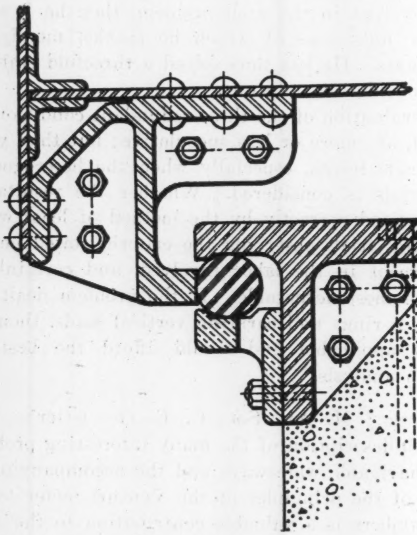

FIG. 21.—SECTION THROUGH SEAL; LOCK-GATE AT WILSON DAM

No doubt the spring seal, described by the author and designed to eliminate pressure between gate and sill, will provide an efficient seal, but experience with similar seals on the Stoney culvert valves at Panama, and at the Lake Washington Canal Lock, at Seattle, Wash., indicates that it may be short-lived. In the cases cited, after operation for a short time, the spring broke near the point of attachment to the valve, presumably due to vibration caused by slight leakage. The springs were replaced by heavy rubber belting and no further difficulty was experienced.

The first attempt to eliminate pressure at the sill that has come to the writer's attention, was in the design of the lock-gates for the double-lift lock at Wilson Dam (Muscle Shoals) on the Tennessee River (see Fig. 21). As the head is increased behind the gate, the water pressure acts through the holes in the lower support and forces the solid rubber bar against the gate. This same method was used by the writer on the gates for Lock and Dam No. 1, Tennessee River, at Florence, Ala., and in both cases was very successful in operation.

Adjustable bearing-plates backed by Babbitt metal, similar to those described by the author, were used on the lock-gates of the Lake Washington Canal. In this case the gates were swung to the closed position, the plates were forced out by set screws until they were in perfect contact, and the molten metal was poured in from the top. In order to insure that the space was completely filled with the Babbitt metal, the plates were removed, and large cavities were found due to the molten metal congealing on the sides near the top and clogging the opening. This difficulty was eliminated by drilling holes about 4 ft. apart and pouring in 4-ft. lifts.

It is interesting to note the effect on the stresses, of placing the sheathing plates on the down-stream side of the gate. Such placing of skin-plate has been considered an advantage in single-skin gates in order to reduce the uplift on the bottom when the gate was closed; but as this puts tension in the rivets that hold the skin-plate, it has seldom been done. One case only has come to the writer's attention and that was in the Tennessee River Lock, at Riverton, Ala. In this case, the girders were I-beams, and it is doubtful whether the designer had any other object in mind than the reduction of uplift. From an operation standpoint such an arrangement is objectionable because in a silt-bearing stream, such as the Tennessee, the girders fill with mud and débris which is difficult to clean out for painting.

The writer agrees with the author that gate anchorages made of eye-bars are difficult to set, but it is his experience that those built of structural members in the shape of a triangle are easily set and held in position, since they are rigid and may be held in place by anchor-bolts. This type of anchorage was used on the Lake Washington Canal Lock. The type described by the author has the advantage perhaps that it does not extend deeply into the concrete and can be placed after the concrete has been poured.

The method of adjusting the rotation axis of the gate as described by the author, was used on Davis Lock at Sault Ste. Marie, Mich., in order to secure the fine adjustment required and, at the same time, to provide the necessary strength of threads, and it has been used on other locks. Observation of the difficulties encountered in removing a gate-leaf of the Poe Lock at Sault Ste. Marie, prompted the writer to introduce the two eye-bars which can be swung out of the way and permit the gate to be lifted off its pintle without changing the adjustment.

Since the change in temperature of a lock-gate may materially change its location when mitered, a spring has been introduced in the spar, in some cases where the bull-wheel is used, to allow full stroke for all conditions. This spring also acts as a cushion to relieve the machinery of undue strain in starting and stopping the gate.

The addition of air chambers to lock-gates is of questionable value since the anchorages and pintle must be made of sufficient size to carry the load of the gate when swinging in air, which is necessary during construction for the purpose of making adjustments and, at other times, when the lock is unwatered for painting or repairs. It is true that the amount of swinging done under these conditions is small compared to the operation conditions, but if the pintle and gudgeon pin are conservatively designed to carry the full load of the gate in air, the wear on these parts will be kept at a minimum.

WALTER M. SMITH,[21] M. AM. SOC. C. E. (by letter).—Since the publication of this paper the Illinois Waterway has been practically finished, except for the completion of some of the bridges.

Owing to the tremendous increase in costs from the time the bond issue was voted for the construction and the time of construction, it was realized, in 1929, that the funds provided by the bond issue would not be sufficient. There were two possible ways to complete the Waterway. The first was for the State of Illinois to vote an additional bond issue. As the Illinois Constitution prohibits the State from lending its credit to the construction of railways or waterways, this would require an amendment. The Illinois Constitution also prohibits more than one amendment being voted on at any election, and there is always some important amendment to be voted on at each election. It was evident that this method would take years for completion, if successful, and there was a strong probability that it would not be successful.

The second method was to ask the Federal Government to furnish the money and complete the Waterway. It was impossible to turn the Waterway over to the possession of the United States Government because it belongs to the people of Illinois and can only be surrendered by a popular vote and very probably it would require another amendment to the Constitution to allow this vote to be taken. However, the President recommended, and Congress provided, the funds to complete the Waterway, except the bridges crossing it. The State, therefore, stopped work in 1930 to retain sufficient funds to construct the bridges.

The United States War Department began work on the project in February, 1931, the State concentrating its work on the bridges. At this time the lock and dam at Starved Rock, the Marseilles Lock, and the Lockport Lock were practically completed, except for the installation of equipment for electrical operation of lock-gates and valves. Brandon Road Lock was finished, except the installation of operating machinery, and the dam and walls through Joliet were about 75% completed. Marseilles Canal was about 90% completed. Dresden Island Lock was about 30% completed, but only a portion of the excavation had been done for the dam.

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[21] Asst. Chf. Engr., Div. of Waterways, State of Illinois, Chicago, Ill.

Marseilles Dam had been designed but work had not been started on it. All metal work had been provided for Dresden Island Lock and Dam as well as the machinery for operating the gates and valves at Dresden Island and Brandon Road Locks.

The Des Plaines and Illinois River Valleys have abundant banks of gravel of a fine quality for concrete. In many of them the gravel does not need to be washed or screened to size. Concrete, with bank-run gravel, was used in all the structures built by the State, because it had been found satisfactory for massive works and was much cheaper than the washed and screened material. The Engineers of the War Department were not accustomed to the use of bank-run gravel in concrete so when they took over the Waterway, they very naturally wished to satisfy themselves as to the quality of the concrete in the structures built by the State Engineers. Therefore, they had many cores, 6 in. in diameter, cut to a depth of 10 ft in the Lockport, Brandon Road, Marseilles, and Starved Rock Locks. Sample pieces, 12 in. long, were then cut from these cores and tested, generally from two to four test specimens from each core. Wherever there were places in the cores that appeared bad, they were included in the test specimens because the Engineers wished to know the worst. In many places one stone would occupy more than one-half the area of the test cylinder. The results are given in Table 2. It is to be noted that from one to five test pieces were cut from each core.

TABLE 2.—Summary of Core Tests of Concrete

Lock	Number of cores	Number of specimens	Breaking Strength, in Pounds per Square Inch		
			Minimum	Mean	Maximum
Lockport	5	19	3 117	5 746	7 874
Brandon Road	4	15	3 915	5 228	6 913
Marseilles	3	11	2 779	4 975	7 666
Starved Rock	4	15	3 510	5 038	7 689
All tests	16	60	2 779	5 306	7 874

This concrete generally contained from 4½ to 5 bags of cement per cu yd, except for the upper part of the lock-walls where the water rises and falls. In this part of the work generally about 6 bags of cement were used per cu yd of concrete, in order to obtain greater density and to prevent spalling of the face from frost action.

Before turning the Waterway over to the War Department Engineers, tests were made on the operation of the gates and valves at Lockport, Marseilles, and Starved Rock Locks.

The specifications called for operating each pair of miter-gates through fifty complete cycles continuously, allowing 2 min each for opening and closing and a ½-min allowance at each end of each swing. This would require each pair of gates to be operated fifty times in 4 hr and 10 min. The lower gates operated under service conditions, but the upper gates had to be operated "in the dry." The lock-gates at Starved Rock and Lockport were swung only five times in order to avoid injuring the pintles. At Lockport, the upper gates were required to open or close in 2 min. No specific test was prescribed. These units are vertical lift-gates, only the lower gates being of the miter type. The leaves are 59 ft high and weigh about 320 tons each. The results of the tests of the miter-gates are shown in Table 3.

TABLE 3.—TESTS OF MITER-GATES

Location	LOWER GATES (150 CYCLES)					UPPER GATES		
	Average time of opening, in seconds	Average time of closing, in seconds	Average time lost, in seconds	Total time elapsed, in hours	Total time allowed, in hours	Average time of opening, in seconds	Average time of closing, in seconds	Average time lost, in seconds
LOCK-GATES								
Starved Rock	118	119	0.45	3.75	4.17	105	105	....
Marseilles	108	108	0.30	3.25	4.17	109	109	0.0
Lockport	109 to 114	109 to 114	....	3.38	4.17	....	....	....
LOCK-VALVES								
Starved Rock	63	57	....	....	....	....	....	....
Marseilles	62	57	....	....	....	61	56	....
Lockport	....	....	....	....	....	92	90	....

Taintor Gates at Starved Rock.—The Taintor gates at the Starved Rock Dam were all completed and tested before the Waterway was turned over to the Government. The War Department Engineers made some tests of the flow through these gates partly open, varying from a small fraction of a foot to about 9 ft. With the water in the reservoir averaging about 4 to 5 in. below the level of the tops of the gates when closed, runs were made with the results shown in Table 4. Confidence can be felt in the reliability of the coefficients obtained.

TABLE 4.—COEFFICIENTS OF FLOW THROUGH TAINTOR GATES AT STARVED ROCK DAM

Item	Number of gates open	Height of opening, in feet	Coefficient
1	9	0.797	0.996
3	8	1.066	0.964
2	7	0.931	0.931
4	6	1.268	0.982
5	1	9.000	0.737

The openings under the gates were entirely submerged, except in the case of Item 5 where the lower edge of the gate was about 7.06 ft above the level of the lower pool.

There is a large pool up stream from the Starved Rock Lock, but the deep water is on the opposite side of the valley where it has just the depth necessary for navigation. Down stream, the water is carried away in the standard channel of the waterway 200 ft wide by 10 ft deep.

The consequence of this is that when the upper valves are opened to fill the lock, the up-stream water drops and does not regain the full head for about 5 min; therefore, for two-thirds of the time of filling, the flow is not under full head. Furthermore, when the lower valves are opened to empty the lock, the first rush of water lowers the down-stream flow for a few seconds. It then surges back and rises higher than normal, decreasing the head for emptying materially; this excess head down stream gradually lowers as the flow from the lock decreases.

In the past, trouble has been caused by surging of water in locks during filling or emptying, making it necessary to limit the rate of rise or fall in the lock. A careful study was made of this subject by Mr. Cornish, and he came to the conclusion, that there should be more ports from the culverts into the up-stream half of the lock than into the down-stream half. He developed a formula for the spacing of the ports.

By referring to Fig. 2 in the paper, it will be seen that there are six ports from each culvert in the up-stream half of the lock, while there are only four in the down-stream half. It will also be noticed that these ports are not equal distances apart, there being a long distance near the center of the lock with no ports.

On account of Mr. Cornish's study, the tests were made to find not only the time of filling and emptying and the corresponding coefficients, but to see whether objectionable surging had been eliminated. The latter is much more important than a small difference in the coefficient of filling and emptying, as it generally takes considerable time to moor a vessel securely if the surging is great. In the tests, therefore, an attempt was made to determine the exact time of fall or rise of each foot at each end of the lock during each filling or emptying. An observer was stationed at each end of the lock with a tape line attached to a float. During the operation each observer would read the time on a stop-watch for each foot of rise or fall. The method was rather crude as the floats were not protected from the rough water in the lock during filling.

In Fig. 22 is shown some typical curves of flow for emptying and filling the lock. Curve 1 is for emptying the upper end and Curve 2 for emptying the lower end. Comparing these two curves, it will be seen that there is a slight surging, but not more than 1 in. from the normal at either end, because of a maximum difference in level of from $1\frac{1}{2}$ to $1\frac{3}{4}$ in. between the water at the up-stream and down-stream ends. The following flow coefficients were determined, from the time the valves were fully open, on the basis of difference in head:

	Emptying lock	Filling lock
Culverts, upper end, $C =$	0.76	0.77
Valves, upper end, $C =$	1.06	1.05
Culverts, lower end, $C =$	0.73	0.78
Valves, upper end, $C =$	1.03	1.06

It is seen that a coefficient of from 0.73 to 0.75 was obtained for the culverts and from 1.03 to 1.06 for the valves.

It was observed that at the beginning of the opening of the valves the water in the canal below the lower gates was first shoved away down stream and the level was lowered about 0.7 ft. It surged back quickly until it rose to 1.5 ft above the level of beginning of flow and then very slowly lowered

so that for almost the entire time of emptying the lock the actual head causing the flow was materially less than the theoretical head used in the computations. Therefore, the actual coefficient should be considerably greater than the one shown.

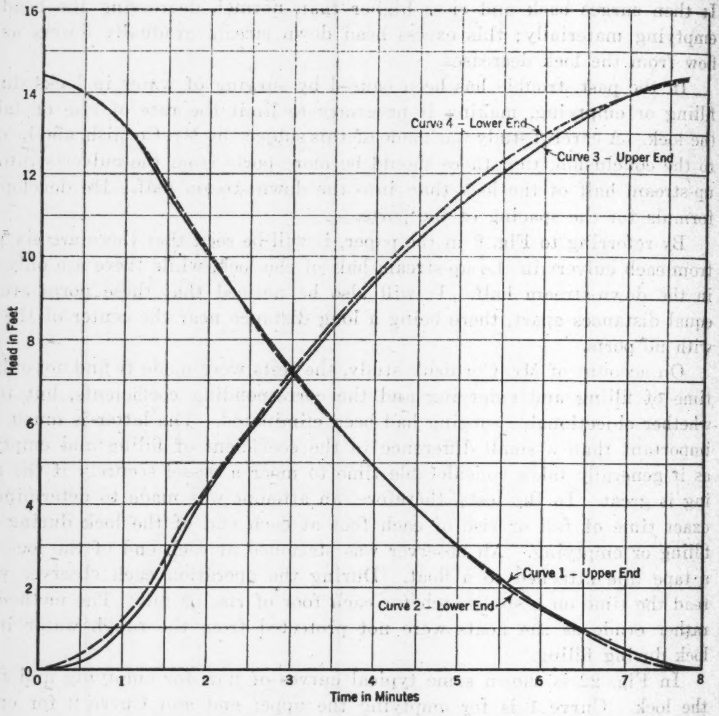

FIG. 22.—STARVED ROCK LOCK OPERATING TEST

Curves 3 and 4 in Fig. 22 are typical of flow for filling the lock, observation being made at both the up-stream and the down-stream ends of the lock the same as in the emptying test.

It will be noticed from these curves that the surface at the up-stream end is consistently from 2 or 3 in. above the surface at the lower end until just before the lock is full, when they practically coincide. It will also be noticed that coefficients of 0.77 and 0.78 are obtained for the culverts and 1.05 and 1.06

for the valves. In this case, also, the actual head causing the flow is considerably less than the head used in the computations from the fact that upon opening the valves the head-water falls 0.8 ft in the canal above the gates before the valves are fully opened, and does not regain its level for 5 min; therefore, the lock is filling under a smaller head than that used in the computations.

When the valves and culverts were designed by the writer, it was hoped to obtain, as a possible maximum coefficient, 0.83 for culverts and 1.15 for the valves. It was realized, however, that seldom is a coefficient of more than 0.75 obtained for the culverts in a lock of this type, and as the valves did not reduce the flow in the culverts below this coefficient, it seems conclusive that they are working as well as larger valves, which answers Mr. Sargent's question about the operation of the valves.

A tow consisting of a large towboat, dredge barge, and derrick barge, coupled end to end and occupying about 300 ft, was locked through between two of the tests. To show how little surging there is during filling at full speed, this tow had out only one stern line and one bow line for the entire tow, and neither of these lines were taut at any time during the filling. Furthermore, the vessels did not move more than a foot away from the lock-wall during filling. The entire time from passing the lower gate going in to passing the upper gate going out was 20 min, and a part of this time was used in taking on water after the upper gate had been opened.

TABLE 5.—COMPARISON OF FLOW COEFFICIENTS THROUGH CULVERTS AND VALVES

Description	Pedro Miguel Lock, Panama Canal	Starved Rock Lock, Illinois Waterway
Flow Coefficients:		
Filling chamber with two culverts	0.65	0.775*
Emptying chamber with two culverts	0.67	0.745*
Filling chamber with side culvert only	0.82	....
Emptying chamber with side culvert only	0.73	....
Filling chamber with center wall only	0.52	....
Emptying chamber with center wall only	0.66	....
Filling lock-chamber with two culverts (for valves)	....	1.055
Emptying lock-chamber with two culverts (for valves)	....	1.045
Areas of Cross-Section, in Square Feet:		
Valves	576	162
Culverts	510	226
Ports	1 285	350
Proportions:		
Valves	1.00	1.00
Culverts	0.88	1.40
Ports	2.23	2.16

* For culverts.

The coefficients obtained in filling and emptying the Pedro Miguel Lock[22] of the Panama Canal are compared with those of the Starved Rock Lock in Table 5, as are also the comparative areas of valves, culverts, and ports in the two structures. It will be noticed that the Panama locks have a culvert area 2.25 times, a port area 3.68 times, and a valve area 3.6 times, as great as the locks of the Illinois Waterway. The Panama locks have a ratio of valve area to culvert area of 1 to 0.88, whereas the Illinois Waterway locks have a ratio of 1 to 1.4, and the Waterway has as high a coefficient of flow as the Panama Canal.

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[22] "Hydraulics of the Locks of Panama Canal," by R. H. Whitehead, Vol. II, p. 200, of "The Panama Canal," by the late George W. Goethals, Hon. M. Am. Soc. C. E.

The spring sealing strips along the lower edge of the miter-gates to form contact with the miter-sill have given some trouble at the up-stream gates at Starved Rock Lock. When filling the lock a slight noise and vibration starts in the gates when the water in the lock has risen to within about 5 ft of the level of the water in the canal above the up-stream gate. This noise and vibration increases slightly for the next 2 ft of rise; then it gradually dies down and stops when the difference of level is somewhat less than 1 ft. Apparently, this vibration is caused by the pressure not being sufficient to keep the spring in close contact with the sill when there is a moderate difference in head. The same noise and vibration between these same levels occurs while emptying the lock. Some of the bolts attaching the spring seal to the gate have been broken by the vibration. It is singular that no noise or vibration occurs in the lower gates.

Bridges.—As stated in the paper, the State was required to replace twelve bridges across the Waterway. It was decided to have two of these bridges constructed by the State Division of Highways and the other ten by the Division of Waterways.

In Joliet, Ill., it was decided to construct four bridges of the rolling-lift bascule type. The fifth bridge in Joliet will be of the trunion bascule type similar to the Brandon Road Bridge. The Brandon Road Bridge was finished and thrown open to traffic in the latter part of 1932.

The cheapest type of bridge that could be used is the simple through-truss fixed bridge and whenever conditions would allow, this type was used. There are two bridges of this type. The fourth type used is a cantilever at Ottawa, Ill. This bridge, while costing slightly more than a simple through truss, was much better suited to this location on account of the long center span, as the Fox River enters the Illinois River immediately above the bridge. Furthermore, the elimination of falsework under the main span is a very desirable feature in the construction of the bridge.

Mr. Sargent has raised various points in discussion. The question as to the success of the future method of valve construction has already been answered. The valves operating under their own weight only, in closing, work with thorough satisfaction as the friction in the roller bearings is very slight.

A skin is used only on one side of the gates and is solely for economy. The reason for putting it on the down-stream side was to make it possible to put the miter-sill along the down-stream side of the gate instead of back beneath the gate.

The emergency dam at Lockport is so simple that there is little prospect of its giving trouble, and it has been operated and thoroughly adjusted. The recess under the up-stream leg is protected so that nothing can get in it but sediment, and this can be sluiced out at any time by connecting the recess with the down-stream pool at one end and the up-stream pool at the other. This will wash the chamber out thoroughly.

Lines of mooring posts have been provided at various levels in recesses in the side-walls of the locks so that lines can be changed as the barges rise or fall. As stated, all appreciable surging has apparently been eliminated in these locks so that very little mooring is needed.

Mr. Walker inquires about the success of the Venturi principle in the valves. He also inquires about the reasons for the adoption of the miter-gates with a rise of 1 to 2 in the sills and with parallel faces.

A very thorough study was made in the design of the gates for the Panama Canal Locks, and actual designs were made upon which prices were obtained. The result was that the type of gate adopted is the one that was shown to be most economical in that investigation. Inasmuch as the Illinois Waterway Locks are the same width as those at Panama, it is evident that the same condition obtains here. A complete account[23] of the investigations pertaining to the Panama Locks was published by Henry Goldmark, M. Am. Soc. C. E.

The detail of the miter-post is exactly the same as the quoin-post, except that the adjustable bearing pieces are alike in both miter-posts and that the convexity of the abutting surfaces are of a much longer radius.

Both Mr. Walker and Mr. Molitor criticize, severely, the method of analyzing the stresses in the side-walls in the part surrounding the culverts. Attention is called to the original paper under "Side-Wall Analysis," where the writer states, "it is not claimed that this method will give exact results, but it is believed to give results nearer the truth in a case of this kind than any other method." He is still of that opinion. Contrary to opinions expressed by Messrs. Walker and Molitor it is believed to be well worth the time spent upon it even if the results are approximations. Engineering structures are designed and built every day under assumptions just as uncertain as any made in this case.

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[23] "Lock Gates, Chain Finders, and Lock Entrance Caissons," by Henry Goldmark, M. Am. C. E., in Vol. II, pp. 87-93, "The Panama Canal" by George W. Goethals, Hon. M. Am. Soc. C. E.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1834

CONSTRUCTION WORK ON A FEDERAL RECLAMATION PROJECT

BY MORRIS MASON,[1] JUN. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. JOHN SANFORD PECK, EDWARD W. BUSH, ORRIN H. PILKEY, CLIFFORD A. BETTS, GEORGE C. IMRIE, ARTHUR RUETTGERS, AND MORRIS MASON

SYNOPSIS

This paper deals specifically with the Kittitas Division of the Yakima Project, in the State of Washington, constructed by the United States Bureau of Reclamation. All irrigation projects undertaken by the Bureau are similar in character, although new ideas of design and new methods of construction are introduced in each new structure. Most of the structures on any one project are small in size, but generally the topography of the ground and the type of structure are such that the method of constructing each unit must be different from any other. There is a great variety in size and type of structures. The choice of a method of construction is placed in the hands of the contractors, subject to the approval of the engineers. In the case of small units, it is uneconomical to install elaborate and expensive set-ups of machinery.

It is the purpose of this paper to give the history and general description of the Kittitas Division of the Yakima Project as a whole, with an enumeration and description of the various structures, and the more common methods of construction. The large quantity of concrete involved (150 000 cu. yd.), varying in strength from mass concrete in the diversion dam to the extremely high-strength concrete in the high-head siphons (all of which must be of high impermeability), made necessary a very strict supervision of the proportioning and mixing of the aggregate. The method by which a concrete of known and uniform quality was attained, will also be of interest to engineers engaged in this class of work.

NOTE.—Published in October, 1931, *Proceedings*.

[1] Engr., Standard Oil Co. (Indiana), Wood River Refinery, Wood River, Ill.

HISTORY

The history of irrigation in the Kittitas Valley, situated in the central part of the State of Washington, dates back to before the appearance of white men in that region. As farming developed in a larger way and more land was put under cultivation, there was more demand for water. Private and community low-level canals were then put in to serve a few farms along the Yakima River and its tributaries. As early as 1889 active survey work was in progress for the construction of a high-line canal to serve the farms on the higher benches and to provide more water for the low farms. Actual construction was begun on some of these canals, but it resulted in failure because of lack of funds. Stretches of the canals are still in evidence. Some survey work was done by the Government on the high-line canal prior to 1910. The Kittitas Reclamation District entered into a contract with the United States for water from Government reservoirs in 1921. In 1923 a board of engineers appointed by the United States Bureau of Reclamation decided that the Kittitas Unit should be selected for first construction, four divisions of the Yakima Project being considered. These were:

1.—Kittitas Division, covering approximately 72 000 acres of irrigable land, near Ellensburg, Wash.
2.—Moxee Division, covering about 35 000 acres of irrigable land, just east of Yakima, Wash.
3.—Rosa Division, covering about 60 000 acres of irrigable land, above the present Main Sunnyside Canal and lying north of the Towns of Sunnyside and Grandview, Wash.
4.—Kennewick Division, covering about 35 000 acres of irrigable land, west of the Columbia River at its confluence with the Yakima River near the Town of Kennewick, Wash.

Surveys were begun in 1924. General authority for the construction of the Kittitas Division was contained in a contract dated December 19, 1925, between the United States and the Kittitas Reclamation District. The active construction period began in April, 1926. Prior to this time the feasibility of the project from an engineering and economic standpoint had been thoroughly investigated. Topographic and soil surveys of the entire area were made, and the value of each 40 acres, or less (according to the legal subdivision lines, and including improvements), had been determined by a Board of Appraisers appointed jointly by the Government and the Reclamation District.

GENERAL DESCRIPTION

The Kittitas Division of the Yakima Project includes the arable lands of the Kittitas Valley in the State of Washington, with the exception of the bottom-lands which have been irrigated from the Yakima River for many years. Figs. 1 and 2 show the part of the Kittitas Valley to be embraced in the District. The main body of the land includes ranges in elevation from 1 700 to 2 100 ft., and surrounds the older irrigated lands in the form of an ellipse, with the Yakima River as the major axis and the City of Ellensburg near the center. (See Fig. 2.)

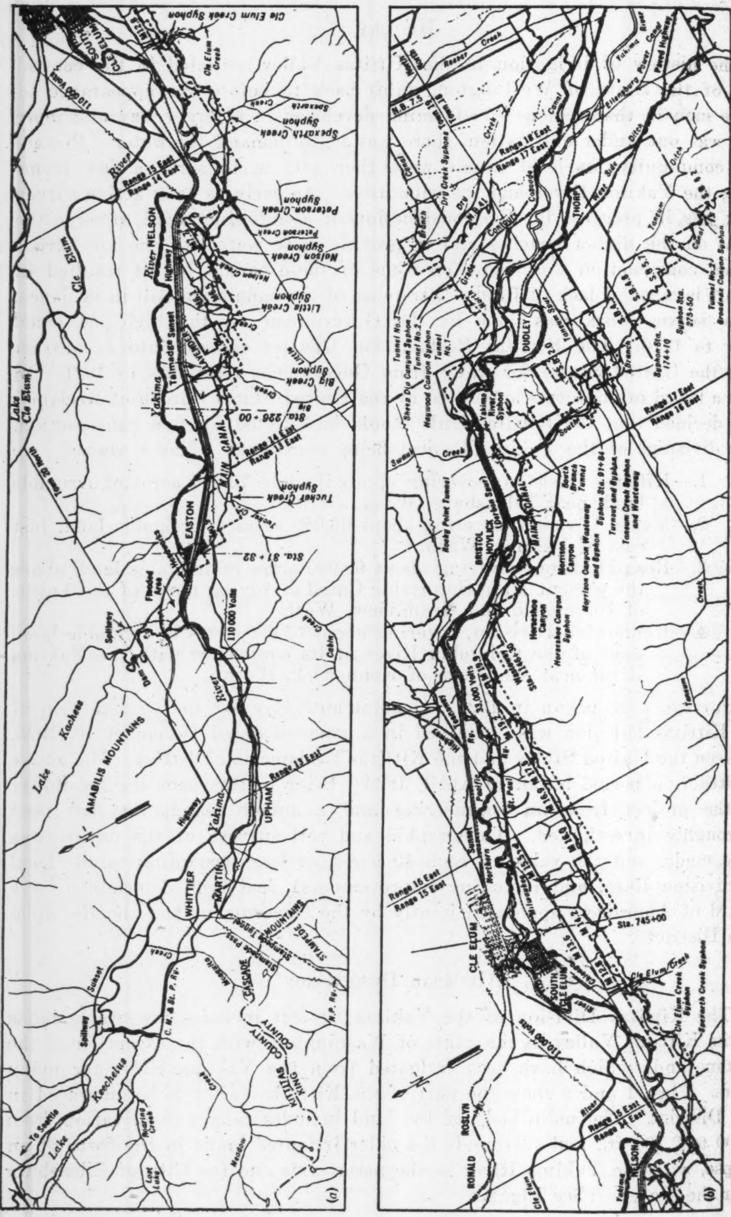

(a)
(b)
Lake Keechelus
Lake Kachess
Lake Cle Elum
AMABILIS MOUNTAINS
STAMPEDE MOUNTAINS
Stampede Pass
Yakima River
Cle Elum River
MAIN CANAL
EASTON
UPHAM
MARTIN
WHITTIER
NELSON
SOUTH CLE ELUM
CLE ELUM
ROSLYN
RONALD
BRISTOL
HOVLICK
DUDLEY
THORP
KITTITAS COUNTY
KING COUNTY
Tucker Creek Syphon
Big Creek Syphon
Little Creek Syphon
Nelson Creek Syphon
Peterson Creek Syphon
Spexarth Creek Syphon
Cle Elum Creek Syphon
Sta. 81+82
Sta. 226+00
Sta. 745+00
110 000 Volts
33,000 Volts
Range 13 East
Range 14 East
Range 15 East
Range 16 East
Range 17 East
Range 18 East
Sunset Highway
Rocky Point Tunnel
Horseshoe Canyon Syphon
Morrison Canyon Wasteway and Syphon
South Branch Tunnel
Taneum Creek Syphon and Wasteway
Tunnel No. 2 Broadway Canyon Syphon
Ellensburg Power Canal
Cascade Canal
West Side Ditch
Town Ditch
Paved Highway
Reecer Creek
Dry Creek
Swauk Creek
Taneum Creek
Chicago, Milwaukee & St. Paul Ry.
Northern Pacific Ry.
To Seattle

FIG. 1.—OVERLAPPING SECTIONS OF THE NORTHWEST AND OF THE KITTITAS DIVISIONS.

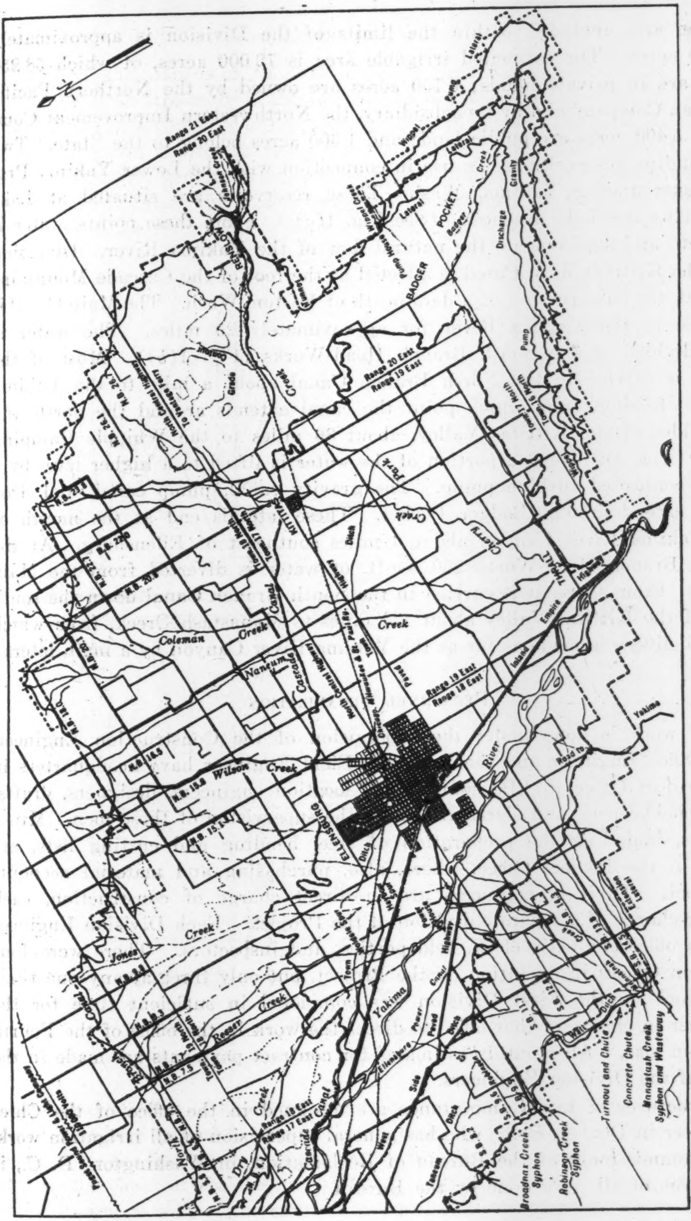

Range 21 East
Range 20 East
Range 19 East
Range 18 East
Range 17 East
Wippel Pump Lateral
BADGER POCKET
Badger Creek Lateral
Discharge
Gravity
Pump
KITTITAS
ELLENSBURG
Wilson Creek
Coleman Creek
Naneum
Cascade Canal
Cherry Creek
Park Creek
Ditch
Highway
Yakima River
Road to Yakima
Empire
Inland
Jones Creek
Reecer Creek
Dry Creek
West Side Ditch
Taneum Ditch
Broadnax Creek Syphon
Robinson Creek Syphon
Turnout and Chute
Concrete Chute
Manastash Creek Syphon and Wasteway
Town Ditch
Ellensburg Power
Chicago, Milwaukee & St. Paul Ry.

FIG. 2.—MAIN BODY OF LAND INCLUDED IN THE KITTITAS PROJECT FARTHER EAST OF, AND OVERLAPPING, FIG. 1.

The area included within the limits of the Division is approximately 96 000 acres. The estimated irrigable area is 72 000 acres, of which 58 250 acres are in private hands; 6 750 acres are owned by the Northern Pacific Railway Company and by its subsidiary, the Northwestern Improvement Company; 5 400 acres are public land; and 1 600 acres belong to the State. Two impounding reservoirs are in use in connection with the Lower Yakima Project, surrounding Yakima, Wash. These reservoirs are situated at Lake Keechelus and Lake Kachess. (See Fig. 1(*a*).) From these points, water is fed into and supplements the natural flow of the Yakima River. Diversion into the Kittitas Main Canal is affected at the foot of the Cascade Mountains through the construction of a dam north of Easton, Wash. The Main Canal is parallel to the Yakima River for approximately 25 miles. The water is then divided at the South Branch Head-Works (Fig. 1(*b*)). Most of the water is carried in the North Branch Canal about a mile to the Yakima River Crossing, from which point the canal extends around the north and east sides of the Kittitas Valley, about 36 miles to the Whipple Pumping Plant (Fig. 2). Here, a portion of the water is lifted to a higher level by a direct-connected turbine pump. The gravity canal, pump canal, and drop canal extend around Badger Pocket. These laterals end at the mouth of the Yakima River Canyon about 6 miles southeast of Ellensburg. At the South Branch Head-Works 250 sec-ft. of water is diverted from the Main Canal. From there, it is carried in the South Branch Canal down the south side of the Kittitas Valley about 14½ miles to Manastash Creek, from which point delivery is made as far as the Yakima River Canyon by a main lateral.

PERSONNEL ORGANIZATION

All work is done under the supervision of the Construction Engineer. The Office Engineer and the Principal Field Engineer have headquarters in the Project Office in Ellensburg. The location engineers, designers, draftsmen, bookkeepers, and clerks are under the supervision of these men. Minor designs, including the preparation of steel bending and cutting lists, are made in the Project Office, where, also, purchasing and material accounts are kept. Division engineers are in direct charge of construction, each having charge of a separate division of the Project. Each Division Engineer has an office engineer, clerk, field parties, and inspectors. There were four Division Offices in operation on the Project, but only three at any one time. The work on the First Division was completed in sufficient time for the engineer in charge of that office to direct the work in the office of the Fourth Division also. All monthly estimates for contract payments are made in the office of the Division Engineers.

Structures of major importance are designed in the office of the Chief Engineer in Denver, Colo., who has general supervision of all irrigation work. The Commissioner of the Bureau of Reclamation, in Washington, D. C., is in charge of all work done by the Bureau.

DIVERSION DAM

The diversion dam is on the Yakima River just above the Town of Easton and a mile down stream from the junction of the Kachess and Yakima Rivers. It is situated in a timbered country on the eastern slope of the Cascade Mountains in a region subject to heavy snowfall during the winter months. The dam is a straight, gravity-type, concrete structure with an over-all length of 248 ft.; and it raises the water surface approximately 43 ft. above the low-water level in the river. Fig. 3 shows the plan and elevation of the dam. The reservoir has an area of about 240 acres and impounds about 4 000 acre-ft. of water. The lake formed is about 1½ miles long by ½ mile wide. The level of the water in the lake must be held practically constant in order to keep the water in the canal at the designed height. Any appreciable rise in the lake level will damage both of the railway tracks that border the lake on the south

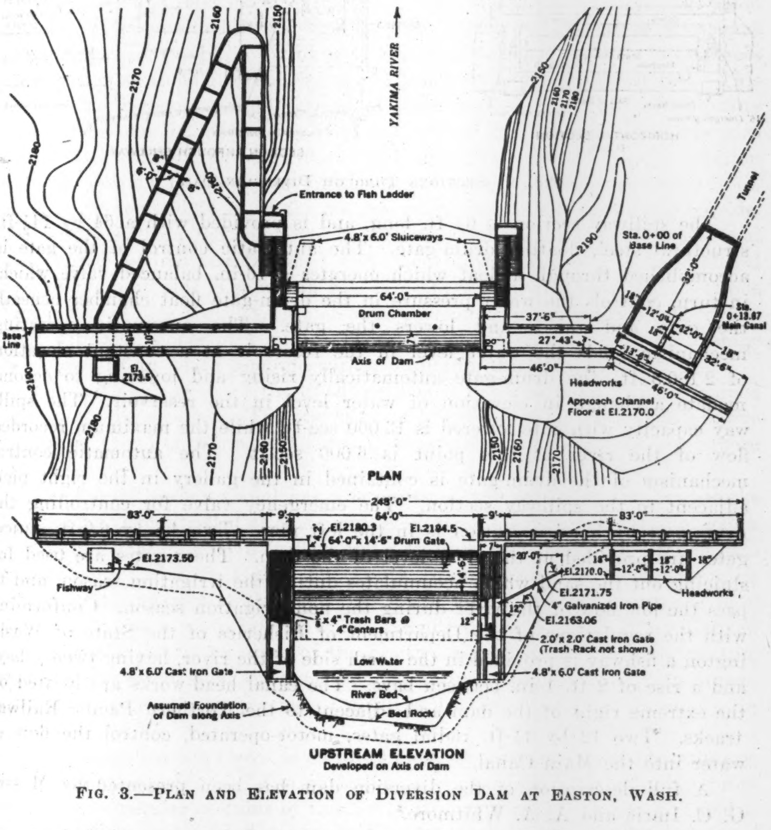
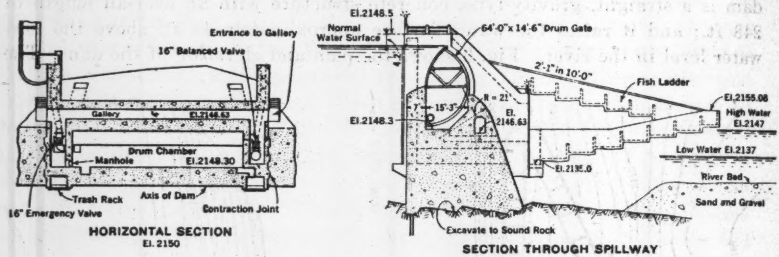

2160
2150
YAKIMA RIVER
2170
2180
2150
2160
2170
2180
8'-0"
8"
8"
2160
Entrance to Fish Ladder
4.8'x 6.0' Sluiceways
2190
Tunnel
Sta. 0+00 of Base Line
32'-0"
64'-0"
Drum Chamber
37'-6"
18"
12'-0"
18"
12'-0"
Sta. 0+13.87 Main Canal
Base Line
10'
27°-43'
13'-6"
18"
32'-6"
Axis of Dam
46'-0"
46'-0"
El. 2173.5
Headworks
Approach Channel Floor at El.2170.0
2190
2180
2170
2160
2150
2150
2160
2170
PLAN
248'-0"
83'-0"
9'
64'-0"
9'
83'-0"
El.2180.3
El.2184.5
64'-0"x 14'-6" Drum Gate
7'
20'-0"
El.2173.50
14'-6"
18"
12'-0"
18"
12'-0"
18"
El.2170.0
Fishway
El.2171.75
Headworks
4" Galvanized Iron Pipe
12"
12"
El.2163.06
⅝"x 4" Trash Bars @ 4" Centers
2.0'x 2.0' Cast Iron Gate (Trash Rack not shown)
Low Water
4.8'x 6.0' Cast Iron Gate
4.8'x 6.0' Cast Iron Gate
River Bed
Assumed Foundation of Dam along Axis
Bed Rock
UPSTREAM ELEVATION
Developed on Axis of Dam

FIG. 3.—PLAN AND ELEVATION OF DIVERSION DAM AT EASTON, WASH.

side. Provisions had to be made for passing by the dam, during the irrigation season, varying quantities of water released from the Kachess and Keechelus Storage Reservoirs for the lower divisions of the Yakima Project. The water level is controlled by an automatic, floating, drum-type gate mounted on the ogee crest of the dam (see Fig. 4).


Entrance to Gallery
16" Balanced Valve
Gallery
El.2146.63
Drum Chamber
Manhole
El.2148.30
Trash Rack
Axis of Dam
16" Emergency Valve
Contraction Joint
El.2148.5
Normal Water Surface
64'-0" x 14'-6" Drum Gate
2'-1" in 10'-0"
Fish Ladder
El.2155.08
High Water El.2147
R = 21'
7'
15'-3"
El.2148.3
El. 2146.63
Low Water El.2137
El.2135.0
River Bed
Sand and Gravel
Excavate to Sound Rock

HORIZONTAL SECTION
El. 2150

SECTION THROUGH SPILLWAY

FIG. 4.—SECTIONS THROUGH DIVERSION DAM

The spillway section is 64 ft. long, and is provided with a 64 by $14\frac{1}{2}$-ft. structural steel, floating drum-gate. The automatic control of the gate is accomplished through a float which operates a 16-in. balanced valve which, in turn, controls the water pressure in the drum-gate float chamber beneath the gate and raises and lowers the gate. The automatic operating mechanism holds the water level in the reservoir at a constant elevation of 2 180.3 ft., the drum-gate automatically rising and lowering to accommodate a change in elevation of water level in the reservoir. The spillway capacity with gate lowered is 13 000 sec-ft. while the maximum recorded flow of the river at this point is 6 000 sec-ft. The automatic control mechanism of the drum-gate is contained in the gallery in the right pier, adjacent to the spillway section. The emergency valve for controlling the drum-gate by hand operation is in the left pier. Two 4.8 by 6.0-ft. sluicegates are provided at the river level of the dam. These gates are used for sluicing out the sand which accumulates during the irrigation season, and to pass the low flow of the river during the non-irrigation season. Conforming with the regulations of the Department of Fisheries of the State of Washington a fishway is provided in the north side of the river, having twenty bays and a rise of 2 ft. 1 in. between bays. The canal head-works are located on the extreme right of the dam and adjacent to the Northern Pacific Railway tracks. Two 12 by 11-ft. radial gates, motor-operated, control the flow of water into the Main Canal.

A full description of the diversion dam has been presented by Messrs. G. C. Imrie and A. A. Whitmore.[2]

THE CANALS

The canal system is composed of a single primary canal and two branch canals, as shown in Fig. 1(*b*).

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[2] *New Reclamation Era,* March, 1931, p. 54.

Location.—From the diversion dam to the South Branch Head-Works, about a mile west of the Yakima River Crossing, the Main Canal of this project extends for a distance of about 25 miles. Its capacity at the point of diversion is 1 320 sec-ft. The variation in dimensions and hydraulic properties of the regular sections of the Main Canal is shown in Table 1. The side slopes in the regular sections of all canals were 1 on 1½ in the unlined sections and 1 on 1¼ in the lined sections. There were a number of special sections. All the reinforced concrete lining in the regular sections was 3 in. thick. About 50% of the Main Canal is concrete-lined although this lining is not all 3 in. thick and reinforced. Due to the topography of the ground, the canal follows along the side-hill slopes. In some places on account of very steep slopes it was impossible to build or to hold the lower banks. In these places combination sections of canal lining and bench flume were constructed. This section has a regular canal section on the inside and a vertical retaining wall on the outside. (See Fig. 5.) About 140 ft. of bench flume was also used. The walls of the bench flume and the outside wall of the combination sections were 11 ft. above the inside of the canal. These walls were 6 in. thick at the top and 14 in. at the bottom, reinforced on the inside and outside. The inside wall on the combination section was 6 in. of plain concrete and was poured against the trimmed rock face of the bluff. The hand-placed rock on the bluff side is shown in Fig. 5. There were also four canal sections termed "lined rock sections," which were excavated in shattered basalt formation, with side slopes of 1 on ½. These sections were lined with an average thickness of 6-in. plain concrete with a minimum thickness of 2 in.

The earth sub-grade of the lined sections of canal is trimmed and tamped to accurate dimensions. The operation of trimming canal sections to prepare for lining (see Fig. 6) is paid for as a separate unit at so much per square yard.

The South Branch Canal has a capacity of 250 sec-ft. at the head-works and the flow is controlled at this point by electric, hoist-operated, radial gates. The Main Canal gate is 15 ft. long and 8 ft. high, and the South Branch gate is 8 ft. long and 8 ft. high. The South Branch Canal is about 14½ miles long and its capacity at the end is 32 sec-ft. and, from that point, the total flow is taken in a main lateral which extends to the Yakima River Canyon (see Fig. 2). About 1¼ miles of the South Branch Canal is lined with 3-in. reinforced concrete slab; the remainder is unlined. The variation in dimensions and hydraulic properties of the regular canal sections is shown in Table 1.

The North Branch Canal is 36 miles long, 3¼ miles of which are ordinary, or combination, section. The variation in dimensions and hydraulic properties of the regular sections of the North Branch Canal is shown in Table 1. A part of the regular lined section is 4-in. reinforced concrete; the remainder of the regular lining is 3-in. reinforced concrete. There is a stretch of combination section between the South Branch Head-Works and the Yakima River Crossing. It has a vertical wall on the outside, 9 ft. high, 6 in. thick at the top, and 11 in. thick at the base, with reinforcement on the inside.

TABLE 1.—DIMENSIONS AND HYDRAULIC PROPERTIES OF REGULAR SECTIONS USED IN ALL CANALS

Dimension	MAIN CANAL		NORTH BRANCH CANAL		SOUTH BRANCH CANAL	
	Unlined sections	Lined sections	Unlined sections	Lined sections	Unlined sections	Lined sections
Number of sections	5	9	19	6	15	4
Water depth, in feet	10.50 to 11.35	8.58 to 9.75	4.15 to 8.20	4.00 to 7.88	1.80 to 4.40	3.10 to 4.30
Lining depth, in feet	....	9.93 to 11.00	....	5.00 to 9.85	....	3.85 to 5.30
Top of bank to water surface, in feet	5.00	4.00	1.0 to 3.0	2.0 to 4.0	1.2 to 2.5	1.5 to 3.0
Bottom width, in feet	30.00	11.0 to 12.0	12.0 to 30.0	6.0 to 10.0	6.0 to 16.0	4.0 to 6.0
Area, A, of stream, in square feet	480.38 to 534.00	189.66 to 236.00	81.84 to 346.86	44.00 to 148.47	15.66 to 76.86	24.41 to 48.91
Velocity, V, in feet per second	2.45 to 2.54	5.53 to 6.63	2.23 to 3.98	5.97 to 7.41	1.49 to 3.22	4.94 to 6.41
Flow, Q, in cubic feet per second	1 210 to 1 320	1 145 to 1 320	200 to 926	260 to 925	32 to 250	120 to 250
Assumed values of n in Kutter's formula	0.0225	0.014	0.0225 to 0.030	0.014	0.0225 to 0.030	0.014
Hydraulic radius, r, in feet	7.08 to 7.49	4.89 to 5.46	2.34 to 5.82	2.34 to 4.34	1.25 to 3.12	1.75 to 2.47
Slope, S, of hydraulic gradients	0.00009 to 0.0001	0.0003 to 0.00047	0.00015 to 0.001	0.0005 to 0.001	0.000315 to 0.001	0.0007 to 0.001

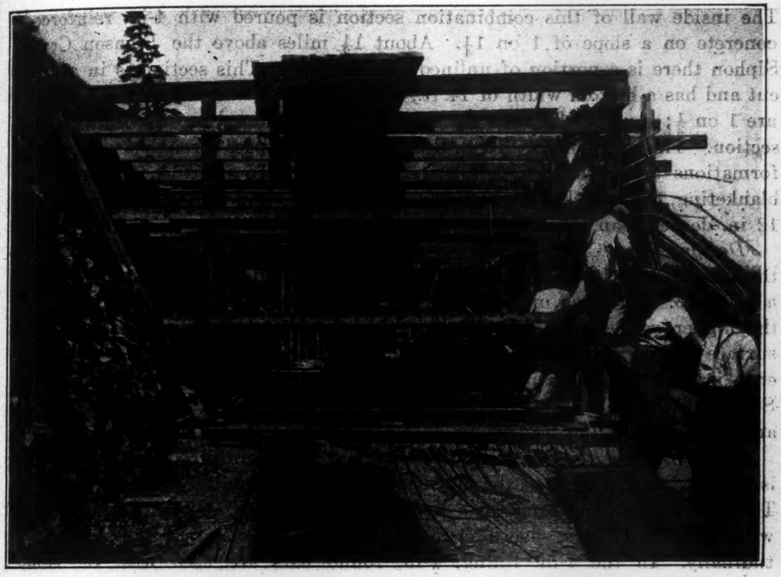

FIG. 5.—PLACING OF FORMS FOR COMBINATION SECTION

FIG. 6.—TRIMMING CANAL SECTION FOR LINING

The inside wall of this combination section is poured with 4-in. reinforced concrete on a slope of 1 on $1\frac{1}{4}$. About $1\frac{1}{2}$ miles above the Johnson Creek Siphon there is a portion of unlined rock section. This section is in a deep cut and has a bottom width of 14 ft., with 4.75 ft. of water. The side slopes are 1 on $\frac{1}{2}$; a value of Kutter's n equal to 0.030 was used in the design of this section. The parts of the North Branch Canal that pass through porous gravel formations are covered with an earth blanket. To provide for the earth blanketing, the gravel is excavated from the bottom and sides to approximately 12 in. deeper than the limits of the normal canal section.

Design.—The Chezy-Kutter formula was used in design throughout. In the lined sections a value of $n = 0.014$ is used; and in the unlined sections $n = 0.0225$. In some unlined sections this was increased to 0.025 and 0.03, the higher values being used in some parts of the canals when the inside surface was very rough due to the presence of cement, gravel, and rock. Reference is made to the paper[3] entitled "The Hydraulic Design of Flume and Siphon Transitions," by Julian Hinds, M. Am. Soc. C. E., which gives plans and elevations of the transitions used by the Bureau of Reclamation.

Canal Lining Reinforcement.—The 3 and 4-in. reinforced concrete lining is provided with a berm at the top that extends horizontally with a 5-in. width. This creates an appearance of a 5-in. lining. The 3-in. lining is reinforced with $\frac{3}{8}$-in. round bars, spaced 24 in. apart transversely and 12 in. apart longitudinally. In the 4-in. lining, $\frac{1}{2}$-in. round bars are used with the same spacing. The $\frac{3}{8}$-in. bars are lapped 18 in. and the $\frac{1}{2}$-in. bars are lapped 24 in. In all other reinforced concrete work the length of splices is 40 diameters. Vertical construction joints extend across the base and up the slopes at about 14-ft. intervals. All longitudinal steel is continued through these joints without providing for expansion.

Canal Sub-Drainage.—All the lined sections of the canals are designed with a system of under-drains. The longitudinal drains are 4 to 10-in. vitrified clay or concrete sewer pipe laid in gravel-filled trenches in the center of the canal base. When side drains, or "finger drains," were used the longitudinal drains were near the upper side and parallel to it. The longitudinal drain-tile was laid with open joints. In Fig. 7 it has been covered with tar paper. The trenches have a minimum depth of 10 in. and drain from 200 to 800 ft. from two directions into the lateral drains, which are 4 to 12-in. vitrified clay or concrete sewer pipe laid with cemented joints. The laterals drain the water from the longitudinal drains and extend beyond the canal prism and the waste banks. These lines have concrete cut-off and head-walls. (See Fig. 8.) The side drains or finger drains were spaced from 5 to 10 ft. These are trenches filled with gravel which extend from the longitudinal drains up the side slopes on the upper side of the canal. When completed they are covered with a thin layer of mortar to hold the gravel in place on the 1 on $1\frac{1}{4}$ slopes until the lining is poured.

Canal Side Banks.—A 14-ft. roadway on one side bank along the entire length of the Main Canal is intended for operation and maintenance purposes

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[3] *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 1423.

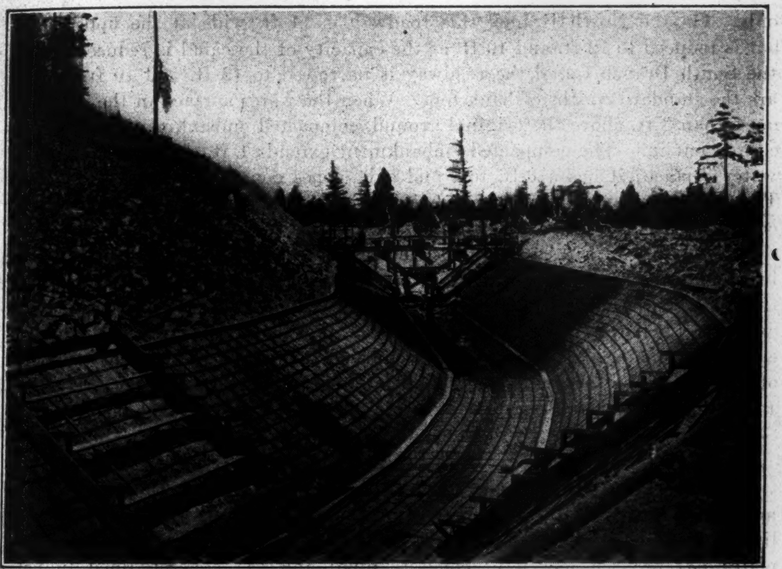

FIG. 7.—REINFORCING STEEL IN PLACE FOR CANAL LINING, SHOWING THE SCAFFOLDS USED IN PLACING STEEL

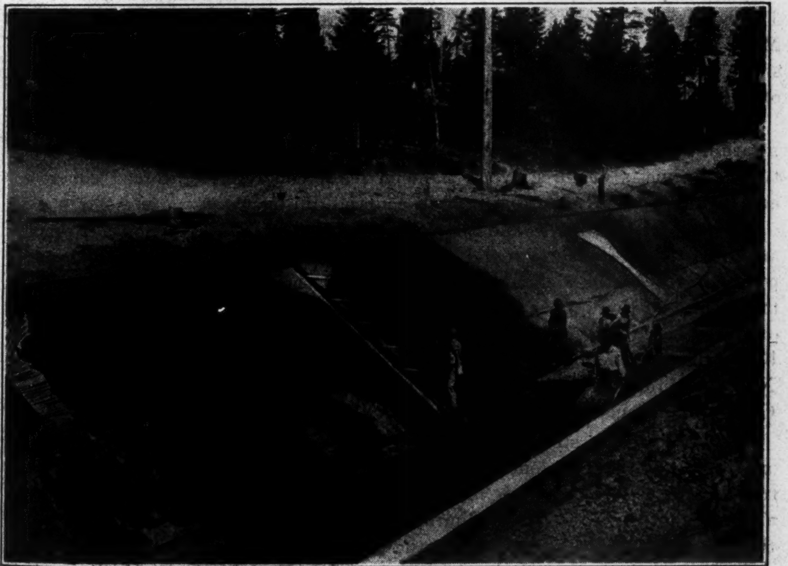

FIG. 8.—SIDE DRAINS, OR FINGER DRAINS, SHOWING LONGITUDINAL DRAINS NEAR BASE OF UPPER SLOPE

only. On the North Branch this roadway is 14 ft. wide at the upper end, but is reduced to 12 ft. and 10 ft. as the capacity of the canal is reduced. On the South Branch Canal the roadway is narrowed to 12 ft. and 10 ft. which are the standard widths of bank tops. When the water surface in the canal is more than 3 ft. above the original ground, compacted embankment or puddled core is put in. The compacted embankment extends 1 ft. above the proposed water surface; it has a 6-ft. top, and side slopes generally equal to 1 on 1½. The compacted embankment is placed adjacent to the canal prism in successive horizontal layers, not thicker than 6 in. A roller weighing not less than 2 000 lb. per lin. ft. does the compacting. These banks are usually built in advance of the regular side bank. Puddled core is placed vertically not less than 3 ft. in width, in the center of the canal bank. This core must extend 1 ft. above the proposed water level and about 3 ft. into undisturbed earth. All material is deposited and churned in water. In the lined portion of the canal the compacted embankment and puddled core are constructed to the top of the lining.

INVERTED SIPHONS

The very rough topography made necessary the construction of a large number of siphons all of which may be classified in four types: (1) Monolithic reinforced concrete; (2) combination reinforced concrete and steel plate; (3) pre-cast lock-joint concrete pipe; and (4) concrete-lined tunnel siphons with both circular and horseshoe barrel. Taneum Siphon on the South Branch (Fig. 1(*b*) and Fig. 9) is one of the siphons constructed of lock-joint pipe; the siphons at Horseshoe and Morrison Canyons on the Main Canal (Fig. 1(*b*)) have steel pipe for the lower part where the hydrostatic head is relatively high; Dry Creek Siphon on the North Branch is one of the monolithic reinforced concrete structures.

The pressure tunnel siphon under the Yakima River passes through a stratum of basalt at an average depth of about 150 ft. below the ground. The tunnel lining is of three types, namely, plain concrete, reinforced concrete, and segmented steel plates with plain concrete lining.

Tucker Creek Siphon and the siphon at Station 275+50, South Branch, are square; the others are round. Table 2 gives the dimensions and hydraulic properties of all the siphons. In Column (3) the maximum head is the distance between the water surface in the inlet and the lowest point of the invert of the siphon. The horizontal distance in Column (4) is measured from the beginning of the inlet transition to the end of the outlet transition. Items Nos. 1 to 9 refer to the Main Canal; Items Nos. 10 to 16, to the South Branch Canal; and Items Nos. 17 to 22, to the North Branch Canal.

The reinforcing steel in the round cast-in-place siphons consists of from one to three layers, depending on the hydraulic head on the siphons. The transverse steel for one layer is bent in the form of an ellipse with the major axis horizontal, and is placed a minimum distance of 3 in. from the inside at the top and bottom and 3 in. from the outside on the horizontal axis. When more than one layer is used, the rings are circular. Each bar is a half ellipse with

TABLE 2.—DIMENSIONS AND HYDRAULIC PROPERTIES OF SIPHONS

		DIMENSIONS				HYDRAULIC PROPERTIES					
Item No. (1)	Location (see Figs. 1 and 2) (2)	Maximum head, in feet (3)	Horizontal length, in feet (4)	Inside diameter, in feet (5)	Thickness of shell, in inches (6)	Area, *A*, of stream, in square feet (7)	Velocity, *V*, in feet per second (8)	Flow, *Q*, in cubic feet per second (9)	Hydraulic radius, *r*, in feet (10)	Slope, *s*, of hydraulic gradient (11)	Assumed values of *n* in Kutter's formula (12)
1	Tucker Creek.........	19.73	148	11.5 (square)	12	132.25	10.00	1 320	2.88	0.00215	0.014
2	Big Creek............	30.55	188	12.5	13	122.71	10.76	1 320	3.13	0.00230	0.014
3	Little Creek.........	30.33	188	12.5	13	122.71	10.76	1 320	3.13	0.00277	0.014
4	Nelson Creek.........	59.67	488	12.0	13 and 12	113.10	11.66	1 320	3.00	0.00280	0.014
5	Peterson Creek........	77.29	363	13.0	14 and 13	132.70	9.95	1 320	3.25	0.00183	0.014
6	Spexarth Creek.......	62.05	424	12.0	12 and 13	113.10	11.27	1 275	3.00	0.00262	0.014
7	Cle Elum Creek.......	125.33	588	12.0	15 to 12	113.10	11.07	1 252	3.00	0.00253	0.014
8	Horseshoe Canyon.....	141.53	862	12.1 12′–1″	18 to 12	114.67	10.29	1 180	3.02	0.00215	0.014
9	Morrison Canyon......	239.99	1 050	12.1 12′–1″	15 to 12	114.67	10.00	1 145	3.02	0.00202	0.014
10	Station 91 +84.......	48.65	406	5.8 5′–10″	6 to 7	26.72	9.36	250	1.46	0.00453	0.014
11	Taneum Creek........	207.10	3 326	4.5 (pipe) 5.0 (pipe)	4⅜ 5¼	15.90 19.63	9.12 7.39	145 145	1.125 1.25	0.00460 0.00265	0.012 0.012
12	Station 174 +10......	76.11	464	4.5 (pipe)	4⅜	15.90	9.12	145	1.125	0.00460	0.012
13	Station 275 +50......	13.68	163	5′–3″ 5.3 (square)	6 and 7	27.44	5.30	145	1.35	0.00164	0.014
14	Broadnax............	54.76	508	4.5 (pipe)	4⅜	15.90	8.53	135	1.125	0.00400	0.012
15	Robinson Creek.......	14.08	191	5.0 (pipe)	5¼	19.63	5.35	105	1.25	0.00140	0.012
16	Manastash............	19.45	486	3.0 (pipe)	3¼	7.07	7.78	55	0.75	0.01473	0.012
17	Hayward Canyon......	158.15	545	11.2 11′–2″	11 to 16	97.93	9.47	925	2.79	0.00200	0.014
18	Sheep Dip Canyon.....	117.89	536	11.2 11′–2″	11 to 14	97.93	9.47	925	2.79	0.00200	0.014
19	Dry Creek...........	52.69	1 637	11.2 11′–2″	11 to 12	97.93	9.47	925	2.79	0.00200	0.014
20	Little Dry Creek......	35.87	415	11.2 11′–2″	11	97.93	9.47	925	2.79	0.00200	0.014
21	Johnson Creek........	115.86	1 815	6.0 (pipe)	6	28.26	9.92	280	1.50	0.00377	0.012
22	Siphon No. 6.........	43.46	356	6.0 (pipe)	6	28.26	9.92	280	1.50	0.00377	0.012

the 40-diameter laps staggered and placed on opposite diagonal quarter-points of the barrel. The 40-diameter laps of the longitudinal steel are staggered by spiraling the joints around the barrel.

Each of the siphon inlets is made up of two transitions, the first from the canal section to a square barrel entrance, and the second from the square entrance to a round barrel. (See Fig. 9.) The outlets reverse the order of the transitions. When the velocity is great enough to carry gravel in suspension, the entrances to siphons on lined canal sections are provided with gravel traps. These traps are 2 to 3-ft. depressions in the canal base, extending 40 to 80 ft. in an up-stream direction.

Construction joints with 3 by 4-in. keyways, are spaced about 32 ft. apart. These joints are furnished with 4-in., 5-in., or 6-in. copper strips which are cast around the entire circumference of the barrel. The strip has a V-shaped crimp, $\frac{5}{16}$ in. deep, in the center, which allows for expansion and contraction of the barrel, without permitting leaks. When steel pipe is used in the lower part of the siphon, expansion is provided for by a sliding joint at each end of the pipe where it meets the concrete part.

The inside forms are supported on 1-in. square reinforcing bars bent in the form of a cradle and embedded at each end in a 1 ft. by 5 by 1.5-ft. concrete pier. As concrete is poured in the barrel section and around these supports they become a part of the structure, acting as anchors. The inside form supports are spaced on 8-ft. centers.

All monolithic siphons are furnished with an 8-in. blow-off valve at the lowest possible point for gravity drainage. The siphons with steel pipe are also furnished with a manhole near the bottom.

WASTEWAYS

On all the canals provision is made for wasting the water during operation, or in case of emergency. In the Main Canal the water is controlled by standard radial gates, one in the canal and one in the wasteway chute. These gates are lifted by motor-driven hoists. The gates in the wasteways of the South Branch Canal are operated by hand. In the Main Canal there are wasteways at Peterson Creek Siphon, at Station 1146+30, and at Morrison Canyon Siphon. In the South Branch Canal, wasteways are located at Taneum Creek Siphon and Manastash Creek Siphon. Ten on the North Branch Canal are located at the following places: Yakima River Crossing, Naneum Creek, Johnson Creek, Cook Creek, Caribou Creek, Whipple Pumping Plant, one on each of the three laterals where they cross Badger Creek, and the wasteway to the Yakima River at the end of the lateral system.

Peterson Creek Wasteway is a 12 by 8-ft. flume, 280 ft. in length. The head-works are located in the inlet transition to the siphon. The flow of water is controlled by a 16-ft. radial gate in the wasteway and a 12-ft. gate in the canal.

The wasteway at Station 1146+30 consists of an 8 by 7-ft. concrete flume, 1 231 ft. long, with a drop in elevation of 340.5 ft. The slopes were 0.1800 for the upper part, and 0.6536 for the lower. The flume extends to the inlet

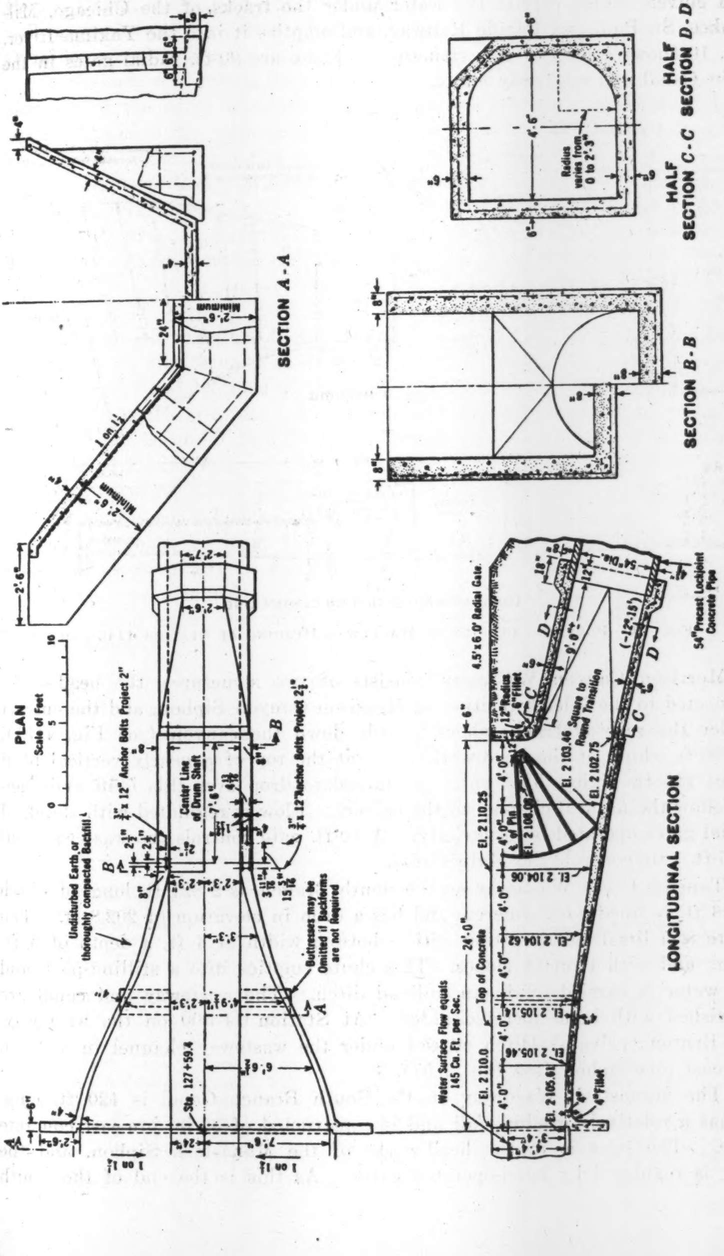
PLAN
Scale of Feet
0 5 10
SECTION A-A
SECTION B-B
HALF SECTION C-C
HALF SECTION D-D
LONGITUDINAL SECTION
Minimum 2'-6"
1 on 1½
24"
Radius varies from 0 to 2'-3"
4'-6"
Undisturbed Earth or thoroughly compacted Backfill
¾" x 12" Anchor Bolts Project 2"
¾" x 12" Anchor Bolts Project 2½"
Center Line of Drum Shaft
Buttresses may be Omitted if Backforms are not Required
Sta. 127+59.4
Water Surface Flow equals 145 Cu. Ft. per Sec.
Top of Concrete
24'-0"
El. 2110.0
El. 2110.25
El. 2105.81
El. 2105.46
El. 2105.10
El. 2104.62
El. 2104.06
El. 2104.00
El. 2103.46
El. 2102.75
4.5'x 6.0' Radial Gate
12" Radius 6" Fillet
Square to Round Transition
54" Precast Lockjoint Concrete Pipe
54" Dia.
4" Fillet
9'-0"
(-12.15')

FIG. 9.—PLAN AND SECTIONS OF INLET TO TANEUM CREEK SIPHON

of a culvert which carries the water under the tracks of the Chicago, Milwaukee, St. Paul and Pacific Railway, and empties it into the Yakima River. Fig. 10 shows a plan of this structure. There are 20-ft. radial gates in the Main Canal and wasteway chute.

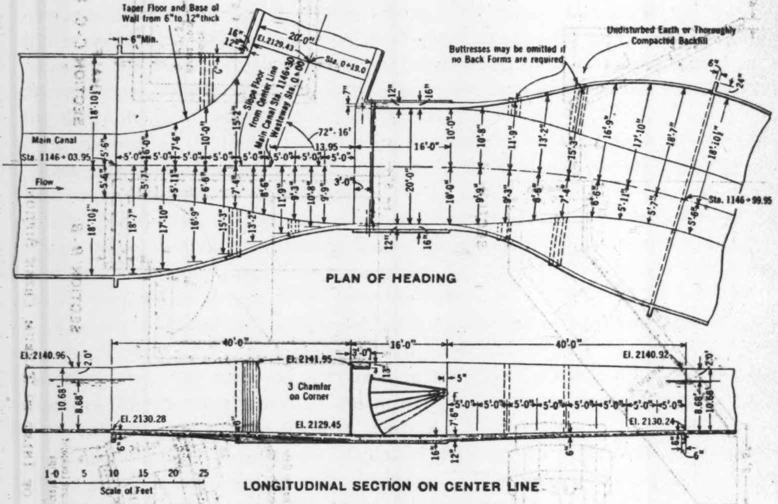
Taper Floor and Base of Wall from 6" to 12" thick
6" Min.
16"
12"
20'-0"
El. 2129.43
Sta. 0+19.0
Slope Floor from Center Line
Main Canal Sta. 1146+30
Wasteway Sta. 0+00
72°-16'
13.95
Main Canal
Sta. 1146+03.95
Flow
Buttresses may be omitted if no Back Forms are required
Undisturbed Earth or Thoroughly Compacted Backfill
Sta. 1146+99.95
16'-0"
20'-0"
3'-0"
PLAN OF HEADING
40'-0"
16'-0"
40'-0"
El. 2140.96
El. 2141.95
El. 2140.92
3 Chamfer on Corner
El. 2130.28
El. 2129.45
El. 2130.24
10.68'
8.68'
2.0'
5"
7'-6"
1-0 5 10 15 20 25
Scale of Feet
LONGITUDINAL SECTION ON CENTER LINE

FIG. 10.—PLAN AND PROFILE OF WASTEWAY HEADING AT STATION 1146+30

Morrison Canyon Wasteway consists of two structures, the head-works connected to the inlet transition of Morrison Canyon Siphon, and the culvert under the railway tracks about ½ mile down the canyon (see Fig. 1(*b*)). A 100-ft. chute at the head-works ends at the top of a nearly vertical bluff about 125 ft. high. The water is allowed to drop over this bluff and then to follow the natural channel to the culvert. Flow is regulated with standard radial gates operated by electricity. A 20-ft. gate controls the wasteway and a 16-ft. gate controls the Main Canal.

Taneum Creek Wasteway on the South Branch, is 2 821 ft. long, of which 1 326 ft. is lined with concrete and has a drop in elevation of 202.8 ft. The chute is a lined canal section with a bottom width of 4 ft, a depth of 3 ft. 3 in., and with 1 on 1½ slopes. This chute empties into a stilling-pool, and the water is carried off in an unlined ditch. The wasteway and canal are furnished with hand-operated gates. At Station 14+00 on the wasteway, the Bruton (private) Ditch crosses under the wasteway channel in a 24-in. pre-cast pipe siphon (see Fig. 2(*b*)).

The Manastash Wasteway on the South Branch Canal is 120 ft. long. It has a relatively gradual fall and is constructed of 36-in. pre-cast concrete pipe. The inlet is in the head-works of the Manastash Siphon, and the flow is regulated by hand-operated gates. As this is the end of the South

Branch Canal, there is an 8-ft. Cipolletti metal-crest weir installed in a baffle wall in the siphon transition. From this point the flow of the South Branch Canal is carried in a main lateral (see Fig. 2).

In the case of Naneum Creek, Cook Creek, and Caribou Creek Wasteways, the base of the canal is approximately at the same level as the bed of the creek. The canal flow can be diverted, but it is not anticipated that it will be necessary to waste the entire capacity of the canals at either of these places.

The Johnson Creek Wasteway is near the inlet of the siphon at that point. The wasteway chute, a 3 by 6-ft. reinforced concrete flume, is 260 ft. long with a drop in elevation of 97.2 ft. Flow into the wasteway is controlled by a hand-operated radial gate.

TUNNELS

On the project there are 16 878 ft. of tunnel work in a total of eleven bores. These vary in size from 5 ft. 0 in. to 12 ft. 3 in. in diameter. They all have concrete-lined sections of the horseshoe type, except the Yakima Crossing Tunnel, which is round. Fig. 11 shows a typical tunnel plan. The two pressure tunnels under the Northern Pacific and the Chicago, Milwaukee, St. Paul and Pacific Railway tracks and parts of the tunnel under the Yakima River are reinforced both longitudinally and transversely. The other linings are not reinforced. Table 3 gives the dimensions and hydraulic properties of all the tunnels. Except for Item No. 6, which is a circular section, all the tunnel sections are of the horseshoe type. The value of Kutter's *n*, in every case, was assumed to be equal to 0.014. The lining of all tunnels is poured without expansion joints, key-way construction joints being provided between pours.

TABLE 3.—DIMENSIONS AND HYDRAULIC PROPERTIES OF TUNNEL SECTIONS

Item No.	Name	Dimensions					Hydraulic Properties				
		Length, in feet	Width and height, in feet	Lining Thickness, in Inches		Water depth, in feet	Area, *A*, of stream, in square feet	Velocity, *V*, in feet per second	Flow, *Q*, in cubic feet per second	Hydraulic radius, *r*, in feet	Slope, *s*, of hydraulic gradient
				Minimum	Average						
				MAIN CANAL							
1	Tunnel under Chicago, Milwaukee, St. Paul, and Pacific Railway	180	12′-3″ 12.2	8	12	Full	124.40	10.61	1 320	3.11	0.00222
2	Tunnel under Northern Pacific Railway	304	12′-3″ 12.2	8	12	Full	124.40	10.61	1 320	3.11	0.00222
				SOUTH BRANCH							
3	Tunnel No. 1........	2 000	6.25	6	10	5.25	29.20	8.59	250	1.91	0.00270
4	Tunnel No. 2........	1 390	5.0	5	8	3.90	17.20	7.86	135	1.53	0.00300
				NORTH BRANCH							
5	Rocky Point.........	965	11.9	6	12	9.86	104.75	8.84	925	3.64	0.00125
6	Yakima River Crossing	3 125	9′-3″ 8.3	9	15 and 30	Full	87.20	13.77	925	2.31	0.00538
7	Tunnel No. 1........	1 686	11.7	6	12	9.66	101.00	9.16	925	3.59	0.00137
8	" " 2........	1 025	11.7	6	12	9.66	101.00	9.16	925	3.59	0.00137
9	" " 3........	2 276	11.7	6	12	9.66	101.00	9.16	925	3.59	0.00137
10	" " 4........	482	11.2	6	12	9.1	90.99	8.51	775	3.42	0.00125
11	" " 5........	3 445	6.7	6	10	5.53	33.08	8.47	280	2.05	0.00240

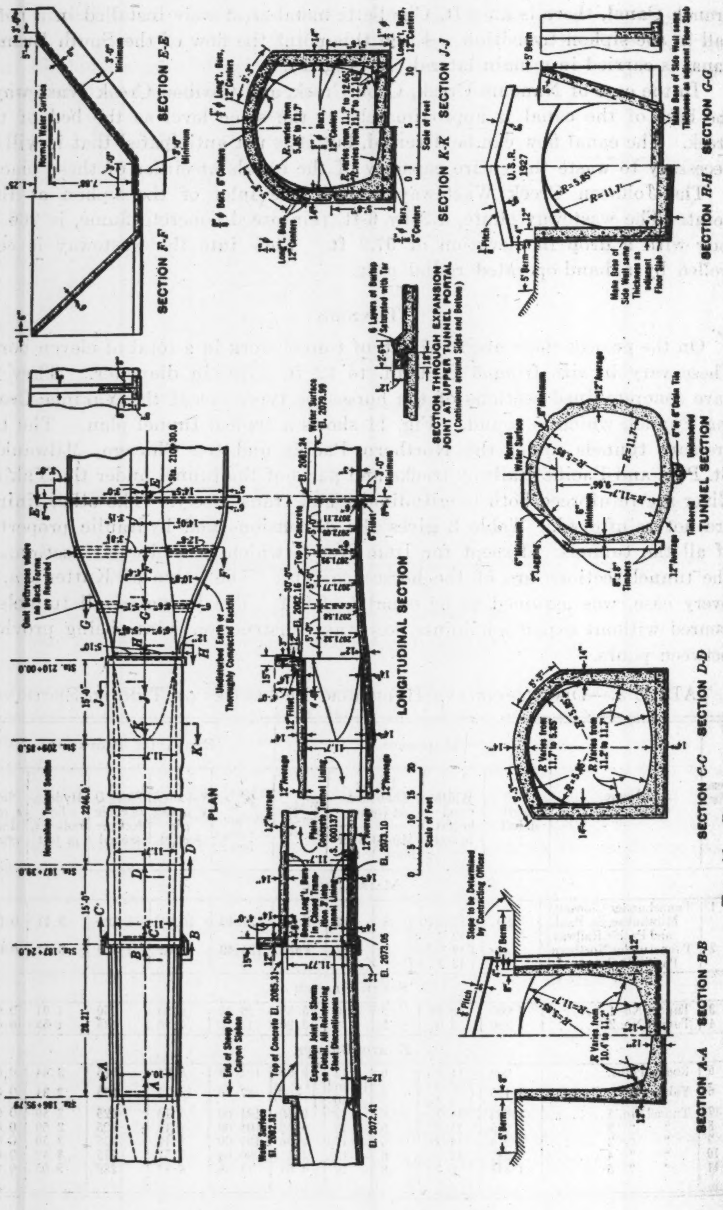
SECTION E-E
SECTION F-F
PLAN
LONGITUDINAL SECTION
SECTION K-K
SECTION J-J
SECTION THROUGH EXPANSION JOINT AT UPPER TUNNEL PORTAL (Continuous around Sides and Bottom)
TUNNEL SECTION
SECTION G-G
SECTION H-H
SECTION C-C
SECTION D-D
SECTION A-A
SECTION B-B
Scale of Feet
Normal Water Surface
Horseshoe Tunnel Section
Undisturbed Earth or Thoroughly Compacted Backfill
Omit Buttresses where no Back Forms are Required
End of Steep Dip Canyon Siphon
Top of Concrete El. 2085.31
Expansion Joint as Shown on Detail. All Reinforcing Steel Discontinuous
Plain Concrete Tunnel Lining
Slope to be Determined by Contracting Officer
U.S.R. 1927
Make Base of Side Wall same Thickness as adjacent Floor Slab
Timbered Tunnel Section
6 Layers of Burlap Saturated with Tar
Sta. 186+85.79
Sta. 187+24.0
Sta. 187+39.0
Sta. 209+65.0
Sta. 210+00.0
Sta. 210+30.0
Water Surface El. 2082.81
Water Surface El. 2079.99
El. 2072.41
El. 2073.05
El. 2073.10
El. 2081.24
El. 2082.16
5' Berm
12" Average
12" Fillet
6" Fillet
R. Varies from 10.4' to 11.2'
R. Varies from 11.7' to 5.85'
R. Varies from 11.2' to 11.7'
R=11.7'
R=5.85'
R=5.85'
R=11.7'

FIG. 11.—PLAN AND TYPICAL SECTIONS, TUNNEL NO. 3, NORTH BRANCH CANAL

Some parts of the tunnels require timbering. These are provided with five-segment, timber sets with 2-in. lagging (see Fig. 11). Where this timbering is permanent, it is used as a back form for the concrete lining. One line of open-joint drain-tile, 4 in., 6 in., or 8 in., is laid in a 12-in. gravel-filled trench in the base of the tunnel. The transition from the canal section to a square section with curved top is accomplished in about 40 ft., the transition from the square section to the horseshoe section is about 15 ft. long. In some tunnels on the North Branch, where they went directly into a siphon, with only a transition between (Fig. 11), the horseshoe base of the tunnel was carried through the closed transition of the tunnel and the open transition, to the closed or barrel transition in the siphon.

MINOR STRUCTURES

There are four types of bridges on the project classified as to use: The highway bridge, the farm bridge, the stock bridge, and the ditch rider bridge. Those used for highway purposes are truss, steel girder, or wood stringer bridges. They all have 4 by 12-in. floors, with $\frac{1}{4}$ in. by 24-in. checkered plate runners. The farm and stock bridges are steel girder or all wood construction. All these bridges are built very sturdily. Those that cross the major canals have reinforced concrete abutments and piers. All the ditch rider bridges over laterals are of timber.

Box culverts are either double or single. They are furnished with apron inlets, and, in most cases, with apron outlets. All box culverts have 6-in. side walls and from 6 to 9-in. bottom and top slabs. There are also a number of reinforced lock-joint concrete pipe culverts, 18 to 60 in. in diameter. These are also fitted with inlet and, ordinarily, with outlet aprons.

Turnouts are 2 by 2-ft., or 2.5 by 2.5-ft., reinforced concrete box structures, or reinforced concrete lock-joint pipe structures. The box turnouts have 6-in. side walls and 7-in. top and bottom slabs. Hand-operated steel gates are used to regulate the flow of water. Dry rock paving is used at the outlet of turnouts and culverts. Fig. 12 shows a typical box turnout.

THE LATERAL SYSTEM

The total length of laterals on the Main Canal is approximately 12 miles, the shortest being 170 ft. and the longest 12 200 ft. The average length is 2 400 ft. in twenty-seven laterals. The estimated quantities for the entire 12 miles, disregarding excavation for structures, is 41 200 cu. yd. The total length of laterals on the South Branch Canal is approximately 25 miles, the shortest being 350 ft. and the longest 17 600 ft., averaging 6 172 ft. in twenty-one laterals. The estimate for the excavation of the 25 miles, disregarding structure excavation, is 87 000 cu. yd. The average excavation for both systems of laterals is only approximately 60 cu. yd. per 100-ft. station. The total length of laterals under the North Branch Canal is 205 miles. The maximum length of a lateral is 19.1 miles, while the minimum length is 90 ft. There are 161 laterals under the North Branch Canal. The total excavation in these laterals, excluding structure excavation, is 931 600 cu. yd., or an approximate average excavation per 100-ft. station of 86 cu. yd.

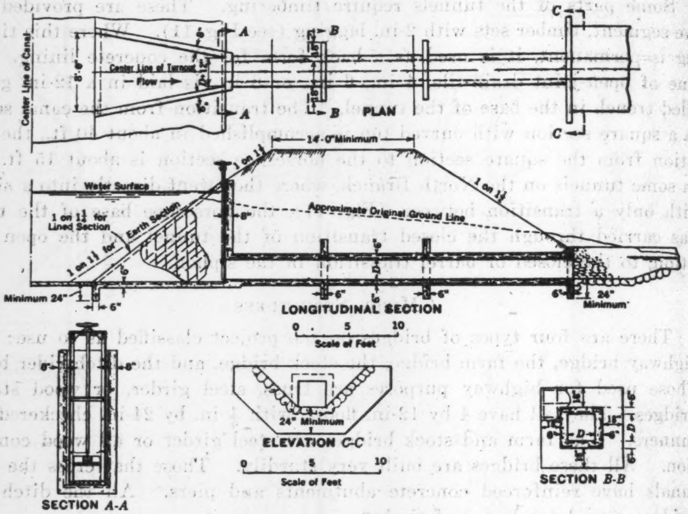
Center Line of Canal
8'-0"
Center Line of Turnout
Flow
A
A
B
B
C
C
PLAN
14'-0" Minimum
1 on 1½
1 on 1½
Water Surface
1 on 1½ for Lined Section
1 on 1½ for Earth Section
Approximate Original Ground Line
8"
D
Minimum 24"
6"
6"
6"
6"
24" Minimum
LONGITUDINAL SECTION
0 5 10
Scale of Feet
8"
8"
SECTION A-A
24" Minimum
ELEVATION C-C
0 5 10
Scale of Feet
D
6"
SECTION B-B

FIG. 12.—TYPICAL BOX TURNOUT

All laterals are ordinary unlined ditches. In most cases the side slopes are 1 on 1½, with two side banks. An attempt is made to balance "cut" so as to use all the material excavated in the side banks. The bottom width of ditches varies from 3 to 12 ft. and the depth from base to ditch to top of bank, from 1.5 to 5.5 ft. The width of bank top is 3 or 4 ft., with 1 on 1½ side slopes on the outside. The water depth varies from 0.4 ft. to 3.0 ft. As a rule, the velocities are about 2.0 ft. per sec., but in a few instances the designed velocities are as great as 3.7 ft. per sec., the higher velocities being in material which will not easily scour. The capacities range from 6 to 75 cu. ft. per sec. In the design of all lateral ditches Kutter's *n* was assumed equal to 0.025.

On account of the very rough topography a large number of structures was necessary and in order to hold the velocities within allowable values, a large number of drops and chutes was constructed. There were fifty-three drops alone in one lateral 2½ miles long, under the South Branch Canal. All lateral structures, except bridges, are constructed of reinforced concrete. The bridges across the lateral ditches are all timber structures with wood mudsills; some of the longer spans have steel girders resting on concrete abutments. The same type of bridge is used in lateral work as in canal work. All weirs are of the Cipolletti type, with metal weir plates (see Fig. 13). In all weir structures, including weir drops and weir chutes, the wooden faceboard, as shown on the plan, was eliminated, the plate being bolted directly on the face of the concrete. The floor of the pool may be raised, if required, to place the maximum elevation of the water surface at the weir crest, or depressed to permit a maximum drop of 3.0 ft. in the water surface. In Fig. 13, $H = 3h + 12$ in.; $H' = D' + 6$ in.; $1 = 3h + 24$ in.; $m = 0.3l$; $n = F + d' + 0.5 - h'$; and $W = \sqrt{Q}$ to the nearest foot. The maximum value permissible for F_w, the total drop in the water surface, was 3 ft. Minimum values specified were: $l = 4$ ft.; $m = 18$ in.; $n = d' + 6$ in.; and $W = 3$ ft.

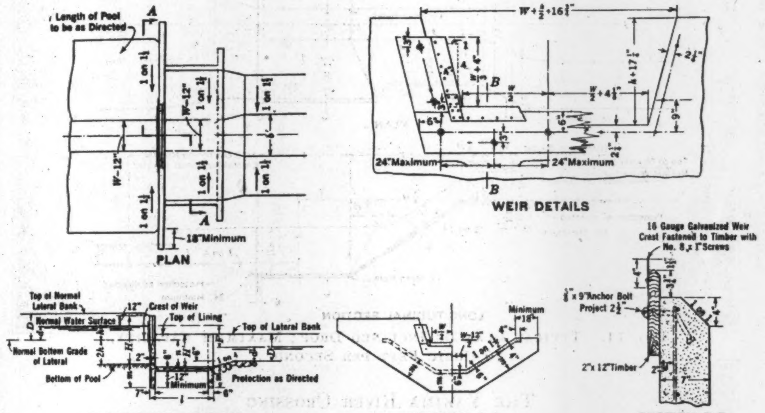
Length of Pool to be as Directed
A
1 on 1½
W-12"
18" Minimum
PLAN
W+½+16½"
B
24" Maximum
24" Maximum
WEIR DETAILS
16 Gauge Galvanized Weir Crest Fastened to Timber with No. 8 x 1" Screws
⅜ x 9" Anchor Bolt Project 2½
2" x 12" Timber
Top of Normal Lateral Bank
12" Crest of Weir
Top of Lining
Normal Water Surface
Top of Lateral Bank
Normal Bottom Grade of Lateral
Bottom of Pool
Protection as Directed
12" Minimum
Minimum 18"
LONGITUDINAL SECTION
SECTION A-A
SECTION B-B

FIG. 13.—PLAN OF STANDARD WEIR, CAPACITY 70 CUBIC FEET PER SECOND

The checks are similar in construction to the weirs, except that the first wall is fitted with grooves for the insertion of the flash-boards. In the weirs this wall has the trapezoidal notch and weir plate. Three types of drops are used: The inclined, the inclined weir, and the inclined drop with flash-board control. The last two are fitted at the upper end as weirs and checks, thus eliminating these separate structures. There are only very minor differences in design of the three structures, except the addition of the wall that carries the weir plate or is notched for the flash-boards. Fig. 14 is a typical plan of the inclined drop. Note that $B' = B + 1\frac{1}{2}d$; $e = \frac{3}{4}(1\frac{1}{4}H' - 1\frac{1}{2}H)$; $H = 1.2d + 6$ in.; $H' = P + 1.2d' + 0.5F + 12$ in.; $l = 1.5(F + P)$; $l' = (F + 1.5d')$; and $P = \frac{1}{4}(\sqrt{d'F} + d')$. The following values are the maximum permissible; $B = b$; $B' = b$; and $F = 15$ ft. The minimum permissible values are $H = 18$ in. and $l' = 4$ ft.

Chutes are either paved with dry rock or are lined with reinforced concrete. The flow through sub-lateral turnouts is controlled by hand-operated gates and the barrels of these turnouts are reinforced "lock-joint" concrete pipe, 18 to 48 in. in diameter. The intakes and pipe collars are monolithic except on one South Branch lateral where, for experiment, they were pre-cast slabs, with mesh reinforcement at the joints arranged so that they joined together when placed, in the same manner as "lock-joint" pipe. The slabs are 3 in. thick and are reinforced with mesh and $\frac{3}{8}$-in. round bars. A number of low-head siphons are used in connection with the laterals as crossings under roads, canals, and railroads. These are all reinforced "lock-joint" concrete pipe with monolithic intakes and outlets.

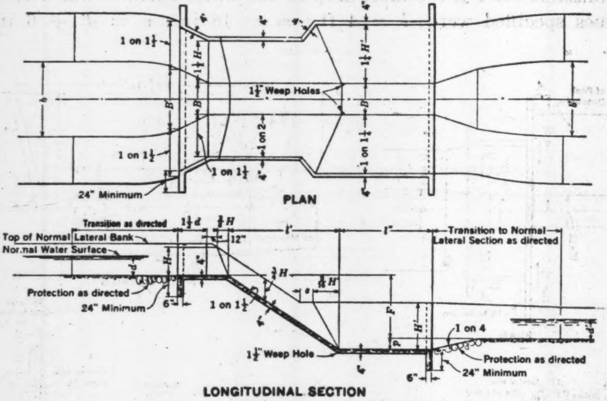
1 on 1½
1 on 1½
1 on 1½
1 on 2
1 on 1½
1½ H
1½ H'
1½" Weep Holes
b
B
B
D
b'
24" Minimum
PLAN
Transition as directed
1½ d
⅔ H
l'
l''
Transition to Normal Lateral Section as directed
Top of Normal Lateral Bank
12"
Normal Water Surface
H
d
¾ H
$\frac{5}{16}$ H'
e
Protection as directed
24" Minimum
6"
1 on 1½
F
H'
P
1 on 4
1½" Weep Hole
Protection as directed
24" Minimum
6"
d'
LONGITUDINAL SECTION

FIG. 14.—TYPICAL PLAN OF INCLINED DROP; MAXIMUM CAPACITY, 70 CUBIC FEET PER SECOND

THE YAKIMA RIVER CROSSING

About 26 miles from the diversion dam, the Main Canal crosses the tracks of the Chicago, Milwaukee, St. Paul, and Pacific, and the Northern Pacific Railways, the proposed location of the Swauk Creek Logging Railway, the Yakima River, Swauk Creek, and the proposed location of the Inland Empire Highway (see Fig. 1). The difference in elevation of the water surface in the canal above the location and the normal water surface in the river is 390.8 ft. The railway tracks are near, and a few feet above, the water surface in the river. Between the 2 100-ft. contours (which is approximately the elevation of the canal) on either side of the river, the horizontal distance is about 3 125 ft. Fig. 15 shows the location of the tunnel. The vertical wasteway shaft in the photograph was also used as a construction shaft.

Thirteen test holes were drilled at the location, along the line of the proposed tunnel. These tests indicate that, in general, the bore will be through shattered basalt formation. The tunnel section is round, 9 ft. 3 in. in diameter, and lined with concrete throughout. Its horizontal length is 3 125 ft. The first 350 ft. has a slope of —1.428, the lower part has a slope of — 0.003 and + 0.003, and the outlet (334 ft.) has a slope of + 1.428. Of course, these are not the hydraulic slopes. The maximum head is 533.2 ft. measured from the water surface at the intake to the lowest point of the tunnel.

Three types of lining sections are to be used varying with the material encountered. The section lined with plain concrete, to be used in stable rock, has an average thickness of 15 in. with a minimum of 9 in. In the steel-lined section the pipe segments are pressure grouted on the outside, with an 18-in. reinforced concrete lining on the inside. The reinforced concrete section

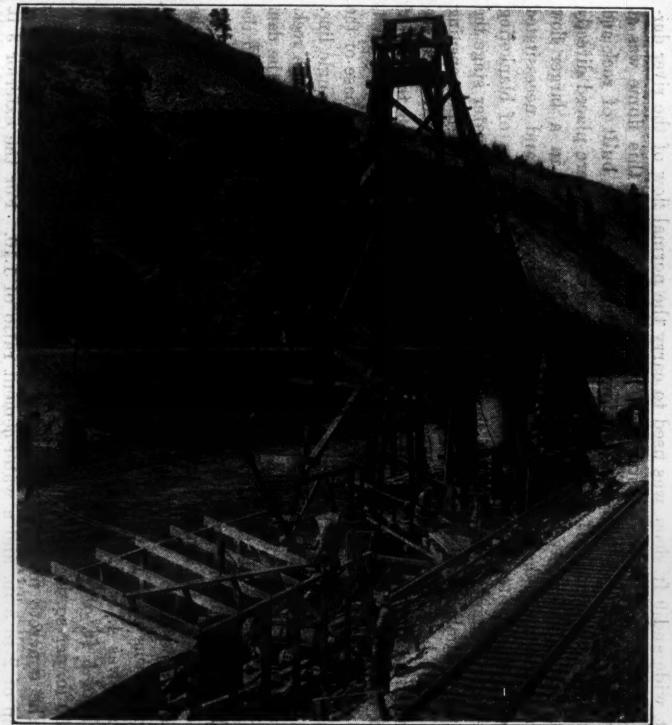

FIG. 15.—LOCATION OF THE YAKIMA RIVER PRESSURE TUNNEL

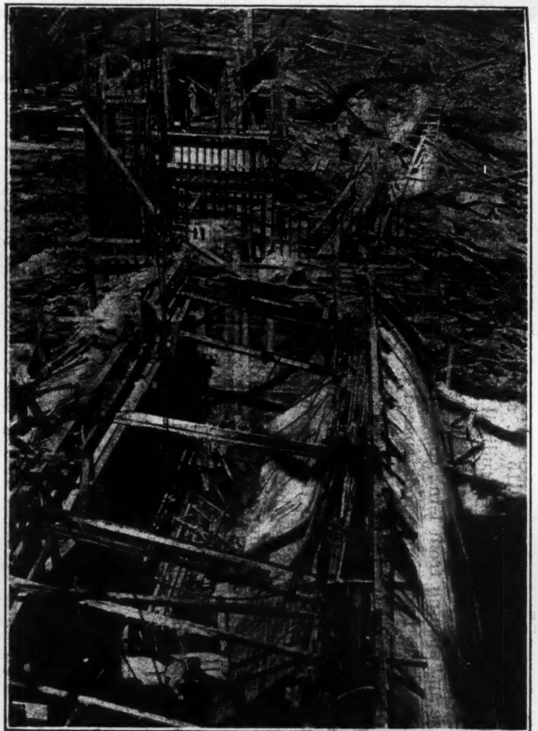

FIG. 16.—CONSTRUCTION VIEW OF DIVERSION DAM

has an average thickness of 15 in. and a minimum, of 9 in. (see Table 3, Item No. 6). For the longitudinal reinforcement, forty-eight ⅝-in. round bars were specified. The transverse reinforcements consist of hoops varying in size and spacing from 1-in., round rods, spaced 10 in. apart, for heads from 30 to 40 ft., to 1-in. square rods, spaced 4 in. apart, for heads from 90 to 100 ft. The steel-lined pipe is used for sections at which the head is greater than 100 ft. where permanent timbering is necessary. The tunnel is provided with a vertical wasteway shaft, 78 in. in diameter. This shaft is lined with ⅜-in. hammer-welded, steel pipe throughout and is grouted with a minimum of 9 in. of concrete outside the pipe. The wasteway outlet is furnished with two 42-in. internal, differential valves discharging into the Yakima River. The structure is also equipped with a motor-driven, direct-flow turbine pump, with a capacity of 600 gal. per min. against a head of 155 ft. The hydraulic properties of the tunnel are given in Table 3.

CONSTRUCTION METHODS

The Diversion Dam.—The diversion dam to the Main Canal was constructed during the spring, summer, and fall of 1928 and 1929. Due to very severe weather very little work was done during the winter months. No serious high water occurred during the construction period. A timber flume, 12 ft. wide and 9 ft. deep, was used to carry the normal flow of the river during the early period of construction. The inlet floor of this flume was at Elevation 2 142.5. The upper and lower coffer-dams were built of rock and gravel, to Elevations 2 155 and 2 143, respectively. They were placed directly on the sand and gravel bed of the river. This resulted in a larger flow through the coffer-dams than could be cared for by pumps, and necessitated the driving of interlocking sheet-steel piling and the addition of blanketing material. When the structure of the dam was above the high-water stage the temporary timber flume was removed, and the river passed through an opening left in the base of the dam near the north side. This was closed when the sluice-gates had been placed. The base of the dam was set in 3 to 4 ft. of solid rock, which was encountered under a stratum of 6 to 15 ft. of sand, gravel, and boulders. The cut-off wall was carried 5 ft. below the base of the dam. In the center of the key-way, grout holes were spaced in a single line, 5 ft. apart. Additional holes were drilled in soft or seamy spots in the rock. The depth of the grout holes ranged from 25 ft. in the main part of the dam to 10 ft. under the abutments. In addition, 20 grout holes were drilled in the south half of the spillway section below the key-way. There were 107 holes drilled with a total length of 2 233 ft. They were 2½ in. in diameter at the top and tapered to 1 in. in diameter at the bottom. As much as 38½ cu. ft. of grout was used in one hole, while some holes would take no grout. The average was 4.75 cu. ft. per hole. The grouting was done after a minimum depth of 5 ft. of concrete had been poured, and forced in under a pressure of 100 lb. per sq. in.

The concrete mix for the interior of the dam was 1 : 2.75 : 6.25, with a slump of 2 to 3 in. and a water-cement ratio of 1.10. For the exterior 3 ft.

of the dam, a mix of 1.5: 2.75: 6.25, with a slump of 3 to 5 in. and a water-cement ratio of 0.80, was used. Of diatomaceous earth, 3 lb. per sack of cement was used for both mixes. The maximum size of aggregate was 3 in. These mixes were quickly interchangeable at the mixer. By the addition of an extra sack of cement to each mixer batch, of the mixer used, the interior mix could be changed to the exterior mix. In the fish ladder, head-works, and other parts, where greater strength was required, the mix was 1: 2.4: 3.6, with a 1¼-in. maximum size of aggregate. An admixture of 3% was used with a slump of 4 to 6 in. and a water-cement ratio of 0.93. In binding successive horizontal lifts about ¾ in. of the old concrete was removed with picks, and the surface was thoroughly cleaned with air, steel brushes, and water. Then a 1-in. layer of 1.5: 2.75 grout was added and brushed with steel brushes. This was followed with a 6-in. layer of 1.5: 2.75: 6.25 concrete. The concreting was then continued with the 1: 2.75: 6.25 mix. An efficient arrangement for pouring concrete was made. Gravel, sand, and cement were chuted to the mixer, and the mixed concrete was chuted to the larger part of the work. The total concrete in the structure was 5 706 cu. yd.

Metal-lined forms, 5 by 6 ft., were used. These were anchored with ½-in. anchor rods. The pours varied in thickness from 4 to 10 ft. and horizontal key-ways were provided between them. These key-ways were formed with two 2 by 12-in. planks standing on edge and placed 18 in. apart at the top and 24 in. at the bottom. They were held in place by a steel rail bent in a U-shape. A richer mix with smaller aggregate than the 3 in. was used in pouring the key-ways.

Great care was exercised in placing the drum-gate in order that this gate would swing freely. Fig. 16 is a construction view of the dam.

Canals.—Earthwork was divided into three classes for payment, both in canal and structure excavation. The specifications are as follows:

"*Class No. 1.*—Material that can be plowed to a depth of six inches or more with a 6-horse or 6-mule team, each animal weighing not less than 1 400 pounds, attached to a suitable plow, all well handled by four men; also material that is loose and can be handled with team-drawn scrapers, and material that can be excavated efficiently, without blasting, by power shovels or dragline excavators of recognized standard manufacture, equipped with suitable dippers or buckets having minimum capacities of ¾ and 1 cubic yard, respectively, well handled by skilled operators; also all boulders or pieces of rock not exceeding 2 cubic feet in volume, that are detached or are embedded in Class 1 material.

"*Class No. 2.*—All material other than Class 1 and Class 3, including, but not restricted to, indurated material of all kinds, such as hard pan, cemented gravel and soft or disintegrated rock, which can not be excavated efficiently by power shovels or dragline excavators of the size and class above described, or by the use of plows and team-drawn scrapers, until loosened by blasting; also, all boulders or detached pieces of rock more than 2 cubic feet and not exceeding one-half cubic yard in volume, and all slide rock or similar loose rock occurring in masses which can not be handled efficiently with team-drawn scrapers, except pieces exceeding one-half cubic yard in volume.

"*Class No. 3.*—All solid rock in place which can not be removed until loosened by blasting, and all boulders or detached pieces of solid rock more than one-half cubic yard in volume. Solid rock under this class, as distinguished from soft or disintegrated rock under Class 2, is defined as sound rock of such hardness and texture that it can not be effectively loosened or broken down by hand drifting picks."

Canal sections were excavated with draglines, shovels being used only in rare instances. The bucket capacity varied from 1 cu. yd. to 2½ cu. yd. Some cuts as deep as 65 ft. were excavated with two draglines, one of them passing dirt to the other higher up.

Concrete for the lining of the Main Canal and the North Branch Canal was mixed in a 3-bag, paving mixer. The slopes were paved by the use of a machine made expressly for that purpose. It had a structural steel framework adjustable to the size and form of the canal section and was equipped with a movable steel form on each side the length of one panel and extending up the slope about 3 ft. The concrete was poured between the form and the trimmed canal slope, the form being held in place so that a lining of the proper thickness was secured. The steel form could be pulled up the slope by a winch mounted upon it, as the concrete was poured and tamped. The bottom was poured last, well tamped, and finished. When the machine had been moved in place for the next panel, the preceding one was finished by hand.

Smaller mixers were used on the South Branch Canal and on lateral chutes and for this work the proper thickness was obtained with screed boards. Fig. 17 shows alternate panels being poured on the South Branch lining. This method was not found to be successful because of the expansion of steel in the open panels. On this work concrete was mixed at a central plant and hauled in trucks. One contractor used this system with some success on lateral structures, but most of them were poured on location, with small ½-bag or 1-bag mixers. Fig. 18 shows a lateral chute being constructed.

The reinforcing steel was cut and bent on the job and was placed and wired ahead of the concreting. It was supported in place by small concrete blocks made to the proper thickness.

In the Combination Section a novel arrangement of panel forms was used (see Fig. 5). The timbers were bolted so that a very rigid structure was obtained, and, at the same time, they could be assembled and taken apart quickly. The lagging was made in large panels bolted together and lined with metal. Arrangements were made so that concrete was poured every third day and the forms were moved up and made ready for the next pour during the intervening days.

Ordinarily, in pouring canal lining on the Main Canal and on the North Branch Canal, the gravel, sand, cement, and admixture were proportioned at the bunkers and hauled dry to the job in batch trucks, three to six 3-bag batches per truck. The proportioning as well as the placing of the concrete in the canal lining was under the direction of an inspector. In most cases both sand and gravel were weighed, but in some instances the gravel was measured by volume.

A full description of the construction of the main canal lining has been presented by Messrs. Arthur Ruettgers and A. A. Whitmore.[4]

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[4] *Journal*, Am. Concrete Inst., October, 1930, p. 117.

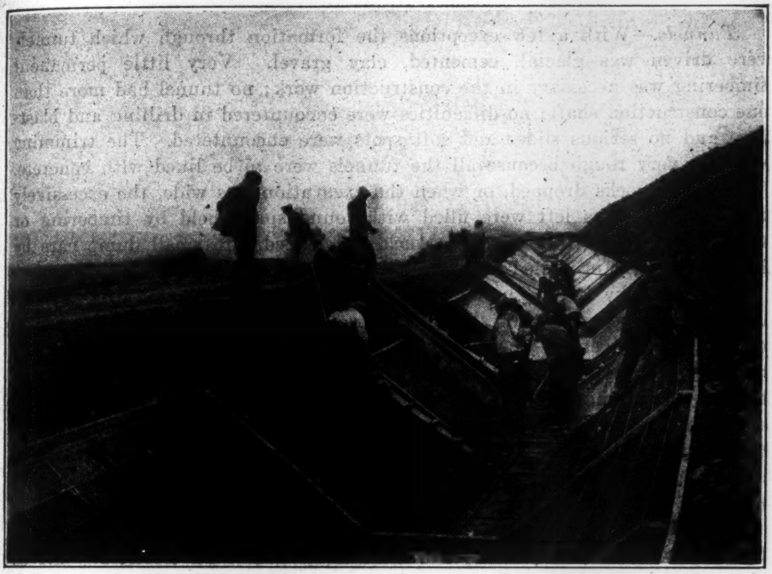


Fig. 17.—Pouring Concrete Lining on South Branch Canal. Alternate Panels Being Poured and Screeded to Proper Thickness

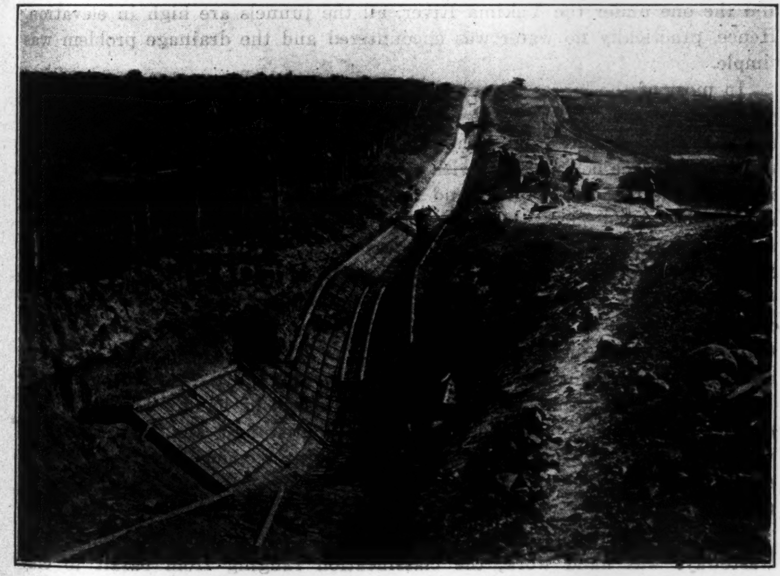


Fig. 18.—Pouring Concrete in a Lateral Chute

Tunnels.—With a few exceptions the formation through which tunnels were driven was glacial, cemented, clay gravel. Very little permanent timbering was necessary in the construction work; no tunnel had more than one construction shaft; no difficulties were encountered in drilling and blasting; and no serious slides and soft spots were encountered. The trimming was left very rough because all the tunnels were to be lined with concrete. When loose rocks dropped, or when the excavation was wide, the excessively large pockets thus left were filled with sound spalls held by timbering or tamped in place with concrete. Muck was loaded into small dump cars by hand and the cars hauled away by horse or mule.

In the Rocky Point Tunnel, an endless electrically operated dragline was used to load the muck train. The cable ran through one pulley connected to an eye-bolt driven in the heading; the other end was wound around a reversible electrically driven drum. The bucket was pulled up an incline to a tramway over the muck train, where it was automatically dumped. The action of the drum was then reversed, the empty bucket being pulled down the muck pile and loaded on the return. The incline and tramway could be moved up as the work progressed. The muck train was hauled by horses or mules on 36-in. gauge track. All permanent timbering was composed of 5-segment timber sets, the size in the smaller tunnels being 6 by 6 in., and in the larger tunnels, 8 by 8 in., with 2-in. lagging. Small electrically driven blowers were used for ventilating purposes and the air was good in all tunnels. Except for the two tunnels under the railway tracks near the diversion dam and the one under the Yakima River, all the tunnels are high in elevation. Hence, practically no water was encountered and the drainage problem was simple.

In most of the larger tunnels the floor was poured by hand and provided a support for the collapsible steel forms. A key-way joint was placed between the floor and the side walls. The forms had timber lagging and the concrete was shot in place behind them with air, the pipe for this purpose being placed inside at the top of the forms and then pushed to the back or to the beginning of the day's pour. Jack hammers were used on the inside face of the forms and no other kind of tamping was done. Open inspection holes were left in the lagging and closed as the concrete pouring progressed. The concrete was mixed outside the tunnel near the most accessible end for delivery of material and was transported to the "gun" by means of a gas locomotive. Only two crews of men were employed, one to set the forms during the night and the other to pour the concrete during the day shift.

The concrete in the smaller tunnels was placed by hand. The same type of forms was used. The side walls were poured first and then the floor, which provided additional head-room. No reinforcement steel was used in the concrete lining of the "no-pressure" tunnels, except in the portal transitions.

Siphons and Wasteways.—The excavation in the majority of siphons and wasteways was hand work, the classification ranging from basalt to clay gravel.

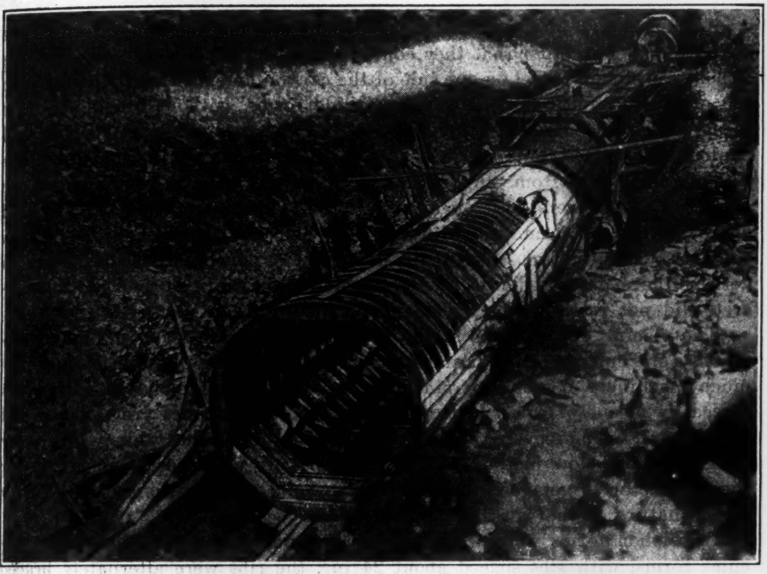


FIG. 19.—PART OF INSIDE FORMS FOR DRY CREEK SIPHON IN PLACE AND SUPPORTED ON INSIDE FORM SUPPORTS.

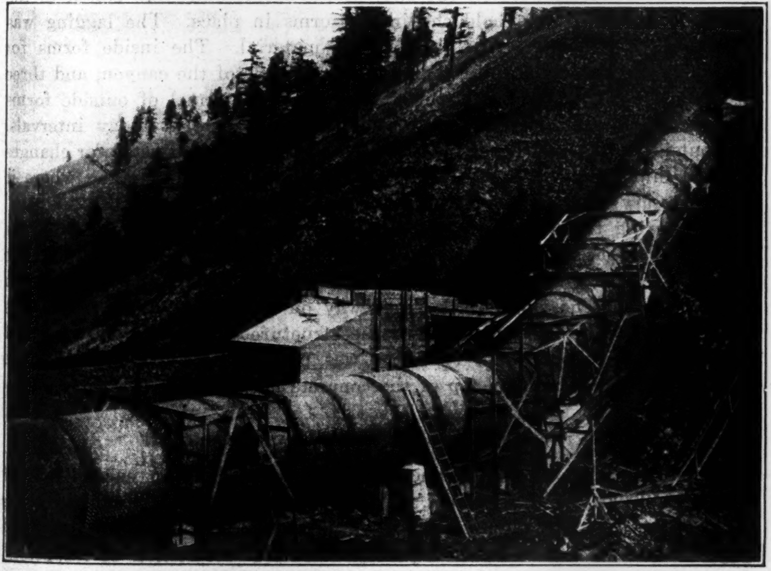


FIG. 20.—PLACING STEEL PIPE ON MORRISON CANYON SIPHON

All form work in these structures was timber, but the siphon forms were bolted together in order that they could be re-used. The usual procedure was to pour the siphon barrel on one side of the canyon and then move the forms to the other side and pour there. The inside forms, without bracing, can be seen in Fig. 19. On most siphons the concrete was mixed at the top and chuted in to place, but on some structures, material could be more easily assembled at the bottom. An aerial cable line was used at two siphons. No elevating towers were used on any structure. The larger siphons contained approximately 1 300 cu. yd., with the exception of Dry Creek Siphon which contained 2 640 cu. yd. The main difficulty in pouring the barrels was in getting the expansion joints properly installed as the men pouring the concrete did not realize the extreme importance of these joints.

The placing of steel pipe at the bottom of Morrison Canyon Siphon presented a difficult problem. Particularly, it involved the question of just how much machinery it was economical to assemble for a small job. Pipe was placed by means of gin-pole rigging, but the canyon sides are so steep and treacherous, being covered with loose, sliding rock, that this was a very dangerous and slow method. (See Fig. 20.) It was nearly impossible for the contractor to get good steel men who would stay on the job, due partly to the nature of the work and partly to the location.

All siphon forms were of timber. The inside forms were 8-segment, circular forms, with ribs spaced about 24 in.; the ribs were alternately braced with two 4 by 4-in. posts at right angles which, in turn, rested on 4 by 4-in. walers. Two walers were in the base of the forms and were bolted to the inside form supports to hold the inside forms in place. The lagging was 2 by 6-in., tongue-and-groove, bevel-edged material. The inside forms for the siphons were usually built complete for one side of the canyon, and three or more, panels of the outside forms were used. A panel of outside forms was moved ahead each third day, pouring being done at 3-day intervals. The outside forms were constructed so that they could be adjusted for changes in shell thickness. Inspection holes were cut in the inside lagging where it was necessary to see that the concrete was thoroughly tamped.

In pouring the lower part of the wasteway at Station 1146+30 and the culvert under the railroad, it was necessary to construct a track on top of the structure. The mixed concrete was carried in small cars controlled by an endless cable. The slope of the wasteway on the lower end is 0.65. The mixed concrete in the remainder of the structure was poured by means of chutes and wheel-barrows. The floor was poured first, and then the side walls, forms being built on top of the concrete floor after it had set.

METHOD OF CONTROLLING QUALITY OF CONCRETE

About 150 000 cu. yd. of concrete were produced at an expenditure of about $4 500 000 on this project. Consequently, the problem of producing concrete of a desired quality and at a minimum expense was considered important. The maintenance of quality was under the control of inspectors. Success in the production of concrete of a desired quality is dependent on the proper performance of all operations from the time the materials are selected and the mixture is designed until the concrete is placed and adequately cured. Failure to perform or control any individual operation properly is likely to result in a product that is non-uniform in character and either inferior in quality or wasteful in cement.

The desire of the engineers in charge to produce concrete of a uniform quality led to the incorporation of all the directions and tables to be used by inspectors in a small blue-print pamphlet entitled, "Instructions for

TABLE 4.—CONCRETE CONTROL FOR THE CONSTRUCTION SEASON, 1928; BASED ON COMPRESSION STRENGTH OF TEST CYLINDERS

Item No. (1)	Structure (2)	Number of cylinders (3)	Average compressive strength (28 days), in pounds per square inch (4)	Average water-cement ratio (5)	PERCENTAGES OF CYLINDERS TESTED THAT HAD COMPRESSIVE STRENGTH EQUAL TO: Within 10% of average strength (6)	Within 15% of average strength (7)	Within 20% of average strength (8)	More than 120% of average strength (9)	Less than 80% of average strength (10)
	MIX: 1: 2.4: 3.6 (DRY RODDED)								
1	Morrison Siphon Wasteway....	40	3 890	0.88	52	65	75	12.5	12.5
2	Wasteway at Station 1146+30	42	3 974	0.89	53	67	79	11	10
3	Rocky Point Tunnel..........	50	4 334	0.87	38	68	80	8	12
4	Tunnel No. 1 on the North Branch Canal..............	69	3 312	0.88	38	52	73	10	17
5	Tunnel No. 1 on the South Branch Canal	60	4 138	0.87	55	75	86	7	7
6	Miscellaneous small canal lining structure (by different contractors)............	145	3 848	0.90	48	65	78	12	10
7	Division 1..................	59	4 359	0.77	65	87	90	5	5
8	" 2..................	91	4 914	0.82	62	82	98	0	2
9	" 2..................	72	4 572	0.83	53	68	92	4	4
10	" 3..................	155	4 466	0.84	51	74	85	8	7
11	" 3..................	*90	3 755	0.84	47	74	87	6	7
12	" 3, Combination Section	*86	3 447	0.88	55	79	91	3	6
13	" 3, Combination Section	117	4 091	0.88	51	68	82	9	9
	MIX: 1: 2.0: 3.25 (DRY RODDED)								
14	Horseshoe Siphon............	23	4 546	0.79	65	74	91	0	9
15	Little Creek Siphon..........	35	5 291	0.76	66	91	94	6	0
16	Spexarth Creek Siphon........	30	4 381	0.81	70	73	90	10	0
17	Peterson Creek Siphon........	44	4 271	0.81	57	77	82	13	5
18	Morrison Siphon..............	30	3 898	0.82	17	37	54	23	23
19	Sheep Dip Siphon............	46	4 578	0.74	54	78	87	4	9
20	Little Dry Creek............	30	4 515	0.83	67	80	90	3	7
21	Dry Creek Siphon............	118	4 742	0.84	57	73	88	7	5
22	Siphon at Station 91+84, South Branch Canal............	17	4 384	0.82	65	65	76	12	12
23	Big Creek Siphon	26	4 864	0.72	54	69	73	8	19
	MIX: 1: 1.7: 2.8 (DRY RODDED)								
24	Cle Elum Creek Siphon.......	17	5 444	0.72	82	88	100	0	0
	MIX: 1: 1.6: 2.9 (DRY RODDED)								
25	Horseshoe Siphon............	19	5 075	0.68	42	63	74	10	16
26	Sheep Dip Siphon............	7	5 150	0.64	43	86	86	0	14
	MIX: 1: 1.6: 2.4 (DRY RODDED)								
27	Pipe plant..................	52	5 704	0.73	58	69	77	6	17

* Sprayed with "process" after removal of mould, no damp sand curing.

Concrete Inspection and Control." Information in this section of the paper was secured from that pamphlet. Its use is not an attempt to supplant the employment of experienced inspectors, but to standardize the method of control. The method of control is the same as that advocated by the Portland Cement Association, but adapted to local conditions. Table 4 gives the results of strength tests on cylinders taken from the various structures on the project during 1928.

Aggregate.—Very complete investigations were made of local sources for sand and gravel, but it was decided to secure the material from a company that had pits near Steilacoom, Wash., on Puget Sound. These aggregates are washed with salt water; the average fineness modulus of the 1¼-in. gravel was 7.15; and the average fineness modulus of the sand was 3.08. The grading of aggregates for the dam and the reinforced concrete pipe differs from these figures.

Concrete.—In the determination of the proper mixes, extensive laboratory tests were made of the aggregates. The assigned proportions were slightly revised to meet the field conditions as the work progressed. The properties required for the concrete were high strength, high impermeability, and good workability. All these qualities were considered equally important. In most of the structures, high strength was an absolute necessity. In siphon structures subjected to high heads, the concrete must be of a known high strength and furthermore must be highly impermeable. In order to be worked around the barrel and between crowded steel bars, the concrete must also be very workable. The mixes as designed had these characteristics, as demonstrated in the field and by laboratory tests with actual specimens under compression and under hydrostatic pressure.

Method of Supervision.—An assumed strength, impermeability, and workability having been decided, and the proper mix to gain these characteristics fixed by test, the problem was to obtain the same results in the field as in the laboratory. On canal lining, for which the assigned mix is 1: 2.4: 3.6 (dry rodded) the percentages of moisture in the sand and gravel, for example, are assumed to be 3.1 and 0.1, respectively.

By consulting the Reclamation Service Instruction Book, previously mentioned, the inspector would find Table (*a*), the proper field mix, proportioned by volume (moist loose, taking into account the bulking of sand); Table (*b*), surface moisture in gravel (per sack of cement); Table (*c*), surface moisture in sand (per sack of cement); Table (*d*), batch weight for sand and gravel per sack of cement (including moisture); and Table (*e*), water-cement ratios for varying quantities of water used per sack of cement. The amount of sand bulking for different percentages of moisture in the sand is also given; no allowance is made for bulking because of moisture content in gravel.

A complete set of these tables is given in the pamphlet for all the assigned mixes used on the project. By their use, it is a simple matter for the inspector to calculate the water to be added to secure the specified water-cement ratio, because the number of gallons of water per sack of cement contained in the sand and gravel can be taken from Tables (*b*) and (*c*), the weight of sand per sack of cement to be used is found in Table (*d*), and the volume of gravel is taken from Table (*a*), or if proportioning by weight, the weight of gravel may be taken from Table (*d*). The number of gallons of water to be added, the weight of sand and the volume or weight of gravel to be used being known, a trial batch is mixed. A slump test is made and more or less water is added to hold the slump below 2 in. which is the maximum allowable for canal lining. No change in the quantity of water or of the weight of aggregate need be made until a noticeable change occurs in the slump, due to more or less moisture in the sand.

Inspectors' Reports.—Form 1 is a sample of a daily concrete report. Each inspector was required to make a report for each structure poured or for the work of each shift. Form 2 is a sample of the report made for each cylinder or group of cylinders. One cylinder was made from the work of each shift. When a patented paint coating method of curing was used, two cylinders were made, one treated with the "process" and one cured in damp sand. All inspectors' reports were checked in the Division offices the day after they were made, and any mistakes corrected and brought to the attention of the inspector. The reports were then forwarded to the project testing laboratory. Cylinders were collected on the same day each week. In this way a very close check was kept of the quality of the concrete that was being poured under each inspector. Any cylinders that tested abnormally high or low were brought to the attention of the inspector, and he was required to make an explanation of the cause. The standard 6 by 12-in. cylinder was used.

The type of report used in the 1928 construction season was improved for the 1929 season. The cylinder report and the concrete report were combined on one 5 by 8-in. card as shown in Form 3. This type of report was found to be better in some respects than Forms 1 and 2, which were mimeographed blanks on separate sheets of 8½ by 11-in. letter paper. The information was more compact and the filing was easier.

Use of Admixture.—It is to be noted that no allowance is made for admixture when computing the water-cement ratio. The usual practice in this respect is to allow 200% absorption, that is, when 3% admixture is used, it is assumed that 6 lb. of water per sack of cement will be taken up by the admixture and is not available for mixing with the cement. In the design of mixes this allowance was not made; thus, there is no difference in the field conditions and the laboratory design. To disregard this feature simplifies the calculations and control of field mixing. The only objection is encountered in comparison of the concrete strength for a certain combined fineness modulus and water-cement ratio on this job with that secured with the same values on any other job. A ratio of $W = 0.90$, computed as described herein, would become approximately 0.80 if the admixture were assumed to absorb 200% of its weight of water. In canal lining and in small and low-head structures 3% of admixture is used. In the parts of siphon barrels under heads of less than 100 ft., 1½% is used; for heads greater than 100 ft., no admixture is used. In the rich mixtures in the lower part of the high-head siphons, it has been found that the addition of admixture is a detriment.

FORM 1.—SAMPLE INSPECTOR'S DAILY CONCRETE REPORT

United States
Department of the Interior
Bureau of Reclamation
Kittitas Division—Yakima Project
Inspector's Daily Report (Concrete)

Contractor: Blank Construction Co. : Day Shift: 10/14 1928

Main Canal, Div. No. 3 : Feature lining : Sta. 1359+20 to 1360+07

Weather Cloudy Warm — 65°

Instrument work performed None

Delays: 1 hour due to making water connections for mixer

Mixer: Type Ransome 13E ; Size 3 sk ; Ran 6 Hrs.; 86 Batches

Cement: Brand Spokane , sax per batch 3 , sax used 258

Admixture: Kind Diatomite ; % by Wgt. Cement 3 %; Lbs. used 774

Mix: Designed 1:2.4:3.6 {Dry / Rodded}; field 1:3.14:3.77 {Moist / Loose}; Slump 2 in.

Sand: Source Pioneer ; 3.1 % moisture; 25 % bulk; 816 lbs. per batch

Gravel: Source Pioneer ; 0.1 % moisture; 1-1/4 in. size; 11.31 C. F. per batch

Water: In sand 2.94 gals. per batch; in gravel 0.15 gals. per batch.

15.81 gals. added; 18.90 gals. total per batch; W/C ratio 84

RECORD OF CURING CONCRETE

Structure	Location	Curing		Water Application		
		Begun	Ended	Method	Applic. per day	Kind of covering
L/MC	1359+20 1360+07	10/14/28		Hunt Process	Once	3 hr. after finishing

John Doe (Inspector)

NOTE: **Use back of sheet to report inspection on placing of forms or reinforcing steel, or for supplying other pertinent information.**

FORM 2.—SAMPLE CYLINDER REPORT

United States
Department of the Interior
Bureau of Reclamation
Kittitas Division—Yakima Project

Record of Concrete Test Cylinders (6″ x 12″) from Field

Structure: Kind L/MC; Location 1359+20—1360+07

Source of concrete for cylinders 1360+00

Cylinders made: Number of 2; Date and hour 10/14/28—9 A. M.

Mixer: Name and type Ransome 13E; Size 3 sk

Cement: Brand Spokane; Sax per batch 3

Admixture: Kind Diatomite; % by wgt. cement 3 %.

Aggregates: Source Pioneer; Size of course agg. 1-1/4 in.

Water: Source Yakima River

Mix: 1:2.4:3.6 {Dry Rodded} 1:2.51:3.77 {Dry Loose} 1:3.14:3.77 {Moist Loose}

Aggregate used per sack cement: Sand 272 Lbs.; Gravel 3.77 C. F.

Time of mixing 1-1/4 min.; Workability Good

Moisture in sand: 3.1%; 0.98 gals. per sack; 2.94 gals. per batch

Moisture in gravel: 0.1%; 0.05 gals. per sack; 0.15 gals. per batch

Water added: 5.27 gals. per sack; 15.81 gals. per batch

Total water in mix: 6.30 gals. per sack; 18.90 gals. per batch

Water-cement ratio .84; Measured slump 2 inches

Weather and temperature Cloudy 65°

Nature and extent of curing cylinders One Hunt Process. One 7 days in moist sand

Remarks:

Signed: John Doe (Inspector) R. Roe (Div. Engr.)

Nature and extent of laboratory curing........

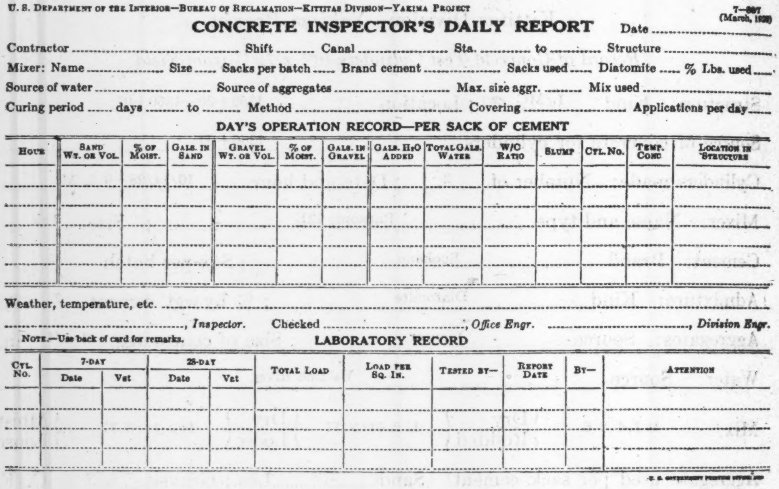
Compressive tests: Date........; Age........; ....lbs.;........lbs./sq. in.

Date........; Age........; ....lbs.;........lbs./sq. in.

Date........; Age........; ....lbs.;........lbs./sq. in.

Broken by..................Reviewed by..............................

Moisture Determination in Sand and Gravel.—Both the displacement, and the burning, methods of determining moisture in sand and gravel were used. The latter was utilized on the smaller structures where the required equipment was not at hand for the former.



U. S. DEPARTMENT OF THE INTERIOR—BUREAU OF RECLAMATION—KITTITAS DIVISION—YAKIMA PROJECT

7—887
(March, 1929)

CONCRETE INSPECTOR'S DAILY REPORT Date ....

Contractor .... Shift .... Canal .... Sta. .... to .... Structure ....

Mixer: Name .... Size .... Sacks per batch .... Brand cement .... Sacks used .... Diatomite .... % Lbs. used ....

Source of water .... Source of aggregates .... Max. size aggr. .... Mix used ....

Curing period .... days .... to .... Method .... Covering .... Applications per day ....

DAY'S OPERATION RECORD—PER SACK OF CEMENT

HOUR	SAND WT. OR VOL.	% OF MOIST.	GALS. IN SAND	GRAVEL WT. OR VOL.	% OF MOIST.	GALS. IN GRAVEL	GALS. H_2O ADDED	TOTAL GALS. WATER	W/C RATIO	SLUMP	CYL. NO.	TEMP. CONC	LOCATION IN STRUCTURE

Weather, temperature, etc. ....

.........., *Inspector.* Checked ...., *Office Engr.* ...., *Division Engr.*

NOTE.—Use back of card for remarks.

LABORATORY RECORD

CYL. NO.	7-DAY		28-DAY		TOTAL LOAD	LOAD PER SQ. IN.	TESTED BY—	REPORT DATE	BY—	ATTENTION
	Date	Vat	Date	Vat						

U. S. GOVERNMENT PRINTING OFFICE

FORM 3.—CONCRETE INSPECTOR'S REPORT USED DURING THE 1929 SEASON

The burning method was used on gravel in all cases. Each inspector was furnished with a pair of balances and an alcohol flask. Each sample was burned with alcohol two or three times, or until all the moisture had been expelled. The prescribed weight of damp material for each analysis was 500 grammes. A table was given showing the percentage of moisture opposite dry weights, assuming a damp weight of 500 grammes. The values in this table were derived by allowing 0.5% for absorbed moisture in the aggregate.

At the sand and gravel bunkers the displacement method of moisture determination was adopted for sand, and the burning method for gravel. Two types of apparatus were used in the displacement method. One consisted of a 3 by 27-in. cylindrical, brass tube container with a water-gauge glass attachment. This tube would accommodate a 2 000-gramme sample of sand or gravel. It was calibrated in cubic centimeters within the range of the gauge glass and a scale graduated to 5 cu. cm. was placed directly behind the glass. The other apparatus was a graduated glass container 18 in. high, designed to accommodate a 500-gramme sample of sand. The flask had a lower and upper chamber or bulb connected by a short tubular section called the lower neck. The upper bulb gradually tapered to a uniform tube about 6 to 8 in. long, called the upper neck. The 200-cu. cm. point was marked on the lower neck, and the upper neck was marked in cubic centimeters throughout that portion used in obtaining the necessary readings for testing sand samples. Tables were given for each type of apparatus showing the percentage of moisture opposite the flask or tube reading, assuming a 500-gramme, damp weight. These tables were compiled by determining the difference in the quantity of water displaced by dry and by damp samples. The volume of water displaced by the dry sand was assumed to be constant as long as the grading, unit weight, or specific gravity of the sand did not change materially.

Removal of Forms.—When the concrete had been poured under normal conditions the forms were required to remain for various periods (days), as follows:

	Days.
Diversion Dam:	
Mass concrete	2 to 3
Thin walls	5 to 7
Tunnels and other openings	14
Transitions, Flumes, and Combined Sections:	
Vertical walls and steel slopes	3 to 4
Flat slopes	2 to 3
Siphon Barrels:	
Outside supports and lagging	3 to 4
Inside diagonal braces	3
Inside horizontal braces	7
Inside vertical braces, arch rings, and lagging	14
Tunnel Lining:	
Tunnels	3 to 4
Beams and Slabs:	
Beams and slabs (subject to flexural stress)	21 to 28
Bridges:	
Piers	3 to 4
Abutments	5 to 7
Turnouts, Culverts, and Ditch Crossings:	
Intake or outlet	3 to 4
Outside of barrel, and cut-off collars	3 to 4
Inside of barrel	14
Lateral Drains:	
(*a*) If puddled immediately	2
(*b*) Otherwise	3 to 4

When concrete was poured at low temperatures the forms were moved at the discretion of the Division Engineer.

Curing.—It has been stated, and apparently with considerable justification, that improper curing is responsible for more defective or short-lived concrete than any other single element in its manufacture. Although curing has had a place in concrete specifications for years past, it is only in recent years that its high relative importance has become generally known, or sufficiently appreciated by construction supervisors to command whole-hearted observance of the specification provisions.

Water can make or ruin concrete. If used too freely in the mixing precess, it weakens the concrete by thinning the cementing fluid. A similar injury results if, after the cement has taken its final set, its affinity for water to effect hydration is not satisfied. Inasmuch as the plastic concrete contains sufficient water for complete hydration of the cement, the process of curing resolves itself into one of retaining the original mixing water, or of supplying additional moisture to counteract evaporation and other losses.

Compressive strength tests of cylinders that were cured under damp sand or were permitted to dry out, indicate increases in strength for sand-cured specimens of about 75% at 10 days, 100% at 14 days, 125% at 28 days, and 145% at 4 months.

The resistance of concrete to abrasive action is similarly increased by moisture curing, since the strength relation has been found to be a reliable index of the efficiency of concrete to withstand wear.

The effect of proper curing in building up the internal structure of the concrete and making it less pervious to water is very marked. Experiments in the laboratory at Ellensburg, have shown that properly cured concrete, made of 1 part cement and 9 parts aggregate, is far less pervious than a rich uncured concrete made of 1 part cement and 3½ parts aggregate, the rich concrete being allowed to dry out, with forms removed, in 24 hours. The importance of proper curing is manifest when it is realized that impermeability, which depends on effective curing, is now regarded as concrete's most effective defense against the forces leading to disintegration, such as frost action, leaching, alternate wetting and drying, and contact with acid or alkali-bearing soil or water.

For best results the curing process should not be interrupted. This is particularly true during the early stage of hardening, because when the interior is moist and swollen, rapid surface drying induces hair cracks (known as crazing) which may subsequently develop into structural cracks. Curing, therefore, should be started as soon as possible, and the concrete should not be permitted to dry out before it is adequately cured.

All concrete, except that cured by patented process, was kept continuously wet for at least 14 days after being poured. Except in cases where small structures could be readily covered with moist earth, the concrete was cured under wet burlap. In the case of canal lining, the first half (7 days) of the curing period was under burlap. The barrels of the concrete siphons were cured inside and outside for a period of 28 days, the outside under burlap. The cost of water-curing the canal lining was stated by one contractor to be approximately $0.15 per sq. yd. The cost of the "process" curing averaged about $0.135 per sq. yd.

CONSTRUCTION COSTS

There is considerable difference in the unit bids received from various contractors. This condition is not common to this project alone. It is difficult to understand why one contractor should receive 100% more payment per unit than another operating on the same work and under the same conditions. It indicates that the competitive system of bidding is at fault.

Table 5 gives a list of unit prices as paid contractors. No attempt has been made to select either the extreme high or the extreme low, but the selections were made at random from the thirty-six contractors operating on the project. Only the more important items are given. On this project the

Government furnished all material that became a part of the completed construction work. The contractor furnished all lumber, nails, wire, etc., used in form work.

TABLE 5.—RANDOM SELECTION OF UNIT PRICES PAID TO CONTRACTORS

Description	Unit	CONTRACTORS										
		MAIN, NORTH AND SOUTH BRANCH CANAL							LATERALS			
		No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11
Clearing	Acre	175.00	......	90.00	150.00	75.00	......	......	......	......	......	......
Grubbing	Acre	250.00	......	150.00	125.00	100.00	......	......	......	......	......	......
Canal Excavation:												
Class 1	Cubic yard	0.16	0.14	0.15	0.17½	0.14	0.15	0.25	0.35	0.29	0.29	0.39
Class 2	Cubic yard	0.55	0.32	0.30	0.42	0.28	0.36	0.50	0.90	0.75	0.60	0.70
Class 3	Cubic yard	1.50	1.25	1.25	1.50	1.50	1.25	1.70	3.00	2.00	2.00	1.00
Compacting embankment	Cubic yard	0.40	0.20	0.40	0.20	0.30	0.60	......	......	......	......	......
Canal Lining:												
Trimming	Square yard	0.18	0.20	0.20	0.15	0.20	0.25	......	......	......	......	......
Concrete	Cubic yard	8.00	15.00	11.50	8.00	12.00	11.50	......	......	......	......	......
Reinforcing	Pound	0.02	0.04	0.014	0.02	0.02	0.015	......	......	......	......	......
Structures:												
Concrete	Cubic yard	24.00	20.00	16.50	15.00	14.00	19.00	14.00	17.50	30.00	22.50	15.00
Reinforcing	Pound	0.02	0.04	0.015	0.02	0.02	0.015	0.03	0.035	0.03	0.04	0.03
Excavation:												
Class 1	Cubic yard	0.75	0.50	0.60	0.55	0.50	0.50	......	1.25	0.75	1.25	0.60
Class 2	Cubic yard	1.50	0.80	1.00	1.00	1.00	1.00	......	1.75	1.00	2.50	0.80
Class 3	Cubic yard	......	3.00	3.00	......	3.00	3.00	......	3.50	......	......	1.25
Back-fill	Cubic yard	0.50	0.50	0.65	0.33	0.25	0.40	1.00	0.75	0.50	0.75	0.45
Puddle back-fill	Cubic yard	0.50	1.00	0.40	0.50	0.60	0.60	0.50	......	......	......	......
Erecting timber in bridges	1 000 ft. board measure	30.00	30.00	28.00	......	16.00	25.00	......	40.00	15.00	......	30.00
Tunnel Excavation:												
11.9 diameter, Horseshoe Section	Linear feet	......	......	......	......	......	......	24.00	......	......	......	......
6.25 diameter, Horseshoe Section	Linear feet	......	......	......	......	......	......	12.50	......	......	......	......
Concrete in tunnel lining	Cubic yard	......	......	......	......	......	......	9.00	......	......	......	......

The cost to the contractors of common labor ranged from $4.00 to $5.00 per 8-hour day, probably the more common price being $4.50. The approximate prices paid to the skilled labor are as follows: Shovel runners, $1.43 per hour; foremen, $1.00 per hour; carpenters, $0.75 per hour; and concrete finishers (in canal lining), $1.00 per hour. Other labor was paid a proportionate price.

GENERAL CONCLUSIONS

Probably the most noticeable difference in the construction work on this project and other projects that have been built by the U. S. Bureau of Reclamation was the use of more scientific methods in the control of concrete manufacture. There was a distinct improvement in the quality and uniformity of the concrete over that poured by the more common method which was used on some of the earlier concrete on these projects. This was shown principally by appearances and the strength of cylinders and samples taken from the cured concrete, and by leakage tests made on siphons. Many engineers have a deep-seated conviction that scientific concrete control is a laboratory procedure alone. In the application of the method of concrete control used on this project, the inspectors and others engaged in directing the operations, were instructed as to the many influences affecting concrete in the making and the necessity for co-ordinating and controlling the various detail processes if concrete of uniformly high quality was to be produced. There is an apparent increase in cost incident to closer control, but a more complete analysis will show a probable decrease in cost for the same quality concrete, due to a more careful check on the cement and aggregate quantities. It is sometimes difficult to convince a practical man on concrete work, who has had some experience, that scientific control has any advantage over the old method.

A study of Table 4 reveals some interesting facts. There were 145 cylinders made from small structures. As a general rule, these were 1 or 2 cylinders per structure. The probable number of structures from which they were taken was about 100. Their average strength was 3 848 lb. per sq. in. and 78% were within 20% of that value. They were made by thirty to forty different inspectors under temperature conditions ranging from nearly freezing to more than 100° Fahr.

The writer is indebted to the Engineers of the U. S. Bureau of Reclamation at the Main Office in Denver, Colo., the Project Office, at Ellensburg, Wash., and the Division Offices at South Cle Elum and Thorp, Wash., for kind assistance in the preparation of this paper. This included the contribution of much of the information and material and the reading and correction of the manuscript.

DISCUSSION

John Sanford Peck,[5] **Assoc. M. Am. Soc. C. E.** (by letter).—In reading this paper, the writer's interest was captured primarily by the sections devoted to the description of the methods used to insure high quality concrete. The success of the project, from a service standpoint, depended entirely upon the quality of the concrete produced on the job. The greatest possible skill in hydraulic design would have been nullified entirely if the concrete had failed under the extremely severe conditions of service to which it was exposed. The results obtained, as judged by the test cylinders taken from the job, are another vindication of scientific methods of concrete manufacture.

The successful results that were obtained depended entirely on the degree of knowledge and skill of the concrete inspectors, and the author's description of the "Instructions for Concrete Inspectors" that were prepared for use on this project, together with the tables that he mentions in detail, point to a handbook of more than passing interest. It must have been successful in teaching the essential features of the control of the water-cement ratio on the job to practical-minded men to whom anything resembling theory is anathema.

With reference to the use of an admixture of diatomaceous earth in the concrete, the writer would like to know what the reasons were for using an admixture at all, and just how much this admixture contributed to the workable properties of the concrete? A description of the laboratory technique used in determining the need for the admixture and in actually measuring the increase in the workability secured by using it, would be of great interest and value.

For some time the writer has been of the opinion that, while admixtures in the proper proportions do no harm to the concrete, they do not appreciably improve a mix, except perhaps to add an indefinite and unmeasurable quality of "fatness," which can only be detected by the eye. The results of several series of laboratory tests made by the writer have confirmed this opinion and have tended to show that the best and cheapest admixture is an increase in the cement of from 5 to 10 per cent.

This part of Mr. Mason's paper again shows the results that can be obtained by scientific methods of concrete control, and it is worth noting that a paper[6] by William M. Hall, M. Am. Soc. C. E., also discusses the practical use in the field of the results that have been obtained and thoroughly proved in the laboratory.

Edward W. Bush,[7] **M. Am. Soc. C. E.** (by letter).—Near the end of this interesting paper, under "Construction Costs," the author makes the statement that:

"It is difficult to understand why one contractor should receive 100% more payment per unit than another operating on the same work and under the same conditions. It indicates that the competitive system of bidding is at fault."

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[5] Instr. in Civ. Eng., Coll of the City of New York, New York, N. Y.

[6] "Manufacturing Concrete of Uniform Quality", by William M. Hall, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1336.

[7] Engr., Aetna Casualty & Surety Co., Hartford, Conn.

The writer offers the suggestion that probably the greater part of these discrepancies were caused by the contractors unbalancing their bid items either by increasing the prices on the items completed during the early part of the work—and thus obtaining money needed to finance the job—or by increasing the amounts bid on items which it was thought might be increased in quantity and, at the same time, decreasing the prices on other items which might be decreased in quantity. Competitive bidding is not to blame for the discrepancies mentioned by the author and, in most cases, these discrepancies can be prevented if the engineer, in the instructions to bidders, includes a condition stating that grossly unbalanced bids will not be given consideration when making the award.

It is thought that most engineers expect a certain amount of unbalancing unless they include in the set-up items like installation of equipment, organization expense, over-head, pumping, coffer-dams, etc., which often are real items appearing in the cost estimate of the bidder and which he naturally desires to load on to the early items if this is possible. It is not unusual to find items like foundation excavation running from $6 to $10 per cu. yd., provided a coffer-dam not elsewhere paid for must be built before the "pay" yardage work is started. In general, it is to be expected that, whenever the engineer has carefully analyzed the work to be done and has set up plenty of items so that each part can be bid separately, rather close bids will be received from the really experienced contenders for the job.

The defense against the gross unbalancing of items that might be increased or diminished in quantity is found in making such thorough preliminary investigations of the site or in working out the details of the design so that little doubt exists as to the quantities that will be required.

Some rather disturbing results may arise from grossly unbalanced bids on a unit price contract. The contractor may accumulate a considerable amount of early profits and expend them on some other jobs being performed at a loss; he may install equipment or an organization on other subsequent contracts; or, he may send his family to Europe, or otherwise expend the profits, and then lack sufficient funds with which to complete the contract on which the unbalancing occurred. Another interesting fact is that during the first part of the work the owner may be holding his own money as the retained percentage instead of any sum that has been earned by the contractor.

Even if the bid items are unbalanced a moderate sum, it is the total price that interests the owner and is considered when making the award; and if the case has been carefully prepared for bidding the totals coming from the experienced bidders will generally be found to compare favorably with one another and only vary because no two persons are apt to think alike on the probable cost or the amount of profit to be added. The average bid seldom has the importance frequently given to it, because there are generally a number of high bidders who only bid because they were asked, or they are bidders who lack an available plant or organization, or possibly a seasoned experience in doing similar work; hence they do not possess reliable cost data and, therefore, bid high. Some high bidders are supplied with enough current work to keep them fairly well occupied and only desire another contract if it can be secured at a favorable price.

Probably in most cases the real contenders for the contract are to be found in the lower third or half of the bids, provided those offering these bids are experienced contractors on the class of work bid upon. Complementary bids are generally high and many of these are offered merely because the engineer or architect sent the plans to the bidder. Others bid merely to see their names among those bidding, thinking that a certain prestige is obtained if the job happens to be a large one. Bankers at times will extend rather large credits for certified checks when it is known that the bidders will put in such high prices that the jobs will not be obtained except at very attractive figures. It is not worth while to give too much consideration to the high bidders if a fairly large number of bids are offered, but when judging the lowest bids before making an award it is desirable to give full consideration to the bidder's experience, financial strength, available plant, and organization; also, whether the new work taken in connection with the other work on hand will be inside the volume which the contractor is considered competent to handle and finance.

ORRIN H. PILKEY,[8] JUN. AM. SOC C. E. (by letter).—It is interesting to note that actual construction was begun on some high-line canals of this project before the U. S. Bureau of Reclamation took part, but that this early attempt resulted in failure because of lack of funds. This is an example of the value of the Federal Bureau. With its greater financial backing it is able to undertake and complete desirable projects which otherwise would not be attempted, or, if attempted, would result in failure.

One of the features of the Kittitas Project worth mentioning is the use of safeguards of several types in the main canal to prevent accidents, especially drowning. Just above each siphon or tunnel intake is a creosoted timber float fastened to a 1-in. guide rope. If a person is being swept down with the current he might be able to catch hold of this timber and hang on to it, thus saving himself from being carried through the siphon or tunnel. If he missed the timber he would scarcely be alive by the time he reached the outlet.

In addition to the timber there are metal ladders placed on both sides of the canal at the siphon entrances. Other ladders are spaced about 250 ft. apart on alternate sides of the lined canal. Their purpose is to provide egress from the canal itself. The side slope of 1 on $1\frac{1}{4}$ of the concrete lining is so steep that it is almost impossible for a person to climb out when the concrete is wet and slippery.

Timber fences of heavy construction border the edges of the canal at both the inlet and outlet ends of the siphons, and metal railings built of angle irons are placed around the operating platforms. The bridges crossing the canal also have angle-iron railings.

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[8] New York, N. Y.

The author has given figures which he states indicate that the competitive system of bidding is at fault. It is certainly interesting to note the great variations in unit prices paid to different contractors operating on the same work and under the same conditions. Some of the bids must have been extremely unbalanced. It is admitted that competitive bidding leaves much to be desired, but other methods of bidding on public work are likely to be far more unsatisfactory. The writer would like to inquire what the author recommends to alleviate the faults of the competitive system.

CLIFFORD A. BETTS,[9] M. AM. SOC. C. E. (by letter).—The description of the scientific control of concrete production and of the up-to-date construction methods and costs on the Kittitas Division of the Yakima Project draws attention to the economies that can be effected by the use of laboratory control and modern machinery. There is a well defined trend in this direction.

The uniformity that can be assured in concrete by effective control permits closer design, lower safety factors, and correspondingly greater savings. On projects involving large yardages, the net savings from this source alone have amounted to as much as 2% of the total cost of the concrete, as evidenced by one case in which the cost of laboratory control and inspection was $0.035 and the saving in cement $0.18 per cu. yd. of concrete. Costly control becomes burdensome; but practical inspection administered with common sense can be made indispensable. Furthermore, on work that is being done on a three-shift program, it becomes highly advantageous to make the proportioning, mixing, and placing as "fool-proof" as possible by standardization of all operations.

In attaining the impermeability, strength, and workability required for long-lived hydraulic structures the following procedure has been found to have merit:

(*a*) Screen both sand and gravel into two or more sizes in order that the screened aggregates may be reblended accurately in such a manner as to produce concrete with a minimum percentage of voids. In this way a troublesome variation in consistency—which inevitably adds to the difficulties of water control and intermittently causes harshness, with water rising to the surface—can be avoided and a satisfactory gradation, which is a vital element, can be obtained.

(*b*) Locate washing and screening plant and arrange the stock-piles so that aggregates will not carry a wide variation of moisture into the mixer bins to interfere with moisture control.

(*c*) Proportion by weight, with graphic corrections for surface moisture and carry-over, the latter requiring attention only when screens are changed. The use of charts at the mixer eliminates interpolation and reduces computations to the subtraction of the surface moisture from the mixing water.

In Fig. 21, a typical illustration, the proportions of each size of aggregate are expressed in terms of parts by weight of cement per batch (diagonal lines marked "stock-pile proportions"). The percentages of surface moisture are ordinates and the weights of aggregates plus moisture, are abscissas. For example, when the surface moisture of a certain aggregate is 5% and the specified proportions are 2 parts to 1 part of cement, the total weight of this moist aggregate required for 1 cu. yd. batch is 1 050 lb. Similarly, the quantity of water to be deducted from the mixing water (in gallons or pounds) can be read from a diagram, such as Fig. 22, having "parts of aggregate by weight" as ordinate, "water to be deducted from total mixing water" as abscissa, and "percentage of surface moisture" as diagonals, with angles of inclination as determined in the laboratory. As an example, if sand contains 4% moisture, the chart shows that 4.8 gal. of mixing water should be deducted per batch.

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[9] Engr., U. S. Bureau of Reclamation, Ontario, Ore.

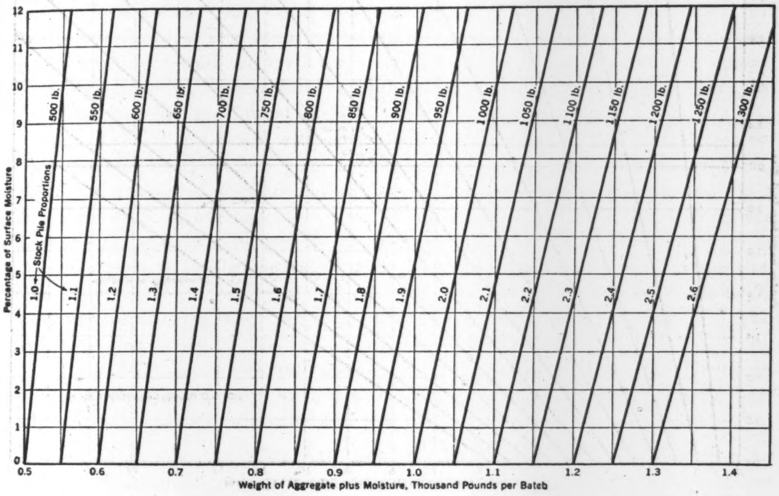

FIG. 21.—WEIGHTS OF AGGREGATES FOR 1 CUBIC YARD BATCH.

In addition to the methods of surface-moisture determinations described in the paper, reference might be made to patented scales reading the percentage of moisture directly on the dial.

Such control applied to a job of sufficient yardage to give a large number of representative samples will permit strength control tolerance so close that no specimens will vary more than 15% above or below the average of all the samples taken, and 90% will be within 10% of the average. As a rule, works involving considerable concrete yardage and extending over several years, show a gradual improvement in quality of concrete produced, as time goes on. The better the control the more nearly all the concrete will approach the perfected product.

The statement by Mr. Mason that "high strength, high impermeability, and good workability * * * were considered equally important," is a reminder that a standard test for impermeability is greatly needed. In fact, it frequently happens in hydraulic structures that water-tightness rather than strength is the criterion controlling the richness of the mix. Unfortunately, there is no well-defined quantitative relation between strength, which is readily tested, and impermeability, which is more difficult to measure, although, in general, both are improved by the same factors. For example, placing and curing, like gradation, have a greater effect on permeability than on strength, whereas wetting a mix may decrease the strength, but increase imperviousness. Pending the perfection of a generally acceptable permeability test apparatus a simple qualitative test can be made by inserting flanged pipes into mass concrete and determining by trial the permissible hydrostatic head.

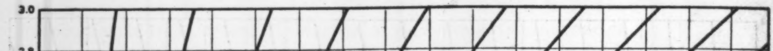

FIG. 22.—SURFACE MOISTURE IN AGGREGATES, 1 CUBIC YARD BATCH.

GEORGE C. IMRIE,[10] ESQ. (by letter).—Taken as a whole this paper gives a true picture of the construction work as conducted on the Kittitas Division of the Yakima Project. Because of conditions encountered as the work progressed, some major changes were made in the construction program. In driving the horizontal leg of the tunnel several sections were excavated through tuffaceous shale, and it was decided to change the type of lining through this part of the tunnel from plain to reinforced concrete, and to increase the thickness of the shell from 15 to 30 in. Where the tunnel was excavated through a substantial covering of basalt, it was lined with 15 in. of plain concrete as covered in the specifications.

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[10] Office Engr., U. S. Bureau of Reclamation, Ellensburg, Wash.

In the 30-in. reinforced sections of the horizontal leg of the tunnel, the transverse reinforcement consists of two layers of 1½-in., square, hard, steel hoops on 8, 9, or 10-in. centers, depending on the classification of the backing. For the longitudinal reinforcement, twenty-two ¾-in., round bars, two 1-in. square bars, and two 1¼-in. square bars were used. The 1-in. square and the 1¼-in. square bars were substituted for the ¾-in. round bars, and were placed to assist in supporting the cages in position. In the upper parts of the inclined raises, the 15-in. concrete lining was reinforced against full hydrostatic head. The longitudinal reinforcement consisted of forty-eight ⅝-in. round bars, and the transverse reinforcement varied from 1½-in. square hoop bars, on 6-in. centers for the highest head, to 1-in. round hoop bars, on 10-in. centers for the lowest head.

The tunnel is provided with a vertical wasteway shaft 78 in. in diameter. This shaft is lined with a ¾-in. copper-bearing, steel plate to a point 17 ft. above the invert of the horizontal leg of the tunnel, and is encased in a minimum of 9 in. of concrete. Two 42-in. internal differential needle-valves discharging into the Yakima River are provided.

The structure is also provided with an electrically driven direct-flow turbine pump with a capacity of 600 gal. per min. against a 155-ft. head, to be used for unwatering the tunnel at the end of the irrigation season. The hydraulic properties of the tunnel are given in Table 3.

Of the 6 750 acres of land shown in the paper as held by the Northern Pacific Railway Company, all but 639 acres have now (February, 1932) been sold at the appraised value to *bona fide* settlers.

ARTHUR RUETTGERS,[11] M. AM. SOC. C. E. (by letter).—This interesting paper is a commendable presentation of the salient construction features of the Kittitas Division of the Yakima Project—a work which included one of the most difficult and extraordinary pieces of canal construction ever undertaken by the U. S. Bureau of Reclamation.

Inasmuch as approximately one-half of the $9 000 000 investment was expended for concrete, the author fittingly emphasizes the importance of the procedure for controlling the quality of this material. The use of diatomaceous earth as a concrete admixture is briefly mentioned in the paper and the writer is taking this opportunity to submit some data on the subject.

The various concrete mixes established by laboratory tests and subsequently modified to some extent by early field experience on the Kittitas work have been presented previously in a paper by the writer and Mr. A. A. Whitmore.[12] No admixture was used in rich mixes having 4½ or less parts of aggregate; 1½% admixture was used in the 1 : 2 : 3.25 mix; and 3% admixture was used in mixes containing 6 parts or more of aggregate.

At first, diatomaceous silica was considered as a means of overcoming a noticeable harshness or lack of plasticity in the leaner laboratory mixes, including those that were over-sanded. This condition was attributed to a natural deficiency of fines in the sand and was alleviated by the admixture treatment, in so far as could be judged by the appearance, "feel," and finishing of the concrete. Before permitting the use of admixture in the field, however, the laboratory tests on plain concrete were extended to determine the effect of admixture on compressive strength for some of the more practicable mixes. Comparisons were made on the basis of equivalent slumps, which resulted in generally higher water-cement ratios for the concrete with admixture. The average results of the 28-day strength tests are given in

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[11] With U. S. Bureau of Reclamation, Denver, Colo.

[12] *Proceedings*, Am. Concrete Inst., Vol. 27, 1931, Table 1, p. 141.

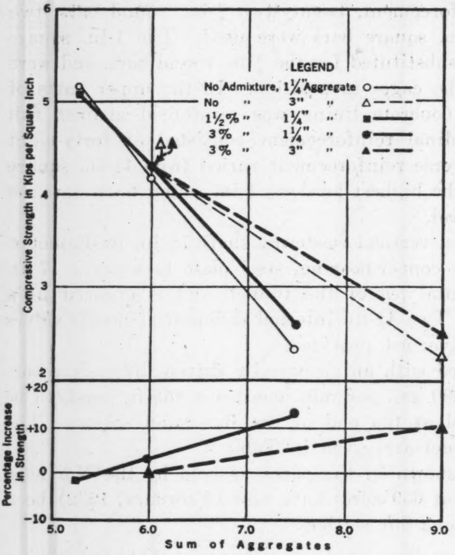

FIG. 23.

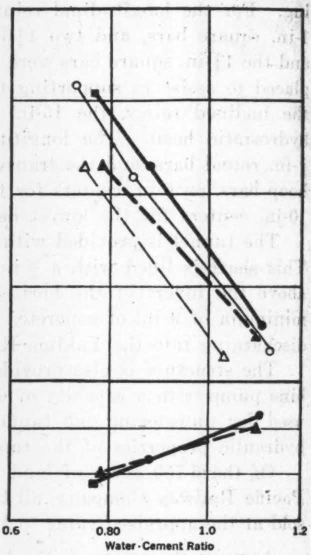

FIG. 24.

TABLE 6.—STRENGTH TESTS

Sum of aggregates	Gravel to sand ratio	Slump, in inches	No Admixture		With Admixture			
			Water-cement ratio	28-day strength in compression	Percentage, admixture	Water-cement ratio	28-day strength in compression	Percentage, increase in strength
FOR CONCRETE OF 1½-INCH MAXIMUM SIZE OF AGGREGATE								
5.25.....	1.84	2	0.70	5 410	1.5	0.74	5 230	−3.8
5.25.....	1.84	6	0.76	4 910	1.5	0.80	4 955	0.9
6.0......	1.62	2	0.83	4 425	3	0.85	4 550	2.8
6.0......	1.62	6	0.87	3 935	3	0.91	4 045	2.8
7.5......	1.35	2	1.09	2 395	3	1.09	2 750	14.8
7.5......	1.35	6	1.17	2 330	3	1.16	2 570	10.3
FOR CONCRETE OF 3-INCH MAXIMUM SIZE OF AGGREGATE								
6.0......	2.28	2	0.75	4 290	3	0.78	4 305	0.4
9.0......	2.28	2	1.03	2 265	3	1.10	2 495	10.2

Table 6 and Figs. 23 and 24 and show that, with one exception, the strengths were increased from 1% to 15% by the use of admixture. It is interesting to note, in this connection, that the greater proportionate increases in strength accompanied the leaner mixes. In other words, the mixes most in need of admixture to counteract harshness and tendency to segregation were, fortunately, also the ones most greatly benefited from the standpoint of strength. In these figures the sum of aggregates is by volume, dry rodded, and the water-cement ratios are by volume.

Permeability tests, conducted principally for the determination of appropriate mixes for high-head concrete siphons, also furnished information on the comparative advantages of concrete with and without admixture. A brief description of these tests may be of interest, due to the fact that the procedure was materially different from that generally used in testing for permeability.

The test procedure consisted in forcing water through concrete disks 12 in. in diameter and 4 in. thick by applying air pressure to a column of water connected to the top surface of the disk. The volume of water entering each specimen was measured by noting water-surface readings on a gauge glass attached to the pipe water column. An ordinary water gauge was used for determining applied pressures. Except at night, on Sundays, and holidays, the flow into the specimens was maintained as nearly as practicable at a fixed rate, sufficient to exceed, slightly, the evaporation losses from the exposed surfaces of the disk, by increasing the pressure slightly whenever the periodically observed gauge-glass readings indicated a rate of inflow less than the established rate. The resulting pressure curve, except for porous specimens, and excluding the effect of periods outside working hours, was a gradually ascending one from zero pressure to (in some instances) 500 lb. per sq. in., the safe limiting capacity of the apparatus. Water outflow was collected and the accretions were measured at frequent intervals. Concrete disks were moist-cured for 14 days and then heat-dried to constant weight before being subjected to test.

A typical graphic test record, with gauge pressures, cumulative flows, and rates of flow plotted against time, is shown in Fig. 25. A photograph of the test apparatus is included in the paper previously cited.[13] The following test data apply to the typical case illustrated in Fig. 25: Real mix, 1:3.84; sum of aggregates (by volume), 4.50; nominal mix, 1:2.25:2.25; ratio of gravel to sand, 1.0; admixture in percentage (by weight) of cement, 0; slump, 6 in.; water-cement ratio (by volume), 0.70; average compressive strength (28 days), 4 645 lb. per sq. in.; maximum size of gravel, 1¼ in.; weight of disk at 14 days (moist), 39.56 lb.; weight of disk at test (heat-dried), 38.58 lb.; loss of water (by weight), 0.98 lb.; and loss of water, 444 cu. cm. The points plotted at breaks in the rate curves (Fig. 25(*a*)) represent average rates for the intervals preceding. Intervening points on the sloping connecting lines do not indicate actual rates. The line indicating the water-absorbing capacity of the disk is approximate, and is based on the loss in weight of the disk after it has been removed from the curing vat and heated. At Point 3, the total absorption was 1.22 lb., or 553 cu. cm., based on the increase in the weight of the disk at the end of the test.

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[13] "Construction of Main Canal Lining on Kittitas Division, Yakima Reclamation Project, Washington," *Proceedings*, Am. Concrete Inst., Vol. 27, 1931, Fig. 20, p. 144.

Approximately fifty concrete disks, ranging from very rich to very lean mixes, were tested for permeability and in only two specimens was the available pressure of 500 lb. per sq. in. insufficient to produce outflow eventually. The average results of the tests, using "pressure to produce outflow" as a permeability index, are summarized in Table 7. As will be noted from the tabular values the resistance to permeation was markedly increased by the use of admixture in concrete having 1¼-in. maximum size of aggregate, especially in the mixes containing a total of 6 parts of aggregate to 1 part of cement. The improvement was inappreciable, however, in concrete made with 3-in.

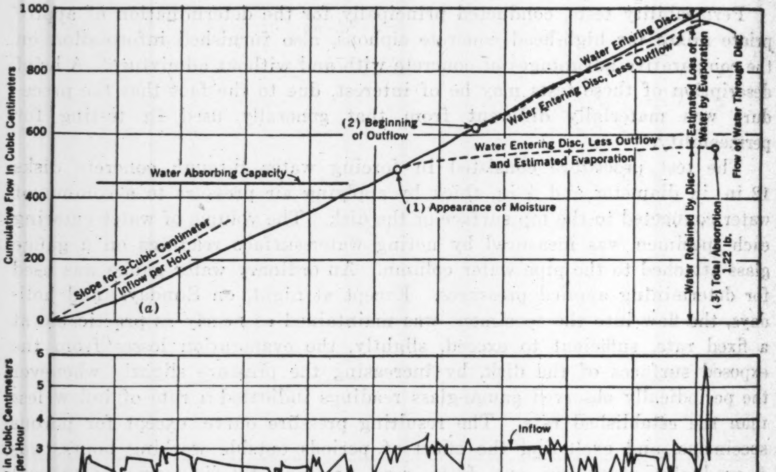

FIG. 25.—TYPICAL GRAPHICAL RECORD OF A PERMEABILITY TEST.

maximum size of aggregate. These results would indicate that admixture is most effective in promoting water-tightness where the voids in the concrete are relatively small in size and where large pieces of aggregate are not present to form direct water channels along their under surfaces.

TABLE 7.—PERMEABILITY TESTS

Sum of aggregates	Gravel to sand ratio	No Admixture: Average water-cement ratio	No Admixture: Pressure required to produce outflow	With Admixture: Percentage, admixtur	With Admixture: Average water-cement ratio	With Admixture: Pressure required to produce outflow
For Concrete of 1½-Inch Maximum Size of Aggregate						
3.5......	2.83	0.575	410	....	....	....
4.5......	1.0 to 2.12	0.69	254	....	....	....
5.25.....	1.84	0.73	78	1.5	0.78	110
6.0......	1.0 to 2.0	0.87	75	3	0.92	310
7.5......	1.0 to 2.5	1.06	13	3	1.13	44
9.0......	1.18	1.39	1	....	....	....
For Concrete of 3-Inch Maximum Size of Aggregate						
6.0......	2.28	0.75	40	3	0.78	45
9.0......	2.28	1.03	2	3	1.06	6

A third series of laboratory tests was conducted on the Kittitas Project in an effort to obtain comparative data on the durability or weather-resisting properties of concrete of various mixes, with and without admixture. The test specimens were made of concrete having 1¼-in. maximum size of aggregate and were basin-shaped to hold water. One set was moist-cured for 14 days before testing and a second duplicate set was simply air-cured. The two sets of specimens were subjected simultaneously to frequent alternations of wetting and heat-drying and, during cold weather, to freezing and thawing for a total period of ten months. In general, the results were as follows:

(1) There was no perceptible difference, for concrete of the same mix, between specimens containing admixture and those made without admixture.

(2) Specimens of richer mix showed more crazing and a larger number of structural cracks.

(3) Specimens of leaner mix were more affected by leaching action.

(4) Air-cured specimens were much more absorptive and affected to a much greater degree by crazing, leaching action, and spalling than companion moist-cured specimens.

It is the writer's opinion that the substitution of cement for diatomaceous earth, or other siliceous admixture, on an equivalent cost basis, may sometimes be (although not always) the best procedure. A large part of the diatomaceous silica used on the Kittitas Division was mined locally and purchased at a price that made the delivered cost approximately 1 cent per lb. The delivered cost of cement was approximately 0.67 cent per lb., which means that 1½ lb. of cement might have been used in lieu of each pound of diatomaceous earth admixture.

Considering the increased strengths obtained from the use of the diatomaceous silica, it seems reasonable to expect that the substitution of cement would have yielded similar strength increases. On the other hand, the permeability test data available indicate that the improvement in water-tightness to be expected from the increased use of cement would have fallen far short of that accomplished by the introduction of diatomaceous silica. This is especially true, as previously pointed out, for the 1:6 mix which was used on most of the concrete work on the Kittitas Division. The substitution of cement would not have imparted those qualities of smoothness and uniform jelly-like consistency so characteristic of admixture-treated concrete and so helpful to a harsh mix in obtaining proper field placement and finishing. In this connection the writer is reminded of a preference expressed by one of the siphon contractors for the 1:2.0:3.25 mix with $1\frac{1}{2}\%$ admixture over the 1:1.7:2.8 mix without admixture, notwithstanding the fact that the latter mix is richer in cement by 14% and that both mixes have essentially the same gravel-to-sand ratio.

In conclusion, the writer does not wish to go one record as advocating the promiscuous use of diatomaceous earth or any other of the innumerable preparations of concrete admixture being promoted. However, some brands or types of admixture have a rightful place in concrete work and the problem is primarily one of determining experimentally their field of application and then applying such knowledge to the advantageous use of the materials in meeting individual requirements.

Morris Mason,[14] Jun. Am. Soc. C. E. (by letter).—It is gratifying to note that there is much unanimity of opinion concerning the advisability of using more scientific methods of concrete control as exemplified by practice on the Kittitas Division of the Yakima Project.

Mr. Peck approves the use of the pamphlet, "Instructions for Concrete Inspectors" and states that it must have been successful in teaching the essential features of the control of the water-cement ratio on the job to practical-minded men "to whom anything resembling theory is anathema." The use of the pamphlet was indeed a success. The education and experience of the inspectors on this work ranged from men of little education and many years of experience, to college professors, out for the summer, with very little experience, but well founded in theory, and to young college men who, in the beginning, were deficient in either or both respects. There were also a number of able engineers among the force of inspectors. The uniformity and good quality of the concrete produced under the supervision of these men speaks well for the method of inspection. The writer is of the opinion that the employment of inspectors of the grade of Assistant Engineers (Civil Service rating) or higher is amply repaid. The effect of employing better inspectors is generally noted in the loss of friction between owner and contractor. For the best work the contractor should respect the ability of the inspector. A chief inspector who is likely to appear on the job at any time, either day or night, proved useful in keeping the inspectors always on the alert.

Mr. Peck questions the use of admixtures and just how much the admixture used contributed to the workable properties of the concrete. It is hoped that the discussion by Mr. Ruettgers has answered this question. The writer agrees with Mr. Peck that, generally speaking, the qualities attributed to the use of admixtures, with the exception of workability, may be attained by the additional use of 5 to 10% of cement. For the most part, it was a question of the relative cost of cement and admixture in place in the concrete. Little unbiased information is to be had on this subject.

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[14] Engr., Standard Oil Co. (Indiana), Wood River Refinery, Wood River, Ill.

The analysis of "unbalanced bidding" which was given by Mr. Bush is concise and to the point. Almost any one with construction experience can cite examples in which the contractor has collected on the "cream" of the work and then has left further worries to the surety or the owner. A. P. Greensfelder, M. Am. Soc. C. E., has emphasized[15] the fact that unbalanced bidding should be guarded against. He states that one is likely to feel that practical experience warrants certain maneuvering of unit bid prices. According to Mr. Greensfelder there is usually enough gambling in construction without this, and it is only warranted through the viewpoint of equity of the owner when the engineers' quantities are themselves unbalanced. On many jobs the retained percentage is retained in name only. As Mr. Bush states during the first part of the work the owner may be holding his own money as the retained percentage instead of any sum that has been earned by the contractor. Where the contractor is responsible and will complete the work, the total amount bid is the important factor. There are exceptional cases in which the work will be in progress a number of years and most of the large items will be completed within the first few years, leaving the miscellaneous low-profit items to be carried for some time. The contract for a large dam is of this type. The contractor may honestly be using the owner's money rather than other money on which interest must be paid, for the larger part of his operating expenses. Mr. Bush believes that most engineers expect a certain amount of unbalancing unless they include in the set-up, items such as installation of equipment, organization expense, over-head, pumping, and coffer-dams. It is becoming general practice to include many of these in the list of contract items.

In most instances the experienced contractor will bid high on the small items that will possibly be increased in amount, and low on the large items that will be reduced in the final estimate. A contractor who has been operating in one particular line for some years may prove a better estimator than the engineer whose work may extend to many lines and locations. The writer agrees with Mr. Bush that the upper 50% of the bidders are of no importance in connection with a true cost analysis. As a general rule, the lower three or four bidders will be found to agree closely on the total bid.

Mr. Pilkey requests the writer's palliative for the faults of the competitive system of bidding. There is no easy solution of the problem; but more intelligent estimating, by only experienced engineers, will induce the submission of more balanced bids by responsible contractors. The safety features used in connection with the canals on the Kittitas Project were discussed by Mr. Pilkey. If a man were sucked into one of the siphons it is scarcely possible that he would reach the outlet alive. It is impossible even for a deer to climb the sides of these canals, in which the slopes are 1 on 1¼. When the canals are empty the deer that accidently happen in the canal must be stunned and removed by hand. As was pointed out by Mr. Pilkey, some of the early canals that were not completed, would not have been abandoned if they had been under Government supervision. The U. S. Bureau of Reclamation has been able to complete work that would otherwise never have been attempted. Construction work on the storage dam at Lake Cle Elum (see Fig. 1) was begun August 18, 1931, and will be completed about October, 1933. This reservoir, with Lake Kachess and Lake Keechelus Reservoirs, stores water for the entire Yakima River Irrigation District. Little of this work could have been done under private supervision. The early canals were constructed when water was plentiful and when any measurements made were in terms of the miner's inch. As more land was put under irrigation it was necessary to prorate the water, and as more water was needed it was necessary to construct storage reservoirs.

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[15] See p. 215.

The exact items used in Mr. Betts' analysis of savings due to effective concrete control are not given, but the writer feels that any concrete job of importance will show a comparable saving. The writer agrees with Mr. Betts that costly control is burdensome and that practical inspection, administered with common sense, can be made indispensable. With the batching equipment now on the market the control of batching quantities, including the allowance for surface moisture in the aggregate, is semi-automatic. The charts mentioned should be useful in a large plant where they might be made to a large scale and tacked to the wall. Mr. Betts quotes some strength tolerances in which no test specimens varied more than 15% and in which 90% of the specimens varied within 10% of the average. That is an additional point in favor of effective control. It is better control than could be expected in scattered small structures on the same job. The writer reported 78% on the specimens within 20% of the average of probably one hundred structures, which were poured under possibly forty different inspectors. Mr. Ruettgers' discussion presents a usable method of testing impermeability of concrete. The writer agrees with Mr. Betts that there is need of a standardized test for impermeability. Thus far, no one has proved a direct relation between strength and impermeability, although, in general, they are both improved by the same factors. The writer does not agree that, as a general rule, wetting the mix will increase the imperviousness, other factors remaining the same.

The writer is indebted to Mr. Imrie for the enumeration of changes made in the design of the Yakima River Pressure Tunnel during construction. This structure is a novel design of the type. It is interesting to note that of the 6 750 acres of land shown as held by the Northern Pacific Railroad Company, all but 639 acres had, up to February, 1932, been sold to *bona fide* settlers at the appraised value. A large part of this acreage was very rough terrain.

Unbiased testimony on the use of admixtures in concrete is scarce. The tests reported by Mr. Ruettgers are an important addition to the subject.

The water-cement ratios given in the paper did not allow for the absorption of mixing water by the admixture. The water-cement ratios reported by Mr. Ruettgers as 0.90 would become approximately 0.80 if allowance were made for 200% absorption by the 3% of admixture; this would possibly account for part of the 1% to 15% increase in strength due to the use of admixture. The method of testing for permeability, and the effect of admixtures on the imperviousness of concrete, as reported by Mr. Ruettgers, is also of interest. Engineers concerned with the design of concrete mixes for hydraulic structures should be much interested in his statement concerning this method and the tests upon which the statement is based. The tests to determine the weathering properties are also interesting and instructive. The evidence of men in charge of actual concrete placing generally substantiates the fact that the concrete containing admixture is much more easily placed; that is, it is much more workable, because it contains the necessary "oiliness" or "fatness" for ease in placing. This characteristic can not be obtained by the addition of extra cement.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 1835

WATER PRESSURES ON DAMS DURING EARTHQUAKES

By H. M. Westergaard,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Theodor von Kármán, Paul Bauman, R. Robinson Rowe, L. Prandtl, Cecil E. Pearce, John H. A. Brahtz and C. H. Heilbron, Jr., Boris A. Bakhmeteff, N. Mononobe, and H. M. Westergaard

Synopsis

Formulas are derived for the changes of water pressures during an earthquake for the case of a straight dam with a vertical up-stream face. The vibrations in the earthquake are assumed horizontal in a direction perpendicular to the dam. A formula is obtained involving a series of sines (Equation (31)). This series, however, may be replaced with satisfactory approximation by a simpler formula (Equation (49)). The results are also expressed in the following manner: The pressures are the same as if a certain body of water were forced to move back and forth with the dam while the remainder of the reservoir is left inactive; Fig. 3 shows approximately the shape of this body of water. The influence of the dynamic action of the water is found to be neither excessively large nor negligible.

Introductory Remarks Concerning Stresses in Dams During Earthquakes

Two causes account for added stresses in a concrete dam during an earthquake: First, the accelerations of the mass of the dam; and, second, the changes of water pressures. If no fault crosses the site of the dam, one may assume for the purpose of determining the accelerations that all points of the foundation have the same displacement, velocity, and acceleration at the same time, because the wave lengths of the main vibrations in the crust of the earth are so great that the differences of accelerations and the tiltings may be considered negligible.

Note.—Published in November, 1931, *Proceedings*.

[1] Prof. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.; Cons. Engr., U. S. Bureau of Reclamation.

For a straight concrete gravity dam with triangular cross-section, with height, H, and base, B, and with a modulus of elasticity of the concrete of 2 000 000 lb. per sq. in., the period of free vibrations (with the reservoir empty) would be approximately $T_0 = \frac{H^2}{2000\,B}\,\frac{\text{sec.}}{\text{ft.}}$ if the base were rigid.[2] With $B = 0.8H$, the formula gives $T_0 = \frac{H}{1600\text{ ft.}}$ sec.; for example, $T_0 = \frac{1}{8}$ sec. when $H = 200$ ft. On account of the elasticity of the base the actual period is longer, but remains a quantity of the same order. In an arch dam the stiffness of the arches reduces the period, as compared with that given by the formula.

Hence, since the period of free vibrations of the dam, T_0, is usually a fraction of a second, while the period of vibration, T, of the earthquake may be assumed to be not less than a second[3] (1.35 sec. measured during the great earthquake in Japan in 1923[4]), resonance—with vibrations of the structure becoming large compared with those of the foundation—need not be expected ordinarily, although the possibility of part resonance may well be investigated in some cases of high dams. When the ratio, $\frac{T_0}{T}$, is small, all points of the dam may be assumed at each time to have the same acceleration as the base. If the maximum horizontal acceleration is $a = \alpha\, g$, in which g is the acceleration due to gravity, the ratio, α, is used as a measure of the intensity of the earthquake. The value, $\alpha = 0.1$, appears to be a reasonable one to use in investigating dams founded on rock in earthquake regions in the United States.

The problem of the dynamic action of the mass of the dam is transformed into an equivalent problem of statics by introducing the inertia forces, expressed as mass times the acceleration reversed (d'Alembert's principle); that is, on each part of the dam weighing 1 lb. there is introduced a horizontal inertia force, opposite the direction of the acceleration and equal to $\frac{1\text{ lb}}{g} \times \alpha\, g = \alpha$ lb. The vertical accelerations, which by the nature of the phenomenon are smaller, either may be treated similarly or may possibly be ignored. If they are ignored, the combined effect of the dead weight and the inertia forces may be described as follows: Each gravity force of 1 lb. is changed into an inclined force, which has a vertical component equal to 1 lb., and a slope relative to the vertical equal to α. If α is not larger than, say, 0.1, the effect is nearly the same as though the dam were tilted so that a line that was horizontal will have a slope, α. The determination of the corresponding stresses involves no new difficulties that have not already been encountered in dealing with static loads.

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[2] See, for example, S. Timoshenko and J. M. Lessells, "Applied Elasticity," 1925, p. 323.

[3] Handbuch der Physik, Vol. 6, 1928, p. 596. The short periods occur only when the center of the earthquake is close.

[4] "Effect of Earth Shocks on Structures," M. de Bussy, *Proceedings*, Am. Soc. C. E., May, 1928, Papers and Discussions, p. 1449.

When the water pressures are known, their influence on the stresses may be computed by the methods applying to static loads. The problem that needs particular attention, therefore, is that of the changes of the water pressures during the earthquake.

NOTATION

The following notation is adopted:

x, y, z = rectangular co-ordinates x and y as shown in Fig. 1; the axis of x is along the undisturbed surface of the water, is directed up stream, and is perpendicular to the plane up-stream face of the dam; the axis of y is vertical downward.

t = time.

h = depth of reservoir.

g = acceleration due to gravity. (See Equations (10).)

w = weight of water per unit of volume. (See Equations (10).)

k = modulus of elasticity of volume of the water (pressure divided by reduction of volume per unit of volume). (See Equations (10).)

ξ and η = displacements of a particle of water in the directions of x and y, respectively.

ξ_0 = value of ξ at up-stream face of the dam.

σ = stress in the water due to the dynamic action, positive as tension (to be combined with the hydrostatic pressure existing beforehand).

α = ratio measuring the intensity of the earthquake = maximum horizontal component of the acceleration of the foundation divided by g.

T = period of horizontal vibrations of the foundation.

r, v = amplitude and maximum velocity of horizontal vibrations of the foundation.

v_s = velocity of sound in water.

l = length of waves of expansion in the water when the period is T.

c_n = value defined by Equations (17), (23), and (32) ($n = 1, 3, 5 \ldots$).

p = maximum pressure of the water on the dam at the depth, y, due to the dynamic action.

p_0 = value of p at the bottom ($y = h$).

Q = horizontal shear produced in a straight gravity dam at the depth, y, by the pressures, p.

Q_0 = value of Q at the bottom ($y = h$).

M = bending moment produced in a straight gravity dam at the depth, y, by the pressures, p.

M_0 = value of M at the bottom ($y = h$).

C, K = coefficients in approximate formulas (Equations (40) to (48).

b, b' = dimensions shown in Fig. 3.

EQUATIONS OF MOTION OF THE WATER

Since the motions involved are small, the relatively simple equations may be used, that apply to sound in liquids.[5] These equations may be interpreted in terms of the theory of elasticity of solids as equations of elasticity without shearing stresses.

The forces acting on an element of volume, $dx\ dy\ dz$, are expressed in terms of the tensile stress, σ, which must be superimposed on the hydrostatic pressure existing before the disturbance. The force on the face with the abscissa, $x + dx$, exceeds that on the face with the abscissa, x, by the amount, $\frac{\partial \sigma}{\partial x} dx$ per unit of area, or, by a total amount of $\frac{\partial \sigma}{\partial x} dx\, dy\, dz$. The mass of the element is $\frac{w}{g} dx\, dy\, dz$, and its acceleration is $\frac{\partial^2 \xi}{\partial t^2}$ (ξ being the displacement in the direction of x). Equating the force to the mass times the acceleration one finds the first of the following two equations:

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[5] "Hydrodynamics," by Horace Lamb, Fifth Edition, 1924, Chapter X.

$$\frac{\partial \sigma}{\partial x} = \frac{w}{g} \frac{\partial^2 \xi}{\partial t^2} \quad \text{.........................} (1)$$

$$\frac{\partial \sigma}{\partial y} = \frac{w}{g} \frac{\partial^2 \eta}{\partial t^2} \quad \text{.........................} (2)$$

Equation (2) refers to the motion in the direction of y, and applies by analogy; a third analogous equation, for the direction of z, need not be written, since only plane motions, parallel to the xy-plane, will be considered.

The stress, σ, in Equations (1) and (2), has the relation,

$$\sigma = k\, \epsilon \quad \text{.........................} (3)$$

to the increase of volume per unit of volume, ϵ, that it produces. The constant, k, is the modulus of elasticity of volume. One may write,

$$\epsilon = \frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} \quad \text{.........................} (4)$$

Consequently,

$$\sigma = k \left(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} \right) \quad \text{.........................} (5)$$

The motion is governed by Equations (1), (2), and (5). A comparison with the general equations of hydrodynamics for compressible fluids[6] shows that Equations (1), (2), and (5) may be assumed to apply when ξ and η are small, and when, in addition, the ratios of the products, $\frac{\partial \xi}{\partial t} \times \frac{\partial \sigma}{\partial x}$ and $\frac{\partial \eta}{\partial t} \times \frac{\partial \sigma}{\partial y}$ to $\frac{\partial \sigma}{\partial t}$, are negligible. In the solutions given in Equations (15), (16), and (20), these ratios are found to be of the order of the amplitude of the motions divided by the depth of the reservoir.

PRELIMINARY STUDY, ASSUMING HORIZONTAL MOTION ONLY

The fact that the water, when pressed by the dam, may escape vertically, is important. A preliminary study in which this vertical motion is assumed to be barred, will nevertheless throw some light on the problem.

With $\eta = 0$, Equations (1) and (5) give:

$$k \frac{\partial^2 \xi}{\partial x^2} = \frac{w}{g} \frac{\partial^2 \xi}{\partial t^2} \quad \text{.........................} (6)$$

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[6] See, for example, Handbuch der Physik, Vol. VII, 1927, p. 291.

Assume that the dam moves during the earthquake in a simple harmonic motion with period, T, and a maximum acceleration, αg, occurring when $t = 0$, T, $2T$, etc. This motion is expressed by the equation,

$$\xi_0 = -\frac{\alpha g T^2}{4\pi^2}\cos\frac{2\pi t}{T} \qquad \text{(7)}$$

The requirement, $\xi_{x=0} = \xi_0$, and Equation (6) are satisfied by the solution,[7]

$$\xi = -\frac{\alpha g T^2}{4\pi^2}\left[\beta\cos\frac{2\pi}{T}\left(t-\frac{x}{v_s}\right)+(1-\beta)\cos\frac{2\pi}{T}\left(t+\frac{x}{v_s}\right)\right] \qquad \text{(8)}$$

in which, β is a constant, and,

$$v_s = \sqrt{\frac{gk}{w}} \qquad \text{(9)}$$

The quantity, v_s, is the velocity of the waves of expansion in the water; that is, the velocity of sound (on account of the low frequency, these waves, of course, would not be audible as sound). The values,

$$\left.\begin{aligned} k &= 300\,000\,\frac{\text{lb.}}{\text{in.}^2} = 21\,600\,\frac{\text{ton}}{\text{ft.}^2} \\ g &= 32.2\,\frac{\text{ft.}}{\text{sec.}^2} \\ w &= 0.03125\,\frac{\text{ton}}{\text{ft.}^3} \end{aligned}\right\} \qquad \text{(10)}$$

give $v_s = 4\,718$ ft. per sec. The value, $v_s = 4\,714$ ft. per sec., was measured by Wertheim in the Seine River at the temperature of 15° cent.[8]

The term in Equation (8) containing the factor, β, in front of the cosine represents waves moving away from the dam, while that containing the factor, $1-\beta$, represents waves moving toward the dam.

Equations (8) and (5) give for $x = 0$ (that is, at the up-stream face of the dam):

$$\sigma = (1-2\beta)\frac{\alpha T}{2\pi}\sqrt{gkw}\,\sin\frac{2\pi t}{T} \qquad \text{(11)}$$

When $\beta > \frac{1}{2}$, that is, when the waves moving away from the dam predominate over those moving toward it, Equation (11) shows that the maximum pressure on the dam, that is, $(-\sigma)_{\max.}$, occurs at the times $t = \frac{T}{4}, \frac{5T}{4}, \frac{9T}{4}, \ldots$, and is equal to,

$$p = (2\beta-1)\frac{\alpha T}{2\pi}\sqrt{gkw} \qquad \text{(12)}$$

It is noted that the greatest acceleration of the dam, and, consequently, the greatest inertia forces due to the moving mass of the dam, occur when $t = 0, T, 2T, \ldots$ The stresses in the dam due to the pressures, p, and the inertia forces, therefore, would have to be combined in the manner of two alternating currents with a difference of phase of one-quarter of the period.

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[7] "Hydrodynamics", by Horace Lamb, Fifth Edition, 1924, p. 452.

[8] Smithsonian Physical Tables, Seventh Edition, 1921, p. 147.

Since, according to Equation (7), the maximum velocity of the dam is,

$$v = \frac{\alpha g T}{2\pi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Equation (12) may be rewritten, by the use of Equation (9), in the form,

$$p = (2\beta - 1)\frac{v}{v_s}k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

With $\beta = 1$, that is, when the waves move away from the dam only, Equation (12) (or, if one prefers, Equation (14)), with the numerical constants stated in Equations (10), and with $\alpha = 0.1$ and $T = \frac{4}{3}$ sec., gives $p = 3.13$ tons per sq. ft. On account of the freedom of the water to escape vertically the pressures are actually much less, and are not distributed uniformly through the depth of the reservoir. A solution which is to be used must show the pressures in the water to be zero at the top.

Solution Considering Horizontal and Vertical Motions of the Water

A solution is desired which satisfies Equations (1), (2), and (5), and, in addition, the following conditions at the boundary:

(1) $\sigma = 0$, when $y = 0$;

(2) $\eta = 0$, when $y = h$;

(3) $\xi = -\frac{\alpha g T^2}{4\pi^2}\cos\frac{2\pi t}{T}$, when $x = 0$; and,

(4) σ converges toward 0 when x becomes large.

In connection with Condition (1), it is noted that when the values of η are small, it makes no appreciable difference whether one specifies $\sigma = 0$ at the undisturbed or the disturbed surface.

The following solution will be shown to satisfy all these conditions:

$$\xi = -\frac{\alpha g T^2}{\pi^3}\cos\frac{2\pi t}{T}\sum_{1,3,5,..}^{n}\frac{1}{n}e^{-q_n}\sin\frac{n\pi y}{2h} \quad \ldots\ldots\ldots\ldots\ldots (15)$$

and,

$$\eta = \frac{\alpha g T^2}{\pi^3}\cos\frac{2\pi t}{T}\sum_{1,3,5,..}^{n}\frac{1}{n c_n}e^{-q_n}\cos\frac{n\pi y}{2h} \quad \ldots\ldots\ldots\ldots\ldots (16)$$

in which,

$$c_n = \sqrt{1 - \frac{16 w h^2}{n^2 g k T^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

and,

$$q_n = \frac{n \pi c_n x}{2h} \quad \text{.........................} (18)$$

Equation (17) gives,

$$\frac{1}{c_n} - c_n = \frac{16 w h^2}{n^2 c_n g k T^2} \quad \text{.........................} (19)$$

Then Equations (5), (15), (16), and (19) lead to the value of the stress,

$$\sigma = -\frac{8 \alpha w h}{\pi^2} \cos \frac{2 \pi t}{T} \sum_{1, 3, 5, \ldots}^{n} \frac{1}{n^2 c_n} e^{-q_n} \sin \frac{n \pi y}{2h} \quad \text{..............} (20)$$

That the solution represented by Equations (15), (16), and (20) satisfies Equations (1) and (2) at all points is verified easily by performing the required differentiations with respect to t, x, and y. Equation (5) is satisfied because it was used in deriving Equation (20) from Equations (15) and (16). Furthermore, inspection of the expressions for η and σ shows immediately that Conditions (1), (2), and (4) are satisfied. That Condition (3), for $x = 0$, is satisfied, is seen by comparing Equation (15) for $x = 0$ with the expression for $\frac{\pi}{4}$, in terms of a Fourier series,[9] applicable when $0 < u < \pi$,

$$\frac{\pi}{4} = \sin u + \frac{1}{3} \sin 3u + \frac{1}{5} \sin 5u + \quad \text{............} (21)$$

In the present application, $u = \frac{\pi y}{2h}$.

Thus, all the specified conditions are satisfied. The applicability of the equations depends, however, on an additional condition: The displacements defined by the solution, as well as their derivatives with respect to x and y,[10] must be small. It is necessary then to examine the possibilities of large displacements. One possibility is identified with resonance in the water; another will be investigated by determining the shape of the wave formed by the surface of the water and the vertical motions along the up-stream face of the dam.

POSSIBILITY OF RESONANCE IN THE WATER

The waves of expansion (or waves of sound) represented by Equations (8) and (9) have the wave length,

$$l = v_s T = T \sqrt{\frac{gk}{w}} \quad \text{.........................} (22)$$

By use of this value Equation (17) may be rewritten in the form,

$$c_n = \sqrt{1 - \frac{16 h^2}{n^2 l^2}} \quad \text{.........................} (23)$$

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[9] See, for example, "Short Table of Integrals", by B. O. Pierce, Second Edition, 1910, p. 95, Formula 808.

[10] Compare the remark following Equation (5).

With $n = 1$, one finds $c_1 = 0$ if $h = \frac{l}{4}$, that is, if the depth of the reservoir is one-fourth of a wave length. This case is analogous to that of an organ pipe open at the top; one may expect the water to vibrate like the air in the pipe. That resonance occurs, is shown by Equations (16) and (20); they give infinite values of η and σ in this case. If h approaches the value, $\frac{l}{4}$, the equations lead to large values of η and σ.

With the period, $T = 1$ sec., $\frac{l}{4}$ becomes $\frac{1}{4} \times 4\,718 \frac{\text{ft.}}{\text{sec.}} \times 1 \text{ sec.} = 1\,180$ ft. Since the main vibrations in the earthquake are not likely to have a period less than 1 sec., this form of resonance need not be expected in any project now contemplated.

SHAPE OF THE WAVE AND VERTICAL MOTIONS AT UP-STREAM FACE OF DAM

In actual cases the values of c_n do not differ greatly from 1. In examining the vertical displacements it will be permissible, therefore, to write $c_n = 1$. In terms of the maximum displacement of the dam according to Condition (3) (stated shortly before Equation (15)),

$$r = \frac{\alpha g T^2}{4 \pi^2} \quad \text{.......................} (24)$$

Equation (16) gives then the following expressions for η at the top ($y = 0$) and at the up-stream face ($x = 0$) when $t = 0, T, 2T, \ldots$:

$$\eta_{y=0} = \frac{4\,r}{\pi} \sum_{1,3,5,..}^{n} \frac{1}{n}\, e^{-\frac{n \pi x}{2h}} \quad \text{.......................} (25)$$

and,

$$\eta_{x=0} = \frac{4\,r}{\pi} \sum_{1,3,5,..}^{n} \frac{1}{n} \cos \frac{n \pi y}{2h} \quad \text{.......................} (26)$$

By use of the formulas,[11]

$$\left.\begin{aligned} &\tfrac{1}{2} \log_e \frac{1+u}{1-u} = u + \frac{u^3}{3} + \frac{u^5}{5} + \frac{u^7}{7} + \ldots\ldots\ldots \\ &\text{and,} \\ &-\tfrac{1}{2} \log_e \tan \frac{U}{2} = \cos U + \frac{1}{3} \cos 3\,U + \frac{1}{5} \cos 5\,U + \ldots \end{aligned}\right\} \ldots\ldots (27)$$

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[11] "Short Table of Integrals", by B. O. Pierce, Second Edition, 1910, pp. 82, 94, and 96, Formulas 681, 797, 813, and 814.

with the substitutions, $u = e^{-\frac{\pi x}{2h}}$, $U = \frac{\pi y}{2h}$, one finds,

$$\eta_{y=0} = \frac{2r}{\pi} \log_e \frac{1 + e^{-\frac{\pi x}{2h}}}{1 - e^{-\frac{\pi x}{2h}}} = -\frac{2r}{\pi} \log_e \tanh \frac{\pi x}{4h} \ldots\ldots\ldots (28)$$

and,

$$\eta_{x=0} = -\frac{2r}{\pi} \log_e \tan \frac{\pi y}{4h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

These formulas give $\eta = \infty$ at the point, $x = 0$, $y = 0$; that is, the solution in Equations (15), (16), and (20) has a singularity (infinite values) at this point, and the condition that the displacements and their first derivatives must remain small, is not satisfied at this point and in the neighborhood of this point. The solution, nevertheless, may very well be adequate (except at and near the singularity), if only the region of large values of η can be shown to be small.

When $x = 0.1h$, one finds $\tanh \frac{\pi x}{4h} = \tanh 0.07854 = 0.07838$. Likewise, when $y = 0.1h$, one finds $\tan \frac{\pi y}{4h} = \tan 0.07854 = 0.07870$. For these and smaller values of x and y, one may accordingly replace Equations (28) and (29) by the simpler approximate formula,

$$\eta_{y=0} \text{ or } \eta_{x=0} = -\frac{2r}{\pi} \log_e \frac{\pi (x \text{ or } y)}{4h} = \left(1.466 \log_{10} \frac{h}{x \text{ or } y} + 0.1538\right) r \ldots (30)$$

Assume, for example, that $\alpha = 0.1$, $T = \frac{4}{3}$ sec.; then Equation (24) gives the amplitude of the motion of the dam, $r = 0.1450$ ft. With $h = 200$ ft., Equation (30) gives the values shown in Table 1.

TABLE 1.—VALUES OF η BY EQUATION (30) FOR $h = 200$ FEET

At distance, x or y (in feet)	0.1	1	5	10	20
Movement, $\eta_{y=0}$, or $\eta_{x=0}$ (in feet)	0.724	0.511	0.363	0.299	0.235

These computations show that the region about the point, $x = 0$, $y = 0$, within which the displacements become large, is so small that the discrepancies introduced by the singularity at this point can have only a negligible influence on the solution as a whole. The solution in Equations (15), (16) and (20) has no other singularities (points at which infinite values are found), and therefore may be accepted as applying to the case.

PRESSURES ON THE DAM

According to Equation (20), the maximum pressures, p $(= -\sigma \text{ for } x = 0)$, of the water on the dam occur when $t = 0, T, 2T$, etc. One finds,

$$p = \frac{8\,\alpha\,w\,h}{\pi^2} \sum_{1,3,5,\ldots}^{n} \frac{1}{n^2 c_n} \sin\frac{n\,\pi\,y}{2\,h} \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

in which, as before, according to Equation (17), and with the numerical constants stated in Equations (10):

$$c_n = \sqrt{1 - \frac{16\,w\,h^2}{n^2\,g\,k\,T^2}} = \sqrt{1 - \frac{0.71889}{n^2}\left(\frac{h \text{ sec.}}{1\,000\,T \text{ ft.}}\right)^2} \ldots\ldots\ldots (32)$$

The greatest value of the pressure, p, occurs at the bottom of the reservoir $(y = h)$, and is,

$$p_0 = \frac{8\,\alpha\,w\,h}{\pi^2} \sum_{1,3,5,\ldots}^{n} \frac{(-1)^{\frac{n-1}{2}}}{n^2\,c_n} \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

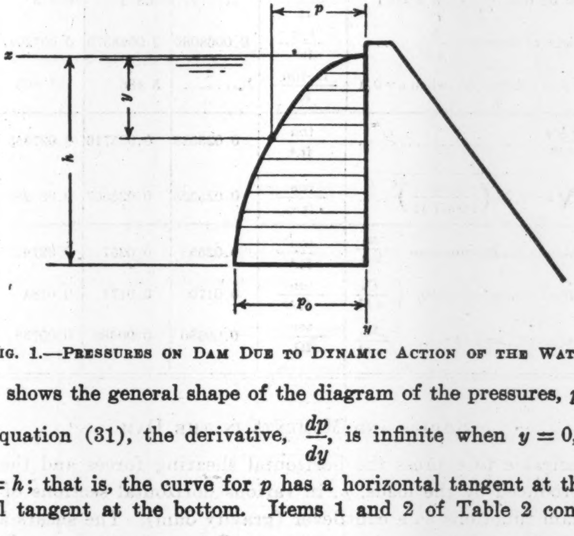

FIG. 1.—PRESSURES ON DAM DUE TO DYNAMIC ACTION OF THE WATER

Fig. 1 shows the general shape of the diagram of the pressures, p. According to Equation (31), the derivative, $\frac{dp}{dy}$, is infinite when $y = 0$, and zero when $y = h$; that is, the curve for p has a horizontal tangent at the top and a vertical tangent at the bottom. Items 1 and 2 of Table 2 contain some numerical values of the ratio, $\frac{p_0}{\alpha h}$, and of p_0, computed from Equations (32) and (33). The ratio, $\frac{p_0}{\alpha h}$, is seen to vary only slowly with h. In the case of $h = 600$ ft., $T = \frac{4}{3}$ sec. (or $h = 200$ ft., $T = \frac{4}{9}$ sec.), the pressure at the depth, $y = \frac{h}{2}$, was found to be $p = 0.814 p_0$.

TABLE 2.—NUMERICAL RESULTS CORRESPONDING TO THE PERIOD, $T = \frac{4}{3}$ SECOND

(The maximum horizontal acceleration equals $\alpha\, g$. When T is varied, the coefficients stated remain constant if h is changed by the same ratio; for example, the coefficients stated for $h = 800$ ft., $T = \frac{4}{3}$ sec., apply to the cases, $h = 600$ ft., $T = 1$ sec., and $h = 200$ ft., $T = \frac{1}{3}$ sec.; the coefficients stated for $h = 200$ ft., $T = \frac{4}{3}$ sec., apply to the case, $h = 600$ ft., $T = 4$ sec.)

Item	Depth of reservoir, h		Ft.	Small	200	600	800
1	Coefficient of pressure, $\frac{p_0}{\alpha h}$		$\frac{\text{ton}}{\text{ft.}^3}$	0.023202	0.023407	0.025254	0.027257
2	Pressure at bottom, p_0, when $\alpha = 0.1$		$\frac{\text{ton}}{\text{ft.}^2}$	....	0.468	1.515	2.181
3	Coefficient of shear, $\frac{Q_0}{\alpha h^2}$		$\frac{\text{ton}}{\text{ft.}^3}$	0.016961	0.017094	0.018286	0.019576
4	Shear at bottom, Q_0, when $\alpha = 0.1$ ..		$\frac{\text{ton}}{\text{ft.}}$	....	68.4	658.3	1 253
5	Coefficient of moment, $\frac{M_0}{\alpha h^3}$		$\frac{\text{ton}}{\text{ft.}^3}$	0.0068089	0.0068576	0.0072947	0.0077666
6	Moment at bottom, M_0, when $\alpha = 0.1$		$\frac{\text{ft.-ton}}{\text{ft.}}$	....	5 486	157 600	397 609
7	$C = \frac{15\, M_0}{4\, \alpha\, h^3}$		$\frac{\text{ton}}{\text{ft.}^3}$	0.025533	0.025716	0.027355	0.029125
8	$K = C\sqrt{1 - 0.72\left(\frac{h \text{ sec.}}{1\,000\, T \text{ ft.}}\right)^2}$		$\frac{\text{ton}}{\text{ft.}^3}$	0.025533	0.025507	0.025282	0.025068
9	Approximate coefficients com-	$\frac{p_0}{\alpha h}$	$\frac{\text{ton}}{\text{ft.}^3}$	0.0255	0.0257	0.0276	0.0296
10	puted from Equations (40),	$\frac{Q_0}{\alpha h^2}$	$\frac{\text{ton}}{\text{ft.}^3}$	0.0170	0.0171	0.0184	0.0198
11	(41), (42), (48), and (49)....	$\frac{M_0}{\alpha h^3}$	$\frac{\text{ton}}{\text{ft}^3.}$	0.00680	0.00686	0.00736	0.00790

SHEARS AND MOMENTS IN THE DAM

It is desirable to express the horizontal shearing forces and the bending moments produced by the loads, p, in various horizontal sections of the dam when the dam functions as a cantilever (gravity dam). The shears are stated as force per unit of width of the cantilever, for example, in tons per foot, and the moments are stated as moment per unit of width, for example, in foot-tons per foot (or simply in tons). The shears, Q, at the depth, y, and Q_0, at the depth, h, and the moments, M, at the depth, y, and M_0, at the depth, h, are obtained from p in Equation (31) by the integrations,

$$Q = \int_0^y p\, dy;\ Q_0 = \int_0^h p\, dy;\ M = \int_0^y Q\, dy;\ M_0 = \int_0^h Q\, dy \quad \text{.....(34)}$$

With the notation,

$$q = \sum_{1,3,5,\ldots}^{n} \frac{1}{n^3 c_n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

one finds by this process:

$$Q = \frac{16\,\alpha\,w\,h^2}{\pi^3}\left(q - \sum_{1,3,5,\ldots}^{n} \frac{1}{n^3 c_n} \cos\frac{n\,\pi\,y}{2\,h}\right) \ldots\ldots\ldots\ldots (36)$$

$$Q_0 = \frac{16}{\pi^3}\,\alpha\,w\,h^2\,q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

$$M = \alpha\,w\,h^3\left(\frac{16q}{\pi^3}\,\frac{y}{h} - \frac{32}{\pi^4}\sum_{1,3,5,\ldots}^{n}\frac{1}{n^4 c_n}\sin\frac{n\,\pi\,y}{2\,h}\right) \ldots\ldots\ldots (38)$$

and,

$$M_0 = \alpha\,w\,h^3\left(\frac{16q}{\pi^3} - \frac{32}{\pi^4}\sum_{1,3,5,\ldots}^{n}\frac{(-1)^{\frac{n-1}{2}}}{n^4 c_n}\right) \ldots\ldots\ldots\ldots (39)$$

Numerical values of the ratios, $\frac{Q_0}{\alpha\,h^2}$ and $\frac{M_0}{\alpha\,h^3}$, and of Q_0 and M_0, computed from Equations (35), (37), and (39), are stated in Table 2 (see, Items, 3, 4, 5, and 6).

Derivation of Approximate Formulas

For the purpose of approximate computations one might replace the diagram of pressures in Fig. 1 by a quadrant of an ellipse (see Equation (50) derived subsequently); the ellipse becomes a circle when drawn to some particular scale for p. A more satisfactory approximate representation is obtained, however, by replacing the curve in Fig. 1 by a parabola with vertical axis, as shown in Fig. 2, even if this parabola has a sloping tangent at the bottom.

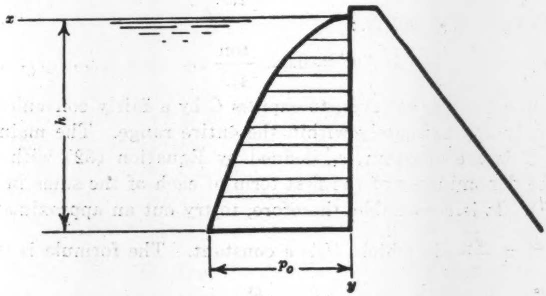

Fig. 2.—Approximate Distribution of Pressures.

The parabola leads to the following simple formulas, in which, C is a coefficient depending on the ratio of h to T:

$$p = C\,\alpha\,\sqrt{hy}\;;\; p_0 = C\,\alpha\,h \tag{40}$$

$$Q = \frac{2}{3}\,C\,\alpha\,y\,\sqrt{hy}\;;\; Q_0 = \frac{2}{3}\,C\,\alpha\,h^2 \tag{41}$$

and,

$$M = \frac{4}{15}\,C\,\alpha\,y^2\,\sqrt{hy}\;;\; M_0 = \frac{4}{15}\,C\,\alpha\,h^3 \tag{42}$$

The coefficient, C, might be determined so that p_0 will have the value computed from Equation (33), but one obtains a better representation of the diagram of pressures as a whole, by determining C so that the moment, M_0, will have the value computed from Equation (39). In terms of a known M_0, one finds then,

$$C = \frac{15\,M_0}{4\,\alpha\,h^3} \tag{43}$$

Table 2 (Item 7) shows some numerical values of C. It is noted that the dimension of C is force per unit of volume; C multiplied by αh gives the pressure as force per unit of area.

The coefficient, C, varies slowly within the range considered. For quick estimates one may very well assume some constant value of C; for example, with $T \geqq \frac{4}{3}$ sec.:

When $h < 310$ ft.:

$$C = 0.026\,\frac{\text{ton}}{\text{ft.}^3} \tag{44}$$

when 310 ft. $< h <$ 540 ft.:

$$C = 0.027\,\frac{\text{ton}}{\text{ft.}^3} \tag{45}$$

and when 540 ft. $< h <$ 680 ft.:

$$C = 0.028\,\frac{\text{ton}}{\text{ft.}^3} \tag{46}$$

It is a simple matter, however, to express C by a fairly convenient formula which will apply approximately within the entire range. The main source of variation of C is the constant, c_1, defined by Equation (32) with $n = 1$; c_1 appears in the denominator of the first term of each of the sums in Equations (35) and (39). It is reasonable, therefore, to try out an approximate formula of the type, $C = \dfrac{K}{c_1}$, in which, K is a constant. The formula is written,

$$C = \frac{K}{\sqrt{1 - 0.72\left(\dfrac{h\ \text{sec.}}{1\,000\ T\ \text{ft.}}\right)^2}} \tag{47}$$

The values of K shown in Table 2 (Item 8) were computed from this formula by using the values of C in Item 7. Inspection of these results shows that one may adopt the value, $K = 0.0255 \text{ ton-ft.}^{-3}$. It is concluded that the coefficient, C, in Equations (40), (41), and (42), is expressed with satisfactory approximation by the formula,

$$C = \frac{0.0255 \text{ ton-ft.}^{-3}}{\sqrt{1 - 0.72\left(\frac{h \text{ sec.}}{1\,000\, T \text{ ft.}}\right)^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (48)$$

RESULTING APPROXIMATE FORMULA FOR THE WATER PRESSURES

The following approximate formula for the maximum water pressures, to be added to the hydrostatic pressures existing before the earthquake, represents the main conclusion from this study; it is obtained directly from Equations (40) and (48):

$$p = \frac{0.0255 \text{ ton-ft.}^{-3}\ \alpha \sqrt{hy}}{\sqrt{1 - 0.72\left(\frac{h \text{ sec.}}{1\,000\, T \text{ ft.}}\right)^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (49)$$

For quick computations, however, Equations (40), (44), (45), and (46) may be used. Values of the coefficients, $\frac{p_0}{\alpha h}$, $\frac{Q_0}{\alpha h^2}$, and $\frac{M_0}{\alpha h^3}$, obtained by use of Equation (49), are stated as Items 9, 10, and 11 in Table 2. A comparison with Items 1, 3, and 5, respectively, shows that the approximation is satisfactory.

By using a quadrant of an ellipse (as suggested under "Derivation of Approximate Formulas") instead of the parabola to represent the pressures, one obtains the following approximate formula by a process similar to that leading to Equation (49):

$$p = \frac{0.0204 \text{ ton-ft.}^{-3}\ \alpha \sqrt{y\,(2h - y)}}{\sqrt{1 - 0.72\left(\frac{h \text{ sec.}}{1\,000\, T \text{ ft.}}\right)^2}} \quad \ldots\ldots\ldots\ldots\ldots (50)$$

This formula gives exactly the same values of M_0 as Equation (49), but gives too small values of p_0 and of the total force, Q_0. Equation (49) gives slightly too large values of p_0 and Q_0, and is preferable.

It is noted that the pressures, p, occur when $t = 0$, T, $2T$, etc., that is, (see Condition (3)), when the dam (during the vibration) is in the extreme position down stream. The stresses due to these water pressures, therefore, must be added to the maximum stresses due to the accelerations of the mass of the dam iself.

When, during the vibration, the dam is in the extreme position up stream, the pressures, p, are reversed and become tensions. These tensions, like the pressures in the extreme position down stream, are to be combined with the hydrostatic pressures existing beforehand. Inspection of the numerical values in Table 2 shows that when the pressure of the atmosphere is included, there is no prospect of the resultant stress in the water becoming either a tension or a very small compression.

Applicability of Formulas to Cases Other than Those Assumed

The formulas were derived under the assumption that there is no overflow. In case of overflow over a spillway it is suggested that Equation (49) be used with the depths, h and y, measured from the plane surface of the reservoir at some distance behind the dam. The parabolic diagram of pressures is then used as before, only the top is cut off.

While the formulas were derived under the assumption that the up-stream face is a vertical plane, they will furnish an indication of the pressures which may be expected in other cases, for example, in the case of arch dams.

The Body of Water Which May Be Considered to Move with the Dam

To visualize the dynamic action of the water on the dam, one may think of a certain body of water as moving with the dam while the remainder of the reservoir remains inactive. For this purpose the body of water moving with the dam may be imagined as frozen into horizontal layers of ice (the expansion by freezing being ignored), while the remainder of the reservoir is emptied. The layers of ice are imagined to support one another by vertical forces only, without friction between the layers; the layers are attached firmly to the dam, so that the dam will exert the horizontal forces necessary to move them back and forth with it. The advantage of picturing the action in this manner is that the forces exerted on the up-stream face of the dam will then be represented as inertia forces like those due to the moving mass of the dam itself.

The shape of the body of water or ice thus considered to move with the dam must be determined so that the inertia forces become equal to the pressures actually exerted by the water due to the dynamic action. With b denoting the dimension of this body in the direction of x at the depth, y, the corresponding mass per unit of area of the up-stream face of the dam becomes $\frac{b\,w}{g}$, and the corresponding inertia force when the acceleration is $\alpha\,g$ becomes $\frac{b\,w}{g} \times \alpha\,g = \alpha\,wb$, which must be equal to p. One finds then:

$$b = \frac{p}{\alpha\,w} \qquad \text{(51)}$$

That is, the shape of this body of water or ice is the same as that of the diagram for p when the pressures are laid off to a proper scale.

It will be sufficient for the present purpose to determine b by one of the approximate expressions for p; for example (compare Equations (40), (44), (45), and (46)),

$$p = 0.02734 \text{ ton-ft.}^{-3}\,\alpha\sqrt{hy} \qquad \text{(52)}$$

which gives, with $w = 0.03125$ ton-ft.$^{-3}$,

$$b = \frac{7}{8}\sqrt{hy} \qquad \text{(53)}$$

Fig. 3 shows the body of water or ice defined by Equation (53).

Instead of the body of water, one may prefer to consider an equivalent body of concrete, having the same mass. If the concrete weighs 144 lb. per cu. ft., the dimension corresponding to b becomes,

$$b' = 0.38\sqrt{hy} \quad \text{(54)}$$

The dotted curve in Fig. 3 indicates this body of concrete. Like the body of ice that was described, it should be imagined as divided into layers resting on one another without friction. The picture then represents no addition to the strength of the dam, but a proper addition to the mass.

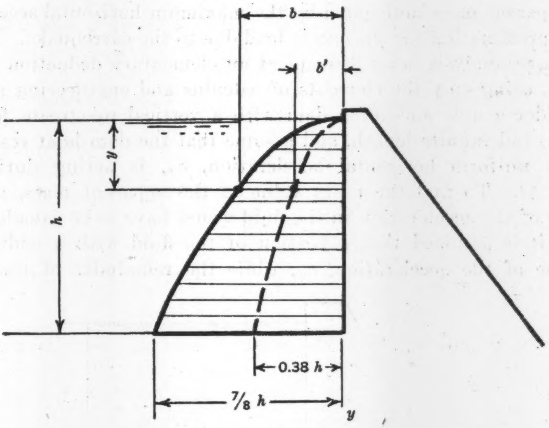

FIG. 3.—BODY OF WATER WHICH MAY BE CONSIDERED TO MOVE WITH DAM. DOTTED CURVE SHOWS EQUIVALENT BODY OF CONCRETE.

ACKNOWLEDGMENTS

This study was made in Denver, Colo., at the United States Bureau of Reclamation, in connection with the design of the Hoover Dam, during a leave of absence of the writer from the University of Illinois. All the important designing work of the Bureau of Reclamation is done in Denver, under the general supervision of J. L. Savage, M. Am. Soc. C. E., Chief Designing Engineer; all engineering and construction are under the general charge of R. F. Walter, M. Am. Soc. C. E., Chief Engineer, with headquarters in Denver; and all the activities of the Bureau are under the general charge of Elwood Mead, M. Am. Soc. C. E., Commissioner, with headquarters in Washington, D. C.

DISCUSSION

Theodor von Kármán,[12] **M. Am. Soc. C. E.** (by letter).—In his paper Professor Westergaard has presented a beautiful and complete analysis of the pressure distribution on a dam undergoing horizontal oscillations of sinusoidal law. He has investigated both the influence of compression waves in the fluid and the effect of the so-called "apparent mass." It is found that, in the range of the frequencies supposed to occur in oscillations due to earthquake, the effect of compressibility is comparatively small, so that the value of the apparent mass multiplied by the maximum horizontal acceleration gives a first approximation for the excess load due to the earthquake. The following approximate analysis is an attempt at an elementary deduction of the apparent mass, using only the elements of calculus and engineering mechanics.

Consider a unit slice of a dam with a vertical up-stream face, with the height, h, and infinite length, and assume that the dam is at rest when $t = 0$, and that uniform horizontal acceleration, a_x, is acting during the time interval, Δt. To find the exact value of the apparent mass, the exact distribution of the acceleration in the fluid would have to be calculated. Instead of that, it is assumed that a portion of the fluid with a width, b, has the full value of the acceleration, a_x, while the remainder of the fluid is not affected in the process. The width, b, varies with the height (see Fig. 4). With this assumption, Conditions (a), (b), and (c) are readily obtained as follows:

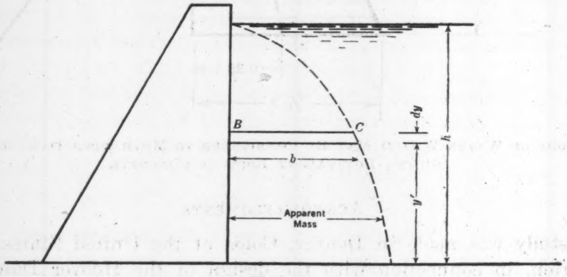

FIG. 4.

(a) *Condition for "Continuity" of Flow.*—Consider the part of the dam between the bottom at $y' = 0$, and an arbitrary height, y'. Then, the fluid mass displaced by this part of the dam must pass through the section, BC. Assume that the vertical acceleration is variable with the height, and has the mean value, a_y', between $x = 0$ and $x = b$, while for $x > b$, the fluid is supposed to be at rest. The displacement of the dam during the time, Δt, is, obviously, $a_x \dfrac{\Delta t^2}{2}$, and the displacement of the fluid through the section,

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[12] Prof., and Director, Aeronautical Laboratory, California Inst. of Technology, Pasadena, Calif.

$BC = b$, is $a_y' \dfrac{\Delta t^2}{2}$, so that the condition of continuity requires that $y' a_x \dfrac{\Delta t^2}{2} = b\, a_y' \dfrac{\Delta t^2}{2}$, or,

$$y'\, a_x = b\, a_y' \ldots\ldots\ldots\ldots\ldots\ldots (55)$$

(*b*) *Equilibrium Between Pressure and Horizontal Mass Forces.*—Let the pressure at an arbitrary height, y', be $p_{y'}$, and let the density of the fluid be ρ. Then, the pressure is obviously equal to $p\, dy' = \rho\, b\, dy'\, a_x$, or,

$$p = \rho\, b\, a_x \ldots\ldots\ldots\ldots\ldots\ldots (56)$$

(*c*) *Equilibrium Between Pressure and Vertical Mass Forces.*—Consider a portion of the fluid body with the variable width, b, between two sections in the height, y', and $y' + dy'$. The resultant of the pressure on the bottom surface of the element is equal to $p\, b$, and that of the pressure acting on the top surface equals $p\, b + \dfrac{d}{dy'}(p\, b)\, dy'$. Now, the difference, $-\dfrac{d}{dy'}(p\, b)\, dy'$, must be equal to the product of the mass of the element, $\rho\, b\, dy'$, and the vertical acceleration, a_y'. Hence, $-\dfrac{d}{dy'}(p\, b)\, dy' = \rho\, a_y'\, b\, dy'$, or,

$$\frac{d}{dy'}(p\, b) = -\rho\, b\, a_y' \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

Substituting $a_y' = \dfrac{y'}{b} a_x$ from Equation (55) and $p = \rho\, b\, a_x$ from Equation (56), Equation (57) becomes,

$$\frac{d}{dy'}\left[\rho\, b^2\, a_x\right] = -\rho\, y'\, a_x \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

or, since p and a_x have been assumed constant, $\dfrac{d}{dy'}(b^2) = -y'$, and,

$$b^2 = b^2_o \ldots\ldots\ldots\ldots\ldots\ldots (59)$$

For $y' = h$, the pressure vanishes, so that $b\ (h) = b^2_o - \dfrac{h^2}{2} = 0$, or $b^2_o = \dfrac{h^2}{2}$.

Substituting in Equation (59), $b^2 = \frac{1}{2}\left[h^2 - (y')^2\right]$, or,

$$b = 0.707\sqrt{h^2 - (y')^2} = 0.707\sqrt{y'(2h - y')} \ldots\ldots\ldots (60)$$

Hence, with this approximation the shape of the fluid body moving with the dam is shown to be a quadrant of an ellipse. One-half the axis equals $b_o = 0.707\, h$, the corresponding pressure equals $p_o = 0.707\, \rho\, h\, a_x$, or, substituting $a_x = \alpha\, g\, l$ and $\rho\, g = w$,

$$p_o = 0.707\, \alpha\, w\, h \ldots\ldots\ldots\ldots\ldots\ldots (61)$$

Professor Westergaard's calculation gives,

$$p_0 = \frac{8}{\pi^2}\left(1 - \frac{1}{3^2} + \frac{1}{5^2} - \ldots.\right) \alpha\, w\, h \ldots\ldots\ldots\ldots\ldots (62)$$

or,

$$p_0 = 0.743\ \alpha\, w\, h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (63)$$

The difference is about 4 or 5 per cent. By the approximate formula (Equation (60)), the total load on the dam is,

$$Q = 0.707\ \frac{\pi}{4}\ \alpha\, w\, h^2 = 0.555\ \alpha\, w\, h^2 \ldots\ldots\ldots\ldots\ldots (64)$$

While the exact solution gives,

$$Q = \frac{16}{\pi^3}\left(1 + \frac{1}{3^3} + \frac{1}{5^3} + \ldots.\right) \alpha\, w\, h^2 = 0.543\ \alpha\, w\, h^2 \ldots\ldots (65)$$

It is apparent that the approximation is close.

PAUL BAUMAN,[13] M. AM. SOC. C. E. (by letter).—The author's excellent treatise enables the designer to analyze accurately the increase of water pressure on the face of dams due to earthquakes. Until quite recently this phase of the subject has been a matter of considerable speculation.

In 1928 the writer was making studies on the effect of earthquakes in connection with the statical analysis of a dam in Southern California. While discussing this analysis with Dr. Fritz Zwicky, of the California Institute of Technology, the writer's attention was called to the fact that the inertia of the water might be more important than its flow, which, according to the analysis, was the only cause of an increase in pressure due to simple harmonic vibrations of the dam. The reasoning was that due to a motion of the dam in an up-stream direction, the water is partly displaced (vertically) and partly compressed. Neglecting for the moment the latter effect and considering the displacement only, the dam acts as a pump, or, better, as a reversed hydraulic ram, forcing the water immediately up stream from it to the surface, as shown in Fig. 5. This displaced water will flow over the top of the dam if the reservoir is full at the beginning of motion, or, if conditions are as shown in Fig. 5, it will flow back into the reservoir.

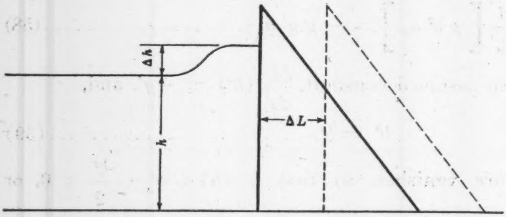

FIG. 5.

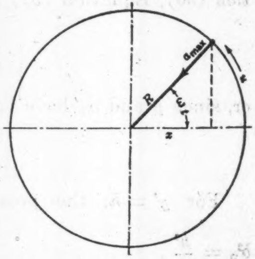

FIG. 6.

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[13] Chf. Designer, Quinton, Code & Hill-Leeds & Barnard, Engrs., Consolidated, Los Angeles, Calif.

Principally, the two cases are identical; that is, Δh is the head required for a discharge equal to the maximum volume displaced per unit of time. Thus, if the dam moves at a velocity, v:

$$vh = \tfrac{2}{3}\,\mu\sqrt{2g}\,\Delta\, h^{\frac{3}{2}} \quad \text{.....................(66)}$$

in which (in addition to Professor Westergaard's notation), v = maximum velocity and μ = discharge coefficient. From the law of simple harmonic motion (see Fig. 6):

$$u = \omega\, R = \frac{2\pi R}{T} \quad \text{.....................(67)}$$

$$v_{\max.} = u \quad \text{.....................(68)}$$

$$a_{\max.} = \frac{u^2}{R} = \frac{4\pi^2 R}{T^2} \quad \text{.....................(69)}$$

and,

$$\Delta L = 2\, R \quad \text{.....................(70)}$$

For a period, $T = 1$ sec. and $a_{\max.} = \frac{g}{10}$:

$$v_{\max.} = 2\,\pi\, R \quad \text{.....................(71)}$$

and R follows from $\frac{g}{10} = 4\pi^2 R$, or $R = \frac{g}{40\pi^2}$, which checks Equation (7). The total motion then becomes $2\,R = \Delta\, L = \frac{g}{20\,\pi^2} = 0.159' = 1.9''$.

Introducing $v_{\max.}$ in Equation (66):

$$h\,\pi\,\frac{g}{20\,\pi^2} = \tfrac{2}{3}\,\mu\,\sqrt{2g}\,\Delta\, h^{\frac{3}{2}} \quad \text{.....................(72)}$$

and,

$$\Delta\, h = \sqrt[3]{\left(\frac{3\,hg}{40\,\pi\,\mu\,\sqrt{2g}}\right)^2} \quad \text{.....................(73)}$$

for $\mu = 0.80$, and $h = 200$ ft., $\Delta\, h = 8.30$ ft.

Due to this head the pressure increase on a 200-ft. dam would be $\frac{8.30 \times 62.50 \times 200}{2\,000} = 52\ \frac{\text{ton}}{\text{ft.}}$, as against $Q_o = 68.4\ \frac{\text{ton}}{\text{ft.}}$, according to the author's Equation (36); and the moment about the base would be $52 \times 100 = 5\,200\ \frac{\text{ft-ton}}{\text{ft.}}$, as against $M_o = 5\,486\ \frac{\text{ft-ton}}{\text{ft.}}$, according to Equation (39).

Although the moments about the base check within a few per cent. for this height of dam, it is obvious that the higher the dam the more the physical difficulties grow, involving the movement of water particles all the way from the bottom to the surface within the time of the up-stream motion of the dam. Therefore, it may be stated that although a displacement of all particles immediately in front of the dam necessarily takes place, this displacement is retarded as the depth increases, thereby increasing the volume of inertia, or, in other words, the mass force.

Based on this cognition, Dr. Theodor von Kármán, of the California Institute of Technology, has derived a simple solution of this problem by neglecting the elastic compression of the water and by considering a body of inertia of water, which must be accelerated with the dam and, therefore, offers a resistance equal to its mass times the maximum acceleration of the dam, as also outlined by the author.

Based on Euler's fundamental equation,

$$dp = \frac{w}{g}\,(X dx + Y dy + Z dz) \quad \ldots\ldots\ldots\ldots\ldots\ldots (74)$$

a result has been derived by the writer which leads to an elliptic distribution of pressure and which checks the one due to Dr. von Kármán.

Applied to the problem of pressures on dams during earthquakes Equation (74) reduces to:

$$dp = -\frac{w}{g}\,Y dy \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

As the acceleration, X, in a horizontal direction is constant, $dx = 0$, and the acceleration, Z, parallel to the dam, is zero.

The vertical acceleration, Y, as previously outlined, follows from the condition that the water which is displaced due to a horizontal, up-stream motion of the dam is forced upward. According to Fig. 6 this may be expressed as follows:

$$y\,X = 2\,b\,Y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76)$$

which is a modification of the condition of continuity as suggested by Dr. von Kármán due to the change of pressure from p at the face of the dam to zero at a distance of 2 b from it. In other words, the body of inertia is actually twice as large as shown in Fig. 6.

The pressure, p, per unit area of dam face follows from the basic law of dynamics, namely:

$$p = m\,X\,b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (77)$$

in which, $m = \frac{w}{g}$ = mass of unit volume of water. Thus, $b = \frac{g p}{w X}$.

Introducing Equation (77) and Equation (75) into Equation (74):

$$2\,p\,dp = \left(\frac{w}{g}\right)^2 X^2\,(-y)\,dy \quad \ldots\ldots\ldots\ldots\ldots\ldots (78)$$

and integrating:

$$p^2 = \left(\frac{w}{g}\right)^2 X^2 \left(\frac{-y^2}{2}\right) + C \quad \ldots\ldots\ldots\ldots\ldots\ldots (79)$$

For $y = h$, therefore, $p = 0$ and $C = \frac{h^2}{2}$.

There follows,

$$p = \frac{w}{g} X \sqrt{\frac{h^2 - y^2}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (80)$$

which is the equation of an ellipse. Numerical evaluations of Equation (80) check those due to the author's Equation (52) within a few per cent.

Finally, due to Equation (77):

$$b = \sqrt{\frac{h^2 - y^2}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (81)$$

The author's efforts throughout the paper to make the same theory comprehensive to engineers who cannot be expected to be "high-powered" mathematicians is quite apparent. It is very thought-provoking indeed, and deserves both commendation and imitation.

R. ROBINSON ROWE,[14] M AM. SOC. C. E. (by letter).—The most useful and interesting theory for the design of structures to resist earthquakes, that has appeared in recent years, is given in this paper. It establishes a criterion applicable to existing as well as to proposed dams.

Although the author does not state positively that his solution is unique, the three differential equations (Equations (1), (2), and (5)) contain only three unknowns other than the dimensional variables, and should lead to a single primitive, made definite by the rigid boundary conditions. The latter should include the condition that ξ and η approach zero as x becomes large. Furthermore, Condition (3) should be modified to exclude the point of singularity; the condition is not met at that point, since Equation (21) is not true for $u = 0$.

That the solution is reasonable can well be judged by a study of the continuity of the dynamic action in the water, as shown graphically herewith. In each of the diagrams assumed the action is shown in the field, $0 < x$ (or y) $< h$, with $c_n = 1$, $T = \frac{4}{3}$, and $\alpha = 0.1$. No corrections have been made for phase difference.

In addition to the author's notation, the following are introduced: $u = \frac{\pi y}{2 h}$; $z = e^{\frac{\pi x}{2h}}$, $\delta = \sqrt{\xi^2 + \eta^2}$ equals the displacement of a particle in the direction of the action; $\theta = \tan^{-1} - \frac{n}{\xi}$ equals the angle between the line of action and the horizontal; and max. is a subscript indicating the maximum value of the variable.

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[14] Civ. Engr. (Allen & Rowe), San Diego, Calif.

Fig. 7 shows lines of equal horizontal displacement for the field, the values shown being maxima for $\alpha = 0.1$ and $T = \frac{4}{3}$ sec., corresponding to $t = \frac{T}{2}, \frac{3T}{2}$, etc. From Equation (15), $\xi_{\text{max.}} = 0.1846 \sum_{1,3,5,\ldots}^{n} \frac{\sin nu}{n z^n}$; and $\xi = 0$ when $x = \infty$ or $y = 0$. When $x = 0$ and $y > 0$, $\xi_{\text{max.}} = 0.1460$; and when $y = h$, $\xi_{\text{max.}} = 0.1846 \cot^{-1} z$.

The curves indicate a gradual change in horizontal displacements, which is more rapid in the vicinity of the top of the dam, where the lines converge at the singular point. For $y = 0$ and $x < r$, the direct interference of the dam would make $\xi_{\text{max.}} > (r - x)$.

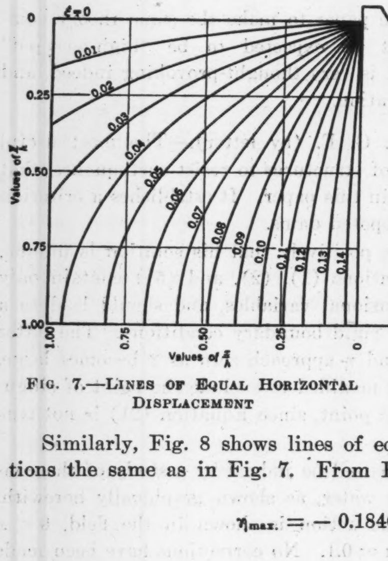

FIG. 7.—LINES OF EQUAL HORIZONTAL DISPLACEMENT

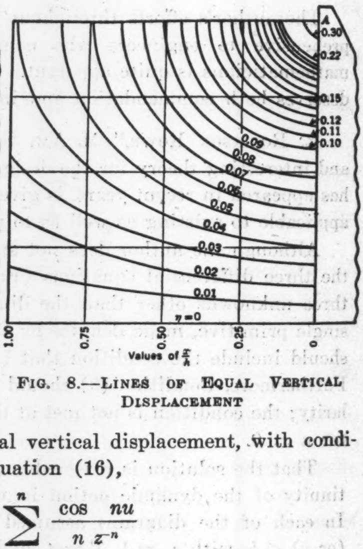

FIG. 8.—LINES OF EQUAL VERTICAL DISPLACEMENT

Similarly, Fig. 8 shows lines of equal vertical displacement, with conditions the same as in Fig. 7. From Equation (16),

$$\eta_{\text{max.}} = -0.1846 \sum_{1,3,5,\ldots}^{n} \frac{\cos nu}{n z^{-n}}$$

and $\eta = 0$ when $x = \infty$, or $y = h$. When $x = 0$,

$$\eta_{\text{max.}} = 0.09295 \log_e \tan \frac{u}{2}$$

and when $y = 0$,

$$\eta_{\text{max.}} = 0.09295 \log_e \left[\left(\frac{z-1}{z+1}\right)\right]$$

At Point A in Fig. 8, $\eta = \infty$.

As in Fig. 7, a gradual change in vertical displacements is evident, the change being more rapid in the vicinity of the top of the dam and infinite at the singular point. The lines are approximately normal to those of Fig. 7

throughout the field. Direct interference of the dam upsets the formula for $y = 0$ and $x < r$.

Fig. 9 shows lines of equal displacements in the direction of the action, being the resultant of values shown in Figs. 7 and 8, since the horizontal and vertical displacements are in the same phase. The conditions are the same as for Fig. 7; and $\delta = \infty$ at Point A. Furthermore, $\delta = 0$ when $x = \infty$; when $x = 0$,

$$\delta_{\text{max.}} = 0.1460 \sqrt{1 + 0.4053 \left(\log_e \tan \frac{u}{2}\right)^2}$$

when $y = 0$,

$$\delta_{max.} = 0.09295 \log_e \frac{z - 1}{z + 1}$$

and when $y = h$,

$$\delta_{\text{max.}} = 0.1846 \cot^{-1} z$$

When x is large, all terms of the summations except the first are negligible, and $\xi_{\text{max.}} = 0.1846 \dfrac{\sin u}{z}$; $\eta_{\text{max.}} = -0.1846 \dfrac{\cos u}{z}$; and $\delta_{\text{max.}} = \dfrac{0.1846}{z}$. Since z is independent of y, the lines of equal displacement approach vertical straight lines. For the singular point, $\delta_{\text{max.}} = \infty$, of course.

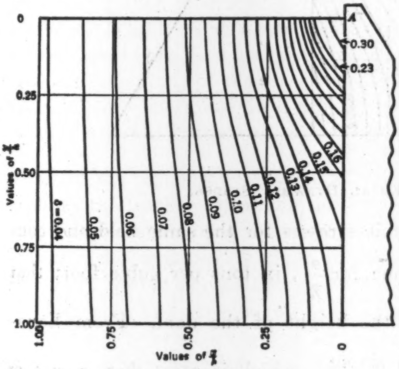

FIG 9.—LINES OF EQUAL DISPLACEMENT.

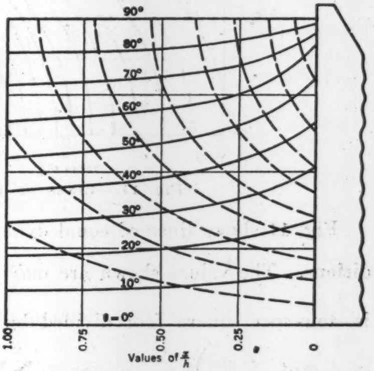

FIG. 10.—LINES OF ESCAPE AND OF EQUAL SLOPE

Dividing Equation (16) by Equation (15), $\tan \theta = F\ (x, y)$, and is independent of t. For any point, therefore, the slope angle of the displacement is constant throughout the cycle, and each particle moves in simple harmonic motion along an essentially straight line of length, $2\ \delta$, and inclined up and away from the dam, making an angle of θ with the horizontal. The solid curves in Fig. 10 are loci of equal inclinations; the action is vertical at the water surface and horizontal at the bottom. As in Fig. 7, c_n (assumed) $= 1$, and $T = \frac{4}{3}$ sec. When $x = 0$, $\tan \theta = 0.6366 \log_e \tan \dfrac{u}{2}$. When $x = \infty$, $\theta = u$.

The broken curves indicate lines of escape; that is, they are drawn tangent to the inclinations, and indicate the direction of the action. Since there is no motion across these lines, the volume of water between two such curves can be considered as acting independently. It is evident, therefore, that about 80% of the impulse finds surface escape within the limits of the field.

In this connection, the writer suggested in 1928 that the thrust due to earthquakes should not exceed that which would be caused by an assumed translation along circular quadrants centering at *A* (Fig. 9). With such an assumption, the surface escape within the field is 100%, and the following are easily computed: $p = 0.00491\,y$; $p_0 = 0.00491\,h$; $Q_0 = 0.00245\,h^2$; and $M_0 = 0.000818\,h^3$. The author's results for the last three, for an 800-ft. dam, are less by 44.6%, 20.1%, and 5.0%, respectively.

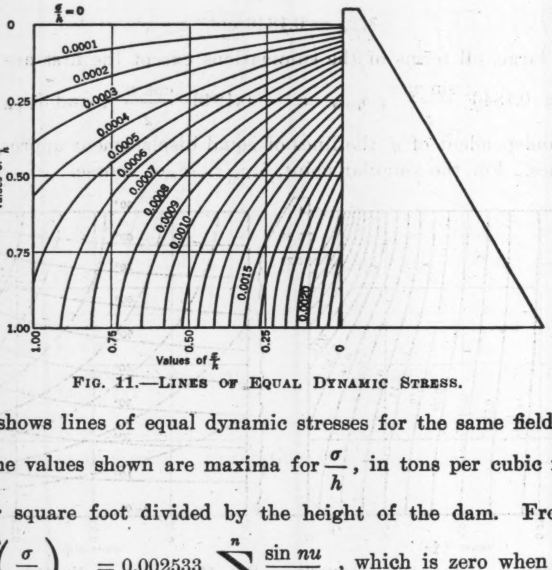

FIG. 11.—LINES OF EQUAL DYNAMIC STRESS.

Fig. 11 shows lines of equal dynamic stresses for the same field and conditions. The values shown are maxima for $\frac{\sigma}{h}$, in tons per cubic foot; that is, tons per square foot divided by the height of the dam. From Equation (19), $\left(\frac{\sigma}{h}\right)_{\max.} = 0.002533 \sum_{1,3,5,..}^{n} \frac{\sin nu}{n^2 z^n}$, which is zero when $x = \infty$ or $y = 0$. When $y = h$, $\left(\frac{\sigma}{h}\right)_{\max.} = 0.002533 \sum_{1,3,5,..}^{n} \frac{(-1)^{\frac{n-1}{2}}}{n^2 z^n}$; when $x = 0$, $\left(\frac{\sigma}{h}\right)_{\max.} = 0.002533 \sum_{1,3,5,..}^{n} \frac{\sin nu}{n^2}$, which reduces to the series,

$$0.001267\,u\,\left(1 - \log_e \frac{u}{2} - \frac{u^2}{36} - \frac{7\,u^4}{7\,200} - \frac{31\,u^6}{181\,440} \;....\right)$$

The change in stress is gradual and continuous, without singularity.

The curves do not represent true pictures of instantaneous conditions, because phase differences have been disregarded. When maxima occur at $x = \frac{h}{2}$, the angular phase difference for $x = 0$, or $x = h$, is $\frac{2 \pi t}{T}$, in which, $t = \frac{h}{2 v_s}$, so that the difference amounts to only $0.0004994\,h$. The distortion of the field at that time amounts to 0.5% for $h = 200$ and 7.9% for $h = 800$. Such distortion does not materially affect the representation intended.

It should be noted that the elastic deformation of the dam and the motion of the bed of the reservoir tend to diminish the stresses, and add to the security of the formulas. The possibility that the dam might be at the focus for compression waves set up in the water by the bed of the reservoir, seems to be exploded, in view of the large percentage of the impulse which escapes to the surface in a relatively short distance.

PROFESSOR DR. L. PRANDTL[15] (by letter).—In connection with this paper by Professor Westergaard, it might be of interest to note that this same mathematical problem is found in the theory of the distribution of lift on an aerofoil that is extended through an air stream of limited width and unlimited height. The water level in the problem by Professor Westergaard corresponds to the free boundary of the jet, and the bottom of the water tank to the middle plane of the jet. The calculations, that lead to exactly the same formulas, are contained in a paper[16] by Joseph Stüper, entitled "Der durch einen Freistrahl hindurchgesteckte Tragflügel."

CECIL E. PEARCE,[17] **ASSOC. M. SOC. C. E.** (by letter).—This paper explains the mathematical solution of a very difficult hydrodynamic problem. Particularly interesting is the fact that the result, although obtained by advanced mathematics, is given in such a convenient and simple form as to be readily usable by the average designer.

The paper and its results are of primary importance to engineers designing dams for localities where earthquakes may be expected, but it is also of use in other structural problems, as, for instance, in determining the horizontal load exerted on a building by the water in a swimming tank on an upper floor of the building.

A brief review of progress and developments in connection with the design of dams to resist earthquake oscillations, and particularly some of the specific findings in connection with the design of one dam to resist earthquakes, will be of general interest, because the next and logical question is: "What effect does it have upon the design of a dam?"

Design of Pine Canyon Dam to Resist Earthquakes.—In the summer of 1928 the writer was instructed to design the Pine Canyon Dam, a concrete gravity type structure on the San Gabriel River, California, to resist the earthquakes which might reasonably be expected to occur in this locality. At that time, as far as the writer knew, no dam had ever been designed to resist earthquakes. Formulas were developed for a theoretical triangular dam designed to resist the inertia effect of the mass of the dam, and, as a result, the writer and S. B. Morris, M. Am. Soc., C. E., presented a paper before the Seismological Society of America in June, 1929, entitled "Design of Gravity Dam in San Gabriel Canyon to Resist Earthquakes."[18] Then, after the World Engineering Congress in Tokyo, Japan, in October, 1929, a copy of Professor Nagabo Mononobe's paper on "Earthquake-Proof Construction of Masonry Dams," was made available to the writer, in which it was discovered that Professor Mononobe had designed and built, or had under construction, seven gravity dams designed to resist the inertia effects of the mass of the dam during earthquakes. The earliest of these dams was completed in 1927. When the proper changes in notation were made, it was found that Professor Mononobe's paper and the paper previously referred to, by the writer and Mr. Morris, contained practically the same equations although derived independently. It is seen, therefore, that Professor Mononobe's work preceded the work done by the writer and Mr. Morris by several years.

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[15] Göttingen, Germany.

[16] Dissertation of Göttingen University.

[17] Chf. Designing Engr., Pasadena Water Dept., Pasadena, Calif.

Hydrodynamic Pressure.—While investigating the forces brought upon dams during earthquake motions, attention was given to the possibility of an increased water pressure against the up-stream face of the dam as it moved toward the water. Mr. Harry O. Wood, in a discussion[19] of the report of the Committee on Arch Dam Investigation, being conducted by Engineering Foundation, stated: "There is a possible action of earthquakes on dams, dikes, etc., which appears to have been generally, if not universally, overlooked." And, still further, he summarized and stated that, if a dam moves back and forth and the water tends to remain still, being frictionless on the bottom, the water might

> "Act as a great sluggish pendulum bob, or a kind of soft-nosed battering ram of relatively great mass, which tends to remain still as the earth and the dam shift rapidly to and fro. Relatively, then, a soft-nosed, but very heavy battering ram may be caused to deliver, in rapid succession, blows or thrusts against the up-stream face of the dam."

It was suggested by the writer that if the dam moves toward the water, it would be just the same as if the water moved or flowed toward the dam, and hence the water would rise in the dam an amount equal to a velocity head, and cause an increase in pressure equal to twice the velocity head, which it was thought was somewhat analogous to the well-known problem of a jet of water impinging upon a flat surface. This was presented to two authorities on hydraulics. One answered that the foregoing viewpoint was correct, and the other thought not, and thought that the problem was a difficult hydrodynamic problem.

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[18] *Bulletin*, Seismological Soc., Vol. 9, No. 3, September, 1929, p. 143.

[19] *Proceedings*, Am. Soc. C. E., November, 1928, Papers and Discussions, p. 2609.

Pursuing the problem further, it was then noted that if a rigid surface or lid could be fixed on the water surface, the problem would be akin to water-hammer in a pipe line. However, if the lid were not there, the bottom of the reservoir would tend to act somewhat like the water-hammer in the pipe line, being restrained, whereas at the water surface there would be instant upward release of the elastic compression in the water. This suggested that the portion of water which moved with the acceleration of the dam, and which offered resistance to that movement, was bounded by some straight or curved surface, such as, for example, a 45° plane, or the parabola given by Professor Westergaard, although no definite shape or value was determined.

The aforementioned velocity-head treatment (which, incidentally, gives a negligible force in any real case), was presented in the paper by Mr. Morris and the writer.[18] However, realizing that perhaps this solution might be bettered, the following statement was made:

"The difficult hydrodynamic problem of the determination of the forces applied to the dam, owing to oscillation of the dam against the body of the water in the reservoir, deserves further investigation both in mathematical analysis and in experimentation with models on a shaking table. There are a number of high masonry dams already constructed in close proximity to active fault lines. Recording devices installed on the up-stream face of such dams placed at intervals from top to bottom of the reservoir should definitely reveal the magnitude and distribution of such stresses. This would undoubtedly afford most certain and accurate answers to the question raised by Harry O. Wood."

In the spring of 1930, a copy of the "Report of Bureau of Reclamation on Boulder Canyon Dam," which contained Professor Westergaard's original treatment of the hydrodynamic force, became available. In all essential respects this treatment was identical to that contained in his present paper.

The writer has thought for some time that the fundamental part of the hydrodynamic force ought to be susceptible of reasonably simple explanation and fairly easy derivation without the somewhat complicated mathematics used by Professor Westergaard. Numerous attempts have been made to do this, and the writer has attempted to trace the elastic actions, following the elastic waves of compression and reflected waves of both compression and dilatation, somewhat in the manner commonly used in water-hammer problems. A graphic method soon became too complicated. The writer's ideas were presented to Dr. Theodor von Kármán, and he suggested the method of images, which is only a convenient way of treating the reflections which the writer was considering. Dr. von Kármán, however, suggested that since the waves of compression radiate from a point, the intensity of the compression, between parallel planes, would vary inversely as the distance from the source, somewhat as the intensity of light varies inversely as the square of the distance from the source, and this, therefore, suggested a method of attack. Following this, Mr. C. H. Heilbron, Jr., worked out the problem along these lines.

Mathematical Check.—In a report to the Pasadena, Calif., Water Department, Eugene Kalman, M. Am. Soc. C. E., derived the same equations as the author and stated that the original Westergaard treatment agrees fully with the general principle underlying hydraulics and that, therefore, an experimental check would not be necessary. Furthermore, he pointed out that it would be difficult because small models are influenced to a considerable extent by the point of singularity at the top of the dam. However, an experimental check has been made by Professor Lydik S. Jacobsen, on the shaking table at Stanford University, a preliminary account[29] of which shows that the experiments check the mathematics of Professor Westergaard. Professor Westergaard assumes that the dam moves in one piece as an integral part of the earth's crust. This is somewhat different from actual conditions since the natural, independent vibrations of the dam have some influence on the motion of the water. Professor Kalman pointed out the great difficulty of considering this effect and concluded that the results as presented must be accepted as a good approximation of the actual conditions.

Following Professor Kalman's report the Pasadena Water Department re-designed its Pine Canyon Dam to resist the hydrodynamic force, using Professor Westergaard's parabolic equivalent body of inertia, as well as the inertia effects upon the mass of the dam.

Effects of Earthquake Loads on the Size of a Dam.—For ease of comparison and calculation a typical triangular gravity dam section with a vertical up-stream face will be used as shown in Fig. 12.

An actual gravity dam design, straight in plan, does not differ materially from a triangular section and, hence, the general effects of any of the factors can be shown by using the simple triangular section. The principles involved in the actual design of a practical dam section with considerable top width are entirely similar to those of the ideal triangular section, except that the inter-relations cannot be expressed so simply. In addition to the symbols used by Professor Westergaard, the following nomenclature is introduced:

$W =$ total weight of the mass of the dam.
$G =$ specific gravity of concrete in mass of dam.
$H =$ total static water load on face of dam.
$U =$ total uplift pressure under dam.
$V =$ total re-active force of foundation upon dam.
$F =$ total force applied to dam by earthquake acceleration upon mass of dam.
$D =$ total force applied to dam by hydrodynamic load.
$h =$ height of dam.
$B =$ base width.
$\alpha =$ ratio of horizontal earthquake acceleration to that of gravity.
$r =$ coefficient of hydrostatic pressure to be used at heel of dam for uplift, which in this study is assumed to diminish uniformly in a straight line to zero at the down-stream toe.
$j_h =$ horizontal acceleration of earth movement during earthquake, in feet per second per second. ($j_h = \alpha g$.)
$j_v =$ vertical acceleration of earth movement during earthquake, in feet per second per second.
$g =$ acceleration of gravity.

All the forces in Fig. 12 are shown to a true scale of masonry units, and, hence, the areas of each loading diagram represent the relative value of each

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[29] *Bulletin*, Seismological Soc. of America, Vol. 21, No. 3, September, 1931, p. 204.

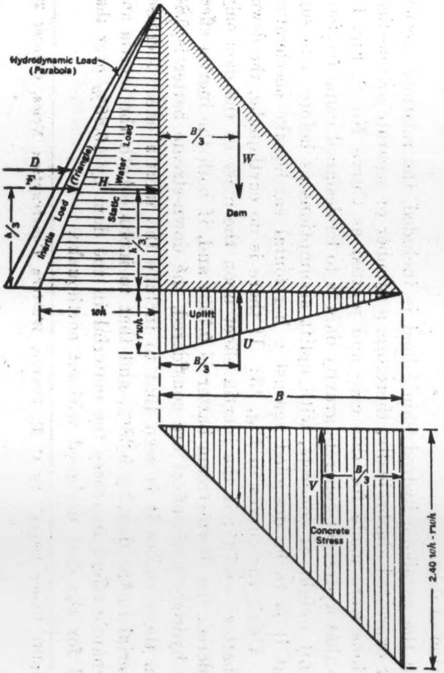
Hydrodynamic Load (Parabola)
D
F
h/3
Inertia Load (Triangle)
Static Water Load
H
h/3
B/3
W
Dam
wh
rwh
Uplift
U
B/3
B
V
B/3
Concrete Stress
2.40 wh - rwh

FIG. 12.—SKETCH SHOWING RELATIVE VALUES OF FORCES ON DAMS.

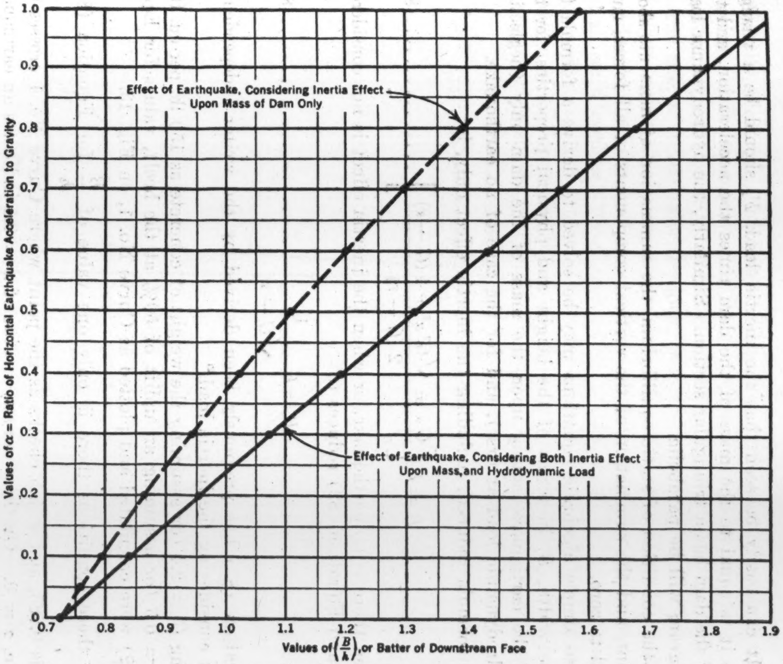

FIG. 13.—CURVES SHOWING EFFECT OF EARTHQUAKE ON SIZE OF DAM.

load. It can easily be seen that the inertia load, F, should be a triangle because it is equal to the mass of the dam times the acceleration, and the mass of the dam has a triangular section. Similarly, the hydrodynamic loading diagram will be parabolic.

For the condition of static equilibrium, the summation of moments about any point, and the summation of the vertical components of all forces, must be equal to zero.

These simple algebraic equations may be solved to derive a formula for the base width, B, in terms of the height and physical properties, for the case of the earthquake acting upon the mass of the dam only (neglecting the hydrodynamic load at first), and for the case of no earthquake.

The formula reduced, including the inertia effect only, is:

$$B = h \left[\frac{G\alpha \pm \sqrt{G^2 \alpha^2 + 4\,(G - r)}}{2\,(G - r)} \right] \ldots\ldots\ldots\ldots\ldots (82)$$

When there is no earthquake, or when the inertia effect is not considered, $\alpha = 0$, and Equation (82) reduces to:

$$B = h \frac{1}{\sqrt{G - r}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (83)$$

Equation (83) has previously been derived by the writer independently without considering the inertia load.[31]

Using $G = 2.4$ (or assuming the weight of concrete as 150 lb. per cu. ft.), and $r = 0.5$ (or assuming an uplift of 50% at the heel), values for Equation (82) were calculated and plotted as Curve No. 1, on Fig. 13.

For these conditions there is only one value of $\frac{B}{h}$ for Equation (83), which is 0.725, and which checks as the point where Curve No. 1 crosses the ordinate, $\alpha = 0$. On these curves, a value of $\alpha = 0.1$ indicates an earthquake acceleration of $0.1g$.

When the parabolic hydrodynamic load is included, the relations are not so easily expressed algebraically, therefore, a number of separate arithmetical calculations were made for this case and plotted as Curve No. 2, Fig. 13. Suppose that a straight concrete gravity dam is to be designed using the foregoing unit weight of concrete, with uplift assumptions as before; and suppose that it is to be designed against a horizontal earthquake acceleration of $0.1g$. Using the curves in Fig. 13: If there is no earthquake, the down-stream batter is 0.725; if the inertia effect upon the mass of the dam only, is considered, the down-stream batter is 0.793; and, if both the inertia effect and the hydrodynamic effect are considered, the down-stream batter is 0.84.

From these values it is seen that to include the inertia effect increases the material in the dam by 9.38%, and that to include both the inertia and hydrodynamic effect increases the material in the dam by 15.85% over that required for the dam designed without earthquake.

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[31] "Uplift Under Dams," by C. E. Pearce, *Western Construction News*, October 25, 1928.

Actually, although Pine Canyon Dam was first designed according to the foregoing uplift assumption (50% at the heel to 0% at the toe) and by assuming an earthquake acceleration of $0.1g$ for both inertia and hydrodynamic effects, it was later designed with the same earthquake assumptions and 100% uplift at the up-stream face, diminishing uniformly to 50% at the galleries near the up-stream face, and then to 0% at the down-stream face, all considered as acting upon 100% of the area. This uplift assumption, together with the parabolic hydrodynamic load, and the proper consideration of all forces for the actual dam (not a theoretical triangle), gave the cross-section shown in Fig. 14, in which, the maximum water level used in the design was 5 ft. below the roadway level. In this dam, vertical drains are 10 ft. apart in a line parallel and near the up-stream face as indicated in Fig. 14, but no allowance for such drainage was made in the design.

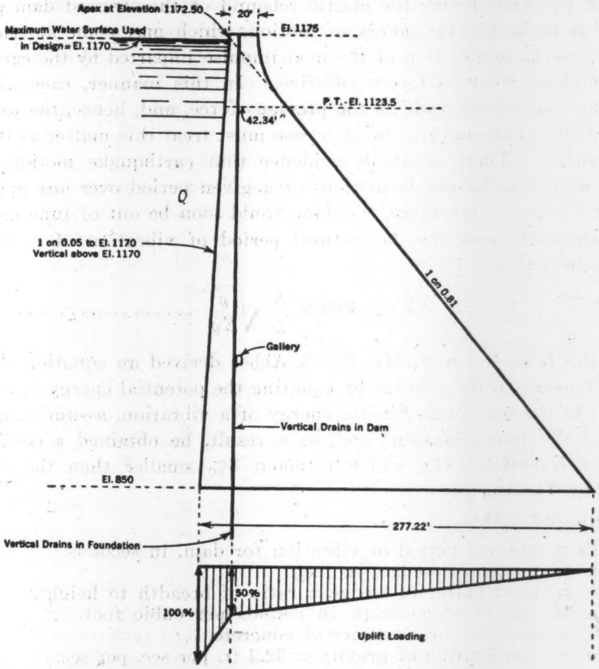
Apex at Elevation 1172.50
20'
El. 1175
Maximum Water Surface Used in Design = El. 1170
P.T.-El. 1123.5
42.34'
Q
1 on 0.05 to El. 1170
Vertical above El. 1170
1 on 0.81
Gallery
Vertical Drains in Dam
El. 850
277.22'
Vertical Drains in Foundation
50%
100%
Uplift Loading

FIG. 14.—ADOPTED SECTION, PINE CANYON DAM.

Vertical Earthquake Acceleration.—In any actual case of dam design, one must investigate all the possible earthquake effects, including vertical acceleration and resonance. It can be shown easily that the vertical acceleration of the earthquake has no overturning effect upon a gravity dam, because the weight of the water, and, therefore, the water pressure on the dam, is increased in the same proportion and at the same time as the mass of the dam during the upward acceleration. In other words, an acceleration of g is acting upon both dam and water for ordinary static conditions, while an acceleration of $(g + j_v)$ is acting upon both dam and water during upward acceleration, and, hence, their moment relations are not changed. In a similar manner, a downward acceleration of the earth does not change the moment relations. The unit stresses in the dam and its foundation are increased or decreased accordingly, but there is no added tendency to overturn the structure. The vertical acceleration, however, does affect the sliding factors, and this must be investigated in each case.

Resonance.—If the natural period of vibration of the dam is the same as that of the earthquake, the elastic rebound of the stressed dam produces a force due to its kinetic energy of motion, which may come at exactly the same time as the application of the next impulse imparted by the earthquake, thereby making the two forces additive. In this manner, each additional earthquake impulse may add to the previous force, and, hence, the total force may build up. The engineer in each case must treat this matter as the situation warrants. There is ample evidence that earthquake motions do not continue as simple harmonic motions of a given period over any appreciable length of time, and, therefore, the dam would soon be out of tune again.

Timoshenko[23] gives for the natural period of vibration of a wedge the following formula:

$$T_d = 4.0952 \frac{h}{k} \sqrt{\frac{\rho}{E g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (84)$$

Working independently, Mr. Frank Abbot derived an equation similar to that of Timoshenko for a wedge by equating the potential energy of maximum deflection to the maximum kinetic energy of a vibration, assuming harmonic motion of the free vibration; and, as a result, he obtained a coefficient of 3.9738 for Equation (84), which is about 3% smaller than the coefficient obtained by Timoshenko.

In Equation (84):

$T_d =$ natural period of vibration for dam, in seconds.
$h =$ height of wedge, in feet.
$k =$ total batter of wedge = ratio of breadth to height.
$\rho =$ weight of concrete, in pounds per cubic foot.
$E =$ modulus of elasticity of concrete.
$g =$ acceleration of gravity = 32.2 ft. per sec. per sec.

From Equation (84), it is seen that for a dam having a constant batter, k, the natural period varies as the first power of h.

For Pine Canyon Dam, using $h = 300$ ft.; $k = 0.86$; $\rho = 150$ lb. per cu. ft.; and $E = 360\,000\,000$ lb. per sq. ft.; Equation (84) gives $T_d = 0.16$ sec.

Since the dam is 300 ft. high only at the center and is fixed and less high along the abutments, it is seen that the higher cantilevers are not only fixed at the bottom, but are also restrained, to some degree, on the sides, and that, therefore, the natural period of the dam as a whole will be even less. Consequently, it will probably not be of much importance for the larger earthquakes having destructive value, during which the period of vibration is generally found to be much longer than 0.16 sec.

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[23] "Applied Elasticity," Timoshenko and Lessells, First Edition, p. 323, Equations (e) and (i).

This seems to indicate that for a gravity dam of this height or less there is little cause for worry regarding resonance. For high slender arch dams, the natural period of vibration may be found to be longer, and to approach dangerously near the periods observed for destructive earthquakes. In such a case, the designer might be justified in assuming two, three, or more vibrations of the earthquake having the same period as the natural period of vibration of the dam. While a high slender arch dam tends to have longer periods, it, in turn, can take an over-stress without failure. The writer does not specifically recommend exactly the foregoing procedure for arch dams, but merely mentions it to show what might be done. As new cases arise, they must be treated as good judgment dictates, and, at the present time, any rational treatment is a forward step.

Earthquake Acceleration for Use in Design.—Since the subject of the hydrodynamic load has brought up the question of its effect on the size of a dam, it is proper, at this point, to discuss earthquake accelerations, because proper results depend upon proper premises.

In any specific case, the engineer designing a dam should investigate the records of past and probable future earthquake accelerations at the locality involved for the kind of material upon which the dam is to rest. Unfortunately, there are comparatively few data at hand that will set a definite value to acceleration for the United States, or for any particular location. However, computations have been made for such values for such major earthquakes as the California earthquake of April 18, 1906, and the Japan earthquake of September 1, 1923.[23] Study of such data is of much value in the selection of a maximum value of acceleration to be used.

Comments.—

(1) By referring to Fig. 12 on which all load areas are drawn to scale, it can be easily seen why numerous dams not designed to resist earthquakes have not failed. The sum of the inertia load plus the hydrodynamic load is small in comparison to the other loads.

(2) By increasing the cost of the dam by only about 15%, the structure may be made safe against earthquakes, and this may be a wise safeguard for a dam in a populous country where earthquakes are likely to occur.

(3) Those who have not given consideration to designing structures to withstand earthquake accelerations may be inclined at first to think that designs purporting to resist those accelerations are flighty mathematical theories. A little investigation, however, will show that the basic part of the problem is really very simple and quite easy of determination. The probable earthquake accelerations which may be expected have been measured by seismologists, and are fairly well known.

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[23] Accelerations which have occurred, or which are recommended for use or have been used by others, are discussed in the *Bulletins* of the Seismological Society, September, 1929, p. 143, and September, 1931, p. 204; in Mononobe's work referred to herein; and in two articles by Henry D. Dewell, M. Am. Soc. C. E., in *Engineering News-Record*, Vol. 100, pp. 650 and 699, entitled "Earthquake-Resistant Construction."

(4) As far as is known by the writer at this time (1932), the following dams in the North American Continent have been designed against earthquakes: The Pine Canyon Dam, on the San Gabriel River; the Hoover Dam, on the Colorado River; and the Madden Dam, in the Panama Canal Zone.

(5) It is suggested that in his closure the author give a simple physical explanation of the difference between his first case, or case of ideal water-hammer, and the true case which he has solved, showing the elastic actions that actually take place in the water.

JOHN H. A. BRAHTZ,[24] ESQ., AND CARL H. HEILBRON, JR.,[25] ESQ. (by letter).—In connection with the design of the proposed Pine Canyon Dam, the writers have been able to follow up lines of thought suggested by Professor Westergaard. The studies made by them are as follows: (1) The author's assumption of an infinitely long reservoir is dropped, and formulas are developed for reservoirs of finite length; (2) the effect of assuming that the water is incompressible is shown; (3) the flexibility of the dam is taken into account and methods of allowing for it in finding earthquake stresses are presented; and (4), an independent method of arriving at the author's principal results is developed.

Effect of Finite Length of Reservoir.—If the reservoir is to be considered as other than infinitely long, several other assumptions may be substituted. Perhaps the simplest is that at a distance, L, from the dam there is an immovable vertical wall. Expressed mathematically, this means that the author's Boundary Condition (4) — σ converges toward 0 when x becomes large — is replaced by a new condition, (5) $\xi = 0$, when $x = L$.

A solution of the author's Equations (1), (2), and (5), which satisfies Conditions (1), (2), (3), and (5), is similar in form to the author's solution as expressed in his Equations (15), (16), and (20), differing only in that the factor, e^{-q_n}, occurring in these equations, is replaced by expressions involving hyperbolic functions. Further, the author's Equations (31), (35), (36), (38), and (39) are modified only in that in each equation the factor, $\coth \dfrac{n\,\pi\,c_n L}{2\,h}$, is inserted under the summation sign.

The effect of the use of Boundary Condition (5) is shown in the following tabulation giving the values of the correction factor for Q_0, for various values of the ratio, $\dfrac{L}{h}$:

Ratio, $\frac{L}{h}$	Correction factor	Ratio, $\frac{L}{h}$	Correction factor
0	∞	1.0	1.095
0.4	1.80	1.5	1.020
0.6	1.37	2.0	1.005
0.8	1.18	∞	1.000

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[24] Teaching Fellow, California Inst. of Technology, Pasadena, Calif.

[25] Asst. Engr., Pasadena Water Dept., Pasadena, Calif.

This correction factor is defined as the ratio of the value of Q_0 computed by using the formula based on Condition (5) to the value computed by Equation (31). It will be seen that Q_0 increases not more than 0.5 of 1% if the length of the reservoir is more than twice its depth.

Inasmuch as in any actual earthquake any surface at the far end of a reservoir would probably vibrate with the earthquake, a better assumption than Condition (5) would be that both ends of the reservoir move with the same amplitude and period. To express this mathematically, the author's Boundary Condition (4) is replaced not by Condition (5), but by another condition, (6), $\xi = -\frac{\alpha g T^2}{4\pi^2} \cos \frac{2\pi t}{T}$, when $x = L$.

A solution of the author's differential equations which satisfies Boundary Conditions (1), (2), (3), and (6), differs from the author's solution as before only in the substitution of hyperbolic functions for the factor, e^{-qx}, in Equations (15), (16), and (20). In this case, Equations (31), (35), (36), (38), and (39) are modified by the insertion under the summation sign of the factor, $\coth \frac{n\pi c_n L}{2h} - \operatorname{csch} \frac{n\pi c_n L}{2h}$.

The effect of using Boundary Condition (6) instead of Condition (4) is shown in Table 3.

TABLE 3.—VALUES OF THE CORRECTION FACTOR FOR Q_0.

Ratio, $\frac{L}{h}$	Correction factor	Ratio, $\frac{L}{h}$	Correction factor
0.5	0.397	2.0	0.921
1.0	0.670	3.0	0.983
1.5	0.835	4.0	0.996
		∞	1.000

It will be seen that if Boundary Condition (6) is considered as correct, the error in assuming the reservoir infinitely long is negligible if the length of the reservoir is more than about three times its depth. Incidentally, if the value of L is small compared with the value of h, Boundary Conditions (1), (2), (3), and (6) represent conditions in an open well in the earth or in a structure during an earthquake, and the expression for the pressure, p, corresponding to Equation (31), reduces to approximately:

$$p = \frac{1}{2} \alpha w L \quad \ldots\ldots\ldots\ldots\ldots\ldots (85)$$

which means that the water pressure due to earthquake on one side of a well is equal to the "inertia force" of one-half the weight of water in the well.

Since the earthquake waves which cause the motion of the dam and of the up-stream end of the reservoir travel with a more or less definite velocity, the motions at the two ends of the reservoir in an actual case would be out of phase. However, it is shown in Table 3 that if the length of the reservoir is not small, one end has no appreciable effect on the other and, therefore, the phase difference is unimportant. On the other hand, if the length is small enough to affect the pressures, the phase difference must be so slight as to be negligible. Therefore, Table 3 gives a sufficiently close approximation to the effect of finite length of reservoir for all practical purposes.

Effect of Compressibility of Water.—If the author had considered the water to be incompressible, he would have written, instead of his Equation (5),

$$\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} = 0 \qquad (86)$$

Using Equation (86), he would have obtained exactly the same Equations (15), (16), and (20), except that the coefficients, c_n, the value of which is given in his Equation (21), would have the value, unity. Continuing, in his Table 2, he would have only one set of values applying to all heights of dam, and these would be the same as he has listed for "small" heights of dam. In other words, therefore, the compressibility of the water is of no importance except when the dam is high.

Effect of Flexibility of Dam.—The author states:

> " * * * since the period of free vibrations of the dam, T_0, is usually a fraction of a second, while the period of vibration, T, of the earthquake may be assumed to be not less than a second, * * * resonance * * * need not be expected ordinarily, although the possibility of part resonance may well be investigated in some cases of high dams."

Approximate formulas for calculating the effects of this part resonance have been developed by the writers. As part resonance is caused both by the hydrodynamic pressures and by the inertia effect of the weight of the dam, and as it adds to both the hydrodynamic pressures and the inertia forces, it is impossible to separate the two effects and both must be considered.

The procedure, in general, is as follows: The hydrodynamic pressures are first taken as those given by the author's formulas and the inertia forces are taken as if the dam were rigid; the deflections of the dam due to these forces are found; the displacement of the dam is now taken as the displacement of its base plus the deflection; the hydrodynamic pressures and the inertia forces are recalculated on the basis of the new displacements; and the process is repeated if a closer approximation is desired.

The most difficult part of the procedure is the solution of the author's differential Equations (1), (2), and (5) for a new set of boundary conditions. Conditions (1), (2), and (4) are unchanged. Condition (3) expresses that the face of the dam moves in a simple harmonic motion with the amplitude of the motion of the earth's crust. If the dam is flexible, only the base moves with the prescribed motion, and Condition (3) is replaced by some relation of the general form of Condition (7), $\xi_0 = f(y, t)$, in which, ξ_0 is the value of ξ for $x = 0$.

If the function, $f(y, t)$, of Condition (7) is considered as known, then a set of expressions for ξ, η, and σ, can be found which satisfy the differential equations and Boundary Conditions (1), (2), (4), and (7). These expressions need not be written here; it can be shown that they lead to equations for p and ξ_0 of the forms:

$$p = \frac{4\,\pi^2}{T^2}\;\frac{w}{g}\sum_{1,3,5..}^{n} E_n \sin\frac{n\,\pi\,y}{2h} \;\ldots\ldots\ldots\ldots\ldots\ldots(87)$$

and,

$$\xi_o = -\,\frac{\pi}{2\,h}\cos\frac{2\,\pi\,t}{T}\sum_{1,3,5..}^{n} n\,c_n\;E_n \sin\frac{n\,\pi\,y}{2\,h}\;\ldots\ldots\ldots(88)$$

in which, the factors, E_n, are constants to be determined so as to satisfy Equation (88). If the values of E_n are found, then p can be computed.

In order to find these values in Equation (88) it is desirable to have the function, $\xi_o = f(y, t)$, in as simple a form as possible; it is impracticable to use an expression for ξ_o involving several different powers of y. In choosing the form of the function, $f(y, t)$, it is convenient first to consider the very special case of a rectangular dam of height, H, the base of which moves according to the author's Equation (7), and which is flexible in shear but not in moment. It can be shown that the motion of such a dam is expressed by:

$$\xi_o = -\,r\,\frac{\cos\nu\gamma y}{\cos\nu\gamma H}\cos\nu t\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(89)$$

in which, $\nu = \dfrac{2\,\pi}{T}, \gamma = \sqrt{\dfrac{\rho}{Gg}}$, ρ is the density of concrete, and G is its elastic modulus in shear. It is seen that in this case the motion of every part of the dam is in phase with that of its base; and it can be shown that in the more general case this must also be true, so that the function, $f(y, t)$, is $\cos \nu t$ multiplied by a function of y.

If ξ_o of Equation (89) is plotted as a function of y for a given time, a curve, as shown in Fig. 15(*a*), is obtained. For a triangular dam, the shear deflections due to either inertia or hydrodynamic forces form curves of similar shape, while the moment deflections due to both effects form curves of the general shape of Fig. 15(*b*). The total deflections form curves of the shape shown in Fig. 15(*c*). This curve is nearly a straight line, and in most cases, little error will be introduced by assuming that it is straight.

Making this assumption, and with δ_o equal to the maximum deflection at the top due to dynamic forces, the expression for $f(y, t)$ must give $r \cos \nu\, t$ for $y = H$, and $(r + \delta_o) \cos \nu\, t$ for $y = 0$; therefore,

$$f(y, t) = \xi_o = \left[r + \delta_o\left(\frac{H-y}{H}\right)\right]\cos\nu\,t\ldots\ldots\ldots\ldots\ldots(90)$$

Substituting this value in Equation (88),

$$r + \delta_o\left(\frac{H-y}{H}\right) = \sum_{1,3,5,..}^{n} E_n\,\frac{n\,\pi\,c_n}{2\,h}\sin\frac{n\,\pi\,y}{2\,h}\;\ldots\ldots\ldots\ldots(91)$$

It can be shown that Equation (91) is satisfied, within the limits, $0 < y < h$, where it is applicable, by,

$$E_n = \frac{8\,h}{n^2\,\pi^2\,c_n}\left[r + \delta_o + (-1)^{\frac{n+1}{2}}\,\frac{2\,h}{n\,\pi}\,\frac{\delta_o}{H}\right] \ldots\ldots\ldots\ldots (92)$$

If $\delta_o = 0$, the substitution of E_n from Equation (92) in Equation (87) gives exactly the author's Equation (31).

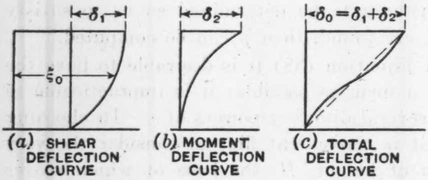
δ_1
ξ_0
δ_2
$\delta_0 = \delta_1 + \delta_2$
(a) SHEAR DEFLECTION CURVE
(b) MOMENT DEFLECTION CURVE
(c) TOTAL DEFLECTION CURVE

FIG. 15.

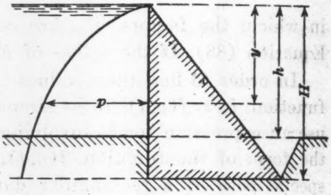

FIG. 16.

The inertia coefficient, a_1, defined as the maximum acceleration of any part of the dam divided by g, is $\frac{1}{g}\left(\frac{\partial^2 \xi_o}{\partial t^2}\right)$ for $t = 0$. Under the straight-line assumption this becomes,

$$a_1 = a\left(1 + \frac{\delta_o}{r}\,\frac{H - y}{H}\right) \ldots\ldots\ldots\ldots\ldots\ldots (93)$$

It will be noticed that a distinction is made between the depth of the reservoir, h, and the height of the dam, H. If the up-stream face of the dam is back-filled to the original stream bed, the depth, h, is probably best measured down to this point. The height, H, is measured down to the foundation of the dam, or lower, if it is desired to allow for foundation deflection (see Fig. 16).

The method and formulas here given have been applied to a straight masonry dam of triangular section, in which, H = 375 ft., h = 295 ft., the base is 0.85 of the height, the concrete weighs 150 lb. per cu. ft., the elastic moduli in bending and shear are 2 000 000 lb. per sq. in., and 900 000 lb. per sq. in., respectively. The reservoir is assumed to be full to the apex of the triangle, and an earthquake is assumed, for which a = 0.1 and T = 1.0 sec.

The deflection, δ_o, at the top of the dam is determined in the first approximation by applying the hydrodynamic forces for an assumed rigid dam in accordance with the author's parabolic distribution and the inertia forces for a rigid dam. The hydrodynamic forces are computed for $h = 295$, and the parabolic distribution is assumed to extend also over the lower part of the dam, where $h < y_1 < H$.

The top deflection due to moment is found to be 0.220 in. and that due to shear, 0.105 in.; whence δ_o = 0.325 in. This compares with r = 0.98 in.

The values of E_n are found by Equation (92) and then p is found by Equation (87) for several values of y. The values of p are plotted in Fig. 17, Curve (2), where they may be compared with the values found by Equation (31) and shown in Curve (1). The inertia coefficients as given by Equation (93) are plotted in Fig. 18.

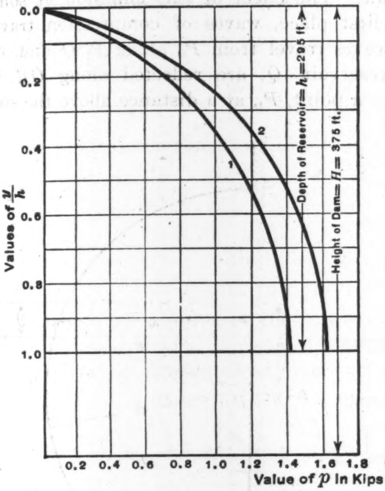

FIG. 17.—HYDRODYNAMIC PRESSURES.

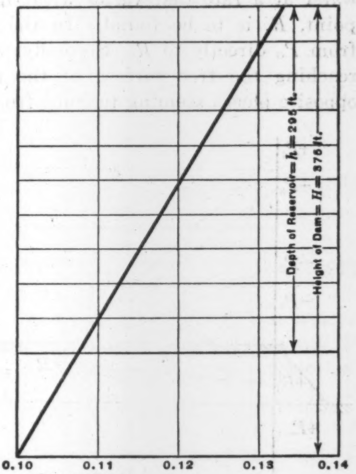

FIG. 18.—INERTIA COEFFICIENTS.

In the case of a dam of this height the increase in pressure due to flexibility is small and it appears unnecessary to carry the computations through another cycle. The shears, moments, and stresses in the dam due to earthquake are to be computed by applying the hydrodynamic pressures represented by Curve (2), Fig. 17, and the inertia forces corresponding to the inertia coefficients in Fig. 18.

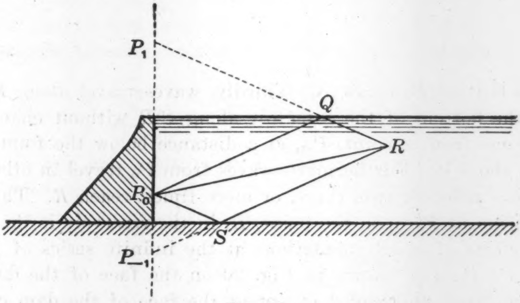

FIG. 19.

In order to place confidence in the author's result, it seems in keeping to explain the true physical picture, and to show that the use of a simple application of it produces the same result and, therefore, strengthens and proves the original result correct.

A point, P_0 (Fig. 19), in the face of the dam is considered to be emitting water at a rate that varies with time. The effect of this emission at some point, R, is to be found. In the first place, waves of compression travel from P_0 directly to R. Secondly, waves travel from P_0 along $P_0\,Q$ and, on reaching the free surface of the reservoir, Q, are reflected along QR in opposite phase, seeming to come from a point, P_1, at a distance above the surface equal to that of P_0 below it. Thirdly, waves travel along $P_0\,S$ and are reflected at the bottom of the reservoir along $S\,R$ without change of phase, seeming to come from a point, P_{-1}, at a distance below the foundation equal to that of P_0 above it. Furthermore, waves from P_0 travel in other directions and after being reflected two, three, or more times, reach R. The total effect of the emanation at P_0 upon the point, R, in the reservoir is the same as the sum of the effects of equal emanations at the infinite series of points $-P_{-3}$, P_{-2}, P_{-1}, P_0, P_1, P_2, P_3— shown in Fig. 20 on the face of the dam extended, acting in a medium unbounded except at the face of the dam extended and in phase or out of phase with P_0, according as they are marked plus or minus in the diagram.

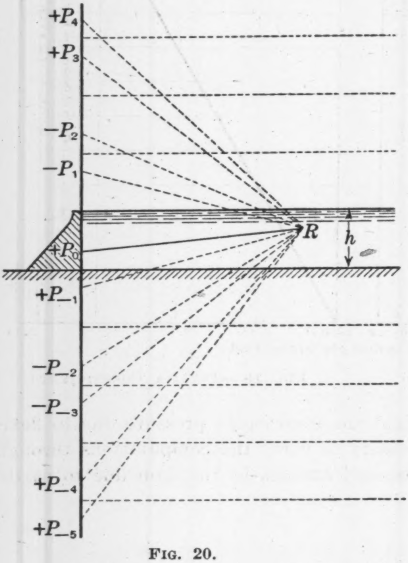

FIG. 20.

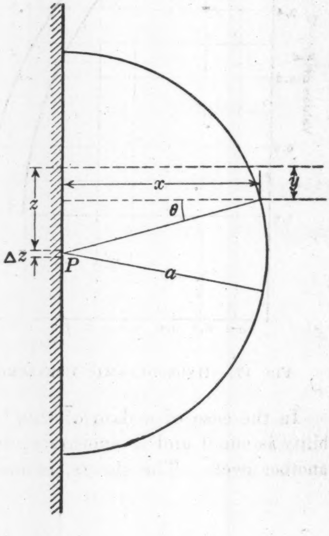

FIG. 21.

The emanation at any point, P (Fig. 21), is now to be considered as due to the motion of a part of the face of the dam of height, Δz, moving with an acceleration, $\alpha\, g$. Then, the rate of change of emission of water is,

$$\frac{dQ}{dt} = \alpha\, g\, \Delta z \ldots\ldots\ldots\ldots (94)$$

If the water is unbounded except along the one line the effect of this emanation will be the same at all points on a semi-circle of radius, a, and center at P, and if the water is considered incompressible, the radial acceleration, $\frac{dv_r}{dt}$, at a point, R, on the semi-circle is,

$$\frac{dv_r}{dt} = \frac{1}{\pi\, a}\,\frac{dQ}{dt} = \frac{\alpha\, g\, \Delta z}{\pi\, a} \ldots\ldots\ldots\ldots (95)$$

The vertical component of this acceleration is,

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{dv_r}{dt} \sin\theta = \frac{\alpha\, g\, \Delta z}{\pi\, a} \sin\theta = \frac{\alpha\, g\, \Delta z}{\pi\, a}\,\frac{z - y}{a} \ldots\ldots\ldots\ldots (96)$$

and,

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{\alpha\, g\, \Delta z}{\pi}\,\frac{z - y}{x^2 + (z - y)^2} \ldots\ldots\ldots\ldots (97)$$

If, now, R is a point in the reservoir, the total effect of emanation at a point, P_0, is as shown, the sum of the combined effects of all the points, P (Fig. 20), being considered as acting in an infinite medium; and the vertical acceleration at R is,

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{\alpha\, g\, \Delta z}{\pi} \sum_{-\infty}^{+\infty} n\,(-1)^n \left[\frac{(2\,n\,h + z) - y}{x^2 + \left\{(2\,n\,h + z) - y\right\}^2} - \frac{(2\,n\,h - z) - y}{x^2 + \left\{(2\,n\,h - z) - y\right\}^2} \right] \ldots\ldots\ldots\ldots (98)$$

If the whole face of the dam is assumed to move with the acceleration, $\alpha\, g$, the value of $\frac{\partial^2 \eta}{\partial t^2}$ at R is given by Equation (98), with Δz replaced by dz and integrated from $z = 0$ to $z = h$.

Performing the integration and reducing,

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{\alpha g}{\pi} \sum_{-\infty}^{+\infty} n\,(-1)^{n-1} \log_e \left[x^2 + (2\,n\,h - y)^2\right] \ldots\ldots\ldots\ldots (99)$$

In order to find the pressure at a point the author's Equation (2) and Condition (1) are necessary; if the former is integrated with the latter as a constant of integration, there results,

$$\sigma = \frac{w}{g} \int_0^y \frac{\partial^2 \eta}{\partial t^2}\, dy \ldots\ldots\ldots\ldots (100)$$

Substituting Equation (100) in Equation (99),

$$\sigma = \frac{w\,\alpha}{\pi} \sum_{-\infty}^{+\infty} n\,(-1)^{n-1} \int_0^y \log_e \left[x^2 + (2\,nh - y)^2\right] dy \ldots\ldots (101)$$

Integrating,

$$\sigma = \frac{w\,\alpha}{\pi} \sum_{-\infty}^{+\infty} n\,(-1)^n \left[2\,nh \log_e \frac{x^2 + (2\,nh - y)^2}{x^2 + (2\,nh)^2} - y \log_e \left\{ x^2 + (nh - y)^2 \right\} \right.$$
$$\left. + 2\,y + 2\,x \left\{ \tan^{-1} \frac{2\,nh - y}{x} - \tan^{-1} \frac{2\,nh}{x} \right\} \right] \ldots\ldots\ldots (102)$$

This is a general expression for σ. In particular, at the face of the dam, $x = 0$, and,

$$-p = \sigma = \frac{w\,\alpha}{\pi} \sum_{-\infty}^{+\infty} n\,(-1)^n \left[2\,nh \log_e \frac{(2\,nh - y)^2}{(2\,nh)^2} - y \log_e (2\,nh - y)^2 + 2\,y \right] . (103)$$

Equations (102) and (103) yield exactly the same numerical results as the author's Equations (20) and (31), respectively, when the C_n values are taken equal to unity.

The foregoing derivation assumes the water incompressible. To follow the same method with compressible water it would be necessary to use Bessel's functions, which are unfamiliar to most engineers.

BORIS A. BAKHMETEFF,[26] M. AM. SOC. C. E. (by letter).—The analysis of water pressure on dams during earthquakes presented in this paper should be considered a pioneer achievement in a realm heretofore unexplored. The engineer is often aided in gaining a grasp of new concepts by comparisons in terms of more familiar objects in other fields. For example, in studying the additional water pressure on dams caused by earthquake shocks, comparisons may be made with the well-known phenomenon of water-hammer in conduits caused when the velocity of flow changes rapidly.

In both cases, the problem deals with acoustical waves. For example, Equations (1), (2), and (5), in the paper, which the author integrates for given boundary conditions, are the general acoustical equations that Joukowsky[27] and Allievi[28] used for a general mathematical study of water-hammer. On the other hand, water-hammer lends itself to elementary treatment along the general lines suggested by St. Venant for the study of elastic impulses in bars[29] and of surface waves in open channels.[30]

Professor Westergaard's "preliminary" case, in which only the horizontal motion of the water layers is considered, is mechanically identical with the model used by Flamant[31] for the study of compression waves in connection with water-hammer. In analyzing the additional pressure, P, created in a conduit by a sudden reduction of the velocity of flow, v, Flamant reverses the movement and determines the pressure, caused by setting in motion a piston sliding in a cylinder that is filled with fluid originally at rest. (See Fig. 22.) By applying the momentum equation in connection with the continuity principle, it is found that the motion of the piston creates a compression wave that is propagated with a speed, c. The pressure exerted on the piston may be stated in terms of the velocity, v, of the piston by the simple equation:

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[26] New York, N. Y.

[27] *Proceedings*, Academie des Sciences (St. Petersburg), Tome IX, 1900; also *Journal*, Am. Water Works Assoc., 1904.

[28] *Annali*, Soc. Ing. ed Arch. Ital., 1901; *Revue de Mecanique*, 1904.

[29] See, for example, Donnell, *Transactions*, Am. Soc. Mech. Engrs., Vol. 52, No. 22, 1930.

[30] Comptes rendus de l'Académie des Sciences. 1870.

[31] *Revue de Mecanique*, 1904; also "Hydraulique," 1923 Edition, p. 478; see, also "Introduction to the Study of Variable Flow" (in Russian), by Boris A. Bakhmeteff, M. Am. Soc. C. E., 1915.

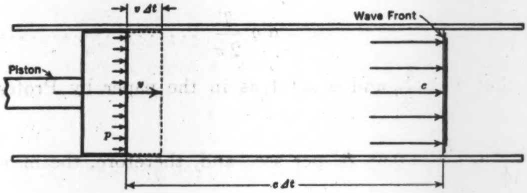

FIG. 22

$$\frac{p}{w} = z = \frac{c}{g}\, v \quad \ldots\ldots\ldots\ldots\ldots\ldots (104)$$

in which, w is the weight of a cubic foot of the liquid; g is the acceleration due to gravity; and z is the pressure head measured by the height of a column of fluid which corresponds to the water-hammer pressure, p. If the walls of the cylinder are assumed to be absolutely rigid, the movement will then be identical with that in the author's preliminary case. The dam will act as the piston, while the speed, c, will be the acoustical velocity, 4 715 ft. per sec. The numerical value of the ratio, $\frac{c}{g}$, will be $\frac{4\,715}{32.2} = 147$ sec. The significance of this value of the ratio, $\frac{c}{g}$, is that for every foot of velocity lost in a penstock or created by the motion of the dam (Fig. 22), a pressure will be exerted, equivalent to a head of $z = 147$ ft. of water, or if $w = 0.03125$ tons per cu. ft., to an equivalent pressure of $147 \times 0.03125 = 4.6$ tons per sq. ft.

The Direct Pressure Impulse.—Assume that in Fig. 23, $v = f\,(t)$ represents the increase in velocity of the piston (or of the dam) during the acceleration period from $v = 0$ to a maximum value of $\sqrt{v_0}$. The pressure equation will be:

$$\frac{p}{w} = \frac{c}{g}\, v = \frac{c}{g}\, f\,(t) \ldots\ldots\ldots\ldots\ldots\ldots (105)$$

In other words, the pressure curve will be identical in shape with the velocity curve, and the scale ratio will be $\frac{p}{v} = 4.6\, \frac{\text{ton}}{\text{ft.}^2} \times \frac{\text{sec.}}{\text{ft.}}$.

The instant of maximum pressure coincides with that of maximum velocity, and is equal to $4.6 \times v_0$. Assuming sinusoidal motion with a maximum acceleration, $\alpha\, g$, at $t = 0$, the maximum velocity will occur at $t = \frac{T}{4}$, and its value will be expressed by the equation:

$$v_0 = \alpha\, g\, \frac{T}{2\pi} \quad \text{.......................} (106)$$

Assuming that $T = \frac{4}{3}$, and $\alpha = 0.1$, as in the paper by Professor Westergaard, $v_0 = \frac{32.2}{10} \times \frac{\frac{4}{3}}{2\,\pi} = 0.68$ ft. per sec., and, therefore, the maximum pressure equals $p_0 = 4.6 \times 0.68 = 3.13$ tons per sq. ft., which is identical with values determined by the use of Equation (14).

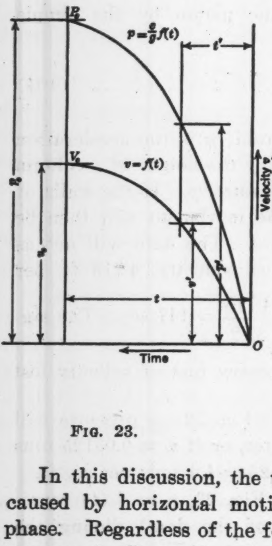

FIG. 23.

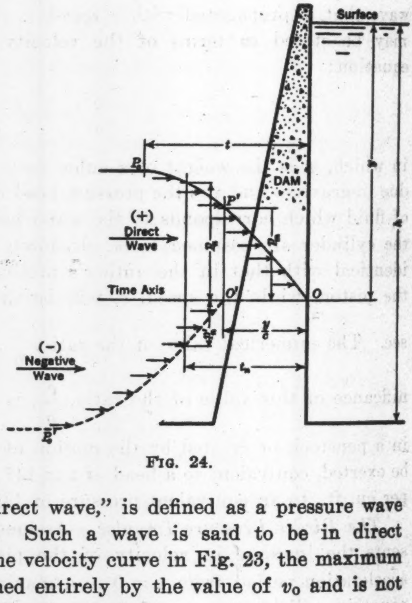

FIG. 24.

In this discussion, the term, "direct wave," is defined as a pressure wave caused by horizontal motion only. Such a wave is said to be in direct phase. Regardless of the form of the velocity curve in Fig. 23, the maximum pressure in direct phase is determined entirely by the value of v_0 and is not affected by the length, τ, of the acceleration period.

Effect of Free Surface; Indirect Phase.—On the other hand, the acceleration period (and, therefore, the manner in which the displacement is imparted to the dam) is decidedly important when one considers the effect of the free surface of the water, or when the phenomenon is in the "indirect" phase.

In the case of water-hammer, the increase in pressure characteristic of the direct phase, is counteracted by the effect of a negative wave reflected from the direct positive wave at the open end of the penstock. The condition in general is identical to that of sound waves at the open end of organ tubes. By way of analogy, the effect of the open surface of the water behind the dam may be visualized as creating a negative pressure impulse, identical in form and coincident in phase with the direct pressure wave, but of opposite sign. This negative impulse, which will account for the vertical displacement of the water layers, will be propagated downward with the accoustical velocity, c, and will reach a layer at a depth, y, in a period equal to,

$$t = \frac{y}{c} \text{ sec.} \qquad \text{(107)}$$

measured from the time the dam first began to move (see Fig. 24).

At the surface ($y = 0$), where the direct and the negative impulses are generated simultaneously and where their phases coincide, the resultant pressure will always be equal to zero. At a depth, y, conditions may be visualized as shown in Fig. 24. The pressure caused by the direct wave which is generated by the movement of the dam is represented by the curve, OP_0. The negative impulse, $O'P'_0$, is identical in shape with OP_0, but opposite in sign, and has a difference of phase, $OO' = \frac{y}{c}$. Obviously, while $t < \frac{y}{c}$, the effect of the surface yield has not had time to reach the layer at a depth, y, and the pressure increase is entirely due to the direct wave. For $t > \frac{y}{c}$, the resultant pressure is the difference between the ordinates of the direct and the negative wave. Thus, for $t = t_n$, the resultant pressure is $p_n - p'_n$.

If the shape of the velocity curve is as given in Fig. 23 (that is, if the acceleration is maximum at the beginning of the motion and then gradually becomes less), the maximum pressure at a layer, distant y from the surface, will be reached at the moment, $t = \frac{y}{c}$. This is the moment when the negative wave reaches the water layer under consideration. The effect of the acceleration period, τ, is now apparent: If it is small, the time, $\frac{y}{c}$, may be greater than τ, so that the negative impulse will not reach the layer in question in time to relieve the direct wave. In that case, the maximum pressure will attain its greatest possible value, $p_0 = 4.6\ v_0$, which corresponds to the direct impulse. A depth or an equivalent height of a dam may be determined for each value of τ by the equation,

$$h_0 = c\tau \qquad \text{(108)}$$

beyond which the increased pressure during the initial acceleration period will not be affected by the surface yield and the maximum pressure will be fully determined by direct impulse. If $c = 4\,715$ ft. per sec., the values of h_0 for corresponding values of τ are as follows:

Values of τ, in seconds	Values of h_0, in feet
0.01	47
0.05	235
0.10	471
0.20	942

For sinusoidal motion, with $\tau = \frac{T}{4}$ and $T = \frac{4}{3}$ sec., $h_0 = 4\,715 \times \frac{1}{4} \times \frac{4}{3} = 1\,570$ ft. This is in excess of the height of any dam thus far built. Therefore, it may be concluded that in actual structures the pressure never reaches the maximum value corresponding to the direct pressure impulse. This conclusion, however, will obtain only if the movement of the dam appears as represented in Fig. 25, in which the displacement begins at $t = 0$ from the extreme position and the velocity reaches its maximum over a period, $\tau = \frac{T}{4}$. On the other hand, conditions would be decidedly different if the motion was generated as shown in Fig. 26 by a sudden impact, such as in a ballistic pendulum, and in which the maximum velocity is reached over a short initial impact period, which is brief in comparison with T, the period of the subsequent oscillations. At least some seismographic records seem to indicate movement of this character.[32]

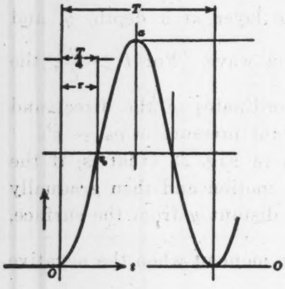

FIG. 25

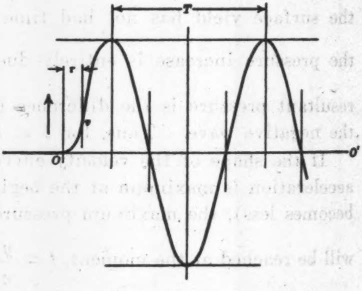

FIG. 26

Case of Sinusoidal Movement Shown in Fig. 25.— To compare data obtained by approximate reasoning outlined in the foregoing discussion, with the results of the exact method introduced by Professor Westergaard, assume the motion to be as in Fig. 25 with $T = \frac{4}{3}$ and $v_0 = 0.68$ ft. per sec., which corresponds to a maximum acceleration of 0.1 g. Assume that the maximum pressure, $P_m\ (y)$, at a depth, y, will correspond to the direct impulse at the moment, $t = \frac{y}{c}$; and that further in view of Equation (108), $\frac{T}{4} = \frac{h_o}{c}$; then,

---

[32] See, for example, the article on "Earthquakes," in the Encyclopedia Brittanica.

$$P_{m(y)} = P_o \sin 2\pi \, \frac{\frac{y}{c}}{T} = P_o \sin \frac{\pi}{2} \times \frac{y}{h_o} \quad \ldots\ldots\ldots\ldots (109)$$

If $P_o = 3.13$ tons per sq. ft., and $h_o = 1\,570$ ft., the value of the maximum pressure, P_m, for different heights of the dam, h, is as shown in Table 4.

TABLE 4.—VALUES OF MAXIMUM PRESSURE, P_m, FOR DIFFERENT VALUES OF h.

h, in feet (1)	$\frac{h}{h_o}$ (2)	$\frac{\pi}{2} \times \frac{h}{h_o}$ (3)	$\sin \frac{\pi}{2} \times \frac{h}{h_o}$ (4)	P_m, in tons per square foot (5)	P_w, in tons per square foot (6)
200	0.127	0.200	0.197	0.62	0.47
600	0.382	0.600	0.565	1.77	1.51
800	0.510	0.800	0.718	2.24	2.18

Column (6), in Table 4, contains the values obtained by Professor Westergaard's equations for comparison. The pressures computed from Equation (109) are somewhat larger, particularly for small values of h. For high dams the difference is small.

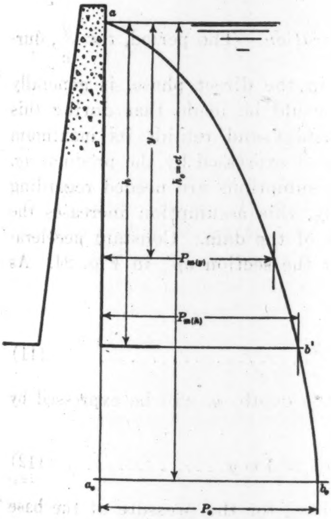

FIG. 27.

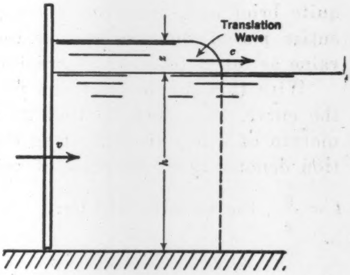

FIG. 28.

Maximum Shear at Bottom.—In computing the total shear, Q, at the base of a dam, the effect of the negative impulse beyond the point, o', in Fig. 24 may be neglected, and instead the simple assumption may be made that the shear is the sum of the pressures as determined for each depth layer by Equation (109). In this case (see Fig. 27):

$$Q = \int_0^h P_{m(y)}\,dy = \int_0^h P_0 \sin\left(\frac{\pi}{2} \times \frac{y}{h_0}\right) dy$$

$$= (p_0 h)\,\frac{2\,h_0}{\pi h}\left[1 - \cos\frac{\pi}{2} \times \frac{h}{h_0}\right] = \beta'\,(P_0 h) \dots\dots\dots\dots (110)$$

Let $h_0 = 1\ 570$ ft. and $P_0 = 3.13$ tons per sq. ft., then values of Q are as listed in Table 5.

TABLE 5.—COMPARATIVE VALUES OF Q

h (1)	$\cos\frac{\pi}{2} \times \frac{h}{h_0}$ (2)	β (3)	$P_0 h$, in tons (4)	Q, in tons (5)	Q_w, in tons (6)
200	0.980	0.100	626	62.6	68.4
600	0.825	0.292	1 878	550	658.3
800	0.697	0.379	2 504	950	1 253

The shears in Column (5), of Table 5, obtained from Equation (110), are somewhat less than those computed by Professor Westergaard, which are given in Column (6).

Simplified Solution with Constant Acceleration.—The period, $t = \frac{y}{c}$, during which the pressure impulse is applied in the direct phase, is generally quite brief and, therefore, the assumption could be made that during this entire period the acceleration remains constant and retains its maximum value as taken from seismographic records and expressed by the product, αg.

With this simple condition no further assumptions are needed regarding the curve, $v = f(t)$, in Fig. 23. Obviously, this assumption increases the margin of safety in computing the stability of the dam. Constant acceleration denotes linear increase of velocity over the section *op'*, in Fig. 24. As $t = \frac{y}{c}$, the velocity will be:

$$v = \alpha g\,\frac{y}{c} \dots\dots\dots\dots\dots\dots\dots\dots (111)$$

and thus the maximum pressure, $P_{m(y)}$, at the depth, y, will be expressed by the equation:

$$P_{m(y)} = v\,\left(\frac{c}{g} \times w\right) = \alpha\,w\,y \dots\dots\dots\dots\dots (112)$$

which, with $w = 0.03125$ tons per cu. ft., gives, for the pressure at the base of the dam,

$$P_h = 0.031\ \alpha\,h\,\frac{\text{ton}}{\text{ft.}^2} \dots\dots\dots\dots\dots\dots\dots (113)$$

The shear, in tons, at the base of the dam, is expressed by,

$$Q = \frac{P_h \times h}{2} = \frac{w}{2}\,\alpha\,h^2 = 0.016\,\alpha\,h^2 \dots\dots\dots\dots (114)$$

and the moment, in foot-tons, is expressed by,

$$M = Q \times \frac{h}{3} = \frac{w}{6}\,\alpha\, h^3 = 0.0052\,\alpha\, h^3 \text{.............} (115)$$

Let $\alpha = 0.1$, then representative values of shear and moment are indicated in Table 6.

TABLE 6.—VALUES OF SHEAR AND MOMENT FROM EQUATIONS (110) AND (111)

h	P_h, in tons per square foot	Q, in tons	M, in foot-tons
200	0.62	64	41.6 × 10^3
600	1.83	575	1 123 × 10^3
800	2.48	1 022	2 662 × 10^3

The numerical coefficients in Equations (113), (114), and (115), should be compared with Items 9 and 10 in Table 2 of Professor Westergaard's paper.

The comparison of values computed by the approximate method and by equations presented by the author are, as follows:

	Approximate solution	Westergaard solution
$\frac{P_h}{\alpha h}$	0.031	0.0255 — 0.0296
$\frac{Q}{\alpha h^2}$	0.016	0.017 — 0.0198
$\frac{M}{\alpha h^3}$	0.0052	0.0068 — 0.0079

The Surface Surge.—It might be desirable to examine the nature of the phenomenon that occurs at the surface behind the dam. If a water-tight barrier (see Fig. 28), formerly at rest in a rectangular canal, were to be set in motion with a uniform velocity, v, a surge or a wave of translation would be formed with a speed,[33]

$$c = \sqrt{gh}\,\sqrt{1 + \frac{3}{2} \times \frac{z}{h} + \frac{1}{2}\,\frac{z^2}{h^2}} \text{.............} (116)$$

The height of this wave when v is not too large compared with c, will be:

$$z = h\,\frac{v}{c}\left(\frac{1}{1 - \frac{v}{c}}\right) \text{....................} (117)$$

For small values of $\frac{v}{c}$ and $\frac{z}{h}$, it is permissible to assume that $c = \sqrt{gh}$ and $z = h\,\frac{v}{c}$, in which, c is the Lagrange velocity of propagation of waves

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[33] St. Venant, Comptes rendus de l'Academie des Sciences, 1870; see, also, Bakhmeteff, "Hydraulics of Open Channels," 1932, p. 255.

of small height in a deep channel. For various depths, *h*, the values of c are, as follows:

h, in feet.	$c = \sqrt{gh}$, in feet per second.
200	80
600	139
800	160

In other words, for a dam 600 ft. high the surge created by an earthquake on the surface of the reservoir will travel with a speed of approximately 95 miles per hour. The velocity of the dam body itself, $v_o = 0.68$ ft. per sec., in the preceding examples, is insignificant by comparison. For a velocity $v_o = 0.68$ ft. per sec., the height, z, of the surge wave would be as follows (from Equation (117):

h, in feet	$\frac{v_o}{c}$	$z = h\,\frac{v_o}{c}$, in feet.
200	0.85×10^{-2}	1.70
600	0.49×10^{-2}	2.94
800	0.425×10^{-2}	3.40

This indicates the height of the surface wave that would be generated if the acceleration of the dam were slow and the velocity, $v_0 = 0.68$, was attained over a comparatively long period of time. The additional pressures in this case, caused by the rise of the water surface, would be the increase in hydrostatic pressure corresponding to the surge height, z. These additional pressures are quite insignificant when compared with the effect of the compression impulse considered in the foregoing analysis. The conclusion to be drawn is, that even with comparatively slow oscillations, $(T = \frac{4}{3})$, the pressures caused by earthquakes on dams are principally the result of elastic impulses within the water, and that the effect of the actual bodily displacement of the fluid is of secondary importance.

N. MONONOBE,[34] M. AM. SOC. C. E. (by letter).—The writer has read this paper with the greatest interest, and the following comments are made in connection with it.

That the solutions obtained by Professor Westergaard have a singularity at the point, $x = 0$, $y = 0$, may be attributed to his Condition (1) that $\sigma = 0$ at $y = 0$. The writer derives a different system of solutions, in which the singularity in question does not occur, by specifying the following boundary condition:

$$\left[\frac{\partial^2 \phi}{\partial t^2} - g\,\frac{\partial \phi}{\partial y}\right]_{y=o} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (118)$$

in which, ϕ is the velocity potential. This condition, which is well known in hydrodynamics, stands for an incompressible fluid, but may be applied to a compressible fluid if y is small.

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[34] **Director, Experiment Station of Public Works, Home Dept., Japanese Govt.; Prof., Civ. Eng. Dept., Tokyo Univ., Tokyo, Japan.**

Let y_0 denote the elevation of the wave above the undisturbed free surface, and p_0, the pressure intensity on the wall. Then, the solutions are:

$$y_0 = -2\delta \sum_{r=1}^{\infty} \frac{1}{n_r \sqrt{1 - \frac{w p^2 h^2}{\kappa g n_r^2}}} \times \frac{1}{1 - \frac{g}{p^2 h} + \left(\frac{g}{p^2 h} n_r\right)^2} e^{-q_r} \cos pt \quad ..(119a)$$

and,

$$p_0 = \frac{w}{g} p^2 \sum_{r=1}^{\infty} \left(\frac{g n_r}{p^2 h} \cos \frac{n_r}{h} y - \sin \frac{n_r}{h} y\right) \frac{2\delta}{n_r^2} \times \frac{h}{1 - \frac{g}{p^2 h} + \left(\frac{g}{p^2 h} n_r\right)^2}$$

$$\times \frac{1}{\sqrt{1 - \frac{w p^2 h^2}{\kappa g n_r^2}}} - \cos pt \quad \ldots\ldots\ldots\ldots\ldots\ldots (119b)$$

in which, $q_r = \frac{n_r x}{h} \sqrt{1 - \frac{w p^2 h^2}{\kappa g n_r^2}}$; $p = \frac{2\pi}{T}$; δ is the amplitude of the motion; and n_r denotes the roots of the following equation:

$$\frac{g}{p^2 h} n_r \sin n_r + \cos n_r = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (120)$$

Inspection of Equation (119a) will show clearly that the series converges at $x = 0$. With data as supplied by the author, $\frac{g}{p^2 h}$ is approximately equal to 1.64×10^{-2} and $\frac{\rho_0 p^2 h^2}{\kappa}$ is approximately equal to 1.8×10^{-3}. Therefore, it is seen that the answers in Equations (119) are nearly the same as those given by the author when r is small, while there is an appreciable difference when h, the height of the dam, is small. The elevation of the wave is found to be $-$ 0.47 ft.

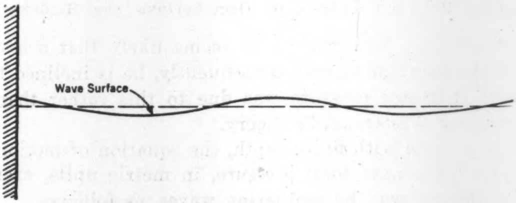

FIG. 29.—WAVES OBSERVED IN TANK.

The writer has investigated the wave motion of water contained in a rectangular tank, 17.9 ft. by 5.97 ft. by 5.97 ft. deep, produced by simple harmonic motions with periods of 0.86 and 1.68 sec. For each period the number of standing waves observed (Fig. 29) was the same as the number of oscillating waves. The period was nearly the same as that of the external force. For the two periods of harmonic motion, 1.68 and 0.86 sec., $1\frac{1}{2}$ waves and $4\frac{1}{2}$ waves, respectively, were observed. According to a theoretical calculation, for $1\frac{1}{2}$ waves the periods of oscillation of the water were found to be 1.59 sec. and 1.53 sec. for depths of 2.9 ft. and 4.5 ft., respectively; and for $4\frac{1}{2}$ waves the period of oscillation was found to be 0.87 sec., nearly independent of the depth.

It is not a simple matter to find a definite relation between the number of oscillations and the heights of waves. Although Fig. 30 suggests that the heights may have a tendency to increase with successive oscillations the reverse tendency was also observed.

It has been stated[35] that experiments by Professor Lydik S. Jacobsen are in fair agreement with the formulas by the author. Because Professor Jacobsen's experimental apparatus was not described in detail the writer hesitates to pass final judgment. Nevertheless, it seems likely that a standing wave was observed in the experiment and, consequently, he is inclined to the belief that the increase in water pressure was due to this rather than to factors involved in Professor Westergaard's theory.

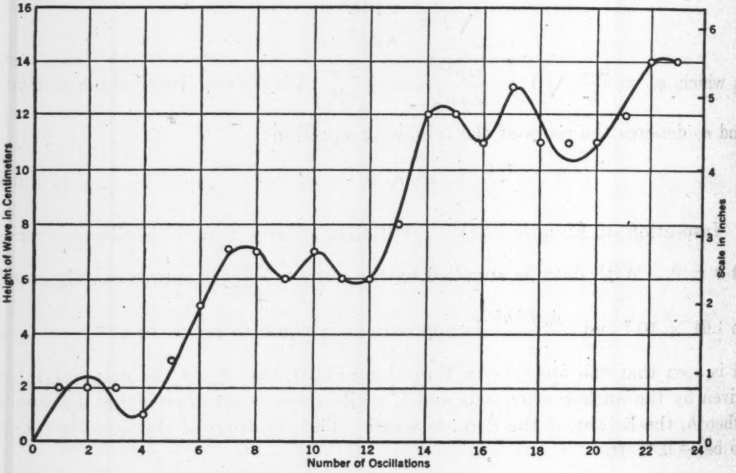

FIG. 30.—RELATION BETWEEN NUMBER OF OSCILLATIONS AND HEIGHTS OF WAVES.

In the case of a basin with finite depth, the equation of motion derived by Flamant gives the maximum total pressure, in metric units, exerted upon a unit width of a vertical wall by oscillating waves, as follows:

$$P = \frac{H^2}{2} + \frac{h_0^2}{2}\left[ln \cosh \frac{\pi H}{L} - \frac{1}{2} \tanh^2 \frac{\pi H}{L} \right] + \frac{h_0 L}{\pi}\left[\frac{\cosh \frac{\pi H}{L} - 1}{\cosh \frac{\pi H}{L} \sinh \frac{\pi H}{L}} \right] \quad ..(121)$$

in which, $2\ L$ is the length and h_0 is the height of the waves. With data obtained experimentally by the writer, $H = 4.53$ ft., $L = 1.97$ ft. and $h_0 = 0.66$ ft., Equation (121) gives $P = 0.3199 + 0.400 = 0.3599$ tons per ft. If the free water surface is assumed to rise only 0.33 ft., the hydrostatic pressure would become 0.3686 tons per ft., an increase of 0 0481 tons per ft. above the original value.

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[35] "Earthquake Forces on Dams" by S. B. Morris, M. Am. Soc. C. E., and C. E. Pearce, Assoc. M. Am. Soc. C. E., *Bulletin*, Seismological Soc. of America, September, 1931.

It does not seem possible to obtain small enough periods of the wave-generating force by the apparatus referred to; moreover, it is evident that the results of experiments in a relatively small tank cannot be applied directly to a very large reservoir without causing considerable error. However, one does observe seiches on a lake during an earthquake and the writer believes that the systems of waves discussed here should be considered in most cases.

H. M. WESTERGAARD,[36] M. AM. SOC. C. E. (by letter).—It is gratifying that the experiments made by Professor Lydik S. Jacobsen (referred to by Mr. Pearce) showed a satisfactory agreement with the analysis. It was desirable that such tests should be made. A full account by Professor Mononobe of his tests and of his theories will be awaited with interest.

The discussions have thrown light on several different phases of the problem. Mr. Rowe, by his diagrams interpreting the theory, has made possible a clear visualization of the action of the whole body of water behind the dam. Mr. Bauman and, especially, Professor Bakhmeteff have connected the problem successfully with practical hydraulics. Messrs. Brahtz and Heilbron and Professor Mononobe have obtained information concerning the effects of certain modifications of the boundary conditions. It is noted that in a majority of important cases the effects of the modifications which they studied will be small.

Particular acknowledgment is expressed to Professor von Kármán on account of his simple approximate solution. It is often more difficult to devise a simple approximate solution than to develop a complete mathematical analysis. Since the results of Professor von Kármán's theory agree so well with those of the more complex analysis, and with Professor Jacobsen's tests, the suggestion is offered that Professor von Kármán's approximate method be applied to the following problem: To determine the influence of the shape of the cross-section of the canyon at the dam when the canyon is fairly narrow.

Let, y = the elevation measured from the bottom of the water, at the dam;

B = the width of the canyon at the elevation, y, at the dam;

$A = \int_0^y B\,dy$ = the area of the cross-section of the canyon between the elevations, 0 and y;

b (as before) = the dimension of the equivalent body of water considered to move with the dam, measured from the dam in the up-stream direction;

h = the maximum depth of the water.

Factors B, A, and b are functions of y only.

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[36] Prof. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.; Cons. Engr., U. S. Bureau of Reclamation.

With the variable width, B, introduced, Professor von Kármán's Equations (55) to (59), inclusive, will be written as follows:

$$A\,a_x = B\,b\,a_y \qquad (122)$$

$$p = \rho\,b\,a_x \text{ (unchanged)} \qquad (123)$$

$$\frac{d}{dy}(\rho\,B\,b) = -\rho\,B\,b\,a_y \qquad (124)$$

$$\frac{d}{dy}(\rho\,B\,b^2\,a_x) = -\rho\,A\,a_x \qquad (125)$$

and,

$$B\,b^2 = \int_y^h A\,dy \qquad (126)$$

The last quantity, Equation (126), may be interpreted, if one wishes, as the bending moment at the elevation, y, in a vertical simple beam with supports at $y = \pm h$ and with a symmetrical load equal to B per unit of length at any elevation, $\pm y$.

Equation (126) gives,

$$B\,b = \sqrt{B\int_y^h A\,dy} \qquad (127)$$

from which, in any given case, the total volume of water which is considered to move with the dam may be computed as,

$$V = \int_0^h B\,b\,dy \qquad (128)$$

As an example, assume,

$$B = \frac{h}{2} + y \qquad (129)$$

This equation applies roughly to the canyon at the Hoover Dam. By Equation (126) one finds, then,

$$B\,b^2 = \frac{1}{12}\,(5\,h^3 - 3\,h\,y^2 - 2\,y^3) \qquad (130)$$

which gives, at the bottom, $b = h\sqrt{\frac{5}{6}} = 0.913h$. Professor von Kármán found $b = 0.707h$ corresponding to a constant, B. Further computations by Equations (127) to (129) give $V = 0.496h^2$, while a constant width equal to the average width, $B = h$, according to Professor von Kármán's Equation (65), gives $V = 0.555h^2$. Thus, with the variable width, the pressure at the bottom, $a\,w\,b$, is greater, but the total horizontal force, $a\,w\,V$, is less than the corresponding values in case of a constant width, $B = h$. By assuming values of b according to Equation (53), $b = \frac{7}{8}\sqrt{h\,y}$, with y denoting the depth of any point and h denoting the maximum depth of the water, one finds in the example of variable width, $V = 0.525h^2$, while with $B = h$, one finds $V = 0.583h^2$. It is concluded that, in the case of a narrow canyon, Equation (53), used as just indicated, leads to a value of V slightly on the side of safety.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1836

WESTERN HIGHWAY PRACTICE, WITH SPECIAL REFERENCE TO CALIFORNIA[1]

By C. S. POPE,[2] M. Am. Soc. C. E.

With Discussion by Messrs. J. S. Bright, R. D. Rader, H. S. Kerr, E. Q. Sullivan, T. A. Corry, Charles T. Leeds, G. S. Paxson, and C. S. Pope

Synopsis

The field of Western highway construction is so widespread and each locality offers so many varied problems that it is only possible to cover a small part of it. The writer, therefore, does not propose to cumber the already voluminous literature of highway engineering with repetitive descriptions of problems or the methods used in the solution of problems already common to highway engineers from their own experience or reading. The paper contains an outline of the methods used in solving certain construction problems of a nature unusual to Eastern terrain and to indicate some of the controls which have been applied in the Western States to those forces of Nature which, unless properly curbed, are destructive to the works of Man.

While the general theme of this paper is devoted to construction designed to insure protection against stream erosion and floods, cloudbursts, and the destructive action of the sea, the writer has also presumed to touch briefly upon the subjects of highway financing and of clearing right of way previous to construction.

Finance

Of all problems facing the Western States, that of finance is probably the most serious. The large areas of public lands, the scant population in many locations, the long stretches of country on which no local need of business or political weight urges the expenditure of funds, and the expensive grading and bridge construction which are often necessary for even the most elemental road construction, place upon Western highway engineers a serious construction problem. Competition in high ideals in road design and construction has necessitated an orderly procedure in the use of such funds as may become available.

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[1] Presented at the meeting of the Highway Division, Sacramento, Calif., April 24, 1930.

[2] Chf. Constr. Engr., California Div. of Highways, Sacramento, Calif.

Western highway engineers now realize that no highway system will ever be entirely completed, but that it will be added to, rebuilt, changed, and improved to the advancing needs of traffic farther into the future than any man can predict. Through the gasoline tax a source of steady income, automatically adjusting itself to the finances required, has been set up, thus enabling construction programs projected from 10 to 50 years into the future to be outlined with a moderate assurance that they will be carried forward. Whether a gasoline tax imposing a moderate tariff on motor vehicle owners will furnish sufficient funds to meet the demands of highway users, without the assistance of bond issues, is a question which time will answer.

The application of the budget plan of expenditure, however, and the assertion of the principle of preferential road construction founded on a study of the economic and engineering justification of each road under consideration has done much to clarify the allotments of funds in a rational manner.

In 1930, California began planning its road expenditures on the basis of the needs of the succeeding ten years, in actual definite projects, and the State authorities have been looking ahead very much farther than that in its prospective highway plans.

Clearing the Right of Way

Present-day highway engineering literature devotes itself quite completely to problems such as soil studies, drainage, paving design, and similar subjects relating possibly more to the highway already graded and constructed under average conditions than to the highway in process of construction. Many Western highways present additional interesting construction problems which have possibly not been given so much attention. Among the first of these is the clearing of the right of way previous to construction.

The problem of clearing brush and trees of medium size and thickness of growth is not unusual in many States. Such conditions as those encountered in the Northern coastal highway construction are, however, somewhat unique. Not only does dense undergrowth occur, but trees of very large size, both standing and prone, which must be felled or cleared away. Normal clearing costs where moderate sized trees and scattered underbrush are to be removed, may be stated as about $250 per acre. In the heavily timbered country of the Northern California coast, the clearing may be as high as $1 200 to $3 000 per acre.

Clearing may include grubbing and the removal of all stumps in the case of trees prone to rotting, but in many cases redwood stumps are not removed because of their known long life. Where stumps are removed, the allowance which must be made in excavation computations for filling stump-holes is often a matter of considerable moment.

Trees of moderate size are pulled over with tractors, but in handling trees up to 18 ft. in diameter, other methods are used. The organization for such

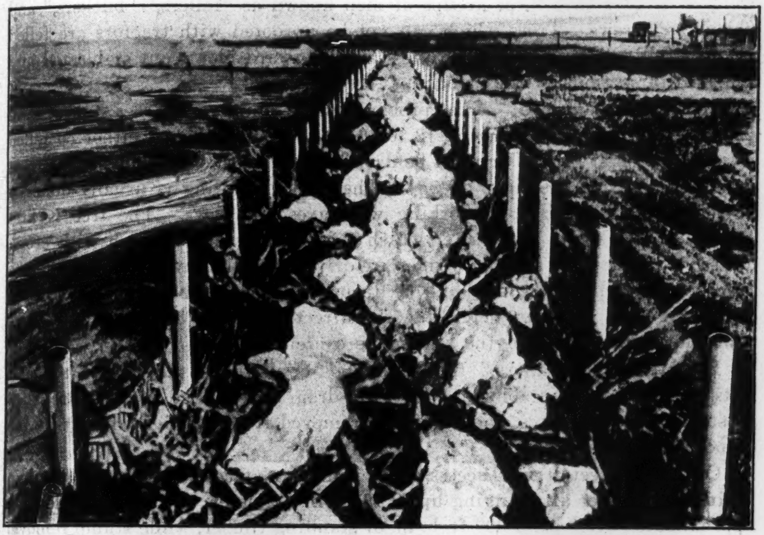

FIG. 1.—PIPE, WIRE MESH, AND BRUSH FENCING FOR BANK PROTECTION

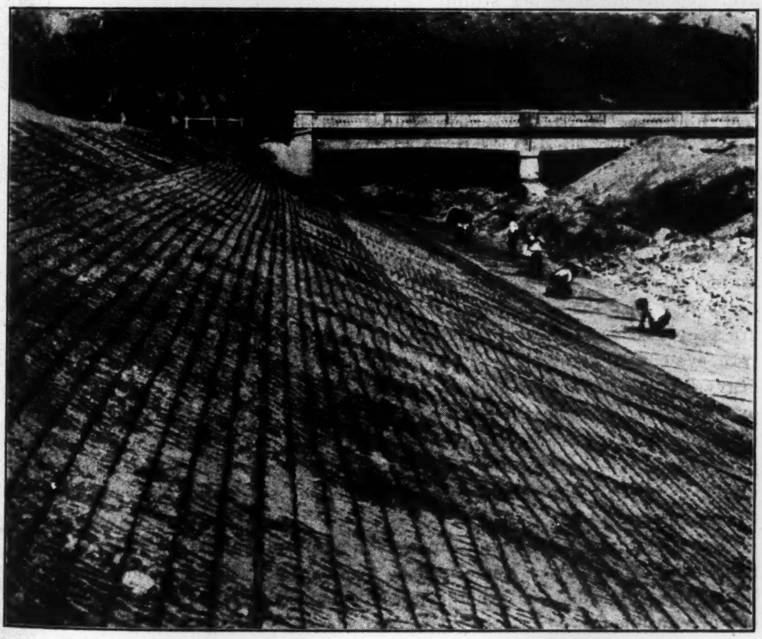

FIG. 2.—WIRE MESH MATS FILLED WITH GRAVEL FOR BANK PROTECTION.

work is divided into three groups of men known as "fallers," "buckers," and "blocking" crews. Trees which can not be uprooted with tractors are felled by the "falling" crew and the stumps are afterward blown out or blasted and removed in sections by the "blocking" crew.

Falling is somewhat of a problem when numerous trees more than 200 ft. in height and averaging 12 to 16 ft. in diameter must either be felled in a 100-ft. right of way or must be dropped so that they will not injure trees which are outside the right of way. The work of falling is usually done by two "fallers" to each tree, working with axes and saws. The tree is first heavily notched with axes, after which the trunk is cut through with crosscut saws. During the sawing, wedges are driven into the saw-cut to prevent the tree from pinching the saw and to insure falling in the proper direction. The final cuts, however, determine the direction of the fall, and it is here that the ultimate skill of the "faller" is exercised.

Following the "fallers," the "bucking" crew trims the felled tree and cuts it into merchantable lengths with drag saws. The stumps are then blasted and logs, stumps, and débris are removed from the right of way by the "blocking" crews.

On one project it was necessary to remove about 1 800 trees larger than 12 in. in diameter and ranging up to 18 ft. in diameter. The cost of clearing approximated $7.00 per 1 000 ft. b. m. of standing timber, while stump removal averaged about $6.50 per stump.

PROTECTION FROM ELEMENTS

Some of the more unusual elements against which it is necessary to protect highways in the Western States are floods, cloudbursts, sea waves, moving sand dunes, and earthquakes.

Floods may be either those caused by the rise of large rivers, or by the sudden flow of water, in otherwise normally dry stream channels, due to cloudbursts.

Structures to furnish protection against cloudbursts consist either of a system of "dips" or depressions in pavements, which permit a temporary overflow across the pavement. By means of these structures the pavements are protected from scouring by means of overflow aprons and cut-off walls; or a system of channels and deflecting dikes which pick up the broad but heavy flow occasioned by cloudbursts and conduct it through control channels over which trestles have been constructed.

Protective structures for sea waves take the form of booms anchored to piles, heavy random rip-rap, hand-placed rip-rap, sheet-piling of wood or concrete, or other bulkheads, groynes of wood or steel, jetties, wave breakers, slope paving, and sea walls.

Protection against moving dunes is generally obtained by a study of their habits and movements. Some may be avoided by a change of location, others by a raise of grade, and others may be removed by wind action or with the use of equipment.

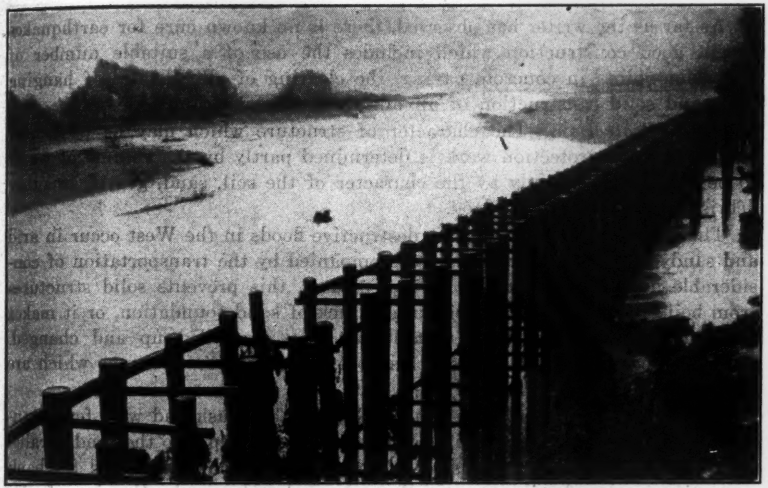

Fig. 3.—Piles and Barbed Wire for Bank Protection on Hard-Bottom Rivers in Timbered Country.

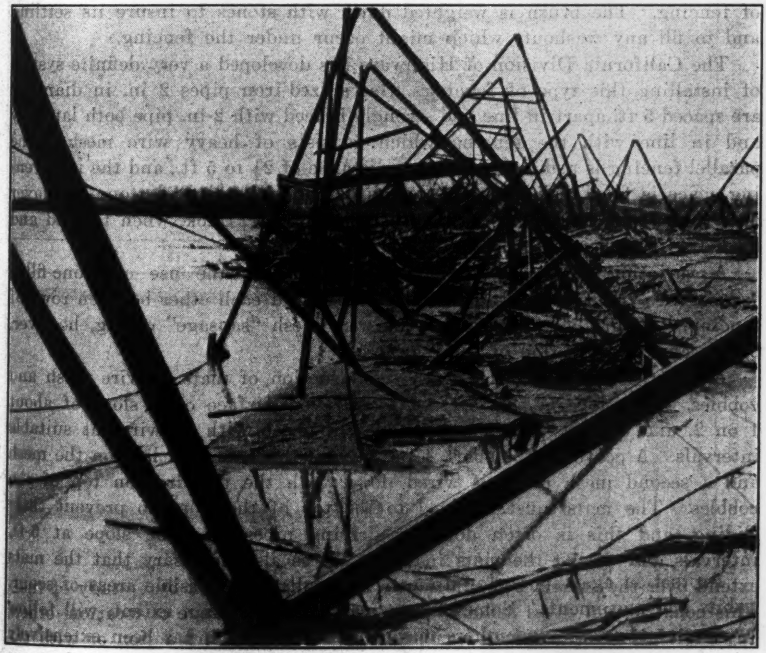

Fig. 4.—Tetrahedrons Fabricated from Steel Rails for Shore Protection.

As far as the writer has observed, there is no known cure for earthquakes except good construction which includes the use of a suitable number of expansion joints in concrete paving, the clearing of all hillsides of hanging rock, and solid construction of all fills in earthquake country.

Flood Protection.—The character of structure which may be used with success in flood-protection work is determined partly by the volume of water to be handled and partly by the character of the soil, sand, gravel, or other débris transported by the flood.

The more common of the most destructive floods in the West occur in arid and sandy regions and are usually accompanied by the transportation of considerable quantities of sand. In many cases, this prevents solid structures from being used successfully because of lack of solid foundation, or it makes them useless for the reason that stream beds are filled up and changed. There has been considerable use, therefore, of structures of a type which are flexible or which may be easily replaced.

Brush and Wire Fencing.--Among these is the brush and wire fence constructed of two rows of galvanized-iron pipe driven deep into the sand parallel with the stream. This type is shown in Fig. 1, which is designed for bank protection in slow-flowing streams through sand and gravel. Fencing of wire mesh is fixed to the pipe to retain the brush which is placed between the rows of fencing. The brush is weighted down with stones to insure its settling and to fill any washouts which might occur under the fencing.

The California Division of Highways has developed a very definite system of installing this type of fencing. Galvanized-iron pipes 2 in. in diameter are spaced 5 ft. apart in line and strongly braced with 2-in. pipe both laterally and in line with the fencing which consists of heavy wire mesh. The parallel fencing is usually placed at a distance of 2½ to 5 ft., and the intervening space is filled with alternate layers of willow brush and stone. A layer of brush, 2½ ft. thick, will compact to a layer, 1 ft. thick, when tamped and loaded with sufficient stone to hold it down.

A variation of the willow and stone filler is the use of stone-filled "sausages" of wire mesh, which are laid on top of each other between rows of fencing. Naturally any break in the wire mesh "sausage" casing, however, brings about destruction of the "sausage."

Another common practice is the construction of mats of wire mesh and cobbles. The slope is first dressed to an even surface on a slope of about 1 on 2, and on it is laid a section of wire mesh with tie-wires at suitable intervals. A coating of cobbles, 4 in. or 8 in. thick, is then laid on the mesh and a second mesh panel is wired down with the tie-wires on top of the cobbles. The mats must be fixed to the top of the slope to prevent their sliding, and this is often done by driving pipes into the slope at 6-ft. intervals and wiring the mats in place. It is also necessary that the mats extend over the stream bed sufficiently to blanket any possible areas of scour. This construction gives a flexible mat which, to a certain extent, will follow any undulations which may occur due to washouts. It has been extensively used in storm-protection work in Los Angeles County, California, but not

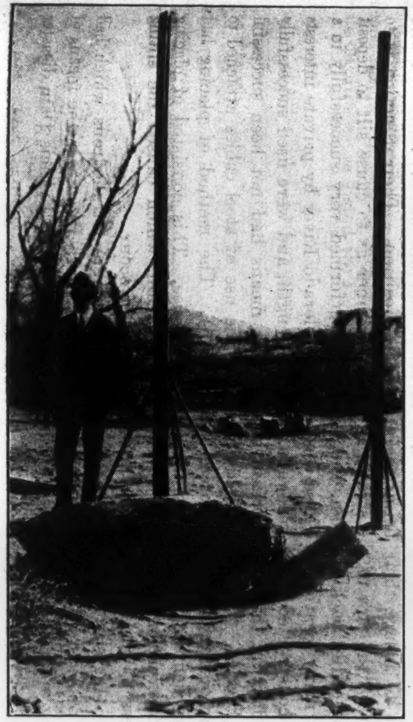

FIG. 5.—CONCRETE MUSHROOM ANCHORS TO HOLD RAIL TETRAHEDRONS IN PLACE.

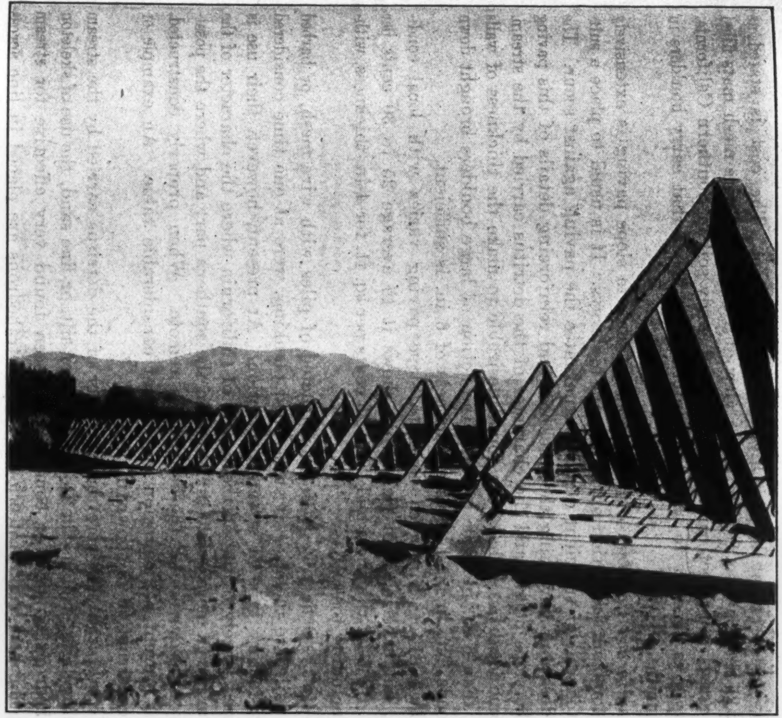

FIG. 6.—REINFORCED CONCRETE TETRAHEDRONS FOR BANK PROTECTION IN A STREAM WITH A WINDING CHANNEL.

to any great extent by the highway organization. The cost is stated as about 25 cents per sq. ft. Fig. 2 is a view of a section of wire-mesh mats filled with gravel to protect banks of gravel and sandy soil in Southern California. The type is effective to withstand intermittent floods that carry boulders in their paths.

Reinforced Concrete Slope Paving.—Concrete slope paving is extensively used for bank protection on California highways. It is usual to place a suitable toe-wall of sufficient depth to guarantee the paving against scour. The considerations governing the thickness and reinforcing details of this paving are the slope height and the character of the detritus carried by the stream. In some localities it has been found desirable to make the thickness of walls more than 6 in. in order to resist the action of large boulders brought down by floods, but in most cases a thickness of 6 in. is sufficient.

The cost of reinforced concrete slope paving varies with local conditions, but experience in California shows it to average 25 to 30 cents per sq. ft. for 6-in. thickness and 20 to 25 cents per sq. ft. for 4-in. thickness, without including toe-walls.

Jetties.—Deflecting jetties constructed of piles with wire mesh, or barbed wire nailed to them, or of stone-filled planking, were at one time considered a satisfactory type of stream protection. At present, however, their use is confined largely to the northern part of California where the character of the stream bed is more stable than that in the southern part and where the possibility of scour under the fencing is remote. When properly constructed, jetties prevent bank erosion and are of considerable value. An example of this construction is shown in Fig. 3.

Tetrahedrons.—Where the character of the detritus carried by the stream is rather light, such as the Colorado River silt, or fine sand, the use of skeleton tetrahedrons of steel or concrete has been found very effective for stream deflection and bank protection. These tetrahedrons are placed in line across a point that is giving trouble due to scour and are usually anchored with cables to insure their being maintained in position. Their function is to interrupt the flow of the stream in such a manner as to cause silt to deposit in the eddies back of them, and they have performed very successfully in a great many locations. Those used on the Colorado River by private interests were constructed from steel rails, 30 ft. in length, and were used successfully in deflecting this large stream where other means had not been successful (see Fig. 4). They were held in line by the use of steel cables attached to mushroom anchors of concrete (see Fig. 5). The method of placing had a very definite influence on their successful use. This consisted of placing the obstructions one after another down stream, which caused the silting process to begin at once and prevented further scour.

On highway work, a row of reinforced concrete tetrahedrons which had been placed to deflect the stream on the Santa Clara River was the means of saving a considerable section of highway during the St. Francis Dam disaster (see Fig. 6).

Stone-Filled Steel Wire Baskets.—In one location, a mountain river in heavily timbered country, metal baskets, 7 ft. square, and of varying depths, filled with stone, were installed under plans made by the U. S. Bureau of Public Roads. A typical plan and elevation are given in Fig. 7 and approximate quantities are listed in Table 1.

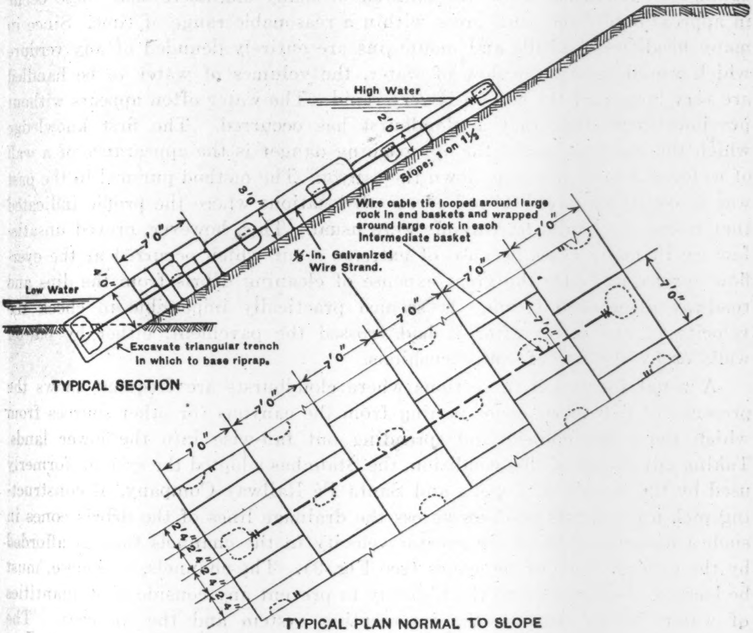
High Water
2'-0"
Slope; 1 on 1½
3'-0"
7'0"
4'-0"
Low Water
Wire cable tie looped around large rock in end baskets and wrapped around large rock in each intermediate basket
⅝-in. Galvanized Wire Strand.
Excavate triangular trench in which to base riprap.
TYPICAL SECTION
7'0"
7'0"
7'0"
7'0"
7'0"
7'0"
7'0"
7'0"
7'0"
2'4"
2'4"
2'4"
2'4"
TYPICAL PLAN NORMAL TO SLOPE

FIG. 7.—BASKET RIP-RAP AND WIRE CABLE TIES.

TABLE 1.—APPROXIMATE QUANTITIES IN RIP-RAP BASKETS

Depth of baskets, in feet	4	3	2
Rip-rap capacity, in cubic yards	7.26	5.45	3.63
Weight of wire mesh, in pounds	150	138	126
Weight of wire ties, in pounds	3.6	3.0	2.4

Slope paving of hand-placed stone with appropriate toe-walls has been extensively constructed in a number of suitable locations. On many streams the use of random rip-rap of considerable weight has been advantageous as a method of protection. Fig. 8 shows two types of slope protection suited to canyons that are subject to boulder-carrying floods.

CLOUDBURSTS

One of the most unusual problems in California is the protection required against cloudbursts. It is the opinion of many engineers that these occur in approximately the same areas within a reasonable range of time. Since in many localities the hills and mountains are entirely denuded of any verdure which would retard the flow of water, the volumes of water to be handled are very large and the run-off is very rapid. The water often appears without previous intimation that a cloudburst has occurred. The first knowledge which the observer has of the approaching danger is the appearance of a wall of water and mud sweeping down the canyon. The method pursued in the past was to construct paved "dips" across all locations where the profile indicated that storm or cloudburst run-offs were usual. This, however, proved unsatisfactory in many cases because of extreme scour which occurred at the overflow aprons, and also the great expense of cleaning débris from the dips and roadway after each storm. It seemed practically impossible to check the velocity of the water after it had crossed the pavement, either by cut-off walls or by the use of water cushions.

A usual formation in sections where cloudbursts are frequent shows the presence of flat débris cones issuing from the canyons (or other sources from which the water comes) and spreading out fan-wise into the lower lands. Taking advantage of this condition, the State has adopted the system, formerly used by the Atchison, Topeka and Santa Fé Railway Company, of constructing pick-up channels or dikes across the drainage lines of the débris cones in such a manner as to secure greater velocity in the channels than is afforded by the general slope of the cones (see Fig. 9). The channels, of course, must be located close enough to the highway to prevent any considerable quantities of water falling between the intercepting system and the highway. The results thus far obtained have been extremely satisfactory in restraining all the water from crossing the highways, except at certain designated points, known as controls, where wooden trestles are constructed along the highway for the passage of streams. The system of drainage thus developed has given rise to a nomenclature for its various parts. The source of the water is usually designated as the "cone," the main drainage channels as "ditches," feeder channels as "sub-ditches," and the outlets at which water is concentrated and carried across the highway as "controls."

In Fig. 10 the highway location is parallel to a desert valley. The main drainage is through a series of dry lakes or sinks. Tributary drainage is shown emerging from the mountains to be intercepted by one or two main channels that break up into interlacing overflow channels. The storm-protection ditches are located so as to obtain the maximum fall. Associated with the channels is often a system of dikes for directing the flow. In other places where large open valleys are encountered in desert locations, and storm flow occurs as broad sheets of water, it has been necessary to construct extensive dike systems and to conduct the water under the highway by means of wooden bridges or trestle structures, as before stated.

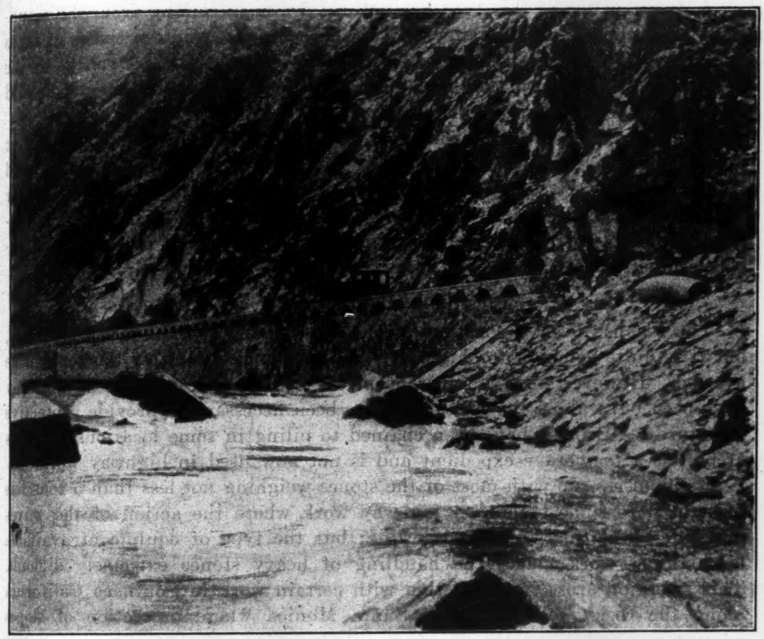

FIG. 8.—RUBBLE WALL AND HAND-LAID DRY SLOPE PAVING.

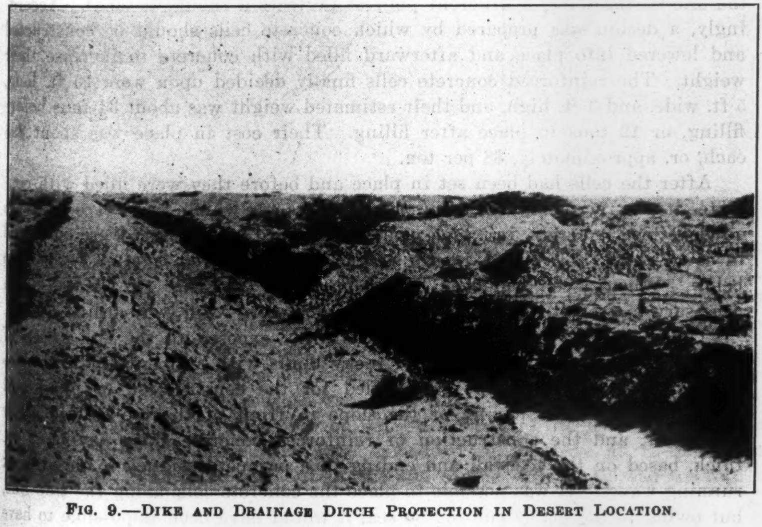

FIG. 9.—DIKE AND DRAINAGE DITCH PROTECTION IN DESERT LOCATION.

A desert valley of this type has natural drainage coming from the mountains on each side of the valley and discharges usually into a series of lakes and sinks. These sinks or dry lakes are formed in most desert valleys by the débris deposits from major side-tributary drainage which has cut off and backed up the main valley drainage.

The cost of ditching for the type of work done on débris cones is about \$1 500 per mile. The cost of the wider ditches with dikes, such as are used in the wide desert valleys, is about \$2 500 per mile. The cost of timber bridges and trestles of 20 to 24-ft. creosoted piles and of redwood superstructure of 24-ft. roadway, varies from \$100 to \$125 per ft. of length. Highway protection by this system costs from \$3 000 to \$3 800 per mile.

Sea Protection

The sea has been a constant menace to construction in many localities, but it is quite common for highways to be located where this danger can be averted. However, in a number of places considerable lengths of highway have been built along the sea, and it has been necessary to provide protective structures. "Booms" have been chained to piling in some locations, but this at best is a temporary expedient and is not now used in highway work.

Random rip-rap with most of the stones weighing not less than 5 tons has been used to some extent on highway work where the action of the waves and tides is normal to the coast line; but the type of equipment available on highway work makes the handling of heavy stones extremely difficult. This situation arose in connection with certain work in Southern California along the rocky coast north of Santa Monica where protection of slopes against sea action became necessary. It was decided that, since large rock suitable for both rip-rap and blanketing the slopes could not be handled at the site of the work, a different plan of protection should be used. Accordingly, a design was prepared by which concrete cells should be constructed and lowered into place and afterward filled with concrete to increase their weight. The reinforced concrete cells finally decided upon were 10 ft. long, 5 ft. wide, and 3 ft. high, and their estimated weight was about 3½ tons before filling, or 12 tons in place after filling. Their cost in place was about \$95 each, or, approximately, \$8 per ton.

After the cells had been set in place and before they were filled with concrete, it was necessary to grout the boulders on which they were set with quick-setting concrete, in which cement and hard wall plaster were combined in equal parts, in order to insure sealing of the concrete against wash from below. When a cell had been filled completely (see Fig. 11), it was again necessary to protect the fresh concrete against wave wash by boarding the top.

The shore line at this location is entirely exposed to the full sweep of the Pacific Ocean and consisted of rocky bluffs with a steep slope into deep water. The design in this case called for the construction of a toe-wall below high tide, the paving of the slope in front of the toe-wall to below low water, and the construction of reinforced concrete slope paving, 9 in. thick, based on the toe-wall and ending in a parapet designed to throw back running waves. It was possible to place the concrete cells along the tide line, but owing to the heavy run of the sea, it would have been impossible to have built the toe-wall or the slope walls without some protection. Accordingly, such leveling and removal of boulders as was possible was done at low tide and the concrete cells were then sunk into line on the roughly prepared foundation and were filled with concrete and heavy stone between tides.

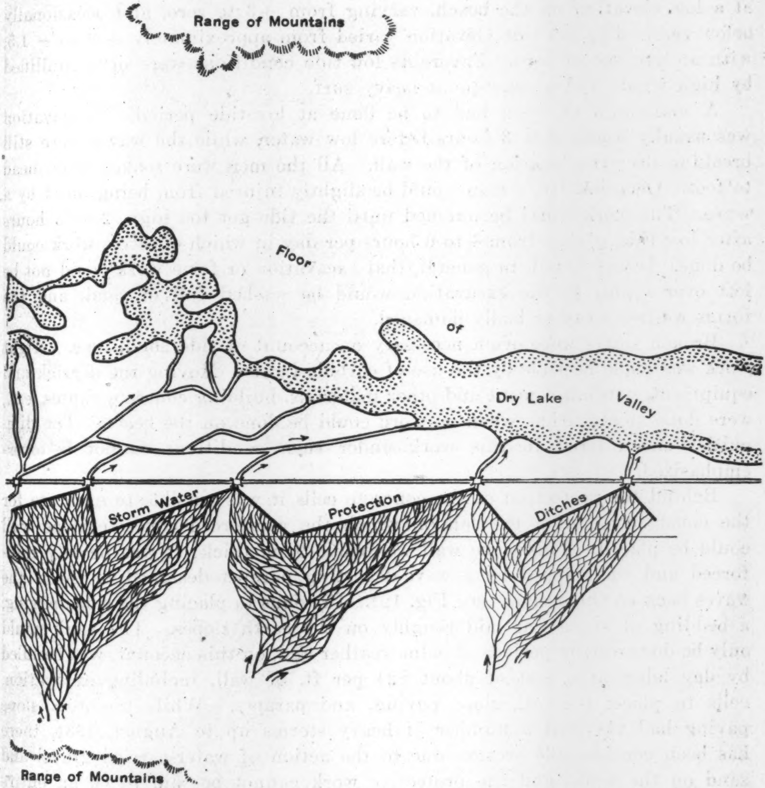

FIG. 10.—TYPICAL HIGHWAY LOCATION, WITH STORM-PROTECTION DITCHES.

When the cells had been poured and the beach boulders grouted, enough protection was afforded so that sufficient time was allowed in spite of the rising tide, to pour the base of the toe-wall to a height of 1½ to 2 ft., leaving from 1½ to 2½ ft. to construct on top of the base, the cells acting as a form on the outside. After the section of base was constructed, the top section was poured into the forms built on the base poured previously. These forms were built while the other work was proceeding. The toe-wall was poured with Class C concrete, 4.2 sacks per cu. yd. Steel bars, ½ in. by 4 ft., were used in the toe-wall. These bars were embedded in the base at 12-in. centers extending up through and out of the top of the wall to tie the toe-wall to the slope paving.

To give a better understanding of the aforementioned procedure, it is necessary to call attention to the great difficulty created by the tides. This work had to be carried on during the winter months, the most unfavorable time of the year, in order not to delay the slope paving. All the work was at a low elevation on the beach, varying from +3 to zero, and occasionally below zero. The low-tide elevation varied from approximately + 2 to − 1.5, with an average of zero. Favorable low-tide conditions were often nullified by high winds and a consequent heavy surf.

A maximum of work had to be done at low-tide periods. Excavation was usually begun 2 to 3 hours before low water, while the waves were still breaking over the location of the wall. All the men were soaked from head to foot. Occasionally, a man would be slightly injured from being upset by a wave. The work could be pursued until the tide got too high, 2 to 3 hours after low tide, giving from 4 to 6 hours per day in which effective work could be done. It was found, in general, that excavation or form work could not be left over night, as the excavation would be washed full of sand, and the forms washed away or badly damaged.

Broken shifts were often necessary on account of tide conditions. Night work was made possible by the use of carbide lights. Moving the derrick and equipment, hauling cement and other materials, building concrete chutes, etc., were done at high tide, when no work could be done on the beach. The difficulties encountered in this work under such conditions cannot be overemphasized.

Behind the protection of the concrete cells it was possible to excavate for the construction of a toe-wall on which the reinforced concrete slope wall could be placed. This slope wall was made 9 in. thick; it was heavily reinforced and was capped by a wave-deflecting parapet designed to throw the waves back on themselves (see Fig. 12). Previous to placing the slope paving, a bedding of stone was laid roughly on the earth slopes. The work could only be done during periods of calm weather and, on this account, was handled by day labor at a cost of about $24 per ft. of wall, including excavation, cells in place, toe-wall, slope paving, and parapet. While the 9-in. slope paving had survived a number of heavy storms up to August, 1931, there has been considerable erosion due to the action of water-carried gravel and sand on the slabs, and the protective work cannot be said to be an entire success.

The use of sheet-piling of either wood, steel, or concrete, on State work in California has never met with much success, and it has not been used extensively in sea protection.

In locations where the run of the tide is parallel to the shore line and where sandy beaches alternately form and depart during storms, groynes have been used extensively along the south coast and in many locations the beach has been restored by their use. Fig. 13 shows a beach immediately after groynes were constructed and Fig. 14 shows their effect some time afterward. The State construction usually contemplates the use of wooden structures, but some of the private beach owners have used steel sheet-piling with considerable success. The usual State construction contemplates the

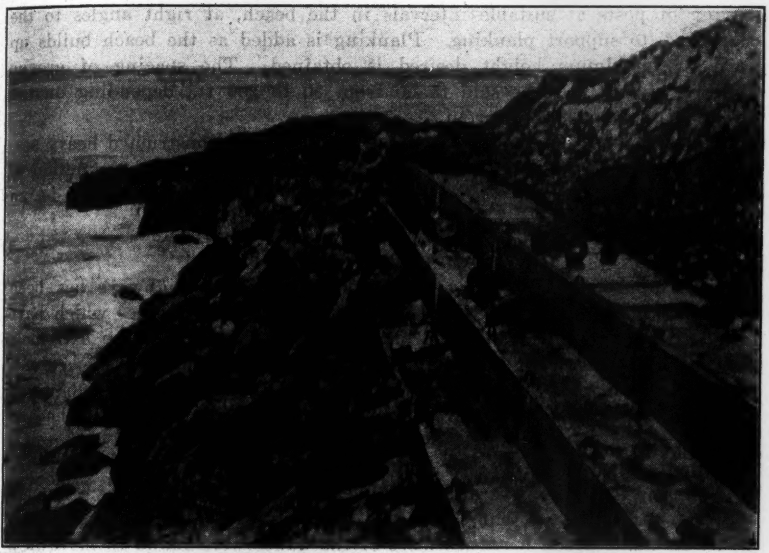

FIG. 11.—REINFORCED CONCRETE CELLS FILLED WITH CONCRETE FOR SHORE PROTECTION.

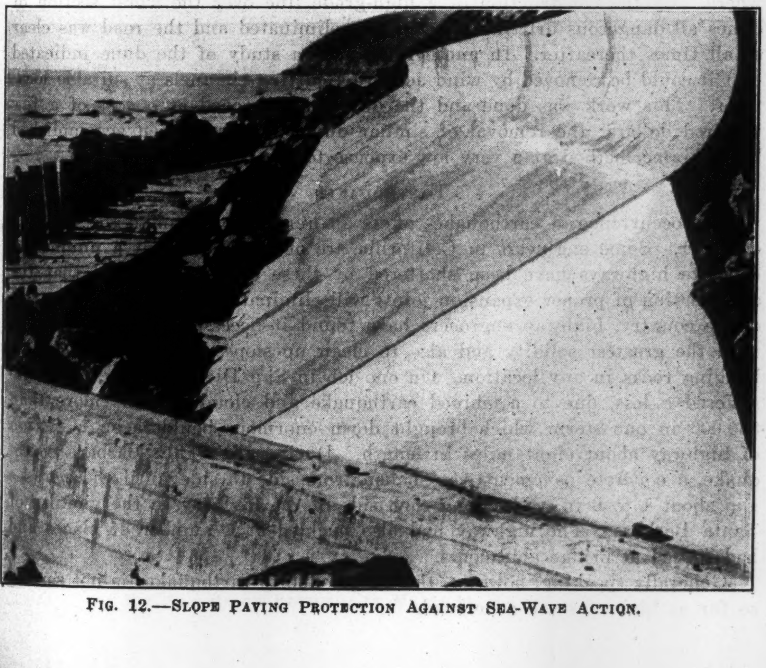

FIG. 12.—SLOPE PAVING PROTECTION AGAINST SEA-WAVE ACTION.

driving of posts at suitable intervals in the beach, at right angles to the coast line, to support planking. Planking is added as the beach builds up until the maximum height desired is obtained. The spacing of groyne lines varies, but may be said to be from 50 to 200 ft., depending on the direction of the coastal current.

Sea-Walls.—In one or two locations, the State has constructed heavy sea-walls of monolithic concrete of sufficient strength to withstand the action of the heaviest waves. These have been built with the usual throw-back design of top which has proved effective wherever tried.

SAND DUNES

In a number of places the California Division of Highways has been faced with the problem of control or elimination of sand dunes which have formed not only a serious burden on maintenance, but have been extremely dangerous to traffic. It is quite usual for a sand storm to place a dune, within a few hours, across the highway, several feet in depth, which would require constant labor to remove. On one particular dune the State had an annual expense of about $7 000 for keeping the road clear over a section of a few hundred feet.

A study of the habits of dunes and their movements, made over a number of years, demonstrated that they could be conquered if proper means were used. In many given localities most of the dunes were found to move in a certain direction and then attain a height which was seldom exceeded elsewhere. By the construction of a high-grade line over the worst section of dunes all dangerous drifting was entirely eliminated and the road was clear at all times thereafter. In another location, a study of the dune indicated that it could be removed by wind action by cutting channels at suitable locations. This work was done and the dune was removed at a cost of a few hundred dollars; the removal of similar dunes has since been accomplished by the same method at a very low expense to the State.

EARTHQUAKES

The occurrence of earthquakes offers an unusual problem to the highway engineer. Some engineers in California are of the belief that a number of concrete highways have been shattered by these disturbances and that the construction of proper expansion joints will eliminate this danger. In earthquake country, highway engineers have found it necessary to construct fills with the greatest solidity and also to clean up slopes so that there are no hanging rocks in any location. On one job in San Diego County, the State suffered a loss, due to combined earthquake and cloudburst, of more than $30 000 in one storm which brought down enormous boulders on a section of highway about eight miles in length. During the Santa Barbara earthquake, a concrete pavement was shaken from side to side until there was a gap about 4 to 6 in. wide along each side of the highway in the vicinity of Santa Barbara. The highway had also buckled at a number of places and had settled at bridge abutments.

Generally speaking, however, the danger due to earthquakes is not serious so far as highways are concerned.

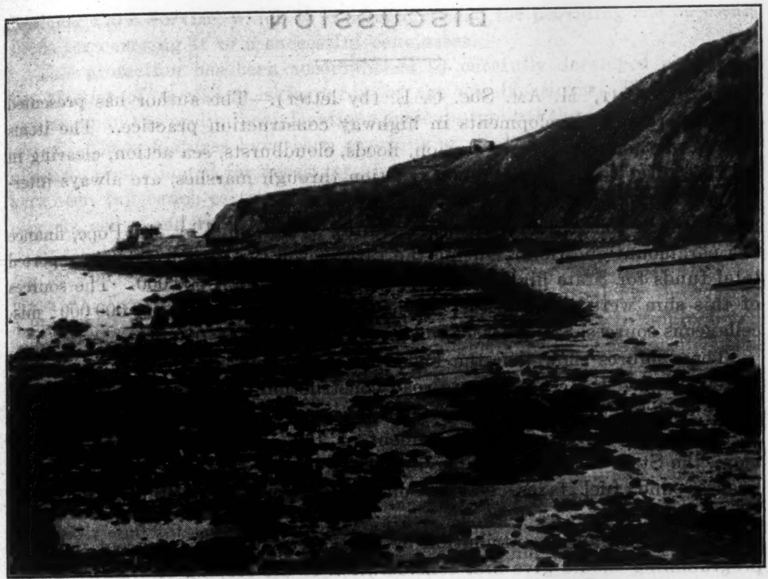

FIG. 13.—WOODEN GROYNES PLACED TO RESTORE A BEACH.

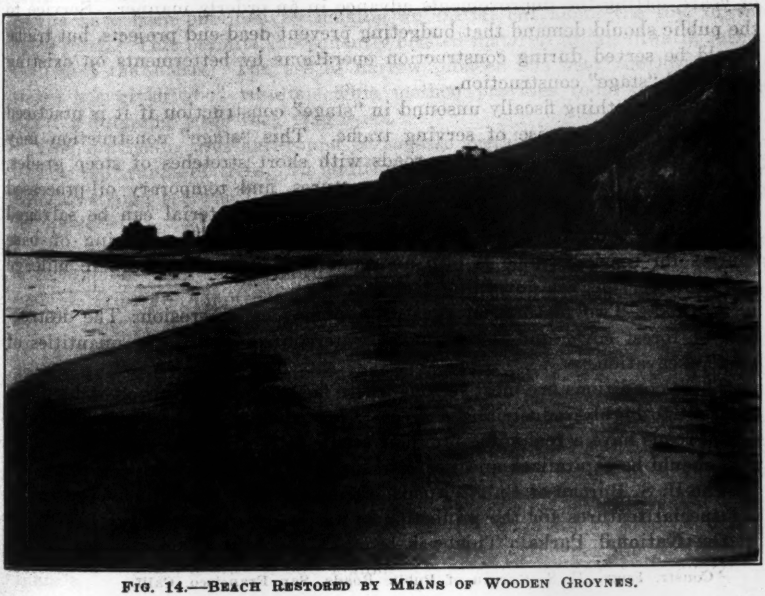

FIG. 14.—BEACH RESTORED BY MEANS OF WOODEN GROYNES.

DISCUSSION

J. S. BRIGHT,[3] M. AM. SOC. C. E. (by letter).—The author has presented some interesting developments in highway construction practice.. The items of highway finance, stream erosion, floods, cloudbursts, sea action, clearing in advance of construction, and construction through marshes, are always interesting due to their general application.

Finance.—In the eleven Western States, as indicated by Mr. Pope, finance is a paramount question. During the 1928 season the Western States reported total funds for State highway purposes aggregating $91 500 000. The sources of this sum were: Motor fuel tax, $40 000 000; plate tax, $18 000 000; miscellaneous sources, $18 500 000; and Federal aid, $15 000 000.

These figures show that the Federal Government is contributing approximately 16% of the cost of State road construction in this area; or, taking it in the reverse order, the State systems are being united into a Federal system at a cost to the National Government of 16% of the programs. Furthermore, the State road users are paying about 76% of the State's share, through licenses and fuel taxes, leaving only 24% for other State interests to contribute.

With such a large contribution by the road users, it is apparent that road programming should give first consideration to the serving of traffic. A well-planned budget is paramount for good administration. It should cover a series of years, letting the improvements advance in an orderly manner. Service to the public should demand that budgeting prevent dead-end projects, but traffic should be served during construction operations by betterments on existing roads and "stage" construction.

There is nothing fiscally unsound in "stage" construction if it is practiced with the major purpose of serving traffic. This "stage" construction may involve narrow surfaces, pioneer roads with short stretches of steep grades, concrete fords in lieu of expensive structures, and temporary oil-processed surfaces on old county roads. Such oil-processed material can be salvaged readily and transported to the final location. The oil-processing of base courses for serving traffic is also an economical expenditure where mileage and short seasons are governing factors.

Erosion.—Two features of construction may cause erosion: The destruction of forest cover and the filling of watercourses with large quantities of waste excavation.

These conditions are most serious where flashy storms occur on water-sheds adjacent to highly industrialized areas. Nevertheless, the borders of all California roads have a real value for their scenic attractiveness. This attractiveness should be capitalized and not disregarded in construction operations.

The U. S. Bureau of Public Roads, during the 1931 season, has been adopting special features for the protection of forest cover on highway operations in the National Parks. This was brought about at the insistence of the

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[3] Constr. Engr., U. S. Bureau of Public Roads, San Francisco, Calif.

National Park Service, which is to be commended for providing the necessary funds for carrying it to a successful conclusion.

This protection has been accomplished by carefully developed plans, and the placing of necessary safeguards in the specifications. The landscape engineers, on the preliminary locations, have designated the features to be preserved. These features have been carefully studied during the location, and measures then adopted for necessary protection. The best protection measures have been full bench sections, balancing fills, tunnels, hand-laid embankment, concrete cribs, and retaining walls.

Specifications for construction in the National Parks have provided clearing by stages, leaving a screen of timber on the lower side until after the heavy shooting has been finished. Prior approval is required for the plan of drilling and loading. It is contemplated that the engineer will approve this plan on its successful demonstration over a small area. The specifications give the engineer, as a last resort, authority to limit the spacing of holes to 2-ft. centers, and a minimum charge of ½ lb. of 40% dynamite per cu. yd. of material. Mats may also be required as a last resort. Penalties consist of gathering the scattered material, the removal and trimming of injured trees, or, in extreme cases, closing down the work. Six contracts under these specifications show an increase of 57% in cost for 25% of the total excavation. The additional total cost, therefore, has been nominal.

The preventive measures adopted by contractors have been mostly in blasting. Quarrying the rock at the end faces of the cuts accomplishes the desired results, but is not adapted to an efficient program for drilling. Better results have been obtained by making an initial cut along the line of the upper ditch. This permits subsequently blasted material to fall inward, rather than over the slopes. The use of narrow pioneer fills near the lower slope stakes has given good results. This method provides room on the upper side for dumping large rock fragments.

One contractor uses half-round corrugated metal culvert sections for protecting the trunks of trees. In addition, the use of heavy woven-wire, guard fencing, strung between the first line of trees, has been proposed for checking rolling rocks.

Hand-laid embankments have proved satisfactory and have involved a cost of $1.40 to $2.00 per cu. yd. more than the cost of rock excavation. Reinforced concrete cribs are adapted to the same purpose, and have cost from $2.00 to $6.00 per sq. ft. of exposed face.

In cliffs, the tunnel is an ideal design to prevent destruction of forest cover below, and also to prevent waste material from falling. The West has become "tunnel minded"; although many projects have been designed with short tunnels, the tendency seems to be for greater lengths. No trouble has been experienced from engine gases on lengths of about 1 500 ft. The ventilation problem in the Western canyons is quite different from that encountered in river tunnels, where the dipping grades generally produce pockets. Tunneling has proved economical, the prices varying from $55 to $100 per ft. for a 23-ft. unlined roadway section, and from $80 to $130 per lin. ft. for lined tunnels.

Advance Clearing.—The author has opened the question of clearing in advance of construction. This practice accomplishes several desirable objects, among which, first and foremost, is the desirability of burning when the conditions of fire hazard are the least. It also allows the slope stakes to be placed well in advance of construction operations and provides sunlight for the early melting of snows with resulting lengthening of the construction season.

Construction Through Marshes.—The problem of construction through muskeg or marsh is a live question in Alaska, and is there solved by ditching and corduroy road building. Logs for corduroy construction should be from 4 to 8 in. in diameter, and should provide a ridged platform. A 6-in. seal of fine roadside material is placed over the corduroy before the gravel sub-base is applied. The seal excludes the air, which causes decomposition. Examined after ten years corduroy roads show no signs of decay where such a seal has been applied, but where gravel has been used directly on the logs the roadway has broken badly in a few years.

A planking to carry construction traffic, even over the completed gravel surface, has been found necessary. This affords time for the adjustment to take place uniformly, while the ditches are drawing down seepage to a constant flow. The ditches should be at least 10 ft. beyond the line of the shoulders, and of generous depth. Through some areas it has been found advisable to construct the ditches well in advance. In general, planking will permit traffic immediately upon the roadway.

Conclusions.—A financial budget for highway construction is absolutely necessary, but the questions of what methods to adopt on problems of erosion, clearing, or wet foundations, are not so well defined. These problems must be considered separately, under existing physical and financial conditions. It is certain, however, that where scientific principles are applied with patience the correct solutions will appear.

R. D. RADER,[4] M. AM. SOC. C. E. (by letter).—This excellent paper covers a number of the problems with which highway engineers are confronted, some of them being peculiar to the Western States.

The eastern half of Montana is a plateau from 3 000 to 4 000 ft. above sea level and is especially subject to so-called "cloudbursts." These occur during the hot summer months, and often cover only a few square miles, but as much as 2 in. of rainfall may occur in less than an hour. Mr. Pope states that it is the opinion of many engineers that cloudbursts occur in approximately the same areas within a reasonable range of time. Eleven years of observation of this phenomenon in Montana leads the writer to believe that there is no possibility of anticipating where cloudbursts will occur except in a general way. Of course, certain large areas are more subject to these torrential rains than others, but it is only occasionally that they are found to cover the same acreage, or that the run-off crosses the highways at exactly the same place. Some of the alluvial "fans" or "cones" spread out several miles in width and unless the highway is located along the foothills so as to bridge across the main channels where they are well defined, it is difficult to control the flow so as to confine it to bridges of reasonable length.

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[4] State Highway Engr., Helena, Mont.

In Montana, such an elaborate system of ditches and dikes as that described by Mr. Pope has not been attempted. The first rush of water, sand, boulders, and débris will fill any ordinary intercepting ditch, and these ditches have not served the desired purpose of directing the flow under the bridges. However, such flow is usually of short duration, and passes over the highway in a broad flat sheet which does little damage, except the scouring and gullying of the shoulder on the down-stream side of the highway. The cost of repairing this damage is less than the interest charge on an adequate system of ditches and bridges. The method followed in Montana, therefore, is to build bridges over the natural channels, designed so as to carry the normal run-off from the usual rains or melting snow. No attempt is made to design and build bridges for the cloudbursts which may not occur more than once in a long period of years. Occasionally, the grade line of the highway is depressed for a few stations so as to allow exceptional floods to flow over, and, where the grade is well compacted, the loss of embankment is found to be small, even if there is no protecting wall or rip-rap.

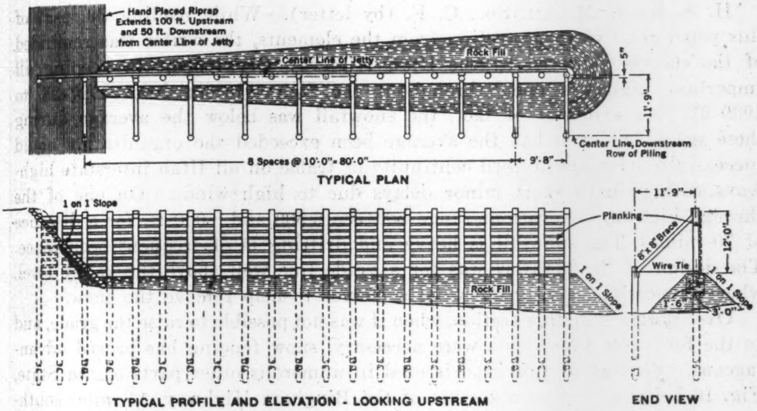
Hand Placed Riprap Extends 100 ft. Upstream and 50 ft. Downstream from Center Line of Jetty
Center Line of Jetty
Rock Fill
5"
11'-9"
Center Line, Downstream Row of Piling
8 Spaces @ 10'-0" = 80'-0"
9'-8"
TYPICAL PLAN
1 on 1 Slope
Rock Fill
1 on 1 Slope
TYPICAL PROFILE AND ELEVATION - LOOKING UPSTREAM
11'-9"
Planking
8"x8" Brace
12'-0"
Wire Tie
1 on 1 Slope
1'-6"
3'-0"
END VIEW

FIG. 15.—PILE AND PLANK JETTIES ALONG THE BIG HORN RIVER, NEAR CUSTER, MONT.

For bank protection of rivers flowing through soil subject to scour, the method of driving double rows of piling parallel to the stream has been used successfully in Montana. Brush and rock are placed between the piling, and are retained by planking or wire mesh attached to it. In such cases, if possible, the piling is driven deep enough to be below the probability of scour. Pile jetties, also, are sometimes used. Fig. 15 shows a layout of pile and plank jetties installed to protect a scouring bank on the up-stream side of a bridge over the Big Horn River, near Custer, Mont. This river carries a large volume of silt in its flood stages, and it is expected that the partly planked, piling jetties will retard the current at flood stage so as to cause the deposition of this silt and thus restore the bank. If this occurs, willows will be planted between the jetties in order further to protect the newly restored bank. The rip-rap is for the purpose of protecting the jetties from scour if a tree or other large drift should lodge against them, and also to prevent damage from floating ice.

H. S. KERR,[5] M. AM. SOC. C. E. (by letter).—While reading the part of this paper relative to protection from the elements, the writer was reminded of the success achieved by the Utah Highway Department in keeping all important highways open throughout the four winters from 1927–28 to 1930–31. As a matter of fact, the snowfall was below the average during these years, but even had the average been exceeded the organization could successfully have maintained continuity of traffic on all Utah interstate highways with perhaps short minor delays due to high winds. On one of the through highways, elevations range between 6 500 and 8 000 ft. for a distance of 70 miles. The snowfall is heavy and drifting is of frequent occurrence. The design of the highway was for a grade line well above the valley level, wherever possible, giving the wind a chance to help remove the snow.

Over portions of this section, when it was not possible to raise the grade, and in the few cases where cuts were necessary, snow fencing has proved advantageous. Fencing of this kind is used in numerous other parts of the State, Fig. 16 being a view of a section of the Bingham Highway, 15 miles southwest of Salt Lake City. This is the common slat fence consisting of laths about 2 in. apart, held together with wire. The posts are old railroad boiler tubes cut to the desired length. The fence is not wired tightly to posts, thus enabling it to be raised as drifts form and to be removed in the spring.

Protection from Floods.—Fig. 17 is a view of the crossing of "West Water" Wash, in Grand County, near the Colorado-Utah State line. This wash is typical of those encountered on the desert of Eastern Utah, across which U. S. Highway No. 50 is located for a distance of 130 miles. Aside from the crossing of the Green River and a small stream—Price River—there are no rivers or creeks for this entire distance; yet there are numerous washes—dry except at times of heavy run-off from the Book Cliff Range—parallel to and north of the highway. Regularly every summer and in the early autumn there are times when some of these washes carry streams as large as from 2 000 to 3 000 sec-ft. for a short time. These are flash floods from mountain lands that are generally devoid of vegetation, continually eroding, and precipitous. Where the land is not bare, it is usually over-grazed.

The State of Utah endeavors to protect its highway bridges from these erroneously termed "cloudbursts," by creosoted piles driven on 5-ft. centers, to which are bolted 4 by 12-in. creosoted planks. Normally, the piles are 25 ft. long, driven 20 ft. into the stream bed. The cost varies from $5 to $8 per ft. of revetment.

A form of basket somewhat different from that described by Mr. Pope has also been used to good advantage. These baskets consist of cedar posts and wire mesh frames, 6 by 6 ft., filled with rock and located along the toe of the slope. Native cedar posts resist decay very well, and the rock gives

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[5] Chf. Engr., State Road Comm., Salt Lake City, Utah.

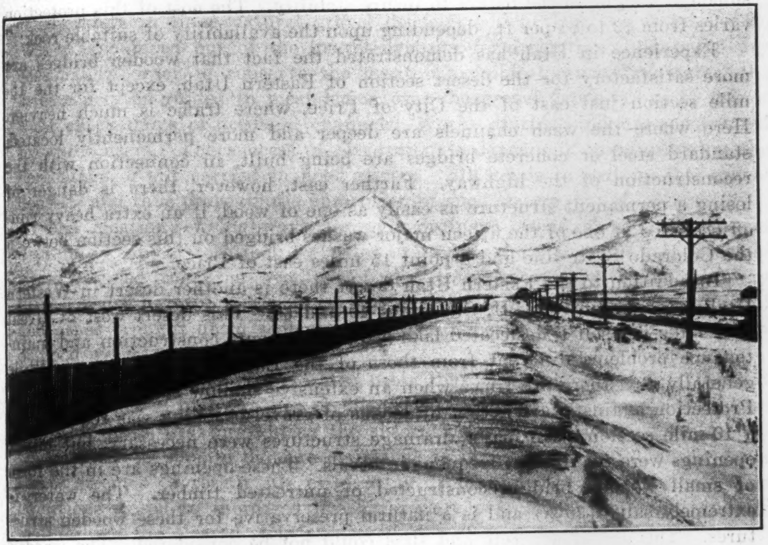

FIG. 16.—FORMATION OF DRIFTS AT SNOW FENCE ALONG BINGHAM HIGHWAY, IN UTAH.

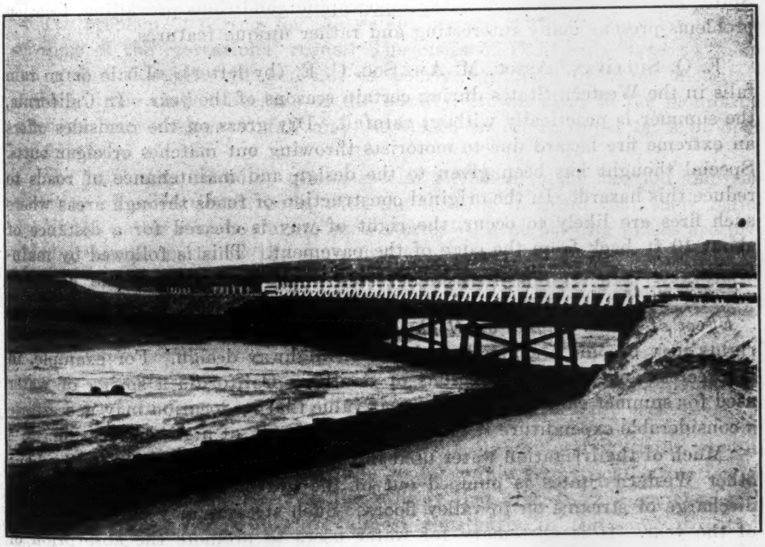

FIG. 17.—BRIDGE OVER COTTONWOOD WASH, IN UTAH.

sufficient weight to the basket to insure stability. The cost of this protection varies from $2 to $4 per ft., depending upon the availability of suitable rock.

Experience in Utah has demonstrated the fact that wooden bridges are more satisfactory for the desert section of Eastern Utah, except for the 15-mile section just east of the City of Price, where traffic is much heavier. Here where the wash channels are deeper and more permanently located, standard steel or concrete bridges are being built, in connection with the reconstruction of the highway. Farther east, however, there is danger of losing a permanent structure as easily as one of wood, if an extra heavy run-off occurred at one of the fifteen major washes bridged on this section between the Colorado State line and a point 15 miles east of Price.

In addition to the Eastern Utah desert there is another desert in Western Utah crossed jointly by U. S. Routes 40 and 50. This desert west of Great Salt Lake is itself a desiccated lake bed and presents construction and maintenance problems different from those of the Eastern Utah desert. While generally dry, there are times when an extensive shallow salt lake is formed. Protection against wave action by means of gravel beaching was required for a 10-mile section. No major drainage structures were necessary, but minor openings were required at frequent intervals. These openings are in the form of small wooden bridges constructed of untreated timber. The water is extremely salty (25%) and is a natural preservative for these wooden structures. The use of concrete and steel could not be considered. The wooden structures were built by the use of wooden pegs in lieu of spikes.

Aside from those noted herein, there are no special problems in this State; however, on account of the topographic conditions prevailing, location problems present many interesting and rather unique features.

E. Q. SULLIVAN,[6] ASSOC. M. AM. SOC. C. E. (by letter).—Little or no rain falls in the Western States during certain seasons of the year. In California, the summer is practically without rainfall. Dry grass on the roadsides offers an extreme fire hazard due to motorists throwing out matches or cigar butts. Special thought has been given to the design and maintenance of roads to reduce this hazard. In the original construction of roads through areas where such fires are likely to occur, the right of way is cleared for a distance of about 10 ft. back from the edge of the pavement. This is followed by maintenance operations, consisting of clearing away all dry grass and brush in advance of the dry season.

Effect of Water Rights on Highway Design.—In the semi-arid West water rights may become a controlling factor in highway design. For example, in the recreational mountain regions of Southern California a spring of water used for summer cabins is of such great value that its location may necessitate a considerable expenditure if it is found in the path of highway location.

Much of the irrigation water used in the valleys of California and in some other Western States is pumped out of the gravel deposits formed by the discharge of streams on to valley floors. Such streams may be dry for most of the year. It is customary for water users to promote the absorption of flood waters in these natural gravel deposits which then act as reservoirs. Water is pumped out of the gravel deposits for irrigation during the dry season. In order to conserve the greatest possible quantity of water, the stream may be guided in meandering over the gravel deposits by the use of light check dams or artificial channels. Works of this kind are constructed by water users. The success of absorption is dependent on minimizing the percentage of silt carried in flood waters. Silt tends to seal up the gravel deposits and thus prevents the absorption of such waters.

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[6] Dist. Engr., State Div. of Highways, San Bernardino, Calif.

The location of highways through drainage areas above such gravel deposits is subject to attack by water users on several grounds. It is sometimes claimed that the opening of the drainage area with the highway increases fire hazard; in the event, the drainage area is burned over, the soil and ashes brought down seal up the gravel deposits and prevent absorption of flood waters.

Claims are also made by water users that earth and rock dust exposed to the weather in highway cuts or fills are washed down the streams to seal up gravel deposits, thus preventing the absorption of flood waters.

In order to obviate damage claims, it is necessary to build check dams across streams to retain silt originating from highway construction. The objections raised to increased fire hazard are overcome by clearing and by maintaining the cleared strip as described.

Control of Snow on Recreational Roads.—The use of recreational roads during and following heavy snowfall has become increasingly popular. This has affected design and maintenance.

The problem of caring for the public is very acute during snow storms on some of the recreational roads. The cities of California have no snowfall and the public, in general, is poorly equipped to enter mountain regions in the snow. Because of the novelty, crowds rush to the mountains whenever snow begins to fall and the roads, therefore, must be kept open continuously.

The problem is solved by giving special attention to the location and construction of recreational roads, with the thought in mind that their major use by the public may be during snow storms. Large parking places are provided at elevations below the snow line where cars can be stopped to put on chains. The highway is located so as to avoid snow slides and it is also built, as far as possible, on southern exposures. Drifting snow is taken care of by snow fences and other devices.

The more popular recreational roads are kept open during snow storms by the continuous use of snow plows, and traffic is aided and controlled to allow the maximum use of the roads by the crowds of pleasure seekers.

T. A. Corry,[7] M. Am. Soc. C. E. (by letter).—Referring to the part of this paper headed "Sand Dunes," the writer would like to call attention to similar problems encountered on the railway that crosses La Joya Desert in Southern Peru. The winds are almost constantly in the same direction and rarely exceed more than 20 miles per hour.

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[7] Los Angeles, Calif.

The sand dunes are all shaped in the form of crescents and they move from 40 to 50 ft. annually, depending on their relative sizes. Their derivation is from the decomposition of granitic rocks.

In Southern Peru the practice is to control this movement by burying the dunes under a thin layer of gravel. Section gangs can cover a dune large enough to be included in a circumscribed square, say 100 by 100 ft., at a cost of about $7.00.

To adapt the same process to California conditions, where the winds are higher, would probably require the use of very coarse gravel.

CHARLES T. LEEDS,[8] M. AM. SOC. C. E. (by letter).—The problems of Western highway construction, particularly in the State of California, and the Southwest, are in many ways so different from those in other parts of the United States that this paper is extremely interesting. The California Division of Highways has much excellent work to its credit in the handling of difficulties such as those described.

Stream Erosion.—The problems of stream erosion, particularly those due to floods and cloudbursts, differ materially in California from those in the central and eastern sections of the country, because of the difference in climate. Many streams may be merely dry beds of sand in summer; and in winter they may become raging torrents. In desert sections of the country, as the author has stated, the water often appears without previous intimation that a cloudburst has occurred. The first knowledge that the observer has of approaching danger is the appearance of a wall of water and mud sweeping down the canyon. The writer cannot agree, however, with "the opinion of many engineers that these [cloudbursts] occur in approximately the same areas within a reasonable range of time." The writer has seen canyons from which no serious flood run-off is known to have occurred within historic times, and yet from which, within the last few years, sudden cloudbursts have brought down a literal wall of water, mud, and gravel, filling up the old channels and creating new ones, and spreading great fans of débris over the highway and the country below.

A serious difficulty of engineers in this Western country has been the paucity of records of rainfall and run-off upon which to base a reasonable estimate of probable flood flow for which provision should be made. In many cases it has been necessary to do as the railroad engineers were forced to do in early years when no records were available; namely, to make a personal inspection of the ground, determine the tributary area, guess at the probable maximum rainfall, and from that deduce a possible value for run-off. If the next flood proved the culvert or bridge inadequate, a larger one was built. This was not slipshod engineering, because no better fundamental data were available. It was the best possible under the circumstances. Now, however, the State Engineer's office maintains careful legal supervision of design and construction of all dams in California, and a painstaking study is being made of the rainfall and probable run-off in all the principal streams. Such data are of great value wherever it is necessary to erect a structure across or over any stream.

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[8] Maj., Corps of Engrs., U. S. A. (*Retired*); Cons. Engr. (Quinton, Code and Hill-Leeds and Barnard), Los Angeles, Calif.

It is probably not out of place here to caution engineers against taking data from one stream or locality and carelessly applying them to another stream where actual run-off records are lacking. At first, the topographic conditions may seem similar in the two cases. Careful study, however, must be given to all available data on topography, wind direction, storm movement, elevation, and temperature, and their influence on precipitation.

Careful study of the geology of a section will often give valuable indication of the magnitude of past floods, and the débris of sand, cobbles, and boulders brought down thereby. The magnitude of a flood as it debouches from a canyon mouth may be very different from the magnitude of the same flood as it passes farther over the débris cone, or as it later reaches more level ground. In the first case, it is not merely a body of flowing water, but a moving mass of combined water, gravel, and sand, the larger and heavier portions of which will be deposited progressively as the flood proceeds downward, until, finally, there remains only the flowing water carrying fine sediment or silt.

Highway Location.—The problem of highway location through a desert region is not always a simple one. The closer to the mountains the location, the easier it will be to determine where the floods shall be carried across the highway, but the greater the mass of débris that will be brought thereto. On the other hand, if the highway is located farther away from the canyon mouths, the bridges or culverts may possibly be smaller, but the length of diversion dikes necessary to concentrate the broad sheets of water at control points may be greater.

An example of the difficulties experienced from wind and sand is that of the California State Highway passing through the Coachella Valley. The old county highway followed along the foot of the mountains on the southern border of the desert. This road was, of necessity, winding and crossed all the canyon mouths, but, at the same time, it was well sheltered from the severe windstorms which are not uncommon in that region. The new State Highway location, which passes down through the center of the valley, permits a much straighter road on a more even grade and requires fewer stream crossings. A saving of approximately five miles in distance and the elimination of practically all dips was effected thereby. Although the number of bridges was unchanged, the general drainage conditions were greatly improved. On the other hand, in time of severe wind and consequent sandstorms, it is sometimes necessary to divert traffic from the main highway and cause it to pass along the old county road at the foot of the mountain where it is well sheltered. This is not so much because of drifting sand as because of wind velocity and sand-blast effect on automobiles.

Earthquakes.—The writer had the good (or bad) fortune to be in Santa Barbara at the time of the earthquake mentioned by the author, and can corroborate his statements that the concrete pavement was shaken from side to side until there was a gap of from 4 to 6 in. wide along each side of the highway. It had also buckled at a number of places and had settled at the bridge abutments. This action took place where the highway had been constructed on filled soft ground. He does not recall seeing any such evidence where the highway had been constructed on solid original ground. The very fact that the highway settled away from the bridge abutments to a distance of from 4 to 8 in. would seem to demonstrate that such settlement does not occur except where there is soft material underneath, and that, therefore, the highway fills should be constructed with the greatest solidity that is economically feasible. Even then some settlement is practically certain to occur.

The size and number of expansion joints in the pavement to prevent damage by earthquake must depend very largely upon the direction in which the probable earthquake wave will be propagated. If it is propagated longitudinally with the highway, expansion joints might take up this motion. On the other hand, if the direction of vibration is at right angles to the highway, it is not seen how expansion joints would be of any advantage. This, again, would seem to be evident from the observations in the Santa Barbara earthquake. Expansion joints would not have taken care of the kind of damage cited by the author, and no serious harm was done. If this earthquake wave (which apparently may have had an amplitude of vibration of from 8 to 12 in. at right angles to the highway axis) had progressed longitudinally of the highway, it is interesting to inquire what width of expansion joints should be inserted in the pavement and at what intervals to resist such action.

The entire question is well summed up by the author in the statement that "generally speaking, however, the danger due to earthquakes is not serious so far as highways are concerned." In view of the probable long intervals between earthquakes in California, it is not believed advisable or economically justifiable, to take undue measures to protect against their occurrence on highway construction.

Foundations.—One phase of the subject upon which the author has not touched is that of foundations for highway structures crossing streams. This phase is particularly important in a country such as California, subject to heavy floods, where stream beds may be composed of sand or silt to a very considerable depth. In the case of pile bridges it is not always adequate simply to drive the piling to a depth sufficient to secure adequate carrying capacity because, in time of flood, the bed of sand in the river may become so saturated with water that the entire stream bed is moving down stream. This has been observed in floods in both the Los Angeles and San Gabriel Rivers. In the flood of 1889–90, the pile trestle bridge that carried the Southern Pacific Railroad across the San Gabriel River moved bodily down stream several feet without breaking or turning over, the entire railroad being simply carried down stream laterally that distance.

It is very common in some streams, as in the Colorado River, for the bed to scour to extreme depth in time of flood and then to refill as the flood peak passes, so that the actual depth of the stream at the time of maximum flood is far greater than any measurements after the flood may indicate. The failure of the highway toll bridge across the Colorado River, from Blythe, Calif., to Ehrenberg, Ariz., in 1928, was largely due to inadequate precaution against scour around the bridge piers, which scour occurred to much greater depth than had been anticipated.

River Crossings.—Because many streams in California, particularly in their higher sections, are to some extent "riding on a ridge," it is very necessary, in designing a river crossing, that study should be made not only of the immediate locality, but of the entire river higher up, in order that adequate measures may be taken to assure that the river shall always pass under the bridge to be erected. In 1906, two bridges were constructed across parallel channels of the San Gabriel River on its débris cone, not far from the mouth of the canyon. They had ample flood capacity, but the flood of 1914 instead of passing under them as had been anticipated, cut new channels for itself, still farther apart, tearing out the outer approaches and thus leaving the bridges "high and dry." Since then these bridges have been replaced by a modern concrete bridge and adequate precautions have been taken to protect the banks thoroughly above the bridge, and a repetition of such a disaster has been prevented.

Bank Protection.—The author has referred to the flood in the Santa Clara River, caused by the breaking of the St. Francis Dam. This was, of course, comparable to a cloudburst and of far greater dimensions. As a result, much damage was done in the Santa Clara Valley and considerable bank protection previously constructed was either damaged or destroyed. As an aftermath, the City of Los Angeles, which had constructed this dam, not only paid for all damages which had resulted, but also appointed a consulting board to work with a similar engineering board, representing Ventura County, in preparing plans for bank protection which should afford to all owners of abutting property the same measure of protection which they had prior to the flood.

In the case of pre-existing protection works, the aim was to repair them to their former efficiency where the protection was not wholly destroyed and, where it was entirely destroyed, to construct new works of similar character to those which existed before the flood, or works of an equivalent efficiency. In the case of new work the problem resolved itself into: (1) The construction of levees to keep the river within reasonable limits of width, the levees to be protected by a system of spurs of different types of construction, depending on the conditions encountered; and (2) where no levees were required, the construction of spurs to protect the existing banks. Wherever levees were constructed, it was required that, in order to prevent the formation of a channel along the toe of the levee, the borrow-pit should not be continuous along the levee. A bar was left between the protecting spurs not less than 20 ft. wide. Where no spurs were required, the bar was to be left at intervals no greater than 200 ft. apart.

Three types of protection for levees or banks were proposed, designated as Types *A*, *B*, and *C*.

Type *A* (see Fig. 18) composed of creosoted timber piles, was used as a replacement for similar protection which had been destroyed. It consisted of timber piles driven in two rows 8 ft. apart, 10 ft., center to center, in rows, and braced between with a 6 by 6-in. by 7 ft. 6-in. timber near the top.

These piles were not less than 12 in. in diameter at the butt, 6 in. in diameter at the tip, 30 ft. long, and were driven to a penetration of not less than 16 ft. and where possible to such penetration that their tops extended 9 ft. above scour grade of the river. This dimension was governed, of course, by the cross-section of the river. The piles were of Douglas fir and creosoted by the 8-lb. empty-cell process. Three 1-in, wire ropes were used in each row to tie the piles together. These ropes were stretched taut against the piles and tied thereto with an independent short piece of wire rope. Each row of piles was also faced on the river side with an 8-ft. width of 6-in. square, electrically-welded, galvanized wire mesh of No. 6 wire, fastened to the piles by heavy staples at each horizontal wire. To hold the piles from spreading at the top, a loop of ⅝-in. wire rope was taken around opposite piles and twisted together tightly.

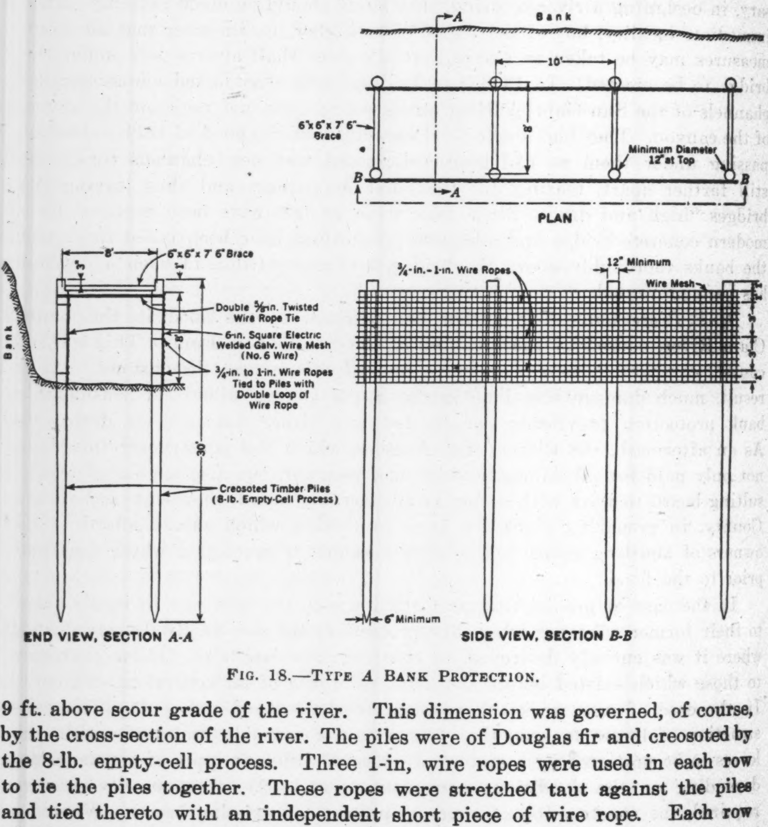
A
Bank
10'
6"x6"x 7' 6" Brace
8'
Minimum Diameter 12" at Top
B
A
B
PLAN
8'
6"x6"x 7' 6" Brace
Bank
Double ⅝-in. Twisted Wire Rope Tie
6-in. Square Electric Welded Galv. Wire Mesh (No. 6 Wire)
8'
¾-in. to 1-in. Wire Ropes Tied to Piles with Double Loop of Wire Rope
30'
Creosoted Timber Piles (8-lb. Empty-Cell Process)
END VIEW, SECTION A-A
¾-in.-1-in. Wire Ropes
12" Minimum
Wire Mesh
6" Minimum
SIDE VIEW, SECTION B-B

FIG. 18.—TYPE *A* BANK PROTECTION.

Type *B* (rail tetrahedrons) which is essentially the same as that mentioned by the author as having been used on the Colorado River, was used

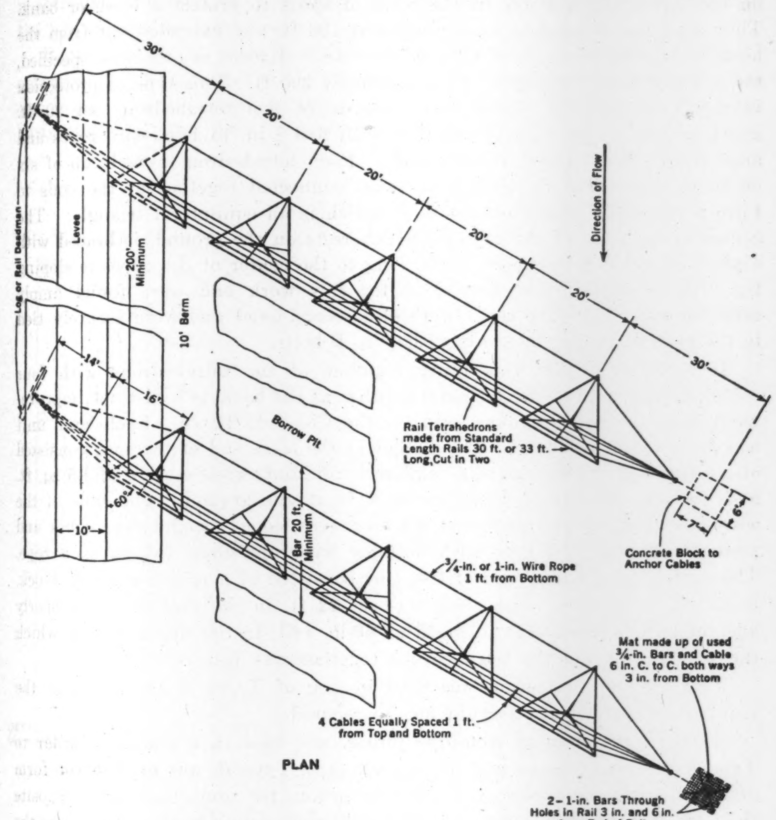
30'
20'
20'
20'
20'
30'
Direction of Flow
Log or Rail Deadman
Levee
200' Minimum
10' Berm
14'
16'
60°
10'
Borrow Pit
Bar 20 ft. Minimum
Rail Tetrahedrons made from Standard Length Rails 30 ft. or 33 ft. Long, Cut in Two
6'
7'
Concrete Block to Anchor Cables
3/4-in. or 1-in. Wire Rope 1 ft. from Bottom
Mat made up of used 3/4-in. Bars and Cable 6 in. C. to C. both ways 3 in. from Bottom
4 Cables Equally Spaced 1 ft. from Top and Bottom
PLAN
2 – 1-in. Bars Through Holes in Rail 3 in. and 6 in. from End of Rail
Top of Levee 9 ft. above Scour Grade
10'
1 on 1.5
6'
3/4-in. or 1-in. Wire Ropes
60-lb. Rails
1 1/2-in. Iron Ring 6-in. Inside Diameter
Slope 1 on 2
10'
Borrow Pit
60-lb. Rail 12 ft. Long
12'
3'
Rail 3 in. from Bottom
ELEVATION

FIG. 19.—BANK PROTECTION TYPE *B*.

on the Santa Clara River in the form of spurs to protect a levee or bank. They were constructed in unit lengths of 100 ft. and extended out from the levee or bank at an angle of 60° down stream. Except as otherwise specified, the spaces between the spurs were invariably 200 ft. This type of protection is shown in Fig. 19. Each unit consists of five tetrahedrons set 20 ft. apart, center to center, tied together with five $\frac{3}{4}$-in. to 1-in. wire ropes and anchored to "dead men" at both ends. Each tetrahedron is made up of six 60-lb. rails from 15 ft. to 16.5 ft. long, connected together at the ends to form a four-sided figure each side of which is an equilateral triangle. The center of each leg of the triangle which rests on the ground, is braced with a piece of rail approximately 12 ft. long to the center of the opposite sloping leg. Second-hand rails were used for this work and were found amply satisfactory. The wire ropes were also second-hand and were securely tied to the rails at proper intervals with $\frac{3}{4}$-in. **U**-bolts.

It is believed that this tying together of the tetrahedrons with four cables, 1 ft. apart, on the face and another at the back, is a distinct improvement over the protection as used on the Colorado River. Each spur unit was anchored at each end. The anchor at the levee end of the spur consisted of a sound log or two logs of a combined minimum cross-section of 1.5 sq. ft. and not less than 12 ft. long, buried 6 ft. in the ground. The ends of the wire rope tying the tetrahedrons together were looped around the anchor and fastened back on the rope with bull-dog wire-rope clips, two to each rope. The anchor at the river end of the spur consisted of a rubble concrete block, 5 cu. yd. in volume. The block was set 12 ft. in the ground and properly anchored in it was a 12-ft. length of 60-lb. rail, to the upper end of which the wire rope tying the tetrahedrons together was fastened.

This type of protection was used in lieu of Types *A* and *C* where the required pile penetration could not be secured.

Type *C*, a system of iron-pipe piling, was used in a manner similar to Type *B* to protect levees and banks and, as in Type *B*, was used in the form of spurs, with one exception. This exception, for protection work opposite the City of Santa Paula, was a location where the river bank and levee on the edge thereof, were protected for a distance of 3 000 ft. by a continuous row of Type *B* construction in addition to spurs.

As in Type *B*, the spurs were each 100 ft. long and extended from the protected levee or bank at an angle of 60° down stream and were spaced 200 ft. apart (see Fig. 20). Each spur is 100 ft. long and consists of a double row of pipe piling, driven 10 ft. on centers in rows 8 ft. apart. The piles used in the lower half or river end of the spur are 6 in. in diameter, while those for the upper half, or levee end, are 4 in. in diameter.

The 6-in. pipe piling, constituting the river end of the spur, was driven to a minimum penetration of 16 ft., or to a penetration of 18 ft. if refusal was not reached before that depth. The tops of the piles were on a straight line sloping downward toward the river on a 2% grade. The 4-in. pipe piling, constituting the levee end of the spur, was driven to a minimum penetration of 14 ft. For economy, second-hand pipe was used throughout the job, subject to inspection and approval.

Between rows the piles were braced by welding a piece of pipe between opposite piles, 2 ft. from the top. The two piles on each end of the spur were also braced longitudinally to the second pile from the end by welding pipe horizontally between them, 2 ft. from the top.

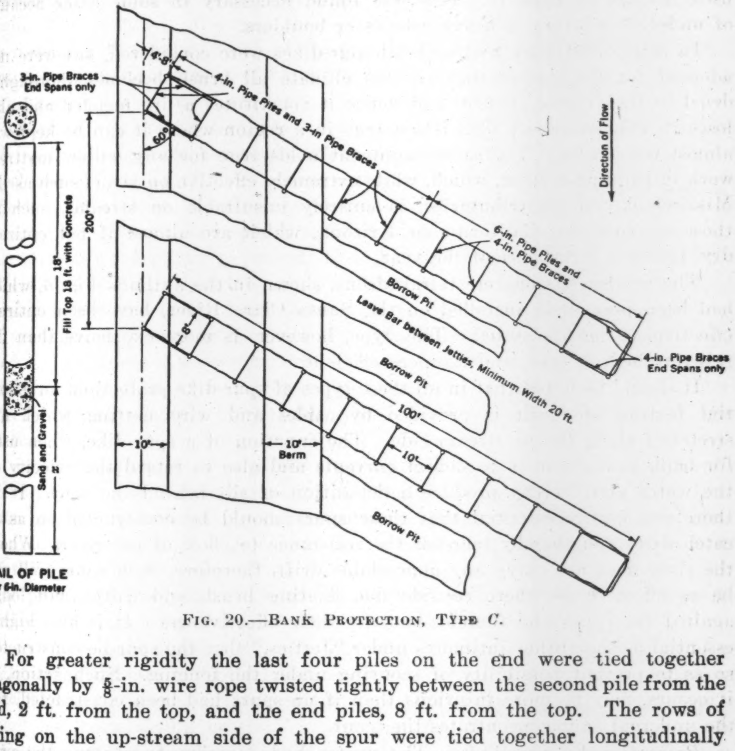
3-in. Pipe Braces
End Spans only
4'-8"
4-in. Pipe Piles and 3-in. Pipe Braces
60°
200'
18'
Fill Top 18 ft. with Concrete
Sand and Gravel
Direction of Flow
6-in. Pipe Piles and
4-in. Pipe Braces
Borrow Pit
Leave Bar between Jetties, Minimum Width 20 ft.
8'
Borrow Pit
4-in. Pipe Braces
End Spans only
100'
10'
10'
Berm
10'
Borrow Pit
DETAIL OF PILE
4-in. or 6-in. Diameter

FIG. 20.—BANK PROTECTION, TYPE *C*.

For greater rigidity the last four piles on the end were tied together diagonally by $\frac{5}{8}$-in. wire rope twisted tightly between the second pile from the end, 2 ft. from the top, and the end piles, 8 ft. from the top. The rows of piling on the up-stream side of the spur were tied together longitudinally with two second-hand wire ropes of $\frac{3}{4}$ in. to 1 in. in diameter, well tied to the piles with No. 6, annealed, galvanized wire.

Each row of piling on the up-stream side was faced with an 8-ft. width of 6-in. square, electrically welded, wire mesh of No. 6 galvanized wire, stretched taut and securely tied to the piles with a double loop of No. 6 annealed galvanized wire at all horizontal wires. The top of the wire mesh was flush with the tops of the piles. The wire mesh on the up-stream row of piles was extended around the river end of the spur, and securely tied to the first pile of the back row. The wire mesh was also tied with No. 6 galvanized wire, at 12-in. intervals, to the wire ropes on the up-stream row of piling.

For greater rigidity the piling was braced diagonally between rows with a double line of $\frac{5}{8}$-in. wire rope, twisted tight, extending from the point of the brace weld 2 ft. from the tops of the piles to a point 8 ft. from the tops of the opposite piles.

Where it was found impracticable to secure the prescribed pile penetration, namely, 16 ft. for 6-in. piles and 14 ft. for 4-in. piles, Type *B* spurs were used instead of Type *C*. This was found necessary in some cases because of underlying layers of heavy cobbles or boulders.

In this work, brush and rock-fill spur-dikes were considered, but were not adopted for the reason that in this climate all brush becomes thoroughly dried in the summer season and hence it constitutes a fire menace and also loses its effectiveness. This is not true in a region where it can be kept wet almost constantly. The same argument holds true for any willow mattress work in bank protection, which, while extremely effective on rivers such as the Mississippi and its tributaries, is entirely unsuitable on streams, such as those of Southern California or Arizona, which are almost if not entirely dry during a large part of the year.

The reinforced concrete tetrahedrons, shown in the author's Fig. 6, which had been previously installed on the Santa Clara River, have been entirely effective, as he has stated. This type, however, is more expensive than the rail tetrahedron type, and no more efficient.

It should be noted that in all these types of spur-dike protection, an essential feature of merit is provided by cables and wire netting which are stretched along the up-stream side. The function of a spur-dike, when used for bank protection, is to deflect currents and also to retard the velocity of the water and thereby produce a deposition of silt behind the spur. It is, therefore, highly essential that these spurs should be constructed so as to catch drift and thereby increase the resistance to flow of the river. Where the river does not carry any appreciable drift, therefore, such spurs will not be as effective as where considerable floating brush and leaves will lodge against the spurs and thereby increase their effectiveness. It is also highly essential as the author intimates under "Jetties," that the spur be constructed so as to prevent possibility of scouring under the fencing. Such action, if it occurs, may be more injurious than if no spurs had been constructed, for the undercut spur concentrates the scour.

Protection by Sea-Walls.—The writer had occasion to observe the protection work of reinforced concrete cells, so well described by the author, while they were under construction by the California Highway Division, and he desires to register his commendation of this excellent device in a difficult situation.

The sea-walls constructed by the State of California of monolithic concrete appear to be of sufficient strength to withstand the action of the heaviest storm waves thus far encountered. The writer believes, however, that the profile adopted has had a tendency, in some instances, to erode the beach, and that with a different design a considerably greater effective beach width could have been secured.

It is true, as the author states, that the use of sheet-piling, of either wood, steel, or concrete, on State work in California has never met with much success, and that it has not been used extensively in sea protection. Except in rare cases, it should not be so used. When a vertical wall is built to offer resistance to the onrush of the waves, the inevitable result is a downward deflection of part of the wave and a throwing into the air of a mass of water which, dropping again with great force, scours out the sand at the toe of the wall. The profile of the wall should be so designed, with slope and curve, as to deliver the return wash of the wave parallel to the beach slope, or still better, to provide a means of breaking up the force of the wave, as with riprap, or with a step-faced sea-wall of adequate design.

Shore Protection by Groynes.—Groynes for the protection or improvement of the beach line are extremely effective where properly used. Groynes of steel sheet-piling or of interlocking concrete blocks have usually been more effective than those of wood, however. Wood groynes, too, are subject to damage by storms. It is often difficult to secure sufficient penetration for the vertical wood piling to prevent their being worked loose in severe storms. Horizontal planking is often torn loose and is quite likely to have the beach eroded below the level of the lowest plank under severe storm conditions. If any opening occurs below the bottom plank, the force of the water flowing under it produces a severe jet action which scours out the beach to a much greater depth than if the groyne had not been in existence. With steel sheet-piling or concrete blocks that can settle into place, there is no danger of this.

In the location of groynes the simplest rule (and the one most generally followed) is to construct them at right angles to the shore line. This, however, is not the scientific procedure, and the groynes will be found to function more effectively if their orientation is carefully determined from a study of the prevalent direction of wave action on the shore. The height, length, and profile of the groyne are also important elements which can only be determined from a careful study of local conditions.

At the present time (1932) the California Division of Highways, and the California Division of State Lands are co-operating in planning a system of steel sheet-pile groyne protection for a section of the State Highway where there has been severe erosion. In this study, the inter-related effects of adjacent parts of the shore line and previous construction thereon have been carefully considered, and the future results will be noted.

This entire problem of shore protection and beach improvement is an intricate one in which no isolated problem can be adequately solved without consideration of adjoining section of the coast. It is probable that in the past adequate attention and study have not been given to this problem in many sections of the country. Much can be done, however, by thoroughly co-ordinating all efforts of State, municipal, and private agencies. This is particularly important in connection with highway construction, as a large measure of the usefulness of the highways is as a means of access to and along the beach and ocean by people of the hinterland.

G. S. Paxson,[9] Assoc. M. Am. Soc. C. E. (by letter).—Except the works designed to withstand earthquake shocks, the problems discussed by Mr. Pope are common to all the Western States. Several interesting and successful structures, designed to protect highways from stream erosion and from the encroachment of sand dunes, have been built in Oregon in recent years.

The State has several rivers of medium size (approximately 10 000 sec-ft. maximum discharge) that flow through flat alluvial plains. The banks of these rivers are composed of sand and gravel and are eroded easily. The rivers are fed by smaller streams flowing on steep gradients and carrying large quantities of sediment. On joining the main river the rate of flow becomes less and the stream contains more sediment than can be carried at the lower velocity. Under these conditions bars are formed that often change the direction of the current causing serious erosion along the banks. To control the current before it reaches the stream bank is usually more economical than to reinforce the banks themselves. A cheap and efficient current-directing structure is shown in Fig. 21. It is built of rock enclosed in wire mesh, and is most easily constructed during the low-water period when the site is dry. A width of wire mesh is laid in position on the stream bed and rock is piled on it to form a dike the section of which is an equilateral triangle. Other sections of mesh are laced to the edges of the bottom section, folded over the sides of the dike, and laced together at the top. The dike is thus completely enclosed by mesh and will not break up even if it is undermined. Dikes of this type have rolled completely over without damage. The crest is built to about the ordinary winter water elevation. During flood periods the current is checked enough to cause sediment to become deposited behind the dike. In a few years an elevated plain is built up, that acts as a buffer between the stream and the threatened bank. This type of current retarder built 5 ft. high, costs about $4 per lin. ft.

This treatment has not been applied to streams that have a rise of more than 8 or 10 ft. during flood periods. For such streams the usual treatment has been a bank protection consisting of piling, brush fascines, and rock. Fig. 22 shows a revetment on the bank of the Willamette River near Harrisburg, Ore. The piles in the front row are spaced at 4 ft. 0-in. centers. The back row is 8 ft. 0 in. from the front row, and the piles are at 8 ft. 0-in. centers. The space between the rows of piles is filled with willows tied in bundles 8 ft. long and 2 ft. in diameter. These fascines are weighted with large blocks of stone. Where the revetment is in contact with the bank many of the willows take root and grow. In a short time a compact and resistant barrier is formed.

In many places, highways have been built through or beside sand dunes that shift on to the roadway during storms. The removal of this sand is a continuous and costly maintenance operation and many attempts have been made to control it. The most successful has been the planting of grass or shrubs. For example, "Holland grass," which grows wild along the Oregon Coast, is planted in bunches of three or four stalks at 2-ft. centers. It has an extensive root system which holds the sand in place and the tops act as

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[9] Asst. Bridge Engr., State Highway Dept., Salem, Ore.

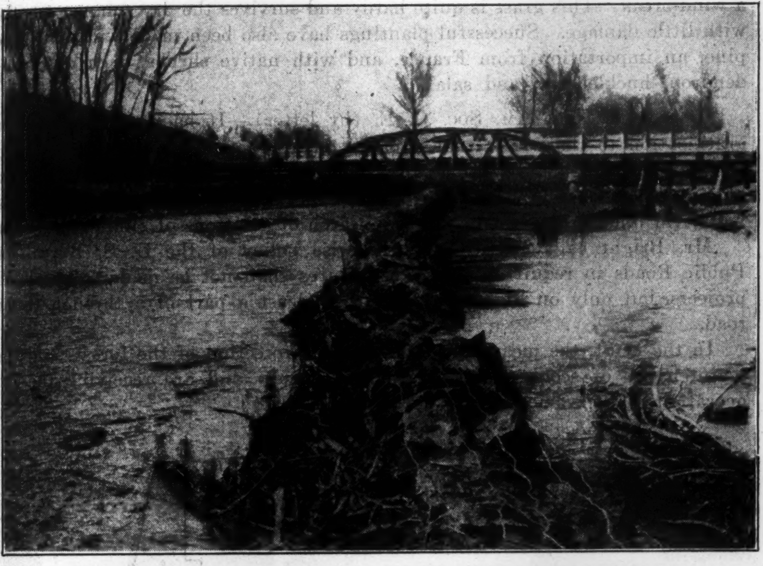


FIG. 21.—CURRENT-DIRECTING STRUCTURE OF ROCK ENCLOSED IN WIRE MESH.

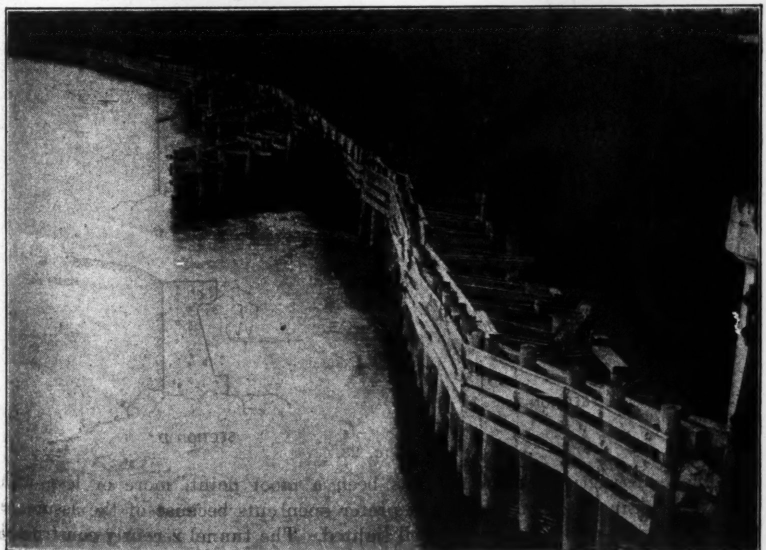


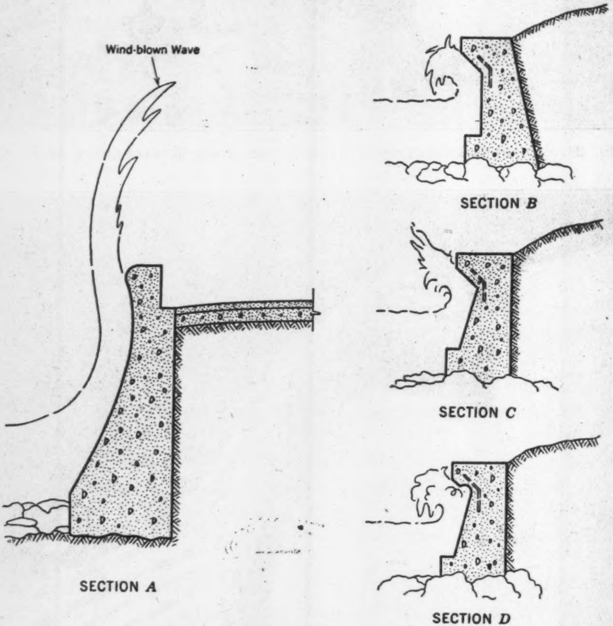
FIG. 22.—BANK REVETMENT OF PILING, BRUSH FASCINES, AND ROCK ON WILLAMETTE RIVER, OREGON.

a wind-break. This grass is quite hardy and survives the dry summer months with little damage. Successful plantings have also been made with maritime pine, an importation from France, and with native shrubs, such as rhododendron, huckleberry, and salal.

C. S. Pope,[10] M. Am. Soc. C. E. (by letter).—It has been a source of gratification to the writer that a paper on a practical subject of this kind should evoke such interesting discussion and cause such a concentration of experience records. A study of the comments of the various discussers will yield much of benefit to those in search of solutions of similar problems.

Mr. Bright calls attention to the wise policy of the U. S. Bureau of Public Roads in requiring that expenditures shall not be made on dead-end projects, but only on highways that are at least a part of a through traffic road.

In the West, the measures taken for the protection of the forests and the maintenance of scenic beauties are most timely and are receiving the co-operation of the various State departments.



SECTION *A*

SECTION *B*

SECTION *C*

SECTION *D*

Fig. 23.

The construction of tunnels has been a moot point, more or less, with highway engineers, many of whom prefer open cuts because of the assurance that the roadway will be always well lighted. The tunnel recently constructed under the supervision of the Bureau of Public Roads into Yosemite Valley

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[10] Chf. Constr. Engr., California Div. of Highways, Sacramento, Calif.

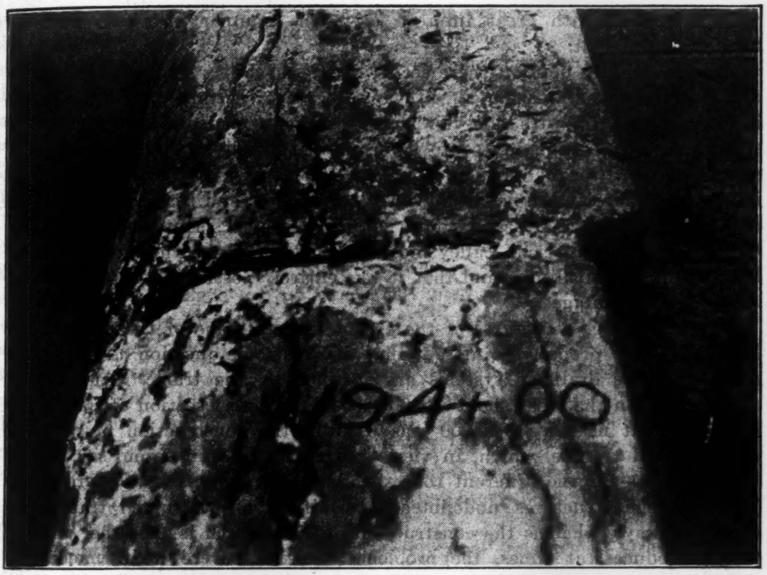

FIG. 24.— COPING OF SEA WALL SHATTERED AT THIN COMPRESSION JOINT DUE TO EXPANSION OF CONCRETE UNDER MOISTURE AND TEMPERATURE.

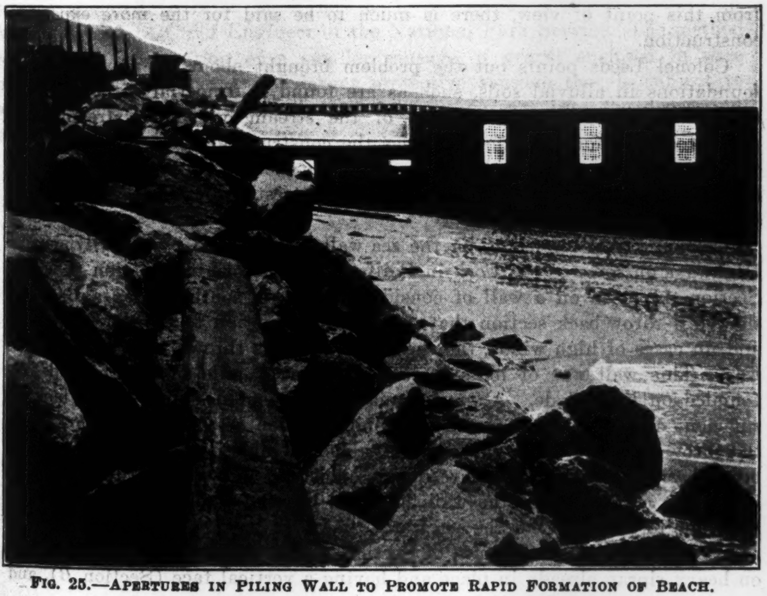

FIG. 25.—APERTURES IN PILING WALL TO PROMOTE RAPID FORMATION OF BEACH.

will be studied with great interest as to the manner in which traffic is handled through it.

In Mr. Rader's opinion there seems little probability that the location of cloudbursts can be anticipated with any degree of certainty, and the writer is inclined to agree with this view, but he makes the mental reservation that studies should be inaugurated to determine, if possible, any laws that may govern the location of cloudbursts. For some years, in California at least, it was held that sand dunes could not be controlled, and that their formation and actions were erratic and could not be predicted. However, a study of the dunes in the Imperial Valley indicated that they conform to very definite laws within certain limits and that it was entirely possible to eliminate some of their disadvantages.

As to drainage structures of the extent used in some of the desert work, Mr. Rader very properly points out that there is some question as to whether such expensive drainage systems are justified by the traffic. An analysis of construction that cost about $3 000 000, in the desert section of California, indicates that nearly $1 000 000 was used in the construction of drainage channels and trestles, which, in view of a traffic running from, say, 200 to 1 200 vehicles for the different routes, might give some room for argument. The drainage structures undoubtedly permit of uninterrupted traffic over these highways, whereas the construction of dips, which is the other alternative, introduces, at times, the problem of rather severe maintenance and also a less safe road. The record of five or six years in one district shows twenty-eight deaths to have occurred due to traffic accidents in dips, so that, from this point of view, there is much to be said for the more expensive construction.

Colonel Leeds points out the problem brought about by moving bridge foundations in alluvial soils, such as are found in Imperial Valley, due to the movement of the entire mass of the stream bed down stream. An attempted cure for this condition was the construction of a submerged dam down stream from one of the bridges, but the method proved unsatisfactory, and it has since been the custom to drive pile trestles and replace them when serious movement occurs.

Colonel Leeds comments on the sea-wall section used by the Division of Highways on some of the work in California. The original section (Fig. 23, Section *A*), used on a wall of considerable length south of Santa Barbara, showed a throw-back section that gave no interruption to the flow of waves, and, in cases of high winds, the sheet of water was thrown over on the highway. This wall was of massive section, about $\frac{3}{4}$ mile in length, and was founded on hard shale. An unfortunate feature of design was the lack of sufficient expansion provision in the coping section of the wall which formed the throw-back, and, as a consequence, a considerable part of this coping has been shattered through pressure due to moisture and temperature (see Fig. 24). The main body of the wall, however, has shown only slight deterioration, due to the action of the sea water.

Along the rocky coast south of Section *A*, Fig. 23, a type of wall founded on heavy riprap already in place and having a vertical face (Section *B*), and a throw-back section projecting at an angle, was constructed over a considerable length of shore, but, later, the front face (Section *C*), was given a batter to break up the direct impact and scouring action of the waves. In one section, also (Section *D*), an effort was made to induce turbulence at the moment the wave struck the throw-back section so that each succeeding wave would be met by a wall of falling water which would tend to break its force. These latter walls have been quite successful in protecting the highway and in preventing spray sheets from being blown over it.

Groynes of steel sheet-piling (Fig. 25) were constructed to a considerable extent during 1931-32, in order to build up shore protection north of Santa Monica, and the technique of building these groynes has shown some advance over past practice. Each groyne was constructed at a pre-determined angle to the shore line, in order to secure the most rapid and stable accumulation of sand, and the grade line was also fixed in such a way as to increase and maintain the deposit. In some cases, certain sections of sheet-piling were made shorter than the adjoining sections (see Fig. 25), leaving openings through the sheet-piling wall through which the water could pour, and this greatly facilitated the depositing of sand in front of the piling. The durability of this type of piling has been questioned, but it has been stated that experience in Germany indicates that a satisfactory economic life may be expected.

Mr. Paxson calls attention to the use of gravel sausages in Oregon, and also to the use of Holland grass for stabilizing sand dunes. Both these methods are valuable in the proper locations. A variation of the gravel sausage enclosed in wire mesh has been developed by F. A. Kittredge, M. Am. Soc. C. E., Chief Engineer of the National Park Service. The method requires the construction of a smaller sausage of gravel enclosed in wire mesh, which is laid beside the larger protective sausage that is subject to scour. In case of a washout, the smaller sausage sags into the depression and supports the larger structure.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1837

LOW-COST BITUMINOUS ROADS [1]

BY WALTER N. FRICKSTAD,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. H. J. SPELMAN, J. T. L. MCNEW, ROGER M. LEE, E. Q. SULLIVAN, HUGH W. SKIDMORE, A. H. HINKLE, CLAUDE P. OWENS, CARLTON N. CONNER, HAROLD L. TILTON, AND L. I. HEWES

SYNOPSIS

In the development of modern highway systems there is a serious need for a low-cost secondary road that will accommodate a limited heavy traffic and an unlimited light traffic. In the western part of the United States a light type of asphaltic binder known as fuel oil has been successfully used to treat crushed aggregate. Whether mixed at a central pug-mill plant, or on the road itself, the costs are about the same, and the result has proved economical in first cost and maintenance charges.

After summarizing the status of this field up to October, 1928, the writer discusses the advances from that time to date (1931), describing construction methods, specifications for materials, and necessary equipment.

INTRODUCTION

Two types of highway work, involving treatment of fine crushed aggregate with a light grade of asphaltic binder known as fuel oil, may be classified as surface treatment and oil-mixing treatment, respectively. In an unpublished paper,[3] entitled "Bituminous Treatment of Fine Crushed Rock and Gravel Roads," Thomas E. Stanton, Jr., M. Am. Soc. C. E., has described these treatments and the extent of their application in the Western United States up to October, 1928. Mr. Stanton's paper is summarized briefly herein.

Surface Treatment.—This is sometimes called penetrative treatment because the binder is absorbed into the upper strata of aggregate. The first step in construction is to prepare the base and this implies repairs, thorough compaction, and thorough cleaning. Approximately, $\frac{1}{4}$ or $\frac{3}{10}$ gal. of asphaltic fuel oil per sq. yd. is spread on this prepared base. The oil should contain 60 to 70% of 80-penetration asphalt having a specific viscosity (Engler) at 50° cent. of 10 to 25. The oil is allowed to penetrate the upper strata of the metal; sometimes stone chips are spread to protect traffic. After a suitable interval another application of asphaltic oil is spread. This may be of the same nature as the primer, or it may be of slightly higher viscosity; it may be much heavier, approaching pure asphalt.

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[1] Presented at the meeting of the Highway Division, Sacramento, Calif., April 24, 1930.

[2] City Engr., Oakland, Calif.

[3] Presented at the meeting of the Highway Division, San Diego, Calif., October 4, 1928.

The next step is to spread stone chips in sufficient quantity to absorb the oil and then the road is rolled. Details vary considerably, particularly in the quantity of oil and stone chips added, which is greater for the heavier oil. Sometimes the second application of oil is kept to a minimum, usually $\frac{1}{6}$ to $\frac{1}{4}$ gal. per sq. yd.; the stone chips are relatively large, as, for example, $\frac{5}{8}$ to $\frac{7}{8}$ in., and a third spread of oil of approximately the same quantity is applied, and followed by still finer stone chips. Heavy oil, applied hot or emulsified, is considered more durable than fuel oil, particularly against weather damage.

Oil-Mixing Treatment.—In this process the asphalt oil binder is thoroughly mixed with the upper 2, 3, or even 4 in. of surfacing material. The first construction operation is to provide the desired quantity of loose mineral aggregate on top of a firmly compacted base. All the aggregate should pass a 1-in. screen and one-third to one-half of it should pass a 10-mesh sieve. It may be obtained by scarifying if a sufficient supply is already on the road, or it may be added for treatment. The oil is usually spread in three applications, aggregating 1.25 to 2 gal., or more, per sq. yd. After each application the materials are mixed by means of a disk or spring-tooth harrow, or by both. After all the oil is applied, the mixing operation is completed by casting the material into a windrow with a blade grader and moving the windrow back and forth over the road, turning the mass at each move. The oil commonly used contains from 60 to 70% of 80-penetration asphalt. The viscosity is variously specified, generally 25 to 45 (Engler specific) at 50° cent., or 45 to 80.

The quantity of oil varies according to the grading and other characteristics of the aggregate. As in the case of asphaltic concrete the higher proportion of oil is required for the finer material. C. L. McKesson, M. Am. Soc. C. E. has suggested[4] the following equation for estimating the quantity of oil required:

$$P = 0.015a + 0.03b + 0.17c \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which, P = percentage of oil required; a = percentage of aggregate retained on the 10-mesh sieve; b = percentage passing the 10-mesh and retained on the 200-mesh sieve; and c = percentage passing the 200-mesh sieve. The quantity of oil is sometimes estimated by Equation (1) and sometimes by appearances, but quite commonly by a modification of the stain test so long used in sheet-asphalt practice. After the mixing operation is completed the material is carefully spread with a blade grader to uniform thickness, and is compacted by traffic. During the compacting period, ruts and other irregularities are eliminated by a drag or blade grader until traffic makes no impression and until the road is as smooth as the most carefully constructed of the higher types of pavement.

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[4] "Light Asphaltic Oil Road Surfaces," by C. L. McKesson and W. N. Frickstad, *Public Roads*, September, 1927, p. 150.

In addition to the foregoing, a method of plant-mixing, first introduced in California in 1928, should be mentioned, in which the aggregate and the oil are mixed at the source in the ordinary type of pug-mill. The aggregate is not heated, but the oil is warmed enough to facilitate pumping. When the mixed material is placed on the road it is spread by a blade grader and compacted in the same manner as the road mixture. It has been found that the cost of pre-mixing when the surfacing material must be brought to the road is little, if any, greater than road mixing under the same conditions. To those acquainted with the price of asphaltic concrete the low cost of fuel-oil mixture may seem surprising, but the explanation lies in the elimination of heating operations at the plant and the simplicity of handling cold material on the road.

At the end of the 1928 working season approximately 3 600 miles of State highways in Washington, Oregon, California, Nevada, Arizona, New Mexico, Colorado, Utah, Idaho, Montana, and Wyoming had been oil-treated. Of this mileage, 2 330 was surface treated, 1 170 was mixed, and 100 was of miscellaneous types. Some of that classified as miscellaneous was in the nature of penetration macadam. During 1929, 730 miles were surface treated, 1 640 miles were mixed, and 130 miles had miscellaneous treatments. Hence, about 2 500 miles were treated in 1929, making the grand total up to that time, 6 100 miles. Statistics for 1930 and 1931 are not available, but at least 3 000 miles were added each year, giving a total at the end of this (1931) season of more than 12 000 miles. The foregoing figures do not include county roads and small town streets, which aggregate a considerable but unknown mileage.

DEVELOPMENT IN SURFACE TREATMENT

The trend of surface treatment practice is now (1931) very definitely toward the use of heavy oils, with an asphalt content of 90%, or more. Fuel oil is used principally as a primer or as a dust palliative. A few of the less traveled roads are being surface treated with fuel oil alone, but generally this application is followed with heavy oil and coarse stone chips. Thus, there is a reversion to the methods by which macadam roads have been treated successfully since about 1910. Although the process has been satisfactory on macadam, it has not heretofore been commonly successful on gravel or fine-crushed rock. The introduction of a primer seems to have overcome the difficulty.

Some authorities have a strong preference for tar rather than oil, reporting better penetration into the road and more careful curing with less annoyance to traffic. The Highway Department of Oregon has used a light tar for this purpose and reports marked success.

Details of the surface-treatment process are far from being standardized. One of the California State Highway methods is among the most elaborate and may be outlined, as follows:

(1) Clean, repair, and compact the base thoroughly;
(2) Spread ⅓ gal. per sq. yd. of 50 to 60 asphaltic fuel oil (Specific viscosity (Engler), 10 to 25 at 50° cent.);
(3) Allow the first application to penetrate and add a small quantity of fine stone chips if necessary to protect the traffic and to absorb oil that may settle in depressions;
(4) Spread ¼ gal. per sq. yd. of 94 + asphaltic oil at a temperature of approximately 350° Fahr.;
(5) Spread sufficient ½ to ¾-in. stone chips to cover the oil. (120 tons per mile of 18-ft. road is the estimated quantity, but this includes considerable waste due to the effort to save labor);
(6) Roll;
(7) Spread ¼ gal. of 94 + asphaltic oil as before;
(8) Cover with ⅛ to ½-in. stone chips. (80 tons per mile is estimated, which includes waste as before); and,
(9) Roll.

The Oregon Highway Department has a similar specification, using slightly smaller quantities of oil. This Department also has a modified method, omitting the third application of oil, but spreading the two sizes of stone chips separately. The Oregon estimate for stone chips is about two-thirds that of California.

The surface-treatment process has made some headway in Washington and in Idaho, in addition to California and Oregon, but the other Western States have not generally adopted this type. The State of Washington, like California, has treated many roads as a temporary expedient, realizing that elimination of dust and conservation of road metal will compensate for a considerable proportion of inevitable failure.

PLANT MIXING

Plant-mixed oil and aggregate presents one of the most interesting possibilities among the developments concerning the oil mixtures. It has spread in an experimental way to several other States, and California is firmly committed to this type of treatment for new construction. When the surfacing material had to be brought to the road, the cost of the earliest project was little, if any, more than road mixing. Plant mixing, of course, permits more complete control than road mixing. Some find that the plant mixture lacks the ideal riding qualities of the road mixture, but if this is true the fault can be corrected by a moderate amount of manipulation while spreading.

To be economical, the mixing must be performed without any extra handling of the material except its passage through the mixing box. Elimination of the need for heating the aggregate and the consequent opportunity for spreading and compacting, without consideration of loss of heat, keeps the mixing cost surprisingly low. It is reported, however, that the power required to operate the mixer is about one-half more than that required for the same quantity of heated aggregate and asphalt.

EQUIPMENT FOR FIELD MIXING

The machinery used for road mixing continues to vary in type. Most of the States are equipped with tractors and blades for snow-fighting and ordinary maintenance, and have simply transferred this equipment to oiling operations during the appropriate season. Use for these other purposes has influenced the purchase of equipment for oil mixing. One State, however, has equipped itself primarily for oil mixing, and has adopted the tractor or dual-drive type of self-propelled blade. A group of four of these motor graders is extremely efficient. The best work is done with blades 10 or 12 ft. long.

Regardless of the type of machinery, workmanship has steadily improved. The first thought was to regard mixing as a rather haphazard process, and too often it was so in fact. Since the beginning of 1929 neatness and system have been notable. Partly oiled material is picked cleanly from the base and turned over and over in accurately lined windrows; during the final turnings, a small quantity is allowed to remain on the base, and the final mixing takes place without the incorporation of uncoated particles. It is then spread quite accurately to uniform thickness (see Fig. 1). Even where the grading of the aggregate varies from station to station, with a consequent necessity of adjusting the quantity of aggregate drawn into the mixture, the windrows become remarkably uniform in color before spreading when properly manipulated. Observing the work of a skillful crew, road mixing can no longer be regarded as haphazard. One step in this process is shown in Fig. 2, which illustrates work on a Nevada project.

Grading of Aggregate

The grading of aggregate has not tended to become standardized, which confirms the opinion that a wide range is acceptable for oil mixing. First experiments indicated that the mixture should conform to that of asphalt concrete. Since the maximum size was fixed at ¾ in., or 1 in., it followed that the proportion of material passing a 10-mesh sieve would approximate 40 to 50%, of which perhaps one-fifth would pass the 200-mesh sieve. However, successful oil-mix projects have been constructed with as much as 80%, or even more, passing the 10-mesh sieve. In fact, there is record of many miles of natural soil mixtures of which practically all the material passed the 10-mesh sieve, and a few in which the greater part passed the 80-mesh sieve. On the other hand, successful oil-mixed projects have been constructed with only 25% passing the 10-mesh sieve. There is good reason, therefore, to believe that gradings which correspond not only to asphaltic concrete, but to Topeka mixture and sheet asphalt, may be successfully mixed with fuel oil.

The proportion passing the 200-mesh sieve is an important element. This has been as low as 3 or 4%, and higher than 20 per cent. Apparently, the 200-mesh material should fill the spaces between the larger particles sufficiently to reduce the size of voids to the minimum and increase their number to the maximum. Studies of the fundamental physical characteristics of asphaltic concrete and sheet-asphalt mixtures have been made and the results have also "shed light" on the subject of oil mixtures.

In particular, it is hoped that further research pertaining to the characteristics of fillers may disclose the quantity and quality of the 200-mesh material desirable for oil as well as asphalt mixtures.

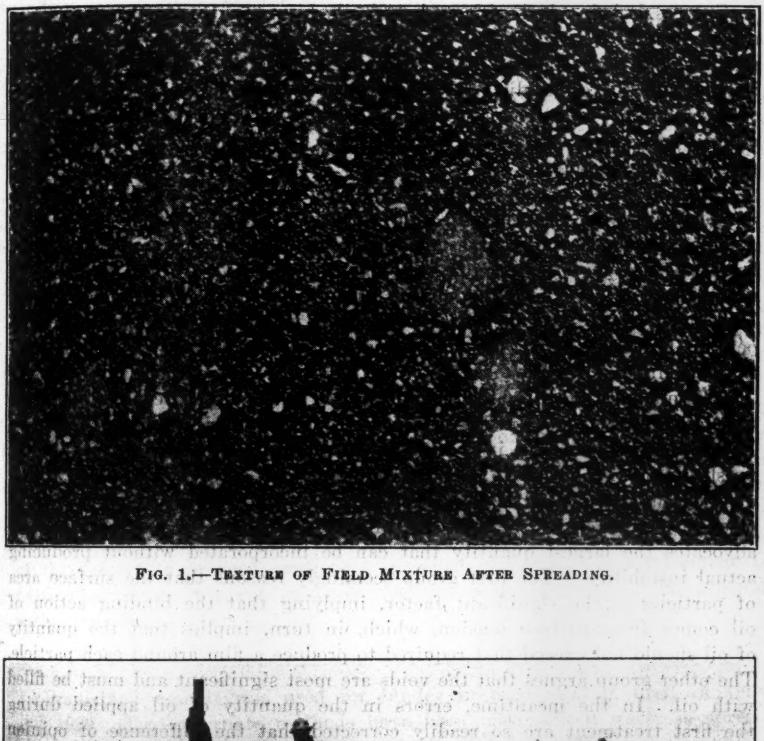

FIG. 1.—TEXTURE OF FIELD MIXTURE AFTER SPREADING.

FIG. 2.—FIELD MIXING, MOVING MATERIAL FROM ONE WINDROW TO ANOTHER.

Quality and Proportion of Oil

Success is being had with asphaltic oils from the Rocky Mountain area, although their characteristics are somewhat different from the California product. So many factors affect the success or failure of mixtures that the influence of the oil is difficult to segregate. Plant mixtures offer a better opportunity to study the effect of oil than road mixtures. Undoubtedly, oils differ in their suitability, and a strong effort should be undertaken to ascertain the reason.

After the first season of oil mixing the medium grade of fuel oil with an Engler specific viscosity of 25 to 45 at 50° Fahr., carrying 60 to 70% of 80-penetration asphalt, was generally recommended. Subsequently, there has been a marked tendency to raise the viscosity to 55 to 85, which is the grade generally used for plant mixing, and the asphalt content to 65 per cent. The higher viscosity generally carries with it higher specific gravity, and perhaps higher ductility, but adds somewhat to the difficulty of mixing, particularly in cool weather.

The quantity of oil which should be used is much debated for the reason that aggregates vary widely in grading and in their absorptive qualities. For estimating purposes it is common to allow ½ gal. per sq. yd. per in. of finished thickness, with 15 to 30% allowance for extra requirements. The quantity used generally falls within the range of the estimate. One group of engineers advocates the least quantity that will coat the particles; another advocates the largest quantity that can be incorporated without producing actual instability. The first group seemingly reasons that the surface area of particles is the significant factor, implying that the binding action of oil comes from surface tension, which, in turn, implies that the quantity of oil should not exceed that required to produce a film around each particle. The other group argues that the voids are most significant and must be filled with oil. In the meantime, errors in the quantity of oil applied during the first treatment are so readily corrected that the difference of opinion is not serious. However, it is obvious that the less oil used, the cheaper will be the construction cost, a statement which applies with particular force to projects distant from the source of oil supply.

Width, Thickness, and Cost

The tendency to use a minimum width of 20 ft. became marked during 1929. Likewise, a compacted thickness of 3 in. is becoming established. Few projects are being constructed less than 2½ in. thick. There is some tendency to use a greater thickness than 3 in., especially on plant-mixed projects.

Cost of treatment exclusive of road metal continues to range rather widely from about $1 200 to as much as $2 000 per mile. The foregoing applies to averages. Individual projects within a State will vary even more widely, depending upon local conditions. The cost of oil is the largest variant. The cost of actual mixing, omitting the cost of oil and the application thereof, shows quite remarkable uniformity at $500, or less, per mile of 18-ft. road.

FAILURES

It is safe to state that the principal failures continue to arise from faulty support. Support may be deficient in thickness of metal or stability of sub-grade. As frequently stated, an impervious oil surface aggravates the effect of moisture in the sub-grade. Since many oil-treated projects are being undertaken upon metal insufficient for untreated conditions, it may be expected they will be found deficient when oil-treated, regardless of the method used. Emulsification of the light fuel oils in the presence of moisture and traffic is frequently reported. If the moisture comes from rainfall, the mixture can be protected by the application of a seal coat of heavy asphaltic oil and stone chips. Even a light application of the fuel oil used in the mixture, approximately $\frac{1}{8}$ gal. per sq. yd., covered with coarse sand, is commonly an effective protection against moisture and a preventative of raveling. Investigation shows that most cases are accompanied by a third condition, namely, the presence of an excessive amount of 200-mesh material, usually clay. Possibly the trouble may be avoided by eliminating clay from the mixture, which, however, is not always practical.

Rain during construction is a source of much annoyance. The immediate effect of free moisture is very similar to that of excess of oil, giving a deceiving appearance of richness and causing the material to gather in lumps while being mixed. If the moisture is retained after spreading, it produces waving and bleeding. The effect of moisture on the mixture during construction and afterward makes it advisable at the present stage of knowledge of the subject to proceed cautiously in moist climates. This statement is particularly true of plant mixtures, because a wet pit cannot be utilized without drying.

EMULSIFIED AND CUT-BACK BINDER

Emulsified asphalt was used for binder in road mixing, first, in 1928 and 1929. Two alternate methods have been used: First, water is spread upon the aggregate prior to and during the mixing to prevent the emulsion from breaking until mixed; and, second, a preliminary application of $\frac{1}{4}$ to $\frac{1}{2}$ gal. per sq. yd. of fuel oil is incorporated into the dry aggregate, improving the mixing qualities of the emulsion. A seal coat of about $\frac{1}{4}$ gal. of emulsion per sq. yd., covered with fine stone chips, is found advantageous to complete either method. Emulsified asphalt has been used in surface treatment with marked success. Cut-backs have also been used, although not extensively in the West.

Following experience with emulsions, it was suggested that cut-back asphalt or heavy road oil would be a suitable binder for a mixing treatment. This type of binder is accepted in California as having passed the experimental stage.

In using emulsions or cut-backs in road mixtures, it is hoped, of course, to produce a harder and more stable surface than can be attained with fuel oil. Engineers and highway authorities are not quite prepared to believe that an asphalt commonly designated "fuel oil" has sufficient binding properties to make a really successful pavement. Oil mixtures admittedly are soft and easily disturbed when first laid, whereas asphaltic concrete hardens at once; hence, it is reasoned that the early hardening of asphalt applied as an emulsion or a cut-back is highly desirable. The truth or error of this reasoning will not be known until a number of projects have been subjected to several years of service, after which a study of their construction and maintenance costs should furnish the answer.

In this connection, much should be learned from a test road in the Truckee River Canyon constructed in 1929 by the California Division of Highways. Thirteen road-mixed sections were built, which were followed in the spring of 1930 by a number of surface-treated sections. In the mixed sections two grades of fuel oil, two of cut-back, and two of emulsified asphalt were used, in combination with crushed gravel and crushed rock of varied grading. Accurate material and cost data were recorded. Although the combinations constitute only a few of the many possibilities, their behavoir and the records of maintenance costs should be very informative.

NEW CONSTRUCTION

First undertaken as a maintenance measure, oil treatments of both types are beginning to have a profound effect upon design and new construction in the Western States. Few Western State highways carrying as many as 500 vehicles per day of summer traffic are being built without a definite program for oiling, and, in many States, the traffic limit is even less.

Approaching the use of oil from the construction standpoint, surface treatment has the disadvantage of requiring a well-compacted base, which ordinarily cannot be had without a tedious wait, or by using an undesirable clay binder. Road mixing requires special equipment that, ordinarily, contractors do not have, and surfacing is quite commonly finished during the season when oiling is impractical. Consequently, many States are restricting oil mixing to their own forces, even on new construction, but are adjusting the grading requirements of new surfacing material to the needs of oiling. Plant mixing has the advantage of being suitable for contract operations, which, perhaps, is one reason for its general adoption upon new projects in California.

The introduction of oiling has brought about a new examination of specifications for low and intermediate types of surfacing. Much material formerly considered too fine has become available. Binder, or filler, is being scrutinized more carefully. On the other hand, the possibilities of penetration macadam and the elimination of fine crushing are attracting attention. Thus far, the facility with which fine-crushed material may be manipulated with machinery, as contrasted with hand labor and other expensive operations for macadam, shows the oil mixture to be much the lower in price.

Oil treatments have never been seriously proposed as a substitute for standard higher types of pavements, but they are actually encroaching upon that field to some degree where freedom from dust and convenience to traffic dominate the selection of type. Heretofore, for lack of a satisfactory or cheap intermediate type, roads carrying from 500 to 1 000 vehicles per day have frequently been paved, but oil treatment is now being found sufficient for at least immediate purposes. Thus, oil treatment is finding its place, mainly at the expense of the lower types, but to some degree as a substitute for the higher types.

DISCUSSION

H. J. SPELMAN,[5] M. AM. SOC. C. E. (by letter).—This paper is an interesting and valuable review of present practice in the Western States in the bituminizing of crushed rock and other traffic-bound surfacings. Mr. Frickstad discusses three classes of such treatments, namely, surface treatments, road mixes, and plant mixes. The total of 12 000 miles completed in those States indicates the importance of the subject.

There would appear to be no reason why the road-mix and plant-mix types should not approach in wearing qualities the asphaltic concrete, Topeka, or sheet asphalt surfaces, the characteristics of which they approach more or less closely.

There are two points in particular, which it is believed should be specially stressed. The first is the importance of sufficient manipulation or spreading, both in plant mixes and road mixes, to insure smooth riding qualities. If plant mixes are to be used more and more, as indicated in the paper, this is particularly important if this type is to give general public satisfaction. The second point is that failures are due principally to lack of sufficient base. It is an old, old story that most failures of pavement or surfacing are due to that cause, and it is well to emphasize that these new types will not take the place of bases of adequate thicknesses.

No mention is made in the paper of the so-called retreads which have been developed quite extensively in some of the States east of the Mississippi River during the time that these oil mixes have been developed in the Western States. As the types covered by Mr. Frickstad compare in characteristics with the older sheet and asphaltic concrete, so do these retreads compare in characteristics with the so-called, mixed bituminous macadams of 20 to 25 years ago, or with the "cold-patch mixtures" used so long for patching almost all kinds of pavements.

These retreads require no material less than $\frac{1}{4}$ in. in size, and the usual size is from $\frac{1}{2}$ in. to $1\frac{1}{2}$ in. Like the oil-mixed surfaces discussed by Mr. Frickstad they are constructed either by the road-mix or plant-mix method. Their increased cost over the Western oil mix is probably due only to the cost of the coarse aggregate which is hauled in and placed on the road just in advance of operations. Cut-back asphalts and tars have both been used successfully in their construction and more recently the development of a slower setting emulsion has enabled that material to be used with excellent results.

This type of construction has proved of great value in the East in the smoothing of old rough macadams, as well as in providing low-cost, dustless, wearing surfaces for the traffic-bound types. It is probable that its development was largely brought about because of the larger number of commercial plants ready to furnish the needed coarse aggregate of crushed stone, slag, or gravel. As in the Western types the riding qualities of this road-mix type are equal to those of any pavement. Failures have been generally attributable to the base, and plant mixes have not produced as smooth surfaces as road mixes. The smooth riding qualities of this type of construction are due primarily to the use of heavy road machines and heavy drags, which has been made possible by the development of tractors of the heavy, crawler type. If plant mixes are to compare favorably in riding qualities with road mixes, the use of similar methods of manipulation is essential.

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[5] Senior Highway Engr., U. S. Dept. of Agriculture, Bureau of Public Roads, Washington, D. C.

The dividing line between surface treatments and retreads is not particularly well defined; nor is the term, "retread," a particularly satisfactory one for this type of construction. The term, "road mix," is a better one. The class of road mix used in the Western States might be called "fine aggregate road mix," and that used in the Eastern States, "coarse aggregate road mix."

Certainly, the dragging of all surface treatments is highly desirable, in order to obtain smooth riding surfaces. A suitable method of differentiation between retreads or road mixes and surface treatments might be to name as "retreads" or road mixes those jobs that involve the use of road machines, harrows, or other mixing equipment; and to call those surface treatments, that involve only dragging and rolling.

Mr. Frickstad's contribution is particularly important and timely because the increasing speeds of all classes of vehicles, as well as the increased number in use, have caused the road users everywhere to demand not merely a usable, all-year road, but a dustless, usable, all-year road.

J. T. L. McNew,[6] M. Am. Soc. C. E. (by letter).—Experiences of Western engineers with so-called "low-cost" bituminous roads have been summarized briefly in this paper. The term, "low cost," may be open to the objection that poor alignment, grade, and workmanship are to be permitted to accomplish "low cost." Invariably, the term is taken by the client to mean small outlay of money, and, in time, it may defeat the real purpose which engineers have had in mind as these surfaces for moderate traffic have been developed. Engineers are agreed that alignment, grade, and structures are probably the most permanent features of a highway; that safety to traffic demands that curves be flattened, widened, and superelevated; and that sight distance must be provided. They also recognize the importance of establishing grades to meet the demands for convenience of the motoring public. These demands must be met almost without regard to surface, and when so met they form a part of the permanent investment. "Low cost," therefore, is not to be applied to these appurtenances, except perhaps in the case of drainage structures themselves.

The term as applied to the surfaces then remains as the subject, and even yet it is somewhat misleading to the client. Much data are available to show both success and failure of so-called "low-cost" surfaces, and usually the promoter who reaches the client first uses the data most convenient to carry the question in favor of his particular product or type of surface.

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[6] Prof. of Highway Eng., Agri. and Mech. Coll. of Texas, College Station, Tex.

Engineers must soon make public their intentions of recommending, designing, and constructing road surfaces to withstand present traffic requirements, plus the requirements of any increase in traffic that may reasonably be expected within a few years. Naturally, then, first cost, maintenance cost, amortization costs, transportation costs, and salvage value all must be considered in type selection to meet the needs of traffic in a locality. When, and only when, engineers accept and begin practicing these fundamental principles will the public begin to get the maximum possible mileage of serviceable roads, and only then will it recognize the fallacy of the idea so often expressed that "this or that surface is permanent." A highway is a structure that must be improved progressively as traffic and maintenance costs demand, and if roads continue to be needed for transportation purposes, they must be rebuilt time and time again in the future.

Bituminous surfaces have been brought to the attention of engineers rather forcibly in recent years and although several different methods are in use they are, in the final result, very similar. Much research has been reported, giving valuable information and guidance, and much more remains to be done.

The mention of the question of asphalt content invariably induces discussion. There are those who regulate the quantity by empirical rule; those who proportion by eye; those who insist that the stain test determines the quantity; and those who proportion the bitumen according to the voids in the aggregate. Naturally and logically the characteristics of the oil have a bearing upon the question as to how much bitumen the surface mixture must contain. Time (or perhaps it might be expressed as "age" of sample) also is a factor, because the light oils or topped crudes must certainly weather or volatilize more than the heavier grade oils. If, by bitumen content, is meant that content at the time the mixture is opened to traffic, and if any rule of proportioning is adopted for two mixtures containing different oils, then these two mixtures are alike in content at the beginning, after which they show variations in bitumen content dependent on the traffic using the surface and the climatic conditions as well. In general, the higher the percentage of asphalt of 80–100 penetration that an oil contains, the more stable it will be under atmospheric conditions and the more nearly true the given proportioning rule will be found.

The difference of opinion between the stain-test advocates and the devotees of the voids theory are not really as great as may appear, for each theory has its correct place in modern practice. Stability tests made on paving mixtures invariably indicate that, when bitumen in excess of that necessary to fill the voids in the compacted mineral aggregate is used, the stability as measured by resistance to shear is decreased. With these results the voids-theory enthusiast draws the conclusion that his theory is correct. On the other hand, the proponent of the stain test (which is no doubt a modification of the pat test as described by Mr. Clifford Richardson[7]), insists that the particles of aggregate should be coated with bituminous binder. Unfortunately, however, if the aggregate is proportioned for low voids in the compacted aggregate this coating of bituminous binder, as determined by the stain test, will prove to be an excess over that required to fill the voids, with the result that the stability test will indicate a weak mixture. If the aggregate contains more voids, then the stain test does not give an excess over that required for voids; hence, stability as affected by bituminous content will not be impaired. Of course, the voids theory must be limited for use with mixtures of aggregate that have sufficient voids to accommodate asphalt in an amount that will impart another property to the surface, which is perhaps more important than any other, namely, that which may be called "flexibility," or, perhaps, "resistance" to suddenly applied forces.

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[7] "The Modern Asphalt Pavement", by Clifford Richardson.

The writer has conducted investigations in an attempt to find the critical bituminous content of certain paving mixtures below which excessive brittleness or lack of flexibility may be expected. The data accumulated from this work indicate that maximum resistance to displacement caused by suddenly applied loads invariably requires more bitumen content than is needed for maximum stability under static or gradually increasing load, as used in the various stability tests.

As the author has indicated, proportioning by empirical rule or formula has value only as a means of estimating bituminous content. In presenting the formula for the percentage of oil (Equation (1)),[8] C. L. McKesson, M. Am. Soc. C. E., stated: "It is not applicable to porous or absorbent materials, such as cinders or lava, but otherwise it appears to give results which are consistent with service obtained from various mixes." The writer wishes to emphasize also that, perhaps, Equation (1) will not apply to oils different in characteristics. Some oils contain fractions of highly volatile compounds, whereas others contain practically no very volatile fractions, and the different oils are absorbed in different quantities by a given aggregate. It must always be kept in mind that the term, "oiled road," may be applied alike to surfaces that may contain crude oil, topped crudes, fuel oil, residual oils, emulsions of any of the heavy residuals, or cut-backs, and that the determination of the proper bitumen content should be predicated upon one of the proper quantity of the particular oil to be used.

Mention has been made previously of the fact that, dependent upon the oil used, the bitumen content will decrease with the passage of time and will be influenced by both the climatic condition and amount of traffic the surface carries. Bituminous surfaces that carry light traffic usually show higher losses of the volatile oils than those under heavy traffic.

Incidentally, the intensity of the maintenance may affect the choice of bitumen content. If a surface is to receive a mixed-in-place treatment of a cut-back, or light topped crude, and provisions are made to maintain the surface continuously, one would be justified in using a slight excess of oil with the knowledge that more maintenance would be required to keep the surface smooth until the oil has reached a more stable condition upon exposure to traffic and weather. The engineer will probably recall instances where "shoves" have occurred in bituminous pavements soon after the surface was opened to traffic and that later these places are the most stable spots in the surface and seem to hold up under the more severe impacts. Not all engineers recognize the fact that a bituminous surface does move about under traffic and that a period of readjustment does take place. During this period the base itself may settle or heave, and careful maintenance in early life is essential to insure a smooth surface. It is also during this period of adjustment that "self-healing" is a desirable characteristic of the mixture. The ability of a surface to rise or fall with the subgrade, and to heal over the crack formed, is dependent upon the content of bitumen in the mixture, that has real binding properties. Experience has taught engineers to evaluate an oil on this score by its asphalt content of 80–120 penetration.

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[8] "Light Asphaltic Oil Road Surfaces," *Public Roads*, September, 1927.

At times, one hears the statement made that the thinner the coating of the gluing agent the stronger will be the bond between two particles stuck together; and then the statement will be used as an argument for extremely lean mixtures. It is admitted that as bitumen content decreases to a given point, stability against shear as measured by static load increases, but the same law does not hold with the same quantities of bitumen if the stability is measured under impact loads. Bituminous surfaces do not have beam strength and no sacrifice of bitumen will impart sufficient strength to allow it to carry load as a beam.

If bituminous surfaces cannot be made to have slab strength sufficient to allow the surface to span soft spots when subjected to load, such facts should be admitted and a proper foundation prepared that will distribute the load over an area of such size that the subgrade will not be over-loaded. A sufficient quantity of oil should always be included, in order that the surface may follow the settlements of the subgrade or foundation course without cracking and subsequent disintegration.

The author calls attention to the fact that no standardized grading for aggregate for low-cost surfaces has been adopted. In the light of present knowledge of bituminous pavement practice perhaps no standard will be adopted. Surfaces falling naturally into the class of "low cost," for the most part, have been made of local materials on account of limited funds. Undoubtedly, the success of the methods of constructing these surfaces with various gradings of aggregate is evidence sufficient to disclose the fact that there are many satisfactory gradings. It is only within recent years that asphalt technicians in the field of hot-mix pavements have disclosed the fallacy in the assumption that any one grading is necessarily the best and that all others must be discarded. To standardize the practice and accept only one grading would defeat the element of "low cost" remaining. Concerning the theory of low-void aggregates there follows the fact that any number of combinations of mineral aggregate can be made with reasonably low total voids and minimum size of individual voids.

Mr. Frickstad states that the part of the aggregate passing the 200-mesh sieve is an important element, and the writer is fully agreed that such material of proper kind is beneficial. At the same time, attention should be called to the fact that this 200-mesh material must be controlled very carefully as to quality, such choice to be dependent upon the climatic conditions to which the surface is to be exposed.

Unfortunately, clay, the worst soil with which highway engineers must deal, passes the 200-mesh sieve. Certain classes of clay have the greatest attraction for water, the largest shrinkage factors, the poorest bearing values, and the highest absorptive properties of soils. Incidentally, some clays have greater affinity for water than for asphalt or tars. An aggregate containing appreciable quantities of such material is of doubtful value for districts where seasons of heavy rain exist. In some sections of the West and Southwest good results have been obtained with aggregates containing clay up to 25 per cent. The success of these surfaces has been due no doubt in part to good drainage for occasional rains and the absence of prolonged wet weather. Excess clay in oiled surfaces, in localities where rainy seasons are prevalent, usually makes its presence known as an extremely slippery surface and one in which the asphalt apparently emulsifies during the rain and immediately afterward.

The quantity of 200-mesh material, as well as the quantity of aggregate passing the 10-mesh, must be used somewhat as a guide as to the choice of oil. Aggregates containing "fines" similar in amount to the Topeka mixtures are much more difficult to mix uniformly with the heavier oils of 80 to 90% asphalt. Aggregates containing such quantities of oil are readily miscible with light oils, emulsions, and cut-backs, and are likely to require more care during the curing period. The writer has used the Topeka class of mixtures with some success on maintenance work by first introducing a distillate to moisten the aggregate in the mixer and then adding the heavier oil. As indicated previously, however, the mix does not reach good stability values until time enough has elapsed to allow a considerable quantity of the volatile fraction to evaporate. This method, therefore, requires careful maintenance until the surface has "set."

The writer feels confident that the experience of the Western and Southwestern States with these so-called "low-cost" surfaces has brought a means of extending the mileage of serviceable roads and has done much to focus the attention of taxpayers and engineers on the undisputable fact that highway-surface betterment is a problem to be solved by progressive improvement wherein each successive stage of development will realize the maximum salvage value of the last surface. It must be admitted, however, that there are yet many problems to be solved with these low-cost bituminous surfaces and that the acceptance of any set rule of mix design, bituminous content, or other construction method, without the proper care to see that such rules are acceptable for the case at hand, is open to numerous pitfalls to the disadvantage of the constructive work that Mr. Frickstad and his Western associates have so successfully carried to the present stage of development.

ROGER M. LEE,[9] ESQ. (by letter).—It is interesting to note that surface treatment of the type described by the author has been applied to more than 12 000 miles of roads in eleven Far Western States. Experience thus gained will be of very great value.

In Brant County, Ontario, Canada, on secondary roads tar and emulsified asphalts have been used, the latter being either plant-mixed or applied in the usual manner. Preferably, a little more money is spent to obtain a retread or, better still, a gravel asphalt is put through a standard asphalt plant and sealed with hot, mixed chips applied along with the remainder of the material so as to prevent the possibility of a slippery surface. With a reasonable base of gravel, 3 in of this surface would apparently provide fairly permanent results.

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[9] County Engr. and Road Supt., Brant County, Brantford, Ont., Canada.

E. Q. SULLIVAN,[10] ASSOC. M. AM. SOC. C. E. (by letter.—One of the first road surfaces constructed by the California State Highway Department, by the oil-mixing treatment, was 44.3 miles between Victorville and Daggett, in San Bernardino County. This road was originally surfaced in 1927 with a 4-in. thickness of crushed gravel, with an average grading approximately as follows:

Sieve	Percentage passing	Sieve	Percentage passing
Fuel oil	3.7	20	45.9
200	10.6	10	56.4
100	19.1	3	72.4
80	22.8	½	82.6
50	28.9	¾	93.3
40	33.5	1	100.0
30	37.8		

The surface was not oiled originally. After this road had been under traffic for one year, there was considerable loss of surfacing material, the exact amount being unknown as the subgrade was sandy and gravelly and some of it had worked into the surfacing. The road surface had not started to fail when it was oiled, but was still in fairly good condition.

Oiling was undertaken by the oil-mixing treatment described by Mr. Frickstad. The cost per mile was, as follows:

Oil furnished and spread	$577.76
Mixing	498.84
Oil for seal coat	94.51
Labor, etc., for seal coat	60.95
Total cost per mile for oil-treating the completed road	$1 232.05

This road has been under an increasing volume of traffic, which has averaged, according to 1931 traffic counts, about 70 trucks per day, plus 850 automobiles, or a total traffic count of 920 vehicles for the 16-hour period between 6:00 A. M. and 10:00 P. M. The average maintenance cost has been as shown in Table 1.

The condition of this road surface has been kept continuously good, including the edges of the surfacing. Since the original gravel surface was only 18 ft. wide, the road is rather narrow for the increasing width of automobile stages and trucks using it. The increased cost of maintenance is due to increased expenditure on the edges of the road surface. Other maintenance has consisted of re-mixing a few short stretches of road surface that showed signs of being too rich in consistency, with resulting corrugations. Probably not more than 1% of the total road surface has been re-mixed as a maintenance measure.

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[10] Dist. Engr., State Div. of Highways, San Bernardino, Calif.

A few broken places developed in this road surface during the first three years, perhaps an average of one such place per mile. These broken places were due, in general, to rodents that excavated burrows, thus causing failures 1 ft. or 2 ft. in diameter. The cost of such repairs is very small. The major maintenance work has been to hold the edges of the mixed surface from breaking.

TABLE 1.—SUMMARIZED COST PER MILE FOR MAINTAINING VICTORVILLE-DAGGETT ROAD

Year.	Road surface cost per mile.	Total cost per mile.
1927–28	$51.54*	$323.63
1928–29	92.46	174.05
1929–30	171.37	293.20

* Estimated; not accurately separated from total cost in this year.

Experience with this route indicates that with the present traffic it can be maintained indefinitely at about the cost indicated. Evidently, the cost of maintenance could be reduced materially if the road surface were widened to eliminate the tendency of wide vehicles to run off and on the edges, breaking them down.

Should the traffic increase on this road and more heavy trucking become established, it might be necessary to re-work the road at some future time, but it has been found by experiment that this road surface can be remixed to a greater thickness at small cost.

The road is constructed in a semi-arid country, on a well-drained sub-base of natural sand and gravel. In view of its success and the apparent permanent nature of the road surface, it was decided to extend the work across similar arid or semi-arid country where good, well-drained sub-base is available.

Alternate bids for field-mix *versus* plant-mix were called for in the first contracts. It was found that contractors bid approximately the same for either method of work, and subsequent calls for bids provided only for plant-mixing of the road surface.

The project provided for the construction of the subgrade by shaping, watering with a sprinkling truck, dragging, and then encouraging traffic to compact it for a period of two weeks. All contract work provided for a similar method of constructing the subgrade. This method is well suited to arid conditions; natural materials in arid regions usually cement and become very hard under this treatment. Exceptionally smooth riding qualities are attained in the profile by this subgrade construction when long drags are used.

The standard of work has been increased in later projects in view of the expectation of heavier traffic. The road constructed under the first contract has a thickness of from 2 to 3 in., but all contracts let have provided for a loose thickness of 5 in., which results in a compacted thickness of about 4 in.

The cost of the Victorville to Daggett project does not include that of crushing and spreading the original crushed gravel surface or the cost of the subgrade, but the cost of the subsequent projects in Table 2 includes that of crushing the gravel, plant-mixing the material, furnishing the oil, constructing the subgrade as outlined herein, spreading the material on the subgrade, windrowing the material, and spreading and laying it down for traffic to compact.

Table 2 is an analysis of the cost of seven typical projects covering 133 miles of road. These were all new roads, the surfacing being placed on new alignment on a newly completed grade. The cost of grading and structures is not included in this analysis. The cost of water used in sprinkling the subgrade has been separated from the remainder, because conditions vary so much in this respect in arid regions. The water for some of these projects was hauled in tank cars on the railroad for a distance of 60 miles. On other projects the contractor was able to develop water by drilling wells.

TABLE 2.—ANALYSIS OF COST OF SURFACING ON SEVEN TYPICAL CONTRACT JOBS COVERING 133 MILES OF ROAD

Location.	Miles.	Cost of Water for Sub-Grade.		Cost of Surfacing in Place, Including Oil and Seal Coat.		
		Total.	Per mile.	Total.	Per mile.	Per ton.
1½ miles northeast of Yermo to 1½ miles southwest of Dunn . .	20.78	$6 885.10	$331.33	$83 791.75	$3 998.04	$1.63
4 miles west of Hector to 2 miles west of Argos..............	14.00	4 232.00	302.29	63 810.50	4 557.89	1.72
Daggett to 4 miles west of Hector.	21.22	4 361.70	205.55	101 728.14	4 793.97	1.78
Barstow to 1 mile east of Yermo.	13.05	7 040.00	539.47	60 659.50	4 648.24	1.78
1½ miles west of Siberia to 6½ miles east of Amboy.........	22.38	17 500.00	782.30	130 437.60	5 830.92	2.19
9½ miles west of Hopkins Well to Black Butte................	22.10	8 542.90	386.56	133 375.78	6 035.10	2.36
2 miles west of Argos to 1½ miles west of Siberia.............	19.47	16 000.00	821.78	123 045.00	6 319.72	2.38
Averages for 133 miles of construction....................	18.998	$9 223.10	$481.32	$99 549.75	$5 169.12	$1.98

It has been found necessary in District VIII of the California State Highway Department, to surface-treat all projects with fuel oil and screenings. In general, it is most desirable to make the mix so dry that there is a tendency to ravel without this surface treatment. The desirable feature in requiring a dry mix lies in increased stability, resulting in elimination of any tendency of the road surface to creep or corrugate; the possibility of bleeding is also eliminated.

The treatment has been applied immediately after completion and compaction of the road surface. The cost of this treatment has averaged $155.46 per mile on day-labor work.

These roads, completed by plant-mixed methods, have given an opportunity to analyze reasons for best results. Experience in California to date indicates that there are three major points governing the attainment of best results, namely, workmanship, quality of oil, and quality of aggregate.

Workmanship.—It has been found that even when the quality of materials is not of the best, reasonably satisfactory results can be obtained if additional skill and effort are put into the workmanship. By this is meant, road-mixing and remixing, with careful adjustments in the quantity of oil.

Quality of Oil.—Plant-mixing has clearly brought to light the fact that, of several oils passing California State specifications, there is a marked difference in results.

Quality of Aggregate.—It has been found that the quality of aggregate influences results, not so much because of qualities of hardness of the particles as qualities of natural binding value in the aggregate. Clay in the aggregate, however, is found to be detrimental. In general, an aggregate having the best mechanical lock, will make the best aggregate for oil-mixing treatment.

It is believed that, if desired, the plant-mixed roads can also be remixed by field methods at any time, with any desired thickness of material added to increase the strength for possible increased stress of heavier traffic.

Conclusion.—The oil-mixing treatment for graveled roads is particularly suited to arid and semi-arid regions, where such roads are subjected to reasonable loads and to moderate traffic. Maintenance costs are low under these conditions. The traffic that these pavements can withstand is as yet undetermined since there is no sign of general failure. The strength can be increased to meet increased future demands.

There were two major developments in construction methods during 1931. The first was the discovery that heating the aggregate on plant-mixed jobs results in a saving in cost. The majority of contractors installed and used heating equipment at the plant, although this was not required under the specifications.

Two major savings are affected by heating the aggregate: (1) The power required to turn the pug-mill is greatly reduced; and (2) plant production is increased because the more rapid coating of the particles of the aggregate with oil reduces the time of mixing.

The second major development of 1931 was the marked improvement in riding qualities of plant-mixed surfaces. Previously, most engineers believed that it was impossible to secure, on plant-mixed jobs, the ideal riding qualities of the road-mixed jobs. It is now found that the same ideal riding qualities can be secured by requiring an appropriate amount of blading of the plant-mixed material. Contractors' bids indicate that there is no appreciable increase in cost in specifying a definite amount of blading with relation to plant output.

Hugh W. Skidmore,[11] Assoc. M. Am. Soc. C. E. (by letter).—Prime desiderata of low-cost road development are: Mileage, serviceability, and cost. The aim is to secure the most miles of the most serviceable road at the least possible cost. A logical development of this idea naturally lends itself to "stage" construction in accordance with the demands of traffic, and entails the utilization of local aggregate materials and, most generally, some one of a variety of bituminous binders.

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Low-cost road construction has developed rapidly during the past few years due to recognition of the economic fallacy of expensive pavements on secondary roads and, as yet, rather sparsely traveled sections of trunk highways. A very great mileage of highway pavement, costing from $25 000 to $50 000 per mile, was unwisely built in many sections of the United States before low-cost road construction reached its present widespread application and before lack of funds forced economies in highway expenditures. One of the benefits of the present financial depression to the highway construction industry is the impetus it has imparted to low-cost road development mainly through the recognized necessity of securing greater road mileage per dollar spent.

The general scope of low-cost road construction is quite broad, both geographically and economically. Every section of the continent is deeply concerned in its application and a considerable number of localities have progressed to a quite well standardized procedure. With respect to cost, a wide variety of types and thicknesses, ranging from $1 000 to $15 000, or more, per mile, may quite properly be included, as compared with expenditures of from $25 000 to more than $50 000 per mile for so-called "permanent" types. In many instances the latter types are not living out the bond issues under which they were constructed, or they have reached a condition in which the maintenance cost is prohibitive. Resurfacing of these originally expensive pavements constitutes a separate field of application of low-cost surfaces which is likewise developing rapidly.

The exact nature of the composition used in low-cost road construction is entirely a matter of locality and the materials available therein, considered in the light of the traffic to be served. Thus, in certain sections of Nebraska, blow sands are the only locally available aggregates, while in other sections of the State a very finely graded gravel (almost a coarse sand, known as Platte River gravel), is the only aggregate available. Some States are blessed with an abundance of both gravel and durable stone, while in some instances very large districts are almost devoid of any kind of suitable aggregate. The Gulf States have available large quantities of oyster shells. Hence, it is clear that the development of pavement or treatment type is peculiarly a strictly local problem, and several varieties may be required within a given State.

A wide variation in aggregates entails a wide range in bituminous binders, from road oils, which in some cases are little more than topped crudes, to highly refined asphalts. Tars also include a wide range of consistencies from very light, volatile, priming tars to heavy, viscous, hot-application binders. Some binders, both tar and asphalt, are cut-back or diluted with a more or less volatile solvent (depending upon the nature and gradation of the aggregate) to enable the use of heavy, viscous binders at air temperatures, or slightly above, allowing the diluent to evaporate rapidly or slowly as the case may be. This evaporation begins during construction as soon as the bitumen is applied to the aggregate and continues for short or long intervals after construction, depending upon the volatility of the diluent used. Likewise, both tar and asphalt are emulsified with water which forms a non-adhesive, mobile liquid, enabling them to be used cold. Some time after application, the emulsion breaks, freeing the water and causing the composition to "set;" that is, the bitumen resumes its normal state of adhesiveness. By this means, quite viscous bitumens may also be utilized without using heat. Emulsions possess the added feature of being applicable to damp aggregate and, in fact, they often function best upon moist surfaces.

While a great variety of aggregates exists, there are also many types of binder available so that practically any aggregate condition may be met. The selection of the most suitable bitumen is not always immediately apparent except in a general way. Traffic, climate, and aggregates are the principal determining factors, but frequently considerable experimenting is required and some modification of existing binders is indicated. Naturally, intimate knowledge of bituminous materials is highly specialized and the application of such knowledge to the problem is essential to economic success.

The simple surface treatment of roads with some form of oil or tar will perhaps always be more or less common, solely as relief from the expensive dust nuisance. Most highway officials will freely state that such treatment is easily warranted for this reason alone, to say nothing of the preservation of road surface and metal afforded thereby. It is not at all uncommon to find gravel and stone road surfaces losing the equivalent of an inch of metal per year from wear, when untreated.

Surface applications are not permanent, however, and require more or less frequent renewals. This may become burdensome and then a more substantial pavement surface is indicated. Road mix or plant mix, utilizing some one of the several cold application binders, quite generally follows surface treatment when it becomes inadequate or too expensive. Conditions may justify even more substantial construction, such as bituminous concrete or sheet asphalt. It is certain, however, that the next decade will witness an increasingly vast mileage of simple, inexpensive, cold bituminous mixtures. Recognition of this fact has resulted in numerous patented types and processes, some of which have merit. Realization of the progress to come in this field has also encouraged manufacturers to apply their engineering skill to the development of equipment which will not only enable more miles to be built for less money, but will insure better pavement by eliminating the human element with respect to uniformity of mixture composition and smoothness of road surface.

Designers, builders, and users of pavements all desire smoothness of surface initially, and as long thereafter as possible. The engineer, aside from professional pride in his creation, realizes that smooth surfaces which retain their smoothness eliminate a costly source of maintenance; the builder, from a plain business point of view, knows that a superior surface is an important aid in securing additional work, and if he is in any degree responsible for maintenance, as is frequently the case for at least the early life of the pavement, he, too, appreciates the effect of initial smoothness upon subsequent expense; the user, obviously, is most intimately and frequently apprised of poor riding quality and he is seriously concerned as a taxpayer in both initial and ultimate cost. Of these three individuals, the engineer, in the main, must assume the burden of improvement and he is, therefore, especially interested in new methods, new materials, and new machinery that may offer possibilities of superior workmanship, greater durability, and reduced cost.

Any bituminous paving composition, regardless of hardness, will flow under pressure in proportion to the load applied and the length of time it is applied. Thus, resistance to distortion is mainly resident in ability to withstand shearing forces. In those compositions utilizing very soft binders, such as oils, very soft asphalts, and tars, stability is principally dependent upon close interlocking of aggregates since the binders are relatively fluid at normal temperatures. The more closely graded and better filled the mineral aggregate, the more effective, from the standpoint of stability, are very soft bitumens. This is another way of stating that as cements, they are more effective in thin films on closely packed particles than in thick films on aggregate masses containing large void spaces and relatively few contacting surfaces.

One of the major types of composition used in low-cost road construction is the rather open, coarse-graded type with mineral (either uncrushed gravel, crushed stone, slag, chatts, or gravel) coated with tar or asphalt, cut-back, or emulsified asphalt or tar, and sometimes a heavy asphaltic oil. There are numerous modifications of this general type, including variations in aggregate sizes from all coarse, large-sized particles; graded coarse particles exclusive of fines; graded stone or gravel plus moderate quantities of fines (sand or stone screenings), to rather well filled graduations, resembling graded bituminous concrete in texture. Binders requiring volatilization necessitate the use of fairly open mixtures with comparatively large void spaces; they are sealed with a surface application of liquid bitumen and chips, sand, or pea-gravel top dressing following an interval of time for evaporation of the diluent. When such binders are used in too dense a composition, volatilization is retarded or prevented to the extent that a crust forms at the surface, preventing complete volatilization in the lower portion of the mixture. Thus, cut-back binders and volatile oils are more or less restricted to open mixtures for the best results. Emulsions, however, can be successfully used in filled mixtures which develop ample stability to permit immediate compression and use by traffic. Suitable oils may be similarly used, but are limited as to the degree of stability developed in the mixture.

Mr. Frickstad confines his discussion to oil-mix and oil-surface treatment in the Far West, recognizing that there are many other similar and entirely dissimilar developments in other sections. Perhaps no section of the country has witnessed such extensive and varied application of low-cost road types as the Central States where practically the whole gamut of road mixes, plant mixes, and surface applications has been run, using such binders as light and heavy oils of asphaltic, semi-asphaltic, and non-asphaltic nature; tars, asphalts, cut-backs, and emulsions; and such aggregates as soils, sands, gravel, stone screenings, slag, chatts, and stone. Furthermore, a great number of proprietary formulas have been developed covering cold-mix, cold-lay; hot-mix, cold-lay; and cold-mix, hot-lay ideas, using all the various aggregates and bitumens. Some of these compositions depend upon the blending of hard, brittle asphalts with soft fluxes (oils) under the action of traffic. The real status of the matter is that this field of construction is just nicely started and outstanding developments, both with respect to cost and durability, may be counted upon during the next few years.

All engineers, and contractors as well, recognize that the greatest single influence in the direction of lower costs is the substantial increase in the production capacity of a given crew of workmen and quota of equipment. Obviously, then, real progress in this direction calls for the application of the inventive ability and mechanical ingenuity of equipment manufacturers. These men have not been slow to realize this fact and have wisely reaped the benefits of co-operation with engineers and contractors in developing machines of practical value. Every engineer realizes that improvements in methods and equipment are vital to real progress; therefore, it is surely proper that sincere recognition be given the designer and builder of machines which will give a better product at a lower cost.

During 1931 machines of the type shown in Fig. 3 have been developed. They have a wide application in the building of better compositions at substantially lower costs than have yet been possible. To the engineer or contractor who observes this equipment in operation, it is immediately apparent that plant mix of the very best quality can be manufactured, spread, and finished mechanically with much less effort and equipment, in greater quantities, and with positive assurance of far superior workmanship, than has been possible by road mix or plant mix to the present time.

The equipment comprises the application of well-known principles which eliminate the personal equation and insure uniformity of composition, perfect coating of aggregates, and a finished pavement of smooth surface and uniform denseness in all portions of the roadway. These very desirable characteristics may be obtained by the use of the full range of aggregates and binders, and upon any subgrade or base which is suitable to function in such capacity. The adaptability and flexibility of the equipment are two of its outstanding features.

The first unit consists of a large capacity, pick-up loader, with spiral feed, discharging into an overhead bin which, in turn, discharges upon a positive-feed, corrugated apron conveyor that has an adjustable, calibrated discharge which gauges, accurately, the continuous flow of aggregate. From the apron feeder, the aggregate drops through a baffled chamber where the bitumen is applied either hot or cold by means of two groups of high-pressure nozzles. One set of nozzles sprays the falling mineral as it leaves the apron, the other group as it leaves the baffles and just before it reaches the large-capacity, twin pug-mill mixer. These sprays are effective in coating at least two-thirds of the total surface.

The flow of bitumen is accurately controlled according to requirements by valves and a recording flow-meter. When the material reaches the mixer, it is already fairly well coated and by the time it has traveled the 8-ft. length of the twin pug-mill, it is thoroughly and uniformly coated. The machine has a capacity of 2½ cu. yd. of aggregate per min., operating at normal speed. The flow of mineral and bitumen is set according to the requirements of the mixture, and remains constant in this ratio until changed to accommodate a new condition. Thus, through continuity of flow of materials and synchronization of elements, exceptional uniformity of product is maintained.

With the loader mixer operating on the subgrade picking up the aggregate from windrows, the full benefit of sun and wind in drying the mineral is secured, thus eliminating costly mechanical drying which, if provided in sufficient capacity to supply the mixer adequately, would be prohibitive in many cases. Operating as a central plant at the source of aggregate, mechanical drying may be readily provided, but always with a substantial increase in cost of mixture.

The second unit is the spreader finisher, which also operates under its own power. Both machines are mounted on crawlers and are adequately powered to provide ample traction under the most adverse conditions. The finisher may operate directly behind the mixer with the mixture discharging directly into the spreader. The latter consists of two horizontal screw conveyors delivering mixture the full width of the road and in any finished depth up to 6 in. The screws operate independently so that the quantity may easily be controlled according to the condition of the subgrade.

Immediately back of the spreader trough is a double-faced tamper operating at 200 rev. per min., with a ¾-in. stroke. The tamper face is 1½ in. wide, divided into two steps of ¾ in., with a ¾-in. riser, with the shallower step forward. Immediately back of this is a screed shoe, 6 in. wide, operating on a half-cycle with the tamper. This ingenious arrangement nicely "tucks" the mixture under the screed and provides substantial compaction without any tearing or pulling effect. A 6 000-lb. force is applied directly upon the tamper, so that if those mixtures contain stable cements, including little or no volatiles, subsequent rolling may be eliminated, thus preventing the slightly "rippled" surface that always results from rolling even under the best of conditions. It has long been recognized that direct vertical compression of bituminous paving compositions is ideal if it can be applied uniformly and adequately.

The third outstanding feature of this unit is the application of a system of levers to smooth out humps and depressions in the subgrade. Long skid-shoes with lever-arm attachments to the main frame, extend ahead and behind the crawlers. The lever arrangement is such that a sharp bump of, say, 2-in. height in the subgrade will result in an imperceptible over-all rise of 1 in. in a distance 12 ft. in both directions therefrom. The principle upon which the subgrade equalizer works is demonstrated by Fig. 4, in which, *A* is a line drawn by straight-edge; *B* is an exaggerated, rough subgrade; *B′* is the same roughness transmitted to the pavement by the wheel; and *A′* shows the roughness removed by means of the equalizer. By this combination of crawlers and subgrade equalizer, the expensive item of forms is eliminated completely and a smooth riding surface is produced.

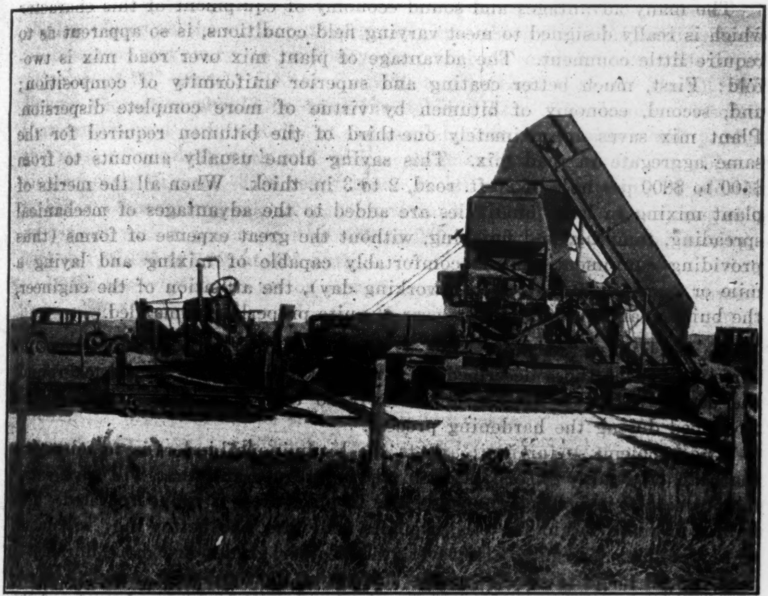

FIG. 3.—A MODERN TYPE OF ROAD-BUILDING EQUIPMENT.

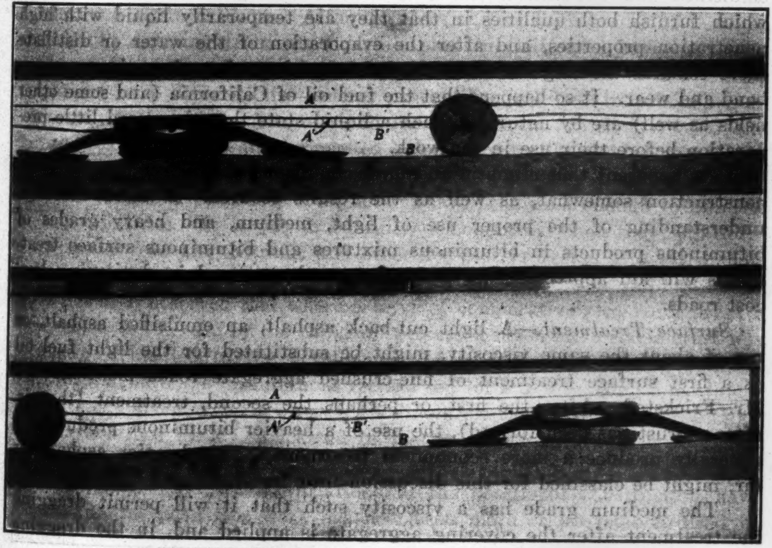

FIG. 4.—PRINCIPLES UPON WHICH A SUBGRADE EQUALIZER WORKS.

The many advantages and sound economy of equipment of this character which is really designed to meet varying field conditions, is so apparent as to require little comment. The advantage of plant mix over road mix is twofold: First, much better coating and superior uniformity of composition; and, second, economy of bitumen by virtue of more complete dispersion. Plant mix saves approximately one-third of the bitumen required for the same aggregate in road mix. This saving alone usually amounts to from $500 to $800 per mile of 20-ft. road, 2 to 3 in. thick. When all the merits of plant mixing in large quantities are added to the advantages of mechanical spreading, tamping, and finishing, without the great expense of forms (thus providing equipment that is comfortably capable of mixing and laying a mile or more of 20-ft. road per working day), the attention of the engineer, the builder, and the tax-paying user is quite properly commanded.

A. H. HINKLE,[12] M. AM. SOC. C. E. (by letter).—Bituminous types of surface, such as those described by Mr. Frickstad, require, as a background, the use of a binder that hardens slowly so that a smooth surface can be developed during the hardening process.

The problem, so far as it affects the bituminous binder, is to select one that can be readily incorporated in the aggregate and that will give satisfactory results in holding the aggregate in place after it is incorporated. The lighter or more liquid the oils, asphalts, and tars, the better they will penetrate the fine aggregate. However, the heavier grades are more durable if they can be made to "stick" to the surface of the aggregate, which is sometimes dirty. Thus, attention is at once brought to the use and study of the emulsified and cut-back asphalts, or similar acting bituminous materials, which furnish both qualities in that they are temporarily liquid with high penetration properties, and after the evaporation of the water or distillate there remains the original heavy, asphaltic product that gives the necessary bond and wear. It so happens that the fuel oil of California (and some other fields as well) are by nature, in such a liquid state that they need little preparation before their use in the work.

It is apparent that climatic and local conditions may affect the details of construction somewhat, as well as the results secured. A more thorough understanding of the proper use of light, medium, and heavy grades of bituminous products in bituminous mixtures and bituminous surface treatments will aid appreciably in the better results secured in developing low-cost roads.

Surface Treatment.—A light cut-back asphalt, an emulsified asphalt, or tar of about the same viscosity, might be substituted for the light fuel oil as a first surface treatment of fine-crushed aggregate roads mentioned by Mr. Frickstad. After the first, or perhaps the second, treatment (that is, after a crust has been formed), the use of a heavier bituminous product will generally produce a more economical treatment. This heavier asphalt, or tar, might be classified for this discussion into "medium" and "heavy."

The medium grade has a viscosity such that it will permit dragging the treatment after the covering aggregate is applied and, in the dragging operation, the aggregate coated with the tar or asphalt will be deposited in the low places on the surface, thus greatly smoothing it. This medium grade of bitumen may vary in viscosity for cut-back asphalt from 50 to 200 (Engler, at 50° cent.); for tar from 30 to 80 (Engler, at 40° cent.). The lighter viscosities should be used in cool weather. The heaviest viscosity that can be used satisfactorily at the available air temperatures will produce the most economical work and will be most satisfactory from the point of view of early drying. The heavy grade of bituminous material, in addition to being a heavy fuel oil, as mentioned in Mr. Frickstad's paper, might be an asphalt with a penetration of 175 to 250, at 25° cent., or a tar having a float test of 120 to 180, at 32° cent.

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[12] Chf. Engr., Maintenance, Indiana State Highway Comm., Indianapolis, Ind.

If the heavy grade of tar or asphalt is used, it should be covered immediately with $\frac{3}{4}$-in. to 1-in. maximum size stone, slag, or gravel (80 to 110 lb. per gal.), and may then be immediately "traveled" without the inconvenience of splashing on machines. This heavy bituminous material, however, does not have the advantage possessed by the medium or lighter grades of permitting the dragging of the surface after treatment.

Most bituminous pavements are subject to slight distortions and wear under traffic which gradually roughens their surfaces. These slight irregularities frequently can be eliminated by proper dragging with a long base drag just after surface treatments with a medium grade of tar or asphalt; such treatments are applied periodically as regular maintenance operations.

Mixing Treatment.—The "mixing treatment" (sometimes called "mulch treatment"), involving a light cut-back asphalt or light tar (specific viscosity, Engler, 40° cent., of 10 to 20), has been used extensively in some of the Central and Eastern States. Although light and medium grades of tars and asphalt oils differing from the fuel oils used in California have been generally used in this work, and although some of the details of the mixing operations have differed, the general process has resulted in producing an intimate mixture of the bitumen and loose aggregate, which is leveled and spread over the surface and compacted by traffic, sometimes aided by a roller. In this work, California fuel and similar oils in climates subject to heavy moisture combined with deep frost action, have been used enough to indicate their possibilities. It is yet to be fully demonstrated that they will form a surface as resistant to traffic and other destructive agents as a tar or cut-back asphalt made from an asphalt base having a penetration of 70 to 110.

It has been observed that where a layer of comparatively coarse aggregate was treated on the road by the "mixing" process, a much more stable crust was produced. This led to the development of what is termed, for want of a better designation, a "bituminous retread top" of 2 to 3-in. compacted thickness of stone about 2 in. in size. This differs primarily from the oil-mixing method described by Mr. Frickstad in that a coarser aggregate is used and the bitumen consists of cut-back asphalt, an emulsified asphalt, or a medium grade of tar. While many variations in the details of construction have occurred in the past, this work has now been more or less standardized in Indiana, as follows.

Any time from one week to six weeks after the oil is applied to the old stone or gravel surface, crushed stone, slag, or gravel (preferably ranging in size from 1½ to 2½ in.), is spread on the surface from trucks to a depth of about 3½ in. This aggregate is leveled by a heavy grader and the edges are "trued up" by hand. To this loose aggregate is applied about 0.5 gal. per sq. yd. of the medium tar, cut-back asphalt, or emulsified asphalt. The aggregate is then harrowed with a spike-tooth harrow, struck off with a grader, and rolled once immediately thereafter; as soon as the bituminous material hardens sufficiently so that the aggregate will compact and remain compacted, it is again rolled. A second application of bituminous material of about 0.4 gal. is applied and, after curing, the surface is again thoroughly rolled. Just preceding the third application of bituminous material, the voids in the coarse aggregate are filled with ¾-in. to ⅜-in size aggregate leaving a uniform and slightly excess layer of covering on the surface. Fig. 5 shows the arrangement of a brush and an old carpet, pulled by a small truck, used for this purpose. This covering is not rolled until after the third application of about 0.35 gal. of bituminous material, after which the surface is planed or dragged and again thoroughly rolled. Equipment for dragging this covering material after the third application is shown in Fig. 6. This process rolls the coated stone chips around over the surface until they drop into the voids or fill the depressions in the surface. (This same process of dragging is used, after re-treatment, in future years, with a medium grade of bitumen to restore the smoothness of a surface that may have been roughened under traffic.) A fourth treatment of about 0.17 gal. per sq. yd. of the bituminous material covered by ½-in. size aggregate may be used if necessary. This process resembles the "mixing treatment" in its first stages (except for size of aggregate) and its finishing operations are like the bituminous (penetration) macadam method.

The work of smoothing and leveling is largely done mechanically, and experience has shown that generally a smoother surface is produced and at a much less cost than with an ordinary penetration macadam. In Indiana, the cost of this top course, 3 in., compacted, including the oiling of the old road, for a 20-ft. width, has been from $3 800 to $6 000 per mile. Experience has shown that where this top is built on a suitable base, it will carry any ordinary traffic that may be found on the average State highway. The surface is maintained the same way as the ordinary bituminous macadam. In this bituminous retread top a low percentage of bitumen is used, which is made possible by means of the coarse aggregate.

While this bituminous retread top has ordinarily been built on an old gravel or stone road, it has also been constructed on old brick and old bituminous concrete surfaces that have become very rough. When this top is built on such surfaces, in lieu of the first oil treatment for the gravel and stone roads, the old surface is given about 0.2 gal. of medium tar or cut-back asphalt and covered with stone chips to prevent the layer of coarse aggregate (to be applied later), from sliding and shoving on the old smooth surface while construction operations are under way.

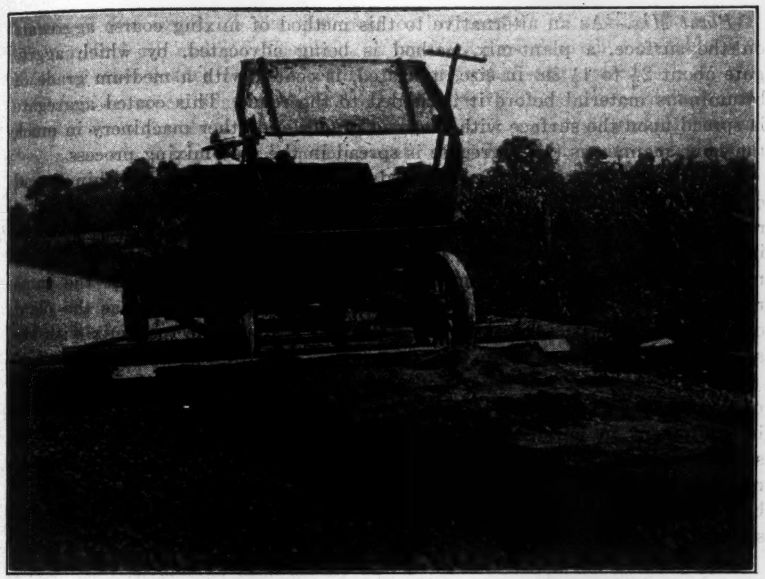

FIG. 5.—EQUIPMENT FOR BRUSHING THE COVERING MATERIAL INTO THE VOIDS OF THE STONE.

FIG. 6.—DRAGGING THE COVERING AFTER THE THIRD APPLICATION OF BITUMINOUS MATERIAL.

Plant Mix.—As an alternative to this method of mixing coarse aggregate on the surface, a plant-mix method is being advocated, by which aggregate about 2½ to 1½ in. in size, unheated, is coated with a medium grade of bituminous material before it is hauled to the road. This coated aggregate is spread upon the surface with the road grader and other machinery in much the same manner as the aggregate is spread in the road-mixing process.

This coated aggregate is compacted by rolling, after which a thin spread of ¾-in. coated aggregate is spread on the surface and the surface is again rolled. A system is now in use by which this aggregate is coated by immersing it in the asphalt or tar as it is unloaded by conveyors from the car. These methods of making a plant mix of coarse aggregate bear about the same relation to the bituminous retread described in this discussion, as the plant oil-mixing method described by Mr. Frickstad bears to the road mix with the same grade of fine aggregate.

Equipment.—Perhaps the use of a long-base heavy drag or planer described under "Surface Treatment" in this discussion, is the greatest development in equipment in building these cheaper types of bituminous roads. With such equipment a surface can be made even smoother than by any hand methods yet devised. This smoothing applies especially to the oil-treated base for the retreaded top where it is necessary to start producing a smooth surface if a smooth-riding, finished, top course is to be obtained.

Generally speaking, the equipment for this type of work may be that used in maintaining an ordinary traffic-bound gravel or stone road. While special machinery may be provided, as Mr. Frickstad states they are doing in some places, fortunately this is not necessary for counties and other smaller governmental units which will have occasion to build short stretches of this type of surface.

Grading of Aggregate.—The method described herein as "bituminous retread top," calls for the use of a comparatively coarse aggregate as compared with the fine aggregate used in methods described by Mr. Frickstad. With the coarser aggregate the question seems to be largely one of sufficient base in order to carry the loads rather than a question of sufficient durability of the top.

This coarse aggregate method has been used on some of the most heavily traveled roads in Indiana, and it can be said that, except for the necessity of treatments as a maintenance operation every year or so (which is some inconvenience to the traffic), the surface is holding satisfactorily.

Bituminous Materials.—It is fully demonstrated that cut-back asphalts, emulsified asphalts, and tars, are perfectly satisfactory for this work. When an asphalt product is used, the writer is inclined to think that best results will be secured by the use of an asphalt having a base with a penetration of about 80 to 120 and that it matters little whether this asphalt is put in a liquid form as a cut-back product, or as a suitable emulsified product.

While the fuel oil of California, and other oils, may not exactly comply with these requirements, they do not seriously depart therefrom; also the lower cost of the fuel oil and similar oils may more than justify their use even if they are not quite so satisfactory under certain conditions. With the use of fine aggregate the first coat of bitumen at least must be a very light tar, fuel oil, emulsified asphalt, or cut-back asphalt with a suitable distillate, as other grades will not readily penetrate such aggregate.

If further experiments with fuel oil mixed with aggregate containing an excess of fines proves successful in all climates, it will have great merit in those areas where much local fine aggregate is available, since, frequently, the local pits will supply this aggregate in about the proper proportions of fines that Mr. Frickstad describes as being used in California.

With the use of a coarse aggregate the percentage of bitumen is greatly reduced. For instance, the quantity used to coat the 1½ to 2½-in. stone will range from 2 to 3 per cent. With this coarser aggregate method there must, of course, be some finer aggregate used to seal the top and this requires a greater percentage of bitumen. Generally, however, the total in the coarse aggregate type, including the seal, is less than 4½ per cent. The low percentage of bitumen contributes to a lower cost and the coarse aggregate, if it makes any difference at all, makes a crust capable of carrying heavier traffic than the fine aggregate.

Due to the heavy base required in a country subject to deep frost action and moisture, it is doubtful whether new construction of this type will make a rapid advancement in such climates. If started from the ground to build up a base, the cost of this type would approach closely that of one of the standard pavements now (1932) being constructed. This is due to the extremely heavy base required on clay soils where the frost action and moisture are pronounced. However, in such climates, there are usually many miles of old stone and gravel roads which have been used for years and which frequently have a well compacted and solid base. These roads (especially where heavy trucking is not likely to develop), are very suitable foundations on which to build this low-cost bituminous surface. Frequently, they will serve the public as well as a higher type of surface.

Standardization of Types.—While a great variety of bituminous mixture is used as a top course (consisting of various sizes of aggregates and numerous kinds and grades of asphaltic oils, cut-back asphalts, and tars), it appears to the writer that the tendency is toward standardization into certain specific types which may be classified under the following five general classes, the first three of which are described by Mr. Frickstad:

1.—Surface treatments on traffic-bound gravel and stone roads wherein a light grade of bituminous material is used as a first application on the clean, compacted surface consisting largely of a fine aggregate; and, then, by successive treatments with medium grades of bituminous materials and the addition of aggregate covering, a bituminous crust is built up sufficiently strong to carry the traffic.

2.—The bituminous "mulch" or "oil mix" method wherein the light grade of bituminous material is mixed with fine aggregate on the surface to a depth of from 1 in. to 4 in.

3.—Coating the road with a plant mix 1 in. to 3 in. deep, wherein the mixture consists of an aggregate containing a large percentage of fines, which is spread on the road from a truck and leveled with a grader and other road machinery and maintained by blading and scraping until it hardens and traffic compacts it. (This mixture is virtually that described under Class 2, except that it is a plant mix instead of mixed in place.)

4.—A bituminous retread top made by mixing on the road an aggregate ranging in size from 2½ in. to 1 in. by successive applications of a cut-back asphalt, emulsified asphalt, or medium grade of tar, the first stages of this method resembling Class 2, and the final stages resembling the penetration macadam type.

5.—A plant mix of the same size of aggregate (unheated) as that used in Class 4, and an emulsified asphalt, a cut-back asphalt, or medium grade of tar made either by mixing the material by ordinary mixing-plant methods or by immersing it in such bituminous materials, the coated material being hauled to the road and spread in the same manner as in Class 3, and which, after thoroughly rolling and curing, is coated with ¾-in., coated aggregated applied and spread in the same manner, using a comparatively lean mix and sealing the surface with an ordinary seal or surface treatment coat if necessary.

In all the foregoing methods the low cost of the finished surface is brought about by the fact that the aggregate is handled, spread, and finished mostly by mechanical means, thus reducing the hand labor and expensive costs to a minimum; also, from the fact that cheaper processes are used in the mixing methods made possible by not heating the aggregate and in using a bitumen with a low viscosity and because frequently a less costly aggregate can be used in some of the work. Inasmuch as the aggregate is unheated in all these cheaper methods, it goes without saying that they are most suitable for use in a dry, warm climate, or during the dry, warm season in other climates.

Base Required Varies with Climatic and Soil Conditions.—Where deep frost action and excess moisture are prevalent, it is necessary to use greater precaution in having a suitable base than is required in the more arid districts and warmer climates. Many of the roads in California may be treated in the manner described by Mr. Frickstad, without developing the base to greater strength; but if that same base were used in other areas, it would prove a complete failure. It is evident that highway engineers are learning much about the construction of these cheaper types of surface and there is no doubt concerning their great value in developing at least a secondary road system. It is likely they will, as he states, encroach in many places upon the higher types of surface, yet it is extremely necessary that a thorough study of the local problems involved be made in order to avoid building such types where they are likely to prove failures. This would tend to discredit them in their legitimate and economical use in other places.

Perhaps there has been no phase of highway development that is more of a "missing link," than the need of a low-cost secondary road which will carry a limited amount of heavy traffic and an unlimited amount of light traffic. It is evident that the bill for the high-priced pavements on all highways will retard the proper development of any highway system unnecessarily.

Claude P. Owens,[13] **M. Am. Soc. C. E.** (by letter).—Few highway officials will question the need for a low-cost secondary road that will accommodate a limited heavy traffic and a fairly large volume of light traffic. The problem of keeping up with the fast-growing demand for increased road mileage and improved service with the limited funds available is one that faces every highway official. Traffic is not only increasing in volume, but also in speed. Certain types of surfaces which, a few years ago, were considered ideal and adequate, are now objectionable and unsafe due to the increase in speed and volume of traffic. Furthermore, maintenance engineers are finding their costs mounting to alarming totals such that road surfaces of the loose gravel and stone types cease to be economical. However, construction investments made only a few years ago cannot be discarded until their value has depreciated. Accordingly, any information as to the best methods of salvaging or treating such surfaces to reclaim the invested values, at the same time rendering improved service to the traveling public, is eagerly studied by highway engineers.

At highway conventions the problems and discussions on low-cost roads always occupy a prominent place on the programs. However, in the exchange of papers and discussions proper progress in solving the problems is seriously hampered by the lack of a common vocabulary, understandable by all. The terms introduced in such technical discussions are not clearly defined or are carelessly used. Even when specifications are quoted the discussion is often difficult to interpret. Different chemists have their own pet methods, and engineers their own taught or adopted practices. Viscosities run at 50° cent. are difficult to compare with those run at 60° or 40° cent. However, the U. S. Bureau of Public Roads has inaugurated a movement to bring the many groups together with a view of simplifying and standardizing terms, specifications, and testing methods. When this work is accomplished, the engineer will find it much easier and safer to compare notes, papers, and discussions.

There seems to be a general opinion that the present specifications and tests are not sufficiently comprehensive to be followed clearly or intelligently. Bituminous materials are derived from several sources and fields, each with varying and peculiar characteristics. An oil may contain 60 to 70% of 100-penetration asphalt, but what is the nature of this residue and what were the characteristics of the solvent or volatile portion? A cut-back asphalt may contain 25% solvent, but most specifications do not show, nor do common tests reveal, the nature of the solvent. Consequently, when reading or discussing technical papers on bituminous materials used in low-cost road construction, the engineer should do so with "his tongue in his cheek" and with a full understanding of the problem, if he desires to avoid costly errors. It is much better that he approach his problem hopefully, but carefully "feeling his way" with the materials and bitumens at hand.

The development of low-cost bituminous road surfaces in the Western States has been watched with considerable interest. The surface-treatment method is probably the oldest and best known. It requires a thoroughly compacted gravel or stone base. Usually, uncompacted or loose maintenance gravel that is swept from the surface, must be wasted on the shoulders. Most gravel and stone roads are none too thick at best to support surface treatments, and material should be added rather than removed. Furthermore, in compacting and cleaning the surface it is difficult to obtain the thoroughly smooth riding surface now demanded by high-speed traffic. Consequently, it would seem that the surface-treatment method has a rather limited field.

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Road Surface Mixes.—The road surface mixes (or plant mixes) seem more nearly to fill all requirements for low-cost bituminous road surfaces. There are several types of mixed-in-place construction, and each type allows the utilization of existing uncompacted road surface metal, or added metal, to build extra thickness and strength to the gravel or stone base. At the same time, these methods permit the leveling and truing up of irregularities of the original gravel or stone surface.

Oil Mix and Fine Aggregate.—In 1928, the Missouri State Highway Department selected a section on U. S. Highway No. 71, in McDonald County, to try out a mixture of road oil and fine aggregate. Methods outlined by C. L. McKesson, M. Am. Soc. C. E., in his "Report of the Co-Operative Work Conducted by the California Highway Department and the U. S. Bureau of Public Roads," were adopted as a guide. The fine aggregate used is known locally as flint, or mine-run chats from the waste piles of the Joplin, Mo., lead fields. Selection was made to obtain as nearly as possible the following specifications:

Screen.	Percentage passing.
¾-in.	95 to 100
No. 4	50 to 80
No. 10	35 to 60
No. 200	5 to 15

The oil used in part met the following specifications:

Specific viscosity (Engler), 50 cu. cm. at 122° Fahr	50 to 65
Percentage of residue, 100 penetration	65 to 75
Percentage of volatilization, in 5 hours, at 160° cent., 20 grammes	15 to 25

Stain tests served as a guide in obtaining the proper mixtures; the mixing was done by blade graders and farm disks; the product was compacted under traffic; and blade graders were used to wipe out traffic marks until the mixture became thoroughly compacted.

The resulting surface texture closely approached sheet asphalt in appearance. Due to the inadequate base some failures occurred during the winter months. The following spring the job was re-worked and more thickness added, the original thickness being about 1½ in. The experimental work was also extended on U. S. Highway No. 71 through Newton and McDonald Counties, using several grades of oil. Oils of lower loss and heavier viscosity were used with fair success.

During construction it was found that to obtain good results, oil should be added in small quantities as the desired total quantity is approached, at the same time avoiding very carefully any excess. The experience with this type of construction has been variable. Aside from weak bases the main difficulty has been due to excess subgrade moisture. Ground-water causes failure apparently because it lifts the oil to the surface of the mix. The bottom portion of the aggregate may be found free of oil, whereas aggregate in the upper portion contains an excess, which results in the familiar corrugated effect. Mixtures of this type are not as adaptable to wet climates as to dry climates.

Road Mix: Retread Type.—The first low-cost bituminous road construction in Missouri was built by the State Highway Department in 1927, on U. S. Highway No. 60, in Greene County. Crushed and screened stone was mixed with several grades of tars and cut-back asphalt to produce a surface thickness of approximately 2 in. Crusher-run stones, 2½ in., 2 in., and 1½ in., in size, with fines out, have all been used. The preference seems to be for the 2-in. size. Stone chips of ¾-in. size are used to fill surface voids. Refined tar of 8–20 viscosity at 40° cent., has been used almost exclusively to prime the old gravel or stone base. Both refined tars and cut-back asphalts are used successfully in the stone mixture.

The equipment in such work consists of a distributor, a 30-h.p., crawler tractor, a multiple-blade maintainer for mixing, a 5-ton roller, a 10-ton roller, and trucks for hauling.

The advantages of this type of construction lie mainly in the fact that it can be built exactly to formula. Aggregate of known quantity and gradation is used. Care is exercised to spread the stone uniformly in a fixed amount per square yard. This is followed by definite alternate applications of bitumen and chips. Excessive mixing is avoided to prevent segregation of stone. An excess of bitumen must be avoided if shoving and corrugations are to be avoided later. In building roads of this type it is easy to add more bitumen if necessary, but very difficult to correct for any excess of bitumen applied. The mixing and leveling process enables the construction of exceptionally smooth-riding surfaces even over fairly rough bases. Large irregularities can be patched before the retread course is laid.

Sufficient time between applications must be allowed for solvents to come off, or for proper curing. The retread type of construction is open in texture due to the large aggregates used. This is an advantage in that proper aeration and curing are readily accomplished, which is necessary when cutbacks and tars are used. A seal may be applied later if necessary or desirable, but must not be applied until the bitumens are thoroughly cured.

During the five years since 1927 there have been no failures in this type of construction, except for lack of support, and these have been limited. Such failures are readily repaired by draining the subsoil and adding thickness to the base, and then patching. If failures are extensive the entire surface may be re-worked by leveling and adding more material. If necessary, another course may be built over the original one to obtain the necessary thickness.

Retread: Crushed-Gravel Type.—Where stone is not available at low cost, crushed, screened, and washed gravel may be used with equal success. If the gravel available is fine, crushed, and screened to pass a 1½-in., circular screen, tar has been found to be preferable. However, when the finer aggregates are used, it is necessary, of course, to increase the percentage of bitumen used.

Road Oil Mats.—During the two years, 1930–31, limited experiments have been conducted in compacting gravel and stone surfaces by the use of road oil. In modern gravel road construction, the road metal is windrowed along the shoulder and fed on to the surface blades as it is compacted by traffic. Maintenance replacement gravel is also placed in windrows and gradually bladed on to the surface. During dry weather, traffic loosens the gravel, and blades are used to return the surplus loose gravel to windrow, where it remains until wet weather will allow its return to the surface for compaction.

During experiments with road oil as a dust palliative, it was found that an excess of oil formed a mat. Under favorable conditions these oil mats gave good service. Consequently, road oil meeting the following specifications is being used experimentally for building oil gravel mats:

	Oil No. 2.	Oil No. 3.
Specific gravity, 60° Fahr./60° Fahr., not less than.........	0.930	0.960
Furol viscosity, at 122° Fahr..	125 to 225	225 to 380
Flash point, not less than......	190° Fahr.	194° Fahr.
Bitumen soluble in CS_2, not less than	99.5	99.5
Volatilization, 5 hours at 325° Fahr., 20 grammes	0 to 18%	0 to 18%
Percentage of residue, 100 penetration	55 to 75	65 to 80
Ductility residue, not less than.	50 cm.	50 cm.

The No. 2 oil is best for first or prime application, whereas the No. 3 oil is used for mixing with the gravel. The mixture is compacted under traffic and kept smooth by motor or blade graders until thoroughly compacted.

This method calls for an excess of oil, which is objectionable to traffic during the process of curing. During the first 2-week period the mixture is "messy," and traffic has just cause for complaint. Recently, a trial was made of the use of powdered asphalt, and early indications are most promising. Powdered asphalt shipped in paper bags was distributed through a lime spreader at the rate of ½ lb. per sq. yd. on a freshly oiled gravel surface. The powdered asphalt has a tendency to hasten the setting period, and possibly adds strength and body to the oil.

Width, Thickness, and Cost.—It would appear that any road carrying sufficient traffic to warrant these surfaces would also justify minimum widths of 20 ft. In some cases greater widths may be justified on account of their water-proofing value to shoulders and edges alone. Experience in Missouri proves that a 2-in. thickness is entirely sufficient to resist shear and scaling where the sub-base support is at all adequate. Greater thickness represents unnecessary waste of funds and possibly invites some trouble due to the difficulty of handling greater quantities in road mixing, and of proper curing which is a most important feature not to be overlooked, especially when tars and cut-backs are used with finer aggregates.

It is difficult to predetermine the strength of an old gravel or stone base. Slight movements under loads during frost or wet seasons are likely to be unobserved on a well-maintained gravel road. These movements are quite likely to become readily noticeable after the bituminous surface is built. Consequently, the type of bituminous construction first used should be of low cost, easy to reconstruct, easy to repair, and easy to add to, if necessary. As a general rule, base failures are confined to small isolated places or to particular portions of the projects. Funds conserved in the building of minimum thickness may be used for special treatment of these places.

Furthermore, in Missouri there is a marked tendency for any road (pavements included) to lose its true cross-section and profile under the action of climatic changes and the pounding of traffic. Therefore, it seems logical to conserve funds by building the required minimum thickness and to use the money so conserved at a later date to add extra thickness where necessary to obtain greater strength or to improve the cross-section and profile, or both.

Costs, exclusive of stone and gravel aggregate which vary greatly, range from $750 to $2 500 per mile for 20-ft. widths, depending upon the bitumens used. Oil-graveled mats, 1½ to 2 in. thick, cost from $750 to $1 000 per mile for oil, oil application, and mixing. Stone retread costs from $1 800 to $2 200 per mile, which includes cut-back or tar applied, spreading stone, mixing, and rolling. Seal coats of $\frac{1}{5}$-gal. application and 15 lb. of sand or chips per sq. yd., cost from $600 to $900 per mile additional. Gravel retread requires more bitumen on account of the greater percentage of fine materials.

If the cost of stone or gravel aggregates is charged against maintenance gravel replacement funds, the foregoing costs are net. Since these treatments conserve road-surface material from loss due to action of traffic and water, further reductions might be deemed proper.

Plant Mixing.—The experience in Missouri with plant mixes on low-cost bituminous road construction has been limited. Advocates of plant mixes claim certain advantages of control over mixtures not obtainable in road mixes. Possibly this is true with certain plants, but the contrary is only too true with many plant mixers. Where better control is obtained in the use of a plant mix without any appreciable increase in cost of production, a plant is justified. To justify its place in the highway program, the low-cost bituminous surface must remain in the low-cost field in fact and not in name only. Elaborate methods and expensive procedures must be avoided, especially if no material benefits are added. This type of construction does not justify frills.

On the other hand, low-cost bituminous road construction should not be haphazard or careless. Care and skill may be used without involving expensive methods.

Where tars and cut-backs are used, the engineer should remember the nature of his materials. These materials contain certain solvents that must come off and allow the body material to stiffen after mixing. Therefore, certain aeration is necessary, which is accomplished in a large measure during the turning and mixing process in the road-mix method.

New Construction.—The highway engineer should not confuse low-cost bituminous surfaces with high type pavements; they are poor substitutes. However, the low-cost bituminous surface has its own field, which lies between the traffic-bound stone or gravel road and the high type pavement designed to carry the combined traffic of high-speed and heavy vehicles.

Carlton N. Conner,[14] M. Am. Soc. C. E. (by letter).—All road builders are indebted to Mr. Frickstad for his successful efforts in developing low-cost roads on the Pacific Coast. Research and practice developed by him and his associates have gone a long way toward solving the problem of building thousands of miles of all-weather roads in the eleven Western and Mountain States at a total cost for surfacing, including base and bituminous surface, of less than $10 000 per mile.

Low-cost construction in the United States has been made possible by the extensive use of tars and asphalts as binders, local materials as aggregates, and improved equipment for the actual work. Developments have been so rapid that the time is now ripe for simplification of methods and specifications, and the standardization of tests for materials, in fact, a consolidation of the rapid and successful advances already made. A concerted effort at this time is particularly important because unmistakable signs and trends show that construction and maintenance of low-cost roads will more completely occupy the attention of road builders for several years to come.

To assist in clarifying the field for standardization and to focus attention on the principal types of low-cost roads, the writer invites attention to the following discussion which is based on present conditions.

Low-cost surfaces, of which there are large mileages, or which are growing in favor, are commonly grouped as: (1) Untreated surfaces; (2) non-bituminous surface treatments; (3) bituminous surface treatments; (4) bituminous road mixes; and, (5) bituminous plant mixes.

In 1931, the State highway systems had 169 000 miles of these low-type surfaces and 95 000 miles of high-type surfacing. Table 3 shows the trends in construction and maintenance of untreated and treated roads.

Untreated Surfaces.—Untreated surfaces are those which consist of mineral aggregates held together by the binding, compacting, or interlocking qualities of the materials themselves, or by the addition of mineral binder, such as clay. These untreated types include sand-clays, gravels, stone, chert, shale, and others less commonly used.

For the purpose of this discussion and to be consistent with tabulations already prepared by the American Association of State Highway Officials and the U. S. Bureau of Public Roads, untreated surfaces are grouped as: (*a*) Sand-clay and top-soil; (*b*) gravel, chert, shale, etc.; (*c*) water-bound macadam; and (*d*) miscellaneous.

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Sand-Clay and Top-Soil.—These types of road surfaces usually consist of clay, silt, fine sand, and coarse sand, with a total sand content of from 65 to 85 per cent. They seldom contain particles larger than ¾ in. in size.

TABLE 3.—STATE HIGHWAY SYSTEMS: TRENDS IN LOW-COST SURFACES BY MILES

State	Built during 1930, untreated and treated	PROPOSED 1931 CONSTRUCTION		EXPECTED 1932 CONSTRUCTION		EXPECTED 1932 MAINTENANCE	
		Untreated	Treated	Untreated	Treated	Untreated	Treated
Alabama	337	125	46	...	...	...	...
Arizona	309	...	295	...	...	...	...
Arkansas	1 378	200	200	200	100	5 450	300
California	317	490*	...	...	119	.....	2 050
Colorado	175	200	75	150	150	1 500	500
Connecticut	124	...	100	...	...	.....	...
Delaware	19	20	...	10	5	....	38
Florida	16	30	...	...	100	235	3 175
Georgia	47	70	80	...	125	500	825
Idaho	576	500	200	50	600	1 694	906
Illinois	...	30	...	...	...	.....	...
Indiana	156	150	...	...	150	1 800	1 600
Iowa	247	240	...	...	...	.....	...
Kansas	853	700	50	200	400	4 000	900
Kentucky	692	459*	...	375	50	2 400	1 650
Louisiana	1 015	1 300	500	...	...	.....	...
Maine	72	8	76	400	35	1 690	2 100
Maryland	71	50	65	35	40	160	1 600
Massachusetts	...	...	...	...	95	.....	255
Michigan	86	77	15	...	...	.....	...
Minnesota	640	150	650	...	500	3 122	1 900
Missouri	893	1 000	...	1 500	50	3 180	150
Mississippi	...	...	...	...	...	.....	...
Montana	195	750	...	...	...	.....	...
Nebraska	489	900	25	1 000	100	6 200	150
Nevada	456	330	...	250	275	1 064	722
New Hampshire	171	50	50	...	90	100	1 940
New Jersey	1	10	...	...	...	.....	...
New Mexico	811	202	198	133	172	1 900	600
New York	144	75	...	...	...	.....	...
North Carolina	303	100	100	...	...	.....	...
North Dakota	454	400	12	700	100	4 319	374
Ohio	154	100	200	...	...	.....	...
Oklahoma	1 020	300	1 000	...	...	.....	...
Oregon	254	100	150	150	100	1 050	1 650
Pennsylvania	610	...	...	...	...	.....	...
Rhode Island	33	...	35	...	...	.....	132
South Carolina	120	75	325	150	325	2 500	900
South Dakota	520	200	300	600	300	4 050	200
Tennessee	600	...	...	...	...	.....	...
Texas	450	300	400	210	426	5 274	4 069
Utah	245	214	215	...	...	.....	...
Vermont	187	100	100	...	...	.....	...
Virginia	357	335*	...	...	...	.....	...
Washington	743	256	157	203	...	380	...
West Virginia	284	125	125	...	...	.....	...
Wisconsin	513	260	120	400	200	2 900	3 000
Wyoming	364	425	520	...	...	.....	...

* Treated and untreated.

The States having from 800 to 3 000 miles of these surfaces are Alabama, Florida, Georgia, Kansas, North Carolina, South Carolina, Texas, and Vermont. In 1930, Alabama, South Carolina, and Virginia laid between 100 and 200 miles each, while Kansas surfaced more than 1 500 miles with sand-clay.

Gravel, Chert, Shale, etc.—Available tabulated classifications do not list separately the miles of treated and untreated roads, but the total at the end of 1930 was 107 000 miles. Only fourteen States have less than 1 000 miles of the gravel, chert, etc., types, while twenty-two have from 2 000 to 6 000 miles of each.

Gravel is used for surfacing in all States, and there is a marked trend toward using gravel passing the 1-in. screen with crushed particles favored. Until recently, traffic-bound surfaces of gravel, stone, or slag were used principally in Ohio, Indiana, Tennessee, Nebraska, and Iowa. This type has proved particularly economical and serviceable, and is meeting with favor in Kentucky, North Carolina, and elsewhere.

Traffic-bound surfaces consist of a layer of clean crushed stone, slag, and crushed or screened gravel evenly spread on a prepared sub-grade. This layer is kept smooth by constant blading. Traffic gradually compacts it from the sub-grade upward. Periodically, new aggregates are added, under traffic, until a surface of suitable thickness is built up.

Macadam.—Macadams of crushed stone, slag, crushed gravel, and Florida lime rock are commonly laid in one course, and, less frequently, in two courses, to form a well compacted surfacing, suitable for improvement with bituminous materials.

Roads of the macadam type, and specifications for them show notable differences in requirements for quality and size of aggregates, thickness of section, and details of construction, but a study of the specifications shows they are written to secure good quality and a practical gradation of the available local materials and that softer stone is permitted in base courses which, later, will be surface treated. In 1930, the State highway departments built 1 300 miles of this type, as compared with a total of 13 000 miles of gravel, chert, and similar types.

The States which have more than 1 000 miles of macadam types in their highway systems are Florida, Kentucky, Maryland, New York, Ohio, Pennsylvania, Tennessee, and Virginia. Kentucky leads the list with about 4 000 miles. Kentucky also built 400 miles of macadam in 1930, which was more than twice that built by any other State; but Florida, Pennsylvania, West Virginia, and Wisconsin each built more than 100 miles.

Miscellaneous Untreated Surfaces.— These include surfaces of various kinds of aggregate which are laid in one or two courses as a type of water or roller-bound macadam, or they are placed in loose layers of 1-in. or 2-in. thicknesses to form a traffic-bound surface.

Materials for them include scoria, decomposed (or disintegrated) granite, stone screenings, mine "chats," iron ore, top-soil, granulated slag, pea gravel, stamp sand, shell, and volcanic cinders.

The use of hand-napped field stone as a base, covered by crushed stone, slag, or gravel, has met with success in Pennsylvania in the surfacing of 1 700 miles of the 20 000-mile program of low-cost roads.

Non-Bituminous Surface Treatments.—The purpose of these treatments with calcium chloride or lignin binder is to lay dust and reduce the loss of aggregates.

Calcium chloride is commonly applied to the road surface in flake or powder form. It has been used quite extensively in Michigan, New York, Vermont, Maine, and, to a lesser degree, in other States east of the Mississippi River. It has not been successful in States having relatively low humidity.

Lignin binder is a liquid concentrate which when mixed with equal parts of water is sprinkled on a road surface. Lignin binders have been used principally in New Jersey and New York.

Bituminous Surface Treatments.—Bituminous surface treatments as commonly designated are two or three applications of bitumen which are covered by sand, gravel, stone, or slag. These successive applications of bitumen and cover make a thin bituminous mat from ½ in. to 1 in. in thickness, which is well bonded to the base; when more than 1 in. thick these dragged or mixed surfaces should be designated as road-mix. The recent and successful innovation of dragging surface treatments which are less than 1 in. in thickness, during construction, makes them approach the road-mix type in texture, appearance, thickness, and smooth-riding qualities.

Bituminous surface treatments have been extensively used in the North Atlantic and South Atlantic States, Ohio, Indiana, Kentucky, and Florida on macadams and, in more recent years, on gravels, sand-clays, and traffic-bound surfaces.

Applications of light asphaltic oils on earth roads have met with little success as compared with heavier bitumens on macadams and other substantial bases. In certain instances on selected soils which were stabilized naturally or artificially by coarse granular particles, such as coarse sand or stone screenings, they have improved the road surface.

Bituminous Road Mixes.—The term, "road mix," is used to designate the surface obtained by mechanical mixing of aggregates and bitumen directly on the road itself. "Mixed-in-place" is another term widely used and equally descriptive. Road mixes are commonly from 1 in. to 3 in. in thickness and are not to be confused with plant mixes or with dragged-surface treatments.

Although there are other types of bituminous road mix, such as the fine aggregate types, which use sand-clay or blow sand for aggregate, most of them are in the experimental stage. There are two distinct types of road mix which are identified by the characteristics of their aggregates. These types are the "graded aggregate" and the "coarse aggregate."

The "graded aggregate" type is usually composed of gravel or stone and sand uniformly graded from a maximum size of about 1 in. to dust. These aggregates are as found in the road, as added to the road surface, or as added and mixed with aggregate already in the road surface. The type is used extensively in the Western States, the Southwest, and parts of the Mississippi and Ohio Valleys, where it is known by such names as "oil mix," "oil process," and other designations.

The "coarse aggregate" type, or "macadam aggregate" type, as the name implies, is composed of aggregates the characteristics of which are similar to those commonly used in macadams. They must be hard, durable, and preferably angular, such as crushed stone, slag, and crushed gravel. The aggregates are uniformly graded from those passing the 1½-in. screen to those retained on the ¼ or ⅜-in. screens. This type, also widely known as "re-tread," is built principally east of the Mississippi River and west of the Atlantic Coast States. Indiana, Pennsylvania, Kentucky, and Tennessee have each several hundred miles.

Still another type of road mix should be mentioned which for convenience may be termed "road-mix—fine aggregate" type. This type includes bitumen mixed on the road with sand-clays, sand-gravels, fine clay-gravel, or blow sands, the aggregates in which are much finer and commonly of poorer quality than those of the "graded" or "coarse aggregate" types.

Work on such roads has been confined to limited areas or has been of an experimental nature. Experimental work on them is still in progress (1932), and they offer a potential field for needed development. Attention is invited to experimental work in South Carolina, Florida, North Carolina, California, Nevada, and Minnesota, and to extensive usage in Long Island, New York.

Bituminous Plant Mixes.—"Plant mix" means the utilization of fixed or portable plants to prepare low-cost surfacing mixtures of bitumen and aggregates. The aggregates for plant mixes in low-cost roads are commonly those of the graded type already described, but they may be of the coarser sizes.

The object of plant mixing is to insure closer control of proportions and uniformity of mix than is possible by mixing in place on the road surface. As a result any plant mixing produces more fully coated aggregates than road mixing, but the requirements in specifications should not be so severe as to discourage contractors and manufacturers from developing plant mixes at low cost.

Whether or not the extra cost of plant mixing is justifiable has not been fully established, but experience and trends indicate its growing popularity where cost of aggregates and hauling is low, or where the road needs surfacing with aggregates which are of better quality than those found in the existing surface.

HAROLD L. TILTON,[15] ASSOC. M. AM. SOC. C. E. (by letter).—To any one outside the Western States this paper emphasizes the fact that while the general methods of construction are quite similar throughout the country, the engineer in each section must adapt these methods and principles to meet the climatic conditions to be encountered in his State and utilize to the best advantage the particular aggregates and bituminous binders that are available to him.

In New England, the use of a light fuel oil which would tend to emulsify in the presence of moisture, would be out of the question in the construction of a road that would be satisfactory at all times of the year. Rain occurs often and at any time. In the winter, the frost penetrates deeply. In the spring, it comes out of the sub-base while the atmospheric temperature is still so low that a bituminous surface lacks the plasticity that it attains later in the season. Aggregates containing fillers, such as silt and argillaceous materials, which possess a greater affinity for water than for bitumen, must be avoided. Surfaces built with such materials would be summer roads only.

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[15] With Vermont Dept. of Highways, Montpelier, Vt.

Engineers in New England are forced to use the heavier oils, and it is interesting to note that the author states that the trend in the West is in the same direction.

For dust-laying purposes only the Vermont Department of Highways ordinarily uses calcium chloride. Low-cost bituminous roads in Vermont are an intermediate type of surface designed to meet traffic conditions where the chloride-treated surface cannot be maintained satisfactorily, and yet where the cost of a high type of pavement is not warranted. One problem is common throughout the country. There are many miles of secondary roads badly in need of an improved surface and there are comparatively few dollars available for the needed improvement. The low-cost bituminous road seems likely to be the solution of that problem.

The use of the drag, the retread mixer, and the blade grader, together with the development of emulsions, retread tars, and cut-back asphalts, has revolutionized the art of constructing and maintaining low-cost bituminous surfaces. In the past most of them were far from being smooth-riding roads. This was due to the fact that hand labor was used so largely in spreading the aggregates. To-day, it is possible to build low-cost bituminous roads by machinery with a minimum of hand labor, that are very smooth riding. Furthermore, by similar methods, they can be kept smooth riding, which is a highly essential factor to be considered.

Mr. Frickstad's statement that the principal failures continue to arise from faulty support, due to deficiency in thickness of metal or stability of subgrade, deserves careful consideration.

It is quite evident that a bituminous-treated surface of gravel or broken stone 2 or 3 in. thick, has little inherent resistance to deflection. Such a surface will retain its true cross-section and grade just as long as it is unyieldingly supported by the foundation on which it rests. An adequate and well-drained sub-base is absolutely essential to the success of such a road. It is a waste of money to construct a bituminous treated surface on a subgrade that will fail to support it.

The importance of a sufficient thickness of metal is the reason why the mixed-in-place method is used on gravel surfaces in Vermont, instead of surface treatment. By the former method an adequate and fairly uniform thickness of metal is obtained at once, which, by the latter method, is usually secured only after several successive years of treatment. The depth of surface obtained by the penetration method varies according to differences in the grading of the gravel, and most of the aggregate particles have a heavier coating of bitumen than is necessary to make them adhere to adjoining particles. A familiar comparable fact is that a thin film of glue will make two pieces of wood adhere more strongly than a thick film. In the mixed-in-place method, the particles are coated with only enough bitumen to bind them together properly and a fairly uniform depth is obtained.

Thus far, experience has led highway engineers in Vermont to prefer the use of tar in the construction of gravel surfaces and the use of cut-back asphalts and emulsions in the construction of broken-stone surfaces.

The author states that "the grading of aggregate has not tended to become standardized, which confirms the opinion that a wide range is acceptable for oil mixing." This bears out experience in tar mixing in Vermont, where aggregates have been used with a range in maximum size from ¾ in. to 2 in. and containing a variation in fineness modulus of the sand from 2.60 to 4.10. Good results have been obtained with the stone content of the aggregate ranging from 40 to 75 per cent. An aggregate containing about 70% of stone produces the most stable and best wearing gravel surface.

In oil mixing the author states that the proportion passing the 200-mesh sieve is an important element. In tar mixing its importance lies in the desirability of its elimination. An excessive quantity of material passing not only the 200-mesh, but also the 100-mesh, sieve, almost invariably is present in surfaces that have failed from lack of stability. Particularly is this true when the fine material is argillaceous, which type possesses a greater affinity for water than for bituminous binder.

A broken-stone mixed-in-place surface is unquestionably stable. Its stability is due to three factors, namely, the keying and interlocking of the angular pieces of stone, the grading of the stone, and the binding qualities of the bitumen. In such a surface there is a considerable percentage of voids both large and small.

A gravel surface containing a coarse sand may contain a small percentage of voids such as would be caused by the absence of material passing the 100-mesh sieve. It may possess high stability due to the fact that so many of the stone and sand particles are quite angular. When the gravel is crushed, this characteristic is even more pronounced. A washed, crushed gravel surface containing 70% of stone is not greatly inferior to a broken-stone road when the stones in each case are of an equal degree of hardness and the proper bitumen is used in each surface. The removal of the very fine material is a decided advantage. As in the broken-stone road, a limited excess of bitumen can be taken care of by the voids. On the other hand, an excess of bitumen with a gravel in which all the voids are already filled with fine material, will be quite likely to develop instability.

A sharp sand, with a fineness modulus of not less than 2.65 and preferably about 3.50, with a color rating of not more than 2, and containing not more than 5% passing the 100-mesh sieve, is most satisfactory for tar mixing. The more closely the characteristics of a gravel surface approach those of a broken-stone surface, the more stable it is likely to be, and the less maintenance it will require.

The author notes the presence of an excess of 200-mesh material, usually clay, whenever failures occur from moisture conditions. This appears to be almost invariably the case. On one project in Vermont the gravel was washed to remove the fine material of an argillaceous nature and the results were decidedly satisfactory. Clay, silt, and loam are no more desirable when mixed with bitumen than when mixed with cement.

The wear of the gravel preferably should be not more than 20%, although gravel with 24% wear may be used if no harder gravel is available. The best results have been obtained by crushing and screening the gravel, separating the sand, and re-combining them in definite proportions. For most gravels a tar of 20 viscosity is advisable for the mix and one of 40 viscosity for the seal, although the lighter tar has produced a good seal. On one project a retread tar of 65 viscosity was used for the seal. It was covered with a coarse hard gravel of 1¼-in. maximum size and dragged and rolled. This produced an excellent non-skid surface. Two additional requirements in tar specifications are now under consideration by the New England States. A requirement for specific gravity will eliminate an excess of water-gas tar in the blending. A lower limit for the softening-point requirement should eliminate a tar unsuitable for cold wet weather conditions. The author's estimate for bitumen, of ½ gal. per sq. yd. per in. of finished thickness (varying, of course, with the grading of the aggregate), seems to be the general experience.

For gravel mixing a 20-ft. drag has been designed, with blades in zigzag arrangement, which has proved the fastest and most efficient mixer. A grader with approximately the same wheel-base is a necessary supplementary piece of equipment for the final shaping.

It is advisable in the construction of both gravel and broken-stone surfaces to apply a prime coat of bitumen to the sub-grade about a week in advance of the surface construction. This serves the double purpose of eliminating the dust nuisance to the public during construction and prevents the inclusion in the mix of undesirable fine and unclean materials from the sub-base.

Broken-stone surfaces may be constructed successfully with an aggregate not larger than 2½ in., which materially increases plant production and decreases costs. It is desirable, as in hot-penetration macadam, to have the maximum size of stone nearly equal to the depth of surface. For the seal, a stone graded from ¼ to ¾ in. is used. In a New England climate the heavier cut-back asphalts are the most satisfactory. The best results have been obtained with the use of a so-called benzol cut-back having a viscosity (Engler, 50° cent.) of 150 to 175, distillate at 680° Fahr., of about 20%, and penetration of the residue of 90 to 100. The surface obtained with this asphalt compares favorably with that produced by the use of a hot-penetration asphalt in macadam construction. In building broken-stone mixed-in-place surfaces, a grader with a wheel-base of 18 ft., or more, is a satisfactory mixing and smoothing agent. The seal is smoothed by the use of the drag or the retread mixer.

Good results have been obtained with the use of a penetration emulsion, the stone being smoothed by a grader, and the emulsion applied in three applications, each properly keyed and rolled. The emulsion has one property very definitely in its favor in a New England climate; it may be used with a wet aggregate. The time lost in construction in Vermont during 1931, while waiting for aggregates to dry, represented no small sum of money.

The average cost of gravel mixed-in-place surfaces constructed in 1931, including the cost of the new gravel, was about $3 600 per mile. On most projects this gravel had to be hauled from 3 to 8 miles. The cost of mixing and rolling alone was about $200 per mile.

The average cost of broken-stone mixed-in-place surfaces constructed in 1931, including the cost of the stone, was about $5 600 per mile. All this work was done by State forces.

The construction of low-cost bituminous roads necessarily means that local aggregates should be used to as great an extent as possible. This requires study as to the best methods of combining them with the bitumens that are available in that particular section of the country.

L. I. HEWES,[16] M. AM. SOC. C. E. (by letter).—That low cost or intermediate types of bituminous road have made substantial contribution to the road developments of the West cannot be questioned. On December 31, 1930, practically 20% of the Federal Aid highway system of the Western States consisted of a surface-treated, plant-mix, or road-mix construction, described by Mr. Frickstad.

Materials.—Aggregate of surprisingly wide range of grading continues to be successful in road mix under varying conditions and with different available local materials. General Western practice in 1932 is to use aggregates of 1-in. maximum size to dust, and a 2½-in. minimum compacted thickness of road-mix or plant-mix surface courses. Oils of higher viscosity are meeting with increased favor, especially where moisture influence is expected. When light oils are used, some engineers are placing a seal coat of heavier oil, with sand or screenings, two weeks or more after completing the road mix. There has been a limited use of "armor coat" treatment approximately ½ in. to ¾ in. thick placed on previously constructed road mixes. All surface treatment requires great care in distribution of material and cannot wholly correct imperfections in the under surface. Cut-back plant mixes are being tried, especially in California. It should be borne in mind that the use of cut-back asphalts or asphaltic oil of higher viscosity increases the cost of scarifying or remixing whenever required during maintenance.

Construction.—No formula yet proposed for the control of the oil and aggregate has received general endorsement. Advance laboratory analyses and tests of the aggregates and bituminous materials will usually furnish valuable indications. It is well to know in advance whether the aggregate is highly absorptive, in order to avoid a lean mix which might ravel. Bituminous mixes that would give unsatisfactory service under unfavorable moisture conditions because of characteristics of the fine material, are often detected in the laboratory by a swell test.

A moist or loose uncompacted base is a frequent cause of distress. Many road-mix or plant-mix projects, with aggregate ranging from 1 in. to 200-mesh material, have shown failures traceable to excess moisture in the materials during the mixing or prior to the final spread and compaction. Most engineers think that an excess of 3% moisture will prevent satisfactory compaction on the road, or possibly cause failure. Accordingly, the practice of heating aggregates in plant mixing is increasing.

Adherents of plant mix claim better control and the possibility of using materials of higher viscosity. Its field is limited, necessarily, to projects on which the mineral aggregates to be treated have not been placed on the road. Plant-mixed material that becomes wet before spreading should be bladed until the condition is corrected. This operation is also frequently used to increase the evaporation of the solvent in cut-back mixes. Contractors do not resort to rolling ordinarily. However, it is advantageous under unfavorable traffic conditions, for compacting the edges, also when heavy rains are expected, or when relatively quick-hardening asphalt products are used in late season work.

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[16] Deputy Chf. Engr., U. S. Bureau of Public Roads, San Francisco, Calif.

Equipment manufacturers are developing several types of self-propelled machines designed to mix the materials while moving along the road. There may be a field for further experimentation in this direction.

Oil road construction commends itself because of low first cost and cheap maintenance. Expensive refinements in plant mixing and road mixing should be avoided unless the results are certain to be commensurate with the increased cost. Unwarranted expenditures, especially when expensive importation of aggregates is involved, also must be watched.

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EFFECTS OF BENDING WIRE ROPE

BY FREDERICK C. CARSTARPHEN[1], M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. ROBERT C. STRACHAN, C. D. MEALS, O. P. ERICKSON, F. W. DECK, O. G. JULIAN AND J. C. DAMON, A. S. RAIRDEN, AND FREDERICK C. CARSTARPHEN

SYNOPSIS

An ever-present suspicion appears in the discussion of wire rope to the effect that the popular formulas for the estimation of the loss of strength due to bending may not be consistent with experiment. This can be settled only by holding the problem up to engineering and analytical scrutiny.

This paper is a start along this path. New and more comprehensive formulas are presented, which may serve a useful purpose in bringing to light data and analyses of the problem that have not been published by those who have given their attention to the subject. Wire rope is one of the most used, and most abused, commodities of industry. Therefore, all its properties should be known, to the end that the public and the wire rope manufacturer may agree upon the increase in its cost, if necessary to provide for more safeguards in the selection of materials and in the process of manufacture, so that its performance may be predicted for a properly supervised service.

Notation.—The symbols used in this paper are defined at the place in which they are first introduced in the text and, in addition, they are completely listed and defined in Appendix I.

HISTORICAL NOTES

The name of the inventor of wire rope is probably lost forever in the mist of antiquity. Samples of wire rope made of bronze have been excavated among the ruins of Pompeii; but, apparently, they consist of three strands twisted without the use of a center. Albert of Clausthal made ropes of both regular and lang lay in 1834, and for some years thereafter, and discussed

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NOTE.—Published in December, 1931, *Proceedings*.

[1] Cons. Engr., Denver, Colo.

haulage technique.[2] In America, credit is given to the late John A. Roebling, M. Am. Soc. C. E., as the pioneer wire rope maker. He manufactured a piece of $\frac{3}{4}$-in., 6 × 7, iron rope, about 300 ft. long, at Saxonburg, Pa., in 1840. This rope, which was used on the canal plane near Portage, Pa., was made on a ropewalk such as was formerly used in the manufacture of fiber ropes. In 1848, the works were moved to Trenton, N. J., and wire ropes were made for use on suspension bridges, for towing canal-boats, and for such other service as required greater strength than could be secured from hemp ropes of reasonable size.

The excellent qualities possessed by wire rope soon won for it deserved popularity and, in 1886, the Trenton Iron Company undertook its manufacture, using machines designed in accordance with the best English practice. As in the case of many other innovations, the market was soon "saturated," and in order to stimulate sales each manufacturer began to offer different types, styles, and grades of rope. This was the period when iron was being abandoned for steel, and it was the proper thing to claim that certain ropes were made of the highest grade crucible cast steel, plow steel, and super-plow steel. It was not sufficient alone to vary the grades of steel, but the construction of wire rope was varied so as to appeal to the whim of the most exacting customer, and each change was always heralded as being a superior product and claims were made that it could not be excelled for the use intended. Although many bizarre constructions of rope have been abandoned, many wire rope catalogs contain more than eighty tables giving the properties of the different styles of wire rope that are manufactured. To quote Mr. Thomas Woodhouse, of Dundee, Scotland:[3]

> "The introduction of aerial ropeways as a means of transportation some 50 or 60 years ago, created a demand for steel wire ropes previously non-existent. * * * Aerial ropeways now provide a big outlet for steel wire ropes of almost all constructions."

TERMINOLOGY

It is reasonable to assume that with so many different kinds of wire rope, confusion sometimes arises concerning its properties and its ability to render service. As a matter of fact, wire rope is nothing more than an assemblage of wires arranged so as to permit it to meet, in the best manner, several general conditions which bound the limit of its use. It may be ideally adapted to discharge a certain duty, but as the limits of its usefulness are extended the border line is reached at which failure is encountered, not through any inherent defect in the wire rope, but because it is not capable of service in an infinite number of fields. There is no other material used for engineering purposes which will stand greater abuse and still function satisfactorily than wire rope; yet it has its limitations and should be used accordingly. It is proposed, therefore, to consider several properties of wire rope which follow from the nature of its construction.

In the remarks that follow, the lines that are of such size and construction that they can be put in motion, will be considered as "ropes," and those that remain at rest, as "cables." The term, "cable," is also applied to ropes of large size, as is also the word, "hawser," when the cable is used for towing purposes. In the electrical field, the words, "cables" and "guys," are in general use.

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[2] *Mining Journal*, London, England, 1837.

[3] Encyclopedia Britannica, Edition XIV, Vol. 23, p. 675.

Wire rope is an assemblage of wires, and the strength of each wire depends upon the composition of the material from which it is drawn, the number of drafts given, the diameter, and the heat treatment which it receives. Hence, it follows that those wires which have been subjected to the greatest number of drafts, and are properly heat-treated, have a higher tensile strength than those of larger size. Unfortunately, wires of small diameter will not stand abrasion and abuse without breaking. It is concluded, therefore, that strong ropes should be built of a number of small wires. Those which are subject to surface wear should be built of larger wires. When both strength and resistance to wear are desired these two properties must be combined, and it is here that the skill of the rope maker is best shown.

Obviously, as long as ropes remain stationary and sustain loads which do nothing more than cause a variation in tension, it is quite sufficient to gather the wires together in a bundle and serve them with wire or tape so as to preserve their grouping and protect them from the weather. Cables of this so-called "selvagee" construction are used on suspension bridges, particularly because this type of cable may be erected in position by stringing the individual wires in place between the bridge towers. To speed the completion of suspension bridge cables, wires are assembled in groups of 200 or more, or as wire cables, and are set into place to make the bridge cable, thus doing in days what formerly required months by "spinning."

CHARACTERISTICS

In addition to strength, wire rope must acquire two other qualities, namely, flexibility and resistance to wear. Since fiber ropes have been made from the earliest times, it is natural to wonder why the development of wire rope did not advance accordingly; but like all great innovations, apparently simple, it was impossible until several associated problems were solved. Just as it is impossible to build an arch without a center, so it was impossible to undertake the construction of wire rope on a commercial scale until sufficient progress had been made in the wire and steel industries, that would insure the production of rods of such quality and quantity that they could be drawn into long wires possessing adequate strength. Prior to about 1850 the iron and steel industry had not made sufficient progress to permit the development of wire rope as it is now known.

Some time in the early part of the Nineteenth Century the important discovery was made, by either Albert of Clausthal, or J. Wilson of Derby, or by both independently, that flexibility could be secured by using fine wires twisted into strands and that the strands could be laid about a fiber center to form ropes. As this process can be done in a great variety of ways, different manufacturers adopted different designs for the symmetrical arrangement of the wires in the strands, the direction of the twist, and its pitch, which is called the lay. Reference to any company's rope list will show the arrangements which are considered to be standard. As experience was gathered in the use of wire rope there has been a gradual shortening of the lay of the wires and strand. In 1890 it was not uncommon to find 6 × 7 wire rope having a lay of wires forming the strand equal to thirty times the diameter of the wire, and a lay of rope equal to nine times the diameter of the strand. In modern practice the corresponding ratios would be nearly 22.5 and 6.5. In other words, with increased tensile strength of wires, it is possible to increase flexibility without a serious sacrifice of strength, although the tables adopted by the Wire Users Association in 1910 and in 1929 each assigned lower values to the breaking strengths of plow and super-steel ropes than formerly.

As an aid in estimating the theoretical number of wires in the diameter of a rope, the lay of the strand, and the lay of the rope, in terms of the diameter of the wires, Table 1 is presented.

TABLE 1.—STANDARD CHARACTERISTICS OF WIRE ROPE

Kind of rope	Lay of strand*	Lay of rope*	Number of wires in the diameter of rope
7-wire, ordinary	$22.4d'$	$58.3d'$	9
7-wire, lang lay	$22.4d'$	$62.8d'$	9
Seale, ordinary	$29.4d'$	$76.6d'$	12
Seale, lang lay	$31.4d'$	$82.6d'$	12
19-wire, ordinary	$35.7d'$	$93.0d'$	15
19-wire, lang lay	$38.1d'$	$100.0d'$	15

* d' = diameter of wire.

The strengths of the various steels used in the manufacture of wire rope (in kips[4] per square inch), are generally accepted, as follows:

Iron wire	85 to 105
Mild cast steel	160 to 180
Cast steel	180 to 200
Extra cast steel	200 to 220
Plow	220 to 240
Super-plow	240 to 280

All the foregoing varieties of steel are used in the manufacture of wire rope. However, it is not sufficient to examine the wires for tensile strength alone; they must also be examined for torsion, bending, and elongation. The tensile strength and elongation are determined in the usual manner in a standard testing machine. When elongations are required an extensometer is used to ascertain the increments in loading. When the yield point is reached the instrument is removed. From the data secured, plus a knowledge of the area and the length of the specimen tested, it is possible to calculate the modulus of elasticity of the steel. This factor varies, of course, depending upon the composition of the steel, but will usually be found to be between the limits of 25 000 to 30 000 kips per sq. in. In other words, the modulus of elasticity is the load required (theoretically) to extend the specimen a distance equal to its own length, and is of considerable importance in solving problems connected with the stresses caused by changes in length of the specimen, whether it be by temperature, or by exterior or interior loading.

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[4] 1 kip = 1 000 lb.

The torsion test is made to determine the number of twists about a longitudinal axis which a wire will stand and, in a way, it is a measure of its ductility. Wires possessing a diameter greater than 0.04 in. are mounted in a machine constructed so that it has two opposed heads fitted with chucks or clamps for holding the wire. These heads are usually spaced 8 in. apart and run 33 rev. per min. in opposite directions, giving an equivalent twisting speed of 66 rev. per min. Means are provided for placing the wire under a known tension, thus standardizing the test. The torsion test quickly reveals brittle wires and is confirmed by the results of the kink and jig tests. Wires which are sufficiently small to be bent by hand into loops such that the diameter is approximately that of the wire, are tested accordingly. This is the kink test and is usually confined to wires less than 0.04 in. in diameter. Larger wires are bent 120° in four directions about dies of a given radius, depending upon their size, by means of a special device called a jig. As an illustration of the variation of the torsion and bending resistance in different grades, Table 2 is presented for a 0.111-in. wire such as would be used in a 1-in., 6 × 7 rope. The torsion specimens are under a tension of 23 lb.

TABLE 2.—TORSION TEST OF WIRES, 0.111 INCH IN DIAMETER

Grade	Breaking strength, in kips	Twists in 8 in.	Jig test (in standard jig)	Percentage elongation
Mild steel	1.57	26	Four 120° bends	2.3 to 3
Cast steel	1.70	26	Four 120° bends	2.3 to 3
Extra strong cast steel	1.96	25	Four 120° bends	1.8 to 2.8
Plow	2.16	24	Four 120° bends	1.8 to 2.6
Super-plow	2.36	22	Four 120° bends	1.6 to 2.6

It is possible to make wire for ropes from both basic and acid steels up to the plow grade, but open-hearth acid steels are preferred. It is understood that steel wire possessing the requisite properties for making suitable wire rope can be secured only by a careful control of the composition of the steel in the furnace, by a proper cropping of blooms, and by the discreet rolling of billets into rods. These steps must be followed by the elaborate and painstaking processes of cleaning, baking, wire-drawing, annealing, and heat treating, or "patenting," all of which combine to make possible the production of the high tensile strengths which characterize wires used for this purpose.

Table 3 illustrates the progressive increase in strength acquired by an acid open-hearth steel rod, $\frac{5}{16}$ in. in diameter, due to wire-drawing. The specimen had the following composition: Carbon, 0.8%; manganese, 0.6%; silica, 0.14%; sulfur, 0.009%; phosphorus, . . . ; and copper, 0.03 per cent. The fact that the elongation in 10 in. had dropped from 4% to less than 1.1% emphasizes the interdependence between the elasticity and strength. In other words, flexibility is not a property of wires possessing high resistance to rupture. Accordingly, wires possessing a strength of 300 kips per sq. in., or more, are usually so brittle as to be considered worthless for wire rope manufacture. This statement is qualified for steels of music-wire grade, and a few alloyed steels, the cost of which precludes them from use in wire ropes.

TABLE 3.—INCREASE IN STRENGTH ACQUIRED BY WIRE DRAWING

Observation	Diameter, in inches	Breaking Strength		Percentage elongation, in 10 in.
		In kips	In kips per square inch	
0	0.3125	4.60	60	(Unstressed)
1	0.191	5.160	180	4.0
2	0.159	3.975	200	3.5
3	0.132	3.145	230	2.4
4	0.093	2.095	308	1.1

MECHANICAL DEVELOPMENT OF MANUFACTURE

When the testing room passes the wire as satisfactory, it is delivered to the rope shop and wound upon bobbins for use in the strand-making machines. The development of wire-rope stranding machines is an interesting one. When Mr. Roebling undertook the manufacture of his first wire rope it was done in a rope walk, similar to the method used in the manufacture of fiber ropes. Albert of Clausthal, the accredited inventor of wire rope, followed a similar method of manufacture. It happens that, in 1838, Mr. R. S. Newall, of Dundee, Scotland, received a letter from a friend commenting upon the method in vogue in Saxony for making wire rope, consisting of four strands of four wires, $\frac{3}{32}$ in. in diameter, twisted together. The friend stated that the process was exceedingly crude and urged Mr. Newall to design a machine for this purpose. Within a month he had prepared the plans of the machine, and, in August, 1840, secured the British patents for the production of wire rope formed by closing strands about a core.

For many years after the introduction of this machine it was felt that means must be used to prevent twisting the individual wires when incorporated into the strand. This resulted in keeping the velocity of the machine at a minimum, 40 to 50 rev. per min. being considered a maximum. An American, Mr. J. B. Stone, of Worcester, Mass., invented a stranding machine which eliminated the gears and other equipment used to prevent torsion in the wires, and, as a result, higher operating speeds were secured.

WIRE-STRANDING MACHINES

The type of machine most favored in America consists of a suitable bedplate, supporting rollers; these, in turn, carry the flyers which are connected by rods. The flyers are placed so as to accommodate a floating cradle which, in turn, carries a bobbin. On the 19-wire stranding machine provision is made for supporting nineteen bobbins. The cradle is constructed with a counterweight and is mounted in the machine so that it does not revolve when the machine is in motion. By giving over to gravity the control of the position of the bobbins, the machine may be operated at high speed. The wires are conducted to the front end of the machine and are led from the ports of the flyers through the slots of the twisting head which wraps them about the wire center.

There must also be longitudinal motion to form the strand. The machine is equipped, therefore, with a draw-out mechanism, which consists of multiple grooved sheaves arranged so as to revolve at a speed which bears a fixed ratio to the number of revolutions of the machine proper. As an illustration, for a certain size of strand the draw-out mechanism handles 1 in. of strand per revolution of the machine. If the draw-out mechanism should cease to function, the strand would soon be twisted in two. In order to vary the velocity of the draw-out mechanism a speed-changing device is used. For instance, two spur gears may be carried in a frame equipped with a handle and mounted upon a spline shaft. When the operator wishes to change the speed ratio it is only necessary to raise the frame, slide it along the shaft until the proper driving gear is encountered, and then drop it into position. Since right or left-lay strands may be required, it is necessary for the draw-out mechanism always to run in the same direction irrespective of the rotation of the machine. This is accomplished by using opposed bevel gears and a pinion.

These machines are rated according to the weight of the wire held by the bobbins, such as 50, 100, or 250-lb. machines. The greater the weight of the bobbin the larger the machine and the slower it must run because of the rotative forces developed. This machine is not rigid, and each flyer is supported on rollers. Both ends of the machine are driven from a countershaft which extends its full length. In practice, it is found difficult to keep a constant pressure on the several rollers. The machine is also equipped with multiple solenoid brakes that act simultaneously and tend to reduce unbalanced stresses in the frame when being brought to rest.

Machines have been built so that strands composed of more than nineteen wires could be laid up in a sequence of continuous operation. Such a machine would be called a tandem compound wire-stranding machine. The first unit would be arranged so as to lay the six wires on the center, the next would place twelve wires, then eighteen, and finally, twenty-four, making a strand of sixty-one wires. Unfortunately, the speed of the assembly depends upon the rate of laying the initial six wires. Since these tandem machines were of considerable length, it was necessary to group each multiple of six flyers in the same transverse plane, giving the planetary arrangement. This arrangement entailed flyers of large dimensions, so that the centrifugal stresses were considerable. The result secured was not satisfactory because of the lack of speed of the assembly. It has been found preferable to prepare the strands as a series of single operations and thereby increase the speed of each one to the limit of economical and satisfactory manufacture. Compound rope machines were also perfected based on the idea of arranging six stranding machines so as to manufacture the strand and close the rope in one continuous operation. These machines have also been superseded because of their inability to give a satisfactory output in competition with superior high-speed stranding machines and a closing machine which run independently at efficient speeds.

The method of assembling the strands into wire rope is very similar to that used in making strand, except that the rope is formed by closing the strands about the fiber or wire rope center. The speed of a closing machine is much less than that of a stranding machine, because of the great increase in weight of the revolving parts. A fair speed is 45 rev. per min.

TYPES OF WIRE ROPE

There are several other classes of cable which are worthy of attention. Locked-coil cables consist of a center of seven or nineteen round wires. When of sufficient diameter the core is covered with several layers of trapezoidal or key wires. The exterior surface consists of a layer of interlocking wires which are placed so snugly as to give a smooth and cylindrical appearance to the cable. Unfortunately, these wires are of large relative cross-section, and as made in America, do not exceed 150 kips per sq. in. in tensile strength (secured by special heat treatment called "patenting").

Half-lock cables are wire strands covered with round and shaped wires to give a smooth exterior. Wires of relatively large diameter are used. In order to improve the tensile properties of the cable, the expedient has been adopted of using many layers of small cross-sectional key wires and covering the assemblage with lock wires. This type of cable is known as locked-wire cable, the word, "wire," conveying the idea of the small key wires in its construction.

Cables formed of nineteen, thirty-seven, and sixty-one round wires as a single strand are known as track strand or smooth-coil cables. All the types aforementioned find their primary use as track cables for aerial tramways and cableways.

Experience in the use of wire rope has demonstrated that one of the most vital factors for long life is interior lubrication. Therefore, the most modern practice of rope construction demands that a center be used which is thoroughly impregnated with a lubricant. Various methods have been tried, such as running the rope, when closed, through a bath of melted petrolatum or a solution of vaseline in fuel oil. The drawback to this system comes from the fact that commercial African or Javanese hemp contains considerable air and moisture which prevents the saturation of the center with the heated oil. To overcome this it is considered good practice to saturate the rope center with hot lubricant in a vacuum chamber. This process impregnates the center with sufficient oil to meet any demand which may reasonably come upon the rope, if properly cared for, during its life.

Practically all ropes of the twisted-strand construction, if more than 200 ft. in length, are wound on wooden reels, which are all built to dimensions that have been standardized by the wire companies.

ANALYSIS OF STRENGTH

Since wire rope, from the inherent nature of its construction, is based upon the twisting of wires, it is proper to investigate the strength of the strand as compared with the sum of the strengths of the individual wires.

Example 1.—Curved Wire About a Straight Center Wire.—Fig. 1 represents a piece of straight wire, l', loaded axially by the tension, T'.

If a piece of the same kind of wire is wrapped about it so as to form a helix (Fig. 2(a)), the stress in the curved wire as compared to that in the straight one may be investigated. It is not so easy to visualize a force acting in curved wires as in straight ones, so the bent wire may be straightened after cutting both the wires by parallel planes a distance of one unit apart, which is assumed to be the lay or pitch, l', of the helix. In twisting the wire its center moves in a curve the horizontal projection of which is a circle whose diameter is equal to $2\,d'$; but the horizontal distance is equal to the circumference of this generating circle, or $2\pi d'$. The vertical distance is l'. Fig. 2(c) shows the right-angle triangle the base of which is $2\pi d'$, and the height, l'; the length of the hypotenuse, l_1, is required.

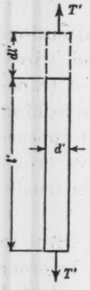

FIG. 1.

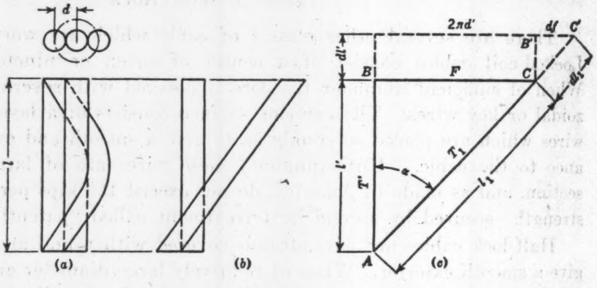

FIG. 2.

Since $l' = 22.5d'$, the hypotenuse equals,

$$\sqrt{506.25(d')^2 + 4\pi^2(d')^2} = d'\sqrt{506.25 + 39.48} = 23.36d'$$

Therefore, the length of l_1 is, $\dfrac{23.36d'}{22.5d'} \times l' = 1.03822\ l'$. Incidentally, 1.03822 is the secant of the angle of lay, which is 15° 35′.

If the two wires are considered to carry a load so that the stress in the straight wire is equal to T' (Fig. 1), and no slipping occurs between them, then the bent wire will be elongated in a direction parallel with the axis of the straight wire an equal amount, or dl'.

The triangles, ABC and $CB'C'$ (Fig. 2(c)), are similar because their sides are parallel; that is, $\dfrac{dl_1}{dl'} = \dfrac{l_1}{l'} = 1.038 = \dfrac{T_1}{T'}$. (The ratio, $\dfrac{T'}{T_1} = \dfrac{1}{1.04} = 0.96$.) The load, T_1, on the twisted wire, is $1.038T'$ If $S' =$ the ultimate strength of the wires and is assumed to be the same for all of them, and S'_s = the effective strength of the strand, then, $S'_s = 0.96 \times 7 \times S' = 6.72S'$, and the strand has a strength equal to 96% of the sum of the individual wires.

If this strand is to be used in a rope the lay of which is $58.3d'$, the diameter of the helix formed by the strand is equal to $6d'$, and the length of the developed strand is equal to,

$$\sqrt{6^2\pi^2(d')^2} = d'\sqrt{355.32 + 3398.89} = 61.2d'$$

Hence, in Fig. 2(*c*), $\sec \alpha = \sec \beta = \frac{61.2}{58.3} = 1.049$, or $\beta = 17° 35'$.

Example 2.—Curved Wire About a Curved Center Wire.—If a curved wire, twisted about a curved center wire, is untwisted, the two will appear as in Fig. 3. The center wire of the strand has a strength equal to $\frac{S'}{1.049}$, and, of the six remaining wires, $\cos(\beta + \alpha)\,S' = \cos 33° 10'\,S' = 0.837S'$. The effective strength of the strand is, therefore, $7 \times 0.837S' = 5.86S'$, since it is evident that the center wire will never be stressed more than the breaking point of the outside wires.

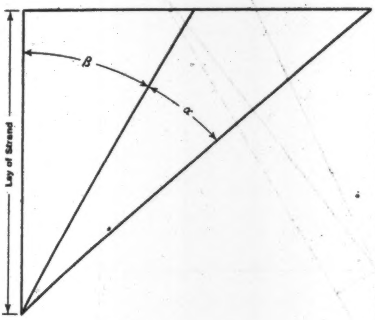

FIG. 3.

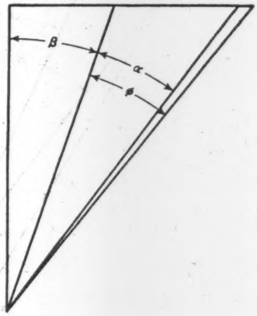

FIG. 4.

The rope will have a strength of $6 \times 5.86S' = 35.16S'$. Since there are 42 wires in the rope, the strength efficiency is $\frac{35.16}{42} = 83.7$ per cent. This value has been established by many tests and may be compared with the rope list for 1-in., 6 × 7, crucible rope which is composed of 42 wires, 0.111 in. in diameter. The area of such a wire is 0.0097 sq. in., and the area of the rope is $42 \times 0.0097 = 0.4074$ sq. in.

If cast steel has an ultimate strength of 180 kips per sq. in., then the strength of the rope will be $83.7 \times 0.4074 \times 180 = 61.370$ kips, or 30.7 tons. The 1910 catalog value is 31 tons, which is a sufficient check. The value in the 1930 American Manufacturers' list is 29 tons, showing the onward decline in the standard wire strengths. Foreign rope lists carry higher breaking strengths, and should be a challenge to American rope manufacturers.

Strength of a 6 × 19 Rope.—The diameter of the wires in a 6 × 19 rope is $\frac{d}{15}$; and these are laid in a strand of 1–6–12 wires, respectively, having lays of $22.5d'$ and $35.7d'$. Let the angle of lay of the outside wires of the strand be ϕ, which is computed to be 19° 22′; the lay of the rope may be $93d'$ for modern practice, as against $112d'$ used so long in the past. To find the angle of the center of the strand, the diameter of the helix is $10d'$, and the circumference is $10\pi d'$. Then, from Fig. 5, $\sec \beta = \frac{d'}{93} \sqrt{93^2 + (10\,\pi)^2} = \frac{98.16}{93} = 1.0554$, and $\beta = 18^\circ\ 39'$. Then, $\cos\,(\beta + \phi) = \cos 37^\circ\ 01' = 0.7986$. The strength of the 6 × 19 cast-steel cable may be $0.7986 \times 114 \times 0.003525 \times 180$, or 57.78 kips.

If slipping occurs between the straight and curved wires the method of computing the strength of the rope is as follows: Assume that a 6 × 19 strand is arranged so that a plane projection is as shown by the solid lines in Fig. 5. The lengths of the wires are $y = X \sec \alpha$ and $z = X \sec \beta$. Then,

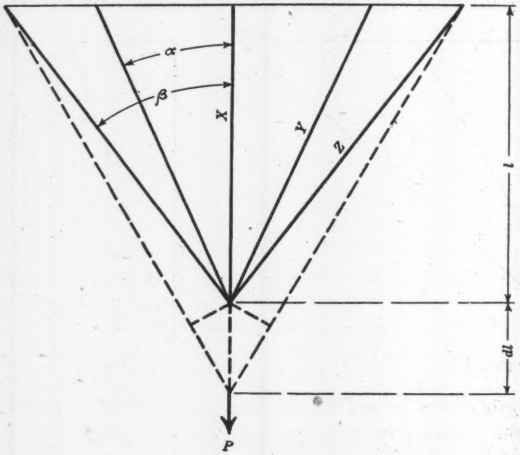

FIG. 5.

$$X + 6Y \cos \alpha + 12\, z \cos \beta = P \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The load on the rope is determined as follows: If $\delta X = \delta l$, then $\delta Y = \delta X \cos \alpha$, and $\delta Z = \delta X \cos \beta$, from which the following elongations may be written:

$$e_X = \frac{\delta l}{l}\,;\ \sigma_X = \frac{E\,\delta X}{l}\,;$$

$$e_Y = \frac{\delta X \cos^2 \alpha}{l}\,;\ \sigma_Y = \frac{E\,\delta X \cos^2 \alpha}{l}\,;$$

$$e_Z = \frac{\delta X \cos^2 \beta}{l}\,;\ \text{and}\ \sigma_Z = \frac{E\,\delta X \cos^2 \beta}{l}.$$

Assuming that all wires have equal areas:

$$X = \sigma_X;\ Y = \sigma_X \cos^2 \alpha;\ \text{and}\ Z = \sigma_X \cos^2 \beta.$$

Substituting these values in Equation (1):

$$X = \frac{P}{1 + 6 \cos^3 \alpha + 12 \cos^3 \beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For the rope in Fig. 6 let $\alpha = 17° \ 39'$ and $\beta = 33° \ 10'$. Substituting in Equation (2),

$$X = \frac{1}{13.2} P = 0.076\ P;\ 6\,Y = 6\,X \cos^2 \alpha = 6 \times 0.076\ P_X \cos^2 \alpha = 0.410\,P$$

and,

$$12\ Z = 12 \times 0.076\ P_X \cos \beta = 0.640\,P$$

The strength equals the sum of 0.076 P, 0.410 P, and 0.640 P, or 1.126 P. The strength of the total wires is $19 \times 0.076 = 1.44\ P$ and the ratio of strengths equals $\frac{1.126}{1.44} = 78.5\%$, as compared with 79.9% obtained in the previous calculation.

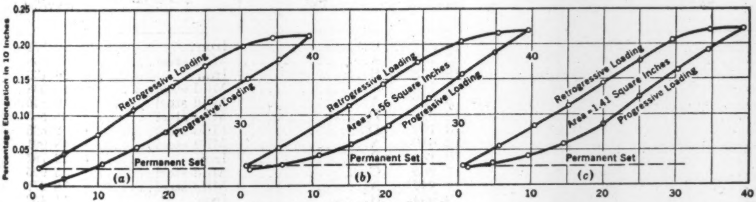

FIG. 6.

Effect of Varying Diameter of Helix.—In making these calculations it is assumed that the diameter of the helix is maintained by the core. It is quite remarkable what profound changes take place in the value of the modulus of elasticity of a rope if the helix diameter varies. Assume the value of E for the steel of the 6 × 19 rope as 27 000 kips per sq. in. The modulus for the rope might be 0.7986 × 27 000, or 21 500 kips per sq. in. By test it may be found to be 10 750 kips per sq. in., the elongations in the latter case being twice the former. If a load of 10 kips is applied to the rope, it is easy to show that a reduction in diameter of the core of 0.007 in. is sufficient to account for the difference in the theoretical and actual moduli of elasticity. Thus, if the lay of the rope is $93d'$, and $d' = 0.067$ in., the lay is 6.231 in.; for 10 kips, the actual elongation (permanent stretch eliminated) equals $\frac{10 \times 6.231}{10\,750 \times 0.4019} = 0.0144$ in. For 21 500 kips, the amount is 0.0072 in. Theoretically, it should elongate to $6.231 + 0.0072 = 6.2382$ in., but actually it elongates to $6.231 + 0.0144 = 6.2454$ in. Because of the modulus of the steel the length of the strand is $6.2382 \times 1.0554 = 6.5838$ in., but it is a question as to how much too short this length is to accommodate the change in length of rope from 6.231 to 6.2454 in. The answer is found thus: $(6.5838)^2 = (6.2454)^2 + C^2$; $C = 2.083$ in.

In this computation, C is the circumference of the helix circle; its radius is 0.3315 in. compared with $5 \times 0.067 = 0.3350$ in. at the beginning. The difference in the diameters of the core to meet the conditions imposed is $2 \times 0.3350 - 0.3315 = 0.007$ in. Of course, this calculation also illustrates the profound easing of the tension in a strand of a bent rope if the core yields ever so little. From experience it is known that the rope centers vary in diameter, and have varying resistances to crushing, through such wide limits that the behavior of rope in service is not a matter of exact prophecy. As an illustration of this fact scan the records of the ropes in Table 4.

TABLE 4.—Service Record of Wire Ropes $\frac{5}{8}$ Inch in Diameter.

(Tension, 1.25 kips; velocity, 550 feet per minute; diameter of sheaves, 34 inches; 6 × 19 construction; regular lay.)

Manufactured by:	Material	Miles traveled	Manufactured by:	Material	Miles traveled
A	Steel	10 247	B	Iron	10 372
C	Swedish iron	7 168	A	Mild steel	42 568
A	Special steel	18 343	B	Iron	26 130
A	Mild steel	9 158	A	Mild steel	13 538
B	Special steel	5 732	A	Mild steel	19 345
B	Mild steel	13 678	D	Mild steel	35 565
A	Mild steel	13 192	D	Mild steel	2 625
B	Iron	12 019	A	Mild steel	48 091
B	Iron	10 184	B	Iron	22 787
A	Mild steel	17 209	C	Mild steel	5 925
B	Iron	11 247	A	Mild steel	18 279
A	Mild steel	19 991	B	Iron	11 452
B	Mild steel	6 963	A	Mild steel	5 723
C	Swedish iron	24 653	B	Mild steel	12 733
B	Iron	13 487	B	Iron	3 508
B	Mild steel	4 714	A	Mild steel	8 115

Properties of Wire Rope

Under tension, wire rope behaves very differently from a homogeneous bar. For instance, a sample of rope may be placed in a testing machine and subjected to stress. If an extensometer is used the strain corresponding to any stress may be noted. If the loading is progressive to a value within the yield point and is then reversed, the stress-strain diagram will fail to close by an amount equal to the permanent stretch. However, if these reversals of stress are repeated a sufficient number of times the stress-strain diagram will become a closed figure, showing that the specimen has reached a condition in which it has completely recovered from the effects of loading. The stress-strain graphs of the progressive and retrogressive loadings do not coincide, but they bound an area which represents work in a manner similar to the well-known indicator diagram. These stress-strain graphs are shown in Fig. 6.

By a proper determination of the average tension and elongation, as shown by the diagram, the amount of work lost in stretching the rope may be estimated. This result indicates the value of the internal friction developed in the interior of the rope and such a stress-strain curve is called a frictional hysteresis diagram. It will be noted that when the loading is relatively light the elongations for a constant increment of loading are not equal and the graph is curved. When the loading becomes relatively high, the stress-strain graph approaches a straight line. The explanation is simple. The modulus of elasticity of wire rope is variable under moderate tensions, but it follows Hooke's law when the loads applied develop internal friction sufficient to compare with the cohesive forces that exist in a solid homogeneous bar. It is evident from the nature of the construction of wire rope that it is not adapted to resist longitudinal compression. Therefore, the modulus of elasticity in compression lengthwise is too small to consider. The neutral axis of a bent rope seldom coincides with the gravity axis of its section.

BENDING STRESS IN WIRE ROPE

The question of bending stresses in wire rope is a mooted subject. It is the usual procedure to rely upon the theory of flexure as developed for homogeneous prisms as an aid in determining these bending stresses. It is admitted by those who have a proper knowledge of the theory of flexure that it is only approximate and its use is permitted in engineering calculations because a great number of physical tests have demonstrated its limitations. It is founded on nine assumptions, three of the most important of which are:

1.—The bending must be slight;
2.—Plane surfaces before, must remain plane surfaces after, bending; and
3.—The modulus of elasticity in tension must be equal to the modulus of elasticity in compression.

In bending wire rope around the usual sheaves these criteria are violated and the use of the accredited formulas yields delusive results. Those who have studied the elastic curve of loaded beams will recall that the theory of flexure gives an approximate formula which may be written, $EI\frac{d^2y}{dx^2} = M$; that is, the modulus of elasticity times the moment of inertia times the rate of curvature of the deflected beam is equal to the external moment. However, the rate of curvature is equal to the reciprocal of the radius, and the external moment (by the theory of flexure) is,

$$E \times \frac{\text{unit stress in the outermost fiber}}{\text{distance from the internal axis to outermost fiber}}$$

Hence, the unit bending stress, b, in a homogeneous circular bar that has been slightly bent, would be given by the formula,

$$b = E\,\frac{r'}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which, r' is the radius of the rod, and R, the radius of bend. When Equation (3) is used to determine the stresses arising from sharp bends, the stresses indicated are of such magnitude that no one of experience would place reliance in the results obtained.

However, a wire rope is not a homogeneous rod, but an assemblage of wires, and, for that purpose, Equation (3) has been arbitrarily modified by substituting the diameter of the wire used in the construction of the rope for the rod, giving the Reuleux formula,

$$b = E\,\frac{d'}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which, D = diameter of sheave.

Since the diameter of a wire in a 6 × 19 rope is one-fifteenth the diameter of the rope, and if the modulus of elasticity of steel is taken at 28 400 kips per sq. in., then,

$$b = 1\,894\,\frac{d}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

which is the well-known Rankine formula. It will be noted that the d' in the Reuleux formula indicates diameter of wire; the d in the Rankine formula is the diameter of the rope.

A large part of the literature on wire rope is devoted to the presentation of tables and diagrams of stresses due to bending, which are based directly, or with slight modification, upon these formulas. In the opinion of many this information is most deceptive and has played a prominent part in blocking the advance of the Engineering Profession to a true knowledge of the behavior of a wire rope in service.

When an engineer purchases his first piece of wire rope he is filled with fear that it will be severely injured if subjected to bends of short radius. When the "rough and ready" rigger immediately forms a bight in the end of the rope by bending it around the usual sized thimble, he is quite sure that the rigger is taking liberties with the rope that are not justified by the tables presented by wire-rope manufacturers. When the rigger proceeds to reeve a set of wire-rope blocks which have sheaves 8 or 10 in. in diameter, with a $\frac{5}{8}$-in. or $\frac{3}{4}$-in. rope, and forthwith uses the blocks to lift weights of many tons without having the rope fall in pieces, the buyer obtains a true insight into the value of the tables of bending stress and minimum sizes of sheaves as published.

Mr. James F. Howe has stated[5] that "bending stress has been confused with the effect of bending or the loss of strength due to bending, two entirely different factors." If the stress under discussion is produced by bending and a designer is trying to determine a suitable allowance to make in his calculations for its effect, the sentence quoted becomes unintelligible; but if it is meant that the tabulated stresses are fictitious, then everything becomes clear.

To put the matter to a test, a $\frac{3}{4}$-in., 6 × 19 rope was cut into six pieces approximately 6 ft. in length. All were fitted with sockets carefully zinced into position. Of the six pieces the even numbered ones were tested in direct tension and failed, respectively, at 35.00, 35.05, and 35.10 kips. The odd numbered samples were bent around a sheave 6 in. in diameter and attached to an equalizing arm, as shown in Fig. 7. The three samples broke within the limits of 63.90, 64.00, and 64.05 kips. Since the rope in this position carries double the load, it broke at 32 kips, or 3 kips less than the 35 kips when tested in direct tension. There can be no other conclusion but that the strength of the rope was reduced 3 kips when bent 180° around a 6-in. sheave.

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[5] *Transactions*, Am. Soc. M. E., December, 1918, p. 17.

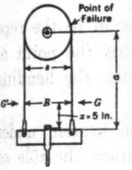

FIG. 7.

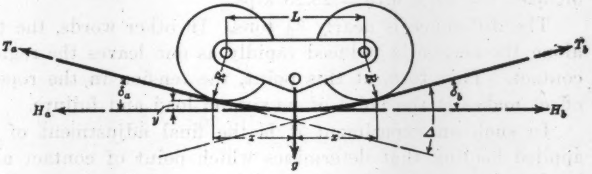

FIG. 8.

The area of the rope was 0.241 sq. in. and the bending stress, therefore, on the usual assumptions, was equal to 12.45 kips per sq. in. If these data are used for determining the constant for the Reuleux formula (Equation (4)), $\frac{E}{15}$ becomes $1\,285\,\frac{d'}{D}$, instead of 1 894 with $E = 28\,400$, as used in the original Equation (5), or the 12 000 as advocated by other authorities. (The answers resulting from these small constants are in kips (1 kip = 1 000 lb.).

This discrepancy between theory and practice is due to the fact that a wire rope, in passing around a sheave, has but one point of distortion, namely, the point of tangency of the rope and the sheave. This is a point of instantaneous bending, provided the rope is in motion. If the bending is performed slowly, the stress developed in the top position of a strand is distributed through its helical winding to the bottom.

The tightening of the top strands of the rope as it passes the point of contact with the sheave, causes a slight twisting of the free portion of the rope, and the resistance to this moment of torsion develops a slight curvature near the sheave. These reactions tend to reduce the magnitude of the bending stress aside from any distortion of the core. This slight rotation of the rope on guide sheaves is clearly shown by the grooving of the tread to the lay of the rope. In most cases the rotation is of the order of 0.05 in., measured on the circumference of the rope.

In testing ropes at rest on sheaves, by breaking them in tension, it has been noted that the failure often occurs in the bent portion near the point of contact of the rope and sheave. If it is admitted that bending increases the strand tension, then the point shown in Fig. 7 is the position of the greatest tension in the rope. Due to the friction of the strand contacts with the sheave in such an experiment, the tension in the rope at the crest of the sheave is a minimum. This may be shown by a simple computation in which the tension, T, in the rope will be taken as 32 kips, the coefficient of friction, f, as 0.15, the angle of contact as $\frac{\pi}{2}$, and T_c as the crown tension,

$$T_c = T e^{-f\gamma} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

or, $32 e^{-0.24} = 32 \times 0.79 = 25.28$ kips.

The difference is nearly 3½ tons. In other words, the tension in the rope along the sheave is reduced rapidly as one leaves the region of the point of contact. Therefore, at this point, the tension in the rope plus the bending effect makes it the point of maximum load and failure.

In such an experiment it is the final adjustment of the system under applied loading that determines which point of contact assumes the rôle of bending or straightening the rope. A revolving sheave determines the question at once. Just as a set of rolls bend a plate on the element of a cylinder, so does a sheave bend a rope at the point of curvature; but unlike the rolls, the sheave supports the rope in the curve developed, and then straightens it at the point of tangency. Accordingly, a sheave is a wonderful device for bending and straightening every portion of a running rope.

The reduction in the distance between the two arms of the rope loop of the test pieces, with increasing tension is noteworthy. A tension of 22.7 kips reduced the distance from $7\frac{1}{8}$ in. to $6\frac{7}{16}$ in., or $\frac{11}{16}$ in. The amount contributed by each rope is not known, but for the sake of making a calculation, assume a span of 1.83 ft., the distance from sheave to equalizing bar, a deflection of 0.029 ft. at a point 0.25 ft. from the sheave, and compute the lateral thrust required to produce the deflection by the formula,[6] $G = \frac{yst}{x\,(s - x)}$. In the present case,

$$G = \frac{0.029 \times 1.83 \times 22.7}{0.25 \times 1.58} = \frac{1.205}{0.395} = 3.05 \text{ kips}$$

It may be significant that this result, derived from the maximum deflection of the system, is in almost perfect agreement with the measured reduction in strength of the rope.

LOSS OF STRENGTH IN A WIRE ROPE DUE TO BENDING

For the purposes of easy reference some of the well-known formulas for bending stress may be summarized. Rankine's formula (Equation (5)) is, $b = 1\,894 \frac{d}{D}$. Since other formulas give total rather than unit stresses, Equation (5) is modified by introducing the area of the rope, say, $0.43\,d^2$. Then,

$$T = 1\,894 \times 0.43\,d^2 \times \frac{d}{D} = 814\,\frac{d^3}{D} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

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[6] *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 916. Equation (30).

For a $\frac{3}{4}$-in., 6 × 19 rope bent around a sheave 6 in. in diameter, Equation (7) becomes, $T = 814 \times \frac{0.75^3}{6} = 56.98$ kips, which is to be reconciled with an ultimate strength of 35 kips.

Hewitt's formula is,

$$T = \frac{E\,A}{2.06\,\frac{R}{d'} + K} \qquad \text{(8)}$$

in which, A is the cross-sectional area of a wire rope, and K, the ratio $\left(\frac{d}{d'}\right)$ of rope diameter to the diameter of the wire.

Reducing Equation (8) by inserting numerical values and substituting $2.06\,R = D$ and $d' = \frac{d}{13}$,

$$T = 910\,\frac{d^3}{D} \qquad \text{(9)}$$

Equation (9) is about the same as the Rankine formula (Equation (7)) corrected for the angle of lay of the rope.

The modified Reuleux formula (see Equation (4)) is,

$$b = E'\,\frac{d'}{D} \qquad \text{(10)}$$

in which, E' is the modulus of elasticity of the rope in tension, say, 12 000 kips per sq. in. Reducing this form to the value of:

$$T = \frac{12\,000 \times 0.43\,(d')^2 \times d'}{13 \times D} = 397\,\frac{(d')^3}{D} \qquad \text{(11)}$$

According to Equation (11), the $\frac{3}{4}$-in., 6 × 19 rope on a 6-in. sheave has a bending stress of 27.9 kips. In other words, the specimen will not fall to pieces before entering the testing machine.

Chapman's formula is,

$$b = \frac{E d' \cos^2\beta \cos^2(\alpha + \beta)}{D} \qquad \text{(12)}$$

in which, β is the angle of helix of a single wire, with the axis of the rope, and $(\alpha + \beta)$ is the angle of helix of a strand, with the axis of the rope (see Fig. 3). Bending stresses computed by Equation (12) fall between the Hewitt formula (Equation (8)), and the modified Reuleux formula (Equation (10)), and may be dismissed without further argument.

A New Formula for Wire Rope

It has been the aim of this paper to show that wire rope is a systematic grouping of wires into helices bent about a core that may be regarded as having the properties of a yielding foundation.

Years ago it was recognized that the curving of a rod into a helix resulted in the setting up of stresses due to bending and twisting.[7] In subjecting a helical spring to axial tension the free end rotates, unless restrained. Accordingly, one must conclude that the wires and strands of a wire rope have been subjected to bending and twisting, and that when they are passed over a sheave, under tension, the rim of the sheave will be cut intaglio, by the rotation of the rope due to torsion. That this is so is a matter of observation, and is well known to users of wire rope. It is the manufacturer's aim to balance these effects, so that, in general, the twist of the rope may be kept at a minimum, but the torsion stresses are always present in the wires. These rim cuts raise the inquiry, "If a wire rope, under tension, passes around a sheave, is it in a state of pure bending? If not, where is the neutral surface?"

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[7] "Natural Philosophy", by Kelvin and Tait, Paragraph 120.

By definition pure bending is the effect produced in a bar by two equal and opposite couples acting upon its ends. In pure bending there is a gravity surface passing through the centroid of the section between the fibers in tension and compression—that is, in the region of neutrality between fibers that are elongated and those that are shortened. In general, it may be said, that as a wire rope under tension approaches the point of contact (curvature) with the sheave, all wires are under tension. When these wires pass on to the sheave surface, those in contact with it remain in tension, because the rim friction prevents the compressive accommodation. Since all wires are in tension, they are on one side of the neutral surface, which, therefore, must be at or near the rim surface of the sheave. The rim is subjected to circumferential and radial compression.

It seems that calculations for loss of strength due to bending wire rope, that are made upon the assumption of pure bending, must lead to results that are not satisfactory. For this reason there has been much speculation as to the wide discrepancy between the theoretical and the actual performance of ropes in service.

Fig. 9 shows a wire rope as an assemblage of open-coiled helical springs, made of high-class steel. It is proposed to investigate the possibility of computing rational values for the loss of strength due to bending a wire rope on this hypothesis. It is granted that the derivation of the several formulas leads to approximate answers, because it is assumed that the radius of rope, of the strands, and the angle of pitch of the several wire springs remain constant, while it is evident that the former becomes less, and the latter larger, when the rope is in service. It is not believed that serious errors arise from this expedient for simplicity. It must be admitted, however, that the bending effect is a function of the tension, if precision of statement is demanded.

The mathematical theory of open-coiled helical springs is well known, and has been tested by numerous investigators, with satisfactory results. The treatment covers axial load, extension, twist, and bending. The axial bending is of importance in this problem, because all the "springs" in the rope have their axes curved about the center of rotation of the sheave.

In Fig. 9, let the heavy line represent the position of the core wire of a strand, in both a straight and a bent section of the rope. Let O be a point for investigation that may be at any point, in projection, upon the circumference of the circle of the helix. Therefore, O is defined by the radius of

the generating circle and the angle, ϕ, between this radius and the principal axial plane. The resistance offered by the open spring to axial bending results in a moment, M, which is to be used to estimate the loss in strength of the wire due to bending it. The axis of the moment, M, will be parallel to the shaft of the sheave. Let O-P be a vector representing the moment, M. This vector may be resolved into two components, O-B producing bending of the wire in the plane of the tangent to the surface of cylinder enclosing the helix, and the moment, O-C, acting in this plane. Therefore, O-B is equal to $M \cos \phi$. The vector, O-C, in turn, may be resolved into two components giving the torque moment, M_t, and bending moment, M_b, in the plane of the coil. If the complement of the angle of lay of the strand, say, is α (equals the angle of the helix), that is, the angle measured from the perpendicular to the axis counter-clockwise to the tangent to the center line of the wire, then these two components may be written in terms of the moment, M: $M_t = -M \sin \phi \cos \alpha$; and, $M_b = M \sin \phi \sin \alpha$.

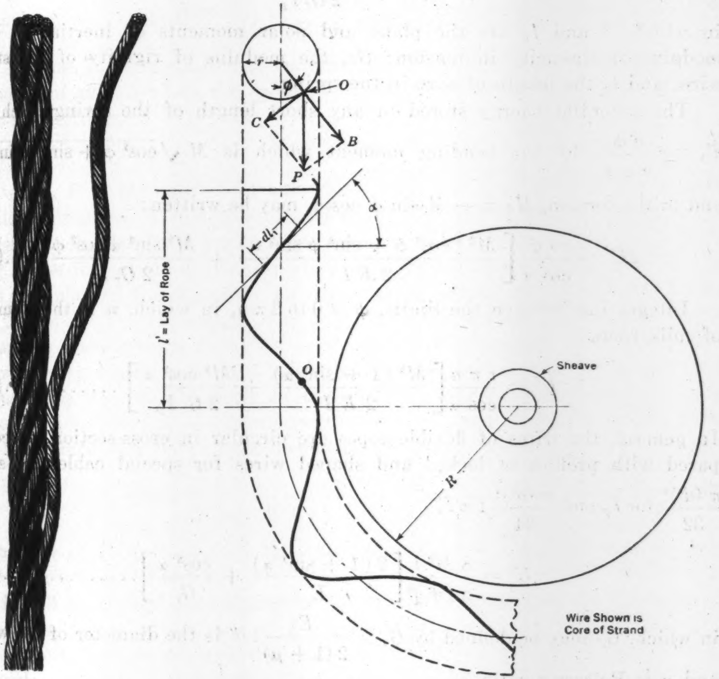

FIG. 9.

The bending due to O-B and M_b must be combined in the usual manner for obtaining maximum and minimum stresses. The bending moment thus considered is measured by $M \sqrt{\cos^2 \phi + \sin^2 \phi \sin^2 \alpha}$.

At this point it is well to recall that the potential energy of bending is given by the expression:

$$U'_b = \frac{M_b^2 l}{2\,E I}$$

Likewise, the potential energy of twist is given by:

$$U'_t = \frac{M_t^2\, l_1}{2\,G_r\, I_p}$$

in which, I and I_p are the plane and polar moments of inertia; E, the modulus of elasticity in tension; Gr, the modulus of rigidity of the steel wire; and l_1, the length of wire in the spring.

The potential energy stored in any short length of the spring, such as $dl_1 = r\,\dfrac{d\,\phi}{\cos\alpha}$, by the bending moment which is $M\sqrt{\cos^2\phi + \sin^2\phi\sin^2\alpha}$, and to the torsion, $M_t = -M\sin\phi\cos\alpha$, may be written:

$$dU' = \frac{rd\,\phi}{\cos\alpha}\left[\frac{M^2\,(\cos^2\phi + \sin^2\phi\sin^2\alpha)}{2\,E\,I} + \frac{M^2\sin^2\alpha\cos^2\phi}{2\,G_r\,I_p}\right]\ldots(13)$$

Integrating between the limits, $\phi = 0$ to $2\,\pi\,n$, in which, n is the number of coils, then,

$$U' = \frac{r\,\pi\,n}{\cos\alpha}\left[\frac{M^2\,(1 + \sin^2\alpha)}{2\,E\,I} + \frac{M^2\cos^2\alpha}{2\,G_r\,I_p}\right]\ldots\ldots\ldots\ldots(14)$$

In general, the wires of flexible ropes are circular in cross-section, as compared with profiles of locked and shaped wires for special cables. Using $\dfrac{\pi\,(d')^4}{32}$ for I_p and $\dfrac{\pi\,(d')^4}{64}$ for I,

$$U' = \frac{8\,M^2\,l_1}{\pi\,(d')^4}\left[\frac{2\,(1 + \sin^2\alpha)}{E} + \frac{\cos^2\alpha}{G_r}\right]\ldots\ldots\ldots\ldots\ldots(15)$$

in which, G_r may be defined by $G_r = \dfrac{E}{2\,(1+\mu)}$, d' is the diameter of the wire, and μ is Poisson's ratio.

Assume that the rope is fixed at, and beyond, the point of curvature (contact) with the sheave; also, that the work done by the moment in bending the axis of the spring to the radius, ρ, will be equal to the energy stored in the spring; that is, if l, the lay of the rope, is equal to $l_1\sin\alpha$,

$$\frac{1}{\rho} = \frac{16\,M}{\pi\,(d')^4\sin\alpha}\left[\frac{2\,(1+\sin^2\alpha)}{E} + \frac{\cos^2\alpha}{G_r}\right]\ldots\ldots\ldots(16)$$

and the moment due to bending a wire shaped as an open helical spring about the center of curvature of the sheave is,

$$M = \frac{\pi\,(d')^4\sin\alpha\,E\,G_r}{16\,\rho\,[2\,G_r\,(1+\sin^2\alpha) + E\cos^2\alpha]}\ldots\ldots\ldots\ldots\ldots(17)$$

which seems to be correct in form, at least, for it meets the well known test that when the elastic curve is an arc of a circle the bending moment is constant. To obtain a value for the stress in the outermost fiber of the wire, in the simplest possible way, for comparison purposes only, $M = \dfrac{S\,I}{\dfrac{d'}{2}}$, or,

$$S = \frac{2\,d' \sin \alpha\, E\, G_r}{[2\,G_r\,(1 + \sin^2 \alpha) + E \cos^2 \alpha]} \text{ .... (18)}$$

The loss in strength due to bending may be evaluated by ascertaining the amount of axial load on the spring that will give an equal moment. This is done by writing,

$$M = P\, r_1 \sin \alpha \text{ .... (19)}$$

in which, P is the equivalent axial load, and r_1 is the radius of the strand. This assumption is somewhat justified by the observation of the effect of tension upon wire ropes having fiber centers. The core is compressed, resulting in the formation of grooves, which thus furnishes a partial restriction to the free play of the strand as an open spring having an axis at the center of the rope. Nevertheless, it does not hinder the action of the wires in the strand as open springs about its own center. Furthermore, it is convenient to work with a dimension that is easily attained; for instance, it is a general assumption that the diameter of the strand is one-third the diameter of a 6 × 19 rope; hence, r_1 equals one-sixth of the diameter of the rope. The axial load sustained by a wire of the strand is determined by,

$$P = \frac{\pi\,(d')^4\, E\, G}{16\,\rho\, r_1\,[2\,G_r\,(1 + \sin^2 \alpha) + E \cos^2 \alpha]} \text{ .... (20)}$$

The value of P multiplied by the number of wires in the rope gives the loss of strength due to bending.

It will be noticed that Equation (20) takes into account the diameter of the wires, the rope, the radius of curvature, the angle of lay, the modulus of elasticity in tension, and the modulus of rigidity, thereby differing from formulas heretofore proposed. In a broad sense, it may be necessary to determine coefficients by actual tests for different kinds of ropes, just as in the practical application of hydraulic and many other formulas, but as a test of its worth, the loss of strength of the ¾-in. rope which was bent around the 6-in. sheave and described in the paper, will next be computed by this formula. It will be recalled that the loss of strength was 3 kips, due to bending.

The actual diameter of the rope as made of cast steel and standard lay, 6 × 19 construction and with hemp center, was 0.79 in. There were 114 wires of 0.054 in. each. (The lay of the rope is 4.94 in., and of the strand, 1.81 in.). The angle of lay of the strand is 18° 40′, and of the wires in the straight strand, 7° 33′; and, hence, of the wires in the rope, 26° 13′; hence, $\alpha = 63° 47'$. The radius of curvature, ρ, is assumed equal to 3.37 in., in the belief that the mean diameter of the rope, when under tension, is 0.74 in. Furthermore,

$$G_r = \frac{E}{1.3 \times 2} = 11\ 000 \text{ kips per sq. in., when } E = 28\ 500 \text{ kips per sq. in.}$$

Therefore, by Equation (20), in pounds per wire,

$$P = \frac{3.14 \times 0.054^4 \times 28.5 \times 10^6 \times 11 \times 10^6}{16 \times 3.37 \times 0.125 [2 \times 11 \times 10^6 (1 + 0.81) + 28.5 \times 10^6 \times 0.19]} = 27.4$$

Multiply by the number of wires, 114 × 27.4 lb. = 3 125 lb., which is in satisfactory agreement with results secured from the test. The writer has experienced considerable satisfaction in estimating the loss of strength due to bending ropes about sheaves and to find the results secured in service were in sufficient agreement so that the riggers and other practical rope users did not smile and turn away when the conversation turned to the strength of ropes, when reeved, say, in the usual wire-rope blocks, an achievement that many sympathetic readers will endorse as being well worth while. It is hoped that the wire rope companies will make the necessary tests to ascertain the general usefulness of this formula, or a comparable one, to the end that the engineer having to do with wire rope may have an accurate estimate of the reduction in strength of the rope due to bending.

The discussion should close at this point because all the foregoing remarks lie in the "battle zone" of wire rope considerations. In order to hold aloft a new target "to shoot at," the following "service factor" formula is offered for discussion. It aims to consider tension and velocity as well as rope-sheave ratios. Since E, the modulus of elasticity of the steel or the rope, is a function of the tension, it is omitted, and the formula is as follows:

$$T_s = k \frac{d^3}{D} (1 + V)^{0.67} \ (10 + T)^{0.50} \ \ldots\ldots\ldots\ldots\ldots (21)$$

in which,

T_s = the total service stress, in kips;
T = the tension on the rope, in kips;
V = velocity, in feet per second; and,
k = a constant for any one type of rope.

Three values may be listed, as follows: For 6 × 7 rope, $k = 300$; for 6 × 19 rope, $k = 250$; and, for 6 × 19 Seale type of rope, $k = 275$.

If V is zero and T is 32 kips, a $\frac{3}{4}$-in. rope on a 6-in. sheave loses 3.6 kips in strength. On a $\frac{3}{4}$-in. sheave, with no velocity, a tension, T, of 12 kips gives a service stress, T_s, of 22 kips; that is, the rope will hold 5 tons when bent on its own diameter without clipping. On the other hand, if a $\frac{3}{4}$-in. rope on a 10-in. sheave has a tension of 5 kips and a velocity of 6 100 ft. per min., the service factor plus the tension will approach the ultimate strength of the rope. In its final form, Equation (21) will probably have a selective coefficient for various conditions of lubrication and exposure. In its present form it is based on the assumption of adequate lubrication and favorable use. If such a service formula is to prevail, the selection of a wire rope for a given duty must be more rational than the guesses of former years.

EQUATION FOR AN ELASTIC CURVE OF A LOCKED-COIL CABLE SUPPORTING A CARRIER

The running of sheaves on standing ropes, or the support of moving ropes on guide-sheaves, presents a variation of the bending stress problem, because the radius of curvature of the rope is, in general, greater than the radius of the carriage wheels, or rollers, as the case may be. The following analysis is based on the theory of flexure, because it contemplates lock-coil track cables operating under sufficient tensions to keep the curvature within the limits demanded by classical theory. Pressures from the helical winding of the several layers of wire give properties to the cable, that are similar but of less magnitude than those of a homogeneous bar.

If a portion of such a cable is supposedly taken from a loaded span, it will present the aspect of a beam with fixed ends, in so far as the direction of the axis is concerned, and an analysis to discover the relation of the tension, moment of inertia, modulus of elasticity, load, and other factors may be made as follows: Assume that the cable has sufficient resistance to flexure to form an elastic curve between the points of tangency of the cable to the right and left carriage wheels. Referring to Fig. 8, the summation of the vertical components equals zero and the summation of the horizontal components equals zero; hence,

$$g = T_a \sin \delta_a + T_b \sin \delta_b \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

and,

$$H = T_a \cos \delta_a + T_b \cos \delta_b \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

If $\delta_a = \delta_b$ in Equations (22) and (23),

$$g = 2\,T \sin \delta = 2\,H \tan \delta$$

and,

$$\tan \delta = \frac{g}{2H} = \frac{g}{2} \times \frac{1}{H}$$

That is, the section represented by Fig. 8 may be replaced by a free body similar to Fig. 10. It will be noted that either side of the curve assumes a position of equilibrium under the conditions of a vertical load and a horizontal load acting simultaneously. It will be assumed that the conditions controlling the theory of flexure in homogeneous bodies can be applied to locked-coil cable, provided the bending modulus is regarded as variable between the limits of $E = 1\,500$ for no tension and $23\,000$ for a tension of 25 kips.

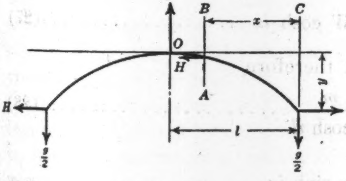

FIG. 10.

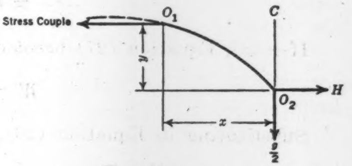

FIG. 11.

Since the curvature is slight under ordinary conditions of tension and loading, the approximate formula, $EI\,\frac{d^2y}{dx^2} = M$, may be used. To apply this equation of the elastic curve consider a free body to the right of Section A-B in Fig 10. Taking the summation of moments about O_1 (Fig. 11),

$$M = -\tfrac{1}{2}\,gx + yH = EI\,\frac{d^2y}{dx^2}$$

and,

$$\frac{d^2y}{dx^2} = \left(y - \frac{gx}{2H}\right)\frac{H}{EI} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(24)$$

For convenience, let $\frac{H}{EI} = i^2$, and $y - \frac{gx}{2H} = u$; then Equation (24) becomes,

$$\frac{d^2y}{dx^2} = i^2\,u = \frac{d^2u}{dx^2}$$

This expression is immediately recognized as a cyclic function because the second differential is a multiple of the function. By integration,

$$\int_0^u \frac{d^2u}{dx^2} = \int_0^u i^2u = Ue^{ix} + We^{-ix}$$

A solution, expressed in hyperbolic functions, is,

$$u = U\cosh ix + W\sinh ix$$

and,

$$y = \frac{gx}{2H} + U\cosh ix + W\sinh ix \quad\ldots\ldots\ldots\ldots\ldots(25)$$

If O_2 is taken as the origin, $x = 0$, $y = 0$, and $U = 0$; let $\frac{g}{2H} = m$; then Equation (25) becomes,

$$y = mx + W\sinh ix \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(26)$$

and,

$$\frac{dy}{dx} = m + i\,W\cosh ix \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(27)$$

If $x = l$, Equation (27) becomes zero; therefore,

$$W = -\frac{m}{i\cosh il} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(28)$$

Substituting in Equation (26),

$$y = mx - \frac{m\sinh ix}{i\cosh il} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

or, for $x = l$,

$$y = \frac{g}{2H}\left[\,l - \frac{\tanh\sqrt{\frac{H}{EI}}\,l}{\sqrt{\frac{H}{EI}}}\,\right] \quad\ldots\ldots\ldots\ldots(30)$$

The general equation may be written,

$$y = \frac{g}{2H}\left[x - \frac{\sinh ix}{i \cosh il}\right] \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

in which, $i = \sqrt{\frac{H}{EI}}$ to simplify typography.

The slope of the tangent at the point, xy (Fig. 8), is,

$$\frac{dy}{dx} = \frac{g}{2H}\left[1 - \frac{\cosh ix}{\cosh il}\right] \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

If the curve (Fig. 8) is assumed to be a circular arc, the radius of curvature can be found for the arc, $O_1\ O_2$, (Fig. 11), for any value of Δ and any slope of cable, provided $\tan \Delta = \frac{g}{H}$.[8]

CONCLUSION

There are many phases of wire rope—its attributes and its manufacture—that may well remind one of the philosopher's "category" and the "unknowable." Modern wire rope represents the best in mining and selecting iron ore and limestone, in the making of coke, in blast-furnace operations, in the selection of scrap, in the control of the "heat" of the open hearth, in the human equation of the melter, in the boss roller, in the soaking shift foreman, in the cleaning house boss, the wire drawer, the tester, the manila rope manufacturer, the annealer, the spooler, the men on the stranding and laying machines, the warehouse, salesmen, and numerous chemists, physicists, executives, and in the use of the power of wealth; all co-ordinated to produce the best product that can be made within the price the purchaser will pay. Every one of these items and more have uncertain factors that may not have been safeguarded against or eliminated, but will be when the art of steel and wire-rope making ceases to be extremely competitive and is directed toward the creation of quality products.

In any symposium for the improvement of the service life of wire the issue must be joined by considering this question: Can the processes involved in the manufacture, the rolling of rods, the drawing of wire, the laying of wire rope, and its uses, be brought under critical technical control to the end that a behavior of wire rope in service may be predicted with assurance? Of course, some ropes fail to render service. The wonder is that so many prove satisfactory in spite of abuse and neglect. It is probable that the improvement of wire rope will be the result of a gradual advance in the control of the basic operations of its manufacture and use.

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[8] For tables of stress in locked-coil cables, see *Transactions*, Am. Soc. C. E., Vol. 92 (1928), pp. 940–948.

APPENDIX I

NOTATION

The following will serve as a general guide for the definition of all symbols used in the paper:

b = unit bending stress.
d = diameter of a wire rope; d' = diameter of a single wire.
e = base of natural logarithms.
f = coefficient of friction.
g = total weight applied by a tramway carriage.
i = substitution factor $= \sqrt{\frac{H}{EI}}$ (Equation (25)).
k = a constant varying with the type of rope (Equation (21)).
l = length along a rope; l' = length of lay, or the pitch of a single wire; l_1 = length of a twisted wire in the straight distance, l' (Fig. 2).
m = a substitution factor $= \frac{g}{2t}$ (Equation (26)).
n = number of coils.
r = radius of a rope; r' = radius of a circular bar, or a single wire. (Distance between the two arms of a wire rope loop.) r_1 = radius of a strand.
s = a sheave.
u = a substitution factor $= y - \frac{gx}{2H}$.
w = suspended weight; w' = unit dead load weight of rope. Distance from the origin of ordinates to a point, $x\,y$ (see Fig. 8).
x = horizontal distance from origin to point, $x\,y$.
y = deflection of a wire rope.
A = cross-sectional area of a rope; A' = section area of a single wire.
B = reduced distance between arms of a wire rope around a sheave, supporting a load (Fig. 7).
C = circumference of a helix circle.
D = diameter of a sheave.
E = modulus of elasticity; E', in tension; E_b in bending.
F = length of circumference of helix circle (see Fig. 2).
G = a force normal to a cable at any point; G_r = modulus of rigidity.
H = horizontal components of tension; H_a and H_b = components of T_a and T_b, respectively.
I = planar moment of inertia; I_p = polar moment of inertia.
K = ratio of rope diameter to wire diameter, $= \frac{d}{d'}$.
L = length of wheel-base of a tramway carriage.
M = bending moment.
P = an axial load on a wire.
R = radius of curvature of bends in rope, or semi-diameter of a sheave; R_w = radius of carriage wheels; R_t = radius of tread of a carriage wheel; R_c = radius of curvature of a bent rope.
S = strength of wire rope; S' = strength of a single wire; S_s = strength of a strand.

$T =$ tension in a wire rope; $T_c =$ tension at the crest of a sheave; $T_s =$ total service strength; $T' =$ load on a single straight wire (Fig. 1); $T_1 =$ load on a single twisted wire (Fig. 2); $T_a =$ rope tension toward the left; $T_b =$ rope tension toward the right.
$U =$ a substitution factor in Equation (25).
$U' =$ potential energy; U'_b, of bending; and U'_t, of twisting.
$V =$ velocity of a wire rope.
$W =$ a substitution factor in Equation (25).
$\alpha =$ angle between a curved center wire and a wire twisted about it (Fig. 3), or the angle of pitch of an open spring.
$\beta =$ angle between the axis of a rope and a curved center wire (Fig. 3).
$\gamma =$ angle of contact between rope and sheave
$\delta =$ angle between forces, T and H.
$\theta =$ angle between principal axial plane and radius to Point O (Fig. 9), on open helical spring.
$\Delta =$ angle of intersection of the tangents of a bent wire rope $= \delta_a + \delta_b$ (Fig. 8).
$\phi =$ angle of lay of the outside wires of a strand of rope.
$\mu =$ Poisson's ratio.
$\rho =$ radius of curvature.

DISCUSSION

Robert C. Strachan,[9] M. Am. Soc. C. E. (by letter).—In many respects, a wire rope is similar to a solid bar. It stretches under tension and recovers when released, in a manner well known through many tests. Short pieces show regularity in measured deflections under lateral forces. Naturally, much attention has been given to the effect of passing a rope around a sheave, since one of the main purposes of a rope is to enable one to change the line of application of a force. As previously noted by the writer,[10] rope action is complicated by the presence of fiber cores, by the twisted state of the wires, and by other factors.

Analogy with a homogeneous bar is entirely lacking when a sheave is present; hence, the diversity in results based on the assumption of such an analogy is not surprising. The fact of extremely large deformations, to which this diversity must be largely attributed, will be easily perceived if values of 35 kips and 30 000 kips for b and E, respectively, are inserted in the author's Equation (3), and R is deduced. It is found thus that at the elastic limit, beyond which the formula is inapplicable, the radius of curvature of a steel specimen is approximately 430 times its depth. This would seem to show conclusively that the result of applying the common theory of flexure to a $\frac{3}{4}$-in. rope, bent around a 6-in. sheave, must be fallacious.

That there are many reasons for conflict between theory and practice is recognized by the author, who grants, in preparing to derive a rational expression for bending stress, that "the derivation * * * leads to approximate answers." One of the reasons not mentioned in the paper is the important fact that many ropes are composed of wires of different sizes. It is evident that any formula such as Equation (20), which assumes uniformity of wires, must be considerably in error for the large class of ropes in which such uniformity does not exist.

The probability of error in numerical application is enhanced by the smallness of the dimensions of the elemental parts. The effect is to make minute absolute differences become large proportional differences. This may be seen in the author's application of Equation (20), where the wire diameter is taken as 0.054 in. So small a difference as 0.001 in this dimension leads to a difference of 6% in the calculated value of P.

The presence of error due to the exclusion of the effect of direct tension in deriving Equation (20) is recognized by the author, but it is altogether probable that this error is of greater importance than his somewhat casual reference to it would indicate.

Taking into consideration the complex mechanics of combined tension and torsion in a compound wire helix, the large deformations, the small dimensions of parts, the internal conditions not discernible by visual inspection, but in fact affecting core resistance, and the variety of materials, as well as the different wire sizes in one rope, it appears to the writer impracticable to derive a satisfactory rational formula for bending stress. In the last analysis, reliance must be placed on the "life-saver" coefficient derived by experiment.

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[9] Cons. Engr., New York, N. Y. Mr. Strachan died on April 16, 1933.

[10] *Civil Engineering*, November, 1930, p. 111.

The comparatively simple Equation (21) would seem preferable in form to Equation (20); but a better line of approach to the problem is that taken by the National Bureau of Standards, in making and recording tests which show how much loss in strength is caused by passing a rope around a sheave. It is to be regretted that the published results do not cover a more extended range of both rope types and sheave sizes. In the writer's opinion, an exhaustive series of tests such as these, would be of far greater benefit than any rational formula for bending stress, however plausible its bases and beautiful its derivation.

C. D. MEALS,[11] ASSOC. M. AM. SOC. C. E. (by letter).—The wire of the ancients was produced solely by hammering and not by the process of "drawing" as practised to-day. The "drawing" of wire was mentioned in an old Latin manuscript by Theophilus, written during the Eighth or Ninth Centuries. Undoubtedly, the art was known and practised before that date. Authentic records are available to show that wire drawing, on a commercial scale, was practised in France, in 1270, in Germany, in 1350, in England, in 1465, and in America, in 1775.

It was a simple beginning of a business of great importance; there are sixteen wire-rope manufacturers in America to-day (1932), capable of producing more than 150 000 tons of wire rope annually, excluding ordinary strand as used by the public utility companies. Of this number, only three make their wire from the open-hearth furnaces to the finished product; most of the others draw the wire from rods purchased from the steel manufacturers.

It is doubtful whether Albert of Clausthal made lang lay ropes, since John Lang, of England, patented this lay in 1879; although in Germany it is called Albert Lay. It is easy to visualize the possibility that hemp rope manufacturing practice influenced the early wire-rope making, and hemp ropes have always been made "regular" lay.

The development of the various constructions and lays of ropes started in 1875. The non-spinning and flattened strand ropes and locked-coil cables were first made and patented in England, and the British manufacturers have developed these ropes to a high degree of efficiency, while American manufacturers have lagged decidedly. American manufacturers, however, have developed the round strand ropes to a much higher degree of efficiency than those made in other countries.

Prior to 1854, all wire rope was made of iron wire, a grade that is relatively soft and easy to handle. The first of the higher grades of steel wire, as used so extensively to-day, was produced in England in 1854; it was a tempered or "patented" wire and represented a great advance in the trade or art.

Mechanical Development of Machines.—The Stone stranding machine is different in construction than is implied from the description given of it, following its mention in the paper. It was found unsuited for high-class wire-rope making and is not used to-day. Instead the "rod and disk" strander, as described by Mr. Carstarphen, and the tubular strander, are the high-speed horizontal stranding machines now in use. The tubular strander is the more rugged and costs less to maintain than the "rod and disk" type machine, but it has the disadvantage of being more difficult to load with bobbins, which slows up production.

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[11] Wire Rope Engr., The B. Greening Wire Co., Ltd., Hamilton, Ont., Canada.

The stranders described herein may have from seven to thirty-seven bobbins arranged in one machine; the latter makes an extremely long unit, and one in which a tubular machine is practically a necessity; or the machine may be split in two units and geared to drive together at the same speed, or at varying speeds.

The tandem horizontal stranders as described by Mr. Carstarphen are of the old English type, which are being supplanted to-day by the high-speed stranders described in the foregoing paragraphs. It is merely a question of production; the high-speed stranders will produce two to three times as much strand per day as the tandem horizontal stranders, at no sacrifice in quality.

Some of the larger companies have vertical stranders in operation, particularly for making the more flexible rope strands. All these machines have a planetary motion imparted to the bobbins, and many are geared to give a back or forward turning motion to the bobbins for making strands for lang lay ropes. It is possible to provide the bobbins in the tubular high-speed stranders with a back-turning motion by a set of ring and internal gearing.

At least six of the American firms have stranding equipment capable of laying 37 wires together in one operation and several have stranders that will lay 41, 43, and 46 wires. Some of these large vertical stranders will make up 20 tons of strand at one setting of the machine.

The "closing" or "laying" machines, in which six or eight strands are laid around the hemp center to form the finished rope, are either horizontal or vertical, with a planetary motion imparted to the spools. They are often geared to give a back turn to the spools for making lang lay ropes and, in addition, may be equipped with a straightening device to neutralize the liveliness of a lang lay rope still further.

Several of the American firms have large vertical closers, capable of making 60 tons of rope at one setting of the machine—a machine that will make 20 000 ft. of 2-in., or 8 000 ft. of 3-in., steel wire rope. It is possible, however, to make 30 tons of rope in one continuous length on a closer of 15-ton capacity.

The compound strander and closer mentioned by Mr. Carstarphen is known as the "Canadian" machine, as it was first built in Canada. It is interesting as an exhibition model; but it was not practical for several reasons, such as, (1) reduced production, and (2) an inferior and cranky rope.

Characteristics of Wire Ropes.—Strands made up "as a series of single operations," as described by Mr. Carstarphen—as, for example, a 37-wire strand made up by laying 18 wires over 12 over 6 over 1 wire—are not offered to the trade by the better rope makers, as it is a well known and established fact that a rope strand of the "one-operation" class is far superior to a rope strand of the "multiple-operation" class. The former has each wire in the strand interlocked with each adjacent wire; the latter has wire crossing wire within the strand, resulting in internal friction and a decidedly inferior rope.

Impregnated hemp centers were an improvement over the old method of running a center through a bath of hot lubricant which did not penetrate, because a hemp center is, next to lariat and yacht rope, the hardest hemp rope made. Impregnation of the centers has some detrimental points; one is that in drying out the centers in a vacuum tank, the fibers lose their resilience to a certain extent and the original diameter of the center is not maintained. Consequently, the progressive makers are now using pre-lubricated centers in which every fiber is given a spraying of hot lubricant before being spun into the yarns. Hemp centers for wire ropes should be uniform in both diameter and hardness. The quality of fiber may well be submerged, within reasonable limits, to these two properties.

Consideration of a wire rope as a machine, having many moving parts and wearing surfaces, should force the realization that it must be lubricated if satisfactory rope service is to result. Proper lubrication retards corrosion and rotting of the hemp center; it decreases external wear of rope and equipment and reduces friction between the component parts of the rope. Varying rope service requires different kinds of lubrication and, primarily, the lubricant should be "built" into the rope during its manufacture, both at the stranding and closing machines, with exterior applications, as required, when the rope is in service.

Admittedly, sheave and drum equipment are generally too small for satisfactory rope service; power and space requirements set a limit to their sizes, but it must be appreciated that tight grooves in sheaves, that wedge or pinch the rope, prevent the unrestricted wire and strand movement that is so necessary to least resistance to bending. Consequently, such grooves subject the rope to greatly increased bending stresses—causing premature breaking of the wires and preventing a new rope from rotating freely—and increase the wear on the wires of the rope.

A new wire rope, operating under varying stresses, will elongate, and this lengthening of the lay is equalized by an untwisting or rotating action of the rope. In a regular lay rope, the wires in the strands are laid in the opposite direction to that of the strands in the rope. Consequently, any untwisting will tighten the wires in the strand and aid in the resistance of the rope to untwisting.

In a lang lay rope, the wires and strands are laid up in the same direction and, consequently, an untwisting action will cause an opening out or "bird-caging" of the individual wires of the strand, because every two and one-half rotations of the rope result in a full rotation of the individual strands. This causes the wires of the strands to "bird-cage," and the strands in the rope will also "bird-cage" or arch away from the hemp center.

An important fact is inferred in the foregoing—never use a lang lay rope where the untwisting action cannot be controlled or limited. Consequently, a lang lay rope should not be used where its ends are not permanently fastened; where a swivel is used; nor where grooves in the sheaves are tight.

There is some limitation to this in the sense that a coarse and stiff rope, as a 6 × 7 or a 6 × 8 flattened-strand lang lay rope, will not "bird-cage" as easily as the more flexible 6 × 19 ropes, because the large wires of the stiff ropes offer considerable resistance to "bird-caging."

Strand and rope lays are not usually given in terms of the wire diameters, as indicated in Table 1, but in terms of the rope diameter, and the outer wires of the strands are fixed by allowance for the angle of twist of the strand and a proper clearance between adjoining wires in the same layer. Accordingly, Table 1 may well be changed to read as shown in Table 5 for a 1-in., or base-size, rope.

TABLE 5.—STANDARD CHARACTERISTICS OF WIRE ROPE

Rope	Outer Wires in Strands		Regular Lay Ropes		Lang Lay Ropes	
	Number	Diameter, in inches	Strand lay	Rope lay	Strand lay	Rope lay
6 × 7	6	0.106	2½ *d*	6¾ *d*	2¾ *d*	7½ *d*
6 × 19 Seale	9	0.080	2½ *d*	6¼ *d*	2¾ *d*	7 *d*
6 × 19 filler wire Seale	12	0.064	2⅝ *d*	6¼ *d*	3 *d*	7 *d*
6 × 19 Warrington	12	0.054 0.072	2⅝ *d*	6¼ *d*	....	....
6 × 37	18	0.046	2⅞ *d*	6¼ *d*	3⅛ *d*	7 *d*
8 × 19	12	0.052	2 *d*	6¼ *d*	....	....
6 × 8 flattened strand	7	0.108	....	....	2¾ *d*	7½ *d*
6 × 25 flattened strand	12	0.073	....	....	2¾ *d*	7 *d*

These values in Table 5 correspond quite closely with those in Table 1, but it must be understood that each manufacturer has his own ideas regarding strand and rope lays. Table 5, however, represents average American practice for high-class operating ropes. For galvanized guy ropes and other stationary ropes as used in suspension bridges, etc., the rope lays may average 8 *d* to 9 *d* lays.

The unit tensile strengths of wire for any grade vary with the wire diameter, as indicated for plow-steel quality in Table 6 (Item Nos. 1 to 4), Mild cast-steel wire is now termed "traction steel" since it is used for elevator rope service, and the super-plow steel ropes are known as "improved plow steel," with the makers' trade designations for this quality of wire.

Torsion requirements for wires are usually given in terms of twists per length of 100 *d′*, even if the tests are made in a gauge length of 6 in. or 8 in. They vary as indicated in Table 6 (Items Nos. 5 to 19). Small wires will show higher torsions than large ones of the same quality, although this fact is not considered in most commercial specifications.

American mills are very reluctant to furnish galvanized wire to a torsion requirement. In fact, some will not accept orders on this basis; but the British and German mills will do so, and the British and Canadian Engineering Standards Associations give torsion values as shown in Table 6 (Items Nos. 11 to 15). Some recent vertical lift-bridge specifications for galvanized improved plow-steel wire are as listed in Table 6 (Items Nos. 16 to 19).

Elongations of rope wire will vary with the diameter of the wire, the process of "patenting" (as "air-patented" or "lead-patented"), and the

TABLE 6.—PHYSICAL PROPERTIES OF WIRE ROPE

Item No.	Identity	Iron	Traction steel	Cast steel	Extra cast steel	Plow steel	Improved plow steel
	TENSILE STRENGTHS, IN KIPS PER SQUARE INCH						
	Diameters of Wire, in Inches:						
1	0.010 to 0.083	....	....	....	....	230 to 255	....
2	0.084 to 0.132	....	....	....	....	225 to 250	....
3	0.133 to 0.170	....	....	....	....	220 to 245	....
4	0.177 to 0.204	....	....	....	....	212 to 235	....
	TORSION REQUIREMENTS FOR STEEL WIRE, IN TWISTS PER 100 *d'*						
5	Typical Rope Manufacturing Practice†	80 to 120	33 to 38	30 to 35	27 to 32	25 to 30	25 to 29
6	American Petroleum Institute*	....	....	32.5	30	27.5	....
7	United States Government*	....	....	33	30	28	25
	Engineering Standards Associations:						
8	British†	....	....	34	32	30	28
9	British*	....	....	26	24	23	21
10	Canadian†	....	....	25	25	25	....
	TORSION REQUIREMENTS FOR GALVANIZED STEEL WIRE, IN TWISTS FOR 100 *d'*						
	Engineering Standards Associations:						
11	British†; 0.104 in. and less	....	....	28	23	15	10
12	British*; 0.105 in. to 0.144 in.	....	....	23	19	11	7
13	British*; 0.104 in. and less	....	....	21	17	11	8
14	British*; 0.105 in to 0.144 in.	....	....	17	14	8	5
15	Canadian†	....	....	15	15	15	....
	Vertical Lift-Bridge Specifications:						
16	0.027 to 0.051 in.	....	....	....	....	....	20
17	0.052 to 0.100 in.	....	....	....	....	....	18
18	0.101 to 0.125 in.	....	....	....	....	....	17
19	0.126 to 0.190 in.	....	....	....	....	....	16
	PERCENTAGE ELONGATIONS OF STEEL WIRES IN 10-INCH LENGTHS						
	Diameters, in Inches:						
20	0.010 to 0.026	....	....	1.5	....	1.25	1.0
21	0.027 to 0.051	....	....	2.0	....	1.5	1.5
22	0.052 to 0.075	....	....	2.5	....	2.0	1.75
23	0.076 and more	....	....	3.0	....	2.0	2.0
24	Galvanized wire, 0.010 in. and more	....	....	5.0	....	4.0	....
	HARDNESS TEST						
25	Brinell Hardness Number	150	250	340	370	390	420
	CHEMICAL PROPERTIES (PERCENTAGE CONTENT)						
26	Carbon	0.06 to 0.12	0.30 to 0.40	0.45 to 0.55	0.55 to 0.65	0.63 to 0.73	0.70 to 0.80
27	Manganese	0.15 to 0.30	0.40 to 0.60	0.40 to 0.60	0.40 to 0.60	0.40 to 0.60	0.30 to 0.50
28	Silicon	0.10 (maximum)	0.10 to 0.20	0.10 to 0.20	0.10 to 0.20	0.10 to 0.20	0.10 to 0.20
29	Phosphorus (maximum values)	0.04	0.04	0.04	0.04	0.04	0.04
30	Sulfur (maximum values)	0.04	0.04	0.04	0.04	0.04	0.04
31	Chromium (maximum values)	0.04	0.04	0.04	0.04	0.04	0.04
32	Nickel (maximum values)	0.04	0.04	0.04	0.04	0.04	0.04
33	Copper (maximum values)	0.10	0.10	0.10	0.10	0.10	0.10

* Tests of wires from finished ropes.

† Tests of wires from coils before being stranded for ropes.

quality of the wire. Values in Table 6 (Items Nos. 20 to 24) are typical of minimum elongations at breaking loads.

There is still another physical property of rope wire that may be of interest, namely, the hardness. Item No. 25 of Table 6 indicates the relative abrasive and wearing resistance of the various grades of wire in terms of the Brinell hardness number (B H N). It will be found that a fairly close relation exists between the tensile strength and the B H N from the formula given by Abbott,[13] in units of 1 000 lb., as follows,

$$S = 0.73\,[\mathrm{B\;H\;N}] - 28 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

The chemical properties of the various grades of rope wire may be of interest; while each manufacturer has his own ideas, the values in Table 6 (Items Nos. 26 to 33) are typical specifications.

Acid or Basic Open-Hearth Steel?—A few years ago the writer conducted a series of bending tests on wire ropes of the various grades and constructions, and it was shown that a good basic steel was as satisfactory as a good acid steel, but the writer favors the acid steel because it is more likely to be consistently good than the basic steel.

The question is occasionally raised, "Why do not the wire-rope manufacturers use alloy steel for wire ropes?" Some research work has been done on nickel, vanadium, molybdenum, and stainless (chromium and nickel) steel wire ropes and some of them have shown considerable merit; but the price of these alloy steels is relatively so prohibitive that they are not commercially practical for wire ropes.

Wire-rope strengths were revised and generally lowered in 1929; on some grades, sizes, and constructions of ropes it was a necessity. In a general sense, it was a move in the right direction since recent research has indicated that wires of lower strength will withstand bending over sheaves better than wires of excessively high strength. Therefore, while ropes can be furnished, in most cases, to the old lists, the new strengths permit of the usage of a better steel for operating ropes.

Foreign ropes are listed at higher strengths than American ropes; but practices abroad differ in that longer lays of ropes prevail. It should be kept in mind that sheaves and drums on equipment abroad are of more liberal proportions than those in the United States. Consequently, each country has developed rope practices to suit its own particular requirements, and the higher strengths of foreign ropes need not be considered as a "challenge" to American manufacturers.

Admittedly, wire ropes made to foreign standards are unsuited for American equipment, but if made to American standards (and the writer has had this done) there is very little, if any, choice between the merits of American and of Canadian, English, and German wire ropes.

Properties of Wire Rope; Analysis of Strength.—For the usual wire-rope problems, detailed calculation of strength is scarcely required, as the approximate breaking strengths as given in the manufacturers' lists will serve most purposes just as the properties of structural shapes given in the steel handbooks serve the structural engineer. However, it may be desirable to make calculations for strengths for special ropes, such as suspender ropes for suspension bridges, vertical lift-bridge ropes, etc.

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[13] **"Strength and Hardness of Steels", *Proceedings*, Am. Soc. for Testing Materials, 1915, p. 42.**

A more accurate method of calculating the strengths of wire ropes than that given by the author (see "Strength of a 6 × 19 Rope"), is, as follows:

$$S = N \cos \beta \left(\sum_{1}^{n} S' \cos^3 \alpha\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

which for a 6 × 19 rope becomes:

$$S = 6 \cos \beta \left(S'_1 + 6\,S'_2 \cos^3 \alpha_2 + 12\,S'_3 \cos^3 \alpha_3\right) \ldots\ldots\ldots (35)$$

Table 7 contains efficiencies of wire ropes listed in Table 5.

TABLE 7.—EFFICIENCIES OF WIRE ROPES

Rope	Equation (34)		Metallic area in 1-in. rope (base size), in square inches	Test Results		British Engineering Standard Association long lay rope
	Regular lay	Lang lay		Regular lay	Lang lay	
6 × 7	86.5%	88.8%	0.3893	87%	90%	92.5%
6 × 19 Seale	84.8%	87.6%	0.4000	85%	88%	87.5%
6 × 19 filler wire Seale	85.5%	88.5%	0.4063	85%	88%	87.5%
6 × 19 Warrington	85.8%	......	0.4034	85%	....	87.5%
6 × 37	84%	86.9%	0.3803	82%	85%	82.5%
8 × 19	83.5%	......	0.3658	82%	....	......
6 × 8 flat; strand	......	88%	0.4766	....	....	92.5%
6 × 25 flat; strand	......	85.4%	0.4782	....	....	87.5%

For the two examples given by Mr. Carstarphen, the efficiencies as given by Equation (34), are as follows:

	Equation (34)	Carstarphen
1-in. 6 × 7 cast-steel rope	86.6%	83.7%
1-in. 6 × 19 cast-steel rope	83.4%	78.5 and 79.9%

For improved plow-steel and for iron ropes, the efficiencies as given by Equation (34) should be reduced by 5 per cent. This value is in conformance with test results and may be explained by the extremely high pressures and wire nicking in the improved plow-steel ropes and the wire nicking in the relatively soft iron ropes. Therefore, for 6 × 19 rope, 85.5% × 95% = 81.2%, etc. A check of Equation (34) is offered by the strengths of ropes used on some well-known bridges, as noted in Table 8.

TABLE 8.—STRENGTHS OF SPECIAL ROPES

Rope	Use	Area, in square inches	By Equation (34)		Average of Tests	
			Strength, in kips	Efficiency	Strength, in kips	Efficiency
$1\frac{3}{4}$-in., 6 × 19	Counterweight ropes, Newark Bay vertical lift bridge Central Railroad of New Jersey.	1.223	255	88.5%	263	91%
$2\frac{1}{8}$-in., 6 × 19		1.729	363	89.7%	367	90.6%
$1\frac{7}{8}$-in., 6 × 19, with 7 × 7 IWRC	Suspender ropes, Ambassador Bridge, Detroit, Mich.	1.668	308	86%	303	84.5%
$2\frac{1}{4}$-in., 6 × 37, with 7 × 7 IWRC	Suspender ropes, Philadelphia–Camden Bridge.	2.226	428	87.3%	436	89.2%

In calculating strengths of ropes with independent wire rope centers (IWRC) the strength of the IWRC is only 80% effective in the rope on account of the heavy pressures on it by the enveloping 6-rope strands. Consequently, a 7 × 7 IWRC will be only 0.865 × 0.80, or 69% efficient

Equation (34) is also of value in calculating the strength of ropes with broken or worn outer wires. In the case of broken wires, it is of value when the breaks are distributed over the various strands and not concentrated in one strand; the latter case leads to an unbalanced distribution of stress which cannot be calculated accurately.

A check of the foregoing is the tests[13] reported by the U. S. Bureau of Mines. In one test, three large and three small outer wires of a 6 × 19 Warrington rope were cut in each of the six strands. Considering metallic area, 68.8% of the rope strength remained, and by test the efficiency was 74.5% and by Equation (34), 72.6 per cent.

Incidentally, it is of interest to note that American manufacturers do not include the strength of the solid triangular wires in the cores of the strands of their flattened strand ropes; the English, Canadian, and German firms do. This fact was disclosed by Equation (34), and it should be kept in mind if a comparison of strengths is made. It is known that these solid triangular wires break up in quite short lengths when such ropes are operated over small sheaves and drums; consequently, it is advisable to neglect the strength of these shaped wires in the strength of flattened strand ropes.

Bending Stress in Wire Rope.—The determination of the bending stresses is in the "battle zone" of wire-rope engineering and no two writers agree on the subject. The writer has stated his views in two previous discussions[14] and still considers that such stresses are not susceptible to mathematical deductions with any reasonable degree of accuracy.

Consider the results of bending tests made of $\frac{5}{8}$-in. ropes on a fatigue bending test machine, in which the ropes were reeved on three sheaves placed closely together. The rope was subjected to a reverse bending operation over and under the three 12-in. tread sheaves, under constant tensions of 1 500 and 2 000 lb. Five different series of tests will be given; after which the reader may decide that bending stresses are an uncertain quantity after all.

Example 1.—In two comparable specimens of 6 × 37 plow-steel rope, $\frac{5}{8}$ in. in diameter, the diameters (in inches), were as follows:

	Rope A	Rope B
One core wire	0.036	0.033
Six inner wires	0.033	0.030
Fifteen inner wires	0.025	0.023
Fifteen outer wires	0.033	0.033

It will be noted that the strand and rope lays were identical as well as the diameter of the hemp center. The outer wires were also identical, and yet the result of the bending tests (in number of bends) was as follows:

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[13] "Rules and Regulations for Metal Mines", *Bulletin No. 75,* U. S. Bureau of Mines, 1915.

[14] "Aerial Tramways", by F. C. Carstarphen, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 92 (1928), p. 966; and "Heavy Duty Wire Ropes and Sheaves", by B. R. Leffler, M. Am. Soc. C. E., *Civil Engineering,* Vol. 1, No. 2, November, 1930, p. 110.

Broken Wires	Rope A	Rope B
First outer wire	87 000	30 000
Forty outer wires per foot	150 000	67 500
Rope pulled apart	425 000	210 000

This was purely a case of proper and improper strand design, most certainly not susceptible to mathematical analysis.

Example 2.—Two samples of two types of $\frac{5}{8}$-in., traction steel elevator ropes, under a tension of 1 500 lb., were tested with the following results (in number of bends):

	First Piece:		Second Piece:	
	First wire	Rope failure	First wire	Rope failure
$\frac{5}{8}$-in., 6 × 27 Seale.......	40 300	244 300	48 000	341 800
$\frac{5}{8}$-in., 6 × 19 Warrington.	40 800	121 600	51 000	170 000

In each case the first and second pieces were cut from the same reel and the tests show a difference of approximately 40% at rope failure.

Example 3.—Samples of $\frac{5}{8}$-in., 6 × 19 Seale traction steel elevator ropes, under a tension of 1 500 lb., were subjected to both reverse bending (three sheaves) and continuous bending (over one sheave in one plane only), with the following results (in number of bends):

	Forty outer wires broken per foot
Reverse bending	102 000
Continuous bending	270 000

The effect is scarcely susceptible to mathematical analysis.

Example 4.—The effect of strand construction for $\frac{5}{8}$-in., 6 × 19 and 6 × 37 plow-steel ropes, under 2 000 lb. tension, is shown in Table 9. The outer wires were identical in each type of wire and the difference in strand construction is seen to be scarcely susceptible to mathematical analysis in comparison to test results.

The writer saw a striking demonstration of Example 4 on a standard type of steam shovel, equipped with $2\frac{1}{8}$-in., 6 × 37 lang lay (IWRC) hoist lines; two ropes made of "three-operation" strand gave only 12 and 13 days' service, respectively; while the 6 × 37 filler wire ropes were giving an average of 33 days' service.

TABLE 9.—RELATIVE BENDING LIFE OF ROPES OF DIFFERENT STRAND CONSTRUCTIONS

Rope	Strand Details		Bends of Rope	
	Construction	Type strand (operations)	Forty wires broken per foot	Rope failure
6 × 19 regular..................................	12/6/1	Two	34 000	59 000
6 × 19 filler wire Seale.........................	12/6/6/1	One	105 600	158 400
6 × 37 ordinary................................	18/12/6/1	Three	82 500	156 700
6 × 37 regular.................................	18/19	Two	104 000	212 000
6 × 37 filler wire..............................	18/9/9/1	One	121 000	312 000

Example 5.—Lubrication, or lack of it, greatly influences the bending life of wire ropes, as Biggart's tests made in 1890 for the Forth Bridge on $\frac{9}{16}$-in., 6 × 19, plow-steel ropes show in the following (number of bends):

	10-in. sheave	24-in. sheave
Dry rope	16 000	74 000
Lubricated rope	38 700	386 000

The foregoing tests (covering strand construction, Examples 1 and 4; reverse bending, Example 3; lubrication, Example 5; and manufacturing variation, Example 2) clearly indicate that bending stresses are not susceptible to mathematical analysis. Nevertheless, such stresses must be considered, even if empirically, and the writer prefers the formula:

$$b = \frac{E_r d'}{D} \qquad \text{......................} (36)$$

in which, values of E_r, the modulus of elasticity of the rope for various rope constructions, are given in the writer's discussions[14] of the two papers, previously mentioned. Equation (36) allows for an increase in Er values for ropes that have been in service some time, which is an important consideration not fully allowed for in other bending stress formulas.

As to the factor of safety on load, acceleration, and bending stresses, experience is the only safe guide, and the late R. C. Strachan, M. Am. Soc. C. E., covered[15] this point in his discussion of the paper by B. R. Leffler, M. Am. Soc. C. E., in which he stated that "the choice of a value for k [a modifier of E] should be governed by the conditions of the particular job and the known results in work of a similar character."

Incidentally, the variables of wire ropes as indicated by tests noted in Examples 1 to 5, will explain the difference in service of the $\frac{5}{8}$-in. elevator ropes in a Cincinnati, Ohio, building, as given in Table 4. Similar variations in elevator rope service were noted[16] by Mr. C. T. Coley and such variations are typical of many wire-rope installations, particularly in elevator service, because it is doubtful whether equal tensions are maintained in a set of six hoisting ropes on an elevator in its travel up and down the shaft, unless some type of equalizer is used on the ropes.

It was thought that Equation (20) would be applicable to wire ropes under static loading, such as suspender ropes on suspension bridges. As the limitations noted by Examples 1, 2, and 4, would not be applicable to such ropes, a check was made with Skillman's tests[17] of ropes over sheaves and of tests[18] of the $2\frac{1}{4}$-in., 6 × 37, suspender ropes of the Philadelphia-Camden Suspension Bridge, and these results, together with tests of the $\frac{3}{4}$-in. ropes noted by Mr. Carstarphen, are arranged in Table 10.

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[15] *Civil Engineering*, Vol. 1, No. 2, November, 1930, p. 112.

[16] Third Conference on Wire Rope Research, under the Auspices of American Engineering Council and American Society of Mechanical Engineeers, September 13, 1929.

[17] "Some Tests of Steel Wire Ropes on Sheaves", by E. Skillman, *Paper No.* 229, U. S. Bureau of Standards, 1923.

[18] Final Report of the Board of Engineers to the Delaware River Joint Commission of Pennsylvania and New Jersey, 1927, p. 118.

TABLE 10.—Tests of Ropes Over Sheaves

Rope	Diameter of sheave, in inches	$\frac{D}{d}$	Breaking Strengths, in Kips		Efficiency	
			Straight rope	Over sheave	Tests	Equation (20)
Skillman:						
¾-in, 6 × 19 plow steel,	10	13.3	49.7	39.8	80.3%	97.3%
1¼-in, 6 × 19, plow steel	10	8	130.7	99.1	75.8%	95.4%
1¼-in., 6 × 19, plow steel	18	14.4	130.7	110.6	84.7%	97.3%
Carstarphen:						
¾-in., 6 × 19, cast steel	6	8	35.0	32.0	91.5%	91%
2¼-in., 6 × 37, with IWRC	32	14.2	436.5	388.4	89%	98.5%

Skillman's tests covered a series of 6 × 19, plow-steel ropes, ⅝ in., ¾ in., ⅞ in., 1 in., and 1¼ in. in diameter, over sheaves, 10 in., 14 in., and 18 in. in diameter, and the results obtained for the various ratios of $\frac{D}{d}$ were quite consistent. They check quite closely with the efficiency given by the 2¼-in., suspender ropes when the difference in construction and flexibility of the latter are taken into consideration, and it is rather difficult to reconcile the great difference in efficiencies between these tests and those reported by Mr. Carstarphen for ¾-in. ropes.

It is obvious from Table 10 that Equation (20) is not satisfactory to use for suspender ropes, slung over cable bands on the main cables of suspension bridges, and for similar reeving of ropes over sheaves. There is a need for a solution of this problem, and the writer has been using the formula given by Mr. Leffler,[19] which is modified by the efficiency factor, ϵ, as follows:

$$S = A\left(t - \frac{E_r d'}{D + d}\right)\epsilon \qquad (37)$$

in which, t is the ultimate tensile strength of the wire, in pounds per square inch, and ϵ is the efficiency of rope in Equation (34).

In the discussion of Mr. Leffler's paper, the writer criticized the use of Equation (37) for bending stresses in operating ropes; but for a non-operating or stationary rope, it gives an approximation that is on the safe side, which results in a lower efficiency than that observed by tests. As examples, for the 2¼-in. rope in Table 10, the value of S becomes 388.6 kips, as compared to test results of 388.4; for the 1¼-in. rope on the 18-in. sheave, it is 104.8 kips, compared to 110.6 by test; and for the 1¼-in. rope on the 10-in. sheave, it is 91.5 kips, compared to 99.1 by test. For the lower ratios of $\frac{D}{d}$, the error is a maximum.

The New Target, A Service Factor Formula, Equation (21).—Much that has been noted in the foregoing discussion of bending stresses, will indicate that so little, really definite, is known about the mechanics of wire rope, that engineers are not ready yet for the advance suggested by Mr. Carstarphen.

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[19] "Heavy Duty Wire Ropes and Sheaves", by B. F. Leffler, M. Am. Soc. C. E., *Civil Engineering,* Vol. 1, No. 2, November, 1930, Equation (4).

At best, all that could hope to be done with such a formula, would be to set up different constants, factors, and exponents in Equation (21) for varying services, as elevators, mines, shovels, tramways, etc., as well as for the many different constructions, lays, and grades of ropes. Then, the "yardstick" would be difficult of application because no two manufacturers make their ropes alike, even for, presumably, the same rope.

Compared to many other engineering materials and products, the knowledge of wire rope is quite limited and, unfortunately, the manufacturers are very reluctant to join in any outside research work to improve the art, such as that sponsored by the Wire Rope Research Board of Engineering Foundation and the American Society of Mechanical Engineers, the last report of which appeared in 1929. The British and German engineering societies have done considerably more research work on wire rope and are being more actively supported by the manufacturers. They have published some excellent reports of their results as accomplished to date (1932), but most of the work is still ahead of them before anything definite on the subject is known.

It must not be considered that the American manufacturers have lagged in research work, such a monumental work as the George Washington Bridge attests to this fact and at least five of the leading manufacturers have research and adequate testing facilities, not only for finished ropes but also for individual wires. The results are kept confidential, however, and until either the National Bureau of Standards or Engineering Foundation equips a laboratory with suitable machines and competent research workers to carry on investigations on wire and wire rope, independent of any aid or encouragement from the wire-rope manufacturers, will anything of definite value be known about the subject.

Had the works of predecessors and contemporaries in the various engineering sciences, such as those of Galileo, Newton, Kelvin, Rankine, St. Venant, Winkler, Whipple, Einstein, etc., been withheld from publication, engineering sciences would have been greatly retarded. Their works have served a twofold purpose: To create an acquaintance with their thoughts and, equally important, to encourage others to think, because the real value of a book is in what it "pulls" out of one rather than what one gets out of it.

Wire-rope engineering owes a debt of gratitude to the writings of Roebling, Hrabak, Vaughan, Epton, Hewitt, Bucknall Smith, Chapman, Adamson, Howe, Sunderland, Carstarphen, and many others. It is to be hoped that in the near future much more will be added to the present meager knowledge of the subject by competent men who are engaged in the industry.

O. P. ERICKSON,[20] M. AM. SOC. C. E. (by letter).—This is a timely paper on a product of wide use in engineering construction and operation activities. That considerable difference of opinion exists regarding the stresses in wire rope is indicated by the many formulas encountered in wire rope literature. As shown by the author,[21] by the late R. C. Strachan,[10] M. Am. Soc. C. E., and others, none of these formulas agrees even approximately with each other.

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[20] St. Augustine, Fla.

[21] See p. 579.

The reason for this is the many variables involved, the most important being lubrication, size arrangement and condition of sheaves and drums, type and size of rope center, length of strand, strand construction, and rope lays. Mr. J. F. Howe[22] states that the modulus of elasticity for high-grade rope wire varies between 25 500 000 and 29 000 000. For stressed ropes, he gives modulus values ranging from 7 000 000 to 13 000 000, depending on the construction of the rope as an average of test results, and states that for new unstressed ropes these values will be somewhat reduced. The foregoing makes the selection of a piece of wire rope seem a formidable task.

Actually, however, rope users can obtain considerable practical information from manufacturers' catalogues. The wire rope user and his consulting engineer can also avail themselves of the rope maker's wide experience regarding sheave and drum diameters, rope leads, and type of rope to use. This information, which may be both theoretical and practical, is freely given, and the rope maker because he is consulted on the problem will make a special effort to furnish the best possible type of rope and service. On the other hand, if the rope user and his consultant specifies all the physical properties of the rope and its application, the rope maker may feel, and perhaps rightly so, that his responsibility ends on delivery of the rope. Therefore, in order to insure better service co-operation between the rope user and the rope maker, the specifications should be made sufficiently flexible to cover the operating life of the rope, and the rope should be paid for according to its hours of service.

As pointed out by rope makers, and confirmed by the experience of most rope users, the importance of proper lubrication cannot be over-emphasized. The proper working load of a well lubricated rope running over a sheave may become a dangerous over-load for the same rope when devoid of lubrication. Ropes subjected to temporary over-loads or under-water usage require much more than the average lubrication due to the lubricant being squeezed out or washed away. In order to penetrate between the wires to the rope core most lubricants must be applied hot. Unless they penetrate between the wires they are of little value in prolonging the rope life.

Other causes for exceedingly short rope life are too small sheaves and reverse bends. The U. S. Government Master Specifications for Wire Rope,[23] gives the following sheave diameters: For 6 × 19 rope construction good practice requires the sheave to be 50 rope diameters, and under no circumstances less than 20 rope diameters. For 6 × 37 or 8 × 19 rope construction good practice requires the sheave to be 30 rope diameters, and under no circumstances less than 15 rope diameters. If reverse bending cannot be avoided the sheave about which it occurs, if possible, should be larger than the other sheaves. This applies to bends of even small angularity.

F. W. Deck,[24] Assoc. M. Am. Soc. C. E. (by letter).—The author's method of attacking the problem of the effects of bending should serve to clear up many of the difficulties that attend the study of wire rope, and all kinds of stranded cables that act under bending similarly to wire rope. A complete understanding of what takes place when a stranded cable is bent is necessary not only in considering the more violent and acute bendings that take place, such as those in wire ropes used for hoisting, cableways, elevators, etc., but also in the study of the high frequency vibration that occurs in ropes and cables. This latter action causes a repetitive bending, usually of low amplitude and high frequency, which, if it occurs at a point of restriction to the free movement of the strands, may soon cause failure through fatigue in the metal of which the strands are composed.

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[22] *Transactions*, Am. Soc. Mech. Engrs., 1918, p. 1061.

[23] No. 297, May, 1925, p. 32.

[24] Structural Engr., Mech. Div., Philadelphia Elec. Co., Philadelphia, Pa.

Failures of this kind are experienced in many cases of cables used in the transmission of electric energy, or under similar conditions of usage, where they may be subjected to the constant action of wind or other forces tending to set up a condition of resonance. Under the condition of free vibration, the cable does not usually sustain serious injury, but where the cable is restrained, as at its terminal or clamp, the free slippage of strands and the stress equalization due to stranding are not allowed to take place, and, consequently, the bending becomes excessive.

In any study of this action, an analysis of the effects of bending, such as is described by the author, is of material service. While the bending in vibration is slight as compared with bending around a sheave, the incipent action of the strands is similar. In the vibrational bending, it is consistent to use the beam elastic curve analysis; while in bending about a sheave, as the author points out, this principle may lead to serious error in calculations.

The experience which some public utilities have had in the use of wire rope bears out many of Mr. Carstarphen's statements. One of the primary uses to which wire rope is subjected in the public utility service is the handling of coal at steam-generating stations. This involves heavy loads, high accelerations, and severe abrasive action. Among the factors that are most important in contributing to rope failures, is friction. Abrasion nearly always precedes the breaking of wires, and practically none of the reduction in size is due to corrosion, which is almost unknown in coal-handling service. The bending, tensile, and shearing stresses are severely augmented by small sheaves, small drums, and reverse bends, together with rapid acceleration and deceleration of the loads and improper reeving of the rope.

Proper size and maintenance of sheaves have naturally been found to be one solution in increasing wire-rope life. Proper lubrication and care in handling are probably the most improtant factors, after care has been taken of the mechanical features of the equipment. The most important of these (care in handling) can be obtained only by careful attention to the operating personnel that handles the rope. It has been found possible to increase the life of rope several times by introducing a spirit of responsibility and competition among the men whose duty it is to use it. Consciousness on the part of the personnel that rope made by reliable manufacturers will give long life, and that early faults can not always be ascribed to poor manufacture, but are usually due to improper usage, will serve to increase such life to a rather remarkable degree.

Specifically commenting on the mathematics used by the author, the writer suggests revision of the substantiating statements in Example 1 as follows: If the two wires are stressed, and if no slipping occurs between them, the bent wire will be elongated and will rotate about A so that its end, C, will take the new position, B'; but if no slipping occurs, there is no increase in length, F (Fig. 12), which remains $2\pi d'$, and hence dl_1 becomes equal to the distance, AB', minus AC, while the angle decreases. The increment, dl_1, will always be less than dl', because it is one of the legs of an approximate right triangle, while dl' is the hypotenuse. Hence, the true ratio of stresses,

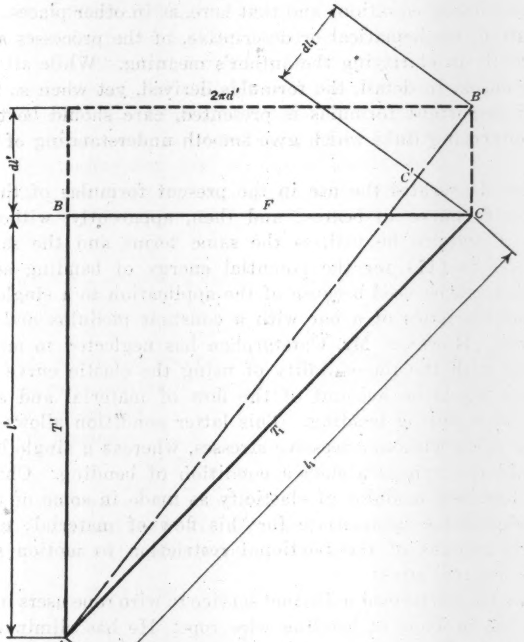

FIG. 12.

$$\frac{T_1}{T'} = \frac{\dfrac{dl_1}{l_1}}{\dfrac{dl'}{l'}} = \frac{l'\,dl_1}{l_1\,d\,l'}$$

; therefore, T_1 is less than T', or the reverse of the condition specified by the author. It is not the inner wire that fails to take its full share of stress to prevent overstressing the outer wires; it is the outer wires, which are not stressed fully to avoid overstressing in the inner wire.

Furthermore with the symbols used by the author, if $\frac{dl_1}{dl'} = \frac{l_1}{l'}$ then $\frac{dl_1}{l_1} = \frac{dl'}{l'}$ and $T_1 = T'$ and, hence, $\frac{T_1}{T'}$ cannot equal 1.038. The results are similar, in giving a strand of lesser strength than the sum of the individual strengths of its wires.

The writer feels that there is some confusion of symbols in Equation (1) and the substantiating equation; and that here, as in other places, a more complete explanation, mathematical or descriptive, of the processes accomplished would aid greatly in clarifying the author's meaning. While all readers may not desire to check, in detail, the formulas derived, yet when so much of the derivation of important formulas is presented, care should be taken not to omit those connecting links which give smooth understanding of the author's meaning.

The author deprecates the use in the present formulas of the expression for the elasticity curve in beams; and then, apparently, without sufficient explanation or reasons, he utilizes the same terms and the same basis in Equations (13) to (17) for the potential energy of bending and twisting. These formulas can be used because of the application to a single wire which takes the characteristics of a bar with a constant modulus and a relatively small diameter. However, Mr. Carstarphen has neglected to make this fact clear, together with the impossibility of using the elastic curve formula for the cable as a whole on account of the flow of material and stress in the wires and strands during bending. This latter condition allows the bending of a stranded cable without excessive stresses, whereas a single bar of larger diameter could not approach such a condition of bending. Changes in the value of the assumed modulus of elasticity as made in some of the formulas in current use cannot compensate for this flow of material; nor can such formulas take account of the frictional restriction to motion and bending which causes the real stress.

The author has performed a distinct service to wire rope users in his method of attacking the problem of bending wire rope. He has eliminated from his mathematical solution of the problem, those features which would tend to lead the analyst astray, and has given a reasonable formula for ascertaining the loss in strength of the wire when bent around a sheave.

The formulas given in this paper for the bending of ordinary stranded cables and the study of the elasticity curve for locked-coil cables, should be of great assistance when combined with tabulated results of experiments along these same lines.

O. G. JULIAN,[25] M. AM. SOC. C. E., and J. C. DAMON[26] ESQ. (by letter).—An interesting account of the development and use of ropes and an analysis of certain problems are contained in this paper. The writers appreciate Mr. Carstarphen's initiative in taking up a subject on which little has been written, and regarding which exact knowledge is scarce.

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[25] Engr., Jackson & Moreland, Boston, Mass.

[26] Engr., Jackson & Moreland, Boston, Mass.

Regarding the modulus of elasticity of steel the author states (in commenting on Table 1) that, "this factor varies, of course, depending upon the composition of the steel, but will usually be found to be between the limits of 25 000 and 30 000 kips per sq. in." This is contrary to the writers' understanding. The late George F. Swain, Past-President and Hon. M. Am. Soc. C. E., has stated[27] that, "notwithstanding the great variation in strength, it is remarkable that the modulus of elasticity of all grades of steel remains nearly constant at about 30 000 000 lb. per sq. in."

The section of the paper entitled 'Analysis of Strength" is interesting and the numerical value obtained for the ratio of the strength of a strand to the sum of the strengths of the individual wires making up the strand (which ratio may be termed the "efficiency" of the strand), agrees well with known values obtained from tests; however, the writers have some qualms regarding the method Mr. Carstarphen uses to obtain these values.

In Example 1, commenting on Fig. 2(*c*), the author states parenthetically that, "the ratio, $\frac{T'}{T_1} = \frac{1}{1.04} = 0.96$." For the case given, $0.96 = \cos\alpha$ (the angle of lay); hence, according to the author, the ratio of the stresses in the center wire, and the curved wire equals $\cos\alpha$. As α approaches $\frac{\pi}{2}$, however, $\cos\alpha$ approaches zero, and the ratio of the stresses in the center and curved wires must also approach zero, according to the foregoing hypothesis. This does not appear reasonable.

According to statics,

$$a'\,T' + a_1\,T_1\cos\alpha = P \qquad (38)$$

in which, a' is the area of the straight wire; a_1, the area of the curved wire; and P, the total applied load. If α is zero, the wires share the load equally (provided $a' = a_1$ and the elastic properties of the wires are identical); but as α approaches $\frac{\pi}{2}$, the curved wire supports a lesser, and the center wire a greater, proportion of the total load, P.

According to Fig. 2 and Hooke's law, provided the proportional limit is not exceeded:

$$T' = \frac{E'\,(dl')}{l'} \qquad (39)$$

while,

$$T_1 = \frac{E_1\,(dl')\sec\alpha}{l'\sec\alpha} \qquad (40)$$

in which, E' and E_1 equal the moduli of elasticity of the center and curved wires, respectively. According to Equations (39) and (40),

$$\frac{T_1}{T'} = \frac{E_1}{E'} \qquad (41)$$

that is, the stresses in the wires are equal if $E_1 = E'$.

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[27] "Structural Engineering, Fundamental Properties of Materials," p. 97.

Combining Equations (38) and (41),

$$T' = \frac{P}{a' + a_1 \frac{E_1}{E'} \cos \alpha} \ldots\ldots\ldots\ldots\ldots (42)$$

and,

$$T_1 = \frac{P}{a' \frac{E'}{E_1} + a_1 \cos \alpha} \ldots\ldots\ldots\ldots\ldots (43)$$

are obtained.

If instead of being surrounded by only a single wire as indicated by Fig. 2, the center wire is surrounded by several wires, Equation (38) becomes,

$$a'\,T' + \Sigma\,(a_1\,T_1 \cos \alpha) = P \ldots\ldots\ldots\ldots\ldots (44)$$

and Equations (42) and (43) become, respectively,

$$T' = \frac{P}{a' + \Sigma\,(a_1 \frac{E_1}{E'} \cos \alpha)} \ldots\ldots\ldots\ldots\ldots (45)$$

and,

$$a'\,T_1 \frac{E'}{E_1} + \Sigma\,(a_1\,T_1 \cos \alpha) = P \ldots\ldots\ldots\ldots\ldots (46)$$

If n_1 equals the number of wires to which T_1, a_1, and α, apply, n_2 equals the number of wires to which T_2, a_2, and β apply, and n_3 equals the number of wires to which T_3, a_3, and λ apply, etc., Equations (44), (45), and (46) may be written in the more general form:

$$a'\,T' + n_1 a_1\,T_1 \cos \alpha + n_2 a_2\,T_2 \cos \beta + \ldots\ldots = P \ldots\ldots\ldots (47)$$

and,

$$T' = \frac{P}{a' + n_1 a_1 \frac{E_1}{E'} \cos \alpha + n_2 a_2 \frac{E_2}{E'} \cos \beta + \ldots\ldots} \ldots\ldots\ldots (48)$$

$$T_1 = \frac{P}{a' \frac{E'}{E_1} + n_1 a_1 \cos \alpha + n_2 a_2 \frac{E_2}{E_1} \cos \beta + \ldots\ldots} \ldots\ldots\ldots (49)$$

$$T_2 = \frac{P}{a' \frac{E'}{E_2} + n_1 a_1 \frac{E_1}{E_2} \cos \alpha + n_2 a_2 \cos \beta + \ldots\ldots} \ldots\ldots\ldots (50)$$

and so on.

If P is the limiting load for the strand and Σp the limiting load for all the individual wires which make up the strand (when the wires are straight), the efficiency is,

$$\frac{P}{\Sigma\,p} = \frac{a'\,T' + n_1 a_1\,T_1 \cos \alpha + n_2 a_2\,T_2 \cos \beta + \ldots\ldots}{a'\,T' + n_1 a_1\,T_1 + n_2 a_2\,T_2 + \ldots\ldots} \ldots\ldots\ldots (51)$$

Then, for the example cited by the author (that is, a center wire surrounded by several others having the same areas as the center wire, and $\alpha = 15° 35'$), $\cos \alpha = 0.96$. Equation (51) becomes,

$$\text{Efficiency} = \frac{1 + n_1 \cos \alpha}{1 + n_1} \ldots\ldots\ldots\ldots\ldots\ldots (52)$$

If $n_1 = 6$, as is usual under these conditions, the efficiency of the strand according to Equation (52) is 96.6%, which differs by only approximately 0.5% from the value given by the author. However, in the extreme case, in which $n_1 = 1$ and $\alpha = \frac{\pi}{2}$, by Equation (52), efficiency $= \frac{1 + 1(0)}{1 + 1} = \frac{1}{2}$, or 50%; whereas by the author's method (as understood by the writers), efficiency $= \frac{(\cos \alpha)\, 2\, S'}{2\, S'} = 0$. The results of the two methods are seen to diverge as α increases. The writers believe that the author is unwarranted in stating that $\frac{dl_1}{dl'} = \frac{T_1}{T'}$.

Regarding "Strength of a 6 × 19 Rope," Equation (1) may be written in the more general form,

$$X + n_Y Y \cos \alpha + n_Z Z \cos \beta + \ldots = P \ldots\ldots\ldots\ldots (53)$$

in which, n_Y and n_z equal the number of wires making angles of α and β, respectively, with the axis of the rope. It will be noted that, except for notation, Equations (53) and (47) are identical.

It is not clear what the author means by the phrase "if slipping occurs," nor is it clear how "slipping" is taken into account in his calculations for strength efficiency. The effect of any individual wire slipping with respect to the others would appear to be a function of: (*a*) The manner in which the ends of the strand of rope are secured; (*b*) the length of the member; (*c*) the coefficient of friction between the component parts; and (*d*) the applied load.

If the ends of the member are secured by wire-rope sockets properly installed so that each wire can assume its due share of the applied load, and if the member is of considerable length, the writers believe that fair approximations of stresses and efficiency may be obtained from Equations (47) to (51). However, it is obvious that if the ends of the member are secured so that certain individual wires may be taut while others are slack, the problem is not susceptible of calculation. It is also evident that the effect of inevitable errors in socketing will have more influence on short members, such as those ordinarily used in tests, than on members of considerable length, such as those actually used in construction.

The analysis given neglects the effect of:

(1) The wires and strands drawing together as the load is increased.
(2) The end sockets or other fastenings and attachments causing compression at right angles to the axis of the member.
(3) Compression resulting from each wire or strand pushing against its neighbors.

(4) Friction between the individual wires or strands.
(5) The change in angle of lay as the load is increased.

On account of the effect of the foregoing, especially Item (1), the writers believe that the moduli of elasticity of strands and ropes are not susceptible of even approximate determination by calculations. They should be determined from tests and, if practicable, the tests should be made on members of lengths comparable to those used in construction. However, Mr. James F. Howe has given[28] formulas for the determination of the modulus of elasticity of strands and ropes, and it is remarkable to note that the results obtained from his formulas agree well with values determined by tests on pre-stressed members.

As pointed out by the author the modulus of elasticity of strands and ropes is not constant for low stresses. It has been found that both the elastic limit and the modulus of elasticity of such members can be raised materially by subjecting them to a cycle or two of comparatively high stress, and that the primitive elastic properties are apt to be quite misleading. It is believed that, when erected, all such members should be pre-stressed to at least the highest point to which they are likely to be subjected while in service, and that by this simple procedure their modulus of elasticity can be made practically constant throughout the range of working stresses.

The author gives Equations (3), (4), and (5), as if they were derived from the approximate formula for beams, $EI \frac{d^2y}{dx^2} = M$. The writers believe that this is an erroneous view. Consider an element of a straight bar of length, ds, and having a radius of cross-section, r'. Let this bar be bent until the radius of the arc is R. Let $d\theta$ equal the angle between two radii subtended by ds. Then, $ds = Rd\theta$, but the unit elongation of an extreme fiber equals $\frac{r'd\theta}{ds} = \frac{b}{E}$; hence, $b = E\frac{r'}{R}$, which is the author's Equation (3). The only assumptions made in deriving this formula are that "plane sections remain plane," and that E is constant and the same for tension as for compression (the stress as well as the strain has plane distribution), which assumptions are substantially true within the proportional limit. The approximate formula, $EI\frac{d^2y}{dx^2} = M$, referred to by the author, is based on the further assumption that the curvature, $\frac{1}{R}$, is small and, therefore, its exact value, $\frac{d^2y}{dx^2}\left(\frac{1}{1+\frac{dy^2}{dx^2}}\right)^{\frac{3}{2}}$, may be taken as equal to $\frac{d^2y}{dx^2}$. This assumption is not involved in the derivation of Equation (3). This equation is not applicable beyond the elastic limit, because then E is not constant. Since the working stress should not exceed the elastic limit the more involved stress conditions prevailing beyond that point are only of academic interest and ordinarily need not be investigated.

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[28] *Transactions*, Am. Soc. Mech. Engrs., Vol. 40, 1918, p. 1058 *et seq.*

The well-known Reuleux formula (Equation (4)) is obtained from Equation (3) by multiplying both numerator and denominator of the right-hand side by 2. The writers fail to see where there is anything "arbitrary" in the argument leading to Equations (3) and (4), and believe them to be rational. From Equation (3) it follows that,

$$b = Er' \, \Delta \frac{1}{R} \quad \text{.........................} (54)$$

in which, $\Delta \frac{1}{R}$ is the change in curvature due to flexure.

It has been shown[29] that for a rope,

$$\Delta \frac{1}{R} = \frac{2 \cos^2\alpha \cos^2\sigma}{D} \quad \text{.........................} (55)$$

in which, α is the angle of lay of a single wire with the axis of a strand, and σ is the angle of lay of a strand with the axis of the rope. Substituting this value of the change of curvature in Equation (54) and substituting d' for $2r'$, Equation (54) becomes[29],

$$b = \frac{Ed' \cos^2\alpha \cos^2\sigma}{D} \quad \text{.........................} (56)$$

It will be noted that Equation (56) differs from Chapman's formula (Equation (12)), as given by the author.

Taking cos α and cos σ as each equal to 0.95 and E as 28 500 kips per sq. in., for a $\frac{3}{4}$-in., 6 × 19 rope bent around a 6-in sheave, Equation (56) gives,

$$b = \frac{28\,500\,(0.95)^4 \left(\dfrac{0.75}{15}\right)}{6} = 193.5 \text{ kips per sq. in.}$$

for the extreme fiber stress due to flexure, in the wires bent on the sharpest curvature. Equation (56) does not state that the total tensile stress on the rope due to bending is the product of b and the area of cross-section of the rope; that is, 193.5 × 0.241 = 46.6 kips, as intimated by the author. Since the extreme fiber stress obtained is above the proportional limit, and since Equation (56) is based on the hypothesis that this limit be not exceeded, the only true conclusion that can be made from the result — $b = 193.5$ kips per sq. in. — is that the proportional limit has been exceeded. However, assuming that the proportional limit of the wires is greater than 193.5 kips per sq. in. and that the radii of bend of all the wires are equal to the radius of the sheave, then,

$$T = 2 \times 6 \times 19 \, \frac{b}{r'} \int_0^{r'} \left[(r')^2 - y^2 \right]^{\frac{1}{2}} y \, dy = 76 \, b \, (r')^2$$

$$= 76 \;\; (193.5) \; (0.000625) = 9.2 \text{ kips}$$

Chapman made a number of experiments to determine the forces required to bend wire ropes and "found a close agreement between the observed and calculated deflections".[30] His calculations were based on Equation (56). In the concluding paragraph of the paper, in which he reports the results of these experiments, Chapman states that, in applying the formulas to any particular ropes, it must be borne in mind that they were established on the assumption that the wires of the rope are not strained beyond the elastic limit. Furthermore, he points out that it is easy to get ridiculous results from the formulas by applying them to ropes bent over too small pulleys. His calculations were applied only to cases in which the wires are not strained to such an extent that they do not completely recover.

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[29] *The Engineering Review* (Lond.), October, 1908, paper by R. W. Chapman.

Since friction between the various individual wires is not taken into account in the derivation of Equation (56), it would appear that the extreme fiber stress, b, obtained from this equation would be low rather than high. If the static friction between the individual wires is sufficient to develop the longitudinal shear required to make the rope act as a beam having a solid cross-section, the following equation,

$$b = E_r \frac{d}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

(in which, E_r is the modulus of elasticity of the rope and d, the diameter of the rope) rather than Chapman's formula, should be applicable. As the ratio of the static friction to this longitudinal shear between the wires approaches zero, the extreme fiber stress due to flexure should approach the values given by Chapman's formula. All cases (for stresses within the proportional limit) lie between these two extremes. Chapman[31] found that the area of the hysteresis loop of the deflection force diagram was materially decreased by lubricating the rope. It would appear, therefore, that b is a function of the degree of lubrication, and since friction can not be entirely eliminated, the value of b is somewhat higher than that given by Equation (56). For a given coefficient of friction, the total friction between the individual wires and strands will be a function of the axial tension, P, on the member and hence, as stated by the author, "the bending effect is a function of the tension" (see "A New Formula for Wire Rope"). It is clear, also, that the friction between the individual wires and strands is a function of the degree of tightness to which the individual wires are laid up in the strands, and the strands in the rope.

It has been shown that for parallel wire cables the force required to produce a given deflection is materially effected by the tightness of the wrapping wire.[32] With a tension of 96 lb. in the wrapping wire, it required an applied central load of only slightly more than 4 000 lb. to cause 1.0-in. deflection in a 9-ft. cable containing 1 904 individual No. 6 wires, while with a tension of 200 lb. in the wrapping wire, it required a load of 23 000 lb. to cause the same deflection. In these tests, the cable was mounted so as to act as a pin-ended beam subjected to a concentrated load at mid-span. It would appear that in ropes and strands the outer layers of wires have an effect on the remainder comparable to the effect the wrapping wire of a parallel-wire cable has on the longitudinal wires.

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[30] *The Engineering Review* (Lond.), October, 1908, p. 264 *et seq.*

[31] *Loc. cit.*, Fig. 3, and adjacent text, p. 265.

[32] "Construction of Parallel Wire Cables for Suspension Bridges," pub. by John A. Roebling's Sons Co., Chart I and p. 10 *et seq.*

The writers do not find the arguments leading up to Equations (13) to (20) clear. It is believed that the value of the paper would be considerably enhanced, if the author would clarify and amplify these arguments. It should be noted that the well-known equations given for the potential energy of bending and twisting, $U'_b = \frac{M^2_b l}{2\,EI}$ and $U'_t = \frac{M^2_t l_1}{2\,G_r I_p}$, are based on the proportional limit not being exceeded and, therefore, calculations depending on them are only applicable for stresses below that limit.

Formulas have been proposed elsewhere[33] for the limiting load on ropes subjected to static and moving loads. These are based on Chapman's findings, the law of conservation of energy, and the obvious fact that if the rope is to last, the limiting stress should be not more than the endurance limit. Of course, ropes used for temporary rigging may be stressed well past this point without serious consequences. However, this does not imply that it is good practice so to stress rope used in permanent engineering structures of any importance.

A. S. RAIRDEN,[34] ESQ. (by letter).—The author has contributed some valuable information on a subject that has been under controversy for considerable time. As pointed out, bending stress formulas on wire rope developed, respectively, by Reuleux, Rankine, and Chapman give bending stress values out of proportion to the breaking strength of the rope. The problem is solved most easily and accurately by the development of empirical formulas by extending breaking-strength tests of wire rope bent to varying radii of curvature.

A series of tests were conducted to determine the breaking strength of $\frac{5}{8}$-in. rope (improved plow steel), of varying constructions bent over sheaves, as given in Table 11. The sheaves contained **U**-grooves conforming to the contour of the $\frac{5}{8}$-in. ropes.

TABLE 11

Diameter, in inches	3	6	8	10	16
Ratio of sheave diameter to rope diameter..................	4.8	9.6	12.8	16.0	25.6

For each test a length of wire rope was carefully socketed on both ends. The wire-rope sample was bent around the sheave (which was free to revolve around its axis), and anchored to the stationary head of a 250 000-lb. vertical testing machine (see Fig. 13). The two ends of rope, leading vertically downward and parallel to each other, were securely attached by sockets to a horizontal bar which, in turn, was attached to the movable head of the testing machine. Gradually the load was applied to the specimen bent around the sheaves until complete rupture occurred.

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[33] *Civil Engineering,* March, 1931, p. 547.

[34] Chf. Engr., Am. Cable Co., Inc., New York, N. Y.

Corresponding to each series of bending tests, a regular straight breaking-strength test of the rope was also conducted. In this test the straight sample of rope was subjected to loads of gradually increasing intensity until it broke.

The load applied to the bent specimen at rupture was divided by two, in order to obtain the load on one end. The value thus obtained was subtracted from the breaking strength of the wire rope obtained by a straight pull, in order to determine the loss in breaking strength of the rope due to being bent around the sheave. In each case this loss is expressed in terms of percentage loss of the breaking-strength value of the rope when pulled to destruction straight. The breaking-strength losses corresponding to each sheave diameter factor are interpreted graphically in Fig. 14, which also shows the cross-sections of the corresponding ropes. The details of construction of these ropes are listed in Table 12.

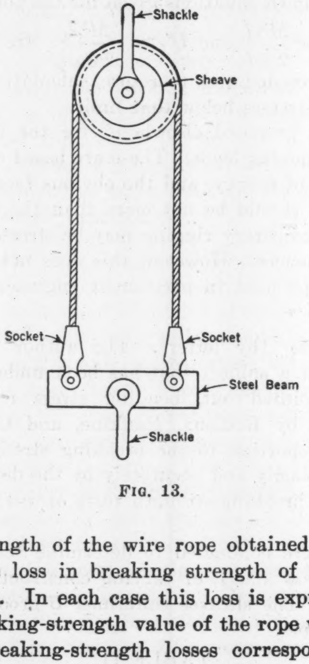
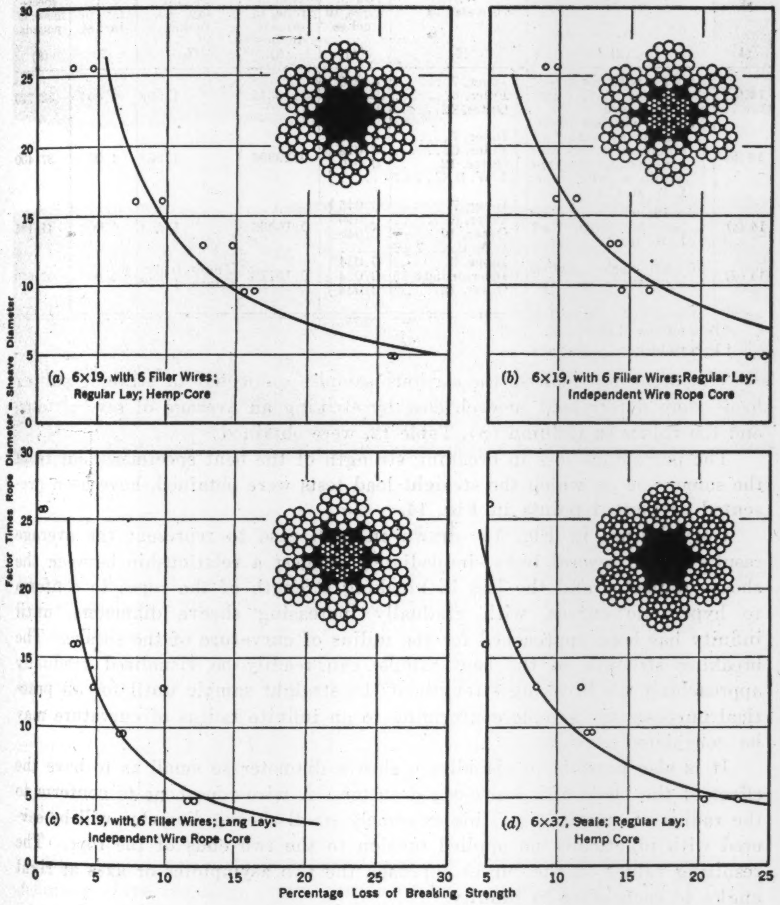


FIG. 13.

The important difference between the sample in Fig. 14(*b*) (regular lay rope) and that in Fig. 14(*c*) (lang lay rope) may be studied in Fig. 15. The details of construction are absolutely identical in all respects except that the direction of twist in the case of the lang lay rope (Fig. 15(*b*)) is the same for the wires in the strands and the strands in the rope. In the case of the regular lay rope the direction of twist of the wires in the strands is opposite to that of the strands in the rope.

The length of lay or pitch of the wires in the strands and the strands in the rope are, in all cases, of such a proportion that the imaginary angles formed by a wire with respect to the strand axis or by a strand with respect to the rope axis are equal and equivalent to 18.5° in magnitude.

In the case of a regular lay rope the angle of lay of the wires is equal and opposite in value to that of the lay of the strands. The resultant angle, therefore, is zero, and the wires on the crowns of the strands run parallel to the rope axis.



(*a*) 6×19, with 6 Filler Wires; Regular Lay; Hemp Core

(*b*) 6×19, with 6 Filler Wires; Regular Lay; Independent Wire Rope Core

(*c*) 6×19, with 6 Filler Wires; Lang Lay; Independent Wire Rope Core

(*d*) 6×37, Seale; Regular Lay; Hemp Core

FIG. 14.—LOSS OF STRENGTH IN WIRE ROPE, DUE TO BENDING AROUND SHEAVES.

In the case of a lang lay rope the angle of lay of the wires is equal in magnitude and direction to that of the strands. The resultant angle is, therefore, 18.5 + 18.5 = 37°, and the axes of the wires on the crowns of the strands form imaginary angles of 37° with respect to the rope axis.

TABLE 12.—CONSTRUCTION DETAILS OF $\frac{5}{8}$-INCH IMPROVED PLOW STEEL WIRE ROPE USED FOR TEST SPECIMENS

Figure No. (1)	Rope (2)	CONSTRUCTION					Breaking load of straight specimens, in pounds (8)
		Wires in strand (3)	Diameter of wires, in inches (4)	Metallic cross-section of rope, in square inches (5)	Strand lay, in inches (6)	Rope lay, in inches (7)	
14 (*a*)	6 x 19, with six fillers; regular lay, with hemp core.	Inner, 7....... Filler, 6....... Outer, 12......	0.045 0.019 0.040	0.16341	1.56*	4.00†	33 720
14 (*b*)	6 x 19, with six fillers; regular lay, with 7 x 7 independent wire rope core (a small wire rope within a wire rope; I. W. R. C.)	Inner, 7....... Filler, 6....... Outer, 12...... I. W. R. C., 7 x 7	0.045 0.019 0.040	0.19396	1.56*	4.00†	37 400
14 (*c*)	6 x 19, with six fillers; lang lay, with 7 x 7 I. W. R. C.	Inner, 7....... Filler, 6....... Outer, 12...... I. W. R. C., 7 x 7	0.045 0.019 0.040	0.19396	1.56†	4.00†	41 175
14 (*d*)	6 x 37, Seale; regular lay, with hemp core.	Inner, 7....... Intermediate, 15 Outer, 15......	0.034 0.024 0.034	0.15779	Inner, 0.64* Main, 1.61*	4.00†	31 660

* In a left-hand direction.

† In a right-hand direction.

The breaking load of the various samples subjected to straight pulling loads were determined in each case by striking an average of several tests, and the values in Column (8), Table 12, were obtained.

The percentage loss in breaking strength of the bent specimens, cut from the same rope on which the straight load tests were obtained, have been presented as plotted points in Fig. 14.

Curved lines in Fig. 14—drawn in each case to represent the average results of the several tests—immediately suggest a relationship between the sheave diameter and the loss in breaking strength of the ropes to conform to hyperbolic curves, with gradually increasing sheave diameters, until infinity has been approached for the radius of curvature of the sheave. The breaking strength of the bent sample can readily be visualized gradually approaching the breaking strength of the straight sample until for all practical purposes the sample conforming to an infinite radius of curvature may be considered straight.

It is also possible to visualize a sheave diameter so small as to have the effect of the blade of a knife-edged cutter. A wire rope bent to conform to the radius of curvature of this extremely small sheave diameter will be severed with practically no applied tension to the two ends of the rope. The resulting values on the curves approach the two asymptotes or axes at right angles to each other in infinity.

The proper definition of a hyperbola is "the locus of a point such that the difference in distances between it and two fixed points is constant." The equation for the rectangular equilateral hyperbola, when the asymptotes conform to the abscissa and ordinate, is,

$$x\,y = \pm \frac{a^2}{2} \quad \text{..........................} \quad (58)$$

(*a*) REGULAR LAY ROPE.

(*b*) LANG LAY ROPE.

FIG. 15.—COMPARISON, SHOWING DIFFERENCE BETWEEN REGULAR LAY AND LANG LAY WIRE ROPE.

in which, a is the distance from the origin or common meeting point of the abscissa and ordinate to the point of inflection of the curve.

For the various types of wire rope in Fig. 14 the following formulas apply:

In Fig. 14(a), $xy = \frac{(\sqrt{12^2 + 12^2})^2}{2} = 144$; in Fig. 14($b$), $xy = \frac{(\sqrt{13^2 + 13^2})^2}{2} = 169$; in Fig. 14($c$), $xy = \frac{(\sqrt{8^2 + 8^2})^2}{2} = 64$; and, in Fig. 14($d$), $xy = \frac{(\sqrt{10.5^2 + 10.5^2})^2}{2} = 110.25$.

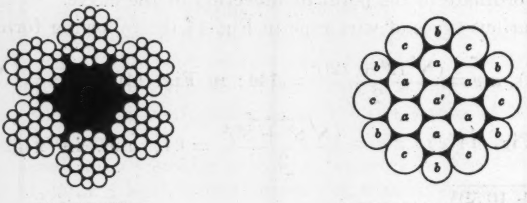
Mr. Carstarphen's static formula and the foregoing empirical formulas definitely prove the large error introduced by the use of the ordinary theory of flexure of solid beams for the development of bending-stress formulas for wire rope. The development of the problem from an angle of considering a wire rope equivalent to a spring is a closer approach to accuracy. However a definite error is introduced by the use of this theory due to the discrepancy between helix characteristics of the outer and inner wires in the strands making up a wire rope.

Frederick C. Carstarphen,[35] **M. Am. Soc. C. E.** (by letter).—It may be in order to remark that this paper was written as a defense of wire rope in the part it plays in industry, and, therefore, must refer to many topics in the use of wire rope that are controversial. It struck at the importance that is attached to the ratio of sheave-rope diameters as a criterion of the service life of wire rope, and called attention to the importance of velocity in such considerations.

It was hoped that the discussion would be extensive, and would cover all phases of the problems considered. Although this hope was not fully realized (more being said about subjects already familiar, than about those that are obscure), the writer is greatly pleased with the data that have been submitted, and feels indebted to those who have taken part in the discussion. The paper and its discussion is a start toward a symposium of the subject and may inspire research on some of the points raised. It is noted that the several discussers have covered and enlightened most of the obscure points in the original paper, so that but little need be said in closing the discussion.

Since the writer does not wish to assume to be true that which is to be proved, it is particularly fitting that the discussion of this paper be closed without serious comment, because the record should stand as it is written, a register of many views on an important subject. Several minor questions may be answered as follows.

Mr. Strachan assumes that there is some inherent difficulty in using Equation (20) to compute the loss of strength due to bending a rope made from different sizes of wire, around a given radius. The contrary is true, and as an illustration of the method, a computation is made of the loss of strength due to bending a $\frac{3}{4}$-in. wire rope around a sheave with a radius of 3 in. Fig. 16 illustrates the grouping of the wires in a $\frac{3}{4}$-in. Warrington, 6 x 19, regular lay rope, with a hemp center. Let the seven center wires of the strand (a, in Fig 16 (b)) be round and have a diameter of 0.054 in., and the outer wires, alternately, b = 0.042 in. and c = 0.058 in. There are six such strands, or 114 wires. The radii from the center of the strand to the center of the several wires are as shown in Table 13.



(a) Section of Rope

(b) Enlarged Section of Strand

Fig. 16.—Cross-Section of 6 x 19, Regular Lay Wire Rope, with Hemp Center.

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[35] Cons. Engr., Denver, Colo.

The diameter of the strand is 0.288 in. and that of the rope, 0.796 in. Therefore, the diameter of the core is 0.220 in.; the lay of the wires in the strand is 1.815 in.; the lay of the rope is 4.94 in.; the angle of the wires in the strand is $\tan^{-1}\left(\dfrac{0.192\,\pi}{1.815}\right) = 18°23'$; and the angle of the strand in the rope is $\tan^{-1}\left(\dfrac{0.508\,\pi}{4.94}\right) = 17°54'$.

TABLE 13.—CHARACTERISTIC DIMENSIONS OF A $\frac{3}{4}$-INCH WARRINGTON WIRE ROPE WITH A HEMP CENTER

Item	Description	Wires, in Fig. 16(*b*)		
		a	*b*	*c*
1	Number of wires per strand	7	6	6
2	Diameter of wire, in inches	0.054	0.042	0.058
3	Distance from center of strand to center of wire, in inches	0.054	0.102	0.096

Applying Equation (20) to the wires forming the centers of the several strands: $r_1 = 0.254$ in.; $\alpha = 72°\ 06'$; $\sin\alpha = 0.95$ $\cos\alpha = 0.308$; and,

$$P_1 = \frac{\pi\,(0.054)^4 \times 28.5 \times 10^6 \times 11 \times 10^6}{16 \times 3.37 \times 0.254\,[2 \times 11 \times 10^6\,(1 + 0.90) + 28.5 \times 10^6 \times 0.0949]} = 13.94 \text{ lb.}$$

The pull on the six center wires (*a'*, Fig. 16(*b*)) at 13.94 lb per wire gives, say, 84 lb. Proceed to the first row of wires (*a*, in Fig. 16(*b*)) about these center wires. Since this is a regular lay rope the angle of the wires of the strand (except the center wire), with the axis of the rope is practically zero, and is so taken in this case. If it was a lang lay rope this angle would be the sum of the angles of the wires and strands, thereby demonstrating what Mr. Rairden so ably proves, that the loss of strength due to bending a regular lay rope is not the same as for a lang lay rope of the same diameter, and size of wire; thus, for Wires (*a*), Fig 16(*b*), $r_1 = 0.054$ in., and,

$$P_2 = \frac{\pi\,(0.054)^4 \times 28.5 \times 10^6 \times 11 \times 10^6}{16 \times 3.37 \times 0.054\,[4 \times 11 \times 10^6]} = 66.8 \text{ lb.}$$

The pull on thirty-six wires, at 66.8 lb per wire, gives 2 404 lb.

Then, proceed to the second row, and consider the six wires (*c*, Fig. 16(*b*)); thus,

$$P_3 = \frac{\pi \times (0.058)^4 \times 28.5 \times 10^6 \times 11 \times 10^6}{16 \times 3.37 \times 0.096\,(4 \times 11 \times 10^6)} = 48.76 \text{ lb.}$$

The pull on the thirty-six wires, *c*, at 48.76 lb per wire, gives 1 755 lb.

Finally, for the six wires, *b*, in Fig. 16(*b*):

$$P_4 = \frac{\pi \times (0.042)^4 \times 28.5 \times 10^6 \times 11 \times 10^6}{16 \times 3.37 \times 0.102\,(4 \times 11 \times 10^6)} = 12.6 \text{ lb.}$$

The pull on the thirty-six wires, *b*, at 12.6 lb per wire, gives 453 lb.

The total loss of strength due to bending the rope on a 3-in. radius (or, with a sheave ratio of $\frac{6}{0.75} = 8$), is the sum of these values or, say, 4 700 lb. If the breaking strength of the straight rope is 35 000 lb, the bending of the rope about such a sheave, statically, amounts to a loss of $\frac{4\ 700}{35\ 000}$, or 13.5% of its strength.

The following formulas for bending in wire rope should be added for completeness:

$$b = \frac{E\,d'}{D}\cos\alpha + \cos\beta \qquad \text{(59)}$$

$$b = \frac{0.44\,E\,d'}{D} \qquad \text{(60)}$$

$$b = \text{Equation (12)} \times \frac{2}{3} \qquad \text{(61)}$$

and,

$$b = \frac{E_r\,d'}{D_1} \qquad \text{(62)}$$

in which, in addition to the notation of the paper, $E_r = E'$ = modulus of elasticity of the rope, and D_1 = diameter of the bend of the center line of the rope. It is to be noted that Equation (60) is a variation of Equation (10).

Mr. Meals always writes well on any subject pertaining to wire rope, and its use, and his opinions are appreciated. The writer believes that little is gained in separating the center wire of the strand from its neighbors in estimating the strength of a wire rope.

The writer appreciates the comments of Mr. Deck and agrees with him concerning the importance of high frequency vibrations, and their effects, on stretched cables and wire ropes. This opinion is based on observations made of the performance of messenger cables in long-span transmission-line crossings of rivers and bays, such, for instance, as the great crossing of the St. Lawrence River, at Three Rivers, Que., Canada.

Messrs. Damon and Julian present a review of much that has been accepted as commonplace in discussions on wire rope and its bending stress, and, therefore, balance the discussion, and their remarks are accepted as a proper addition to it. From personal contact with the late Professor Swain, the writer is certain that he did not intend any sweeping generality, that may have been said concerning the constancy of the modulus of elasticity of steel, to apply to rope wire, because it is well known that the value may range between 23 000 000 and 30 000 000 lb per sq in.

The test data submitted by Mr. Rairden are most instructive in that they show conclusively, the difference in the loss of strength due to bending regular lay rope as compared with lang lay. This important fact has never been given the attention that it merits by writers on the bending stress of wire ropes, and, of course, is ignored in the simple generalities that pass for engineering formulas of merit.

There is need for more tests of wire rope to determine the loss of strength due to bending, statically and kinetically, and no doubt these tests will be made by competent authority, but until such tests are made, there will be many opinions concerning the service factors of wire rope. Sometimes, it seems to the writer that the design of wire rope is like that of a mansion of many windows, porches, and blinds, that are shuffled around at the owner's will. There are many styles of wire rope, but all are built on the same old foundations of a hemp or wire rope center. Since such centers have little or no resiliency, it is believed that the next step forward in wire-rope construction will be in the adoption of an elastic foundation for the strands.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1839

LAKE CHAMPLAIN BRIDGE[1]

By Charles M. Spofford,[2] M. Am. Soc. C. E.

With Discussion by Messrs. Robert W. Abbett, Henry W. Troelsch, Jacob Feld, Clarence W. Hudson, and Charles M. Spofford

Synopsis

A chronological narrative of the planning, design, and construction of the bridge that crosses Lake Champlain from Crown Point, N. Y., to Chimney Point, Vt., is given in this paper. After an historical summary outlining the preliminary legislative steps necessary, the subjects of traffic counts, estimates, finances, and proposed toll rates are discussed.

A comparison of various types of bridges studied for this crossing and the reasons for adopting the continuous truss type are presented, followed by a description of the finished bridge itself, surveying methods of field location, specifications, loadings, materials, concrete, etc. Construction problems encountered on the substructure and superstructure are treated under separate headings.

Historical

Since early times Lake Champlain has been an important traffic route between the valleys of the St. Lawrence and the Hudson Rivers, and long before the construction of railways or highways the lake carried considerable freight traffic. During the struggle between France and England over control of the North American Continent the possession of the lake became of utmost importance both to the French and to the English, and strong forts were erected by both nations at Crown Point and at Ticonderoga. In Revolutionary times, equal importance attached to the control of the lake, and the capture by Ethan Allen and the "Green Mountain Boys" of Fort Ticonderoga by a night crossing of the lake forms a notable event in American history. While the lake has been a traffic route for centuries, it has also been a serious barrier to free intercourse between the State of Vermont, which was formerly a part of New York State, and New York State itself—a barrier, 120 miles in length, without a bridge. Communication across it has been by infrequent ferry lines operated in the season of open waters only, approximately 8 months per year. In many cases the ferries were not operated during the night and, in some cases, the approaches were hazardous. Occasionally, they were prevented from operating by bad weather. During the period between the formation of thick ice and its breaking up, occasional routes across the ice are available for vehicles.

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[1] Presented at the meeting of the Structural Division, Boston, Mass., October 10, 1929.

[2] Hayward Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge; Cons. Engr. (Fay, Spofford & Thorndike), Boston, Mass.

The bridge described in this paper is the first and only vehicular bridge crossing the lake in a distance of 120 miles. Its westerly approach is located a few miles above the Village of Crown Point, N. Y., at the Crown Point Reservation. The latter is a State Reservation containing the ruins of Fort St. Frederick and Fort Amherst, and the Champlain Memorial Lighthouse. It is being developed as a public park with bathing facilities, recreation piers, and park sites.

The bridge forms a connecting link between the main north and south highway routes on either side of the lake and also establishes a new traffic route between the Adirondacks, the White and the Green Mountains, and the Maine, New Hampshire, and Northern Massachusetts cities and coast resorts.

Popular demand for a bridge across the lake to replace the ferry-boat service, existed for a long time and finally "came to a head" in the Vermont General Assembly by the passing in 1923 of an Act creating a commission to inquire into the feasibility and possibility of joint action with New York State to bridge the lake. In February, 1925, the New York Assembly also established a commission to investigate such a bridge from the standpoint of New York and thereafter the two State Commissions acted together in making a preliminary investigation. The report[3] of this Joint Commission, submitted in 1926, discusses, in a general way, several possible bridge locations. It gives statistics as to traffic, points out the inconvenience of ferries, and recommends that the New York Commission be continued to complete its investigations and that additional funds be provided for borings and other studies. As a result, the New York Commission was continued and given a grant of $25 000. The Vermont Commission was automatically continued.

The 1927 report[4] of the Joint Commission contained data giving the results of borings at five possible locations, a report of the geologic conditions at these locations, and preliminary designs with estimates of cost of bridges at the proposed sites. The engineering features of this report were prepared by Roy G. Finch and J. A. L. Waddell, Members, Am. Soc. C. E. The geologic report was made by C. A. Hartnagel, Assistant Geologist of the State of New York.

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[3] Legislative Document (1926) No. 93, of the State of New York, entitled, "Preliminary Report of the Joint Legislative Commission on Bridge Connections Between the States of New York and Vermont Across Lake Champlain."

[4] Legislative Document (1927) No. 59, of the State of New York, entitled, "Final Report of the Joint Legislative Commission upon Bridge Connections Between the States of New York and Vermont."

As a result of its investigation the Joint Commission finally recommended the immediate construction of a highway bridge crossing the lake from the vicinity of Fort St. Frederick, at Crown Point, on the New York shore, to Chimney Point, in the Town of Addison, Vt. (see Fig. 1), to be built, owned, and maintained jointly by the two States and the cost divided between the States approximately in proportion to the length of bridge located in each State, that is, 60% for the State of New York and 40% for the State of Vermont. It was recommended that the bridge be operated as a toll bridge and become free for the use of the public when the revenue collected should prove sufficient to pay the complete cost of construction, together with interest and maintenance charges. It was further recommended that necessary legislation be passed by both States to permit the two States to enter into a compact to carry out the project.

Following the recommendations of the Joint Commission the Legislatures of New York and Vermont early in 1927 passed the legislation necessary to establish a temporary Joint Commission to negotiate a compact between the two States to permit their joint action in constructing the bridge. The report of this Commission, which consisted chiefly of a proposed compact between the two States, was promptly approved by the Legislatures of both States, and the necessary legislation was passed by each. The compact was signed by the Commissioners on May 11, 1927, and was ratified by the Congress of the United States on February 16, 1928.

This compact creates the Lake Champlain Bridge Commission as a body, both corporate and politic, with full power to purchase, construct, own, maintain, and operate a highway bridge, including approaches, across Lake Champlain between the State of New York and the State of Vermont. It has power to make charges for the use thereof, and for any of such purposes to purchase, take, hold, own, and operate, real or personal property; and to borrow money and secure it by bonds or mortgages upon any property held or acquired by the Commission. In other words, this compact sets up an independent body similar to the Port of New York Authority.

In accordance with the terms of the compact a permanent Commission was appointed by the Governors of the respective States in the spring of 1927 and the Commission had its first meeting on June 20, 1927. On August 2, 1927, the writer's firm was appointed as Engineers, and proceeded immediately to make preliminary studies and estimates of various types of bridges and the necessary surveys required for construction, and to place contracts for additional borings. On November 15, 1927, and prior to the ratification of the contract by Congress in February, 1928, the final design was adopted by the Bridge Commission. After public hearings a license for construction was issued by the Secretary of War on March 23, 1928. Bids were received for the construction on May 8, 1928, and the contracts for construction of the main portions of the bridge and approaches were awarded on May 18, 1928. The bridge was opened to traffic on August 26, 1929, fifteen months after the award of the principal contracts. The appearance of the completed structure is shown by Figs. 2 and 3.

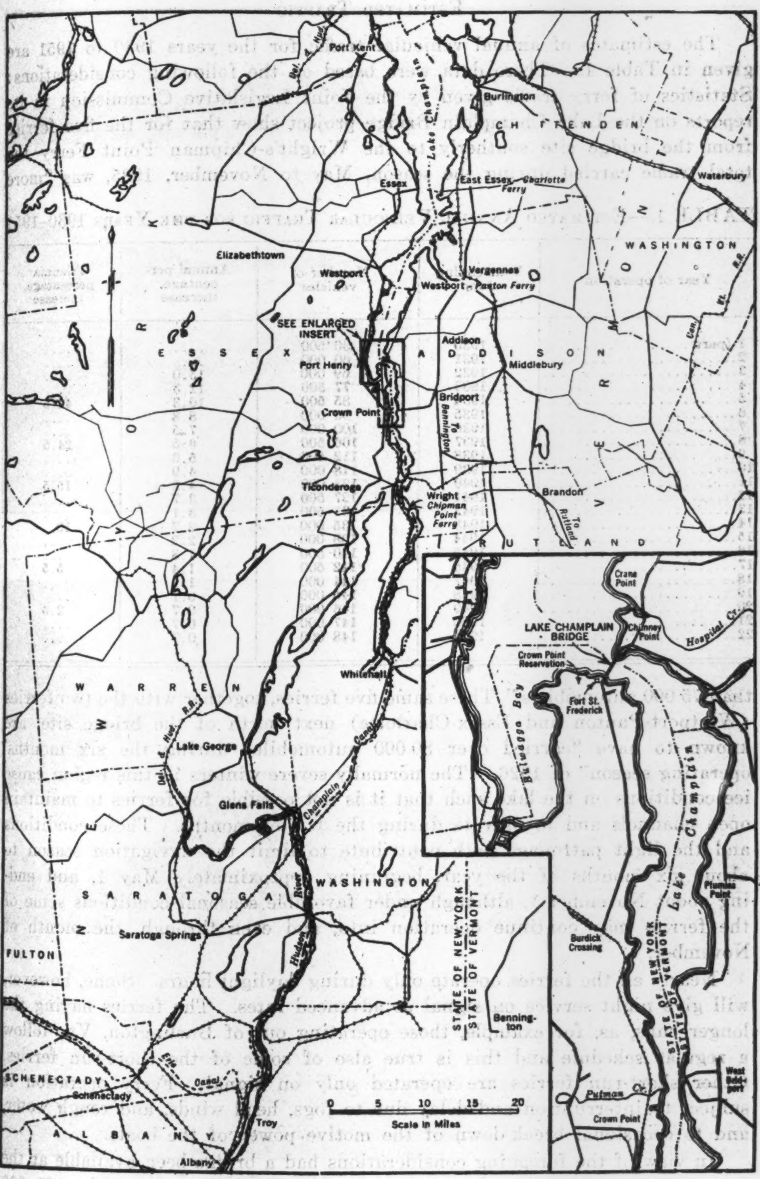
Port Kent
Del. & Hud.
Lake Champlain
Burlington
CHITTENDEN
Waterbury
Essex
East Essex
Charlotte Ferry
WASHINGTON
Elizabethtown
Westport
Vergennes
Westport
Paston Ferry
SEE ENLARGED INSERT
Addison
ESSEX
ADDISON
Port Henry
Middlebury
Bridport
Crown Point
To Bennington
VERMONT
Ticonderoga
Wright
Chipman Point Ferry
Brandon
To Rutland
RUTLAND
Rut. R.R.
Rut.
Rut.
Whitehall
WARREN
Del. & Hud. R.R.
Lake George
Champlain Canal
Glens Falls
WASHINGTON
SARATOGA
Saratoga Springs
Hudson River
FULTON
SCHENECTADY
Schenectady
Erie Canal
Troy
ALBANY
Albany
0
5
10
15
20
Scale in Miles
Crane Point
LAKE CHAMPLAIN BRIDGE
Chimney Point
Hospital Cr.
Crown Point Reservation
Fort St. Frederick
Bulwagga Bay
Lake Champlain
Plumley Point
STATE OF NEW YORK
STATE OF VERMONT
Burdick Crossing
Benning-ton
Putnam Cr.
Crown Point
West Addi-port

FIG. 1.—LAKE CHAMPLAIN BRIDGE: KEY MAP AND LOCALITY MAP.

Estimated Traffic

The estimates of annual vehicular traffic for the years 1930 to 1951 are given in Table 1. These data were based on the following considerations: Statistics of ferry traffic given by the Joint Legislative Commission in its reports on the Lake Champlain Bridge project show that for the five ferries from the bridge site southerly to the Wright's-Chipman Point Ferry the total traffic carried during the season, May to November, 1925, was "more than 75 000 automobiles." These same five ferries, together with the two ferries (Westport-Panton and Essex-Charlotte) next north of the bridge site, are known to have "carried over 80 000 automobiles during the six months' operating season" of 1926. The normally severe winters in this region cause ice conditions on the lake such that it is not possible for ferries to maintain open channels and to operate during the winter months. These conditions and the light patronage both contribute to limit the navigation season to about six months of the year, beginning approximately May 1, and ending about November 1, although under favorable seasonal conditions some of the ferries may continue operation into, and even through, the month of November.

TABLE 1.—Estimated Annual Vehicular Traffic for the Years 1930–1951

Year of operation	Year ending June	Number of vehicles	Annual percentage, increase	Triennia percentage, increase
1 (part)	1930	30 500	....	....
2	1931	60 000	....	....
3	1932	69 000	15.0	....
4	1933	77 500	12.3	....
5	1934	85 500	10.3	42.5
6	1935	93 000	8.8	....
7	1936	100 000	7.5	....
8	1937	106 500	6.5	24.5
9	1938	112 500	5.6	....
10	1939	118 000	4.9	....
11	1940	123 000	4.2	15.5
12	1941	127 500	3.7	....
13	1942	131 500	3.1	....
14	1943	135 000	2.7	10.
15	1944	138 000	2.2	....
16	1945	140 500	1.8	....
17	1946	142 500	1.4	5.5
18	1947	144 000	1.1	....
19	1948	145 000	0.7	....
20	1949	146 000	0.7	2.5
21	1950	147 000	0.7	....
22	1951	148 000	0.7	....

Nearly all the ferries operate only during daylight hours. Some, however, will give night service on signal at advanced rates. The ferries having the longer runs, as, for example, those operating out of Burlington, Vt., follow a regular schedule and this is true also of some of the short-run ferries. Other short-run ferries are operated only on signal. Ferry operation is subject to interruption and delay due to fogs, head winds, and rough water, and to occasional break-down of the motive power of the boats.

In view of the foregoing considerations had a bridge been available at the proposed site in 1926, it was estimated: (*a*) That of the more than 80 000

FIG. 2.—LAKE CHAMPLAIN BRIDGE FROM THE AIR.

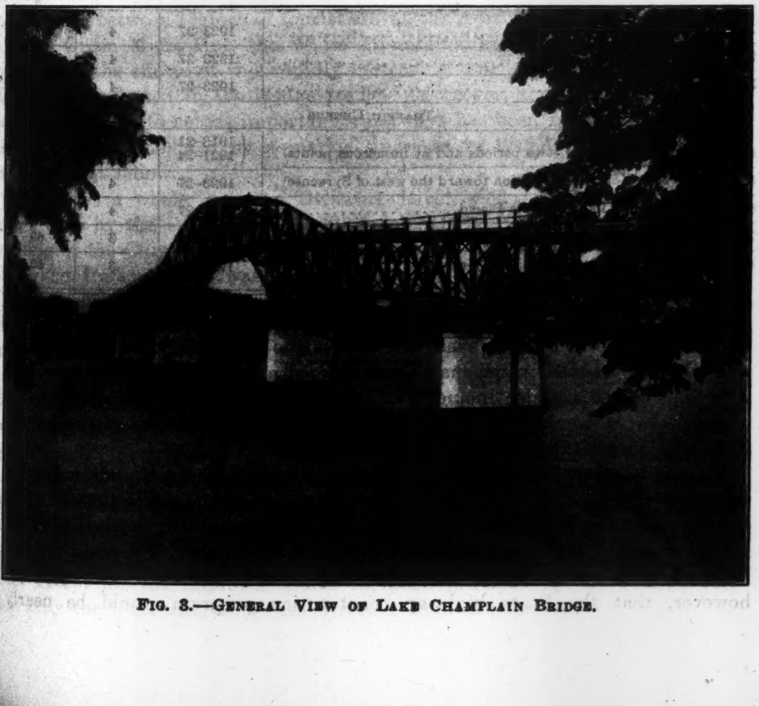

FIG. 3.—GENERAL VIEW OF LAKE CHAMPLAIN BRIDGE.

cars carried by these seven ferries during that operating season (all of which ferries are within about twenty miles of the bridge site), at least 60% or fully 50 000 cars would have used the bridge; (*b*) that other traffic would have been diverted from more distant points (the Burlington ferries and from around the ends of the lake) amounting to, at least, 10 000 cars; and (*c*) that 10 000 additional cars would have used the bridge during the winter and at other times when the ferries were not operating. Hence, a total of 70 000 cars was assumed as the probable traffic which would have crossed a bridge at the Crown Point-Chimney Point site in 1926; but to be conservative, 60 000 cars were taken in making up financial estimates as the number which would cross the bridge in its first normal operating year ending June 30, 1931.

No reliable data were available as to the growth from year to year of the ferry traffic across Lake Champlain. It was necessary, therefore, to base estimates of the probable growth of traffic upon statistics of traffic in general and in the districts bordering the bridge, and also to allow for effect of changed conditions resulting from the construction of the bridge.

TABLE 2.—GROWTH OF MOTOR VEHICLE TRAFFIC ON STATE HIGHWAYS

Item No.	Location	Years	Length of period, in years	Percentage increase
	MOTOR VEHICLE REGISTRATION			
1........	United States....................................	1923–27	4	53
2........	New York State..................................	1923–27	4	61
3........	Vermont...	1923–27	4	51
	TRAFFIC CENSUS			
4........	Massachusetts (two periods and at numerous points)..	1918–21 1921–24	3 3	66
5........	New York State (section toward the west of Syracuse)..	1923–27	4	63
6........	Saranac Lake to Lake Placid......................	1923–27	4	79
7........	Warrensburg to Chestertown.......................	1923–26	3	69
8........	Saratoga Springs to Lake George..................	1923–26	3	60

Table 2 shows the growth in automobile registration, and in traffic on the Massachusetts highways as well as on the New York State highways westerly of Syracuse, which may be considered as representing normal traffic growth. Values are also included in the tabulation for certain of the highways in the Adirondack Region which are more representative of the growth in tourist travel (Items Nos. 6, 7, and 8).

Statistics of traffic on the New York highways in the Adirondack Region must be viewed in the light of the fact that, during the years 1925 to 1930, extensive road construction brought about sub-normal traffic conditions in many instances and super-normal conditions in others. By the time the bridge was to be opened to travel, on September 1, 1929, it was expected, however, that the State highway construction program would be nearly finished; and that the automobile highway from New York City to Plattsburg, N. Y., and Montreal, Que., Canada, along the westerly shore of Lake Champlain, the most direct route from New York City to these northern points, would be entirely completed. It was assumed that these improvements would be reflected in an increased traffic in the districts served by the bridge and also on the bridge itself.

Another factor which was expected to have a decided influence upon the bridge traffic was the fact that the Crown Point Reservation—within which is located the New York approach to the bridge, and which contains the ruins of the historic forts, Amherst and St. Frederick—is being developed as an historic shrine and public park by the State Conservation Department. The development program provides for the building of a recreational pier and boat landing on the shore of the Reservation of the Champlain Memorial Lighthouse, the building of bathhouses and the improvement of the bathing beach, the extension of the museum, the construction of adequate buildings for the shelter of visitors, and provisions of ample parking facilities for motorists.

It was also confidently expected that the opening of the bridge would itself stimulate travel in this region such that the growth of traffic across the bridge would proceed, at least for the first few years, at a substantially higher rate than that on long established highways where for many years conditions have remained substantially unchanged. Tourist travel in general is increasing faster than normal travel, and as this bridge would form a connecting link between regions of tourist travel, the Adirondacks on the one hand and the Green Mountains, the White Mountains, and Maine resorts on the other hand, the effects of this rapidly increasing tourist travel would unquestionably be reflected in the bridge traffic. Moreover, the opening of a new and improved route across important waterways has been found in numerous cases to be accompanied by a marked increase in traffic, as shown by the examples that follow.

Traffic as a whole over the Delaware River was increased 70% in one year's time by the opening of the Philadelphia-Camden Bridge. With the opening of the Carquinez Strait Bridge across an arm of San Francisco Bay the bridge traffic of 1927 was 71% greater than the traffic on that important California highway route carried by ferry in 1926, whereas the normal growth in traffic over the 7-year period prior to the opening of the bridge showed an average increase of only 15.6 per cent.

At the Peace Bridge over the Niagara River at Buffalo, N. Y., where the ferry traffic had been 350 000 cars per year, the bridge traffic during the first year's operation was 1 407 272 vehicles, or four times the traffic on the ferry which the bridge replaced.

In the case of the Holland Tunnel beneath the Hudson River between New York and New Jersey, during its first year of operation ending November 12, 1928, it is estimated by the Chief Engineer that approximately 45% of the year's business was new traffic which would not have crossed the river except for the improved traffic facilities provided by the tunnel, and that only the remaining 55% was diverted from the ferries.

In view of the probable stimulation in traffic due to the foregoing causes, it was assumed that for a few years after its opening the travel over the bridge would increase rapidly and at a rate greater than the average increase in traffic on the highways generally of Vermont and New York. After a time this increase would be less rapid and in eight or ten years the increase might be expected to correspond more closely with the growth of traffic on the highway systems of these two States.

Financing and Insurance

The estimated and actual receipts and expenses from the time of opening the bridge to December 31, 1932, are shown in Table 3.

TABLE 3.— ESTIMATED AND ACTUAL TOLL RECEIPTS

Month	Actual Earnings by Month		Earnings as Estimated by Engineer*	
	Monthly	Total from August 27, 1929 (cumulative)	Monthly	Total from September 1, 1929 (cumulative)
1929:				
August (5 days)	$3 062.75		....	
September	16 393.35		$6 000.00	
October	7 726.25		4 000.00	
November	4 116 50		2 000.00	
December	2 110.00	$33 408.85	2 000.00	$14 000.00
1930:				
January	$821.25		$1 000.00	
February	921.25		1 000.00	
March	1 934.75		1 000.00	
April	3 292.50		2 000.00	
May	4 981.25		4 000.00	
June	7 562.50	$52 922.35	7 500.00	$30 500.00
July	12 967.50		11 000.00	
August	18 234.75		11 000.00	
September	10 616.00		9 000.00	
October	6 582.00		6 000.00	
November	3 987.75		3 000.00	
December	2 438.00	$107 748.35	2 000.00	$72 500.00
1931:				
January	$1 286.00		$1 000.00	
February	881.00		1 000.00	
March	1 704.00		1 000.00	
April	3 390.00		2 000.00	
May	5 247.25		5 000.00	
June	6 964.75	$127 221.35	8 000.00	$90 500.00
July	12 563.75		12 500.00	
August	17 138.25		12 500.00	
September	11 162.50		10 500.00	
October	6 996.25		7 000.00	
November	4 224.00		3 500.00	
December	2 660.25	$181 966.35	2 000.00	$138 500.00
1932:				
January	$2 044.50		$1 250.00	
February	2 125.25		1 250.00	
March	2 190.75		1 500.00	
April	2 911.50		2 500.00	
May	4 355.25		6 000.00	
June	5 309.75	$200 903.35	8 500.00	$159 500.00
July	10 125.25		14 000.00	
August	12 617.50		14 000.00	
September	9 092.25		12 000.00	
October	5 090.00		7 500.00	
November	4 000.00		2 687.00	
December	2 500.00	$244 328.35	2 293.25	$211 980.25

* Engineer's report of May 24, 1928, used as a basis of financing.

Serial bonds, bearing interest at 4¼% and redeemable at any interest date upon thirty days' notice, at varying premiums, were issued to finance this project. They are legal and tax exempt in New York and Vermont for public officials and bodies of each State and of its municipal sub-divisions, for insurance companies, savings banks, and other fiduciary bodies. In the opinion of counsel they are also exempt from the Federal income tax.

Estimates of gross revenues and net earnings, together with payments of interest at the rate of 4% and retirement on the bond issue of $1 000 000, are shown in Table 4.

The revenues are based on an average toll of $1 per car, including all passengers. This rate is less than the average toll rate charged by the several Lake Champlain ferries. The estimates indicate that by 1952 the accumulated surplus will be sufficient to retire the bonds then outstanding ($420 000) and to pay principal and interest on the $200 000 advanced by the States of New York and Vermont.

TABLE 4.—ESTIMATED FINANCIAL SHOWING FROM BRIDGE OPERATIONS

Year ending June 30	Gross revenues from tolls	Operating expense	Net earnings	Bond interest and retirement	Bonds outstanding after retirement paid July 1 of year in question
1929	....	....	....	$40 000	$1 000 000
1930	$30 500	$14 000	$16 500	40 000	1 000 000
1931	60 000	15 000	45 000	40 000	1 000 000
1932	69 000	15 500	53 500	40 000	1 000 000
1933	77 500	16 000	61 500	40 000	1 000 000
1934	85 500	16 500	69 000	40 000	1 000 000
1935	93 000	17 000	76 000	40 000	1 000 000
1936	100 000	17 500	82 500	40 000	1 000 000
1937	106 500	18 000	88 500	40 000	1 000 000
1938	112 500	18 500	94 000	40 000	1 000 000
1939	118 000	19 000	99 000	40 000	1 000 000
1940	123 000	19 500	103 500	60 000	980 000
1941	127 500	20 000	107 500	89 200	930 000
1942	131 500	20 500	111 000	87 200	880 000
1943	135 000	21 000	114 000	85 200	830 000
1944	138 000	21 500	116 500	83 200	780 000
1945	140 500	22 000	118 500	81 200	730 000
1946	142 500	22 500	120 000	79 200	680 000
1947	144 000	23 000	121 000	77 200	630 000
1948	145 000	23 500	121 500	75 200	580 000
1949	146 000	24 000	122 000	73 200	530 000
1950	147 000	24 500	122 500	71 200	480 000
1951	148 000	25 000	123 000	79 200	420 000
1952	149 000	25 500	123 500	....	....
Total	$2 669 500	$459 500	$2 210 000	$1 381 200	....

The bridge (except for its filled approaches) is insured for use and occupancy and for damages by fire, lightning, flood, tornado, earthquake, collapse, explosion, riot, malicious action, and other insurable hazards.

TYPE OF BRIDGE AND REASONS FOR ITS ADOPTION

While no limit was set upon the capital cost of the proposed bridge, in order to sell the bonds it was necessary that the bridge should show a probable revenue from tolls sufficient to cover carrying charges and also to amortize the investment in a reasonable time, which was fixed at thirty years by the Commission. The width of the roadway was fixed by the compact, and the clear width and rise of the channel span were determined by navigation requirements as established by the United States Army Engineers.

The Commission and its engineers were in agreement that the bridge should have as pleasing an appearance as possible consistent with the foregoing. The historic importance of the site and the fact that the bridge would be conspicuous for many miles on account of its height made its appearance of special importance. Borings and test piles disclosed that the soil overlying the bed-rock was extremely soft and that the bridge piers would have to be carried to bed-rock which over a part of the site occurs at a depth of 100 ft. below low-water level. After consultation with Army officials, it was decided to provide a vertical clearance at the channel span of 90 ft. above standard low water (Elevation 92.5) for a width of 186 ft., and a clearance of 73 ft. above the same level for a width of 300 ft. Fig. 4 shows the clearance at various points in the channel span. The total width of the lake at this water level is approximately 1 500 ft.

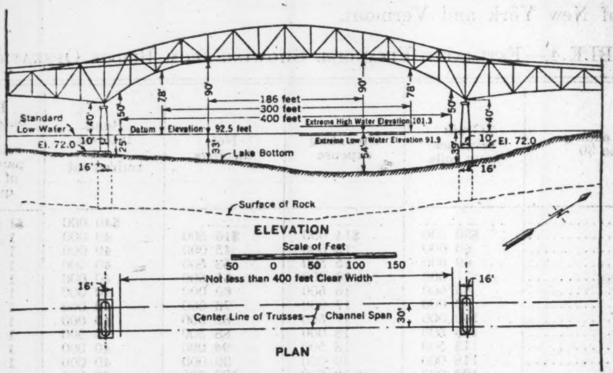
78'
90'
90'
78'
50'
40'
50'
40'
186 feet
300 feet
400 feet
Standard Low Water
Datum Elevation 92.5 feet
Extreme High Water Elevation 101.3
Extreme Low Water Elevation 91.9
El. 72.0
10'
25'
33'
Lake Bottom
54'
39'
10'
El. 72.0
16'
16'
Surface of Rock
ELEVATION
Scale of Feet
50 0 50 100 150
Not less than 400 feet Clear Width
16'
16'
Center Line of Trusses
Channel Span
30'
PLAN

FIG. 4.—CHANNEL SPAN, LAKE CHAMPLAIN BRIDGE.

In selecting the type of bridge consideration was given to the relative advantages and disadvantages of the following types:

(*a*) End-supported truss bridge;
(*b*) Cantilever truss bridge;
(*c*) Continuous truss bridge; and
(*d*) Suspension bridge.

An arch bridge either of reinforced concrete or steel was not considered economically possible in view of the expense of carrying unbalanced arch thrusts to the bed-rock at its great depth below the arch springing lines.

In comparing the availability of the various types of truss bridges the method of erection had to be given careful consideration. Navigation requirements made it essential that the channel span should be erected without the use of falsework. In some truss bridges this has been done by erecting a complete span on a lighter and hoisting it in place, but such a method was not seriously considered for this bridge in view of the lack of suitably located anchorage basins in the vicinity of the site. In consequence, all studies of truss bridges were based upon erecting the center span by cantilevering each half of it from the adjoining span. This method would not involve special difficulty, or greatly increased expense in the case of the cantilever or continuous type of structure; but for an end-supported center span, some of the truss members would have to be increased in size.

Moreover, in the engineer's judgment, the end-supported type, if designed with due regard to economy (with a shorter center span than that actually used for the Lake Champlain Bridge), would prove much less pleasing in appearance than either the continuous or the cantilever type and, also, would require somewhat more expensive center piers, owing to the greater pier width necessary to provide landing space for abutting trusses than that required for the support of a continuous truss. If end-supported spans were to be given the same outline as the continuous spans adopted, the trusses would be far from economical in outline. A further complication affecting the appearance of a bridge with end-supported trusses was the gradient which would require deck trusses for part of the crossing and through trusses over the channel. The writer found it impossible to sketch any simple span design that was at all satisfactory in appearance.

A cantilever bridge of the usual statically determined type did not seem especially appropriate, particularly in view of the necessarily short suspended span which would be proportionably shorter than is generally considered advisable for cantilever bridges. Some study was given to this type, but it was not possible to discover any important advantages as compared with a continuous structure while it would have the disadvantage of additional expansion joints in the floor and of greater and more rapid deflection particularly in the center span. Noticeable deflection of any bridge, especially when the deflection occurs rapidly, is objectionable both from the standpoint of the users of the bridge (a particularly important factor in a highway bridge), and of maintenance. It was, therefore, the engineer's desire to produce as rigid a bridge as possible with due regard to first cost and maintenance charges. A cantilever bridge that is not statically determined, such as the Queensboro Bridge in New York City, is, in reality, continuous. The comparisons of this paper do not refer to such a bridge, but only to the usual type of statically determined cantilever bridge.

The continuous truss type has all the advantages of the cantilever type except that of statical determination, and is also economical of material, especially when the dead stresses are large compared with the live stresses as in the case of a highway bridge with a concrete floor and with spans as long as those of this bridge. The lack of statical determination requires additional mathematical investigations on the part of the designers, but involves no special theoretical difficulties, merely increasing the labor of making the necessary computations. The deflection of a continuous span is less in amount and occurs with less rapidity than that of a cantilever span, this being an important element in favor of the continuous bridge. Moreover, the continuous type can be given a more pleasing appearance, consistent with economy, than any of the other types of truss bridges.

One objection sometimes raised to continuous bridges is that settlement of foundations causes serious changes in truss stresses, but with piers supported on bed-rock, as in the Lake Champlain Bridge, this objection does not exist. Moreover, in the case of yielding foundations most of the settlement occurs during the construction of the piers themselves, and does not affect the truss stresses. Unequal settlement of the piers after the erection of the trusses and the establishment (by jacking) of the desired end reactions would affect only that part of the truss stresses due to the dead loads carried by the trusses at the time the ends are jacked. These loads would include the weight of the steel trusses and bracing and such portions of the floor system as might be found convenient for erection purposes; that is, unless the unequal settlement is so great as to leave the trusses suspended over one of the piers or abutments, a contingency which should be guarded against by proper design.

With both end-supported and cantilever truss spans rejected in favor of continuous truss spans, it became necessary to compare this type of structure with the suspension span type.

Detailed estimates were made of a continuous span structure and two suspension bridges. One suspension bridge had a center span of 700 ft., with two flanking spans of 350 ft. each and two 185-ft. approach spans, and the other had a center span of 1 000 ft., with two flanking spans each with a length of 350 ft. The estimated cost of the shorter of these suspension spans proved to be somewhat greater than that of the continuous truss type. Moreover, its appearance was not entirely satisfactory since the main structure, consisting of the center span and the two flanking spans, would not have sufficient length to extend from shore to shore at all stages of the lake level thus giving it the effect of not being quite long enough to accomplish its purpose. The Commission and the engineers agreed that such a structure would not look as well as the continuous span finally adopted. The other suspension bridge was long enough to extend from shore to shore without the use of approach spans, and would have made an attractive bridge, except for the fact that such a span appeared of unwarranted length. The cost of such a structure, however, would be so much greater than that of a bridge with continuous trusses that its adoption could not be considered.

A design involving the use of continuous span trusses for the greater part of the structure, therefore, was adopted finally with the approval of the Commission and the engineers. The writer and his associates believe that the structure as designed has a pleasing appearance, is economical, and will prove satisfactory in operation and maintenance. The outline of the trusses and the general dimension of the entire structure are shown in Fig. 5.

GENERAL DESCRIPTION

The bridge is a high-level structure approximately 2 190 ft. in length, of which 1 500 ft. more or less is over the water. The ascending gradients of 5½% upward from each end to the channel span are connected by a parabolic vertical curve. The main part of the bridge consists of one end-supported riveted-truss structure; two continuous riveted-truss structures

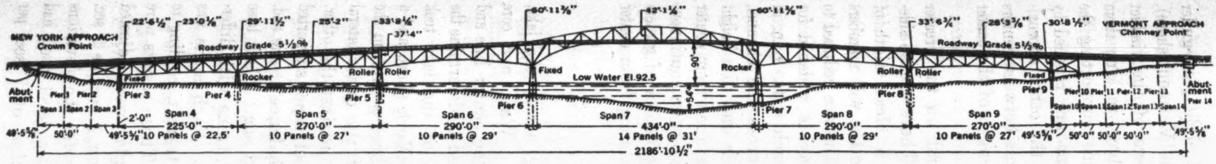
NEW YORK APPROACH
Crown Point
22'-6½"
23'-0⅛"
29'-11½"
25'-2"
33'-8⅛"
37'-4"
60'-11⅜"
42'-1¼"
60'-11⅜"
33'-6¾"
28'-3⅜"
30'-8½"
VERMONT APPROACH
Chimney Point
Roadway Grade 5½%
Roadway Grade 5½%
Fixed
Rocker
Roller
Roller
Fixed
Low Water El. 92.5
90'
54'
Rocker
Roller
Roller
Fixed
Abutment
Pier 1
Pier 2
Pier 3
Pier 4
Pier 5
Pier 6
Pier 7
Pier 8
Pier 9
Pier 10
Pier 11
Pier 12
Pier 13
Abutment
Pier 14
Span 1
Span 2
Span 3
Span 4
Span 5
Span 6
Span 7
Span 8
Span 9
Span 10
Span 11
Span 12
Span 13
Span 14
2'-0"
225'-0"
270'-0"
290'-0"
434'-0"
290'-0"
270'-0"
49'-5⅝"
50'-0"
49'-5⅝"
10 Panels @ 22.5'
10 Panels @ 27'
10 Panels @ 29'
14 Panels @ 31'
10 Panels @ 29'
10 Panels @ 27'
49'-5⅝"
50'-0"
50'-0"
50'-0"
49'-5⅝"
2186'-10½"

FIG. 5.—ELEVATION OF LAKE CHAMPLAIN BRIDGE SHOWING DEPTHS OF TRUSSES

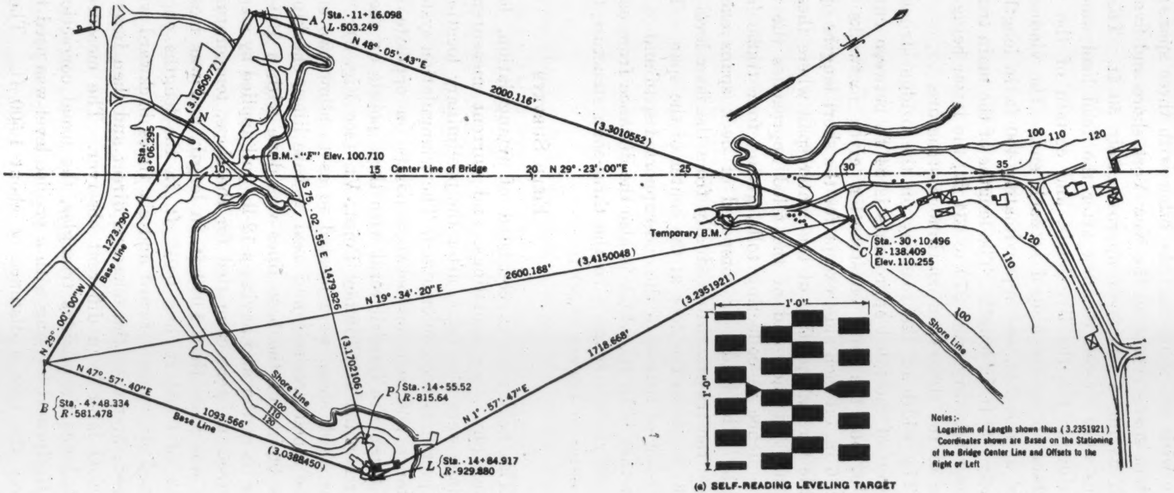
A {Sta. -11+16.098 L-503.249
N 48°-05'-43"E
2000.116'
(3.3010552)
(3.1050977)
N
Sta. 8+06.295
B.M. "F" Elev. 100.710
10
15
Center Line of Bridge
20
N 29°-23'-00"E
25
30
35
100
110
120
Temporary B.M.
C {Sta. -30+10.496 R-138.409 Elev. 110.255
120
110
100
Shore Line
1273.790'
Base Line
N 29°-00'-00"W
S 75°-02'-55" E
1479.826
(3.1702106)
2600.188'
(3.4150048)
N 19°-34'-20"E
(3.2351921)
1718.668'
N 1°-57'-47"E
B {Sta. -4+48.334 R-581.478
N 47°-57'-40"E
1093.566'
Base Line
(3.0388450)
Shore Line
100
110
120
P {Sta. -14+55.52 R-815.64
L {Sta. -14+84.917 R-929.880
1'-0"
1'-0"
(a) SELF-READING LEVELING TARGET
Notes:-
Logarithm of Length shown thus (3.2351921)
Coordinates shown are Based on the Stationing of the Bridge Center Line and Offsets to the Right or Left

FIG. 6.—TRIANGULATION SYSTEM, LAKE CHAMPLAIN BRIDGE

(one with two spans and the other with three spans); and eight plate-girder viaduct spans (three on the New York shore and five on the Vermont shore), each having a length of approximately 50 ft. The girders of the viaduct spans were riveted together after the dead load was in place, making them continuous for the live loads, and certain of the columns supporting the viaduct spans are hinged at each end. The viaduct spans are reached by filled approaches, one approximately 400 ft. in length and the other approximately 300 ft. in length. The lengths of the main truss spans between centers of bearings vary from 270 to 434 ft., the latter being the length of the center span of the three-span continuous structure.

The width of the roadway on the bridge structure and on the greater portion of the filled approaches is 24 ft. between curbs. There are no sidewalks, but the curb construction gives footwalks with a clear width of 2 ft. 6 in. on each side, except for the short lengths opposite the web members of the through portion of the center span where these widths are reduced to 15 in. At each end of the filled approaches the roadway is widened for convenience in collecting tolls, giving four traffic lanes each 8 ft. 9 in. in width. The bridge is composed of deck spans except across the channel, where the trusses rise gradually from the floor level at each end to a height of 40 ft. above the floor at the center of the span. The distance from center to center of trusses of the end-supported span and of the two-span continuous structure is 18 ft., which is also the distance from center to center of girders on the viaduct spans. In the three-span structure the distance from center to center of trusses is 30 ft.

FIELD SURVEYS

The field surveys consisted of triangulation, leveling across the lake, topography, lake soundings, and current measurements. Wash and core borings were also taken after the preliminary location of the center line and the piers had been determined. The triangulation system used to determine the distance and relation between points on opposite shores of the lake took the form of a quadrilateral with three points on the New York shore and a fourth point at Chimney Point, Vt. (see Fig. 6). The three points on the New York shore were located so as to allow two sides of the quadrilateral to be taped precisely and used as base lines, checking one against the other. The taping of the base lines was done with a 100-ft. steel tape, supported near its center and having a 12-lb. pull applied by a spring balance at one end. Corrections were computed for the slope, temperature, and sag. Each base line was more than 1 000 ft. in length, and was measured twice with differences of 0.001 ft. and 0.012 ft. in the lengths obtained. Each angle was turned six times forward and six times backward with a transit, reading to 10 sec., first with the telescope direct and then inverted. These results were checked later by a different observer. The measured angles were adjusted to balance the various triangles, the usual correction being 1 sec. or 2 sec.

Reciprocal leveling with a precise level was used to transfer the elevations across the lake a distance of about 1 500 ft. The precise level used had three leveling screws and a bubble with an angular value of 7 sec. per division. Bench-marks were set close to each shore, at about the same elevation. The level was set near one of the bench-marks, and a series of readings were taken across the lake. The position of the level and target was then reversed, and another series of observations were taken in the opposite direction.

A self-reading home-made target, shown by Fig. 6 (*a*), was used for this work. It had four rows of alternate black and white spaces each 0.1 ft. high, the two outer rows being offset 0.05 ft. vertically from the two middle rows, thus aiding the observer in interpolating to hundredths. The two sets of measurements taken in opposite directions were then averaged to eliminate the effect of refraction and curvature of the earth. One set of rod readings was also taken on the surface of the water close to the instrument at each shore as a check on the results. The results from the readings on the water checked the average of the results from the reciprocal leveling more closely than the water level could be read without a hook-gauge. The latter was considered for use in place of the reciprocal leveling, but was not adopted owing to the uncertain effect of wind and the need of building a stilling-box at each shore. The curvature and refraction indicated by the readings were about 0.05 ft. during the morning readings, but this amount was reduced by afternoon. No check on the results of either the triangulation or the leveling has been made other than an approximate check made before the construction of the center spans and tests made by running lines across the ice which gave a check in both distances and elevations within a few hundredths of a foot.

Soundings were taken about 300 ft. to each side of the center line and near the shores and docks. These were located mostly by one transit cutting across range lines. The directions and velocities of the currents were determined by floats made from 2 by 2-in. wooden studs, 13 to 16 ft. long, weighted so as to float vertically and to submerge, except for approximately 1 ft.

LIVE LOADING AND UNIT STRESSES

The live loading used in the design of the main members of trusses and girders was Loading *H*-15 of the Standard Specifications for Steel Highway Bridges issued by the U. S. Department of Agriculture,[5] and as revised by mimeographed supplement. This loading gave a uniform load of 525 lb. per ft. per girder or truss, combined with a concentrated load of 24 500 lb. applied at a point where it would produce maximum stress in the member under consideration. The live loads used in the design of the floor system and columns or hangers supporting the floor-beams consisted of one 20-ton truck moving along the bridge in any part of the roadway, or two 20-ton trucks side by side moving in the same or opposite directions. The dimensions and load distributions of these trucks are those given in the specifications previously mentioned.

Impact and wind stresses were also computed in general in accordance with the foregoing specifications. The unit stresses used in the design of the main truss members were 16 000 lb. per sq. in. for axial tension and for bending on the extreme fiber, and $15\,000 - \frac{50\,L}{r}$ for axial compression, with corresponding values for other unit stresses. In designing the members of the floor system allowable unit fiber stresses of 18 000 lb. were used instead of the 16 000 lb. used for the main truss members.

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[5] *Bulletin 1259*, U. S. Dept. of Agriculture.

MATERIALS

Steel.—Rolled steel, rivet steel, cast steel, and forgings were required to conform to the standard specifications of the American Society for Testing Materials for carbon steel, as follows:

Structural and Rivet Steel: Open-hearth (Serial Designation A7-24);
Cast Steel: Open-hearth, electric, or crucible (Serial Designation A27-24);
Steel Forgings: Open-hearth, or electric (Serial Designation A20-27);
Reinforcing Trusses for Concrete Floor: Open-hearth billet structural steel (Serial Designation A15-14); and
Other Reinforcement Bars: Either the same as for reinforcing trusses or rail steel (Serial Designation A16-14).

Paint.—Exposed metal work was given one shop coat of red lead and two field coats of tinted white lead. The coloring pigment used in the first field coat consisted of a small quantity of lampblack giving this coat a brown color. Considerable study was given to the second field coat to ensure a color that would harmonize with the landscape and prevent the bridge from being too conspicuous. The color finally selected was made by mixing from 13 to 13½ lb. of raw umber in oil with 100 lb. of white lead, and adding a small quantity of ferrite yellow until the exact color was obtained.

The red lead, white lead, linseed oil, and drier used in the paints were required to conform with the specifications of the Federal Specifications Board, as follows:

Red Lead: U. S. Standard Specifications No. 11, as corrected and revised by mimeograph circular of December 15, 1897, for grade containing not less than 95% Pb_3O_4.
White Lead: U. S. Standard Specifications No. 5.
Linseed Oil: U. S. Standard Specifications No. 4.
Driers: U. S. Standard Specifications for Liquid Paint Drier No. 20.
Turpentine: American Society for Testing Materials (Serial Designation D13-24).

Concrete.—Various grades of concrete were required for different parts of the work, and approximate parts by volume for these various grades were stated in the specifications as well as the maximum gallons of water, inclusive of the water contained in the aggregate per bag of cement. Slump tests for the various grades were also specified, depending upon the proportions of the mix, these tests to be made in accordance with the Tentative Method of Tests for Consistency of Portland Cement Concrete of the American Society for Testing Materials (Serial Designation D138-25T). These various requirements are stated in Table 5.

Coarse and fine aggregate used in the concrete came from iron mines at Mineville, N. Y., and were obtained at a depth of from 1 500 to 2 000 ft. below the surface. This material consisted of crushed stone from which the iron had been separated by a magnetic separator, less than 10% of the iron remaining in the rock after separation. The aggregate was transported to Port Henry by rail and thence to the bridge site in scows. Test cylinders of concrete were made regularly throughout the progress of the work and tested in the Cambridge Laboratories of the Massachusetts Institute of Technology. These tests showed the concrete to be unusually strong.

In view of the fact that much of the pier concrete had to be deposited in water at a considerable depth, the question arose as to what effect the hydrostatic pressure would have both on the time of setting and the strength of the concrete. An investigation of this question in 1928 and 1929 revealed that concrete similar to that used on this bridge, deposited in water at a pressure of 50 lb. per sq. in. (corresponding to a head of 115 ft.) develops a somewhat higher compressive strength in periods between 3 and 28 days than similar concrete setting under atmospheric conditions.

TABLE 5.—SLUMP TEST. REQUIREMENTS FOR VARIOUS CLASSES OF CONCRETE.

Designation (Class)	Maximum water per sack of cement	APPROXIMATE PARTS BY VOLUME			Maximum slump, in inches
		Cement	Fine aggregate	Coarse aggregate	
A	7.0	1	2	4	4
A (in walls)	...	...	...	...	6
B	7.7	1	2.5	5	4
C	7.0	1	2	4	3
D	6.5	1	1.8	3.6	3
*E**	...	1	1.5	2	3

* The proportion of water for this mixture to be determined in the field.

SUBSTRUCTURE

The character and depth of the material overlying the bed-rock are shown by the record of borings, given in Fig. 7. These borings were made in 1927. In addition, the data from borings made in 1926 were available to contractors, for examination. Five test piles were also driven. The overlying material was found to be so soft that a pile foundation would not be feasible; hence it was necessary to sink the piers to bed-rock, the depth of which reached a maximum of approximately 100 ft. below the low-water level of the lake.

Geological reports, core borings, and outcropping ledges showed the bed-rock to be of Chazy limestone formation of homogeneous structure and well fitted to carry any loads which could be put upon it by the bridge. The investigations indicated that the surface of the rock would be reasonably level and free from disintegration, and these conclusions were verified by the conditions found during construction.

All abutment piers and viaduct footings are of concrete. The piers and certain of the viaduct footings bear on bed-rock. The abutments and other viaduct footings are supported on good material without piling. The general type of all the piers (except Pier 9, which is on shore), is shown by the channel pier illustrated in Fig. 8. Pier 9 was laid "in the dry."

Piers 3 to 8, inclusive (see Fig. 5), had to be built in water, and it was specified that all of them, except the two channel piers (Piers 6 and 7), should be built by the open coffer-dam method. Bids were obtained for building the two latter piers by either the pneumatic caisson or open coffer-dam process.

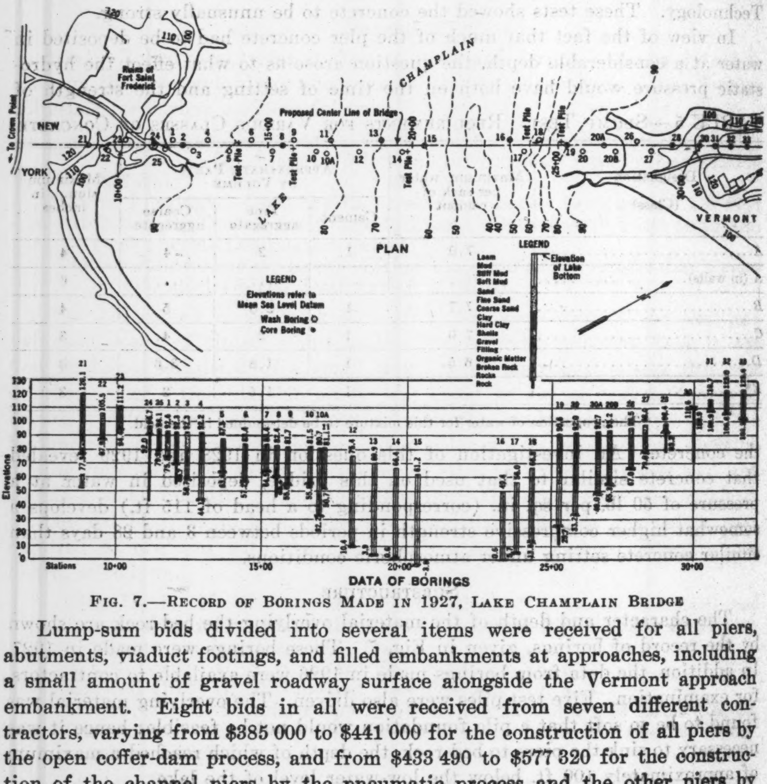
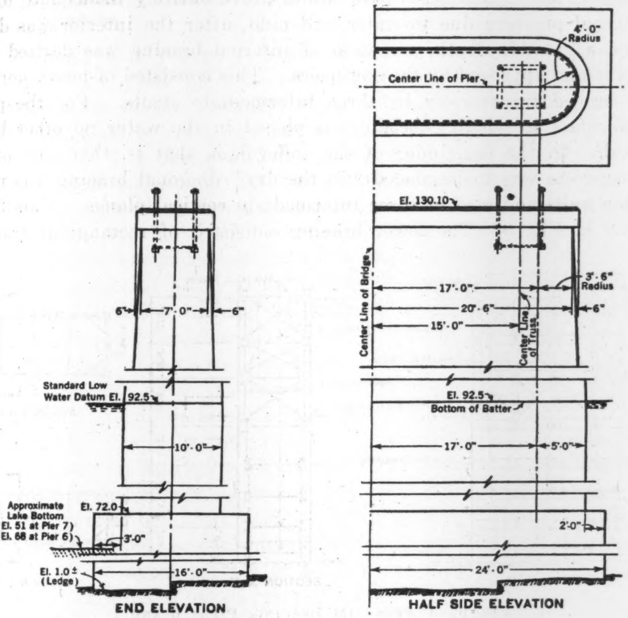

LAKE
CHAMPLAIN
NEW
YORK
VERMONT
To Crown Point
Ruins of Fort Saint Frederick
Proposed Center Line of Bridge
Test Pile
PLAN
LEGEND
Elevations refer to Mean Sea Level Datum
Wash Boring ○
Core Boring ●
LEGEND
Loam
Mud
Stiff Mud
Soft Mud
Sand
Fine Sand
Coarse Sand
Clay
Hard Clay
Shells
Gravel
Filling
Organic Matter
Broken Rock
Rocks
Rock
Elevation of Lake Bottom
Elevations
Stations
10+00
15+00
20+00
25+00
30+00
DATA OF BORINGS

FIG. 7.—RECORD OF BORINGS MADE IN 1927, LAKE CHAMPLAIN BRIDGE

Lump-sum bids divided into several items were received for all piers, abutments, viaduct footings, and filled embankments at approaches, including a small amount of gravel roadway surface alongside the Vermont approach embankment. Eight bids in all were received from seven different contractors, varying from $385 000 to $441 000 for the construction of all piers by the open coffer-dam process, and from $433 490 to $577 320 for the construction of the channel piers by the pneumatic process and the other piers by the open coffer-dam process.

The itemized bids for Piers 6 and 7 showed a variation from $176 444 to $295 000 by the open coffer-dam process and from $249 340 to $349 000 by the pneumatic process. The accepted bidder's figure for these two piers was $220 653, approximately $27 000 lower than the lowest bid for the same piers by the pneumatic process. The second lowest bidder submitted the lowest bid for these two piers, but put in a much higher bid for the other piers, thereby making his total bid $4 485 higher than that of the accepted bidder. It should be noted that the lowest bid for Piers 6 and 7 by the coffer-dam method was $72 896 lower than the lowest bid for these piers by the pneumatic process.

Consideration of the bids for Piers 6 and 7 show clearly the financial advantages of the open coffer-dam process for the particular conditions which existed at these piers, namely, soft material overlying bed-rock at an approximate distance below extreme low water of 100 ft., and below extreme high water of 109 ft., in a fresh-water lake without rapid changes in water level and with comparatively little current. As far as the writer is aware, these bridge piers are the deepest yet constructed by the open coffer-dam process with a single-wall coffer-dam and with a deep concrete seal deposited under water.

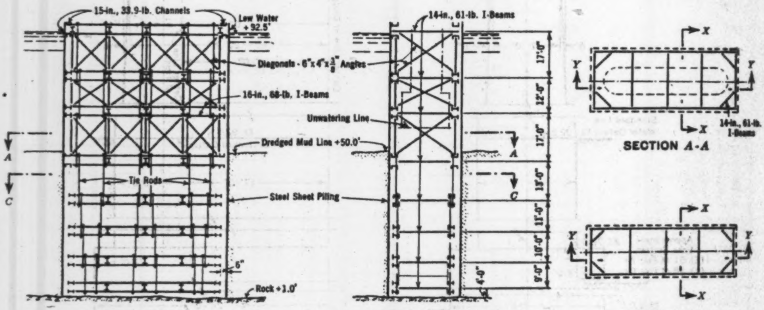

END ELEVATION

HALF SIDE ELEVATION
6"
7'-0"
6"
Standard Low Water Datum El. 92.5
10'-0"
Approximate Lake Bottom (El. 51 at Pier 7) (El. 68 at Pier 6)
El. 72.0
3'-0"
El. 1.0 ± (Ledge)
16'-0"
4'-0" Radius
Center Line of Pier
El. 130.10
Center Line of Bridge
3'-6" Radius
17'-0"
20'-6"
6"
15'-0"
Center Line of Truss
El. 92.5
Bottom of Batter
17'-0"
5'-0"
2'-0"
24'-0"

Fig. 8.—Dimensions of Channel Piers.

Wooden piles varying in length from 60 to 80 ft. were driven around the site of each pier and capped with 12 by 12-in. timbers placed approximately 8 ft. above the low-water level. On this capping a rough, working floor was placed. Waling pieces, 12 by 12 in., were fastened on this floor and similar ones were fastened approximately at the water level. These waling pieces served as guides for the steel sheet-piling forming the coffer-dam and were located horizontally so as to provide a clearance of approximately 4 ft. all around the concrete required by the plans for the portion of the pier below the lake bed. The sheet-piles formed a continuous wall weighing approximately 25 lb. per sq. ft. The individual piles had a width of 16 in. and a depth of 4 in. The maximum length of pile used was 98 ft., consisting of two pieces spliced by bolting in lengths of 40 ft. and 58 ft. Piling was driven to bed-rock without difficulty by a steam hammer weighing 7 000 lb. In no case were obstacles encountered, such as boulders or driftwood.

Inasmuch as such a coffer-dam would prove entirely inadequate to resist the external pressure due to water and mud, after the interior was dredged to any considerable depth, a system of internal bracing was devised by the contractor and approved by the engineers. This consisted of heavy horizontal frames braced transversely by three intermediate struts. For the part of the coffer-dam in which concrete was placed in the water no other bracing was used. In the remainder of the coffer-dam, that is, that part of it in which concrete was to be placed "'in the dry," diagonal bracing was used in the outer walls and also in three intermediate vertical planes. This bracing is shown in Fig. 9. The lower bracing consisted of rectangular frames of double 16-in., 68-lb. I-beams riveted together at the corners by knee-braces with three cross-struts in each frame, each consisting of a single 14-in., 61-lb. I-beam. The horizontal frames for the upper part of each coffer-dam was similar, but made of somewhat lighter members.



SECTION *Y-Y* SECTION *X-X* SECTION *A-A* SECTION *C-C*

FIG. 9.—COFFER-DAM BRACING, PIERS 6 AND 7.

Horizontal frames had to be left in place in the part of the pier in which concrete was laid in water. The 4-ft. clearance between coffer-dam walls and the specified dimension lines of the pier was sufficient to permit these frames to be placed entirely outside the required cross-section of the pier. The three cross-struts in each frame, however, passed through the piers.

For the upper part of the pier the horizontal frames were removed during construction, but the cross-struts and diagonals were embedded in the concrete. The diagonals, however, were cut off at the upper ends of each panel

The contractor's plan for placing the bracing below the mud line (which was accepted with slight modification for practically all piers), was as follows:

(*a*) After excavating approximately 12 ft. of mud from the coffer-dam, place all the horizontal bracing in this space in the form of a nest, and wedge the upper horizontal frame in place against the sides.

(*b*) Proceed with the excavation and, when conditions permit, lower the remaining frames until the upper one is at its desired level, and then wedge it in place. Proceed in this manner until all the bracing is in place, connecting the various sets of horizontal frames by tie-rods to act as suspenders and spacers during the process.

The foregoing plan proved satisfactory on the whole except that in the case of one of the piers the contractor excavated more than was planned before wedging one of the frames in place and, as a consequence, the sheet-piling buckled slightly—just enough to make it impossible to lower the remaining frames into place without cutting them apart and re-assembling them. This work had to be done in a considerable depth of water and required the services of six divers for several weeks. It would have been less expensive and the results would have perhaps been equally satisfactory had all the frames been assembled and bolted in place by divers.

The sheet-piling was easily pulled, except in the pier in which buckling occurred, where it was burned off under water at the mud-line. The mud was readily excavated by clam-shell buckets, except portions of it which adhered to the steel work of the coffer-dam and the bracing, which had to be loosened by jetting. After the removal of the mud, the water within the coffer-dam was found to be freer from sediment than the lake water outside.

Following the completion of the excavating and jetting, soundings were made within, and jet borings without, the coffer-dam to determine whether the bed-rock was reasonably level, not only over the site of the pier itself, but for a reasonable distance outside, and the bottom within the coffer-dam was examined by divers to determine whether it was clear of sediment and whether any of the upper surface of the rock required removal. The condition of the rock surface on all piers was found to be such that no further preparation of these surfaces was necessary. As a matter of fact the condition of these surfaces was especially satisfactory since they had been scored somewhat by glacial action and, in some cases, had pot-holes, thus ensuring good bond between concrete and bed-rock. In no cases were the surfaces tilted appreciably.

The concrete was placed in position by a patented, bottom-drop bucket with a capacity of 1 cu. yd. and made of ½-in. steel. At the bottom of the bucket double flap doors were attached to the bail by a chain. When suspended by the bail these doors were closed. When the bucket reached bed-rock, or the upper surface of the concrete previously laid, the bail would slide down, releasing the flap doors. This also released a clutch which engaged a lug on the side of the bail, preventing the doors from closing when the bucket was raised to permit the concrete to flow out. It was found that the cubic yard contained in the bucket spread on a level surface in a circle of 10 to 12 ft. in diameter.

Experience with depositing concrete in this manner shows that for the most satisfactory results the bucket should be level full; otherwise, when the bucket submerges, the rush of water will wash the cement from the upper portion of the concrete in the bucket. Moreover, the concrete should not be too dry. For the concrete in these piers a slump of 3 to 4 in., as determined by the standard slump test, was required. The concrete was laid from the center of each space between cross-braces toward the sides, in order to allow the laitance to flow toward the outer sides of the coffer-dam beyond the pier dimension lines.

After the under-water concrete that extended to grade, Elevation 72 (about 20 ft. below the low-water line), was placed, the coffer-dam was unwatered, and the remainder of the concrete laid "in the dry," the sheet-piling providing a satisfactory dam for this stage of the work.

SUPERSTRUCTURE

General Description and Unit Prices.— The superstructure consists of riveted truss and plate-girder viaduct spans all of carbon steel, supporting a concrete floor. The make-up of the various main members of the superstructure involves no unusual features and will not be described. The maximum stress in any truss member occurs in the bottom chord of the 270-ft. end-supported span. In the continuous spans the maximum stress in any truss member is tension and occurs over any intermediate pier.

Itemized bids for the steel, concrete floor, curbs, and broken stone on approaches were received. The bids on important items varied as follows:

Structural steel trusses, etc., erected and painted, per pound....	$0.06925	to	$0.084
Floor system, viaduct, etc., per pound...	$0.066	to	$0.0788
Bridge flooring complete	$98 000	to	$141 840

The successful bidder for the superstructure presented the lowest aggregate bid on the basis of estimated quantities, at $0.06925 per lb. for all structural steel and $98 000 for the bridge flooring.

The amount of structural steel as estimated by the engineers in asking for bids was 6 000 000 lb. The actual weight, as shown by final estimates for payment, was 5 883 727 lb., or approximately 60 tons less than the original estimate.

Stress Computations.—In designing the continuous trusses preliminary stress computations were made, using the values given in Griot's tables[*] of influence data for shear and moment for continuous beams of constant moment of inertia.

The principal dimensions of the structural members in Spans 6, 7, and 8, are shown in Fig. 10. For these spans, the values of the dead reactions, as obtained at Piers 5 and 8 from the preliminary design, were used as final values, a clause being introduced into the specifications stipulating that these predetermined reactions should be secured by the use of hydraulic jacks fitted with gauges and with the ratio between inches raised and reactions applied determined by actually raising the ends of the trusses before the laying of the concrete floor. Prior to the final establishment of the reactions by jacking, the Method of Least Work was used to determine the

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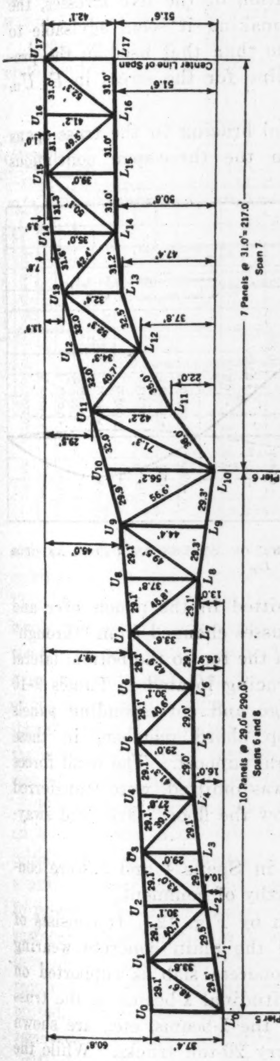
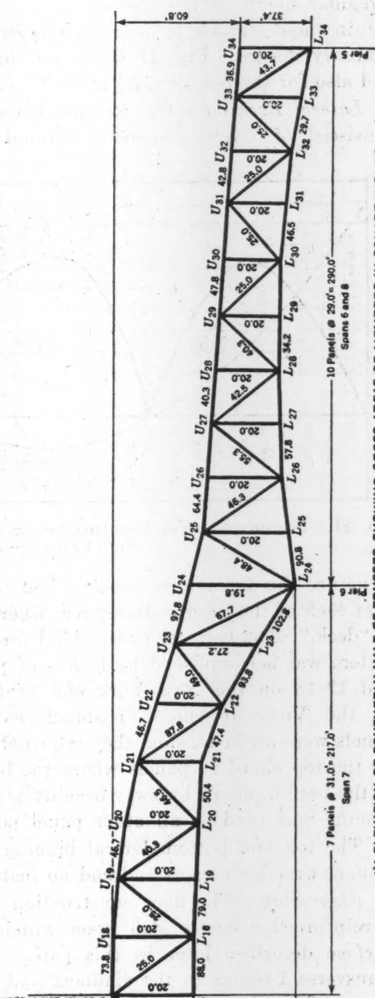
[*] "Kontinuierliche Trager Tabellen Griot", Second Edition, pub. by von Aschmann & Scheller, Zurich, Switzerland.


U0
U1
U2
U3
U4
U5
U6
U7
U8
U9
U10
U11
U12
U13
U14
U15
U16
U17
L0
L1
L2
L3
L4
L5
L6
L7
L8
L9
L10
L11
L12
L13
L14
L15
L16
L17
Pier 5
Pier 6
Center Line of Span
10 Panels @ 29.0' = 290.0'
Spans 6 and 8
7 Panels @ 31.0' = 217.0'
Span 7
60.8'
37.4'
42.1'
51.6'
49.7'
45.0'
37.8'
47.4'
50.6'
22.0'
13.0'
10.4'
16.9'
16.0'
29.1'
29.0'
31.0'
32.0'
56.6'
56.2'
71.3'

(a) ELEVATION OF NORTHERLY HALF OF TRUSSES, SHOWING LENGTHS OF MEMBERS


U17
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L28
L29
L30
L31
L32
L33
L34
Center Line of Span
Pier 6
Pier 5
7 Panels @ 31.0' = 217.0'
Span 7
10 Panels @ 29.0' = 290.0'
Spans 6 and 8
42.1'
51.6'
60.8'
37.4'
73.8
88.0
79.0
46.7
50.4
47.4
97.8
93.8
102.8
90.8
64.4
57.8
40.3
34.2
47.8
46.5
42.8
29.7
36.9
43.7
67.7
27.2
19.8
20.0
25.0

(b) ELEVATION OF SOUTHERLY HALF OF TRUSSES, SHOWING GROSS AREAS OF MEMBERS IN SQUARE INCHES

FIG. 10.—PRINCIPAL DIMENSIONS, SPANS 6, 7, AND 8.

dead-load reactions due to the steel work and the temporary tracks and concrete forms which were on the span prior to the jacking. The method of least work was also used for a final determination of the live stresses, the irregular depth of the trusses in these spans making it seem advisable to obtain these stresses by a method more accurate than that used in the preliminary design. Fig. 11 shows an influence line for the stress in U_9 U_{11}, and also for the reaction, R_L at Point L_0.

Lateral Bracing.—The top and bottom lateral bracing in the truss spans consisted of riveted members throughout. In the three-span continuous structure the top lateral bracing had to be omitted in the panels over and near each of the intermediate piers where the trusses changed from "through" to "deck." The transfer of the wind forces from the top to the bottom lateral system was accomplished by means of portal bracing located in Panels 9–10 and 12–13 on the New York end of the bridge and corresponding panels on the Vermont end. Inasmuch as the top chord members in these panels were all in tension, they required no lateral support. The wind forces on the top chord in panels where the bracing was omitted, were transferred to the bottom chord by sway-bracing placed below the floor. Overhead sway-bracing was used at all other panel points.

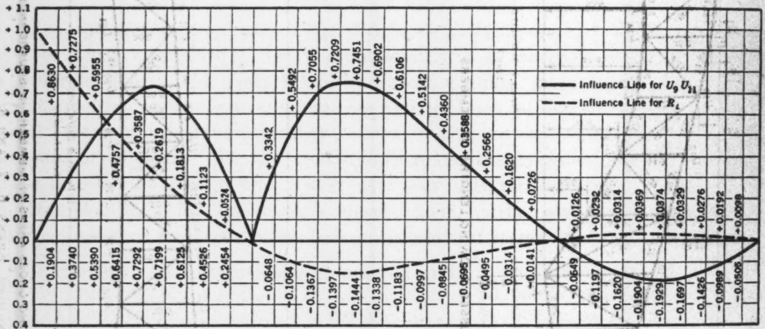
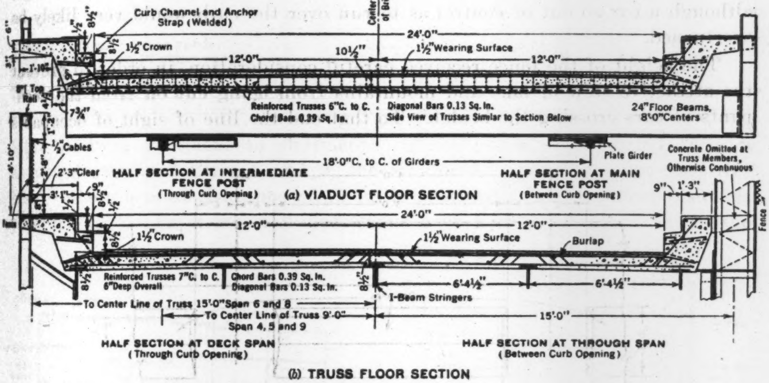


FIG. 11.—INFLUENCE LINES FOR HORIZONTAL COMPONENT OF STRESS IN U_0 U_{11}, AND FOR THE REACTION AT POINT L_0.

The top and bottom lateral bracing system in Spans 4 and 5 were continuous over both spans and had no features worthy of comment.

Floor-Slab.—The floor construction is shown by Fig. 12. It consists of a reinforced concrete slab upon which is laid the plain concrete wearing surface described later in this paper. The concrete slab is supported on transverse I-beams in the viaducts and on longitudinal I-beams in the truss spans. The dimensions of the slab, spacing of the I-beams, etc., are shown in the diagram. The floor was designed to carry 20-ton trucks. While the slab had sufficient strength in bending, with the depth used, it was found to be weak in shear; hence shear reinforcement was provided in the form of unit trusses. It was at first intended to use trusses with web reinforcement along the entire length of each unit, such as have been used in other bridges, but it was found that a material saving in cost could be made by omitting the diagonals in the central portion of the span where the concrete alone was of ample strength to carry the shear.

Concrete Wearing Surface.—The Commission decided in favor of a concrete roadway surface corresponding to the surfacing on the State roads at each end of the bridge. The question then arose as to the desirability of making use of the reinforced concrete slab itself as the wearing surface. This idea was abandoned, however, in order to protect the slab from wear, involving reduction in strength and possible exposure of the steel reinforcement. These factors seemed especially important for this bridge with its steep gradients and long periods of snow and ice, requiring the frequent use of chains on motor cars. After careful consideration it was decided to construct a special patented wearing surface. The Commission itself secured the right to use this surface and paid all royalties therefor, as well as the cost of special supervision of the laying by a representative of the owner. The wearing surface was made by obtaining a flush mortar upper surface on the concrete floor-slab and embedding in it (before it had acquired its initial set) a special cleavage fabric consisting of burlap with strands having a uniform spacing of not less than $\frac{1}{4}$ in., or more than $\frac{3}{8}$ in., small enough to prevent the large aggregate of the slab from bonding with the wearing surface. Then the concrete wearing surface was poured before the upper surface of the slab had obtained its initial set. The cleavage fabric was required to extend to within 2 in. of the curb, and lap-joints were not permitted. The object of the cleavage fabric is to permit the top layer to be removed if it wears out, without disturbing the integrity of the floor-slab.


Center Line of Bridge
Curb Channel and Anchor Strap (Welded)
6"
$\frac{1}{2}$"
$8\frac{1}{2}$"
$8\frac{1}{2}$"
$1\frac{1}{2}$" Crown
24'-0"
12'-0"
$10\frac{1}{2}$"
$1\frac{1}{2}$" Wearing Surface
12'-0"
3"
1'-10"
8"I Top Rail
$4\frac{1}{2}$"
$7\frac{3}{4}$"
Reinforced Trusses 6" C. to C.
Chord Bars 0.39 Sq. In.
Diagonal Bars 0.13 Sq. In.
Side View of Trusses Similar to Section Below
24" Floor Beams, 8'-0" Centers
1'-2"
$\frac{1}{2}$" Cables
4'-10"
18'-0" C. to C. of Girders
Plate Girder
Concrete Omitted at Truss Members, Otherwise Continuous
2'-3" Clear
HALF SECTION AT INTERMEDIATE FENCE POST
(Through Curb Opening)
HALF SECTION AT MAIN FENCE POST
(Between Curb Opening)
9"
1'-3"
3'-1"
9"
$8\frac{1}{2}$"
$\frac{1}{2}$"
24'-0"
12'-0"
12'-0"
Fence
$1\frac{1}{2}$" Crown
$1\frac{1}{2}$" Wearing Surface
Burlap
$8\frac{1}{2}$"
Reinforced Trusses 7" C. to C.
6" Deep Overall
Chord Bars 0.39 Sq. In.
Diagonal Bars 0.13 Sq. In.
$8\frac{1}{2}$"
6'-$4\frac{1}{2}$"
6'-$4\frac{1}{2}$"
I-Beam Stringers
To Center Line of Truss 15'-0" Span 6 and 8
To Center Line of Truss 9'-0" Span 4, 5 and 9
15'-0"
HALF SECTION AT DECK SPAN
(Through Curb Opening)
HALF SECTION AT THROUGH SPAN
(Between Curb Opening)

(a) VIADUCT FLOOR SECTION

(b) TRUSS FLOOR SECTION

FIG. 12.—CROSS-SECTIONS OF FLOOR, LAKE CHAMPLAIN BRIDGE

Curbs and Fencing.—In view of the height of the bridge above the lake every effort was made to safeguard automobiles from riding over the curb and through the fence. To that end the double curbs shown in Fig. 12 were provided. From abutment to abutment the fencing was made of steel wire mesh attached to wire cables at top and bottom, the cables being stretched into position between fence-posts. In addition, a structural steel channel top rail was used, this being connected to the fence-posts, except in the through portion of Span 7 where it was connected to the truss web members. The writer believes that the protection afforded by the double curb and the wire fencing and cables is ample to protect cars from going off the bridge, although a car so out of control as to run over the curb would very likely be overturned.

The height of the fence received careful consideration, in order to prevent the attractive view of lake and mountains from being cut off from the occupants of cars crossing the bridge. To that end the line of sight of occupants

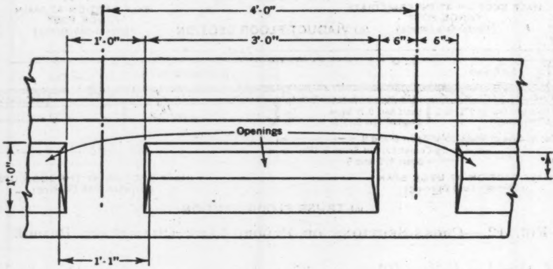


FIG. 13.—ELEVATION SHOWING TYPICAL CURB ELEVATIONS.

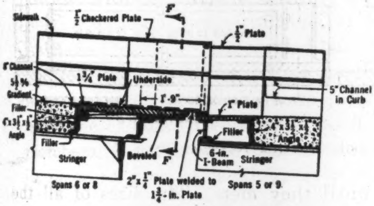
of motor cars of average height was studied, and the decision was finally reached to locate the top of the rail at a height of 4 ft. 6 in. above the concrete curb, with the top of the fencing 14 in. below the top of the rail, thus affording a clear space between bottom of rail and top of upper cable of approximately 11½ in. Views can also be obtained through the meshes of the fence. The fencing is of the chain-link type and consists of a fabric of galvanized aluminum wire slightly less than No. 6 gauge, with 2-in. mesh and with both edges knuckled. The shop coat of paint was applied by dipping the fencing while still warm from galvanizing.

Drainage and Snow Removal.—In view of the fact that the curb which also forms a footwalk on either side of the bridge would add considerably to the weight of the bridge if solid, the curb openings shown in Fig. 13 were provided. These serve not only to reduce the weight of curb and amount of concrete, but they also give ample provision for drainage and improve materially the appearance of the bridge as seen in side view. It is also expected that they will help keep the bridge floor free from snow which, if not too damp, will to some extent be blown off the bridge through these openings. The openings were made uniform in size throughout the length of the viaduct and the truss spans to simplify the form work. Unit quantities in the floor are as given in Table 6.

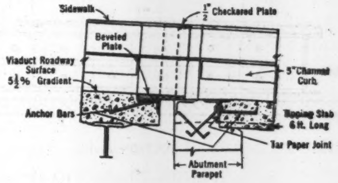
Expansion Joints.—Because of the continuity of the truss and viaduct spans, it was possible to reduce the number of expansion joints (which are always troublesome in highway bridges) to four in all in a total length of 2187 ft. from abutment to abutment. These joints were located at each abutment and over Piers 5 and 8, the maximum expansion occurring over Pier 8 where a total clearance of 15 in. is provided. The same expansion is provided (although it was not necessary) over Pier 5. Fig. 14 shows the details of the expansion joints at these two points. Attention may be called to the fact that the expansion plates consist of flat plates roughened by transverse strips welded thereto instead of the checkered plates commonly used.

TABLE 6.—QUANTITIES OF MATERIAL IN THE FLOOR PER LINEAR FOOT OF BRIDGE

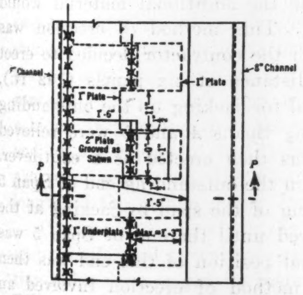
Material	Truss spans	Viaduct spans
Wearing surface, in cubic yards	0.11	0.11
Other concrete floor, in cubic yards	0.87	0.97
Reinforcing trusses, in pounds	153	168
Reinforcement rods, in pounds	97	141


Sidewalk
½" Checkered Plate
F
½" Plate
8" Channel
5½% Gradient
1¾" Plate
Underside
1'-9"
5" Channel in Curb
1" Plate
Filler
6"x3½"x½" Angle
Filler
Filler
Angle
Stringer
Beveled
F
6-in. I-Beam
Stringer
Spans 6 or 8
2"x½" Plate welded to 1½-in. Plate
Spans 5 or 9

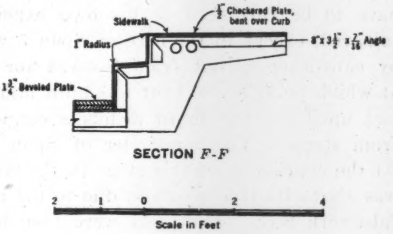
LONGITUDINAL SECTION THROUGH ROADWAY
SECTION *P-P*


Sidewalk
½" Checkered Plate
Beveled Plate
Viaduct Roadway Surface 5½% Gradient
5" Channel Curb
Anchor Bars
Approach Slab 6 ft. Long
Tar Paper Joint
Abutment Parapet

LONGITUDINAL SECTION THROUGH ROADWAY
DETAIL OF EXPANSION JOINTS AT ABUTMENTS


8" Channel
8" Channel
1" Plate
1" Plate
1'-6"
2" Plate Grooved as Shown
P
P
1'-5"
1" Underplate
Max.=1'-3"

DETAIL PLAN OF MAIN EXPANSION JOINT


½" Checkered Plate, bent over Curb
Sidewalk
1" Radius
8"x3½"x 7/16" Angle
1½" Beveled Plate
2
1
0
2
4
Scale in Feet

SECTION *F-F*

FIG. 14.—EXPANSION JOINTS, LAKE CHAMPLAIN BRIDGE.

Rocker and Roller Bearings.—In order to reduce the friction in the truss bearings and to prevent rusting, the rollers and rockers are enclosed in steel boxes filled with Albany grease and with sides that can be removed for replacing the grease if this becomes necessary. This method of protecting the rollers was used on the Queen City Bridge across the Merrimac River at Manchester, N. H., and an examination several years after its construction showed that the Albany grease had not hardened and that no rusting of the rollers had occurred. Fig. 15 gives details of one of the rocker nests on Pier 7.

Erection.—In designing the superstructure it was assumed that Spans 4, 5, 6, 8, and 9 would be erected on falsework and that the projecting ends of Span 7 would be cantilevered out until they met. The sizes of all the truss members were determined on this basis, with the specification provision that if the contractor wished to use another method of erection requiring any of the members to be increased in size the additional material would have to be furnished at his own expense. This method of erection was followed, except in the case of Span 5 which the contractor decided to erect by cantilevering out from Span 4 for a distance of six panels (162 ft.), at which point a pile bent was built and used for jacking up the outstanding end until the top chord members connecting Spans 4 and 5 were relieved from stress. The remainder of Span 5 was then erected as a cantilever. At the conclusion of this stage in the erection the outstanding end of Span 5 was above its true position, due to the raising of the span by jacking at the falsework bent. The jacks were then lowered until the end of Span 5 was supported on its appropriate pier. The final position of this end was then established by jacking at the pier. This method of erection involved an increase in the size of a few of the main members of Span 4, but the cost of this additional material was comparatively small.

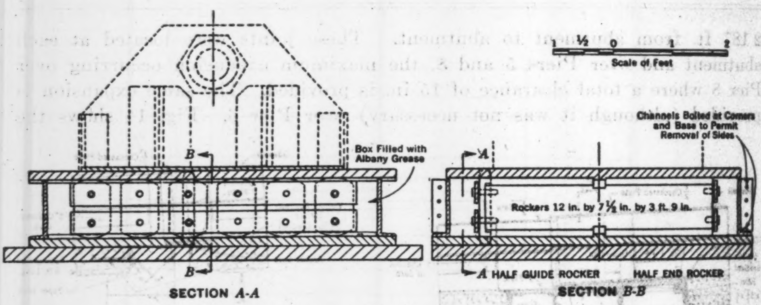

1 ½ 0 1 2
Scale of Feet
Channels Bolted at Corners and Base to Permit Removal of Sides
Box Filled with Albany Grease
B
B
A
A
Rockers 12 in. by 7½ in. by 3 ft. 9 in.
HALF GUIDE ROCKER
HALF END ROCKER
SECTION A-A
SECTION B-B

FIG. 15.—LAKE CHAMPLAIN BRIDGE, ROCKER NEST AT PIER 7.

The method of erection adopted for Span 7 required careful computations of the horizontal movements of Panel Points U_{17} and L_{16} at the cantilevered ends of Span 7 due to cambering and to the weight of steel and erection track. The deflection due to the traveler when located at one of the projecting ends was also determined. Computations also had to be made of the position of the bottom panel point, L_{34}, under varying conditions of load and temperature to determine the proper location of the rocker nests at this point with the rockers vertical. These computations were made by the contractor, and the decision was reached to set the rockers at this pier vertical and 2 in. forward (south) of their position under dead load and normal temperature.

The horizontal deflections of Panel Points L_0 and L_{34} were also computed to determine the proper position of the base plates at these points, which were specified to be located so that the roller nests should be central at mean temperature. The horizontal movements of U_0 and U_{34} were also computed to determine the amount of expansion to be provided in the expansion joints at these points.

The rollers, wedges, and shims provided at L_0 and L_{34} of Spans 6, 7, and 8 were temporarily omitted, and the truss shoes were supported directly on the base plates, planed surfaces being required for the sole plates on the shoes and the base plates to permit sliding during erection. In consequence, these panel points were located 20 in. below their final height. Panel Point L_{34} was also located 3.25 in. forward (south) of its final position.

For closure, Panel Points L_0 and L_{34} were both raised by jacking, and Span 8 with the projecting end of Span 7 was moved forward horizontally. As these processes were carried out it became possible to insert the previously omitted truss members in an unstressed condition. After these members were in place, L_0 and L_{34} were raised farther by means of calibrated jacks, until the calculated end reactions due to the steel trusses, bracing, and floor members, together with the erection track and such concrete forms as were in place to time of jacking, were obtained.

LIGHTING AND OTHER ELECTRICAL WORK

The bridge is lighted by incandescent lamps which are alternated on the two sides and are spaced longitudinally from 100 to 150 ft. apart. From the standpoint of traffic, lights on the bridge seemed even less needed than on the country highways giving access to it. The bridge is straight—horizontally at least—and every reasonable precaution has been taken to prevent vehicles from going over the side. Lights were necessary, however, at each end of the bridge to facilitate the collection of tolls and navigation lights are required by law; the Commission decided, therefore, to light the entire length of the bridge partly for advertising purposes.

Current is received at the New York end of the bridge at 2 300 volts and transformed to voltage suitable for lighting units. Two sets of navigation lights were originally installed—electric and oil—the latter for emergency use only. It was found, however, that the oil lights could not be readily lighted in a high wind, and these have been changed to electric lights operated by a storage battery, to prevent interruption in case of line or station trouble; these lights have proved entirely satisfactory.

Steel lamp posts consisting of 6-in. I-beams bolted to the structural steel fence-posts are used on the truss and viaduct spans. Reinforced concrete posts are used on the approach spans. The lamps in both cases are supported on structural steel brackets consisting of a single 4-in. car-building channel with flanges turned upward.

Toll Collection and Rates

The facilities for toll collection are on the approach at the New York end of the bridge. The traveled way is here divided, by an island 8 ft. wide, into two roadways, each 20 ft. 6 in. wide. A shelter for toll collectors, with signs giving the rate of tolls, is provided on this island. By means of smaller islands, each of these two roadways is again divided, through a small portion of its length, into ways 8 ft. 9 in. wide for the passage of single lines of vehicles. Any or all of these four narrow ways may be shut off by structural steel roadway gates, and each is equipped with a pavement unit, electrically connected to a counter in such a manner that the passage of a vehicle over any one of the pavement units is recorded and a substantially correct traffic count is made automatically. The toll collection area is well lighted and "mushroom" pavement lights and headlight reflecting signals are used where needed. The toll rates are as follows:

Passenger automobile (including passengers)	$1.00
Foot passengers and bicycles	0.25
Motorcycle, or passenger car trailer	0.50
Trucks, tractors, and trailers:	
1-ton, or less, truck or trailer (carrying capacity), or tractor (by weight)	1.00
1-ton to 2½-ton truck or trailer (carrying capacity), or tractor (by weight)	1.25
2½-ton to 5-ton truck or trailer (carrying capacity), or tractor (by weight)	1.50
5-ton truck or trailer (carrying capacity), or tractor (by weight)	2.00
Motor buses:	
Motor bus, seating capacity 16 passengers, or less	$2.00
Motor bus, seating capacity more than 16 passengers	3.00
Horse-drawn vehicles or saddle horses	0.50

Steam rollers, steam shovels, cranes, traction engines, and unusual vehicles of all descriptions, as well as cattle, sheep, and small animals, are permitted on the bridge only upon application to the Toll Collector, and at rates of toll published by the Commission or fixed by the Toll Collector.

A lighted sign is provided at the Vermont end, giving the toll rates, and a guard is stationed there during periods of considerable traffic. A private telephone line connects the two ends of the bridge.

The tolls may be collected at any of four points—the easterly side of the approach, each side of the large island, and the westerly side of the approach, and, at times of heavy travel, collectors are stationed at these four points. When traffic is light, it may be diverted to two lanes, or even to one lane if desired. At each collection point a pedestal is provided upon which a toll register may be placed for the use of the collector in recording tolls. These toll registers record the number of tolls taken by each collector under each of several classifications, and show to the traveler, at time of registering, the amount of toll paid. Different registers are used for northbound and eastbound traffic. The receipts are taken from the collectors by bank messengers at regular intervals.

At the easterly side of the New York approach is the toll collector's dwelling, in which are located living quarters for the collector and an office from which a view of the bridge to the north and of the New York approach to the south may be had. The switchboard from which all navigation and street lights are controlled is in the office. The dwelling also contains an office for the Commission in which is located the electric traffic counter, enclosed and under lock and key. By means of the roadway gates all vehicles can be diverted, if desired, to the lane immediately adjacent to the dwelling; hence, during the winter and at other times of light travel the toll collector may remain in his dwelling, stepping to the curb to collect tolls when a vehicle approaches.

Total Cost

The total construction cost of the bridge, including foundations, superstructure, approaches, lighting, toll houses, and equipment, was $967 800. Other items, including interest during construction, administration, legal and engineering charges, and real estate damages brought the grand total up to $1 149 000.

The contract included an allowance by the foundation contractor of $100 per day for failure to complete abutments, viaduct footings, and Piers 3 and 4 on or before October 20, 1928, and the entire work on or before December 1, 1928. It also provided a penalty or bonus of $250 per day for the superstructure contractor for every day lost or saved in the work to or after August 24, 1929, with the provision that the maximum amount thus deducted or added should not exceed $15 000. While certain details, including the final coat of paint on the part of the superstructure below the floor, was not completed by August 24, it was possible to open the bridge to traffic on the date assigned for its dedication, Wednesday, August 26, 1929, and no penalty was imposed upon the contractor.

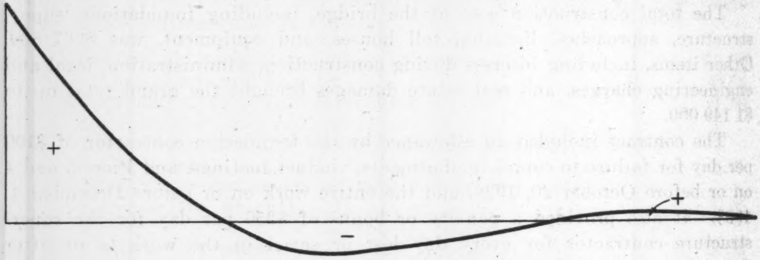
Acknowledgments

The effect of hydro-static pressure in depositing concrete at considerable depths, as reported in this paper, was studied in a series of tests by Mr. J. Wayne Couter, at the writer's suggestion.

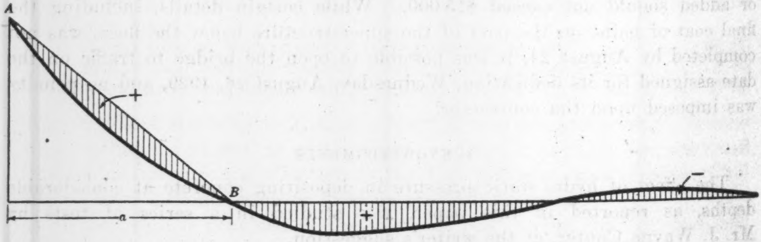
The Merritt-Chapman and Scott Corporation was awarded the contract for construcing the piers. The contract for the superstructure was awarded to the American Bridge Company.

DISCUSSION

Robert W. Abbett,[7] Assoc. M. Am. Soc. C. E. (by letter).—The author is to be commended for his thoroughness and originality in planning the Lake Champlain Bridge. It stands as another monument to the death of what has been a bugaboo to American bridge engineers for the past century. Continuity in bridge construction has been scrupulously avoided in design and vigorously attacked by many of the most eminent designers for several reasons; notably because it is statically indeterminate and because a slight pier settlement has been thought to introduce critical stresses in various parts of the structure. That the weakness and inaccuracy of these reasons have been recognized is proved by the increasing popularity of this type of construction, and by the large number of continuous trusses that have been built since Gustav Lindenthal, Hon. M., Am. Soc. C. E., proposed the Sciotoville Bridge.



THE ELASTIC CURVE FOR LAKE CHAMPLAIN BRIDGE



INFLUENCE LINE FOR STRESS IN U_9-U_{11}

Fig. 16.—Influence Line for Lake Champlain Bridge.

The layout of the Lake Champlain Bridge is most ingenious. The writer can think of no other arrangement which would have provided the necessary clearance at mid-span with such economy in approaches. In the high-level viaduct from Newark to Jersey City, N. J., a somewhat similar layout was necessary for the river spans. Here, however, the cantilever principle has been used with comparatively long suspended spans. Since the approach conditions for the two sites are quite similar, the writer cannot hold with the author in his unsupported statement that a short suspended span was necessary in the cantilever type. An economic comparison between the continuous truss, as built, and a cantilever truss with a longer suspended span similar to the Hackensack and Passaic River Bridges, would be interesting.

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[7] Asst. Prof., Bldg. Constr., Thompson-Starrett Foundation, Dept. of Civ. Eng., Union Coll., Schenectady, N. Y.

It appears to the writer that the method of consistent deflections might have been utilized in the stress analysis to some advantage over the method of least work. This method seems to be more simple and less laborious. The elastic curve for the truss may be found by removing the redundant, R_L, and drawing the deflection diagram for a unit load at its point of application. The influence line for the stress in any member in an end span is easily constructed using the elastic curve as a foundation. Thus, the influence line for Member, U_9–U_{11} is constructed by simply drawing a straight line across the elastic curve from A to B (Fig. 16) and multiplying each ordinate of the resulting diagram by $-\frac{a}{r}$, in which, $r =$ the lever arm of Member U_9–U_{11} about its center of moments in the truss, and $a =$ the distance of R_L from the same center of moments. For members in the central span the construction of influence lines is a more lengthy process, but fundamentally it is no more complicated.

HENRY W. TROELSCH,[8] M. AM. SOC. C. E. (by letter).—Some of the circumstances encountered in carrying out the field work in connection with the erection of this bridge should be of general interest.

The spans erected on falsework were supported at alternate panel points on towers, each of which consisted of two bents braced together. The erection travelers, running on the floor of the bridge, were used to place the falsework and to erect the spans in one pass, working from the shore to the middle of the through span.

The mud of the lake bottom was very soft, and no difficulty was encountered in driving piles to a bearing on the underlying rock. In some cases, however, the rock was 100 ft. below the water level, and piles, 140 ft. long, were required. The piles consisted of 12 by 12-in. timbers, and were spliced by four angles, 4 by 4 in. by $\frac{5}{16}$ in., fastened with $\frac{3}{4}$-in. through bolts. Squared timber piles, being free from the initial curvature usually present in round piles, are less subject to bending from column action. They also have a larger area than round piles of the usual dimensions, and can carry a greater load, especially when they are driven to bearing on rock. They can usually be withdrawn, and have a high salvage value.

The piles were driven by a steam hammer suspended from the boom of the traveler. No leads were used; instead, wooden guides were lashed across cables extending from the completed part of the span to the next pier.

The closing of the through span was simplified by the shape of the structure and by the presence of a roller shoe at the north intermediate support, L_{24}. The end supports, at L_0 and L_{34}, were erected 20 in. below their normal position, which was equivalent to rotating the two halves of the structure about the intermediate supports, L_{10} and L_{24}. Since the closing joints, U_{17} and L_{18}, were well above the centers of rotation, the rotation caused a separation of the two halves of the structure at these points, leaving a gap at each closing joint. The gap in the upper chord was about an inch larger than that in the lower chord.

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[8] Designing Engr., Am. Bridge Co., New York, N. Y.

The closing operation consisted of rolling the north half of the structure southward until the gap in the lower chord was closed, and then jacking up the end supports, L_0 and L_{34}, until the upper chord was closed. The closure was thus accomplished without temporary or adjustable members, such as are ordinarily used in closing cantilever bridges, and without drilling any of the connections in the field. No special equipment was required other than four 300-ton hydraulic jacks.

Jacob Feld,[9] Assoc. M. Am. Soc. C. E. (by letter).—The writer is chiefly interested in the description of the substructure design and construction. Although given in great detail there are several questions which might be answered in the closing discussion to complete the record.

The contractors were asked to bid on the construction of all piers by the open coffer-dam process with an alternate choice of constructing the channel piers by the pneumatic process. In accordance with information shown by the borings, with approximately 50 to 60 ft. of fine uniform soft materials to penetrate, it seems that a design of pre-cast concrete or steel-shell caisson might have proved just as economical, if not more so, than the open coffer-dam. Such a design would have given a better concrete surface than can be guaranteed to result from concrete placed under water in spite of all precautions that were taken.

There is no mention of any special precautions in protecting the surface of the piers at the water-line to take care of ice pressure and wear during the winter seasons. Undoubtedly, this item was considered in the design, and the omission of special protection for the concrete, an unusual procedure, must have resulted from definite reasons. It would be interesting to have those arguments on record.

An item often overlooked in the preparation of construction procedure for such piers is the accumulation of laitance on the top of each pour of under-water concrete. In Piers 6 and 7, the total height of under-water concrete was approximately 60 ft. A considerable thickness of soft material must have been exposed after unwatering. The chemical tests of this soft material in a number of projects has shown that the quantity of cement removed from the concrete is between 10 and 15%, so that the total amount of soft material which can be expected is about one-twenty-seventh the total height. In these piers the amount of soft material on top of the under-water concrete should have been somewhat more than 2 ft. It would be interesting to know what depth of soft material was actually removed from the top of under-water concrete, and whether any tests were made as to the strength of the concrete below the removed material.

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[9] Cons. Engr., New York, N. Y.

Clarence W. Hudson,[10] M. Am. Soc. C. E. (by letter).—This paper is valuable for several reasons. It gives a comprehensive description of the engineer's service to his client from the inception of the project to its completion; it describes the work of the engineers preliminary to their actual engineering service; it begins with certain traffic statistics, from which estimates of traffic on the proposed bridge and of the financial returns from such traffic were made; and the amount of money that could justifiably be spent for the construction of a bridge was determinable from the estimated return from traffic.

The following comments apply to the various subjects described by Professor Spofford:

(*a*) The author describes the service of the engineers after the financial feasibility of the project was established.

(*b*) The description of the field survey is very concise and yet comprehensive and clear.

(*c*) The engineer's conclusions with respect to the type and details of the structure to be adopted, were sound.

(*d*) The reasons given for the elimination of simply supported and cantilever type truss spans, from their design for the central portion of the structure, the writer believes are correct.

(*e*) The writer does not believe, however, that the difference in cost due to using simple truss spans, cantilever spans, or continuous spans, for the principal part of a structure, would, in general, be large enough in itself to be a controlling factor as against a better appearing or more satisfactory structure inherent in any one of the types, for use in a definite location.

(*f*) The description of the method of building the deep piers is excellent and the word or two with reference to the engineer's effort to eliminate the harmful effect of laitance (which the writer has always found to be existent) in concrete deposited under water, is evidently good.

(*g*) The writer feels, however, that the longitudinal grade adopted was about the maximum permissible and that the use of higher unit stresses in the floor system than in the main trusses is a variation from general practice.

(*h*) The resulting structure is very pleasing in appearance.

Charles M. Spofford,[11] M. Am. Soc. C. E. (by letter).—The writer has been gratified by the favorable comments on this paper, and takes this opportunity to express his thanks for such comments.

As stated by Mr. Abbett the method of deflections might have been used advantageously in computing the stresses in the trusses. Many years ago, in connection with an investigation made in behalf of the Comptroller of the City of New York, the writer used the method of deflections in computing the stresses in all the members of the Queensboro Bridge. This is a continuous structure statically indeterminate to the second degree (as is the Lake Champlain Bridge). Later experience in dealing with statically indeterminate structures has convinced him that the method of least work is simpler to apply, largely because the signs of the various expressions take care of themselves, requiring the computer to give attention to the numerical computations only. As a matter of fact, both methods are identical when reduced to their simplest terms.

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[11] Hayward Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge; Cons. Engr. (Fay, Spofford & Thorndike), Boston, Mass.

As far as graphical methods go the writer's experience indicates that analytical methods are more satisfactory in cases where the computations may be divided among several staff members. One of the chief advantages of an analytical method is the fact that the results may be checked with more precision than by any graphical method known to the writer, thus giving a greater feeling of security than would otherwise be the case. At all events the computations form such a small factor as far as the expense of designing goes that any theoretically correct method may be applied without affecting materially the expense or time required for the design.

With respect to laitance in the concrete piers, the concrete was properly proportioned, thoroughly mixed, and carefully and continuously placed in position by bottom dump 1-yd. buckets. As the pouring progressed the upper surfaces of the piers were inspected by divers from time to time, who reported little or no laitance. At the conclusion of pouring, the amount of laitance that had to be removed was a matter of only 2 or 3 in. instead of the 2 ft. suggested as a possibility by Mr. Feld. Whether the strength of the concrete was reduced by the formation of laitance during pouring was not investigated, but tests of the concrete as it came from the mixer showed it to be of high strength; and as the stresses to which it was subjected were not severe, no doubt existed as to its having adequate strength. Inspection of the character of concrete deposited in sea water in the piers of the Charlestown Bridge, in Boston, Mass., was made after the concrete had been in place for several years. Samples were taken by diamond-drill borings, under the direction of Frederic H. Fay, M. Am. Soc. C. E. This inspection showed the concrete to be of excellent quality throughout the depth of the pier.

In reply to the query by Mr. Feld as to the reason for not protecting the pier concrete against abrasion and deterioration, it may be pointed out that the piers are in a fresh-water lake with little current and are practically free from danger of abrasion from ice and floating objects. The concrete piers of the Queen City Bridge, across the Merrimac River (designed and constructed in 1922 at Manchester, N. H., under the supervision of the writer's firm), which are located in fresh water and a short distance below a dam, have undergone no deterioration, either from the action of the current or from other causes, as far as the writer knows. Had the Lake Champlain Bridge piers been built in tidal water, granite encasement between high and low water would have been used in accordance with the writer's usual practice.

As to the relative cost of piers constructed in open caissons as compared with those built by the pneumatic process, the writer can only judge by the actual bids received, which were mentioned in the paper. The specifications stipulated that the design of the caissons should be left to the contractor who might use timber, steel, or concrete, and presumably some of these contractors bid on caissons designed with either pre-cast concrete or steel shells, if they found such a construction economical as suggested by Mr. Feld.

In reply to Mr. Hudson's comment on the longitudinal gradient, it should be borne in mind that the bridge is in a country where steep hills are frequent and where the $5\frac{1}{2}\%$ gradient used in the bridge is moderate and substantially less than that on some of the main highways in the vicinity. No difficulties from the gradient have yet been reported to the engineers.

As stated in the paper, the bridge was designed for the *H*-15 loading given in the Standard Specifications for Steel Highway Bridges of the U. S. Department of Agriculture, with the exception that 20-ton trucks were used instead of the 15-ton trucks specified in the loading. The engineers believe that, in view of the probable infrequency and possible non-appearance of such trucks on the bridge, it was entirely logical to apply a higher unit loading for the floor system than had the 15-ton trucks conforming to the *H*-15 loading been used.

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CONSTRUCTION PLANT AND METHODS FOR ERECTING STEEL BRIDGES.[1]

BY A. F. REICHMANN,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. HOWARD P. TREADWAY, BENJAMIN WILDER GUPPY, R. A. VAN NESS, J. G. TRIPP, A. A. EREMIN, AND A. F. REICHMANN

SYNOPSIS

This paper is concerned with the erection, only, of steel bridges, and, in consequence, classification of the structures will be made more from a standpoint of the problems facing the erection engineer than from type of structure. The physical conditions at the site, time permitted for erection, particular season during which erection is to take place, quantity of river flow, and similar factors have a greater bearing on the way the structure is to be handled than the type of structure itself.

In general, the construction plant for erecting a steel bridge is determined by the type and size of the bridge and the method by which it is to be erected. In consequence, the problems involved by the type and size, which determine the method of erection, will be discussed before the equipment necessary to develop this method of erection, is taken up.

MAJOR CONDITIONS TO BE ANALYZED

The method of erection depends upon so many conditions that it is impossible to use any set plan of procedure for a given type of structure. Each project should be considered by itself, and the choice which is the simplest and which requires the least equipment is a problem for an experienced erection engineer. In attacking the problem of erection, it might, at first thought, be considered possible to divide the structures into two general classes; that is, railway and highway. However, there is such a wide divergence in both classes of structures, from light single-track railroad structures to the most up-to-date three and four-track railroad bridges, and from the small 20-ft. highway span to the tremendous structures built in the larger cities of the country, that the two classes are interlocking so far as erection problems are concerned. Both highway and railway bridges might be divided into groups as to type of structure. The more important of these types are I-beam spans, girder spans and viaducts, simple truss spans, arches, cantilever spans, continuous spans, suspension spans, and movable bridges. In determining and developing problems of erection and in attempting to arrive at the proper method of procedure, it will be found that the following major conditions will require consideration and analysis:

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[1] Presented at the meeting of the Construction Division, New York, N. Y., January 16, 1930.

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1.—Type of Structure.

2.—Location of Structure:
  (*a*) The character of the terrain to be crossed.
  (*b*) The distance from the center of supply for labor and materials.
  (*c*) Available space for storage yards and necessary shops.
  (*d*) Connection with railroad facilities.
  (*e*) Power and fuel supply.
  (*f*) Height above the water or ground.

3.—Maintenance of Traffic:
  (*a*) River.
  (*b*) Railroad or highway beneath the structure.
  (*c*) Railroad or highway on the structure proper.

4.—Effects of the Elements:
  (*a*) Floods and high water.
  (*b*) High winds.
  (*c*) Extremely hot or cold weather.

5.—Time Allowed for Construction:
  (*a*) As provided in the contract.
  (*b*) As governed by the seasonal effect on the work.
  (*c*) As governed by schedule of foundation construction.

TYPE OF STRUCTURE

In attempting to discuss the various problems listed in the foregoing outline, it will be found that in a great many cases the items are "interlocking" and cannot be treated independently. If the bridge consists entirely of I-beam spans, girder spans, or viaducts, no great consideration need be given to many of the subdivisions following. However, in cases of most other types of structures—especially truss spans that will require a temporary supporting structure—consideration of conditions beneath the latter must be given.

Practically all types will require consideration of the traffic over the bridge, accessibility for delivery of materials, etc. For that reason, no further discussion of the type of structure under that particular heading will be considered, but the various types as affected by conditions under the other following classifications will be mentioned.

LOCATION OF STRUCTURE

The first consideration under this subject will be the character of the terrain to be crossed. The amount of attention to be given to this problem depends on the type of structure. If the type of bridge is such that no temporary supporting structure is required, little consideration will be necessary. For truss spans of every description, this particular condition will be of major importance. If the structure is over dry land or shallow water, it will probably be found simplest to use timber falsework as a support; if it is over a deep stream, subject to wide variations in flow, where there is danger of driftwood, scour, and ice, or if there is river traffic to be considered, it may be found impracticable to use much, if any, temporary supporting falsework, and the bridge can best be erected by cantilevering, regardless of whether or not it is the intended permanent type. The erection of light girder spans and viaducts and of suspension bridges does not require major consideration under this heading as, in most cases, the supporting structure is eliminated.

When giving consideration to the problems connected with the distance from center of supply for labor and materials, all types of structures will be involved. If the structure is a railroad bridge, the problems in general entering into the delivery of materials will be simple because only in rare instances are attempts made to erect railroad bridges before the roadbed and track are completed up to the site. In highway bridges, however, the reverse is the case; and the question then develops as to whether it is most economical to build a temporary track to the site, or to arrange for transportation of equipment and materials by the use of trucks or teams. Under this consideration comes also the question of labor supply. If housing conditions are available only at a considerable distance away, it must be determined whether to supply this housing close to the job by the construction of labor camps or to supply transportation for labor to and from the available housing facilities. The latter is of considerable importance in the erection of steel bridges for the reason that the major part of the organization is made up of semi-skilled workers, and to be able to maintain an efficient organization, these workers must be kept satisfied as to living conditions.

Available space for storage yards, office, and shops will require consideration. In general, it is found that the bridges will be approached by earth fills through low lands, or in the case of bridges in cities, by streets in congested districts. Consequently, it is frequently necessary to arrange for yards, shops, etc., along the fills or streets, and often very little space is available, so that careful planning is necessary to take care of supplies and equipment. Otherwise, it may require going a considerable distance away from the structure for this space, with consequent increased cost of handling and transportation.

The subject of available railroad facilities is to a certain extent connected with the preceding discussion regarding distance from center of supply and conditions at the site and affects this consideration very materially. There may be good railroad facilities at one end of the bridge, but due to the nature of the stream, the method and order of placing the foundations, traffic under the structure, etc., it may be necessary to start the construction of the bridge from the opposite end, in which case it may be a saving to transport the material from the point of delivery on the railroad by hauling, by ferrying in barges, or by transfer on cableways. On some of the larger streams, such as the Ohio River and the Mississippi River, recent developments in transportation have sometimes made it possible to eliminate the railroad problem entirely by delivering the material from the fabricating plants directly to the site by water. Where this can be readily arranged, it is of considerable advantage in many cases because it puts the material directly under the point where it is to be erected and saves one or more handling charges.

As a rule, the question of available power and fuel supply does not have any great bearing on the method of erection. Its greatest effect will be on the equipment used. If there is electrical power connection, electrically driven machinery is usually found to be most economical. This condition, however, is rather rare, and usually it is found that power must be supplied by the contractor, in which case, steam-driven or gasoline-driven units can be provided to best advantage. The question of whether steam or gasoline will be selected depends on the available water supply, the facilities for obtaining fuel, and sometimes the type of structure, as, in general, gasoline equipment is somewhat lighter in weight and more easily transported than steam-driven units.

The height of the structure above the water or ground is of major importance and will not only affect the method of erection; it goes even further back than the erection itself, and often affects the design of the structure. When it comes to consideration of the method of erection after the design is definitely decided upon, then this question of height determines whether falsework will be used or whether necessary alterations in the design can be economically made to permit of cantilever erection. As the height of the structure increases, the cost of temporary falsework increases very rapidly, due to extreme wind loads which require additional main supports and much extra bracing. Often, if the structure is low, it is possible to erect truss spans at remote locations more advantageously and then float them on barges to their final location. This will especially merit consideration if the structure is close to the surface of the water, and the water is excessively deep at that point.

MAINTENANCE OF TRAFFIC

Under this heading, the maintenance of river traffic is of first importance for the reason that traffic on navigable streams in the United States is controlled by the Federal Government, and in case of interference thereto, permission from the United States War Department is necessary. In many cases there is only one available channel for river traffic, and, therefore, no supporting structure can be placed in the river at that point. As a consequence, it may be necessary to arrange for the erection of the span by cantilevering or by floating the structure in place, or by some such means, although other considerations would indicate that falsework would be most economical. In the northern part of this country it is sometimes possible to arrange the time of erection, so that the setting of steel over channels can be prosecuted when navigation is closed by ice. In any cases where it becomes absolutely necessary to do any construction that in any way affects river traffic, arrangements must be started for this well in advance of the time it is needed because the Federal Government, in granting permission to do this, will require various transporting companies to be notified and warned of any obstruction to navigation that may be contemplated, and, in addition, notices must be published in local newspapers.

In the case of railroad or highway traffic beneath the structure, it is usually possible to restrict this traffic partly, by narrowing the lanes of traffic, so that they can be crossed without great difficulty. If this cannot be done, it may be possible to detour the traffic temporarily, so that this problem is not of as great moment as the maintenance of river traffic.

The maintenance of traffic over the structure may be very important, as in the case of replacing an old span. In a problem of this kind, the maintenance of traffic will have a great bearing on the method of erection. In some cases it will be found possible to carry the traffic on the old structure while building the new one around it, or it may be possible to place temporary supporting falsework and carry traffic on this falsework at the same time a new structure is carried on it. It does not have as great effect on the method of erection as it does on the cost. Where there is any traffic to be maintained, it will have considerable effect on the cost of the work for the reason that it cannot fail to cause many interruptions, and consideration should be given to a method which will involve the least interruption to the progress of the work by traffic. The maintenance of traffic over the structure is usually more easily arranged than that under the structure because, in arranging this traffic, the erector is dealing with parties vitally concerned with the completion of the structure, and, in consequence, can more easily arrange for the necessary interruptions in traffic.

EFFECTS OF THE ELEMENTS

The most important considerations under this heading are floods and high water. In erecting bridges over most of the major streams of the United States, it will be found that it is of most importance to place temporary supporting structure during the periods when low water may be expected. Therefore, from necessity, the entire program will be predicated on the time when the water will be low. Even if the fluctuation between high and low water is small, the periods of high water will bring down driftwood or floating ice, or it will cause scour in the river bottom that may prove disastrous.

A high wind must be anticipated at any time and provision must be made for it. This means consideration of extra bracing and guys to the structure. There are localities in the United States where, at certain times of the year, high winds are especially prevalent, which may affect the time when the work is to be prosecuted. This requires consideration not only from a standpoint of safety, but also from a standpoint of economy. However, there may be long periods when it is impossible for men to work on a high structure due to the wind.

There are few localities in the United States where extremes of hot or cold weather will be a great problem, except as it affects the cost of the structure. It will not be necessary to suspend work entirely on account of hot or cold weather, but progress may be seriously impaired, with consequent increase in cost.

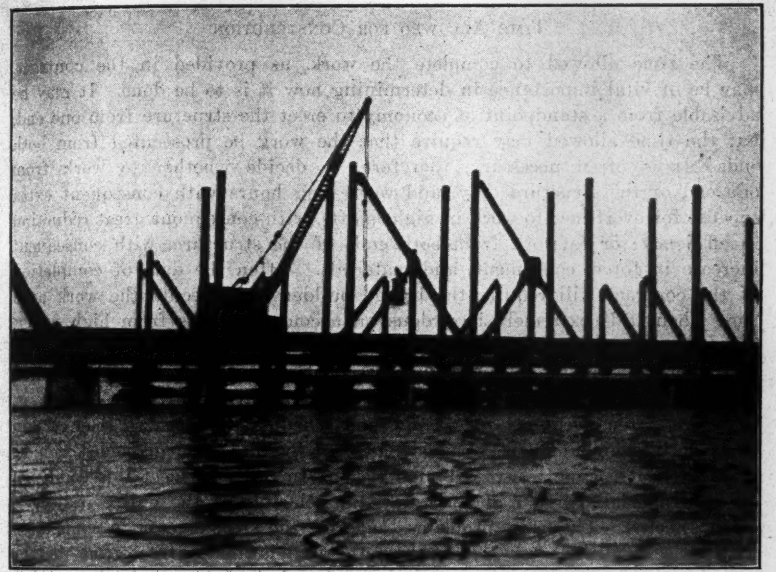

Fig. 1.—Locomotive Crane Erecting Bottom Chord and Web System on Falsework.

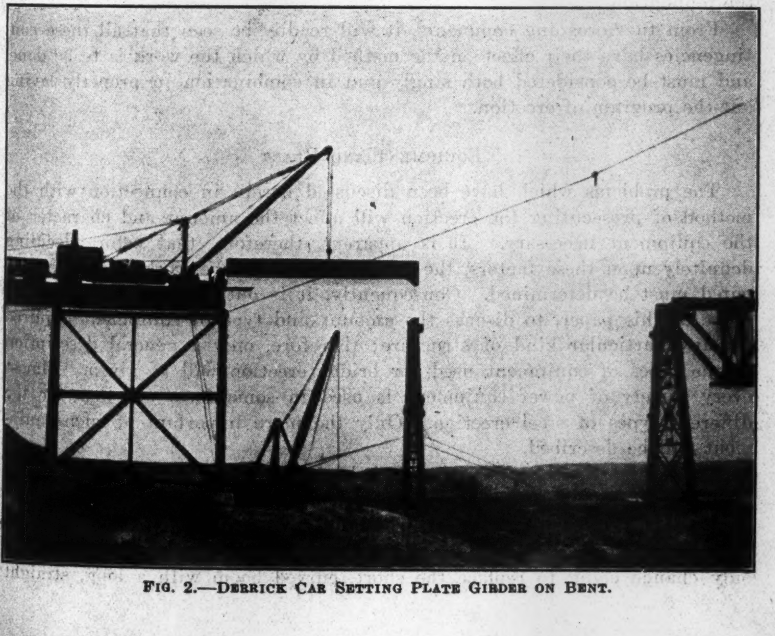

Fig. 2.—Derrick Car Setting Plate Girder on Bent.

TIME ALLOWED FOR CONSTRUCTION

The time allowed to complete the work, as provided in the contract, may be of vital importance in determining how it is to be done. It may be advisable from a standpoint of economy to erect the structure from one end, but the time allowed may require that the work be prosecuted from both ends It is often necessary, therefore, to decide whether to work from one end of the structure only, and work long hours with consequent extra expense for overtime; to work in night shifts, with consequent great reduction in efficiency; or to work from both ends of the structure, with consequent increase in force, equipment, and materials. Often the date of completion of the contract will require the bridge builder to prosecute the work at a time when it is extremely hazardous on account of risks from high water, ice, etc.

There are certain seasons of the year when one can almost be assured that he must contend with ice, driftwood, current, scour, etc., and it is, therefore, often necessary to arrange and schedule the work so that certain parts of it, which may be seriously affected by these contingencies, can be prosecuted at a time to avoid undue risks. In order to meet this condition on even a small portion of the work, the schedule of the entire project is often affected. The same problems that face the steel erector in connection with river conditions very often affect the foundation contractor, so that he may be unable to alter his foundation schedule and it will be imperative, therefore, for the steel erector to adapt his schedule of erection to fit that of the foundation.

From the foregoing comments, it will readily be seen that all these contingencies have their effect on the method by which the work is to be done, and must be considered both singly and in combination in properly laying out the program of erection.

EQUIPMENT AND PLANT

The problems which have been discussed herein in connection with the method of prosecuting the erection will affect the amount and character of the equipment necessary. It is apparent, therefore, that before deciding definitely upon these factors, the method by which the work is to be prosecuted must be determined. Consequently, it is out of the question in the scope of this paper, to discuss the amount and type of equipment required for any particular kind of structure; therefore, only a general description of the types of equipment used for bridge erection will be given. Almost every variety of power equipment is used in some form or other for the different types of steel erection. Only the more important of such equipment will be described.

Locomotive Erecting Cranes.—Probably the most universal type of equipment used for erecting bridges is the locomotive crane (see Fig. 1). This is an adaptation of the machines originally designed for wrecking cranes for use by the railroads. Originally, the railroad wrecking cranes were used, the only change being to replace the short, curved boom with a long, straight boom. Later, these cranes were developed more specifically for erection work by the addition of special auxiliary hoists, increased counterweights, etc. These locomotive cranes consist of a short, heavy railroad car carrying a heavy steel body mounted on a turn-table so that it will rotate 360° in either direction. This heavy cab contains the power plant and all the hoisting machinery. In addition, the hoisting engine is connected through proper gear trains to the trucks of the carrying car so that the car is self-propelling. It is from this feature that it gets its name, "locomotive crane". In order to increase the radius at which heavy loads can be handled by these machines, they are equipped with jack-beams at each end of the car body. These jack-beams are heavy I-beams which can be pulled out of the frame of the car (usually a distance of about 8 ft. from the center line of the track) and when blocked up, give the crane a base of about 18 by 24 ft. These cranes are usually also provided with booms so made that when handling excessively heavy lifts a short boom is used, while for setting lighter loads at a greater height above the track, additional sections can be added.

Due to the fact that these locomotive cranes—in order to have stability for handling loads at any advantageous distance from their center of rotation—must be excessively heavy, their use on bridge erection is confined to those structures which, by their design and method of construction, will support heavy loads. In the case of I-beam spans, girder bridges, and viaducts within their capacity, and truss spans the dead weight of which requires heavy falsework, these machines make ideal rigs for erecting. Due to their rotating features, they can be used with equal readiness at all points of the compass from where they are located. In most cases of truss spans erected by cantilevering and of suspension bridges and for all highway bridges, except the unusually heavy ones, these cranes are not economical because their excess weight would require too much additional material for the structure to make it capable of carrying it.

Derrick Cars.—The derrick car (Fig. 2) consists of a rather long heavy frame car on the front of which is mounted an A-frame for providing the mast of the derrick rig, and on the back end the hoisting equipment and sufficient additional counterweight to take care of the uplift. In addition, it is often augmented by a "counterweight car" behind it, to provide stability against extraordinary uplift. It has the advantage, therefore, over the locomotive crane in that it will pick heavy loads at a greater distance in front of the car; but it has the decided disadvantage in that the swing of the boom is limited to slightly less than 90° each side of the front.

The derrick car, like the locomotive crane, is usually provided with proper driving mechanism so as to make it self-propelling. Its principal use is for the erection of I-beam and girder spans, viaducts, etc., the character and size of which make the loads to be handled and the distance to be reached beyond the capacity of the locomotive crane. It is particularly advantageous for setting heavy girder spans 80 ft. long, or more, which, under present-day railroad traffic, often run from 75 to 100 tons in weight.

Derricks.—These are stiff-legs or guy derricks made in various designs and capacities. The difference consists in the bracing of the mast. If there are two stiff legs, the derrick is a "stiff-leg derrick"; if braced with guy ropes, it is a "guy derrick". A combination derrick has one stiff leg braced with guy ropes at right angles.

Derricks can be used for practically every type of structure, but as they are usually difficult to move from place to place, they are not often used except for the erection of small isolated structures, or for handling the steel from cars to storage at the storage yard. They are, however, often of distinct advantage for the erection of bascules which are to be built in open position and for the towers of vertical lift bridges. In both cases, this amounts to erecting a truss span standing on end, and, consequently, the structure is not spread over a great deal of ground and is readily reached without moving the equipment. Derricks are often mounted on barges forming derrick boats which are particularly serviceable for the placing of temporary supporting structures for bridges, such as driving piles, setting falsework, etc., and if the permanent structure itself is not too high above the water, they can be used to advantage in erecting the steel.

Travelers.—Briefly, travelers may be described as derricks with their hoisting equipment mounted on a movable platform. They are of almost every conceivable type and are usually designed and built to fit the requirements of each individual structure. Depending on the particular type of structure for which they are designed, they may be operated on the floor of the bridge or span from one truss to the other on the top chords. If they are operated on the floor of the bridge, some special provision usually has to be made to move the material either through them or around them in order to get it within reach of the booms. They are of particular advantage when the material is to be delivered to the site by water; therefore, the question of getting the material through or around them does not enter into the problem. They have a further advantage in that as they operate over the floor of the bridge, there is no special provision necessary to get them from one span to the other as the erection progresses over a series of spans. This is offset by the disadvantage of having them sufficiently strong and heavy to carry the long boom necessary to reach to the upper parts of the structure.

If the travelers are operated on the top chord of the structure (Fig. 3), they must be constructed to span from one chord to the other and require the necessary special construction on the trusses to permit moving them from one span to the next. Offsetting this they have the advantage, since they are entirely above the steel to be erected, that the booms may be much shorter, permitting lighter construction. Furthermore, placing them on the top chords leaves the floor system entirely unobstructed for bringing forward the materials required.

Travelers are used for the erection of almost every type of bridge (see Fig. 4). In cases where truss bridges—either due to their type or to some of the conditions heretofore discussed—are to be erected by cantilevering, some type of traveler is ordinarily used because less weight is involved for their capacity than with cranes or derrick cars. They can be modified in innumerable ways to adapt them to special needs. In the case of towers for suspension spans and extra long bascule spans to be erected in the open

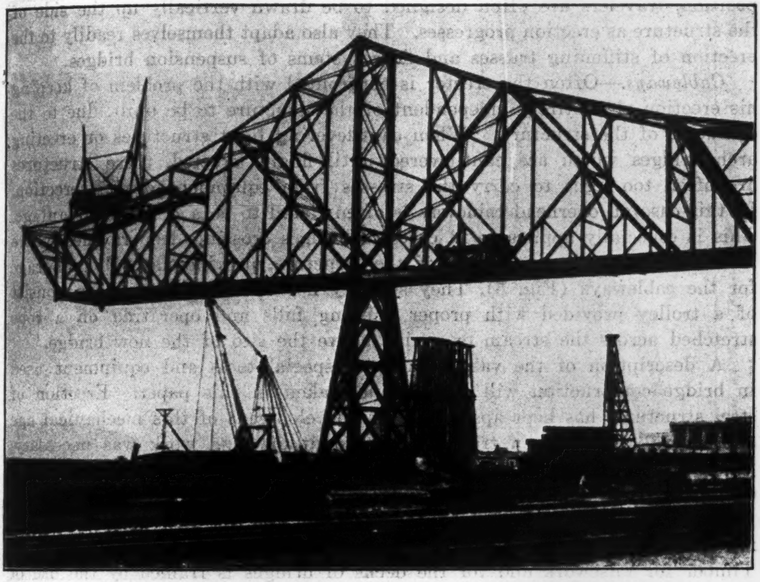

Fig. 3.—Top Chord Two-Boom Traveler, Cantilever Erection.

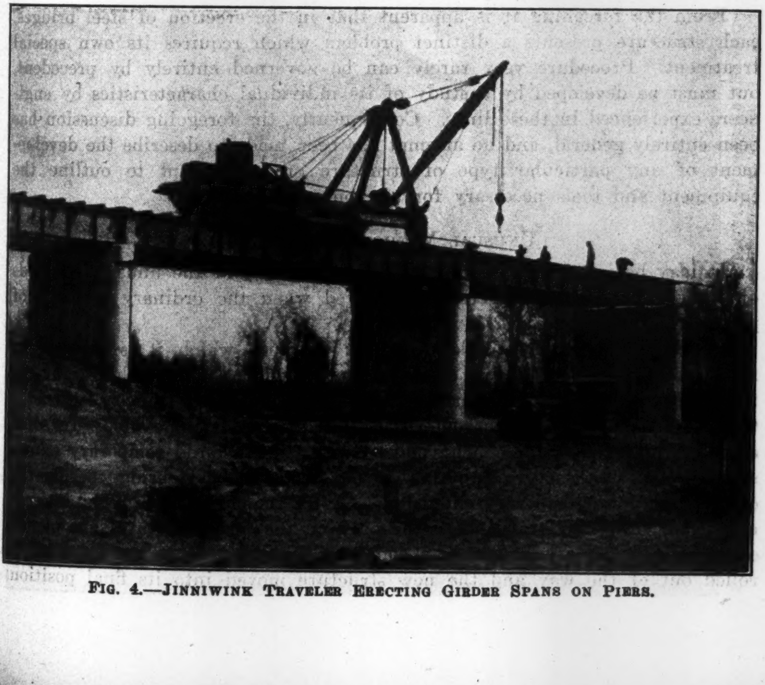

Fig. 4.—Jinniwink Traveler Erecting Girder Spans on Piers.

position, travelers are often designed to be drawn vertically up the side of the structure as erection progresses. They also adapt themselves readily to the erection of stiffening trusses and floor systems of suspension bridges.

Cableways.—Often the erector is confronted with the problem of keeping his erection rig entirely independent of the structure to be built, due to the character of the structure. When cantilevering light structures or erecting arch bridges which are cantilevered until finally erected, these structures are often too light to carry the stresses from equipment during erection. In this case an overhead cableway is often found to be a distinct advantage. This is especially applicable in cases of bridges crossing deep ravines where hillsides, adjacent to each end of the structure, provide natural anchorage for the cableways (Fig. 5). They scarcely need description as they consist of a trolley provided with proper hoisting falls and operating on a rope stretched across the stream or ravine above the site of the new bridge.

A description of the vast number of special tools and equipment used in bridge construction will not be undertaken in this paper. Erection of steel structures has kept apace with the development of this mechanical age and where, years ago, a tremendous amount of hand work was necessary, practically all operations are now done by mechanical appliances. For example, rivets are now almost universally driven by air or electricity; and the oxy-acetylene cutting and welding flame and the electric arc are used for the removal of old structures and for alterations to new ones in the field. Timber for falsework and for the decks of bridges is framed by the use of electrically and air-driven portable tools.

From the foregoing it is apparent that in the erection of steel bridges, each structure presents a distinct problem which requires its own special treatment. Procedure very rarely can be governed entirely by precedent, but must be developed by a study of its individual characteristics by engineers experienced in these lines. Consequently, the foregoing discussion has been entirely general, and no attempt has been made to describe the development of any particular type of structure or to attempt to outline the equipment and tools necessary for any one type.

UNUSUAL METHODS OF ERECTION

It may be interesting to touch briefly on a few of the unusual methods of erection which are sometimes employed when the ordinary methods of procedure will not apply.

Generally, when the traffic over the structure which it is planned to replace is exceedingly heavy, every effort is made to change the alignment for the new structure so that it may be built without interference with traffic. Often, however, physical characteristics will make this impossible. In some cases of this kind, arrangements have been made to build temporary extensions to the substructure and then to construct the new bridge beside the old one on these temporary extensions. After the new structure is entirely completed ready for traffic, both the old and the new structures are placed on a rolling system and, within a comparatively short time, the old span is rolled out of the way and the new structure moved into its final position

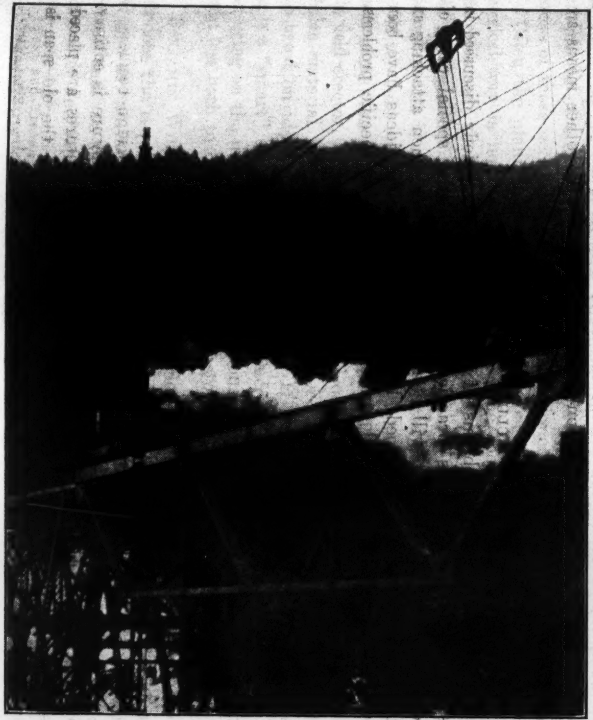

FIG. 5.—ERECTING TRUSS WITH CABLEWAYS; LOCOMOTIVE CRANE IN BACKGROUND SUPPORTING ONE END OF TRUSS.

FIG. 6.—HYDRAULIC JACK RAISING SPAN TO RELEASE FALSEWORK.

ready for service. To handle a structure in this way requires—in addition to the rolling system—some means of raising the structure to release the rollers. For this purpose, hydraulic jacks (Fig. 6) have been developed with capacities as high as 2 500 tons per jack. Water is generally used to operate them except during periods of low temperature when mixtures of alcohol in water or even pure alcohol are used.

In the erection of permanent cantilever spans, it is usually possible to erect the suspended span by temporarily cantilevering one-half of it from each permanent cantilever arm. Occasionally, however, the stresses developed by this method make it impractical to extend the cantilever beyond the permanent cantilever arm. In a few cases, therefore, the suspended span has been erected at a remote location, transported on barges to the site, and raised into final position. In some cases the span has been raised by the use of hydraulic jacks; in others, where the height was considerable, the suspended span has been raised by the use of cables running over sheaves mounted on the ends of the cantilever arms. To reduce the power required to lift the span, counterweights are used to balance the lifted span.

The erector will sometimes be confronted with the problem of constructing a single truss span, where, because of exceedingly deep water, swift current, or because of navigation, it is an impossibility to place any supporting structure under the span. In such cases it has been found necessary to erect temporary anchor arms at each end of the structure in order to make a temporary cantilever of this simple truss span. Such a method requiring, as it does, the erection and later removal of a large quantity of material, is, of course, expensive and is only resorted to where no other means are available.

CONCLUDING REMARKS

The more important problems of erection are only briefly discussed in this paper, because there is practically no limit to the special cases of erection and equipment that will confront the engineer when attacking an individual problem. It is hoped, however, that sufficient ideas have been advanced to serve as a general guide in the solution of specific problems.

DISCUSSION

Howard P. Treadway,[3] M. Am. Soc. C. E. (by letter).—This paper is a distinct addition to the literature on the subject of construction plant and methods for erecting steel bridges. It is a comprehensive general statement of this interesting subject, and, as such, little can be added by way of discussion. It naturally leads, however, to a discussion of details entering into the various problems at hand and could very well be used as the opening statement of an extensive treatise on definite problems that have been solved.

It is quite probable that almost any steel bridge can be erected by several different methods of procedure and by using different types of erection equipment. Ordinarily, the designing engineer submits, to erecting contractors, plans showing a finished structure, and it is then the problem of the erecting engineer to determine the most economical plan for the field work. Usually, it is only the cheapest one that ever has any publicity, because under ordinary circumstances it is the cheapest procedure that warrants the awarding of the contract. Occasionally, there are conditions in which special service or responsibility of the contractor is considered, and a more elaborate scheme submitted by such a contractor is given the preference at a higher price.

The erecting engineer makes a careful study of the plans, the location, and local conditions, as set forth by Mr. Reichmann. It frequently happens that he has to re-design certain members of a proposed structure in order to erect it safely, because the designing engineer has overlooked the fact that during the process of erection his structure will have stresses in these members in excess of those occurring in the finished balanced structure.

Often the plant necessary to do the work in the easiest and most economical way is not available, or it may be too expensive to acquire, and the equipment that the erector has on hand has to be adapted to the problem. All these conditions having been considered a scheme is worked out, and an estimated cost is made, which bid is submitted to the customer.

If the field engineer develops a method that is safe under all conditions he is more than likely to be a high bidder. Therefore, it often happens that the field erector, in considering the flood conditions of a stream or the foundation material for the falsework, takes a long chance and offers to do the job based on ideal weather conditions. If these weather conditions prevail he is a "smart" man, and his performance is applauded. If, however, he is bordering on the unsafe side and conditions go against him, he possibly incurs great loss both to himself and to the customer.

Benjamin Wilder Guppy,[4] M. Am. Soc. C. E. (by letter).—The author mentions the requirement for the maintenance of traffic when replacing an old span as one factor that determines the method to be adopted for erecting the new one. In such a case it is necessary to remove the old span and sometimes alter and repair the existing substructure.

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[3] Pres. and Treas., Kansas City Bridge Co., Kansas City, Mo.

[4] Engr. of Structures, B. & M. R. R., Boston, Mass.

The type of superstructure to be removed and its condition and the repairs and alterations required to adapt the substructure to carry the new span and the heavier loads are factors that must also be considered in determining the erection method to be used. The writer recalls one case—the renewal of the superstructure of an important double-track railroad bridge over a wide river—where these were found to be the governing factors for determining the method to be used for the erection of the new superstructure and also for developing its design.

R. A. VAN NESS,[5] M. AM. SOC. C. E. (by letter).—The author has compiled a paper which condenses the subject in such form as to make it of real value as permanent information for the files of interested engineers, as well as educational matter for structural student engineers. To those who are closely in touch with bridge erection in one field or another, the paper revives a general appreciation of the entire subject.

The Company with which the writer is connected has maintained a steel bridge erection plant for many years. At present (1932), it comprises rolling equipment to house a normal force of 30 persons; one derrick car capable of setting a 106-ft. Cooper E-65 girder ahead; one locomotive crane with a capacity of 75 tons; two gasoline-operated air compressors, capacity 310 cu. ft.; one two-unit, electric arc (300 amperes) welding machine; an oxy-acetylene torch; and miscellaneous small equipment.

Most of the Company's bridge steel is erected with this plant, and no work is contracted except that which must be erected before Company forces would be available and other work of such magnitude that it requires a large outside plant for the most economical erection.

J. G. TRIPP,[6] M. AM. SOC. C. E. (by letter).—Only one addition is suggested for Mr. Reichmann's paper. The author does not stress sufficiently the fact that the solution of construction and erection problems should precede the design of a bridge. Mr. Reichmann follows this practice in his organization, but too often such considerations are passed by and left for the erector to make the best of poor "construction symmetry"—that ogre of the construction man. For example, it is not an uncommon occurrence in inaccessible territory to be faced with the fitting of the design to such a simple thing as the available plant of the hauling contractor in order to determine the sizes that can be handled into the location of the structure. Natural obstructions, road grades, etc., often cause complications difficult to overcome. Such structures as the Marble Canyon Bridge, in Arizona, is a case in point. The designers well knew that their problem was not so much stress, strain, and connections, but such simple things as maximum sizes of pieces to be transported from the railroad to the job, a matter of 120 miles over rough terrain. Design must accommodate erection methods and available plant to obtain economical results.

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[5] Bridge Engr., A. T. & S. F. Ry. System, Chicago, Ill.

[6] Constr. Mgr., Merritt-Chapman & Scott Corporation, New York, N. Y.

Mr. Reichmann's tabulation of erection conditions is particularly well stated. In the writer's opinion, he might have added:

"6.—Specification Limitations:
(*a*) Eight-hour laws.
(*b*) Inspection routine."

If the specifications are inelastic and their effect unweighted in the estimate to erect, costly results and even defeat of the plans based entirely on the other five major conditions might follow.

As the author states, equipment and plant types are legion. The writer has found that little refinements on standard equipment pay big dividends. Simple devices enabling men to reach into difficult places; tool kits in basket slings; visual and manual signal assistance; proper power cord attachments, etc., are examples. Special refinements in cableway control apparatus and power take-ups on stay-lines pay handsomely.

In the design of structural steel falsework—another bridge problem—an important point is to build something that is easily dismantled without injuring the value of the dismantled parts. In this case, the study of erection methods and plant is paramount, and the design of the members is hedged in with limitations rarely found in permanent structures. In such cases all Mr. Reichmann's conditions must be evaluated for choice of method and plant, but final selections are tempered by considerations for easy and quick dismantling. For example, it might prove economical to erect a set of concrete arch-bridge centering from a cableway, but it might also be very expensive to dismantle such centering with the same tool. The net result would be to develop some plan, such as trestle and locomotive cranes, or floating equipment and jacks, to displace the cableway. If the only consideration was to erect for permanence, the cableway might be chosen without further thought as being the most economical for the case in hand.

Mr. Reichmann's conclusion that there are no limits to the special cases of methods and equipment used in bridge erection, is fully seconded by the writer. If the designer, as well as the erector, will follow Mr. Reichmann's condition chart, few, if any, mistakes in judgment will result.

A. A. EREMIN,[7] ASSOC. M. AM. SOC. C. E. (by letter).—With the advancement in modern bridge design considerable attention is generally given to the problem of erection. Often the equipment available and the economic considerations of the erecting methods decide the selection of the type of bridge.

An interesting type of semi-permanent bridge with a half-through truss of uniform depth was developed by M. Pigeaud,[8] for the French Army, during the World War, which deserves careful consideration of military engineers.

Parallel cables are swung over some steel-framed square towers, which are composed of members that can easily be handled. Cross-beams are suspended from the cables, and tracks are placed on top of the beams. The trusses are made in sections about 15 ft. long which are rolled into position in the span by means of a car. After completing all connections the entire span is raised with jacks to release the cables and is lowered on the bridge seats. This method of erection is applicable to a half-through bridge with any number of spans. From November, 1914, to May, 1919, 152 bridges of this type were constructed in France.

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[7] Asst. Designing Engr., Bridge Dept., State Highway Com n., Sacramento, Calif.
[8] *Le Genie Civil,* June 14, 1919.

The Ludwigshafen-Mannheim Bridge over the Rhine River, in Germany, was designed to be erected in two steps:[9] (1) Erect the bridge as a three-span, through-type structure with the trusses of uniform depth and continuous over the intermediate piers; and (2) construct a steel arch over the three spans, and reinforce the trusses, and suspend them to the arch. The result, after removing the intermediate piers, is a steel arch bridge with stiffening trusses, which forms an exceedingly graceful structure.

Mr. Reichmann has given a valuable summary of the construction plans and methods used in erecting steel bridges.

A. F. REICHMANN,[10] M. AM. SOC. C. E. (by letter).—The discussion has added some interesting cases of specific erection problems to a paper which was limited to a generalization of major considerations in the determination of the most feasible and economical equipment and methods of erection.

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[9] *Die Bautechnik*, September 16, 1930, p. 624.

[10] Asst. Chf. Engr., Am. Bridge Co., Chicago, Ill.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1841

FULTON STREET, EAST RIVER TUNNELS, NEW YORK, N. Y.[1]

By Miles I. Killmer,[2] M. Am. Soc. C. E.

With Discussion by Messrs. H. J. King, S. M. Swaab, Jacob Feld, Ole Singstad, John F. O'Rourke, H. H. Hatch, and Miles I. Killmer

Synopsis

This paper treats the specific problems encountered by the contractors in constructing the two tubes of the Fulton Street, East River Tunnels, in New York, N. Y. Useful information is given concerning the size and general types of equipment necessary and the manner of using it to the best advantage. The paper supplies a general picture of the contractor's power house, required on a job of this scope, and the assembly and erection methods of various phases of the work, especially the shields, are described in detail. The paper closes with an interesting account of construction problems met during the building of the High Street Station.

Introduction

A description of the plant used on the Fulton Street Tunnel, which is situated on what is known as Route 101, Section 1, of the new subway of the City of New York, is given in this paper. The contract began at Church Street about 100 ft. south of Park Place in Manhattan (see Fig. 1). Local and express tracks run south on Church Street to Fulton Street, where the local tracks terminate at a station next to the Hudson Terminal Building. The two express tracks turn east, pass under the southbound tube of the Brooklyn-Manhattan Transit (B. M. T.) Company in a short section of concrete tunnel, and continue east in separate cast-iron tubes to the end of the contract at Jay and Nassau Streets, in Brooklyn, N. Y. Following the cast-iron tubes from west to east, one passes under the northbound "B. M. T." tube, St. Paul's Churchyard, Fulton Street, the East River, Cranberry Street, High Street, and under private property to the end of the job.

The tubes were constructed by the shield method under compressed air. The working shafts were at Fulton and Front Streets, Manhattan, and on

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[1] Presented at the meeting of the Construction Division, New York, N. Y., January 16, 1930.

[2] Mgr., Mason & Hanger Co., Inc., New York, N. Y.

Furman Street, Brooklyn. Shields started east and west from each of these shafts in each tunnel, making eight shields in all. From January until May, 1929, all eight of the shields were under way together. In Brooklyn, the ground pierced by the shields was mostly sand, gravel, and boulders. In Manhattan, it was a fine sand with clay admixture and was very favorable for tunneling. The Manhattan shields passed under the tracks of the Interborough Rapid Transit (I. R. T.) Company, in William Street and in Broadway, and the easterly "B. M. T." tube near Church Street; and the Brooklyn shields passed under the east approach to the Brooklyn Bridge. Along "the line of march" through Fulton Street were many tall buildings the footings of which were close over the shield, vertically, and only 2 or 3 ft. away from the outside of the tunnel, horizontally. These buildings were not underpinned. The air pressure used in driving the shields and in putting down the mid-river sumps ranged all the way from normal to 48 lb. per sq. in.

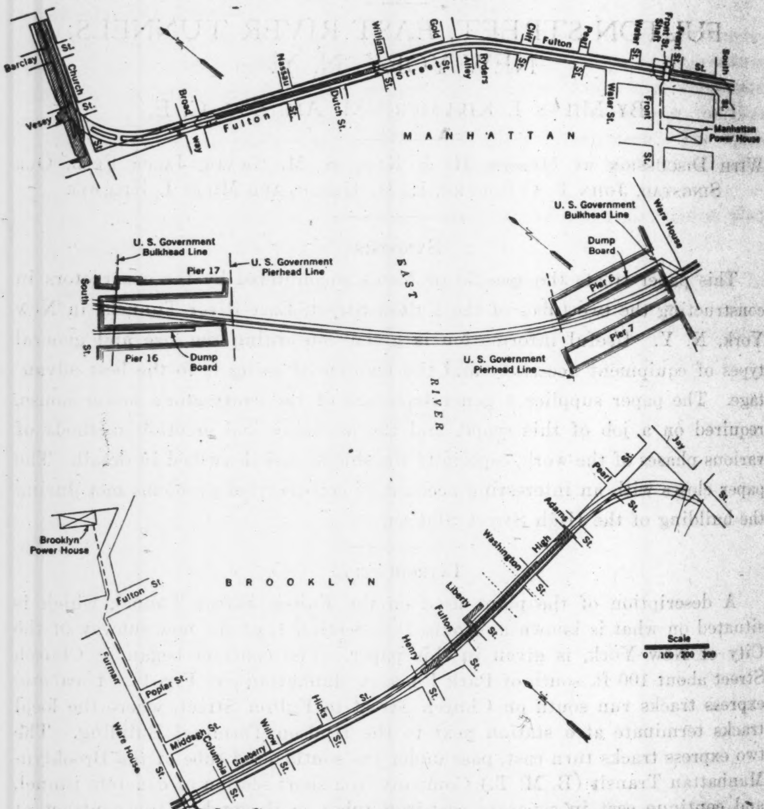
Barclay
St.
Church
St.
Vesey
St.
Broad
way
Fulton
Nassau
St.
Dutch St.
William
St.
Gold
St.
Street
Alley
Ryders
Cliff
St.
Fulton
Pearl
St.
Water St.
Water St.
Front St.
Front St. Shaft
St.
South
St.
MANHATTAN
Manhattan Power House
U. S. Government Bulkhead Line
U. S. Government Pierhead Line
Pier 17
South
St.
Pier 16
Dump Board
EAST
RIVER
U. S. Government Bulkhead Line
Ware House
Dump Board
Pier 6
Pier 7
U. S. Government Pierhead Line
N
Jay
St.
Pearl
St.
Adams
St.
High
Washington
St.
Liberty
Fulton
St.
Henry
St.
Scale
0 100 200 300
BROOKLYN
Brooklyn Power House
Fulton St.
Furman
Poplar St.
Ware House
St.
Middagh St.
Columbia
Willow
Hicks
St.
Cranberry
St.
St.
St.
Furman St. Shaft

FIG. 1.—GENERAL PLAN OF FULTON STREET, EAST RIVER TUNNELS.

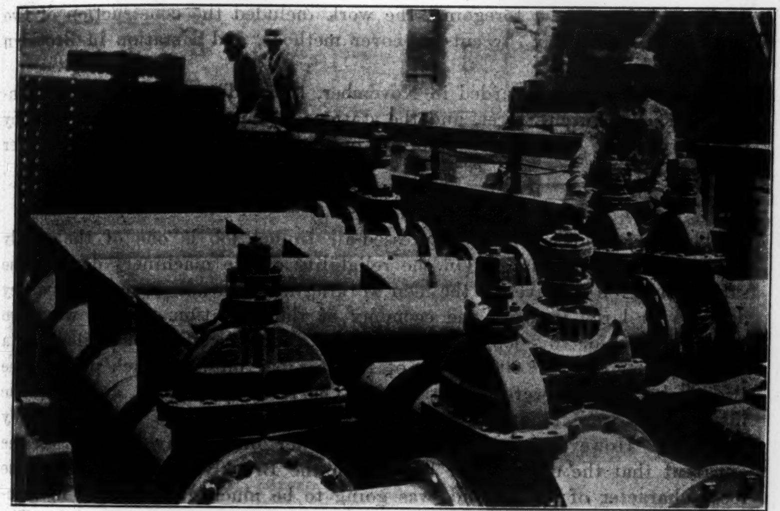

FIG. 2.—MANIFOLDS AND VALVES AT TOP SHAFT IN MANHATTAN.

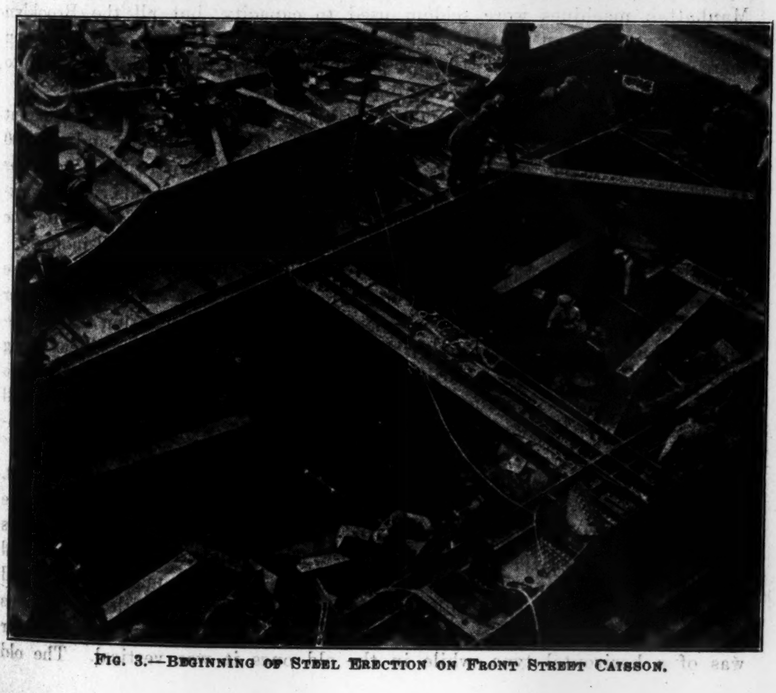

FIG. 3.—BEGINNING OF STEEL ERECTION ON FRONT STREET CAISSON.

In addition to the foregoing, the work included the construction of two stations in Manhattan, by cut-and-cover methods, and a station in Brooklyn by tunneling.

The contract was awarded in November, 1927. The two shafts, the open-cut work on Church Street, and the station on Fulton Street, were built by sub-contractors. This paper describes only the plant used on the remainder of the contract.

POWER HOUSES

The power house on a compressed-air tunnel job is one of the vitally important items of plant. On the reliability of the machinery depends the safety and even the lives of the men working in the heading; on its adequacy depends in large measure the economy of the operation. There were two power houses for this job, one at each end. The specifications called for a low-air capacity of 28 000 cu. ft. of free air per min. for each plant. The two plants as built were almost alike. The low-air, high-air, and hydraulic installations were identical, but the electrical installations were necessarily different. However, the work was no sooner under way than it became apparent that the demand for low air on the Brooklyn side, because of the loose character of the ground, was going to be much greater than in Manhattan. This condition persisted throughout the duration of the job. The Manhattan machines were seldom used to capacity, but all the Brooklyn machines were almost continuously being pushed to their limit, and even then there were times when it was not possible to keep up the air pressure to what was desired in the river headings.

All the machinery was operated by electricity. In Manhattan, the current was received at the power house at 13 200 volts from one feeder and at 7 500 volts from the other two feeders, and it was stepped down to 2 200 volts for the larger motors and to 220 volts for smaller power demands. In Brooklyn, the current was received at 4 300 volts and used at that voltage for the large machines; it was stepped down to lower voltage for the small motors.

Power was supplied by two commercial companies as sub-contractors. The electric current reached each power house by three entirely independent feeder lines, coming from three different sub-stations. This precaution was necessary, of course, to insure power in the event of a break-down or accident along one of the feeders. The power-house engineer and electrician always had to be on the alert. In a number of instances, break-downs did occur and all machines came to a stop. It usually took from 3 to 5 min. to start the compressors again.

In each power house there were five low-air machines of 5 600 cu. ft. capacity. Four in each plant were new, and one was a used machine. The new machines were 29 by 29 by 21 in., and were operated by synchronous motors of 625 h. p., for 2 200-volt, 3-phase, 60-cycle current with a full-load speed of 180 rev. per min. The motors on the Brooklyn machines operated at 4 300 volts. The used machines were 28 by 28 by 24 in., with synchronous motors of 600 h. p., at 164 rev. per min. In the new machines the after-cooler was of a horizontal type, while in the old ones it was vertical. The old

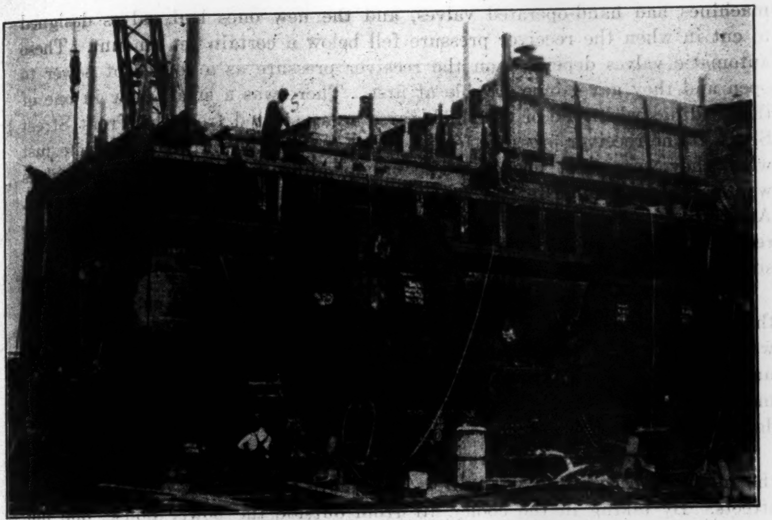

FIG. 4.—STEEL ERECTION AT LATER STAGE.

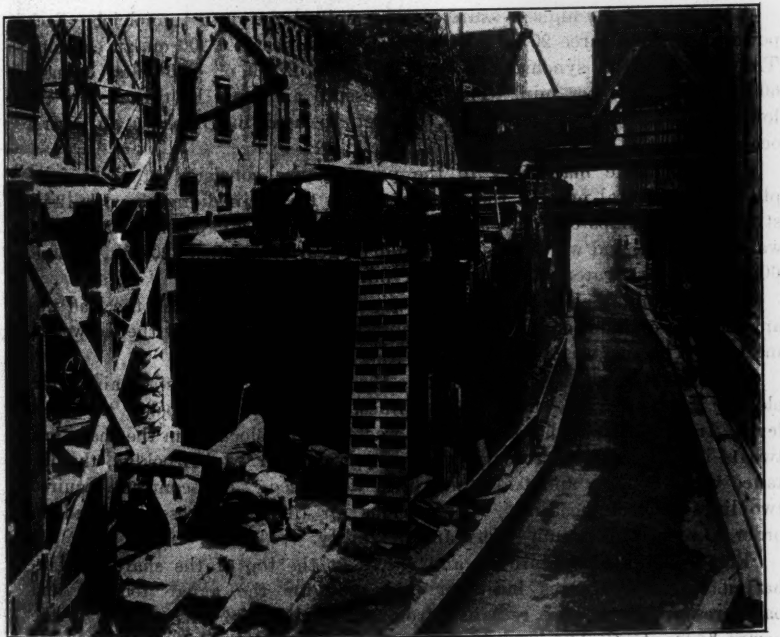

FIG. 5.—SINKING FURMAN STREET SHAFT.

machines had hand-operated valves, and the new ones had valves designed to cut in when the receiver pressure fell below a certain set amount. These automatic valves depended on the receiver pressure as a source of power to open and they gave some trouble at first. There was a small blow in one of the headings when one of the shields was being started from the Front Street Shaft. The receiver pressure fell so low that the valves failed to operate just when they were most needed. The old machine with hand-operated valves was started and sufficient air was obtained from it to meet the emergency. After this incident, all the automatic valves were connected to a separate receiver which was kept supplied with the necessary air at the desired pressure by one of the high-air machines.

The compressors gave admirable service; the operators, as well, proved their mettle; dramatically, on one occasion, when the Brooklyn power house was struck by lightning and less conspicuously through many months of unfailing service. In Brooklyn, there were weeks and weeks when all five machines were compressing air at maximum load, 24 hours per day and 7 days per week.

The air for the compressors was taken in above the roof. In Manhattan, the inlets were equipped with filters to remove some of the dust of the city streets. By taking in the cooler air from outside the power house, one not only secures better air, but better compression economy.

The demand for high-pressure air is usually large on a tunnel job. Each power house had three 20 by $12\frac{1}{2}$ by 14-in. machines of 1 300 cu. ft. capacity. The motors were synchronous and rated at 225-h. p., 220-volts, 3-phase, 60-cycle, 257 rev. per min. The high-pressure air could be turned into the low-air lines if need be and was, in fact, so used in Brooklyn on several occasions.

The hydraulic machinery in the power house consisted of three four-plunger horizontal pumps; the plungers were $1\frac{3}{4}$ in. in diameter with 12-in. stroke. The pumps operated at 50 rev. per min., had a capacity of 24 gal. of water per min., and were driven by 100-h. p., gear-connected, 220-volt, 3-phase, 60-cycle motors, rated at 700 rev. per min.

The accumulator was of the hydro-pneumatic type in which the load was applied by a cylinder charged with compressed air instead of by the old weight method. The size was 8 by 42 in.

The power houses were connected to the shafts by the necessary air, water, electric, back-pressure, and telephone lines, laid in trenches below the street level. In Manhattan, the power house was only one block from the shaft, and two 14-in. lines sufficed to carry the low-pressure air. In Brooklyn, the distance was 1 600 ft., and two 16-in. lines were used. The contract required two lines to each heading. The various air lines are cross-connected, in order to secure maximum safety and flexibility.

Fig. 2 shows the manifolds and valves at the top of the shaft in Manhattan. A shanty was built on this spot for the gauge tenders, and back-pressure lines were returned from all headings to this shanty, to the power house, to the engineers' office, and to the superintendents' office.

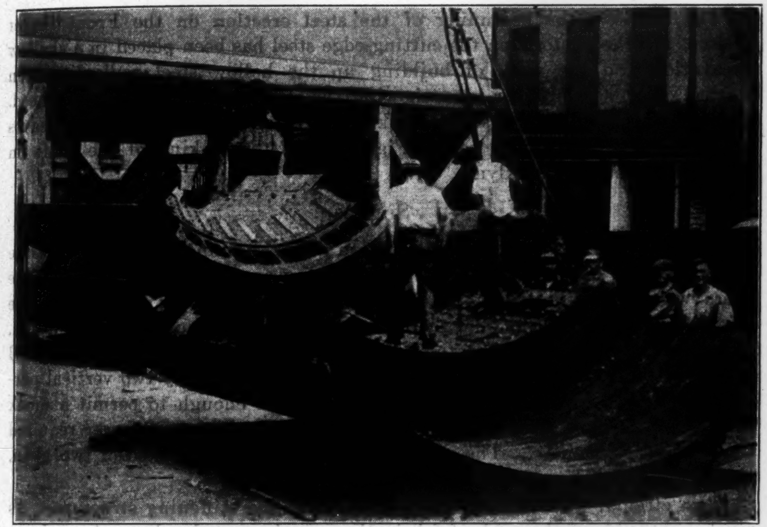

FIG. 6.—TWO SECTIONS OF SHIELD.

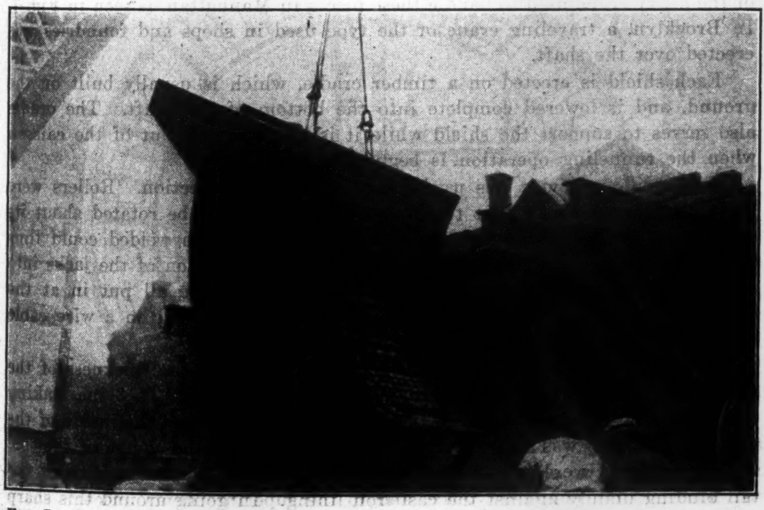

FIG. 7.—ONE OF LOWER-QUARTER SECTIONS OF A SHIELD IN THE SLINGS OF A HEAVY DERRICK, READY TO BE LOWERED.

CAISSONS

Fig. 3 shows the beginning of the steel erection on the Front Street caisson. In the photograph the cutting-edge steel has been placed in a shallow excavation and the work of building up the hollow steel walls has been started. Fig. 4 is another view showing the steel erection further advanced. The circular openings (closed by bulkheads) through which the shield leaves the caisson can be plainly seen. Fig. 5 shows the sinking of the shaft on Furman Street, in Brooklyn.

SHIELDS

The contract required the construction of eight tunnel shields. Since the material to be encountered in all the tunnels was sand or gravel and, since there was no possibility of "shoving blind" at any stage of the job, the shields were entirely open, and there were no unusual features in their design. One platform extended across the horizontal diameter. The spaces above and below the platforms were each divided into three pockets by two vertical partitions. The lower middle pocket was made large enough to permit a muck car to pass through to the face, but the conditions of the job never required the use of this facility. The skin-plates of the shield consisted of two 1¼-in. plates, riveted and welded together.

Two circular girders or diaphragms formed the stiffening structure. As much as possible of the work of fabrication was done at the shop. A quadrant of the skin and quadrants of the stiffener girders with cutting-edge or hood castings attached, were riveted together in the shop. Fig. 6 shows two of these sections. The largest of them weighed about 20 tons. Part of the boom of the heavy crane used to handle these pieces in Manhattan, is seen in Fig. 7. In Brooklyn, a traveling crane of the type used in shops and foundries was erected over the shaft.

Each shield is erected on a timber cradle, which is usually built on the ground, and is lowered complete into the bottom of the shaft. The cradle also serves to support the shield while it is being shoved out of the caisson when the tunneling operation is begun.

An ingenious device was used to speed up shield erection. Rollers were built into the cradle so that the assembled shield could be rotated about its axis. The seams along which the skin-plates were to be welded could thus be brought "into the clear" for easier access. The insertion of the jacks into their places was made much simpler, because they were all put in at the bottom. The shield was rolled by the derrick which pulled on a wire cable wrapped several times around the outside.

The outside diameter of the shield was 18 ft. 10 in., the thickness of the skin was 2½ in., and the outside diameter of the tunnel, 18 ft. 3 in., making the theoretical clearance between the outside of the iron and the inside of the tail 1 in., radially. This clearance served very well in the course of the job. In Manhattan, two shields went around curves of 335-ft. radius without the tail binding unduly against the cast-iron lining. In going around this sharp curve, special efforts had to be made to build the iron so as to overcome the

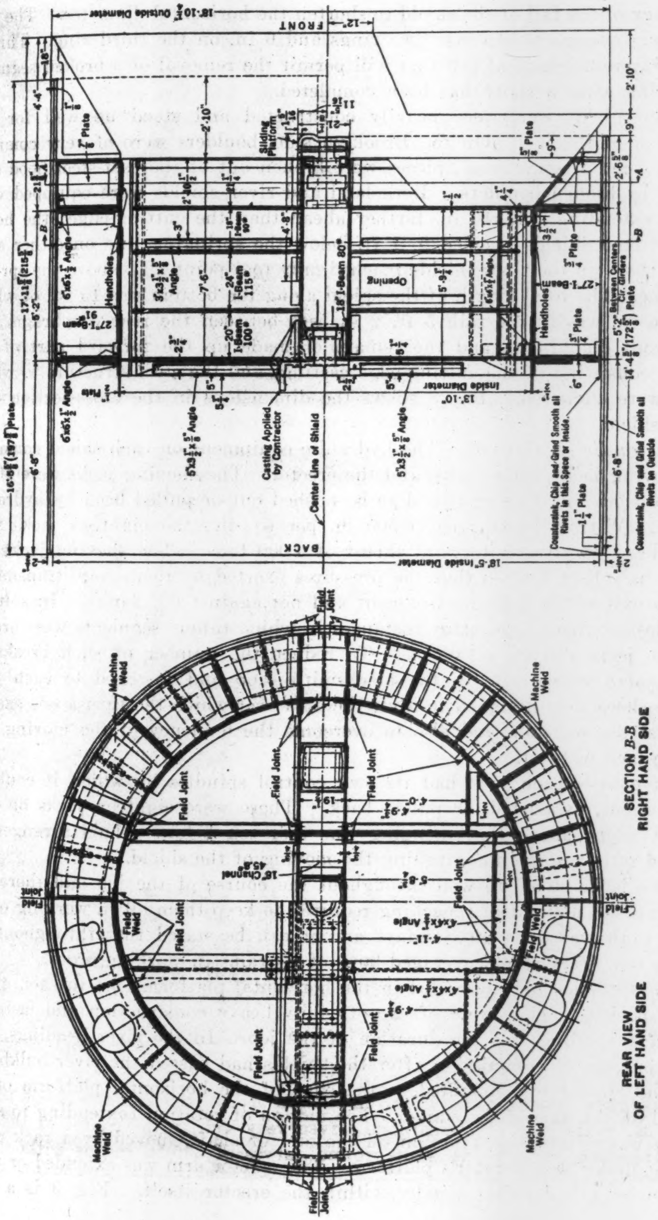
FRONT
18'-10 3/4" Outside Diameter
12"
18"
4'-4"
1/8" Plate
21 1/4"
6"x6"x1/2" Angle
Handholes
27" I-Beam 91#
17'-11 3/8" (215 3/8")
8'-2"
2'-10"
Sliding Platform
7'-6"
24" I-Beam 115#
20" I-Beam 100#
3"x3 1/2"x5/8" Angle
18" I-Beam 80#
Opening
5'-9 1/2"
27" I-Beam
13'-10" Inside Diameter
16'-5 1/2" (197 1/2") Plate
6'-6"
5"x3 1/2"x5/8" Angle
Casting Applied by Contractor
Center Line of Shield
BACK
18'-5" Inside Diameter
Countersink, Chip and Grind Smooth all Rivets in this Space or Inside.
Countersink, Chip and Grind Smooth all Rivets on Outside
1 1/4" Plate
3/4" Plate
5'-2" Between Centers Cir. Girders
14'-4 5/8" (172 5/8") Plate
2'-6 3/4"
9"
SECTION D-D
Field Joint
Machine Weld
Field Weld
18" Channel 45.8#
4'-0"
4'-9 1/2"
5'-8 1/2"
4'-11"
4"x4"x1/2" Angle
SECTION B-B RIGHT HAND SIDE
REAR VIEW OF LEFT HAND SIDE

FIG. 8.—TYPICAL CROSS-SECTION OF RIVER SHIELD.

tendency of the tail of the shield to shorten the horizontal diameter. The tail was long enough to lap over two rings and 6 in. on the third ring. This is the minimum length of tail that will permit the removal of a broken segment in a ring after a shove has been completed.

The shields were very heavily constructed and stood up well for the duration of the job. On the Brooklyn side, boulders were of very common occurrence; in some cases, pieces were broken out of the cast-steel hood segments by hidden boulders. Both land and river shields were equipped with hoods extending 2 ft. 10 in. farther ahead than the cutting-edge; the hoods on the land shields reached to 1 ft. below the springing line on either side, while those on the river shields reached only to a point 1 ft. above the springing line. The total length of the shield along the bottom was 15 ft., of which 6 ft. 6 in. was in the tail, 5 ft. 2 in. was between the rear diaphragm and the front diaphragm, and the remainder made up the forward part of the shield consisting of the cutting-edge castings and the steel structure to which they were anchored. Fig. 8 shows the dimensions in the cross-section of a river shield.

Hydraulic Equipment.—The hydraulic equipment on each shield consisted of shoving jacks, table jacks, and the erector. The shoving jacks were 10 in. in diameter and were equipped to be pushed out or pulled back by hydraulic power. With a pressure of 6 000 lb. per sq. in., the eighteen jacks were capable of exerting a forward thrust of 4 000 tons. The shoving jacks had the usual offset foot so that the pressures exerted by them were transmitted to the web of the cast-iron segment and not against the flange. In spite of this design, there were many instances in which tunnel segments were broken by the jacks during a "shove." To reduce the number of such breakages, steel shoes were made in the blacksmith shop and fastened to each jack. These shoes were designed to limit the area over which the thrust was exerted against the segment, and also to overcome the tendency of the moving part of the jack to twist.

Of course, each jack had its own control spindles by which it could be pushed out, exhausted, or pulled back. There were eighteen jacks on each shield, eight above the springing line and ten below. This arrangement proved satisfactory in controlling the moving of the shield.

The jacks stood up well throughout the course of the job and there was only a small amount of repacking required to keep them all in working order. This work was done on Sundays and it can be stated that throughout the entire job, every shield averaged 90% of its rams in working order.

Three tables were provided in the horizontal platform, one in each of the three pockets. They were of exceptionally heavy construction and lasted in the land headings for the duration of the job. In the river headings, they were put out of commission after the shields had passed the river bulkheads.

The erector was mounted at the edge of the horizontal platform of the shield on the axis of the tunnel. The motion of rotation (extending to about 380°) was effected by a pinion on the erector shaft, moved by a rack above it just under the operator's platform. The erector arm was extended or withdrawn by a hydraulic cylinder within the erector itself. Fig. 9 is a view

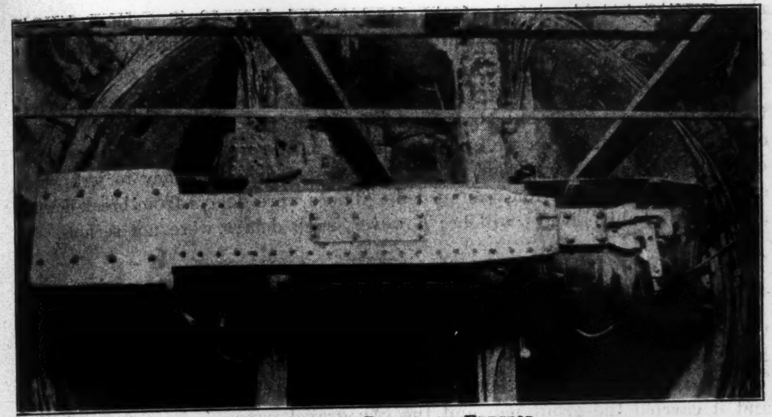

Fig. 9.—Outside Detail of Erector.

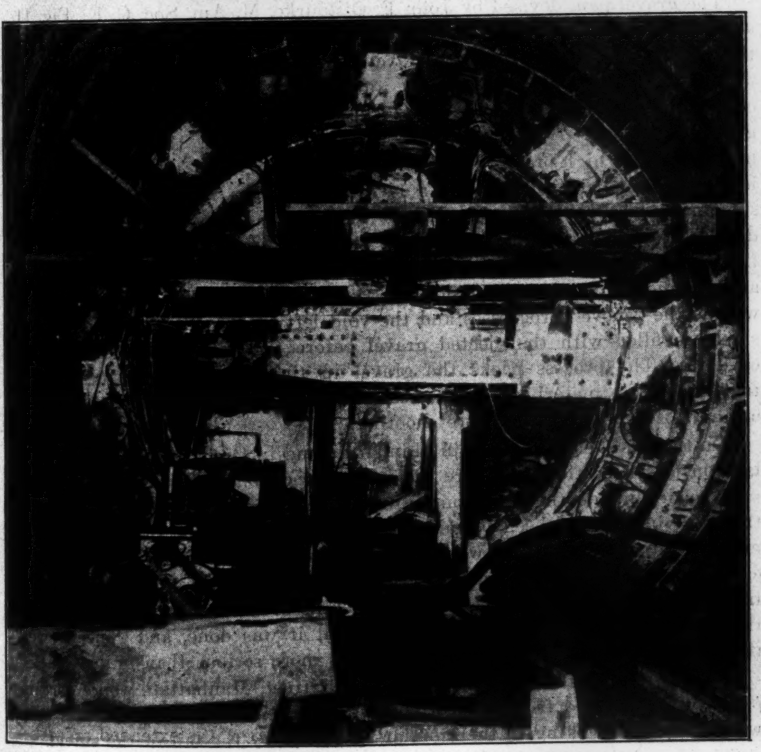

Fig. 10.—North River Heading in Manhattan.

showing the outside detail of the erector, and Fig. 10 is a more general view showing erector, jacks, shield platforms, and the tunnel rings in the foreground.

The erectors were built to operate at the same pressure that it was planned to use for shoving the shields, namely, 5 000 lb. per sq. in.; but this system offered disadvantages as well as advantages. The wear and tear on erector control valves was very high, and the erectors had a tendency to jerk when a slow and definite motion was needed. There were delays from break-downs, as there probably always will be; however, such delays were not serious.

Tunnel Plant in Rear of Shield

The traveling platform was dragged ahead by the shield and slid on the tie-rods, just above the springing line of the tunnel. It was originally 20 ft. long, but was extended in the field; it was the full width of the tunnel. The traveler served as a bolting platform in re-tightening the back rings, and it carried the grout pan and the two gravel pans.

The gravel pan, devised by John F. O'Rourke, M. Am. Soc. C. E., Fig. 11, was used for ejecting pea gravel outside the cast-iron lining, although in the photograph it is not on a traveling platform. It is a tank holding about 33 cu. ft. of gravel, with a nipple and valve at the bottom of its conical floor. There is an air-tight trap-door at the top through which the gravel is loaded. When the tank has been charged, the door is closed by admitting high-pressure air to the tank. Below the bottom nipple is a tee, on one side of which is a 1½-in. hose with a valve and on the other side an armored rubber hose leading to the grout hole in the segment. Immediately after the valve in the air line is opened, the valve at the bottom of the tank is opened. The gravel falls into the air jet and is carried along the armored hose and ejected outside the tunnel lining. This process is under way as the "shove" progresses and the void left by the advancing tail of the shield is filled with the ejected gravel before the ground has a chance to cave in. The process packs the gravel in tightly and almost cements it together. The material may be ejected dry or the charge in the tank may be inundated with water before shooting.

On the land headings, both in Manhattan and in Brooklyn, great stress was laid on this operation to insure the instant and complete back-filling of the void made outside the cast-iron lining by the advance of the shield. Had the overlying ground been permitted to fall into this space, disastrous settlement of adjacent buildings would have resulted. In fact, the success of the entire tunneling procedure was linked to the success of this operation. It is impossible, in this paper, to go into the details of the campaign waged to get this work done quickly and thoroughly; but it was done, and done so well that no building settled or suffered injuries more serious than minor cracks.

Fig. 12 is a photograph taken at Nassau Street, Manhattan, where certain plates were removed from the bottom of the cast-iron lining. It shows very well the gravel packing in the annular space left by the tail of the shield.

FIG. 11.—VIEW SHOWING GRAVEL PAN.

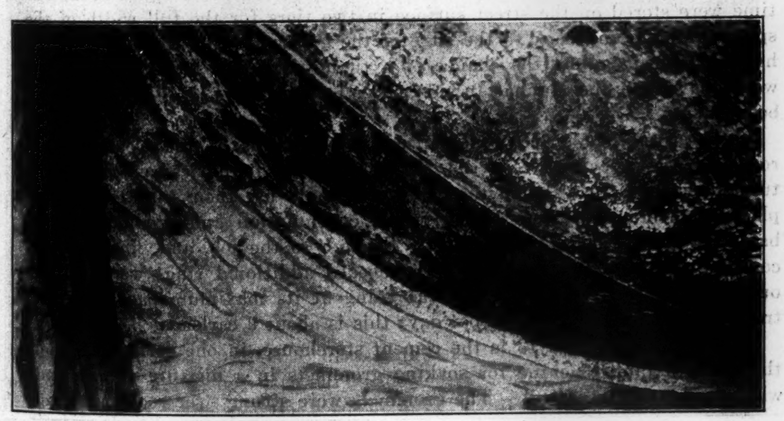

FIG. 12.—GRAVEL PACKING UNDER BOTTOM OF CAST-IRON LINING.

The ejection of grout outside the cast-iron lining is a second line of defense in the prevention of settlement and is no less important than the gravel process. While graveling is done in the first ring behind the tail, grouting is done seven to twenty rings back. Grout penetrates farther than gravel. It spreads out into seams that may have opened up in the overlying ground. It sometimes spreads out in such a seam over a wide area and, being under pressure, can actually lift up the ground that has slumped down. The grout pan is practically the same as that used in the construction of the Pennsylvania Tunnels.[8]

The machine is nothing but a tank into which sand, water, and cement are dumped. These ingredients are mixed by paddles turned by a small air motor at the end of the machine. The liquid grout is ejected by admission of compressed air after the tank has been closed.

Tunnels are grouted to check the loss of air while the shield is passing through loose ground even when the question of settlement of buildings does not enter. Thus, grouting was extensively used in passing under the dock bulkheads and, also, out under the river. Fig 18 shows typical shoring beneath an elevated railway structure in Brooklyn, which was required while the tunnel was passing beneath it. The ground settled about 1 in. and the columns were jacked up and held by wedging.

DOCK PLANT AND GANTRIES

Practically all the materials used for construction were delivered to the job by water transportation and excavated material was disposed of in the same way. The right to use certain property on the East River was given in the contract. In Manhattan, a gantry was built from the shaft, east along Fulton Street to South Street, thence east along the northerly half of Pier 16 (see Fig. 1) to dumping boards erected along the edge of the pier.

The dock space available in Manhattan for handling segments, grouting sand, pea gravel, and cement is shown in Fig. 1. Tunnel segments at one time were stored on the street surface in two tiers for the full width of the space available and back to the edge of the gantry. This iron was kept on hand to guard against the chance of a tie-up of harbor transportation. It would have kept the Manhattan side going for four days if all deliveries had been cut off.

A large storage space was utilized in the yards of the Central Railroad of New Jersey, in Greenville, N. J. When the foundries were producing tunnel segments faster than the contractor could use them, the surplus was placed in storage in New Jersey; when consumption and production were balanced, the segments came directly through to the job, and, finally, when consumption exceeded production, the needed additional rings were ordered out of storage. When the work was going at its maximum pace, the contractor erected about 48 rings per day; this is about 9 carloads.

At the right in Fig. 13 is the cement storehouse, in one corner of which there was a tank machine for soaking grommets in a mixture of red lead, white lead, and linseed oil. The grommets were strung over rods that fitted

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[8] *Transactions*, Am. Soc. C. E., Vol. LXVIII (September, 1910), p. 466.

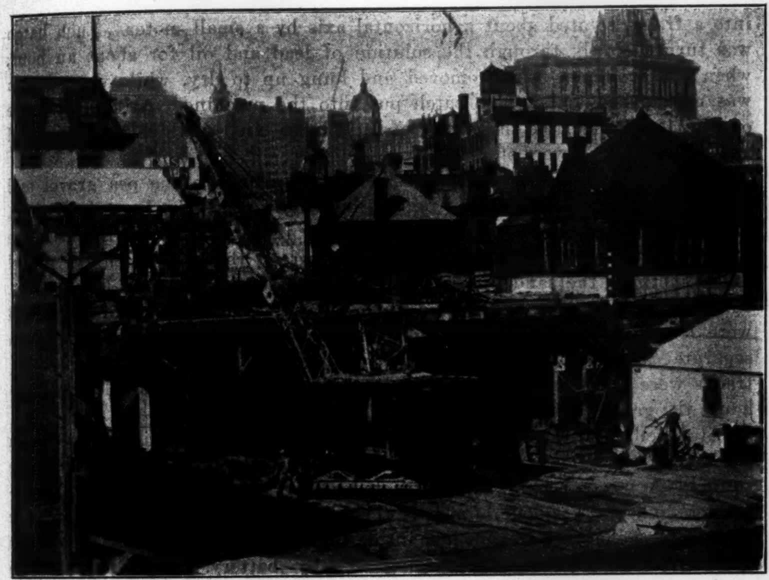

FIG. 13.—STORAGE YARD AT BULKHEAD IN MANHATTAN.

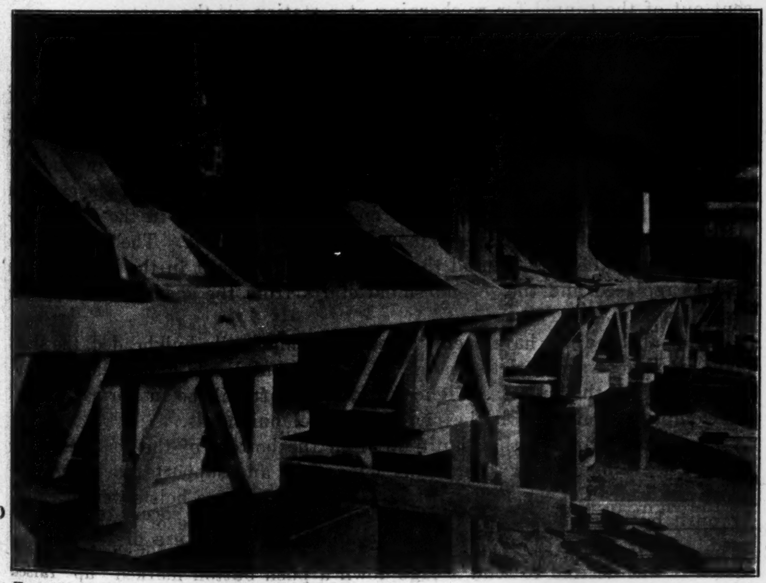

FIG. 14—HOPPERS FOR SCREENING, MEASURING, AND BAGGING GROUTING SAND AND PEA GRAVEL, IN MANHATTAN.

into a frame rotated about a horizontal axis by a small motor. Each batch was turned slowly through the solution of lead and oil for about an hour, when the grommets were removed and hung up to dry, while the solution was replenished and a new batch put into the machine. At the left edge of Fig. 13, one can see the corner of the timber bins in which the pea gravel and grouting sand were stored.

The detail of the measuring hoppers through which the pea gravel and grouting sand were drawn off from the bins and bagged, is shown in Fig. 14. The storage bins were equipped with steam coils for winter conditions. The measuring and bagging device included a screen through which the gravel and sometimes the sand was passed. It was essential to avoid large stones, which choked the hose and caused much delay. The bagged sand and gravel were loaded into muck cars which were elevated to the gantry level in a separate cage constructed near the cement shed. In Brooklyn, the gantry tracks were laid through the third floor of a warehouse. Outside the warehouse the gantry extended out on Pier 6 (see Fig. 1).

Storage-battery locomotives were used throughout the entire job—both on top and in the tunnels. In Manhattan, the batteries were re-charged in the tunnel. A power room was constructed at the top of the shaft with transformers and a motor-generator set. The direct current was carried down the shaft to a re-charging station just inside the locks in each tunnel. Automatic devices prevented the overcharging of any battery. In Brooklyn, the re-charging was handled differently. The batteries were put on flat cars and sent out of the tunnel for re-charging at a station on the gantry.

Shaft Plant

The shaft plant on a tunnel job is required to take care of two very different shaft conditions. There is, first, the period when the shields are being started from the shaft. The deck is in place in its upper position and the space below is under compressed air. All materials pass in and out through locks above the caisson deck. After one tunnel has been extended about 150 ft. from the shaft, a concrete bulkhead is built in the face of the shield, and the air pressure is removed from the heading. The deck is then removed from the caisson, and the second shield is built in the bottom. The deck is replaced and the second shield starts from the shaft in the opposite direction.

After all headings have been started and temporarily bulkheaded, the shaft plant undergoes a complete change. Concrete bulkheads with locks are constructed in all the tunnels. Henceforth, the shaft will be in normal air and the compressed air will be beyond the tunnel bulkheads.

Steel framework is then built up from the bottom of the shaft to above the gantry level. This frame forms a hoistway and supports the cage hoists above the well. On the Fulton Street Tunnels, the cage hoists were operated by push buttons. This was the first time that this type of equipment was used, and it worked out very well. The cages were of the usual design. When it was desired to send a cage down a push button marked "up" raised the cage about 1 ft., freeing the "landing dogs." The "down" button was

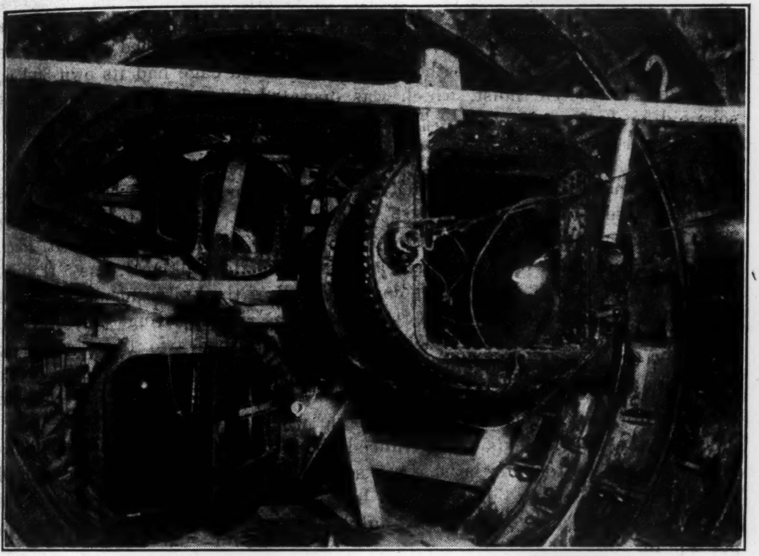

FIG. 15.—BULKHEAD AND LOCKS IN RIVER HEADING.

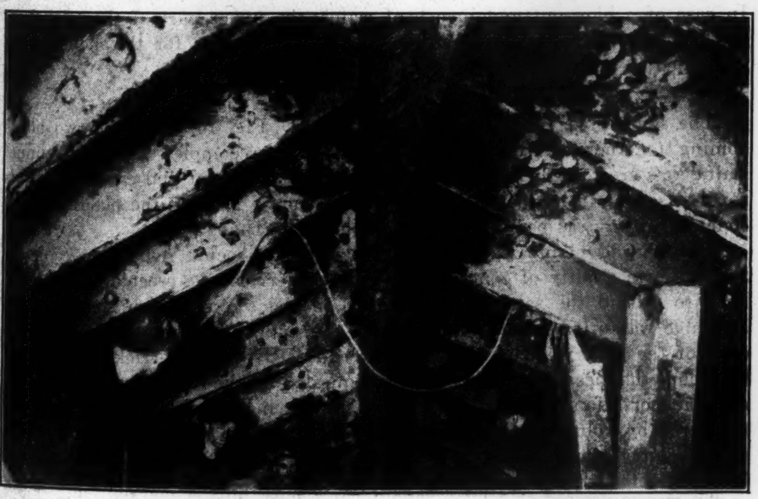

FIG. 16.—MEETING OF TWO HOODS, MAY, 1929.

then pushed and the cage descended to about 2 ft. above the bottom where it stopped automatically. A second button was then pushed and the cage was slowly "inched" down to the final level. Each cage had its own 30-h.p., single-drum, electric hoist, which was operated by a slip-ring motor, with solenoid brakes on 220-volt, 3-phase, 60-cycle current at 860 rev. per min.

Tunnel Bulkheads and Safety Screens

The tunnel bulkheads presented no novel features. Because of the large sized cars used, the muck locks had to be 8 ft. 6 in. in diameter. They were 38 ft. long. The man locks were 6 ft. 6 in. in diameter and 34 ft. long. The emergency lock, placed as high as possible in the bulkhead, was 5 ft. 0 in. in diameter and 20 ft. long. The concrete wall of the bulkhead was 6 ft. 6 in. thick in the land headings and 8 ft. thick in the river headings (see Fig. 15).

In addition to its locks, each bulkhead contained two 10-in. air lines; two 6-in. blow lines; one 4-in. high-pressure, air pipe; one 3-in. water line; two 1½-in. hydraulic lines; a back-pressure pipe; and several iron conduits for electric cables and telephone wires.

After the river shields had advanced about 1 000 ft. from the tunnel bulkhead near the bottom of the shaft, a second bulkhead was built close behind the shield. After this bulkhead had been put in use, the air pressure in the section of tunnel between the two bulkheads was maintained at about 15 lb. per sq. in.

The safety screen was required, by specification, in the river headings after the shield had passed the river bulkhead. This is an air-tight bulkhead closing the upper part of the tunnel. Two were used in each river heading so that one would always be in place while the other was being moved ahead. In case of a blow-out and inrush of water, the upper part of the tunnel on the landward side of the screen would be a safe haven for the men if they succeeded in reaching it.

Completion

The south river shields met in May, 1929. Fig. 16 shows the two hoods coming together. After all the excavation was completed and all tunnel lining was erected, there was the big job of cleaning out and caulking. This work was done in normal air. Toward the end of August, the placing of the concrete lining was begun; it was completed in June, 1931.

The mixing plants were carefully planned to secure maximum economy and uniform quality of concrete. A stiff-leg derrick was erected at the bulkhead to unload the sand as it came in on the scows. Such space as was available was utilized for storage piles. A mixing plant was set up near the edge of the gantry. The aggregates were raised by the derrick to a storage bin about the measuring hoppers. As requested by the engineers of the Board of Transportation, the aggregates were measured by weight. The water was measured by volume. The mixer discharged into a car which was elevated to the gantry level and then taken to the form in a train of two or three cars. The concrete cars were made by substituting a concrete body on the muck cars. The mixing plant and methods of placing were about the same in Brooklyn as in Manhattan.

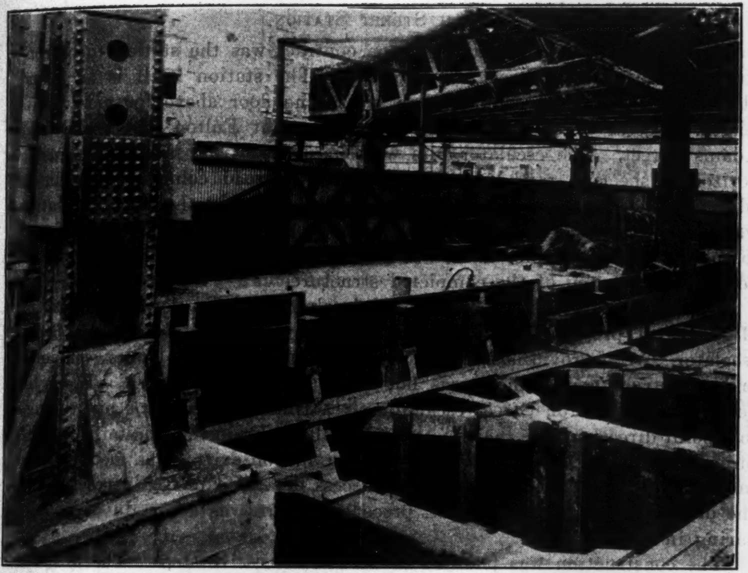

FIG. 17.—EXCAVATION FOR ESCALATOR PITS, HIGH STREET STATION, BROOKLYN.

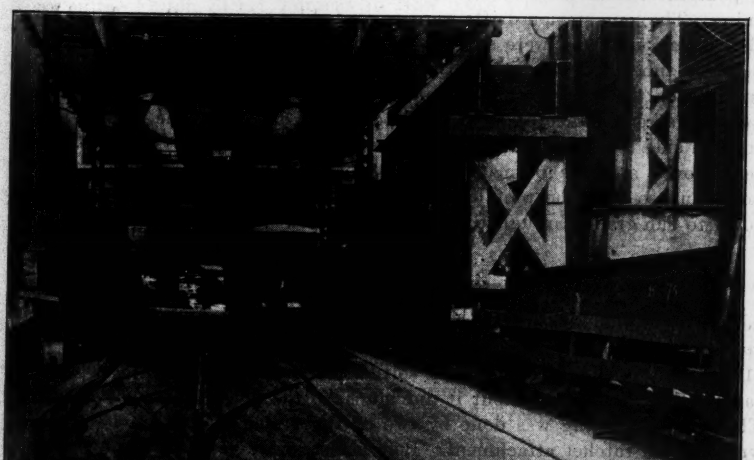

FIG. 18.—SHORING UNDER ELEVATED RAILWAY ON HIGH STREET, BROOKLYN.

HIGH STREET STATION

A somewhat unusual feature of the contract was the station to be constructed under High Street, in Brooklyn. The station itself is an island platform about 700 ft. long, with a mezzanine floor above for most of its length. Access to the station is by escalators at Fulton Street and near Adams Street. The escalator pits were excavated from the surface, using horizontal sheeting methods. The elevated railroad columns immediately adjoining the pits were underpinned to sub-grade before the main excavation was made. The column in the foreground of Fig. 17 was first underpinned to sub-grade, but the pier had to be partly removed and the column given a new bearing on top of the completed structure.

Fig. 18 shows the shoring of the steel columns of the elevated railway on High Street preparatory to making the excavation for the Adams Street escalator. After providing for such column shoring, and for trolley, vehicular, and pedestrian traffic, there was not much room left to work. From the bottom of the Fulton Street escalator pit a timbered drift was put through to a point directly above the two cast-iron tubes. From the sides of this drift, mining gangs broke in and started the long station drift. The methods used in this work were new. Since their success depended on the correctness of the designer's figures and not on precedent, there was a certain element of daring in them.

The construction was planned in several successive steps. The first was to pour the tunnel inverts and erect thereon timber bracing for the cast-iron lining. The long station drifts were excavated by miners. Pressed-steel liner plates were used and were supported by arch ribs resting on the tops of the cast-iron tubes and strengthened by a bowstring. The first heading was large enough for the central or bowstring section of the arch rib; side headings followed at a distance of 50 ft. and permitted extending the arch rib down on each side to a firm bearing on the cast-iron tubes. All posting could then be removed and the permanent steel erected. The arch ribs were finally jacked up so that the roof load was taken from the tubes and transmitted down to sub-grade through the permanent steel.

CONCLUSION

The foregoing covers the major items of plant used on this work. Several new items would be welcome, but have not yet been devised. A number of attempts have been made to develop a bolt-tightening machine but, to date (1930), this problem had not been solved. A small, air-driven tightener that ran up the nut was used, but it did not have the power to complete the job. The final tightening was done by two or three men pulling on a 5-ft. hand wrench with ratchet attachment. Belt loaders were used for a time, but the available space was so small that the advantages of the loader were outweighed by its disadvantages.

ACKNOWLEDGMENTS

The construction work described in this paper was done by the Mason and Hanger Company, Incorporated.

DISCUSSION

H. J. KING,[4] M. AM. SOC. C. E. (by letter).—While comparatively few engineers are faced, during their professional careers, with the problem of driving tunnels under such difficult conditions as those described by Mr. Killmer, nevertheless this extremely valuable contribution can be studied with great profit by tunnel men engaged in work in normal air.

The writer is acquainted with several tunnels driven under more or less populous centers over which subsequent subsidence of the ground surface has been the basis of actions brought to recover losses for heavy damages to adjacent buildings and other improvements. In certain types of ground this subsidence may not be evident for several years after the completion and acceptance of the tunnel by the owner.

In the writer's opinion the application of the apparatus for ejecting pea gravel, so clearly and interestingly described by Mr. Killmer, could well be extended to include many liner tunnels driven under areas where surface subsidence is objectionable. In many cases the gravel treatment alone would be entirely sufficient to prevent subsidence, and the necessity for the more costly grouting treatment could be obviated.

S. M. SWAAB,[5] M. AM. SOC. C. E. (by letter).—The work done by Mr. Killmer on the Fulton Street Tunnels in New York City represents the highest development of the art of subaqueous tunneling. While shield tunneling is an English invention and the first shield-driven tunnels were built in England, still the art has received its maximum development in methods, safety, and economy, in the region of the Port of New York since the date of the building of the Hudson and Manhattan Tunnels by the late C. M. Jacobs and J. V. Davies, Members, Am. Soc. C. E.

On each of the several tunnels as it was built, certain innovations were made until the process of building subaqueous tunnels can now be said to be fairly well established, the size of the tunnel and the nature of the ground being the same.

On the Fulton Street Tunnels there has been made the most consistent, and, at the same time, the greatest and best progress (except that made on those tunnels where the material of the face practically flowed into the tunnel through the openings in the shield, or where the shield merely displaced the material) on any of the subaqueous tunnels in the neighborhood of New York; and for workmanship, alignment, and grade, they have not been surpassed.

The specifications issued by the Board of Transportation for this work left to the contractor the choice of electric or steam power for the operation of the compressor plants, and the contractor chose to use electric power. This was made possible by improvements in compressor machinery. The plate valves used to-day are lighter in weight and, at the same time, provide greater area for the air to pass through and cause less restriction than the poppet-valves formerly used. Consequently, the machines can be operated at much higher speeds than formerly, being, in fact, direct-connected to the motors and operating at full motor speed without restriction. However, in adopting electric power, one has to be alert because of accidents to the feeder lines to which there is always some liability (such as that to which Mr. Killmer calls attention), even if the power lines come from different sources. Proper protection of the circuits by lightning arrestors can readily be provided at comparatively little expense. The expense of supplying this protection is, in fact, so small that one cannot readily afford to be without it.

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[4] Pasadena, Calif.

[5] Cons. Engr., Philadelphia, Pa.

The automatic valves for controlling the output of air from the low-pressure compressors were, undoubtedly, designed to be operated by moderately high, low-pressure air. Consequently, when the low-pressure air was reduced materially by a blow-out in one of the headings, there was not sufficient pressure available to operate the valves. Mr. Killmer immediately overcame this by cutting into the line of the old compressor with hand-operated valves, and the pressure was thus boosted to provide for the emergency. Subsequently, a recurrence of this condition was prevented by taking the air supply for valve operation from a receiver connected with the high-pressure machines.

The plant which is described in detail in Mr. Killmer's paper was ample for the purpose. Three of the low-pressure compressors would have been sufficient, at times, on the Manhattan side, and six might have been used on the Brooklyn side to advantage, owing to the fact that the material on the Manhattan side held the air better than that on the Brooklyn side of the river.

The connection between the high-pressure and the low-pressure air lines is very advantageous and, as Mr. Killmer states, was availed of several times in connection with this work.

The shields which were used on Fulton Street fulfilled every purpose and stood up so well that in the subsequent work done by Mr. Killmer on Rutgers Street, this same shield design was used without any substantial change.

The innovation made in the method of tunneling under atmospheric air in the station on High Street was, undoubtedly, a very desirable one, since there has been no material left in the ground which, in the nature of things, will deteriorate.

The method of tunneling in soft ground without air (previously used on the stations built in connection with the several subaqueous tunnels, prior to the building of the Fulton Street Tunnels), permitted a large quantity of heavy timber to remain in the ground, but in the substitute method used by Mr. Killmer, this is not so. The advantages of this method over that formerly used on similar work are readily recognizable and have been highly satisfactory. In any event, it is quite possible that there will be less disturbance of the utility structures, buildings, and street paving, in the future than has occurred in connection with any of the other work heretofore built.

In Fulton Street, Manhattan (which is about 55 ft. in width between the houses), the tunnels were driven under compressed air through sand below the level of the ground-water, in many instances below, and in all cases extremely close by, the foundations of the buildings, without causing injury worthy of the name, and, in fact, without appreciable settlement. This was done in advance of the building of the station between Broadway and William Street. The latter work was done from the surface after the tunnels had been driven. It was originally intended to underpin the buildings on both sides of the street in advance of the driving of the tunnels. This was not done, however; but for the purpose of protecting and securing the buildings during and after the construction of the station, permanent bulkheads, built piecemeal, in isolated sections, in individual pits, were constructed in front of them on either side of the street and just beyond the station walls, in advance of the building of the station. These bulkheads, when completed, were solidly braced across the cut provided for the station, as the excavation was made, and the station itself was subsequently built within the lines of the bulkheads thus provided.

The work on the station in Fulton Street, Manhattan, which was one of the most difficult operations to be performed in connection with this contract, was distinctly illustrative of what could be done by way of tunneling in close proximity to high and heavy buildings. After the completion of the tunnels and immediately over them, the earth prism between the surface and the top of the tunnels was removed in a bulkhead-protected trench, without permanent injury to any of the buildings, and with a minimum of disturbance during their occupancy. Incidentally, there was as little or possibly less settlement of the individual buildings than would have occurred had they been previously underpinned.

The method of following up the erection of the iron close by with pea gravel and of subsequently injecting into this cement grout had the desired effect of preventing surface settlements to the largest extent that they have ever been prevented in any work of this character in New York City, or elsewhere.

Indeed, it would be difficult, to say the least, for one to conjecture what might possibly happen on work of this character. One must always be on the alert. For the ordinary things that happen, a man may be, and is to some extent, prepared. It requires an ingenious, resourceful, and quick-witted man, however, to be prepared to meet unexpected contingencies as they arise, and then there must be a liberal allowance of "good luck," or its equivalent, no matter by what name it is called. In addition, a good organization properly directed, adequate financial resources, and a properly designed plant, such as was here available, are required to complete a contract, such as this, successfully.

JACOB FELD,[6] ASSOC. M. AM. SOC. C. E. (by letter).—The author has omitted mention of the methods for checking line and grade in the two sections of each river tunnel by which such remarkable coincidence as that shown in Fig. 16, was obtained. The writer has given line and grade in rock and earth tunnel construction under structures, and appreciates how much detailed work is required in providing accurate control points in blind headings to permit accurate meeting of such headings. The same operation must be considerably more difficult in tunnels under water, because of the longer sections and because of the movements which must occur in each section due to such conditions as change in tide levels and possibly motion of the soil surrounding the tunnel shield.

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[6] Cons. Engr., New York, N. Y.

OLE SINGSTAD,[7] M. AM. SOC. C. E. (by letter).—This paper is a concise and well prepared account of an important and interesting construction job, well executed. Two outstanding features of this work are most impressive: (1) The rapid progress attained in shield driving, which has established a new record for subaqueous tunneling in this kind of ground; and (2) the driving of the shields through Fulton Street below and very close to the foundations of buildings of considerable height, without underpinning the buildings and without causing serious settlement or damage to them. This result, of course, could not have been attained without the effective measures taken, as described in the paper, to insure the instant and complete backfilling of the void created outside the cast-iron lining in advancing the shield.

It is also interesting to note certain improvements over previous practice in the details of the plant, such as the use of automatic or push-button elevators in hoisting materials up and down the shafts (an arrangement which no doubt was prompted by the scarcity of labor and the high wages paid at the time); and the use of a simple mechanical device for saturating the hemp grommets with red lead. This device not only effected a saving in labor, but unquestionably resulted in a more thorough saturation of the grommets, thereby improving their water-proofing properties.

The erection of the shield on a cradle equipped with rollers also is a novel feature, resulting in greater ease in erecting and equipping the shield and a saving in the time required for this work.

The rapid progress made on the work and the excellent results obtained are due largely to the fact that the contractor, as well as the City, had organizations of construction men and engineers in charge, with extensive experience in tunnel work, and that the job was equipped with an adequate and up-to-date plant.

JOHN F. O'ROURKE,[8] M. AM. SOC. C. E. (by letter).—In addition to this paper there are only two papers[9] in the publications of the Society on the subject of under-river shield tunnels and their approaches. The papers are all well written, fully illustrated, and treat the subject from two different points of view; namely, that of the owner's engineers who supervised the work in the field and that of the engineer who was in actual charge of the work for the contractor. It would be well if descriptions of the construction record of the numerous other shield tunnels under the Hudson River and the East River had been published by the Society so that the growth of the art of shield tunneling in New York City could be followed from one to another, including the views of the engineers and contractors. At any rate the facts concerning them are generally known and short references to some of them may be of value in showing the successive improvements by which shield tunneling has reached the high state of development disclosed in Mr. Killmer's extremely lucid and important paper.

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[7] Chf. Cons. Engr. on Tunnels, The Port of New York Authority, New York, N. Y.

[8] Cons. Engr., New York, N. Y.

[9] "The New York Tunnel Extension of the Pennsylvania Railroad—The North River Tunnels," by B. H. M. Hewett and W. L. Brown, Members, Am. Soc. C. E.; and "The East River Tunnels," by J. H. Brace, Francis Mason, and S. H. Woodard, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXVIII (1910), pp. 152 and 419, respectively.

The first under-river shield tunnels in New York City were the two under the Hudson River (The Hudson and Manhattan Tunnel) from Morton Street, Manhattan, to Jersey City, N. J. (begun in 1874). These were started as brick tunnels constructed without shields by the Haskins System, which were later changed to shield tunnels lined with cast iron. The brick tunnels were driven through the clay river bed with 34 lb. of compressed air by excavating the neat size of the brickwork and lining the roof and sides of the excavation, as it progressed, with light steel rings, 30 in. wide, in 3 and 6-ft. segments, that broke joints with those of adjoining rings. This steel lining was braced radially with timbers from a pilot tube at the center, 6 ft. in diameter, which was driven a few feet in advance of the heading. The 24-in. brick lining was built as each 15-ft. length of excavation was completed. The muck from the next 15-ft. length was filled into the last length of brickwork to a little below the springing line (upon which the construction tracks were extended), after which the remainder of the excavation for the length was completed and hauled to the shaft in muck cars drawn by mules. It is significant that the material from the heading was sufficiently firm to support the construction track and give a footing for the mules without planking, and confirms what follows on that subject.

A paper[10] on this tunnel by the late Arthur Spielmann and Charles B. Brush, Members, Am. Soc. C. E., relates to the work at the beginning and before the method of construction described herein had been fully developed. However, the paper has many facts concerning the steel-plate rings. The material of the river bed through which the tunnel passed, is described as so stiff under the air pressure that it could be cut in benches, with steps rising from one bench to the other. Another paper[11] by the late William Sooy Smith, M. Am. Soc. C. E., gives further information regarding the river bed and describes the Haskins tunneling methods in detail. This comparative solidity of the so-called silt, which is in reality a sedimentary deposit of clay in horizontal layers, was subsequently confirmed by Messrs. Aims and Fitzgerald, Resident Engineer and Superintendent, respectively, in numerous personal interviews which contained a wealth of corroborative detail, and also by the writer's own observations in front of a shield where these tunnels were being driven through part rock and clay near the New York side of the river. He also had opportunities to observe the clay of the river bed at the Pennsylvania Railroad tunnels; at Haverstraw, N. Y., where the clay for brick-making is taken in great quantities from behind coffer-dams in the river, the clay banks being like those on shore; and also at the Poughkeepsie Bridge during the sinking of the pier foundations. In all these cases the clay was similar to that previously described, through which the Hudson and Manhattan Railroad Company's tunnels were driven so that the entire river bed from New York to Poughkeepsie may be said to consist of a fairly firm and dependable clay with a shallow covering of mud where the clay has been disturbed. Attention is called to these facts as showing the reliable nature of the bed of the river where undisturbed for the firm support of tunnels and other structures with properly designed foundations, because there is a widespread belief that the river bed is soft and affords only insecure support. The direct opposite is true, particularly as to tunnels, which weigh less than the material they replace, so that the clay beneath them has less load on it than before their construction.

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[10] "The Hudson River Tunnel," *Transactions*, Am. Soc. C. E., Vol IX (1880), p. 259.

[11] "The Hudson River Tunnel," *Transactions*, Am. Soc. C. E., Vol. XI (1882), p. 314.

It is a fact that at both the Pennsylvania Railroad tunnels and the Holland Tunnel the clay surrounding the tunnels directly behind the shields became soft and slushy for a few feet out from the tunnel lining and that some settlement as well as flotation of the tunnels occurred. This was occasioned by the ground not closing in on the lining at the tail of the shield, but maintaining itself for some time as a dirt tunnel of the diameter of the shield. The weight of the overlying ground and water caused arch stresses in the surrounding parts greatly in excess of the ordinary compression stresses when directly supported from below, and squeezed the firm clay composing the face of the dirt tunnel into a watery mud.

This same clay when forced during shoves into the tunnel through openings in the bulkheads, by hydraulic pressures exceeding 100 lb. per sq. in. on the heading in front of the shield, required a foot on the spade to cut into it during removal. This shows how firm the clay surrounding the tunnels naturally is and that if immediate support were provided for the tunnel and surrounding ground as the shield was shoved (as can easily be done), the destructive arch stresses would be prevented and the tunnel would be as free of either settlement or flotation as were the brick tunnels in the same kind of ground, where immediate support by the steel lining plates preserved the firmness of the clay unchanged.

The Hudson and Manhattan Railroad Company's tunnels are also of interest because in them cast-iron shield tunneling was introduced in New York City through the advice of the late Sir Benjamin Baker, Hon. M. Am. Soc. C. E. Certain London bankers retained Sir Benjamin to report on these tunnels when they were considering a loan to the Tunnel Company. He learned that on two occasions when the air pressure in the northerly brick tunnel, extending from Jersey City to the middle of the Hudson River, was reduced from 34 lb. to 20 lb., because of trouble with the steam boilers, the heading began to move into the tunnel. This was stopped both times by rapid and effective bracing. The possibility that, on some future occasion, when the air pressure might be lowered or lost, the bracing might fail and the tunnel be filled with silt, seemed to Sir Benjamin too great a risk to ignore, and he advised the bankers to make the use of shields a condition of the loan, which was done. This involved abandonment of the brick lining and the substitution of cast-iron lining, which was the only kind known at that time that could resist the thrust of shield jacks. The existing brick tunnels were entirely satisfactory in his judgment and subsequent events have justified this opinion. The northern tunnel (the western half of which was built of brick at that time and the eastern half subsequently built of cast iron) has been in constant and satisfactory use to its full capacity for twenty-three years (since 1907), and the brick and cast-iron parts are both in equally fine condition.

The next shield tunnels undertaken (1903) were for the Rapid Transit Commission of New York from South Ferry, Manhattan, to Joralemon Street, Brooklyn, with approaches in Battery Park, Manhattan, and Joralemon Street, Brooklyn. These tunnels were of cast iron, as are all subsequent river tunnels and approaches in New York at the present. Shield tunneling being still quite undeveloped, there were difficulties and stoppages of work, but after some delays and much discussion, the troubles were corrected, the tunnels were finished, and have been giving satisfactory service for more than twenty years.

The tunnel construction along Joralemon Street in Brooklyn caused immediate settlement of the street over the tunnels as the shields advanced, a little later followed by settlement of the stoops and areas, outward from the buildings. Later, the fronts of the buildings began to settle and crack, the settlement of the ground under them progressing slowly to the rear for several years, until the angle of rupture of the ground was finally reached when the settlements stopped. By this time some of the buildings had moved out several inches into the street, spur shores against the fronts were frequently used until the settlements stopped, and a number were so badly cracked that the front walls had to be rebuilt. After years of litigation, the Court of Appeals affirmed the verdicts for damage to the buildings and disallowance of the much larger claims for contingent damages, such as losses of rentals and interference with business. The opinion stated in support of this that the work had been done without negligence, that the grout used to minimize settlement, while it was the best known method, had not entirely succeeded in doing so, and that the damages were incidental to the character of the work. This means that the best known method of preventing settlement must be used if consequential damages are to be avoided.

The shield tunnels undertaken after the Pennsylvania Railroad tunnels, were those across the East River, two from Whitehall Street, Manhattan, to Montague Street, Brooklyn, and two from Old Slip, on the New York side, to Clark Street, in Brooklyn. It was on these contracts that the writer introduced gravel packing to fill the space left around the tunnel lining by the shields. The contracts specified grout for this purpose, paid for per barrel of cement used. After gravel packing had been tried for a short time and was found to prevent visible settlement of streets or buildings the contracts were modified to substitute it for the grout used immediately behind the shields.

The gravel packing was first used in the river headings from the shaft at Old Slip and Front Street on the New York side. Figs. 19 and 20 show Old Slip on January 29, 1916, about two months after the tunnels were driven beneath. Gravel packing was supposed to be used from the beginning, but the street settled about 6 in. for nearly 75 ft. When the writer went into the tunnel it was found that, although the blowing apparatus and pea gravel were provided, the men did not understand the method and had confined their activities to grouting. The writer instructed them in getting the apparatus in working order and in blowing the gravel into the space around the tunnel at the rear of the shield, so that in a few hours the tunnel and ground at the rear of the shield were firmly supported for the length of a ring. When the next shove was made the operation was repeated and from that time on gravel packing the space left by the shield was started with the shove and continued until the entire space was packed, in the order of top, sides, and bottom, and there was no further settlement.

In Fig. 19 a white arrow shows where the settlement was stopped at the tail of the shield. The line marks the end of a piece of curb where the shield had not moved, of which the other end had settled 5 in. From this point the curb did not settle perceptibly, as the gravel packing was regularly used with a success that was warmly appreciated by the tunnel men, many of whom had worked on the Joralemon Street tunnels and believed shield tunneling and settlements to be inseparable.

Fig. 20 is a view from the corner of South Street looking backward to the shaft at Front Street, Manhattan. The sidewalk and roadway in this picture as far as the white arrow, remained without settlement. The line of the curb shows this, and any unevenness in the sidewalk and pavement was there before the tunnel was driven. Neither view shows settlement in the adjoining building as its pile foundations were carried on solid rock.

The same successful avoidance of visible settlement followed the construction of the Brooklyn approaches on Clark and Montague Streets where the gravel packing was faithfully performed. There was equal success in preventing settlements in driving the tunnels on Fulton and Willoughby Streets, Brooklyn.

Grout was used in all these tunnels from one to three days after the gravel packing had been placed. The gravel packing was so dense that grout could not be forced through it and a hole had to be punched through the gravel at each grout hole before the grout hose nipple was attached. In excavations subsequently made around the tunnels for inter-track stations, the grout was usually found in large lumps in the ground where it emerged through the gravel as there were no holes or cracks into which it could flow. It appeared that when the grout came in contact with the surrounding ground that the water was filtered from it, the sand and cement forming a lump that grew in size until the resistance of the ground to further expansion of the lump caused a refusal of flow from the grout mixer under a pressure usually about 100 lb. per sq. in. These lumps greatly compacted the ground surrounding them.

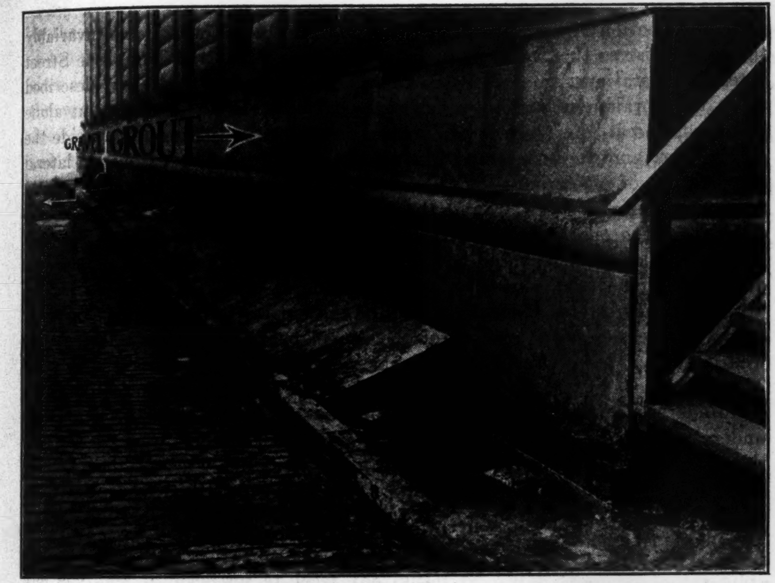

Fig. 19.—North Side of Old Slip, Showing Where Grout Was Used Outside of Lining Allowing Settlement of Street Over Tunnel.

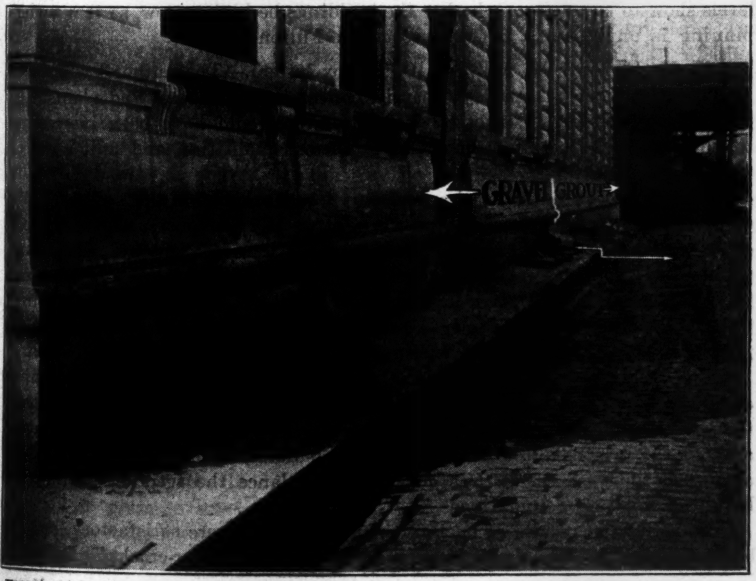

Fig. 20.—North Side of Old Slip, Showing Where Gravel Was Used Outside of Lining Preventing Settlement of Street Over Tunnel.

The collection of grout into lumps around the tunnel, which was invariably the case so far as the excavations disclosed at the Clark and Montague Street inter-track stations, goes far to explain the settlements already described when the Joralemon Street tunnels were driven. In that case grout alone was used, and it must have made pockets for itself in the ground outside the grout holes and formed lumps like those described. The extent of lateral compression of the ground by these lumps was limited, so that the greater part of the empty space around the tunnels was left unsupported. In any event were the character of the ground to permit the entire empty space to be filled with grout, it would be too soft for at least an hour to support the overlying ground which usually starts to settle at once.

Mr. Killmer states that the success of the entire tunneling procedure was linked to the successful placing of the gravel packing, and that this was so well done that only a few minor cracks were caused in driving the tunnels. This latter fact is very important regarding that contract as none of the high buildings was underpinned, while all buildings of seven stories or more were underpinned before the Montague and Clark Street tunnels were driven past them, because the security afforded by gravel packing was not then known. An examination to-day of underpinned buildings and those adjoining them on Montague Street would show the joints between them to be undisturbed although in some cases the walls of the small buildings extend only 10 ft. below the streets while the underpinning of the high buildings near the river extends 100 ft. below the street. The fact is that gravel packing prevents all but a very slight settling of the ground as a whole along the tunnels, which levels show to be uniform whether the buildings be high or low, so that underpinning is unnecessary, as was proved on Fulton Street.

H. H. Hatch,[12] M. Am. Soc. C. E. (by letter).—The author has covered his subject-matter thoroughly; that is, contractors' problems and plants. In connection with his instructive description, it would have been interesting to know also about the engineering problems, quantities, and unit costs of this work.

It always pays to make the necessary precautions for the safety of life and the expedient progress of the work, especially in constructions of more or less risky nature. Sometimes a contractor tries to "get by" with plant on hand and without making the necessary preparations for emergency. At times, he succeeds, and often he has "hard luck" which results in loss of life, time, and money. Mr. Killmer, however, seems to have taken the necessary precaution for any emergency with respect to his equipment and personnel.

It is interesting to note that the many tall buildings near-by the tunnel could be held in place without settlement and without any support other than a maximum air pressure of 48 lb. per sq. in. Disregarding the disturbance of the original ground in driving the shields, it means that a force of about 3.5 tons per sq. ft. was sufficient to balance the reaction of the tall buildings on their foundations.

Evidently the results of back-fill with pea gravel were satisfactory, as "no building settled or suffered injuries more serious than minor cracks." It occurs to the writer, however, that the more uniform the material sizes the higher is the percentage of voids; and that if there is any variation in the particle sizes of the gravel, or if pea gravel is mixed with sand, before being shot into the openings outside the shield, the back-fill would become more compact due to a smaller percentage of voids. It is desirable to know the maximum and minimum pressures used in shooting the pea gravel back of the shields.

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[12] Engr. in Chg., Cobble Mountain Reservoir, Springfield Water-Works, Westfield, Mass.

More information on the grouting would have been welcome. Practical experience on various tunnel and foundation grouting has proved that better work can be obtained with neat cement than with an admixture of sand. The thinner the grout the farther is its penetration. Often, with sand-cement grout it appears that the sand, being heavier, will stop short during the discharge of the grout and let the fluid travel past it. This process gradually will form sand pockets near the discharge of the grout pipe and eventually will prevent the passage of more grout.

In a rock tunnel with concrete lining, it is possible to compute the minimum quantity of grout necessary to fill the voids back of the concrete. To this amount, of course, should be added the estimate of grout necessary to seal up the rock seams, if any. It has been found that the grout will shrink and that the excess water, especially in thin mixtures, will occupy space, provided the pressure does not drive it away. Often it is this excess water that will find weak spots in the concrete lining and seep through, thus guiding the engineer in locating at least the outlet of water from voids for additional grouting. It is questionable whether, in shield tunneling, the quantity of grout can be estimated in advance. It would be instructive to know just what methods were used to make positively sure that the grouting was complete. Were there any investigations or attempts to determine whether additional grout could be forced back of the shields after the completion of the first grouting?

MILES I. KILLMER,[13] M. AM. SOC. C. E. (by letter).—Mr. Feld comments on the omission of any description of the methods of running line. This phase of the work was done by the engineers of the Board of Transportation of the City of New York. It involves triangulation, base-line measurement, dropping line down the shafts, prolonging line through the lock bulkheads, and many other special methods. It is an extensive subject, and it could not be treated adequately, except in a separate paper. The work was done with great accuracy by the Engineers of the Board, and the Manhattan and Brooklyn survey lines checked at the junction of the shields within $\frac{1}{4}$ to $\frac{1}{2}$ in.

The discussion by Mr. Hatch requires some comment. The air pressure in a sand heading does not support the ground. The pressure used is such as to balance the head of ground-water at the bottom of the tunnel and is not influenced by the presence or absence of near-by building loads. The actual pressure in the land headings was generally not more than 18 lb. The 48-lb. pressure was required only in putting down the sumps below the tunnels at the lowest point under the river. The ground is supported by the cast-iron tunnel lining surrounded by its envelope of ejected pea gravel and grout, by the exterior of the skin on the shield, and, at the face, by breast-boards held in place by braces against the shield. When the shield is to be advanced, vertical timbers ("soldiers") are placed against the breast-boards. From these "soldiers," horizontal stretchers extend back through the pockets of the shield to cross-timbers ("walking sticks") wedged into the last cast-iron ring erected. The braces that had originally supported the breast against the shield are then knocked out, and the shield can be advanced without disturbing the support of the face. Under certain conditions, the horizontal tables can be used to support the face. They are shoved out by means of their rams until they make contact with the "soldiers." When the shield is advanced the hydraulic pressure behind the rams is released slowly, and the tables come back as the shield goes ahead.

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[13] Mgr., Mason & Hanger Co., Inc., New York, N. Y.

All the foregoing applies to a sand or gravel heading. If the face is of semi-plastic clay, a condition arises in which the pressure of air in the tunnel may be utilized to support the face, or at least to supplement other methods of supporting it.

The possibility of using sand with the pea gravel to secure an enveloping back-fill having a smaller percentage of voids, is mentioned by Mr. Hatch. This idea was successfully put in practice in the construction of the traffic tunnel from Boston to East Boston, Mass., recently completed. It was found that about one-third sand could be used with the pea gravel; a larger percentage caused stoppages in the tank or in the hose. In the Boston Tunnel, the surrounding ground was a stiff clay that did not fall into the void left by the advancing tail of the shield. It is the writer's opinion that where the surrounding ground is sand or gravel and tends to fall almost immediately into the annular space, pea gravel without sand will prove more effective as its granular nature makes it more fluid or mobile. The air pressure used in ejecting the gravel was the same as that used in grouting, that is about 80 lb.

Mr. Hatch also suggests the use of neat-cement grout to secure greater penetration. This was tried on a subsequent job with which the writer was connected. In driving two shields up Jay Street, Brooklyn, a very loose running sand and gravel was encountered. Some heavy buildings had to be passed. The quantity of 1 to 1 grout ejected was not fully satisfactory and neat-cement grout was resorted to for a distance of several hundred feet. The computed volume of the grout actually ejected, proved to be about equal to the computed volume of the 1 to 1 grout previously ejected, per foot of tunnel. In this case, therefore, the extra expense was not warranted. Of course, in grouting rock seams, neat-cement grout would probably be superior to the 1 to 1 mix.

Although the question of supporting buildings along a street seems to have been solved from an engineering point of view, there still remains the problem of defending suits brought by owners who, in many instances, will seize upon the occasion of the presence of a contractor to bring suit even when the physical damages are so slight as to require magnifying glasses to be seen.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 1842

WIND-BRACING CONNECTION EFFICIENCY

By U. T. Berg,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. David Cushman Coyle, William R. Osgood, O. G. Julian, Harold S. Richmond, Robins Fleming, L. E. Grinter, C. R. Young, C. H. Sandberg, N. A. Richards, Jacob Feld, Dana Young, A. J. Wilcox, R. D. Dalton, John J. Gould, W. C. Huntington, and Paul Nielsen, A. Toriggino and Erle L. Cope, R. A. Caughey, and U. T. Berg

Synopsis

This paper is intended as a warning, especially on wind connections in which negative moments due to gravity loads are neglected. Very high stresses in the connection members may be relatively harmless in themselves, but dangerous as far as the tension rivets are concerned, especially under alternate loading, where a gradual loosening may result.

Introduction

Wind resistance in structures is divided between resistance exerted by steel, walls, partitions, etc., in an unknown ratio. The mass inertia creates additional resistance against gusts of wind. The modern tendency toward increased height and relatively lighter weights intensifies the deflection and brings the question of rational design to the front.

What is the use of providing thousands of rivets that can not possibly do the work expected, and other rivets stressed to such a point that a few years of service will loosen them and increase the sway? To make architects "kneebrace-minded" would go far toward increased strength, as a conscientious design of the present-day types of connection meets many difficulties due to the large number of rivets required.

The term, "educated moments," is sometimes used in reinforced concrete design to denote that the reinforcement is placed more to suit the practical condition than the elastic theory can justify. Instead of providing for a

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Note.—Published in January, 1932, *Proceedings*.

[1] Structural Engr., Brooklyn, N. Y.

support moment, $\frac{W\,L}{12}$, a smaller value, such as $\frac{W\,L}{16}$, or $\frac{W\,L}{24}$, is substituted, assuming that a re-adjustment of stresses will take place.

Tests substantiate the claim that, at working stresses and intermediate stages, re-adjustments take place only to a slight degree, but, at ultimate loads, re-adjustments become very marked.

Fig. 1 illustrates the relations between computed and measured moments obtained from tests[2] on continuous beams. The conditions are that the beams are completely fixed at the ends; 47% of the bottom reinforcement steel is extended over the support. Steel designers generally take for granted that stresses will be distributed according to the connection; that the weaker parts will "give;" and that there is a balancing of stresses from the weaker to the stronger member. This practice may be justified within certain limits.

A beam with standard connections, as shown in Fig. 2, is designed as simply supported. If the column is unyielding, the connection angles must be sufficiently elastic to permit the beam ends to rotate. Angles, $\frac{3}{8}$ in. thick, with 4-in. legs, will readily satisfy this requirement. The computed stresses in the angles are high, but harmless, because at no time is there any danger of excessive pull on the rivets. However, if the legs are made very thick the rivets, which are generally considered as taking shearing stresses only, will be called upon to take heavy tensile stress in addition, due to the increased strength of the connection.

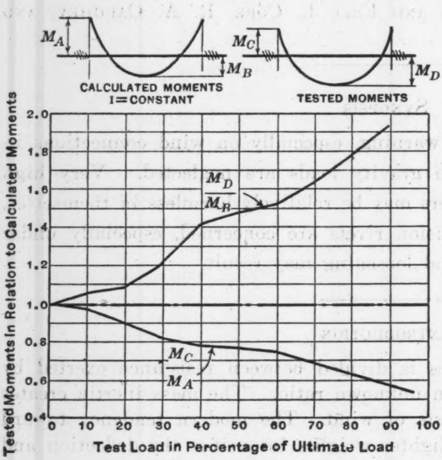

FIG. 1.

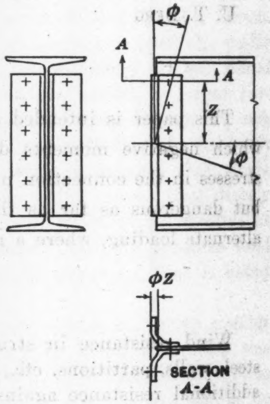

FIG. 2.

Notation.—In addition to definitions given directly in the text, the notation in this paper is arranged for convenience in Appendix I.

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[2] Deutschen Ausschuss für Eisenbeton, 41–47, "Versuche mit eingespannten Eisenbetonbalken," Technischen Hochschule zu Stuttgart, by Dr. Ing. C. Bach and O. Graf.

TENSION RIVETS

It is logical to base the design on the stage in which the initial tension is reached on the rivet, and to express the factor of safety, as follows:

$$\text{Factor of safety} = \frac{\text{Initial tension value of rivet}}{\text{Total tension stress on rivet}}$$

It is readily seen that the I-beam stub connection (Fig. 3 (*a*)), with a tension force of the value, 2 *S*, cannot deform as illustrated in Fig. 3 (*b*). The initial tension in rivets would have to be overcome to permit the rivets to elongate. A force, $R - S$, is developed which maintains the edge in position as shown by Fig. 3 (*c*). When the initial tension is reached, it is not unreasonable to assume that the force, $R - S$, is distributed as a triangular load, zero at the rivet center, and maximum at the edge. This would locate the center of gravity of the force at a distance equal to $\frac{2}{3}$ *C* from the rivet center. (Actually the pressure diagram is parabolic due to the deformation of the plate. However, the corresponding shorter arm is compensated for by the fact that the rotation due to the variation of pressure between the plates reduces the fixing moment at the rivet.)

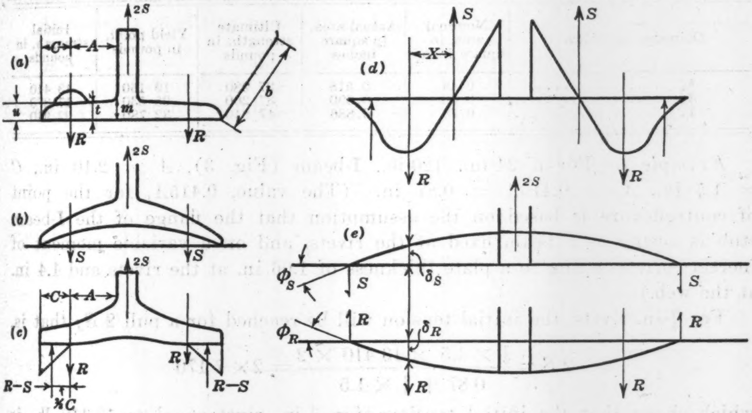

FIG. 3.

If *X* denotes the distance from the point of contraflexure to the rivet center, the following equations can be written:

$$S X = (R - S) \tfrac{2}{3} C \dots\dots\dots\dots\dots\dots\dots (1)$$

or,

$$R = \frac{S(X + \frac{2}{3} C)}{\frac{2}{3} C} \dots\dots\dots\dots\dots\dots\dots (2a)$$

$$S = \frac{\frac{2}{3} C R}{X + \frac{2}{3} C} \dots\dots\dots\dots\dots\dots\dots (2b)$$

Reliable tests are available on tension rivets.[8] For red-hot driven rivets, the ultimate strength was found to be 54 000 lb. per sq. in.; the yield point,

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[8] *Bulletin No. 210*, Eng. Experiment Station, Univ. of Illinois, by Wilbur M. Wilson, M. Am. Soc. C. E., and William A. Oliver, Assoc. M. Am. Soc. C. E.

37 000 lb. per sq. in.; and the initial stress at least 70% of the yield-point strength, or 25 900 lb. per sq. in. Table 1 gives the foregoing values applied to the rivets commonly used.

There is some element of doubt relative to the value, C, in Equation (1). It is possible that a reduced value of the distance from rivet center to the effective bearing edge should be entered in the equation when used for thinner plates, so that $C_{\text{max.}}$ is a function of the thickness of the plate plus one-half the diameter of the rivet head. However, in most practical cases, the plate thicknesses are such that this question does not have any importance.

TABLE 1.—DIMENSIONS AND STRENGTHS OF RIVETS

Diameter, in inches	Nominal area, in square inches	Actual area, in square inches	Ultimate strength, in pounds	Yield point, in pounds	Initial strength, in pounds
¾	0.44	0.518	27 980	19 160	13 410
⅞	0.60	0.690	37 260	25 530	17 870
1	0.78	0.886	47 840	32 780	22 940

Example 1.—For a 24-in., 120-lb., I-beam (Fig. 3), $A = 2.10$ in., $C = 1.5$ in., $X = 0.415A = 0.87$ in. (The value, $0.415A$, for the point of contraflexure is based on the assumption that the flange of the I-beam stub is acting as a beam fixed at the rivets, and on a variable moment of inertia corresponding to a plate thickness of 1.06 in. at the rivets and 1.4 in. at the web.)

For $\frac{3}{4}$-in. rivets, the initial tension will be reached for a pull $2\,S$; that is,

$$2\,S = \frac{\frac{2}{3} \times 1.5 \times 13\,410 \times 2}{0.87 + \frac{2}{3} \times 1.5} = 2 \times 7\,170$$

which shows that the initial tension of a $\frac{3}{4}$-in. rivet equal to 13 410 lb. is overcome by the application of a force of only 7 170 lb. acting as described in this example.

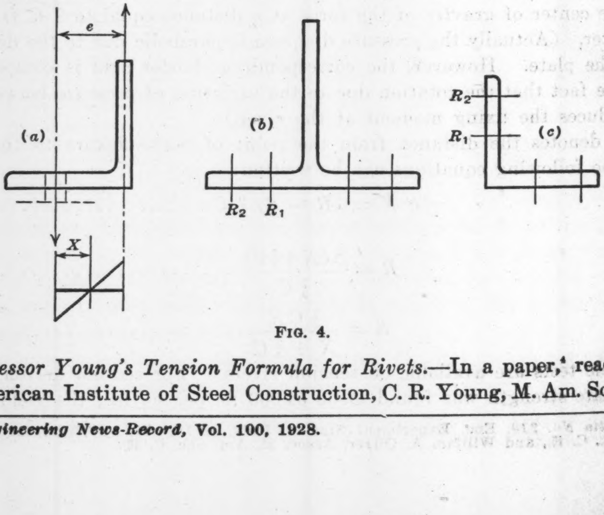

FIG. 4.

Professor Young's Tension Formula for Rivets.—In a paper,[4] read before the American Institute of Steel Construction, C. R. Young, M. Am. Soc. C. E., presented the following equation for tension stress in rivets:

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[4] *Engineering News-Record,* Vol. 100, 1928.

$$f_r = 21\,000 - 8\,000\,d - 5\,500\sqrt{e}\ldots\ldots\ldots\ldots\ldots(3)$$

in which, f_r is the permissible tension stress on the rivet (factor of safety of 4); d, the diameter of the rivet, in inches, before driving; and, e, the eccentricity of loading on the rivet.

Equation (3) is based on tests using 4 by 4-in. angles. In comparing the values from it with the values derived from Equation (2*b*), it becomes important to use the right judgment relative to the eccentricity, e (Fig. 4(*a*)). If the test angles were free to move sidewise without any restraint on the outstanding leg, the values, e and X, should correspond. However, this condition is unlikely.

Assuming a condition as shown in Fig. 5 (*a*) and Fig. 5 (*b*), and expressed by $A - X = \frac{2\,A^2}{4\,A + B}$, the point of contraflexure of the angle would be located at a distance of about 0.65 e from the rivet center. (The foregoing equation is derived subsequently.) Table 2 has been prepared giving permissible pull on the connection per rivet on the following assumptions:

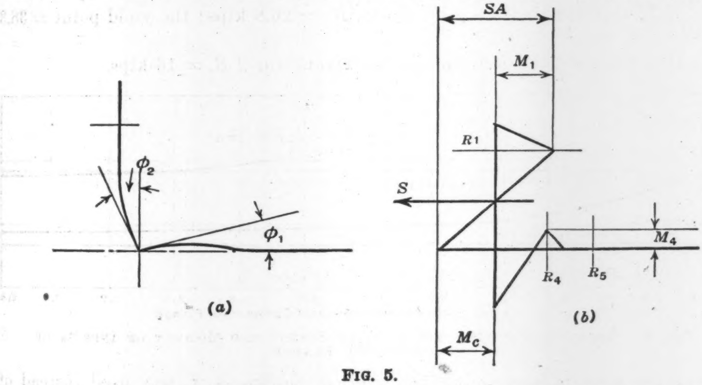


FIG. 5.

For values from Equation (2*b*):

Factor of safety of 2 on initial stress on gross area of rivet, $C = 1.5$ in.

For values from Equation (3):

Factor of safety of 4 on ultimate strength, $e = \frac{X}{0.65}$.

TABLE 2.—COMPARISON OF WORKING STRESSES

Nominal size, in inches	X, in inches	Eccentricity, $e = \frac{X}{0.65}$ in inches	PERMISSIBLE TENSION PER RIVET, IN POUNDS	
			Equation (2*b*)	Equation (3)
3/4	0.75	1.15	3 840	4 000
3/4	1.50	2.31	2 690	2 930
7/8	0.75	1.15	5 100	4 850
7/8	1.50	2.31	3 560	3 400
1	0.75	1.15	6 500	5 500
1	1.50	2.31	4 600	3 630

Equation (1) applies only for stages at the initial tension of the rivet. Lower values are meaningless because the rivet stress is never less than the initial tension value. From this point on, the rivet elongation must be taken into account.

The following equation can be derived from the loading condition, as shown in Fig. 3(*e*):

$$\delta_S - \delta_R = \Delta_R = (R - R_i)\,\frac{g}{EF} \qquad (4)$$

in which, R is the total stress on the rivet; R_i, the total initial stress on the rivet; g, the rivet grip; F, the area of the rivet; E, the modulus of elasticity; Δ_R, rivet elongation; δ_R, rivet elongation due to the force, R; and δ_S, rivet elongation due to the force, S.

Fig. 6 illustrates the relation between the total rivet stress (for two rivets) and the moment of inertia of the connection flange for a condition of loading as shown in Fig. 3(*a*). The values used in plotting the curves were: $I = \frac{1}{12}\,bt^3$; and, for the two $\frac{3}{4}$-in. rivets, $R_i = 26.8$ kips; the yield point $= 38.3$ kips; and, the total tension, $2R$, on two rivets, for $2\,S, = 16$ kips.

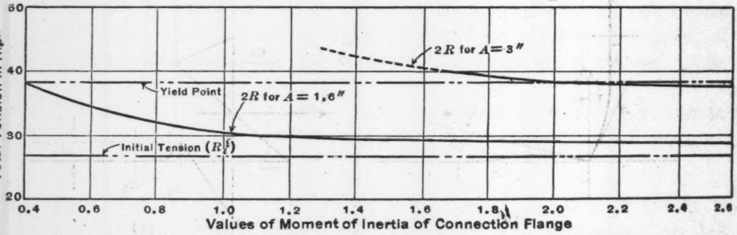


FIG. 6.—RELATION BETWEEN TOTAL RIVET STRESS AND MOMENT OF INERTIA OF CONNECTION FLANGE

For the sake of simplicity, the average thickness, *t*, was used instead of variations from *u* to *m* (Fig. 3(*a*)). From a study of Fig. 6, it is forcibly shown that the total tension developed on connection rivets under a constant force, 2 *S*, varies materially with the gauge distance, *a*, and with the thickness of the connection member up to a certain limit, after which increased thickness does not materially affect the tension developed.

Connections with more than one line of rivets, such as shown in Fig. 4, have merit only because of increased shearing value. Analyzed on the elastic theory, using commercial sizes, it is not possible to rely on any tension on the second line of rivets, R_2, unless the stress in the first line of rivets, R_1, exceeds not only the initial tension of the rivet, but also the yield point.

FLANGE ANGLE CONNECTIONS

A bending moment, *M*, at the joint, shown in Fig. 7 (*b*), is resisted by a couple, of force, *S*, acting on an arm, *Y*. The compression force, *S*, is transmitted directly into the column. The only bending effect at the lower angle is the influence of an eccentric action for the vertical load.

At the top, the tensile force would tend to pull the angle away from the face of the column, introducing bending moments in both legs of the angle.

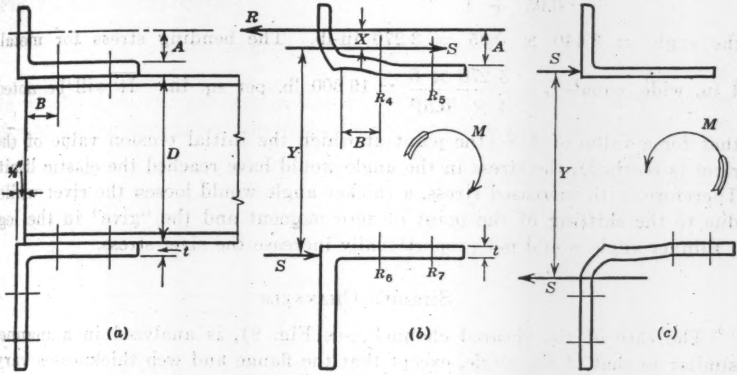


FIG. 7.

The elastic line of the deformed angle remains vertical at Rivet R_1, and horizontal at Rivet R_4. Rivet R_5 has no other effect than horizontal shearing resistance.

Referring to Fig. 5, the angle is assumed to be completely fixed at R_4. Hence, $M_4 = -\frac{1}{2} M_c$; and, $\phi_1 = \frac{1}{4} M_c \frac{B}{EI} = \phi_2 = \frac{SA^2}{2EI} - \frac{M_cA}{EI}$.

The corner moment (M_c in Fig. 5(*b*)), is,

$$\frac{2SA^2}{4A + B} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

and the distance from the upper face of the angle to the point of zero moment equals,

$$(A - X) = \frac{2A^2}{4A + B} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The total moment at the joint, M, equals $S\,Y$. Let the thickness of metal equal t, and the depth of the beam, D; then (Fig. 7),

$$Y = D + \frac{t}{2} + t + A - X \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

For a moment in the opposite direction, the effect on the angles is reversed. The combined effect of bending on the lower angle from a force, S, plus eccentricity from the application of the direct load, would make a more serious condition than is the case for the top, except for the fact that, in most cases, the moment from the gravity loads is deductible from the wind moment, reducing tension at the bottom.

Example 2.—For a top angle, 6 by $3\frac{1}{2}$ by $\frac{1}{2}$ in., $A = 2 - \frac{1}{2} = 1\frac{1}{2}$ in.; $B = 2\frac{1}{2} - \frac{3}{8} = 2.13$ in.; and, $C = 1.5$ in. From Equation (6), $A - X = 0.55$, and $X = 0.95$ in. The permissible pull per rivet with a factor of safety of 2 is, $S = \frac{1}{2} \times \dfrac{13\,410}{0.95 + 1} = 3\,440$. The maximum bending moment on the angle $= 3\,440 \times 0.95 = 3\,270$ in-lb. The bending stress for metal, 4 in. wide, equals $f_s = \dfrac{3\,270 \times 6}{4 \times (0.5)^2} = 19\,600$ lb. per sq. in. It will be noted that for a value of 2 S (the point at which the initial tension value of the rivet is reached), the stress in the angle would have reached the elastic limit. Therefore, with increased stress, a thicker angle would loosen the rivet while, due to the shifting of the point of zero moment and the "give" in the leg, a thinner angle would not proportionally increase the rivet stress.

SHEARED CHANNELS

The case of the sheared channel (see Fig. 8), is analyzed in a manner similar to that of the angle, except that the flange and web thicknesses vary. If the average moment of inertia of the flange is I_F and that of the web, I_W, Equation (6) can be re-written for channels:

$$(A - X) = \frac{A^2}{2A + \dfrac{I_F}{I_W} \times \dfrac{B}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

SHEARED I-BEAMS

This case has been covered under the section, "Tension Rivets." Note that $Y = D + t$. For a constant thickness, t, of the flange, $X = \frac{1}{2} A$, and for a straight-line variation from u (the thickness at the rivet) to m (the thickness at the web), the distance, X, is equal approximately to,

$$X = \frac{u}{u + m} A \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

BEHAVIOR BEYOND THE ELASTIC LIMIT

The equations developed for the location of points of contraflexure of connections have been based on stresses within the elastic limit. Above the elastic limit, re-adjustments take place. The writer is not in a position definitely to determine these re-adjustments. Probably tests would be required. However, some approximate values can be derived. A redistribution of the moments in relation to the section modulus does not seem unreasonable. In that case, the distance, X, equals, approximately,

$$X = \frac{u^2}{u^2 + m^2} A \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The same reasoning applied to angles would make $X = \frac{1}{2} A$ for all cases above the elastic limit.

ROTATION FROM DEFORMATION OF CONNECTIONS

The rotation of the beam end, due to deformation of the connection (see Fig. 9), can be expressed:

$$\phi = \tan \phi = \frac{(\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4)\ S}{D + t} \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which, Δ_1 is the elongation due to bending in the top connection from $S = 1$; Δ_2, the elongation due to tension in the top connection only, for $S = 1$;

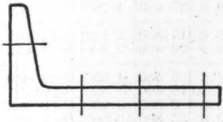


FIG. 8.

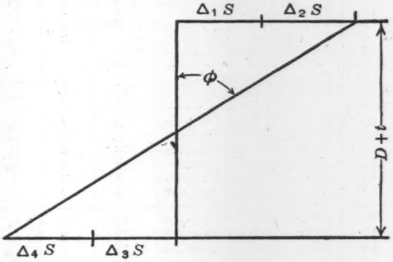


FIG. 9.

Δ_3, the shortening in the bottom connection due to $S = 1$; and, Δ_4, the shortening due to compression when $S = 1$.

For a case as shown in Fig. 7(*b*):

$$\Delta_1 = \frac{1}{3\ E\ I}\left[X^3 + (A - X)^3 + (A - X)^2\ B\ \tfrac{3}{4}\right] \ldots\ldots\ldots (12)$$

and,

$$\Delta_2 = \frac{l}{E\ F'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

in which, l is the effective length of connection, and F', the total area of web for sheared I-beams or horizontal leg for angles. Δ_3 = shortening in the bottom connection due to bending from $S = 1$. In most cases this value is zero, as the connection butts up against the column or adjacent connection. Δ_4 = shortening due to compression from $S = 1$. For most cases $\Delta_4 = \Delta_2$.

For a total moment, M, at the joint, ϕ is the angle of rotation due to deformation (see Fig. 9), and can be expressed: $\phi = k\ M$, or,

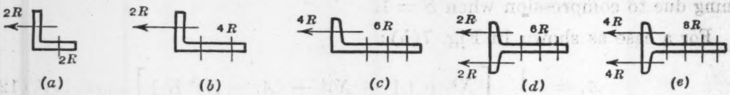
$$k = \frac{\phi}{M} = \frac{\Sigma\ \Delta\ S}{(D + t)\ Y\ S} = \frac{\Sigma\ \Delta}{(D + t)\ Y} \ldots\ldots\ldots\ldots\ldots (14)$$

Table 3 gives values of Δ_1 and Δ_2, when S equals unity, for typical commercial connections. This table also gives the total pull, S, which stresses rivets up to 75% of the initial tension value, and the corresponding stress on flange and web. In Column (1), the various types correspond with those shown in Fig. 10.

TABLE 3.—CHARACTERISTICS OF TYPICAL COMMERCIAL CONNECTIONS

Type of member (see Fig. 10.) (1)	Item No. (2)	Size (3)	Length, b, in inches (4)	TENSION RIVETS Number (5)	TENSION RIVETS Diameter, in inches (6)	Total pull on connection, in kips (7)	UNIT STRESS, f, IN POUNDS PER SQUARE INCH f_F (8)	UNIT STRESS, f, IN POUNDS PER SQUARE INCH f_W (9)	Lever arm, y, in inches (10)	ELONGATION FACTOR $E\Delta_1$ (11)	ELONGATION FACTOR $E\Delta_2$ (12)
a	1	4 by 3 in. by 5/16 in.....	6	2	¾	5.3	37 000	2 800	....	.....	.....
	2	4 by 3 in. by ½ in.....	6	2	¾	10.3	33 400	3 400	$D+1.42$	13.2	0.85
	3	6 by 3½ in. by ½ in...	6	2	¾	7.1	27 000	2 340	$D+1.05$	8.40	1.50
	4	6 by 3½ in. by ⅝ in...	.....	2	¾	10.7	24 000	2 880	$D+1.11$	3.26	1.20
	5	6 by 3½ in. by ¾ in...	6	2	¾	11.1	16 000	2 460	$D+1.19$	1.46	1.00
	6	6 by 3½ in. by ¾ in...	.....	2	⅞	14.8	21 400	3 300	$D+1.19$	1.46	1.00
	7	6 by 3½ in. by ¾ in...	6	2	1	19.0	27 400	4 200	$D+1.19$	1.46	1.00
c	8	15-in., 50-lb. channel...	12	4	¾	18.0	14 800	2 120	$D+1.47$	2.50	0.70
	9	15-in., 50-lb. channel...	12	4	⅞	24.2	20 000	2 800	$D+1.47$	2.50	0.70
d	10	20-in., 81.4-lb. I-beams.	8	4	¾	22.4	6 550	4 650	$D+0.60$	0.24	1.25
	11	20-in., 56.0-lb. beams*.	8	4	¾	22.50	19 400	7 500	$D+0.37$	1.25	2.00
	12	20-in., 81.4-lb. I-beams.	8	4	⅞	30.0	8 800	6 200	$D+0.60$	0.24	1.25
	13	20-in., 56.0-lb. beam*.	8	4	⅞	30.0	25 800	10 000	$D+0.37$	1.25	2.00
	14	20-in., 81.4-lb. I-beams.	8	4	1	38.4	11 250	8 000	$D+0.60$	0.24	1.25
	15	20-in., 56.0-lb. beams*.	8	4	1	38.50	33 000	12 800	$D+0.37$	1.25	2.00
e	16	20-in., 81.4-lb I-beams.	12	8	¾	44.8	8 700	6 250	$D+0.60$	0.16	1.04
	17	20-in., 56.0-lb. beams*.	12	8	¾	45.00	25 800	10 000	$D+0.37$	0.84	1.78
	18	20-in., 81.4-lb. I-beams.	12	8	⅞	60.00	11 700	8 320	$D+0.60$	0.16	1.04
	19	20-in., 56.0-lb. beam*.	12	8	⅞	60.00	34 200	13 300	$D+0.37$	0.84	1.78
	20	20-in., 81.4-lb. I-beams.	12	8	1	76.8	15 000	10 700	$D+0.60$	0.16	1.04
	21	20-in., 56.0-lb. beam*.	12	8	1	77.0	44 000	17 000	$D+0.37$	0.84	1.78

* Bethlehem Section; C (assumed) = $\frac{1}{2}$ rivet head + $\frac{3}{2}$ thickness, u, at the rivet.



(a) (b) (c) (d) (e)

FIG. 10.

JOINTS

Gravity Loads Only.—For joint equilibrium the following rotations must balance:

(*a*) The rotation due to a gravity load on a freely supported span is,

$$\phi_{GA} = \frac{W L^3}{24 E I} \qquad \text{.........................(15)}$$

(*b*) The rotation due to negative moments induced by gravity loads is,

$$\phi_{MA} = \frac{L}{E I}\left[\frac{M_{GA}}{3} + \frac{M_{GB}}{6}\right] \qquad \text{..................(16)}$$

and, for a typical symmetrical case, in which, $M_{GA} = M_{GB} = M_G$,

$$\phi_{MA} = \frac{L}{2 E I} M_G \qquad \text{.........................(17)}$$

(*c*) The rotation due to the connection is, $\phi_k = k\, M_{GA}$; thus,

$$\frac{W L^3}{24 E I} = \frac{L}{2 E I} M_G + k\, M_G$$

and,

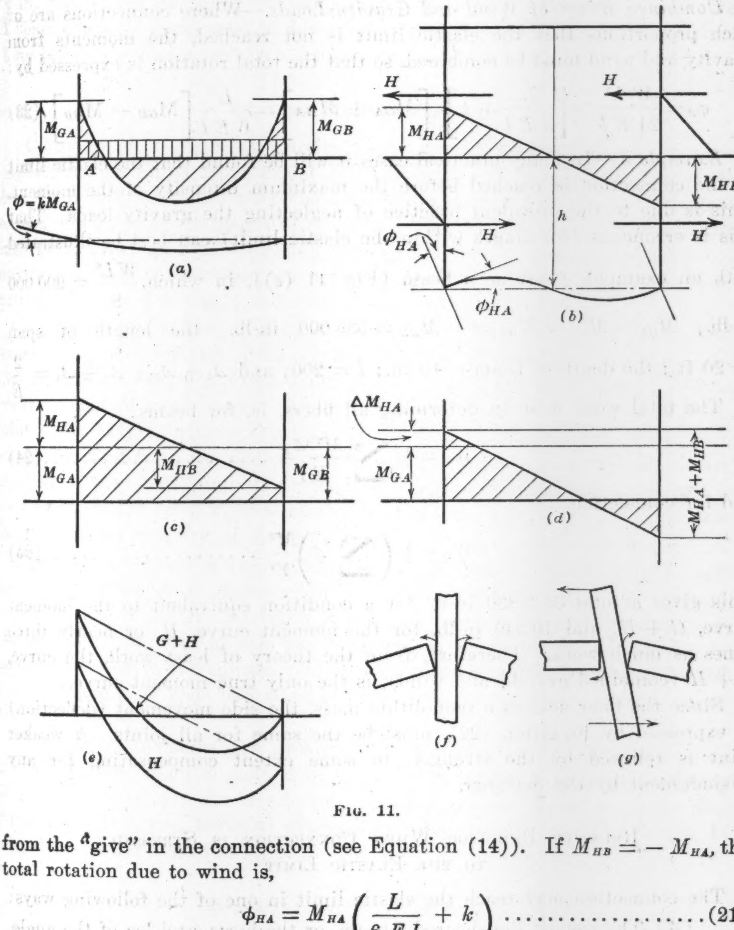
$$M_G = \frac{1}{L + 2 E I k} \times \frac{W L^3}{12} \qquad \text{..................(18)}$$

Wind Forces Only.—For wind forces, as shown in Fig. 11(*b*), and for the corresponding moments, M_{HA} and M_{HB}, the column rotation can be expressed:

$$\phi'_{HA} = \frac{L}{6\,E\,I}\,(2\,M_{HA} + M_{HB}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

from beam deformation, and,

$$\phi''_{HA} = k\,M_{HA} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$



M_{GA}
M_{GB}
A
B
$\phi = kM_{GA}$
(a)
H
H
M_{HA}
M_{HB}
h
H
H
ϕ_{HA}
ϕ_{HA}
(b)
M_{HA}
M_{GA}
M_{HB}
M_{GB}
(c)
ΔM_{HA}
M_{GA}
$M_{HA}+M_{HB}$
(d)
G+H
H
(e)
(f)
(g)

FIG. 11.

from the "give" in the connection (see Equation (14)). If $M_{HB} = -M_{HA}$, the total rotation due to wind is,

$$\phi_{HA} = M_{HA}\left(\frac{L}{6\,E\,I} + k\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Equation (21) expresses the rotation of the joint due to the wind forces, including the effect of the "give" in the connection. This rotation, it should be noted, also expresses the unit horizontal deflection, as the influence from the gravity loads is small and can be neglected, so that the horizontal deflection from one story to the next can be assumed equal to:

$$q = \phi_{HA}\, h + \left\{ \frac{H\, h^3}{12\, E\, I_c} \right\} \quad \text{.....................(22)}$$

in which, h is the story height; H, the wind force (Fig 11(b)); and I_c, the moment of inertia of the column.

Combined Effect of Wind and Gravity Loads.—Where connections are of such proportions that the elastic limit is not reached, the moments from gravity and wind must be combined, so that the total rotation is expressed by:

$$\phi_A = \frac{W\, L^3}{24\, E\, I} - \left[\frac{L}{3\, E\, I} + k \right] \left[M_{GA} + M_{HA} \right] - \frac{L}{6\, E\, I} \left[M_{GB} - M_{HB} \right] . \text{(23)}$$

Example 3.—In many practical cases it will be found that the elastic limit of the connection is reached before the maximum intensity of the moment. This is due to the prevalent practice of neglecting the gravity loads. That this is erroneous (for stages within the elastic limit) can best be illustrated with an example. Assume a beam (Fig. 11 (e)), in which, $\frac{W L^2}{8} = 900\,000$ in-lb.; $M_{GA} = M_{GB} = M_{HA} = - M_{HB} = 585\,000$ in-lb.; the length of span $= 20$ ft.; the depth of beam $= 12$ in.; $I = 200$; and $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 = \frac{2}{E}$

The total work done in deforming all fibers, is, for beams:

$$W_1 = \tfrac{1}{2} \sum \frac{M^2\, \Delta_x}{E\, I} \quad \text{.....................(24)}$$

and for connection,

$$W_2 = \tfrac{1}{2} \left(\sum \Delta \right) \frac{M^2}{Y^2} \quad \text{...................(25)}$$

This gives a total of 3 830 in-lb. for a condition equivalent to the moment curve, $G + H$, and 10 940 in-lb. for the moment curve, H, or nearly three times as much work. Therefore, from the theory of least work, the curve, $G + H$ (combined gravity and wind), is the only true moment curve.

Since the floor acts as a monolithic mass, the side movement (deflection) as expressed by Equation (22), must be the same for all joints. A weaker joint is relieved by the stronger, to some extent compensating for any misjudgment by the designer.

RELATIVE BEHAVIOR WHEN CONNECTION IS STRESSED TO THE ELASTIC LIMIT

The connection may reach the elastic limit in one of the following ways:

(A) The web of the sheared I-beam, or the horizontal leg of the angle, reaches the elastic limit from tension.
(B) The flange of the sheared I-beam, or the vertical leg of the angle, reaches the elastic limit from bending.
(C) The rivet stress reaches the elastic limit.

If it is not possible to design the connection within the elastic limit, the writer believes that care should be taken at least to protect the rivets from being overstressed.

A study of the relative stresses as shown Table 3 will help to balance the design. For example, for flange-angle connection, the rivets are relatively stronger than the vertical leg up to and including angles, $\frac{5}{8}$ in. thick, for $\frac{3}{4}$-in. rivets, and angles, $\frac{3}{4}$ in. thick, for $\frac{7}{8}$-in. rivets. Many of the sheared I-beam connections as used to-day would reach the elastic limit on the rivets long before the flange and web stresses are nearly that high.

Bending Moment Effects

Consider for a moment gravity loads alone. Comparing the bending moment as computed from Equation (18), with the rigid frame (in which, $k = 0$), it will be found that the connection does not lower, to any great extent, the value of the negative moment. The effect of an angle connection may be as great as 10% reduction, while the effect of the sheared I-beam is nearer 4 per cent. Therefore, the moments as obtained from monolithic frame action are from 4 to 10%, and even more, on the safe side.

Designers should not neglect checking the connection for gravity loads only. Especially for the upper stories, under relatively smaller wind forces, the negative moments induced from gravity loads will predominate. These gravity negative moments are serious because they exist on both sides of the joint (Fig. 11(*f*)), with approximately the same intensity, and there is no balancing effect from the weaker to the stronger connection; while in a wind moment, if the elastic limit is reached on one side of the column, the bottom connection on the opposite side of the same column (Fig. 11(*g*)), will immediately take its capacity of the remainder.

Fig. 11(*d*) illustrates the case in which the gravity loads do not carry the stresses to the elastic limit. Not all the wind moment, M_{HA}, but a part of it, can be added before the limit is reached. A redistribution takes place whereby the remainder of the wind moment is taken by the connection on the opposite side of the column. (The wall column is an exception. In this case the monolithic floor will re-adjust the stresses, transferring them to the interior joints.)

If the wind moments are greater than the gravity moments, the main objection to the common method of using wind moments only in designing the connection, is that the elastic limit is likely to be reached on some part of the connection and harmful effects may arise, particularly the danger of loose rivets under repeated stress.

Summary and Conclusions

Formulas have been developed for the determination of tension in rivets, effective lever arm of connection, rotation of joint, joint equilibrium, horizontal deflection (sway), etc., for the commonly used connections, such as flange angles, sheared channels, and sheared I-beams. The insecurity in the present method of rivet design has been illustrated, showing particularly how a stress of 18 000 lb. per sq. in. on the rivet as used by many designers, may develop stresses actually twice that amount.

The importance of analyzing the relative strength of the various parts of the connection has been stressed, and a warning against making the rivets the weakest link has been given. Finally, the designer has been cautioned against neglecting gravity loads, particularly under the condition when the wind forces are comparatively small.

Discrepancies between current practice and rational analysis emphasize the need for further experimentations along these lines.

With the introduction of lighter forms of construction, making buildings more vulnerable to wind, the influence of riveted connections on the life of structures becomes increasingly important, and should be reflected in better balanced details.

APPENDIX I

NOTATION

In the paper, the following notation is adopted:

a = gauge distance.
b = length of a structural element in a connection.
d = diameter of rivet before driving.
e = eccentricity of a load acting on a rivet.
f = unit stress; f_s, due to bending; f_F, stress in the flange, or in the vertical leg of an angle; and f_w, tension stress in the web, or in the horizontal leg of an angle; f_r, stress in rivet.
g = rivet grip.
h = story height.
k = a coefficient in Equation (14) $= \frac{\phi}{M}$.
l = effective length of a connection.
m = thickness of a flange at the face of the web.
q = horizontal deflection.
t = average thickness of a flange between the center of a rivet and the face of the web.
u = thickness of the flange at the center line of the rivet.
w = work (w_1) in beams, or (w_2) in connections.
z = a polar ordinate (see Fig. 2).
A = distance from face of web to center line of rivet.
B = rivet distance.
C = distance from center line of rivet to effective bearing edge of flange.
D = depth of beam.
E = modulus of elasticity.
F = area of a rivet; F' = total area of web for sheared I-beams, or horizontal leg for angles.
H = a wind force.
I = moment of inertia; I_F = average for the flange; I_W = average for the web; and I_C = average for the column.
L = span length.

M = bending moment. (A subscript, c, denotes moment in the corner of an angle; Subscripts g and h denote moments due to gravity and wind loads, respectively; and Subscripts A and B denote moments at the ends, A and B, respectively, of a beam.)

R = a tensile force on a rivet. (A subscript, i, denotes initial force.)

S = an eccentric force opposing R.

W = total weight per foot.

X = distance from the point of contraflexure to the center line of rivet.

Y = a moment arm.

δ = the elongation of a rivet; δ_S, due to the force, S; and δ_R, due to the force, R.

Δ = total elongation, or shortening of a member. Numerical subscripts denote cause of elongation, as follows: (1) Bending in the top connection due to $S = 1$; (2) tension in the top connection due to $S = 1$; (3) bending under $S = 1$, causing shortening in bottom connection; and (4) compression under $S = 1$, causing shortening.

ϕ = rotation of the end of a beam due to the deformation of the section, ϕ_k, due to the connection; and ϕ_M, due to bending moments induced by gravity loads. The significance of the subscripts, G, H, A, and B, is as defined under M.

DISCUSSION

David Cushman Coyle,[5] **M. Am. Soc. C. E. (by letter).**—The author is quite correct in stating that it is sometimes unsafe to disregard the dead load negative moments which are to be added to the wind moments usually figured on the beam connection. His analysis in detail is, however, open to some qualification.

Consider the connection shown in Fig. 3 (*a*), regarding the possibility of loosening the rivets. Assume two ¾-in. rivets for which A equals 1.6 in., t is 1.4 in., and $C = 1\frac{1}{2}$ in. Let the total pull equal the yield-point strength of the two rivets, 38 320 lb. Then, by Equation (2*a*):

$$2R = 38\,320 \times \frac{(0.67 + 1)}{1} = 64\,000$$

As this stress is far above the yield point, the rivets will yield until they are as in Fig. 3 (*b*), when the rivet stress will have become $2R = 2S = 38\,320$ lb. Let $b = 3$ in. so that the flange is just strong enough to hold this load at its own yield point. Then, at the rivet, C being $1\frac{1}{2}$ in.:

$$\frac{dy}{dx} = -\frac{W\,(A - 0.2)^2}{2IE} = -\frac{19\,160 \times 1.4^2}{60\,000\,000 \times 0.69} = 0.00091 \times 1.5 \text{ in.} = 0.0014 \text{ in.}$$

that is, at the point when R becomes equal to S the total elongation of the rivet is 0.0014 in. Meanwhile, from Equation (4), if $g = 2$ in.:

$$\Delta_R = \frac{(19\,160 - 13\,410) \times 2 \text{ in.}}{30\,000\,000 \times 0.518} = 0.00075 \text{ in.}$$

which is the elastic stretch.

On removal of the load, the rivet contracts 0.0014 in., reducing the tension in the rivet; thus:

$$\frac{-0.0014 \times 30\,000\,000 \times 0.518}{2} = -10\,800 \text{ lb.}$$

After this experience the rivet has a permanent or "initial" stress of about $19\,160 - 10\,800 = 8\,360$ lb., instead of the original 13 410 lb. If the flange is less stiff it is obvious that it will not allow $2S$ to reach 38 320 lb., so the total elongation of the rivet will be slightly less in order to bring it down to its yield point, and its final permanent stress slightly more.

If the flange is stiffer, $\frac{dy}{dx}$ will be less, and, again, the final permanent stress will be more. It seems, therefore, that there is no way to reduce the permanent stress to zero, and loosen the rivet, without carrying the total pull beyond the yield point of the rivets. For this reason it would seem that the factor of safety against loosening the rivets is the ratio of the yield strength of the rivets to the actual pull on the connection.

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[5] Cons. Engr., New York, N. Y.

Another point has to do with Table 3. Consider, for instance, a 15-in., 50-lb. channel, with a 24.2-kip pull (see Fig. 12). Assuming the dimensions shown, the uplift on the first two rivets in the channel web is more than $\frac{24.2 \times 2}{1.7} = 29.5$ kips. The bending in the web of the channel is about 29.5×1.2 in. $= 35.5$ in-kips. With a section modulus of $\frac{12}{6} \times 0.716^2 = 1.02$, the bending stress is 35 kips; also, the direct stress is 2.8 kips per sq. in., making a total tension in the upper surface of the web of about 37.8 kips per sq. in., which is high.

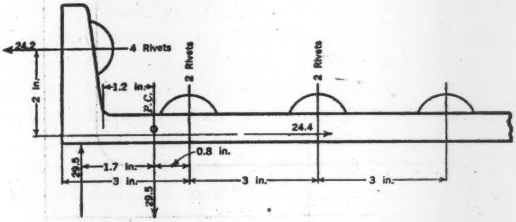

FIG. 12.

These rough figures indicate that 4-rivet angle or channel connections are likely to have such high stresses in tension in the first pair of rivets and in the horizontal leg that they have little theoretical safety factor. Their greatest use is on jobs which, on account of the shape of the building and the number of walls and partitions, need no wind-bracing, except to comply with legal requirements.

Beam stub connections, therefore, would seem to be stronger, and angle connections weaker, than is claimed by the author. This fact does not in any way detract from his main point, the comparative lack of stiffness in top-and-bottom connections, and the immense superiority of the knee-brace.

WILLIAM R. OSGOOD,[6] ASSOC. M. AM. SOC. C. E. (by letter).—The author points out a sore spot in structural practice, which well deserves the attention he gives it. The fact that premature failures of many connections of the kind he discusses do not occur, may be attributed to the "cleverness of the material"[7] in adjusting itself to conditions which overstress it. The reserve of strength in such connections is necessarily very small.

It is interesting to apply Mr. Berg's method of analysis to the tests made by Professor C. R. Young, at Toronto, Ont., Canada, in 1927.[8] After the initial tension in the rivet has been exceeded, the force, $R - S$, begins to move outward; and at failure, if the material is stiff enough, this force will have moved close to the outer edge of the connection, as shown in Fig. 13. Professor Young's test angles were free to move sidewise without any restraint on the outstanding leg, so that the values of e and X do correspond. From Fig. 13:

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[6] Engr., U. S. Bureau of Standards, Washington, D. C.

[7] "Dynamische Bruchversuche mit Flugzeugbauteilen," von Heinrich Hertel, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Munich, September 14, 1931. Hertel says "Schlauheit des Materials."

[8] "Tensile Working Stress for Rivets," by C. R. Young, Address before the Fifth Annual Convention, Am. Inst. of Steel Construction, Inc., Pinehurst, N. C., October, 1927; also, *Engineering and Contracting*, December, 1927.

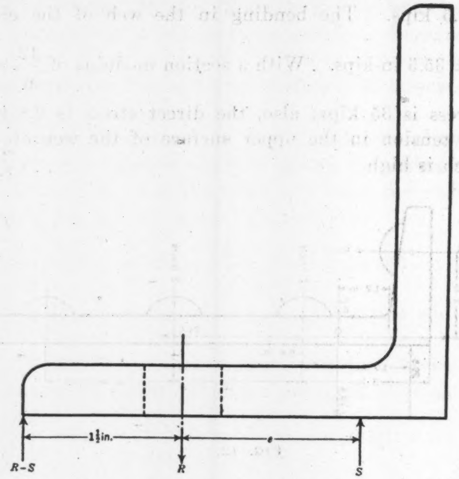

FIG. 13.

$$S = \frac{1.5\,R}{e + 1.5} \quad \text{..........................} (26)$$

and if this equation is used to compute the strengths of the rivets in Professor Young's tests, the results in Table 4 are obtained, with a factor of safety of four. The agreement between the computed values and the test values is better than that obtained by either Professor Young's formula or the formula proposed by the writer.[9]

TABLE 4.—COMPARISON OF FORMULA FOR PERMISSIBLE TENSILE STRESS ON RIVETS, WITH RESULTS OF TESTS

University of Toronto, 1927	MINIMUM TEST VALUES DIVIDED BY FOUR, IN POUNDS PER SQUARE INCH			Permissible tensile stress by formula, in pounds per square inch
	Diameter of Rivets Before Drive			
	⅞-in.	¾-in.	⅝-in.	
Pure tension, $e = 0$....................	16 460	17 110	13 790	....
Tension and Flexure:				
$e = 1\frac{1}{4}$ in........................	9 690	9 750	8 260	9 410 9 780 7 880
$e = 2\frac{1}{2}$ in........................	6 550	7 200	5 790	6 580 6 840 5 530

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[9] *Engineering and Contracting*, March. 1928.

O. G. JULIAN,[10] M. AM. SOC. C. E. (by letter).—This paper is an able presentation of a much neglected subject. It is believed that too often the stresses resulting from eccentricity of connections are given insufficient attention and that at times the rigidity of joints is neglected altogether.

It is clear without mathematical analysis that if all the joints of a rigid frame rotate through equal and reasonably small angles no error in the computed moments is introduced thereby. However, if the relative rotation of the joints is other than unity, errors of considerable magnitude in the computed moments may be introduced, because the less rigid connections and portions of the main members that intersect at a joint are relieved of, and transfer moments to, the more rigid portions of the frame. Tests by Professors W. M. Wilson and H. F. Moore[11] indicate that the relative rotation of well-designed rigid joints equipped with brackets, may introduce errors of about 8% in moments computed on the basis that the connections prevent, entirely, the relative rotation of the main members intersecting at a joint. Furthermore, the relative rotation of joints built without special consideration as to absolute rigidity may cause large errors in the computed moments. Due to deformation of the connections, the total deflection of a rigid frame equipped with well-designed and bracketed joints may be increased by as much as 20%, while the total deflection of frames equipped with strong but comparatively less rigid joints may be more than doubled. It is apparent that, in the design of rigid frames, the absolute and relative rigidity of the connections should be given careful consideration, as well as the stresses.

The author introduces the important but controversial question of the use of rivets in tension. Many specifications do not mention tension rivets, while some others state that rivets shall not be subjected to applied tensile stress. It appears to the writer that the latter provision places an undue restriction upon the design of structural connections. Tests made at the University of Toronto[12] and at the University of Illinois[13] indicate that tension rivets are reliable. Several recent specifications permit tensile working stress on rivets varying, with the specification, from 30 to 100% of the allowable stress in shear. Tension rivets are practically always also stressed in shear. Some authorities claim that the direct shearing and tensile stresses are independent of each other and need not be combined. Professor C. R. Young, suggests the following tentative formula for rivets in combined tension and shear:[14]

$$p_t = 21\,000 - 8\,000d - 6\,750\left(\frac{V'}{T'}\right)^2 \quad\ldots\ldots\ldots\ldots\ldots(27)$$

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[10] Engr. with Jackson & Moreland, Boston, Mass.

[11] "Tests to Determine the Rigidity of Riveted Joints of Steel Construction," by Wilbur M. Wilson, M. Am. Soc. C. E., and Herbert F. Moore, *Bulletin No. 104*, Eng. Experiment Station, Univ. of Illinois.

[12] "Permissible Stresses on Rivets in Tension," by C. R. Young, M. Am. Soc. C. E., and W. B. Dunbar, *Bulletin No. 8*, Section No. 16, Univ. of Toronto.

[13] "Tension Tests on Rivets," by Wilbur M. Wilson, M. Am. Soc. C. E., and William A. Oliver, Assoc. M. Am. Soc. C. E., *Bulletin No. 210*, Eng. Experiment Station, Univ. of Illinois.

[14] *Bulletin No. 8*, Section No. 16, Univ. of Toronto, Formula 6, p. 418.

in which, p_t is the permissible tensile stress on rivets, in pounds per square inch of rivet before driving; T', the total tension on the rivet due to all causes; V', the total shear on the rivet due to all causes; and, d, the diameter of the rivet, in inches, before driving.

Equation (27) is based on a factor of safety of 4 with respect to the ultimate strength, and was devised to take account of variations in the ratio of V' to T' from zero to 0.85, the latter being the highest value covered by the tests.

The tensile stress in a rivet must of necessity be at least equal to the initial stress due to driving and cooling. It has been claimed by some that the applied tension is additive to this initial tension. However, both theoretical considerations and tests show that if fillers are not used between the active parts, this view is fallacious,[15] and that the initial tension is not increased by an applied tension of lesser magnitude. If fillers are used between the active members, an applied load, P, which is less than the initial tension, increases the total tensile stress to,

$$\frac{R}{A_R} = \frac{R_t}{A_R} + \frac{P}{A_R + A_p} \text{ .... } (28)$$

in which, A_R is the area of the rivet; and, A_p, the effective area of the plate compressed by the rivet in question.[16]

It is believed that in most cases, $A_R + A_p$ may be taken as equal to the area of the plate (normal to the axis of the rivets) between adjacent rivets or, in the case of isolated rivets, as at least 9 sq. in. Then, for a ⅞-in. rivet, with $\frac{R_t}{A_R} = 26\,000$ lb. per sq. in., and an applied load, P, of 15 600 lb. (26 000 lb. per sq. in.): $\frac{R}{A_R} - \frac{R_t}{A_R} = \frac{15\,600}{9} = 1\,730$ lb. per sq. in.; that is, an applied load equal to the initial tensile force on the rivet in this case increases the tensile stress only 6.7 per cent.

It has been shown[17] that the limiting (elastic limit or yield point) stress of members subjected to direct shear (by applying torsion) is approximately 60% of the limiting stress in tension or compression. Furthermore, it has been shown that the limiting stress produced by simultaneously applying tension and shear at right angles to each other, may be determined according to the maximum strain theory up to the point where the limiting stress according to the maximum shear theory[18] (with the limiting stress in shear taken as 60% of that in tension), is reached. Beyond that point the maximum shear theory applies. The limiting stresses according to these two theories are represented graphically by Curves (1) and (2) of Fig. 14. The equations of the curves in this diagram are, as follows:

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[15] "Permissible Stresses on Rivets in Tension," by C. R. Young, M. Am. Soc. C. E., and W. B. Dunbar, *Bulletin No. 8*, Section No. 16, p. 405, Univ. of Toronto; also "Tension Tests of Rivets," by Wilbur M. Wilson, M. Am. Soc. C. E., and William A. Oliver, Assoc. M. Am. Soc. C. E., *Bulletin No 210*, Eng. Experiment Station, Univ. of Illinois, Section (6) of "Conclusions," p. 35.

[16] *Engineering News-Record*, June, 1928, Vol. 100, No. 23, p. 903.

[17] "The Strength and Stiffness of Steel Under Biaxial Loading," by Albert J. Becker, *Bulletin No. 85*, Eng. Experiment Station, Univ. of Illinois.

[18] *Loc. cit.*, p. 8 *et seq.*; also, "Strength of Materials," by Dr. S. Timoshenko, Pt. II, p. 705 *et seq.*

Curve (1), the maximum strain theory:

$$f_v = \tfrac{1}{4}\,(90\,000 - 600\,f_t - 3\,f^2_t)^{\frac{1}{2}} \quad\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Curve (2), the maximum shear theory:

$$f_v = \tfrac{1}{2}\,(14\,400 - f^2_t)^{\frac{1}{2}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

Curve (3), the strain-energy theory:

$$f_v = 61.2\left[1 - \left(\frac{f_t}{100}\right)^2\right]^{\frac{1}{2}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

Curve (4), Hancock's ellipse:

$$f_v = 60\left[1 - \left(\frac{f_t}{100}\right)^2\right]^{\frac{1}{2}} \quad\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

in which, f_t is the limiting tensile stress in percentage of limiting stress in direct tension; and f_v is the limiting shear stress, in percentage of limiting stress in direct tension.

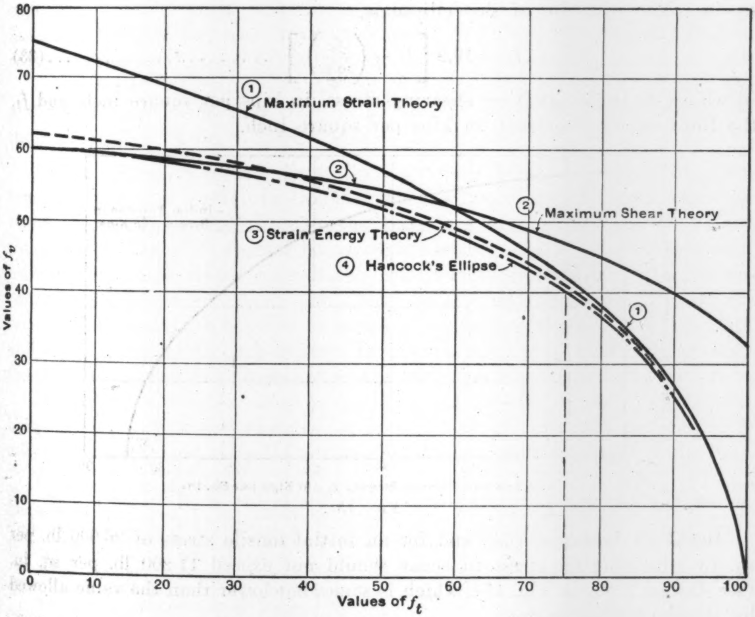

FIG. 14.

Curve (2) has been drawn with the limiting stress for direct shear taken as 60% of the limiting stress for direct tension. Curve (3) indicates the limiting stress computed according to the strain energy theory.[19] In determining Curves (1) and (3), Poisson's ratio has been taken as one-third. The writer believes that, for practical purposes, Curves (1), (2), and (3), of Fig. 14 may be replaced by Hancock's ellipse[20] which is shown by Curve (4). The application of this curve may be illustrated by the following example: If a member is stressed to 75% of the limiting stress in direct tension, an additional shearing stress, amounting to 40% of the limiting value in direct tension, will result in the total limiting stress being reached. (See dashed lines on Fig. 14.)

It is evident that all properly driven rivets are subject to initial tension. The magnitude of this tension for hot-driven rivets has been found to be at least 70% of the yield point strength of the rod from which the rivets are manufactured[18] (average, 37 184 lb. per sq. in.); that is, the initial tension is at least 26 000 lb. per sq. in. From the foregoing considerations, it would appear that the working stress in shear should be limited to such a value as to insure that the elastic limit in combined tension and shear is not exceeded. Fig. 15 is based on Hancock's ellipse and is drawn for a limiting tensile stress of 32 kips per sq. in. (which is assumed to be the elastic limit), and a limiting shearing stress of 60% of this value; that is, 19.2 kips per sq. in. The equation of this ellipse is,

$$f_v = 19.2 \left[1 - \left(\frac{f_t}{32}\right)^2\right]^{\frac{1}{2}} \dots\dots\dots\dots\dots\dots (33)$$

in which, f_v is the limiting shearing stress, in kips per square inch, and f_t, the limiting tensile stress, in kips per square inch.

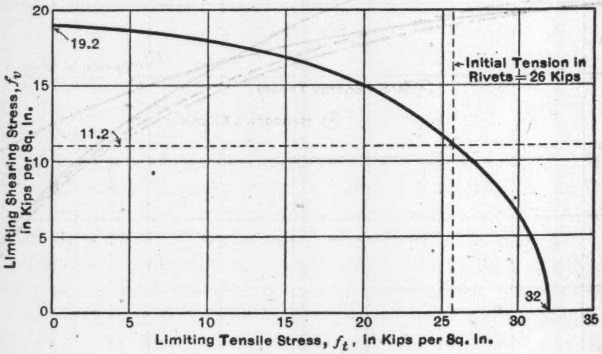

FIG. 15.

Based on Equation (33) and for an initial tensile stress of 26 000 lb. per sq. in., the working stress in shear should not exceed 11 200 lb. per sq. in. (see dashed lines on Fig. 15), which is somewhat lower than the value allowed by standard specifications.

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[19] "Strength of Materials," by Dr. S. Timoshenko, Pt. II, p. 708.

[20] "Results of Tests of Materials Subjected to Combined Stress," by E. L. Hancock, *Proceedings*, Am. Soc. for Testing Materials, 1908, p. 373 *et seq.*; also, "The Strength and Stiffness of Steel under Biaxial Loading," by Albert J. Becker, *Bulletin No. 85*, Eng. Experiment Station, Univ. of Illinois.

Regarding the connection shown by Fig. 2 it is evident that the rivets above the neutral axis of the connection between the hitch angles and the column are in tension as well as in shear. However, it is not clear that this neutral axis coincides with that of the girder itself. Four texts[21] consulted by the writer on this point give three materially different answers. It would appear that this question is not as simple as it is usually represented, the location of this neutral axis and, hence, the tensile stress in the rivets, being dependent upon the rigidity of the bearing surfaces, the initial tension in the rivets, and the axial load on the girder.

The author mentions the "prevalent practice of neglecting the gravity loads." It is difficult to understand how such a practice can be defended or how it became prevalent. Standard textbooks[22] give methods of determining the moments resulting from gravity as well as from wind loads, and the corner moments resulting from gravity and wind loads can be determined by means of the Cross method of distributing fixed-end moments.[23] It appears obvious that frames should be proportioned so that when the structure is subjected to the most unfavorable reasonable combination of dead, live, and wind loads, no part, including details and rivets, will be stressed beyond the endurance limit for the range of stress to which the part is subjected. The endurance limit for 0.2% carbon steel subjected to complete reversal of stress has been shown to be about 25 000 lb. per sq. in.[24] Dr. Timoshenko gives a corresponding limit of 27 000 lb. per sq. in. for mild steel.[25]

A number of interesting and useful formulas have been presented by the author. It is believed that their value would be considerably enhanced by the amplification of their derivations.

Harold S. Richmond,[26] Esq. (by letter).—Much of value has been brought out by the author in emphasizing the important problems involving rivets in tension acting through the thin legs of lug-angles. The point illustrated in Fig. 4(*b*) cannot be over-emphasized, that is, in a tension connection only those rivets immediately adjacent to the pulling stem of the connecting lug are effective. In a case sometimes encountered, two such lugs or diaphragms are opposed to each other on opposite sides of a web-plate and at right angles to each other. In such a case, it is obvious that only those rivets nearest the intersection are effective in tension.

In dealing with this class of problem the writer makes a distinction between static loads and variable or reversible loads and also between self-relieving stresses and those of a definite nature. For example, if the connection illustrated in Fig. 2 is considered to be merely a shear connection and not of use as a bracing connection, then thin lug-angles will make those stresses resulting from static loads self-relieving, but the stresses resulting from variable or reversible loads will result in fatigue unless held within the elastic range. Mr. Berg's comment that "the computed stresses in the angles are high, but harmless," therefore, will be taken as safe counsel only to the extent that the detail is subject to deflection from static loads. The danger in this case is that the design when put into use seems to be a successful one; yet the structure cannot be considered permanent unless all variable stresses lie within the elastic range. The writer sometimes satisfies his conscience by considering that the variable-load stresses are self-relieving to the extent of ranging themselves between a positive and a negative stress of equal magnitude after self-adjustment, rather than between zero and a higher stress, but always of the same sign.

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[21] "Strength of Materials," by the late George Fillmore Swain, Past-President and Hon. M. Am. Soc. C. E., p. 392 *et seq.*; "Structural Engineers' Handbook," by Milo S. Ketchum, M. Am. Soc. C. E., Third Edition, p. 730; "Structural Members and Connections," by Hool and Kinne, Section 3, by C. A. Wilson, Assoc. M. Am. Soc. C. E., p. 339; and "Design of Wind Bracing," by Edward Smulski, *Journal*, Boston Soc. of Civ. Engrs., Vol. XVII, No. 9, p. 509 *et seq.*

[22] For example, "Modern Framed Structures," by Johnson, Bryan, and Turneaure, Tenth Edition, Pt. II, Section V, p. 530 *et seq.*

[23] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E. *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[24] "A Study of Slip Lines, Strain Lines, and Cracks in Metals Under Repeated Stress," by Herbert F. Moore and Tibor Ver, *Bulletin No. 208*, Eng. Experiment Station, Univ. of Illinois.

[25] "Strength of Materials," by Dr. S. Timoshenko, Pt. II, p. 682.

[26] With Gibbs & Hill, Cons. Engrs., New York, N. Y.

In a case encountered by the writer, of a girder connection in a railroad viaduct, similar to that shown in Fig. 2, the lug-angle was split at the root, due presumably to fatigue. Whether or not the structure was built of wrought iron or steel, the criticism will apply with sufficient force, and designs of this kind will call for extremely careful calculation should wrought iron ever come back into vogue as a structural material.

While it is important to calculate and limit the tension in the rivets, it is at least equally important to limit the bending stresses in the legs of the connection lugs in the case of variable stresses. If the author's calculations are used, the tension in the rivet will range between the initial stress and the yield point—a range of only 11 100 lb. per sq. in.—and this only, needs to be considered in relation to fatigue effects, whereas, the variable bending stresses in the lugs will usually run much higher. There is, of course, a bending stress in the rivet due to the "ripping" effect of the lug. The theory would be difficult to develop and designers might possibly assume that this bending stress is partly self-relieving.

In brief: Connection lugs of thin material are indicated for constant loads, while thick material with the design carefully analyzed to keep stresses within the elastic range is indicated for variable loads. There would be danger of overstressing the rivets in the case of high moments due to constant loads and danger of fatiguing connection lugs in the case of variable loads.

Robins Fleming,[27] Esq. (by letter).—By his painstaking paper Mr. Berg has again brought to the attention of structural engineers the importance of the proper design of wind-bracing connections. In addition to Mr. Berg's paper three others lie before the writer, namely: "Design of Wind Bracing," by Edward Smulski;[28] "Connection Angles Subjected to Bending," by Norman B. Green;[29] and "Details for Wind Connections in Tier Buildings," by Walter H. Weiskopf,[30] Assoc. M. Am. Soc. C. E.

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[27] With The Am. Bridge Co., New York, N. Y.

[28] *Journal*, Boston Soc. of Civ. Engrs., Vol. XVII, p. 491, November, 1930.

[29] *Western Construction News*, Vol. 5, p. 137, March 10, 1930.

[30] *Engineering News-Record*, Vol. 99, p. 396, September 8, 1927.

The opinions of these writers and those of other engineers show at times a variation of 100% in the strength of connections. In actual practice the variation is even greater. It would thus seem that tests are necessary before definite and convincing conclusions can be reached. In fact, the theories put forth from any set of assumptions should be substantiated by a series of full-sized tests covering a wide range of connections as well as a number of connections of the same type.

Mr. Berg states,

> "The insecurity in the present method of rivet design has been illustrated, showing particularly how a stress of 18 000 lb. per sq. in. on the rivet as used by many designers, may develop stresses actually twice that amount."

Others will doubtless examine, critically, the reasoning that leads to such a sweeping claim. The writer will call attention to a few points.

Mr. Berg locates the point of contraflexure by the theory of flexure. In his analysis of the flange of a 24-in. 120-lb. beam, he assumes a span of 2½ in. and an average depth of about 1¼ in. The theory of flexure is scarcely applicable to a beam of these proportions. Moreover, the deformation from shear would be nearly as large as that from flexure.[31]

Mr. Berg refers to a paper by Professor Young and states, "* * * it becomes important to use the right judgment relative to the eccentricity, e (Fig. 4(*a*))." In the paper to which reference is made[32] Professor Young states,

> "In the eccentric loading tests, the rivet connecting the cross-piece to the longitudinal member was tested by applying the load to the cross-piece at a distance of 1⅛ and 2¼ in. from the gage line of the angles by means of knife-edges attached to the ends of the inverted U-shaped casting."

This clearly defines e as the distance from the center of the rivet to the knife-edge or point of contraflexure, which is the same as that defined by Mr. Berg. The author, however, takes e as $\frac{X}{0.65}$, which is too large. This changes the results of his Table 2.

Sometimes e is taken as $\frac{1}{2A}$, A being the distance from the face of the web to the center line of the rivet. This, however, assumes that the vertical leg is fixed at the root. With thin flange angles or channels there is a tendency to bend about the rivets in the horizontal leg.

The question of rotation from deformation of connections should be determined from tests. The structural engineer is not often concerned about "Behavior Beyond the Elastic Limit." His efforts are directed to keeping stresses within the elastic limit. Nevertheless, it is well the subject should be understood. Mr. Berg has written an able paper but experimental data are needed to confirm his results. In conclusion, a valued correspondent will be quoted. He writes:

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[31] "Horizontal Shear and Shearing Deflection of Beams," by Robins Fleming, *Engineering News-Record,* Vol. 107, p. 896, December 3, 1931.

[32] "Tensile Working Stresses for Rivets Investigated," by C. R. Young, M. Am. Soc. C. E., *Engineering News-Record,* Vol. 100, p. 128, February 2, 1928.

> "The only wind detail that has been used up to date that is susceptible of an exact mathematical analysis is the detail where there was a common inserted web plate between the column and the wind girder. This detail has not been used for a long time it having for commercial reasons passed out of the picture."

L. E. Grinter,[33] Assoc. M. Am. Soc. C. E. (by letter).—This paper opens for discussion an important division of the general field of structural design. Its publication is timely in that there seems to be an unusually wide interest in the subject of wind-bracing at the moment. The extension of the tall building frame to the 1 000-ft. tower naturally has led engineers to conclude that a review of standard methods of design is desirable.

Mr. Berg undoubtedly has many followers in his belief that the simple approximate methods that have characterized such design, need to be replaced by more exact processes. The writer accepts this viewpoint wherever he is convinced that the resulting complication with its attendant possibility of error can be justified by greater reliability in the finished structure. Unless such increased reliability can be demonstrated beyond reasonable doubt, there is always an important advantage in reducing analysis and design to the simplest possible processes.

The mere fact that hundreds of buildings have survived heavy wind storms, many of hurricane intensity, is a reasonable indication that wind-bracing as evolved in the past cannot be radically wrong, although such observations do not preclude improvement in the future. Where (as Mr. Berg shows) an application of the theory of elasticity indicates stresses twice as great as those calculated by the designer, one may well investigate either the assumptions upon which the analysis is based, or the hypothesis of elasticity.

The author discusses the action of tension rivets through the flange of an I-beam stub connection, and by use of certain assumptions, shows that the tension stress in a rivet may be nearly twice the stress found by dividing the total pull by the number of rivets. His major premise is that the flanges are held flat against the face of the column by the initial tension in the rivets, as shown in Fig. 16(*a*). However, it is important to realize that when stressed above their initial tension the rivets will elongate, and the deflected shape of the connection may approximate the condition shown in Fig. 16(*b*) even before any permanent elongation occurs in the rivets. Certainly, only small permanent distortions of the rivets are necessary to permit the action shown in Fig. 16(*b*) and to change the stress per rivet to the total load divided by the number of rivets.

The question then arises as to whether a situation such as this does not justify the neglect in design of the relatively high rivet stresses which Mr. Berg shows may exist at lower loads. The writer would make an important distinction between this case and another which is also discussed in the paper; that is, the case of two lines of tension rivets through one leg of an angle. In the latter case, one line of rivets must be stressed far past the yield point in order to stress the second line appreciably, and it seems impossible to justify the neglect of this fact.

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[33] Prof. of Structural Eng., Agrl. and Mech. Coll. of Texas, College Station, Tex.

The author's discussion of the stresses that exist in the rivets of wind-bracing angles is of particular interest. As in the case of the stub I-beam, there is a possible reduction in the tension which he would calculate in the rivets, from the effect of elongation of the rivets after the initial tension has been exceeded (see Fig. 17). When considered from this viewpoint, the advantage of using thick angles (to reduce stress and deflection) becomes apparent. Again, there is a clear argument to support the contention that such high rivet stresses at low loads are of less importance than their values may indicate, since an increase in the load does not increase the stresses in direct proportion.

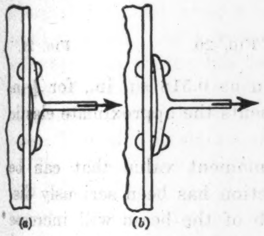

FIG. 16.

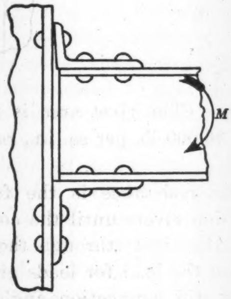

FIG. 17.

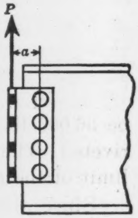

FIG. 18.

Mr. Berg discusses standard end connections for beams, and mentions an objection to the use of heavy connection angles, namely, that their stiffness may be sufficient to overstress the tension rivets. His point is well taken, but the discussion is not complete until one has investigated the actual stresses in the rivets through the web of the beam. Standard handbooks give the value of this connection as the direct shear or bearing value of the rivets through the web of the beam. Actually, there is a bending moment to be resisted, and, if the angles can be made sufficiently flexible to reduce the stress in the tension rivets to a low value, the bending moment is approximately equal to the load, *P*, times the lever arm, *a*, as shown in Fig. 18. On the other hand, one then finds that the stresses in the rivets through the web of the beam are increased greatly by the moment of the eccentric force.

As an example, consider the simple end connection shown in Fig. 19. The value of the connection is 11 250 lb., which is the resistance of the rivets in bearing on the web.[34] Considering the moment of the eccentric force, one finds that the maximum stress per rivet is increased from 5 625 lb. to approximately 20 500 lb. Since the working stress in the rivet is only 5 625 lb., the connection is seriously overstressed. Clearly, the tension resistance of the rivets through the outstanding legs of the angles is required to produce a resisting moment that will oppose the moment of the eccentric force. If the effective lever arm for this tension is taken as one-half the length of the angle less $\frac{1}{4}$ in., or 1.0 in., the rivet stress to produce a resisting moment equal to the moment of 37 300 in-lb. produced by the eccentric force, would be 36 000 lb. per sq. in. (The rivet area is taken as 0.518 sq. in., for $\frac{3}{4}$-in. rivets.) The stress of 36 000 lb. per sq. in., represents the approximate elastic limit of the rivet.

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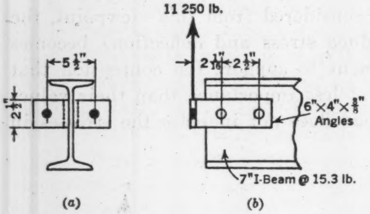
[34] Handbook of Steel Construction, Am. Inst. of Steel Construction, First Edition (1932), p. 277.



(a) (b)

FIG. 19.

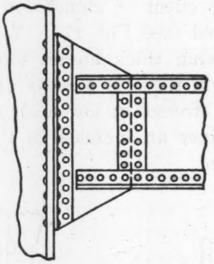


FIG. 20.

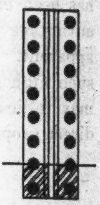


FIG. 21.

Since the foregoing resistance is the full moment value that can be obtained from the tension rivets until the connection has been seriously distorted, the stresses in the rivets through the web of the beam will increase much more rapidly than the load for loads greater than those used in design. The use of reasonably stiff connection angles to insure moment resistance, therefore, seems necessary, even if their effect may be to stress the tension rivets highly. To obtain a thickness of angle that would produce a nicety of balance so that the two groups of rivets would approach their elastic limits at the same time, would be desirable, but with standard connections one probably must depend upon some plastic deformation to accomplish essentially the same purpose.

It appears expedient to mention here a misconception that seems common in regard to the proper calculation of the tension stress produced in a line of rivets by flexure. Such rivets occur through the outstanding leg of a clip angle, such as may be used on a wind-bracing gusset (see Fig. 20). Since the stiffness of the gusset in its own plane is extremely high, it is assumed that its connection to a relatively stiff column remains essentially a plane section. The beam formula, $S = \frac{Mc}{I}$, may be used, therefore, to calculate the stresses in the tension rivets, but it is necessary first to locate the neutral axis. When the connection is unstressed by moment, there is a uniform pressure between the clip angles and the face of the column, caused by initial tension in the rivets. Ordinary negative moment in the beam will reduce the bearing pressure at the top of the connection and increase it near the bottom. Until the maximum reduction in pressure equals the initial pressure, the true neutral axis lies outside the section, but the reduction in pressure can be determined correctly by assuming a neutral axis at the mid-height of the vertical angles.

After the initial tension in the upper rivet is overcome by the applied moment, an entirely different condition prevails, and the true neutral axis gradually moves downward from the top toward the bottom of the angles. At failure, the connection undoubtedly 'rides" on the bottom of the angles, the neutral axis having reached its lowest possible position.

During the initial period when the angles are in bearing on the face of the column, the neutral axis for flexure lies at the mid-height, but since the neutral axis actually approaches the bottom of the angles at the point of failure, it seems highly conservative to compute the stress from the first position of the neutral axis. On the other hand, to assume that the neutral axis is at the lower edge of the connection angles actually assumes that failure is imminent, since the upper rivets, being placed perhaps 2 or 3 ft. above the neutral axis, would be severely distorted by even an extremely small rotation. As a reasonably conservative procedure between these two extremes, the writer has been accustomed to take the neutral axis at the center of gravity of the effective section, considering the cross-sectional area of tension rivets above, and the entire area in bearing below, the neutral axis. This area is shown cross-hatched in Fig. 21.

The author discusses the need for considering gravity forces (dead load plus live load) in the design of wind connections. He indicates, without definite recommendation, that wind-bracing connections should be designed for dead load, live load, and wind, although he states that gravity moments at the joints may be reduced as much as 10%, or even more, by elastic distortion of the connection angles. It seems desirable to reconsider the possible effects that may reduce the bending moment as caused in a wind connection by gravity loads. These may be listed as follows:

(1) Elastic distortion of the connection angles;
(2) Slip in the riveted connection;
(3) Elastic elongation of tension rivets when stressed above their initial stress; and
(4) Plastic deformation of the rivets, connection angles, end of beam, or face of column, when the stresses exceed the elastic limit.

Only the first item of this list is considered by Mr. Berg. The second item, slip in riveted connections, has been treated[35] by Professors Wilson and Moore. They have demonstrated that a wind connection of the type considered by the author may be expected to show an angle change due to slip of nearly 0.003 radian when the rivets through the beam flanges are stressed to one and one-half times their normal working stress in shear. It is of interest in a study of this effect to note that the end rotation of a simple beam uniformly loaded, where the span is ten times the depth, is approximately 0.004 radian when stressed to a normal working stress. However, a wind connection of the angle type would not be designed capable of developing the moment resistance of the beam, and, hence, the slip alone might readily be sufficient to release practically all end restraint in so far as dead load is concerned.

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[35] *Bulletin No. 104*, Eng. Experiment Station, Univ. of Illinois.

It is to be noted that the foregoing discussion does not consider end adjustments that may occur as the connection approaches its ultimate resistance. The ultimate strength of the connection tested by Professors Wilson and Moore corresponded to a load of more than two times the load that produced a slip of 0.0028 radian, and the maximum rotation was approximately 0.015 radian.

At several places Mr. Berg mentions the fact that "marked re-adjustments" of moments may be expected to occur when stresses pass the elastic limit. Undoubtedly, re-adjustments do occur, and perhaps the usual tendency is for the re-adjustment to relieve distress in the structure by preventing rapid increase of stress at points of weakness. Nevertheless, one should not depend too seriously upon this phenomenon. Consider a panel of a building in which wind connections are identical at the two ends of the girder. If the point of contraflexure in the girder moves sufficiently toward the right to overstress the left-hand connection so that a partial failure develops, there will probably be an immediate shift of the point of contraflexure to the left-hand side of the center with a resultant over-stressing of the right-hand connection. If this damage is started by a faulty joint, the resistance will then be transferred entirely outside this panel, possibly without serious damage to the structure, but if the over-stress is general throughout several panels of the structure, failure is impending.

Mr. Berg's paper was quite properly restricted to riveted joints. On the other hand, the structural engineer now faces an entirely new problem in this field, the design of welded wind-bracing connections. A service can be performed by any one who will study the design of welded connections from the viewpoint adopted by Mr. Berg. Where welded joints are designed properly, essentially perfect continuity is obtainable, and the design of the joint for both gravity loads and wind loads becomes necessary. An analysis of welded joints by the theory of elasticity would not be open to the criticisms that have been mentioned in this discussion.

C. R. Young,[36] M. Am. Soc. C. E. (by letter).—The distinctive weaknesses of "knuckle" connections are treated in a definite and quantitative manner in this commendable paper by Mr. Berg. That knuckle connections have proved satisfactory for low buildings is probably due to the fact that the force of the wind never reaches the intensity prescribed in the specifications. If, in addition to the negative moments due to gravity loads, there were anything like the expected wind moments, a less satisfactory story of these connections would have to be told.

Recognition of the existence of high stresses in connections due to end moments is not new in structural literature. In discussing web connections of beams to girders or columns, Jacob Friedland[37] pointed out twenty years ago the existence of high tensile stresses in the rivets connecting the beam to the girder or column. For characteristic I-beams, tensile stresses in the rivets, based on the area of the rivet as driven, were found to be as high as 29 100 lb. per sq. in., and on the basis of the area of the rivet before driving, as high as 34 200 lb. per sq. in. There is no doubt that in countless cases the tension rivets, under gravity loads only, have been stressed close to the elastic limit of the material in them, and probably beyond the initial stress in the rivets.

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[36] Prof. of Civ. Eng., Univ. of Toronto, Toronto, Ont., Canada.

[37] *Engineering News*, April 25, 1912, p. 786.

In asserting in the first paragraph that "the mass inertia creates additional resistance against gusts of wind," the author is correct if he means that an addition to mass lessens the frequency; but he is in error if he means that it lessens the deflection or amplitude of vibration that would follow the application of wind force. As has been pointed out by H. V. Spurr,[38] M. Am. Soc. C. E., and D. C. Coyle,[39] M. Am. Soc. C. E., the deflection and amplitude of vibration of a building are independent of mass. This is evident if, for simplicity, the building be considered as a simple cantilever. In the common deflection formula no quantity appears representing the mass of the structure. However, the weight of a building affects the frequency of vibration since, other things being constant, the frequency varies inversely as the square root of the mass.

There is merit in basing the allowable load on a rivet on its initial tension value. While against failure there is the entire margin between actual load and ultimate strength, the increased emphasis now being placed on rigidity makes it particularly desirable, under all circumstances, to keep below the point at which the deformation of the joint will begin to be affected by tension rivet yield. Based on work done at the University of Toronto in 1928, the writer has held that within the initial tensile value of the rivet there is no appreciable elongation.[40] He consequently concurs in the author's assumption to that effect.

The soundness of the choice of representative test values for tension rivets made by Mr. Berg from the results of Professor Wilson and Mr. Oliver may not be entirely obvious. The average ultimate strength of rivets in tension was not 54 000 lb. per sq. in., as might appear from the statement following Equations (2), but considerably greater. This value was the minimum mean observed and was for $\frac{3}{4}$-in. rivets (effective diameter, $\frac{13}{16}$ in.), driven with air-hammer at a cherry-red heat and having a grip of 5 in. For a 2-in. grip the mean was 63 900 lb. per sq. in. In low buildings, and for the upper stories of tall buildings, where the column metal is thin, the grip is likely to be nearer 2 in. than 5 in.

The yield point assumed—that is, 37 000 lb. per sq. in.—is for the material from which the rivets were made. It is not the yield point of the rivet as driven, since that quantity was not observed by Professor Wilson and Mr. Oliver, or in the University of Toronto tests.

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[38] "Wind Bracing", pp. 114–115.

[39] *Civil Engineering*, May, 1931, p. 700.

[40] "Permissible Stresses on Rivets in Tension", by C. R. Young and W. B. Dunbar, *Bulletin* 8, Section 16, School of Eng. Research, Univ. of Toronto.

As the the initial tensile stress would probably be a smaller fraction of the yield point of the rivet as driven than it is of the bar stock, or the manufactured rivet, an assumed initial tensile stress of 70% of the yield point of the bar is fair.

The author has applied Equation (3) in a manner not intended by its proposers. In the presentation of the results upon which this formula was based,[41] it was recommended that the eccentricity, *e*, should be taken as about two-thirds the value of the gauge for the outstanding leg. This distance was supposed to give, in typical cases, the position of the point of inflection with respect to the rivet center. As far as the rivet under consideration is concerned, the application of a force parallel to the rivet at the point of inflection would produce the same result as a load applied by a knife-edge at a corresponding distance from, and parallel to, the gauge line. It was in this manner that the specimens tested in combined tension and flexure were loaded, and the formula should be used assuming that the applied force has an eccentricity equal to the distance from the rivet center to the point of inflection.

On this basis the values of f_r from Equation (3) would be as given in Table 5. Examination of this table shows that the two equations compared, give results that are very close for 1-in. rivets, and that Equation (3) gives somewhat higher values than Equation 2(*b*) for the smaller rivets.

TABLE 5.—COMPARISON OF WORKING LOADS ON RIVETS SUBJECTED TO A COMBINATION OF TENSION AND FLEXURE

Nominal size of rivet, in inches	Assumed X, in inches	Eccentricity, $e = \frac{X}{0.65}$ as used by author, in inches	Eccentricity, $e = X$, as used by writer, in inches	PERMISSIBLE TENSION PER RIVET, IN POUNDS	
				Equation 2 (*b*), gross area	Equation (3), nominal area
¾	0.75	1.15	0.75	3 840	4 520
¾	1.50	2.31	1.50	2 690	3 660
⅞	0.75	1.15	0.75	5 100	5 550
⅞	1.50	2.31	1.50	3 560	4 370
1	0.75	1.15	0.75	6 550	6 480
1	1.50	2.31	1.50	4 680	4 940

Fig. 6 is apparently based on the application of an equation similar to Equation (8), although the author does not so state. Insufficient data are given in the text to permit the checking of the curves. In order to do this it would be necessary to know the values of B and b used by the author. In establishing Equation (11) for the angular rotation imposed upon the beam by the connection, apart from the effect of tension rivet yield, the author has neglected two factors.

To make this clear, let it be assumed that for purposes of measuring rotation, or tilt, a plane is passed transversely through the beam at the outer edge of the connection and parallel to the face of the column. The movement of this plane away from the column at the tension face of the beam would be influenced (neglecting tension rivet yield) by bending in the tension connection, by elongation of the material of this connection, by shear deformation or slip of the rivets through the beam flange, and by the extension of that part of the beam between its end and the section plane. Similar influences would be operative at the compression face of the beam. The result of including the two factors not considered by Mr. Berg would be to permit a greater rotation than he calculates and a correspondingly smaller moment of restraint.

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[41] *Bulletin No. 8*, Section 16, School of Eng. Research, Univ. of Toronto, p. 408.

Another factor not discussed in detail by the author is the effect of repeated and reversed loading. For the rivets attaching the connection to the beam flanges, for example, there would be under stress a deformation which, in part, would be non-elastic and, in part, elastic. The ratio of these will change with the repetition of stress, and what might be called a steady elastic state is not reached until several repetitions or reversals have occurred.

If one were to make a general criticism of Mr. Berg's paper, it would be on the ground that it is rather too greatly condensed. It would have made for clarity if he had indicated his processes a little more in detail. For example, in Table 1 the values of the rivets given in the last three columns are based on areas of rivets as driven and not on the nominal area. In the second paragraph under "Flange Angle Connections," it should be made clear that it is the elastic line of the deformed angle that remains vertical at the rivet, R_1 and horizontal at the rivet, R_2 and not the rivets themselves that preserve these directions.

C. H. SANDBERG,[42] ASSOC. M. AM. SOC. C. E. (by letter).—This paper brings to the attention of the Engineering Profession a greatly neglected phase of wind-bracing design in tall steel buildings. As pointed out by Mr. Berg, since connections of the type shown in Fig. 2 are designed for shear only, on the assumption that the details are so flexible that only a negligible end moment is developed, there is danger that excessive tensile stress will be developed in the rivets unless this phase is specifically investigated. The author's analysis showing the possibility of high tensile stress in this and other types of connections, therefore, is considered timely.

Since 1927 or 1928 there has been an abundance of printed matter on wind-bracing analysis, but very little has been written on the equally important subject of the moment resistance of riveted joints. The few papers that have been published on the subject have been based on assumptions unsupported by experimental data. In the writer's opinion Mr. Berg's paper is the best one that has appeared on this subject; yet his results also are based on unverified assumptions. These seem reasonable enough, but designers have no means of knowing how closely they approach actual conditions. In view of this lack of knowledge on the subject, it is surprising that this important design feature has not been studied experimentally long ago.

In 1928, Professor C. A. Hughes and the writer began a series of tests at the University of Minnesota to determine the resistance of different types of riveted joints. The types of connections selected for this investigation are typical of those commonly used in present-day construction.

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[42] Asst. Engr., Bridge Dept., A. T. & S. F. R. R., Chicago, Ill.

The deformation of connection details is not quite as simple as indicated by the author in Figs. 3 and 7, in which they are parallel to the plane of the web of the beam only. In addition to these, there are deformations of the connection details and the column flanges due to bending in a direction normal to the web of the beam. This results in a marked difference in the distribution of stress on the rivets adjacent and remote to the web of the beam or the web of the column. This point has been considered by Sub-Committee 31, Committee on Steel, of the Structural Division, of the Society, as is evidenced by the statement, "* * * rivets in the column should be kept in single gauges near the column web."[43] If the rivets considered effective for moment are limited to gauge lines adjacent to the web of the sheared I-beam and the web of the column, the safe moment of resistance of this type is limited and is much less than has been attributed to it in many designs.

The author cautions against neglecting the combined effect of wind and gravity loads in determining the moment at the end connections, especially where the rivets are in tension. This point is well taken as there are cases, in which brackets are used for end connections, where the negative moment can be larger than $\frac{WL^2}{12}$, due to the fact that the common assumption of constant section throughout the beam does not hold. This may also be true when the spans are unequal.

Connections have been and are being designed, however, for wind moment only, and there are some justifications for this practice. Bracket connections with tension rivets are usually designed too conservatively. One of the common methods is to assume the rotation of the joint at the center of gravity of the tension rivets (rivets into column), while actually the point of rotation is considerably lower. Beams and columns encased in concrete will also add greatly to the rigidity of the connection. A large part of the wind resistance is usually designed to be taken through the spandrels and, in some cases, one or two additional lines. This arrangement makes it necessary to select the wind-resisting lines of beams for wind loads in all except the upper stories, and the actual moment due to the gravity loads in the end connections is a small part of the total. Another factor in support of ignoring the moment due to gravity load is that if a connection on one side of the column is over-stressed the connection on the other side of that column will be under-stressed, and an adjustment will take place. There will also be a distribution through the floor which will cause a stronger joint to relieve a weaker one.

It is common knowledge that since about 1927 the use of the sheared I-beam type of connection has been overdone. This is due largely to the fact that it gives minimum interference with architectural features, and also that some designers undoubtedly do not realize that the bracket connection is the next in effectiveness and economy to diagonal cross-frame bracing.

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[43] *Proceedings*, Am. Soc. C. E., February, 1932, p. 222.

In the design of end connections there are as many ramifications as in the problem of analyzing and distributing the wind moments in the frame of the building, and this cannot be done satisfactorily or in a hurry by an inexperienced designer.

The writer wonders whether it is not being unduly conservative to base the factor of safety for tension in rivets on the initial tension rather than on the yield point. Until the yield point is reached, the deformation of the extreme tension rivets in a connection and the consequent "give" or "slip" of the joint will be so slight as to be practically negligible. Another factor that should be considered in this connection is that the measured values of initial tension are probably too high. Information now available was obtained from tests of shop-driven rivets on comparatively small specimens, on which driving and bucking conditions were ideal, and it is reasonable to expect that the average initial tension of field-driven rivets in large joints will be somewhat less.

Considerable work has been done and much progress made in determining the horizontal loads on structures and in developing rational and relatively simple methods of analysis for frames. It is hoped that further tests of riveted connections will be undertaken so that the design of joints of adequate resistance may be placed on an equally satisfactory basis.

N. A. RICHARDS,[44] M. AM. SOC. C. E. (by letter).—The action of usual types of angle, channel, and beam-section connections is treated in an interesting and stimulating manner in this paper.

Building laws and conservative practice require that beams and girders be generally proportioned on a simple span basis for dead and live loads, but it is quite obvious that, as heavy wind-bracing connections are added, the members cannot so act, and in wind analyses, they are assumed to be restrained at the ends. The added stresses resulting in the rivets and connections, due to live load and dead load action of the members, should receive more thought and attention.

One feature that might add interesting further study is the prying action resulting from the pivoting of a beam or girder near the outer edge of the supporting seat, and the corresponding lift at the heel of the upper angle as the member deflects under load.

In reviewing the paper the writer found difficulty in following some of the rather abruptly stated mathematical steps; as, for instance, under "Flange Angle Connections," referring to Fig. 5, $M_4 = \frac{1}{2} M_c$, and below Equations (5) and (6). This is probably due to the rustiness in pure mathematics and mechanics acquired in many years of general practice; but, inasmuch as many interested readers will no doubt find similar difficulty, the author would do well to explain the mathematics of the study with a little less abruptness.

The paper as a whole is a distinct contribution, and is well worth study by those engaged in the design of structures involving connections of the types considered.

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[44] Vice-Pres., Purdy & Henderson Co., New York, N. Y.

Jacob Feld,[45] Assoc. M. Am. Soc. C. E. (by letter).—The author gives a timely warning of a frequent error. The design of wind-bracing has become involved in so much mathematical detail that engineers have lost sight of rational and logical carrying out of stress analysis. Too often, a careful analysis of stress is completed and then scrapped because clearance or head-room requirements insisted on by the architect do not allow space for the necessary details.

If the engineer insists on the required space he very often finds himself displaced by another engineer who, according to the architect, is a more efficient designer because he can design wind-bracing without using up more space than can be permitted.

As a result, and possibly also because they do not know any better, some designers believe in an extreme theory of "educated stress methods." In practice, the method consists of summating the resisting moments of all beam-to-column connections on a floor and if such sum is greater than the wind moment of that floor, calling it a sufficient design. If not, a few additional rivets put in here and there make up the necessary total. The method is really based on the principle of least work, or its special statement known as the Moseley principle of least resistance, which is that, if a series of resistances consistent with the conditions of stability can be determined, that series is sufficient to take care of all external forces, although it may not be the most efficient system that can be devised. The application to the foregoing method of wind-bracing design is in error because the major premise is forgotten; namely, that the assumed resistances must obey the conditions of the problem. One condition is that the summation of forces and moments acting on each unit member and connection must vanish.

The universal assumption that a steel beam is simply supported as far as vertical loads are concerned and that the same beam is rigidly connected as far as lateral loads are concerned, is not consistent with the truth or even with logic.

That similar assumptions are sometimes made in reinforced concrete design is true, but the results appear almost immediately. The large reserve of unused strength which exists in steel structures does not exist in reinforced concrete improperly reinforced because of the low tensile strength of plain concrete. When improper assumptions of steel distribution are made, cracks appear, and the cracks are not always caused by poor workmanship, temperature, or shrinkage during setting. Disregard of moments in columns due to unbalanced loads gives cracks in the exterior faces of exterior columns, especially in flat slab buildings; disregard of the two-way action of panels gives cracks in the slabs along girders, unless special reinforcement is provided across the top of the girders; disregard of the rigidity of connection between beams and columns when unequal spans are supported by the same columns, gives cracks in the tops of the beams when full loads occur on the longer spans; and disregard of the torsion in exterior beams always loaded on one side only gives the diagonal cracks visible in so many concrete lintels. These are only a few of the more common errors of details visible in concrete structures. In flexible steel frames similar errors are not visible; yet the effects exist, and the true factors of safety of the steel frames are much lower than is usually assumed. If the basic assumption of wind-bracing design is that connections in a steel frame are rigid, they must certainly be rigid before the wind starts to act, and the author's recommendations are timely and important.

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[45] Cons. Engr., New York, N. Y.

DANA YOUNG,[46] JUN. AM. SOC. C. E. (by letter).—The analysis of wind-bracing connections given in this paper is important in calling attention to various factors often neglected in design. Theoretical formulas are developed, which indicate that present design practice is seriously in error. The question arises naturally as to which is more nearly correct, theory or practice. This can be settled satisfactorily only by tests. The formulas given are based on various assumptions, the effect of which, while perhaps slight, cannot be definitely determined analytically. As the author states tests are necessary to establish their accuracy.

A number of tests of tension rivets have been made. However, before fully accepting the formulas given in this paper for tension rivets, it is important to consider the assumptions upon which the analysis is based and how these compare with the test conditions.

As shown in Fig. 3, a section of the angle or sheared beam is taken through the rivet, and bending only in the plane of this section is considered; but, since the rivets are not continuous along the length of the member, sections between rivets are not restrained in the same manner as the section considered. The stress conditions will be different for all sections. This has some indeterminate effect on the rivet value.

The assumption that the connection angle or beam is rigidly fixed (that is, that it has a horizontal tangent) at the center line of the rivet is not strictly true. Since both the rivet and the member to which the connection is fastened are elastic there must be rotation at this point.

As the author states, both the distance, C, and the pressure distribution under that part of the connection are uncertain. For accurate results, it is necessary to make reasonably correct assumptions for these factors. The importance of these assumptions is shown by Equation (2*b*), from which it is seen that any variation in C or its coefficient will affect the value of S in almost direct proportion.

With a flange-angle connection the rivets are also subject to shearing stress due to the end moment. In addition, the rivet may receive shearing stress from the end shear of the beam. These stresses, in combination with the direct tension, produce a principal tensile stress somewhat greater than the direct tensile stress. In some cases this may affect the strength of the rivet appreciably.

It is practically impossible to determine analytically the effect of these various assumptions, either singly or in combination. Designers must rely on tests to establish the accuracy of the formulas.

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[46] Washington, D. C.

The tension rivet tests[47] of Professor Wilson and Mr. Oliver, furnish authoritative data in regard to rivets in pure tension, but this condition does not exist for the rivets used in wind-bracing connections. The tests reported[48] by Professor C. R. Young, are better for this case because they not only include rivets in pure tension, but also rivets with an eccentrically applied load. For this latter condition, 4 by 4-in. clip angles were riveted to a supporting piece and the load was applied through knife-edges bearing on the riveted leg of the angle (the riveted leg was horizontal) at a distance, e, from the center line of the rivet. Part of the tests were with $e = 1\frac{1}{8}$ in. and part with $e = 2\frac{1}{4}$ in. This method of loading is equivalent to establishing the distance, X, used in the author's formulas at $1\frac{1}{8}$ in. and $2\frac{1}{4}$ in., respectively, from the center line of the rivet. In computing Table 3 the author assumes that this eccentricity is equal to $\frac{X}{0.65}$, but from the method of loading with knife-edges there could be little restraint and, therefore, e must equal X.

TABLE 6.—COMPARISON OF TEST AND COMPUTED VALUES FOR RIVETS IN TENSION

Rivet size, in inches (1)	ULTIMATE STRENGTH OF RIVET, IN POUNDS PER SQUARE INCH				
	Pure tension, test value (2)	Tension with Eccentricity = $1\frac{1}{8}$ Inches		Tension with Eccentricity = $2\frac{1}{4}$ Inches	
		Test (3)	Computed (4)	Test (5)	Computed (6)
$\frac{5}{8}$..........	74 400	41 500	35 000	27 400	22 900
$\frac{3}{4}$..........	74 600	36 500	35 100	26 000	23 000
$\frac{7}{8}$..........	63 100	30 300	29 700	21 000	19 500

While the formulas in this paper do not apply to cases beyond the yield point, it is interesting to compare Professor Young's results with values computed from Equation (2b), in the same manner that the flexure formula is used for computing the modulus of rupture. In this case $C = 1.5$ in. From Equation (2b), for $e = X = 1\frac{1}{8}$ in., the value of S is 0.47 R, and $S = 0.308\ R$ for $e = X = 2\frac{1}{4}$ in. Table 6 shows the ultimate strength of the rivets as found in the tests and the values as computed. Each test value given is the average of six results. For the computed values, R was taken equal to the pure tension strength as given in Column (2) of the table. Everything considered, the computed and experimental values check very closely. The maximum difference is 20%, and the minimum, 2 per cent. However, it must be noted that these tests do not give any data as to the accuracy of Equation (6) for determining X; nor is it necessarily true that tests with angles of other sizes, or with sheared beams, will show equally close agreement.

A flange-angle connection is subject to forces perpendicular to the direction of the pull, S. This is shown in Fig. 22. As a result there is shear on the column rivet and tension on the beam rivet, in addition to the forces generally considered. In most cases these will be small and probably may be neglected. There are also the shearing stresses set up by the end shear of the beam to consider. An idea of the relative magnitudes of the forces involved, may be gained by noting that, in Fig. 22, when $a = b$ and $e = d = \frac{1}{2} a$: $R = 2.2\ S$; $V = 0.6\ S$; $T = 1.0\ S$; and $X = 0.6\ A$.

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[47] *Bulletin No. 210*, Eng. Experiment Station, Univ. of Illinois.

[48] Address before the Fifth Annual Convention, Am. Inst. of Steel Construction, October, 1927, (Abstract), *Engineering News-Record*, February 2, 1928, p. 188.

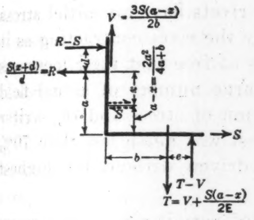

FIG. 22

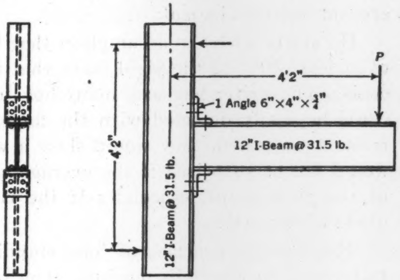

FIG. 23.

Equation (12) may be simplified. Since X is a function of A and B, as given by Equation (6), B may be eliminated and Δ_1 expressed as,

$$\Delta_1 = \frac{A^2}{6\,EI}(3\,X - A) \quad \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

For $X = 0.5\ A$, this reduces to $\Delta_1 = \frac{A^3}{12\,EI}$, which is the deflection formula for a beam fixed at both ends, but displaced perpendicularly to its length.

Equation (11) is subject to the various assumptions used in the preceding theory and besides it does not take into consideration the slip in the beam rivets and bending in the column flanges. Rivet slip, especially, will greatly increase the angle of rotation. This may be demonstrated by referring to a test made by Professor W. M. Wilson and H. F. Moore[49] to determine the rigidity of riveted joints. The test piece and method of loading are shown in Fig. 23. For a load, $P = 4\,500$ lb., the rotation at the joint as computed by Equation (11) is $\phi = 0.000156$ radian. The test showed a rotation of $\phi = 0.0028$ radian. This is a large difference and shows the need of further experimentation.

However, regardless of the possible error in the formula for joint rotation, the effects of this rotation, as pointed out by the author, are important. The fact that many buildings have been designed neglecting joint rotation and the effect of gravity moments on wind-bracing connections, does not mean that these factors can always be neglected. The factor of safety, the bracing effect of walls, and the fact that the full design loads are seldom realized, have all helped to cover up poorly designed details.

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[49] "Tests to Determine the Rigidity of Riveted Joints of Steel Structures," by Wilbur M. Wilson M. Am. Soc. C. E., and H. F. Moore, *Bulletin No. 104*, Eng. Experiment Station, Univ. of Illinois.

A. J. Wilcox,[50] M. Am. Soc. C. E. (by letter).—The design of wind-bracing connections is a subject on which there is a wide difference of opinion although until one has made a considerable study of them, it appears to be a very elementary problem. The author has pointed out some of the conditions that make these connections difficult to design along undisputable theoretical lines. His solution is based on certain assumptions which the writer believes are not entirely logical.

He starts with the assumption that hot driven rivets have an initial stress of at least 70% of the yield point that is set up by the rivet contracting as it cools. The writer has seen many hot driven rivets so free that their looseness could be easily detected with the fingers. If a large number of actual field rivets were tested, they would show a wide variation of stress and the writer would not be surprised if the average of such a test was much less than 70% of the yield point, especially if the rivets were driven without the highest grade of inspection.

Engineering practice of long standing does not permit any such stresses to be used in rivets in tension. The practice, until recent years, was to use the shearing value of 12 000 lb. per sq. in., increased 50% for wind stresses. When this unit stress was increased to 13 500 it was not considered safe to increase the stress for wind 50%; this was changed to 33⅓%, the result being no increase over the 18 000 lb. per sq. in. formerly used. This limit is not definitely set by any building code or standard specifications, but as the result of many years of experience by the leading men in the profession. Many of them were keenly interested in producing the most economical structural steel to meet competition from other forms of construction. This experience must have convinced them that it was not safe to take advantage of an apparent defect in the codes.

The point is often argued that since it is impossible to drive hot rivets consistently without occasionally setting up cooling stresses much in excess of the standard limits, and since it is impractical to relieve them, it is entirely proper to make use of such stresses. However, after considerable designing experience and study of the subject of distribution of wind stresses, the writer feels that the day is far distant when these stresses can be distributed accurately. The more general methods used by many engineers, because of their simplicity, appear to give results that are not approximately correct. Furthermore, the common practice of to-day is to neglect the effect of the continuity of the dead and live load moment. These factors would most certainly alter the designed stress in wind-bracing connections.

The author used the diameter of the hole and not the nominal diameter of the rivets in determining his rivet areas; again, this is against standard practice. This assumes that when separately punched pieces are fitted together, the holes match exactly. Such workmanship requires machine-shop practice. Because of the cost involved in the use of holes punched so accurately that the rivets or bolts would have a tight fit throughout, larger holes are a more practical solution. This assumption requires the rivet to fill the hole completely.

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[50] Cons. Engr. (H. G. Balcom), New York, N. Y.

With a combination of good detailing, workmanship, and inspection, the writer believes this result can be approached. However, when the grip is long, it is more uncertain. Practically all codes and standard specifications further reduce the standard stress on the nominal area for long-grip rivets. The designer should also be governed on important work by the quality of workmanship he may expect.

Inspection of material and workmanship requires as much skill as design, although along a somewhat different line. The chief inspector and his assistants could be selected with as much care as the designing force. No workman in any trade cares to be made to do his work over again, and many workmen are more skilled in covering up defective work than inexperienced inspectors are in detecting it.

The author also assumes the connecting member fixed at the rivet as shown in his Fig. 3(*c*). He arrives at this by assuming for the distance, C, that there is no deformation over the distance, X. In his Example 1, $C = 1.5$ in. and $X = 0.415\ A = 0.87$ in. If there is a deformation in the length, X, there most certainly is a deformation in the length, C, which is almost twice the value of X. The writer believes an eccentric force must be applied to the rivet, and its amount depends on the stiffness of the connection.

In Example 2, Mr. Berg uses an angle 4 in. long and does not deduct the rivet hole, although he assumes the maximum stress at that point. Besides using the gross length, he assumes a uniform stress over the full length. With moment arms of about 1 in., there is considerable question concerning the proper length to use.

Going further into the design, the members to which the connection is made, should be considered. Four rivets in a horizontal line can be used in a 14-in. column in either web or flange. If moment arms of less than 1 in. are used to design a connection, certainly the outside rivets in such connections are not stressed the same as the inside pair. In investigating the strength of the column at the point of the connection, through what vertical length does the stress act in the flange or web and with what variation of stress?

The principal point, the writer wishes to bring out is that the application of theory to wind-bracing connections is a complicated problem. This also applies to ordinary shear connections. No liberty should be taken with rivet values and reasonable efforts should be made to favor rivets in tension until some time as sufficient tests have been made, and made public, to indicate to the designer just what distribution of stresses takes place.

Until such time, the single shearing unit stresses should be used for tension in rivets; the connections should be designed using a thickness of metal equal to the nominal diameter of the rivet; and the gauge should be the minimum driving clearance, riding the fillet $\frac{1}{8}$ in. There is also doubt in the efficiency of channel stubs because of their thin webs. This recommendation is based on considerable study, and the results of such meager tests as the writer has seen.

R. D. DALTON,[51] ESQ. (by letter).—The assumption that the force, $R-S$, in Fig. 3(*c*), is distributed as a triangular load on the length, C, is untenable. The surface upon which this force is presumed to be distributed is steel against steel and not steel against any other softer material. The overhanging length, C, is a cantilever and will take the form of a cantilever depending upon its load. It is inconceivable that the surface of the column steel would assume a shape such as to cause this distribution of load. It seems to the writer that the force, $R-S$, should be considered a concentrated load and that there must be some elongation of the rivet due to the lever action of the T-flange.

In Example 1, the assumption that the flange of the **T** acting as a beam is fixed at the rivet presumes an infinite force in the rivet or a deflection of the overhanging length, C, in the direction of the force, $R-S$. This deflection of the overhanging end seems impossible of attainment under the conditions.

In Fig. 7(*b*), if the heel of the upper angle is pulled away from the column face as indicated, the structural beam must have rotated about the point where the lower force, S, is applied, and the horizontal leg of the upper angle would have turned downward through the same angle with the top surface of the structural beam. Then, ϕ_1 does not equal ϕ_2 and Equations (5) and (6) do not hold true.

Also, if the structural beam is rotated, the horizontal leg of the lower angle is bent downward and the vertical leg of this angle is a cantilever. Therefore, the lower force, S, will be moved downward a certain distance, depending on the stiffness of the angle leg, and the value of Y will be somewhat greater than is given by Equation (7).

In Equation (11), Δ_1 is the only Δ that needs to be considered since the structural beam is assumed to extend at least to the face of the column in

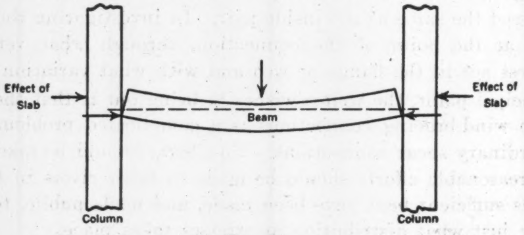

FIG. 24.

moment equations, and the horizontal leg or stem of the connections is substituted for, and is approximately equal to, the immediately adjacent parts of the top and bottom flanges of the structural beam. If the horizontal parts of the connection are approximately equal in area to the adjacent parts of the top and bottom flanges, Δ_3 and Δ_4 are already taken care of in the I and L of the structural beam.

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[51] Structural Engr., City Bldg. Dept., Oakland, Calif.

There is one action, where a continuous-slab floor is used, that may tend to relieve the bending in the connections due to gravity loads, and especially that due to live loads. When a beam is bent the tension side becomes longer and the compression side shorter. Therefore, if the slab will hold the columns a constant distance apart, a force is set up at the column face in line with the lower flange of the beam. (See Fig. 24.) Whether this action actually exists and, if it does, to what extent it may be depended upon, is very problematical and the idea is only presented here with the thought that it may or may not be found useful after investigation.

Lest this discussion appear too critical, the writer agrees with Mr. Berg in his conclusions that designers should not neglect the effect of gravity loads on the connections, and should strive for better balanced connections and a better analysis of joints. Although he believes more logical basic assumptions might be made, the paper is a forward step in breaking away from the rule-of-thumb methods of the wrought-iron age.

Further tests on typical joints should be made with special reference to the action of connecting angles and **T**'s under stresses below the elastic limit. Tests beyond the yield point or to failure do not give much information on the distribution of moments in a structure if the members of the structure are working at a stress below the elastic limit.

John J. Gould,[52] **Assoc. M. Am. Soc. C. E.** (by letter).—In the Synopsis of his paper the author states that where wind connections are provided, the effect of vertical loads on these connections as well as their adjoining members should not be neglected. It is common practice, however, to pay no attention to the effect of the vertical loads on the relatively rigid wind connections. Generally speaking, the structural engineer proceeds along the following routine.

Step 1.—Design of a Structure for Vertical Loads.—Beams and columns are designed as a statically determinate system; that is, the connections between beams and columns are assumed to be hinged and frictionless. The columns take vertical loads only; no moments are transmitted from the beams. The elastic behavior of the material according to Hooke's law is accepted.

Step 2.—Design of a Structure for Horizontal Loads.—Beams and columns are designed as a statically indeterminate system; that is, the connections between beams and columns are assumed to be rigid. The columns and beams take moments as well as shears. Hooke's law is accepted as under Step 1. It is usually assumed that the masonry takes no loads.

In comparing the fundamental principles underlying the design analysis as stated under Steps 1 and 2, the logic of the routine appears to be highly questionable.

Both analyses are based on Hooke's law that the strain is proportional to the stress. In one design for one kind of loading, a joint is assumed to be able to rotate freely, while in the next step for another kind of loading this same joint is converted into a rigid connection.

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[52] Structural Engr., with L. H. Nishkian, M. Am. Soc. C. E., San Francisco, Calif.

Some engineers are raising the question as to what is happening to this joint when gravity alone is at work, and are led to assume that the conflict between the different assumptions can be reconciled because steel has an additional quality, namely, ductility. It is also claimed that the steel frame adjusts itself.

In looking honestly into these problems the explanation is given that the "adjustment" referred to, takes place in the connections and that "ductility" implies that steel becomes malleable when subjected to high stress.[53]

The writer believes that at present the problems of adjustment of connections have been placed within reach of a sound analysis. Tests made by Professor W. M. Wilson and Professor H. F. Moore,[54] have demonstrated that rigidly built connections fulfill their duty for any kind of loads if designed within the elastic limit. It is shown that errors on the frame due to adjustments are either negligible or can be properly analyzed and guarded against.

Ductility, however, still remains one of the magic forces. So far as the writer understands the present building practice, it is quite sufficient to overstress some particular part of a structure, generally the wind connections, make the steel malleable, thereby raising the elastic limit, and the structure will stand up.

The proponents of the ductility theory have not availed themselves, during the past half century, of the opportunity to clarify their case.[55] In order to take definite advantage of ductility in a design, systematic research, together with a rational theory, should form the basis of an analysis rather than speculative assumptions. Tests of connections, built according to the ductility theory, and subjected to reversible dynamic forces, are also suggested.

It appears to the writer that the major adjustment of a structure does not take place in the connections or in the property of the material, but in the actual factor of safety of a structure. In other words, if a typical structural steel frame were built without any load-resisting walls of fireproofing; if it were then loaded vertically and horizontally to the extent of the assumptions and designed along the routine as outlined under Steps 1 and 2, it would have an actual factor of safety considerably smaller than that calculated.[56] Theoretically, such a frame would be inefficient and uneconomical because of its non-uniform distribution of stress and strength.

In regions where a structure is never called upon to deliver its ultimate strength capacity, this condition is apparently of little consequence to the investing public. It also does not seem to bother, seriously, the conscience of the professional structural engineer.

The problem, however, becomes magnified in countries where either earthquakes or hurricanes are expected. In such countries the building public is vitally interested in maximum protection for a minimum cost. It should be apparent that the owner of a building is entitled to a structural frame in which the elastic limit of the material is not exceeded before the disaster occurs, and that a safety margin of a building should not be assumed in the steel frame while actually a great part of it is in the fire-proofing and masonry walls.

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[53] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1.

[54] "Tests to Determine the Rigidity of Riveted Joints," *Bulletin No. 104*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

[55] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1213.

[56] *Loc. cit.*, p. 1485.

In the action of earthquakes on buildings one of the few definitely known facts is that a shock will produce bending moments and shears at the beam and column joints. In designing a steel structure as a continuous elastic frame for vertical loads, the savings made in the beams are placed in the connections and columns. This will make a structure far more immune from earthquake damage with about the same amount of investment, or very little more, than could be done according to the present design practice.

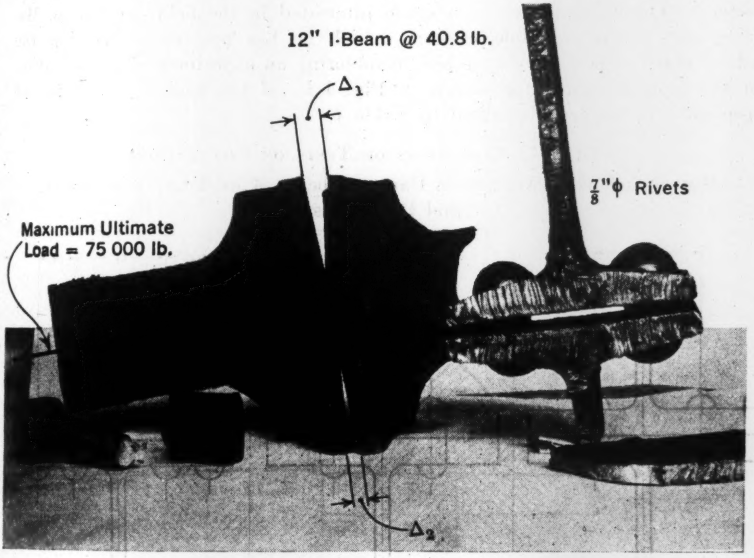


FIG. 25.

It is not intended to suggest that an engineer should apply to all structures a tedious analysis, but rather that he should be aware of the relative merits to the building owner of his fundamental assumptions.

As an addition to Mr. Berg's commendable paper, Fig. 25 illustrates a case in which the relative value of rivets and bolts in tension, of the same diameter, were tested. These tests were made by L. H. Nishkian, M. Am. Soc. C. E. Of all the tests made, the rivet had an ultimate strength of about 40% (without breaking) greater than the bolt. It is interesting, furthermore, to note that the half I-beam suffered great deformations in the direction parallel to the web (Δ_1 Δ_2, Fig. 25). Reference is also made to the author's Fig. 3(*b*), concerning which the statement is made that the flange cannot act as a cantilever.

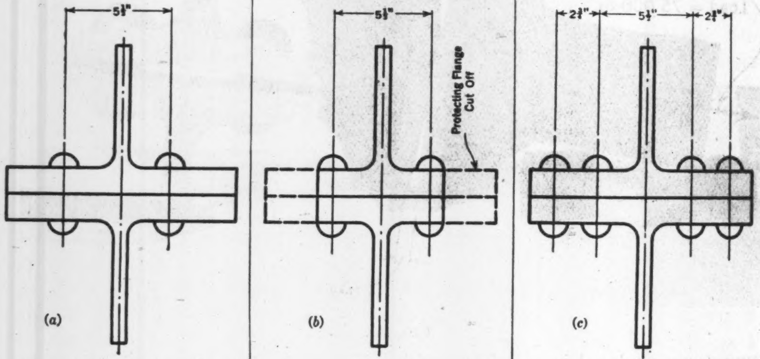
Undoubtedly, this condition is true for most of the cases. It is possible, however, that where long rivets are used, the elongation of the rivet under normal stress condition, will prevent the outer edges of the flange from bearing on the column flanges. In these cases, the flange will act as a cantilever.

The writer would recommend that in conjunction with the theoretical discussion of the author, full-sized tests of half I-beam connections be made, in order to verify their safe loading capacity so as to keep their efficiency factor within that of the structure as a whole.

W. C. Huntington,[57] M. Am. Soc. C. E., and Paul Nielsen,[58] Esq. (by letter).—The writers have been much interested in the field covered by Mr. Berg and in the valuable material which he has presented. During the school year, 1931-32, they have been conducting an experimental investigation of the type of connection shown in Fig. 4(*b*) of the paper. The types of specimens tested are illustrated in Table 7.

TABLE 7.—Conditions of Tests on Connections

(Material: One 30-in., 240-lb. Carnegie beam, 4 in. long; $\frac{7}{8}$-in. rivets; and $1\frac{5}{8}$-in. bolts.)



(a)	(b)	(c)
Specimen *A*: Rivets; initial tension	Specimen *B*: Rivets; initial tension	Specimen *C*: Rivets; initial tension
Specimen *F*: Bolts; initial tension	Specimen *H*: Bolts; initial tension	Specimen *E*: Bolts; initial tension
Specimen *G*: Bolts; no initial tension*	Specimen *I*: Bolts; no initial tension*	Specimen *D*: Bolts; no initial tension*

* In order to hold these specimens firmly together it was necessary to have some initial tension in the bolts.

The riveted specimens were made by first cutting longitudinally along the center of the web a 240-lb. Carnegie beam of sufficient length to provide the desired number of specimens. The flanges of the two parts of the beam were then riveted together, as illustrated in Table 7, the arrangement of the rivets being such as to yield the desired specimens when the member was cut into 4-in. lengths. The rivets were driven by hand with a gun riveter and are equivalent to those obtained in field riveting. It is hoped that the investigation can be extended to include shop rivets.

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[57] Prof. of Civ. Eng., and Head of Dept., Univ. of Illinois, Urbana, Ill.

[58] Instructor in Gen. Eng. Drawing, Univ. of Illinois, Urbana, Ill.

Plugs are inserted in the ends of each rivet and bolt in such a manner that the deformation of the shank can be measured accurately. This arrangement is illustrated in Fig. 26. All deformation readings are taken with an extensometer, making use of an Ames dial. Each division on the dial corresponds to a deformation of 0.0002 in., and the fractional part of each division is estimated.

A comparison of the results secured from Specimens *A* and *C* (Table 7) indicates the effect of the outside row of rivets. The effect of the bearing between the flanges on the area outside the rivets is indicated by a comparison of Specimens *A* and *B*. The effect of initial tension on the stress distribution among the rivets is indicated by comparing the results of a riveted specimen with those of a similar, bolted specimen in which the bolts are tightened only enough to hold the specimen firmly together. In Specimens *E, F,* and *H* the bolts are tightened to give the desired amount of initial tension as indicated by their deformation. These specimens give a quantitative measure of the effect of initial tension which can be varied as desired.

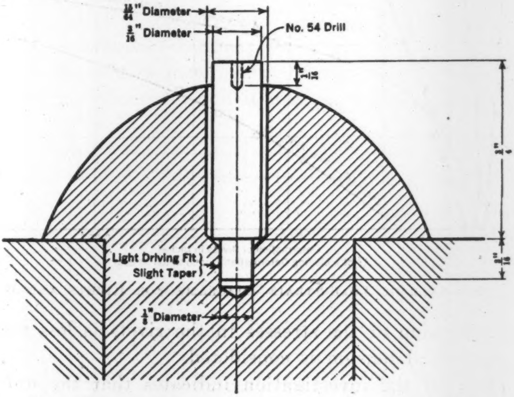

FIG. 26.—SECTION THROUGH RIVET HEAD SHOWING GAUGE-PIN.

A part of the data secured in this investigation is shown on the load-strain diagrams in Fig. 27. The deformations given are in addition to the initial deformations due to the initial stress in the rivets or bolts. These diagrams will not be discussed in detail, but it may be readily noted that the outer rivets add materially to the strength of a connection of these dimensions. For instance, for the rivet stress corresponding to a deformation of 0.001 in. in excess of the initial deformation, the load carried by the *A* specimen that has no outer rivet is 30 000 lb., while the load carried by the *C* specimen with outer rivets is 49 000 lb. That the effectiveness of the

C specimen is due to initial tension in the rivets is apparent when the load-strain diagrams for this specimen are compared with those for the *D* specimen in which the initial tension is only sufficient to hold the specimen firmly together. The effect of even this small initial tension is important, however, when calculations involving these diagrams are made. For instance, in some cases, the sum of the stresses in the rivets is less than the applied load, unless the initial tension is considered.

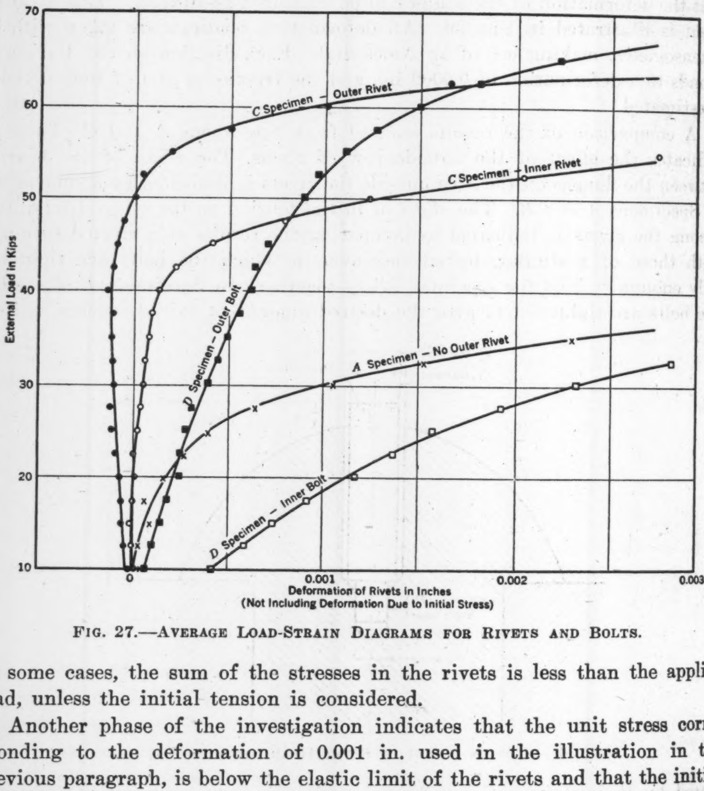

FIG. 27.—AVERAGE LOAD-STRAIN DIAGRAMS FOR RIVETS AND BOLTS.

Another phase of the investigation indicates that the unit stress corresponding to the deformation of 0.001 in. used in the illustration in the previous paragraph, is below the elastic limit of the rivets and that the initial stresses in the rivets are about the same in the *A* and *C* specimens. The data given in Fig. 27 are the averages obtained from three *A* and *C* specimens and two *D* specimens. These tests confirm other investigations which have indicated that rivets can be relied upon to carry tensile stresses.

A. TORIGGINO,[59] ESQ., and ERLE L. COPE,[60] M. AM. SOC. C. E. (by letter).—Some tests of full-sized wind connections, of cut I-beams, were made in connection with the design of the Mills Tower in San Francisco, Calif., in 1930. Fourteen specimens were tested, with rivets from $\frac{3}{4}$ in. to $1\frac{1}{4}$ in. in diameter, each having four rivets in tension. Detail dimensions of the heavier specimens, measured before testing, are given in Table 8, and the key Fig. 28, from which it is seen that the accuracy of the shop work was no greater than that in ordinary building construction. All the tension rivets were driven with an ordinary pneumatic hammer.

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[59] Cons. Engr., San Francisco, Calif.

[60] Cons. Engr., San Francisco, Calif.

TABLE 8.—DIMENSIONS OF GIRDER SPECIMENS

Specimen No.	Dimensions, in Inches (see Key Fig. 28)									
	a	*b*	*c*	*d*	*e*	*g*	*A*	*B*	*C*	*D*
Bethlehem Sections										
Bethlehem *A* — 1⅛-in........	6.31	2.73	2.68	6.28	4.40	4.30	3.98	4.27	4.29	3.99
Bethlehem *B* — 1⅛-in........	6.09	2.65	2.93	6.50	....	....	3.98	4.29	4.27	3.99
Bethlehem *A* — 1¼-in........	6.17	2.57	2.72	6.32	4.50	4.50	3.99	4.28	4.27	3.96
Bethlehem *B* — 1¼-in........	6.74	2.95	2.54	6.27	....	....	3.93	4.21	4.21	3.96
Carnegie Sections										
Carnegie *A* — 1⅛-in*.........	6.50	2.95	2.38	5.91	4.76	4.08	4.46	4.49	4.48	4.41
Carnegie *B* — 1⅛-in..........	6.96	2.28	2.74	6.28	....	....	4.53	4.58	4.56	4.54
Carnegie *A* — 1¼-in..........	6.40	3.80	2.85	6.67	4.57	4.68	4.52	4.51	4.51	4.52
Carnegie *B* — 1¼-in..........	6.45	3.06	2.61	6.18	....	....	4.52	4.52	4.53	4.60

* Dimensions of Carnegie *A*, 1⅛-in. are with respect to center of beam web which was 0.50 in. off center of column web.

The jaws of the testing machine would not grip a width as great as the desired width of the specimen, and it was necessary to reinforce the webs of the 24-in. I-beam sections to insure them against failure. The webs of the girder-beam sections were forged to dimensions to suit the testing machine. The largest beam sections in use at the time the tests were outlined, were the 36-in. 300-lb. Carnegie beams and the 36-in. 260-lb. Bethlehem beams. Before the design of the Mills Tower was completed a 36-in., 300-lb. Bethlehem beam was produced, that has greater thickness of flange at the web than either of those tested, and this heavier section was used in the building. A

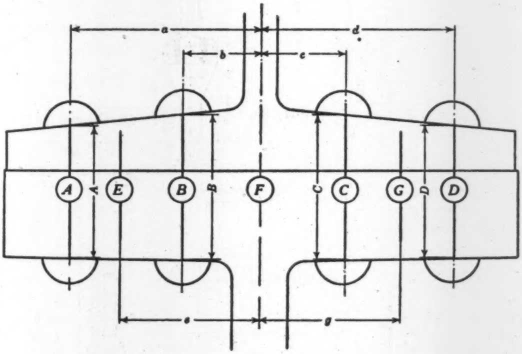

FIG. 28.—POSITIONS OF STRAIN GAUGES *A*, *B*, *C*, *D*, *E*, *F*, AND *G*.

TABLE 9.—LOADS THAT CAUSE DEFORMATIONS OF 0.005 INCH, 0.05 INCH, AND 0.5 INCH AT THE CENTER

Item	Specimen	Rivets		Beam from which connection was cut	Loads, in Kips, Causing Deformation Shown at Center			Deformation at ultimate strength, in inches	Ultimate strength, in kips
		No.	Diameter, in inches		0.005 in.	0.05 in.	0.5 in.		
1	1*A* — ¾-in.*.........	4	¾	24-in. I; 100-lb.	56.0	84	.....	0.46	123.0
2	1*B* — ¾-in.*.........	4	¾	24-in. I; 100-lb.	60.0	85	120.0	0.48	120.0
3	1*A* — ⅞-in.*.........	4	⅞	24-in. I; 100-lb.	.....	100	174.0	0.60	176.0
4	1*B* — ⅞-in.*.........	4	⅞	24-in. I; 100-lb.	.....	.....	180.0	0.60	181.5
5	1*A* — 1-in.*.........	4	1	24-in. I; 100-lb.	35.0	130	200.0	0.82	206.5
6	1*B* — 1-in.*.........	4	1	24-in. I; 100-lb.	62.5	133	207.6	0.65	212.0
7	Bethlehem *A*—1⅛-in.*	4	1⅛	36-in. B; 260-lb.	63.7	125	173.0	1.46	209.0
8	Bethlehem *B*—1⅛-in.†	4	1⅛	36-in. B; 260-lb.	102.0	121	195.0	0.80	205.0
9	Bethlehem *A*—1¼-in.‡	4	1¼	36-in. B; 260-lb.	53.5	157	225.0	0.90	253.4
10	Bethlehem *B*—1¼-in.‡	4	1¼	36-in. B; 260-lb.	126.0	140	220.0	0.95	251.0
11	Carnegie *A*—1⅛-in.§..	4	1⅛	36-in. C; 300-lb.	50.0	128	197.0	0.74	205.4
12	Carnegie *B*—1⅛-in.‖..	4	1⅛	36-in. C; 300-lb.	110.0	145	213.0	1.05	237.0
13	Carnegie *B*—1¼-in.†..	4	1¼	36-in. C; 300-lb.	76.0	154	258.0	0.94	293.6
14	Carnegie *B*—1¼-in.†..	4	1¼	36-in. C; 300-lb.	130.0	165	262.0	0.60	281.6

* Failed by "necking."
† Manufactured head snapped off.
‡ Manufactured head of Rivet *C* snapped off.
§ Rivet *B* failed in shear, in field-driven head.
‖ Rivet *B* failed by "necking."

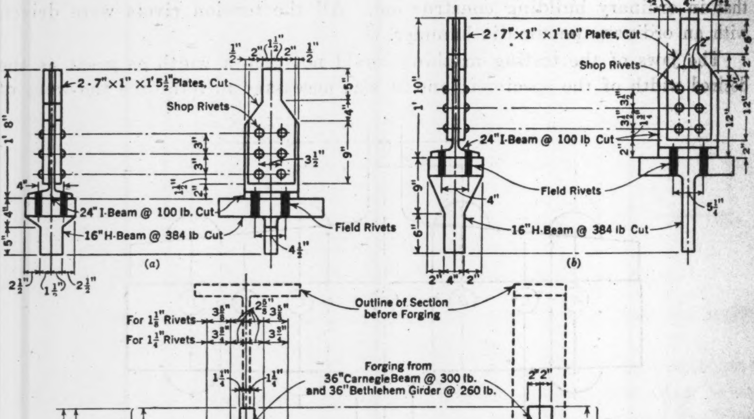
2 · 7"×1"×1' 5½" Plates, Cut
Shop Rivets
1' 8"
24" I-Beam @ 100 lb. Cut
16" H-Beam @ 384 lb Cut
Field Rivets
(a)
2 · 7"×1"×1' 10" Plates, Cut
Shop Rivets
1' 10"
24" I-Beam @ 100 lb Cut
Field Rivets
16" H-Beam @ 384 lb Cut
(b)
Outline of Section before Forging
For 1⅛" Rivets
For 1¼" Rivets
Forging from 36" Carnegie Beam @ 300 lb. and 36" Bethlehem Girder @ 260 lb.
Minimum
Finished on 4 Sides after Forging
Not Less than 2' 11" Not More than 4' 0"
(All Field Rivets)
Finished on 4 Sides after Forging
Forging from 16" H-Beam @ 384 lb.
(c)

FIG. 29.—MAKING OF TESTS SPECIMENS IN TABLE 9.

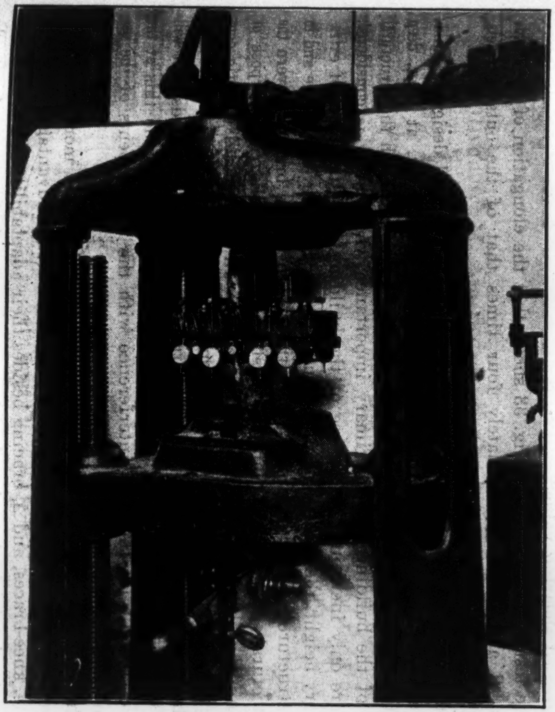

FIG. 30.—SPECIMEN, BETHLEHEM *A*, 1⅛-INCH, BEFORE LOADING, SHOWING DIAL DEFORMETERS ON RIVETS AND FLANGES.

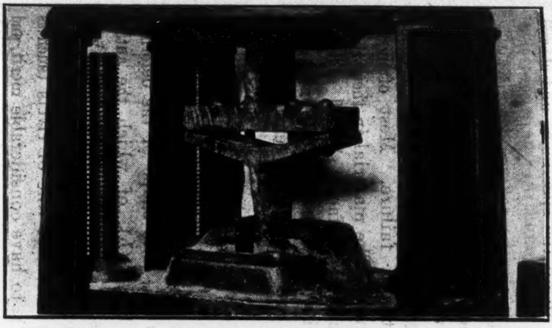

FIG. 31.—SPECIMEN BETHLEHEM *A*, 1⅛-INCH, AFTER FAILURE AT 209.8 KIPS.

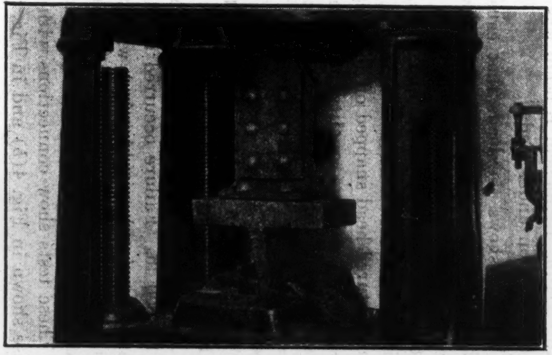

FIG. 32.—SPECIMEN 1 *B*, 1-INCH, AT LOAD OF 175 KIPS. FAILURE OCCURRED AT 212 KIPS.

16-in., 384-lb., Bethlehem **H**-beam was used in all the specimens and showed no deformation under load.

Table 9 shows the loads that caused deformation of 0.005 in., 0.05 in., and 0.5 in. at the center. It also shows the ultimate strength in each case. Fig. 29 shows the typical make-up of the various sections. While the ultimate strength of the $1\frac{1}{8}$-in. and $1\frac{1}{4}$-in. rivets (Items 7 to 14, Table 9) was satisfactory, their elongation varied greatly at failure. Most often the manufactured head snapped off to cause failure. The material in these rivets was not of uniform quality and the fractured surfaces showed a coarse crystalline structure. The tests show that rivets can be depended on to resist tension, and the writers believe that they are stronger in tension than in shear.

Test Specimen Bethlehem *A*, with $1\frac{1}{8}$-in. rivets (Item 7, Table 9), is shown in Figs. 30 and 31.

Fig. 32 shows the deformation of a cut **I**-beam connection under a load of 175 000 lb. Failure occurred at 212 000 lb. The shape of the deformed connection is almost identical with that Fig. 3(*b*), of which the author states: "It is readily seen that the **I**-beam stub connection (Fig. 3(*a*)), with a tension force of the value, 2 *S*, cannot deform as illustrated in Fig. 3(*b*)."

These tests show connections with more than one line of rivets (such as those shown in Fig. 4(*b*) and in Fig. 29(*c*)), to have considerable merit, and the writers cannot agree with the author's statement that, "connections with more than one line of rivets, such as shown in Fig. 4, have merit only because of increased shearing value."

The test record plotted in Fig. 33 shows that the elongation of the inside rivets, *B* and *C*, is approximately four times that of the outside rivets, *A* and *D*.

Many engineers are of the opinion that the architectural design of buildings should always permit the use of diagonal bracing or, at least, deep gussets or knee-braces. It should be remembered that although the structural parts of the building are of primary importance when safety is considered, they are only incidental to the use of the building. In structures of extraordinary height, the bracing assumes much greater relative importance and in such structures it may be proper to let the design of the bracing govern the architectural arrangement, but in most cases the utilitarian purpose of the building should be the controlling factor. In other cases esthetic considerations do not permit deep connections or *X*-bracing.

The structure must have sufficient strength to resist the loads that will come upon it and it is the function of the engineer to provide this at the least cost and with the least interference with the requirements of the use of the building.

The shallow types of connections are not so rigid, are more costly, and are generally less efficient; however, they have some advantages over deep gussets, knee-braces, and *X*-bracing besides their adaptability. One important advantage is the high ratio of ultimate strength of the connection to the working stress. In structures subject to forces of unknown magnitude, such as tornadoes and earthquakes, this is a very desirable characteristic. Frames with connections that will deform when highly overstressed, without failing (thereby relieving the bending stresses in the beams and columns) are much more likely to stand up than those with stiffer and more unyielding connections.

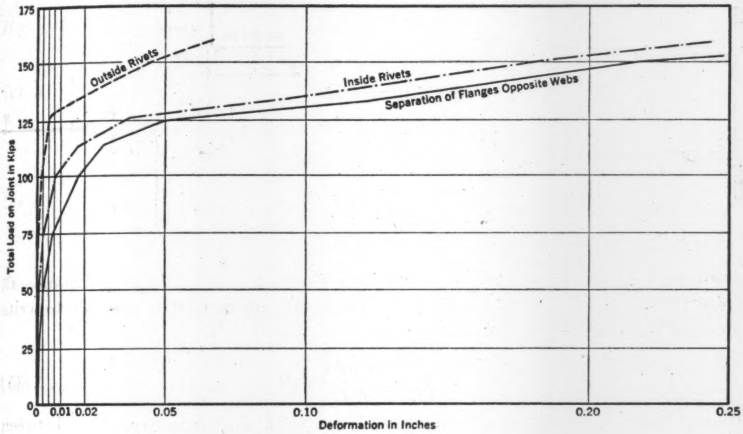

FIG. 33.—DEFORMATION OF SPECIMEN BETHLEHEM *A*, 1⅛-INCH RIVETS UNDER LOAD.

R. A. CAUGHEY,[61] M. AM. SOC. C. E. (by letter).—A problem that has been generally recognized as difficult, but which has not received much attention as far as definite solutions are concerned, is attacked in a commendable manner in this paper. It will be recognized by any one who has given the problem any thought that the author's solution is not by any means the final word on the subject, but that he has made many steps in the right direction. It is not with the idea of criticism that the writer has prepared this discussion, but rather with the hope that some of his suggestions may help in a small way in arriving at a generally accepted solution.

Angle Connections.—Referring to the derivation of Equations (5) and (6) the writer would call attention to the fact that they have been derived in such a manner as not to call attention to the shear on the rivets in the outstanding legs of the angles, caused by the tendency of the angles to straighten. Fig. 34 shows two common types of connection. The author's notation is used, except that the center line of the angle is taken as the working line of a rigid frame. The shear referred to is denoted by the symbol, *H*, in Fig. 34(*c*). This diagram shows only one angle, which is one of a pair of angles such as is used in Fig. 34(*a*), or Fig. 34(*b*).

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[61] Prof. of Structural Eng., Iowa State Coll., Ames, Iowa.

As the unit pressure on the column caused by the bending of the outstanding leg of the angle is rather small considering the material, it will be assumed that the bending is resisted by a force, $R - S = D$, at a distance, C, from the rivet. Since the distance, A', does not change, it is possible to write an elastic theory equation as follows:

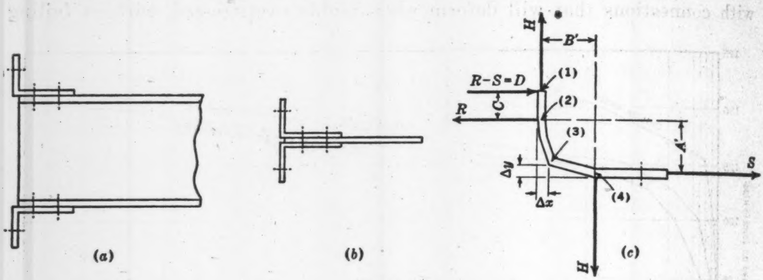

FIG. 34.

$$\int_{(2)}^{(4)} \frac{M\,x\,ds}{E\,I} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

Using Point (2) as an origin, it is seen that the quantity, $M\,x$, between Points (2) and (3) is zero, since $x = 0$. Between Points (3) and (4), $M = -D\,(A' + C) + R\,A' - H\,x$ and $ds = dx$. Then,

$$\int_{(3)}^{(4)} \frac{M\,x\,ds}{E\,I} = \frac{1}{E\,I}\int_{0}^{B}\Big[-D\,(A' + C)\,x + R\,A'\,x - H\,x^2\Big]\,dx$$

$$= \int_{0}^{B}\Big[-D\,(A' + C)\,\frac{x^2}{2} + \frac{R\,A'\,X^2}{2} - \frac{H\,X^3}{3}\Big]\frac{1}{E\,I} = 0$$

from which,

$$H = \frac{3\,R\,A' - 3\,(A' + C)\,D}{2\,B'} \quad \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

Assuming the change in angle between tangents to the curve of the angle at Points (2) and (4) equals zero, the following elastic theory equation may be written:

$$\int_{(2)}^{(4)} \frac{M\;ds}{E\,I} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

Again, using Point (2) as an origin, M between Points (2) and (3) $= -\;D\,(y\;+\;C)\;+\;R\;y$ and $ds\;=\;dy$. Between Points (3) and (4) $M = -\,D\;(A' + C) + R\,A' - H\,x$ and $ds = dx$.

Using the values of moment specified, the elastic equation is developed as follows:

$$\int_{(2)}^{(4)} \frac{M\,ds}{E\,I} = \frac{1}{E\,I}\int_{0}^{A'}\Big[-\,D\;(y + C) + R\,y\Big]\,dy$$

$$+\,\frac{1}{E\,I}\int_{0}^{B'}\Big[-\,D\,(A' + C) + R\,A' - \,H\,x\Big]\,dx = 0 \ldots\ldots\ldots (38)$$

from which,

$$\frac{-D\,(A')^2}{2} - D\,C\,A' + \frac{R\,(A')^2}{2} - D\,A'\,B' - D\,C\,B' + R\,A'\,B' - \frac{H\,(B')^2}{2} = 0$$

or,

$$H = \frac{-D\,(A')^2 - 2\,D\,C\,A' + R\,(A')^2 - 2\,D\,A'\,B' - 2\,D\,C\,B' + 2\,R\,A'\,B'}{B^2} \quad\text{..(39)}$$

Equating the values of H given in Equations (36) and (39) and solving for R,

$$R = D\left[\frac{C\,B' + A'\,B' + 2\,(A')^2 + 4\,A'\,C}{2\,(A')^2 + A'\,B'}\right] \quad\text{............ (40)}$$

Substituting for D the quantity, $R - S$, in Equation (40) and solving for R,

$$R = S\left[\frac{C\,B' + A'\,B' + 2\,(A')^2 + 4\,A'\,C}{+\,C\,B' + 4\,A'\,C}\right] \quad\text{............(41)}$$

or,

$$S = \frac{R\,(4\,A'\,C + C\,B')}{[2\,(A')^2 + 4\,A'\,C + A'\,B' + C\,B']} \quad\text{............(42)}$$

Having adopted a value of R, and having computed S by Equation (42), the value of H may be obtained from Equation (36), remembering that $D = R - S$.

In order to illustrate the method of procedure that the writer has in mind, Example 2 of the paper will be solved by use of the formulas proposed. The angle in question with applied forces and moment diagrams is shown in Fig. 35.

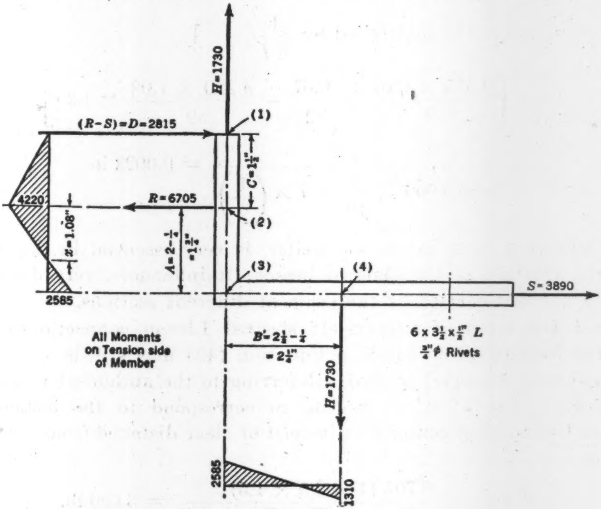
H=1730
(1)
(R−S)=D=2815
C=1½"
4220
R=6705
(2)
x=1.08"
A=2−¼
=1¾"
(3)
(4)
S=3890
2585
All Moments
on Tension side
of Member
B'=2½−¼
=2¼"
6 x 3½ x ½" L
¾" ϕ Rivets
H=1730
2585
1310

FIG. 35.

Taking into account the difference between A' and B' as defined by the writer and A and B used by the author, $A' = 2 - \frac{1}{4} = 1\frac{3}{4}$ in.; $B' = 2\frac{1}{2} - \frac{1}{4}$

$= 2\frac{1}{4}$ in.; and $C = 1\frac{1}{2}$ in. Using $\frac{3}{4}$-in. rivets and allowing tension to the amount of one-half the initial tension as does the author, $R = \frac{13\,410}{2}$ $= 6\,705$ lb. per rivet. By Equation (42),

$$S = \frac{6\,705\,(10.50 + 3.37)}{(6.12 + 10.50 + 3.94 + 3.37)} = 3\,890$$

Substituting proper values in Equation (36),

$$H = \frac{3 \times 6\,705 \times 1.75 - 3 \times 3.25\,(6\,705 - 3\,890)}{2 \times 2.25} = 1\,730$$

Fig. 35 shows the moment diagrams for the legs of the angle from Point (1) to Point (4). The value of S is 3 890 against a value of 3 440 computed by the author, and the maximum moment is 4 220 against a value of 3 270 computed by the author. The fiber stress due to bending, using 4 in. as the width of metal as did the author, is $\frac{4\,220 \times 6}{4 \times (0.5)^2} = 25\,300$ lb. per sq. in., against 19 600 lb. per sq. in. obtained by Mr. Berg. The value of X is 1.08 in. as compared to 0.95 in. obtained by the author.

The deflection, Δx, of Point (3), Fig. 34(c), may then be found from the moment diagram by the moment-area method, remembering that the deflection of Point (3) with respect to Point (2) equals the moment of the moment-area diagram about Point (3) multiplied by $\frac{1}{E\,I}$,

$$\Delta x = \left[\frac{2\,585 \times 0.67}{2} \times \frac{0.67}{2} - \frac{4\,220 \times 1.08}{2} \times 1.39\right]$$

$$\times \frac{1}{30\,000\,000 \times \frac{1}{12} \times 4 \times \left(\frac{1}{2}\right)^3} = 0.0023 \text{ in.}$$

This deflection, it seems to the writer, is very essential in the computation of the rotation of the ends of beams; it influences, very directly, the amount of moment carried by the beam at different sections.

Sheared **I**-*Beam Connections*.—If sheared **I**-beam connections are considered, the formula given for S in Equation (42) may still be used by making the distance, B', equal to zero. Referring to the author's Example 1 and changing the value of A' to 2.5 in. to correspond to the distance from web center line to rivet center line instead of clear distance from web to rivet center line,

$$S = \frac{6\,705\,(4 \times 2.5 \times 1.5)}{(2 \times 2.5 \times 2.5 + 4 \times 2.5 \times 1.5)} = 3\,650 \text{ lb.}$$

The author's value is 3 585 lb. For the flanges of the sheared **I**-beam, moments and deflection may be computed from a moment diagram as noted in the case of the outstanding legs of angles in a previous paragraph.

This method gives the results desired in a direct way and everything is taken into account that can be accounted for conveniently in a theoretical way. There may be cases in which the horizontal reaction, H (Fig. 35), on the rivets of the outstanding leg of an angle connection, with a resulting tension in the rivets of the other leg, would be very heavy. It is with the idea of taking this reaction into account and assisting in the development of a general method for the design of this type of construction (either angle or sheared I-beam connections) that the writer submits these formulas.

It becomes evident, however, as one studies this problem that more experimental data are greatly needed. Granted that there is no appreciable deformation in rivets under initial stress and that the deflection of the connection pieces is possible of close computation, there still remains the effect of the behavior of column flanges or webs to which the beam is connected, the deflection in connection pieces due to shear, etc.

U. T. BERG,[62] ASSOC. M. AM. SOC. C. E. (by letter).—High ultimate strength for connections, as given by Messrs. Toriggino and Cope, are not surprising; nor is the shape of the deformed connection anything but natural at the later stages of loading. However, in the modern steel tower, stiffness rather than ultimate strength is the important factor. H. V. Spurr, M. Am. Soc. C. E., has shown[63] that steel tonnage may have to be sacrificed to keep deformations within desirable limits; therefore, a careful study of each connection becomes imperative.

The Wilson-Oliver tests mentioned in the paper, showing uniformly high values for the initial tension of rivets, seemed to the writer to give promise of a definite basis for the design for stiffness. With rivet stresses less than the initial tension, and, consequently, practically no rivet elongation, the joint deformation would be confined mostly to the connection to the beam. Unfortunately, the tests reported by Professor Huntington and Mr. Nielsen, do not appear to substantiate initial tension values as high as 70% of the yield-point strength. This obviously jeopardizes any practical application of the initial tension values, unless the "factor of safety" is lowered proportionally.

The test results shown on Fig. 27 undoubtedly prove that connections with more than one line of rivets have merit, but the curve for the C specimen shows clearly that the inner rivets reach the yield point at a load approximately only 18% greater than 43 000 lb., which is the standard working stress for $4\frac{7}{8}$-in. rivets.

That there may be inferior workmanship, as mentioned by Mr. Wilcox, does not appear to be a sound reason why rational design methods should not be attempted. Surely, the cost of the riveting work is a small part of the total cost of a building, so that time spent for inspection could be increased many times without appreciable extra expense.

The writer has used the actual area of the rivet hole rather than the nominal area, in order to conform to the test values used as a basis for the paper and given in Table 1. Of course, a hot driven rivet can not fill the hole completely after cooling. This leads directly to the question of rivet slip and its effect on the rigidity of the joint.

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[62] Structural Engr., Brooklyn, N. Y.

[63] In his book on "Wind Bracing."

Several discussers have pointed out that the value, Δ_3, is incomplete and that an additional item should be included for the rivet slip. The reason for omitting it is based on the report of the Committee of the American Railway Engineering and Maintenance-of-Way Association[64], which concluded that:

(1) The resistance of a rivet joint against deformation by shearing forces, up to the yield point, is due to the friction between the surfaces held in contact by the rivets.

(2) The yield point of a riveted joint is reached when the shearing forces are equal to the friction of the surfaces held in contact by the rivets.

(3) The deformation of a riveted joint at the yield point is caused by the slipping on each other of the surfaces held in contact by the rivets. This slipping, is due to the "diametral" contraction of the rivets in cooling after they are driven, which leaves a space between the body of the rivets and the edge of the rivet hole.

(4) After the slip at the yield point has occurred and the rivet is brought to bear against the edge of the rivet hole, a deformation of the body of the rivet takes place with an accelerating increase in the resistance until the entire side of the rivet has been brought to bear against the edge of the rivet hole. This deformation continues beyond this point with a diminishing increase in the resistance until the ultimate strength of the rivet in shear has been reached.

An examination of the effect of a tensile force, S, acting at each end of a connection, as shown in Fig. 36, suggests that the shearing stress can not be distributed evenly among the rivets. The upper plate between Rivets R_4 and R_5 would be stressed five times as much as the lower plate with corresponding elongations. A gradual movement must take place to adjust these differences in deformations. The tests mentioned previously indicate that the friction at the plates is overcome by a force equivalent to a shearing stress on the rivet equal to, say, 10 000 lb. per sq. in., after which a gradual slipping takes place.

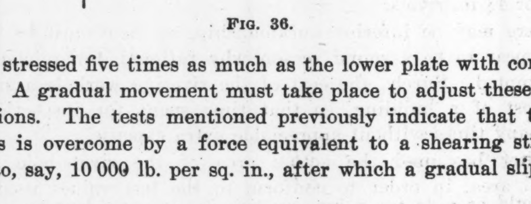

FIG. 36.

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[64] *Proceedings*, Am. Ry. Eng. and Maintenance-of-Way Assoc., Vol. 5, 1905.

The force, S (Fig. 36), would first have to overcome the friction at the rivets, R_4 and R_9. A slip would then take place, but this is limited to the amount of elongation between the rivets, R_4 and R_5 and, also, R_8 and R_9. The friction at Rivets R_5 and R_8 has not yet been overcome, and the elongations are fully elastic in spite of the gradual slipping at the first line of rivets. Subsequently, the friction at Rivets R_5 and R_8 is overcome, permitting Rivets R_6 and R_7 to come into action. It is not until the friction at all rivets has been overcome that any slipping not included in the elastic elongations will enter into the value of the total elongation.

The approximate value of the combined elongation of both plates can be expressed as follows:

Effect from Rivets R_4 and R_9 only:

$$\frac{S}{3}\,\frac{1}{E}\left(\frac{2\,B}{F} + \frac{5\,B}{2\,F}\right) = \frac{S}{3}\,\frac{B}{E\,F}\ (4.5) \quad\ldots\ldots\ldots\ldots (43)$$

Effect from Rivets R_5 and R_8 only:

$$\frac{S}{3}\,\frac{1}{E}\left(\frac{4\,B}{F} + \frac{3\,B}{2\,F}\right) = \frac{S}{3}\,\frac{B}{E\,F}\ (5.5) \quad\ldots\ldots\ldots\ldots (44)$$

Effect from Rivets R_6 and R_7 only:

$$\frac{S}{3}\,\frac{1}{E}\left(\frac{6\,B}{F} + \frac{B}{2\,F}\right) = \frac{S}{3}\,\frac{B}{E\,F}\ (6.5) \quad\ldots\ldots\ldots\ldots (45)$$

The sum of these elongations is:

$$5.5\,\frac{S\,B}{E\,F} \quad\ldots\ldots\ldots\ldots (46)$$

Equation (46) applies to both plates. However, in a practical case, the lower plate would be the flange of the I-beam and this elongation is included in Equation (17), except that the approximation is made that the moment, M_G, is constant. Actually, the value of M_G increases from zero at the face of the support, to the full value at the last line of rivets (R_9 in Fig. 36). The angular rotation corresponding to this new form for the negative moment can be expressed as:

$$\phi = \frac{L\,M_G}{E\,I}\left(\frac{1}{2} - \frac{l}{2\,L}\right) \quad\ldots\ldots\ldots\ldots (47)$$

This actually increases the value of M_G compared with the straight-line assumption.

Assuming that the elongation of the upper plate is the same as that of the lower one (actually this condition may vary; on some structures the plate may be the stiffest and, on others, the I-beam flange), and keeping in mind that the lower plate, corresponding to the flange of the I-beam, is included in the deformation as expressed in Equation (17), the net additional elongation to be included in Equation (11) would be one-half the total expressed by Equation (46), namely, $2.75\,\frac{S\,B}{E\,F}$.

This would provide for values up to a force equivalent to a shear on all rivets of about 10 000 lb. per sq. in. An examination of the aforementioned elongations[64], suggests to the writer that if the full length of the connection is included in the elongation ($6\,B$ for a condition, as shown in Fig. 36), full provision is made for a slip up to an average shearing stress on all rivets equal to, say, 18 000 lb. per sq. in. Applying the full length does not materially alter the fact that as far as determining the amount of rigidity of the connection, the reduction in the fixing moment due to the connection is small and had better be neglected.

The writer believes that if the value for Δ_2 is based on an effective length of the connection equal to the full length (to the center of the last rivet), full provision would be made for any deformation that may come into play within the practical limit of the connection (up to the point when the yield point of the tension rivet is reached), and that no additional value should be included for slip of rivet or elongation of any part of the **I**-beam, as these conditions are all included.

(The values, $E\,\Delta_2$, in Table 3, are based on an effective length assumed to be equal to one-half the distance from the first to the last rivet plus 3 in.)

Mr. Fleming points out the importance of shear deformations. Unquestionably, they would increase the values, $\Delta_1 E$, as given in Table 3—in some cases considerably—but the influence in reducing the fixation value of the joint is not appreciable. The effect on the rivet elongation is also unimportant. The writer has computed many variations of the general Equation (4), but in all these cases the effect on the rivet elongation was small. However, the influence is important on cases similar to that given by Mr. Coyle.

In his example (in which, $2\,R = 2\,S = 2 \times 19\,160$ lb., $A = 1.6$ in., $t = 1.4$ in., $C = 1.5$ in., $I = 0.69$ in^4., and $b = 3$ in.), the bending effect is: $\frac{dy}{dx} = -\frac{W A^2}{2\,EI} = 0.00118 \times 1.5 = 0.00178$ in. The shear effect is expressed by, $\frac{dy}{dx} = -\frac{3\,W}{bt\,E} = 0.00045 \times 1.5 = 0.00068$ in., and the combined effect is, $-0.00178 + (-0.00068) = -0.00246$ in. When the load is removed the tension is reduced an amount equal to $\frac{0.00246 \times 30\,000\,000 \times 0.518}{2} = 19\,200$ lb. Therefore, the permanent initial stress is equal to $19\,160 - 19\,200 = -40$ lb., which indicates a loose rivet.

For a connection such as that of Fig. 3(a), with a flange thickness varying from $u = 1$ in. to $m = 1.4$ in., the corresponding values are, as follows: The bending effect is approximately 33% more than for $t = 1.4$ in., that is, $0.00178 \times 1.33 = 0.00237$ in. The shear effect is approximately 40% more than for $t = 1.4$ in., or, $0.00068 \times 1.4 = 0.00095$ in. The combined effect is $0.00237 + 0.00095 = 0.00332$ in. In this case, when the load is removed the tension is reduced an amount equal to $\frac{0.00332}{0.00246} \times 19\,200 = 25\,800$ lb. Therefore, the permanent initial stress is $19\,160 - 25\,800 = -6\,640$ lb., which indicates the presence of a loose rivet even on a stiffer connection.

Mr. Dalton questions the assumption relative to the location of the force, $R - S$. The effective value of C is influenced, of course, by the plate thickness, hence the value, $C = \frac{1}{2}$ rivet head $+ 1\frac{1}{2}\,u$, was adopted as the maximum when computing Table 3.

Within this limit, and at the point at which the initial tension is reached on the rivet, the distance, $\frac{2}{3}\,C$, does not seem unreasonable if one is willing to assume that the initial tension presses the plates together approximately uniformly at the stage when no outside force is acting. However, for exceptional cases, such as Test Specimen B, Table 7, in which the value of C is approximately 0.75 in., and that of A approximately 2.3 in., the pressing down of the edges would increase the force under which initial rivet tension is reached.

The shortening due to plate pressure is approximately $\frac{R_i\,t}{2\,C\,b\,E}$. This leads to the following equation for the rotation at rivets:

$$\frac{R_i\,t}{2\,C\,b\,E} \times \frac{1}{2\,C} = \frac{S\,A^2}{2\,E\,I} \qquad \text{.....................(48)}$$

in which, $I = \frac{b\,t^3}{12}$. If this rotation is doubled, an approximately correct allowance would be made for the shear influence. Add this value to the original Equation (2*b*) and,

$$2\,S = 2\,R_i \left[\frac{t\,I}{A^2\,b\,C^2} + \frac{2\,C}{3\left(X + \frac{2}{3}\,C\right)} \right] \qquad \text{..............(49)}$$

This makes allowance for exceptional small values of C in relation to A.

It is correct, as stated by Mr. Dalton, that the beam rotation is not included in Equations (5) and (6). At Rivet R_4, the dip in relation to the corner would be, $\Delta_1\,\frac{B}{D}$, and the revised Equation (5) would be:

$$M_c = \frac{S\,A^2\,(2\,A + 3\,B)}{3\,A^2 + 1.5\,B\,D + 6\,A\,D} \qquad \text{...................(50)}$$

The revised value for M_c will be found to vary in most cases very slightly, and certainly not sufficiently to justify the more elaborate form.

Equation (5) is based on the design of a beam, continuous over three supports: The first fixed at R_1; the second support at the corner; and the third at R_4. The second and third supports are at the same elevation and the first support is below this line a distance, Δ_1, but this value does not influence the equations, except in deriving Equation (50). The unknown moments are: M_c, which is constant from the first to the second supports (otherwise, the shear in this span would not be S); and M_4, the fixed-end moment at R_4. Then, by slope deflection and the theory of area-moments (applying the force, S, as a triangular moment load varying in value from $S\,A$ at R_1 to zero at the second support):

$$S\,\frac{A^2}{2} - M_c\,A - E\,I\,\phi = 0 \ldots\ldots\ldots\ldots\ldots\ldots (51)$$

and,

$$-\,M_c\,\frac{B}{2}\,\frac{B}{3} - M_4\,\frac{B}{2}\,\frac{2}{3}\,B = 0 \ldots\ldots\ldots\ldots\ldots\ldots (52)$$

from which, $M_4 = -\,\frac{M_c}{2}$, and,

$$M_c\,\frac{B}{2}\,\frac{2}{3}\,B + M_4\,\frac{B}{2}\,\frac{B}{3} - E\,I\,\phi\,B = 0 \ldots\ldots (53)$$

Professor Caughey's solution should give the same result as the continuous-beam method. Equations (6) and (2*b*) give all the information supplied by Equations (39) to (42). The value, *H,* introduced by Professor Caughey, is the shear on the second span of the continuous beam. Its value is:

$$H = \frac{M_c - M_4}{B} = \frac{M_c - \left(-\,\frac{M_c}{2}\right)}{B} = \frac{3}{2}\,\frac{M_c}{B} \ldots\ldots\ldots (54)$$

This force does not seem of much importance. It is not a shear on any rivet, but pressure from the corner of the angle to the beam. Another force, equal, but in the opposite direction, produces a tension on Rivet R_4.

The writer prefers Equation (6) because it depends on the shape of the section only. It should be kept in mind that the values, *X* and *Y,* must be solved before the true value of *S* can be determined. There is absolutely no discrepancy between Professor Caughey's formulas and those of the writer, if identical values for *A, B,* and *C* are used. However, the force, $R - S$, should not be applied on an arm, *C* unless a stage beyond the initial tension value of the rivet is investigated, but then Equation (37) no longer holds true, because the elastic line ceases to be fixed at the rivet, R_1.

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STEREO-TOPOGRAPHIC MAPPING[1]

BY C. H. BIRDSEYE,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. THERON M. RIPLEY, O. S. READING, LOWELL O. STEWART, F. H. PETERS, W. H. CROSSON, DWIGHT F. JOHNS, DOUGLAS H. NELLES, R. E. BALLESTER, T. P. PENDLETON, OTTO LEMBERGER, E. R. POLLEY AND LEON T. ELIEL, AND C. H. BIRDSEYE

SYNOPSIS

This paper begins with a general introductory statement of the relative advantages of aerial and ground-survey methods in topographic mapping. A discussion of the fundamental principles of stereoscopic measurements is given to prepare the reader for a better understanding of the mechanical features involved in the construction of measuring stereoscopes. A description is given of some of the instruments in use at present for making topographic maps from photographs taken in the air. The theory of the aerocartograph is described in detail because that is the instrument with which the writer is most familiar. However, it is intended that this description be considered merely the "ground work" for a general theory and that discussion will produce comments on similar instruments in so far as they vary from the aerocartograph, on the advantages and disadvantages of the various types, and on general suggestions for improvement based on experience.

INTRODUCTION

It is recognized generally that the surface of the ground can be depicted by a proper use of aerial photography with a greater degree of faithfulness, especially as regards minor detail, than can be accomplished by the exclusive use of ground-survey methods. Modern aerial-survey methods will provide the essential information required on many engineering works, such as railroad, highway, and transmission-line location, and will eliminate many trial ground surveys at a saving of both time and money.

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[1] Presented at the meeting of the Surveying and Mapping Division, Sacramento, Calif., April 24, 1930.

[2] Eng. Asst. to Director, U. S. Geological Survey, Washington, D. C.

These modern methods are particularly well adapted to aerial mapping of regions in which the drainage, culture, or relief is complex and full of minor detail. Too often the engineer has seen the results of surveys by transit and stadia or by plane-table methods in which he knows that the features "under the feet" of the instrumentman or his rodmen are located with a high degree of accuracy, but other features which are of equal if not more importance are interpolated or "sketched in." It is not an exaggeration, but a truth that should be apparent to every engineer, to state that a map properly made by adequate photographic surveying methods must be more consistent throughout than one made by what may be called the old-fashioned ground-survey methods.

Of course, there are exceptions, and the use of the aerial camera can not be cited as a solution for all surveying problems. For example, the present-day limitations of the airplane in flying low enough to secure photographs adequate for extremely large-scale mapping seems to rule out the use of these methods for maps on scales much larger than 1 in. = 200 ft. It would be foolish to attempt to use aerial photography in a property boundary survey of an area for which the measurements of directions and lengths of the courses are required with a high degree of precision. Moreover, areas covered by dense and tall timber hiding the surface of the ground, areas in which the ground has little or no photographic contrast, and areas which are continually covered by fog, mist, or cloud shadows often make impracticable and sometimes prohibit the use of aerial photographic mapping.

Nevertheless, these and other exceptions comprise only a small part of the mapping activities which confront the engineer and in most mapping projects the aerial camera offers the engineer a new tool which will give him more help than all the other surveying and mapping instruments combined. This statement also must be qualified, for the aerial camera can not be expected to solve many problems without the aid of ground-surveying instruments, and even the most enthusiastic exponent of its use does not believe that it will ever entirely displace the transit and the plane-table.

A properly designed aerial-surveying camera may be considered as a combination of transit and plane-table. With the aid of relatively little ground-survey control and proper laboratory instruments, triangulation can be extended from the photographic data at least with tertiary accuracy sufficient for the details of most mapping projects. Photographs taken with such a camera afford a wealth of detail that could not be recorded by means of either the transit or the plane-table without an expenditure of time and effort often not justified. In fact, the writer believes that there are many inaccessible areas which can be mapped adequately by no other method than an aerial one; for instance, low-lying swamp lands or coastal areas.

Fundamental Principles of Stereoscopic Measurement

The examination and use of photographic data require a clear understanding of the principles of perspective and a knowledge of certain controlling factors, such as the position, elevation, and angle of tilt of the camera, and the positions and elevations of certain ground-control points.

However, the use of such photographs without some practical mechanical means of viewing them stereoscopically is likely to lead the observer to miss much minor detail which he should have at his disposal. In fact, it is as foolish for an observer to attempt to study overlapping aerial photographs without the use of at least some kind of simple stereoscope as it would be for a ground surveyor to attempt to eliminate the use of a protractor or a plotting scale.

The general. principles underlying the stereoscopic use of photographs as practiced in all the modern methods are much the same. A detailed description of these general principles will not be given in this paper as they have been adequately demonstrated elsewhere.[3] In the simple stereoscope, two photographs of the same object or area, taken from the ends of a base line and with the camera negatives inclined to the horizontal plane by approximately the same angle, can be adjusted to form a single plastic image of the object or area. This gives the same impression of distance and relief as is perceptible to the human eye, except that the impression of distance derived from the stereoscopic examination of a pair of photographs taken with the axis of the camera in a horizontal position, or of relief derived from examination of a pair taken with the camera axis in a vertical position, is, of course, only relative. For mapping purposes, certain absolute control data are required, such as the positions and elevations of the camera stations or the positions and elevations of at least three points in the field of view. With adequate ground-control data and with photographs viewed in a properly designed measuring stereoscope, the positions and elevations of all the features shown in common on the stereoscopic pair can be determined.

Of course, the problem is not as simple as the foregoing statement may indicate, and both the theory and practice of the stereoscopic operations vary with the nature of the problem and the accuracy desired. One of the best discussions of the stereoscopic use of photographs published in English is that by Capt. M. Hotine, R. E. The following description is prepared from Captain Hotine's paper.[4]

To demonstrate the stereoscopic fusion of two images of the same object as viewed from two different positions, hold a sheet of cardboard between the two dots in Fig. 1, so that one dot only is seen by one eye. The other dot will be hidden by the cardboard and will be in the view of the other eye. Place the head opposite the dots in the "square on" reading position, but not too close to the dots, and attempt to look past the dots and through the paper. If any difficulty is experienced in performing this feat, make two similar marks on a window pane and look actually beyond them to some object out-

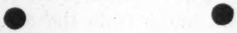

FIG. 1.—STEREOSCOPIC FUSION OF ONE PAIR OF IMAGES

side. The two dots will appear to move inward and will finally come together, appearing as a single dot in stereoscopic fusion. If this dot appears blurred maintain it in fusion by an effort of will and look back at the paper, thus bringing the fused image into visual focus.

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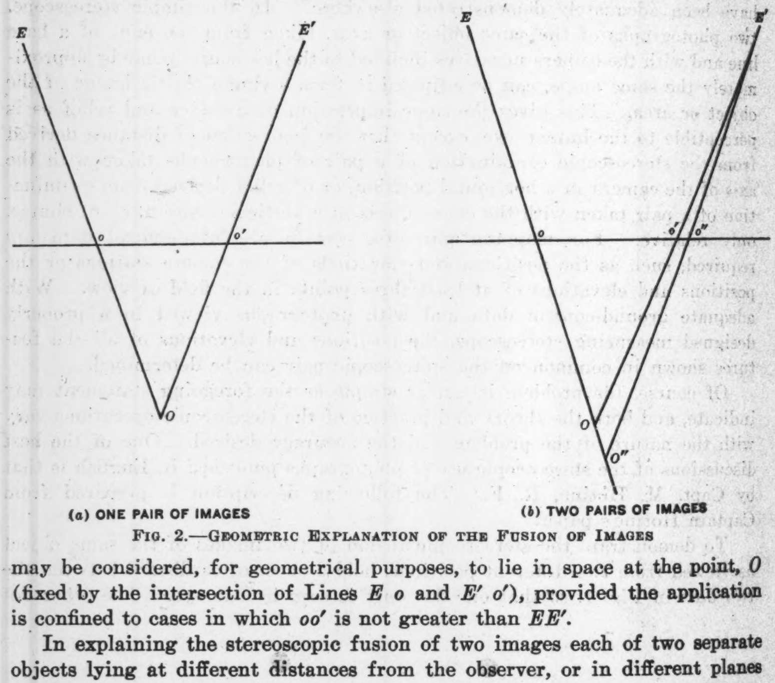
[3] "Aeroplane Topographic Surveys," *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 627-655; also, *Bulletin 788*, U. S. Geological Survey, pp. 384-398.

[4] *Professional Paper No. 4*, British Air Survey Committee; see, also, "Surveying from Air Photographs," by Capt. M. Hotine, pub. by Richard R. Smith, N. Y.

By means of the foregoing procedure one is able to appreciate the existence of only one dot where two actually exist. The question of where the third dot is, in relation to the original two, is answered by the geometrical principle of ortho-stereoscopy. In Fig. 2(*a*) let Points *o* and *o′* be the actual positions of the two dots and Points *E* and *E′*, the positions of the two eyes. Then, according to the principle of ortho-stereoscopy, the single fused dot may be considered, for geometrical purposes, to lie in space at the point, *O* (fixed by the intersection of Lines *E o* and *E′ o′*), provided the application is confined to cases in which *oo′* is not greater than *EE′*.



(*a*) ONE PAIR OF IMAGES (*b*) TWO PAIRS OF IMAGES

FIG. 2.—GEOMETRIC EXPLANATION OF THE FUSION OF IMAGES

In explaining the stereoscopic fusion of two images each of two separate objects lying at different distances from the observer, or in different planes of elevation, Captain Hotine used two pairs of dots, as shown in Fig. 3, which, with a little practice and the use of a cardboard strip, can be fused into two separate dots, provided the two horizontal pairs are not separated too far from each other.

If proper fusion is obtained, the observer will at once see that the upper fused dot is appreciably nearer than the lower. Captain Hotine explains this with the statement that there is a binocular parallax between the two images equal to the distance, *o′ o″*, on Fig. 2 (*b*), which is the difference in the separation of the component dots. As in the case of normal binocular vision, it is the existence of this parallax that gives rise to a perception of relative distance.

In Fig. 2 (*b*), the angle, EOE', is the "parallactic angle" or the "angle of convergence" of the visual directions from the eyes to the fused image, O. Similarly, the angle, $EO''E'$, is the convergence to the image, O'', formed by the fusion of the two corresponding points, o and o''. The angle, $OE'\,O''$, represents the difference of these two angles of convergence, and corresponds

Fig. 3.—Stereoscopic Fusion of Two Pairs of Images

to the parallax, $o'\,o''$. Consequently, the existence of such a difference of convergence is associated with a perception of relative distances.

The writer has repeated these illustrations almost in Captain Hotine's identical words because they express so clearly and so simply the fundamental principles of stereoscopic measurement of distance and depth and because it is important actually to see the fused image of one pair of dots floating in space above the fused image of the other pair. It is then evident that, if one can measure the difference in elevation between the fused dots with precision and have some means of recording that difference, an entirely new field in map construction has been opened.

Measuring Stereoscopes

From the stereoscopic impression of binocular parallax, which is a factor that can be measured, the observer is able to evaluate his estimation of relative depths in measurable terms, provided he has at his disposal an adequate measuring stereoscope. If the aerial photograph be considered as made up of a large number of dots, and two overlapping vertical photographs are viewed stereoscopically, it is obvious that the observer needs only the relative elevations of a sufficient number of dots or features to control his measurements in order to determine the elevations of all the dots or features represented in the stereoscopic pair. It is equally obvious that if all the elevations can be determined, half the problem of drawing contour lines is solved, and there remains only the need of some device to draw these contour lines to some definite and uniform scale.

In all measuring stereoscopes, there are two index marks (crosses, circles, triangles, etc.), one in each binocular telescope, and the stereoscopic fusion of these two marks forms a pointer which is commonly called a floating mark. This takes the place of the cross-hairs in a transit telescope, and enables the operator to measure and trace in the third dimension.

Ability to "see stereoscopically" requires both training and practice. Some possess remarkable stereoscopic power, but the majority of engineers have had neither the training nor the time to investigate the subject. The exercise, in so far as the eyes are concerned, is not harmful, and experience has shown that although progress may seem to be quite slow at the start, persistent effort will result in increasing ability to obtain stereoscopic fusion easily and without effort. This fusion is difficult, if not impossible, to obtain if the observer is at all under the influence of alcoholic drink. Jocular references to the sensation of "seeing double," therefore, may have some basis in fact; however, it should be stated that the effect is only temporary.

After ability to see clearly and rapidly with an ordinary stereoscope has been acquired, practice will be necessary to accustom oneself to the use of the floating mark. This pointer provides a reference mark by means of which the varying parallax caused by changes in the relief of the terrain can be made apparent. The fused image, or pointer, such as is herein mentioned, will appear to float in space relative to the landscape. By moving the plate-holders, it may be placed farther away or closer to the image of the ground by varying the difference in parallax existing between the mark and the image of the ground, shown as a plastic model in the fused aerial photographs.

It is not always possible to secure immediately a perfect stereoscopic effect in a measuring instrument having a floating mark or pointer. A person who can obtain a good plastic effect from an ordinary stereoscopic pair of photographs often becomes confused while observing the image moving behind a fixed pointer. The untrained eye wanders from image to pointer, and when the apparent level of the pointer is changed, has difficulty in restoring the stereoscopic effect. This lack of co-ordination may be gradually overcome by training and practice, and when finally secured, remains indefinitely. The time required for this training varies with different individuals, but it is safe to say that two weeks' application would be sufficient for one having normal sight and ability to learn to see well with the ordinary stereoscope.

There are several different types of measuring stereoscopes, ranging from instruments of the stereo-comparator type—which permit the measurement of differences of elevation, but require independent plotting—to instruments which combine the functions of measuring and plotting into one operation.

A stereo-comparator is an instrument aptly described by its name. A rather crude description is an instrument which accommodates a pair of stereoscopic views of the same object or area taken in, or rectified to, the same plane and, by measuring parallactic displacements between fused images of points, permits the evaluation of the three co-ordinates of the points. This is accomplished by comparing measured parallaxes of "unknown" points with those of points the three co-ordinates of which are known. The mechanics of the operations involves the movement of one photograph toward or away from the other in the direction of the line of flight and the accurate measurement of these movements. Such instruments permit "point to point" plotting in a three-dimensional system. Contour maps can be made by their use, but the process is slow and laborious. Instruments of this type are made by most firms that specialize in photogrammetrical instruments (see Figs. 4 and 5). An interesting but somewhat theoretical discussion of point plotting by an instrument of this type has been issued by Syracuse University.[5]

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[5] "Topographic Mapping from Aerial Photographs by Measurements with the Stereo-comparator," by Earl F. Church, M. Am. Soc. C. E., *Bulletin*, Syracuse Univ., June, 1930.

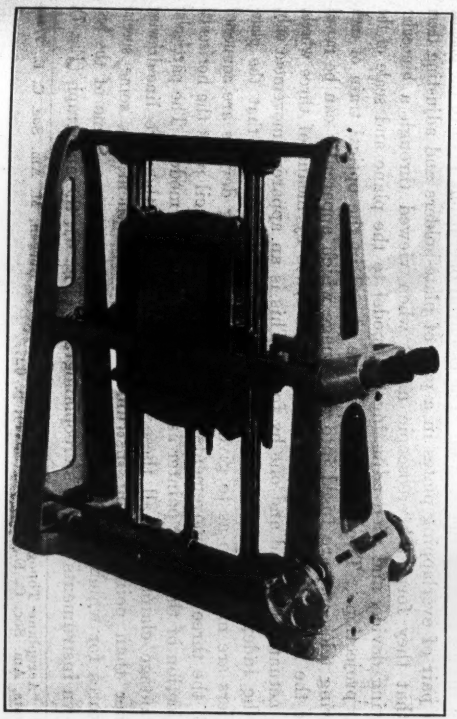

FIG. 4.—VERTICAL TYPE OF STEREO-COMPARATOR, EUROPEAN DESIGN

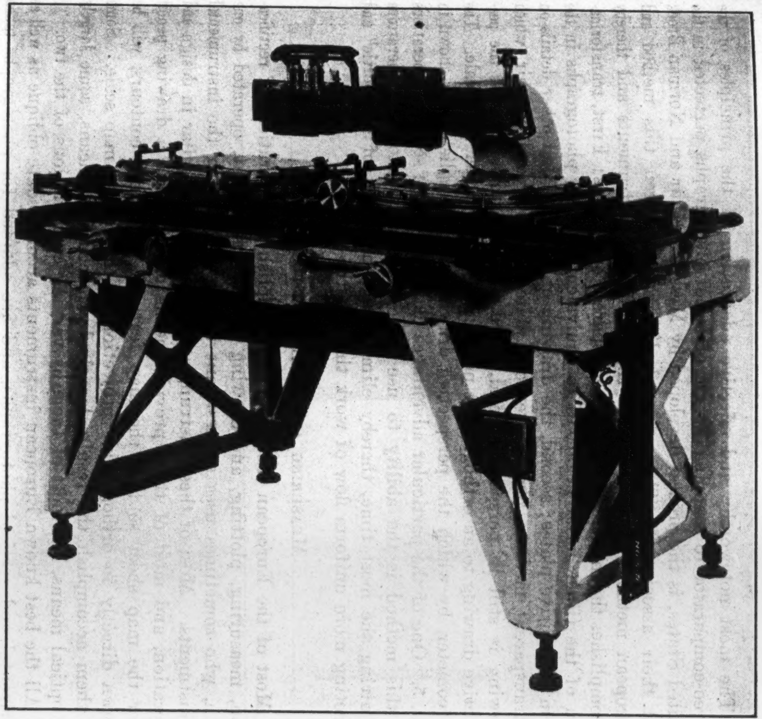

FIG. 5.—AN AMERICAN TYPE OF STEREOMETER

The most noteworthy and effective adaptation of the principles of the stereo-comparator to mapping by use of aerial photographs perfected in the United States, is the process developed by Messrs. Arthur and Norman Brock and their associates.[6] The essential difference between this method and European methods is that it uses three different instruments and thereby accomplishes the map drawing in three separate operations: First, transformation of the tilted aerial negatives to equivalent vertical photographs in the form of positive plates; second, the drawing of contours and other details on a transparent medium placed over one plate of a stereoscopic pair, which drawing is still in perspective; and, third, the transformation of the perspective drawing to an orthographic projection to the proper map scale. The stereometer, by which the perspective drawing is accomplished, is shown in Fig. 5. One of the particular advantages claimed for the separate operations of this method is the ability to use more instruments for the operations requiring the most time, thereby eliminating the "neck of the bottle" and effecting more uniform flow of work through the plant.

Measuring and Drawing Stereoscopes

Most of the European instruments accomplish the operations of rectification, measuring, plotting, and drawing in one instrument, operated by one man, who sometimes uses an assistant to help in making the instrumental adjustments. Most of these instruments are somewhat similar in design and operation, and most of them provide means of actuating a drawing pencil over the map sheet so that the map features, including contours, will be drawn directly in orthographic projection to the proper map scale. Some of them accomplish the results largely by mechanical means, some largely by optical means, but most of them by varying combinations of the two.

All the best known European instruments accommodate oblique as well as horizontal and vertical photographs, and are adapted for photographs taken from the air or from the ground. The essential operations involve the placing of a pair of overlapping plates in a pair of plate-holders and adjusting them so that they form a stereoscopic model when viewed through a binocular viewing device, and then adjusting the model to the plane and scale of the map projection. Two index marks, one placed in the optical train of each viewing telescope, are fused into one image which apparently can be moved over the model in the three co-ordinates of space by means of three wheels, two actuated by hand and one by foot. This is an apparent movement only, as the index marks are fixed, and what really happens is that the plate-holders are moved so as to give this result. Plotting devices are connected with the three space movements so that a drawing pencil plots the horizontal projection of the fused pointer as it is moved over the model. The particular advantage claimed for all these instruments is that they are line-drawing rather than point-plotting instruments. Each of them has some special features for which the maker claims specific advantages. Some of the best known instruments are the stereoplanograph (Fig. 6), the autograph (Fig. 7),

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[6] "Aeroplane Topographic Surveys," by George T. Bergen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 627.

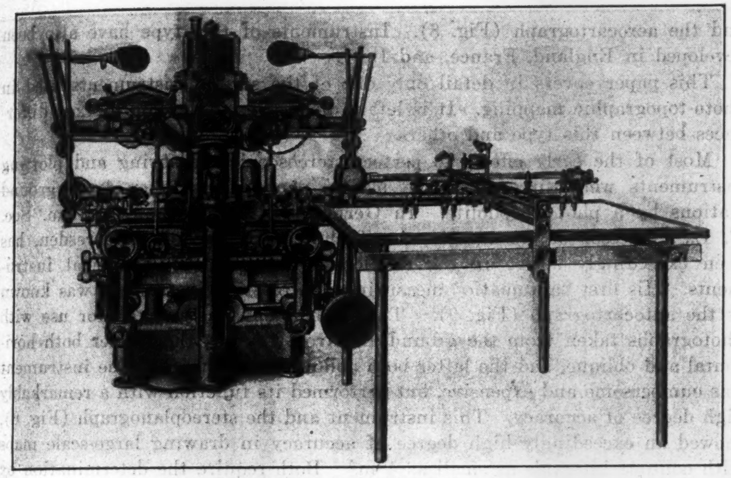

FIG. 6.—THE STEREOPLANOGRAPH

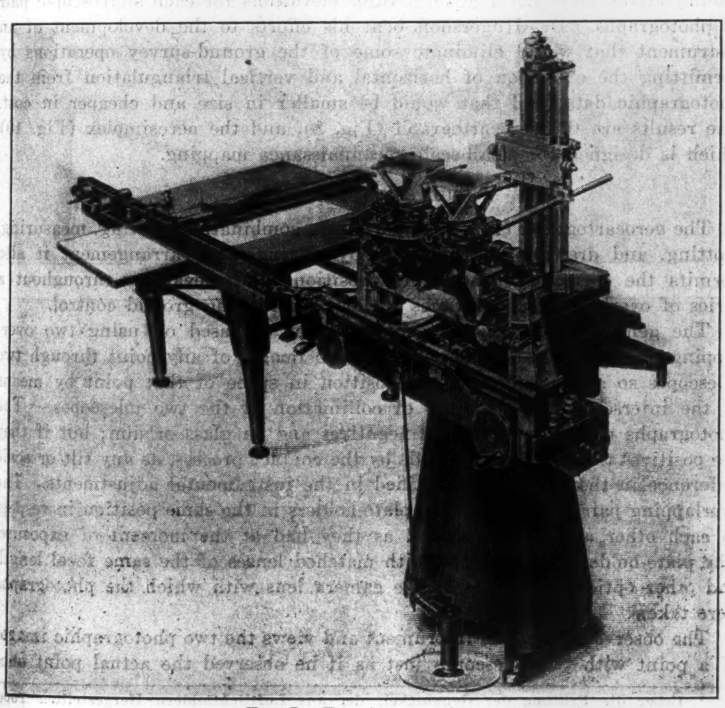

FIG. 7.—THE AUTOGRAPH

and the aerocartograph (Fig. 8). Instruments of this type have also been developed in England, France, and Italy.

This paper covers in detail only one of the several instruments used in photo-topographic mapping. It is left to discussers to develop marked differences between this type and others.

Most of the early efforts to perfect stereoscopic measuring and plotting instruments were directed to the use of photographs taken from ground stations by a photo-theodolite. In Germany, R. Hugershoff, M. Am. Soc. C. E., Professor of Geodesy in the Technical Hochschule at Dresden, has been exceedingly active in the development of photogrammetrical instruments. His first "automatic" measuring and plotting instrument was known as the autocartograph (Fig. 9). This instrument was adapted for use with photographs taken from the ground and from the air, the former both horizontal and oblique, and the latter both oblique and vertical. The instrument was cumbersome and expensive, but performed its function with a remarkably high degree of accuracy. This instrument and the stereoplanograph (Fig. 6), showed an exceedingly high degree of accuracy in drawing large-scale maps with contour intervals as small as ½ m.[7] Both require the determination by ground-survey methods of at least three elevations for each stereoscopic pair of photographs. Dr. Hugershoff bent his efforts to the development of an instrument that would eliminate some of the ground-survey operations by permitting the extension of horizontal and vertical triangulation from the photographic data, and that would be smaller in size and cheaper in cost. The results are the aerocartograph (Fig. 8), and the aerosimplex (Fig. 10) which is designed for small-scale reconnaissance mapping.

THE AEROCARTOGRAPH

The aerocartograph (Figs. 8 and 11) is a combination viewing, measuring, plotting, and drawing instrument. By an ingenious arrangement it also permits the carrying of horizontal positions and elevations throughout a series of overlapping photographs without intervening ground control.

The general theory of the aerocartograph is based on using two overlapping photographs and observing the two images of any point through two telescopes so as to determine the position in space of that point by means of the intersection of the lines of collimation of the two telescopes. The photographs may be positives or negatives and on glass or film; but if they are positives they are usually made by the contact process, as any tilt or scale difference in the negative is rectified in the instrumental adjustments. The overlapping pair is placed in two plate-holders in the same position in respect to each other and to the ground as they had at the moment of exposure. The plate-holders are equipped with matched lenses of the same focal length and other optical properties as the camera lens with which the photographs were taken.

The observer sits at the instrument and views the two photographic images of a point with two telescopes just as if he observed the actual point once

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[7] "Ueber die Prüfung der Genauigkeit der aus Luftlichtbildern Hergestellten Topographischen Grundkarte 1:5 000 von Amrun und ihre Wirtschaftlichkeit."

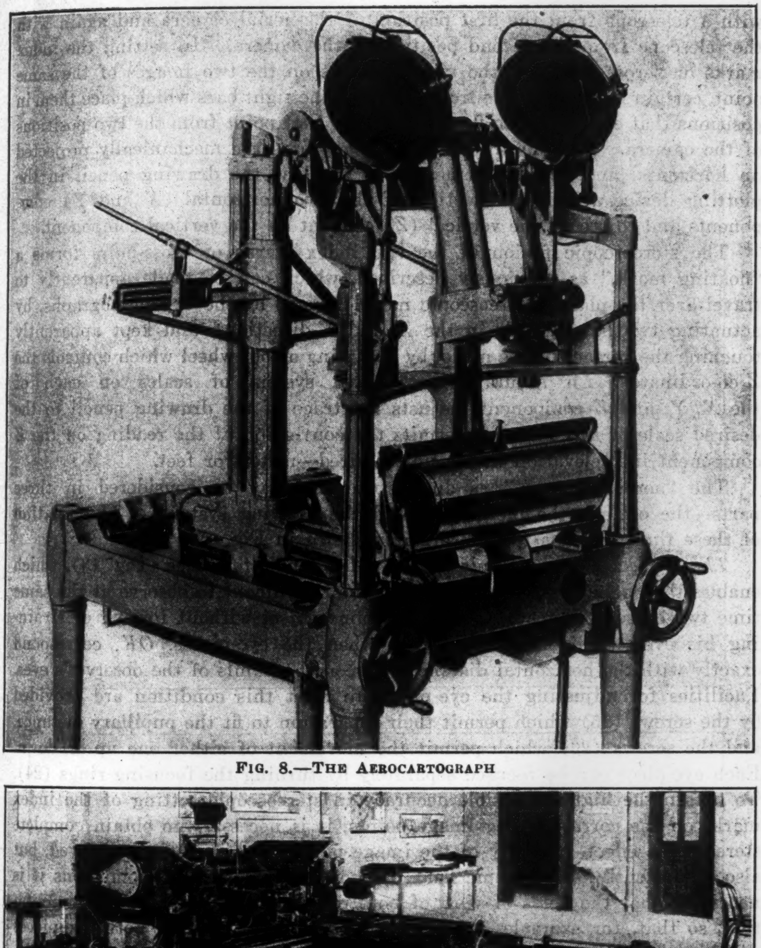


FIG. 8.—THE AEROCARTOGRAPH

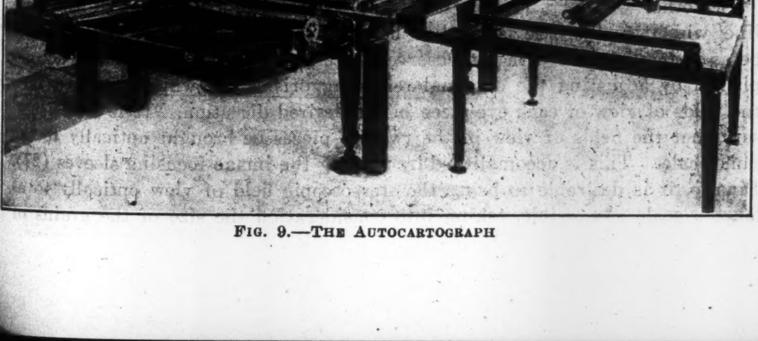


FIG. 9.—THE AUTOCARTOGRAPH

with a telescope from the first position of the aerial camera and again with the telescope from the second position of the camera. In setting the index marks or "cross-wires" in the two telescopes on the two images of the same point, certain movements are transmitted to the sight bars which place them in positions that correspond to lines of sight to that point from the two positions of the camera. These positions of the sight bars are mechanically projected on horizontal and vertical planes so as to place the drawing pencil in the plotting device at the intersection of the two horizontal (X and Y) components and to record the vertical (Z) element on the vertical component.

The stereoscopic fusion of the two index marks or cross-hairs forms a "floating mark," as previously described, which can be made apparently to travel over the plastic stereoscopic model formed by the two photographs, by actuating two hand-wheels in the X and Y-directions, and kept apparently touching the surface of the model by actuating a foot-wheel which controls the Z-co-ordinate. An accurately graduated system of scales on each of the X, Y, and Z components adjusts the trace of the drawing pencil to the desired scale of the map and permits the conversion of the reading on the Z component into elevation in the unit desired—meters or feet.

The "aerocartograph," as an instrument, may be considered in three parts—the optical, the measuring, and the drawing systems. The relation of these three systems to each other is shown in Fig. 12.

The Optical System.—This consists of a double telescope (Fig. 13), which enables the operator sitting in front of the instrument to observe at the same time two corresponding images of the same object without forcing or straining his eyes. This is possible only when the eye-pieces, *OK*, correspond exactly with the horizontal distance between the pupils of the observer's eyes. Facilities for adjusting the eye-pieces to meet this condition are provided by the screws (25) which permit their separation to fit the pupillary distance, and the screws (26) which permit the movement of either one up or down. Each eye-piece can be focused separately by turning the focusing rings (24). To obtain the highest possible accuracy in stereoscopic setting of the index marks on the corresponding image points, it is necessary to obtain complete stereoscopic effect, not only of the image points that are being observed, but also of the entire stereoscopic field. In order to meet these conditions it is necessary that the fields of view of both eyes have the same optical orientation, so that, for example, the images of a straight road in both eye-pieces will be parallel.

In other words, it is necessary that the vertical projection of the base line, which in this case is the line between the two camera stations, should be as nearly parallel with the observer's eye-base as possible. This is accomplished by adjusting the diagonal reflecting prisms, *D*, which serve to turn the fields of view of each eye-piece in the desired direction. It is also necessary that the fields of view in the two eye-pieces be brought optically to the same scale. This is accomplished by turning the image focusing sleeves (27). Finally, it is desirable to bring the stereoscopic field of view optically to as large a scale as possible, taking into consideration the size of the grains of

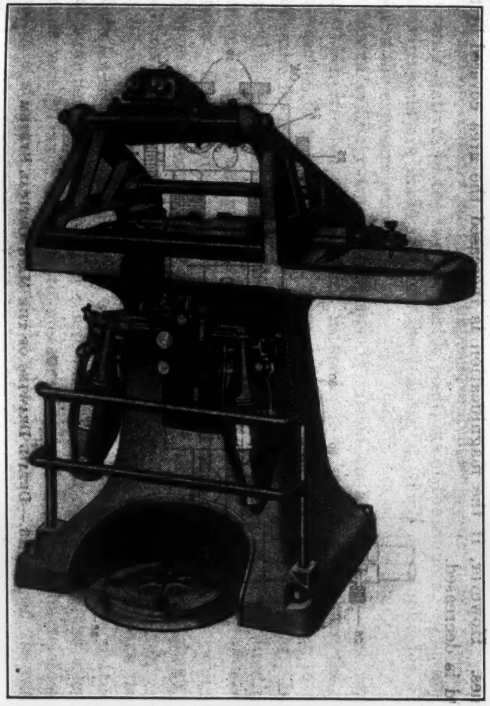


Fig. 10.—The Aerosimplex

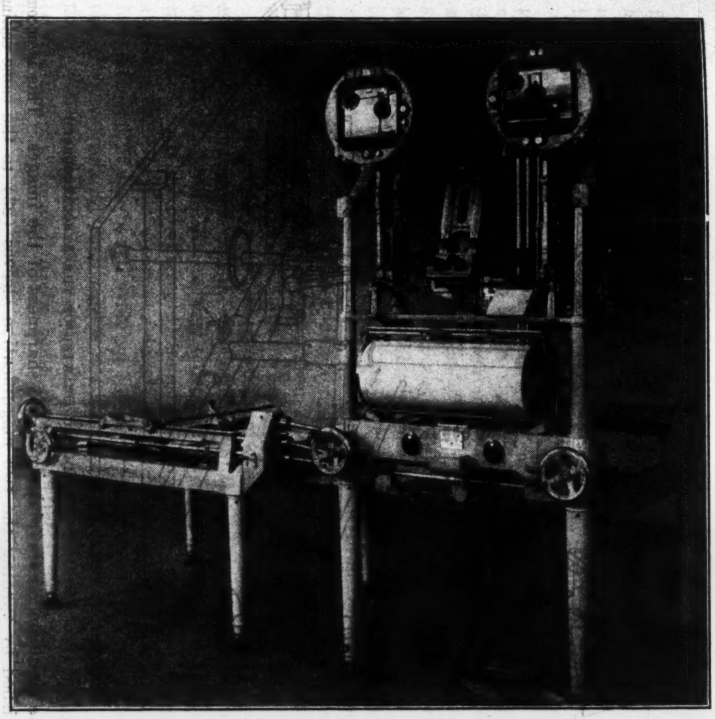
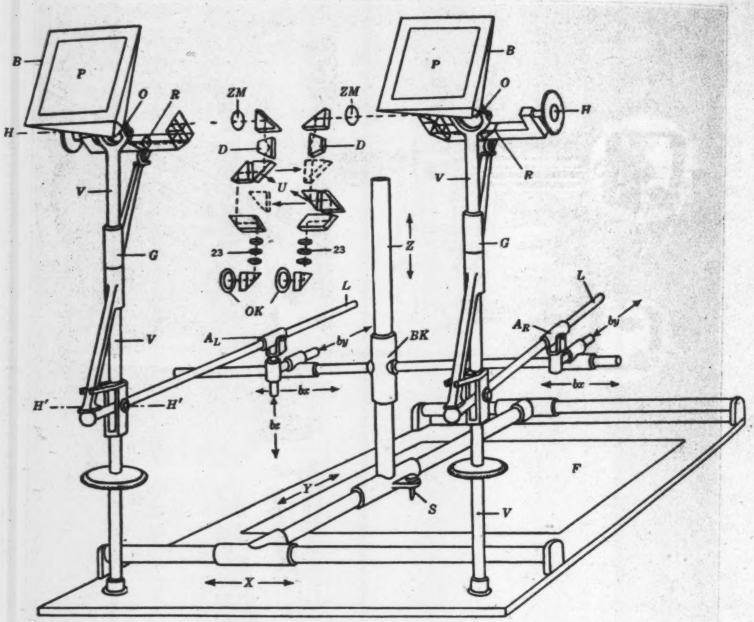


Fig. 11.—The Aerocartograph with Co-Ordinator


B
P
O
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ZM
D
D
U
23
23
OK
L
A_L
by
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G
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H'
bx
bz
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S
X
F
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L
A_R
by
bx
V

Fig. 12.—Schematic Drawing of the Aerocartograph

the emulsion on the plates and the sharpness of the images. This is accomplished by turning the two magnification sleeves (23) to the same setting. These permit changing the magnification from two to four and one-half times. However, if the magnification is increased the area covered by the field is decreased.

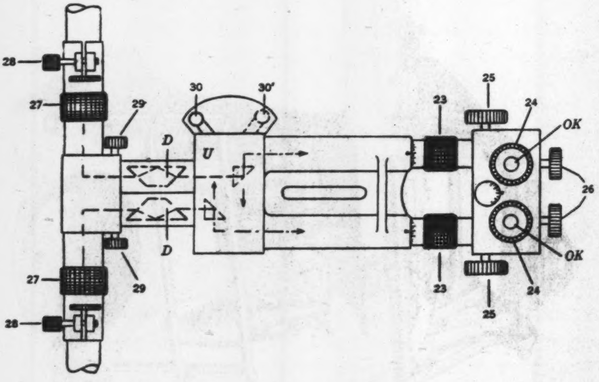
28
27
29
30
30'
D
U
D
29
27
28
25
23
24
OK
26
OK
23
24
25

Fig. 13.—Detail Drawing of the Main Optical System

Besides fulfilling all conditions required by modern stereoscopic practice, the optical system contains a special installation, *U*, which, by means of a lever (30–30′), permits the switching of a prism in each telescope to a corresponding position in the other telescope so that the right-hand plate can be viewed with the left eye, and *vice versa.* This switching of prisms has two advantages: First, the changing from normal stereoscopic vision to pseudo-stereoscopic observation, in which mountains appear as valleys and *vice versa,* and trees appear to be growing into the ground instead of above the ground, permitting duplicate measurement of their height, once in normal and once in pseudo-position; and, second, the changing of only one plate at a time in a sequence of overlapping photographs, so that after the operator has completed his observations on Plates 1 and 2 in the series, Plate 1 can be removed and Plate 3 inserted in its place and adjusted by images alone to Plate 2, so as to form a second stereoscopic model properly conformed to the map projection. The last mentioned operation makes it possible to carry horizontal and vertical control from one photograph to another without any ground control except in the first and last plate pair of the series.

The Measuring System.—The recording of directions in space by the aerocartograph is analogous with the measurement of these directions by a transit, but instead of sighting the natural object with a transit, the aerocartograph operator observes the photographic image of this object. In order to locate a point in space by a transit, two directions in space must be measured, one from each end of the base, involving the use of two transits or one transit placed, successively, at each base end station (aerial or ground camera stations). The aerocartograph operator locates this point in space by sighting two photographic images of the same point, one taken from each camera station, by a binocular telescope.

The photographic images are contained in the diapositive plates, *P,* in the plate-holders, *B* (Fig. 12). These plate-holders are optical substitutes for the exposure camera and the diapositive plates are reproductions of the camera negatives in the two successive positions of the camera. The angular measurements are made through the lenses, *O,* which are duplicates of the lens in the exposure camera.

As an ordinary transit telescope is turned on a horizontal and vertical axis when directed at any point in space, it is necessary to have the same movements, or substitutes for them, in the aerocartograph in order to cover the entire field of view in a stereoscopic pair of photographic plates. In the operation of a transit the direction in space of the line of collimation is resolved into two components which are recorded as horizontal and vertical angles. In the same manner the line of collimation of each observing telescope in the aerocartograph, when directed at the photographic image of a point, is resolved into two components.

However, the viewing of the photographic image of the landscape instead of the landscape itself permits a substantial simplification in the construction and movements of the sighting telescopes. Instead of rotating the telescope on its vertical axis in order to measure horizontal angles, the telescope is kept in a stationary position and the plate-holder is rotated on its vertical axis, *V*. Instead of inclining the telescope on its horizontal axis in order to measure vertical angles, a set of reflecting prisms, *R*, is rotated on a horizontal axis, *H-H*, in front of the objective lens of the telescope. The use of a fixed and stationary binocular telescope simplifies the simultaneous operation of two telescopes and permits stereoscopic examination of two corresponding images of the same natural objects. The optical intersection of the two lines of sight determines the location of an object in its three co-ordinates in space. This imaginary intersection, of course, must be rectified to the scale of the co-ordinate system.

The mechanical expressions of the two lines of collimation are the guide rods, *L*, which act like two transit telescopes. These move up and down on their horizontal axes, *H′-H′*, and rotate about the vertical plate-holder axis, *V*. An azimuthal change of their direction, with respect to the co-ordinate system of the instrument, causes the plate-holders to turn simultaneously through the same horizontal angle. A change in the inclination of the guide-rods is transmitted through the sliding members, *G*, to the revolving reflecting prisms, *R*. This is accomplished with angular precision. Thus, the guide-rods, when once made parallel to the lines of collimation of the corresponding telescopes, will be the true representation of the lines of collimation of the telescopes and will permit directing them toward any point in the photographic field of view. The line of sight in each telescope is defined by an index mark, *ZM*, which corresponds to the cross-wire in an ordinary telescope. These index marks are engraved on the inside face of a lens as indicated in Fig. 12. The rays of light originating from image points on the photographic plates are brought into coincidence with these marks in their plane. By forming this coincidence in this place it is possible to avoid measuring errors which would have been introduced if the plane of coincidence had been in other parts of the optical system. Binocular observation with both telescopes properly adjusted brings these two index marks into fusion and produces a single pointer or floating mark which seems to touch the object sighted.

Different lenses used in exposure cameras, although of the same specifications, may have slightly different focal lengths from those in the plate-holders. This difference of focal length must be compensated for in the plate-holders by changing the position of the plates with respect to the lenses. This is accomplished by special focusing rings on the plate-holders.

The horizontal position of an image point is recorded in the drawing system either graphically or on the two horizontal co-ordinate scales. The elevation is recorded on the vertical co-ordinate scale and also on a rotating recorder something like a speedometer placed at the right side on the front of the instrument so that the operator can observe the elevation reading without moving from his seat. The graduations on all the recording scales are in millimeters and can be read to tenths by means of verniers. Conversion of these readings to actual dimensions in space in the units desired is simple and depends on the map scale.

The Drawing System.—This consists of a three-dimensional co-ordinate system (Fig. 14) functioning with the guide-rods so as to draw a horizontal projection of the photographic images to the map scale desired. In comprehending this system, one must keep in mind that the instrument is designed for use either with vertical (or oblique) photographs taken from the air, or with horizontal (or oblique) photographs taken from the ground. (It is understood that references to vertical and horizontal photographs mean photographs taken with the optical axis of the camera in a vertical or horizontal position.) One must also keep in mind that the plane of the drawing-board in the base of the instrument is approximately at right angles to the plane of the plates in the plate-holders so that the axes of co-ordinates in the drawing system differ in drawing from vertical and from horizontal photographs. In the case of horizontal photographs, the Z-co-ordinate, as shown on the diagrams in Figs. 12 and 14, is the elevation element, while in the case of vertical photographs the Y-co-ordinate is the elevation element.

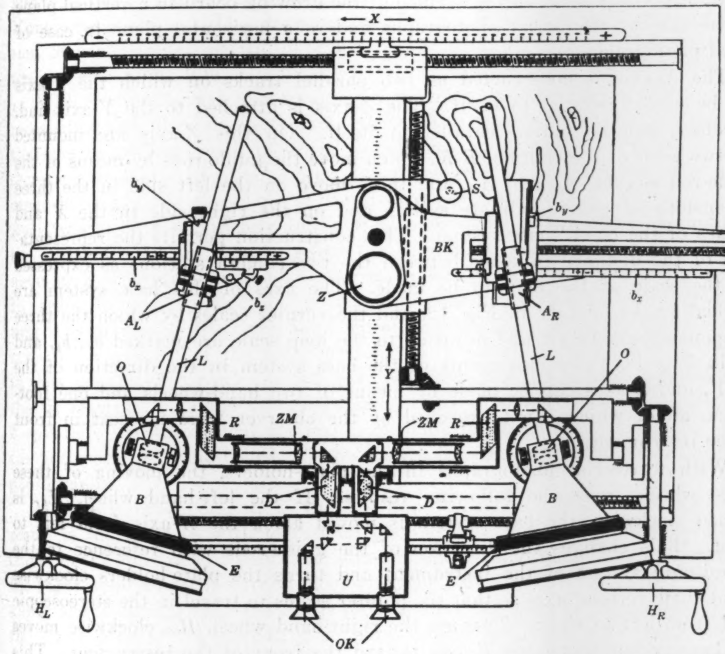

FIG. 14.—THE AEROCARTOGRAPH AS SEEN FROM ABOVE

It is easier first to consider the use of the instrument with horizontal photographs taken from ground stations, because these photographs are really taken in a plane at right angles to the plane of the map to be made, which planes correspond to the planes of the plates and the drawing-board in the instrument, respectively. It is then easy to understand how, by a simple shifting of gears, the movements are adapted to the proper co-ordinates in drawing from vertical aerial photographs. The axes of the drawing-board, *F*, are the *X* and *Y*-axes of the instrument. The *X*-axis is parallel to the observer's eye-base in a vertical plane in the case of terrestrial photographs, and in a horizontal plane in case of aerial photographs. The *Y*-axis is at right angles to the eye-base in a horizontal plane in the case of terrestrial photographs and in a vertical plane in case of aerial photographs. The *Z*-axis is perpendicular to the surface of the drawing-board in a vertical plane in the case of terrestrial photographs and in a horizontal plane in case of aerial photographs.

The *X*-axis is constructed as two parallel tracks on which the *Y*-axis can be moved parallel to itself. The *Z*-axis is attached to the *Y*-axis and, therefore, can be moved parallel to itself. On this *Z*-axis are mounted the two arms of the column, *B K,* which move the guide-rods by means of the guide-rod sleeves, A_L and A_R, and these move on the left side in the three dimensions of the co-ordinate system and on the right side in the *X* and *Y*-axes of the co-ordinate system. This construction permits the representation of the distance in space between the two camera stations as expressed on the scale of the map to be made. The axes of this base system are marked *bx, by,* and *bz* in Fig. 12, and the vernier scales by which the three components can be set and adjusted to the map scale are marked b_x, b_y, and b_z, in Fig. 14. The movements of the base system in the direction of the *X, Y,* and *Z*-axes can be made by means of two hand-wheels and one foot-wheel, all of which can be reached by the observer from his seat in front of the instrument.

With terrestrial photographs in the plate-holders, the moving of these three wheels gives the following results. If the left-hand wheel, H_L, is turned clockwise, the base system is moved along the *X*-axis from left to right. This changes the direction of the guide-rods with reference to the co-ordinate system of the instrument and turns the plate-holders clockwise about their vertical axes so that the pointer seems to travel in the stereoscopic field from left to right. Turning the right-hand wheel, H_R, clockwise moves the base system along the *Y*-axis toward the front of the instrument. This causes the guide-rods to converge so that the intersection of their directions is closer to the eye-base and the pointer seems to move to the foreground of the stereoscopic field. Turning the foot-wheel clockwise moves the column, *B K,* downward along the *Z*-axis, tilts the guide-rods, and raises the intersection of their directions, so that the pointer is moved toward the upper edge of the stereoscopic field.

These three movements, all being translated to the guide-rods, are so interrelated that one affects the others, and they must be made simultaneously. By means of these movements the pointer can be traced in any direction over the model and kept apparently touching the surface at all times. If the drawing pencil, *S,* is connected with the sliding collar at the base of the *Z*-component, it will plot on the drawing-board the horizontal projection of each optically intersected point or of the trace of the pointer over the stereoscopic model. The vertical component can then be read on the graduated scale on the Z-column, in millimeters and decimal parts, which can readily be transposed to elevations above the datum plane in the units desired — feet or meters.

If the foot-wheel is turned so that the reading on the Z-co-ordinate scale represents a certain contour and is left in that position so that the elevation reading will always be the same, the pointer can be forced over the model and apparently kept in contact with its surface by means of the X and Y-movements alone. The drawing pencil then traces the horizontal projection of a continuous series of points of the same elevation which must be the proper contour line.

With vertical aerial photographs in the plate-holders the relation of the measuring and drawing systems to the optical system must be changed so that the co-ordinates will conform to the length, breadth, and depth of the stereoscopic field. In other words, the plane of the plate-holders should be made horizontal, or the plane of the drawing-board should be made vertical. As it would be mechanically impracticable to make either of these changes, the simplest solution is to to select the X and Z-co-ordinates of the instrument as a theoretical map plane corresponding to a vertical drawing-board. In order to accomplish this, a second Z-spindle is thrown into gear so that it is driven by the right-hand wheel and the same gear-switching transmits the movement of the foot-wheel to the Y-spindle. The drawing pencil is then connected directly to the gear which drives the second Z-spindle so that it receives the same movement from the right-hand wheel as the second Z-spindle. The theoretical vertical map plane has thus been turned 90° and the horizontal drawing-board, *F*, now serves for drawing from vertical aerial photographs.

With this arrangement, if the left-hand wheel is turned clockwise, the result is the same as with terrestrial photographs, and the pointer seems to travel in the stereoscopic field from left to right parallel to the line of flight. If the right-hand wheel is turned clockwise, the column, *B K*, is moved upward along the Z-axis, tilting the guide-rods so that the pointer is moved toward the lower edge of the stereoscopic field — in the same plane as the X-movement, but at right angles to it, the two movements being in the horizontal plane in space. The drawing pencil, however, moves in the direction of the Y-axis toward the front of the instrument. Turning the foot-wheel clockwise moves the base system along the Y-axis toward the back of the instrument. This causes the guide-rods to diverge so that the intersection of their directions is farther away from the eye-base and the pointer seems to move downward or into the ground. The drawing pencil will then plot in the X and Y-co-ordinates the horizontal trace of the movement of the pointer over the model, and the vertical component or elevation can be read on the graduated scale on the Y-axis.

The aerocartograph has two other drawing surfaces so that, if desired, three maps can be drawn at the same time on the same or on different scales. The drawing drum shown in Fig. 8 is placed on the front side of the instrument just below the observer's eyes. This can be covered by tracing paper and, by an ingenious gear arrangement, the same movements in the plane of the map are transmitted to a second drawing pencil and a second copy of the map is made. The particular advantages of this arrangement are that the observer can see what he is drawing without moving his position, and he can remove at any time the tracing paper and compare his work with the plotted positions on a master sheet.

The third drawing surface is on a separate table known as the co-ordinatograph, which is geared to the co-ordinate spindles of the map plane and on which a third pencil traces the apparent movement of the pointer in the same way and with the same accuracy as the pencil on the base drawing-board. The particular advantages of the co-ordinatograph are that the drawing-board is more accessible and the gear ratio permits a larger change in scale from the photo-scale than can be accomplished by the drawing system in the base of the instrument. The co-ordinatograph also permits the plotting of projections and control points from known co-ordinates with more facility and accuracy than can be accomplished by the usual drafting mears.

Effect of Relief.—The pointer provides a reference mark by which the varying parallax caused by the changing relief of the terrain may be made visible. It is understood, of course, that the term, "pointer," refers to the fused image of the two index marks, one in each eye-piece, which are rigidly fixed in the optical axes.

If identical photographs be used in each plate-holder, and viewed as one, there would be no stereoscopic effect, and the pointer would remain in apparent contact with the ground at all points in the area, no matter how great the relief. If a stereoscopic pair be viewed and so adjusted that the pointer coincides with the image of a hilltop, and the plate pair moved as a whole, so that the pointer covers a lower area, it will appear to float above the ground, owing to the parallax.

The fixed positions of the two index marks may be represented by the points, A and A' (Fig. 15), on a line representing the plane of coincidence where it intersects the traces of the rays from a hilltop to the camera in its successive exposure Stations 1 and 2. The two index marks are then in coincidence as seen with each eye-piece and, therefore, are fused into one image which appears to rest on the hilltop. If the left-hand wheel is turned, effectively moving the index marks in the fixed plane of coincidence to B and B', respectively, the ray from the image, x, of the lower point in the left plate intersects the plane of coincidence at B, but owing to the parallax, $B'B''$, the ray from the corresponding image, x', in the right plate does not intersect the plane at B' but at B'', and, consequently, monocular vision of the corresponding images of the lower point (seen by alternately closing the two eyes) will show a displacement between the two index marks and the two respective images of the same point on the ground. Stereoscopic vision (with each eye focused on its respective index mark) will show stereoscopic fusion of the index marks into one pointer which appears to float above the point on the ground.

This parallax, $B'B''$, is eliminated by turning the foot-plate, which diverges the plate-holders, thus bringing the pointer to rest on the ground at the lower point. If the left-hand wheel is turned so as to move the index marks back to A and A', the parallax is shifted to the hilltop, and at that point the pointer will appear double, or deeply buried below the ground.

It will now be seen that in drawing cultural detail and drainage, the foot-plate must be used simultaneously with the hand-wheels, as roads and streams necessarily change their level. If the foot-plate is not used, it is, of course, possible to use the eye-pieces independently and cause the index marks to follow a road or a stream. The two projections of a road or a stream thus drawn would not coincide, and neither would be a true projection. While drawing contours, the foot-plate, of course, is locked.

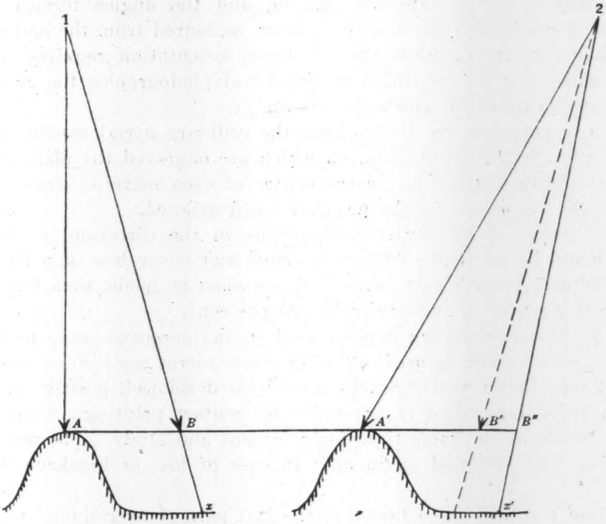

FIG. 15.—DIAGRAM OF PARALLAX CAUSED BY RELIEF

AERIAL PHOTOGRAPHY

Since in an ordinary stereoscope, the photographs must be set in a certain position in order to get perfect stereoscopic fusion, it is evident that the same procedure must obtain in the orientation of the photographs in the plate-holders of the aerocartograph. The stereoscopic field must be oriented in a plane which coincides with the proper plane of the co-ordinate system, since it is desired actually to take measurements in this stereoscopic field and such measurements must be obtained through the co-ordinate system of the aero-cartograph. The stereoscopic field must also be so oriented that the eyes of the observer will see it as if he were at the stations where the exposures were made. This is only possible when the photographs in the plate-holders have the same relative positions to the lenses in the plate-holders that the negatives had to the lenses in the exposure camera. Also, the photographs must have a position in the drawing system of the instrument in which the relation to the horizon in the original exposure is maintained. It is essential that there be a true reproduction of the angles of the light rays as they come into the exposure camera in order to obtain the interior orientation required. The interior orientation is accomplished with the help of collimating marks in the exposure camera, and the angles formed by these collimating marks and the negative center, measured from the optical center, determine the camera constants. Exterior orientation requires only three points, which can be identified in the aerial photographs, the ground positions and elevations of which are known.

The exposure camera differs from the ordinary aerial camera in that it has an optically flat glass plate, on which are engraved the plate center and four collimating marks, one in the center of each margin, arranged so that these marks are shown on the negative when exposed.

The overlap of the aerial photographs in the direction of the line of flight should be as nearly 60% as possible and never less than 50 per cent. When the area is such that parallel flights must be made, such flights should overlap at least 10%, and preferably 20 per cent.

In the aerial surveying camera used in the aerocartograph method, uniformly shrinking film is used, although the cameras are also equipped to use glass plates. After the negatives have been developed, positive glass plates of each individual negative are made by contact printing. Contact prints are also made at the same time for reference and study. The negatives are then filed, and are used again only in case of loss or breakage of positive plates.

Ground Control.—The first and the last pair of overlapping photographs in each flight or strip are used to establish ground control. The length of such a strip or the number of photographs over which aero-triangulation can be carried safely without additional ground control depends on the scale of the map, the amount of relief, and the contour interval; but, under average conditions, it is from ten to fifteen photographs. Three points, and preferably one additional for a check, which are clearly identifiable in the overlap portion of the first two photographs are chosen, and the position and elevation of these points are determined by ground-survey methods. These three points form the vertices of a triangle and if it can have the shape of an equilateral triangle with as long sides as the overlap will permit without being too close to the edges of the photographs, the control approaches the ideal condition. A similar set of terrestrial control points should be established in the overlap of the last pair of photographs in a strip flight, although these points may be in an approximately straight line at right angles to the line of flight instead of forming a triangle. For ease of identification of selected points, it is advisable, if time permits, to photograph the area previous to the establishment of the ground control. Such points as road corners, intersecting fences, stream forks or crossings, or points easily referenced by lone trees or houses, are proper control points for aerial photographic work. The usual ground-survey triangulation points are seldom identifiable in a photograph, and it appears that, in this method of the production of maps from aerial photographs, one must change the ground surveyor's ideas in regard to what constitutes a proper set of control points just as further investigation of the manner in which these points are located in the field may result in an entirely new technique of ground operations. Experience has shown that the aerocartograph is capable of a remarkable degree of accuracy in extending horizontal control by aero-triangulation, provided that, at least for large-scale work, the ground control is established with a high order of precision.

Mapping

After the ground-control points have been located and plotted on the map sheet, the positive plates are placed in the plate-holders of the aerocartograph, and the interior, stereoscopic, and exterior orientation is secured by optical and mechanical means in a definite routine. A considerable advantage in this method is due to the fact that in the aerocartograph instrument, there are a minimum of moving parts as well as operations or steps in the securing of the final product. When this orientation has been perfected, the operator has before him a stereoscopic picture or plastic model of the landscape. The flexibility of the method is clearly apparent, since it is now possible for him to produce at once, without intermediate operations, a map of any kind to suit any definite purpose.

To obtain the scale, it is not necessary to have ground-control points in every pair of photographs. The first pair of plates (called, for example, Plates 1 and 2) are oriented and adjusted by comparing an aerocartograph plotting of the positions and the reading of the elevations of the stereoscopic images of the ground-control points with the actual plotted positions and known elevations of those same control points. Three new picture points are then selected by images alone (and without locating them on the ground) so that they lie in the advanced edge of the stereoscopic model and will also appear on the third plate. (This is the reason for the 60% overlap.) These points are selected so that the central point is about in the center of the strip and the other two points are on a line at right angles to the line of flight and near the sides of the strip of photographs.

Plate 1 is then removed from its plate-holder and Plate 3 is substituted for it and oriented and adjusted to Plate 2 by images alone, using the advanced half of the undisturbed Plate 2 in forming the stereoscopic model. The crossed vision resulting from this arrangement of plates produces a pseudo-stereoscopic effect in which hills appear as valleys and *vice versa.* This pseudo-stereoscopic effect can be corrected by shifting the prisms in the optical train so that the two plates are viewed in their normal relation and appear in proper sequence. The orientation of Plate 3 is thus accomplished without the use of any additional points located on the ground. This process is continued for all the successive plates in the series of photographs taken in any one strip flight. By means of the control points in the last pair of plates, the error of closure, if any, becomes apparent, and the entire flight is adjusted. Adequate provision is made for the correction of the difference of elevation due to the curvature of the earth, and the adjustment is made by the use of formulas which have been proved by comparison with test strips of aerial photographs on which many ground-control points have been located for each overlapping pair of plates.

The corrected setting of each plate is then computed so that upon re-insertion in the plate-holders, the sequence of stereoscopic models can be developed in detail with the assurance that there will be perfect closure in position and elevation at the end of the strip. (In short strips, or in cases where intermediate elevations are available, the corrections are made as the work proceeds, thus avoiding duplicate insertion of the plates.) As each plate pair is inserted in the plate-holders, the map details are drawn directly from the model with the aid of the floating mark. The culture and planimetry, as well as the contours, are traced by means of the floating mark as a series of continuous lines and not as lines formed by interpolation between plotted points, as in ground-survey methods. The movements of the floating mark are plotted directly on the map sheet in the base of the instrument, or on the co-ordinatograph, these details being shown in true orthographic projection. Inasmuch as this plotting is of a continuous series of points of intersection of rays projected in a true horizontal plane, there is no pantographic principle involved in the construction and operation of the aerocartograph.

Information as to land boundaries, names, and other features not appearing on the photographs is usually determined in the field during the progress of obtaining the ground control. Any features that could not be drawn with the aerocartograph because of natural obstructions or obscure parts of photographs are added by ground-survey methods, but such operations are usually of very limited extent.

The economic value of good maps has long been apparent to the map producer. It is believed that the value of maps constructed according to proper standards is being recognized more and more by the profession, by those in authority in the development of projects involving the use of natural resources, and by the general public. This recognition will result in an increased demand for better maps, and it is belived that aerial photographic methods will serve more and more as an important aid to the map producer in meeting this demand.

DISCUSSION

Theron M. Ripley,[8] **M. Am. Soc. C. E.** (by letter).—In the "Synopsis" of his paper Colonel Birdseye states that it is to be considered as covering the aerocartograph and brings out discussions of it and similar instruments. In the "Introduction" to his paper he compares ground surveys with aerial surveys and shows certain advantages of one over the other.

After the field work is done and the negatives are in the office a trained organization is required to produce a proper map from them by the use of the aerocartograph or any other similar instrument. Except war and other emergency work the engineer is interested only in securing, economically, a survey and map of such accuracy that a reasonable preliminary design and estimate can be made therefrom. For final location and construction purposes ground surveys must be made.

Colonel Birdseye covers the questions of topographic detail, such as drainage, culture, etc.; the usual method of "sketched in" features not "under the feet"; and the mapping of "inaccessible areas"; but he makes no statement of the probability of time saving. The time which can be saved will often be the important factor in determining the use of aerial surveys.

A mountain survey, for sixty miles of railroad line, following a creek in a gorge, required six months with a full party, including camping personnel and equipment. This was in the Rocky Mountains where flying and photographic days are many. An aerial map of that creek and its gorge, with the necessary control, could have been put into the hands of the engineer for projection and estimate in one-quarter the time required for the ground survey and, at the present time, at no greater cost.

An exhaustive estimate of the cost of an aerial survey covering a large territory was made by the writer in 1928. It was approximately the same per mile as the cost of ground surveys in the same country. The great advantage of the aerial method was found to be in the time saved and the mass of accurate detail obtained. The estimated personnel to accomplish this work by ground methods was about equal to that estimated for the aerial method, but the time required for the latter was one-third that of the former.

The two preceding examples are from the writer's personal experience and confirm the statement relative to the importance of stressing the time element.

O. S. Reading,[9] **Esq.** (by letter).—A remarkably clear and comprehensive statement of stereo-topographic mapping has been presented in this paper. It may well serve to acquaint the profession generally with the subject as well as supply a basis for further study to those especially interested.

It is difficult to convey the power of the stereoscopic method of mapping without some background of experience on the part of the reader, or at least the possession of a simple stereoscope with some suitably arranged stereoscopic pairs of photographs. The writer recalls a pair which he examined in the author's office. The lines of sight of two figures of triangulation across a wooded valley were shown by fine white lines. The data for one of the stations were in error, but enough other stations appeared on the photographs to orient them correctly. The manner in which the lines of sight that were in error intersected at a point above the ground, while the correct lines intersected precisely at the stations, gave a striking demonstration of the power of the stereoscope and the infallibly uniform accuracy of the photographic record. At the same time the unobstructed view of the undulations of the wooded hills showed the advantages of aerial photographs under a stereoscope over the limited views which would have been available to ground methods on such terrain.

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[8] Cons. Engr., Buffalo, N. Y.

[9] Hydrographic and Geodetic Engr., U. S. Coast and Geodetic Survey, Washington, D. C.

The literature of surveying a few centuries ago abounded with statements hailing the plane-table, which permits the drawing of the map in the field with the earth before it as a model instead of the compilation of more or less inadequate notes in the office. Surveyors of the Twentieth Century may well hail aerial photographs and the stereoscope in the same manner. These devices permit the transportation of a model of the earth instead of more or less inadequate maps to any number of offices for the study of problems as they occur. It is no longer necessary to survey and record a vast amount of data in the effort to anticipate all the problems that may arise, or, later, to make expensive extra trips to the field. There is a growing appreciation among engineers of the value of the almost unlimited detailed information given by aerial photographs under the stereoscope as a supplement to the quantitative information of topographic maps and line surveys.[10]

The writer has yet to examine an aerial re-survey which did not bring out some faults in previous ground surveys, due to mistakes in traverses or to interpolation and sketching between instrument set-ups and rod-readings. In contrast, carefully controlled and plotted aerial photographs are as self-checking as a layout of quadrilaterals in triangulation. If the detail is carefully traced all the well-defined objects of a photo-survey will be shown as accurately on the map as the control stations. This uniform precision of detail is particularly valuable for basic surveys intended for use in the general development of an area. The amount of detail that is traced from the photographs guarantees an ample number of common points for compiling partial surveys of the area, while its uniform precision permits of adjusting, without question, the partial surveys or revision photographs to the base map. Heretofore, it has usually been less expensive to make a complete re-survey rather than attempt to reconcile the discrepancies of the former survey with the new data. This uniformity of precision in mapping, backed by the permanent record of the photographs, is a second great contribution of aerial photography to surveying.

Regarding the question as to which method or machine for stereo-photographic mapping is best suited to a given problem, the art is being developed too rapidly and is too much in the controversial stage to permit of positive statements at this time. Aside from the process developed by Arthur and Norman Brock and their associates, and the aerocartograph, which have been successfully used in the United States, it has been reported that the stereoplanograph (Fig. 6) has been adapted to plotting quadruple-lens photographs having a field of 83° without resetting. Such a wide field might well increase accuracy and reduce costs, if successful, and deserves the consideration of any one actively interested. The British have developed a stereogoniometer for determining the successive orientation of the photographs to each other along a flight strip, the map relations being determined later by calculation. This method would seem to have practical advantages for extensive mapping on scales smaller than 1:10 000, and has been described by Capt. M. Hotine[11] who also reviews other machines and methods critically. For accessible country, apparently it has been found satisfactory and somewhat less costly in some cases[12] to take planimetry only from the aerial photographs and to contour the map sheets on the plane-table. This method is likely to be used frequently on account of the high first cost of the machines. Since the various stereo-plotting machines cost from $15 000 to $30 000 to construct, or to import into this country, and also require specially trained operators, considerable mapping is required to underwrite their use. Present economic conditions have undoubtedly retarded the development and use of such machines considerably.

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[10] "Aerial Survey for Transmission Line Location," by F. G. Dana, *Civil Engineering*, Vol. 1, No. 4, January, 1931, p. 249; also, "Aerial Photographic Survey and Mapping," by Harry Brighton, *Michigan Engineer*, March, 1931, pp. 42–43.

Certain inaccessible areas of considerable relief, such as the back canyons of Zion National Park, or the wooded mountains of the Olympic Peninsula, cannot be adequately mapped, except at prohibitive costs unless the aerocartograph or some other method of stereo-photographic mapping be used. For mapping terrain of considerable relief on scales from 1:3 000 to 1:10 000, undoubtedly stereo-photographic mapping methods give superior accuracy at lower costs. It would be appreciated, if, in closing the discussion, Colonel Birdseye would give some comparison of the relative costs and technical advantages of mapping various types of terrain at different scales by means of the aerocartograph, the photo-plane-table method, and the plane-table alone. In making this request, the writer is aware that it is almost impossible to discuss costs intelligently without reference to particular localities and map specifications, but Colonel Birdseye is believed to be better qualified than any one known to the writer to define the present economic and technical limits of the different methods.

Lowell O. Stewart,[13] **Assoc. M. Am. Soc. C. E.** (by letter).—Machines that are used to make maps from aerial photographs are described in an interesting and lucid manner in this paper. The author's statement, "that a map properly made by adequate photographic surveying methods must be more consistent throughout than one made by what may be called the old-fashioned ground-survey methods," is a point that deserves emphasis.

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[11] "Surveying from Air Photographs," by Capt. M. Hotine, R. E.

[12] "Developing the Upper Mississippi—Surveying and Mapping Methods," by L. Ylvisaker, *Civil Engineering*, Vol. 1, No. 15, December, 1931, pp. 1357–1360.

[13] Asst. Prof., Civ. Eng., Iowa State Coll., Ames, Iowa.

The question of relative accuracy always enters the discussion when the merits of various topographic methods and instruments are being considered. In that connection the surveyor should keep in mind that a map is a representation of the original to a certain scale and contour interval, and must be a compromise between absolute accuracy and time and cost. Certain features must be omitted. Others that do appear on the map are designated by conventional symbols. Therefore, in judging the quality of any map the scale, the cost, and the time required to make it, must be considered. High standards of precision in the detailed field work cannot be written into the finished map, because of plotting limitations. Probably the limit of precision in scaling lies between 0.01 in. and 0.02 in., and ordinary protractors are good only to the nearest 10′ or 15′.

A matter of greater importance has to do with the detailed field work. Any one who has taken "shots" on the ground for contours knows how important a part judgment plays in the selection of those points. Later, after the points have been plotted, the contour lines must be interpolated. This requires more or less generalization, the amount depending upon the minuteness with which the ground was covered by "shots." Therein lies an important factor in determining the accuracy of the finished map. In rugged country the lines may be very close to their proper positions, while on easy slopes they may be far from that ideal.

Engineers are prone to accept locations as correct on a map in spite of the fact that such maps differ widely from one another in accuracy. It is time that surveyors develop standards for checking topographic maps, and apply them properly. Some progress has been made along that line on extensive city surveys and on the recent surveys of the Upper Mississippi River. Two tests have been applied. The first, for position of general features, usually requires that all features shall be located with such accuracy that no error is measurable at the map scale, which means that a given feature must be within 0.01 to 0.02 in. from its correct position on the map; the second, a test for contour accuracy, specifies that none (sometimes 10%) of the test points may be in error on the map more than one-half a contour interval. The latter test seems insufficient in flat country.

Likewise, a sound field technique for testing should be developed after the standards of accuracy for the finished map have been set up. Different types of terrain would not require the same detailed field procedure; but surveyors should have those requirements and differences so clearly set forth that any one could test a map and grade it as to its acceptability according to the recognized standards. When that is done, and the requirement for testing is included in the specifications for every topographic mapping project, the pseudo-maps that are now frequently made, will disappear. Accurate maps will cost more than hurried and inaccurate maps; but the assurance of their reliability is worth much more than the added cost.

These standard specifications should have an important bearing on the development of aerial mapping. After all, a map is a map until it has been proved otherwise. The advocates of aerial methods may state, with justification, that better and more consistent maps may be made from aerial photographs than from ground surveys alone. The facts may be learned through experience with maps that have been made by the two methods and used for identical purposes; but a better and more scientific way seems to be through the use of standard tests.

F. H. PETERS,[14] M. AM. SOC. C. E. (by letter).—In Canada the ground survey camera has been continuously used for many years in mapping the rugged mountainous areas where numerous commanding positions make this method of survey economically practical for small-scale mapping. The method followed is that introduced many years ago and consists of exposing photographs in vertical planes in several directions, usually covering the entire horizon from the particular station. Each photograph serves as a record of the horizontal and vertical angles read from the particular station to the features shown therein. The line joining two stations thus serves as a long base and the map positions and heights of features shown in the area common to the views taken from each of the base terminals are determined by the usual graphical intersection methods. Surveyors mapping in this manner frequently experience difficulty in identifying the same feature or object viewed from different camera stations, and, at times, the intersection angle may be so small as to introduce errors in the graphical intersected positions.

A solution to these objections was offered in the application of the stereoscope for identification purposes and provision for accurate stereoscopic measurement on the photographs exposed in a common vertical plane from the ends of a short measured base. The stereo-comparators devised by Dr. Carl Pulfrich, of Germany, and H. G. Fourcade, of South Africa, were constructed to this end and served for point-by-point fixations. At about the same time the late Dr. E. Deville, Surveyor General of Canada, was experimenting on the construction of a stereoscopic plotter such as might serve for reconnaissance detail mapping from vertical ground views taken in parallel or slightly converging directions. He constructed an experimental model of his plotter ("stereoplanograph") which owing to certain disadvantages he did not fully develop.

The viewing apparatus consisted of a reflecting stereoscope equipped with a pair of half-silvered mirrors which served as a means of seeing through the eye-holes of the apparatus both photographs of the pair at once, and thus presenting a vertical stereoscopic image of the common area while, at the same time, a "floating mark" or spatial point of light behind the mirrors could be seen directly by the observer. The photographs—diapositives provided with illumination—were adjusted in their holders and with reference to the observing position of the eyes so as to reproduce the internal perspective conditions of the field camera. By further adjusting both the mirrors and the photographs and the inclination of the instrumental eye base the external orientation was reproduced, and a true-to-scale model or plastic image was formed apparently behind the mirrors. The "floating mark" or "cross-wires" consisted of an adjustable black screen, perforated with a hole or slit through which light was transmitted, carried on a movable vertical standard with a pencil at its base. With the "floating mark" at a definite setting for a particular contour, this contour line was plotted by guiding the tracer by hand over the paper so that it remained in apparent contact with the stereoscopic model.

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[14] Surveyor-General, Dept. of the Interior, Ottawa, Ont., Canada.

In 1907, Lieut. F. V. Thompson modified the stereo-comparator by adding to it a simple lever system and parallax drum so that, instead of a point-by-point measuring apparatus, it became capable of plotting plan positions automatically, but not contours directly.

The desire to make full use of the camera field of view in covering land areas, to eliminate the waste portions occupied by sky areas prevalent in pairs of ground photographs taken in the same vertical plane, and to obviate the mistakes in point-by-point plotting, resulted later in the design and experimental construction of automatic plotters with facilities for the rapid and economic setting of the photographs in their correct positions for plotting to correspond with the measurements made at exposure, and for the automatic plotting of plans and contours therefrom. In this way the evolution came about of a number of machines among which are found:

In 1908–10, the stereoautograph by Lieut. von Orell and Carl Zeiss—an instrument still in use and the first to have incorporated in its construction the "Zeiss parallelogram."

In 1919, the autocartograph, (Fig. 9), a large and expensive instrument that served for the precise plotting of inclined or convergent photographs tilted up to 30° below the horizon and exposed on any orientation relative to the base line.

In 1922, the stereoplanograph (Fig. 6), also a large and expensive machine plotter capable of precise plotting, which allowed for setting photographs exposed on any orientation.

In 1922, the stereotopometer by M. Predhumeau, an instrument of cheaper construction for plotting planimetry and contours from terrestial views exposed in a common vertical plane.

In 1924 the autograph, (Fig. 7), which embodied complete stereo-photogrammetric plotting machinery and was of considerably smaller size and weight than the other plotters of its class. As reconstructed in 1928, it represents one of the best machine plotters at present in practical use with terrestial and aerial photographs.

In 1925, the "stereotopographe," by M. Poivilliers, a plotting machine of the intersection photogoniometric type modeled after the stereoautograph and autocartograph and recently improved.

The introduction of aerial photography with its problems of unknown exposure conditions—position, tilt, and orientation—started considerable research, resulting in improvements being made to existing plotters in order to take full advantage of the new field as well as the design and construction of new machines for use with aerial photographs.

The aerocartograph (Fig. 8), which has been so well described by Colonel Birdseye, is a much more portable instrument than the autocartograph (Fig. 9). It will accommodate aerial views taken at any orientation, is capable of turning out precise plotting, and costs less than its predecessor.

In 1925 an Italian "photocartograph" appeared and is still in extensive use, chiefly for large-scale mapping from aerial photographs where a close net of control is supplied. It is constructed on the camera plastica principle and consists of two projectors, each containing a lens, negative carrier, condenser, and light source, all of which are centered on the optical axis of the projector and are rigidly supported by a rod parallel to the optical axis. The rod can be tilted and can be rotated in the tilted position, both movements being centered on the front node of the lens. In addition, the rod is movable in three dimensions relative to the plotting plane and the photograph is rotatable in its own plane. Each projector lens is of similar design, but of shorter focal length than the field lens, and is correctly set for principal distance and principal point.

The four ground-control points of the common overlap of a pair are set out to scale in the plane of the ground glass by means of control markers, and are adjusted in height. Each projector is then set individually by trial (consuming about 20 min.), so that the images of the control points fall in their correct position. When both negatives are set, the control markers are removed and replaced by a ground glass screen, the matte side of which is nearer the projection lenses. A pointer or index marker actuated by the *x, y,* plotting controls and connected to a pantograph moves over and in contact with the projection surface, and coincidence plotting is effected by "the flicker" system.

The instrument will plot satisfactorily from vertical aerial photographs for a range of height within the permissible depth of focus of the projecting lens, and this range may be increased by extending the instrument base for various plotting zones to bring the point of coincidence within the depth of focus (involving a re-calculation of the height scale and re-setting the pantograph). Photographic extension of control is possible through a few overlaps.

In 1925 Dr. Archibald Barr and Dr. William Stroud began the construction of a "photogrammetric" plotter and later produced one model that was used on experimental work. It embodied the camera plastica projection principle with the depth-of-focus problem obviated by means of an auxiliary lens system. To speed up the setting of the photographs in the plotter, a "tilt-finder" or the "stereogoniometer" designed by H. G. Fourcade was used in locating the photo-plumb points of the pair from the ground-control points, and glass positives printed from the original negatives were perferably used in the projectors. The setting movements of each projector allowed for the re-establishment of the internal perspective of each photograph in relation to the projector lens, and of the external perspective, by a rotation of each photograph in its own plane, the tilting of the projector as a whole about an axis through the front nodal point of the projector lens, and the rotation of the projector as a whole about the normal from the nodal point of the projector lens to the screen. One projector could be moved independently of the other in the *X*-direction. Furthermore, one projector could be given an offset movement in the *Z*-direction equal to the difference in altitude of the two air stations.

A stereoscopic comparator was used for the examinations of the separated screen images projected from each of the two plate-holders, which were spaced apart a distance equal to the length of base to the required scale of plotting plus the length of the comparator base.

In this machine the two projectors are moved bodily in the three co-ordinate *x, y, z,* movements, two of which (*x* and *y*) are also imparted to the drawing-board. The auxiliary lens system served to maintain sharp focus between each projector and the screen. The separation of the two projectors in the base direction was to provide ample room necessary for their movements in plotting directly to small scales. Two floating marks at the same separation which serve as real marks in the plane of the screen are contained in the eye-pieces of a stereoscopic comparator used for the examination of the screen images. This comparator could be moved over the screen, but it always maintained the line joining the floating marks parallel to the base direction.

Attached to the comparator is an arm carrying the drawing pencil, and points were easily plotted by simply following, by a single movement, the course of detail as the observer saw it. The drawing-board is close to the operator where he can watch the progress of his work. The screen may be moved bodily up and down, and clutches are provided to enable the drawing-board to be moved independently of the projectors. Both projectors and board are moved by the *x, y,* and *z,* control wheels to throw a fresh patch of detail on the screen. Scales for *x, y,* and *z* are provided to enable the relative co-ordinates of any point in observed coincidence with the comparator floating marks to be read directly. The first model of this plotter was made for use with lenses of various focal lengths which made it unduly large, but the construction of the machine was quite simple. It was not, however, well adapted to correspondence setting by the Fourcade system and only one model was built.

One of the most recently constructed instruments for use in plotting aerial photographs is the "stereogoniometer." This instrument was designed by Mr. Fourcade using the principle of "correspondence setting" devised by him about 1925. One model of this instrument has been constructed and used for experimental work in the determination of the unknown exposure conditions of stereoscopic air photographs.[15]

W. H. CROSSON,[16] ESQ. (by letter).—The author has closely covered the principles of stereoscopic measurements and the operation of modern instruments for making topographic maps by the utilization of aerial photographs in connection with ground control. The application of stereo-topographic principles and the practicability of using stereoscopic instruments on military mapping projects is being studied by Army engineers.

If an instrument can be developed that is capable of rapid delineation of topography from air photographs, the task of the military engineer in time of war will be made easier.

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[15] For complete description, see *Professional Paper No. 7*, British Air Survey Committee.

[16] Capt., Corps of Engrs., U. S. Army, Washington, D. C.

An excellent description of the principles and operation of the aerocartograph is given in Colonel Birdseye's paper. Its operation or rather the amount of map work that could be turned out on this machine would be materially increased if it were possible to indicate on each photograph its exact angle and direction of tilt so that each plate could be set and oriented mechanically in its proper relative position.

DWIGHT F. JOHNS, ESQ.[17] (by letter).—Officers of the United States Army who have been concerned with the question of the provision of adequate military maps, recognize that the principles of stereo-topographic mapping, and practical methods of their use under military field conditions, have a definite marked application to the problems of mapping for military purposes.

Topographic maps are necessary for modern military operations. If they are not available for the theater of operations at the outbreak of hostilities, time and space factors will preclude their being produced by the usual ground survey methods—time factors because war will not wait for the topographer, and space factors because the terrain for which maps are most needed, will be that occupied by the enemy. It is a principle of war that offensive action is the only means by which a decision is gained, and offensive action is not secured by staying on one's own side of the fence. Mapping the terrain accessible to topographers is not enough for the necessary offensive operations. A certain amount of it will be necessary, of course, just as a temporary assumption of the defensive may be necessary to become better prepared for the offensive, but the ground that the military forces need to know most about is that occupied by the enemy. This knowledge in past operations has been limited to such pre-war maps as existed, likely to be out of date with respect to much of the information needed, to other pre-war intelligence data, and to what could be gained, in a fragmentary way only, by limited ground reconnaissance methods such as patrols, raids, reconnaissances in force with a considerable portion of the command, and by general attacks. At best, these methods could reach only a shallow depth within the enemy lines, and they were usually costly in human life.

Now, the airplane and its subsidiary, the aerial camera, furnish the means by which the necessary information can be obtained, in the form of photographs, within the enemy lines to a depth limited only by the practical photographing radius of the plane, which is many miles.

The aerial photograph itself and the photographic mosaic, are of very material value in such a situation for planimetric study of limited areas and for limited distribution. They do not permit topographic relief to be observed, however, except through stereoscopic observation of overlapping pairs, which would be entirely impossible of course for general use in military operations. Neither do they lend themselves to ready interpretation and study by the general user. Topographic maps which can be quickly and cheaply reproduced in quantity are necessary, and there must be some practical means by which the map can be quickly produced from the air photograph in order that the tremendous advantage of the latter may be retained.

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[17] Maj., Corps of Engrs., U. S. Army, Fort Leavenworth, Kans.

It would appear, then, that the principles of stereo-topographic mapping and such practical field methods of their application as have been and may be developed, will furnish an answer to this important military problem. Colonel Birdseye's clear explanation of these principles, and of the design and operation of the aerocartograph as one instrument embodying them, is a very valuable contribution to military, as well as to civil, engineering literature on this subject.

DOUGLAS H. NELLES,[18] ESQ. (by letter).—Photo-topographic mapping from land stations was used for practically the entire 30 000 sq. miles mapped in the vicinity of the South-Eastern Alaska-Canada International Boundary Line; for a large portion of the territory adjacent to the 49th Parallel, International Boundary, in the Rocky Mountains; and for the mapping of the great Canadian National Parks in the Rocky Mountains.[19]

When the territory to be mapped has low relief and mapping points are difficult to identify with the ordinary photo-topographic method, good work can be done by the stereo-photogrammetric method. In this method the base between plotting photographs is only a few hundred feet long and points seen in one photograph, can always be identified in those from the other end of the base.[20]

In Canada, mapping from air photographs has been carried on extensively by the Federal Departments in producing what is called "The National Series Maps" on scales of 1, 2, 4, 8, and 16 miles to the inch. These maps have a very fine numbering system, by which, from a small index map, the number of a map on any of these scales, covering any territory in Canada, can be quickly picked out. This system will bear examination by other countries with reference to its application to their own territory. The Royal Canadian Air Force, Civil Operations Section, takes the aerial photographs for the Civil Government Departments under special instructions for each operation. The Air Force has taken approximately to date (1932), 280 000 sq. miles of oblique photographs and 125 000 sq. miles of vertical photographs. In addition, private aircraft companies have taken between 5 000 and 6 000 sq. miles of obliques, and approximately 285 000 sq. miles of verticals for private companies and Provincial Governments. Special mapping methods have been developed in Canada for using aerial photographs.[21]

An accurate survey of the crest of Niagara River Falls, using both ground photo-topographical stations and a special aerial photographic method that is interesting to a student of the subject, has been described by the Geological Survey of Canada.[22]

Colonel Birdseye's paper shows that European methods and instruments are very complicated, and that the instruments are costly. This fact limits their use on the continent of America to a great extent. The same trend is noticed in European instruments for land photo-topography. On the other hand, surveys in Canada have developed practical methods and simply constructed instruments, made to stand hard usage of day after day of mountain packing but which, at the same time, are capable of giving the highest degree of accuracy. In aerial mapping, Canadians are developing methods and instruments along the same practical lines.

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[18] Ottawa, Ont., Canada.

[19] "Photographic Surveying," by M. P. Bridgland, *Bulletin No. 56,* Topographical Survey of Canada, Ottawa, Ont., Canada.

[20] By Messrs. B. J. Woodruff and D. H. Nelles, in *The Engineer* (London), October 30, November 6, and November 13, 1925.

[21] "The Use of Aerial Photographs for Mapping," *Bulletin No. 62,* Topographic Survey of Canada, Ottawa, Ont., Canada.

[22] "The Niagara Falls Survey of 1927," by W. H. Boyd, *Memoir 164,* Geological Survey of Canada, Ottawa, Ont., Canada.

R. E. BALLESTER,[23] ESQ. (by letter).—This paper deals especially with map production by stereo-topographic methods and by photographs taken either from the air or from the ground. For the designing engineer who uses such maps the precision or accuracy that it is possible to attain with these methods is of primary importance.

The writer had the opportunity to order the mapping, by the stereo-topographic method from ground stations, of a stretch of river with steep slopes, and to compare the precision attained with that of precise leveling with the usual topographic instruments.

The area mapped covered 28 hectares (69 acres) at the Primero River at Córdoba, Argentine Republic.[24] The map was to be used for design purposes or for correction of the location of a dam and appurtenant works. The extreme difference in elevation was 100 m. (328 ft.). The work was performed by the "Instituto Geográfico Militar" (Army Department of Argentina). While the field work was prepared and executed to make a map on a scale of 1:1 000, the final map was traced with the stero-comparator to a scale of 1:500, with contour intervals of 2.00 m. (6.56 ft.).

An independent field party staked out three profiles crossing the river at the proposed site. These were compared with profiles taken graphically at the same location using the stereo-topographic map. Comparing the three sets of profiles, from direct leveling and from the map, the writer derived the following values, in metric units, for the mean error in altitude and in plan:

$$m_h = \pm (0.06 + 0.49 \tan \alpha)$$

and,

$$m_p = \pm (0.49 + 0.06 \tan \alpha)$$

in which, m_h is the mean error in altitude; m_p, the mean error in plan; and, α, the slope of the ground.

Taking for α, the mean value of 35°, corresponding to a slope of 1 on 1.5, the mean error is: $m_h = \pm 0.41$ m. and $m_p = \pm 0.57$ m. both for the 1:500 scale.

This precision was quite sufficient and satisfactory for the design of the dam, spillways, plant layout, etc. The writer believes that a topographic survey with transit and level would not have resulted in the same detail and precision, especially in view of the fact that the slopes of the river gorge were so steep as to be almost inaccessible to the rodmen.

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[23] Prof. of Applied Hydraulics, Univ. of Buenos Aires, Buenos Aires, Argentine Republic.

[24] A detailed description is given in *La Ingenieria*, Buenos Aires, Vol. 35, No. 7, p. 222, July, 1931.

It will be interesting to learn the author's opinions concerning the precision attainable by the different methods of stereo-topographic mapping, for different scales. This will encourage engineers who cannot take the time to study the subject in detail to select this method, which, in many instances, presents advantages of cost, and accuracy over the usual topographic mapping.

T. P. PENDLETON,[25] M. AM. SOC. C. E. (by letter).—A new method of mapping is described in some detail in this paper. It calls for consideration on the part of engineers and others having need for topographic maps, and it may be that increasing familiarity with it will aid in clarifying an unfortunate situation.

Consideration of Colonel Birdseye's paper will reveal that the introduction of stereoscopic methods of mapping into modern practice emphasizes the fact that topographic mapping is a task for the specialist. The principles on which such methods depend have been well developed, and although the various instruments differ widely in their manner of solving the problems involved, it can be stated that when properly used they are capable of producing excellent maps. The cost of the equipment is relatively high as none but the highest grade of workmanship can be tolerated and this quite effectively bars the small engineering organization from possession of instruments of this type. The method also demands personnel trained in its use and, at present, such men are not generally available.

A few statements regarding the differences in the American and European point of attack on the problem may not be out of place. The aerocartograph, which is described in considerable detail, has many points of resemblance to some other European stereoscopic mapping instruments. It combines in itself all the functions necessary to transform information contained in the aerial photographs to map form. This makes for an instrument of considerable complexity, with many optical and mechanical parts in motion when it is in use. Thus far, the only American method in use was devised with the idea of breaking up the work into its various essential steps and designing for each one a simple rugged instrument with few moving parts. It was felt that this method of attack was more satisfactory from a production point of view as it would enable the production engineer to localize causes of delay readily and speed up the output of that operation by assigning additional men for the purpose. It also assured freedom from instrumental troubles caused by wear and the need for frequent adjustment.

Another essential difference is in the character of the various optical parts. It is desirable that the two objective lenses in the aerocartograph match closely in their characteristics the lens used in the aerial mapping camera. When this is accomplished any faults in the camera lens are largely compensated for by similar faults in the objective lenses of the aerocartograph. This frees the manufacturer of the necessity of securing lenses highly corrected in the matter of distortion and enables him to use objectives of a quality readily obtainable on the market. On the other hand, the process introduced by

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[25] Long Beach, Calif.

Messrs. Arthur and Norman Brock makes use of lenses in the aerial cameras and office instruments that are selected largely for their freedom from distortion. Consequently, every photograph made is practically a true perspective projection of the original and no compensation for errors in the original photographs is necessary.

The matter of lens perfection must be considered in connection with the type of photographs to be used. It is quite useless to seek lenses in which the distortion has been eliminated if the aerial negatives to be used are not true point-to-point representations of the earth's surface. The only method thus far devised of grasping and retaining such a view is in the use of glass-plate negatives on which any motion of the emulsion that may occur is so slight that it can be ignored. It is for this reason that the Brock process makes use of glass plate negatives rather than roll film. Thus far, it has not been possible to coat a film base that would retain its shape satisfactorily. Attempts have been made with more or less success to secure a base that would change uniformly with changing atmospheric conditions and it is probable that some improvement has been made in this regard. No information has come to the writer, however, to indicate that success in this direction has been sufficient for the purpose. It is probably true that for mapping work in which the accuracy requirements are not great, aerial film negatives can be used satisfactorily, but at the most it is scarcely to be hoped that they will excel glass negatives in so far as freedom from change in shape is concerned.

Since aerial photography will play an important part in mapping methods of the future, it introduces another controversial point into the discussion of what constitutes a good topographic map. There is a wide divergence of opinion on the part of engineers and the faculties of technical colleges on this subject. Indeed this difference of opinion extends further, as there is the same lack of agreement as to the manner in which the actual mapping surveys should be carried out and the type of instrument best adapted to the purpose. Many colleges and universities give instruction in plane surveying and geodesy, but only a few devote enough time to these subjects to enable the student to judge of the merits of the various mapping methods.

Many and varied questions are continually arising which demand for their proper solution information that only a topographic map can give. These problems may be entitled to no greater weight, however, than many others involved in some major engineering study. The engineer in charge is frequently at a loss to know what type of map is most suitable and the method to be followed in making the necessary surveys. This is not surprising, as topographic mapping, in common with many other branches of civil and military engineering, is a specialized subject demanding instruction not commonly given in colleges, and which the engineer engaged in general practice has but little chance to acquire. It frequently happens that when the need for the map is felt a transitman will be diverted from his usual work of staking out structures, or of subdividing real property, and will be assigned to map work calling for training only acquired by a topographic engineer.

As a result, he uses such instruments as are at hand regardless of their suitability for the purpose and applies methods differing but little from what he would use ordinarily.

The result is frequently unsatisfactory because the map suffers from improper choice of methods and a lack of understanding on the part of the engineer as to what constitutes a good map. Sometimes, it suffers from faults that can be traced to false economies and, frequently, it must bear the burden of unnecessarily high cost due to the employment of untrained personnel and improper equipment and procedure. Skill in the use of the proper instruments and the development of what may be termed the engineer's topographic sense can only be acquired by working with and subject to the friendly criticism of men who have developed this ability to a high degree. If personnel of this training is not available it is advisable to request the aid of engineering companies making a specialty of topographic mapping, that are properly equipped and in a position to decide on the best method for any particular case, whether it be by one of the new stereoscopic methods or by methods with which all are familiar.

OTTO LEMBERGER,[26] ASSOC. M. AM. SOC. C. E. (by letter).—That surveyors and engineers in the United States have finally begun to use European methods and apparatus for stereo-topographic surveying purposes is particularly interesting to the writer. In confining his remarks to a description of the aerocartograph, Colonel Birdseye treats only a limited part of the broad field of stereo-photogrammetry, without giving the general theory upon which this apparatus is based. The introductory remarks relating to parallax which is the fundamental principle of stereoscopic measurements, should have been extended to include the fundamental equation of parallax.

As applied to this subject parallax is expressed as a difference of abscissa. The writer has attempted to clarify this point previously.[27] The principles may be demonstrated as follows: Look through two tubes at a pair of sharply pointed pencils, or one finger of each hand, held at the opposite ends. Only one pencil, or one finger, will be noticeable. Next move one of the objects across the end of its tube toward or away from the other object, and the impression is created to the eye that the motion is along the axis of the tubes rather than transverse. In stereoscopic surveying the term, parallax, indicates two conditions. In terrestrial surveying it indicates horizontal distance, as explained by the foregoing experiment, and in aerial surveying it indicates vertical height.

In general, only three types of stereoscopic plotting instruments have thus far proved to be sound, either as to theory or construction, for use in constructing maps on a scale of 1: 5 000 to 1: 30 000. These are the aerocartograph described by Colonel Birdseye (Fig. 11), the stereoplanograph (Fig. 6), and the autograph (Fig. 7). These three are known in Europe as space autographs and their construction is based on the principles of

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[26] Vienna, Austria.

[27] *Die Wasserwirtschaft*, 1919.

Porro and Koppe as developed in the "Bildmesstheodolit," known in English as the photogoniometer. The photogoniometer, shown in Fig. 16, is an apparatus that enables one to measure points on a photograph as one measures them with a theodolite in the field. The same lens must be used in the photogoniometer as in the camera with which the photographs were taken, or at least the lens in the photogoniometer must have the identical optical qualities and the same focal distance as the lens of the aerial camera. Only in this way can the algebraic sum of the errors of the aerial camera and of the photogoniometer be made equal to zero.

With an instrument constructed on the foregoing principles, it is possible to measure the tilt of the optical axis of the aerial camera as well as its lateral inclination, which is called the "swing" of the horizontal line of the photograph. When the tilt and the swing of an aerial photograph are known, measurements can be made directly on the photograph the same as with a theodolite on the ground. The idea of the space autograph originated with Dr. C. Pulfrich, but it was Gustav Heyde, following the principles of Dr. Reinhard Hugershoff, who developed the first practical space autograph. He combined two photogoniometers and named the apparatus the autocartograph (Fig. 8). The process of adjusting the plates used in the three space autographs was developed by Dr. Otto von Gruber. While plates as well as films may be used in the aerocartograph and the stereoplanograph, only plates are used in the autograph. In the writer's opinion, the latest model of the stereoplanograph developed in 1930 can be considered as the most generally adaptable instrument of the three. The writer has described them in detail elsewhere.[28]

In all these instruments the photographs accommodated show only a comparatively small area, generally not larger than 20 sq. km. (7 sq. miles). This is one of the principal arguments against their use in surveying larger tracts of land. The space autographs described in the foregoing comments, used in connection with corresponding aerial cameras, have not proved their economic superiority in preparing small-scale maps of large territories in which there is no triangulation network. Captain Hotine has criticized European space autographs for this reason. He based his judgment on the many investigations made by the Geographical Section of the British General Staff. Two instruments constructed in England have been described by Captain Hotine[29] and Lieut. T. S. A. Salt.[30] One of these, the photogrammetric plotter, was said to be specially adapted to surveying larger areas, but apparently it has not achieved any considerable popularity. The instrument described by Lieut. Salt is the stereogoniometer, perfected by H. G. Fourcade. This instrument is used in combination with "image triangulation," a process explained subsequently in this discussion, to obtain a contour map. Photographs for use in this instrument must have scarcely no tilt and must be taken in a strip with an overlap of 60 per cent. With the stereogoniometer, it is possible to produce a map on a scale of 1:20 000 at the rate of 4 sq. miles per hour. This progress requires a computer, a force of five draftsmen, and ground control equivalent to that of an ordinary secondary triangulation network.

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[28] "La Fotogrametria Terrestre y Aerea. Su applicacion en la formacion de planos tecnicos y cartas geograficas," *Memorias y Revista,* Sociedad Cientifica "Antonio Alzate," City of Mexico, Vol. 50, 1928.

[29] *The Geographical Journal,* February, 1930.

[30] *Internationales Archiv für Photogrammetrie,* VII Band, 1930.

In the stereogoniometer overlapping photographs placed in two cameras are observed through a fixed binocular telescope system fitted with floating marks. The line connecting the two cameras may be called the air-base line; that is, the line joining the positions of the camera at the moments of successive exposure. This base line is called the polar axis. All movements of the two cameras as well as of two movable mirrors placed between the cameras and the telescope are referred to this axis. The axes of the mirrors coincide with that of the telescope system. One characteristic of this apparatus is that the devices for moving both cameras are entirely independent of those that govern the observation and the plotting. This simplified the design of the instrument which is similar in theory to that previously accredited to Dr. Koppe in which two movable cameras are observed from a fixed observation system, as in the case of the autograph.

In the autograph, however, all movements are referred to a horizontal plane in such a manner that the air-base line is considered as projected on the three space planes. In the stereogoniometer the error of projection of a point on the photograph of each camera is corrected by means of mirrors that can be moved and rotated, whereas, in the autograph, corrections are made automatically by turning the plates around their principal points, and in their planes, by a small swing angle. This error of projection in the autograph is due to the fact that the guide-rod that coincides with the camera axis, includes with the sighting axis of the fixed observation telescope other angles than the light ray includes with the axis of the air camera. The use of mirrors in the stereogoniometer to effect perfect correspondence (parallelism to eye-base) of two images of a point when viewed in a stereoscope was first introduced in 1924 in the stereoplanograph.

The stereogoniometer has only a limited field of application. It requires photographs taken vertically, the maximum tilt being 2 degrees. It is practicable for producing maps on the photo-scale only; for example, 1:20 000, whereas in extensive territorial surveys a scale of 1:100 000 or less is desired.

Contrary to the opinions of Captain Hotine and Lieut. Salt, the writer considers that the plotting of contours and topography by means of plotting instruments equipped with hand-wheels and foot-operated disks is entirely feasible. He has instructed quite a number of topographical draftsmen (most of them lacking scientific education entirely) in the operation of automatic plotting machines, and the results have been excellent as long as the stereoscopic sight of the operator was good. Plotting with such an instrument and keeping it in good working order are necessarily two different functions, but the latter also is made possible by explicit instructions. Special radial triangulators should be used in "image" triangulation for good results. Their use reduces the number of draftsmen and makes it possible to do the work faster and better.

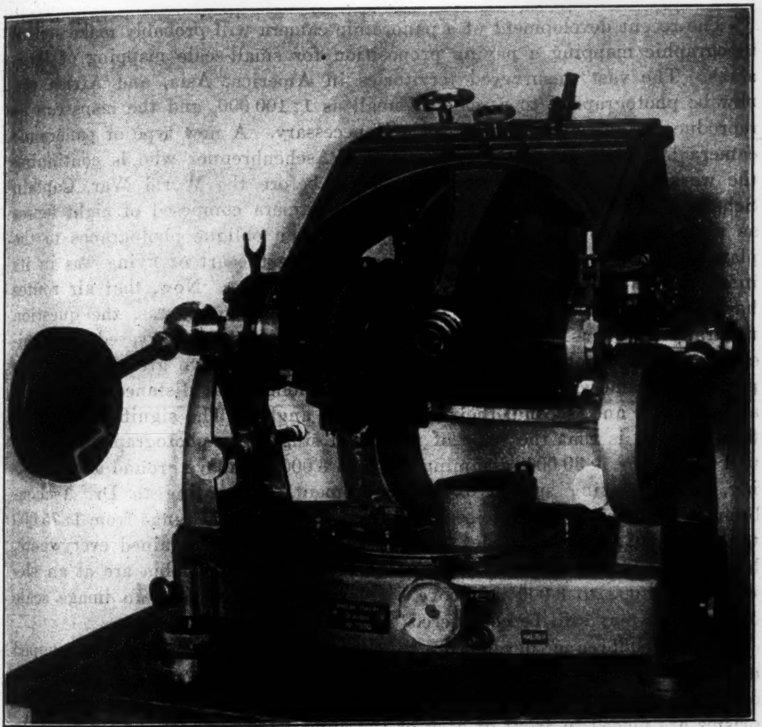

FIG. 16.—"BILDMESSTHEODOLIT," OR PHOTOGONIOMETER.

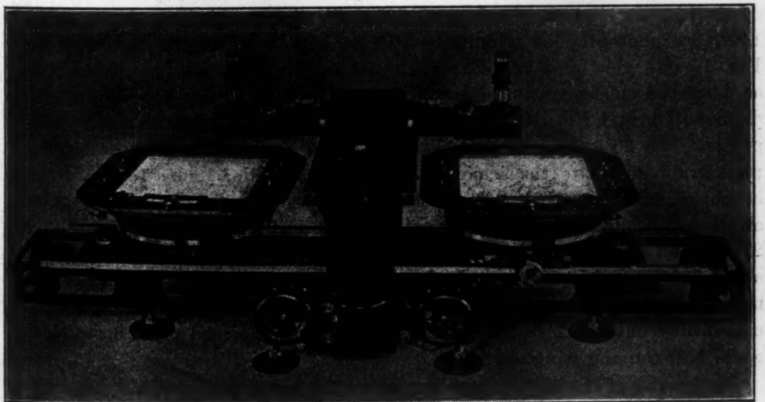

FIG. 17.—THE RADIAL TRIANGULATOR.

The recent development of a panoramic camera will probably make stereo-topographic mapping a paying proposition for small-scale mapping of large areas. The vast unsurveyed territories in America, Asia, and Africa can now be photographed to a scale as small as 1: 100 000, and the maps can be reproduced to an even smaller scale if necessary. A new type of panoramic camera has been developed by Dr. Claus Aschenbrenner who is continuing the work of Capt. T. Scheimpflug. Long before the World War, Captain Scheimpflug had developed a multiple-lens camera composed of eight lenses as well as an apparatus to transform the seven oblique photographs to the plane of the central vertical one. At that time the art of flying was in its infancy and Captain Scheimpflug used free balloons. Now, that air routes have been established over so many large unsurveyed areas, the question naturally arose as to how these large areas could be surveyed economically. Dr. Aschenbrenner considered that there were two great obstacles to perfecting cameras for such use: First, the long focal distance of existing air cameras; and, second, their small image angles. The significance of the first obstacle is that the scale of the usual single-lens photograph is limited to 1: 20 000 or 1: 30 000, assuming 5 000 to 6 000 m. above ground as the ceiling, without using special oxygen equipment. According to Dr. Aschenbrenner an economical photo-scale for such surveys would range from 1: 75 000 to 1: 100 000, or even smaller. Such scales cannot be obtained everywhere, however. For example, the plateaus of Mexico and Colombia are at an elevation of 1 000 to 3 000 m. above sea level. In such cases an image scale of 1: 60 000 may still be considered the limit.

Dr. Aschenbrenner developed a nine-lens camera with eight lenses grouped around the central one, all with focal lengths of 54 mm., or 2.1 in. The optical axes of these eight lenses are parallel to that of the central one, but prisms are placed in front of each of the eight lenses to gather light rays 54° from the vertical. By means of this disposition of lenses and prisms and by operating one shutter, the images from all nine lenses, covering a field of 140°, are photographed simultaneously on one 18 by 18-cm. section of film. Fig. 17 is a view of a radial triangulator used in determining control points as described subsequently. Fig. 18 shows the camera with the film magazine dismounted, and Fig. 19 shows the transformer used to make the assembled prints. Fig. 20 is an exact reproduction of the segments of an untransformed photograph taken to the scale of 1: 60 000 on a film, 18 by 18 cm., from a height of 4 800 m. above the ground. Fig. 21 shows corresponding segments of the same photograph after transformation and assembly in a print, 27 by 27 cm., on a scale, of 1: 90 000, covering an area of about 590 sq. km., or about 212 sq. miles. The particular advantages claimed for this camera over the American type of five-lens camera are the much shorter focal length, the use of only one shutter, and the fact that the transformed print has no blank gores between the wing segments.

By means of the instrument known as the radial triangulator the number of control points necessary in the panoramic process devised by Dr. Aschenbrenner is greatly reduced. The radial triangulator shown in Fig. 17 serves

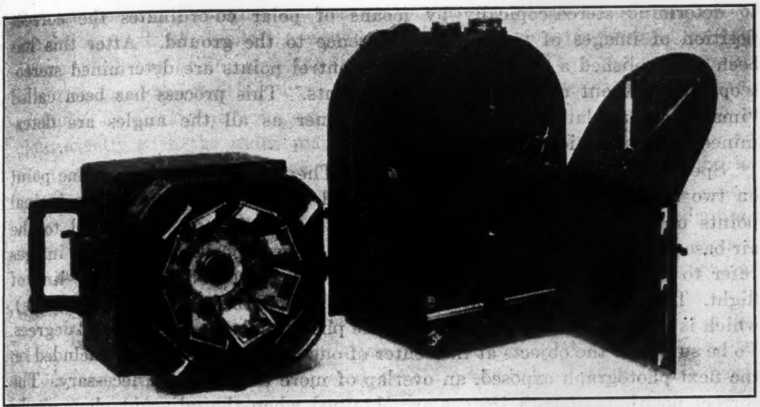

FIG. 18.—NINE-LENS CAMERA AND MAGAZINE.

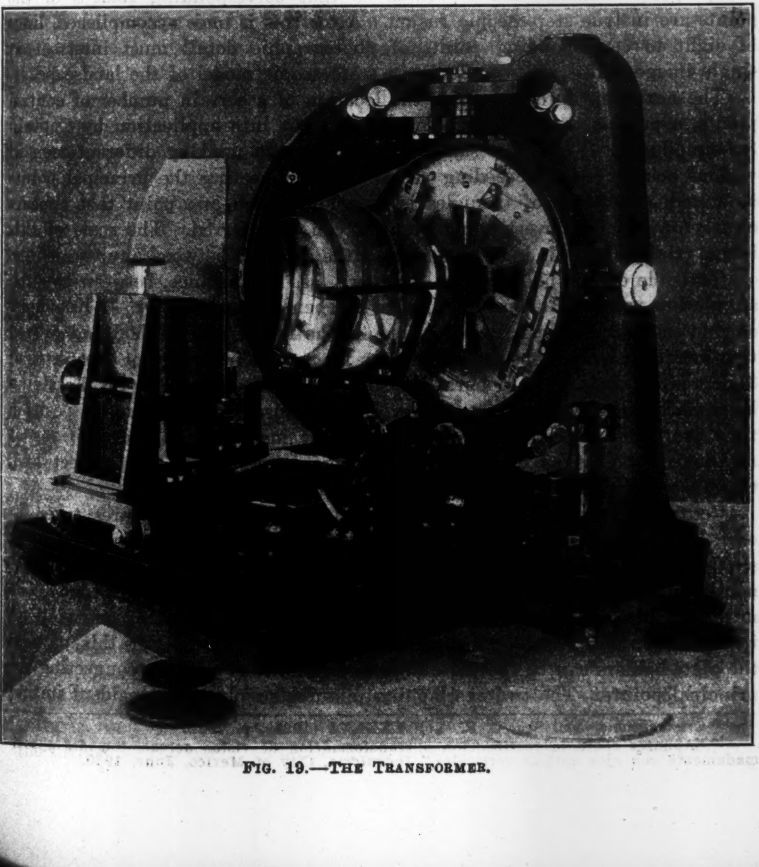

FIG. 19.—THE TRANSFORMER.

to determine stereoscopically by means of polar co-ordinates the correct position of images of points with reference to the ground. After this has been accomplished a certain number of control points are determined stereoscopically without any ground measurements. This process has been called "image" triangulation by Dr. Aschenbrenner as all the angles are determined by observation of the images.

Specifically, the method is as follows: The two images of the same point on two overlapping photographs are rotated about their centers or principal points until they lie with their corresponding images on or parallel to the air-base line or the line connecting the two camera positions. If both images refer to the same flight strip, the air-base line corresponds with the line of flight. The principal points are assumed to replace the plumb-points (nadirs), which is permissible as long as tilts of the photographs do not exceed 3 degrees. To be sure that the objects at the center of one photograph are also included on the next photograph exposed, an overlap of more than 50% is necessary. The correct positions of both images are obtained when the adjacent photographs are properly oriented and separated so that corresponding images of the points are in true stereoscopic fusion. After this is once accomplished, lines of sight to corresponding points of photographic detail must intersect at single image points, thus producing a stereoscopic model of the landscape.

The next step is to determine the location of a certain number of control points, a process that was first devised and put into application by Captain Scheimpflug in 1898. The same method has been used by different men in various countries in succeeding years.[31] By connecting the principal points as stated, a network of triangles is drawn to which any point that appears clearly on different overlapping pictures can be referred. The scale of this image triangulation is known as soon as the distance between any two points, quite visible on the photographs, is known, as, for instance, between two triangulation points. Independently, Dr. Aschenbrenner and Dr. J. Koppmair conducted mathematical investigations of the radial line method and proved that vertical photographs can be used as long as their tilt is not greater than 3 degrees.[32] In aerial topographic surveys some differences in the flying height are permitted as long as they do not cause greater errors of position than are consistent with the accuracy of the map itself. The positions of the control points as determined by image triangulation are correct and, therefore, independent of difference in the flying height.

Control points are determined, as follows: First, the pictures taken at each exposure of the nine-lens camera are transformed and printed on a single plate, 30 by 30 cm. Then, the principal points are marked permanently on the plate by a special device known as the cross straight-edge. The next step is to transfer the position of the principal point of one plate to its corresponding image position on the adjacent plate. For this purpose the plate-holders can be rotated and moved parallel to the line connecting the principal points. The two overlapping pictures supply the coincident images that are observed through a stereoscope by means of the floating mark. First, the index mark of the left eye-piece of the stereoscope is brought to coincidence with the principal point in the left picture. Then the right picture is moved until the floating mark observed by both eyes touches the apparent ground. To mark this point on the right-hand picture the needle that coincides exactly with the index mark of the right eye-piece is moved down and the point on the ground below the image of the principal point is marked permanently on the plate. Similarly, any other point identified by a pair of images is marked permanently on both plates. This is a special advantage of the radial triangulator over other similar instruments, because even on very even ground the point looms up like a cairn. Micrometer screws are provided for different movements.

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[31] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 792.

[32] "Apuntes sobre la rectificacion o transformacion de vistas aereas, tomades aproximadamente con ejes opticos verticales," *Ingenieria*, City of Mexico, June, 1930.

The next step is to determine the direction from the principal point of each plate to the images of the triangulation points shown on the plate. Each plate of an overlapping pair is adjusted in its plate-holder so that its center (principal point) coincides perfectly with the rotating center of the holder. This adjustment may be made by means of one of the eye-pieces of the double microscope or by means of a special microscope. The stereoscopic image of each triangulation point is then brought to coincidence with the floating mark by turning one plate-holder and manipulating one side-wheel at the same time. When this operation is complete, the angle to this image from the principal point of each plate is read to minutes. The same general process can be done graphically.

Stereoscopic fusion of these pictures reveals a surprisingly sharp relief due to the long base. With an overlap of 65%, the proportion of air-base line to height of camera is 5:3. The good results are due to the fact that both picture planes are parallel to each other and, at the same time, to the air-base line.

To check this method of image triangulation, Dr. Aschenbrenner made the following test: An area 30 km. long was selected, which had been covered by an accurate cadastral map on a scale of 1:5 000, and in which there were a number of triangulation stations which could be accurately identified on the photographs. A strip of six pictures was taken and used as a traverse line, the centers of each plate being the traverse stations. A number of points identifiable on the cadastral map were intersected from adjacent traverse stations. In addition, a number of clearly visible points were chosen, located on either side of the traverse line. Intersection angles were measured from the traverse stations to these points. In this way overlapping trapezoids were formed over the entire traverse line. A method of computation was chosen such that each trapezoid was calculated by itself, assuming a length for the first side. Different methods may be used to refer the traverse line to the surface of the earth, the co-ordinates of two primary triangulation points at either end of the traverse line being known in the test. These two points were marked on the plates and were intersected from the traverse line, in order to enable the determination of its true position on the surface of the earth. The distance between the two points was then com-

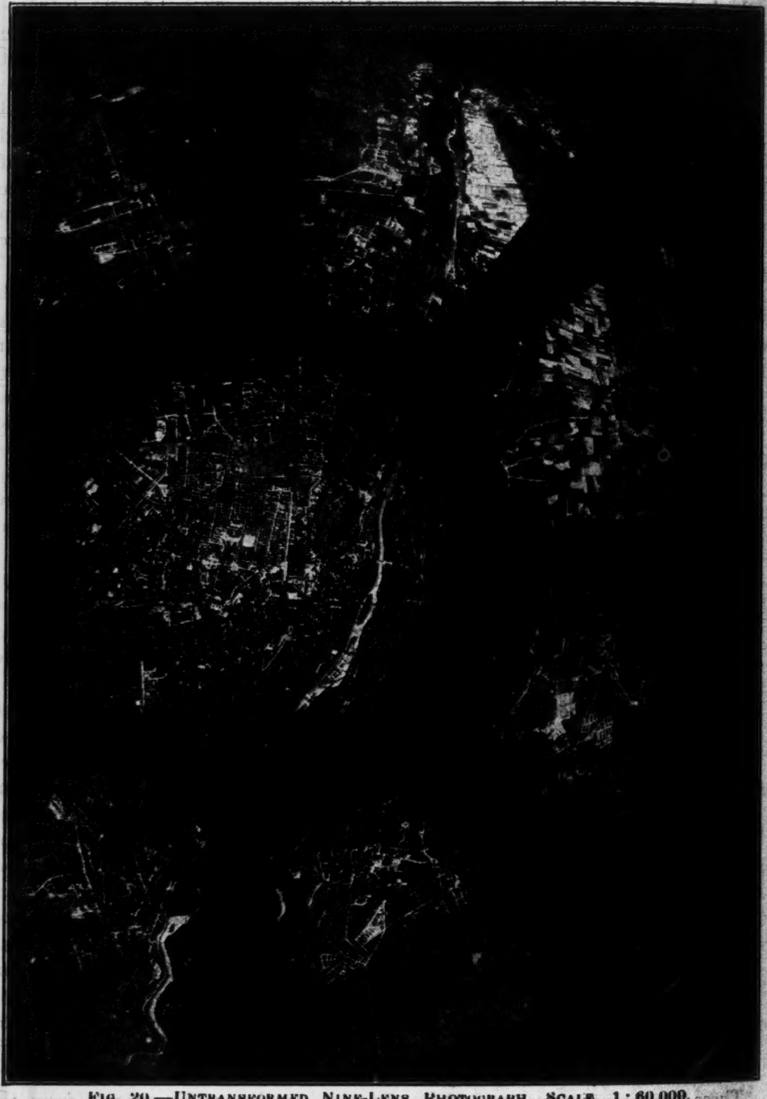

FIG. 20.—UNTRANSFORMED NINE-LENS PHOTOGRAPH, SCALE, 1 : 60 000.

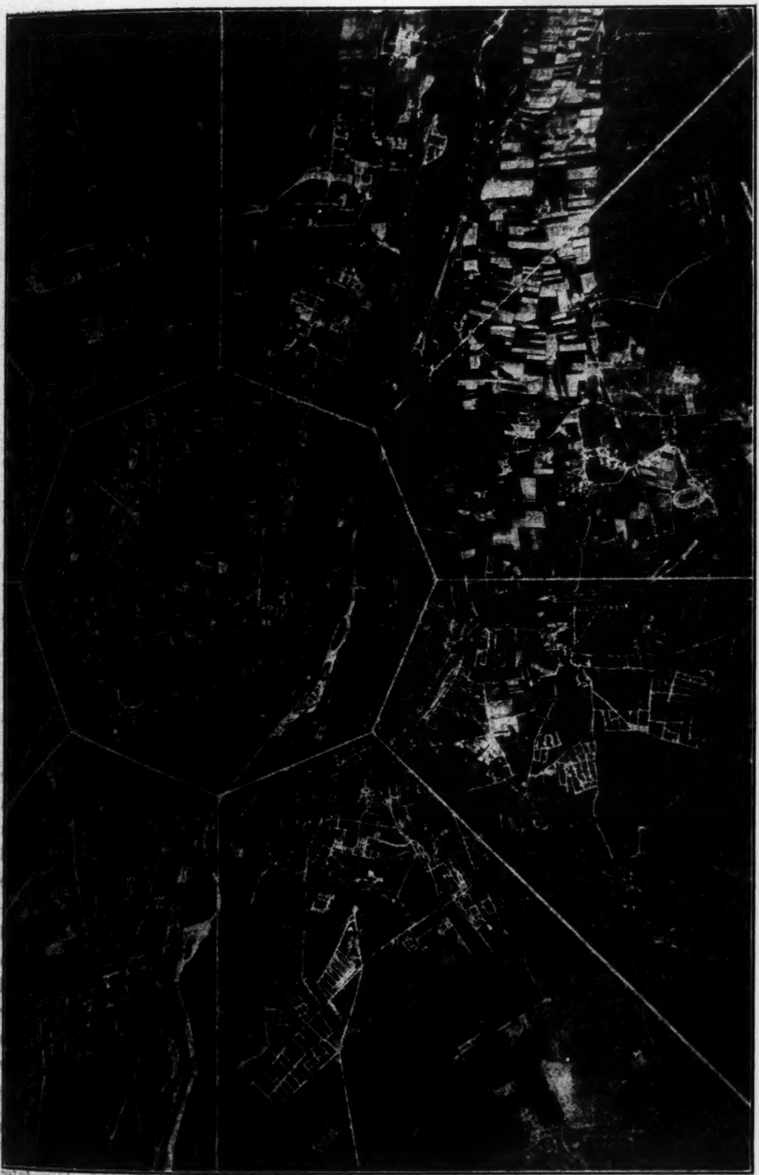

Fig. 21.—Transformed and Assembled Nine-Lens Photograph of Same Area as Shown on Fig. 20, Scale, 1 : 90 000.

puted from the photographic traverse on the assumed scale and on the earth, and, in this way, the scale of the traverse was obtained. By computing the azimuth of the connecting line between the two points, the orientation of the traverse line was obtained. In all, twelve control points were located on the cadastral map. A comparison between the computed co-ordinates of these control points and those taken from the map resulted in a mean error of 34 m. on the ground, which on a scale of 1: 100 000 represents 0.034 mm.

In countries in which no triangulation network has been established, different methods may be applied. For example, on each end of the traverse line a base can be measured and its azimuth determined. A better method is to lay out two parallel traverse lines and connect them by photographic triangulation. To determine the bearing of such chains of triangles it would be sufficient to measure on the ground a base for each 100 km. In this way, with four base lines that could be checked or located by astronomical observations, a territory of about 10 000 sq. km. (3 600 sq. miles) could be surveyed. Dr. Aschenbrenner has reported[33] that a territory of 10 000 sq. km., in which only two known points were available, has already been surveyed by means of image triangulation, and good line maps prepared from the surveys.

Since 1930 Dr. Aschenbrenner has adapted his methods to the preparation of contour maps by stereoscopic plotting. The photographs are transformed to horizontal image planes by means of image triangulation. This may be done by calculation,[34] or by transforming the aerial photographs by means of the control points determined by image triangulation, at the same time reducing the aerial photographs to sizes that can be used for plotting in a space autograph. Thus, it is seen that stereoscopic pairs of transformed multiple-lens photographs may be plotted by either of the three space autographs mentioned at the beginning of this discussion. Even the stereometer or the stereogoniometer, could be used.

At least six advantages may be stated for plotting small-scale contour maps from stereoscopic pairs of transformed panoramic photographs, in a space autograph: (1) Necessary adjustments can be made more quickly than by means of oblique pictures; (2) the optical relief of the stereoscopic model is accentuated and magnified as many times as the focal distance of the transformed reduction is contained in the focal distance of the space autograph; therefore, depth and height in the stereoscopic model are observed to an exaggerated scale, so to speak, so that contours can be drawn with greater accuracy even on flat terrain; (3) since the field of a wide-angled stereoscopic pair is many times greater than that produced by any other aerial camera, the plotting can be done much more quickly; (4) the area included in each exposure, is many times greater than that of an ordinary aerial camera; therefore, the time required to change the stereoscopic pairs is

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[33] *Internationales Archiv für Photogrammetrie*, Vol. VII, II Band, 1931: a short informal report about his paper read at the Third International Congress of Photogrammetry, in Zurich, Switzerland, September 5–10, 1930.

[34] "Hohenkarten aus weitwinkligen Luftaufnahmen." von Dr. S. Finsterwalder, *Internationales Archiv für Photogrammetrie*, Vol. VII, 2d Pt. 1930–31.

greatly reduced; (5) for the foregoing reasons the number of control points is reduced considerably; and (6) the multiple-lens photographs are already generalized, while in the other processes the first map is the product of large-scale picture pairs and, therefore, must be redrawn to produce a generalized map on the scale of 1 : 15 000, or smaller, just as if plane-table sheets prepared on a scale of 1 : 15 000 were used to produce a final map on a scale of 1 in. equals 1 mile, or smaller. Every topographer knows what that means in terms of reproductions, redrawings, etc., because it takes time and personnel and reduces the exactness of the original map.

It is to be emphasized that by means of the panoramic camera and auxiliary equipment, areas as large as 400 to 500 sq. km. (144 to 180 sq. miles) can be plotted from one stereoscopic pair of photographs in comparison with 20 sq. km. that are obtained as a maximum by means of other aerial cameras.

E. R. POLLEY,[35] M. AM. SOC. C. E., and LEON T. ELIEL,[36] ESQ. (by letter).—This paper describes, excellently, the principles and methods involved in the making of topographical maps by the utilization of aerial photographs and the aerocartograph-measuring stereoscope. The introductory remarks of this paper indicate that Colonel Birdseye has assumed that the basic principles underlying all photogrammetric methods, have been demonstrated adequately elsewhere. Unfortunately, the writers find that these principles are not well known except by specialists. Until an engineer is able to visualize clearly how photographs are used to replace certain fundamental operations of conventional survey practice, stereoscopic methods are rather confusing at best.

This principle can best be explained from the standpoint of the familiar operations of an ordinary ground survey.

The fundamental principle upon which measuring stereoscopes operate is simply triangulation. This principle remains the same regardless of whether terrestrial photographs or aerial photographs are used, but it is demonstrated most easily by reference to terrestrial photographs.

Imagine that a survey by triangulation is contemplated of a dam site, such as that shown in Fig. 22. Many points on the north side of the canyon are to be located by triangulation from a base line on the south side. For example, assume Point *X* to be any point on the surface of this ground.

The ordinary procedure would be first to lay off a base line at a convenient location on the south bank, possibly 100 ft. in length. Then angles would be turned successively from the extremities of this base line to the various points (of which Point *X* is an example), and the positions of each point would be calculated from the resultant data.

Next, assume that two transits are set up simultaneously at the extremities of this base line and that by a system of prisms the transitman stands in the middle of the base line, looking through the right transit with his right eye and the left transit with his left eye. Thus, the transits, when directed to the same object give an effect similar to that observed through a pair of binoculars. The observer is not conscious of looking through two separate instruments. He enjoys a marvelous capacity for relief perception, because effectively his eyes are separated 100 ft. instead of 2½ in., as is the case when observing with the unaided eye. To understand the remarkable stereoscopic effect he secures by this means, consider similar effects from every-day experience.

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[35] Vice-Pres., Fairchild Aerial Surveys, Inc., Los Angeles, Calif.

[36] Pacific Mgr., Fairchild Aerial Surveys, Inc., Los Angeles, Calif

When looking through a pair of binoculars, human vision and depth perception (stereoscopic vision) are greatly improved for two reasons: First, the binoculars give a high degree of magnification; and, second, they make it possible to judge differences in distance with much greater accuracy. This latter phenomenon exists because the binoculars increase the base line of the eye from about 2½ in. to possibly 5 in. This practically doubles the accuracy with which relative distances can be judged. Therefore, a superlative effect can be secured, if, instead of looking through binoculars with a 5-in. base, the observer looks through binoculars with a 100-ft. base, such as is secured with the two transits. The transitman perceives the most minute differences in distance. If one of two objects is only a few inches closer than the other, they appear to be separated definitely. This ability to judge easily slight differences in distance is effective up to distances of 1 000 to 2 000 ft., with transits separated 100 ft.

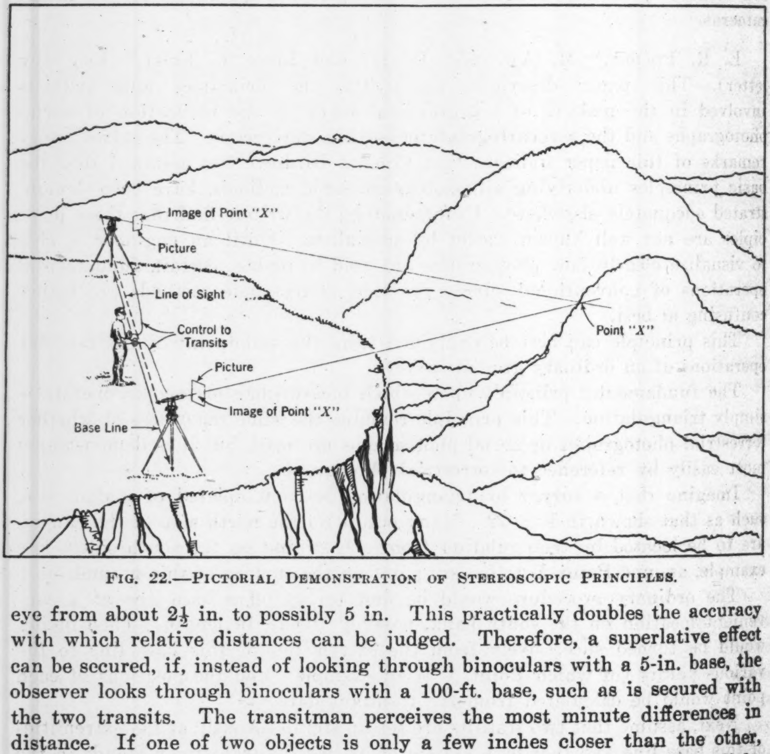

FIG. 22.—PICTORIAL DEMONSTRATION OF STEREOSCOPIC PRINCIPLES.

The two cross-hairs of the transits appear to merge into a single cross-hair, the center of which, lying in the lines of sight of the two instruments, appears to rest exactly at the point of intersection. In fact, the cross-hairs appear to stand in space as a vertical grid, the center intersection of which appears to rest exactly at the point of intersection. If the two transits are directed at Point *X,* the center of this grid (the cross-hairs) will appear to rest exactly on that point. Below this, the grid appears to have receded under the ground. Now, if the convergence of the two transits is increased slightly, so that the intersection occurs perhaps 1 ft. nearer the base line, the transitman will still see Point *X* stereoscopically. The only change which this increase in the convergence angle of the two transits causes, is an apparent movement of the vertical grid (the cross-hairs) so that it appears to stand nearer the transitman than Point *X.* If the angle of the two transits is diverged so that the point of their intersection occurs behind Point *X,* the grid will appear to have receded into the ground. To the observer it will appear as if the side hill were composed of transparent jelly with a grid of wire appearing first in front of the surface of the jelly and subsequently buried in it.

In similar fashion the transits can be directed at any point or object and the apparent coincidence between this object and the cross-hair grids will permit precise setting of the transits on this object.

Next, assume that the observer replaces each of the transits temporarily with a camera and takes a picture with the center of the camera lens exactly at each transit station. Then, assume that he replaces the transits and supports the pictures directly in front of them, taking extreme care to make the distance between the pivot point of the transits and the principal point of the photograph precisely the same as the focal length of the camera that took the pictures.

Now, if the observer directs the transit at the tiny dot on the picture, that is the photographic reproduction of Point *X,* and then removes the photograph from in front of the transit, he will find the transit pointing directly at Point *X* on the hillside. Therefore, the observer can direct his transits to Point *X,* or to any other point he wishes, by observing the corresponding image point on one photograph through one transit and the same point as shown on the other photograph through the other transit.

The application of the stereoscopic principle in the measuring stereoscope might be described very broadly by stating that the pictures are brought to the office and set up before the two observing systems of the aerocartograph instead of in front of two transits.

The foregoing description refers to photo-theodolite or ground pictures for which length of base and angles at which pictures are taken, are known. Therefore, it is relatively simple to reproduce the relative positions and separation (all to scale) in the aerocartograph. In the case of aerial pictures, the camera is supported by a rapidly moving airplane; the base line is the distance traveled by the airplane in the interval between exposing two photographic plates. This distance is not measured; camera angles are unknown. Therefore, it is necessary to know the elevation and horizontal positions of several points on the ground photographed in order that the base-line distance and camera angles can be reproduced in the aerocartograph.

There is no other essential difference; the fundamental principle is simply photographic triangulation regardless of whether aerial photographs, ground photographs, or combinations of the two, are used.

C. H. BIRDSEYE,[37] M. AM. SOC. C. E. (by letter).—Since the presentation of this paper there have been many new developments in stereoscopic mapping instruments and methods, and some changes in the construction and use of the aerocartograph. A German book on photogrammetry,[38] has been translated into English. It is unfortunate that another book[39] has not been translated, because the text is a valuable contribution to the literature on this subject. In the United States, Earl F. Church, Assoc. M. Am. Soc. C. E., has extended his papers on topographic mapping from aerial photographs to five, all published by Syracuse University, Syracuse, N. Y. These publications have not been mentioned in the discussion of this paper and they add to the bibliography available on the subject.

As stated in the Synopsis of the paper, the writer described the aerocartograph in detail because it is the instrument with which he is most familiar. The only American-made instrument of this type was described in a previous paper[6] and, therefore, the writer made only brief mention of it. Mr. Pendleton cites the principal differences between the American and European instruments and comments effectively on the relative merits of film and plates for accurate work. The writer agrees with him in so far as large-scale work is concerned, but realizes that use of glass plates on small-scale mapping of large areas is relatively slow and expensive.

Messrs. Polley and Eliel call attention to the fact that many engineers may not be familiar with the basic principles underlying photogrammetric methods. The writer acknowledges the inadequacy of his reference to another source[3] for demonstrations of the general principles underlying the stereoscopic use of photographs, and he should have expanded other references given in the paper,[4] in the discussion by Lieutenant Reading,[11] and in this closing discussion.[38, 39] However, the writer is glad that Messrs. Polley and Eliel have added such a clear and simple demonstration.

Mr. Lemberger comments briefly on the other European space autographs and presents a most interesting description of Dr. Aschenbrenner's nine-lens camera and accompanying small-scale mapping instruments and methods. The writer has not seen these instruments, but believes that the nine-lens camera and accompanying small-scale mapping instruments and methods. of the filling of the gores between the wing pictures. He is doubtful, however, as to the sharpness of the images on the negative gathered by prisms into eight of the lenses. At any rate, the multiple-lens idea is certainly the solution of small-scale mapping.

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[37] Eng. Asst. to Director, U. S. Geological Survey, Washington, D. C.

[38] "Photogrammetry—Collected Lectures and Essays," Edited by O. von Gruber, translated from the German original, by G. T. McCaw and F. A. Cazalet, pub. by Chapman & Hall, London, England, 1932.

[39] Handbuch der Wissenschaftlichen und Angewandten Photographie, Band VII. "Photogrammetrie und Luftbildwesen," by Dr. R. Hugershoff, pub. by Julius Springer, Vienna, Austria, 1930.

The Corps of Engineers, U. S. Army, is now (1932) conducting experiments with stereoscopic plotting from five-lens photographs and, also, with the Geological Survey, is interested in tests that are being made with the new German stereoplanograph and quadruplex (four-lens) camera. The latter development has occurred since this paper was written. The camera has four magazines, all taking oblique negatives, and the stereoplanograph has been equipped with two quadruplex plate-holders so that each set of four negatives, without any transformation to the horizontal or even making glass positives, is placed in one of the plate-holders with exactly the same relations between the four negatives that were maintained in the camera. The field of each of the four negatives taken at each exposure overlaps the field of each of the others, so that there are no gores between them. The stereoscopic operator moves the pointer from one segment to another as if the composite negative was made by a single lens. The short focal length of the lenses ($13\frac{1}{2}$ cm.) permits covering a relatively large area in a single exposure. On the test of an area in Pennsylvania, a single exposure taken at an altitude of about 14 000 ft. above ground, covered an area of about 20 sq. miles on a scale of 1:30 000.

Mr. Ripley comments on the lack of a statement as to the saving of time by the use of an aerial survey. It is difficult to make a general statement because of the great difference in projects as regards scale and detail desired, and because of great differences in types of aerial surveys. The writer assumes that Mr. Ripley refers to contour maps in the two examples cited, in which case the time saved should depend largely on the scale of the map. Single-lens stereoscopic equipment heretofore used in the United States has been restricted to scales of 1:24 000, or larger. With this equipment the writer would hesitate to compete with a skilled ground topographer on a topographic survey of an easily accessible area on a scale of 1:48 000, or smaller. However, if multiple-lens equipment will permit accurate delineation of relief, the photo-topographic engineer should be able to give the ground surveyor "cards and spades" and come out ahead in a small-scale survey of an inaccessible area in which the culture is not obscured by timber cover.

Photographs completely covering a project may be taken in a day, but the photography itself is a relatively small part of the job. To secure the necessary ground control usually takes longer than the photographic work; but the total time consumed on an average project for which the use of aerial photographs is suitable, should be considerably less than that required to map the area solely by ground-survey methods.

Lieutenant Reading raises much the same question but specifies relative costs and technical advantages and asks for comparisons. The writer will attempt to give these from a combination of actual and estimated data.

In mapping Great Smoky Mountains National Park to a scale of 1:24 000, and with a contour interval of 20 ft., adequate ground control was available. A heavily timbered area of about 100 sq. miles was mapped by means of the aerocartograph under contract, at a price of $125 per sq. mile, resulting in a small profit. Similar areas in the same region were mapped by the U. S. Geological Survey by the plane-table method at an average cost of $148 per sq. mile for topography alone, after the fundamental ground control had been established. Had the Geological Survey used the multiple-lens camera to delineate the culture and drainage and then added the contours by means of the plane-table, it is estimated that the cost would have been much less than with the plane-table alone. With such a method gaps in trails obscured by the timber would have been added by the topographer in the field, but the difficulty the topographer had in forcing his way through the dense brush along the ridges and creeks would not be eliminated, and elevations of features off the traverse lines would still be sketched in with less exactness than resulted from aerocartograph mapping.

In mapping Zion National Park, in Utah, to a scale of 1: 24 000, with a contour interval of 50 ft., adequate control was also established in advance. The area mapped was 305 sq. miles. Of this area, 157 sq. miles were covered by a topographer extremely skilled in mapping cliff areas by what is known as the station method; that is, plane-table intersections with vertical angle elevations. Climatic conditions prevented the aerial photographic contractor from delivering the photographs until near the end of the field season, but the topographer was able to identify enough control on the prints so that the remaining area of 148 sq. miles was mapped in the office on the aerocartograph. An interesting result was that the contours of the two surveys joined with practically no adjustment. The average cost per square mile of the field mapping by plane-table was $44 per sq. mile and the cost of the aerocartograph mapping was $51 per sq. mile. This area was ideal for stereoscopic mapping, but the project was somewhat experimental because the stereoscopic operators had had no previous experience in mapping cliffs. If an automatic plotting instrument had not been available, the writer would still have used single-lens photographs and executed the field plane-table work in much the same way, except that he would spend no time whatever in the field in delineating all the minute detail of slopes and cliffs. He would not prepare a base map in advance because of the delay in securing the required picture control after the photographs had been taken.

Another photo-plane-table method is that of sketching the details on the photographic prints and adjusting the data into map form later.[40] This method was used very successfully on a contract survey of Flathead Lake, Montana, on which 2-ft. contours, both under and above water, were required between extreme low and high-water limits. The preparation of a base by the radial-line method would have required waiting for field control and picture point identification work, and even with these data available the topographer would have only the shore line and the few roads, houses, and streams on his base sheet. Use of the method permitted the topographers to leave Washington as soon as the photographer wired that the area had been photographed and, by the use of air mail both ways, the topographers had a few prints available to work on the day they reached the project. The particular advantage of using this method was the elimination of almost all traversing and the substitution of the use of two short base lines on each print for scale and rectification, and the use of image points to which only vertical angles were read. The under-water forms showed perfectly in the photographs, so that the two 2-ft. under-water contours were delineated by means of a very few soundings on each print. The writer does not have at his disposal the unit cost of this survey, but he is confident that it could not have been executed as quickly or cheaply, or as well, by any other method.

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[40] "Aerial Photography as an Aid in Map Making," by Gerard H. Matthes, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E. Vol. 86 (1923), pp. 779–802.

The discussions by Major Johns, Captain Crosson, and Messrs. Ballester and Stuart bring up points that are answered at least briefly in the foregoing comments. Mr. Peters gives an excellent outline of what has been done in this field by means of other instruments and methods.

From the preceding discussion it is apparent that the writer does not believe that any one method or instrument is best adapted to all kinds of mapping projects. There are so many different types of mapping projects that an organization engaged in general topographic mapping should have at its disposal as wide a range of equipment as is practicable and, above all, should have on its staff experts able to select the method best adapted to each particular project. There are few areas in the United States in which some use of photography, either aerial or terrestrial, will not produce a better map than can be obtained by ground-survey methods alone, and, in most areas, there will be a distinct saving in time and cost by using some combination of photographic and instrumental survey methods. An extremely valuable by-product of the survey will be the availability of the photographs for other uses.

print for scale and verification, and the use of image points to which only vertical angles were read. The under-water forms showed perfectly in the photographs, so that the under-water contours were determined by means of a very few soundings on each line. The writer does not have at his disposal the unit cost of this survey, but he is confident that it could not have been executed as quickly, as cheaply, or as well, by any other method.

The discussions by Major [illegible] Cutting, Lawson, and Messrs. [illegible] and Stuart bring up points which are answered at least briefly in the foregoing comments. Mr. [illegible] gives an excellent outline of what has been done in this field by means of [illegible] instruments and methods.

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PUBLIC SUPERVISION OF DAMS[1]

A SYMPOSIUM

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[1] Papers presented at the meeting of the Power Division, New York, N. Y., January 16, 1930, and at the Technical Session, Sacramento, Calif., April 23, 1930, respectively.

RECOMMENDATIONS FOR LEGISLATION AND FOR APPLICATION OF LAW

By A. H. Markwart,[2] M. Am. Soc. C. E.

Synopsis

Provided that State supervision of dams is to obtain, this paper contains recommendations upon the means to be taken for the regulation of design, construction, and maintenance. Characteristics of the best types, the extent, exemptions, personnel, and costs of supervision are discussed, and the paper concludes with comments on the feasibility of building codes for dams.

Popular Interest in Dams

The failure of an engineering structure, unless attended by loss of life or property, does not usually attract the attention of the layman. When such a failure results in damage to property only, it is of interest to the owner or owners of the structure and to the party or parties damaged. When, however, the failure involves loss of life as well as damage to property, it becomes a tragedy which is the subject of great concern to the general public.

There are probably no engineering structures which the public fears more than dams. While the layman, as a rule, cannot visualize the effects of failure of many engineering works, the destruction which would follow the failure of a dam is to him most apparent. For this reason, there is perhaps more popular interest in dams than in other engineering construction.

The failure of the St. Francis Dam in Southern California was a tragic occurrence which has prompted many to question seriously the stability of existing dams. It does not follow, however, that the building of dams must be discontinued any more than that mining activities should be discontinued because of loss of life in coal-mining operations.

The conservation of fuel resources by the building of storage works for power purposes must be continued. The irrigation of lands from storage resources must also be continued, and flood-control reservoirs must be built, unless other and better means are devised for handling flood flows. What the failure of the St. Francis Dam and other ill-fated dams teaches, is the necessity of introducing greater conservatism and sounder judgment, particularly the latter, in the design and construction of such structures.

Necessity for Supervision

It is being realized more and more, as intelligent investigations of dam failures are made, that the critical defects have been almost invariably avoidable, had proper judgment and foresight been exercised in the design and construction. Such avoidable defects can be eliminated almost entirely with effective regulation and control.

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[2] Vice-Pres., in Chg. of Eng., Pacific Gas & Elec. Co., San Francisco, Calif.

In exerting supervision over dams, the State is merely exercising one of its police functions in the interest of the public. State supervision of dams is desirable if properly done, and undesirable if not properly done since it then gives a false security to those who place their trust in its protection.

State interference in private matters is always to be deplored, but protection to life, health, and property is in the public interest. It is regrettable that, where these are endangered, resort to Government is invariably had to effect correction. The reform movement which always follows finally centers in further governmental pressure restricting the individual or local group. The same result might have been achieved by action from within the group, if those in responsible positions would voluntarily exercise the necessary control over dangerous practices. Individuals, corporations, and local governments should know of themselves what is essential for the protection of life and property and should impose the necessary self-discipline to eliminate the risks. It is apparent, however, with human nature constituted as it is, that this control from within is not to be expected.

The public has a right to the protection of life and property from the possible failure of dams which impound large volumes of water, and in the absence of self-imposed regulation, it is the duty of Government to provide legislation to that end. This will not necessarily provide a perfect assurance of safety, however, because the criticism may be made that governmental agencies are no more infallible than private agencies, and that all the engineers responsible for the design of dams which have failed, would have been qualified to act as consulting engineers in the service of the State. The value of State supervision does not consist in requiring of the owner the employment of a consulting engineer, but rather in requiring an independent review of the design, insuring the application of more than one mind to the problem. In the words of the findings on the St. Francis Dam disaster, "the construction and operation of a great dam should never be left to the sole judgment of one man, no matter how eminent."[8]

LEGISLATION

Considerable legislation has been enacted in the United States, most of it conservative and well considered. The States of Colorado, Idaho, Massachusetts, New Jersey, New York, Pennsylvania, and Washington have statutes bearing on this subject, some of them of long standing. California enacted a statute, to take effect July 1, 1929.

Other Countries.—All the more important foreign countries have some form of regulations looking toward the safety of dams. These, however, differ fundamentally from those in force in the United States, in that they stipulate the theoretical design methods and the working stresses to be used, rather than limit their authority to the approval of the work of others.

France, since 1897, has had in force, regulations applicable to gravity dams and is formulating similar regulations for other types of dams. Great

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[8] Verdict filed by Jury Drawn by the Coroner of Los Angeles County to Consider Testimony on St. Francis Dam Failure, *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, p. 2162.

Britain, only during the last decade (1920–1930), has undertaken to stipulate precautions to be taken in the construction and operation of dams, and to clarify the law with respect to liability for damage, in the event of a failure. The Italian regulations, as revised December 31, 1925, are perhaps the most comprehensive in force at present, in any country.

It is of interest that, in nearly every instance, the regulations were prompted by some dam catastrophe which impressed the necessity for such regulations upon the minds of the people.

CHARACTERISTICS OF BEST TYPE OF SUPERVISION

Essential as the protection of life and property is, legislation to that end must be well considered along sound lines and must be neither difficult of compliance nor needlessly extravagant in its effect upon costs.

It is apparent that the failure of the St. Francis Dam caused governmental agencies concerned with the approval of dams to be considerably more exacting than they had been theretofore. In view of the apprehension engendered it is not improbable that the tendency will be to require dams to be constructed stronger than is actually necessary. Such excess strength can be had only from capital expenditures greater than have been required in the past, which in the case of hydro-electric power will impose even greater burdens than those which now obtain. Water powers are constantly growing more costly and one of the factors having much to do with this, is the growing State and Federal authority over water and water powers. It is conceivable that if this continues, water powers will finally become too costly to develop. Even now (1932), a water power must be most favorably situated to compete with the modern, highly efficient steam plant.

Engineers, public utilities, and other organizations interested in the building of dams, however, should take a rational, rather than an opposed, view on legislation, because there appears to be some justification for governmental control. Engineers, and particularly civil engineers, should determine for themselves the kind and extent of legislation desired. They should study the problem with a view to directing public opinion along lines not subversive of the best public interests. They should discourage anything in legislation which would tend to hamper the fine progress in the development of dam construction which has characterized the art.

As to the statute itself, it would seem desirable to foster the maximum use of existing governmental machinery—State engineers' offices for administration and the Courts for enforcement—rather than to create new agencies, except as providing for the services of consulting experts in major cases. Consulting boards acting during the design and construction periods only are not sufficient. Operation and maintenance must be checked by the routine inspections of some standing authority, after the consulting boards are discharged.

The statute, in so far as possible, should confine itself to the basic principles, leaving the details to be covered by regulations and rules which may be formulated by the State engineer or other administrator, as occasions develop. Few limitations should be placed upon the supervisory agency as to the technical aids which it may employ in the discharge of its functions. By these means necessary revision could be more readily effected than would be possible through legislative amendment that would otherwise be required. Furthermore, the statute should require promptness of action in the matter of approvals, either from State engineers' offices or other supervisory bodies which may be charged with its administration, in order that there may be no undue delay in gaining decisions on proposed construction. For instance, the Idaho law requires that approval be either given or refused within forty-five days.

The elements which should characterize the best type of supervision will fall naturally into certain groups which may be discussed more or less independently, bearing in mind that they all should have the common but limited objective of safeguarding life and property. Any governmental regulation which does not conduce to that end is needless and extraneous.

Extent of Supervision.—It seems that supervision must be had and it is probable that there is no way of getting it except through some governmental agency. If State supervision is to be established, it should be inclusive of the entire undertaking. It should involve, in so far as safety alone is concerned, the approval of the location, the design, the construction, and the maintenance of the dam. Such supervision should not involve the independent selection of the location or the design, the superintendency of construction, or the direction of maintenance on the part of the State, for the reason that such a procedure would result in a tremendous duplication of effort and expense and an unwarranted interference in the affairs and plans of the owners.

The supervision should be limited, therefore, to the approval of the location for the dam as selected by the builder, the checking of the design which he submits, and such occasional inspection as will insure the carrying out of the approved design and its maintenance thereafter. In keeping with this procedure, the builder of the dam should be required to submit a report of his investigation of the geological conditions of the dam site, a study of the flood run-off conditions of the water-shed behind the dam, as such study would affect the capacity of the spillway, together with plans and specifications of the dam proper, in sufficient detail to describe the nature and type of structure upon which approval is sought. He should be permitted to file supplemental plans and specifications with the same sufficiency of detail, to cover major changes in design which may seem desirable in the interests of safety, as subsequent information is revealed by the construction operation, or for any other good reason.

In addition to its responsibilities over new construction, the State authority should also have the power to condemn unsafe structures, to order water removed or lowered, or to limit permanently the height to which water may be stored, where existing dams are found to constitute a menace.

Exemptions from Supervision.—If the supervision of the design, construction, and maintenance of dams by the State Government is deemed essential in the public interest, there should be no exceptions because of ownership. Dams built by individuals, private corporations, quasi-municipalities, municipalities, counties, and the State itself, should be subject to the same supervision. It is manifestly unreasonable to exempt the structures of municipalities and quasi-municipalities, on the score that they maintain organized engineering departments, and then to require the public utilities which also maintain engineering departments, to submit to supervision by State agencies. Dams built by the departments and bureaus of the Federal Government may not be subject to review by State authorities, due to lack of sovereignty.

Some exemption should be made for unimportant structures, the failure of which would not involve loss of life and property. This exemption, while governed largely by the volume of stored water, should also take into consideration the drainage area contributory to the reservoir and the height of the dam. The menace from bursting dams is largely a question of the volume of water released.

Supervision Personnel.—Those in supervisory capacity on behalf of the State Government must obviously be the peers in technical experience and judgment of those engineering, designing, and constructing the dam, in order that the decision of the supervisor may be unquestionably sound. State authority would accept no responsibility in case of a failure even if the structure had received its approval. Consequently, as authority and responsibility cannot be associated in such cases, authority must associate with itself unquestioned technique, experience, and judgment.

It is unfortunate that the compensation allowed engineering officers and consulting engineers in the service of the State imposes a hardship upon men of the best type when they forsake private practice for public appointments. It is even more difficult for engineers on standing consulting boards, who, in exchange for a small fee, find themselves precluded from practicing their specialty in the State which they serve.

It is patent that such State control should be vested in one office, possibly a bureau of the Department of Engineering. Not to centralize this responsibility would result in confusion as to authority, in lapses in jurisdiction over which no authority would exist, and in divided responsibility.

BURDEN OF COST

It is proper that the applicant for a permit to construct a dam should pay something into the State Treasury to help defray the expenses of the supervisory board; however, it would seem that the public at large should also pay something toward the security which it desires and receives, just as it does in connection with any other health and safety measure, within the police powers of the State. To this end, the cost of supervision and inspection of dams and the expenses of the bureau, board, or department that exercises the authority, should be divided as nearly as possible between the State on one hand and the owners and operators of dams on the other.

Some provision should be made respecting the burden of cost for inspections and investigations in the case of complaints alleging that the person or the property of the complainant is endangered by the construction, maintenance, or operation of a dam. While every consideration should be given to meritorious complaints, the State must safeguard itself against nuisance complaints and must not permit itself to become an instrument of malicious persecution. To this end many States require deposits of complainants, returnable in the event that the complaint is well founded.

APPEAL PROVISION

There are bound to be cases where the builder of a dam would be reluctant, in the interest of safety or economy, to follow the decision of the supervisory control. It is conceivable that an individual, corporation, or municipality may actually regard the dam required to be built by the supervisory control as a less safe structure, or one for which the expenditure would be more than need be, to safeguard the public, compared with that which they propose and for which a permit might have been denied. Therefore, in order to avoid situations which would leave no alternative to those contemplating the building of dams, but to build that dam which would be approved by the State Supervisor, provision should be made in any legislation on the subject for an appeal to the Courts from the decision of the State agency, by any party in interest who may be dissatisfied with any decision. The issues on appeal should be heard at the earliest practicable time and determined summarily without a jury. The decision of a Lower Court should be subject to appeal or review as in other cases.

In addition to the right of appeal to the Courts, an alternative method of review could well be provided, namely, the right to present the questions at issue to a board of engineers acting as arbitrators, one of such board to be selected by the State Engineer, one by the builders of the dam, and a third by the first two named. The decision of this board should be final. The expense of the hearing before this board should be divided in every case between the State and the party making the reference, since the State would never make the reference in its own behalf, and, otherwise, the cost of the arbitration would fall, invariably, upon the private appellant.

EXISTING DAMS

The fact that a given structure has been standing for a period of years and during that time has successfully met unusually severe conditions, generally should be taken as conclusive evidence of satisfactory construction. However, investigation of the operating history of the many existing dams as to their performance during their lives may be desirable. Should investigation disclose structures that have been on the border line during periods of stress, or if there are found those which should be replaced because of depreciation, the owners might be called upon to show cause why such structures should not be replaced. Consequently, any legislation that may be enacted in any State looking to the supervision of dams should be broad enough to cover the inspection of existing dams and confer power on the supervisory agency to compel owners of such dams as are not now safe, or which may become unsafe, to repair or reconstruct them to any reasonable extent that may be found necessary to insure their safety.

It is probable that it would be unnecessary to inspect all the many existing dams, some of which are small and impound only small quantities of water.

Every owner of a dam, however, could well be called upon to furnish a description of it, of all works connected with it, an estimate of the water impounded, and a statement of the tributary drainage area. After such reports are received, those particular dams—the failure of which would be likely to endanger life or property—should be inspected, and if any are found unsafe the owners should be compelled to repair or reconstruct them as may be necessary. Plans for the reconstruction or modification of existing dams would be subject to the same approvals as are required for new structures.

MAINTENANCE OF DAMS

While a structure, when built, may be of satisfactory design and construction, its continuance in a good state of preservation is, of course, another matter, and while a well-constructed dam should remain in safe condition under intelligent operation and with reasonable maintenance, it would doubtless be advisable to establish a routine inspection, perhaps biennial, over all important structures. Such an inspection would operate to guard against malicious and insincere complaints respecting existing structures.

The very massiveness of earth, rock-fill, and masonry dams prompts the belief that they are of everlasting life. Lacking definite data respecting the erosive and destructive action of the elements on dam materials over long periods of time, the safe working life of such construction cannot be established as yet. It is possible that serious deterioration, more or less unsuspected, has been taking place in some of these structures built several decades ago. State supervision, due to its perpetuity, offers a continuing control over structures of this type, outlasting the lives of the designers and builders.

BUILDING CODE FOR DAMS

In the foregoing no mention has been made of a code to cover the building of dams, because it is probable that legislation will revolve around the idea of supervisory personnel rather than rules of good construction. However, in other types of structure, experience has dictated the wisdom of building codes. While it is recognized that there are many disadvantages in the codification of anything, and that a code for the building of dams would be more difficult to devise than a code for buildings, it is not impossible, should a code be considered otherwise desirable, that a competent group of engineers could fashion a comprehensive and workable one, at least as far as certain important features for the building of dams are concerned. Such a code could be administered by the department of engineering of a State desiring one. This idea suggests itself because of the possibility of outlining the conditions under which the generally accepted types of dams would be best suited. The well-known types of dams include: (*a*) Earth dam (rolled or hydraulic-fill); (*b*) rock-fill (dropped rock or rubble rock); (*c*) rock masonry; (*d*) concrete (gravity, arch, multiple-arch, or buttressed slab); (*e*) crib (generally wood and occasionally concrete); and (*f*) frame (steel, wood, or concrete). All these types have their peculiar adaptability, taking into consideration location (topography, geology, and altitude), height, availability of material, desired life, foundations, disposal of floods, and other factors.

It would be possible to prepare a code to cover the construction of dams, which would describe the conditions under which each of the types could be built, from which a builder could select one, providing the economics in the particular situation did not make the structure entirely unfeasible. If it did, nothing could be built. Codification probably would not dispose of differences of opinion, in individual cases, as to the safety of the structure which might be required by a board administering such a code. An arbitration board would be valuable here also, but it is probable that the action of such a board would be infrequently required. The code could be in the nature of a minimum requirement, which responsible builders would tend to exceed in their own interests, having in mind the protection of their investment.

This suggestion of codification is offered for what it is worth. It might prove, after consideration, that the conditions to be covered would be too diverse to permit of practicable definition in such a code.

CONCLUSION

Any one who visited the site of the St. Francis Dam after its failure could not but be profoundly impressed by the death-dealing effect of the released flood. Those who were in the path of this flood, unconscious martyrs as they were, gave their lives for the benefit of mankind. Nothing great in the world has ever been accomplished without sacrifice; it is to be hoped that the hundreds in this case have not given their lives in vain.

The loss of life through the failure of any dam is deplorable. It is not more deplorable than the loss of life which occurs through modern instruments of production and transportation. No one would consider for a moment abolishing automobiles because of the tremendous loss of life which takes place through their existence. Consequently, a disaster like that of the St. Francis Dam must not occasion unnecessary capital investments and of such an order to prohibit the conservation of water for the various uses to which it is put.

The fears which have been raised throughout the United States must not be allowed to prohibit the construction of dams. Engineers must not be stampeded because of unfortunate happenings, because they know that very high dams to impound large volumes of water can be built safely, provided proper engineering principles are used. Technique of engineering design need not cause concern because of the abundance of high-class technical knowledge and skill that is available. Common sense and judgment, however, are rare qualities, the one being inherent and the other to be had only through experience.

The general subject of dams and the kind of legislation relating to supervision and control which will safeguard the public on the one hand, and not impose conditions which would cause unnecessary delay, excessive cost, or prohibition on the other, would be a valuable subject for further study by engineers. It would perhaps be timely for such a study to include an outline of a type of uniform legislation which it would be best for the States to enact, since the conditions which it is desirable to control in the various States are not so diverse as to preclude the successful operation of such a uniform act.

NECESSITY FOR, AND PENALTIES FOR LACK OF, SUPERVISION

By M. C. Hinderlider,[4] M. Am. Soc. C. E.

Synopsis

It is not the purpose of the writer to go into the question of the economics of dam construction, nor to discuss the many intricate, albeit fascinating, questions involved in theories of dam design, but rather to endeavor to discuss the subject more from the viewpoint of the public which, after all, is the party most vitally interested in the design, construction, maintenance, and operation of dams. This superior interest arises from two causes: First, that having to do with the element of hazard; and, second, that having to do with the economic advantages to be secured through the construction and operation of storage reservoirs.

The paper contains a tabulated summary of 293 dams that have wholly or partly failed, and a classification of the causes of these failures; it also contains a general comment on existing laws in the various States, and a suggestion for framing a code for the design and construction of dams.

Fifteen items are included in the paper to indicate subjects that should, or should not, be included in State supervision; twelve items portraying the advantages and disadvantages of State control of dams, and twelve items defining the ideal qualifications of supervising officials.

Basic Need for Supervision

Every dam, regardless of its size, is in some degree a potential menace to everything below it. There is nothing so relentless in its immediate destructiveness, so uncontrollable and deadly as a huge volume of water suddenly released. The effects of cyclones, earthquakes, and even volcanoes are generally local, and their occurrence is infrequent, and such menaces usually furnish warning of their approach. Great conflagrations are subject to control by modern methods, and science has developed means for successfully combatting the ravages of diseases and epidemics; but no means will ever be contrived for overcoming the ruthless destructiveness of huge bodies of water suddenly released from restraint.

The number of recorded failures of dams, however, will doubtless compare favorably with the failures of other engineering structures similar in magnitude and in menace to life and property. Nevertheless, such failures have been all too frequent, and constitute a reflection upon the ability of the Engineering Fraternity. Fortunately, or unfortunately, the psychological result of such failures is of short duration; the effects are soon forgotten until the tragedy is re-enacted.

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[4] State Engr., Denver, Colo.

Table 1 is a summary of reported partial or complete failures, since 1799, of 293 dams located throughout the United States and in some foreign countries. Such failures represent various types ranging in height from 10 to 190 ft. The list includes 159 earth dams—9 of which were of the hydraulic-fill type; 12, rock-fill; 67, gravity masonry type; 7, single and multiple-arch; 7, reinforced concrete; 1, steel; and 7, timber; and 23 other dams of unclassified types. It includes failures of spillway, abutment, cut-off, or any other appurtenant part of a dam that may have jeopardized life or property. A critical study of these data should be made by every engineer who assumes the responsibility of designing or supervising the construction of a dam, no matter of what type or magnitude. Table 1 discloses that inadequate spillway facilities and defects in outlet works were the causes of most failures of earth dams, while foundation troubles were the principal causes contributing to the failure of masonry dams, or earth dams with masonry cores.

A tabulation and bibliography of the failure or partial failure of 293 dams of different types have been placed in the Engineering Societies Library, 33 West 39th Street, New York, N. Y. Photostatic copies may be obtained at cost.

The State of Pennsylvania heads the list with thirty-three recorded failures; twenty-six failures of dams are reported from the State of California; twenty-four from Colorado; and twenty-five from New York State. Eleven failures are reported from Connecticut; thirteen from Massachusetts; twelve from Missouri; seven from Ohio; six from each of the States of Michigan, Minnesota, and Texas; five from each of the States of New Mexico, North Carolina, Arizona, and Utah; four from each of the States of Montana, New Hampshire, New Jersey, Rhode Island, South Carolina, and Maine; three from each of the States of Georgia, Oregon, and Tennessee; and a total of thirty from all other States. The record also includes forty dam failures in foreign countries.

Doubtless this tabulation does not include all total or partial failures of dams in such States and countries, and in the absence of a complete record showing the relative percentages of failures, significance attaching to State supervision is lacking. However, in so far as the analysis is partly complete, it discloses that the largest number of failures have been in Pennsylvania, California, Colorado, New York, Wisconsin, Massachusetts, and Connecticut, all of which States have exercised supervision over the construction and maintenance of dams for many years. The largest number of failures, however, might be expected from those States which have, by far, the largest number of dams in operation. In California, divided or complete lack of State authority probably has had something to do with so many failures. Colorado, which has more than 1 000 dams under the supervision of the State Engineer, has been a pioneer in such construction, which has provided

TABLE 1.—LIST OF TOTAL, PARTIAL, AND INCIPIENT DAM FAILURES, 1799–1931, INCLUSIVE.

(Compiled December, 1931.)

Reference No.	Cause of Dam Failures.	Reference No.	Cause of Dam Failures.
1	Inadequate spillway.	8	Ice pressure or disintegrating effect of.
1(*a*)	Inadequate spillway. Overtopping by flood wave due to failure of dam above.	9	Improper operation or inadequate maintenance.
2	Inadequate cut-offs. Porous foundation allowing leakage and erosion under earth dam, and/or sliding in rigid types.	10	Burrowing rodents.
3	Faulty construction; material not properly compacted in earth dams; poor construction in masonry dams.	11	Poor materials, including soluble salts.
4	Inadequate cut-offs around conduits in earth and rock-fill dams.	12	Unstable or structurally weak foundation.
5	Faulty design of section; slopes too steep in earth dams; section too light in masonry dams.	13	Conduits through earth or rock-fill dams not properly supported to prevent settlement or failure. Improper location of valves.
6	Inadequate means for stream control during construction.	14	Insufficient provision against erosion from back-wash below dam or spillway.
7	Excessive quantities of clay or other classes of fine material.	15	Earthquakes.
		16	Miscellaneous and undetermined.
		17	Failure of bottom in small water-works reservoirs.

SUMMATION OF DAM FAILURES FROM DETAILED LIST WHICH FOLLOWS. NUMBERS IN PARENTHESES DENOTE PERCENTAGES OF THE TOTAL IN EACH CASE.

Type.	Height, in feet.	NUMBER OF FAILURES UNDER EACH CAUSE. REFERENCE NUMBER OF CAUSE REFERS TO ABOVE LIST.																		Total, percentage.	
		1.	1(*a*).	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.		
Earth............	0– 25	15	4	4	1	5	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	4	.....	34	(21)
	25– 50	16	1	5	.....	5	2	2	2	.....	1	.....	.....	.....	.....	1	1	1	.....	37	(23)
	50– 75	3	.....	5	4	2	3	2	2	.....	.....	1	.....	.....	.....	1	.....	3	.....	26	(16)
	75–100	.....	.....	.....	.....	1	.....	.....	2	.....	.....	.....	.....	.....	.....	.....	1	2	.....	6	(4)
	100–125	.....	.....	.....	1	.....	1	1	1	.....	.....	.....	1	.....	.....	.....	.....	1	.....	6	(4)
	125–150	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	1	1	.....	.....	.....	.....	3	(2)
	150+	.....	.....	.....	1	.....	.....	.....	2	.....	.....	.....	.....	.....	.....	.....	.....	2	.....	5	(3)
	Not given	10	3	4	1	5	.....	.....	2	.....	1	1	2	.....	1	.....	1	5	6	42	(27)
Total............................		44	8	18	8	18	6	5	12	0	2	2	3	1	2	3	3	18	6	159	.....
Percentage............................		(28)	(5)	(11)	(5)	(11)	(4)	(3)	(8)	(0)	(1)	(1)	(2)	(1)	(1)	(2)	(2)	(11)	(4)	(100)	

TABLE 1.—LIST OF TOTAL, PARTIAL, AND INCIPIENT DAM FAILURES.—(*Continued.*)

Type.	Height, in feet.	Number of Failures Under Each Cause. Reference Number of Cause Refers to Above List.																		Total, percentage.	
		1.	1(*a*).	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.		
Rock-fill	0– 25			1			1											1		3	(25)
	25– 50	2										1								3	(25)
	50– 75				1															1	(8)
	75–100																			0	(0)
	100–125	1			1															2	(17)
	125–150	1																		1	(8)
	150+							1							1					2	(17)
Total		4	0	1	2	0	1	1	0	0	0	1	0	0	1	0	0	1	0	12	
Percentage		(33)	(0)	(8½)	(17)	(0)	(8)	(8½)	(0)	(0)	(0)	(8)	(0)	(0)	(8½)	(0)	(0)	(8½)	(0)	(100)	
Masonry gravity type	0– 25			7	2		3			5	1							3		21	(31)
	25– 50	2	1	3	5		4			1	1	1						2		20	(30)
	50– 75			2	1									1				2		6	(9)
	75–100	1		1																2	(3)
	100–125	2																		2	(3)
	125–150						1											1		2	(3)
	150–200			1																1	(1½)
	200+							1												1	(1½)
	Not given	1		7				1						1		1		1		12	(18)
Total		6	1	21	8	0	8	2	0	6	2	1	0	2	0	1	0	9	0	67	
Percentage		(9)	(1½)	(31)	(12)	(0)	(12)	(3)	(0)	(9)	(3)	(1½)	(0)	(3)	(0)	(1½)	(0)	(13½)	(0)	(100)	

TABLE 1.—LIST OF TOTAL, PARTIAL, AND INCIPIENT DAM FAILURES.—(*Continued.*)

Type.	Height, in feet.	NUMBER OF FAILURES UNDER EACH CAUSE. REFERENCE NUMBER OF CAUSE REFERS TO ABOVE LIST																		Total percentage.	
		1.	1(*a*).	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.		
Arch—single and multiple	0–50 50–100 100–150 150+				 1 1 		 1			1 1 				 2 						1 4 1 1	(14) (57) (14½) (14½)
Total		0	0	0	2	0	1	0	0	2	0	0	0	2	0	0	0	0	0	7	......
Percentage		(0)	(0)	(0)	(28½)	(0)	(14½)	(0)	(0)	(28½)	(0)	(0)	(0)	(28½)	(0)	(0)	(0)	(0)	(0)	(100)	......
Reinforced concrete	15–51	(0)	(0)	6 (86)	(0)	(0)	(0)	(0)	(0)	(0)	1 (14)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	7 (100)	......
Steel	70	(0)	(0)	1 (100)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	1 (100)	......
Timber	1 dam, 100 16 dams, less than 100	2 (12)	(0)	4 (23)	5 (29)	(0)	(0)	1 (6)	(0)	(0)	2 (12)	(0)	(0)	(0)	(0)	(0)	(0)	3 (18)	(0)	17 (100)	......
Not given	Not given	2 (9)	(0)	1 (4)	1 (4)	(0)	1 (5)	(0)	(0)	1 (4)	(0)	(0)	(0)	1 (4)	(0)	(0)	(0)	16 (70)	(0)	23 (100)	......
Grand total		58	9	52	26	18	17	9	12	9	7	4	3	6	3	4	3	47	6	293	......
Percentage		(20)	(3)	(18)	(9)	(6)	(6)	(3)	(4)	(3)	(3)	(1)	(1)	(2)	(1)	(1)	(1)	(16)	(2)	(100)	......

ample opportunity for mistakes. However, with the exception of the failure of two small earth dams, each 17 ft. in height, no failures have occurred in the State of Colorado in the eight years since 1923.

Every dam is in some degree a menace to life and property, yet these structures are indispensable to the welfare of the people. As such, their safety is a matter of such concern to the general public as to bring their supervision properly within the scope of the police prerogatives inherent in all civilized nations.

Legislative Trends

A digest of the laws of all the States, and the principal countries of the Old World, pertaining to dam design, as might be expected, strongly indicates the close relation between the economic importance attaching to the regulation of water supplies of such countries, and the degree of public supervision exercised over the design and construction of dams therein.

This police power or authority is defined[5] as "the power vested in the legislatures to make such laws as they shall judge to be for the good of the Commonwealth and its subjects. The power to govern men and things, extending to the protection of lives, limbs, health, comfort, and quiet of all persons and the protection of all property within the State." The exercise of this power of a State or Nation to protect the health and lives of its citizens from all kinds of evils and dangers has spread rapidly during the last fifty years.

Such police powers are very broad, as they must necessarily be in order that the objective sought shall be attained. Such protection is thrown around the citizens of practically all civilized countries and extends to factories, mines, packing plants, dispensaries, transportation systems, buildings and structures for whatever purpose used, and within comparatively recent times, to lines of agricultural development, food supplies, and for the control and eradication of contagious and infectious diseases, and, in fact, wherever the safety and well-being of the citizen and his property is beyond his individual ability to insure. Such supervision on the part of a State is the gradual outgrowth resulting from increase in population and the complexities incident to the development of civilizations the world over. It is a function of Government which is becoming increasingly great and of apparent necessity in the evolution of the social, economic, and commercial development of a nation. Its beneficial aspects are readily recognized, although having the effect of subordinating individual liberty in the greater interest of public welfare.

Public supervision over the building of dams is exercised in some degree in most of the States. The degree of such supervision is found to be generally in the proportion that regulation of stream flow has progressed in any particular State. The centralization or co-ordination of such public control varies widely in many of the States and in foreign countries as well. Centralized control is being more and more recognized as a public necessity. Such tendency is evidenced in the States of Pennsylvania, New York, California, and Colorado, which report the largest number of dams. Until 1928, public supervision of dams in California was vested in four State and Federal Departments; in addition, municipalities under certain conditions were exempt from any public supervision whatever.

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[5] Bouvier's Law Dictionary.

For many years the supervision of the design, construction, maintenance, and operation of all dams in Colorado more than 10 ft. in height, has been vested exclusively in the State Engineer, whose decisions in such matters are practically final.

In matters of such vital concern as the safety of a dam, even of nominal height, theories and ideas of a single mind should not be considered conclusive. Regardless of the extent of the study and experience of the individual engineer or geologist, each has had different problems to meet and overcome in traveling the rough road of a professional career. It is the well co-ordinated results of such schooling that contributes in the greatest degree to the safety of the public. The conditions surrounding the design, location, and construction of every dam, has its own peculiarities which should be studied, analyzed, and weighed from all points of view and under all the probable conditions under which the structure is likely to function. It is rarely possible to find in a single individual a combination of experience involving full knowledge of design, construction methods, and materials, geology, hydrography, effects of temperature changes, disintegration, and the many other elements which contribute to the safety or weakness of a dam. The writer is convinced that the average practicing civil or hydraulic engineer is generally unfamiliar with many of the fundamental requirements essential to the rational design, supervision of construction, and maintenance of such structures, and that the usual "factor of safety" in design customarily adopted is indeed a saving factor to those whose lives or property may be placed in jeopardy by such structures.

In the interest of public welfare, many States have legalized the practice of engineering in its various branches by statutory provision, but, unfortunately, such provisions fall short of affording the degree of protection contemplated. Contrary to a reasonable assumption on the part of the general public, the reverse may even be true. With rare exceptions has the writer felt justified in approving either the plans or specifications for dams built in Colorado until they had been revised in whole or in part, and it is presumed that a similar condition may exist in other States. Just how far the authority of the approving official should extend in indicating the type of dam and appurtenant structures, character of construction, etc., is a mooted question.

In Colorado, the State Engineer follows the practice of not only suggesting the type of structure indicated by the site and geological formation, run-off conditions, availability of construction materials, etc., but for the purpose of conserving the time of, and expense to, the owner, he recommends to the engineer that a preliminary draft of his plans and specifications be submitted for criticism prior to the preparation of the final draft for approval. This plan has seemed to work well in practice. Study and practice relating to the design of practically all conservative types of dams have resulted in the development of principles generally recognized as being sound, although there is still some diversity of opinion with respect to important theories affecting the safety of huge structures of the gravity and lighter-than-gravity types of masonry dams. This is to be expected since much remains to be learned concerning the physical and chemical properties of building materials, their behavior under combinations of loading, temperature changes, chemical reactions, and variations in construction details, all of which are impossible of complete control.

As in the evolution of the development of all the arts and sciences, the proof of a theory is largely dependent upon its successful application in practice. In the earlier history of dam construction, such structures were usually built of materials to which scientific principles, even, if they had been understood, were impossible of application. In general, the evolution of scientific knowledge in the field of engineering has not provided a greater degree of safety in the construction of dams of modern height, but it has vastly increased the range, not only in the magnitude of such structures, but in the ability of the engineer to overcome the deficiencies of Nature by the adaptation of proper types of structure, to any particular condition.

Doubtless, the greatest value attaching to present-day knowledge of the principles of dam design and construction, is one of economics. Such knowledge has made it possible to utilize storage sites formerly considered infeasible, and through such use of basins of great capacity, stream regulation upon a large scale has been made feasible. Under the urge for development of natural resources the world over, structures of ever-increasing magnitude are coming into existence and are no sooner completed than even larger ones are conceived and planned for the future. The engineer is called upon to meet this urge in a way that bids fair to tax to the limit his ingenuity, good judgment, and conservatism. He must of necessity be something of a dreamer, or he would be unable to originate new ideas and evolve the great undertakings demanded by modern civilization. He is inherently a conceiver and builder. He is generally an optimist, at times prone to allow himself to be unduly influenced by a desire to achieve great undertakings, sometimes under conditions in which safety to life and property is dependent upon his good judgment. Probably in no field of endeavor are the responsibilities to the public so great or exacting as in that of the design, construction, and supervision of structures for the impounding of large bodies of water. It is well, therefore, that those charged with such obligations should proceed with caution, be guided by a high degree of conservatism, and withal imbued with an exalted sense of their responsibility. The public is prone to defer to the judgment of those, presumably competent, to look after its interests, and to go about its own business of fighting the battles or seeking the pleasures of life. It is lulled into a sense of security through its aversion to being bothered, until some great catastrophe is precipitated upon it, and then it usually acts under the stimulus of stress and hysteria.

The responsibility resting upon a public official in charge of dams is indeed a heavy one, not only as a result of his approval of the plans for such structures, but because of possible internal weaknesses in dams with which he may have had no former connection. Fortunately, as in the case of human diseases, certain symptoms usually warn the careful observer of trouble ahead, and therein lies one of the best arguments for centralized responsible public supervision over such structures, which enables some one to act promptly and with authority.

A review of the statutes of all the forty-eight States, including the District of Columbia, discloses that, with the exception of Delaware, Louisiana, Minnesota, Nebraska, Oklahoma, and the District of Columbia, they all exercise some form of public supervision over dams, ranging from that relating to damages which may result from the action of back-water above a dam, and the imposing of regulations in the interest of fish propagation, navigation, and sanitation, to the higher purposes of protection to life, property, and industries which may be placed in jeopardy. Supervision is vested in various agencies, ranging from Courts, municipalities, commissions, and counties, to the State, and departments of the National Government. As is to be expected, the degree of such regulation increases with the need for dams in the economic and commercial development of the State or country. In many of the States where the topography of the country and gradient of the streams are conducive to power development in only a minor way, the need for public regulation is not so urgent, while in those States and countries in which conditions and demands are favorable to the greatest utilization of their water resources, will generally be found the greatest degree of public supervision.

The aforementioned review of the laws of the forty-eight States disclosed that, in nine States, supervision of dams is vested in the Courts; in six States, in the Board of County Commissioners or Supervisors; in fourteen States, in State Commissions or Boards; and in fourteen States, it is placed in the hands of a State Engineer or his equivalent. In several of the Public Lands States, some Government agency, such as the Bureau of Reclamation, Federal Power Commission, Débris Commission, Indian Service, or Forestry Department, exercises partial or joint supervision over the design, and, at times, over the administration of dams. The Army Engineers exercise supervision over all dams where navigation is affected, or in the interest of the protection of post routes.

Public supervision extends to structures for practically all purposes, and of all heights, ranging from 5 ft. upward. Supervision relates in many States not so much to the safety features of such structures as it does to matters of navigation, power development, stream pollution, and flood control. From the standpoint of safety, supervision is usually exercised by the State in which the structure is located, although, in several States, dams built and operated by the Federal Government are exempt from State supervision.

Analysis of the laws of all the States discloses that inspection and supervision over dams in the interest of public safety is exercised only in nineteen of them. Of these, fourteen have regulatory laws which may be considered reasonably effective. In only twelve of the States may existing laws relating to State supervision be considered ample for the safguarding of the public's interest where dams of considerable magnitude are contemplated. In more than one-half of all the States there are either no laws relating to public control of dams which may be considered a menace to life and property, or such laws as exist are for the minor purposes heretofore mentioned. Accordingly, it would appear desirable that in such States careful study of this subject of public supervision of dams should be made. Practically all States of the Rocky Mountain region and the Pacific Coast, have quite effective laws relating to this most important matter. This condition also applies to several of the Atlantic Seaboard States, such as New York, New Jersey, Pennsylvania, Maine, New Hampshire, Vermont, Connecticut, and Rhode Island.

THE ENGINEER'S RESPONSIBILITY AS REGARDS LEGISLATION

Legislation affecting the construction of dams invites the close attention of engineers. It is of far-reaching importance and should not be passed preliminary to careful and mature consideration of the objectives to be attained and the difficulties to be overcome. The engineer, who is trained in the theory of stresses and strains and the characteristics and frailties of the materials with which he must work, is fully appreciative of the real magnitude of such undertakings and of the tremendous responsibilities they represent. Activity on the part of engineers, who by study and experience are justified to speak on this important subject, is a public service which the engineer could, and should, render. No one is so qualified as he, to point out the dangers and to indicate the way.

Some of the disadvantages resulting from public supervision over the design and construction of dams are attributable to the fact that much of such legislation was enacted without adequate information, or that it is the result of sporadic attempts to correct, from time to time, deficiencies in previous legislation. This has frequently resulted in contradictions, ambiguities, or complete lack of some essential, and too often has resulted in the objectionable features of divided authority. Consequently, the laws of many States relating to this most important subject are a "hodge podge," making for inefficiency in their administration, with the resultant unwarranted sense of security which the public has every right to expect under State supervision. The success of any law depends largely upon the degree of public approval back of it, and upon the confidence reposed in the official charged with the duty of administration.

It is also apparent that the beneficial effects of any law are measured by the degree of efficiency with which such law may be executed or administered and, regardless of the safeguard which a law may seek to establish, the effectiveness will depend very largely upon its enforcement. It is just as apparent also that the actions of a public official are circumscribed by the limitations and weaknesses of the law which he is charged with enforcing. The first requisite, therefore, are laws as nearly ideal as may be, for the attainment of the desired objective. Hence, the desirability that laws relating to the design and construction of dams should reflect the best engineering thought and experience.

CODES FOR DESIGN AND CONSTRUCTION OF DAMS

A comprehensive code covering the design and construction of dams may well be included in such studies. It is realized that constitutional provisions in many States may render the application of a general code difficult, if not impossible, but doubtless it would be invaluable as a background for the framing of needful legislation applicable to conditions in some particular State or country.

It is believed that fundamental principles relating to stresses, strains, reactions, and the behavior of materials used in the construction of masonry dams of various types, and modern methods relating to construction, are sufficiently well understood and demonstrated, to justify the formulation of a code of procedure which would receive the approval of the great majority of engineers familiar with this subject. A uniformity of practice based upon such a code would go a long way toward minimizing the number of failures of dams, which would certainly justify the endorsement of the profession. No type of structure involves the degree of hazard to the public as that of storage dams. The public is entitled to every consideration in this regard, but can obtain the necessary degree of protection only through proper laws, which should embody the consensus of the best thought and experience available. Within recent years, and as a result of intensive studies by the constituent bodies of Engineering Foundation and other similar organizations, uniformity of practice and procedure, with respect to building materials, many types of structures, and related matters, has been adopted and recognized in almost all lines of engineering endeavor. A code of requirements relating to the art of dam building would be invaluable in the drafting of legislation providing for public supervision over the design and construction of dams.

A code[6] proposed by the late Fred A. Noetzli, M. Am. Soc. C. E., provides an excellent foundation upon which to predicate a practical and effective set of rules which, with certain additions or modifications, could be embodied in future legislation on the subject of dam construction.

Of scarcely less importance in the public interest, is the matter of inspection and maintenance of dams. The evil effects of faulty design and construction may generally be met successfully by constant and careful inspection, operation, and maintenance, even under trying conditions. The authority of the public official in this regard covers many acts which must necessarily be of a discretionary nature. Such acts directly affect the property rights of the owner and, at all times, the degree of safety to lives and property below a dam. Just what depth of water it may be safe to store back of a dam under certain given conditions, or how long such storage should be permitted in the case of earth dams, or what margin of free-board should be required on many dams (particularly during periods of high wind velocities peculiar to the arid regions), the proper size of spillway to maintain, and many similar items are problems which continually arise to perplex the administrative official. He is charged with the duty of safeguarding life and property below a dam, and, at the same time, he must recognize and permit the maximum utilization of the investment represented by such structure.

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[6] *Western Construction News*, September, 1929.

Since the foregoing problems are quite complex in their nature and are continually in a state of flux, it does not appear practicable to formulate a code of rules or laws applicable to the maintenance or operation of dams except along general lines. The effectiveness of administration very largely depends upon the authority, experience, and sense of duty of the public official, and, therefore, cannot be fixed by any given set of regulations.

The Act[7] passed by the Legislature of California in 1929, providing for State supervision over all dams other than those built by the United States Government, is an excellent piece of legislation on this subject. The California law is the result of much study and discussion and, with minor exceptions, will doubtless be found to be highly effective and practical in its application. This law is largely the result of the crystallization of public opinion occasioned by the St. Francis Dam disaster and reflects quite generally the most modern thought on the subject of State supervision of dams.

QUALIFICATIONS OF SUPERVISING OFFICIALS

The qualifications of the official charged with the important duties of State supervision of dams should include:

(*A*) An understanding of the fundamental principles affecting the stability of all types of dams. (This requirement, in general, pre-supposes that he is a technical graduate or, because of previous preparation, is competent to understand the mathematics of engineering design.)

(*B*) An appreciation of the advantages of the science of geology and its adaptation to the art of dam building. (He should also be thoroughly grounded in the knowledge of climatology and hydrography.)

(*C*) Broad experience in the problems of construction (which is conducive to a discriminative sense of values and dangers), and of materials, their properties, and uses.

(*D*) A willingness to improve his knowledge of the art by consulting with and studying the methods and theories of others. (He should be imbued with an honest desire to learn the lessons taught by the failures of the past.)

(*E*) An exalted sense of his duty to the public and the ability to visualize the grave responsibilities with which he is charged.

(*F*) Complete freedom from any influences other than the motivating ones of his official duties.

(*G*) Tenacity of purpose.

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[7] Chapter 766, Laws of 1928–29.

(*H*) An appreciation of the value of systematic methods in the conduct of his office and the essential need for reliable data at all times.

(*I*) Progressive ideas, albeit leavened with that sense of responsibility which his duties impose.

(*J*) The confidence of his fellow engineers and the public.

(*K*) A genuine desire to co-operate, but the stability of purpose not to allow his enthusiasm to get the better of his sober judgment; and, finally,

(*L*) That rare combination of natural ability and experience commonly known as "horse sense."

ITEMIZED CONCLUSIONS

The following suggestions are offered by the writer with the hope that they may elicit further discussion of this most important subject of public supervision of the design and construction of dams.

A law for State supervision of dams should include authority to:

1.—Pass on the design, construction, maintenance, and operation of all dams and reservoirs of a certain minimum height and capacity.

2.—Require (preliminary to beginning of construction), complete data on the geology, topography, rainfall and run-off characteristics of the drainage basin, samples of materials from foundations and those to be used in construction, and all other information needful for a proper determination of the suitability of the dam site.

3.—Require complete plans, specifications, and analyses of design, and also authority to modify such plans and specifications prior to and during the progress of construction when conditions would seem to justify such changes.

4.—Require continuous inspection of new construction and all materials entering into it, preferably at the expense of the State.

5.—Employ consulting engineers and geologists either upon the authority of the State, or upon the request of the owner, such authority to apply not only to contemplated construction and dams under construction, but also to all existing dams.

6.—Exercise supervisory control during construction and operation of the dam following construction.

7.—Regulate at all times the quantity of water stored back of any dam.

8.—Require all dams to be maintained in a safe condition.

9.—Enforce all provisions of the law with respect to the construction, operation, and maintenance of dams, with recourse to the Courts only under exceptional conditions which would require a prompt determination of the matter by the Court.

10.—Provision for ample funds to carry out State supervision as herein noted.

State supervision should not include:

I.—Financial obligation on the part of the State in case of failures.

II.—Imposition of unnecessarily drastic regulations not essential for safety.

III.—Unnecessary interference with methods of construction.

IV.—Authority to assume the functions of the practicing engineer with respect to the preparation of plans and specifications and related data required in connection with the design and construction of a dam.

V.—Authority for State supervision in too minute detail. In regard to Item V, State supervision should be limited to the approval or disapproval of the plans and data presented, and the authority to require essential amendments thereto, which, in the judgment of the State official, appear to be needful for insuring the proper degree of safety, and to provide for adequate inspection during the period of construction.

State control of the design, construction, and maintenance of dams has the following advantages:

(*a*) Centralized authority with its singleness of purpose and responsibility to the public.

(*b*) Minimum of effects of local and political influences.

(*c*) Prompt and concerted action in anticipation of dangers and the forestalling of failures under most conditions.

(*d*) Uniformity of procedure in design and inspection.

(*e*) Greater assurance of disinterested inspection.

(*f*) Co-ordinated control in administration of the uses of stored water and stream flow, especially in arid regions.

(*g*) Continuity and permanency of records and public accessibility to them.

It has the following disadvantages:

(*h*) Danger of incompetence of, or prejudice on part of, State official.

(*i*) Lessening of the sense of responsibility and precaution on part of owner.

(*j*) Possibility of injustices to owner resulting from unwarranted requirements of State official.

(*k*) Erroneous assumption on the part of property owners below a dam that State supervision is tantamount to State responsibility in a pecuniary way.

(*l*) Dangers accompanying increase in bureaucratic authority.

DISCUSSION

H. de B. Parsons,[8] M. Am. Soc. C. E. (by letter).—If there is to be State supervision of the design and construction of dams, it is the writer's opinion that the laws, enacted by the several States, should be uniform, or as nearly alike as may be possible. Engineers are the ones best qualified to furnish information for legislation that will be effective and not be burdensome. As the art of designing dams is in a progressive state, there should be no statutory limitations on design or method of construction. The writer does not favor a building code, as it probably would increase the cost of construction without providing the protection intended.

The following suggestion for supervision is offered for criticism. The basic idea is akin to arbitration, which has been proven fair, quick, and inexpensive, and has the approval of the Courts. The act should define the "owner" as the individual, corporation, or public body desiring to erect a dam, and the "State Engineer" (or a State bureau or department), as the representative of the general public; and should recite, in a clear manner, that approval does not convey any State guaranty of the safety of a dam, but that the public welfare has been safeguarded by bringing outside judgment to study the development.

The "owner" should apply to the State Engineer (or to the equivalent State bureau or department) for permission to construct. This application should be referred to a supervisory board composed of three commissioners, the first being appointed by the State Engineer, the second by the appellant (the engineers responsible for the design of the dam or development should not be eligible), and the third selected by the first two. The third commissioner should be the chairman and an engineer having no connection, direct or indirect, with contracting. Should a State be the "owner," then the State would be the appellant and should apply to the highest Court having jurisdiction, which Court should appoint the first commissioner. In case of disability, any vacancy should be filled by the appointment of another commissioner in accordance with the procedure outlined.

The "owner" should submit complete plans, with specifications and all his data regarding flood flows, rainfall, geologic formation, and borings. Additional data should be filed by the "owner" if requested by the board, provided such data are obtainable. The board should be empowered to retain a geologist, but must do so for all important or high dams, and for those in earthquake or volcanic localities. An important dam could be defined briefly as a dam located above and near enough to habitations or to cultivated or developed lands, that a complete failure would cause the risk of loss of life or heavy property damage. Similarly, a high dam could be defined as a dam, of any form, more than 100 ft. in height, above the bed of a stream or valley, which impounds, say, 6 000 acre-ft., or more. The decision of the board, in which two of the commissioners shall concur, should be made promptly, and a maximum time limit could be fixed by the act, with a proviso for an extension. The decision should be limited to either an approval or a rejection of the plan (including foundation) submitted. The commissioners' report should not make recommendations, but should simply state the reasons for approval or rejection.

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[8] **Prof. Emeritus, Rensselaer Polytechnic Inst.; Cons. Engr., New York, N. Y.**

The total cost of the board, including all fees, traveling expenses, and disbursements, should be approved by the State Comptroller, and then divided equally between the "owner" and the State. If the State is the "owner," then the entire cost should be paid by the State. If more than one State is affected, then the States should divide the cost *pro rata* as their interests may appear.

Either party should have the right to appeal from the decision of the supervisory board. The appeal should be to the highest Court having jurisdiction. This Court should hear arguments on all legal questions and render its decision, but should refer all technical questions of engineering, scientific data, or facts to an appeal board, composed of three commissioners to be appointed by the Court. The Court should select one from a list of at least two named by the "owner," and one from at least two named by the State Engineer. These two should submit a list of at least two (or more if asked by the Court) from which the Court should select the third commissioner. The act should fix the time in which this board should file its report, with a proviso for an extension; and also should provide for filling a vacancy through the disability of a commissioner. The appeal board should have power to retain its own geologist. The decision of this board, in which two of the commissioners shall concur, should be accepted by the Court as its decision on all questions of engineering and scientific fact. The cost of the appeal should be paid by the party losing the appeal.

State supervision for the construction of dams could be made a function of the State Engineer, or the equivalent bureau or department. All matters in dispute could be referred to the supervisory board for settlement, with the right of appeal as described in the foregoing remarks. If deemed advisable, the "owner" could be made to file at the State Engineer's office, on completion of the work, a revised set of plans, with specifications, showing the dam as actually constructed.

The writer concurs with Mr. Markwart that there should be no exceptions because of ownership, except for unimportant structures the failure of which would not involve loss of life or property. Dams built by the departments and bureaus of the Federal Government, when erected on State lands, should not be exempted, if existing laws will so permit. All dams should be treated alike. Federal or State engineers are no more infallible, as Mr. Markwart states, than municipal, corporate, or private engineers.

The writer submits this suggestion for criticism and betterment. He believes, if State supervision is adopted, that legislation as briefly outlined herein would tend to secure a supervisory personnel that would be, as nearly as practicable, the equal in technical knowledge of those responsible for the design and construction of a great dam.

WILLIAM P. CREAGER,[9] M. AM. SOC. C. E. (by letter).—Messrs. Markwart and Hinderlider have covered the subject so completely as to leave little opportunity for further constructive data. There remains, then, only the opportunity for an expression of opinion in the matter of details. The writer thoroughly agrees with Mr. Markwart's statement under "Burden of Cost" that,

> "While every consideration should be given to meritorious complaints [of danger], the State must safeguard itself against nuisance complaints and must not permit itself to become an instrument of malicious persecution."

However, his next statement to the effect that "to this end many States require deposits of complainants, returnable * * *," if taken literally would seem to indicate that, in such cases, the possibility of a property owner obtaining immediate and adequate police protection in this respect is contingent upon his possession of liquid assets. The only cases, to the writer's knowledge, in which such deposits are required are when the complaint, after adequate gratuitous investigation by the State agency, is considered by it not to be meritorious and the complainant demands further investigation or inspection.

It is the evident desire of all laws governing the supervision of dams to provide for State approval in all cases where failure would involve loss of life and property and to exempt those which do not. It is fallacious to make the assumption, as has been done in many statutes, that a dam having a certain height or a certain size of pond is on the border line between peril and security, omitting entirely the consideration of length and other important influences.

The damage done by a flood is a function of the velocity, the depth, and the duration of the discharge. The first two factors, ordinarily, have the preponderating influence on the extent of destruction. For a given channel below the dam, the velocity and depth are determined by the rate of discharge. The rate of discharge is governed not only by the height of the dam, but also by the width of the break, which is frequently the length of the dam. Furthermore, in addition to the characteristics of the dam, the degree of utilization and occupancy of the valley below it has, in many cases, a great influence. It seems difficult, if not impossible, therefore, to fix definite limits on the size of dams not subject to State approval.

If thought to be feasible, it might be desirable to require the approval of all dams of any size or nature, or perhaps those limited to an unusually small minimum size; but with the provision that, if in the opinion of the owner the failure could not possibly cause loss of life or property, a preliminary application could be presented accompanied only by sketches and written descriptions. The State agency would then notify the owner whether or not a final application in prescribed form would be required.

The writer believes that, with the provision of appeal only to the Courts from the decision of the State agency, Mr. Markwart's hearing "at the earliest possible time" would ordinarily not be soon enough. The writer is inclined to favor his alternative provision for presentation of the case to a board of engineering arbitrators. However, there still remains the question as to whether the provision of an appeal to the Courts from the decision of the board would not be justified.

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[9] Chf. Engr., The Power Corporation of New York, Buffalo, N. Y.

If it could be proved to be practicable, the writer is very much in favor of a code. However, it is a lamentable fact that the divergence of opinion among engineers respecting the proper design of dams is so great as to preclude the possibility of universal agreement. On the other hand, without a code for the guidance of the State agency, the requirements for approval of design might be altered radically at each change in tenure of such agency. This condition might seriously affect the possibility of making accurate estimates of cost.

A complete code in the sense used in building construction would require several hundred pages. In fact, it would be a treatise on the design and construction of dams, with explanations and proofs of theory omitted. The new California regulations allot about three pages to "General Information" which is, in effect, a miniature code. One cannot deny that it is a help, but to such an insignificant degree as to cause the writer to wonder why it was included. It is believed that a code will not be feasible until engineers have standardized the theory of design to a much greater extent.

The writer is of the opinion that there will be little or no opposition to State regulation in all States. Owners of dams should welcome the opportunity for an independent review of their engineering designs at a cost much less than is customarily paid for consulting services.

M. M. O'SHAUGHNESSY,[10] M. AM. SOC. C. E. (by letter).—The St. Francis Dam disaster in Southern California was a tragic event, which has created an hysteria on dam construction in California and the West. This catastrophe was of such an appalling extent that citizens have practically lost their heads on the subject of control of dam design and construction and have enlisted the aid of the Legislature of the State of California to pass a most drastic law covering the subject.

The writer was personally opposed to the passage of this law in the interest of the City of San Francisco, because he did not desire to see State authorities intrude themselves into local projects. On October 3, 1928, he protested to the State Engineer against this legislation, making the statement that after every disaster to a dam there is great hysteria among the newspapers and the public to vest supreme authority in some outside source so as to guarantee the safety of all future structures. The great problem is to find that "some one" who has gathered enough wisdom from his experience and who has adequate force, technical knowledge, and authority to fill the job. The commission of sins regarding dam building will not be cured by delivering the problem into the hands of young Civil Service employees who may pass adequate examinations after coming freshly from universities, qualifying them to act as aides to the State Engineer.

The amended Act controlling dam construction in California has adequate provision for supervising this work. In some ways the Act is unfair because the owner of the property may not submit his engineer, as a member of the supervising board vested with authority, to pass on the engineering structure. That responsibility is entirely assumed by the State Engineer and his assistants. Hence, unless the field of the State Engineer is well established, engineers on dam construction are precluded from rendering valuable service in designing large industrial projects.

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[10] Cons. Engr., San Francisco Public Utilities Comm., San Francisco, Calif.

Mr. Hinderlider's recommendation for a revision of the code will safeguard the construction of dams. Due care should be exercised in formulating this code because a very unhappy solution might result if an undesirable code is adopted, such as the Italian regulations governing the construction of rock-fill dams. The suggestion that engineers should study the problem of directing public opinion along sane lines that would lead to constructive measures for their protection, is a good one.

The Society is indebted to Mr. Markwart for his clear exposition of the problem and also to Mr. Hinderlider for the exposition of practice in Colorado. The Society, through the engineering press, should not hesitate to expose the construction conditions underlying all dam failures, as it is only by an intense study of those disasters that safety in all future dam building will be developed. The best account of dam failures that the writer has seen is included in a paper by Mr. F. C. Uren, entitled "The Design of Reservoir Dams, with Some Accounts of Failures."[11]

It is not unlikely that defective dams are fewer in number than defective buildings. On being filled with water they are very quickly brought to a supreme test of endurance and their weaknesses exposed, whereas defective buildings may survive a long time without being discovered. Great credit is due to the American engineers in the past thirty years who have developed the art of safe building of high dams with real fortitude and discretion.

N. A. ECKART,[12] M. AM. SOC. C. E. (by letter).—The subject of this Symposium is one that should induce a widespread discussion, because the Engineering Profession is somewhat divided as to the desirability of having the supervision and authority over the design, construction, and operation of all dams in a State vested in a public official or body. Those who advocate State supervision will differ among themselves as to the extent of the authority to be vested in that official, and those who oppose it will express many varied and worthy arguments therefor.

Much of the value of Mr. Hinderlider's paper lies in the fact that the author has set forth concisely his views as to what State supervision should include, its advantages and disadvantages, and the qualifications which the official charged with supervisory control should possess.

The question as to whether or not supervision of dams should be vested in the State Engineer, or other public official, is not one which will be decided by the Engineering Profession, and any discussion of that phase of the subject would be more or less academic. The right of the State to vest supervision and control of dams in the State Engineer can not be questioned as a part of its police power looking to the public safety, and legislation providing for such supervision no doubt in most instances is the result of a public demand following some disaster.

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[11] *Transactions*, Institution of Civ. Engrs. of Ireland, Vol. XXXVI, 1910, p. 52.

[12] Gen. Mgr., San Francisco Water Dept., San Francisco, Calif.

The Engineering Profession, however, is deeply concerned in the form such legislation will take, the extent of the authority vested in the State official, the manner in which the authority is exercised, and the procedure involved, but even more than in the form of legislation practising engineers are interested in the individual who is chosen to exercise this power, not only as to his qualifications from the viewpoint of training and technical ability, but even more as to his executive ability, the broadness of his view, and the soundness of his judgment, or, as Mr. Hinderlider terms it, the amount of his "horse sense." The right man in charge of State supervision can make almost any legislation workable and can secure the desired results in soundness of design and construction, with the minimum of interference with the work of competent engineers. The wrong man can play havoc with the best engineering plans, construction methods, and program; petty interference, incompetence, indecision, or delay in approving proper plans or foundations, may add hundreds of thousands of dollars to the cost of a project, as the delay of a few weeks at some stage may result in throwing the work into the flood period and possibly may cause the loss of a season's run-off. In an Eastern State, approval of the design for an arch dam was withheld, pending the publication of the report of Engineering Foundation on the "test dam" at Stevenson Creek.

The California law has not been in effect long enough to permit judgment to be passed as to the workability of the procedure, but it has gone into effect under the most favorable auspices as to personnel. The Engineering Department has a large task ahead in reviewing the design and construction of all the dams in the State (some of which were built 70 or 80 years ago), in addition to checking the designs and construction of current work.

The greatest advantage of State supervision is in insuring disinterested and independent review of all dam design and foundation conditions and independent inspection during construction. The value of this independent check is dependent on the competence of the reviewer. If he is incompetent the review is more than worthless, for, in many cases, it will result in preventing a competent review and check by independent consulting engineers and geologists who otherwise would have been called in. It behooves engineers to use their best endeavors to have the high standard of the officials maintained.

As the first advantage Mr. Hinderlider cites "centralized authority with its singleness of purpose and responsibility to the public." This is only realized in part in California since dual authority, as between the State Engineer and the Federal Power Commission, still exists in connection with projects involving the use of lands in the Federal Reserve. This is a condition which should be corrected, either by delegation of authority or adoption of uniform requirements and standards.

Another important advantage is the constructive work that will be done by the State departments toward the development of a code of practice in the design, analysis, and construction of dams. This will be done as a matter of convenience and necessity for the uniform administration of the law with respect to these matters. With the vast number of projects coming before the Department for review and check, a uniform procedure must be adopted, with standard requirements as to factors of safety, methods of analysis, and principles of design. The adoption and promulgation of such a code of practice by the State department will avoid the necessity of much delay and unnecessary work in re-design and re-submission of plans, which might be necessary to meet the requirements of the Department, and which might not otherwise be definitely known.

The writer can not agree as to all the advantages claimed for State control noted in Mr. Hinderlider's paper. There is no assurance, for instance, of the minimum effects of local and political influence; this is no doubt true as between county and State control, but it does not hold as between State control and no State control. Co-ordinated control in administration of the uses of stored water and stream flow does not necessarily go with the State supervision of dams. Such power might be considered as an advantage for the general public interest, but might result in serious detriment to the development of the resources of a State; it would depend largely on how it was administered. Undoubtedly, the disadvantages of State control, as set forth, will not meet with any disagreement.

As to the qualifications of the official charged with State supervision of dams, in addition to those which Mr. Hinderlider has specified, the official should have a good fighting heart, the ability to stand on his feet and forcibly and clearly express himself to defend his views, and, above all, he should have the moral courage to accept full responsibility for the failures as well as the successes in his administration of the office.

R. C. JOHNSON,[13] ASSOC. M. AM. SOC. C. E. (by letter).—The feasibility of a building code for dams is discussed in this Symposium. Not only is such a code feasible, but it would be practicable, a great aid to safer designs, and a very definite step toward an ultimately uniform practice in building dams.

No doubt such a code would of necessity be very involved, due to complications of conditions and different types of structures to be considered; but this would not make the undertaking impossible. At present, a great number of differences of opinion exist in relation to certain features of design. Uplift pressure under masonry structures, expansion and contraction joints, and the distribution of stresses in gravity arch dams, are only a few of the important features actually considered in a variety of ways. The existence of this diversity of opinions alone offers sufficient grounds for the formulation of a code, however difficult that may be. Ultimately, codification would also result in a solution of many of the existing problems and uncertainties. Authenic investigations and tests would, of necessity, be made.

It is commonly granted that the responsibility for the safety of the structure must rest with the designers, owners, and the Engineering Profession in general. It is of paramount importance, therefore, that the Engineering Profession do all within its power to perfect such safety. The formation of a code to promote uniform design and construction should be equally as important as the promotion of legislative action. Should the designers and owners themselves be able adequately to provide for the safety of such structures, no further State legislation and regulations would be necessary.

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[13] Associate Prof., Civ. Eng., School of Eng., Univ. of South Carolina, Columbia, S. C.

The writer agrees with Mr. Hinderlider in that a building code enacted as law does not appear practicable at the present time, due to the complexity of the problems involved and the subsequent continual changing that will be necessary. Nevertheless, no doubt, such a guide would prove valuable to State engineering staffs engaged in checking and approving general designs.

F. W. HANNA,[14] M. AM. SOC. C. E. (by letter).—The early establishment of sound legislative and engineering principles in the laws, rules, and regulations governing the public supervision of dams is most necessary. Unless sound principles are included in the original laws many unwise and unjust acts are certain to result.

All jurists, and indeed all engineers, must agree with the authors that the State, in assuming supervision over dams, is merely exercising one of its police functions in the interest of the public. The States of the Union have been slow in proclaiming authority over the design, construction, and maintenance of dams, because the necessity for it has not been so apparent, until increase in population and in construction of dams, coupled with recent failures, has made it evident. The danger now is that the pendulum may swing from no control to superfluous control, which will result in loss of time and money to investors and, consequently, to the public in general. This, the Society and the Engineering Profession in general, can help to prevent by proper consideration, action, and advice.

The first essential is to secure State statutes of the proper character, which is not an easy task. The statute granting authority for the supervision of dams should not only be simple, clear, and specific, as should all statutes, but it should be economically reasonable and comprehensive. It is essential, of course, that the act be drawn in simple and clear language to avoid endless disputes and lawsuits, and that it be specific as to what is to be supervised and as to who is to perform the supervision. It should be reasonable in order to prevent unnecessary expenditures in the construction of dams, that would cripple the industries depending upon dam construction. The law must be so comprehensive as to cover all features involved in the safety of the dam, the location, the foundation materials, the design (including outlet works and spillways), the materials of construction, and the method of construction. Unquestionably, the law should also give supervisory control over the maintenance of dams after their construction and should extend to the approval of past as well as to future construction.

In setting up State machinery to handle the act, it is desirable to use existing governmental machinery so far as practicable, but it should be remembered that not all the States of the Union have governmental machinery suitable for the supervision of the construction and maintenance of dams. This supervisory control is essentially and wholly an engineering feature and should not be left to quasi-engineers and politicians. As a matter of fact, the greatest danger of lodging supervisory control of the design, construction, and maintenance of dams in the hands of the State is that of making it a political rather than an engineering problem. Every effort should be made in the statutes and in their control to eliminate politics, professional jealousy, factional disputes, and other evils that are likely to creep into the administration of an act of this character.

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[14] Chf. Engr. and Gen. Mgr., East Bay Municipal Utility Dist., Oakland, Calif.

The builder of a dam may find it unsatisfactory, in his opinion, to comply with the requirement of the supervisory control, and, there should be opportunity, therefore, for appeal to the Courts or to a board of arbitration. Such a board should be composed of engineers and geologists qualified to pass on the feasibility of the dam and its appurtenances.

In the Western States of the United States, where public lands are still held by the National Government, the Federal Power Commission has authority over dams for power development, and the builder should not be subjected to duplicate authority, nor should he be compelled to thrash out the differences of opinion between State and National authorities. Where both State and National Governments are interested in any dam, these agencies should either agree on an appropriate division of their authorities, or they should thrash out their differences of opinion and delegate one or the other agency to deal with the builder.

In addition to duplication of authority in intrastate projects, there may be conflicts between interested States on interstate projects on rivers passing through two or more States. A State supervising the construction of a dam may be so fortunate in the location of its population as to make the failure of a dam on an interstate river entirely without danger to persons or property within its boundaries, while the persons and property in an adjoining State through which the river passes may be in dire danger from the failure. In such cases it would seem that the State in which the persons and property are in danger should have supervisory control over the dam constructed in the sister State. Cases of this kind might be quite common since rivers pay little attention to State boundary lines.

The status in interstate problems may be still further complicated in the Western States by the introduction of National authority where federally controlled power projects are designed, constructed, and operated under the approval of the Federal Power Commission. This conflict of interests of neighboring States and National Government may not be an easy one to handle in connection with State supervision of the design, construction, and maintenance of dams. Possibly the solution of the problems in these cases is National supervision. There are also instances both on the northern and southern borders of the United States where not only duplication of State authorities and of State and National authorities may arise, but where duplication of National authorities may be involved. Such situations cannot readily be handled by State supervision of dams within their borders, because international questions are involved. Thus, it appears that the supervision of dams by States is not simple, at least in some of the States.

In the matter of the codification of dam-building principles, the writer believes that it is entirely practicable to codify methods of design and construction of dams for specific foundations. However, there are so many diverse foundation materials to be encountered in the construction of a dam that the multiplicity of code requirements to meet all of them scarcely seems practicable. Nearly every foundation involved in the construction of a dam has some peculiar and specific problem that must be solved in order to determine the character of design and the character of construction to be used, and it will be difficult to meet all these conditions fully and properly in the formulation of a code. Therefore, only general principles that are applicable to each type of dam should be included in the code.

JOEL D. JUSTIN,[15] M. AM. SOC. C. E. (by letter).—Universal State supervision of the design and construction of dams is almost a certainty of the near future, and the writer believes that such supervision is necessary and desirable. On the other hand, it is conceivable that it might lead, in some cases, to unnecessary and unproductive expense, or might even so increase the cost of a project that it could not be executed. Furthermore, if State supervision is merely perfunctory or incompetent, the approval of the project by the State authority would be no index as to its safety. It is highly desirable, therefore, that legislation on the subject be of such a nature as to help to secure the safety of the public without placing financial burdens on the owner which might "sink" the project.

Although open to conviction, the writer is doubtful of the efficiency and desirability of a code which, supposedly, would be a series of specifications covering in detail the design and construction of every type of dam on every conceivable kind of foundation. Such a code might even contain attempts to specify the types of design which would be considered permissible for certain specified foundation conditions.

Under the United States system of government, it would probably not be difficult to get the code, but without provision for a personnel to see that it is carried out efficiently. The State engineering authority would have to certify that the design and construction complied with the code. For the State engineering authority to do this honestly would require nearly as great an expense as that incurred for engineering by the owner. Most, if not all, of this additional expense would almost surely be assessed against the owner.

A code would lead to the attempt to standardize. Standardization in design is not applicable to dams to the extent that it is applicable to some classes of building construction, for instance. Any attempt to reach standardization through a code would undoubtedly lead to much unnecessarily costly construction, and would make uneconomic the undertaking of many desirable projects.

The writer, therefore, favors Mr. Markwart's first suggestion, namely, that the State engineering authority should be charged with passing on the safety of the design of the dam as submitted for the given foundation conditions, and should also be charged with passing on the safety of the structure as built. This latter requirement should mean that for important projects the State engineering authority would have an inspection engineer on the job throughout its construction.

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[15] Cons. Engr., Philadelphia, Pa.

Such an inspector should have no authority to give instructions, but should report his observations to the State engineer, who would take such action as might be necessary. He should confer constantly with the owner's resident engineer and bring to his attention all cases of faulty design and workmanship which he notices.

There are often cases in which such State supervision may be of the greatest help to the owner's engineer. Thus, in the case of a project in which the financial backing is not of the highest class, the pressure on the owner's engineer to design a cheap structure is often great and difficult for the average man to resist. If, in addition, the owner is also doing the construction, the position of the owner's engineer may prove most unfortunate. Precautions which, to the engineer, are of the greatest importance, often seem foolish and expensive refinements to the uninitiated and, therefore, may be omitted in such cases by the owner's orders. In such cases, State supervision proves a veritable Gibraltar of strength to the owner's engineer, enabling him to get a conservatively safe design adopted and constructed with due regard to safety instead of being compelled to resign in order that he should not be connected with an unsound project.

As Mr. Markwart points out, it is necessary that a method of appeal should be provided from the decisions of the State engineering authority. This is necessary because a State authority interested only in safety might insist on structures which, although safe, were not the most economic which could be designed for the particular conditions. Mr. Markwart's suggestion of a board of engineers appeals to the writer as more desirable and less likely to involve delays than an appeal to the Courts. The decision of the board should be final, of course, and the expense of the appeal should be borne by the owner.

Additional laws providing for closer supervision of the design and construction of dams will be passed by the various State legislatures. If the formulation of these laws is not guided by competent engineers, serious errors will be made.

I. C. STEELE,[16] and WALTER DREYER,[17] MEMBERS, AM. SOC. C. E. (by letter).—The essential and desirable features that should be included in legislation providing for State supervision of the design and construction of dams, has been clearly outlined by Mr. Markwart. The writers are in accord with all that is stated under the headings "Extent of Supervision," "Exemptions from Supervision," "Supervision Personnel," "Burden of Cost," "Appeal Provision," "Existing Dams," and "Maintenance of Dams." It is extremely undesirable to incorporate a building code as part of the law because of the continual change in the art of dam construction, and because of the difficulty of making legislative revisions. The adoption of a minimum requirement code as a regulation of the supervising body might not be objectionable, since such a code can be changed from time to time without passing through the State Legislature. The danger then would be that after some years of operation under a tentative code a desire would develop to place the code itself on the statutes, in which case progress in the art would stagnate more or less until a revision was adopted.

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[16] Chf., Div. of Civ. Eng., Pacific Gas & Elec. Co., San Francisco, Calif.

[17] Asst. Chf., Div. of Civ. Eng., Pacific Gas & Elec. Co., San Francisco, Calif.

Prior to August 14, 1929, dams erected in California by privately owned public utilities were subject to the approval of the State Railroad Commission. This Commission had no jurisdiction over the so-called "publically-owned" utilities. Dams erected by private agencies other than utilities were subject to the approval of the State Engineer. Federal authority exercised supervision over débris dams, dams owned by the Government, and those occupying Government rights of way. Municipalities or quasi-municipalities with regularly organized engineering departments were not subject to State supervision. When no such department existed the dams were subject to the approval of the State Engineer.

Public opinion following the failure of the St. Francis Dam directed the attention of the State legislative body to the necessity for concentrating the supervision of dams under one central control. As a result, the 1929 Legislature, under the police powers of the State and for the purpose of safeguarding life and property, invested the duty of the supervision of the construction and maintenance of all dams, with the exception of those owned by the United States, in the Department of Public Works to be administered by the State Engineer.[18] This law repeals all other State Acts governing the supervision of dams. Its principal features are, as follows:

(1) Dams under jurisdiction are only those structures which are either 15 ft., or more, in height measured from ground level to the spillway, or which impound 10 acre-ft., or more.

(2) The law invests the Department of Public Works with authority under the police power of the State and directs it to supervise the construction, enlargement, alteration, repair, maintenance, operation, and removal of dams for the protection of life and property. This includes all dams and provides for co-operation between State and Federal authorities when both have jurisdiction. In controversy between State and Federal authorities, the Federal jurisdiction predominates.

(3) Owners of existing dams are required to obtain a certificate of approval and all structures must be examined by the State within a period of three years from the date of the Act. Costs of such examination are borne by the State.

(4) An application, accompanied by complete engineering data and a filing fee, must be presented before a new dam can be constructed or an existing dam enlarged. Certain penalties are provided in the nature of addi-

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[18] Senate Bill No. 723, Chapter 766, Section 1, Statutes of 1929.

tional fees when the actual cost exceeds the estimated cost by more than 10 per cent.

(5) An application must be filed and written approval secured before commencing the repair, alteration, or removal of a dam. When repairs are of an emergency nature, work may be started at once, but the Department must be notified and the work made to conform to such orders as the Department shall issue.

(6) No application will be approved in less than 10 days from the date of filing, and the Department must be given 10 days' notice before construction is begun. Approval becomes void after one year unless the time is extended for cause.

(7) During construction the Department may order the owner to change his plans or methods of construction if necessary to insure the safety of the structure.

(8) The Department is empowered to supervise the maintenance of dams and to take any remedial means it deems necessary in an emergency.

(9) The Department acts on complaints alleging that person or property of complainant is endangered by the dam. The complainant must deposit cash when an inspection is required. If the complainant is sustained, the money is returned, otherwise it is paid into the State Treasury.

(10) No action can be taken against the State due to failure or to the operation of a dam.

(11) The owner is not relieved of any legal obligations or liabilities.

(12) Persons wilfully obstructing the Department are guilty of misdemeanor. If work is knowingly done without an approval, or in violation of an order, the owner or the contractor is also guilty of a misdemeanor.

(13) The law provides for a mandamus or injunction when the orders of the Department are being violated or violation is threatened. Action is brought in the Superior Court of the county in which the work is being done; the owner must answer the petition within twenty days, and after he has answered, or defaulted in answering, the Court investigates and either dismisses the action, makes the injunction or writ of mandamus permanent, or so modifies it as to afford appropriate relief.

It is to be noted that the law includes most of the characteristics desirable in this type of legislation. The only dams that are exempt from supervision are those owned by the Federal Government, and this is due to the superiority of Federal over State jurisdiction. Existing governmental machinery is used to administer the duties created by this Act. Supervision has been properly limited to that necessary to safeguard life and property. The rights of the owner to select the site, to design the dam, to superintend the construction, and to maintain the structure, are in no way impaired, as long as the dam is deemed safe by the State authorities.

The State is empowered to act in an emergency, but this should be unnecessary where the owners are in a financial position to take action themselves. The regulations issued by the Department under the Act require the formal approval of plans before construction can proceed, and do not delineate the procedure for changes initiated by the owner after the work has been started. It is probable that it will be difficult to give formal approval to a set of plans and then have construction conform exactly to those plans. It is usually necessary to make changes as construction proceeds, and it is quite likely that the Department will find it advisable to permit construction to proceed along lines which, in general, follow the original plans, and, at the same time, give the owner latitude to make such changes as he finds advisable. With close and co-operative contact between the owner and the Department, there should be no difficulty in administering this part of the work.

The law properly specifies that the State shall pay the cost of examining existing structures. However, it provides for the payment of fees for the construction of new dams, or for the enlargement of existing dams, that apparently place the burden of the cost of supervision by the State upon the owner of the dam. This is evident when one considers the magnitude of the fees; for example:

Cost of dam	Fee	Cost of dam	Fee
Minimum	$20	$1 000 000	$5 500
$100 000	1 000	5 000 000	9 500
500 000	3 000	10 000 000	12 500

Appeal Provision.—The Act provides that any owner shall have such recourse to the Courts as he may be entitled to under the laws of the State. When requested in writing to do so by the owner, the Department appoints a consulting board of two, or more, which reports to the Department on the safety features involved. The cost and expense of such a board, if appointed on the request of an owner, are paid by the owner.

While the language of the Act does not make the decision of the Consulting Board binding on the State Engineer, it may be presumed that he would only appoint such a board if he were willing to abide by, or at least be influenced by, its decision or report. If the owner cannot find relief from what he may consider unwarranted interference or unnecessary expenditures through the medium of the Consulting Board, he must seek relief in the Courts. Recourse to the Courts will greatly hamper the progress of construction, and it is unlikely that an owner will adopt this means of relief unless the burden imposed on him by the Department is, in his opinion, unbearable.

It is evident from the foregoing analysis that the new California statute, made necessary by public opinion following the St. Francis Dam disaster, has in general been wisely prepared and contains many of the desirable features that should be included in such a law. While the burden of the cost of supervision has apparently been placed on the owner, after all it is the owner who creates the risk to life and property, and the objection to this means of providing the funds necessary to insure safe construction and operation may be subject to argument. Provision has been made, rather loosely it must be admitted, for relief for the owner in the event of a difference of opinion, and this feature might have been more clearly outlined in the terms of the Act. It would seem reasonable to have the decision of a consulting board appointed by the State Engineer at the request of the owner, binding on both parties. Presumably, however, this can be done by agreement between both parties prior to the appointment of such a board.

In all other respects, the California law seems to be entirely satisfactory to the owners of dams, as well as to the public; and if wisely and judicially administered, with both parties willing to co-operate to the interests of the public safety, its operation should be smooth with few, if any, controversies of importance.

FRED A. NOETZLI,[19] M. AM. SOC. C. E. (by letter).—Structural engineers have realized for a long time the chaos that would exist without a code for the design and construction of buildings. Dam engineers are slowly approaching the point of recognizing the necessity of a code for dams. The excellent papers by Mr. Markwart and Mr. Hinderlider are certainly steps in the right direction.

Table 1, presented by Mr. Hinderlider, records the failures or partial failures of 253 dams in the United States, and of 40 dams in all European and other foreign countries. This record would be rather appalling if it did not include, as properly pointed out by Mr. Hinderlider, so-called "partial" failures which merely required some repair work to put the dams in operating condition again. A number of foreign countries, notably France, Germany, and Italy, have adopted rules and regulations for the design, construction, and maintenance of dams. These rules are sufficiently strict to preclude unsafe designs or constructions. On the other hand, they are sufficiently flexible so as not to restrict progress in the art unduly.

The writer is informed that Arizona was the first State in this country to take steps to adopt a definite code for dams. This code was compiled by J. A. Fraps, Assoc. M. Am. Soc. C. E. California enacted a statute in 1929 placing jurisdiction over all dams in the State, except those of the Federal Government, in the State Engineer. In California, an applicant is required to submit the plans and specifications for a proposed dam to the State Engineer for approval, and to pay a certain fee, depending on the estimated cost of the dam. This fee is intended to cover the State's expenses for passing on the plans and specifications of the dam; further, for the geologic and engineering investigation of the dam site, and for occasional supervision during the construction of the dam. If the applicant should desire to submit the plans for an alternative type of dam at the same site, a second fee of the full amount must be paid, although a relatively small amount of work is involved in passing on a second set of plans.

It may also happen that at a certain site the foundation conditions as revealed by the excavations indicate the desirability of a change of the type, for instance, from a massive concrete dam to a type that is more flexible and less subject to uplift. If such a change should be made a new filing fee in the full amount would have to be paid.

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[19] Cons. Hydr. Engr., Los Angeles, Calif. Mr. Noetzli died May 24, 1933.

At certain sites the type of dam best suited for the conditions can be determined only by comparative designs. Actual costs of construction are best ascertained by bids from responsible contractors. In the interest of economy and progress, therefore, the asking for bids on alternative designs approved by the State Engineer should be facilitated by requiring only a relatively small additional filing fee for such designs.

It is interesting to note that according to Mr. Hinderlider the State Engineer in Colorado follows the practice of suggesting to applicants for permits for dams the type of structure indicated by the site and geological formation. Inasmuch as dams of different types may be feasible at certain sites—such as, for instance, a gravity dam, an arch dam, a cellular type of dam, or a structure of one of the buttressed types—the problem is decidedly one of economics. While the cheapest type may not be the one best suited for given conditions, in all cases, there is no good reason why the most expensive type, namely, the gravity dam, should receive first and sometimes sole consideration.

GEORGE N. CARTER,[20] M. AM. SOC. C. E. (by letter).—It appears that some agency, clothed with sufficient authority, is necessary as a watchman over lives and property situated below dams impounding quantities of water, that would be dangerous if suddenly unloosed. The State is better adapted to perform this function than any other agency. The ruling hand is thus sufficiently local in its nature to acquire first-hand information; yet it is general enough in character to afford some uniformity. By placing this duty with the State, there is assurance of responsibility, because it is definitely placed by legislative enactment. It would be preferable if every owner of a potentially destructive dam could be charged with responsibility for its safe design, construction, maintenance, and operation; but this is not possible. In the interest of public welfare, therefore, a definite and fixed responsibility should lodge some place and the State Government seems the most logical agency for such a protective measure.

No corporation, company, individual, or any other agency wants to build and maintain a dam that is a hazard to life and property. Whenever the supervision of the design, construction, and operation of a dam, which may create potential danger, has been approached from the standpoint of co-operation, rather than as a police function, considerable progress has been made in establishing harmonious relations between the owner of such a structure and the public official whose duty it is to safeguard equally well the interests of all. Generally, it is a sound premise that only through ignorance will an unsafe dam be built or operated. In most instances, if failure occurs, the major loss falls upon the owner; therefore, only in a case of bad faith or inclination to minimize the cost of a dam (which the ownership of, and responsibility for, will pass in title immediately upon completion of construction), could it logically be argued that a hazardous dam was being wilfully maintained. Thus, the majority of cases of questionable safety will fall into the category of ignorance. Others may arise through honest differences of opinion.

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[20] Civ. Engr., Boise, Idaho.

Where the shortcoming is the result of ignorance, there is little to be said. Fortunately, these cases generally involve dams or water storage of small magnitude and, by the same token, often limited financial resources and credit. To impose a comparatively large repair or reconstruction charge arbitrarily upon such an owner frequently works a hardship. The owner is beset with two difficulties, he may lose the structure and may also become liable for damages, or he must suffer a financial outlay for improvement he can ill afford. In such cases the public official charged with the duty of correcting the condition has a delicate problem to adjust the requisite repair measures to the owner's financial ability. True, the necessary construction or reconstruction thus thrust upon the owner is a kindness and a favor, but the public officer doing this must have, and keep, a fine sense of balance in values and not become imbued with a non-governable zeal to do a surpassingly fine job regardless of the cost burden that may be imposed. In some cases, enthusiasm may be the cause of unjustified expense, and in others, inexperience may prevent the application of more economical but equally adequate safety measures. If the public officer, saddled with the responsibility of preserving safety, is over-cautious in his purpose and does not know that the function of an engineer is "to do with one dollar what any one can do with two or three," he may easily go far beyond the exigency and possibly be the cause of virtual confiscation of the property.

If the alleged defect in a dam comes down to differences of opinion between the owner's engineers and those of the State as to adequacy of foundations, sufficiency in design, thoroughness of construction, or existing operating hazard, then there is nothing to do but "thresh out" the matter, and, failing in agreement, call in more and, if possible, superior engineering talent.

The extent or amount of statutory direction for dam supervision that will give the best results is debatable. Some States have seen fit to attempt almost completely to define, by statute, the principles of location, design, and construction of dams; others have enacted only a broad general law, authorizing scrutiny and requiring approval of locations and design that shall be safe. The best results are obtainable by the broader method. The subject of suitable foundations and design of dams is such a broad one, covering as it does so many different conditions, no two of which are often alike, that the formulation of complete statutory rules is almost impossible. It would indeed be a keen and far-sighted engineer or group of engineers who could write a dam code to cover every condition that might be met in the future. The many arguments and controversies always current over the principles of dam design, illustrate the futility of ever writing a code not subject to dispute, and the action that would result from the efforts of a board of engineers would be a compromise in which, possibly, the best judgment of any one would not be expressed. Furthermore, the officer supervising such work can function more efficiently if he is not circumscribed by some statute which, while not applicable, still governs. If the State Engineer has not the ability and judgment to act wisely in each case before him, neither will he be capable, in all probability, of interpreting and following a complex code.

The State of Idaho has a law covering this matter, enacted in 1895. It is short and adequate, the only two objects stressed being safety to life and property, and the authorization granted the State Engineer to do whatever is necessary to provide or restore safety. In case of existing danger an officer of the peace may enforce his orders, if necessary, and the right of appeal to a Court is provided.

Administration of this law has produced a record of every dam in the State that may be potentially dangerous and this record is readily available for any emegency. Its value, however, would be considerably enhanced by filing therewith record drawings made after completion of construction, showing in all instances departures from the original plans. Cases have developed in which the plan at hand did not conform to the structure as built, and some delay was occasioned thereby when time was precious. A dam failure of any serious consequence has never occurred in the State and Idaho has many dams of size and importance.

Mr. Hinderlider states that the practice in Colorado is for the State Engineer to suggest the type of structure indicated by physical conditions. It is doubtful if this is a function of a State Engineer or other public official charged with supervision. He should only approve or disapprove the plans and specifications laid before him; possibly after disapproval he might appropriately suggest what would be acceptable.

Supervision and control of dams will be accomplished with the greatest satisfaction to a State as a whole when the function is exercised with the least notice and interference, commensurate with absolute safety to life and property.

GEORGE W. HAWLEY,[21] M. AM. SOC. C. E. (by letter).—In 1929, the State of California enacted what is perhaps the most complete and far-reaching legislation governing the supervision of dams, which has been effected by any governmental agency for a similar purpose. Prior to this legislative enactment there existed in this State, as in many other States, several regulatory agencies with varying degrees of authority and responsibility. These agencies exercised jurisdiction in differing degrees from complete control over some dams to partial, ineffective, divided, or no supervision over others. Lack of centralized authority and responsibility could result only in confusion of authority, divided responsibilities, ineffective supervision, and a condition of insecurity in the minds of interested parties.

The continued economic growth and prosperity of California, in common with that of any semi-arid region having variable stream flow, is in large part dependent on complete economic utilization of its water resources and the degree of flood protection afforded. The natural stream flow of California will admit of very limited increase over present use and, consequently, the future water supply will be obtained chiefly by storage. This storage can be developed only by the construction of dams, which is indicative that as time progresses dam building will be materially extended both in size and number. Since the most favorable sites, topographically, geologically, and economically, are first in order to be developed, it follows that, as requirements for storage and flood control increases, the suitability of the available sites will be less favorable. As a complement of this proposition, the property values and number of lives that might be jeopardized through the failure of any dam, or affected by the construction of a dam, are becoming increasingly great.

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[21] Deputy State Engr. in Chg. of Dams, Sacramento, Calif.

There are at present about 800 dams in California, many of which are structures of outstanding magnitude and importance. In many instances dams are constructed at strategic locations as regards potential hazard to populous and rich communities in the event of partial or total failure. When consideration is given to the fact that the size and number of dams are of necessity materially increasing, that the property values and lives which would be involved in the event of failure of a dam are becoming increasingly greater, and that inherent public fear and apprehension attaches to dams more, perhaps, than to other engineering works, centralized, authoritative, and competent supervision of dams becomes imperative.

Public fear in California has been accentuated by two major failures, involving loss of life, together with the question of safety and integrity that has been raised as to other dams.

Accordingly, in response to widespread public demand for more comprehensive, adequate, and thorough protection from the recurrence of loss attendant upon these failures, the California statute govering the supervision of dams (which it is held is a sovereign duty of the State), was enacted to safeguard the life and property of its people. The constitutionality of the California law has been upheld by two Appellate Court decisions and one Supreme Court decision.

The California law vests authority over all dams in California (other than those Federally owned) that have a capacity of 10 acre-ft. or more, or a height of 15 ft., or more—regardless of ownership or supervisory control—exclusively in the State Engineer. The law was carefully worked out in conjunction with the owners and builders of dams and is believed to be the most complete and effective statute of any State on this subject. The State control is limited to that necessary to safeguard life and property and embraces three main features:

1.—An examination into the safety of all existing dams and their approval for use if found safe or after such repairs as the State Engineer may find necessary are made;

2.—Approval of plans and inspection of construction of new dams, followed by their approval for use; and,

3.—Supervision of maintenance and operation of all dams in so far as necessary in the interests of safety.

Provision is made in the Act whereby the State Engineer is authorized to co-operate with agencies having joint jurisdiction, such as the California Débris Commission, Federal Power Commission, and U. S. Forest Service.

The State Engineer is invested with authority under the police power of the State and directed to supervise the construction, enlargement, alteration, repair, maintenance, operation, and removal of dams for the protection of life and property. All dams in the State whether heretofore or hereafter built or under construction at the date the legislation was effected are under the jurisdiction of the State Engineer and application for their approval must be filed.

Every owner of a dam completed prior to the effective date of the Act is required to file an application for approval of the dam, this application to be accompanied by such available and appropriate information concerning the dam as may be required by the Department.

Subsequent to the effective date of the Act, construction of any new dam or the enlargement, repair, or alteration of any dam shall not be commenced until the owner has applied for and obtained from the Department written approval of the plans and specifications. It is required that the application for approval of plans and specifications for a new dam shall set forth the location, type, size, and height of proposed dam and appurtenant works; contemplated use and storage capacity of the reservoir; and such other pertinent data as the Department may require concerning foundation conditions, drainage-basin area, precipitation, flood flow, and other appropriate data. It is also required that a filing fee, based on the estimated cost, shall accompany the application. The State Engineer is empowered to approve or disapprove the application or require the modification or revision of any incomplete, defective, or insufficient application.

The action of the Department is restricted to approval or disapproval of the application submitted by the owner and to assurance of proper execution of the work in accordance with the approved plans and specifications. If an application is disapproved the reasons for so doing are set forth. In the case of one major dam, the only dam for which application has been denied to date, the State Engineer disapproved the application on the grounds that the dam, if built as proposed in the application, would be unsafe and a serious menace to life and property in the populous valley below.

During the construction, enlargement, repair, or alteration of any dam the Department is required to make or cause to be made such continuous or periodical inspections, investigations, or examinations as may be necessary to secure conformity with approved plans and specifications; and in order to insure safety, the State Engineer has authority to order revisions or modifications in the plans and specifications, or, if conditions are revealed which will not permit the construction of a safe dam, the approval may be refused or revoked.

To accomplish the desired objective, namely, the determination and establishment of safety of each of 800 existing dams, in addition to supervising new construction, an experienced personnel has been organized to cope with the many involved technical and practical problems.

The personnel and activities of the Department, of necessity, must permit of extreme flexibility to meet the wide field of activities to which it is necessary that they apply themselves, in order to conclude a sound and proper determination of safety. This variation in duties results from the facts that:

1.—The art of design and construction of dams which is in the evolutionary stage is highly technical and advancing with rapid strides;
2.—There is a wide range in type and location of structure;
3.—The variation in geological formations throughout the State makes the study of foundation conditions of each dam an independent but important problem; and
4.—The volume of work involved in reviewing in excess of 800 dams within the time limit allowed necessitates efficient and competent prosecution of the various activities.

The activities of the Department have been directed along six general lines of endeavor: (1) Field examination and investigation of existing dams; (2) geological examination and report on dam sites; (3) analytical study of the structural features of all dams and appurtenant works; (4) hydrographic study of the drainage-basin area tributary to the dam; (5) hydraulic study of the spillway of each dam; and (6) competent and regular inspections of all new construction and repair work.

As a result of investigations of the physical conditions of existing dams it has been possible to segregate these dams, for purposes of expediting the work, into three general classes, namely, (*a*) dams needing repair; (*b*) dams requiring further study; and (*c*) dams ready for approval. Approximately one-third of the total constructed prior to the effective date of the Act falls in each class. The action taken in each of the three classes my be summarized, as follows.

Dams Needing Repair.—Owners are informed in what respect their dams are in need of repairs and are requested to take whatever action is necessary to place them in a satisfactory condition. Plans for the necessary repairs are prepared by the owner and submitted to the State Engineer with an application for approval. After approval of the application the repairs are made by the owner under the supervision of the State Engineer's Office. The Department is meeting with a ready response and has received more than 200 applications for such repair to date (1932).

Dams Requiring Further Study.—There are numerous dams concerning which there are insufficient data to base a decision as to safety. The plans may be too meager in detail for a complete analysis, in which case actual measurements are made. The materials placed in a dam, or the geology of the foundation and abutments, may be questionable, in which case test borings or other means of examination must be made of the dam or site. If the first inspection was made at a low stage of the reservoir another may be necessary when the reservoir is full in order to determine the amount of leakage. Examination into the safety of these dams, must, by law, be completed prior to August, 1932.

Dams Ready for Approval.—Formal certificates of approval are being prepared for those dams for which the investigations have been completed and which in the judgment of the Engineer Department are ready for approval. That a dam has been approved once does not relieve the owner or the State from constant vigilance to assure proper maintenance. Supervision over maintenance and operation is exercised at all times. Applications for repair or enlargement of a dam, even if the dam has been previously approved, must be approved by the State Engineer. A new certificate of approval will be issued upon the completion of the subsequent work.

In dams of magnitude, or where the technical features involved are such as to require it, or when major controversial technical issues are involved, the State Engineer avails himself of the services and advice of experienced consultants especially qualified in the particular phase under consideration to examine into and report upon the safety matters involved, in order that a proper and sound solution of the problem, from a safety viewpoint, may be reached.

It is the aim and endeavor of the Department, rigidly adhered to, to require that the personnel refrain from forming conclusions on the basis of local or prejudiced influence; refrain from imposing unwarranted or dictatorial conditions beyond the requirements of safety; refrain from exerting unnecessary or onerous influence over construction; refrain from assuming engineering direction; and refrain from expressing opinions or influencing decisions on economic considerations, such as choice of type, location, etc. In other words, the feasibility and economics of any proposed construction rest solely within the discretion of the owner and wholly without the scope of the State Engineer's jurisdiction. On the other hand the personnel stands at all times willing to discuss informally with the engineer (acting for the applicant) controversal problems of design or construction relating to the dam under consideration to the end that the dam, when built, will be a safe structure.

The mandatory duties placed upon the State Engineer in exercising jurisdiction over the dams in this State and the grave responsibility exacted of the State Engineer in the interests of safety merit the co-operation of the owners of dams, engineers, geologists and the public alike. It is most gratifying to find that, to date, the State Engineer of California has enjoyed a splendid spirit of co-operation, willingly and generously given by all interested parties.

To a limited extent only, has approval of a proposed structure been sought, prior to filing formal application with the Department, for the announced purpose of financing, consummating contracts, selection of design, avoiding payment of professional fees, or shifting responsibility. The Department is not willing nor can it lawfully accede to such requests.

The Department, in so far as may be possible and advisable, is endeavoring to program its methods and policies to the end that uniformity of procedure in accordance with accepted engineering practice may be obtained. Only in the event a sound and workable code for the design and construction of dams can be devised and agreed upon by engineers will the Department be willing to formulate prescribed design criteria for the determination of the safety of every dam which, after all, is a problem for individual study. In the interim it seems desirable that concerted action be taken by engineers to establish minimum requirements for design purposes, such as allowable unit stresses, length of path of percolation, slopes of earth and rock dams, uplift pressures, etc.

An analysis of records discloses that a major number of dam failures has resulted from inadequate spillway provisions and foundation or abutment insufficiencies. Where ordinary care has been exercised in design and construction instances of failure within the structure itself have been extremely rare. This indicates not that less thought and effort should be directed to technical design, but rather that more thought should be applied to geologic and hydraulic features.

State supervision of dams, if properly and competently administered, and if directed aggressively to the proper requirements of safety for dam design and construction, merits the support of the Engineering Profession for the establishment of public confidence in, and for the technical advancement that will accrue to, the Engineering Profession. State supervision if competently administered assures the public of unbiased and uninfluenced engineering opinion; dispels inherent public fear and apprehension of the safety of dams; centralizes, and makes uniform, co-ordinated control; and makes available in condensed form recorded technical information and data of inestimable value. State supervision of dams, however, should be ever cognizant of the fact that the advancement of any community depends upon the development of its water resources through the construction of dams. Obviously, this program must not be retarded through over-cautious and unwarranted functioning of the office having jurisdiction beyond reasonable requirements for assurance of safety.

H. K. Barrows,[22] M. Am. Soc. C. E. (by letter).—The writer has read with interest the excellent and full outline by the authors of the various phases of this subject. The list of dam failures is of especial value and very pertinent to this subject.

Recent legislation in Vermont (1929) under which new dam construction has had State supervision is of interest. This Vermont Act provides that no dam impounding more than 500 000 cu. ft. of water in any stream or at the outlet of any body of water shall be constructed without the approval of the Public Service Commission, and requires that application to the Commission for such construction shall include the filing of plans and specifications therefor.

The Commission, with the approval of the Governor, may employ a competent hydraulic engineer "to investigate the property, review the plans and specifications and make such additional investigation as such Commission shall deem necessary."

Under this Act several new projects have been supervised, including the Fifteen-Mile Falls power development, with a dam 170 ft. high. The cost of engineering supervision is paid by the party constructing the dam, through the office of the Commission.

Any existing dam that is unsafe can also be investigated under this Act, and caused to be properly repaired, by the Commission.

In many States the method of supervision through a competent consultant is undoubtedly effective and economical as this class of work is usually not continuous, but nevertheless requires the highest degree of professional judgment and experience. In States where enough work of the kind warrants the continuous employment of an expert it is advisable to have such an engineer under salary by the State.

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[22] Prof. of Hydr. Eng., Mass. Inst. Tech.; Cons. Engr., Boston, Mass.

As regards the extent of inspection during construction, when the work is under the direct supervision of competent engineers for the party building the project, it does not appear necessary or desirable to duplicate inspection. When adequate engineering supervision by the owner is lacking, however, the State inspection should be as complete and adequate as necessary to obtain the desired results.

The writer questions the advisability of incorporating any code relating to plans or type of construction in legislation upon this matter. Legislation should be broad and comprehensive rather than detailed. Such details should be left to the executive authority and its engineer.

HARRY W. DENNIS,[23] M. AM. SOC. C. E. (by letter).—The paper by Mr. Hinderlider presents a strong case in support of the proposition that dams are potential sources of danger, that they should be designed with skill and care, and that they should be reviewed by some independent agency. The supervision of the work by the State provides such review by an agency that is outside the domination of the owner, and this is a strong argument.

Under "Legislative Trends," Mr. Hinderlider states that while sound principles have been developed in the design of almost all conservative types of dams, there is still some diversity of opinion regarding theories that affect the safety of huge masonry dams. This emphasis and that expressed in connection with the dam failures in Table 1, show that although the supervision by the State presupposes that the engineer "knows all about it," there are still many, many things in connection with the design and construction of dams that the engineer (speaking broadly) does not know as yet. If the State is to provide supervision of design and construction and of maintenance and operation of dams, is it not the duty of the State to provide the necessary additional research whereby its officials can carry on and can develop research problems that are far beyond the financial ability of any individual corporation, or technical society, or any agency except the public treasury?

Who knows what the behavior of large masses of concrete may be? It is well known that the temperatures created in the setting of concrete dissipate over a long period of time, depending, of course, on the mass. In gravity dams in which the thicknesses are in excess of, say, 100 ft., the fact has been established that many years pass before the ineherent temperatures, due to the setting of the concrete, will have been entirely dissipated. Engineers do not know what is going on in huge masses of concrete.

If the State Engineer is to be the supervising official, with the necessary facilities to determine some of these things about which so little is known, his status and power in this connection should be established within the same legislation that creates the State as the supervising agency.

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[23] Chf. Civ. Engr., Southern California Edison Co., Los Angeles, Calif.

Effective legislation in connection with this subject seems to have required the invocation of the theory of the police power of the State. That, therefore, makes the legislation in the interest of the public safety. From that premise it follows that the only authority that can be given to the supervising officials, is that which comes in administering their duties in the interest of the public safety. Their authority is limited. The writer takes some exception, therefore, to Mr. Hinderlider's recommendation that the law should include authority to "pass on the design, construction, maintenance, and operation of all dams and reservoirs of a certain minimum height and capacity" (see Conclusion 1). Many of the recommendations are acceptable, if there is added after each one, "so far as the public safety is involved," but the writer cannot agree that the supervisory body should be given blanket authority to exercise control over these things regardless of whether or not they involve the public safety.

Consider the recommendation (Conclusion 6) that the law should include authority to exercise supervisory control during construction and operation of the dam following construction.

The financier who is asked to approve an application for the necessary funds for a new project examines the estimates of cost and cannot take exception to the condition whereby the State may impose changes in plans and specifications before the work is begun. Revised estimates may be made as a result of such changes and examined before the project is authorized. If, on the other hand, the State official has the authority to change those plans and specifications from time to time, at his own will, and for reasons other than the public safety, there will be difficulty in financing large and important structures.

Mr. Hinderlider's comments on Item V, of what State supervision should not include, contains the gist of the writer's attitude, namely,

> "State supervision should be limited to the approval or disapproval of the plans and data presented, and the authority to require essential amendments thereto, which, in the judgment of the State official, appear to be needful for insuring the proper degree of safety, * * *."

Dr. Ing. N. Kelen,[24] (by letter).—Convincing proof of the necessity of the licensing and supervision of the design, construction, and operation of dams by the State is advanced in the Symposium. State supervision is necessary for two main reasons. In the first place hydraulic establishments are public utilities; each artificial interference with the natural stream flow touches public interests and can have consequences that are more or less noticeable both above and below it. For this reason in most countries the State is the proprietor of all bodies of water. In the second place a dam creates a certain menace to life and property in the region below it because, as correctly emphasized in the Symposium, dams differ from other engineering structures inasmuch as their failure may cause an enormous catastrophe. Some years ago it was estimated that the failure of a dam designed by the writer would involve the certain death of 10 000 people and (since densely populated industrial districts would have been destroyed by the ensuing flood wave) a property loss impossible to estimate. Considering such possibilities, the engineer hesitates to take over the sole responsibility for the structure especially since many points in theory, geology, and construction of dams are not yet quite clear. Mr. Hinderlider's Table 1, concerning the failures of 293 dams and their causes, is valuable and highly interesting.

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[24] Cons. Engr.; Privatdozent, Technische Hochschule, Berlin-Charlottenburg, Germany.

If a central governmental bureau takes over the supervision of all dam construction beginning with the approval of the general design and including operation after completion, the technical safety of dams is guaranteed. Furthermore, it is possible for such a bureau to collect many data on the subject, which could be used successfully in further construction. The systematic collection and application of these experiences are necessary details that cannot be recommended too strongly. The writer could enumerate many instances in which the same errors were repeated because the engineer who designed the dam, or the manager of building operations, was not aware of the experiences already accumulated.

For example, it may be that in order to save money the geological examinations were not sufficiently thorough and, consequently, when excavation began the proper foundation could not be found or was found only at a greater depth than was expected, so that the costs for the dam were considerably higher than calculated; or, possibly, the dam site had to be reflooded and abandoned because the foundation was totally inadequate. Other cases have occurred in the writer's experience, in which the dam was finished and operation was begun, but in which it was impossible to raise the water to the necessary level because the reservoir was not tight enough and could not be made water-tight artificially. In these cases the result was merely a useless dam, an empty reservoir, and considerable sums of money were spent for nothing. Unfortunately, such experiences are gained only through one's own practice or indirectly, as in most cases the most valuable experiences of this nature are not published. Only a continuing State control would enable the collection, systematic use, and dissemination of information concerning the design, construction, and operation experiences of this nature. After all dams of a country have been placed under State control, it is possible to arrange an international exchange of experiences. Thus, every country is served and, finally, all mankind.

State control is also an absolute necessity with regard to a correct calculation of the flood-discharge capacity. The reliable calculation of the spillway requires years or even decades of systematic measurements of rainfall, water, sediment, and evaporation, and it is in the nature of things that these have to be made by State agencies. This is also necessary because all measurements must be based upon the same point of view. The correct dimensions of the spillway are among the most important requisites for the safety of dams. This is of special importance for earth dams, which are never allowed to be overflowed. In Table 1, Mr. Hinderlider shows that 159 of the 293 dam failures (that is, more than 54%), are earth dams, and the principal cause of such failures is found to be insufficient dimensions of the spillway; 44% of the earth dams failed for this reason.

State approval and supervision of dams are necessary in order that their construction may be made according to a consistent plan. Unregulated construction of dams can eventually impair the entire future development in rational use of streams and thus create considerable loss to the nation.

By requiring that measuring instruments be arranged in all newly constructed dams the State can contribute considerably to the safety of dams and the future development of dam construction. To clarify some of the moot questions that affect the safety and economy of dams, it will be necessary not only to install such instruments, but to arrange for continuous observation. The deformation of the structure must be observed constantly; the trend of the interior temperature must be followed; strain-gauges must be arranged for measuring stresses; hydrostatic uplift must be measured, etc. A sufficient number of such observations and measurements can clarify theoretical questions still pending, with the result that dams can then be erected more safely and more economically.

Of course, State supervision should not be allowed to lead to a bureaucratic rigidity and to hamper the free development of personal initiative. Under strict observance of the principles of safety, each person who has anything to do with design and construction must be privileged to make independent proposals, because this is the only way to promote a living technical agency. Out of their cumulative experience, the personnel of the central bureau for dam control will provide the facts that will furnish the basis of the real principles of safety.

One can conclude from the foregoing that it is reasonable to summarize these principles and to publish them as a building code. The opinions of colleagues differ as to whether or not such a code is necessary. Indeed, different opinions are justified by different local conditions. A small country, such as Switzerland, which can be easily surveyed and where conditions are rather homogeneous, can most probably dispense with a building code for dams. Larger countries, such as the United States, however, with greatly varying conditions, can scarcely do without such codes. They should be made uniform for all States, and such special factors as climatic conditions, earthquake hazard, etc., that typify the different parts of the States, can be easily taken into account in the specifications. It would be best perhaps to publish uniform regulations that are valid for all States and to issue supplements for States or State groups in which special conditions require them.

In Germany engineers have long been of the opinion that such regulations are absolutely necessary since they are of special importance, with regard to National economy as well as to safety. The responsibility for their publication comes under the domain of the various States, because the dams are regulated by the various agricultural ministeries. Administratively, this may be justified, but technically it would be more reasonable to issue a code for the entire country. In effect, the new Prussian regulations for dams will be adopted as authoritative for all Germany. The old Prussian regulations are dated 1913 and consequently are antiquated, because dam construction, especially since the World War, has undergone a great, unprecedented development.

State control in Germany has been so effective that (although the dams do not have excessively large dimensions) thus far, not one structure has failed. There are fifty dams more than 25 m. (82 ft.) in height. Dimensions of a few of the more representative types are shown in Table 2. The highest are: The Bleiloch Dam (Item 1); the Edertal Dam (Item 2); and the Schwarzenbach Dam (Item 3). The highest earth dams are: The Sorpe Dam (Item 4); and the Söse Dam (Item 5). Thus far, Germany has no arch dams because the valleys are too flat and, therefore, unfitted for the erection of such dams. German engineers, however, recognize the fact that arch dams are the safest of all types. The Vöhrenbach Dam (Item 6, Table 2) is the only multiple-arch dam constructed to date.

TABLE 2.—Dimensions of Characteristic Dams in Germany

Item	Dam	Total Height		Reservoir capacity, in acre-feet	Type	Date constructed
		In meters	In feet			
1	Bleiloch	65	213	174 000	Concrete	1931
2	Edertal	52	171	162 000	Masonry	1914
3	Schwarzenbach	67	219	12 000	Concrete	1925
4	Sorpe	69	226	66 000	Earth	1932
5	Söse	57	187	20 000	Earth	1931
6	Vöhrenbach	25	82	900	Multiple-arch	1924

In spite of this, the new regulations for dam building include all the principal types of dams, in anticipation of future development and they take into account all experiences that become available, from whatever source.

The German expression, "Talsperre," applies only to "storage dams." The others are called "Wehre," which is translated "overflow dams." The exact distinction between these two types causes great difficulties in practice and cannot always be made absolute. With regard to their construction movable dams are typical overflow dams. To avoid all possibility of misunderstanding, the Prussian Water Law defines storage dams as structures with a height above stream bed of more than 5 m. (16.4 ft.) and impounding more than 100 000 cu. m. (81 acre-ft.) of water. Other structures are considered as storage dams if their failure would involve considerable damage. This supplementary definition shows clearly the purpose of the regulations for dam building; that is, the guaranty of the public safety. Certain exceptions, especially regarding the foundation, are admitted for non-storage dams. Whereas a masonry dam must be founded on unyielding rock, masonry overflow dams may be founded on alluvial soil, sand, gravel, etc., with sufficient precautions.

It must be emphasized that the new Prussian regulations for dam building are not strictly specifications, but rather instructions. The official title is, therefore, "Instructions for the Design, Construction, and Operation of Dams." They were developed by a committee consisting of ten members, including the writer. The work required a session of fifty full days. While these instructions are all based on experience in Germany and elsewhere, the fact is stressed that they are meant neither as textbooks, nor as inviolable specifications, but only as instructions to which the officials who have the power of supervision and approval of the projects, may hold.

The instructions include thirty-five pages and are divided into the following eight main parts: Definition of a dam; preparation, form, and subject of the design; technical requirements; construction, operation, and maintenance by the owner; State supervision of construction and approval; State supervision of operation and maintenance; and example of instructions to the dam superintendent.

The technical requirements are the most voluminous, comprising twenty-six pages. The following types are considered: Gravity dams, arch dams, buttressed dams, and earth dams. These types are considered as main types; of course, the construction of other, newer, dam types is not excluded; it was only impossible to consider them because in the course of the last few years a number of newer types have appeared, the construction of which had to be judged separately in each case.

Regarding foundation conditions the "Code" stipulates that masonry dams must be founded on rock and it requires, in order to secure increased safety against sliding, that the toe of the dam be propped directly against a steep step of the solid rock. The usual grouting is recommended and instruments for the measurement of the hydrostatic uplift must be installed.

Arch dams are permitted only in mountains in which sites may be selected on geologically reliable, sound, and inflexible rock. For arch dams with a height of more than 30 m. (98.4 ft) the "Code" recommends that the computations be checked by experiment with a model. Suggestions are offered for the calculation of stresses due to hydrostatic pressure, dead weight, temperature, shrinkage, etc. The highest allowable compression stress amounts to one-fourth the compression strength; and the maximum tension stress to one-third the tension strength. When reinforced concrete is used, the latter value is raised to one-half the tension strength. Compression and tension strength must be ascertained by 90-day test cubes. It is specified that vertical, radial joints be left open during construction and that they be grouted, after the principal shrinkage is complete, possibly in cold weather. In this way most of the disagreeable stresses caused by shrinkage and decrease of temperature can be avoided.

The "Code" also contains descriptions of calculations and detailed laboratory tests for the higher dams. Construction of earth dams by the hydraulic-fill method common in the United States is not mentioned in the instructions because German engineers are not quite convinced that dams constructed in this way are sufficiently safe.

Ford Kurtz,[25] M. Am. Soc. C. E. (by letter).—The writer wishes to record his complete agreement with the following salient ideas in Mr. Markwart's paper:

1.—There is necessity for introducing greater conservatism and sounder judgment, particularly the latter, in the design of dams;

2.—Critical defects in dams that have failed, have been avoidable in many cases;

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[25] Hydr. Engr., The J. G. White Eng. Corp., New York, N. Y.

3.—Such avoidable defects can be eliminated almost entirely with effective regulation and control;

4.—With human nature constituted as it is, control from within is not to be expected;

5.—The value of State supervision will lie only in requiring an independent review of the design, thus insuring the application of more than one mind to the problem. The construction and operation of a great dam should never be left to the judgment of one man, no matter how eminent; and,

6.—Those in supervisory capacity on behalf of the State Government must obviously be the peers in technical experience and judgment of those designing and constructing the dam.

The requirements set forth in Statements (5) and (6), which are the indispensable elements of "effective regulation and control," can be met only through a permanent State Commission of competent engineers, not less than six in number, appointed by the Governor of the State and approved by the State Senate, the State Engineer to be an *ex officio* member of the Commission. Decisions should probably require a two-thirds vote, those dissenting being allowed to present minority opinions. All administrative and routine work would be carried out by existing agencies, but the absolute authority would rest with the Commission.

No useful purpose will be served unless the State Government brings to the solution of these problems men of skill equal to or greater than the skill of those employed by the owners. Therefore, it is of the utmost importance that authority be given to a commission rather than to a single individual, such as the State Engineer. The Commission must be chosen from competent men active and prominent in their profession. Obviously, such men cannot be precluded from practicing their specialty in the State which they serve, but the writer can see no reason why they need be thus precluded. If a judge is interested in a case directly or indirectly, he does not resign from the bench. He simply withdraws from his judicial post as far as that case is concerned. There is no reason at all why a member of the Commission should not act in the same manner when he is interested directly or indirectly in a project before the Commission.

Of the three features—design, construction, and maintenance—the first and last without question can be handled efficiently by a competent Commission. The second feature presents much greater difficulties and the "occasional inspection," which is suggested by Mr. Markwart, would not only be inadequate, but might be dangerous. Proper supervision of construction would require (at least on major projects) the residence of a Governmental Inspector. Construction problems must often be settled quickly. Unless the Inspector is a man of qualifications equal to those of the Resident Engineer and the Construction Superintendent, his decisions will be of doubtful value.

Care must be taken to see that the Commission is not led insidiously into designing the dam. It must pass upon designs only and, if they are not approved, it must indicate the general basic reasons for disapproval. Details of methods of correcting disapproved features should be left to the owner's engineer.

"Effective regulation and control," such as outlined briefly herein, will be valuable to the owner, and therefore he should pay for it and pay liberally. Only by the payment of adequate compensation to members of the Commission and to the Resident Inspector can men of proper skill be induced to accept these duties. The public should pay some, but certainly not the major, portion of the cost.

As pointed out by Mr. Markwart, appeal to the Courts will always remain as the owner's right. That cannot be taken away from him. It would be his great safeguard against an incompetent or "political" Commission. It is not practicable to codify rules for the design, construction, and maintenance of dams.

WILLIAM W. TEFFT,[26] M. AM. SOC. C. E. (by letter).—In the ideal State every activity affecting the safety of life and property should, and would, be subject to State supervision. The ideal condition presupposes availability of infallible public servants having expert knowledge of the subjects under their supervision. If these ideal conditions could be attained, no doubt State supervision of dams would be beneficial.

Unfortunately, it will be difficult, if not impossible, even to approximate this under present political conditions. The great difficulty will be to find supervisors who have adequate experience and knowledge to fill the requirements and who, if found, would be willing to accept a subordinate position in some State Engineer's office. The safety of dams will not be guaranteed by entrusting supervision to "young Civil Service employees who may pass adequate examinations after coming freshly from universities, qualifying them to act as aides to the State Engineer.

Under the practical application of such an act, instituting public supervision of dams, plans prepared by some consulting engineering office having command of a lifetime of experience, would be placed under criticism, and subject to amendment, by junior employees having only occasional or book knowledge of the problems under consideration. This would not necessarily occur always, but undoubtedly it would happen in many cases. It would lead to disputes and delays and would tend to abandon altogether the construction of otherwise meritorious projects.

State engineers must exercise authority in many branches of engineering. It is possible, but not often the case, that the State Engineer is also an expert in the design and construction of dams. The problem would be referred to a special department in the State Engineer's office. Disputes would arise in which the promoter's or owner's engineers would be overruled by State authority.

Such conditions already exist in large corporations, and will be accentuated under State supervision. The Corporate or State authority, if desiring not to act on its own responsibility, is quite apt to call in some outside expert perhaps less familiar with the subject under consideration than its originators or designers, with the result that there are more delays, changes of unwarranted nature, extra expense, etc. In a recent case a $250 000 spillway was added by an outside expert and dictated solely by general considerations wholly inapplicable to the region in which the amended structure was located. It did not increase safety, and its cost was wasted.

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[26] Cons. Engr.: Vice-Pres., Fargo Eng. Co., Jackson, Mich. Mr. Tefft died June 24, 1932.

Difficulties of supervision are increased by the present state of knowledge regarding the design of dams. A great many elements entering into such design are matters of experience, and wide diversity of opinion exists. Structures built for private undertakings are necessarily planned with the utmost economy. The expert consulting engineer who specializes all his life in dam design knows, or thinks he knows, how far economy in design can be carried without endangering the structure. The State supervisory body on the other hand would be inclined to be ultra-conservative, partly because of lack of long and exclusive experience, partly because the very object of its interference is to guarantee safety only. Economical considerations would not be the primary guide of the public supervisor. Disputes would be inevitable. Projects would be hampered and possibly entirely abandoned.

On the other hand, changes ordered for the purpose of increasing safety might have exactly the opposite effect, as previously mentioned. In a given State there might be two outstanding consulting engineers having experience with dams on earth foundation. One is widely known and in all probability would be chosen to supervise the dams of that State partly because of his political activities. He has designed eight or ten dams, two of which have failed. The other consultant has had to do with about seventy economically designed dams built mostly on earth, no one of which has failed. The point is that there is more risk to safety in choosing the supervisor than in the dams themselves as now being built throughout the United States. On a percentage basis the failure of dams is very small—less than for almost any other business in which money is invested or lives are jeopardized.

The design of dams is at present not an exact science. The National Electric Light Association appointed a sub-committee to gather "experience with different types of dams." This Committee compiled replies on a questionnaire pertaining to the problems involved in the design and construction of dams. The answers received from about twenty-six of the leading designers of dams in the United States showed the wide diversity of opinion among experienced engineers relative to many questions of a fundamental nature involved in the design of dams of various types.[27]

It will be quite impossible to formulate a State code for guidance in dam design, similar to the city building codes in existence. The principles followed in dam design are still very much in a state of flux; each individual case offers a variety of possible solutions. If a code cannot be made to cover the design of dams successfully, the critical examination of projects by a State Examiner will be based solely on personal judgment. This contains so many elements of uncertainty that the net effect will be to delay and hamper the successful launching of new projects.

Many projects for hydraulic power as well as irrigation are at present (1932) just within the "economic limit." Possible extravagant changes in design in behalf of unnecessary safety will cause them to be abandoned altogether. Dams for domestic water supply are more often able to bear additional cost that can be charged for in a higher rate for water. A large number of private irrigation and power projects, which have been in successful operation for many years, if required to be designed according to Government standards, would never have been built. The old Bear Valley Dam is a case in point.

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[27] Progress Rept. of Sub-Committee, Hydraulic Power Comm., National Elec. Light Assoc. (not published).

The cry for governmental supervision of dam design is largely a matter of public hysteria, unwarranted on the basis of aggregate losses caused by failures among these structures. In the last fifty years there have been only two cases of dam failures that have attracted nation-wide attention, namely, the Johnstown and the St. Francis disasters. Such cases should not be permitted to recur, but State supervision can scarcely be expected to act as a guaranty. Under an ideal Government it might so act, but under existing political conditions in State Government, the public supervision of dams would merely impose additional hardships on a branch of enterprise already hard pressed by competitive and economic conditions, without offering the compensating guaranty of any better results than are now attained under private initiative.

A. H. MARKWART,[28] M. AM. SOC. C. E. (by letter).—Consideration by the Society of the subject of Public Supervision of Dams has invoked liberal discussion from its members. This discussion, which represents the opinions of engineers interested or engaged in the design and construction of dams, should be of much value in forming a basis for the preparation of future legislation.

The writer's opinions, as expressed in his paper, were briefly as follows:

(1) While it is to be regretted that resort to Government is necessary to effect correction, State supervision of dams is apparently to be had.

(2) Supervision should be conducted through the medium of existing governmental machinery—the State Engineer, or equivalent—for administration, and the Courts for enforcement.

(3) Administrative duties should be vested in one office, and the statute should be confined to basic principles, leaving the details to be covered by regulations and rules.

(4) Supervision should be limited to the approval of the location, design, construction, and maintenance of the dam in so far as safety alone is concerned.

(5) No dams should be exempt from supervision because of ownership.

(6) Small dams that cannot become a menace should be exempt from supervision.

(7) The burden of the cost of supervision should be divided between the owner and the State.

(8) Provision should be made for appeal either to the Courts, or to a board of arbitrators, in the event that the owner is reluctant to follow the

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[28] Vice-Pres. in Chg. of Eng., Pacific Gas & Elec. Co., San Francisco, Calif.

decision of the supervisory official. Such appeals should be expedited to avoid delay.

(9) Legislation should provide for the inspection of existing dams and should confer power on the supervisory agency to order necessary remedial measures.

(10) Routine inspection should be made of all important dams.

(11) It is suggested that a "minimum requirement" code be developed, although this may prove to be difficult or impracticable.

It is interesting to comment on the manner in which the foregoing points impressed the members who discussed the paper. In general, State supervision was considered desirable, although Mr. O'Shaughnessy and the late Mr. Tefft commented adversely on this point. Mr. Hanna believes that State supervision will not meet all problems, and suggests some method of National control for intra-State projects.

In the matter of supervisory personnel, Mr. Parsons suggested for important structures the appointment of a supervisory board, to approve or reject the design and foundations of the dam. In the event of dispute, an appeal would be made directly to the highest Court having jurisdiction. Technical matters under appeal would be referred to a new board appointed by the Court. Mr. Kurtz suggested the appointment by the Governor of a permanent State Commission of not less than six competent engineers, and the vesting of all authority in the Commission, which would use existing agencies for carrying on routine duties.

In the matter of appeal, most of the comment was in favor of an appeal to a board of arbitration rather than to the Courts, in order to minimize delays.

The subject of the practicability and desirability of a code was discussed with considerable difference of opinion. Several thought a code desirable and practicable; others thought it desirable, but either difficult or impracticable of formulation at this time; while still others considered it undesirable in any event. If the consensus of opinion is correctly interpreted, it seems that some kind of minimum requirement code, general in its terms, but not incorporated in the law, is considered desirable, and will ultimately be developed.

In conclusion, the writer desires to reiterate that, in his opinion, the real values of State supervision lie in the independent review of the original design and construction, so the problem is not entrusted entirely to one man, and in the continuity of technical inspection and control long after the interest of the designers and constructors in the structure have terminated.

M. C. HINDERLIDER,[29] M. AM. SOC. C. E. (by letter).—As one of the authors of this Symposium, the writer desires to express his deep appreciation of the interest shown by the profession in this important subject, as manifested by the numerous interesting and valuable discussions. Such discussions have brought out many valuable points pertinent to the subject, and not covered by the authors. With but one or two exceptions, they disclose a unanimity of thought as regards the necessity for public supervision of dams and the essentials to be incorporated in such supervision.

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[29] State Engr., Denver, Colo.

Regulatory laws covering public supervision of dams, as suggested by Mr. Parsons, would be too unwieldly and unnecessarily burdensome, would result in great delay in initiating construction, and would be impracticable of efficient administration. No distinction should be drawn as between high dams impounding large volumes of water, and those of nominal heights impounding comparatively small quantities of water. Experience, particularly in the Western States, shows that even nominal volumes of water, when suddenly released, create momentary discharges far in excess of the capacity of the natural channels to carry them, and hence cause great devastation to areas lying below such structures. A dam of only nominal height may create a greater menace, under certain conditions, than one of much greater height and impounding capacity.

Since 1925 the only two dam failures in Colorado (where there are about 1 200 dams under public supervision) have been small earth structures, each about 17 ft in height, impounding less than 100 acre-ft; yet in each case a life was lost and considerable property damage resulted. Hence, there should be no distinction drawn as regards the degree of public supervision of dams which impound even inconsiderable quantities of water, since such impoundment may be located immediately above habitations, arteries of transportation, and valuable property rights. In this connection, the writer agrees most heartily with the views expressed by Mr. Creager.

The writer is in accord with the objections of Mr. Eckart concerning the danger of political influence entering into State supervision of dams. As in all functions of public officials vested with police authority, the opportunity is present for the transgression of the rights of the public. Apparently, the only remedy for this danger is reliance upon Court review of the acts of the administrative official, which right is generally conceded, whether or not specifically provided by the laws clothing the administrative official with his authority. Undue restriction upon the authority of the administrative official only tends to nullify the objective sought, that is, the protection of the public as to life and property, which may be placed in jeopardy by the erection of works of the nature under consideration. Where the public official charged with the supervision of dams is either incapable or over-conservative in the interests of the public, to the detriment of the owner of a dam, and where the situation is sufficiently justifiable, the owner can always secure redress through the Courts.

The suggestions of Mr. Hanna respecting public supervision of dams that may be constructed on an interstate or international stream, are very pertinent, and are food for much careful thought, especially since in the future the construction of dams of material size will be, in many instances, on stream systems, interstate or international in character. This problem is twofold: First, in its effect upon the relative rights of the States or nations, with respect to the utilization of the waters impounded by such structures; and, second, as regards safety of life and property in an adjoining State below such structures. In the arid regions of the West, the actual consumptive use of the natural water supplies is essential to existence and habitation, while the use of this all-important resource for domestic, sanitary, and manufacturing purposes, is equally vital to the inhabitants of the humid regions. The writer believes firmly that, prior to the construction of any regulatory dam of any considerable size, on any interstate stream, a determination should first be made of the relative interests of each State affected, and proper provisions made for safeguarding such interests and rights, both in the actual uses of the waters thereof, and for the protection of life and property below such structure. Adjustments as regards water utilization may best be effectuated through interstate agreements or compacts. Failure to attain this end results eventually in suits in equity between the States, which can only be decided by the Supreme Court of the United States. Such procedure results in long drawn out litigation of a most expensive nature, and such decisions, of course, cannot be mutually satisfactory. Questions of design and safety may best be determined by joint high commissions appointed by the sovereignties whose rights are affected.

Discussions of the new law of California, covering the construction and supervision of dams, by Messrs. Steele and Dreyer, and by Mr. Hawley, are most interesting and valuable, especially to those in other States, where more rigorous laws for public supervision of dams are being considered.

Plans and specifications for two of the largest dams ever contemplated (San Gabriel and Pine Canyon), were presented to the State Engineer of California immediately following the adoption of the present California law, and, in this connection, the writer had the privilege of acting as a member of a commission appointed by the State Engineer of California to pass upon the adequacy of the plans submitted and the suitability of the sites for such dams. As a result, the functioning of this outstanding attempt by the people to apply complete supervision to works of this nature has been watched with interest.

Dr. Kelen's discussion of the laws governing public supervision of dams in Germany is also most interesting and profitable, and indicates rather definitely that a code embodying the essentials for safety, is workable. The new regulations covering public supervision of dams in Prussia, entitled "Instructions for the Design, Construction, and Operation of Dams," is a most valuable addition to the common fund of information on this important subject. It is interesting to note Dr. Kelen's comments that thus far "German engineers are not quite convinced that dams constructed by the hydraulic method are sufficiently safe." The writer quite agrees that, in the absence of the application of proper engineering knowledge and experimental data pertinent to such design, such types of dams are pregnant with danger. Nevertheless, Terrace Dam, across the Alamosa River, in Colorado, has withstood the test of time most successfully. This dam is about 175 ft high, and all but the top 32 ft was constructed by the hydraulic method. The materials used, however, lent themselves admirably to such type of construction. Such requirements, of necessity, limit the presence of clayey materials to the minimum necessary for providing the requisite degree of imperviousness.

The completion of the new Eleven-Mile Canyon Dam by the City of Denver may be mentioned as an instance of effective co-operation between the owners of such a structure and the public officials charged with the duty of supervision over both design and construction methods. This dam is of the semi-gravity arched type, of a maximum height of about 150 ft., constructed at a cost of about $1 250 000. This is one of the best constructed and most beautiful structures of the kind to be found in America. The plans were prepared by the engineers of the Board of Water Commissioners of the City of Denver, and submitted to the State Engineer of Colorado, for approval. They were then referred to the Chief Designing Engineer of the U. S. Bureau of Reclamation for consideration and criticism. Following such studies, the writer felt that a design involving a length of radius between the two limits suggested by the City and the Government Engineers would best fit the topographic conditions at the dam site, and the final draft of the plans was predicated upon this determination.

Following the preliminary stages of excavation of the foundations for the dam, experienced geologists were retained to pass upon the geological features of the foundations. Fortunately, the foundations are in granite of excellent character. The only undesirable feature discovered was a fracture extending diagonally across the right abutment near the top of the dam. Prior to the placing of concrete, the writer made a critical examination of every foot of the foundation area and designated points where pressure grouting was to be applied. All materials that entered into the construction of the dam were manufactured from the granite excavated from the spillway section. The City provided a well-equipped laboratory on the ground for testing all aggregates and concrete. Tests were made under the immediate supervision of a representative of the State Engineer, who remained constantly on the work and sampled every day's pour of concrete, and also the pressure grouting in the foundations. Records of each day's pour of concrete, and the gradation and other characteristics of the concrete aggregate, were compiled by the State Inspector, and they constitute a complete and valuable record of the history of the construction of this dam. Throughout the period of construction the finest spirit of co-operation prevailed between the City's engineers and inspectors, the contractor, and the representatives of the State Engineer.

During the same time the Penrose-Rosemont Dam was also under construction in Colorado. The designing engineer for this rock-fill dam kept constantly in touch with the work. Prior to the commencement of construction he conferred many times with the State Engineer in the preparation of his designs, and during the construction period, retained on the work an engineer qualified through experience to supervise the work carefully. As in the case of the Eleven-Mile Canyon Dam, competent geologists were retained to pass upon the foundation conditions, which were much more complicated than at the Eleven-Mile Canyon Dam, but which were treated successfully through deep pressure grouting and drainage. This latter dam involves some unique features, in that the water face is rendered impervious by means of a steel diaphragm laid over the up-stream face of rubble masonry, and electrically welded in place. Contacts between the diaphragm and the foundations consist of concrete emplacements overlying sound but fractured bed-rock. The seams and rock fractures were sealed later by heavy pressure grouting, through two lines of holes drilled at suitable depths through the concrete toe-wall.

No little part of the discussions of the paper related to a proposed code to cover the design and construction of dams, the opinions being about equally divided concerning the desirability of such a code.

It was not the writer's thought that a general code covering the entire field of dam design and construction could be devised, which would be applicable to all conditions that might arise, since it is realized that each dam and the controlling conditions appertaining thereto are problems within themselves, and accordingly must be treated as such. It was the writer's feeling, however, that a code covering the fundamentals that generally govern the design for any type of dam, might well be adopted for the guidance of State officials charged with the duty of passing upon such matters. These fundamentals would include such items as perviousness and permanency of foundation and structural materials, limitations on unit stresses and pressures, factors covering uplift and drainage, temperature changes, effect of earth movements, or temblors, etc. It is realized that a public official charged with the duties of passing upon the adequacy of design and foundation conditions, may not be fully versed in such matters, and, as a result, he might defer to the presumably superior knowledge and experience of the designing engineer. If, however, he was required by law to take into consideration all the fundamentals appertaining to the final safety of the structure, it would, to this extent, eliminate many probabilities of faulty design and construction, and to that extent would be in the interests of safety to the public. There are innumerable instances in which the magnitude of the proposed work does not justify the employment of expert advice, other than that furnished by the approving State official. Where his duties are not specifically enumerated in sufficient detail, the State official is prone to depend upon his own experience and judgment, which may not be justifiable. As a result of his official approval under such conditions, the best interests of the public, as well as of the owner of the structure, are not properly conserved. To this end the writer believes that a limited code, involving the fundamentals of dam design and foundation requirements, could be incorporated in a code with great advantage to the public, even if, with the advancement of the art of engineering, such code would probably have to be amended from time to time.

Acknowledgment.—The writer also desires to acknowledge the use of the report of "Hydraulic Power Committee, Engineering Section, J. E. Stewart, M. Am. Soc. C. E., Chairman, Presented at the Twenty-third Annual Convention of the Pennsylvania Electric Association," and Mr. Stewart's tabulation of dam failures, and to extend to Mr. Stewart full credit for the use of such information in the writer's compilation of the failures of 305 dams in the United States and foreign countries. In the bibliography of such failures noted on the tabulated list incorporated in the writer's paper, due credit for the foregoing reference was, unfortunately, omitted in the writer's compilation of the list of failures.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPER NO. 1845

ECONOMIC PROPORTIONS AND WEIGHTS OF MODERN HIGHWAY CANTILEVER BRIDGES

BY J. A. L. WADDELL,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. HENRY C. TAMMEN, ROBERT W. ABBETT, FRANK W. SKINNER, CHARLES M. SPOFFORD, R. WAYNE LINCOLN, HAROLD E. WESSMAN, HERSCHEL H. ALLEN AND WILSON T. BALLARD, AND J. A. L. WADDELL.

SYNOPSIS

During the last decade there has been quite a development in the construction of long-span cantilever highway bridges; and a number of such structures have been designed in the writer's office. As the computations have always been made and recorded systematically, it was practicable to obtain weights of steel per linear foot of structure for the various divisions of the metal, namely, trusses, floor system, lateral system, anchorages, and on piers; and with these, by some established empirical formulas of transition, it was feasible to pass from known weights per foot in any layout to the corresponding weights per foot in a different but somewhat similar layout, and, in that way, and by means of moment-area diagrams, to determine all the economic functions for this type of highway bridge.

In 1897 the writer made a similar, but much less extensive, research for single-track railway cantilever bridges, and published the results in his little treatise, "De Pontibus," now out of print, but absorbed in his larger work, "Bridge Engineering."

As was anticipated, the economic proportions proved to be somewhat different in the two types of bridges; because in the first case the structures investigated were narrow, their live loads were comparatively large, and their dead loads were small; whereas in the second case the structures investigated were generally wider, their live loads were comparatively small, and their dead loads were great, owing to the heavy highway flooring.

NOTE.—Published in February, 1932, *Proceedings*.

[1] Cons. Engr., New York, N. Y.

The investigations thus made permitted the writer to prepare diagrams of total weights of metal per square foot of floor for modern, highway cantilever bridges up to main openings of 1 200 ft., beyond which it is not worth while to go, because, under all but very exceptional conditions, a suspension highway bridge is cheaper for that length of span.

To round out the paper there have been added diagrams of total weights of metal per square foot of floor for simple-truss spans, in both silicon steel and carbon steel. It is shown that silicon steel for cantilever bridges is always cheaper than carbon steel, and that for present (1932) unit prices the dividing length in simple-truss spans ranges from 200 to 300 ft., according to the width of the structure.

From these various diagrams and the computations involved in their preparation, some interesting and useful deductions have been drawn.

INTRODUCTION

During the last fifteen years, the building of large and costly bridges over the navigable waters of the United States has been gradually on the increase, until, to-day (1932), there are projected, under construction, and completed, numerous structures of this type scattered throughout the entire country. When the spans required are excessively long, suspension bridges are adopted; and when they can be made reasonably short, simple-truss spans are used; but, in the majority of cases, where the rivers crossed are wide, the cantilever type of bridge is found to be the most suitable. Such being the case, an economic investigation concerning this class of structure is certainly in order; hence, this paper, upon which the writer has been working (in a rather desultory manner, it must be confessed), during the five years, 1926–1930, inclusive.

In addition to the writer's studies of the long ago (to be described later) concerning the economics of railway cantilever bridges, investigations have subsequently been made on the subject by the late Edgar Marburg, M. Am. Soc. C. E., and Henry S. Jacoby, William H. Burr, and D. B. Steinman, Members, Am. Soc. C. E., but their computations generally consisted of theoretical analyses based upon approximate assumptions, in order to simplify the mathematics involved. These assumptions do not represent with sufficient accuracy the actual governing conditions; hence the results of the various researches are not in accord. They need a check by some practical method; and it has been the object of this research to furnish such a check. The principal aim of these investigators was to determine the economic length of the suspended span in the usual type of three-span cantilever bridges; but, as indicated hereinafter, there are a dozen or more variables involved in the solution of the problem, and these, with their varying importance, must unavoidably produce divergent results.

HISTORY

About thirty-five years ago, while preparing the manuscript of "De Pontibus," the writer had his office force compute, for Chapter V thereof, a number of complete designs and estimates of quantities of materials, in order to determine the economic functions and weights of metal for single-track railway, pin-connected, carbon-steel, cantilever bridges. The principal results of the investigation, as indicated in that little treatise, were as follows:

(*A*) The economic length of the suspended span, in an ordinary three-span structure, is three-eighths of the length of the main span, measuring from center to center of piers.

(*B*) Given the total distance between centers of anchorages in such a layout and *carte blanche* as to the location of the two main piers, the economic length of each anchor arm is two-tenths of the total distance between centers of anchorages.

(*C*) When given a fixed distance between centers of main piers, to determine the economic length for the anchor arms, it was found that, for æsthetic reasons, the length of each of the said arms should not be less than 20% of that of the main opening, or, say, 15% of the total distance between centers of anchorages, greater lengths than this minimum being preferable for appearance.

(*D*) The economic truss depth or tower height over main piers is about 15% of the length of the main span.

PRESENT STUDY

The *vera causa* of this paper was a statement made about five years ago by the writer's partner, Shortridge Hardesty, M. Am. Soc. C. E., to the effect that the economic functions of modern, highway cantilever bridges with their comparatively great dead loads, small live loads, and wide roadways, are likely to differ materially from those of old-time, single-track railway cantilever bridges with their comparatively great live loads, small dead loads, and narrow roadways. In his various designs for long-span, highway cantilever structures, Mr. Hardesty had not been governed at all closely by the "De Pontibus" findings; hence, the writer's decision to make a new set of economic investigations for highway cantilever bridges instead of railway cantilever bridges and for modern, instead of old-time, conditions.

The principal economic functions that required determination were the following:

(*a*) Proportionate length of the suspended span, for an ordinary three-span layout, in terms of the main-span length, when the anchor arms are short.

(*b*) Proportionate length of the suspended span, for an ordinary three-span layout, in terms of the main-span length, when the anchor arms are long.

(*c*) The economic depth of truss over main pier in terms of the main-span length.

(*d*) For the ordinary, three-span, cantilever layout, given the positions of the two main piers, what is the economic length for the anchor arms? In comparing total costs of a group of layouts, the "out-to-out" lengths of all contrasted structures must be made equal by the addition of steel trestle approaches.

(*e*) For the ordinary, three-span, cantilever layout, given the locations of the anchor piers and *carte blanche* as to the two main pier locations, what is the economic proportionate length for the anchor arms?

(*f*) For a layout consisting of a suspended span, a cantilever arm, an anchor span, a like cantilever arm, and a like suspended span, given the positions of the end piers and *carte blanche* as to the two main-pier locations, what is the economic proportionate length for the anchor span?

BASIC DATA

A search of the writer's office records provided the necessary data from seven complete studies with estimates of quantities of materials and costs, namely, two Arthur Kill Bridges for the Port of New York Authority, each having a 42-ft. clear roadway and two 5-ft. clear sidewalks; one Mississippi River Bridge, and one proposed Ohio River Bridge, at Cairo, Ill., each having a 20-ft. clear roadway and no sidewalks; two Cooper River bridges, at Charleston, S. C., each having a 20-ft. clear roadway and no sidewalks; and a proposed Mississippi River Bridge, at New Orleans, La., having a 40-ft. clear roadway and two 6-ft. sidewalks.

The main-span, side-span, and suspended-span lengths of these structures are as shown in Table 1.

TABLE 1.—DIMENSIONS OF CANTILEVER BRIDGES

Structures.	Main span, in feet.	Side spans, in feet.	Suspended span, in feet.
Perth Amboy (N. J.) Bridge over the Arthur Kill	750	375	301
Elizabeth (N. J.) Bridge over the Arthur Kill	672	240	336
Cairo (Ill.) Bridge (as first designed) over the Mississippi River	675	450	245
Cairo (Ill.) Bridge, as later designed	700	450	254.5
Proposed Cairo Bridge over the Ohio River	730	444	265
Charleston (S. C.) Bridge over the Cooper River	1 050	450	437.5
Charleston (S. C.) Bridge over the Cooper River, Town Creek Span	640	256	320
Proposed New Orleans (La.) Bridge over the Mississippi River	1 754*	493	714

* Since increased by the War Department to 1 760 ft.

Besides these data, there were available computations of quantities of materials made for the Perth Amboy Bridge of the Port of New York Authority, with a main opening of 800 ft., and estimates for some other spans of various lengths. It is fortunate that two of the recorded structures have main spans of almost exactly the same length, and that one of these has long anchor arms and the other short ones, because the writer has come to recognize that the economic length of the suspended span may be affected materially by the lengths of the anchor arms. On the other hand, however, it is unfortunate for this investigation that the width of roadway in three of the bridges should be about 40 ft., plus two sidewalks, and in the other five only 20 ft., without sidewalks, because there is more theoretically excessive material in narrow bridges than in wide ones; and, in addition, the wind loads add to the chord sections in a narrow structure, whereas usually they do not in a wide one.

EFFECT OF SUBSTRUCTURE

In determining the comparative economics of different layouts for any cantilever bridge, the variation in dimensions of both the main piers and the anchor piers will affect somewhat the proportionate economic length of the suspended span; but these effects usually cannot readily be forecast or properly evaluated. If the piers rest on bed-rock, the changes in loading on pier tops, even for large variations of length in the suspended span, may cause no increase or decrease of importance in the total cost of substructure, because, in highway bridges, the sizes of piers are often governed by a sense of fitness rather than by the superimposed loads; but where the piers rest on piles or comparatively soft material, their cost will generally be an almost direct function of the superimposed loading.

SHORT-CUTS IN MAKING CALCULATIONS

If, at the outset of this investigation, it had been necessary to prepare a complete detailed estimate of cost for the superstructure of each layout used in locating points on the economic curves, the labor and expense involved would have been too discouraging. Therefore, some "short-cuts" were adopted, so as to secure, with sufficient accuracy for general purposes, the results needed for plotting at least four points on each of the curves. To this end the writer made the following assumptions that he deemed close enough to theoretical correctness for the object in view:

First.—Variations in metal weights from bending and from shear, when passing from known values in one layout to the corresponding desired values in another and somewhat similar layout, are almost in the same proportion; hence no great error would be involved in assuming that bending moments alone will constitute the working criteria for economic investigation.

Second.—After all the bending moments for any layout of spans are plotted to scale, and the greatest direct and the greatest reverse moments for dead load, live load, and impact are computed, then, on the principle of adding one-half of the smaller of two moments of opposite sign to the larger one, the sum of the three moment areas for the layout, when compared with the corresponding sum for another similar but slightly different layout, will give a ratio nearly identical with the ratio of the weights of truss metal in the two layouts.

On this basis the economic length of the suspended span, as far as the superstructure is concerned, can be determined by plotting the moment areas; and from these, all the needed weights of truss metal can be found rather expeditiously, and with sufficient accuracy for the purpose contemplated, by using ratios of average moments.

Third.—While, in computing the dead-load moments over the main pier, due cognizance is taken of the correct center of gravity of the dead load in the cantilever arm, in order to avoid complicated plotting of moments, these are assumed to vary according to the ordinates of a triangle, as are also the moments for the live load on the cantilever arm. This is equivalent to assuming that the entire moment over the main pier is due to an undetermined loading placed at the end of the cantilever arm, and that the latter is stressed accordingly. While the error involved by this assumption may not be small, it applies with about equal seriousness to not too different layouts; hence, the differential of the error may be considered negligible in computing the moment areas.

Fourth.—Similarly, in computing the dead-load moments on the anchor arm, cognizance is taken of the center of gravity of the dead load; nevertheless, the amount of the maximum moment is calculated somewhat as if the dead load were uniformly distributed. The moments are plotted by means of two half parabolas of the same height, but of different lengths, the axis of division passing through the center of gravity. As in the last case, the effect of the incorrectness of this assumption upon the comparison of total areas is not large.

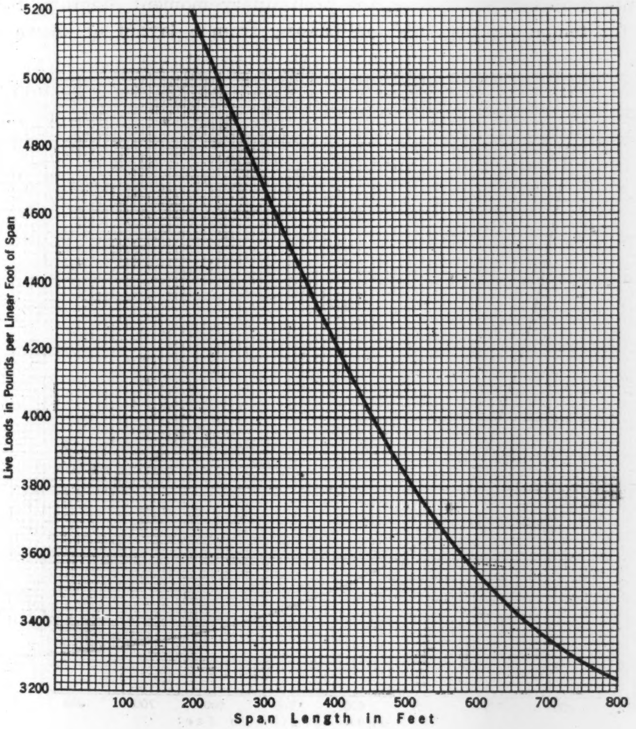

FIG. 1.—TOTAL LIVE LOADS PER LINEAR FOOT OF SPAN FOR THE ELIZABETH AND PERTH AMBOY BRIDGES, IN NEW JERSEY.

ECONOMIC LENGTHS OF SUSPENDED SPANS

In order to determine the economic lengths of suspended spans, two sets of computations were made, one for short anchor arms, and one for long anchor arms, the moment areas were plotted, and the economic span lengths were determined, all on the basis of "guessed" weights of metal for trusses in the cantilever arms and the anchor arms of the uncomputed layouts; but accurate data were available from office records for the corresponding weights in the suspended spans.

A check of the assumed weights of metal showed them to be fairly accurate, but a few of the moment plots had to be slightly modified to suit the final, corrected dead loads. In making these new assumptions, the average moments were computed from the preliminary moment diagrams that were made for temporary use; and, in passing from a layout of known weights of truss metal to a closely similar layout, the corresponding new weights were found by a modification of the writer's long established equation,

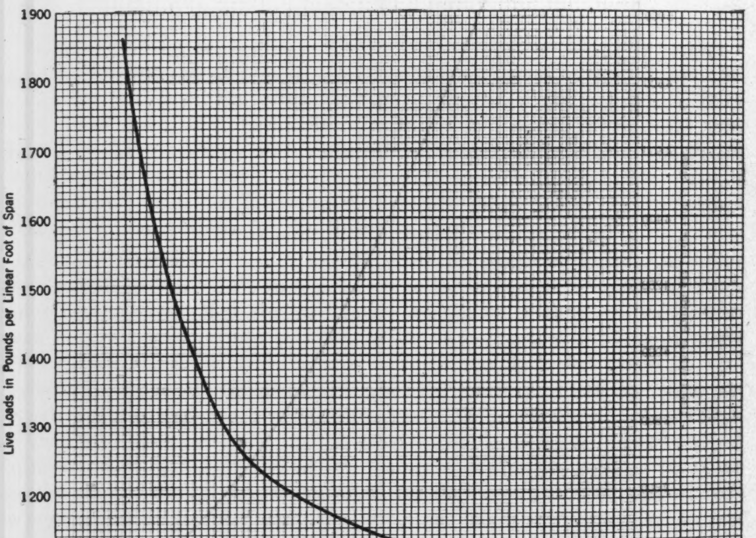

FIG. 2.—TOTAL LIVE LOAD PER LINEAR FOOT OF SPAN FOR THE MISSISSIPPI BRIDGE AT CAIRO, ILL.

$$W' = \frac{W}{5}(1 + 4r) \qquad (1)$$

or,

$$W' = \frac{W}{7.5}\,(1 + 6.5r) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which, W' is the new average weight of truss metal per linear foot of span, W, the corresponding old weight, and r, the ratio of the average moments for their spans.

This correction made little difference in the economic curves first determined; for, as before indicated, the variations from the assumed dead loads were comparatively small.

DATA FROM COMPUTED SPANS

The data for several of the computed structures previously mentioned are shown in the following diagrams and tables: Fig. 1 gives the live loads per linear foot of span for the two Arthur Kill Bridges; and Fig. 2 furnishes that information for the two Cairo Bridges over the Mississippi and Ohio Rivers, and for two of the spans of the Cooper River Bridges, at Charleston. Fig. 3 shows the impact percentages for all these structures.

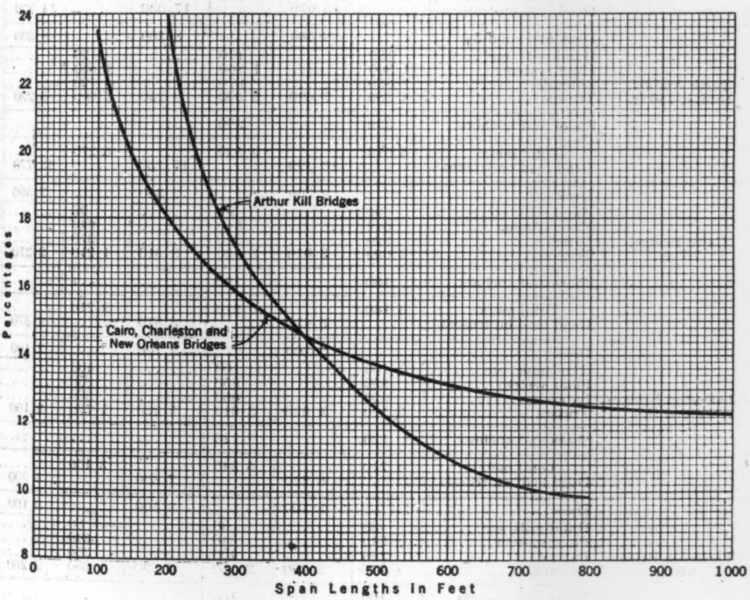

FIG. 3.—IMPACT PERCENTAGES

Weights of metal per linear foot of span, and the dead loads that were used as a basis in making the computations for this paper, are recorded in Table 2.

New Computations

Using the data from Table 2, the writer computed from the Elizabeth Bridge statistics numerous curves of bending moments similar to Fig. 4, for various ratios, r, of suspended span to main span. The total moment area for the anchor arm, the cantilever arm, and one-half the suspended span was marked at the bottom of each sheet. These total moment areas were then plotted in Fig. 5. From that diagram it is seen that the economic length of the suspended span, as far as superstructure metal alone is con-

TABLE 2.—Distribution of Weights of Materials, in Pounds per Linear Foot of Span, in Highway Cantilever Bridges

Name of structure and its main-span length	Portion of structure	Anchor Arm		Cantilever Arm		Suspended Span	
		Metal	Flooring	Metal	Flooring	Metal	Flooring
Elizabeth Bridge, 672 ft.	Deck and conduits	......	8 500	......	8 500	......	8 500
	Hand-rails, etc	120	......	120	......	120	......
	Floor system	2 000	......	1 950	......	1 950	......
	Laterals	810	......	900	......	650	......
	Trusses	4 540	7 470	5 560	8 530	3 050	5 770
	Shoes and anchors	560	......	280	......	......	......
	Total metal	8 030	......	8 810	......	5 770	......
	Dead load on trusses	......	15 970	......	17 030	......	14 270
Perth Amboy, Bridge, 750 ft.	Deck and conduits	......	8 500	......	8 500	......	8 500
	Hand-rails, etc	120	......	120	......	120	......
	Floor system	1 860	......	1 860	......	1 830	......
	Laterals	830	......	970	......	720	......
	Trusses	4 880	7 690	6 300	9 250	2 600	5 270
	Shoes and anchors	320	......	280	......	......	......
	Total metal	8 010	......	9 530	......	5 270	......
	Dead load on trusses	......	16 190	......	17 750	......	13 770
Cairo Bridge, 700 ft.	Deck	......	2 360	......	2 360	......	2 360
	Hand-rails, etc	90	......	90	......	90	......
	Floor system	640	......	640	......	640	......
	Laterals	320	......	380	......	270	......
	Trusses	2 000	3 050	2 400	3 510	1 210	2 210
	Shoes and anchors	100	......	100	......	......	......
	Total metal	3 150	......	3 610	......	2 210	......
	Dead load on trusses	......	5 410	......	5 870	......	4 570
Charleston Bridge, 1 050 ft.	Deck	......	2 100	......	2 100	......	2 100
	Hand-rails, etc	100	......	100	......	100	......
	Floor system	850	......	850	......	850	......
	Laterals	550	......	660	......	380	......
	Trusses	3 120	4 620	3 610	5 220	1 770	3 100
	Shoes and anchors	120	......	120	......	......	......
	Total metal	4 740	......	5 340	......	3 100	......
	Dead load on trusses	......	6 720	......	7 320	......	5 200
Charleston Bridge, 640 ft.	Deck	......	2 100	......	2 100	......	2 100
	Hand-rails, etc	80	......	80	......	80	......
	Floor system	640	......	640	......	640	......
	Laterals	330	......	420	......	250	......
	Trusses	1 740	2 790	2 150	3 290	1 280	2 250
	Shoes and anchors	100	......	100	......	......	......
	Total metal	2 890	......	3 390	......	2 250	......
	Dead load on trusses	......	4 890	......	5 390	......	4 350

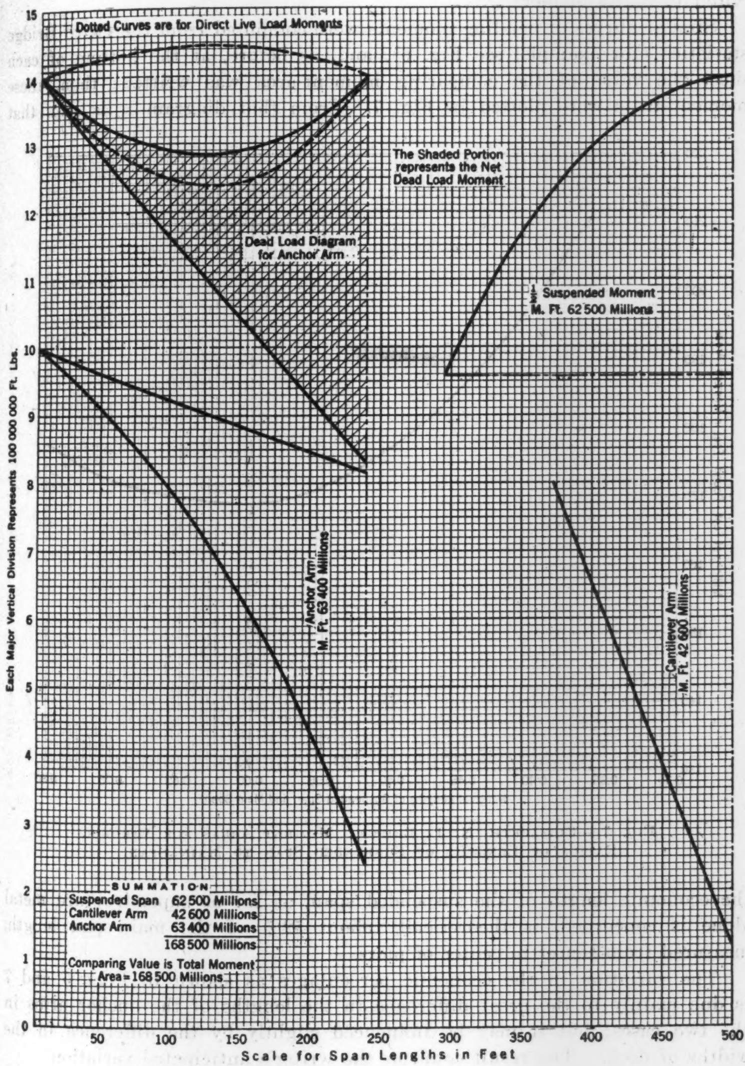

FIG. 4.—MOMENT AREAS FOR ELIZABETH, N. J., BRIDGE (r=0.625).

cerned, is theoretically about 60% of the main-span length, measured from center to center of piers.[2]

Similar curves of bending moments were computed from the Cairo Bridge statistics (for example, see Fig. 6), and, as before, at the bottom of each sheet the value of the comparing moment-area was written; then, these moment-areas were plotted in Fig. 7. From that diagram it is seen that the economic length of the suspended span, as far as superstructure metal alone is concerned, is theoretically about 52% of the main-span length, measured from center to center of piers.

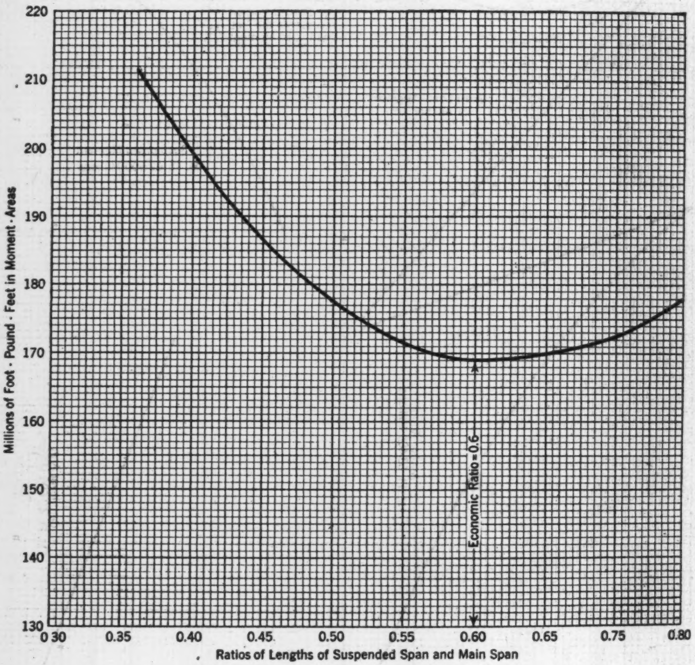

FIG. 5.—ELIZABETH, N. J., BRIDGE: MOMENT AREAS FOR VARIOUS RATIOS OF LENGTHS OF SUSPENDED SPAN TO MAIN SPAN.

The variation in the economic percentages as shown in Figs. 5 and 7 is due mainly to the great difference in the lengths of the anchor arms in the two cases; but it may be influenced slightly by the difference in the widths of deck. The result confirms the writer's anticipated variation.

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[2] As this set of curves has already served its purpose and can be of no further use to the profession, except to afford an opportunity for checking the writer's accuracy and good faith, only a single specimen thereof is herein published; but a complete set of blue prints of all the diagrams of the paper has been placed on file in the Engineering Societies Library, 33 West 39th Street, New York, N. Y.

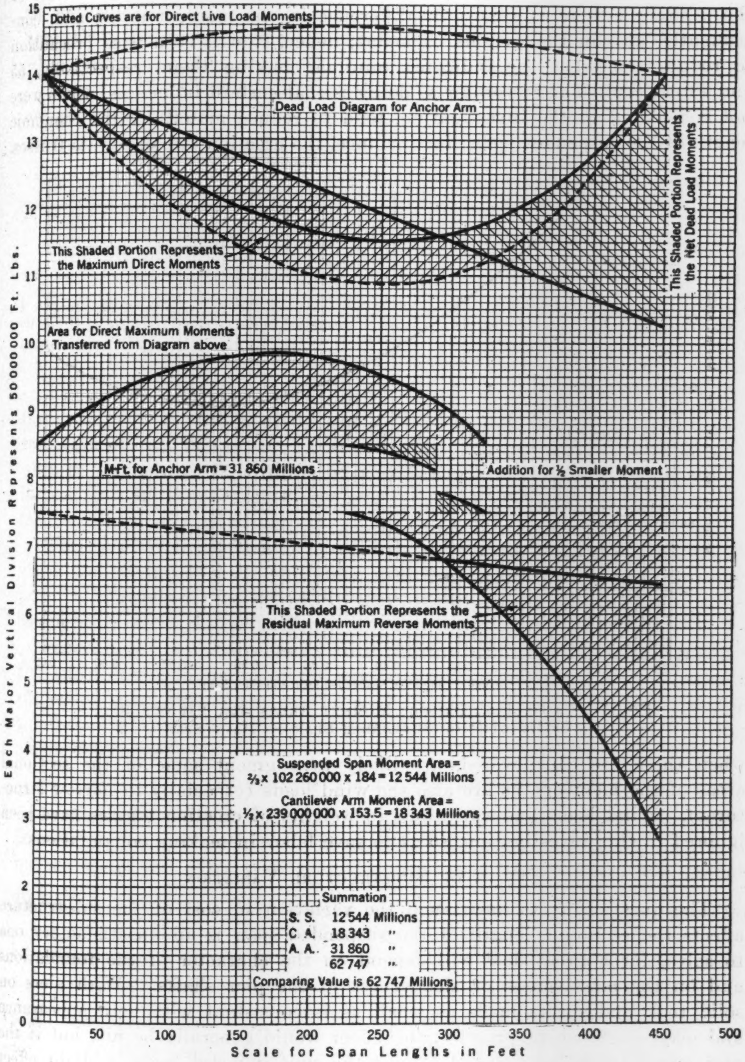

FIG. 6.—MOMENT AREAS FOR CAIRO, ILL., BRIDGE ($r = 0.545$).

It will be noticed that the economic curves of Figs. 5 and 7 are very flat at the bottom, so that for a variation of several points on each side of the minimum there is almost no difference in the moment areas, and, consequently, also in the weights of metal; hence, it will be a wise precaution to reduce somewhat the indicated minima, making them, respectively, 55 and 48 per cent. The reason for so doing is that certain influences, which were not considered in the investigation, tend to reduce materially the economic length of the suspended span; for instance, the erection stresses in its trusses when these are cantilevered, are likely to augment some of the sectional areas of the members, as are also the wind loads (especially in narrow structures). Wind loads and variations in weights of erection toggles have been ignored in this investigation, so as to save labor in making computations.

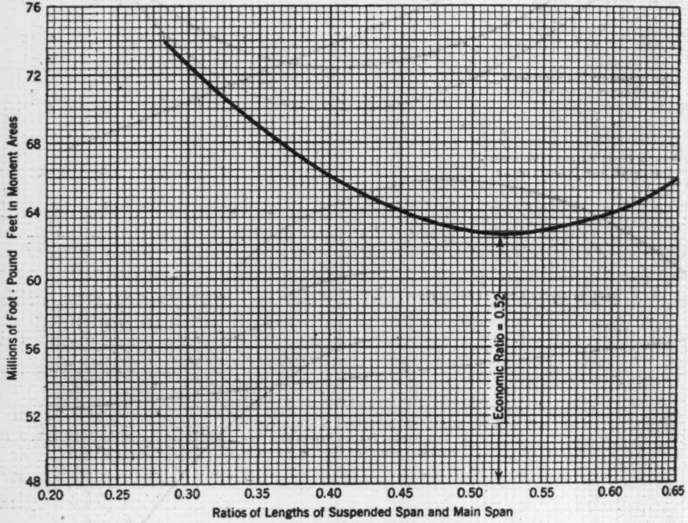

Fig. 7.—Cairo, Ill., Bridge: Moment Areas for Various Ratios of Lengths of Suspended Span to Main Span.

Effect of Substructure Variation

The question as to how much the variation in cost of the substructure affects the economic length of the suspended span is an unsatisfactory one to solve, for a great deal will depend on the character of the foundations and on the massiveness or slenderness of the pier shafts. With piers on solid rock and shafts made as small as is consistent with both appearance and good engineering practice, the effect would generally be nil; but if the pier tops have to be changed to suit the superimposed loads, a slight effect would be found, even with pier bases on solid rock. When, however, the bases are supported on piles, or on soil that is only moderately hard, the effect undoubtedly must be considered.

It is recognized, of course, that lengthening the suspended span reduces the superimposed loads on the main piers, decreases the uplift on the anchorages when the anchor arms are short, and increases the direct load thereon when the anchor arms are long.

Data for the Elizabeth Bridge from the calculations made for this paper are given in Table 3.

TABLE 3.—SUPERIMPOSED LOADS, IN POUNDS, ON PIER TOPS.

Layouts.	ANCHOR PIERS.		MAIN PIERS
	Greatest uplift	Greatest direct loading	Greatest direct loading
For 336-ft. suspended span	1 684 000	None	13 100 000
For 420-ft. suspended span	1 159 000	146 000	12 200 000
Ratios	$r = \frac{1159}{1684} = 0.69$	....	$r = \frac{122}{131} = 0.93$

For rock foundations the saving of 7% in the total superimposed load on the main pier generally would not reduce its size or cost, although under some exceptional conditions it might effect an economy of 3 or 4%; but, when the foundation is on piles, or on only moderately hard soil, the economy is real, and its percentage value may run nearly as high as the variation in the superimposed loading.

The economy in cost of anchorages is truly real for all cases, except only when the minimum size, consistent with appearance and good engineering practice, is reached; and especially when the foundation is on piles or on soil having a small bearing capacity.

While it is not feasible to make a general determination of the quantitative effect of the influence of the substructure on the economic length of the suspended span, this much can be said in relation to the present investigation—that, if there is any such influence, it will tend to augment the length herein determined.

EFFECT OF METHOD OF ERECTION

It should be remembered that the preceding superstructure investigation is based on the assumption of "floating in" the suspended spans, thus avoiding the use of any extra truss metal to resist erection stresses. The probable effect on the economic length from erection by cantilevering would be to reduce the ratio by about five points, thus making it 43% for structures with long anchor arms and 50% for those with short anchor arms. The final results of the preceding dissertation are given in Table 4.

TABLE 4.—ECONOMIC PERCENTAGES FOR SUSPENDED-SPAN LENGTHS COMPARED WITH MAIN-SPAN LENGTHS

Comparative length of anchor arm	For cantilever erection	For flotation erection
Long	43	48
Short	50	55

In the case of very long suspended spans floated into place, it may be economical to shorten them considerably below the theoretical economic length, because of the great expense for barges and hoisting machinery. This is likely to make a difference of as much as five points, or even more under extreme conditions.

Again, in the case of the erection of the suspended span by cantilevering, when eye-bars are used in the bottom chords, it will be necessary to stiffen temporarily these purely tension members; hence a reduction of the percentages given in Table 4 would be logical, say, about five points.

VERIFICATION BY SECTION DIAGRAMS

Fearing that the conclusions from the preceding calculations (because of the use of the moment-area method) may not prove acceptable to the profession, the writer decided to make a test that is reliable beyond any doubt, namely, the comparison, for the trusses of two layouts having different lengths of suspended span, of $\Sigma\, l\, a$ for the anchor arm, the cantilever arm, and one-half the suspended span, in which l is the axial length, in feet, of any truss member and a is the gross area of its cross-section, in square inches, Σ, of course, being the summation sign, in order to observe how the comparison agrees with that obtained by the moment-area method.

TABLE 5.—COMPARISON OF ECONOMY OF POLYGONAL TOP CHORDS AND PARALLEL CHORDS.

Span.	SUSPENDED SPAN, RATIO=0.5		SUSPENDED SPAN, RATIO=0.625	
	Parallel chords.	Polygonal top chords.	Parallel chords	Polygonal top chords
Anchor arm	107 418	107 418	94 192	94 192
Cantilever arm	113 796	113 796	88 733	88 733
One-half suspended span	63 130(*a*)	62 374(*b*)	95 039(*c*)	90 549(*d*)
Totals	284 344(*A*)	283 588(*B*)	277 963(*C*)	273 474(*D*)

Applying this to two of the layouts for the Elizabeth Bridge, in both of which the chords of the suspended span are parallel, there resulted a saving of $2\frac{1}{4}\%$ by the longer suspended span. This result is not absolutely conclusive, however, because for true economy the top chords of the latter should be polygonal; consequently, the writer had one of his assistant engineers recompute the two layouts, using polygonal top chords and adopting the most economic truss depths for all spans in both cases, as closely as the experience of the office force and himself could determine. The results of these various calculations, in pounds of metal, were as shown in Table 5.

The savings indicated are, as follows:

$$(1)\ldots\ldots \frac{a - b}{a} = 1.2 \text{ per cent.}$$

$$(2)\ldots\ldots \frac{c - d}{c} = 4.7 \text{ per cent.}$$

$$(3)\ldots\ldots \frac{A-C}{A} = 2.2 \text{ per cent.}$$

$$(4)\ldots\ldots \frac{B-D}{B} = 3.6 \text{ per cent.}$$

A comparison of Item (1) with Item (2) indicates how rapidly the economy of polygonal top chords over parallel chords in simple-truss spans increases with the span length.

Referring to Fig. 5, the ratio of moment areas for $r = 0.625$ and $r = 0.5$, is $\frac{169}{178} = 0.952$, showing a saving of 4.8% instead of the 3.6% found in Item (4). This indicates that, although the theoretical economic span length determined by the moment-area method is closely correct, the actual variations in economics for longer and shorter spans are not quite as rapid as those found by that method of computation. In any event, it has been proved that the moment-area method of comparison is accurate enough for practical purposes; consequently, it will be used for the solution of the remaining economic problems of this research.

Tabulation of Results to Present Stage

Before passing to such additional problems, however, it will be well to compare the ratio results already found with those of other investigators, as follows:

Henry S. Jacoby...... .... = 0.68
Edgar Marburg .... 0.6 to 0.7, average.......... = 0.65
William H. Burr...... 0.5 to 0.55, average.......... = 0.53
D. B. Steinman....... .... = 0.60
J. A. L. Waddell...... 0.43, 0.48, 0.50, 0.55, average... = 0.49

The average of the averages for the four other investigators is 0.615, while that of the writer is 0.49. The general practice of the writer's firm is to use ratios somewhat smaller than those indicated in Table 4, one reason being the addiction of his office force to the use of the Warren or triangular truss rather than the Pratt or Petit truss. An investigation of this question was made as a check, by M. N. Quade, Assoc. M. Am. Soc. C. E., using the 1 050-ft. main span of the Cooper River Bridge as a basis. Mr. Quade computed six curves, each having three points, using not only the 20-ft. clear roadway of the structure, but also an assumed one of 40 ft., taking carefully into account the effects of wind loads, erection stresses, and minimum sections. He found the economic lengths of the suspended span to vary from 0.44 to 0.46 of the length of the main span, with an average of 0.45; but his curves were very flat between the 0.42 and the 0.5 ratios, the average excess weight of metal for the latter minimum being only 1.2 per cent. This shows that, in adopting 0.5 as a working value for the subsequent economic investigations, no error of importance was made. The Warren truss does not accommodate itself readily to changes in span length. For instance, in the Cairo

Bridge over the Mississippi River, as finally designed, the ratio is 0.36, there being eight panels of 32 ft. each; but if the same style of truss and the same panel length had been used for a longer suspended span, the governing conditions would have forced the adoption of twelve panels and a ratio of 0.54, which did not produce a pleasing layout—at least, to the eye of the designer.

From what precedes, one might be inclined to draw the deduction that the economic length of the suspended span, in respect to total weight of metal in a structure, is a rather uncertain quantity; and in so doing he would be perfectly right, for it is more or less dependent upon the following conditions: Width of roadway; method of erection; type of truss; width of main opening; æsthetics of layout; relative importance of wind loading; proportion of minimum sections involving theoretically excessive metal; proportion of heat-treated metal used, and where; proportionate utilization of alloy steel or steels; proportionate length of anchor arms; anticipated difficulties in erection; effect of substructure costs; use or non-use of eye-bars; and idiosyncracies of the computer.

In view of all these variables, it certainly is difficult to determine for any particular crossing the exact length of suspended span that will involve the absolutely smallest weight of metal for the entire structure; but it will be helpful to remember that it will generally be from 45 to 50% of the length of the main span.

ECONOMIC CONSIDERATIONS OTHER THAN METAL WEIGHTS

It must not be forgotten that there are important factors other than total weight of metal, that influence the economics of a cantilever layout, especially those relative to cost of erection, which usually have a tendency to shorten the suspended span. There is also the variation in costs of substructure, which generally has the opposite effect. Each case as it arises will have to be considered as a special economic problem, the data of this paper being used as a guide rather than a hard-and-fast rule in determining the layout of spans.

ECONOMIC DEPTH OF TRUSS OVER MAIN PIERS

As previously mentioned, in some special investigations for "De Pontibus," the economic truss depths over the main piers for railway cantilever spans was ascertained to be 15% of the main-span length. Dr. Yun Tieng Chang found[3] exactly the same result for modern, highway cantilever bridges as is shown in Table 6.

It will be seen from Table 6 that a variation of unit percentage on either side of the minimum makes little difference in the total weight of metal; and, as a short column is easier to erect than a long one, it might be well to adopt 14% of the main opening as the economic truss depth. The writer's office force has found the best depth to vary between 12 and 15% with an average of 13.5 per cent.

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[3] In his post-graduate thesis for his Doctor's degree, which was prepared in the writer's office.

The longer the suspended span the smaller will be the maximum moment over the main pier, and, consequently, the less will be the best depth there for the trusses. Furthermore, the wider the roadway and the heavier the bridge, the greater, most likely, will be the best truss depth. Again, the Petit truss generally calls for a slightly greater height than the Warren or triangular truss, appearance often being the governing factor.

TABLE 6.—WEIGHT OF BRIDGE WITH DIFFERENT TOWER HEIGHTS

Ratio: $\frac{\text{Tower height}}{\text{Main-span length}}$	WEIGHT, IN KIPS			
	Anchor arm	Cantilever arm	Suspended span	Total
0.13	1 757	610	710	3 077
0.14	1 500	580	710	2 790
0.15	1 425	570	710	2 705
0.16	1 495	565	710	2 770
0.17	1 773	610	710	3 093

ECONOMIC LENGTHS FOR ANCHOR ARMS WITH FIXED LENGTH OF MAIN SPAN

No development in the thirty-three years since 1898 has given the writer any reason for changing his conclusion concerning the best length for the anchor arms of any ordinary, three-span, cantilever bridge, when the main span length is fixed, namely, that for economy in total metal weight of structure it should be made as short as the governing conditions of layout will permit; but that for appearance it should not be less than one-fifth of the main opening or, preferably, a little longer.

ECONOMIC LENGTHS FOR ANCHOR ARMS WITH FIXED DISTANCE BETWEEN ANCHOR PIERS

As before stated, the "De Pontibus" research determined the economic length for anchor arms, when the distance between the anchor piers of an ordinary, three-span, cantilever railway bridge is fixed, to be one-fifth of the total length of the structure, or one-third of the main-span length. Dr. Chang found a somewhat different result, as shown in Table 7.

TABLE 7.—WEIGHTS OF METAL FOR DIFFERENT RATIOS OF ANCHOR ARM TO MAIN SPAN

(Weight, in kips.)

Ratio: $\frac{\text{Anchor arm}}{\text{Main span}}$	Total.	Minimum	Difference	Weight increase, in percentage
0.3	4 345	2 710	1 635	60.3
0.4	3 081	2 710	371	13.7
0.5	2 726	2 710	16	0.59
0.6	2 758	2 710	48	1.77
0.7	2 912	2 710	202	7.45
0.8	3 133	2 710	423	15.6
0.9	3 393	2 710	638	25.2

This shows that, for economy, the anchor arm should be about one-quarter the total length of the structure, or one-half the main-span length. This variation from the writer's old finding is undoubtedly due to the fundamental difference between the two types of structure investigated. The appearance of the layout would certainly be more pleasing with the 0.25 ratio than with the 0.2 ratio originally found for railway bridges.

The problem, however, is one that is not likely to arise often in actual practice, as there are usually conditions of ground or stream that determine the location of the main piers; nevertheless, it is a possibility worthy of consideration, for the ruling limitations might be such as to give the designer *carte blanche* in respect to main-pier locations.

STRUCTURE HAVING A CENTRAL ANCHOR SPAN, TWO CANTILEVER ARMS, AND TWO SUSPENDED SPANS

As generally the "most economic possible" type of cantilever layout consists of a structure having a central anchor span, two cantilever arms, and two suspended spans, it is likely that there will occasionally arise in a bridge engineer's practice the problem of determining the economic layout for such a bridge, when the positions of the outer piers are fixed, and when entire liberty is given as to the location of the two intermediate piers. In such a case the length of the suspended spans should be determined from the already ascertained percentage of the length of an assumed main span in an ordinary three-span cantilever having a length equal to $S + 2C$, in which, S is the length of the side suspended span and C is the length of the cantilever arm to which it is hung. The total length of the structure, L, is $A + 2C + 2S$, in which, A is the length of the anchor span, all as shown in Fig. 8, in which (for convenience in computation) S is taken equal to $2C$, which is nearly, if not quite, the most economic proportion.

The question for solution is: "Given the length, L, what is the economic length for A"? Before attempting to solve it, there is a preliminary investigation that can be made advantageously, that is, "having a layout of spans resembling that illustrated in Fig. 8, and with a fixed length for the side opening ($S + C = 3C$) and certain standard live loadings with all the corresponding truss-metal weights per linear foot of span shown for S and C, what are the weights per linear foot for adjacent A spans of various lengths"? At present, these proportionate weights are unknown to the profession and are sometimes needed.

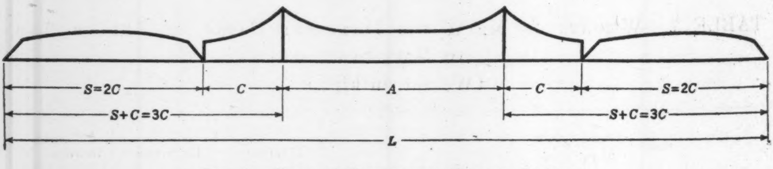

FIG. 8.—TYPE *C* CANTILEVER BRIDGE LAYOUT.

For the investigation, the suspended span and cantilever arms of the Elizabeth Bridge can be used, for which the weights of metal have been already recorded, and in which, $S = 2C = 336$ ft. and $C = 168$ ft. Assume that $A = 300$ ft., $A = 400$ ft., $A = 500$ ft., $A = 600$ ft., and $A = 700$ ft., and find by the moment-area method the weights of metal in the trusses per linear foot of span, the weight per foot of floor metal being constant, and that of the lateral system increasing directly with the span length.

ANCHOR SPAN: 300-FOOT

Using the weights of metal per linear foot of span for the Elizabeth Bridge, as recorded in Table 2, and the live loads and the impact percentages shown in Figs. 1 and 3; assuming a trial truss weight of 12 400 lb. for the anchor span; computing the average moments for both the anchor span and the suspended span, and finding their ratio, the truss weight for the anchor span was ascertained to be 12 250 lb. per lin. ft., which is close enough to the trial weight. It is almost needless to state that it took more than one assumption to obtain a coincidence even as close as that.

ANCHOR SPANS: 400, 500, 600, 650, AND 700-FOOT

Similar calculations resulted in finding the following weights of truss metal per linear foot of span for longer anchor spans:

Length of anchor span, in feet	Weight of metal in trusses, in pounds per linear foot	Length of anchor span, in feet	Weight of metal in trusses, in pounds per linear foot
300	12 250	600	7 850
400	10 300	650	11 115
500	8 320	700	13 970

The standard yardstick for cantilever bridges is the main-span length, or the distance between centers of inner piers, in an ordinary three-span structure, which consists of two anchor arms, two cantilever arms, and a suspended span. Hence, it will be well to refer all truss weights to the average of those of the suspended span and the two cantilever arms, it being understood that the weights of the columns over the main piers are equally divided between the cantilever and anchor arms. It will facilitate computations, both for this research and for future bridge computers, if a satisfactory relation can be established between main-span lengths and the average weights of metal per linear foot for the trusses of the main spans. From data obtained in designing the Cairo and the Charleston Bridges (all spans being of the same general type and loading, and proportioned by the same specifications), the writer has evolved the equation:

$$w' = w \frac{(r + r^2)}{2} \qquad (3)$$

in which, w and w' are the average weights of metal per linear foot, for the trusses of the two main openings considered, namely, l and l', in feet, and

$r = \frac{l'}{l}$. Equation (3) is fairly accurate, provided the lengths, l and l', do not differ too greatly.

Again, for layouts like that of Fig. 13, when changing the length of the main span from l to l', the new weight of truss metal per linear foot in the suspended span, w'_s, is given by the equation,

$$w'_s = r\,w_s \qquad (4)$$

In the same type of layout when making the change, the new average weight of truss metal per linear foot in the cantilever arm, w'_c, is given by the equation,

$$w'_c = w_c\,\frac{(r + 3r^3)}{4} \qquad (5)$$

Returning to the layout in Fig. 8, if w_m is the average weight of metal per linear foot in the trusses for a completed main span, equal in length to

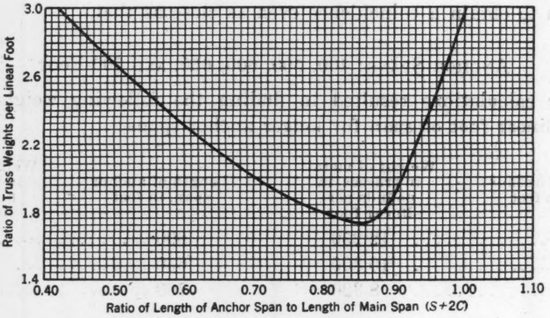

FIG. 9.—RATIOS OF WEIGHTS OF METAL PER LINEAR FOOT IN TRUSSES OF ANCHOR SPANS AND MAIN SPANS.

$S + 2C$ (or, in this particular case, to $4C$, or 672 ft.), a diagram can be made of ratios of weights of truss metal per linear foot in that main span and in anchor spans of various lengths. This was done and is recorded in Fig. 9, which is used later in preparing Figs. 11 and 12.

Continuing the investigation of the economic ratio, $\frac{A}{L}$, in Fig. 8, there is a very simple method that can be used, namely, to find the average truss weights per linear foot of total structure with values of A varying from 300 to 700 ft., still using the Elizabeth Bridge as a basis, in which, $L = A + 6 \times 168 = A + 1\,008$, and in which, the truss weights per linear foot are, as follows:

Suspended span	3 050 lb.
Cantilever arm	5 560 lb.
Main span ($S + 2C$)	4 300 lb.

If $A = 300$ ft., the total weight is:

Suspended spans......$2 \times 336 \times 3\ 050 = 2\ 050\ 000$ lb.
Cantilever arms......$2 \times 168 \times 5\ 560 = 1\ 868\ 000$ lb.
Anchor span..............$300 \times 12\ 250 = 3\ 675\ 000$ lb.

Total weight........................ $= 7\ 593\ 000$ lb.

The total length $= 300 + 1\ 008 = 1\ 308$ ft., and the average weight of truss metal per linear foot $= 7\ 593\ 000 \div 1\ 308 = 5\ 805$ lb.

Similar calculations for nine other cases gave the curve shown in Fig. 10, the economic ratio of A to L evidently being for $A = 565$ ft. In this the total length of structure is $565 + 1\ 008 = 1\ 573$ ft.; hence, $\frac{A}{L} = \frac{565}{1\ 573} = 0.36$. The economic length of the anchor arm is well defined; and the sharpness of the **V**-curve indicates that any material variation from this ratio would be decidedly uneconomic. This economic function of $\frac{A}{L} = 0.36$ holds good for all total lengths of three such openings, as far as cost of superstructure is concerned. If in any case of a very long crossing there is no restriction as to the location of the piers, the best length, L, for three such openings will be determined by an economic adjustment of the costs of substructure and superstructure on the basis of the ratio of the several span lengths being as already ascertained, $S = 2C$, and $A = 0.36L$.

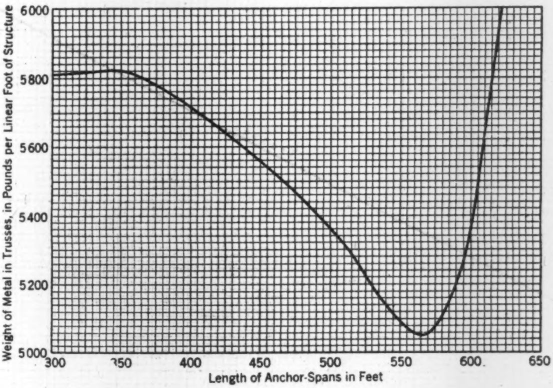

Fig. 10.—Weights of Metal per Linear Foot of Total Structure in Cantilever Bridges of Type C, Having the Span Lengths and the Loadings of the Elizabeth Bridge with Different Lengths for the Anchor Span. Prepared Solely for the Purpose of Determining the Economic Ratio of Length of Anchor-Span to Total Length of Structure, Namely, $R = 565 \div 1\ 573 = 0.36$.

This last investigation has led up to and permitted the preparation of Fig. 12, in which are recorded the weights of metal per square foot of floor for modern highway cantilever bridges of Type C^4. The layout is indicated in Fig. 8, and the metal is mainly silicon steel.

The *modus operandi* of computing these weights was, as follows: In Fig. 8, let $A = xC = 0.36\ L$. Then, $L = \frac{xC}{0.36}$; but $6C + xC = L = \frac{xC}{0.36}$. Therefore, $6 + x = \frac{x}{0.36}$, and $x = 3.37$. Hence, $L = 9.37\ C$.

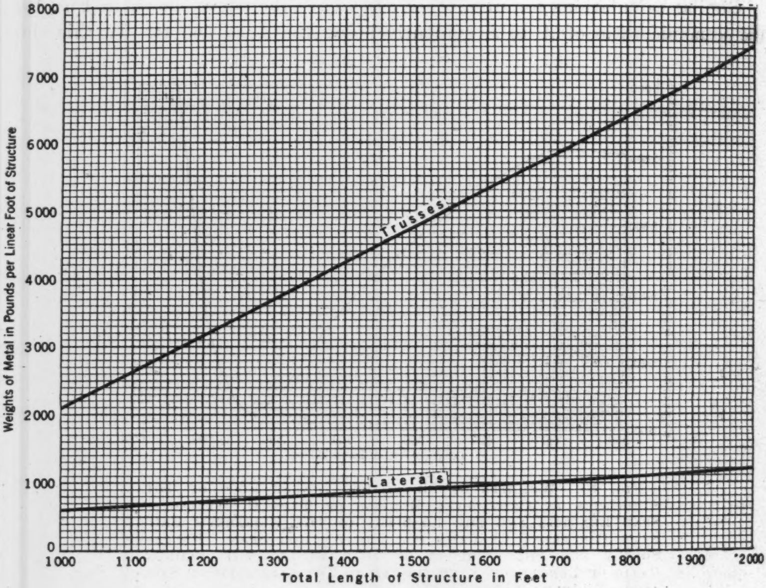

FIG. 11.—PRELIMINARY DIAGRAM OF WEIGHTS PER LINEAR FOOT FOR CANTILEVER HIGHWAY BRIDGES OF TYPE *C*

Still using the Elizabeth Bridge, for which, $C = \frac{672}{4} = 168$ ft., let the weights of metal per linear foot in trusses be, as follows: For the suspended span, w_s; for the cantilever arm, w_c; and for the main opening, $\frac{w_s + w_c}{2}$.

Using Fig. 9 for the ratio, R (on the abscissa line), $\frac{A}{4\,C} = \frac{3.37\ C}{4\,C} = 0.84$. For this, the ratio of truss weights per foot is 1.76; therefore.

$$W_a = 1.76\ \frac{(w_s + w_c)}{2} = 0.88\ (w_s + w_c).$$

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[4] "Bridge Engineering," p. 1271.

Let w_l = the average truss weight per linear foot for the entire structure; then,

$$w_l\,L = w_s \times 2C + \frac{w_s + w_c}{2} \times 4C + \frac{w_s + w_c}{2} \times 1.76 \times 3.37\ C$$

or,

$$w_l\,L = 2\,w_s\,C + 2C\ (w_s + w_c) + 2.97\ (w_s + w_c)\ C \quad \ldots\ldots..(6)$$

Taking a new total length, L', with $\frac{L'}{L} = r$, Equation (6) can be written:

$$w'_l\,L' = 2rw_s\,C' + 2C'\ (w_s + w_c) \times \frac{r + r^2}{2} + 2.97C'\ (w_s + w_c) \times \frac{r + r^2}{2}$$

From the Elizabeth Bridge:

$$w_s = 3\,050 \text{ and } w_c = 5\,560 \ldots\ w_s + w_c = 8\,610$$

then,

$$w'_l\,L' = 2r \times\ 3\,050 \times rC + 2rC \times 8\,610 \times \frac{r + r^2}{2}$$
$$+ 2.97 \times rC \times 8\,610\,\frac{(r + r^2)}{2}$$

or,

$$w'_l\,L' = 6\,100\,r^2C + 8\,610\ Cr\ (r + r^2) + 12\,786\ Cr\ (r + r^2) \ldots\ldots(7)$$

From the Elizabeth Bridge layout, $C = 168$, and $L = 9.37 \times 168 = 1\,574$. From Equation (6):

$$w_lL = 2 \times 3\,050 \times 168 + 2 \times 168 \times 8\,610$$
$$+ 2.97 \times 8\,610 \times 168 = 8\,214\,000 \text{ lb.}$$

Therefore,

$$w_l = \frac{8\,214\,000}{1\ 574} = 5\,219 \text{ lb.}$$

Trying a 2 000-ft. total length:

$$r = \frac{2\,000}{1\,574} = 1.27;\ r^2 = 1.61;\ \text{and, } r + r^2 = 2.88$$

Using Equation (7),

$$w'_l\,L' = 6\,100 \times 1.61 \times 168 + 8\,610 \times 168 \times 1.27 \times 2.88$$
$$+ 12\,786 \times 168 \times 1.27 \times 2.88 = 14\,798\,000 \text{ lb.}$$

from which,

$$w'_l = \frac{14\,798\,000}{2\ 000} = 7\,399$$

Similar calculations were made for several other total lengths of structure, and the results were plotted in Fig. 11, all points falling almost exactly on a right line.

Another right line on the same diagram gives the weights per linear foot for laterals. Combining these and adding 2 280 lb. for weight of metal in the floor system gave the Class *A* line shown in Fig. 12, which includes also weights of metal on piers. Similar calculations determined the Class *B* line.

Fig. 13, compiled from the writer's office data, with considerable manipulation for additional cases, gives a similar record for the corresponding Type *A* cantilever bridges, that type being the ordinary one of three spans, namely, two anchor arms, two cantilever arms, and a suspended span. (See Fig. 13.) These curves contain a proper allowance for weights of metal in anchorages and on piers.

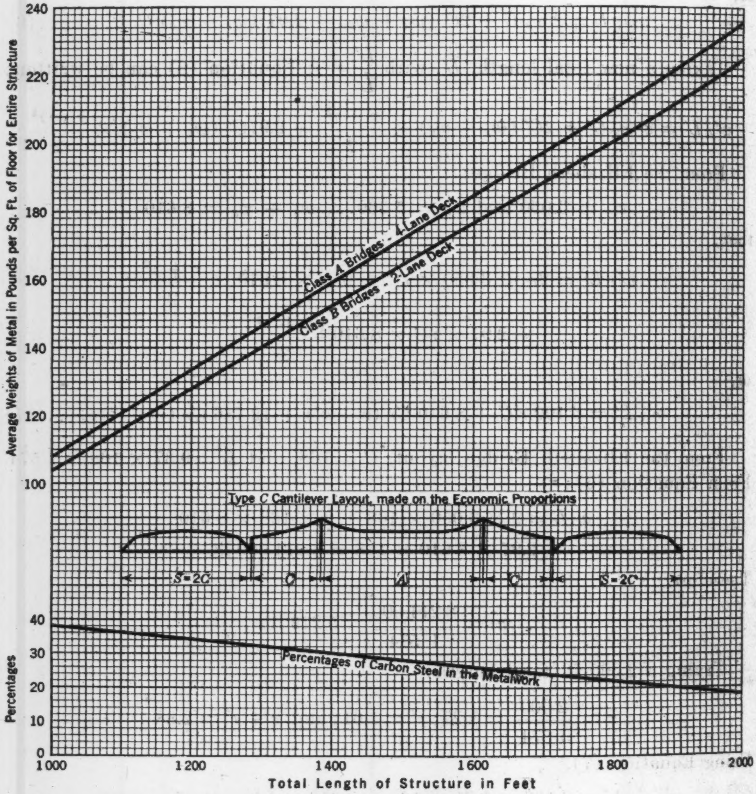

FIG. 12.—DIAGRAM OF WEIGHTS OF STEEL PER SQUARE FOOT OF FLOOR FOR CANTILEVER HIGHWAY BRIDGES OF TYPE *C*.

In Fig. 14 are given the weights of metal per square foot of floor for simple-truss spans, built mainly of silicon steel; and in Fig. 15 are shown the corresponding values of similar bridges built entirely of carbon steel. In Figs. 14 and 15, if the structure has sidewalks, only one-half their area is to be added to the area of the roadway when computing the total area of the floor. These curves do not apply to structures that carry electric railway tracks.

Figs. 14 and 15 will be found useful in determining the comparative economics of cantilever and simple-truss structures; they will also be found useful in determining, for any conditions of the metal market, the comparative economics of carbon steel and silicon steel in bridge work. Each silicon-steel diagram contains a sub-diagram giving the varying proportions of carbon steel in spans of different lengths, because there are always certain minor truss parts, as well as a part or even the whole of the floor and lateral systems, in which carbon steel can be used advantageously.

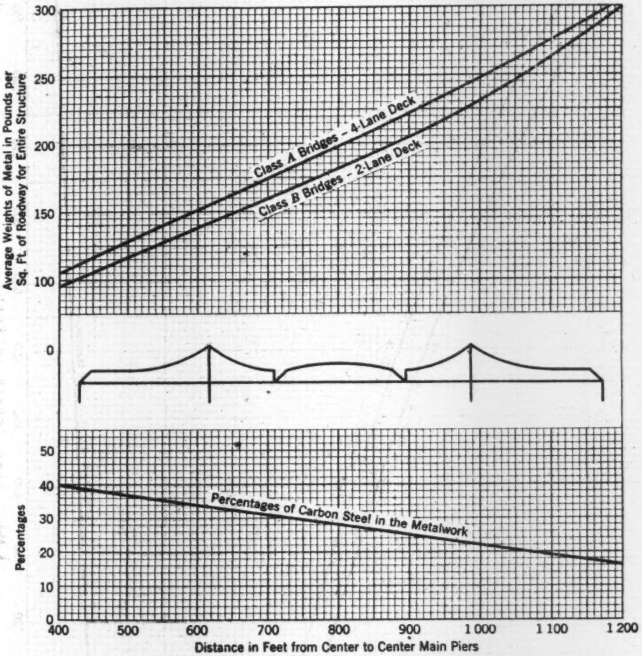

FIG. 13.—WEIGHTS OF METAL PER SQUARE FOOT FOR MODERN HIGHWAY CANTILEVER BRIDGES OF TYPE *A*, OF SILICON STEEL, CARRYING NO ELECTRIC RAILWAY LOADINGS.

The first of these two problems, that is, at what span length it pays to change from the simple-truss to the cantilever type, can be solved by a comparison of Figs. 13 and 14. Adopting the economic proportion, already established, for length of anchor arm equal to one-half the length of main opening, it is necessary to find from Fig. 13 the length, L, of the latter, for which the weight is the same as that of a simple-truss span of $\frac{2L}{3}$. Inspec-

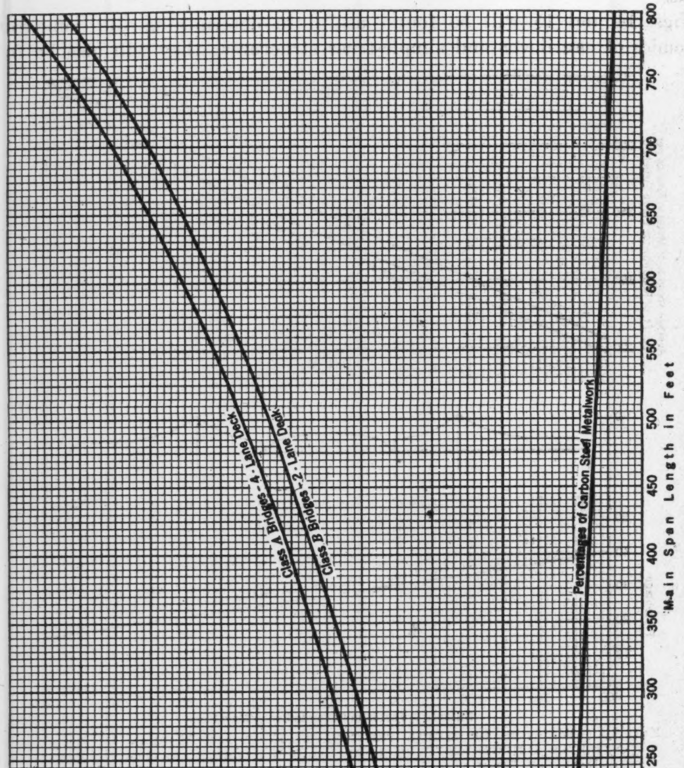

FIG. 14.—WEIGHTS OF METAL PER SQUARE FOOT OF FLOOR IN SIMPLE-TRUSS HIGHWAY BRIDGES OF SILICON STEEL.

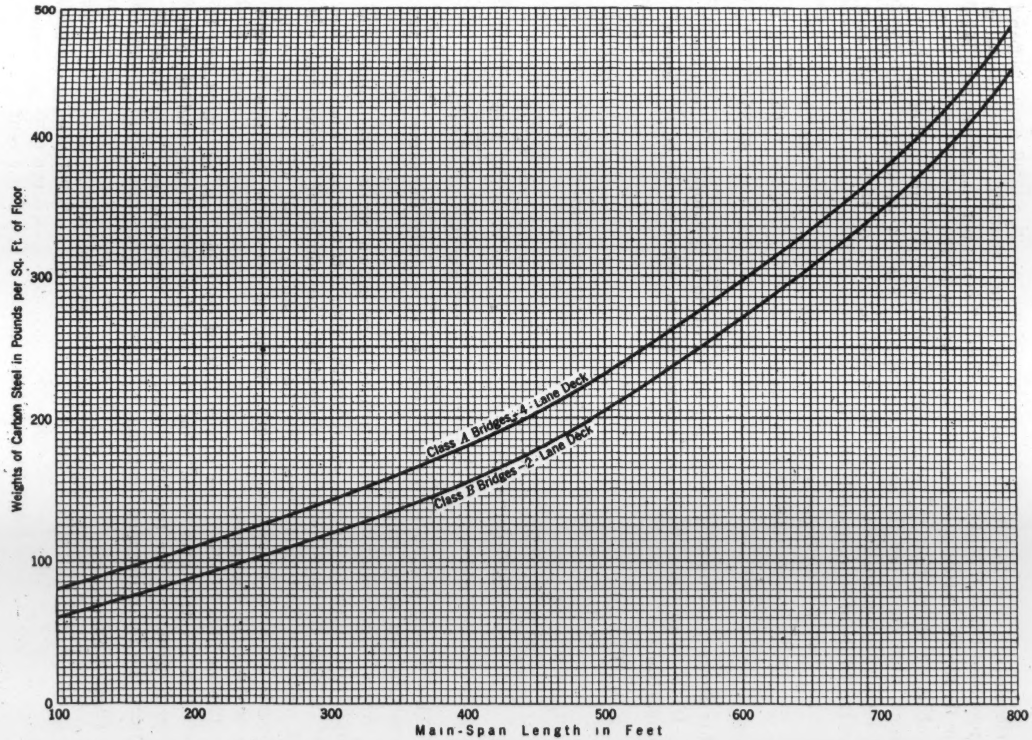

FIG. 15.—WEIGHTS OF METAL PER SQUARE FOOT OF FLOOR IN SIMPLE-TRUSS HIGHWAY BRIDGES OF CARBON STEEL.

tion shows that this condition exists for spans of 400 ft.; beyond that length the cantilever type will be economic.

As an example of determining the economics of silicon steel compared with carbon steel for simple-truss spans, assume that the former erected in a proposed Class *B* highway bridge is worth 7 cents per lb., as against 6 cents per lb. for the latter, and that the span length is 420 ft.

Fig. 15 shows that the weight of carbon steel per square foot is 164 lb., which, at 6 cents per lb., would cost $9.84. Fig. 14 shows that the weight of metal per square foot is 140 lb., of which, 32%, or 45 lb., is carbon steel, leaving 95 lb. of silicon steel; that is,

45 lb., @ 6 cents.......................... = $2.70
95 lb., @ 7 cents.......................... = 6.65
Total .... = $9.35

This shows a saving of 49 cents per sq. ft. of floor by adopting silicon steel.

If the span were 200 ft., as against 420 ft., what would be the economic comparison?

From Fig. 15,
88 lb. @ 6 cents.......................... = $5.28
From Fig. 14,
32 lb. @ 6 cents.......................... = $1.92
48 lb. @ 7 cents.......................... = 3.36
Total .... = $5.28

This demonstration shows the costs to be exactly the same for the two metals in this span length.

As cantilever highway bridges have been shown to be uneconomic compared with a succession of equal-length, simple-truss spans, each less than 400 ft. long, the question of the comparative economics of the two steels will not arise for cantilever structures, because silicon steel will always be the cheaper for such long-span bridges; hence, no provision has been arranged for making the comparison.

The weights of metal per square foot of floor in Figs. 12, 13, 14, and 15 for Class *A* structures were computed for four-lane decks, and those for Class *B* structures for two-lane decks. To use them for any other lanes will require the following modifications:

Class *A* Structures:
For a two-lane bridge add.............. 6 per cent.
For a three-lane bridge add..............2.5 per cent.
For a five-lane bridge deduct .... 2 per cent.
For a six-lane bridge deduct .... 4 per cent.

Class *B* Structures:
For a three-lane bridge deduct...........2.5 per cent.
For a four-lane bridge deduct .... 6 per cent.

There are several reasons for this variation:

A.—According to the latest and most approved specifications, the live load per square foot of floor decreases as the width of roadway increases.

B.—The effect of wind loads on truss weights decreases as the width of roadway augments.

C.—There is less waste metal in heavy trusses than in light ones.

D.—The weight of metal per square foot of floor in the lateral system decreases somewhat as the width of roadway becomes greater.

E.—On the other hand, the weight of metal per square foot in the floor system augments as the width of deck increases, because of the longer and much heavier cross-girders.

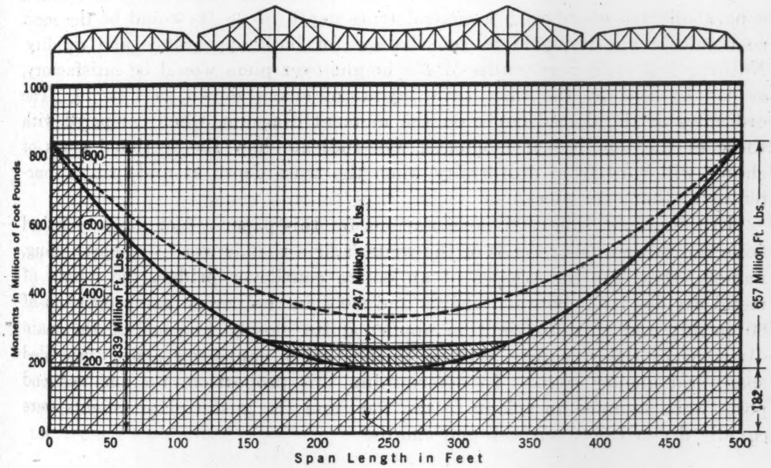

FIG. 16.—MOMENTS AND TRUSS DEPTHS FOR ANCHOR SPANS OF A TYPE *C* CANTILEVER BRIDGE.

There is a case in which these diagrams (Figs. 12, 13, 14, and 15), even with the foregoing modification, will not apply at all accurately, namely, in a very long span having a narrow roadway, where there is a wide uncovered deck space between the trusses.

From the preceding diagrams and the calculations made during their establishment, there can be drawn a deduction of value; that is, the economic layout for the top chord of the anchor arm of any Type *C* cantilever highway bridge. It is evident that the theoretically economic layout would be one making the maximum chord stresses constant from end to end of span; but, of course, that desideratum is not exactly attainable, because it would cause the top chord to be too irregular. However, it might be feasible to approximate it, as can be seen in Fig. 16, which represents the bending moments for a 500-ft. anchor span of a Type *C* cantilever bridge, 1 508 ft. long. This was prepared from the calculations used in determining some of the preceding results of this investigation.

It will be noted from Fig. 16 that, for any economic layout of a Type *C* cantilever highway bridge, the net dead-load bending moment at the middle of the anchor span is very small, in this case too small to be visible on the diagram.

Three different sets of hatchings will be observed: That for the parabola being for net dead-load moments; that of the rectangle being for extraneous live loading; and the small crescent representing one-half the net, direct, live-load bending moments. The ratio of compensated maximum bending moments at the center and ends is $\frac{247}{839} = 0.3$, nearly.

The economic truss depth over piers for this case is about 94 ft.; hence, a parabolic top chord with a central truss depth of 28 ft. would be the most economical; but it would be too shallow for both appearance and rigidity. Nothing less than four-tenths of the height over piers would be satisfactory, as can be seen by the small-scale layout of spans shown in Fig. 16. The ordinates of the dotted curve on the moment diagram, when compared with those of the top line of moments, will indicate how the sectional areas of chords will vary from uniformity when the truss depth at mid-span is four-tenths of that over piers.

In the preparation of the calculations for this paper a fact was established from the voluminous records of the writer's firm that is worthy of chronicling, namely, that, for the trusses of an entire cantilever bridge, the weight of metal divides itself between chords and webs in the percentage proportion of 60 to 40; and this distribution applies quite closely to each of the main divisions of the structure, that is, anchor arms, cantilever arms, suspended span, and anchor span. In determining this proportion, the inclined end posts were assumed to belong to the web, and the gusset-plate weights were equally divided between web and chords.

Résumé of Results

The findings of this research are, as follows:

First.—The economic ratio of length of suspended span to length of main opening in an ordinary three-span cantilever highway bridge varies from 0.45 to 0.50.

Second.—The economic truss depth over main piers in that type of structure varies from 12 to 15% of the distance between centers of main piers, with an average of 13.5 per cent.

Third.—The economic length for the anchor arms in that type of structure, when the distance between centers of main piers is fixed, is as short as the æsthetic appearance of the bridge will permit, say 25 or 30% of the main opening.

Fourth.—When, for the same type of layout, the distance between anchor piers is fixed, with no restriction as to the locations of the two main piers, the economic length for each anchor arm is about one-quarter of the total length of structure.

Fifth.—In a highway bridge having at one end a suspended span of length, l, a cantilever arm of one-half that length, and an adjoining anchor span of a length, y, that may be varied, the ratio of truss weights per linear foot for the spans, y and l, are as follows:

Values of $\frac{y}{l}$	Ratio of truss weights	Values of $\frac{y}{l}$	Ratio of truss weights
0.8	3.10	1.6	1.80
1.0	2.70	1.7	1.75
1.2	2.30	1.8	1.90
1.4	2.00	2.0	3.00

Sixth.—In a structure consisting of a suspended span, a cantilever arm, an anchor span, a like cantilever arm, and a like suspended span, the economic length of the anchor span is about 36% of the total length of the layout; and even a small variation from this economic proportion involves a material excess in the total weight of truss metal.

Seventh.—In highway bridges it is economic to change to cantilever construction from three simple truss spans of equal length, when that length is about 400 ft.

Eighth.—At ordinary current unit prices for metals erected, in simple-truss spans it begins to be economic to use silicon steel at a span length of about 200 ft. for four-lane bridges and 300 ft. for two-lane bridges; but, for truly warranted, cantilever structures, silicon steel will invariably be found to be less expensive than carbon steel.

Ninth.—The economic truss depth at mid-length of an anchor span of economic proportions in a Type *C* cantilever highway bridge is about three-tenths of that over the piers; but such a depth would be too small for both appearance and rigidity. This inferior limit should be about four-tenths of the depth over piers.

Tenth.—In the trusses of highway cantilever bridges, the weight of metal generally divides itself between chords and web in the ratio of 3 to 2; and this division applies to the different spans as well as to the structure as a whole.

DISCUSSION

HENRY C. TAMMEN,[5] M. AM. SOC. C. E. (by letter).—The author set out to determine the economic lengths of suspended span, cantilever arms, and anchor arms (or anchor span), for two types of cantilever bridges and in order to lighten his labors has used certain "short-cuts." A check is given for an ordinary cantilever bridge (the author's Type *A*) to show that the short-cuts used do not introduce any consequential error in determining the economic proportions for this type of bridge. The writer is satisfied that this is true for Type *A* cantilever bridges and that, while the increased weights of metal for departures from the most economic lengths of suspended spans shown on Figs. 5 and 7, may not be as large as there indicated, the errors in the economic length of the suspended span can be only trivial. There is doubt as to whether the conclusions regarding Type *C* cantilever bridges are equally as close to the truth and still more doubt whether the curves given in Figs. 12, 13 and 14, correctly show the relative weights of cantilever spans and of simple spans.

In determining the weights of anchor spans for Type *C* bridges the author apparently made a comparison of the average moments in anchor spans of various lengths with the average moment in the 336-ft. suspended span in the Elizabeth Bridge, and assumed that the weights per linear foot of the spans would be proportional to these average moments. This would be approximately true only if the average truss depth remained the same, but it results in errors increasing in magnitude as the length of the anchor span and its average depth depart from these dimensions for the Elizabeth suspended span.

Inasmuch as Fig. 10 shows such a decided increase in weight of metal for any variation from the most economical length of anchor span there indicated, it seems probable that the errors cited will have little influence in changing this economical length. They may be of considerable consequence, however, in determining economy of type.

To show clearly the economy of type of bridge, the writer has taken from Figs. 12, 13, and 14, the weights of metal for cantilever bridges and for simple spans, reduced them to equivalent silicon weights, and then calculated their ratios for Type *A* and Type *C* cantilever bridges to the weights for simple spans. Table 8 gives these data and Fig. 17, the plotting of the ratios.

An inspection of the curves for the Type *C* cantilever bridge (Fig. 17) shows that its weight is less than that for the Type *A* bridge for all span lengths. Its weight is also less than a simple span structure, except that for Class *B* two-lane bridges (Fig. 17(*b*)) the curve indicates equality of weight between the Type *C* bridge and a simple span bridge with a span length of about 500 ft. It is surprising to find that the curve has a downward trend to the left for all simple span lengths less than about 500 ft., indicating greater economy by the use of the Type *C* bridge for short span lengths than for long span lengths. The writer would expect the curve to slope upward to the left throughout the range of span lengths considered and he doubts whether the curves for Type *C* shown on these two diagrams represent the true state of affairs. It would appear that the assumptions made in computing the weights to which attention has been called, have introduced errors of such magnitude that the curves shown cannot be considered reliable and that they do not warrant reaching any conclusions as to economy with this type of cantilever bridge.

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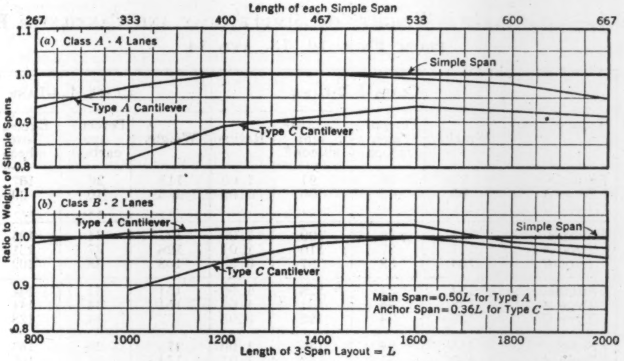

FIG. 17.—COMPARATIVE WEIGHTS OF METAL PER LINEAR FOOT.

The same downward trend to the left for spans less than 500 ft. is shown for the curves representing the Type *A* cantilever bridge, but in this case the downward trend is not so sharp. If any conclusion as to the relative economy of simple span and Type *A* bridges is to be drawn from these curves, it is that the two types have substantially equal weights for all spans up to about 600 ft. Clearly, the author's conclusion that it is economical to change to a cantilever structure at a span length of about 400 ft. is not justified by data presented in this discussion.

It might be emphasized that the comparison being made is for a three-span crossing composed either of three simple spans of equal length or of a cantilever structure in which the anchor arm is equal to one-half the main span. The comparison is, therefore, for a crossing where there is no restriction on the location of the piers. If there is any restriction requiring the main piers to be placed farther apart than one-third the distance between the end piers, the economy of the cantilever structure will be increased, and a cantilever structure of Type *A* under such conditions may well prove economical for lengths less than the 600-ft. span indicated herein.

For equal weights of a simple span layout and a cantilever layout, the matter of economy in total cost will, of course, have to take account of the substructure, as the author points out, and also of the unit cost of fabrication and erection of the metal work. Due to duplication, the fabrication cost of the simple span should be less than for the cantilever span, so that for equal weights, in order to secure equality in superstructure costs, it follows that the cantilever structure must show a saving in erection cost.

For the Type *C* bridge the author has reached the conclusion that a central span length equal to 0.36 of the total length of the three-span crossing gives the greatest economy. The weights given in the paper are for such a layout. This arrangement of spans corresponds to a ratio of 0.89 between the end span and the center span, whereas the most economical Type *A* layout has a ratio of 0.50 between the side span and the center span. Considering the rapid increase in weight for the Type *C* layout as the center span is lengthened (see Fig. 10), it may be expected to show economy over the Type *A* layout only for ratios of side span to center span of 0.75 or more.

TABLE 8.—COMPARATIVE WEIGHTS OF SIMPLE SPAN AND CANTILEVER BRIDGES FROM FIGS. 12, 13, AND 14

Total length of three-span bridge, in feet	Type of span	CLASS *B*, 2-LANE				CLASS *A*, 4-LANE			
		Weight	Percentage, carbon	Equivalent silicon*	Ratios	Weight	Percentage, carbon	Equivalent silicon	Ratios
800.....	Simple.....	96	36	91	1.00	113	36	107	1.00
	Type *A*....	95	40	90	0.99	105	40	99	0.93
	Type *C*....	...	..	...	....	...	..	...	....
1 000.....	Simple.....	114	34	109	1.00	131	34	125	1.00
	Type *A*....	116	37	110	1.01	128	37	121	0.97
	Type *C*....	104	38	97	0.89	108	38	102	0.82
1 200.....	Simple.....	134	31	128	1.00	151	31	144	1.00
	Type *A*....	138	34	131	1.02	151	34	144	1.00
	Type *C*....	128	34	122	0.95	134	34	128	0.89
1 400.....	Simple.....	155	28	149	1.00	173	28	166	1.00
	Type *A*....	160	31	153	1.03	174	31	166	1.00
	Type *C*....	153	30	147	0.99	159	30	152	0.91
1 600.....	Simple.....	177	26	170	1.00	198	26	191	1.00
	Type *A*....	182	28	175	1.03	197	28	189	0.99
	Type *C*....	177	26	170	1.00	185	26	178	0.93
1 800.....	Simple.....	205	23	198	1.00	226	23	219	1.00
	Type *A*....	203	25	196	0.99	222	25	214	0.98
	Type *C*....	201	22	195	0.98	210	22	203	0.92
2 000.....	Simple.....	235	20	228	1.00	259	20	252	1.00
	Type *A*....	230	22	223	0.98	248	22	240	0.95
	Type *C*....	224	18	218	0.96	236	18	230	0.91

* Approximate weight of silicon metal that would give same cost as combined carbon and silicon metal n the bridge.

Simple Spans of Part Silicon Steel.—The author also gives conclusions as to the minimum lengths of simple spans for which the use of silicon steel becomes advantageous. Such determinations, of course, are dependent upon manufacturing processes and conditions, and within one year or five years the quantitative determination of such values might be quite incorrect. Even for conditions as they have been within the last few years, the author's use of a fixed difference between prices of carbon and silicon steels leads, in the writer's opinion, to erroneous conclusions, fixing the span lengths for which silicon steel should be used somewhat too low.

In arriving at proper unit prices to use for carbon and silicon steels, in what is called a silicon steel bridge, due consideration should be given to the fact that the total weight of metal in the silicon steel bridge is somewhat less than in a bridge composed entirely of carbon steel. In such a silicon steel bridge many of the items included in the fabricating cost and the erection cost will remain unchanged in total amount even if the total weight of metal is decreased. To make allowance for this, it is necessary either to apply an increased cost per pound to the carbon steel and to the silicon steel in the silicon steel bridge, or to use the same unit price for the carbon steel in the silicon steel bridge and raise the silicon steel price correspondingly higher. The writer has had occasion to study this problem and submits the following analysis.

The economy of silicon steel for any structure as compared with carbon steel will depend upon the following considerations:

(1) The relation between the allowable unit stresses for silicon steel and carbon steel;
(2) The effect of the lighter weight of silicon steel members in decreasing the dead load stresses;
(3) The effect of the lighter dead load on the substructure, and in the case of movable spans, also on the counterweights, machinery, and other special parts;
(4) The relation between the base costs of silicon and carbon steels; and
(5) The relation between the fabrication and erection costs of silicon and carbon steels.

Of these five items, the third can be ignored for this discussion as the author is considering only superstructure costs. The first and second items also need no consideration herein as they affect only the weights of metal which are already given in Figs. 14 and 15. Some comments, however, will be given later on the second item. This leaves only the fourth and fifth items for consideration.

Base cost of metal here refers to the average cost per pound of plates, shapes, and beams for the bridge, delivered at the site. This cost would include the cost delivered at the shop and the freight to the site. The difference between base costs of silicon and carbon steel has been given by the manufacturers repeatedly during recent years as $15.00 per ton, or 0.75 cent per lb. At present (1932), this difference may be smaller, but for this discussion it can be assumed as given.

If C_1 is the base cost per pound of carbon steel, in cents; and S_1, the base cost per pound of silicon steel, in cents; then,

$$S_1 = C_1 + 0.75 \qquad (8)$$

The difference between the cost of a pound of metal in place on the structure and the base cost of metal as defined herein will represent the cost of fabrication, the cost of erection, insurance, bond, and contractor's profit. For convenience, this difference will be referred to as the "production" cost. Comparing the relative "production" costs on alternate members of silicon steel and of carbon steel, it will be found that a part of this cost is constant for a member (giving, therefore, an increased cost per pound as a change is made from a carbon member to a lighter silicon member), while the remainder of the cost is variable for the member and will be the same per pound for the carbon member as for the lighter silicon member.

Considering the fabrication cost of two alternate members, the first designed in silicon steel and the second in carbon steel, the same number of rivets would be required for connections, for tie-plates, and for lacing, and, generally, also for riveting together the component parts of the member, such as riveting between angles and plates and stitch-riveting between plates. On heavy members where two plates would be required in carbon steel, frequently one plate will serve in silicon steel, giving a reduction in stitch-riveting; and also where more than two plates are used in carbon steel one less plate would frequently be used in silicon, giving a reduction only in punching. In erection, the field riveting would be practically identical for either member. The lighter silicon members might permit somewhat lighter equipment and perhaps somewhat lighter falsework. The cost of handling completed members either at the shop or in the field will be substantially the same per member. The cost of handling individual plates and angles in the shop prior to assembly in the member might show a saving in total dollars in favor of silicon steel because of somewhat fewer pieces, but this would be offset by the additional attention and inspection necessary to avoid interchange of silicon and carbon parts. The total painting cost, shop and field combined, should be substantially the same for either silicon or carbon steel members. The labor, insurance, bond, and profit can all be taken proportional to labor and material, some parts of which are constant for the job regardless of whether silicon or carbon metal is used and some parts of which will vary with the weight of the metal.

It is difficult to say what part of the "production" cost can be taken as equal per pound of metal for both carbon and silicon steels, but the writer is of the opinion that for span lengths at which it just becomes economical to change to silicon steel (which span lengths would not involve members of great weight), a reasonable value would be 25 per cent. For larger and heavier structures it no doubt would be increased due largely to the substantial reduction in the number of individual plates and shapes making up the silicon steel members. For the present discussion the value of 25% will be used, but consideration will be given to the possibility that 50% might be more appropriate.

Let E = production cost of carbon steel per pound;
E_1 = production cost of silicon steel per pound;
C = total cost of carbon steel per pound;
S = total cost of silicon steel per pound; and
R = number of pounds of carbon steel required to replace 1 lb. of silicon steel.

Then,

$$C = C_1 + E \qquad (9)$$

and,

$$S = S_1 + E_1 = C_1 + 0.75 + E_1 \qquad (10)$$

but, $E_1 = 0.25\,E + 0.75\,E\,R$; therefore,

$$S = C_1 + 0.75 + 0.25\,E + 0.75\,E\,R \qquad (11)$$

Applying this to the example given by the author for a 420-ft. span, Class *B*, 2-lane:

Metal in carbon-steel span	164 lb. per sq. ft.
Metal in silicon-steel span:	
Silicon	95 lb.
Carbon	45 lb.

Then, $R = \dfrac{164 - 45}{95} = 1.263.$

Assume that the carbon-steel price of 6 cents is proper and that it is made up as follows: $C_1 = 2.25$, and $E = 3.75$. Then,

$$S = 2.25 + 0.75 + 0.25 \times 3.75 + 0.75 \times 3.75 \times 1.263 = 7.49$$

The relative costs of the span are then:

Carbon-steel span:	
164 lb. @ $0.06	$9.84
Silicon-steel span:	
Carbon, 45 lb. @ $0.06	$2.70
Silicon, 95 lb. @ $0.0749	7.12
Total	$9.82

This shows the costs to be substantially equal. If 50% of the "production" cost is taken as constant per pound of metal, then,

$$S = 2.25 + 0.75 + 0.50 \times 3.75 + 0.50 \times 3.75 \times 1.263 = 7.25$$

and the cost of the silicon-steel span would be:

Carbon, 45 lb. @ $0.06	$2.70
Silicon, 95 lb. @ $0.0725	6.89
Total	$9.59

This shows a small economy by use of silicon steel, and that for Class *B*, 2-lane bridges, it would be economical to change to silicon steel when the span length is about 400 ft. This compares with the 300-ft. length given by the author. For Class *A*, 4-lane bridges, similar calculations show that it would be economical to change to silicon steel when the span length is about 350 ft., which compares with a length of 200 ft. given by the author.

The author does not give the general design specifications that were used, nor the ratios that were used between allowable carbon and silicon unit stresses. Presumably, carbon-steel unit stresses are on a 16 000-lb. basis and silicon on a 24 000-lb. basis. Using the latest design specifications for highway bridges of the American Assocation of State Highway Officials, or the 1929 Conference Specification, both of which put dead load stresses on a 24 000-lb. basis, the span lengths at which it would be economical to change to silicon steel would be slightly increased, due to the smaller amount of truss metal required to carry the dead load of the structure.

At best, such determinations cannot be followed blindly. There may well be cases in which other practical considerations will make the plain carbon-steel span more economical than the carbon-silicon span for lengths greater than those determined by such analyses. The delay which may be caused in securing rollings of a variety of shapes and plates for a comparatively limited tonnage of silicon steel, and like elements, may prove determining factors.

The general conclusion may be offered that while such economic studies as guides to conclusions are desirable tools and are often of considerable use to the engineer who devises them and has an understanding of their limitations, they must be applied by others with caution. Formulas cannot be devised to take the place of ripened experience and judgment.

ROBERT W. ABBETT,[6] ASSOC. M. AM. SOC. C. E. (by letter).—This study together with Mr. Waddell's previous papers on cantilever bridges for railways should practically settle the question of ideal economic proportions for that type of structure.

The author has placed as the limiting factors for the cantilever bridge the simple span or a series of simple spans on the one side and the suspension bridge on the other. Thus, two important bridge types are omitted from consideration, namely, the arch and the continuous truss. The economics of the arch is so vitally affected by local foundation conditions that it does not lend itself to a *carte blanche* comparison with other types. The continuous bridge on the other hand seems to be rapidly living down its unsavory reputation of the Nineteenth Century, and it now demands recognition.

Given ordinary satisfactory foundation conditions, such as any important structure would require, and with no restrictions as to the location of piers, it is a question in the mind of the writer whether the selection of a series of simple spans is ever justifiable from the point of view of the metal involved. Theoretically, the inherent features of a continuous truss should always cause it to show an economy in material over simple spans. If this is true, where does the continuous type stand with reference to the cantilever? Giving it consideration would certainly alter the point at which the cantilever should be adopted as determined by comparison with simple spans, and, as the relative economy of the continuous type increases with an increase in span length, a thorough study might possibly reveal facts which would place a new light on the economic selection of bridge types.

FRANK W. SKINNER,[7] M. AM. SOC. C. E. (by letter).—Too many engineers content themselves with designing safe structures of standard type and ordinary dimensions, with little or no regard to the economic influence of variations in relative proportions. Such variations may involve large differences in efficiencies and total costs that can be determined, almost at a glance, from the graphical data presented in this paper. No engineer, or any other authority, has ever investigated the great field of the economics of steel bridges in a manner comparable with Mr. Waddell's researches; and this paper is a highly important application of its broad principles. It is presented so clearly and simply that, in connection with the author's other books and papers, it gives the most up-to-date instructions for the design of important steel bridges in usable shape for any qualified designer. It provides for a great economy of time and labor for him, and an important reduction of cost of construction, thus notably fulfilling one of the true engineer's most sacred obligations, that of efficient, reliable service to the public, to his clients, and to his chosen profession.

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[6] Asst. Prof., Bldg. Constr., Thompson-Starrett Foundation, Dept. of Civ. Eng., Union Coll., Schenectady, N. Y.

[7] New York, N. Y. Mr. Skinner died December 24, 1932.

For fifty years engineers have differed widely as to the best relative lengths of the suspended span and its supporting cantilever arms, and the demonstration (under "Tabulation of Results to Present Stage") of the fact that the cantilever arms may each be made as nearly as convenient, about one-quarter the length of the distance between their piers, without materially affecting the most economic results, will certainly be a greatly appreciated fact. This feature will also be in accord with other economic elements, as in the development of field construction methods, where the slow, expensive cantilever erection can be reduced by the erection, under suitable conditions, of longer suspended spans assembled on sheltered falsework, and erected as single hoisted units.

A careful consideration of the clear and forceful demonstrations in this paper will assure most complete reading and confidence in the accuracy of the résumé; and the acceptance and beneficial use of the ten important points thereof will be of great practical and theoretical benefit to designers and critics alike.

CHARLES M. SPOFFORD,[8] M. AM. SOC. C. E. (by letter).—This paper is an interesting and useful contribution to the literature on highway bridges, and the conclusions reached should prove of distinct value to engineers engaged in making preliminary studies of long-span bridge projects of the cantilever type.

In his Seventh Conclusion, the author states "it is economic to change to cantilever construction from three simple truss spans of equal length, when that length is about 400 ft." Without questioning the validity of this conclusion, the writer wishes to suggest that it is quite probable that a continuous truss bridge under the same conditions, would be found more satisfactory than, and probably equally economical to, a cantilever bridge provided reasonably good foundations are available. One important advantage which the continuous highway bridge has, as compared with either an end-supported or cantilever highway bridge, is the fewer number of expansion joints in the floor. Such joints are expensive and also frequent sources of maintenance troubles in highway bridges; while the interruptions in the roadway surface due to them are objectionable to motorists. Moreover, the deflection of a cantilever truss is greater in magnitude and rapidity than that of a continuous truss and in one unusually short-span cantilever bridge with which the writer is familiar, the vibration at the ends of the anchor arms under the passage of heavy motor trucks and trolley cars is most annoying to pedestrians. In a continuous structure, also, the hinges which are necessary to make a cantilever structure discontinuous are not required, and the lateral bracing may be made continuous throughout.

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[8] Hayward Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.; Cons. Engr. (Fay, Spofford & Thorndike), Boston, Mass.

From the standpoint of appearance, the writer believes that a continuous structure can be given more pleasing lines consistent with an economical profile than the cantilever with its top chord lines interrupted at the suspended span, as in the cantilevers shown in the paper.

The writer has no comparative figures as to the cost of cantilever and continuous highway spans with modern concrete or other solid floors, as the only continuous structure of moderately long spans with which he is familiar—the Lake Champlain Bridge—was made of the continuous type in order to secure a more pleasing outline than any cantilever or simple span design that could be developed, and to eliminate expansion joints and other expensive and troublesome details. He believes that the construction cost was no more and was probably less than would have been the case with a cantilever design and is quite sure that the maintenance charge will be less and the roadway surface more satisfactory for traffic. One feature of the continuous bridge referred to is the practical elimination of stress reversal in the truss members, but the writer cannot state whether such reversal would be more or less than in a cantilever bridge with the same type of floor and the same loading requirements.

As far as stress computations are concerned, no difficulty occurs in the continuous bridge; a little more labor is required, but the cost of this has no appreciable influence upon the cost of the bridge. No concern need be felt from the effect of pier settlement unless the foundations are so bad that it would probably not be wise to construct any type of long-span bridge upon them. Moreover, much of the settlement of the bridge piers occurs during construction and is due to the dead weight of the piers themselves and the superstructure rather than to the live load. Hence, by establishing the dead end reactions in a three or two-span structure by jacking at the piers before connecting the trusses, the effect of construction settlement upon the dead stresses may be eliminated.

The writer hopes that in his closure Mr. Waddell will give his views on the relative merits of continuous and cantilever highway bridges and state his opinion as to whether the conclusions reached by him on the economical proportions of cantilever bridges would apply also to continuous bridges.

R. WAYNE LINCOLN,[9] ASSOC. M. AM. SOC. C. E. (by letter).—The writer has always maintained, that designers are prone to apply the data prepared for railroad bridges formerly published by the author, to any and all kinds of bridges, regardless of loading. It is both surprising and gratifying to note that the economic proportions, as shown in the present paper for highway bridges, are not materially different from those formerly set up for railway bridges.

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[9] Structural Engr., Seattle, Wash.

As the author discusses through trusses exclusively, it would be interesting and instructive if he would make a brief statement concerning the application of these results to deck trusses. In the latter type, the depth of the suspended span generally affects the height of the structure, as was the case with the Lake Union Bridge, in Seattle, Wash., on which the suspended span was made exceptionally short in order to reduce the depth to a minimum and thus save height throughout the entire length of the structure, as the required channel clearance occurred at this span.

Harold E. Wessman,[10] **Assoc. M. Am. Soc. C. E.** (by letter).—The author is to be congratulated for presenting to the Engineering Profession another extensive paper dealing with the economics of an important type of highway bridge structure. In past years, Mr. Waddell has given much study to the economics of various kinds of railroad bridges, as a result of which many valuable data are now available. The intensive building of long-span highway bridge structure. In past years, Mr. Waddell has given much study economics to the fore. In view of Mr. Waddell's previous researches, no one is better fitted to present authoritative information on the economics of spans of this type.

One of the important points brought out is that previous investigators found values entirely too high for the economic length of suspended span. When differences of opinion exist, it is necessary to examine the mode of procedure leading to certain conclusions. Maximum moments and shears due to dead load and live load, erection stresses, wind stresses, minimum sections, number and kinds of steels used in the same structure, and width of roadway, must all be considered in a series of designs in order to determine exactly the least total cost of metal possible for a single crossing. In other words, exact and complete detailed designs and weight estimates are necessary in order to draw rigid conclusions as to the most economic length of suspended span. This, of course, involves an immense amount of labor. The author used an approximate method in first making his deductions; but he established his point by having one of his computers make complete designs and weight estimates, using the 1 050-ft. Cooper River Bridge as a basis. A series of designs were made in which the length of suspended span was varied. The effect of varying the roadway widths was also ascertained. The results demonstrated conclusively that the economic ratio of suspended-span length to main-span length was less than 0.50 for that particular crossing.

In the making of economic studies, it must be remembered, however, that variations in certain factors over which the engineer has little control, such as the difference between estimated unit prices and contractor's unit prices, the ratio of cost of alloy steels to carbon steel for a structure using two or more kinds of metal, the method of erection, etc., may affect conclusions considerably. Consequently, rigid laws of economic proportion cannot be established. That, however, does not detract one whit from the worth of an investigation such as this. The general limits for the length of suspended span, 45 to 50%, established in this paper, are of great value in obtaining an economic layout for any highway cantilever bridge.

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[10] Supt., Water Dept., Rockford, Ill.

Aesthetic requirements often tend to conflict with true economy in a bridge structure. In the case of the cantilever bridge, however, æsthetics and economy "walk hand in hand." Better outlines are obtained by keeping the length of the suspended span to less than 50% of the length of the main span. The writer has made several scaled layouts of a cantilever bridge with a 900-ft. main span, varying the lengths of the suspended span from 40 to 55% of the length of the main span. In each case, the economic proportions for the suspended span accorded with Mr. Waddell's practice for simple truss spans.[11] It was found that the lower limits for the lengths of suspended span gave the best structure from an artistic standpoint. With a suspended span, 420 ft. long (47% of the main span length), very pleasing outlines were obtained. In fact, there was little to choose between this and a layout with a suspended span, 360 ft. long (40% of the main span length).

TABLE 9.—RELATION BETWEEN MAIN SPAN AND SUSPENDED SPAN

Bridge	Main span, in feet	Suspended span, in feet	Percentages
New Orleans Bridge, over the Mississippi River.......	1 760	714	41
Longview Bridge, over the Columbia River...........	1 200	440	37
Carquinez Straits Bridge............................	1 100	433	39
Cooper River Bridge................................	1 050	438	42
Wabash Railway Bridge, over the Monongahela River.	820	360	44
Ohio River Bridge, at Beaver, Pa....................	760	285	37
Outerbridge Crossing, over Arthur Kill..............	750	300	40
Sewickley Bridge, over the Ohio River...............	750	350	47

It is interesting to note that most of the important cantilever bridges, proposed or built, have relatively short suspended spans. Some of these bridges are listed in Table 9.

As a by-product of his major investigation, Mr. Waddell found that it was economical to use silicon steel in simple truss highway spans as short as 200 ft., when the bridge carried four lanes of traffic When only two lanes were carried, the economic limit was 300 ft. These limits depend, of course, on the ratio of market prices of silicon steel and carbon steel, and the percentage of carbon steel in the silicon truss spans. The significance of this part of the author's investigation lies not, however, in the actual figures given as limits, but in emphasizing the fact that it is worth while to investigate alloy steel designs for short-span structures. Many designers have the idea that an alloy steel is something to be used only in a long-span bridge. This mistaken idea may be due in part to the fact that many technical schools teach steel design in terms of ordinary carbon structural steel, without emphasizing the economic possibilities of high-strength steels. At any rate, it is an expensive idea to cherish. There are cases in which it might even be economical to use alloy steels in girder spans.

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[11] "Economics of Bridgework," by J. A. L. Waddell, Chapter XX.

Herschel H. Allen[12] and **Wilson T. Ballard**,[13] Members, Am. Soc. C. E. (by letter).—The author, in his paper, has provided the Engineering Profession with a contribution of great value. The writers have long felt the need of such a study in order that all the essential basic facts pertaining to cantilever bridges might be crystallized into a form readily available for use in fixing the economic proportions and quickly estimating the weights of such structures.

The writers concur in the statement by Mr. Hardesty (under "Present Study," in the main paper) to the effect that the economic proportions that governed the design of early single-track railroad cantilevers do not apply to modern highway cantilever structures.

The practice of the writers' office confirms the author's first and second conclusions under "Résumé of Results." The third conclusion states without qualification that the economic length for anchor arms of his Type *A* cantilever bridges, when the main span length is fixed, is as short as æsthetic consideration will permit. Such a conclusion does not take into account the economy of the entire structure. The topography of the site must be taken into consideration, as illustrated in the case of the alternate designs for the Ashland Bridge with long and short anchor arms hereinafter described, wherein, by lengthening the anchor arms and eliminating uplift on the anchor piers, it was possible to more than offset the additional superstructure cost by a reduction in pier costs.

Table 10 shows in detail the distribution of weights of materials in nine highway cantilever bridges and Table 11 shows the dimensions of cantilever bridges. The Huntington, W. Va.; Bellaire-Benwood, Ohio; Ashland, Ky., with long anchor arms; and the Point Pleasant-Henderson, W. Va., bridges have been constructed and are in service. Designs and stress sheets have been prepared for the other five bridges listed, and detailed estimates of weights have been made, but to date (1932) these latter five bridges have not been constructed.

All the designs, except those for the long and short anchor arms at Ashland, and the Norfolk, Va., design, were based on the use of medium carbon steel, with a basic working stress of 18 kips per sq. in. The Ashland Bridge, as built, contains 49% of silicon steel with a basic working stress of 24 kips per sq. in., and 51% of medium carbon steel with a basic working stress of 18 kips per sq. in. Included with the carbon steel are heat-treated eye-bars with a basic working stress of 24 kips per sq. in. The Norfolk design was based on the use of silicon steel. Tables 10 and 11 have been prepared in the same manner as Tables 1 and 2 of the author's paper in order to facilitate comparison.

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[12] Vice-Pres., The J. E. Greiner Co., Baltimore, Md.

[13] Vice-Pres., The J. E. Greiner Co., Baltimore, Md.

TABLE 10.—DISTRIBUTION OF WEIGHTS OF MATERIALS, IN POUNDS PER LINEAR FOOT OF SPAN, IN HIGHWAY CANTILEVER BRIDGES

Name of structure and its main-span length	Portion of structure	ANCHOR ARM		CANTILEVER ARM		SUSPENDED ARM	
		Steel	Floor	Steel	Floor	Steel	Floor
Huntington, W. Va., over Ohio River, 700 ft.	Floor, track, etc	......	1 575	......	1 575	......	1 575
	Hand-railing	70	......	74	......	70	......
	Floor system	914	......	947	......	860	......
	Lateral bracing	528	......	535	......	303	......
	Trusses	3 115	4 627	3 390	4 946	2 110	3 343
	Shoes and anchors	131	......	273	......	......	......
	Total steel	4 758	......	5 219	......	......	......
	Dead load on trusses	......	6 202	......	6 521	3 343	4 918
Bellaire, Ohio, over Ohio River, 700 ft.	Floor, conduits, etc	......	1 265	......	1 265	......	1 265
	Hand-railing	78	......	78	......	78	......
	Floor system	850	......	829	......	780	......
	Lateral bracing	327	......	629	......	262	......
	Trusses	2 600	3 855	4 350	5 886	2 070	3 190
	Shoes and anchors	73	......	376	......	......	......
	Total steel	3 928	......	6 262	......	3 190	......
	Dead load on trusses	......	5 120	......	7 151	......	4 455
Point Pleasant–Henderson, W. Va., over Kanawha River, 600 ft.	Floor, conduits, etc	......	2 300	......	2 300	......	2 300
	Hand-railing	91	......	87	......	83	......
	Floor system	520	......	488	......	514	......
	Lateral bracing	476	......	461	......	303	......
	Trusses	2 290	3 377	2 945	3 981	1 540	2 440
	Shoes and anchors	93	......	292	......	......	......
	Total steel	3 470	......	4 273	......	2 440	......
	Dead load on trusses	......	5 677	......	6 281	......	4 740
Ashland, Ky., over Ohio River (long anchor span), 739 ft.	Floor, conduits, etc	......	2 718	......	2 718	......	2 718
	Hand-railing	76	......	76	......	77	......
	Floor system	532	......	535	......	527	......
	Lateral bracing	390	......	770	......	377	......
	Trusses	2 140	3 138	3 640	5 021	1 774	2 755
	Shoes, anchors, baffles	46	......	298	......	......	......
	Total steel	3 184	......	5 319	......	2 755	......
	Dead load on trusses	......	5 856	......	7 739	......	5 473
Portsmouth, Ohio, over Ohio River, 700 ft.	Floor, conduits, etc	......	1 255	......	1 255	......	1 255
	Hand-railing	75	......	75	......	75	......
	Floor system	700	......	700	......	635	......
	Lateral bracing	450	......	650	......	329	......
	Trusses	2 450	3 675	3 490	4 915	1 816	2 855
	Shoes and anchors	73	......	286	......	......	......
	Total steel	3 748	......	5 201	......	2 855	......
	Dead load on trusses	......	4 930	......	6 170	......	4 110
Cabin Creek, W. Va. over Kanawha River, (long anchor span), 450 ft.	Floor, conduits, etc	......	2 510	......	2 510	......	2 510
	Hand-railing	100	......	100	......	100	......
	Floor system	440	......	440	......	440	......
	Lateral bracing	275	......	440	......	195	......
	Trusses	1 525	2 340	2 340	3 320	1 000	1 735
	Shoes and anchors	187.5	......	480	......	......	......
	Total steel	2 527.5	......	3 800	......	1 735	......
	Dead load on trusses	......	4 850	......	5 830	......	4 245
Cabin Creek, W. Va., over Kanawha River, (short anchor span), 450 ft.	Floor, conduits, etc	......	2 510	......	2 510	......	2 510
	Hand-railing	100	......	100	......	100	......
	Floor system	437	......	440	......	440	......
	Lateral bracing	330	......	440	......	195	......
	Trusses	1 636	2 503	2 340	3 320	1 000	1 735
	Shoes and anchors	287	......	480	......	......	......
	Total steel	2 790	......	3 800	......	1 735	......
	Dead load on trusses	......	5 013	......	5 830	......	4 245

TABLE 10.—(*Continued*)

Name of structure and its main-span length	Portion of structure	ANCHOR ARM		CANTILEVER ARM		SUSPENDED ARM	
		Steel	Floor	Steel	Floor	Steel	Floor
Ashland, Ky., over Ohio River (short anchor span), 739 ft.	Floor, conduits, etc.	......	2 718	......	2 718	......	2 718
	Hand-railing	76	......	76	......	76	....
	Floor system	621	......	519	......	528	....
	Lateral bracing	418	......	665	......	268	....
	Trusses	2 345	3 460	2 760	4 020	1 460	2 332
	Shoes and anchors	350	......	514	......	......	....
	Total steel	3 810	......	4 534	......	2 332	....
	Dead load on trusses	......	6 178	......	6 738	......	5 050
Norfolk, Va., over Elizabeth River, 1 198 ft. 6 in.	Floor, conduits, etc.	......	7 970	......	7 970	......	7 970
	Hand-railing	100	......	100	......	100	....
	Floor system	1 912	......	1 912	......	1 912	....
	Lateral bracing	1 640	......	1 645	......	1 175	....
	Trusses	10 080	13 732	9 300	12 957	4 400	7 587
	Shoes and anchors	473	......	1 420	......	......	....
	Total steel	14 205	......	14 377	......	7 587	....
	Dead load on trusses	......	21 702	......	20 927	......	15 557

Fig. 18 shows the weight per square foot of roadway of the foregoing nine designs, superimposed on the curve of Fig. 13. Bridges 7, 8, and 9 (Fig. 18), are of silicon steel; all the others are carbon steel. The writers are of the opinion that the author's weights per square foot, as shown for Type *A* bridges, with main span lengths of less than 700 ft., are too light even if they were adjusted to equivalent weights of bridges of all carbon steel in order to make them fully comparable with the writers' values for such main span lengths.

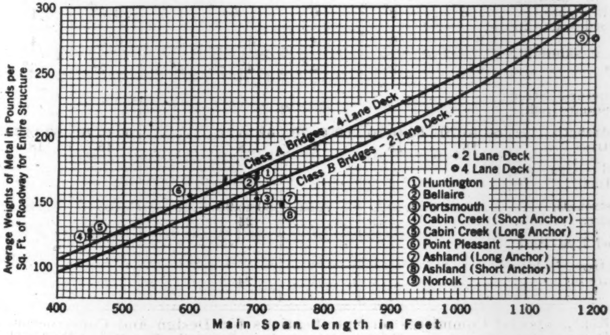

FIG. 18.—WEIGHTS OF STEEL FOR NINE MODERN HIGHWAY CANTILEVER BRIDGES.

The writers agree with the author's seventh conclusion, but it is not substantiated by the data presented in his paper. The discrepancy between his seventh conclusion and his supporting data may be the result of error either in the weights of simple spans or in the weights of Type *A* cantilever spans as presented. It is the writers' opinion that his weights pertaining to simple spans are too high for short spans, and that the weights for cantilever bridges, as hereinbefore stated, are too low for spans of less than 700 ft., and too high for spans greater than 700 ft.

The primary features of the specifications that affect the weight of steel in seven of the nine bridges are also given in Table 10. The writers feel that such data are of vital importance in making the entire paper and discussions of practical value. It may be noted that the type of floor used on the roadway materially affects the weight of steel and that, in the cases of the Huntington and Bellaire Bridges, the live load used in the design included 50-ton trolley cars.

TABLE 11.—DIMENSIONS OF CANTILEVER BRIDGES AND SPECIFICATION DATA

Description	Huntington, W. Va.	Bellaire, Ohio	Point Pleasant, W. Va.	Ashland, Ky. (long anchors)	Ashland, Ky. (short anchors)	Portsmouth, Ohio	Cabin Creek, W. Va.	Norfolk, Va.
River crossed	Ohio	Ohio	Kanawha	Ohio	Ohio	Ohio	Kanawha	Elizabeth
Lengths of Bridge, in Feet:								
Main span, *L*	700	700	600	739	739	700	450	1198.5
Side spans	275	275	219.33	431	251	275	{ 200 148.5	423
Suspended span, *l*	350	350	300	350	350	350	200	493.5
Ratio, $L:l$	0.5	0.5	0.5	0.473	0.473	0.5	0.444	0.412
Specification	*	*	†	‡	*	*	*	*
Live Load:								
Roadway (truck loading)	*H*-15§	*H*-15§	*H*-15	*H*-15	*H*-15	*H*-15	*H*-15	.....
Sidewalk loading, in pounds per square foot	500¶	500¶	100	100	100	100	200	.....
Roadway:								
Width, in feet	22	22	20	22	22	22	20	.....
Thickness of deck, in inches	6 to 6.5[a]	4 to 5[a]	7[b]	7[b]	7[b]	6 to 6.5[a]	7.5[b]	.....
Thickness of wearing surface, in inches	1.5[c]	2[c]	0	1.5c	1.5[c]	2.5[d]	1[c]	.....
Width of sidewalk, in feet	8	6	4	5	5	6.5	4	.....
Thickness of sidewalk, in inches	4[e]	2[f]	4[e]	4[e]	4[e]	2[f]	3[e]	.....
Medium Structural Steel (Kips per Square Inch):								
Ultimate strength	60 to 70	60 to 70	60 to 70	62 to 72	......	60 to 70	60 to 70	.....
Working stress	18	18	18	18	......	18	18	.....
Silicon Steel (Kips per Square Inch):								
Ultimate strength	....	......	....	80 to 95	......	......	....	.....
Working stress	....	......	....	24	......	......	....	.....
Heat-Treated Eye-Bars:								
Working stress, in kips per square inch	....	......	....	24	......	......	....	.....

* Final Rept., Special Committee on Specifications for Design and Construction of Steel Highway Bridge Superstructure, *Transactions*, Am. Soc. C. E., Vol LXXXVII (1924), p. 1273. † Specifications, State Road Comm. of West Virginia, 1928. ‡ Specifications, Kentucky State Highway Dept., 1926. § Two 50-ton trolley cars in tandem on each track, abreast, preceded, and followed by *H*-15 trucks. ¶ Pounds per linear foot. [a] Laminated redwood. [b] Reinforced concrete. [c] Black top. [d] Asphalt. [e] Reinforced concrete slab. [f] Plank.

The value of this paper can be increased materially by the addition of similar data pertaining to cantilever bridges to be constructed in the future.

J. A. L. WADDELL,[14] M. AM. SOC. C. E. (by letter).—The writer has given due thought to the discussions of his paper and has endeavored to answer every important question raised.

While Mr. Tammen is satisfied that the approximate "short-cuts" used in preparing the paper have done no harm in the treatment of the Type *A* cantilevers, he is not so sure about their being innocuous in the case of the Type *C* cantilevers. Without these short-cuts the writer would have had neither the time nor the patience to make the investigation because he would have had to revert to his customary method of preparing complete calculations of weights of metal for four or more structures in the case of each plotted curve—a tremendous job in cantilever-bridge computation. It is true that, in some cases, the errors involved by the short-cuts were a little greater than had been anticipated; but they were not large enough to nullify the conclusions of the résumé. Moreover, the most important of them were corrected in the preliminary results, when the approximate values of the small variations from accuracy were determined; and these corrections were incorporated in the paper.

Mr. Tammen doubts whether the curves in Figs. 12, 13, and 14 correctly show the relative weights of cantilever spans and simple-truss spans. These curves were compiled partly from old records of weights that had to be adjusted to care properly for variations in loads and differences in specifications; hence, on that account, the results involve the possibility of some slight inaccuracies. Again, the personal equations of the different designers cause small divergences that are unavoidable. Their effects are partly, but never entirely, evaded by drawing a smooth curve between the plotted points instead of an irregular line joining them.

Practically nothing was known about the weights of metal for the Type *C* highway cantilevers prior to the issuance of the paper; hence, if there is any material variation from correctness in these three diagrams, it is likely to be in Fig. 12, which records the weights for Type *C*. That type has seldom been adopted hitherto, although known to be economic, because usually it fails to fit a crossing that calls for a cantilever layout. Such a crossing generally demands an extra long main span, because of navigation interests, or because of deep and swift water, or a dip in the bed-rock that requires a wide opening to straddle it, in order to avoid costly piers. Whenever it is feasible to use either Type *A* or Type *C* cantilevers for a proposed crossing, it will be more economical to adopt the latter, provided the length of the central span can be made about 36% of the total length of the triple-opening structure. The designer can begin with the weights of metal given in the diagram, using them for determining his dead loads, then compute his stresses, sections, and final weights of metal. It is highly improbable that any recomputing will have to be done because of serious variation between the assumed and the final weights.

The slight dropping of the cantilever curves for comparatively short spans, shown by Mr. Tammen, is curious but unimportant, because it would not be good engineering practice to adopt cantilever construction in preference to a succession of simple-truss spans of 400 ft., since the latter layout is the more rigid, and, therefore, preferable. The longer the spans the smaller is the variation in rigidity, and, consequently, the less objectionable on that account is the cantilever construction.

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[14] Cons. Engr., New York, N. Y.

If in any case the probable weight of metal in the cantilever layout is not much less than that in the simple-truss layout, it is better to adopt the latter for the following reasons:

1.—The superior rigidity of the simple-truss spans, as already mentioned.

2.—The lower pound price, due not only to greater duplication, but also to simpler details.

3.—The cost per pound for erection is often greater for cantilevers than for simple-truss spans, but not always. The reason is that cantilevers are erected in a piecemeal manner, while simple-truss spans are not, unless it is found necessary to use either the method of semi-cantilevering or that of protrusion. The steadily increasing shortage of timber in this country, with the corresponding augmentation of its unit cost, has a tendency to reduce the use of wooden falsework, and, therefore, to increase the use of cantilevering, semi-cantilevering, and protrusion. For high-level bridges, especially over deep and navigable streams, the writer, of late years, has found timber falsework to be far too uneconomical; hence, he has had to adopt the use of widely spaced steel bents carried on temporary pile piers, and to cantilever between them.

The writer's practice has indicated that the average span length at which it usually pays to change from simple-truss to cantilever construction is between 400 and 500 ft. Generally, however, navigation interests call for a long main span, and sometimes also for long flanking spans, the other spans (if there are any) being of the simple-truss type, and, if feasible, of economic lengths. Other things being equal, or nearly so, the writer has made a practice of giving preference to simple-truss spans over cantilevers, and has not countenanced, for solely economic motives, the building of the latter with main openings for Type *A* less than 600 ft. This replies to Mr. Tammen's anticipated economy for Type *A* cantilever bridges having main spans shorter than that length.

As to the effect of cost of substructures on the comparative economics of simple-truss and cantilever layouts, each case will have to be solved as an independent problem, involving generally some close detailing. As a rule, however, it will be found that this consideration will not be of paramount importance because, although the sum of the total loads on all pier tops is greater for the cantilever than for the simple-truss layout, the latter requires two small pedestals per truss, whereas the former has only a single large one per truss over each main pier. If the pier foundations are piles, the cantilever construction will be at a disadvantage; but, otherwise, the contrary condition generally holds.

In respect to the economic proportionate length of anchor spans of Type *C* cantilever bridges, the writer would state that he made two sets of calculations by different methods (although both were only approximate), and found in each case the percentage of the total length of the three-span structures to be almost exactly 36. This is a matter that hitherto has received little consideration in technical print.

Four layouts illustrating Type *C* cantilever bridges have been discussed by the writer elsewhere,[15] namely, two over the Mississippi River, at Memphis, Tenn., one over the same river at Thebes, Ill., and one over the Danube, at Cernavoda, Rumania. In each case the ratios of anchor-span length to total length vary from 0.333 to 0.34, with an average of 0.335. These ratios were probably selected for economic reasons. As these layouts are for railway structures, it is evident that the economic ratio for highway bridges should be somewhat greater, because the reason the anchor span is heavy is that it has to care for reversing stresses. These stresses are much less for modern highway bridges with their weighty floors and small live loads than for railway bridges with their light, open floors and their large live loads. For this reason it is logical to conclude that the 0.36 economic ratio found for highway bridges is just about right, especially in view of the fact that, for a structure carrying dead load alone, such as a water-pipe-line bridge, the economic ratio would be 0.4.

Mr. Tammen's dissertation about silicon steel *versus* carbon steel is interesting. The subject can be approached in several ways, and the deductions will be influenced greatly by the initial assumptions that are made. No conclusion reached would be permanent, because of the large fluctuations that are constantly occurring in the unit prices (manufactured) of the two steels, and on account of the fact that the unit prices do not necessarily increase and decrease in the same ratio.

A solution of this economic question, based upon reasonable present-day unit prices is as follows: Consider a Class *A*, simple-truss span of 300 ft., having a 42-ft. clear roadway and two 5-ft. sidewalks, and assume the manufactured steel unit prices, f. o. b. cars at shops, to be 3 cents for carbon steel and 3.5 cents for combined silicon and carbon steels—also a 0.5 cent per lb. freight rate, and a 1.5-cent cost per lb. for carbon-steel erection.

The total cost of erection for the silicon-steel structure is almost, but not quite, as great as for the carbon-steel structure, the pound price for erecting the alloy steel being about $\frac{1.5}{5}(4r + 1)$, in which, r (> 1) is the ratio of weights of metal. From Figs. 14 and 15, $r = \frac{142}{122} = 1.164$. The remaining computations are:

Cost of erecting alloy steel $= 0.3\,(4 \times 1.164 + 1) = 1.7$ cents.
Cost of carbon steel erected $= 3 + 0.5 + 1.5 = 5.0$ cents.
Cost of silicon steel erected $= 3.5 + 0.5 + 1.7 = 5.7$ cents.
Carbon steel, 142 lb., at 5 cents = \$7.10 per sq. ft. of floor.
Silicon steel, 121 lb., at 5.71 cents = \$6.90 per sq. ft. of floor.

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[15] "Bridge Engineering," pp. 597 to 601, inclusive.

Hence, the alloy steel is the cheaper. If the span length were 200 ft., $r = \frac{110}{97} = 1.134$. and,

Cost of silicon steel erected = 3 + 0.5 + 1.7 = 5.7 cents.
Carbon steel, 142 lb., at 5 cents = $7.10 per sq. ft. of floor.
Silicon steel, 121 lb., at 5.71 cents = $6.90 per sq. ft. of floor.

This shows that 200 ft. is the span length for equal cost under the assumptions made, agreeing exactly with the limit stated in the paper.

Although these two costs are alike, the choice should be in favor of the carbon-steel bridge, because of the slightly thicker metal that would naturally be used and the consequent longer life of structure, as far as resistance to rust is concerned.

The variation between Mr. Tammen's conclusions and those of the writer in regard to silicon steel *versus* carbon steel is due to the differences in the initial assumptions. The only way to settle the matter would be by open competition among a large number of bidders; and even then the idiosyncrasies of the competitors' computers and cost adjusters would tend to prevent a truly valid comparison.

Mr. Tammen asks what general specifications were used and what was the ratio used between allowable carbon and silicon unit stresses. The answer is "those of 'Bridge Engineering' as far as they would apply, and $\frac{16\,000}{24\,000}$."

Mr. Tammen is quite right when he says:

> "* * * while such economic studies as guides to conclusions are desirable tools and are often of considerable use to the engineer who devises them and has an understanding of their limitations, they must be applied by others with caution. Formulas cannot be devised to take the place of ripened experience and judgment."

Nevertheless, information of the kind furnished in this paper can be of great use to both young and old engineers who are studying or practicing in bridge work.

It is truly gratifying to note that Mr. Skinner in his discussion recognizes the importance and potential usefulness of such investigations. An endorsement by an engineer of his ripe age, wide experience, and high standing, ought to be convincing, even to the most "hard-boiled Doubting Thomas."

Both Professors Abbett and Spofford have suggested a comparison of the economics of continuous-truss and cantilever bridges. A dozen years ago the writer, in collaboration with H. Malcolm Priest, M. Am. Soc. C. E., made an exhaustive investigation of "Comparative Economics of Continuous and Non-Continuous Trusses."[16] The summary of conclusions reads as follows:

First.—For long spans the divided-triangular trussing is decidedly superior to the Petit trussing for bridges with continuous-truss spans, but not much so, if at all, for those of simple-truss spans.

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[16] Presented as a memoir to the Engrs. Soc. of Western Pennsylvania, and, later, incorporated as Chapter XI of the writer's treatise, "Economics of Bridgework," pub. in 1921.

Second.—For long spans there is an important saving of metal by the adoption of continuous trusses, and this saving is nearly twice as great for standard highway bridges as for modern, double-track railway bridges.

Third.—For long-span bridges the method of treating the matter of stress reversal has practically no effect upon the comparative economics of continuous and non-continuous trusses.

Fourth.—For comparatively short-span, steam-railway bridges, the continuous truss has a small advantage over the simple truss only when the divided-triangular trussing is used and stress reversals are ignored. In all other cases the comparison is either a stand-off or in favor of the simple truss.

Fifth.—For comparatively short-span, steam-railway bridges, the divided-triangular trussing is generally more economic of metal than the Pratt trussing.

Sixth.—In no case should either the Pratt or the Petit truss be used for continuous spans, because in these the divided-triangular truss is more economic.

Seventh.—In case of a structure requiring erection by semi-cantilevering, the continuous truss will possess an advantage over the non-continuous one, at least to the extent of the weight of extra metal required by the toggles for connecting temporarily over the center pier the inner hips of the two simple-truss spans. This advantage may even extend to the reinforcement for erection stresses, but if it does, the gain thus involved can never be very great.

About the only worth-while economic comparison that can be made between continuous-truss and cantilever spans is by the adoption for the latter of Type *C*, in both of which all spans are either of the same length, or nearly so. When the effect of reversion is duly considered (as it always ought to be), continuous trusses and non-continuous ones in highway bridges weigh alike for spans of 387 ft., but for spans of 775 ft., the ratio is 1.22 in favor of continuity.[17] Assume that for intermediate span lengths the ratios can be interpolated directly. Figs. 12 and 14 of the paper and those varying ratios furnish the information given in Table 12.

TABLE 12.—COMPARATIVE WEIGHTS OF METAL

Total lengths of structure, in feet	Simple-span lengths, in feet	WEIGHTS OF METAL, IN POUNDS PER SQUARE FOOT OF FLOOR FOUR-LANE STRUCTURES		
		Type *C* cantilever	Simple truss	Continuous truss
1 200	400	138	150	150
1 400	467	159	170	166
1 600	533	190	197	188
1 800	600	210	225	208
2 000	667	236	260	233
2 200	733	262	295	258
2 400	800	292	340	287

It is evident that, in comparison with a layout of equal-length, continuous-truss spans of less than 500 ft., the Type *C* cantilever structure is somewhat more economic, but for greater lengths, the reverse is the case, the advantage of continuity augmenting very slowly with the span length. However, as simple-truss spans of 450 ft., for the sake of rigidity, would probably be adopted instead of cantilever construction, and as the percentage of variation for greater span lengths up to 800 ft. is quite small, it is fair to conclude that in regard to the comparative economics of Type *C* cantilevers and continuous-truss structures, there is practically no choice between them.

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[17] "Economics of Bridgework," p. 80.

Mr. Abbett is right in his statement that "the economics of the arch is so vitally affected by local foundation conditions that it does not lend itself to a *carte blanche* comparison with other types." It might be pointed out that the height of roadway above water level still further complicates the problem.

Replying to Mr. Lincoln's question as to whether the economic functions determined for the paper will apply to deck structures as well as to through bridges, the answer is in the affirmative, except in so far as a small height of deck above water level may prevent the adoption of economic span lengths or of economic truss depths.

Mr. Wessman's remarks are directly to the point; and the writer takes no exception to any of them. Whether the ratio of length of suspended span to that of main opening is 0.45 or 0.5 makes very little difference in the total weight of metal, but it may in the æsthetics of the layout. It is conceivable that a case might arise wherein a greater ratio than 0.5 would be advisable, in order to meet some unusual condition, but this is not likely to occur very often.

The average of the eight actual cases quoted by Mr. Wessman is 0.41, while that of the four theoretical investigators mentioned in the paper is 0.615. Averaging these two averages gives 0.51, while, as therein stated, the writer's average of four results was 0.49; hence it may be concluded that the findings of his investigation of this economic function agree quite closely with a compromise between theory and past practice.

Answering the comments of Messrs. Allen and Ballard, the writer deems the data they have presented to be of great value to the Engineering Profession, as well as a fine supplement to his paper.

As for the disagreement between their weights of metal and his, that is easily accounted for by the variation in specifications, especially in the items of live load and impact, which would tend to make the two plotted lines of weights cross. Furthermore, the weight per square foot of the flooring adopted would have a powerful influence in causing differences in weights of metal.

There can be no mistake of any importance in the writer's weights for Type *A* cantilever bridges, because they were based largely on actual constructions. A valid explanation of the alleged lightness in the weights of the writer's cantilever bridges with main-span openings of less than 700 ft., is that for many years both he and his associates have studied deeply into the matter of economy in bridge designing, and have applied their findings in all their work.

The writer's statement in his third conclusion that "the economic length for the anchor arms in that type of structure [Type *A* cantilever bridges], when the distance between piers is fixed, is as short as the æsthetic appearance of the bridge will permit," should have had this limitation—"provided that trestle approaches are economically feasible." In the recapitulation of "the useful findings of the entire investigation," given at the end of this closing discussion, this restriction is indicated, and the possible economics of lengthening the anchor arms is discussed.

There is another economic function that has developed as an afterthought, but which should have been included in the original paper; that is, the length of the anchor arm in a Type *A* cantilever giving zero reaction on the anchor pier when the maximum live load is on the main span. This condition occurs when one cantilever arm and the suspended span are fully loaded, with no live load on the anchor arm. The live load per linear foot under this condition is that given in Fig. 1, plus the impact allowance indicated in Fig. 3, for a length of $C + S$.

The dead load for an anchor arm of any assumed length can be obtained by proportion from the dead loads of the cantilever and anchor arms already computed for the case under consideration; and no error of any account is made by assuming that both these dead loads are uniformly distributed.

With these assumptions, using the data in Table 2, and assuming various lengths in each case for the anchor arm, the results were as follows:

Name of structure.	Ratio of lengths of anchor arm to total main opening for zero reaction.
Perth Amboy Bridge	0.50
Elizabeth Bridge	0.50
Cairo Bridge	0.50
Charleston Bridge, for 1 050-ft. span	0.507
Charleston Bridge, for 640-ft. span	0.50

The slightly larger ratio for the 1 050-ft. span of the Charleston Bridge is possibly due to the great length of the main span; but the difference is so slight that it will be proper to conclude that, for zero reaction at the anchor pier, the length of the anchor arm will be exactly one-half that of the main opening in all cantilever bridges of Type *A*.

In making these computations, the effect of wind pressure was ignored; but with the almost impossible distribution of live loading that causes the zero uplift, it would be unreasonable to assume any wind pressure acting. Its effect would be to increase very slightly the 50% ratio found.

With a comparatively long anchor arm, as in the case of the Perth Amboy Bridge, an expensive pendulum support at the anchorage is not necessary. As a matter of precaution under such conditions, a slight holding-down detail can be used with a roller-bearing; or, if there is a flanking span, its end can be supported on the rolling pedestal. If the flanking span is quite long, the length of the anchor arm can be made much less than one-half that of the main opening without the necessity for adopting a pendulum support.

Before closing this résumé of the discussions, it will be well to recapitulate the useful findings of the entire investigation. They are, as follows:

Type A Cantilevers.—(A Suspended Span, Two Cantilever Arms, and Two Anchor Arms).—

First.—The economic ratio of length of suspended span to length of main opening may generally be taken at 0.5, or a little shorter if the appearance of the structure would be improved thereby. The layout involving the smallest weight of metal is not always the cheapest, because sometimes the erection conditions affect the economic length of the suspended span.

Second.—The economic truss (or tower) depth over main piers varies from 12 to 15% of the distance between centers of those piers, with an average of 13.5 per cent.

Third.—The economic length for the anchor arms, when the distance between main piers is fixed, is as short as æsthetic considerations will permit, say, 25 or 30% of the main opening, provided the depths of water beyond the anchor arms are not too great for steel-trestle construction. Should the water conditions necessitate truss flanking spans instead of the trestle work, it might prove economical to lengthen the anchor arms in lieu of using simple-truss flanking spans. This would undoubtedly effect a reduction per linear foot in the quantities of materials for piers, but might increase the weight of metal per linear foot in the anchor arms. Using the data recorded in the paper, and with due recognition of the law of variation of truss weights when augmenting the length for the anchor arms, it is a question requiring for solution a very few minutes of a competent computer's time.

Fourth.—When the distance between anchor piers is fixed, with no restriction as to the locations of the two main piers, the economic length for each anchor arm is about one-quarter of the total length of the structure.

Fifth.—The length of anchor arm giving a zero reaction on the anchor pier when the spans of the main opening are subjected to their greatest live load, and when the effect of wind pressure is ignored, is exactly one-half the length of the main opening.

Type C Cantilevers.—(An Anchor Span, Two Cantilever Arms, and Two Suspended Spans).—

Sixth.—In Type *C* cantilever bridges, the economic length of the anchor span is almost exactly 36% of the total length of the three-span layout; and even a small deviation from this economic proportion involves a material excess in the total weight of truss metal.

Seventh.—The economic truss depth at mid-length of any anchor span of economic proportions is about three-tenths that over the piers; but such a depth would be too small for appearance, and probably also for rigidity. The inferior limit should be about four-tenths the depth over piers.

General.—

Eighth.—In highway bridges it is not economical to change to cantilever construction from three simple-truss spans of equal length, unless that length is in excess of 400 ft.

Ninth.—Concerning the comparative economics of cantilever and continuous-truss bridges, there is little difference. It has been found that, in comparison with a layout of equal-length, continuous-truss spans less than 500 ft. long, the Type *C* cantilever is somewhat more economical; but that for greater span lengths the reverse is the case, the advantage of continuity augmenting very slowly with the span length.

Tenth.—In the trusses of highway cantilever bridges, the weight of metal generally divides itself between the chords and the web in the ratio of 3 to 2; and this division applies to the different spans as well as to the structure as a whole.

Eleventh.—At ordinary unit prices for metals erected in simple-truss spans, it begins to be economical of material to use silicon steel at a span length of about 200 ft. for four-lane bridges and 300 ft. for two-lane bridges; but for truly warranted cantilever structures, silicon steel will invariably be found to be less expensive than carbon steel. For spans slightly greater than these limits, especially in light structures involving thin sections of metal, it is good practice to adhere to the use of carbon steel, so as to make better provision against deterioration by rust. If the variation in unit prices, erected, for carbon steel and silicon steel is reduced in the future, as ought to be the case, the economic superior limits herein stated for carbon-steel construction in simple-truss spans will be increased.

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TRANSACTIONS

PAPER NO. 1846

DESIGN CHARACTERISTICS OF THE READING OVERBUILD TRANSMISSION LINE

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WITH DISCUSSION BY MESSRS. E. E. R. TRATMAN, PERCIVAL S. BAKER, AND FREDERICK W. DECK

SYNOPSIS

The usual type of transmission-line structure that supports high-tension lines across open country is of so frequent occurrence, that little comment is aroused when such a line is constructed, unless it is of unusual length or carrying capacity. Sometimes, however, the approach of a power line to the boundaries of a city requires the use of unusual types of supporting structures in the effort to minimize right-of-way costs and provide additional safety in regions where the damage attendant upon a failure of the line would be disastrous.

This was the case at Philadelphia, Pa., where the power from the Conowingo Hydro-Electric Plant and interconnection lines is brought into the city on wires supported by structures built over the tracks of the Reading Railroad. Certain features of the structural designs are most unusual and interesting, and are described briefly in this paper.

GENERAL DESCRIPTION OF PROJECT

Power generated at the hydro-electric plant[2] of the Philadelphia Electric Company, on the Susquehanna River, at Conowingo, Md., is transmitted about 57 miles at 220 kv. to the large distributing sub-station at Plymouth Meeting, Pa., about 10 miles outside of Philadelphia. This sub-station is also the terminus of 220-kv. lines connecting to adjacent large systems in Pennsylvania and New Jersey. From Plymouth Meeting, the power is carried at 66 kv. to sub-stations within the city.

NOTE.—Published in February, 1932, *Proceedings*.

[1] Structural Engr., Mech. Div., Philadelphia Elec. Co., Philadelphia, Pa.

[2] For description of this plant, see *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 970.

For this latter section of the power-transmission scheme it was desirable to provide avenues of entry to the city as independent of each other and as capable of expansion as possible. Over a part of the distance from Plymouth Meeting to Philadelphia, this was accomplished by building parallel tower lines carrying two circuits each, on a wide, corporately owned, right of way. On account of the closely settled condition of the district within the environs of the city, and the consequent high cost of right of way, the transmission lines were brought in over two separate railroad rights of way. A co-operative agreement between the Electric Company and the Railroad Company on the basis of mutual benefit, permitted the adoption of this method of transmission within the city limits.

One of these lines was built in 1928, with two circuits in operation.[3] It is the purpose of this paper to describe the design of the second of these lines, about 4½ miles in length, embodying unusual features in the design of the transmission structures and their foundations. A similar problem of overbuild or collinear construction was solved in the first overbuild line; but in the line described herein, the difficulties of design and construction were far greater than those encountered in the earlier work.

LAYOUT OF TRANSMISSION LINE

This transmission line is built over and along a passenger railroad, through a city street with buildings on each side, and with many of its supporting structures located on a concrete and steel viaduct (see Fig. 1). In general, the limitations of the right of way made necessary the use of some type of structure occupying a minimum of area at the supports, while carrying the power wires as nearly as possible over the center of the trackage, in order to provide adequate lateral clearance from the boundaries of the railroad right of way.

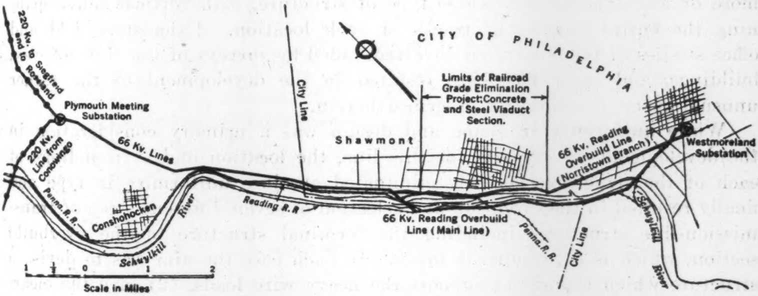
220 Kv. to Siegfried and to Roseland
Plymouth Meeting Substation
220 Kv. Line from Conowingo
66 Kv. Lines
Penna. R. R.
Conshohocken
River
Schuylkill
City Line
CITY OF PHILADELPHIA
Limits of Railroad Grade Elimination Project; Concrete and Steel Viaduct Section.
Shawmont
Reading R. R.
66 Kv. Reading Overbuild Line (Main Line)
66 Kv. Reading Overbuild Line (Norristown Branch)
Westmoreland Substation
Penna. R. R.
City Line
Schuylkill
River
1
½
0
1
2
Scale in Miles

FIG. 1.—ROUTES OF THE CONOWINGO TRANSMISSION LINES FROM PLYMOUTH MEETING TO PHILADELPHIA, PA.

One section of the railroad lies on a tangent, but the remainder of the line curves through a city street; and in this latter part the Railroad Company rebuilt its surface line as an elevated section and eliminated grade crossings.

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[3] *Engineering News-Record*, May 3, 1928, p. 686.

The problem of layout, therefore, involved first, the determination of the most economical span length between structures, providing adequate vertical clearance above the rails and lateral clearance for the side swing of the wires, while using a tension in the wires that would allow an economical tower design. In order to obtain this economical structure and maintain the required vertical clearance, it was necessary to reach a balance between the low-wire tension usable with high structures and a high-wire tension usable with low structures. For the straight line, a basic span of 600 ft. was chosen, dependent on the previously mentioned considerations of vertical and lateral clearance, and economical structure design. The actual maximum span used was 803 ft., and the minimum, 280 ft.

On curves the structures were located with shorter intervening spans; and over the elevated viaduct section the transmission-line construction was restricted as closely as possible to the width actually used by the two-track viaduct. This restriction forced the incorporation of the footings for the transmission structures as a part of the railroad viaduct structure. In this section, also, it was not always possible to locate the structures where they would be most suitable from the theoretical layout point of view, and frequently the locations were those at which the supports for the power wires would interfere least with the railroad facilities, which were already restricted.

General Description of Structures

The original estimates of cost for supporting structures were based on a more or less hypothetical bridge type of structure, with vertical sides, spanning the entire number of tracks at each location. Exhaustive field and office studies of the conditions involved, aided by surveys of the right of way, buildings, and other obstacles, resulted in the development of the rather unusual types of structures described herein.

While uniformity in shape and design was a primary consideration in the development of supports for this line, the location and surroundings of each of the structures speedily eliminated absolute uniformity in type and finally resulted in the evolution and selection of seven different types of transmission-line structures including the terminal structure of the overbuild section, which is a two-circuit tower. In each case the aim was to devise a structure which would (1) support the heavy wire loads, (2) give the clearances required, and (3) afford a maximum simplicity of design and ease of construction, all at a minimum cost.

The height of all structures was determined by the required clearance of wires at the lowest point of the sag curve from the rail, and a standard height of approximately 68 ft. above the rail for the lowest cross-arm on bridge structures and towers was adopted. Where greater clearance was required in the adjacent spans for crossing over other railroads, or streets, this height was increased to 78 ft., or to the maximum of 90.5 ft.

A vertical clearance of 45 ft. from wires to rail, with the maximum sag in the wires under normal weather conditions, is maintained throughout, with a minimum diagonal clearance of 20 ft. between wires and the nearest portion of any building.

The railroad right of way for a large part of the line is 66 ft. wide and, where possible, 65.5 ft. was adopted as the over-all width of bridge structures, giving space for four tracks, ample clearance on each side of the outside track, and a framed post, 3 ft. 3 in. in width, outside the clearance line on each side (see Fig. 2). Modification of this width was required at points where structures were located along and over the elevated viaduct section of the railroad, and elsewhere, as demanded by right-of-way conditions. Typical dimensions are shown in Fig. 2.

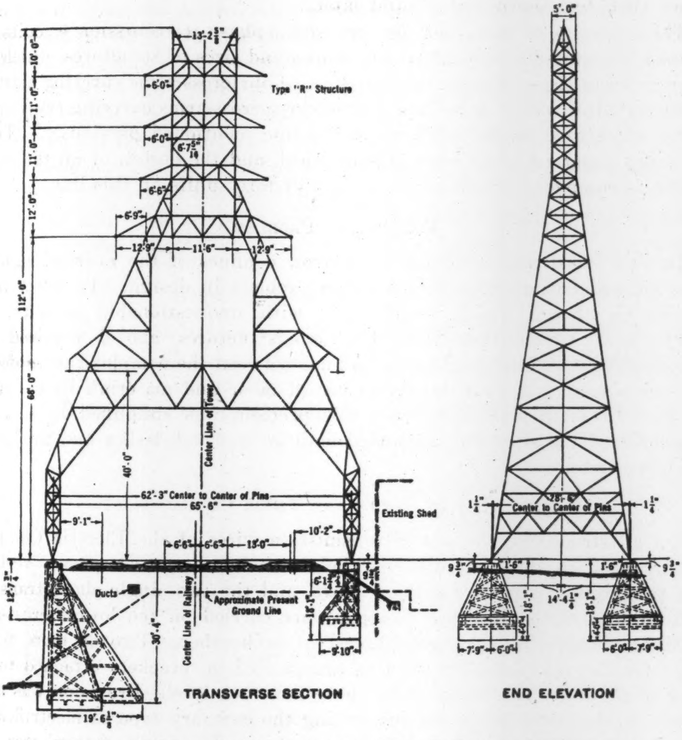

13'-2 5/8"
Type "R" Structure
10'-0"
11'-0"
11'-0"
12'-0"
112'-0"
68'-0"
6'-0"
6'-0"
6'-7 5/16"
6'-6"
6'-9"
12'-9"
11'-6"
12'-9"
Center Line of Tower
40'-0"
62'-3" Center to Center of Pins
65'-6"
9'-1"
10'-2"
6'-6"
6'-6"
13'-0"
Existing Shed
12'-7 3/4"
Ducts
Center Line of Pins
Center Line of Railway
Approximate Present Ground Line
30'-4"
6'-1 3/4"
9 3/4"
18'-1"
9'-10"
19'-6 1/8"
TRANSVERSE SECTION
5'-0"
28'-6"
1 1/4"
Center to Center of Pins
1 1/4"
9 3/4"
1'-6"
1'-6"
9 3/4"
18'-1"
14'-4 1/4"
18'-1"
4'-0"
4'-0"
7'-9"
6'-0"
6'-0"
7'-9"
END ELEVATION

FIG. 2.—GENERAL DIMENSIONS OF A TYPE *R* BRIDGE STRUCTURE.

The cross-arm arrangement for carrying wires was determined by the required lateral and vertical clearances between conductors, and by the height, width, and slope of the sides of the structures. In general, there are two types of tops, one comprising the tower and the eccentric bridge structure, and the other (see Fig. 2) covering the remaining four types. The design of these types was based on the desirability of maintaining certain features of symmetry in loading with a minimum of change in the slope of the posts.

Many changes from the original plans were made as the result of the development of unforeseen features. The early construction of the elevated viaduct section of the railroad to eliminate grade crossings had the effect of forcing the completion of the designs of most of the foundations before the design of the structures was finished, since it was desirable to incorporate this work in the course of construction simultaneously with the railroad work rather than tear down and rebuild later.

The necessity of providing for two single-phase transmission circuits for railroad use caused a revision of the shapes and sizes of structures, which are different from those used on the first line. Four cross-arms carrying thirteen conductor wires were required as against three cross-arms carrying twelve conductor wires used on the other overbuild line mentioned previously.[3] These and many other revisions were accomplished, and the design of all the structures was completed, resulting in the use of forty units on this line.

FOUNDATION PROBLEM

In such construction within the narrow confines of the railroad right of way, the foundations presented a major problem in design. To select those foundation shapes which would bear, with unquestionable security, the heavy loads transmitted from the superstructures above, required the utmost care in maintaining a nice balance between the complexities in design and consequent cost, and the lavish use of labor and materials in a simpler design. In some places, however, this problem was simplified in that the exigencies of the situation demanded both an unusual design and the use of much concrete.

ARRANGEMENT OF WIRES AND THEIR CHARACTERISTICS

On all structures, the power-transmission wires of the Electric Company are arranged in three circuits, parallel to each other, with phases vertical. The two ground wires are carried above, and the two single-phase transmission circuits of the Railroad Company are carried on the lowest cross-arm, the four wires being arranged abreast of each other. Two or more feeder wires for the railroad electrification are carried on brackets attached to the sides of the structures about 40 ft. above the track level. Provision was made on all "bridge" structures for suspending the catenary type of electrification wires of the Railroad Company from points on the inside of the lower arch of the structure. On tower structures, the catenary wires are supported from a cross-member one end of which is attached to a pole on the other side of the tracks, and the other end to the tower.

The conductor wires for the Electric Company are 500 000 cir. mils, bare, stranded, hard-drawn copper, with an ultimate strength of 23 450 lb., and an allowable working tension of 11 770 lb. They are strung so as to have a maximum tension of 8 000 lb. under the maximum loading of $\frac{1}{2}$ in. of ice measured radially, and 8 lb. of wind per sq. ft. of longitudinal cross-section of the loaded wire exposed to the wind. The ground wires are a composite cable consisting of seven strands of a copper, aluminum, and tin alloy, and an outer layer of twelve strands of a cadmium-copper alloy. This cable has a diameter of 0.477 in., with a breaking strength of 12 500 lb. and an elastic limit of 7 500 lb. Under maximum loading of $\frac{1}{2}$ in. of radial ice, and 8 lb. of wind per sq. ft., it has a maximum tension of 3 900 lb., and thereby closely conforms in sag to the conductor wires.

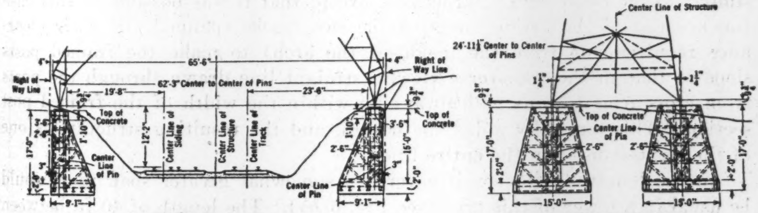
The conductor wires for the Reading Railroad are smaller than those for the Electric Company. The other wires supported by the structures described are for the purpose desired by the Railroad Company in its electrification plan. All the wires used for the Electric Company are attached to the cross-arms of the supporting structures through strain-insulator assemblies of a double string of nine standard strength insulator units, with a total ultimate strength of the assembly of 24 000 lb.

The pull of the conductors is thus applied directly to the cross-arms, imposing a large load on the structures. This method of attachment was chosen because it afforded advantages in clearance, a saving in the heights of structures, and additional security of the line against failure in this important section over the railroad.

Detailed Description of Structures

The structures have been classified into seven types—*R, T, U, V,* and *W* bridges and *M* and *C* towers, respectively, and, for convenience, these designations will be used in the paper.

Type R Bridge.—The bridge structure shown in Fig. 2 is 65.5 ft. wide at the base in the transverse view; it spans four tracks, is 68 ft. high to the lowest cross-arm, and 112 ft. high over all. This is the type used most frequently on this work. It is a symmetrical structure, the lowest section being an arch portal, and framed throughout. The posts are pin-connected to the footings through standard cast-steel shoes which are secured to the upper section of the steel framework of the footings by anchor-bolts.


4"
65'-6"
4"
Right of Way Line
Right of Way Line
62'-3" Center to Center of Pins
19'-8"
23'-6"
Top of Concrete
Top of Concrete
3'-6"
2'-6"
17'-0"
7'-10"
12'-2"
Center Line of Siding
Center Line of Structure
Center Line of Track
3"
Center Line of Pin
9'-1"
1'-10"
3'-6"
2'-6"
15'-0"
3"
2'-0"
Center Line of Pin
9'-1"
Center Line of Structure
24'-11⅛" Center to Center of Pins
1¼"
1¼"
Top of Concrete
Center Line of Pin
2'-6"
2'-6"
17'-0"
2'-0"
2'-0"
17'-0"
15'-0"
15'-0"

TRANSVERSE VIEW LONGITUDINAL VIEW

FIG. 3.—MODIFIED END POSTS OF A TYPE *R* BRIDGE STRUCTURE.

This structure was modified as shown in Fig. 3, to enable it to meet the conditions of high banks on both sides of the tracks without excessive excavation, by cutting off the lower posts at different levels. The cross-arms are maintained at the standard height of 68 ft. above the rail, and the structure is the same as that shown in Fig. 2, except for the shortened legs. This innovation sometimes resulted in a two-hinged arch, with hinges at different levels.

Type T Bridge.—Possibly the most interesting of all the bridge structures is that shown in Fig. 4. It was designed for the purpose of carrying the transmission wires offset from the center of the structure, in order to remove them as far as possible from high buildings or other obstacles on one side of the tracks. Space limitations for the base required the use of some kind of eccentric structure, and in this case it was desired to retain as much symmetry and simplicity as possible.

It was found possible to make the base width just twice that required between the posts of the standard cross-arm, and by carrying one post down from the cross-arm section to the ground vertically, the other post was exactly in the center of the structure, with the consequent advantage in distribution of the load from this side of the structure to the two post bases. The load on the straight side is carried down mainly through the vertical post to the pin. The various loading combinations, including torsional effects, naturally modified this to some extent, but the results of this symmetry of arrangement where possible in an eccentric structure were shown in its consequent comparative lightness. The structure spans two tracks; it is pin-connected to the footings in the manner described for the Type *R* Bridge; framed throughout; and, as in the case of the other bridges, the lower section is an arch portal in its stress diagnosis.

Type U Bridge.—Fig. 5 shows a type of structure that was arranged for special use on the steel portion of the railroad viaduct. It is symmetrical in shape, spans two tracks, is 44 ft. wide from center to center of pins, and measures approximately 68 ft. from top of rail to the lowest cross-arm. It is similar to the other bridge structures, except that it was possible in this case (on account of the arrangement of the two tracks spanned and their clearance requirements from the inside of the arch) to make the framed posts slope so that in the transverse view, a straight line drawn through the posts from cross-arms to pins will always lie within the width of the framed post section. This naturally aided the design, and the resulting structure is one of the lightest used on the entire line.

In the longitudinal view it covered a somewhat greater span than would be usual in a tower of this type (see Fig. 6(*b*)). The length of 40 ft. between pins was used to allow them to rest directly above the posts of the steel viaduct, thus reducing the complications in loading. This structure is attached directly to the steel of the railroad viaduct and, therefore, requires no separate footings. A standard shoe was used.

The Type V Bridge.—The structure shown in Fig. 7 is similar to those shown in Figs. 2, 4, and 5. Due to clearance requirements the inside outline of the steel arch does not follow the same lines as those of the simpled *U* type (Fig. 5). This tower is 122 ft. high over all. It was devised primarily to fill the special requirements of a high railroad-crossing structure with a narrow base. In the one location where it was used, it was necessary, for clearance purposes, to make the two posts on one side, of solid girder-beam sections.

Type W Bridge.—Fig. 8 shows a Type *W* Bridge, 112 ft. high over all. It is similar in analysis to the other bridge structures and was devised to span two tracks, furnishing a narrow base structure for those locations not

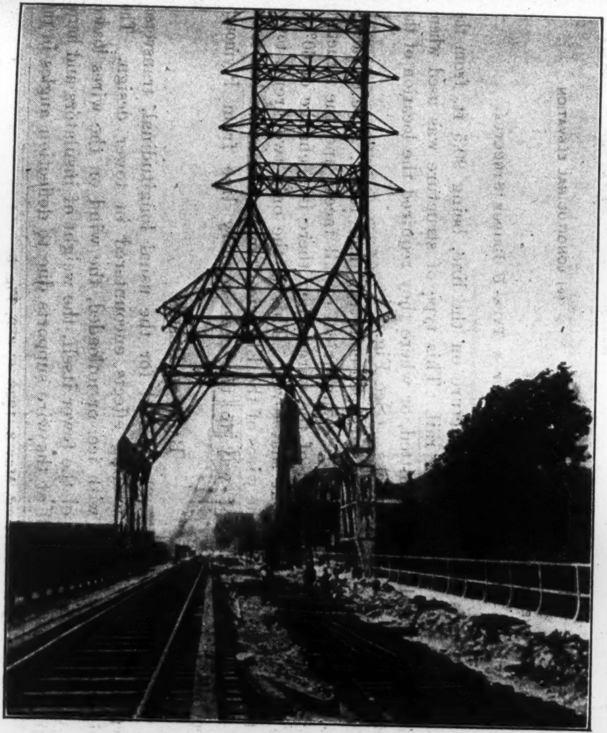


FIG. 4.—TYPE *T* BRIDGE STRUCTURE.

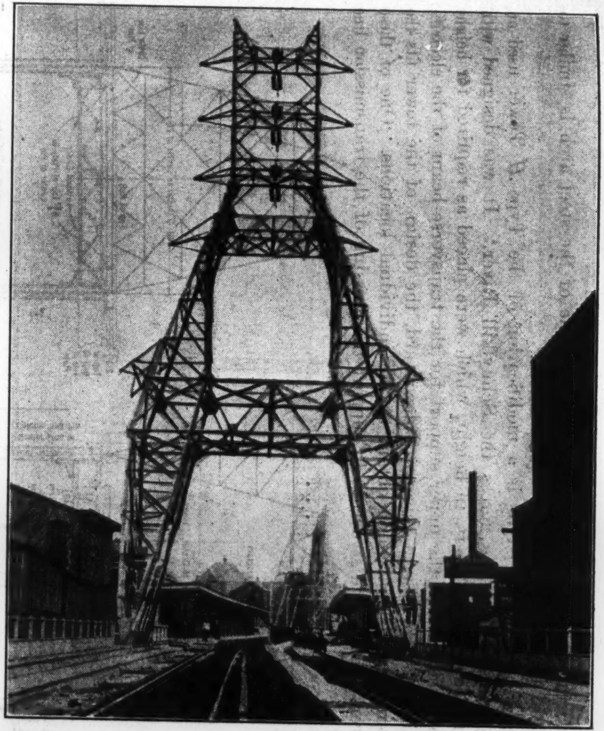
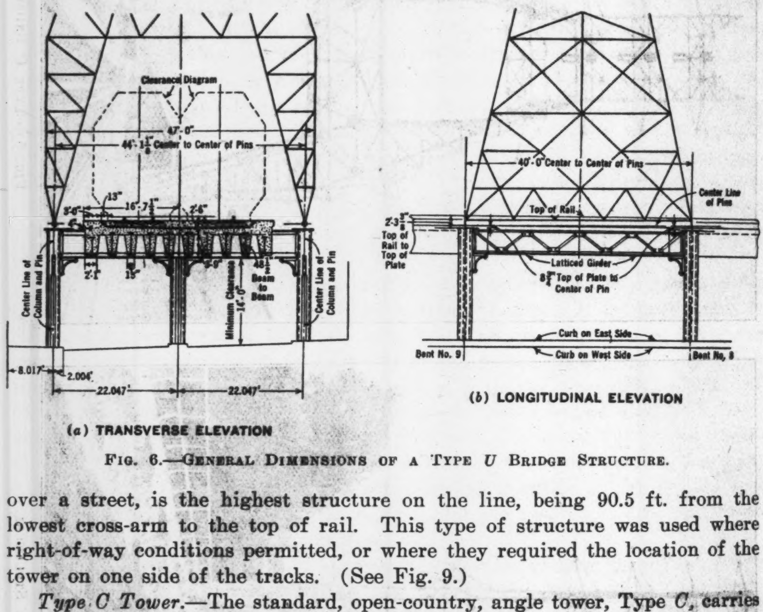


FIG. 5.—TYPE *U* BRIDGE STRUCTURE.

requiring special height. The inside outline of the steel arch is similar to that of the Type *V* Bridge (Fig. 7).

Type M Tower.—This is a modification of the Type *H* Tower used on the transmission line across the Schuylkill River.[4] It was designed with standard sections 7.5 ft. in height, which were placed as required for height above the level of the upper support for the transverse beam of the electrification system. From this point downward, the design of the tower fits the ground conditions which obtain at the individual locations. One of these towers used as the structure to support the crossing of the transmission line over a street, is the highest structure on the line, being 90.5 ft. from the lowest cross-arm to the top of rail. This type of structure was used where right-of-way conditions permitted, or where they required the location of the tower on one side of the tracks. (See Fig. 9.)


Clearance Diagram
47'-0"
44'-1⅛" Center to Center of Pins
13"
3'-0"
16'-7½"
2'-6"
6"
2'-1"
15"
3'-0"
48½" Beam to Beam
Minimum Clearance 14'-0"
Center Line of Column and Pin
Center Line of Column and Pin
8.017'
2.004'
22.047'
22.047'
40'-0" Center to Center of Pins
Center Line of Pins
Top of Rail
2'-3⅜"
Top of Rail to Top of Plate
Latticed Girder
8¾" Top of Plate to Center of Pin
Curb on East Side
Curb on West Side
Bent No. 9
Bent No. 8

(*a*) TRANSVERSE ELEVATION

(*b*) LONGITUDINAL ELEVATION

FIG. 6.—GENERAL DIMENSIONS OF A TYPE *U* BRIDGE STRUCTURE.

Type C Tower.—The standard, open-country, angle tower, Type *C*, carries two circuits only; the sides are symmetrical on all faces; and the structure is designed to stand the pull of the wires when there is a change of 60° in the direction of the line at that tower. This is the only two-circuit tower on the line, and carries the wires of the Electric Company from the last tower on the railroad right of way to the cross-country line from Plymouth Meeting.

BASIS OF DESIGN

These structures are all planned for the usual longitudinal, transverse, vertical, and torsional loading effects encountered in tower design. The weight of wires loaded with ice, or unloaded, the wind on the wires loaded or unloaded, the weight of the tower itself, the weight of insulators and hardware, the transverse pull at the wire supports due to deflection angles in the

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[4] *Engineering News-Record*, May 3, 1928, p. 689, Fig. 5.

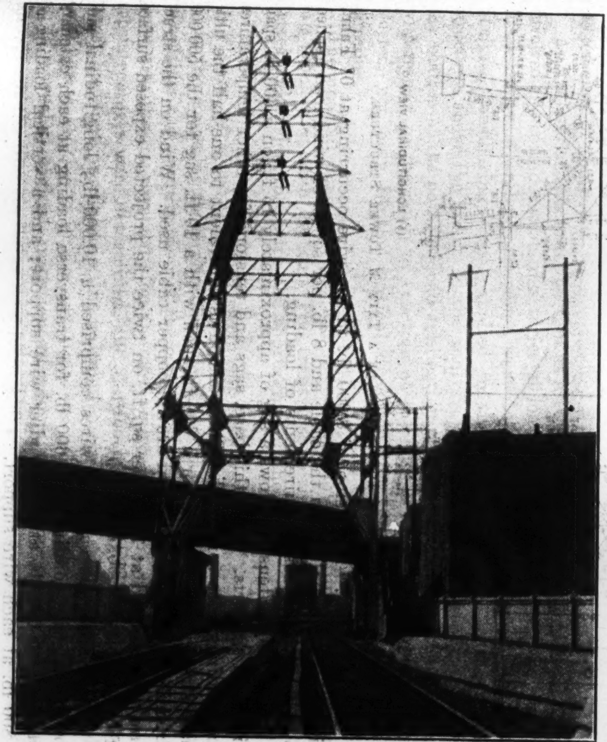


FIG. 7.—TYPE *V* BRIDGE STRUCTURE.

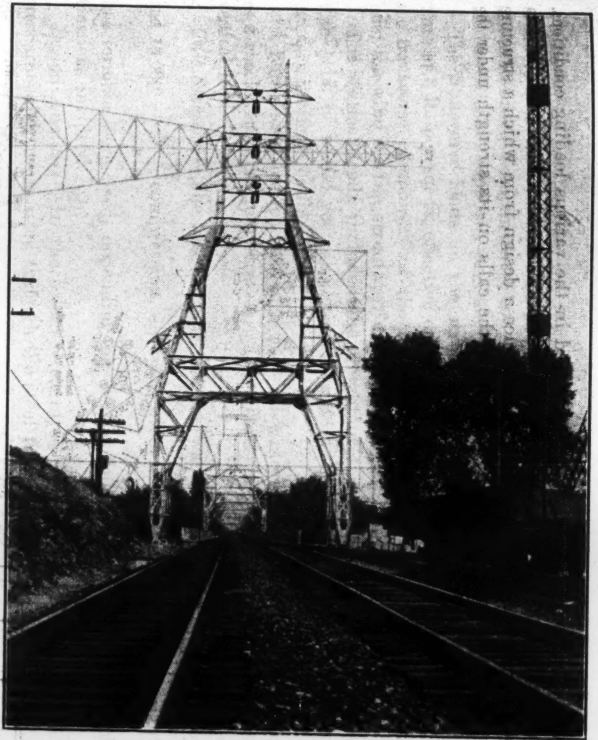
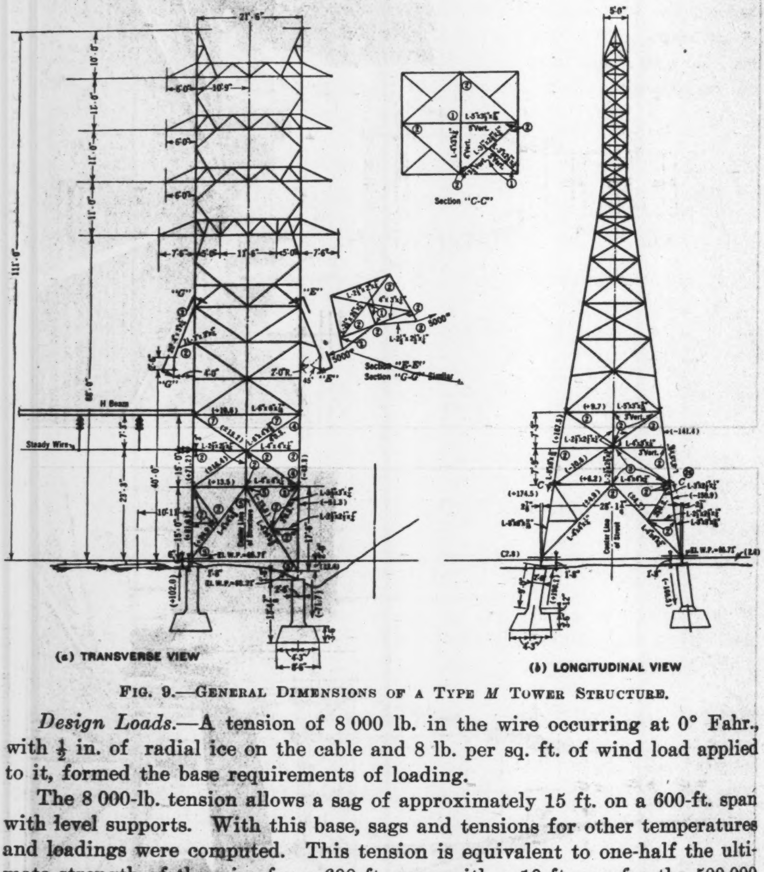


FIG. 8.—TYPE *W* BRIDGE STRUCTURE.

line, broken wires of the number specified in the various loading conditions, and wind on the tower itself, are among the factors which were considered in the several combinations, in order to produce a design from which a structure could be fabricated, adequate to meet all the calls on its strength under the most adverse conditions.



(a) TRANSVERSE VIEW

(b) LONGITUDINAL VIEW

FIG. 9.—GENERAL DIMENSIONS OF A TYPE *M* TOWER STRUCTURE.

Design Loads.—A tension of 8 000 lb. in the wire occurring at 0° Fahr., with $\frac{1}{2}$ in. of radial ice on the cable and 8 lb. per sq. ft. of wind load applied to it, formed the base requirements of loading.

The 8 000-lb. tension allows a sag of approximately 15 ft. on a 600-ft. span with level supports. With this base, sags and tensions for other temperatures and loadings were computed. This tension is equivalent to one-half the ultimate strength of the wire for a 600-ft. span with a 10-ft. sag for the 500 000 cir. mils, bare, stranded, hard-drawn, copper cable used. Wind on the structure was assumed at 13 lb. per sq. ft. on twice the projected exposed surface of one face.

Loads from the catenary wires comprised a 10 000-lb., longitudinal pull for broken wire conditions; 3 600 lb. for transverse loading at each catenary support; 10 000 lb. at the steadier wire support; and a vertical loading of 1 800 lb. at each wire support.

Conditions for loading the tower included the longitudinal pull of nine conductors of the Electric Company at 8 000 lb. each, and four conductors of the Railroad Company at 5 000 lb. each; or 11 770 lb. each at any four of the eleven cable supports, or 7 500 lb. each at the four lower conductors.

The transverse loads on the structure consisted of the wind load on the wires at 8 lb. per sq. ft. effective on the cable loaded with $\frac{1}{2}$ in. of radial ice; the transverse component of the longitudinal tension due to angles in the line; the wind on the structure; and the catenary loadings.

The vertical loads included the weight of the cable unloaded, or loaded with $\frac{1}{2}$ in. of radial ice; the dead weight of the structure itself; and the catenary vertical loadings.

A 2-in. radial ice loading with no wind was used in the load applications as a check for maximum stresses. This represents the condition of extreme vertical load.

Unit Stresses.—The base for unit stresses was 18 000 lb. per sq. in. tension and $18\,000 - \frac{60\,l}{r}$ compression, although other values based on the straight-line formula were used, depending on the loading conditions and the combinations involved.

Design Analysis.—In the bridge structures the loads were applied at the cross-arms and transmitted through the steel framework to the arch portal in the lower section. Here, they represented loads on the crown of the arch and resulted in the consequent thrusts in the transverse direction at the pin-connected ends of the posts.

An analysis of this lower section as a two-hinged arch was necessary. This involved the consideration of a series of load combinations which are unusual in the design of arches, and which were due to the requirements of the stress analysis, considering forces applied in three planes to that part of these structures above the arch. In the analysis, to find chord stresses in this arch, the location of the point of load application toward the center of the structure with respect to the point of reaction at the base of the posts had to be considered, not only in the vertical direction, but also in the longitudinal direction.

The unequal length of the legs of one of the types further complicated this problem of arch design. The transverse horizontal arch reactions were increased considerably with the shortening of the posts.

GENERAL DESCRIPTION OF FOUNDATIONS

In the course of the preliminary design of the structures it was necessary to consider the co-ordination of the construction work on some of the foundations with the improvements on the new railroad viaduct. Reactions for the superstructures were determined, and the footings designed for all the structures that were involved in the railroad program for elimination of grade crossings.

It was the original intention to adopt a uniform design of footings throughout the line, as far as possible. The policy of maintaining uniformity in design was followed as closely as possible throughout, but in this overbuild of the masonry and steel viaduct section of the railroad, it was found necessary to locate the foundations in masonry walls, in fresh earth-fills, and as integral parts of the framing of the steel viaduct; and these restrictions of location rendered absolute uniformity impossible. It was usually found more economical to design a special footing for the case in question than to extend the so-called standard to cover the requirements at that location.

Footings that formed integral parts of the retaining walls, or other railroad construction, were placed as the walls were built. Expansion joints were eliminated as often as possible from the short section of wall that contained the footings.

In order to remain within the right-of-way boundaries of the railroad, and still provide adequate support for the superstructures at points very close to the right-of-way lines, most of the footings were necessarily eccentric in shape in the transverse view.

All these footings were designed to support the superstructure loads, not only under its normal dead-weight condition, but also under the loadings imposed by extreme weather conditions or unusual, unbalanced wire pulls. Thus, they were called upon to withstand direct down thrust and uplift with or without horizontal, longitudinal, or transverse shear at the top of the foundation. The adoption of pin connections in the case of the bridge type of structures between the superstructure framework and the footing frame, eliminated the bending in the transverse direction at the top of the footing, which would otherwise have been present due to the loading of the wide transverse arch. The transverse and longitudinal shear, down thrust, and uplift were cared for as described for each of the several types, maintaining the unit pressures on the soil, concrete, or other supporting medium, within very conservative limits.

It will be noted that in all the foundations described herein, a steel framework was provided, which was structurally competent to sustain all the superstructure loads placed upon it, independent of the concrete in which it was invariably encased. Actually, the bond existent between the steel and the surrounding concrete is active throughout the footing in the distribution of stress. The use of a steel frame instead of reinforced concrete footings was deemed to be safer, easier to construct in accordance with the office design, and more economical, in these cases of unusual loading.

The footing as a unit—concrete and steel—was always designed to sustain all loads, including those due to the tower, retaining wall, train loads, or other influences that could act upon it. The steel was designed to distribute the loads from the tower throughout the footing, and such reinforcement as was required was placed to bond the concrete where the steel frame was inadequate for this purpose.

SOIL CONDITIONS

Borings had been taken in many cases, and these, together with contemporary local construction experience, were found of assistance in the footing design. It was impossible to rely entirely upon these aids, however, since soil conditions were often found to vary with extreme abruptness. About $2\frac{1}{2}$ miles

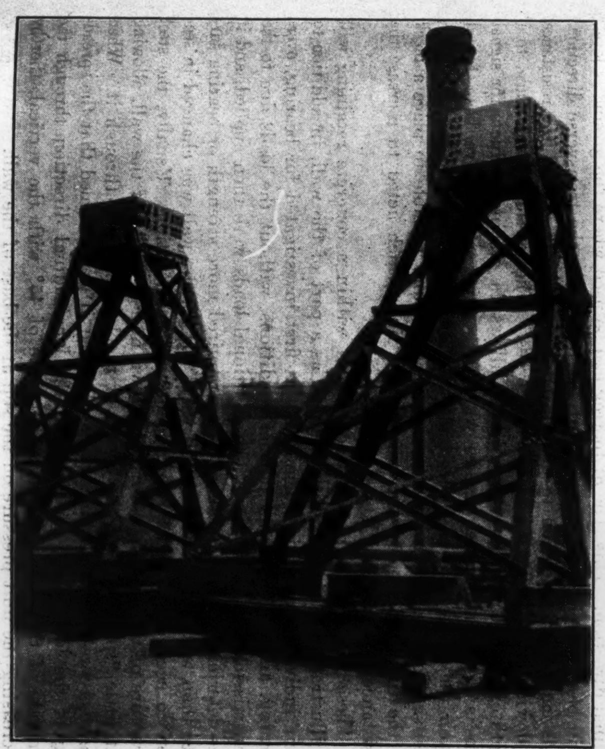


FIG. 10.—STEEL FOOTINGS BEFORE BEING ENCASED IN CONCRETE.

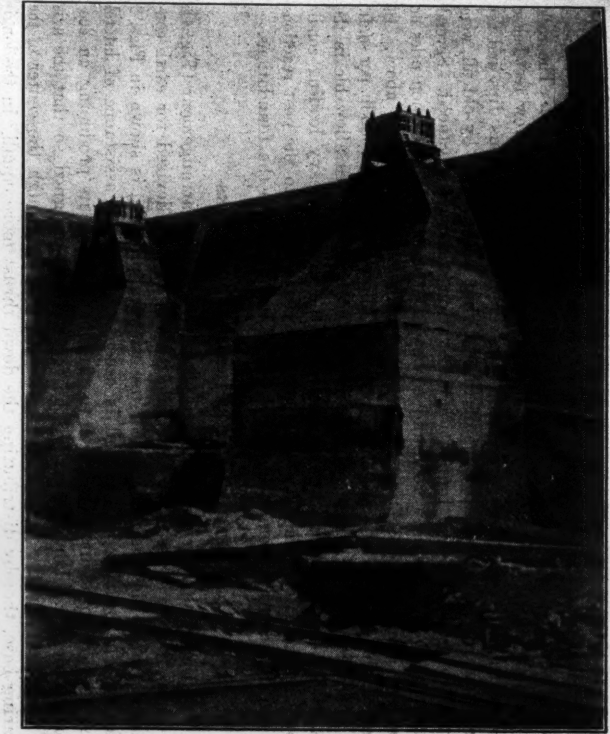


FIG. 11.—STEEL FOOTINGS ENCASED IN CONCRETE.

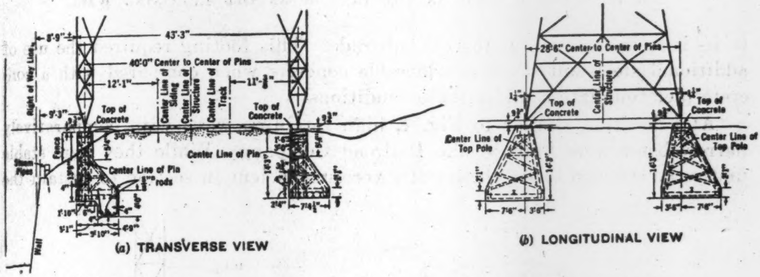
of this line over the railroad is located along a hillside, where the slope is transverse to the line toward the Schuylkill River (see Fig. 1). The rock stratum frequently is outcrop and, at other times, it is just a few feet below the natural surface. It is traversed, transversely, by numerous dips and fissures toward the river, and some of these, being well covered by old fill, were not discovered until the foundation excavation was well advanced. Several times these depressions were found to be old stream beds, and then piles or concrete piers were used to support the walls and foundation above. In general, the soil was of such character as to allow the use of 4 000 lb. per sq. ft. as a bearing value, a base conforming to the unit stresses allowable in the steel when the structure was subjected to the normal heavy loading condition. An additional factor of 10%, or more, was added to the post reactions of the structures in computing the loads to be applied to the foundations.

Detailed Description of Foundations

For the Type *T* bridge structure with its eccentric arrangement (Fig. 4), and also for the Type *R* bridge (Fig. 2), the footing devised for earth conditions and as the basic design for use where applicable, is shown in Figs. 2 and 3. The bearing on the soil is eccentric, and the assistance of lateral passive earth pressure was used to great advantage in producing an economical design. Longitudinally, the footing was designed so that the axis of pressure from the post of the tower would pass through the center of the bearing base, thereby distributing the down thrust more equally in the longitudinal direction. This type of footing can be used where the ground is level, or in banks sloping to either side of the footing in the transverse direction. It is capable of extension to provide additional depth needed for resistance to down thrust, uplift, or transverse forces. The vertical inside face, to a depth of about 6 ft. below the rail level, allows the placing of ducts for signal lines and communication cables which otherwise would have been forced nearer the tracks into an undesirable position of proximity to the rail movements. For some cases of extension it was found possible to place a block on the inner side, below the normal base line, which acted to increase the resistance to uplift and lateral thrust.

These footings (Fig. 10), when placed within a concrete retaining wall as illustrated in Fig. 11, were designed as a part of the wall, in addition to their individual analysis. The wall was first investigated for bearing, overturning, and sliding under normal conditions, without the loads due to the transmission-line structures. The additional loads were then applied, and it was determined whether this addition required more strength or bearing surface than was existent in the wall itself. If so, the wall was changed in section to accommodate the requirements at that location. Usually, the steel structure of the footing was carried down near the base of the wall, allowing but little "spreading" of the effects of the loads applied through it. When the steel was at some distance above the base, it was assumed that the spreading of the load effect downward in the longitudinal direction through the concrete of the wall occurred at an angle of 45° with the vertical, thereby decreasing the unit pressure on the soil at the base of the wall.

At those locations where the right-of-way width permitted, the eccentricity of the footing was reversed, the toe pointing outward from the center of the structure. In this manner the shape of the footing was directly adapted to the support of the loads placed upon it, and its size and weight were reduced considerably. The resultant of combined horizontal arch thrust, and down thrust applied at the top of the footing, was inclined closer to the axis of the footing through the center of its base, and thereby an equalizing of the bearing pressure on the soil along the base was affected. This is illustrated in Fig. 12.


8'-9"±
43'-3"
40'-0" Center to Center of Pins
12'-1"
Center Line of Siding
Center Line of Structure
Center Line of Track
11'-3/8"
Right of Way Line
9'-3"
Top of Concrete
Top of Concrete
9 3/4"
3'-0"
Fence
Steam Pipes
Center Line of Pin
Center Line of Pin
rods
1'-6"
3'-0"
16'-3"
2'-4"
7'-4 1/2"
1'-10"
9'-1"
9'-10"
4'-9"
Wall
28'-6" Center to Center of Pins
Center Line of Structure
1 1/2"
Top of Concrete
Top of Concrete
9 1/2"
Center Line of Top Pole
Center Line of Top Pole
2'-1"
16'-3"
7'-6"
3'-6"
3'-6"
7'-6"

(a) TRANSVERSE VIEW (b) LONGITUDINAL VIEW

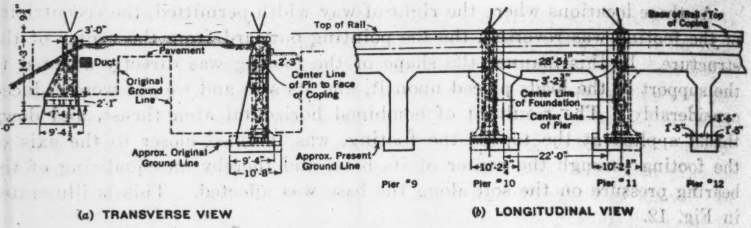
FIG. 12.—FOOTINGS REVERSED TO SUIT SPECIAL RIGHT-OF-WAY CONDITIONS.

It will be seen that a generally similar shape of footing was used in many of these special cases, modified in each location to the extent required, in order to support the loads at that point and to conform to the outlines of the walls or abutments in which the steel frames were embedded. In all cases the steel framework was covered by a minimum of 3 in. of concrete.

At one section of the work, the Railroad Company had decided to raise its tracks about 20 ft. out of a cut, faced by retaining walls, to the level of the adjacent parallel street. The old railroad cut was filled in, with the tracks placed on the new fill. In order to support the shallow boundary wall above the surface, between the street and the tracks, piers in the line of the old retaining wall were carried down to spread bases below the railroad grade.

Footings constructed for the transmission structures in the usual manner would have been mainly in new fill. Hence, a steel column was devised to go down to the old subgrade under each pin of the superstructure on this side and bear directly upon a steel grid encased in concrete (see Fig. 13). Sufficient bearing support on firm soil was thus obtained, and the resistance to uplift was entirely satisfactory. Longitudinal thrust was taken at the tops of the columns through the reinforced concrete wall-beam resting on the concrete piers of the railway. The earth, with the stability of the base itself, insured the footing against lateral movement due to transverse thrust. On the outside, the firmly packed subgrade of the street afforded resistance to the greater of these transverse pressures.

The columns of the railroad construction had to be enlarged to contain the steel framework, and they were reinforced with steel where necessary. All the load of the superstructure was carried through the steel substructure


9"
14'-3"
3'-0"
Duct
Pavement
Original Ground Line
2'-1"
7'-4½"
4'-0"
9'-4½"
2'-3"
Center Line of Pin to Face of Coping
Approx. Original Ground Line
9'-4"
10'-8"
Top of Rail
Base of Rail = Top of Coping
28'-5½"
3'-2¾"
Center Line of Foundation
Center Line of Pier
3'-6"
1'-5"
1'-5"
Approx. Present Ground Line
22'-0"
10'-2¾"
10'-2¾"
Pier #9
Pier #10
Pier #11
Pier #12

(a) TRANSVERSE VIEW (b) LONGITUDINAL VIEW

Fig. 13.—Special Piers in the Line of an Old Retaining Wall.

to its base in firm soil at the old subgrade. This footing required the use of additional steel, but saved considerable concrete when compared with a concrete pier foundation under these conditions.

At a location similar to Fig. 4, high retaining walls with comparatively narrow bases were built by the Railroad Company. While they were stable under the railroad loads, these walls were insufficient in section to sustain the

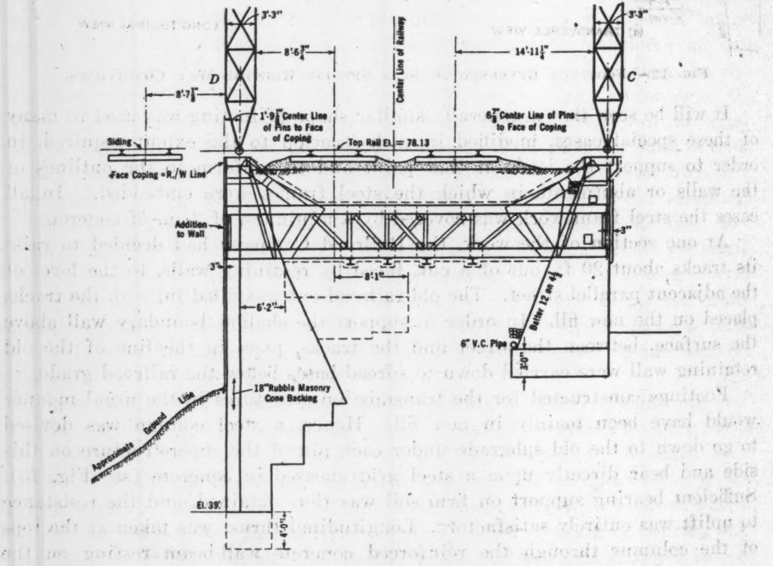

3'-3"
3'-3"
8'-6¾"
Center Line of Railway
14'-11¼"
D
C
8'-7⅛"
1'-9⅞" Center Line of Pins to Face of Coping
6⅞" Center Line of Pins to Face of Coping
Siding
Top Rail El. = 78.13
Face Coping = R./W Line
Addition to Wall
3"
3"
8'-1"
6'-2"
Batter 12 on 4½
6" V.C. Pipe
3'-0"
18" Rubble Masonry Cone Backing
Approximate Ground Line
El. 39.
4'-0"

Fig. 14.—Special Footings for Narrow Clearances.

additional loads imposed by the tower structure above. It would have been necessary to place considerable plain or reinforced concrete at or near the base of the walls and also to carry the steel of the footing framework to the base of the wall. The situation was made more acute due to the over-

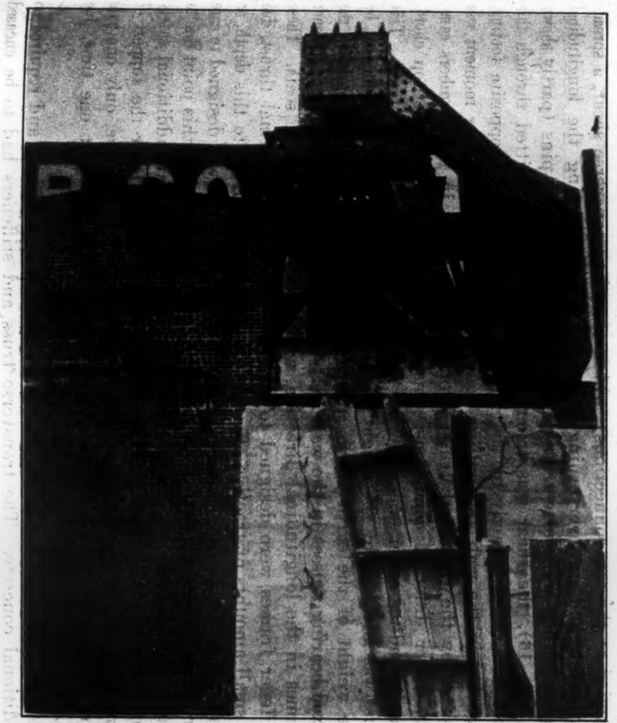


FIG. 15.—VIEW OF SPECIAL PIER DESIGNED FOR NARROW CLEARANCES.

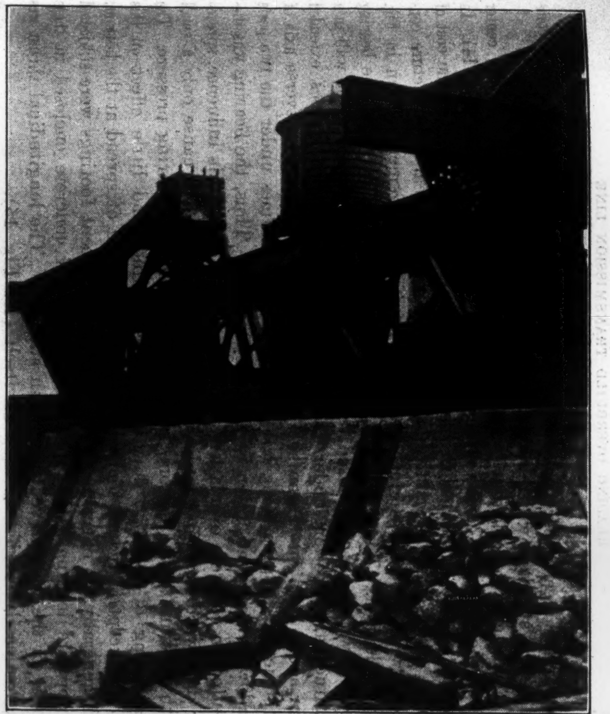


FIG. 16.—VIEW SHOWING CONNECTION OF LONGITUDINAL AND TRANSVERSE TRUSSES, ON SPECIAL PIERS.

hang of the steel superstructure at the top of the wall, in order to accommodate the standard width and maintain the required clearances.

In the case of all four footings, the vertical axis through the center line of the pins was close to the outside face of the wall (see Fig. 14). In view of these facts (and with the desire to save concrete even at the cost of a little more steel, so as to obtain the most economical substructure consistent with the best engineering), the footing arrangement shown in Fig. 14 was devised. The principles involved, consisted in carrying the bearing due to down thrust to a boxed truss, steel framework, placed in the wall in a longitudinal direction under each footing point. These frames extended from about 7 ft. on the outside of the pin to the center transverse axis of the structure, a space being left here between the frames under the two posts to allow independent action by the two footings. Thus, the bearing was distributed on concrete over a length of about 21 ft., and this influence, spreading downward at 45°, was dissipated in the concrete to cause only a small earth pressure at the bottom, in addition to the wall-bearing pressure. The overlapping of the 45°, downward pressure lines had little effect on this theory, except to reduce somewhat the effective area of spread at the bottom; and, generally, no effect at all when the two adjacent footings were subjected to opposite loadings. In uplift, the weight of concrete enclosed in these trusses in the wall proved more than adequate. The longitudinal thrust was likewise easily carried by the wall itself (see Fig. 15).

It remained necessary to provide for the large horizontal transverse thrust (applied at the pin connections), which caused a transverse shear, a spreading effect, and an overturning moment. By connecting the longitudinal trusses with a transverse truss placed directly under the pins (partly shown in Fig. 16), the shear was taken care of, being transmitted through this truss and resisted partly by the longitudinal truss in the opposite footing, anchored in the wall and pulling against the fill. Overturning moment was resisted by the transverse truss through direct stress in the members causing, under different conditions, tension or compression in the top chord. The spreading effect was likewise resisted by the transverse truss. This truss was anchored by the weight of the concrete in which it was encased, by the weight of the earth above, and by its connections at the ends to the boxed bearing trusses in the walls. It was provided with triangular stiffener frames in the longitudinal direction, which rested on concrete seats. These stiffener frames were designed to take care of any longitudinal thrust, due to small amounts of tractive effect on the rails, transmitted to this depth, or other distortioning influences. The main transverse truss was designed to care for all tendencies of the walls to overturn, or fail outward at this point due to loads applied above its lower chord. It thus afforded an additional safety factor for the wall itself. This truss was also designed for the support of concentrated axle loads of 75 000 lb. each, on the track above, only one axle per track being applicable (longitudinally) to the truss at one time, due to its comparatively narrow section.

The bearing trusses were entirely within the normal wall and required no additional concrete. The transverse truss and stiffeners had to be encased

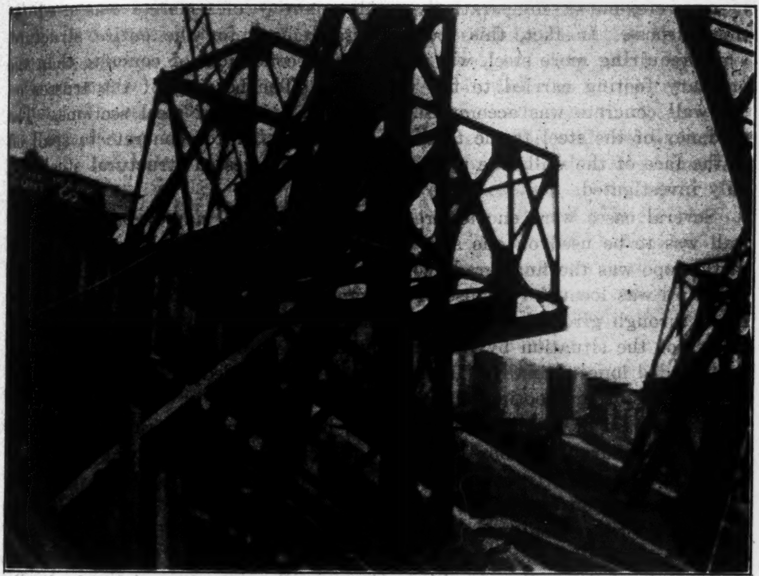


FIG. 17.—CANTILEVER RIB DESIGNED TO FURNISH SUPPORT IN A TRANSVERSE DIRECTION.

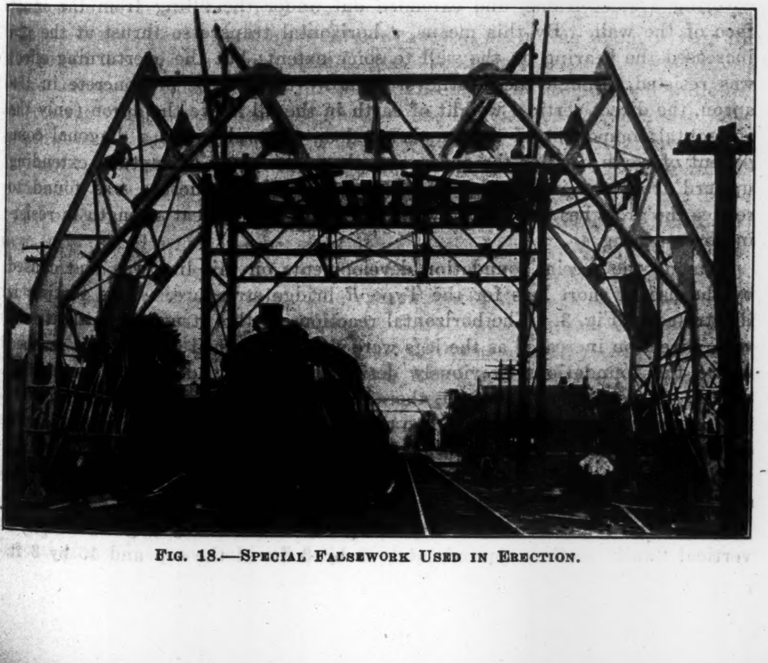


FIG. 18.—SPECIAL FALSEWORK USED IN ERECTION.

in concrete, but a comparatively small quantity of concrete was used for this purpose. In fact, this footing arrangement for the entire structure, while requiring more steel, was much more economical of concrete than the ordinary footing carried to the wall base. The bearing of the trusses on the wall concrete was accomplished through a grid of steel sections. The tendency of the steel frame to shear out locally, or for concrete to spall off at the face of the wall, due to the proximity of stressed structural steel, was duly investigated.

Several cases were encountered in which a high, narrow-base, retaining wall was to be used on one side of the tracks, while, on the other side, an earth slope was the final grade condition. In one of these places, the tower structure was located (because of clearance demands), close to an abutment for a through girder bridge over a street. This greatly increased the difficulties of the situation by limiting the distance in which the footing could be extended longitudinally. In this case it was decided to use a modification of the cross-bridge footing.

In the case of the two wall footings, bearing trusses, resting well up in the wall similar to the box trusses of the cross-bridge structure, were used. For support in the transverse direction, a cross-bridge would have been a useless expense on account of the earth slope on the other side. Therefore, a cantilever rib or bent was used as shown in Fig. 17. This bent was placed exactly at the pin, and projected in the rear of the normal wall section down to the base of the wall. At this point a small apron, braced longitudinally and transversely, was attached to the bent. The apron was made of concrete covering a steel frame, and extended out under the filling from the inner face of the wall. By this means, a horizontal transverse thrust at the pin increased the bearing on the wall to some extent; but the overturning effect was resisted through the cantilever bent by the weight of concrete in the apron, the direct vertical weight of earth in the fill above the apron (only the horizontal component of this earth acted on the wall), and a diagonal component of earth back of the apron and above the plane of rupture, extending upward from the inner edge of the apron. This structure was found to reduce the gross bearing on the wall materially by its great strength in resisting overturning.

Among the special foundation developments on this line was that caused by the use of short legs for the Type *R* bridge structure. This footing is illustrated in Fig. 3. The horizontal reactions of this structure in the transverse direction increased as the legs were shortened, until a point was reached where the foundations previously designed did not adequately resist this thrust. A footing was devised, therefore, with a large area of bearing surface in the vertical plane parallel to the right-of-way line, utilizing the lateral earth resistance. This type was found more economical than one which would resist the eccentricity of reaction by the addition of weight of concrete and increased bearing surface.

This type really consisted of three parts, as shown by Fig. 3, that is, a vertical "wall" section approximately 8 by 3 ft. at the top and 15 by 3 ft.

at the bottom. The concentrated load from the shoe of the bridge structure on the 3-ft. square section at the top, was spread by beams to the entire 8 by 3-ft. surface of the top. This load was distributed by the frame with battered ends to the 15 by 3-ft. area at the bottom, and also by the transverse sloping rib, about 13 ft. high and 3 ft. wide, to the mat at the base. This mat was about 6 by 15 ft. in area and 2 ft. thick. The wide section of the "wall" in the longitudinal direction near the top afforded a large area for lateral bearing on the soil near the transverse reaction at the pin. The large base, a total of 15 by 9 ft. in area, amply resisted vertical bearing pressure, and the rib combined the vertical "wall" and horizontal mat sections. In supporting the passive weight of the earth above it, the mat also resisted the overturning tendency due to the large transverse reaction. This proved to be an economical and simple solution of this difficulty. The steel frame was comparatively simple in design, little bending of angles being required. The usual grid of I-beams and channels was placed at the bottom of the framework, and the whole encased in concrete.

CONSTRUCTION

As stated previously, it has been the intention to confine the scope of this paper to design features. A brief description of the construction is interesting, however, in view of the close relation between it and the structural design.

The foundations were built in groups, primarily to facilitate a co-ordination of this construction work with that of the Railroad Company in its elimination of grade work and change of signal system. Accordingly, more than one-fourth of the footings were constructed, beginning about a year in advance of the superstructure work. These footings were at those locations where the steel framework was directly connected with the railroad walls or other structures. Approximately another one-fourth were constructed well in advance of the superstructure work, because they were in the path of or adjacent to conduits built for the new railroad signal system. The remainder were built as a part of the final operation in the construction of the line.

In several places in the foundation construction, unforeseen difficulties were encountered, which neither close inspections of the site, local experience, or borings had disclosed. The case of one of the cantilever types of wall footings is an example of this. The excavation of the base of the wall disclosed a soil condition entirely different from that for which the wall and footings were designed. Although at short distances on either side (less than 50 ft.) solid rock had been exposed, it was found necessary to place the wall and footing in this section on piles driven to refusal, the pile-heads being encased in a thick mat of reinforced concrete. Other cases of similar difficulty developed and were met by a corresponding change in the office design or by modification of the construction in the field.

In nearly all cases, construction operation was confined to the railroad right of way, thereby causing some restrictions as to methods of excavation, disposal of surplus material, and deliveries.

In many sections, access to the work was along the railroad tracks, or over ground inaccessible to trucks. It was possible to use trains on part of this job, but this type of construction auxiliary was distinctly hampered by the frequent passing of high-speed passenger trains.

Much of the excavation was in rock, although all types were experienced in the foundation construction. Concrete for the footings was partly mixed at the site or delivered from central mixing plants, along the job, and partly delivered ready-mixed from more distant sources.

The erection of the Type *M* Towers (Fig. 9) presented no peculiar difficulties; standard methods, a gin-pole, and hoist were all that was necessary. For the bridge structure, however, the problem was somewhat more involved. Usually, no large space was available for assembling the structure in sections; and even if there had been, the required locomotive cranes would have proved uneconomical agents in this construction, due to lack of space for their operation and to their interruption of passenger-train service. Therefore, steel bridge falsework as illustrated in Fig. 18 was used. This framework was easily assembled, dismantled, and moved from site to site. The lower sections of the bridge post bents in the longitudinal view—that is, up to the first bend in the posts—were assembled, propped, and guyed in place. With the falsework in place over the tracks, light derricks were erected on it, and the lower transverse cross-bridge of the structure was quickly assembled. Then, using this as a falsework, the upper part of the structure was erected at will, the falsework having been moved in the meantime to another site. By this method, erection unit costs on this line were kept surprisingly low, quite comparable to unit costs of open-country, transmission-line construction.

With the exception of the great additional care required because of the location of this work over a railroad, the methods of carrying out the other phases of the work on this job were similar to the usual practice in ordinary transmission-line construction.

CONCLUSION

It has been the writer's intention to present a general picture of the design of this rather unusual type of transmission-line construction. The problem of power transmission has become increasingly acute with the development of widespread suburban residential districts around the manufacturing load centers. It becomes necessary to purchase right of way at high cost or to adopt costly design for a transmission line to bring power into a city from outside generating sources.

The line herein described presents one solution of this problem. The future may develop other and better solutions, all resulting in increased economies to power users and closer co-ordinating relations between utilities of all types.

The design of structures and foundations was done by the Engineering Department of Philadelphia Electric Company, and the construction of all superstructures and line erection and part of the footings by the United Engineers and Constructors, Inc. Other contractors built the remaining footings.

DISCUSSION

E. E. R. TRATMAN,[5] Assoc. M. Am. Soc. C. E. (by letter).—The interesting construction of a transmission-line structure on and over a railway right of way, as described in this paper, inevitably calls to mind the occasional or perennial suggestions for the double-decking of railway lines in city and outlying districts. Some of these projects have been for the purpose of providing additional trackage, as in separating suburban and main-line traffic. More generally they aim at bringing some other railroad (suburban or interurban) into the terminal station in order to improve the service of the secondary line. In at least one case, it has been proposed to build a toll-road viaduct for high-speed automobile traffic above the railroad.

As most of these projects have been put forward by outside interests and would be more or less competitive with the railroad company whose property is to be utilized, it is natural that the railways have not been disposed to regard them favorably. In fact, as a rule such projects have not advanced much beyond the paper or publicity stage. So far as the writer now recalls, none has reached the construction or even the design stage. Nevertheless, the idea or principle has value, and it is quite conceivable that a railway with growing traffic on narrow right of way through a built-up district or through high-priced property might find it economical to double-deck its line, or to co-operate with some other concern in doing this.

One objection that is generally made to such propositions is the danger of damage to the upper structure by derailments on the main level; but accidents of this kind are not frequent enough to constitute a valid objection. Apparently, this objection is given little weight in regard to the Philadelphia power line, since the photographs reproduced in the paper do not indicate that the railroad considered it necessary even to provide guard-rails along the tracks spanned by the tower structure.

As a rule, strong objections are made by railroad companies to structures spanning or crossing their tracks. This applies particularly to crossings by high-tension electric cables, for which stringent protective requirements have been formulated. It would be of interest, therefore, to have some particulars of the relations and agreements between the Railway Company and the Electric Company as to the occupation of the former's right of way and the protection of its traffic. Undoubtedly, the co-operation between the two companies was made unusually easy in this case by the fact that the transmission line will serve the electrification of the Philadelphia suburban lines of the Reading Company.

PERCIVAL S. BAKER,[6] M. Am. Soc. C. E. (by letter).—The author deals principally with the overbuild on the Norristown Branch of the Reading Company, where the greatest difficulties were encountered. Within the section concerned, there were formerly numerous grade crossings and the Reading Company, at the time the overbuild of the Philadelphia Electric Company was first considered, was engaged in the preparation of detail plans for the elimination of all these crossings through the section of Philadelphia known as Manayunk. The elimination of these crossings involved the raising of the tracks for a total distance of 7 200 ft. For part of this distance, the tracks were carried on an embankment supported by retaining walls and for part on a steel viaduct.

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[5] Civ. Engr., Wheaton, Ill.

[6] Engr. of Bridges, Reading Co., Philadelphia, Pa.

Detail plans of the viaduct and retaining walls were furnished to the Philadelphia Electric Company, which Company then located its towers to conform to the general design.

It was found that the additional tower loads to be supported on the viaduct section were small in comparison with the dead load of the structure and the moving loads of the trains, so that comparatively little additional metal was required in the viaduct and at no place was it necessary to change the type of members as originally designed, although, in some cases, it was necessary to change the details. This is shown particularly in the author's Fig. 6.

Throughout the deck girder part of the viaduct the curb columns were stopped at the bottoms of the transverse girders which were seated on them. Where the overbuild towers occurred, the curb columns were lengthened to the tops of the transverse girders to form direct bearings for the overbuild structures and the transverse girders were framed into the sides of the columns.

On the part of the line where the elevated tracks are carried on earth embankment supported between retaining walls, the tower loads were carried down to the bottom of the walls by means of structural steel foundations, which were embedded in the walls. These steel foundation frames were furnished by the Electric Company and then encased in the concrete wall by the railroad contractor. A careful accounting was kept of all additional costs caused by the overbuild, and this addition was paid by the Philadelphia Electric Company.

The method of erecting the overbuild structures as described by the author proved entirely satisfactory to the Railroad Company.

FREDERICK W. DECK,[7] ASSOC. M. AM. SOC. C. E. (by letter)—In his discussion of the paper, Mr. Tratman has mentioned the objection which has frequently been made by railroad companies to overbuild propositions, concerning the chance of a derailment of trains. Accidents of this kind, as he states, are relatively of such infrequent occurrence that generally this objection must be given little weight. In the case of the power lines of the Reading Overbuild there is a further safeguard to the structure in the distance which the posts are removed from the nearest track. On a tangent section about $1\frac{1}{2}$ miles in length, the bridges that span the tracks have their posts, under present conditions, approximately 23 ft. from the center line of the nearest track. If the Railroad Company should ever find it necessary to build the third and fourth tracks along this section, the nearest track would then be only 10 ft. from the posts. That, however, is probably a remote possibility. On many of the curves where the bases of the structures are much closer to the tracks than on tangents, trains are operated at less speed and possibly with somewhat greater care.

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[7] Structural Engr., Mech. Div., Philadelphia Elec. Co., Philadelphia, Pa.

From the point of view of the Railroad Company, there is not much more hazard in this overbuild transmission line than there would be with the slender post bents which otherwise would support the transmission line for railroad-electrification purposes.

In order to provide maximum security, it was found desirable, in this case, to make the structures and the other elements of the transmission line as strong as was reasonably necessary to stand up under the most adverse conditions of loading likely to be encountered. The design of the structures was conservative, involving comparatively low unit stresses with heavy loading assumptions. Double strings of insulators were used in strain positions, thus adding another safety factor; and the structures themselves were designed so as to stand up under the complete dead-end pull with all the transmission wires broken.

Naturally the unity of interest of the Railroad Company and the Electric Company in the provisions for railroad-electrification facilities as well as for power-line facilities, made the arrival at an agreement somewhat easier to accomplish. However, in this particular case, the two companies were especially fortunate in being able to view the matter from a relatively clear and impartial standpoint, unencumbered by hidebound adherence to fictitious principles of safety. Every effort was made to render this transmission line safe from the standpoint of either the Railroad Company or the Power Company, but useless and obsolete design restrictions were not thrown in the path of the negotiations leading up to mutual agreement.

For more or less obvious reasons, it was not possible to quote the terms of the agreement between the Electric Company and the Railroad Company. It may be stated, however, that the Railroad Company was amply protected through a liability clause in case of accident due to the presence of electric facilities.

Mr. Baker has mentioned the work of eliminating grade crossings that was done by the Railroad Company at about the same time that the transmission line was constructed. The Electric Company was particularly fortunate in being able to co-ordinate the design and construction of the transmission line with the design and construction of the elevated viaduct of the railroad. This simultaneous process of design and construction by the two companies was a means of saving considerable time, labor, and expense, that otherwise would have been necessary had the transmission-line construction been delayed until after the railroad work was completed.

The comment may be made by some engineers, that these structures appear to be unusually heavy for the type of loads they are called upon to sustain. Upon a casual inspection of the line this certainly appears to be true, but a closer study will reveal, inevitably, the fact that under complete loading the structures are subjected to such a variety of heavy and unusual pulls, far removed from their natural points of support, that the steelwork will no longer seem too heavy. The conditions under which the work was accomplished, the limitations of the right of way, and the prerequisites of safe clearances dictated in large measure the unusual shapes that were adopted; and, consequently, large quantities of steel and concrete were required.

The Electric Company, realizing the necessity for providing structures of maximum safe design, both for its own sake and for that of the Railroad Company, did not hesitate to incur any reasonable expense for steel and concrete for arriving at this end. This attitude was rewarded by furtherance of the good relationship existing between the two companies. The spirit of co-operation and co-ordination of the work on the part of the two utilities involved in this project is worthy of note. It stands as an example of what may be accomplished and what benefits all parties may receive by the attainment of a common plane of thought and action.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 1847

DETERMINATION OF PRINCIPAL STRESSES IN BUTTRESSES AND GRAVITY DAMS

By W. H. Holmes,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. W. J. Stich, Hakan D. Birke, Dirk A. Dedel, Fred A. Noetzli, Eugene Kalman, I. M. Nelidov, H. E. von Bergen, Howard L. Cook, Herman Schorer, A. Floris, W. C. Huntington, Calvin V. Davis, and W. H. Holmes.

Synopsis

This paper presents the results of a study of stresses in buttress and gravity type masonry dams and develops formulas for the analytical determination of stresses at any point on a horizontal section through dams. The structures treated in this paper are divided into three groups:

I.—Gravity dams, straight in plan.
II.—Gravity dams, arched in plan.
III.—Buttresses of flat slab or multiple-arch dams having rectangular horizontal sections that taper with the height.

It is shown that the stress equations for Groups II and III become identical with the equations for Group I when the radius is infinite and the taper is zero, respectively. Examples of several section types are given.

Introduction

The formulas derived in this paper are based on the numerical solution of principal stresses in a gravity dam, straight in plan, as presented[2] by the late William Cain, M. Am. Soc. C. E. If Professor Cain's analysis had been made more general and had been extended to include the vertical component of the water load on gravity dams either arched in plan, or to include tapering buttresses of flat slab and multiple-arch dams, the arithmetical work would have been extremely laborious.

Note.—Published in January, 1932, *Proceedings*.

[1] Hydr. Engr., State Dept. of Public Works, Los Angeles, Calif.

[2] "Stresses in Masonry Dams," *Transactions*, Am. Soc. C. E., Vol. LXIV (September, 1909), p. 208, and "Practical Designing of Retaining Walls," Seventh Edition, 1914.

Observations made by the writer and experience gained in an endeavor to obtain the principal stresses in solid and buttress dams have indicated that the ordinary method of determining the vertical normal stress at the heel and toe of high dams is not sufficient for the complete analysis of the stresses. Although engineers in the past have given thought to the existence of principal stresses, no great advancement in the art of the stress determinations has been made.

Vertical or inclined cracks, due to shrinkage of concrete and temperature changes, occur in large gravity dams even before any water load is applied, and an analysis of stress distribution is not complete if these cracks or inclined contraction joints are not included in the analysis. The suggested use of inclined joints in a high gravity dam arched in plan, caused the writer to develop the second group of equations for stress distribution of gravity dams arched in plan.

The analysis of the 130-ft. buttress of a multiple-arch dam indicated considerable tension at the up-stream face although the vertical normal compression stress at the point was 121 lb. per sq. in. Because the buttress cracked approximately on the line of maximum principal stress an analysis was made assuming complete severance of the structure along that line. The result of this analysis indicates the possible advantages of inclined contraction joints in this type of structure and caused the writer to derive the following formulas for the determination of internal stresses.

In the analysis used to compute the principal stresses in the structure the stresses caused by shrinkage of concrete, temperature changes, earthquake shocks, settlement of foundations, etc., and the effect of the foundation upon the distribution of stresses, are not considered. The summation of the vertical forces is equal to zero; the summation of the horizontal forces is equal to zero; and the accepted formulas, $\frac{P}{a} \pm \frac{M c}{I}$, for the determination of the vertical unit stress, are the basic equations used.

The determinations of the vertical normal stress, shearing stress, and horizontal normal stress for gravity dams straight in plan, are derived first. The first and second principal stress are computed and Professor Mohr's[3] graphical method is briefly described. The basic checks are shown and numerical solutions of the principal stresses for two triangular dams and a rectangular dam are presented in Group I.

In Group II, the vertical, horizontal, and shearing stresses and the principal stresses are derived for gravity dams arched in plan, and it is shown that with infinite radius these equations are identical to the equations of Group I. Examples of the principal stresses for a gravity dam arched in plan (assuming that the arch does not carry any of the load) is shown.

In Group III, the vertical, horizontal, and shearing stresses and the principal stresses are derived for buttresses of flat slab or multiple-arch dams having rectangular horizontal sections that taper with the height. The stress distribution in the buttress of a multiple-arch dam has been computed with and without the introduction of an inclined joint on two assumptions: First, as a monolithic base; and, second, as containing an inclined joint completely severing the base. Some of the results of these computations are tabulated herein. Other tables and graphs have been omitted to conserve space.

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[3] "Technische Mechanik," by Otto Mohr, 1914.

Notation.—For convenience, the symbols introduced by Professor Cain are used. When considered necessary these are defined in the text, as introduced, and are grouped in Appendix I for convenient reference. All stresses and loads are in terms of masonry units.

GROUP I.—STRAIGHT GRAVITY DAMS

Vertical Normal Stresses.—The vertical normal stress on any plane computed by the usual formula is,

$$\frac{P}{a} \pm \frac{M c}{I} = p_a \text{ or } p_b \quad \dots\dots (1)$$

in which, p_b is the vertical normal stress at the up-stream face, and p_a, the vertical normal stress at the down-stream face. (Equation (1) is often written: $\frac{P}{a} \pm \frac{6 M}{L^2} = p_a$ or p_b).

The vertical normal stress at any point, x distance from the up-stream face, equals,

$$p = p_b + \frac{(p_a - p_b)\, x}{L} \quad \dots\dots (2)$$

Let $S = \frac{p_a - p_b}{L}$ be the rate of change of intensity of the vertical normal stress. Then, the unit vertical pressure, p, at Point x is,

$$p = p_b + S\, x \quad \dots\dots (3)$$

The symbol, p_u, p_o, or p_d will be used for the vertical pressure at the up-stream face. The subscript, b, is omitted as p_a does not appear in the following formulas, but the subscripts, u, o, and d must be used to denote the plane on which the stress is applied.

The vertical normal stress at any point on the plane, u, is,

$$p = p_u + S_u\, x_u \quad \dots\dots (4)$$

and the summation of the vertical stresses on the plane, u, is,

$$P_u = \sum p = p_u\, x_u + \frac{S_u\, x_u^2}{2} \quad \dots\dots (5)$$

Let the vertical stresses on the planes, o and d, at the up-stream face be, respectively, p_o and p_d; then the summations of these vertical stresses on the middle plane and the lower plane are,

$$P_o = p_o\, x_o + \frac{S_o\, x_o^2}{2} \quad \dots\dots (6)$$

and,

$$P_d = p_d\, x_d + \frac{S_d\, x_d^2}{2} \quad \dots\dots (7)$$

Shearing Stresses.—The vertical shear, q_1, at any point, x, in the section between the planes, u and o, is obtained from the basic assumption that the summation of the vertical forces is equal to zero; thus, $P_u + V_1 + W_1 - P_o - q_1 = 0$, or,

$$q_1 = P_u + V_1 + W_1 - P_o \qquad (8)$$

in which, q_1 is the total vertical shear on the end of the section (see Fig. 1).

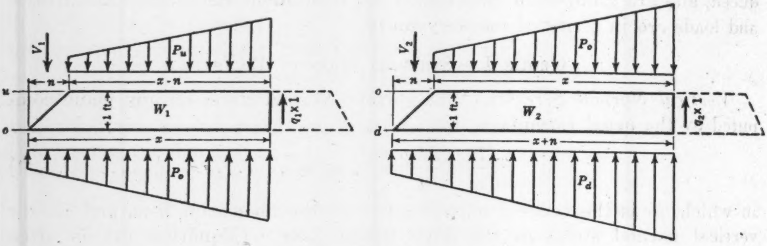

FIG. 1.

Substituting values for P_u and P_o, remembering that $x_u = x - n$, $w = 1$ = masonry unit, and $W_1 = wx - \frac{wn}{2}$, and solving Equation (8):

$$q_1 = \left(V_1 - \frac{n}{2} - p_u n + \frac{S_u n^2}{2}\right) + (p_u + 1 - p_o - S_u n)\, x + \left(\frac{S_u}{2} - \frac{S_o}{2}\right) x^2 \qquad (9)$$

Let,

$$A_1 = V_1 - \frac{n}{2} - p_u n + \frac{S_u n^2}{2} \qquad (10)$$

$$B_1 = p_u + 1 - p_o - S_u n \qquad (11)$$

and,

$$C_1 = \frac{S_u}{2} - \frac{S_o}{2} \qquad (12)$$

Then,

$$q_1 = A_1 + B_1 x + C_1 x^2 \qquad (13)$$

The unit shear, q_1, on the end of the cross-section at x is assumed to be the average shear at a point midway between u and o on the vertical plane at x distance from the origin.

The shear on the end of the cross-section between o and d is obtained in the same manner as shown on the section between u and o. The sum of the vertical forces on the lower section is,

$$q_2 = P_o + V_2 + wx + \frac{wn}{2} - P_d = \left(V_2 + \frac{n}{2} - p_d n - S_d \frac{n^2}{2}\right)$$
$$+ x\,(p_o + 1 - p_d - S_d\, n) + x^2 \left(\frac{S_o}{2} - \frac{S_d}{2}\right) \qquad (14)$$

Let,

$$A_2 = \left(V_2 + \frac{n}{2} - p_d n - \frac{S_d n^2}{2}\right) \dots\dots\dots\dots\dots (15)$$

$$B_2 = (p_o + 1 - p_d - S_d n) \dots\dots\dots\dots\dots (16)$$

$$C_2 = \left(\frac{S_o}{2} - \frac{S_d}{2}\right) \dots\dots\dots\dots\dots (17)$$

Then,

$$q_2 = A_2 + B_2 x + C_2 x^2 \dots\dots\dots\dots\dots (18)$$

The average of q_1 and q_2 is the average shear on the vertical plane at x distance from the origin, or, $q = \frac{q_1 + q_2}{2}$.

Substituting the values of q_1 and q_2 from Equations (13) and (18):

$$q = \frac{A_1 + A_2}{2} + \left(\frac{B_1 + B_2}{2}\right) x + \left(\frac{C_1 + C_2}{2}\right) x^2 \dots\dots\dots (19)$$

As the horizontal shear at a point is equal in magnitude to the vertical shear at the same point, q is the horizontal shear on the plane, o. The summation of the horizontal shear on the plane midway between Planes u and o (see Fig. 2), is,

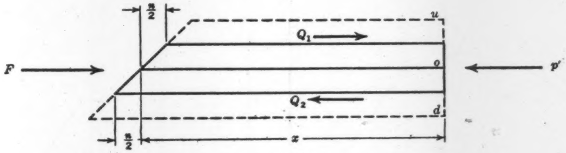

FIG. 2.

$$Q_1 = \int_{0.5n}^{x} q_1 \, dx \dots\dots\dots\dots\dots (20)$$

and, similarly, for the plane midway between Planes o and d,

$$Q_2 = \int_{-0.5n}^{x} q_2 \, dx \dots\dots\dots\dots\dots (21)$$

Horizontal Normal Stresses.—The horizontal normal stress at any point, x, is obtained by the use of the basic assumption that the summation of the horizontal stresses is equal to zero, thus,

$$p' = F + Q_1 - Q_2 = F + \int_{0.5n}^{x} q_1 \, dx - \int_{-0.5n}^{x} q_2 \, dx \dots\dots\dots (22)$$

Inserting the values of q_1 and q_2 from Equations (13) and (18), respectively, and integrating:

$$p' = F - \frac{n}{2}(A_1 + A_2) + \frac{n^2}{8}(B_2 - B_1) - \frac{n^3}{24}(C_1 + C_2) + x(A_1 - A_2) + \frac{x^2 (B_1 - B_2)}{2} + \frac{x^3 (C_1 - C_2)}{3} \dots\dots\dots (23)$$

First and Second Principal Stresses.—Equations (3), (19), and (23) are the general equations necessary to determine the principal stresses at any point on the plane. After obtaining the stresses, p, p', and q, at any point, the principal stresses and their direction may be computed or obtained graphically by Mohr's diagrams. Professor Cain's method of computing the principal stress results in the derivation of the formula[4]:

$$\tan 2\,\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2\,q}{p - p'} \qquad (24)$$

and,

$$f = \tfrac{1}{2}\,(p + p' \pm \sqrt{(p - p')^2 + 4\,q^2}) \qquad (25)$$

A graphical determination of the magnitude of the first and second principal stress by the construction of a Mohr diagram[5] checks Professor Cain's derivation of stresses. To construct a Mohr diagram for stresses in one plane draw three parallel lines, a, b, and c, distant q units apart, to any convenient scale (see Fig. 3(a)). Draw a reference line, o, perpendicular to c. Let distances to the left of o be compression and to the right, tension. Draw Ao' equal to p' and Bo'' equal to p. Draw the circle, $ACBC'$, with AB as the diameter. The distance, C', to the line, o, is the maximum or first principal stress. The direction of the first principal stress is shown by AC. It is to be noted that no tension can occur if $\dfrac{p + p'}{2}$ is greater than the radius of the circle.

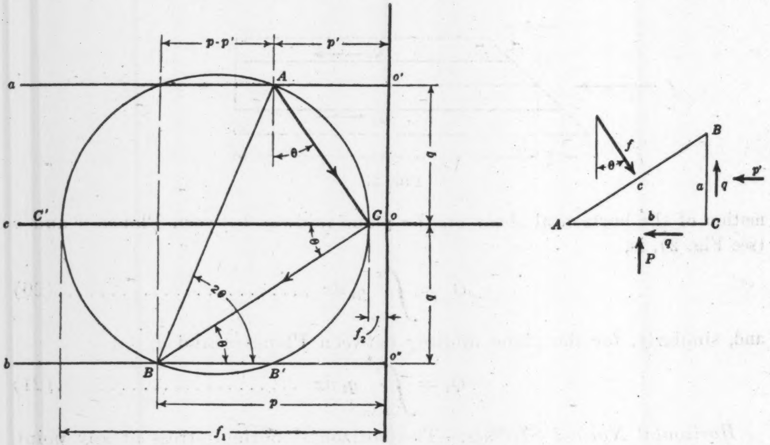

FIG. 3.

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[4] "Practical Designing of Retaining Walls," Seventh Edition, 1914.

[5] "Abhandlungen aus dem Gebiete der Technischen Mechanik," by Otto Mohr, 1914: also, "Design and Construction of Dams," by Edward Wegmann, M. Am. Soc. C. E.

Maximum Shear.—The maximum shear is equal to the radius of the circle (Fig. 3(*a*)). The diameter of the circle is equal to the difference of the two principal stresses and the radius is equal to the maximum shear, or $q_{\text{max.}} = \dfrac{f_1 - f_2}{2}$.

Checks for Shearing Stress, Horizontal Normal Stress, and Principal Stresses at Up-Stream and Down-Stream Faces of the Section.—The following checks should be made of the stresses obtained from Equations (19), (23), and (25):

(1) By substituting values of x for the up-stream and down-stream faces in the equations for q and p' (Equations (19) and (23)), the following equations are obtained:

For the up-stream face:

$$q = V - pn \quad \text{.........................}(26)$$

and,

$$p' = F - q\,n \quad \text{.........................}(27)$$

For the down-stream face:

$$q = mp \quad \text{.........................}(28)$$

$$p' = m^2 p \quad \text{.........................}(29)$$

and,

$$f_1 = \frac{p}{\cos^2\theta} = p\sec^2\theta \quad \text{.....................}(30)$$

A check of Equation (30) at $x = L$, is as follows: In Equation (25) let $q = mp$; $p' = m^2p$, and $m = \tan\theta$; then,

$$f_1 = \tfrac{1}{2}\left(p + m^2p + \sqrt{(p - m^2p)^2 + 4\,m^2p^2}\right) = p + m^2p = p\,(1 + m^2)\ .(31)$$

but, $\tan^2\theta + 1 = m^2 + 1 = \sec^2\theta$, or $f_1 = p\,(1 + m^2) = p\sec^2\theta$, which checks.

(2) The total internal shear on any horizontal plane must be equal to the horizontal component of the total water pressure above that plane, or,

$$\int_0^L q\,dx = Q = H \quad \text{.....................}(32)$$

(3) The sum of the first and second principal stress is equal to the sum of the normal stresses at any point, or,

$$f_1 + f_2 = p + p' \quad \text{.....................}(33)$$

(4) The secondary principal stress, f_2, on the down-stream face is equal to zero.

For special cases:

(5) When the up-stream face is vertical, the shear is equal to zero at the up-stream face, or, when $n = 0$, $q_{(x=0)} = 0$.

(6) When the up-stream face is vertical, the horizontal normal stress at the up-stream face is equal to the unit water pressure, or when $n = 0$, $p'_{(x=0)} = F = \frac{h}{k}$.

(7) When the down-stream face is vertical, the shear is equal to zero at the down-stream face, or, when $m = 0$, $q_{(x=L)} = 0$.

(8) When the down-stream face is vertical, the horizontal normal stress at the down-stream face is equal to zero, or, when $m = 0$, $p'_{(x=L)} = 0$.

Stress Distribution in Gravity Dams Straight in Plan—

Example 1.—Given a triangular dam with water load to the top, with the up-stream face vertical, and with the resultant of forces falling at the middle-third point. Let $n = 0$; $m = \frac{1}{k^{\frac{1}{2}}}$; $h =$ height; and $L = hm =$ base length.

The up-stream vertical normal stress is zero at any elevation, and $S = \frac{p_a - p_b}{L}$ is a constant $= \frac{h}{L}$, or $P = S\,x = \frac{x}{m} = k^{\frac{1}{2}}\,x$.

As the up-stream face is vertical, $n = 0$ and $V_1 = 0$, and from Equations (10), (11), and (12), $A_1 = 0$; $B_1 = 1$; and $C_1 = 0$.

Similarly, by substituting the values of $n = 0$ and $V_2 = 0$ in Equations (15), (16), and (17), $A_2 = 0$; $B_2 = 1$; and $C_2 = 0$. Substituting in Equation (19), $q = x$; and, in Equation (22), $p' = F + Q_1 - Q_2 = F + 0 - 0 = F$.

The three equations for the determination of stress are:

$$p = k^{\frac{1}{2}}\,x \quad \text{.........................} \quad (34)$$

$$q = x \quad \text{.........................} \quad (35)$$

and,

$$p' = F \quad \text{.........................} \quad (36)$$

The vertical or horizontal shear stress for a triangular dam varies along a horizontal section as a straight line and the horizontal normal stress is a constant, but the principal stresses do not vary as a straight line along a horizontal section. The numerical computations for a dam 100 ft. high, with a 66-ft. base, that meet the requirements of Example 1, follow. For convenience L is chosen as a whole number and as the resultant falls on the one-third point, the weight of the concrete is determined as 143.48 lb. per cu. ft.

The value of k is $\frac{143.48}{62.50} = 2.296$; $L = \frac{h}{\sqrt{k}} = 0.66\,h$; $W = \frac{0.66\,h^2}{2} = 0.33\,h^2$; and, $H = \frac{62.50}{143.48} \times \frac{h^2}{2} = \frac{h^2}{2.296 \times 2} = 0.2178\,h^2$.

Taking moments about the down-stream toe (Fig. 4): $0.44h \times 0.33h^2 - 0.2178h^2 \times \frac{h}{3} = 0.1452h^3 - 0.0726h^3 = 0.0726h^3$.

The point of application of the forces $= \dfrac{0.0726h^3}{0.33h^2} = 0.22h$, or, at the one-third point. By Equation (1),

$$p = \frac{P}{a} \pm \frac{6\,M}{L^2} = \frac{0.33\,h^2}{0.66\,h} + \frac{6 \times 0.11\,h \times 0.33\,h^2}{(0.66\,h)^2} = 0.5\,h \pm \frac{0.2178\,h}{0.4356} = 0.5h \pm 0.5h$$

In this example, $p_b = 0$, and $p_a = h$; therefore, $S = \dfrac{h}{L} = \dfrac{h}{0.66\,h} = 1.51515$ = constant.

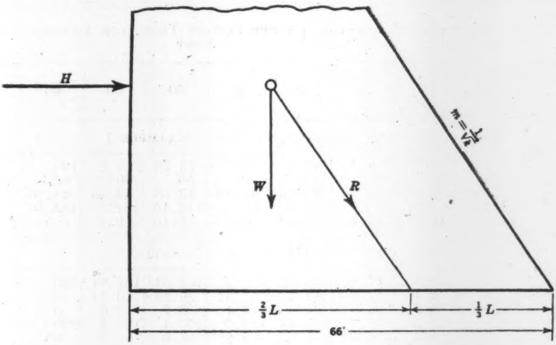

FIG. 4.

For the same distance, x, on any plane, $P_u = P_o = P_d$, and, by Equations (34), (35), and (36).

$$p = 1.51515\,x \quad \text{.......................} (37)$$

$$q = P_u + x - P_o = x \quad \text{....................} (38)$$

and,

$$p' = F + Q_1 - Q_2 = F = 0.4356\,h \quad \text{..............} (39)$$

For a 100-ft. dam, $h = 100$; $L = 66$ ft., and,

$$p = 1.5151\,x \quad \text{......................} (40)$$

$$q = x \quad \text{........................} (41)$$

$$p' = 43.56 \quad \text{.........................} (42)$$

Substituting $x = 0$ and $x = L$ for the up-stream and down-stream faces, respectively, in Equations (40), (41), and (42), $p' = 43.56$ at both faces; at the up-stream face, both p and q are equal to zero; and at the down-stream face, $p = 100$ and $q = 66$.

Applying the eight itemized checks to Equations (40), (41), and (42), for the up-stream face, $q = 0$ by Equation (26) (Table 1, Item No. 2), and $p' = F - qn$; for the down-stream face, $q = 66$ by Equation (28), $p' = 43.56$

by Equation (29) (Table 1, Item No. 3); and $f_1 = 143.56$ by Equation (30) (Table 1, Item No. 4). By Equation (32), $Q = H = \frac{66^2}{2} = 2\,178$; but $H = \frac{h^2}{2k} = \frac{10\,000}{4.59136} = 2\,178$, which checks. Equation (33) is checked at all values of x in Table 1 (Items Nos. 1 to 5).

TABLE 1.—STRESSES IN VARIOUS TYPES OF GRAVITY DAMS

Item No.	Factors	STRESS IN MASONRY UNITS PER SQUARE FOOT, FOR VARIOUS VALUES OF x, IN FEET									
		0	10	20	30	40	50	60	66	70	80
TRIANGULAR GRAVITY DAM — EXAMPLE 1											
1........	p	0	15.15	30.3	45.4	60.6	75.76	90.9	100	.....	......
2........	q	0	10	20	30	40	50	60	66	.....	......
3........	p'	43.56	43.56	43.56	43.56	43.56	43.56	43.56	43.56	.....	......
4........	f_1	43.56	46.7	58.0	74.5	93.6	112.16	131.7	143.56	.....	......
5........	f_2	0	12	15.8	14.5	10.6	7.16	2.7	0	.....	......
RECTANGULAR GRAVITY DAM — EXAMPLE 2											
6........	p	0	30.30	60.60	90.91	121.21	151.51	181.81	200	.....	......
7........	q	0	25.46	41.84	49.12	47.31	36.40	16.41	0	.....	......
8........	p'	43.56	40.9	34.0	24.8	15.1	6.7	1.4	0	.....	......
9........	f_1	43.56	67.2	91.2	116.8	139.0	160.1	183.1	200	.....	......
10........	f_2	0	4.0	3.42	—1.15	—2.8	—1.9	+0.1	0	.....	......
GRAVITY DAM, ARCHED IN PLAN — EXAMPLE 4											
11........	p	33.62	37.88	42.14	46.40	50.66	54.92	59.18	.....	63.44	67.70
12........	q	0	6.75	13.52	20.30	27.1	33.9	40.7	.....	47.4	54.156
13........	p'	41.67	42.	42.2	42.6	42.6	43.0	42.8	.....	43.2	43.325
14........	f_1	41.67	47.01	55.7	64.9	74.1	83.4	92.5	.....	101.8	111.0
15........	f_2	33.62	32.87	28.7	24.1	19.1	14.5	9.5	.....	8.4	0

The secondary principal stress, f_2, at the down-stream face is equal to zero (see Table 1, Item No. 5), and the stress at the up-stream face is equal to zero, in accordance with Checks (4) and (5). Finally, the horizontal normal stress at the up-stream face is equal to the unit water pressure, which is in accordance with Check (6).

A common method showing the distribution of the first and second principal stresses is to draw lines of equal stress or contours of principal stress, as illustrated in Fig. 5 for Example 1. The method of showing the stress distribution by means of the ellipse of stress (Fig. 6) was introduced by Messrs. J. S. Wilson and William Gore.[*] The major and minor axes indicate both magnitude and direction of the principal stresses.

Example 2.—Assume a restangular dam with the up-stream face vertical and with the resultant of forces falling at the middle-third point at a height, $h_1 = 100$ ft. Let $L = 66$ ft., and let n, m, and $V = 0$ (see Fig. 7). Then, $P = 66\ h$, or 6 600 masonry units, and, $H = \frac{62.5}{143.48} \times \frac{h^2}{2} = 0.2178\ h^2$.

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[*] *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII, Pt. II, 1908.

Taking moments about the down-stream toe, $M_L = 145\,200$. The point of application of the resultant of forces $= \dfrac{145\,200}{6\,600} = 22$ ft., which is the third point. By Equation (1), $p_o = 0$; $p_a = 200$; and $S_o = 3.0303$. For the section 1 ft. above the base: $P_u = 66 \times 99 = 6\,534$. The moment about the down-stream toe equals 145 198. The point of application of the resultant of forces is 22.22 ft. from the down-stream face, or 10.78 ft. from the center of the base. Equation (1) gives, $p = \dfrac{6\,534}{66} \pm \dfrac{6\,534 \times 10.78}{66} \times \dfrac{6}{66} = 99 \pm 97.030$; that is, $p_u = 1.97$; $p_a = 196.03$; and $S_u = 2.940$.

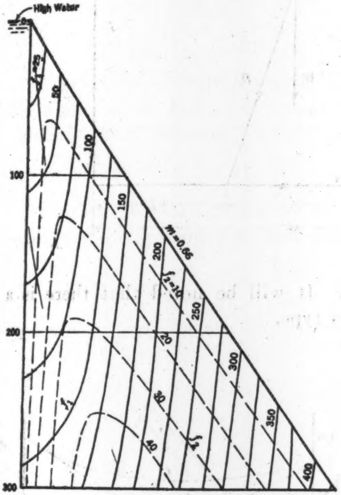
High Water
0
25
50
100
150
200
250
300
350
400
m=0.66
$f_2=10$
20
30
40
f_1
f_1
100
200
300

FIG. 5.

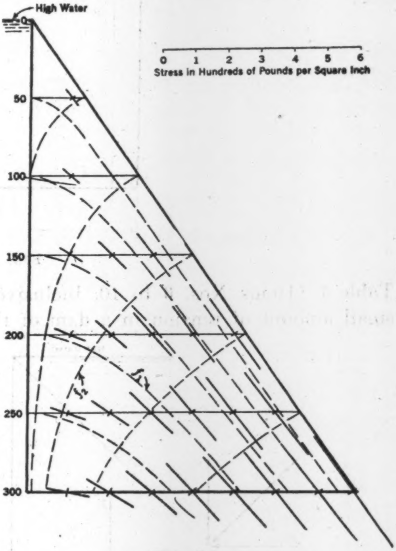
High Water
0
0 1 2 3 4 5 6
Stress in Hundreds of Pounds per Square Inch
50
100
150
200
250
300
f_2
f_1

FIG. 6.

Similarly, for the section 1 ft. below the base, $P_d = 6\,666$; the moment about the toe is 145 178; the point of application of the resultant of forces is 21.78 ft. from the down-stream face; and $p = 101 \pm 103.030$; that is, $p_d = -2.03$; $p_a = 204.03$; and $S_d = 3.12$.

Solving Equations (10), (11), and (12), for these conditions, $A_1 = 0$; $B_1 = 2.97$; and $C_1 = -0.045$. Similarly, for Equations (15), (16), and (17), $A_2 = 0$; $B_2 = 3.030$; and $C_2 = -0.0459$. It follows from Equation (3) that,

$$p = 3.0303\,x \qquad (43)$$

With the foregoing values determined, make the proper substitutions in Equations (19) and (23); thus,

$$q = 3\,x - 0.045\,x^2 \qquad (44)$$

and,

$$p' = 43.56 - 0.03\,x^2 + 0.0003\,x^3 \qquad (45)$$

The first and second principal stresses are determined by substituting Equations (43), (44), and (45) in Equation (25). The results are listed in Table 1 (Items Nos. 6 to 10, inclusive). It will be noted that there is a small amount of tension in a dam of this type.

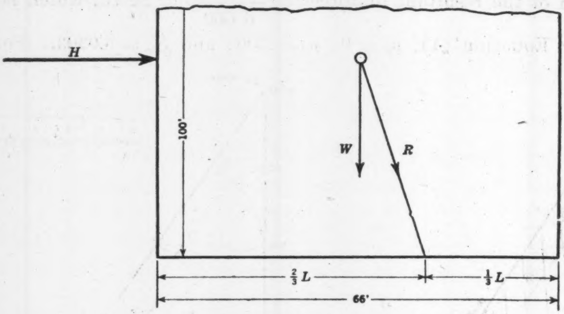

FIG. 7.

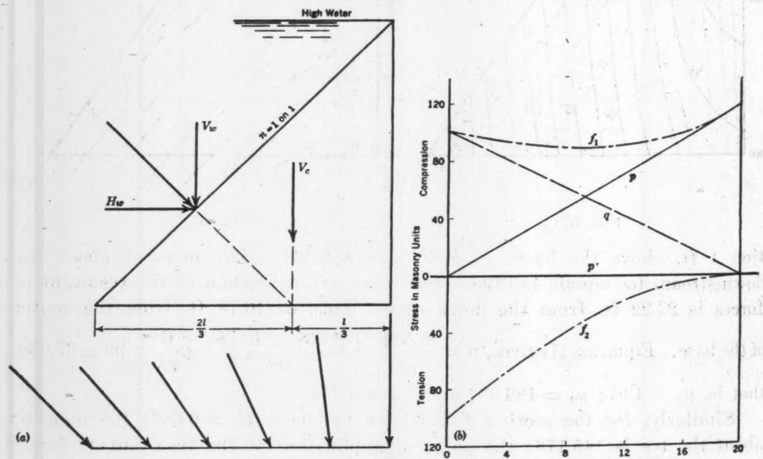

FIG. 8.

When plotted graphically the shear distribution is parabolical in shape. As in Example 1, the total shear, determined by Equation (32), is $Q = 2\,178$. The average shear is $\frac{2\,178}{66} = 33$, and the maximum vertical shear is $1.5 \times 33 = 49.5$. The results are checked, item for item, as demonstrated in Example 1.

Example 3.—Assume a triangular buttress type dam with the up-stream face on a 1 to 1 slope, or $n = 1$; the down-stream slope is vertical, or $m = 0$; the buttress spacing is twelve times the buttress thickness; and, for simplicity, the density of the deck slab is the same as the density of water, and the weight of 1 cu. ft. of masonry is assumed as 150 lb. (See Fig. 8(a)).

The three equations for the determination of stresses are:

$$p = 6\,x \quad \text{(46a)}$$

$$q = 5\,h - 5\,x \quad \text{(46b)}$$

and,

$$p' = 0 \quad \text{(46c)}$$

A graphical presentation of the principal stresses is shown in Fig. 8(b) for a dam 20 ft. high. Tensions would become excessive if this design were carried to any reasonable height.

Group II.—Gravity Dams Arched in Plan

Vertical Normal Stress.—The vertical normal stress on the up-stream edge of the trapezoidal section computed by the usual formula is:

$$\frac{P}{a} - \frac{Mc_1}{I} = p_a \quad \text{(47)}$$

and the vertical normal stress on the down-stream edge of the trapezoidal section (Fig. 9), is:

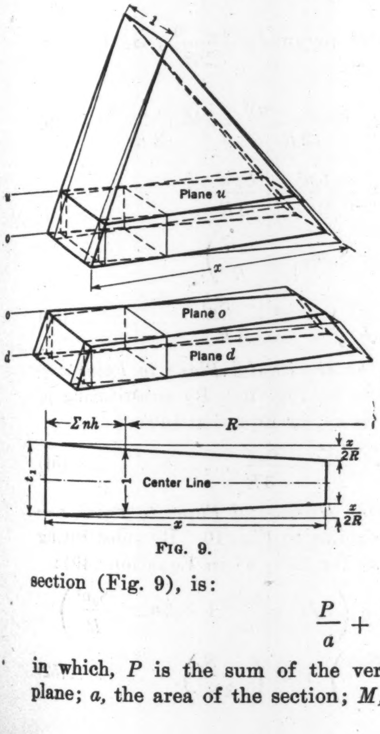

FIG. 9.

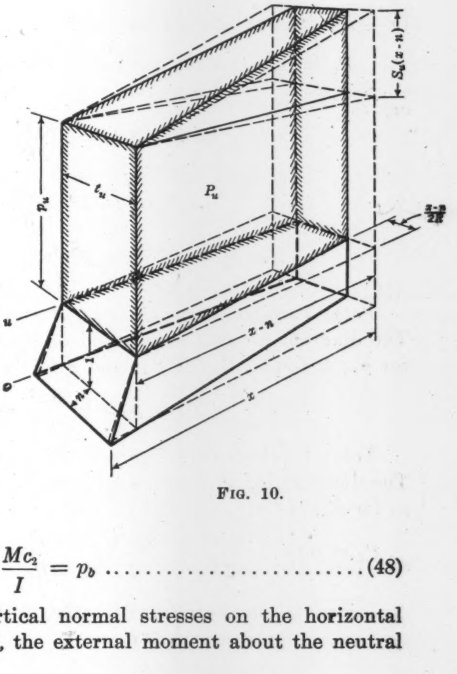

FIG. 10.

$$\frac{P}{a} + \frac{Mc_2}{I} = p_b \quad \text{(48)}$$

in which, P is the sum of the vertical normal stresses on the horizontal plane; a, the area of the section; M, the external moment about the neutral line; t_1, the thickness of section at the up-stream face; t_2, the thickness of section at the down-stream face; $c_1 = \frac{L}{3}\left(\frac{t_1 + 2t_2}{t_1 + t_2}\right)$, the distance from the neutral line to the up-stream face; $c_2 = \frac{L}{3}\left(\frac{t_2 + 2t_1}{t_1 + t_2}\right)$, the distance from the neutral line to the down-stream face; and I, moment of inertia of the sections about the neutral line; that is,

$$I = L^3 \frac{(t_1^2 + 4t_1t_2 + t_2^2)}{36\,(t_1 + t_2)}$$

Since t_2 will be written in terms of t_1 and x, it does not appear in the formulas, and t will be used for the width at the up-stream face omitting the subscript, 1; but the subscripts u, o, and d, will be used to denote the plane on which t_1 is measured.

Total Vertical Normal Stress, P_u, on the Horizontal Plane to Point x.—The volume of the solid, P_u (shown in shaded lines in Fig. 10) is equal to the volume of the parallelopiped, p_u, t_u, $(x - n)$, plus the volume of the wedge, t_u, $(x - n)$, $S_u\ (x - n)$, minus the volume of the two wedges, $\frac{x - n}{2R}$, p_u, $(x - n)$, minus the volume of the two pyramids, $\frac{x - n}{2R}$, $S_u\ (x - n)$, $(x - n)$. Stated algebraically,

$$P_u = p_u t_u (x - n) + \frac{S_u (x - n)^2 t_u}{2} - \frac{p_u (x - n)^2}{2R} - \frac{(x - n)^3 S_u}{3R} \quad \text{..(49)}$$

or,

$$\begin{aligned} P_u = & -p_u t_u n - \frac{p_u n^2}{2R} + \frac{S_u t_u n^2}{2} + \frac{S_u n^3}{3R} \\ & + x\left(p_u t_u + \frac{p_u n}{R} - S_u n t_u - \frac{S_u n^2}{R}\right) \\ & + x^2\left(\frac{-p_u}{2R} + \frac{S_u t_u}{2} + \frac{S_u n}{R}\right) + x^3\,\frac{(-S_u)}{3R} \quad \text{...........(50)} \end{aligned}$$

Total Vertical Normal Stress, P_o, on the Horizontal Plane to Point x.—The diagram for this case would be similar to Fig. 10. By substituting p_o for p_u; t_o for t_u; S_o for S_u; and x for $(x - n)$, in Equation (49):

$$P_o = p_o t_o x - \frac{p_o x^2}{2R} + \frac{S_o t_o x^2}{2} - \frac{S_o x^3}{3R} \quad \text{...............(51)}$$

Total Vertical Normal Stress, P_d, on the Horizontal Plane to Point x.—The diagram for this case would also be similar to Fig. 10. By substituting p_d for p_u; t_d for t_u; S_d for S_u; and $(x + n)$ for $(x - n)$ in Equation (49):

$$\begin{aligned} P_d = & \; p_d t_d n - \frac{p_d n^2}{2R} + \frac{S_d n^2 t_d}{2} - \frac{S_d n^3}{3R} + x\left(p_d t_d - \frac{p_d n}{R} + S_d t_d n - \frac{S_d n^2}{R}\right) \\ & + x^2\left(\frac{-p_d}{2R} + \frac{S_d t_d}{2} - \frac{S_d n}{R}\right) + x^3\left(\frac{-S_d}{3R}\right) \quad \text{.........(52)} \end{aligned}$$

Volume, W_1, of the Section Between Planes u and o.—The volume of W_1 (shown in solid line in Fig. 11) is equal to the volume of the parallelopiped, t_o, x, 1, minus the two wedges, t_o, n; minus the two wedges, $\frac{x}{2R}$, x; plus the two inverted pyramids, n, $\frac{n}{2R}$, 1, which has been subtracted with the two wedges, t_o, n. Stated algebraically,

$$W_1 = t_o\, x - \frac{x^2}{2R} - \frac{t_o\, n}{2} + \frac{n^2}{6R} \qquad (53)$$

Volume, W_2, of the Section Between Planes o and d.—The volume of W_2 (shown in solid lines of Fig. 12) equals,

$$W_2 = t_o\, x - \frac{x^2}{2R} + \frac{t_o\, n}{2} + \frac{n^2}{6R} \qquad (54)$$

Shearing Stresses.—The vertical shear, q_1, at any point, x, on the section between Planes u and o is obtained from the basic assumption expressed by Equation (8).

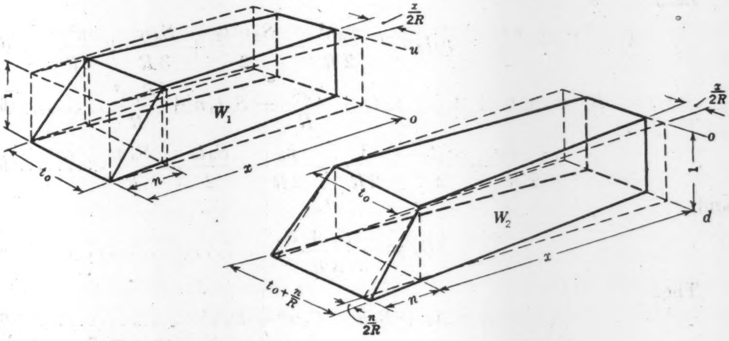

FIG. 11.

FIG. 12.

Substituting the values of P_u, P_o, and W_1 from Equations (50), (51), and (53), Equation (8) becomes,

$$\begin{aligned} q_1 = V_1 - \frac{n\, t_o}{2} - p_u\, t_u\, n - \frac{p_u\, n^2}{2} + \frac{S_u\, t_u\, n^2}{2} + \frac{S_u\, n^3}{3R} + \frac{n^2}{6R} \\ + x\left(p_u\, t_u + t_o - p_o\, t_o + \frac{p_u\, n}{R} - S_u\, n t_u - \frac{S_u\, n^2}{R}\right) \\ + x^2\left(\frac{-p_u}{2R} + \frac{S_u\, t_u}{2} - \frac{1}{2R} + \frac{p_o}{2R} - \frac{S_o\, t_o}{2} + \frac{S_u\, n}{R}\right) + x^3\left(-\frac{S_u}{3R} + \frac{S_o}{3R}\right) . (55) \end{aligned}$$

Let,

$$A_1 = V_1 - \frac{n\, t_o}{2} - p_u\, t_u\, n - \frac{p_u\, n^2}{2R} + \frac{S_u\, t_u\, n^2}{2} + \frac{S_u\, n^3}{3R} + \frac{n^2}{6R} \qquad (56)$$

$$B_1 = p_u\, t_u + t_o - p_o\, t_o + \frac{p_u\, n}{R} - S_u\, n t_u - \frac{S_u\, n^2}{R} \qquad (57)$$

$$C_1 = \frac{-p_u}{2R} + \frac{S_u t_u}{2} - \frac{1}{2R} + \frac{p_o}{2R} - \frac{S_o t_o}{2} + \frac{S_u n}{R} \quad\ldots\ldots(58)$$

and,

$$D_1 = \frac{-S_u + S_o}{3R} \quad\ldots\ldots\ldots\ldots(59)$$

Then,

$$q_1 = A_1 + B_1 x + C_1 x^2 + D_1 x^3 \quad\ldots\ldots\ldots\ldots(60)$$

Shear in the section between Planes o and d is obtained from $\Sigma \Gamma = 0$, or,

$$q_2 = P_o + V_2 + W_2 - P_d = V_2 + \frac{nt_o}{2} - p_d t_d n + \frac{p_d n^2}{2R} - \frac{S_d n^2 t_d}{2} + \frac{S_d n^3}{3R}$$

$$+ \frac{n^2}{6R} + x\left(p_o t_o + t_o - p_d t_d + \frac{p_d n}{R} - S_d t_d n + \frac{S_d n^2}{R}\right)$$

$$+ x^2\left(\frac{-p_o}{2R} + \frac{S_o t_o}{2} - \frac{1}{2R} + \frac{p_d}{2R} - \frac{S_d t_d}{2} + \frac{S_d n}{R}\right) + x^3\left(\frac{-S_o + S_d}{3R}\right) \ldots(61)$$

Let,

$$A_2 = V_2 + \frac{n t_o}{2} - p_d t_d n + \frac{p_d n^2}{2R} - \frac{S_d n^2 t_d}{2} + \frac{S_d n^3}{3R} + \frac{n^2}{6R} \quad\ldots\ldots(62)$$

$$B_2 = p_o t_o + t_o - p_d t_d + \frac{p_d n}{R} - S_d t_d n + \frac{S_d n^2}{R} \quad\ldots\ldots\ldots(63)$$

$$C_2 = \frac{-p_o}{2R} + \frac{S_o t_o}{2} - \frac{1}{2R} + \frac{p_d}{2R} - \frac{S_d t_d}{2} + \frac{S_d n}{R} \quad\ldots\ldots\ldots(64)$$

and,

$$D_2 = \frac{-S_o + S_d}{3R} \quad\ldots\ldots\ldots\ldots(65)$$

Then,

$$q_2 = A_2 + B_2 x + C_2 x^2 + D_2 x^3 \quad\ldots\ldots\ldots\ldots(66)$$

The average of q_1 and q_2 is the average shear on the vertical plane at a distance, x, from the up-stream face, or,

$$q_t = \frac{q_1 + q_2}{2} = \frac{A_1 + A_2}{2} + \frac{(B_1 + B_2)\,x}{2} + \frac{(C_1 + C_2)\,x^2}{2} + \frac{(D_1 + D_2)\,x^3}{2} \ldots(67)$$

and for horizontal normal stresses,

$$p'_t = F + Q_1 - Q_2 = F + \int_{0\cdot 5n}^{x} q_1\,dx - \int_{-0\cdot 5n}^{x} q_2\,dx = F - \frac{n}{2}\,(A_1 + A_2)$$

$$- \frac{n^2}{8}\,(B_2 - B_1) - \frac{n^3}{24}\,(C_1 + C_2) - \frac{n^4}{64}\,(D_2 - D_1) + x\,(A_1 - A_2) + x^2\,\frac{(B_1 - B_2)}{2}$$

$$+ x^3\,\frac{(C_1 - C_2)}{3} + x^4\,\frac{(D_1 - D_2)}{4} \quad\ldots\ldots\ldots\ldots(68)$$

The stresses, q_t and p'_t in Equations (67) and (68) are not unit stresses at a point, but for the total width of a section. To obtain unit stress the values must be divided by the width of the section at that point. When $R = \infty$; $\frac{1}{R} = 0$, and $t_u = t_d = 1$; Equations (56), (57), (58), (59), (62), (63), (64), and (65) reduce to Equations (10), (11), (12), (15), (16), and (17), respectively.

First and Second Principal Stresses.—The general equation (Equation (25)) for the first and second principal stresses for the case of a straight gravity dam, also apply for this case of gravity dams arched in plan.

Checks of Equations.—All checks for shearing stress, horizontal normal stress, and principal stresses at up-stream and down-stream faces of straight gravity dams apply for this case of gravity dams arched in plan.

Stress Distribution in Gravity Dams Arched in Plan—

Example 4.—Consider a triangular dam with up-stream face vertical, or $n = 0$. Other dimensions are: $m = 0.8$; $R = 1\,400$ ft.; height of dam, $h = 100$ ft.; the weight of 1 cu. ft. of masonry = 150 lb.; and the weight of 1 cu. ft. of water = 62.5 lb. Values for p and S are obtained by the usual rules of geometry and mechanics. The results are summarized in Table 2.

TABLE 2.—UNIT NORMAL PRESSURES, GRAVITY DAM ARCHED IN PLAN

h	p_b	p_a	S
99u	33.2957	66.9918	0.42545
100o	33.6189	67.6955	0.42595
101d	33.9418	68.3998	0.42646

Substituting values of Table 2, together with values of R, n, and t, the values of the factors, A, B, C, and D, in Equations (56), (57), (58), (59), (62), (63), (64), and (65) are obtained. Using these factors in Equations (3), (67), and (68), there follows: $p = 33.6181 + 0.42596\,x$; $q_t = 0.67696\,x - 0.0004932\,x^2 + 0.0000001197\,x^3$; and $p'_t = 41.666 - 0.0001362\,x^2 - 0.00000010579\,x^3 - 0.0000000000126\,x^4$.

In Table 1, Items Nos. 11 to 15, inclusive, are the stresses in a gravity dam, arched in plan, at a height of 100 ft. The radius is 1 400 ft., the up-stream face is vertical, and the down-stream slope has a batter of 1 on 0.8.

GROUP III.—TAPERING BUTTRESSES

The formulas developed for the stress distribution in a gravity dam either straight or arched in plan are not directly applicable for the stress distribution in the buttress of slab and buttress or arch and buttress types of dams that taper in thickness with the height. The taper of the buttress must be included in any formula for the stress analysis. The assumption is made that the slab or arch loading and the normal component of slab weight are applied normal to the face of the buttress, but the weight of the arch is applied vertically to the buttress. These assumptions enable the distribution of the vertical normal stress on the buttress to be written algebraically. With a reduction of the vertical normal stress near the up-stream face because of distribution over the greater area of deck and buttress, the shearing stress would be greater. Any increase in shear or decrease in the vertical normal stress, or both, decreases the second principal stress (increases the tension). The late Fred A. Noetzli, M. Am. Soc. C. E., discussed this point in detail.[7]

Vertical Normal Stress.—The vertical normal stress on any rectangular plane computed by the usual formula is,

$$\frac{1}{t}\left(\frac{P}{L} \pm \frac{6\,M}{L^2}\right) = p_a \text{ or } p_b \qquad (69)$$

For a tapering buttress, t must vary with the height. For convenience, the section generally analyzed is taken as a unit width and in this case the unit section is taken on Plane o. (See Fig. 13.)

Let t_u, t, and t_d be the total thickness as of buttress at upper, middle, and lower planes, respectively. The thickness of the upper section compared to that of the center section is in the ratio of $\frac{t_u}{t} = b_u$.

A stress, p, on Plane u, when distributed on Plane o is carried by a greater area; hence, the stress on Plane u may be considered as a smaller stress acting on a unit area on Plane o; thus, on Plane u:

$$p = p_u + S_u\,x_u \qquad (70)$$

To apply p of Plane u on Plane o, Equation (70) must be multiplied by the ratio of the thickness of Planes u and o, or,

$$p = p_u\,b_u + S_u\,b_u\,x_u \qquad (71)$$

and the total vertical normal stress on Plane u is,

$$P_u = p_u\,b_u\,x_u + \frac{S_u\,b_u\,x_u^2}{2} \qquad (72)$$

The thickness of the lower section compared to that of the center section is in the ratio of $\frac{t_d}{t}$. Let $\frac{t_d}{t} = b_d$; hence, Equation (73) is similar to Equation (72):

$$P_d = p_d\,b_d\,x_d + \frac{S_d\,b_d\,x_d^2}{2} \qquad (73)$$

The total vertical normal stress on the center plane is,

$$P_o = p_o\,x + \frac{S_o\,x^2}{2} \qquad (74)$$

Let W_1 and W_2 be the weight in masonry units of the unit width of concrete at Plane o between Planes u and o and between Planes o and d, respectively. From an inspection of Fig. 13 it is seen that,

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[7] "Design and Construction of Masonry Dams," by Edward Wegmann, M. Am. Soc. C. E., Eighth Edition.

$$W_1 = \frac{(1 + b_u)\,x}{2} - \frac{b_u\,n}{3} - \frac{n}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots(75)$$

and,

$$W_2 = \frac{(1 + b_d)\,x}{2} + \frac{b_d\,n}{3} + \frac{n}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots(76)$$

Shearing Stresses.—The vertical shear, q_1, at the point, x, on the section between Planes u and o is obtained from the assumption that the summation of the vertical forces is equal to zero, or, $\Sigma V = 0$. In Equation (8) substitute values of P_u, P_o, and W_1, from Equations (72), (6), and (75), and for x_u, substitute $x - n$; then, combining terms,

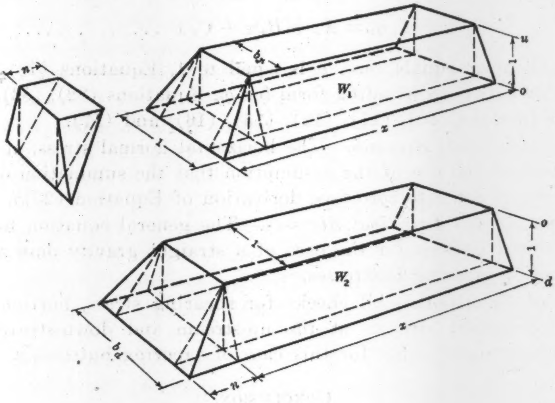

FIG. 13.

$$q_1 = \left(V_1 - \frac{n}{6} - \frac{b_u\,n}{3} - p_u\,b_u\,n + \frac{S_u\,b_u\,n^2}{2}\right)$$

$$+ x\left(-p_o + \tfrac{1}{2} + \frac{b_u}{2} + p_u\,b_u - S_u\,b_u\,n\right) + x^2\left(\frac{S_u\,b_u}{2} - \frac{S_o}{2}\right) \quad ..(77)$$

Let,

$$A_1 = V_1 - \frac{n}{6} - \frac{b_u\,n}{3} - p_u\,b_u\,n + \frac{S_u\,b_u\,n^2}{2} \quad \ldots\ldots\ldots(78)$$

$$B_1 = -p_o + \tfrac{1}{2} + \frac{b_u}{2} + p_u\,b_u - S_u\,b_u\,n \quad \ldots\ldots\ldots\ldots\ldots(79)$$

$$C_1 = \frac{S_u\,b_u}{2} - \frac{S_o}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(80)$$

Then, Equation (80) may be reduced to the form of Equation (13), or,

$$q_1 = A_1 + B_1\,x + C_1\,x^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(81)$$

Likewise, by a similar process, q_2 reduces to the form of Equation (18) and,

$$A_2 = V_2 + \frac{n}{6} + \frac{b_d\, n}{3} - p_d\, b_d\, n - \frac{S_d\, b_d\, n^2}{2} \quad \text{.............(82)}$$

$$B_2 = p_o + \tfrac{1}{2} + \frac{b_d}{2} - p_d\, b_d - S_d\, b_d\, n \quad \text{................(83)}$$

$$C_2 = \frac{S_o - S_d\, b_d}{2} \quad \text{....................................(84)}$$

or,

$$q_2 = A_2 + B_2\, x + C_2\, x^2 \quad \text{....................(85)}$$

When the taper equals zero, b is equal to 1, Equations (78), (79), and (80) for q_1 and its corresponding form for q_2, Equations (82), (83), and (84), reduce to Equations (10), (11), (12), (15), (16), and (17).

Horizontal Normal Stresses.—The horizontal normal stress, at any point, x, is obtained by the use of the assumption that the summation of the horizontal stresses is equal to zero (see derivation of Equation (23)).

First and Second Principal Stresses.—The general equation for first and second principal stresses for the case of a straight gravity dam also applies for this case of tapering buttresses.

Checks of Equations.—All checks for shearing stress, horizontal normal stress, and principal stresses at the up-stream and down-stream faces of straight gravity dams apply for this case of tapering buttresses.

CONCLUSION

Middle-Third Theory.—The numerical examples indicate the fallacy of the middle-third theory as a safe criterion for dam design.

In rectangular gravity dams the volume of material is twice that in triangular dams and the up-stream half of the dam has very limited resistance to uplift, while the down-stream half has a tension in the second principal stress making the former a less stable structure than the latter (compare Items Nos. 5 and 10, Table 1.)

Many unreinforced buttresses of the type shown in Fig. 8(*a*) are standing and are considered safe, but high tensions exist in the second principal stress, and provisions should be made for this stress in the design of this type. By making only a slight change in the quantity of material in the buttress (changing the down-stream slope), considerable tension is eliminated.

New Design—Buttress and Gravity Dams.—In the design of dams of the buttress type the tension at the up-stream face should be held at a minimum. From an inspection of Mohr's diagrams it is evident that the tension is reduced if the shear, q, is reduced. The vertical shear at the up-stream face is,

$$q_y = n\left(p - \frac{V}{t}\right) \quad \text{......................(86)}$$

in which, t is the thickness of the buttress; V, the unit water pressure plus the vertical component of the weight of the deck on a 1-ft. section at any elevation. An increase of the up-stream slope decreases n, the tangent of slope measured from the vertical, and, hence, decreases the shear, and if the up-stream slope were vertical ($n = 0$) the shear would be zero. However, as the slope decreases the resisting moment, due to the vertical component of the weight of water, is decreased, and the ratio of horizontal forces to vertical forces (sliding factor) is increased. Fredrik Vogt, Assoc. M. Am. Soc. C. E., has shown[8] that the most economical design is that in which $n = 0.5\pm$. (The average slope for most American multiple-arch or slab-and-buttress dams is about 1.) The use of these slopes, while giving good stress distribution results, involves a sliding factor in excess of that generally accepted by engineers as adequate for some types of foundation footings. It is probable that the most economical design for buttress dams will have an up-stream slope between 1 on $\frac{1}{2}$ and 1 on 1 and one or more contraction joints.

The writer made an effort to determine the stresses in the San Gabriel Dam with inclined contraction joints, but was unable to decide upon any satisfactory assumption of loading or pressures that might exist on the contraction joints as designed. It is his opinion, however, that the derived formulas will indicate a system of joints located so that satisfactory assumptions for determining a design can be made.

Distribution of Stresses.—It is well known that the vertical normal stress distribution of gravity dams is not linear, although for triangular dams of thicknesses of 50 to 60 ft., the error probably is small. When and if a more exact distribution of the vertical normal stress is expressed in terms of the base length, it should be inserted in the foregoing formulas and the shear and horizontal normal stresses will not be distributed linearly. Some large-scale experimental work on stress distribution in mass concrete would be valuable to the profession, although a more intimate knowledge of other physical properties of concrete is more essential.

By the use of methods now in practice the principal stresses at the heel and toe of small gravity or buttress dams, either arched or straight in plan, are easily obtained after finding the vertical normal stresses. The shear and horizontal normal stresses may be assumed as being distributed linearly for only a monolithical triangular dam with water to the apex of the triangle (see Table 1 and Fig. 8). The shear and horizontal normal stresses for rectangular and irregular sections are not distributed linearly (see Table 1) and an analytical method of analysis is necessary to determine their magnitude and direction.

The gravity dam as designed must transmit shear from one portion of the section to another portion. If the section is jointed or ruptured from any cause, when the load is applied, shear can not be transferred completely across the line of rupture. As temperature and shrinkage stresses cause partial or total failure, dividing the base into irregular sections, the section modulus of the entire base should not be used in Equation (1). Any gravity dam higher than, say, 100 ft., cannot be made monolithically; hence, each monolithic block into which the dam is divided by cracks or joints (which may not be triangular) should be analyzed.

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[8] "Economical Design of Buttresses for High Dams and of Cellular Gravity Dams."

The analytical method described in this paper, when used for gravity or buttress dams, has at least five advantages:

(1) It provides an exact and concise method of determining maximum stresses;
(2) It gives complete internal stress analysis;
(3) It indicates that the monolithic section only should be used when determining the section modulus of the base;
(4) It suggests a method of design of buttresses eliminating tension; and,
(5) It makes available a basis for design of construction joints.

APPENDIX I

Notation

The symbols used in this paper are as follows:

a = area of section analyzed.
b = a ratio of thicknesses, t; that is, $b_u = \frac{t_u}{t_o}$.
c = the distance of the neutral line from the faces: c_1, from the down-stream face; c_2, from the up-stream face.
d = the horizontal plane 1 ft. below Plane o.
f = unit stress: f_1 = first principal stress; and f_2 = second principal stress.
h = height.
k = ratio of the weights of a unit of masonry to a unit of water.
m = batter at the down-stream face = horizontal distance: 1 unit vertical.
n = batter at the up-stream face = horizontal distance: 1 unit vertical.
o = the horizontal plane through the section considered.
p = unit vertical normal pressure at any point, x, on a plane; p_b = vertical pressure at up-stream face; p_a = vertical pressure at down-stream face.
p' = unit horizontal normal pressure at any point, x, on a plane.
q = unit vertical or horizontal shear at any point on a plane; q_1 = shear between Planes u and o; and q_2 = shear between Planes o and d.
t = thickness of the section analyzed; in a straight gravity section, $t = 1$, and in a tapering buttress section, t = the total thickness at the middle plane; t_1 = thickness at up-stream face; t_2 = thickness at down-stream face.
u = a horizontal plane, 1 ft. above Plane o.
w = weight of one cubic unit of masonry.
x = horizontal distance from an origin of ordinates to the point to be analyzed; x_o = distance from the up-stream face of the section cut by Plane o; x_u = distance from the up-stream face of the section cut by Plane u; x_d = distance from the up-stream section cut by Plane d; etc.
A = a substitution factor, with appropriate subscripts.
B = a substitution factor, with appropriate subscripts.

C = a substitution factor, with appropriate subscripts.
D = a substitution factor, with appropriate subscripts.
F = horizontal component of external forces applied on a section 1 ft. thick midway between Planes u and d.
H = horizontal component of the resultant of all external forces applied to the structure above any plane.
I = moment of inertia.
L = the length of any horizontal section.
M = bending moment.
P = the sum of vertical normal stresses on any horizontal plane under consideration; P_u refers to the summation of stresses on Plane u, etc.
Q = horizontal shear: Q_1 on the plane midway between Planes u and o; Q_2 on the plane midway between Planes o and d.
R = radius of dam.
S = the rate of change of intensity of the vertical normal stress; S_u refers to any point on Plane u, etc.
V = the vertical component of the water or deck pressure on the up-stream face of a section between two limiting planes; V_1 is between Planes u and o; V_2 is between Planes o and d.
W = weight of concrete included between two horizontal planes; W_1 is between Planes u and o; W_2 is between Planes o and d.
θ = direction of first principal stress measured from the vertical.

DISCUSSION

W. J. Stich,[9] Assoc. M. Am. Soc. C. E. (by letter).—An interesting adaptation of stress analysis in gravity dams, as first formulated by the late William Cain,[2] M. Am. Soc. C. E., is given in this paper. In addition, the subject of inclined joints and a means of locating these joints have been discussed. As long as dams of only moderate height were proposed, the ordinary methods of stress determination, either graphical or analytical, satisfied the requirements of design. In general, it was considered that: (*a*) If vertical normal stress at the up-stream and down-stream faces of the dam, with an adequate allowance for uplift, appeared to be free from tension; (*b*) if vertical contraction joints were specified at the proper spacing; (*c*) if the sliding factor was kept within accepted limits; and (*d*) if water-tightness and drainage were given due consideration; then, the dam was assumed to be safely designed, and that the selection of the proper profile was a relatively simple matter. With the advent of the high gravity dam, however, and with the resulting limitations imposed on the profile by the bearing power of foundations and the effectiveness of mass concrete in withstanding vertical or inclined cracking parallel to the axis of the dam, the matter of tensile stress within the dam and the provision of inclined joints became of prime importance.

The author's paper appears to be particularly timely in view of the impending construction of dams of great magnitude, such as the Hoover Dam. Furthermore, the progress being made in the art of dam design as a result of experimental research should re-act as a stimulus to the engineer in his efforts to formulate equally advanced methods of theoretical analysis.

The writer has been unable to check some of the formulas presented. For instance, in order to obtain Equations (26) and (27), it was found necessary to assume S_u equal to S_d and p_u equal to p_d. This, however, is only true for the particular case of $n = 0$ and $m = \frac{1}{\sqrt{k}}$.

In July, 1927, the writer was assigned the problem of analyzing the stresses in the proposed San Gabriel Dam, in Southern California. The dam was of the gravity type, arched in plan, with a radius of 1 400 ft. at the reference line and with the cantilevers assumed to take all the load. A tentative profile had been decided upon, based on a limiting foundation pressure of 40 tons per sq. ft. and absence of tension at either the up-stream or the down-stream face. In order to reduce the maximum foundation stress, a rather unusual reverse curve was selected for the down-stream batter near the base; and this batter, together with the height of the dam (492.5 ft.), raised the question as to what objectionable stresses, if any, might exist in the interior.

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[9] Civ. Engr., City Board of Education, Los Angeles, Calif.

As a means of calculating these internal stresses, the only available method appeared to be that of Professor Cain. However, this method, when made applicable to the case at hand, was found to be extremely laborious and involved the use of coefficients of eighteen significant figures for the calculation of stresses at the lower elevations of the dam.

In order to expedite the calculations, an approximate method, requiring only the same degree of accuracy as the ordinary calculation of vertical normal stresses and involving only a fraction of the time necessary to apply Professor Cain's analysis, was suggested by the late Fred A. Noetzli, M. Am. Soc. C. E. By means of this method it was possible, in a reasonable length of time, to make a complete stress analysis and calculation of principal stress trajectories for the dam in question. Subsequently, the writer made a check calculation of the results obtained for two critical sections of the dam (Elevation 450 and Elevation 492.5) by applying Professor Cain's analysis; and, as a further verification of the results, Fredrik Vogt, Assoc. M. Am. Soc. C. E., computed the stresses at these two elevations by means of a more exact method. Both checks confirmed the results obtained by the approximate method.

As the author points out, the necessity of vertical or inclined joints parallel to the axis in high gravity dams is now recognized. The provision for joints at 50-ft. centers in the Hoover Dam is a striking example. This specification has also been adopted for buttress dams, as evidenced in the design of the Coolidge Dam, Big Dalton (multiple-arch) Dam, in Los Angeles County, California, and in the Rodriguez Dam, on the Tijuana River, for the Mexican Government. If joints are placed along the lines of maximum principal stress, then, theoretically, for reservoir full, the plane of the joint will be one of zero shear, and the calculated stresses will remain unchanged. If, in addition, the joint is grouted adequately and liberal keys are provided, other stages of the reservoir which may produce moderate shear along this plane need not be a source of concern. The grouted joint will transfer the compression, and the keys will resist the shearing stresses developed.

HAKAN D. BIRKE,[10] ASSOC. M. AM. SOC. C. E. (by letter).—The marked trend of recent years toward the use of high buttress dams has made necessary the development of more accurate and comprehensive methods of stress analysis than were used previously for designing this type of structure. These improved methods of design, which are founded on the work[2] of the late William Cain, M. Am. Soc. C. E., involve an accurate determination of the principal stresses and their directions. Any further developments based on the fundamentals established by Professor Cain should be pursued with caution as such procedures in certain cases may result in misleading conclusions or even erroneous and dangerous results. It is obvious that Mr. Holmes has fallen into some of these pitfalls. The methods developed in his paper are not applicable to the design of buttress dams, and they give incorrect results which are not on the side of safety.

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[10] Designer, Ambursen Dam Co., New York, N. Y.

The method, referred to by Mr. Holmes, of determining the vertical normal stress at the heel and toe of a dam is entirely obsolete and of recent years all important dams have been analyzed by the method of principal stresses. For example, in designing the Big Dalton Dam, the Coolidge Dam, the Stony Gorge Dam, and the Rodriguez Dam, methods were developed for determining the principal stresses, which were practical in application and which gave theoretically correct results. Furthermore, in three of these structures a system of inclined joints placed along trajectories of principal stress was used. Relating the location of joints to the direction of principal stresses is, therefore, well established in practice and is not new, as suggested in Mr. Holmes' paper.

The methods developed by the author which involve the determination of stress equations on planes 1 ft. apart vertically have long been discarded in practical designing work. In the first place these methods are entirely too cumbersome for structures with simpler outlines, such as the triangular gravity dam. In the second place these methods give results that are incorrect and not on the side of safety for structures having variable sections, such as buttress designs with flaring sections, haunches, pilasters, arches, etc. Grave error will be introduced if these variations are not included in the computations and if the horizonal sections are reduced to simple rectangles as required by Mr. Holmes' methods.

In the case of the gravity dam of triangular section the final procedure of analysis will be greatly simplified if the method presented by the author is slightly modified and carried to its logical conclusion. If the equilibrium Equations (9), (14), and (23) of Mr. Holmes' paper were developed on the basis of taking the three horizontal planes of analysis a distance, *dy*, apart instead of 1 ft., the intensity and directions of the principal stresses may be found directly by relating these stresses to vectorial planes. The equilibrium Equations (9), (14) and (23) given by Mr. Holmes may be expressed for an infinitely small particle in the following manner:

$$\frac{\partial p}{\partial y} + \frac{\partial q}{\partial x} - k = 0 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (87)$$

and,

$$\frac{\partial p'}{\partial x} + \frac{\partial q}{\partial y} = 0 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (88)$$

The state of stress in any structure may be completely determined if any of the stress functions of Equation (87) or Equation (88) are known, provided it is practical to express the variations of structural dimensions by definite mathematical laws. Consider, for example, the triangular gravity dam shown by Fig. 14. The forces acting on any horizontal section, *A-A*, are:

W = weight of masonry $= \frac{1}{2}(m + n)\,k\,y^2$,
V = vertical component of water pressure $= \frac{1}{2}\,n\,y^2$,
H = horizontal component of water pressure $= \frac{1}{2}\,y^2$,

in which, $m = \tan\alpha$ = slope of down-stream face, and $n = \tan\beta$ = slope of up-stream face.

These forces may be combined as follows:

P = sum of vertical forces (masonry + water) on Section A-A, or,

$$P = [(m + n)\ k + n]\ \frac{y^2}{2} \qquad \text{(89)}$$

Q = sum of horizontal forces on Section A-A, or,

$$Q = \tfrac{1}{2}\, y^2 \qquad \text{(90)}$$

M = moment of all forces about the center of gravity of Section A-A, or,

$$M = [2 - (3\,m + n) - (m^2 - n^2)\ k]\ \frac{y^3}{12} \qquad \text{(91)}$$

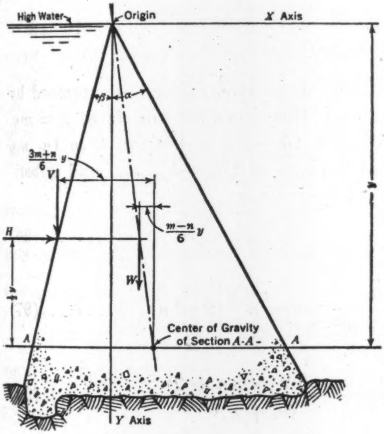
High Water
Origin
X Axis
β
α
$\frac{3m+n}{6}y$
V
H
$\frac{m-n}{6}y$
W
$\frac{1}{3}y$
y
A
A
Center of Gravity of Section A-A
Y Axis

FIG. 14.

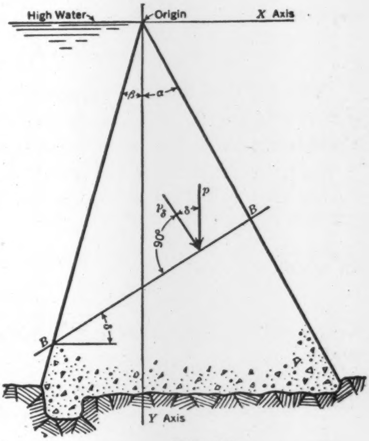
High Water
Origin
X Axis
β
α
p
p_δ
δ
90°
B
B
δ
Y Axis

FIG. 15.

These expressions may now be substituted in the well-known stress formula, $p = \dfrac{P}{a} + \dfrac{Mc}{I}$, in which, c = the distance from the center of gravity of the section to the fiber $= x - \dfrac{m - n}{2}\,y$, and I = moment of inertia $= \dfrac{y^3\,(m + n)^3}{12}$.

Substituting these values in the expression for p and simplifying,

$$p = a_1\,x + b_1\,y \qquad \text{(92)}$$

in which,

$$a_1 = -\ \frac{k\,(m - n)}{(m + n)^2} - \frac{(n^2 + 3mn - 2)}{(m + n)^3} \qquad \text{(93)}$$

and,

$$b_1 = \frac{k\,(m^2 + n^2)}{(m + n)^2} - \frac{m - n - 2\,m^2\,n}{(m + n)^3} \qquad \text{(94)}$$

After determining the normal stress functions from Equations (92), (93), and (94), the shearing stresses may be found by substituting the value of p in Equation (87) and solving for q; thus,

$$q = -\int_x \left(\frac{\partial p}{\partial y} - k\right) dx + \phi_1(y) \dots\dots\dots\dots (95)$$

in which, $\phi_1(y)$ is an arbitrary function of y that may be determined from the boundary conditions at the up-stream or down-stream face. Substituting the value for p in Equation (95), and integrating, $\frac{\partial p}{\partial y} = b_1$ and,

$$q = \int_x (b_1 - k)\, dx = (b_1 - k)\, x$$

or,

$$q = (k - b_1)\, x + \phi_1(y)$$

The relation between the stresses at the down-stream face is expressed by $q = m\,p = m\ (a_1\ my + b_1\, y)$ because, for the down-stream face, $x = my$. Combining these two values for q will give $(k - b_1)\ x + \phi_1(y) = m\ (a_1\ my + b_1\, y)$. Solving for $\phi_1(y)$ gives $\phi_1(y) = [a_1\, m + 2\, b_1 - k]\ my$, and substituting this value for $\phi_1(y)$, in the expression for q:

$$q = a_2\, x + b_2\, y \dots\dots\dots\dots (96)$$

in which,

$$a_2 = \frac{2\,kmn}{(m+n)^2} + \frac{1}{(m+n)^3}\,(m - n - 2\,m^2 n) \dots\dots\dots\dots (97)$$

and,

$$b_2 = -\,\frac{kmn\ (m - n)}{(m+n)^2} + \frac{m\,n\,(m^2 - m\,n + 2)}{(m+n)^3} \dots\dots\dots\dots (98)$$

The horizontal normal stress may now be determined by solving Equation (88); thus,

$$p' = -\int_x \frac{\partial q}{\partial y}\, dx + \phi_2(y) \dots\dots\dots\dots (99)$$

in which, $\phi_2(y)$ is an arbitrary function of y. Substituting the value for q as found before, integrating, and determining the function, $\phi_2(y)$, will result in the following expression:

$$p' = a_3\, x + b_3\, y \dots\dots\dots\dots (100)$$

in which,

$$a_3 = \frac{kmn\,(m - n)}{(m+n)^2} - \frac{1}{(m+n)^3}\ mn\,(m^2 - mn + 2) \dots\dots (101)$$

and,

$$b_3 = +\,\frac{2\,k\,m^2 n^2}{(m+n)^2} - \frac{1}{(m+n)^3}\ m^2\,(2\,mn^2 - m - 3\,n) \dots\dots (102)$$

Equations (92), (96), and (100) are of prime importance as they show that the vertical and horizontal normal stresses (p and p') and the shearing stresses, q, will have straight-line distribution on any plane.

From the foregoing it may now be proved that the stresses normal to any inclined plane making an angle, δ, with the horizontal will be straight and linearly distributed.

The stresses normal to an inclined plane (Fig. 15) will have the value,

$$p\delta = p \cos^2 \delta + p' \sin^2 \delta + q \sin 2\,\delta \quad \ldots\ldots\ldots\ldots\ldots\ldots (103)$$

in which, $p\delta =$ stresses normal to the inclined plane, *B-B;* and $\delta =$ the angle between the inclined plane and the horizontal.

Combining Equations (92), (96), (100), and (103):

$$p\delta = (a_1 x + b_1 y) \cos^2 \delta + (a_3 x + b_3 y) \sin^2 \delta + (a_2 x + b_2 y) \sin 2\,\delta \quad ..(104)$$

Equation (104) expresses the condition that $p\delta$ varies linearly with the co-ordinates, x and y, along a plane making the angle, δ, with the horizontal. Therefore, the normal stresses along any plane, horizontal or inclined, will be distributed according to the law of the trapezoid. Any section in the dam, that is a plane before the loading, will remain a plane after the deformation, if the deformations due to shearing stresses are neglected.

From the foregoing procedure may be derived an easy method of determining the stress trajectories. The direction of the first principal stress at any point is determined by the equation, $\tan 2\,\theta = \dfrac{2q'}{p - p'}$. Substituting the values for q, p, and p',

$$\tan 2\,\theta = \frac{2\,(a_2 x + b_2 y)}{(a_1 - a_3)\,x + (b_1 - b_3)\,y} \quad \ldots\ldots\ldots\ldots\ldots (105)$$

or,

$$[(a_1 - a_3) \tan 2\,\theta - 2\,a_2]\,x + [(b_1 - b_3) \tan 2\,\theta - 2\,b_2] = 0 \ldots (106)$$

If a certain constant value for the angle, θ, is chosen for the inclination of the first principal stress, for example, $\theta = \theta_1$ (see Fig. 16(*a*)), and substituted in Equation (106), the result will be the equation for a curve which the stress trajectories will intersect under a certain constant angle, θ_1, with the vertical. Equation (105) shows that this curve is a straight line through the origin and can easily be located. By determining a few of these straight lines the stress trajectories are fully located. (See Fig. 16(*a*).)

Carrying the analysis further, if the direction of the principal stress is known for one point on a radius vector it will be known for any point on that radius vector. For example, consider Fig. 16(*b*), in which the triangular gravity dam analyzed by Mr. Holmes (Fig. 6) is shown with the stress trajectories and two radius vectors. From this illustration it will be noted that the directions of principal stress are constant along each of these radius vectors.

It will now be proved that principal stress intensities will have straight linear distribution along the vectorial planes as well as the normal stress intensities.

Let the equation for the plane, *O-C* (see Fig. 16(*a*)), be $x = sy$. Substitute this value for x in Equation (104) and let δ be equal to θ_1. This will give,

$$p_{\theta_1} = [(a_1 s + b_1)\cos^2\theta_1 + (a_3 s + b_3)\sin^2\theta_1 + (a_2 s + b_2)\sin 2\,\theta_1]\,y_m \quad (107)$$

Now, p_{θ_1} will be the first principal stress along the plane, *O-C*, and according to Equation (107) it varies linearly along that plane from the value, 0, at the top. The same thing applies, of course, to the second principal stress.

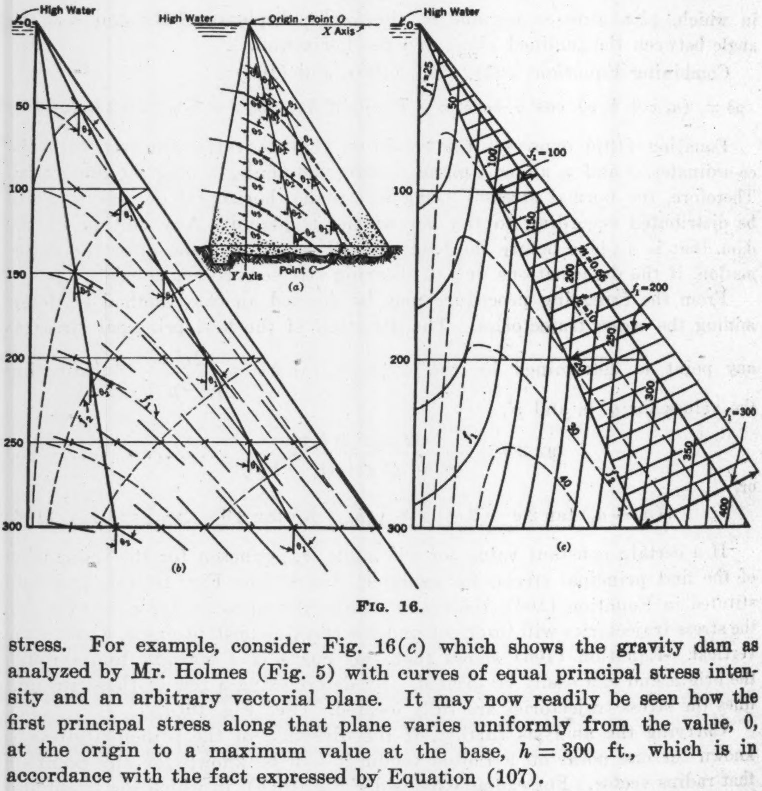
High Water
O
50
100
150
200
250
300
θ_1
θ_2
f_2
f_1
(b)
High Water
Origin - Point O
X Axis
Y Axis
Point C
θ_1
θ_2
(a)
High Water
O
f_1=25
50
f_1=100
100
150
m=0.66
200
f_2=10
f_1=200
250
20
300
f_1=300
350
400
f_1
30
40
f_2
(c)

FIG. 16.

For example, consider Fig. 16(*c*) which shows the gravity dam as analyzed by Mr. Holmes (Fig. 5) with curves of equal principal stress intensity and an arbitrary vectorial plane. It may now readily be seen how the first principal stress along that plane varies uniformly from the value, 0, at the origin to a maximum value at the base, $h = 300$ ft., which is in accordance with the fact expressed by Equation (107).

The foregoing has proved that the methods presented in Mr. Holmes' paper may be developed somewhat farther with the fortunate result that the final application to the triangular gravity dam is greatly simplified. All that is necessary is to find the principal stresses on any single horizontal plane. The principal stresses on other horizontal planes in the dam may be determined by straight linear interpolation of principal stress values along vectorial planes, these principal stresses varying from the values determined along the first plane of analysis to zero at the apex. The great amount of labor involved in analyzing the stresses on many planes, as presented in Mr. Holmes' paper, is entirely unjustified as by simple modifications of these methods stresses in the entire dam may be obtained from a single plane of analysis.

Analysis of Buttress Dams.—In developing the formulas for stresses in buttresses, Mr. Holmes has not included horizontal projection of the arches or haunches in the load-carrying section of the buttress. This procedure gives incorrect results as the haunches and arches materially affect the stress distribution.

For dams of the buttress type only the general equilibrium equations may be set up for elementary prisms, but no expressions for the stresses as functions of the co-ordinates can be developed for the buttress as a whole. The analysis must be made step by step, first determining the shear stresses for a great number of points by taking the sum of the vertical forces on each elementary prism and then, when the shear stresses are known, determining the horizontal normal stresses by summing up all horizontal forces. From the results of the foregoing procedure the principal stresses may be found either graphically by Mohr's circle of stress or analytically by the principal stress equations.

The contraction due to shrinkage and temperature in a gravity dam, or in the buttresses of a buttress type dam, should be taken care of by properly arranged contraction joints. If these joints are placed along the trajectories of the first principal stress and if the second principal stress is zero or some value in compression then there will be no tendency for these joints to open up and as the shear is zero there will be no sliding tendency. Any movement along these joints due to temperature changes or shrinkage will not impair the structure as a whole because the joints still are able to transmit the stresses necessary for the monolithic action of the structure. The writer does not agree, therefore, with Mr. Holmes' statement (see "Distribution of Stresses") that no gravity dam higher than about 100 ft. should be analyzed as a monolith. Since, at all points, the structure is able to take the stresses necessary for monolithic action, why should it not be designed as such? However, as a check, the separate columns into which the structure is divided by the contraction joints may be analyzed independently and then, of course, the second principal stress as found by analyzing the structure as a monolith along the contraction joints should be taken into account as loading.

If the contraction joints (or cracks) have some location other than on a trajectory of principal stress the analysis of the structure will be a difficult problem. For analyzing each monolithic column the forces transmitted through the joints have to be known; but these forces can be determined only by considering the elasticity of the entire structure. When applying Mr. Holmes' equations to the monolithic columns the forces between the columns, that is, the stresses along the joints, must be known. These forces, however, are not known beforehand and, therefore, in the writer's opinion, there is no possibility of using equations of the type set up by Mr. Holmes.

DIRK A. DEDEL,[11] ASSOC. M. AM. SOC. C. E. (by letter).—It is interesting to note the advances that have been made in dam design, especially in the last decade. The development of the theory of principal stresses applied to dam analysis has shown that the determination of only the vertical normal pressures on horizontal planes does not tell the story of stress distribution.

Mr. Holmes has endeavored to develop a series of formulas for the stress analysis of dams of the gravity and buttress types with the assumption that they could be readily used in solving the principal stresses in these two types. However, in the development of the formulas for the dam of the buttress type, an important factor has been overlooked, making them purely academic and useless in practical design problems. The formula used by the author for vertical normal pressures includes the assumption that the plane on which the stresses are to be determined is rectangular. Of course, this condition never exists in any buttress dam due to the necessity of widening the up-stream end of the buttress to support the flat-slab deck, or to support the arch barrels in the multiple-arch type. It is to be noted also that the round-head buttress type of dam has an enlarged up-stream buttress section to transmit the forces of the water load directly to the buttress proper.

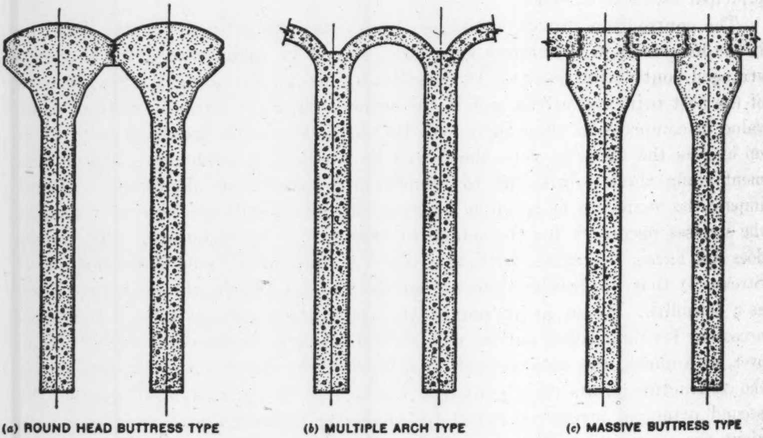
(a) ROUND HEAD BUTTRESS TYPE (b) MULTIPLE ARCH TYPE (c) MASSIVE BUTTRESS TYPE

FIG. 17.

From Fig. 17 it can be plainly seen that no horizontal buttress section in any of the three types can be assumed as rectangular. The irregular shape of the horizontal buttress section makes it necessary to compute the center of gravity and moment of inertia of each section. With these two factors known and the true eccentricity of the resultant of the vertical loads determined, the vertical normal stresses at the up-stream and down-stream ends of the section under investigation may be obtained by substituting in the usual formula:

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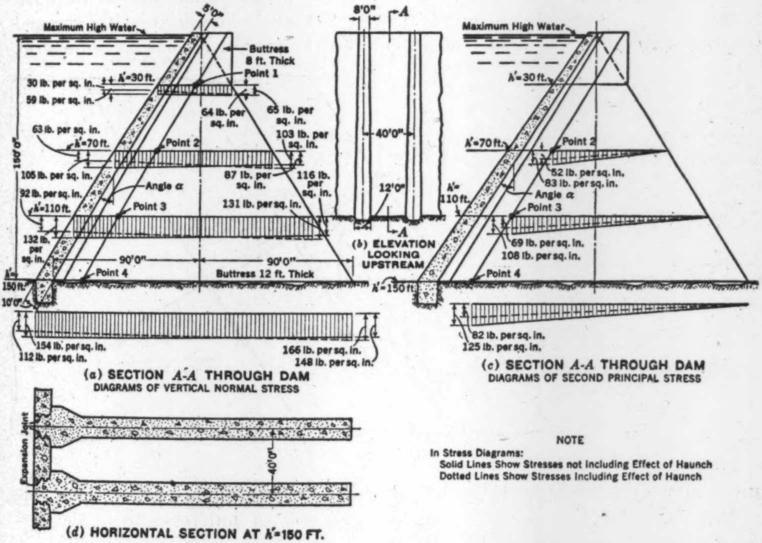
[11] Designing Engr. Ambursen Constr. Co., New York, N. Y.

$$\frac{P}{a} \pm \frac{Pec}{I} = p_a \text{ or } p_b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (108)$$

in which, e, the eccentricity, is equal to the distance between the point of application of the vertical load and center of gravity of the section.

The fact that the buttress is irregular in shape makes it impossible to use the author's equations for shear, normal stresses, and principal stresses which include unit buttress thickness and also the factor, b (ratio of buttress thickness). The total area of the section including the haunch area must be used if correct results are to be obtained. To show the large errors in stresses that can be made by the use of an incorrect procedure the writer has analyzed the critical stresses in a massive buttress dam, shown in Fig. 18, using both the correct and incorrect assumptions.

Mr. Holmes' assumptions give entirely too conservative results. Fig. 18 shows an increase of more than 50% in the second principal stress at the maximum height of the dam. It is necessary, therefore, to approach the


Maximum High Water
5'0"
Buttress 8 ft. Thick
Point 1
30 lb. per sq. in.
h'=30 ft.
59 lb. per sq. in.
64 lb. per sq. in.
65 lb. per sq. in.
63 lb. per sq. in.
h'=70 ft.
Point 2
103 lb. per sq. in.
150'0"
105 lb. per sq. in.
87 lb. per sq. in.
116 lb. per sq. in.
Angle α
92 lb. per sq. in.
h'=110 ft.
Point 3
131 lb. per sq. in.
132 lb. per sq. in.
90'0"
90'0"
Point 4
Buttress 12 ft. Thick
h'=150 ft.
10'0"
154 lb. per sq. in.
112 lb. per sq. in.
166 lb. per sq. in.
148 lb. per sq. in.
8'0"
A
40'0"
12'0"
A
h'=150 ft.
Maximum High Water
h'=30 ft.
h'=70 ft.
Point 2
52 lb. per sq. in.
83 lb. per sq. in.
Angle α
h'=110 ft.
Point 3
69 lb. per sq. in.
108 lb. per sq. in.
Point 4
82 lb. per sq. in.
125 lb. per sq. in.
Expansion Joint
40'0"

(*a*) SECTION *A-A* THROUGH DAM
DIAGRAMS OF VERTICAL NORMAL STRESS

(*b*) ELEVATION LOOKING UPSTREAM

(*c*) SECTION *A-A* THROUGH DAM
DIAGRAMS OF SECOND PRINCIPAL STRESS

(*d*) HORIZONTAL SECTION AT h'=150 FT.

NOTE

In Stress Diagrams:
Solid Lines Show Stresses not Including Effect of Haunch
Dotted Lines Show Stresses Including Effect of Haunch

FIG. 18.

subject of principal stresses in a somewhat different manner from that presented by the author. The auxiliary stress planes necessary in the computation of these stresses must be a comparatively small distance above and below the plane under investigation, but the use of 1 ft. for this distance is obsolete. Sufficient accuracy may be obtained if a value of 5 to 30 ft. is used, the exact distance depending entirely upon the height of the structure. However, if computations are made, using the total area of the horizontal section, together with small distances between sections, small differences in large numbers are the result. This makes the work laborious and requires the use of a calculating machine or a many-place logarithmic table to obtain reliable results. Preliminary investigations of stress distinction in a dam do not warrant this vast amount of work. For this reason, a simpler procedure can be used to calculate the critical stresses, which gives sufficient accuracy for most cases.

In the preliminary stages of the design when the proportioning of the structure is under investigation, sufficient accuracy is obtained if it is considered that the component of pressure normal to the deck and directly under the haunch is the maximum principal stress. This component is obtained by taking the water pressure acting on a strip of deck, plus the component of the weight of a strip of masonry, acting normal to the deck, and considering this combined force to be concentrated directly under the haunch on a strip of buttress, all three elements being 1 ft. wide. If the value of this stress together with the known value of p is combined in Mohr's circle the values of p', q, and f_2 can be obtained, assuming that the direction of the first principal stress is normal to the under side of the deck.

Let N be the buttress spacing; h', the distance below high-water surface; w_w, the weight of a cubic foot of water; W_c, the weight of masonry of a strip, unity in width, between the points, a and b; α, the angle of the under side of the deck with the vertical; and, t, thickness of buttress; then, the first principal stress at Point b, Fig. 19, is,

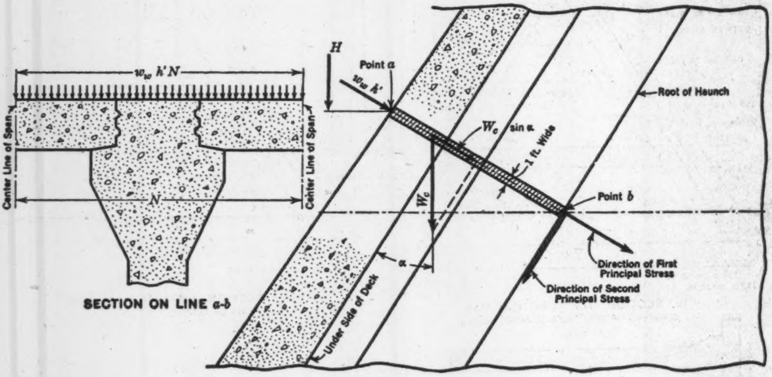


FIG. 19.

$$f_1 = \frac{w_w h' N + W_c \sin \alpha}{t} \qquad \text{(109)}$$

Knowing the vertical normal pressure and the first principal stress at oint b, the remaining stresses, q, p', and f_2, may be found graphically by tructing Mohr's circle of stresses, keeping in mind the assumption that tion of the first principal stress is normal to the under side of the

The stresses may also be expressed analytically by the following equations:

$$q = (f_1 - p) \tan \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (110)$$

$$2\, q_{\max.} = (f_1 - p)\,(1 + \tan^2\alpha) \ldots\ldots\ldots\ldots\ldots\ldots (111)$$

$$f_2 = f_1 - 2\, q_{\max.} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (112)$$

and

$$p' = f_1 - q \tan \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (113)$$

Much time can be saved by making these preliminary investigations, especially at the beginning of the design when the general proportions of the structure are to be determined and the economies compared with other designs. However, if the final stresses in the entire structure are to be investigated and the true direction of principal stress trajectories located for the purpose of placing inclined buttress joints, the exact method of analysis similar to the author's method must be used. It is unnecessary to complicate the work shown in Mr. Holmes' paper by introducing various other factors into the stress equations, making them very difficult to use in a practical design problem. Taking into account all the variations in section a few simple equilibrium equations may be constructed, conveniently expressing the stress conditions in the buttress. Briefly stated, consider the buttress to be divided into a number of elementary prisms by a series of horizontal and vertical planes. Each prism is then considered in equilibrium by the forces acting upon it and equations may be written expressing the unknown stress in terms of the vertical normal stress, directly determined by Equation (108). If the vertical normal, horizontal normal, and shearing stresses are thus determined, both principal stresses and their direction may be found.

Assume, for example, that it is desired to determine the intensities and directions of the principal stresses on the plane, *B-B*, Fig. 20. An enlarged section of this diagram is shown in Fig. 21. Horizontal planes, *C-C* and *D-D*, are then plotted equal distances above and below the plane, *B-B*, and the values of *p* are then computed for each plane by Equation (108). Next, construct any vertical plane intersecting the three horizontal planes at Points *E*, *F*, and *G*. The elementary prism, *E-C-B-F*, can now be assumed as separated from the buttress as shown in Fig. 21 and to be held in equilibrium by the external forces acting upon it. These forces are the vertical normal

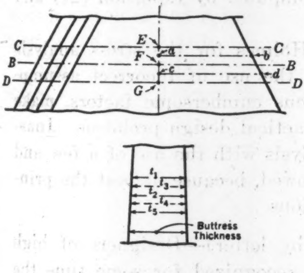


FIG. 20.

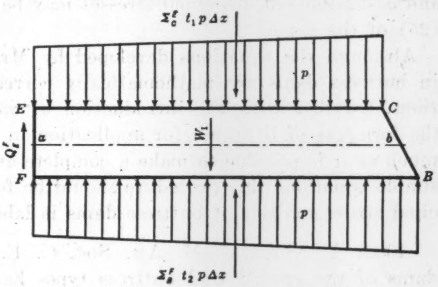


FIG. 21.

pressure on the plane, *E-C*, $\sum_C^E t_1 \, p \, \Delta x$; the weight of the masonry in the prism, w_1, and the total vertical normal pressure on the plane, *F-B*, $\sum_B^F t_2 \, p \, \Delta x$. The total shear, Q_{E-F}, across the plane, *E-F*, may now be determined by writing the equation for vertical equilibrium for the prism, *E-C-B-F*, thus,

$$Q_{E-F} = \sum_C^E t_1 \, p \, \Delta x + w_1 - \sum_B^F t_2 \, p \, \Delta x \quad \text{...........(114)}$$

The average intensity of shear, q_a, on the section, *E-F*, is equal to Q_{E-F}, divided by the sectional area between Points *E* and *F*. By a similar procedure the average shearing stress, q_c, between Points *F* and *G*, may be determined. The shearing stress at Point *F* on Plane *B-B* is equal to $\frac{q_a + q_c}{2}$.

Next, assume the prism, *a-b-c-d*, to be separated from the buttress and consider it in horizontal equilibrium under the shearing forces acting upon it. As the horizontal and vertical shearing stress intensities are equal, the total shear acting on the planes, *a-b* and *c-d*, may be determined by the foregoing procedure. The difference between these total shearing forces will equal a normal force, N_y, distributed over an area, *a-c*. These forces are expressed in the following equation:

$$N_y = \sum_a^b t_3 \, q \, \Delta x - \sum_c^d t_4 \, q \, \Delta x \quad \text{...............(115)}$$

The average intensity of horizontal normal pressure, p', equals N_y divided by the sectional area between the planes, *a* and *c*. With the stresses, p, p', and q, determined, principal stresses may be computed by Equation (24) and (25) of the paper.

Although the equations developed by Mr. Holmes for the stress analysis in buttress dams are mathematically correct, the use of incorrect assumptions, together with the introduction of various cumbersome factors, make the formulas of little use for application in practical design problems. Inasmuch as it is possible to make a complete analysis with the use of a few and simple equations this procedure should be followed, because at best the principal stress analysis of buttress dams is laborious.

FRED A. NOETZLI,[12] M. AM. SOC. C. E. (by letter)—Designers of high dams of the gravity and buttress types have recognized for some time the importance of the determination of the principal stresses in these structures. Mr. Holmes' paper is a valuable contribution to the literature on this subject.

The author's equations are based on the assumption of a linear distribution of the vertical normal stresses upon horizontal planes. Although it is known that the so-called trapezoidal rule is theoretically not quite correct, especially close to the foundation of a dam, this assumption nevertheless may be justified in most cases for the following reasons: First, the difference between the vertical stresses, computed in the ordinary manner by the trapezoidal rule, and the stresses calculated on the basis of non-linear distribution, is relatively small, except perhaps for very thick dams; second, shrinkage and temperature stresses in the interior of a dam may amount to several hundred pounds per square inch in tension (at least in some dams these stresses have produced open cracks, so that theoretical refinements regarding distribution of vertical stresses would appear to be of questionable value); and, third, the desirability, if not the necessity, of relieving internal tension stresses by vertical (as in the Hoover Dam), or inclined joints (as in the Coolidge Dam), offers the possibility of an artificial re-adjustment of stresses whereby the validity of the trapezoidal law can be approached relatively closely for the individual units or columns and thereby also for an entire section of a dam. A clear demonstration of a simple method to accomplish this result was described by C. V. Davis, M. Am. Soc. C. E.[13]

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[12] Cons. Hydr. Engr., Los Angeles, Calif. Mr. Noetzli died May 24, 1933.

Mr. Holmes emphasizes the desirability of providing contraction joints in buttresses and in the vertical slices of gravity dams. The vertical joints in the Hoover Dam and the inclined joints in the Coolidge Dam are striking examples. These joints are not used merely for the sake of introducing a novelty in dam construction, but are dictated by the necessity of providing for shrinkage in large masses of concrete. Even if no open cracks have developed in a dam, high tension stresses may exist nevertheless. The writer knows of one dam in which vibrations due to blasting operations in the adjoining spillway increased the shrinkage stresses beyond the tensile strength of the concrete, and thereby caused an appreciable extension of cracks which previously had remained stationary for a number of years.

As a remedy against irregular cracks the author recommends contraction joints parallel to the direction of the principal stresses. Such joints divide a dam into inclined columns each of which is supporting its proper proportion of the water load.

The use of inclined columns or struts for supporting the deck of a dam is quite old. The wooden dam shown in Fig. 22 may serve as an illustration.

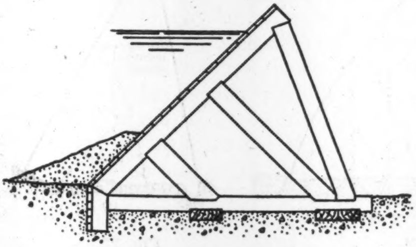

FIG. 22.—TIMBER DAM.

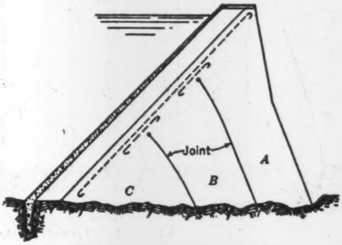

FIG. 23.—BIG DALTON DAM.

Fig. 23 is a typical cross-section of the Big Dalton (multiple-arch) Dam, in California, in which the "inclined columns" are parts of the buttress walls separated by inclined joints.

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[13] "Economies in Dams Designed for Increase in Height," *Engineering News-Record*, February 25, 1932, p. 292.

In the Hoover Dam the joints are vertical (see Fig. 24). The columns are interlocked by key and groove connections. When the concrete has cooled the joints will be grouted in an effort to make the entire dam act again monolithically. A slight modification of joints is shown by the dotted lines in Fig. 24. This arrangement is believed to offer certain advantages over the actual design in that at least the upper portions of the joints extend in planes of little or no shear (practically normal to the down-stream face). In Fig. 24, shear keys are provided in all joints.

Fig. 25 shows another arrangement of joints suitable for dams of lesser height. Rounded openings at the upper end (Fig. 25(*a*)), or at the intersection of the joints (Fig. 25(*b*)), will minimize the danger of an extension of the joints. This expedient has been used with surprising success for stopping cracks in steel castings, etc. In dams, these openings are preferably made to coincide with inspection tunnels.

The author raises the question of how to design a dam which is divided by joints into several columns. In the case of dams illustrated by Fig. 22 the proper method of designing the inclined struts is at once evident. There is no reason why the inclined "columns" *A*, *B*, and *C*, of the dam shown in Fig. 23 may not be designed in a similar way. If the "open type of construction" is safe, which it undoubtedly is, then the jointed type of buttress illustrated in Fig. 23 should be equally safe, or even more so, by reason of the shear keys and reinforcement tying the columns together. If the monolithic character of the structure is more or less re-established by grouting the joints

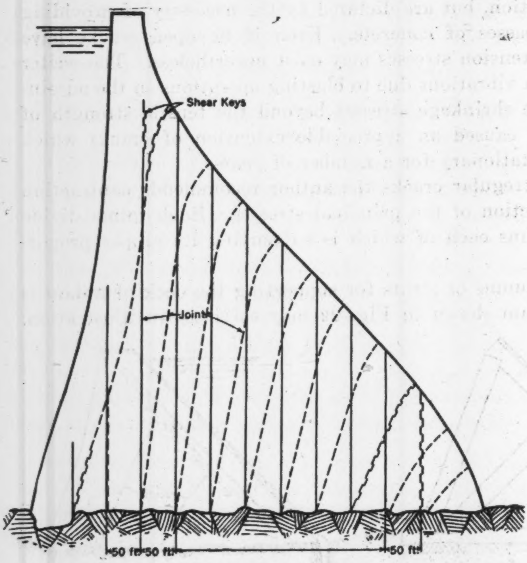

FIG. 24.—HOOVER DAM.

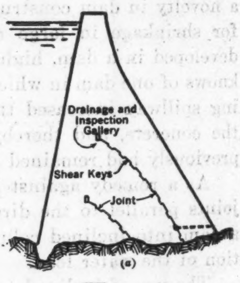
Drainage and Inspection Gallery
Shear Keys
Joint
(a)
Drainage and Inspection Gallery
Shear Keys
Joint
(b)

FIG. 25.

the original assumption of monolithic action under load may be quite close to the actual conditions of the dam.

The designers of the Hoover Dam are satisfied that the vertical joints, properly interlocked and grouted, will not materially affect the size and distribution of the stresses in the dam as computed for the structure without joints. Inclined joints coinciding with planes of zero shear, keyed and grouted for additional safety, should affect still less the size and distribution of the stresses in the dam as a whole. The writer, therefore, is of the opinion that it will be quite proper to design a buttressed or a gravity dam as if it were monolithic, and determine the shears and principal stresses by any of the methods available, for instance, that presented so ably by the author. Joints should then be located at such intervals, say, 50 to 70 ft., as to avoid subsequent shrinkage cracks. The joints are preferably interlocked so that shear forces which may develop in the plane of the joints due to partial or full load are transmitted safely from one column to the other. As a final step of the analysis the stability and safety of the "inclined columns" should be investigated by means of force and string polygons.

EUGENE KALMAN,[14] M. AM. SOC. C. E. (by letter).—This paper is stimulating from several points of view. Nevertheless, it is doubtful whether the results convey much that is really new or useful to that part of the profession interested in dam engineering.

With commendable industry, the author derives a series of formulas for stresses in the interior of straight and curved gravity dams, and for tapering buttresses of multiple-arch or slab dams; but straight dams of triangular and rectangular profile—as treated in the author's examples—were analyzed successfully more than thirty years ago by M. Lévy,[15] who gave explicit formulas for the stresses at any point of the profile. These formulas, by the way, are more correct than those presented by the author. Furthermore, tapering buttresses were analyzed successfully by Carl Boegh in conformity with the theory of elasticity.[16]

As for the author's formulas for arched gravity dams, the writer thinks that their treatment is a logical contradiction contrary to common sense. Indeed, one may or may not join the group of engineers professing that there is no arch action in an arched gravity dam owing to the fact that vertical radial cracks reduce the dam to a definite number of isolated blocks. If one does join this group, he simply will not design such a dam or derive formulas pertaining to it, and, *vice versa*, if one does not believe in an arched gravity dam cracking up into isolated blocks, he must believe in some arch action which will securely compensate for the tapering of ideal blocks delimited by vertical radial sections. The effect is to eliminate the necessity of taking into consideration the very moderate differences in the behavior of tapering blocks in arched dams and normal rectangular blocks in straight gravity dams.

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[14] Visiting Prof., Civ. Eng., California Inst of Technology, Pasadena, Calif.

[15] "Sur l'equilibre élastique d'un barrage en maçonnerie a section triangulaire," also, "Sur la légitimité de la regle dite du trapère dans l'etude de la resistance des barrages en maçonnerie," Comptes rendus de l'Academie des Sciences, Paris, 1838.

[16] "Beitrag zur Berechnung der Spannungsverteilung in Stüttpfeilen von Stauwehren aufgelöster Bauweise," *Beton und Eisen,* Berlin, 1927.

If some arch action were not expected, why should a gravity dam be arched, a layout involving increased cost? Conversely, if there is some arch action, why not take advantage of it and omit considering the tedious features of tapering blocks?

The author concludes that his "numerical examples indicate the fallacy of the middle-third theory as a safe criterion for dam design." As a matter of fact, experts on dam design dropped the middle-third criterion almost as far back as the epoch of Rankine; and in recent times no one versed in the subject would ever dream of claiming that the middle-third theory gives a safe criterion for dam design.

The writer views with doubt and surprise the industry displayed in this paper and elsewhere with a view to determining stresses in all points of the interior of a dam section. There is legitimate doubt that maximum stresses may be found at points other than on the faces of the dam. Since the publications dealing with the stresses in the interior of gravity dams fill volumes, the writer wishes to show, at least in the case of a triangular profile and with reservoir filled, that there can be no maximum stress in the interior.

First, consider the following formulas derived by Lévy:

$$\sigma_x = -(\gamma - \gamma' \text{cotan}^2 \alpha)\, x + (\gamma - 2\gamma' \text{cotan}^2 \alpha)\, y \ldots\ldots (116a)$$

$$\sigma_y = -\gamma' x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (116b)$$

and,

$$\tau_{xy} = -\gamma' \text{cotan}^2 \alpha\, y \ldots\ldots\ldots\ldots\ldots\ldots (116c)$$

σ_x, σ_y, and τ_{xy}, being the vertical and horizontal normal stress and the shearing stress, respectively; x and y, the vertical and horizontal co-ordinates; cotan α, the slope of the down-stream face—the up-stream face being vertical; and γ and γ', the specific weights of masonry and water, respectively. These formulas give the stresses at any point of the triangular profile.

The normal and shearing stresses related to an arbitrary direction having an angle, δ, with the horizontal direction, may be obtained from the stresses, σ_x, σ_y, and τ_{xy}, by the aid of the following well-known formulas:

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \sin 2\delta + \tau_{xy} \cos 2\delta \ldots\ldots (117a)$$

and,

$$\tau = -\frac{\sigma_x - \sigma_y}{2} \cos 2\delta + \tau_{xy} \sin 2\delta \ldots\ldots\ldots\ldots (117b)$$

It is known that the principal stresses correspond to that direction for which $\tau = 0$, and that one of these principal stresses will be the maximum stress at the point under consideration.

Suppose, now, that there is a maximum stress at a certain point, P, of the interior, so that its intensity is greater than that of the principal stresses at any other point in that region. Suppose, also, that this particular maximum stress is the principal stress at that particular point, in relation to a direction defined by the angle, θ.

Draw a straight horizontal line through the point, P, and at any point of this line, the vertical and horizontal normal and shearing stresses will be given by Equations (116), in which, x (the distance of the assumed straight line from the top of the dam), will be a constant, while y will be an independent variable. All these stresses, thus, will be linear functions of y only.

Next, consider the intensities of the normal stresses in the direction, θ, at the different points of the assumed horizontal straight line. They will be expressed by Equation (117a), if the angle, δ, is substituted by θ and σ_x, σ_y, and τ are replaced by their values from Equations (116). Since the resulting formula is a linear function of y, it can not yield a maximum value in the interior of the section. If one considers the fact, that at any of the near-by points the stress along the direction, θ, is not a principal stress, while admittedly it is a principal stress at the point, P, under consideration, one sees that the maximum stress at the latter point is surpassed in intensity by other stresses in near-by points, which are not even the principal stresses at those points.

It thus follows that there can be no maximum normal stresses in the interior of the dam; and the same statement holds for the maximum shearing stresses, as can be shown by repeating the foregoing reasoning. If the profile is not triangular, there will be some stress distribution other than linear along straight line sections; but the maximum stress will probably appear at some point on the faces, and probably at some of the lowest points.

In certain cases, structures must be analyzed at every point. A little hole in the center of a sphere subjected to hydraulic pressure causes in that region, a stress three time as great as would be the case if there were no hole. In ordinary beams in reinforced concrete, shearing stress conditions must be studied in the interior to provide for a sufficient number of stirrups, bent-up bars, etc. Dams, however, are supposed to be continuous and homogeneous structures, and, therefore, the writer never believed in searching their interior for extreme stresses.

For the same reason, the generous display of trajectories in modern engineering and especially in dam literature, appears as slightly over-emphasized, when related to their usefulness. Years ago, when he was a young engineer the writer derived much thrill from trajectories of principal stresses; but as yet he has failed to derive from them any practical profit of consequence.

Two statements in the paper should be discussed to some extent, because they are characteristic for a certain peculiar trend of thought among engineers at large. The author claims that a triangular profile with vertical up-stream face is safer than a rectangular profile having the same up-stream face and base. He also states that, in the latter, tension will develop, while there is no tension in the triangular profile.

These statements, of course, are not true. They are the consequence of erroneous assumptions, and there would be little to discuss about them if it were not for the question of why engineers accept similar results. If a farmer, alone, or with the help of a bricklayer, had to build a small dam on his farm, and he had to choose between the triangular and rectangular profile, with identical base and up-stream face, he would consider the rectangular profile as safer. Why is it, now, that the engineer accepts the triangular profile as the safer, while the farmer and the bricklayer do not accept it? The reason must be that during his college career the engineer develops a kind of inferiority complex. The bricklayer is a kind of engineer relying on the authority of his common sense. The engineer is a kind of bricklayer, or builder, who, relying on the authority of formulas and mathematical developments and examining professors, relinquishes part of his common sense.

If there is no tension in a triangular profile, and if another triangular mass is superimposed upon the first so as to complete the rectangle, the bricklayer would not believe that such addition would create tension at the base of the dam. On the contrary, he would think that such addition of mass would eliminate or diminish tension if tension existed in the triangular profile; but the engineer believes the incredible.

The author states that in some isolated internal region of a profile tension may develop, while everywhere around that spot the material is compressed. The writer is not able to visualize a mass of homogeneous and continuous concrete of reasonable shape, in which all circumferential portions are compressed, while some central portion is subjected to tension.

Some time ago, Dr. Nicolosi, of the Institute of Technology in Rome, Italy, published a paper showing that in some cases additional haunches between beams and slabs may cause a decrease of strength. Every one familiar with reinforced concrete structures is fully aware of the substantial increase of strength developed by such haunches; and unprejudiced minds would agree that any addition to the section of a beam involves increasing its strength. The bricklayer would agree, and the farmer hewing primitive beams out of logs would agree; but an engineer, or an engineering professor, is often a slave of his formulas, a victim of his acquired inferiority complex.

In a paper[17] discussing Dr. Nicolosi's result, the writer showed that the perfectly correct mathematical operations and deductions were based on a false assumption; that he did not believe in the result even if the assumptions could not be proved to be false; and that the basis of the science of construction would have to be revised if they yielded such results contrary to common sense.

Numerous examples and cases of a similar type could be added. The fact is that many college graduates, in arriving at some paradoxical result, simply limit themselves to a check of the correctness of the mathematical operations, and if the latter are found to be correct, the paradoxical result is admitted as an indisputable fact. Any doubt by outsiders and critics provokes the solemn statement accompanied with an Olympian frown: "You cannot doubt mathematics, can you?"

In the writer's opinion, there is an uneven apportionment in the university curricula between too much mathematics and too little or no effort to apply it to engineering problems, with an emphasis upon the unavoidable presence of physical assumptions and consequent necessity of submitting the final results to the judgment of common sense, or to the verdict of experimental tests.

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[17] "Su un Paradosso Statico," *Il Cemento Armato*, Milano, May, 1926.

To return to the author's results under consideration, they are based on the assumption that the vertical normal stress in a horizontal section of a rectangular profile of dam, with reservoir filled, is distributed according to the law of the trapezoid. This is not the case, as shown by Lévy. Consequently, results based on the author's assumption should be faced from the beginning with a certain critical attitude.

The aforementioned result reported by Dr. Nicolosi is based equally on the usual assumption of the validity of the law of the trapezoid. Prompted by a legitimate curiosity as to the validity of that law, the writer undertook some time ago a simple investigation of that problem.[18] He found that the elastic stresses consistent with the postulate of the linear distribution of normal vertical stresses along all horizontal sections of a given profile are algebraic functions of the fourth degree of the co-ordinates, x and y. Hence, the number of such possible elastic conditions is very limited. He showed, particularly, that in no dam profile delimited by straight up-stream and down-stream faces can the stress distribution be linear, except in the triangular profile with the water level at the apex.

In his "Introduction" the author states that "the ordinary method of determining the vertical normal stress at the heel and toe of high dams is not sufficient for the complete analysis of the stresses." This statement is somewhat vague. If Mr. Holmes wishes to convey the idea that the vertical normal stress at the heel and toe (or any other point of the profile), of high or low dams, differs from the principal stress at those points, every one will agree with him; but not if he means that it is necessary to review the stresses everywhere in the interior of the profile, as well as at the heel and toe. If Mr. Holmes has at hand some observations to support such a statement with more authority than his numerical result based on the validity of the law of the trapezoidal, such data would be valuable. The author is to be commended for the clear exposition of his results.

I. M. Nelidov,[19] Assoc. M. Am. Soc. C. E. (by letter).—A development of the method of computing stresses in hydraulic structures is presented in this paper. It consists essentially of evaluating certain coefficients of a polynomial stress function and of solving for the value of this function with various values of x. The author deserves special credit for the application of this method to dams curved in plan and to tapering buttresses. His method is based on the linear distribution of one of the three unknown stresses.

The problem of the internal stress distribution arises mostly in connection with the following structural members, of which the cross-sections are large in relation to the length: (1) Beams fixed at one end, or cantilevers; (2) beams fixed at both ends; and (3) beams restrained at both ends, either straight or curved. The first class will be treated in the present discussion.

The writer wishes to complete the author's study by outlining briefly the present condition in the art of stress determination in gravity dams and also by arriving at equations developed by the author from the general equations of equilibrium.

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[18] "On Two-Dimensional Linear Elastic States," *Philosophical Magazine*, London, Vol. XII, August, 1931.

[19] Senior Engr. of Hydr. Structure Design, State Dept. of Public Works, Sacramento, Calif.

Equations for stress distribution in cantilevers, triangular in shape and loaded with a uniformly increasing pressure beginning at the crest, were derived by M. Levy[20] and by P. Fillunger.[21] For the uniformly increasing load indicated, the stresses are linear as shown also in the author's Examples 1 and 3. The stress distribution in the cantilever of triangular and trapezoidal shapes for the uniform and uniformly varying pressures was determined by Professor B. Galerkin.[22] The formulas for the trapezoidal profile are very complicated, so that for practical purposes the linear stress distribution may be used. When the sides of the cantilever are not parallel, or when its profile is curvilinear, the linear stress distribution for one of the three unknown stresses is the only one that can be used. In all the foregoing cases the influence of the foundation on the stress distribution is disregarded.

The general equations of equilibrium of an element within a homogeneous body of a thickness, t, in rectangular co-ordinates are:

$$\frac{\partial s_y}{\partial y} + \frac{\partial v}{\partial x} - w_c\, t = 0 \qquad \text{(118)}$$

and,

$$\frac{\partial s_x}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(119)}$$

in which, w_c is the unit weight of the element acting parallel with the vertical axis, Y; $t = f\,(x, y)$; s_y is the vertical normal stress; s_x, the horizontal normal stress; and, v is the shearing stress—all for the total width of the body.

Normally, a third equation of compatibility should be added in order to solve for the three unknown stresses. With the assumption of linear stress distribution for one of the unknown stresses the compatibility equation is not satisfied and the resulting solution for stresses becomes only approximate.

Integrating Equations (118) and (119) and neglecting the stresses along the plane, $X\,O\,Y$:

$$v = qt = -\frac{\partial}{\partial y}\int s_y\, dx + \int w_c\, t\, dx + f_1(y) \qquad \text{(120)}$$

and,

$$s_x = p't = \frac{\partial^2}{\partial y^2}\iint s_y\, dx - \frac{\partial}{\partial y}\left[\iint w_c\, t\, dx + \int f_1(y)\, dx\right] + f_2(y) \qquad \text{(121)}$$

The values of the functions, $f_1(y)$ and $f_2(y)$, are obtained from boundary conditions. If the stress, $s_y = pt$, is a comparatively simple function of x

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[20] Comptes Rendus de l'Academie des Sciences, Vol. 127, p. 10, 1898.

[21] *Zeitschrift für Mathematik und Physik,* 59–60 Band, 1910–12.

[22] "On the Problem of Stresses in Dams and Retaining Walls with Trapezoidal Profile," by Prof. B. Galerkin, reprint from *Proceedings,* Leningrad Inst. of Ways and Communications, 1929.

and y, Equations (118) and (119) can be integrated directly; if not, the approximate integration must be used. For the linear stress distribution for s_y, Equations (120) and (121) become:

$$v = -\frac{\partial}{\partial y}\int\left[\frac{P}{a} - \frac{M}{I}\,(i - x)\right] t\,dx + \int w_c t\,dx + f_1\,(y) \;\ldots\ldots (122)$$

and,

$$s_x = \frac{\partial^2}{\partial y^2}\iint\left[\frac{P}{a} - \frac{M}{I}\,(i - x)\right] t\,dx$$
$$- \frac{\partial}{\partial y}\left[\iint w_c t\,dx + \int f_1\,(y)\,dx\right] + f_2\,(y) \ldots\ldots\ldots\ldots (123)$$

In these equations, i is the distance from the origin of co-ordinates to the center of gravity of the base section.

For the approximate integration, p stresses should first be computed on three consecutive planes. Then Equations (122) and (123) become:

$$v = \frac{1}{2\,\Delta y}\Biggl[\biggl(\frac{P}{a}\int t\,dx\biggr)_u - \biggl(\frac{P}{a}\int t\,dx\biggr)_d + \biggl(\frac{M}{I}\int (i - x)\,t\,dx)_d$$
$$- \biggl(\frac{M}{I}\int (i - x)\,t\,dx\biggr)_u + \frac{1}{2}\Biggl[\biggl(\int w_c t\,dx\biggr)_1 + \biggl(\int w_c t\,dx\biggr)_2\Biggr]$$
$$+ \frac{V_1 + V_2}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (124)$$

and,

$$s_x = \frac{1}{\Delta y^2}\Biggl[\biggl(\frac{P}{a}\iint t\,dx\biggr)_u - 2\biggl(\frac{P}{a}\iint t\,dx\biggr)_o + \biggl(\frac{P}{a}\iint t\,dx\biggr)_d$$
$$- \biggl(\frac{M}{I}\iint t\,(i - x)\,dx\biggr)_u + 2\biggl(\frac{M}{I}\iint (i - x)\,t\,dx\biggr)_o$$
$$- \biggl(\frac{M}{I}\iint (i - x)\,t\,dx\biggr)_d\Biggr]$$
$$+ \frac{1}{\Delta y}\Biggl[\biggl(\iint w_c t\,dx\biggr)_1 - \biggl(\iint w_c t\,dx\biggr)_2 + \frac{V_1 + V_2}{2}\Biggr] + F \ldots\ldots (125)$$

After substitution of all the numerical data and consequent integration Equations (124) and (125) become polynoms with numerical coefficients and with x as an independent variable. These polynoms are exactly the same as those derived by the author.

The first of the following two examples demonstrates the case in which the direct integration is possible. The second example is made with the aid of the approximate integration.

Example 1.—Using the data in the author's Example 3, let the ratio of buttress spacing to buttress thickness be $J_1 = 12$. The origin of co-ordinates is at the crest, the X-axis is directed down stream, and the X-axis is vertical or downward. Then (in pounds per square inch):

$$\frac{P}{a} = \left(1 + \frac{J_1\,n}{k\,(n+m)}\right)\frac{w_c\,y}{2} \times \frac{1}{144} = J_2\,y = 3.125\,y$$

$$\frac{M}{I} = \frac{\frac{1}{k}\,J_1\,(2 - 3\,m\,n - n^2) + (n^2 - m^2)}{(n+m)^3} \times \frac{W_c}{144} = J_3 = 6.252$$

$$i = \frac{m-n}{2}\,y = J_4\,y = -\,0.5\,y$$

$$c = \frac{m+n}{2}\,y = J_5\,y = 0.5\,y$$

$$q = -\frac{\partial}{\partial\,y}\left[(J_2 - J_3\,J_4)\,y\,x + J_3\,\frac{x^2}{2}\right] + w_c\,\frac{x}{144} + f_1\,(y) = -\,5.21\,x$$

and,

$$p' = -\frac{\partial}{\partial\,y}\int q\,dx + f_2\,(y) = 0$$

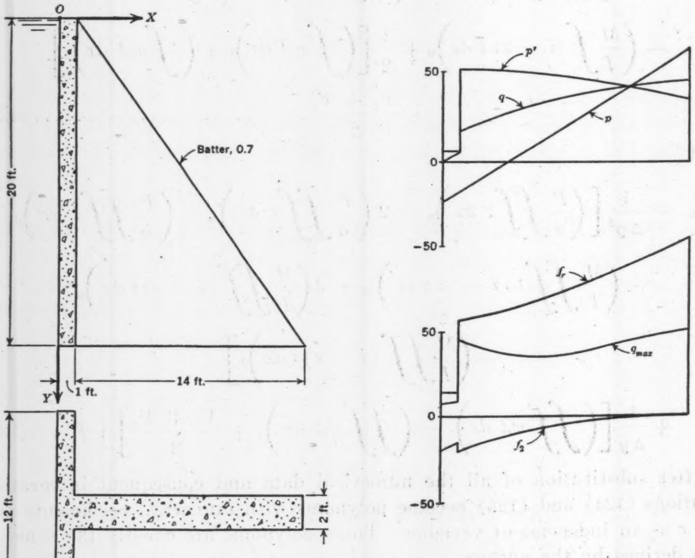

FIG. 26.

Example 2.—In a slab and buttress dam (Fig. 26) let the vertical face be 1 ft. thick; the down-stream batter, 0.7; the water surface at the crest; the height, 20 ft.; the spacing of the buttresses, 12 ft.; the thickness of the buttresses, 2 ft.; the unit weight of concrete, $w_c = 144$; and the unit weight of water, $w_w = 62.5$.

By ordinary calculations the stresses (in pounds per square inch) are: At the up-stream face, $p = -22.1$, $q = 0$, and $p' = 8.7$; and at the down-stream face, $p = 69.5$, $q = 48.6$, and $p' = 34.1$. Using Equations (124) and (125), the following expressions for stresses are obtained:

For $x<1$ ft.: $q = 3.28\,x - 0.0738\,x^2$, and $p' = 8.71 - 0.0831\,x^2 - 0.0017\,x^3$.

For $x>1$ ft.: $q = 16.05 + 3.328\,x - 0.0738\,x^2$, and $p' = 51.92 - 0.0831\,x^2 + 0.0017\,x^3$.

Table 3 gives the values of the stresses and the computed principal stresses.

TABLE 3.—PRINCIPAL STRESSES, EXAMPLE 2.

x	p	q	p'	f_1	f_2	$q_{max.}$
0	—22.1	0.0	8.7	8.7	—22.1	15.4
1	—16.0	3.2	8.6	9.0	—16.4	12.7
1	—16.0	19.3	51.8	56.9	—21.1	44.9
5	8.5	30.6	50.1	66.3	— 7.7	37.0
10	39.1	40.9	45.3	83.3	1.2	41.0
15	69.5	48.7	35.0	103.9	0.0	52.0

It follows from the foregoing computations that the point at which the slab and the buttress join is dangerous from the stress viewpoint. The stresses at the "neck" change to high values rapidly. This is especially true of the case with the inclined face. Therefore, either reinforced haunches should be provided or the sliding support for the face must be used. This also shows the importance of the investigation for the internal stresses with the structures having sudden changes in the area of the base.

H. E. VON BERGEN,[23] JUN. AM. SOC. C. E. (by letter).—The author deserves much credit for his industry in deriving formulas for determining the principal stresses in gravity and buttressed dams. These formulas are rather tedious to develop as well as to apply and, therefore, it is interesting to note that, for a strictly triangular profile with water surface at the apex, the author also has shown (assuming a linear distribution of vertical normal stress) that the horizontal normal stress as well as the horizontal and vertical shear are distributed linearly. For this case, then, which occurs frequently, it is a simple matter, knowing the stresses at the up-stream and down-stream toes, to find the direction and magnitude of the principal stresses in the interior of a dam section or buttress by the application of Mohr's circle.

For the usual buttressed dam with a considerable inclination of the up-stream face, tension is found to exist at or near that face and is approximately parallel to it even if the vertical normal stress is of considerable magnitude in compression. This tension and the stress produced by the difference in shrinkage in the deck and in the supporting buttress, due to water-soaking, setting variation in temperatures, etc., are probably the two main causes for the typical "diagonal" cracks so frequently found in structures of this type.

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[23] Sacramento, Calif.

It may be shown mathematically that, other things being equal, the tension due to the external loads on a buttressed dam—that is, the second principal stress—is a function of the slope of the up-stream face. Consider a buttressed dam and for simplicity assume that it has a weightless deck. The second principal stress at the up-stream face is:

$$f_2 = p_u (1 + n^2) - f_1 n^2 \qquad \text{(126)}$$

in which, p_u is the vertical normal stress at the up-stream face; f_1, the first principal stress at this point; and n, the slope of the up-stream face, or the cotangent of the angle the up-stream face makes with the horizontal.

By substituting in the equation:

$$p_u = \frac{\Sigma V}{A} - \frac{\Sigma M}{S} \qquad \text{(127)}$$

the physical dimensions of the buttress and the water loads, and by placing this value of p_u in Equation (126) and setting its first derivation with respect to n equal to zero, the following equation will result:

$$n^3 - A n^2 + B n - C = 0 \qquad \text{(128)}$$

in which, $A = \frac{6SL - 18 yztLK}{-4hS}$; $B = 1 + \frac{SL^2 - ytkL^2(1 + 3z)}{2h^2S}$; and $C = \frac{A}{3}$. Furthermore, k = ratio of weights of a unit of masonry to a unit of water; h = height of buttress and depth of water; S = span of buttress; L = length of base of buttress; t = thickness of base of buttress; y = coefficient which, when multiplied by $h L t$, gives the volume of the buttress; and z = coefficient, which when multiplied by h, gives the vertical distance from the center of gravity of the buttress to its base.

For a numerical example consider a buttress in which the height and length of base equal 100 ft., the thickness is uniformly 3 ft., and for which $y = \frac{1}{2}$, $z = \frac{1}{3}$, and the span of the buttresses is 30 ft. For the water surface at the crest the solution of Equation (128) for this example will give a value of n equal to 0.5 which is the slope, as far as the water load and weight of buttress are concerned, that will produce a minimum tension parallel to the up-stream face. For the case in which the buttress thickness decreases to zero at the apex, n is equal to 0.48. (Fredrik Vogt, Assoc. M. Am. Soc. C. E., states[24] that the best slope is probably 65 to 70°; that is, n = 0.37 to 0.47.) These slopes will be altered slightly upon inclusion of the deck weight or alteration of physical dimensions. In the writer's opinion the foregoing consideration points more directly toward the "design of buttresses eliminating tension," than the argument in the author's paper.

To determine tensile stress in the concrete caused by the water-soaking of the deck and the shrinkage of the adjacent buttress due to dissipation of setting heat is a matter of conjecture. (This consideration applies only to the case in which the deck is fixed to the up-stream face of the buttress.) As an example, assume a 40° drop in temperature in the buttress after setting, with the accompanying shrinkage and a swelling of the deck due to water-soaking of 0.005%;[25] and assuming a modulus of elasticity at 2 000 000 and one-half the deformation as compression in the arch, the tensile stress in the buttress parallel to the up-stream face is 270 lb. per sq. in. In summer, when the temperature of the buttress is considerably higher, the stress will be less, and *vice versa.*

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[24] In "Economical Design of Buttresses for High Dams and Cellular Gravity Dams."

That temperature greatly affects the deformation of a buttress is shown by data obtained on variations in width of the diagonal cracks in the buttresses of Lake Hodges Dam, near San Diego, Calif. In one instance, neglecting the gradual increase in the width of the crack, it was found that a drop in temperature of 20° Fahr. was accompanied by an increase in width of the crack, in a direction normal to it, of 0.058 in., while the reservoir stage was fairly constant. In other instances the data showed no obvious variation in width of crack with the fluctuation of the water surface. There did seem, however, to be some slight deformation parallel to the crack with this fluctuation. This is probably due to the fact that a drop in temperature produces contraction along the three major axes, whereas stress induced by loading does not necessarily cause such deformation.

From the foregoing discussion, it seems obvious that unsightly cracks can not be eliminated by simply eliminating tension due to external loading. Either there must be sufficient reinforcing steel to force the buttress to act as a structural monolith, or contraction joints must be provided similar to those in Figs. 24 and 25.

The writer believes another matter worthy of discussion is the quantity and manner of placing steel reinforcement in a buttress. Some buttressed dams have been built with vertical and horizontal steel, some with inclined and horizontal steel, and some with no steel at all. After a few years of service, the buttress usually shows into which class it falls.

Textbooks on the subject of reinforced concrete usually indicate that, in a homogeneous beam, ideal reinforcement follows the trajectories of the second principal stress. This same reasoning might well hold good in a buttress.

The percentage of steel necessary in this case, if the direction of the steel and tension coincide, may be computed simply by dividing the computed tension by the allowable stress in steel, or $p = \frac{f_2}{f_s}$; or, if the steel lies in two different directions (see Fig. 27), the stress in the horizontal steel is:

$$f_h = \frac{f_2}{p_h \cos\alpha\,[\cos\alpha + \sin\alpha \cot(\beta - \alpha)]} \quad \ldots\ldots\ldots\ldots\ (129)$$

and the stress in the inclined steel is:

$$f_i = \frac{f_2}{p_i \cos(\beta - \alpha)\,[\cos(\beta - \alpha) + \sin(\beta - \alpha)\cot\alpha]} \quad \ldots\ldots\ldots (130)$$

It will be seen that if $\beta = 90°$ and $\alpha = 45°$, the steel is just one-half as effective as when it coincides with the direction of the tension and $\beta = \alpha = 0$.

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[25] Rept., U. S. Bureau of Reclamation, Vol. 1, Section 1, Compilation of Data on Concrete Problems.

These equations are based on the assumption that the concrete has cracked normal to f_2, and that the steel carries the tension. The percentages of the horizontal and inclined steel are represented by p_h and p_i, respectively.

An analogy may be made between web reinforcement in a concrete beam and reinforcement in a buttress; assume that the vertical stirrups carry the total shear.[26] Actually, the vertical stirrups are assumed to carry the vertical component of the diagonal tension which, numerically, is of the same magnitude at the neutral plane. In Equation (127) the $\frac{2}{3}$ is omitted since steel carries all the tension; thus, $A_s = \frac{V_s}{f_s j d}$; $V = v' b j d$; and $A_s = \frac{v' b j d s}{f_s j d}$. In terms of p, the ratio of steel area to concrete area is $A_s = p b s$; and substituting f_2, the "diagonal tension," for v', its numerical equivalent,

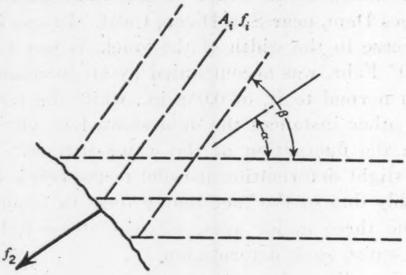

FIG. 27.

$$f_s = \frac{f_2}{p} \quad \text{.........................} \quad (131)$$

For inclined stirrups[27], or bars bent up at 45° (omitting the $\frac{2}{3}$), $A_s = \frac{0.7\ (V_s)}{f_s j d}$. In terms of p, again, $A_s = p b s \sin 45°$, and,

$$f_s = \frac{f_2}{p} \quad \text{.........................} \quad (132)$$

as in Equation (131).

It will be found that by substituting in Equation (130), $\beta = 90°$ and $\alpha = 45°$ (which corresponds to vertical stirrups), that Equation (131) will result; and by substituting in Equation (130) $\beta = 45°$ and $\alpha = 45°$ (which corresponds to bars bent up at 45°) that Equation (132) is the result. It will also be found in the latter case that the inclined steel carries the total tension, leaving no horizontal component to be carried by the horizontal steel and, therefore, Equations (129) and (130) are consistent for web reinforcement. In the former case, involving vertical steel, it will be found that the horizontal steel carries the horizontal component of the diagonal tension produced by shear, but that it is considered as bond stress in reinforced concrete beams. Thus, the analogy is complete.

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[26] "Reinforced Concrete Handbook," by Hool and Johnson, p. 285.

[27] *Loc. cit.*, Equation (4), p. 286.

Temperature steel either may be considered separate or, if convenient, may be combined with other steel to resist stresses due to the combination of temperature and external loads. The steel necessary to prevent large cracks that are likely to occur, due to shrinkage of a member restrained at both ends, ranges up to about 0.4%, depending on the maximum tensile strength of the concrete.

As shear always resolves itself into tension and compression it seems more sensible to reinforce against the tension directly instead of its components because concrete is by far the weakest in tension, whereas its pure shearing strength is from 60 to 80% of the compressive strength. In other words, with the diagonal tension adequately provided for, shear steel or shear reinforcement is unnecessary and irrelevant.

HOWARD L. COOK,[28] JUN. AM. SOC. C. E. (by letter).—The formula, $\frac{P}{a} \pm \frac{Mc}{I}$, for the normal stress over a section of a bent and compressed beam (Equation (1)) is applicable when: (1) The dimensions of the sections are small in comparison with the length of the beam; (2) these dimensions are small compared to the radius of curvature of the central line; (3) the cross-sections considered are planes perpendicular to the line of their centroids both before and after bending; and (4) Hooke's law applies to all the material affected, and this material is homogeneous and isotropic.

A masonry dam is a thick, short beam, formed so that Conditions (1), (2), and (3), are not met. Condition (4) may be allowed for purposes of discussing the action of such a beam. Obviously, the formula given does not apply to a horizontal, or any other, section of such a structure.

Equation (1) gives a linear distribution of normal stress over a section of the dam. If it does not apply, the so-called law of the trapezoid does not hold for a dam. Mr. Holmes recognizes that this is so in his statement (under "Distribution of Stresses") that, "it is well known that the vertical normal stress distribution of gravity dams is not linear, although for triangular dams of thicknesses of 50 to 60 ft., the error is small." Evidently, he thought the law held closely enough to enable a complete internal stress analysis to be based upon it. The question is: Does it hold with sufficient exactness to support an analysis of any scientific utility whatsoever?

There is only one way of getting at the answer to this question; that is, by experiments on model dams. The late William Cain, M. Am. Soc. C. E., in the paper upon which Mr. Holmes bases his work,[2] refers to a number of these experiments, all of which happen to have been made in England. Professor Cain's conclusion was that these investigations substantiated the view that the stress distribution is closely linear for horizontal sections not too close to the foundation.

In a paper by Mr. B. F. Jakobsen,[29] the linearity of the stress distribution at horizontal sections is not admitted. Indeed the attempt is made to find the true distribution. Mr. Jakobsen also refers to the experiments cited by Professor Cain, calling them the "English" experiments. It is from these experiments that Mr. Jakobsen arrives at the conclusion that substantial departures from linearity of stress distribution actually occurs.

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[28] Asst. Engr. with Robert E. Horton, M. Am. Soc. C. E., Cons. Engr., Slingerlands, N. Y.

[29] "Stresses in Gravity Dams by Principle of Least Work," *Transactions*, Am Soc. C. E. Vol. 96 (1932), p. 489.

If the hypothesis of linearity of stress distribution is substantially in error for gravity dams, it is misleading to base a complete internal stress analysis upon it. If Mr. Holmes errs in doing this, it must be said that he only follows a popular fashion among engineers.

A careful investigation of the internal strains and stresses in models of dams was made in 1907 by Messrs. Karl Pearson and A. F. C. Pollard.[30] It demonstrated more clearly than any other, the alarming discrepancy between actual normal stress distributions and the hypothetical linear distribution. A peculiarity of this investigation is the lack of notice it receives from engineers. In 1904, Professor Pearson, with Mr. L. W. Atcherley[31] made some elementary experiments on models of dams. These experiments were, and still are, widely discussed in engineering texts and papers dealing with the design of dams. The 1907 series of experiments under the supervision of Professor Pearson, while of much greater value, are seldom mentioned.

At the suggestion of the late Sir Benjamin Baker, Hon. M. Am. Soc. C. E., Messrs. Pearson and Pollard adopted a jelly as the material for their models. It consisted of gelatine, glycerine, and coloring matter, and was cast into shape in moulds. The models adopted were about 18 in. through the base and 14 in. high. The length of a section was about 4 in. The models were of shapes similar to the Vyrnwy Dam, in Wales, and the Assuan Dam, in Egypt.

Two kinds of experimentation were pursued. A first series of models were ruled with a grid of lines 2 cm. apart. Forces were applied to the models and full-sized photographs obtained under various conditions of loading. Angle changes in the grid system were then measured on the photographs by a specially designed instrument called a direct microgoniometer.

The final experiment of the investigation was made with extreme care. A model of the Assuan Dam was ruled while just floating in a bath of glycerine, thus eradicating stresses due to the weight of the dam. This model was tested directly at various loads by an optical microgoniometer developed for the work.

Normal water loads on the models were supplied by means of an elastic bag of liquid resting against the up-stream face. Exaggerated loadings were made by means of a board pressing on the up-stream face of the jelly model and loaded normally at a depth of two-thirds the total.

If the hypothesis of linear distribution of stress were true, the shearing stresses would be distributed parabolically. The experimenters simply had to compare the actual shear distribution of their models with a parabolic distribution, to determine the applicability of the linear hypothesis. As the shearing stress is proportional to the slide or change of angle in the grid system at any point, measurement of this angle change was all that was required. Details of methods and results may be had by referring to the paper[30] of Messrs. Pearson and Pollard.

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[30] "An Experimental Study of the Stresses in Masonry Dams," Drapers Company *Research Memoirs*, Tech. Series V, Cambridge Univ. Press.

[31] "On Some Disregarded Points in the Stability of Masonry Dams," by L. W. Atcherley, Assisted by Karl Pearson, Drapers Company *Research Memoirs*, Tech. Series II.

As a result of their work the experimenters arrived at conclusions which, in substance, follow:

(1) There is no approach to a parabolic distribution of shear, at least up two-thirds, and probably up the entire height, of masonry dams. Linearity of distribution of vertical stresses on horizontal planes is a wholly unjustifiable hypothesis.

(2) The present theory of the stability of masonry dams is erroneous. The line of resistance being within the middle third does not prevent the material from being in tension. The customary theoretical analysis adds an appearance of safety not warranted by experiment, for the stresses obtained thereby are probably not within 100% of those in the actual dam.

(3) Tension exists in masonry dams of the types commonly constructed, but is not necessarily a source of danger. It seems desirable to admit that masonry can and does undergo tension and to set some limit not to be exceeded by these stresses in any dam.

(4) It is possible by experimenting upon a model dam to determine stresses within 10% of those in a theoretical dam and within 30 to 40% of those in the actual structure. Model investigations should be made the basis of the design of high masonry dams.

Having determined the approximate shape of the shear curves a semi-empirical determination of the stress in the body of masonry dams was attempted. The undulating character of the curve of shear stress required that, at the very least, a quartic distribution of shear be assumed. This would make the normal stress-distribution curve a cubical one. After considerable labor on the solution of the complicated equations involved, the problem still remained without a satisfactory answer. It was found, however, that the quartic shear curve would: (1) Satisfy the internal equations of elasticity; (2) satisfy the stress conditions at the up-stream face; and (3) bring into equilibrium the resultant force and stresses on horizontal sections at top, mid-height, and base of dam. The method does not account for the true distribution of stresses over the down-stream face, however.

Compare the merits of the foregoing solution with that of the paper under discussion. The solution given by Mr. Holmes may be made under certain conditions: (*a*) To satisfy the surface stress conditions at the up-stream face; and (*b*) to satisfy the equilibrium of stress and resultant force on the base section only. The stress distribution on any horizontal section by this solution will be totally unlike that found by experiment. In comparison with the old middle-third theory, however, the analysis outlined by Mr. Holmes is an advance. The middle-third hypothesis fails to satisfy stress conditions at the up-stream or the down-stream face, or at the top of the dam. It also gives linear distributions of normal stress on horizontal sections.

Even if Mr. Holmes' analysis is preferable to the use of the old rule of the middle third, it seems far-fetched to extend it to the point of predicting completely the stresses throughout a dam. Such an elementary treatment should never pretend to the dignity of a complete analysis. In view of the departures from a linear distribution of stress shown by experiment, it is doubtful whether such an analysis is even useful. It would seem dangerous to locate joints, as suggested, along the stress paths calculated by the methods given.

Messrs. Pearson and Pollard, in 1907, recommended the experimental investigation of a model of any proposed dam. To-day, the Engineering Profession is just entering the era of model investigation.

HERMAN SCHORER,[32] ASSOC. M. AM. SOC. C. E. (by letter).—The basic equation, accepted and used by the author throughout his paper, is the ordinary column formula for eccentric loading, which, in turn, also includes the assumption of linear, vertical, normal stress distribution. In his "Conclusion," under the heading "Distribution of Stresses," Mr. Holmes states that it is well known that the vertical normal stress distribution of gravity dams is not linear, although for triangular dams of thicknesses of 50 to 60 ft., the error probably is small. When, and if, a more exact distribution of the vertical normal stress is expressed in terms of the base length, he declares it should be inserted in the formulas presented in his paper, and the shear and horizontal normal stresses will then not be distributed linearly.

These assumptions and statements are rather vague and even contradictory. The application of the ordinary column formula is scarcely adapted to the production of any new results. For the special case of a triangular dam slice of constant thickness, unlimited height, and with the water level at the crest, Levy showed[15], that the trapezoidal stress distribution conforms to the exact theory of elasticity. This result has since been confirmed by numerous other investigators.

The determination of principal stresses in tapering buttresses, based on the assumption of linear stress distribution in normal sections, was first treated[33] by Mr. B. F. Jakobsen, who also gave a full description of the application of Mohr's stress circle.

It seems rather doubtful that any further progress in the analysis of dams, of the type treated by the author, can be realized without the aid of the fundamental elastic equations. The writer has derived,[34] the general differential equation of a symmetrical buttress or slice of variable thickness, subject to gravity action and external loads, acting along the rims in the direction of the middle plane. This equation indicates that the buttress shape has a pronounced influence on the stress distribution law.

It can be shown that the assumption of a linear stress distribution will satisfy the differential equation in the case of very simple disk shapes, with correspondingly simple loading, such as the triangular gravity dam or buttress shapes of constant thickness. In these special cases, the author's results can be given in simple and general equations by means of an Airy stress function.

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[32] Chf. Engr., Thebo, Starr & Anderton, Inc., San Francisco, Calif.

[33] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 276.

[34] *Loc. cit.*, Vol. 96 (1932), p. 725.

A. Floris,[35] Esq. (by letter).—In this paper an attempt is made to derive algebraic expressions for the stress components in buttress and gravity dams by the aid of Professor Cain's analysis. In other words, the author utilizes this numerical method for the purpose of obtaining general algebraic formulas. The question now arises, whether such a method of procedure is preferable to that of Professor Cain.

It is quite common among engineers in practice to use numerical expressions when the algebraic ones become too cumbersome. In fact, in such cases, it is the only method that leads to quick and easy answers. From this point of view, therefore, it seems that the author's efforts are in vain. His formulas are lengthy and awkward, while the corresponding numerical expressions in specific cases are simple and easy to handle. For this reason the writer doubts the usefulness of this method of approach.

W. C. Huntington,[36] M. Am. Soc. C. E. (by letter)—The writer has been interested in this paper in so far as it applies to the calculation of the principal stresses in dams of gravity section. The method presented by the author is a valuable extension of that developed by the late William Cain, M. Am. Soc. C. E., for determining the principal stresses in dams of gravity section[2] and is not limited in its usefulness to profiles in which the up-stream face is vertical, as is the case with the elegant solution formulated by Mr. E. C. Hill.[37] In deriving an expression for the unit shear at any point in a gravity dam, Mr. Holmes might have simplified his problem by making use of one plane instead of two. Furthermore, by an extension of the method suggested[38] by the late W. C. Unwin, Hon. M. Am. Soc. C. E., the value of the unit horizontal stress at a point on a vertical plane could have been obtained by using two planes rather than three as in the author's solution. This procedure, demonstrated herein, reduces the amount of calculation.

The cross-section of a straight gravity dam is illustrated in Fig. 28(*a*). A section with a length of unity perpendicular to the page is considered. The normal downward pressure on the horizontal section, $A\,B$, at a distance, h, below the water surface and the normal upward pressure on the horizontal section, $A'\,B'$, at a distance, $h + \delta h$, below the water surface are also shown in Fig. 28(*a*). The total shear on the vertical area, $X\,X'$, is then, $q_x \delta h$ = Area $A'\,X'\,E'\,C'$ − Area $A\,X\,E\,C$ − weight of prism, $A\,A'\,X'\,X$, considering shear as positive, which tends to move upward the part to the right of the point under consideration with reference to the part to the left. As demonstrated in mechanics, the unit horizontal shear at a point equals the unit vertical shear at that point.

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[35] Civ. Engr., Los Angeles, Calif.

[36] Prof. of Civ. Eng., and Head of Dept., Univ. of Illinois, Urbana, Ill.

[37] *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII, p. 134.

[38] *Engineering*, Vol. LXXIX (1905), p. 593.

The part of the dam under consideration is shown on a larger scale in Fig. 28(*b*) in which, the pressure diagrams are plotted in masonry units and the diagram for the upward pressures on $A' B'$ is placed above that plane instead of below it, as in Fig. 28(*a*). The area of the trapezoidal block, $A A' X' X$, represents the weight of that block in masonry units. Since the upward and downward pressure diagrams have been superimposed, the parts of the diagrams that overlap cancel each other. From the parts that do not overlap, the shear at any point can be determined. For instance, the shear at the down-stream toe, A, is equal to the area, $A A' C' G$, and, discarding the products of differentials, is $q_a \, \delta h = p_a \, m \, \delta h$, or,

$$q_a = p_a \, m \qquad (132)$$

Similarly, the shear at the up-stream toe is equal to the water load on $B B'$ minus the area, $B B' D' I$, or, $q_b \, \delta h = w_w \, h \, n \, \delta h - p_b \, n \, \delta h$. Solving,

$$q_b = (w_w \, h - p_b) \, n \qquad (133)$$

the weight of water per unit of volume being $w_w = w \div k$.

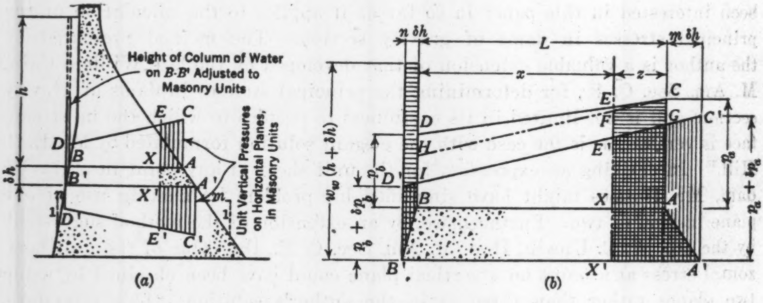

Height of Column of Water on B-B' Adjusted to Masonry Units
Unit Vertical Pressures on Horizontal Planes, in Masonry Units
h
δh
D
B
B'
n
1
D'
E
C
X
A
A'
X'
m
1
E'
C'
(a)
n δh
L
m δh
x
z
C
E
D
F
G
C'
H
E'
$w_w(h+\delta h)$
p_b
D'
I
B
X
A
p_a
$p_a + \delta p_a$
$p_b + \delta p_b$
B'
X'
A'
(b)

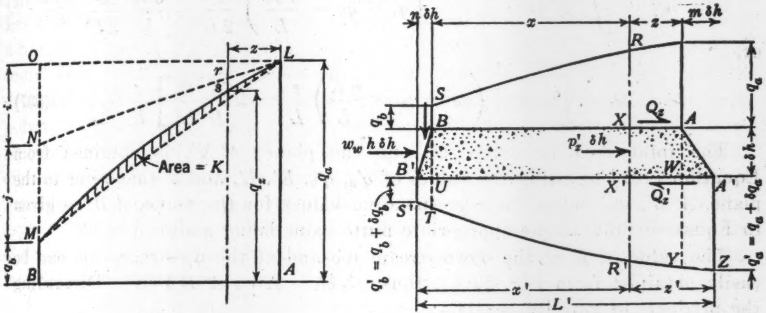
FIG. 28.

The shear on any section, $X X'$, at a distance, x, from the up-stream toe, or z from the down-stream toe, is equal to the algebraic sum of the force on the right of that section, upward forces being considered as positive. Therefore, referring to Fig. 28(*b*), $q_z \, \delta h =$ Area $A A' C' G -$ Area $E' G C E$. If $C H$ is drawn parallel to $C' I$, the shear is expressed by $q_z \, \delta h =$ Area $A A' C' G$ $-$ Area $E' G C F -$ Area $F C E$, or,

$$q_z = q_a - \frac{\text{Area } E' G C F}{\delta h} - \frac{\text{Area } F C E}{\delta h} = q_a - r - s$$

As illustrated in Fig. 29(*a*) the area of the parallelogram, $E' G C F$ (designated by r), increases directly with the first power of z; hence, in the shear diagram in Fig. 29(*a*), the values of r for various values of z will fall on the straight line, $L N$.

The area of the triangle, $F\,C\,E$, designated by s, varies as the square of z; so it is represented by the parabola, $L\,M$, in Fig. 29(a) which is tangent to the straight line, $L\,N$, at L since the values of s are measured from $L\,N$.



(a) (b)

FIG. 29.

The value, q_z, of the shear at any point can be determined if r and s are evaluated, the line, $L\,O$, being parallel to $A\,B$.

From Fig. 29(a), $\frac{r}{i} = \frac{z}{L}$, or $r = i\,\frac{z}{L}$, and $\frac{s}{j} = \frac{z^2}{L^2}$, or $s = j\,\frac{z^2}{L^2}$. From geometry, the area included between the parabola, $L\,M$, and the straight line, $L\,M$, is $K = \frac{1}{2}\,j\,L - \frac{1}{3}\,j\,L = \frac{1}{6}\,j\,L$. Therefore, $j = \frac{6\,K}{L}$. Furthermore, the area of the shear diagram equals the total shear on the section, $A\,B$, and, therefore, must equal the total horizontal water pressure above $A\,B$. Using this relation and referring to Fig. 29(a), the area under the parabola is:

$$K = \tfrac{1}{2}\,w_w\,h^2 - \tfrac{1}{2}\,(q_a + q_b)\,L \quad\ldots\ldots\ldots\ldots\ldots(134)$$

The value of K may be either positive or negative depending on the dimensions of the section under consideration. From Fig. 29(a), $i = q_a - q_b - j = q_a - q_b - \frac{6\,K}{L}$, and $q_z = q_a - r - s = q_a - i\,\frac{z}{L} - j\,\frac{z^2}{L^2}$; or,

$$q_z = q_a - \left(q_a - q_b - \frac{6\,K}{L}\right)\frac{z}{L} - \frac{6\,K}{L}\,\frac{z^2}{L^2} \quad\ldots\ldots\ldots\ldots(135)$$

The quantity, K, can be eliminated from Equation (135) by substituting its value as given in Equation (134), but the form given in Equation (135) is probably preferable. The factor K, must be used with the proper sign.

The horizontal stress on the vertical plane through a point can be obtained by considering the horizontal forces acting on the trapezoidal element, $A\,X\,X'\,A'$, of Figs. 28(a) and 29(b) in a manner similar to that which was used in obtaining the shearing stress from the vertical forces acting on this element. Then, $\Sigma\,H = p'_z\,\delta h + Q_z - Q'_z = 0$, and,

$$p'_z = \frac{(Q'z - Qz)}{\delta h} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(136)$$

The total shearing force on the plane, *A B*, from *A* to *X* is, from Equation (135),

$$Q_z = \int_0^z q_z \, \delta z = q_a \, z - \left(q_a - q_b - \frac{6\,K}{L}\right)\frac{z^2}{2\,L} - \frac{6\,K}{L}\,\frac{z^3}{3\,L^2}$$

or,

$$Q_z = \left[q_a - \tfrac{1}{2}\left(q_a - q_b - \frac{6\,K}{L}\right)\frac{z}{L} - 2\,\frac{K}{L}\,\frac{z^2}{L^2}\right]\frac{z}{L}\,L \text{......(137)}$$

The total shearing force, Q'_z, on the plane, *A′ X′*, is obtained from Equation (137) by using the values of q'_a, q'_b, K', L', and z' that refer to the plane, *A′ B′*, instead of the corresponding values for the plane, *A B*, as given in Equation (137), some appropriate finite value being assigned to δh.

The value of p' at the down-stream toe and at the up-stream toe can be easily obtained from Fig. 29(*b*), thus, $p'_a \, \delta h =$ Area *A′ Z Y W*. Discarding the products of two differentials:

$$p'_a = q_a \, m \text{ ....(138)}$$

Similarly, $p'_b \, \delta h = w_w \, h \, \delta h -$ Area *B′ S′ T U*, or,

$$p'_b = w_w \, h - q_b \, n \text{ ....(139)}$$

In applying Equations (135), (136), and (137) to obtain the principal stresses at a point, two horizontal planes are used, one at some finite distance above the point and another at the same distance below. Equation (137) gives the values of Q_z and Q'_z for these two planes for use in determining p' from Equation (136). Following the author's procedure, the value of q at the point is taken as the average of the values at points on the horizontal planes, just above and just below the point in question. Of course, a value of q that is exact, within the limits of the assumptions on which the analysis in this paper is based, can be found directly by passing a horizontal plane through the point and using Equation (135). This procedure is unwarranted, however, because the average value of q is sufficiently accurate. A satisfactory value for p at the point is obtained by averaging the values for p at corresponding points on the planes just above and just below the point. If the up-stream face of a dam is vertical, the calculations are simplified by taking the origin at the up-stream face as in the author's solution. The changes that must be made in Equations (135), (136), and (137) to suit the new origin are obvious.

If the horizontal planes are taken 1 ft. apart, as suggested by Mr. Holmes, it is necessary to carry all calculations out to a large number of significant figures, because the calculations for p' involve the difference of two large quantities that are nearly equal. Considerably larger intervals will give results that are sufficiently accurate for practical purposes, especially if there is no pronounced change in the profile in the region being investigated. The refinement represented by small intervals between the horizontal planes does not seem justified when the approximate nature of the assumptions, on which the entire analysis is based, is considered.

Calvin V. Davis,[39] M. Am. Soc. C. E. (by letter).—This paper has directed the attention of engineers to the basic principles of stress analysis for concrete dams. The need for such fundamental thinking in dam design has been forcibly impressed upon the writer during the past few years. In a number of designs prepared by independent consulting engineers the writer found that investigations had been carried previously no further than the determination of the vertical normal pressures on horizontal planes. This procedure indicated that, theoretically, extremely thin buttresses would be safe, although actually impractical to construct. A complete analysis of the principal and shearing stresses, however, indicated that the intensity of the shearing and tensional stresses was sufficient to cause structural failure. The experience of the writer with these designs substantiates the conclusions of Mr. Holmes as to the great importance of making a complete analysis of the internal stresses in dams.

In analyzing the principal stresses in buttress dams the writer prefers to divide the buttress into elementary prisms and to determine the stresses on each prism through the solution of simple equilibrium equations.[40] This method has been applied successfully to the analysis of most of the important buttress dams constructed in recent years. Its advantages over the equations which are fully developed in Mr. Holmes' paper lie in the ease in which the effects of the irregularities of section commonly encountered in buttresses may be included in the analysis. Fundamentally, however, the methods are the same; the difference is one of procedure rather than of basic principle. The same results should be obtained by both procedures if applied to structural sections that are identically the same.

The writer is not greatly concerned, however, about the technicalities of precedure. Far more important than these are the improved types that are now being developed as a result of a better understanding of the stress distribution in dams. Such important and far-reaching improvements of type as the round-head buttress dam,[41] the buttress dam of uniform strength,[42] as well as the massive buttress dam previously described[43] by the writer, resulted directly from fundamental studies of the distribution of principal stresses in the several structural members of a dam.

Another example of structural improvement that resulted from a thorough analysis of internal stresses is afforded by a dam, near Prescott, Ariz. Maximum economy in concrete was attained in this dam by proportioning the structure in such a way that both the intensities of shearing stress and first principal stress were substantially uniform on any horizontal plane. The diagrams shown by Fig. 30 indicate the variation of the first principal stress in this dam. Further economy was attained through keeping the second principal stress in compression, thus eliminating the necessity for excessive inclined steel. The writer found that the uniformity of these stress functions was controllable almost entirely through the proper selection of the up-stream and down-stream buttress slopes. This important relation of buttress slopes to principal and shearing stress distribution would not have been revealed if the analysis had been carried no further than the determination of the vertical normal stresses on horizontal planes.

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[39] Chf. Designer, Ambursen Dam Co., Inc., New York, N. Y.

[40] "Safeguarding the Lake Pleasant Dam," Fred A. Noetzli, M. Am. Soc. C. E., *Western Construction News*, April 25, 1929.

[41] "The Design and Construction of Dams," by Edward Wegmann, M. Am. Soc. C. E., Eighth Edition.

[42] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 666.

[43] "Recent Advances in Buttress Type Dams," *Civil Engineering*, February, 1931.

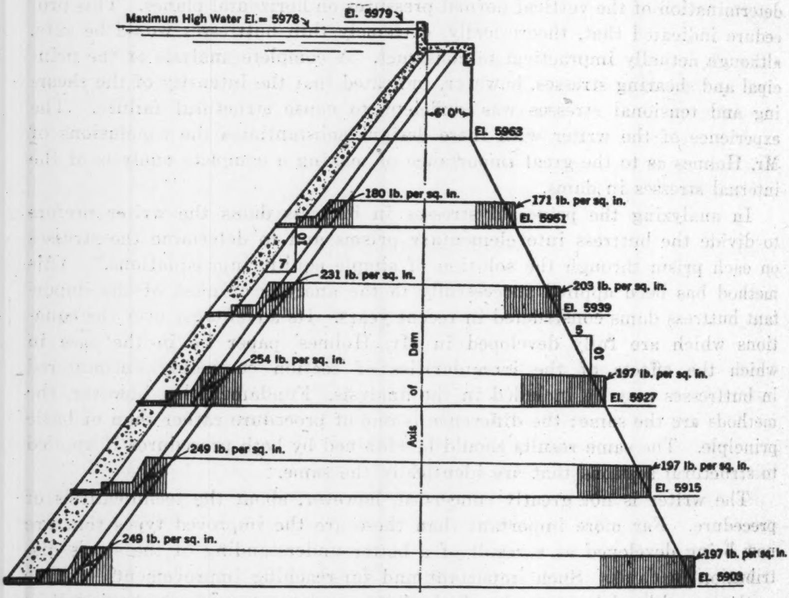
Maximum High Water El.= 5978
El. 5979
6' 0"
El. 5963
180 lb. per sq. in.
171 lb. per sq. in.
El. 5951
10
7
231 lb. per sq. in.
203 lb. per sq. in.
El. 5939
5
10
Axis of Dam
254 lb. per sq. in.
197 lb. per sq. in.
El. 5927
249 lb. per sq. in.
197 lb. per sq. in.
El. 5915
249 lb. per sq. in.
197 lb. per sq. in.
El. 5903

FIG. 30.—DIAGRAMS OF PRINCIPAL STRESSES IN DAM NEAR PRESCOTT, ARIZ.

It may be seen from this example that both increased safety and economy may result from the practical application of any of the methods of determining principal stresses. If Mr. Holmes' paper results in a more thorough understanding of the internal stress distribution in dams, a widespread use of the improved structural types is certain to follow.

W. H. HOLMES,[44] ASSOC. M. AM. SOC. C. E. (by letter).—As stated in the "Introduction," the writer's study of the cause of cracks in the buttresses of a multiple-arch dam led to the development of a formula for determining the principal stresses at any point on any horizontal plane through the buttress. When inclined contraction joints were proposed for a high curved gravity dam, the formulas were expanded to include gravity dams arched in plan. These formulas greatly reduced the tedious work involved in the study of the stresses in the interior and along construction joints parallel with the axis of buttresses and gravity dams. The logical method of presenting the subject was first to show the development of certain formulas for the straight gravity dam along the lines proposed by the late Professor Cain,[3] and then to derive the more general cases of gravity dams curved in plan and buttresses with tapering sections.

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[44] Hydr. Engr., State Dept. of Public Works, Los Angeles, Calif.

The discussions of the paper covered five main questions, or subjects, namely:

1.—The necessity for any analysis of stresses other than at the faces;
2.—The practicability of the formulas;
3.—The accuracy of any formula based on the trapezoidal law;
4.—Past use of Professor Cain's theories in practice; and,
5.—The selection of economic sections.

The personal opinion of several writers was expressed as to various details of design, but cases for which no general proof was given, will not be discussed.

Professor Kalman has questioned whether any study of the stresses at some interior point was necessary. He then derives equations to prove for a triangular profile, and with reservoir filled, that there can be no maximum stress in the interior of the section. Mr. Birke also demonstrates that the vertical and horizontal normal stress and shearing stresses for a triangular section are a linear function of x. The writer's Equations (34), (35), and (36) prove that, for the triangular profile, the stresses are a maximum at the downstream face. Professor Kalman concludes his discussion of maximum stresses with the statement that if the profile is not triangular, there will be some stress distribution other than linear along straight-line sections, but that the maximum stress will probably appear at some point on the faces. However, he states that since dams are supposed to be continuous, homogeneous structures, he has never believed in searching the interior for extreme stresses.

The writer concurs with Professor Kalman that for continuous and homogeneous triangular profiles the stress will be a maximum at the faces and any further analysis based on the trapezoidal loading theory is useless, because the principal stresses at the faces can be computed directly.

Mr. Birke has derived certain formulas (Equations (87) to (107)) for determining principal stresses in a triangular section. These discussions only apply to a continuous triangular section, and have no application other than checking the writer's general formulas when applied to this particular case. It is erroneous to extend his deductions to an irregular section.

Many dams are not triangular, and a brief inspection of many of the larger gravity dams and buttresses would indicate that open cracks prevent the structures from being continuous; hence, some analysis should be made to determine the stresses in the irregular sections, and also to determine whether the stresses are a maximum at some point other than at the faces. Physical evidence indicates that there is some stress not found by analyzing stresses at the faces, that is causing buttresses to crack.

Professor Kalman questions the use of any analysis for adjustment of stresses in a section arched in plan, and has raised the old question of arch action in a gravity dam, curved in plan. He concludes that a designer who does not believe in arch action of a gravity dam will not design the dam as an arch, "a layout involving increased cost." Conversely, if a designer believes in some arch action, he should assume, according to Professor Kalman, that it compensates for "very moderate differences in the behavior of tapering blocks in arched dams and normal rectangular blocks in straight gravity dams," and therefore, should not consider "the tedious features of tapering blocks."

His arguments are not in accord with the facts, as may be illustrated by an example. The proposed San Gabriel Dam was to have been a gravity section, 492 ft high, arched in plan on a 1 400-ft radius, and was to have a crest length of 2 100 ft. The designer stated[45] that the primary reason for arching the dam was that the cost was less, as better rock at less depth was found at that location.

A dam of this length cannot be considered as a monolith; hence, it will crack or open at the construction joints. It is logical to make such construction joints on radial lines. The distance between any two radial lines at the down-stream toe is 73% of the distance between the same lines at the up-stream face. This reduction of 27% of the base width at the toe could not be considered as a "very moderate difference." There is valuable property and a dense population below this proposed dam, and any reasonable mathematical study promoting safety should not be omitted because it is too "tedious."

Professor Kalman believes the writer's statement to be untrue that "a triangular profile with vertical up-stream face is safer than a rectangular profile having the same up-stream face and base." A triangular dam of such height that the maximum compression or shear equals the safe working strength of the material used, would not be safe if, later, the section was changed to a rectangular one having the same base.

For examples of triangular and rectangular dams as shown in Examples 1 and 2, the maximum shear in the rectangular profile is 39% greater than in the triangular profile. The first principal stress is also 39% greater in the rectangular profile than in the triangular profile.

Mr. Floris could not find any advantage in using an algebraic formula for deriving stresses in buttresses. Any one attempting to extend Professor Cain's numerical method, for other than the simplest cases, would find that a systematic grouping of certain factors as in the algebraic method would be a saving of time, and, what is of more importance, an assistance to greater accuracy.

The practicability of formulas using 1-ft increments was questioned by Messrs. Stich, Birke, and Dedel. Mr. Stich found it necessary by his method to use coefficients of eighteen significant figures for the calculations of stresses in the lower elevation of the San Gabriel Dam. The writer did not have the difficulty expressed by Mr. Stich in making the stress analysis of this dam. Results of these computations were given in Example 4, for the height of the dam equal to 100 ft. For a height of 492½ ft, no additional difficulty was found in confirming each of the checks, 1 to 4. Mr. Birke concurs with Mr. Stich that using planes 1 ft apart vertically is too cumbersome. These discussers appear to have overlooked the fact that if the moment in one plane is known to any desired degree of accuracy, it is a simple matter to determine the moment of a section 1 ft above or 1 ft below that section to the same degree of accuracy.

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[45] *Western Construction News*, July 10, 1927, p. 48.

Both Messrs. Birke and Dedel criticized the methods developed, because these methods did not cover the one particular case in which they were interested, namely, buttresses having an enlarged up-stream face, as shown in Fig. 17. Mr. Birke found that the omission of these sections gave "results that are incorrect and not on the side of safety for structures having variable sections, such as buttress designs with flaring sections, haunches, pilasters, arches, etc."

If the rectangular base section for any dam were safe, it is impossible to conceive that by the addition of a small amount of material (haunch, etc.) near the up-stream face the structure would become unsafe. The writer is unable to follow Mr. Birke's arguments. Even his colleague, Mr. Dedel, concurs with the writer that the omission of the flanges produces a conservative design, instead of an unsafe design.

Mr. Dedel criticized the writer for proposing a "too conservative design" and, in Fig. 18, attempts to prove that his own designs are less conservative. The writer believes that Mr. Dedel's assumptions are less conservative, even to the point of being unsafe. It is time that more conservative thought is given to the design of buttress dams, because the record of some of the larger buttress dams is not very satisfactory. Fredrik Vogt, Assoc. M. Am. Soc. C. E., has described the cracks in a number of multiple-arch dams,[46] and a repetition is unnecessary.

Both Mr. Birke and Mr. Dedel appear to have difficulty in analyzing a section having flanges or haunches. If they had examined a few of the many buttresses that have cracked they would have found the proper procedure to follow in determining what part of the base to include in the analysis. Many cases could be cited to show where open cracks exist in the arch ring and extend through the buttress near the up-stream toe of the dam, thereby eliminating the arch or haunch from any part of the analysis.

If the stress at the down-stream toe is called the first principal stress, continuing a fiber of stress to the up-stream face, the external force (water pressure) must be the first principal stress. If the buttress has an open crack at the foundation, the first principal stress at the up-stream edge of the block down stream from the crack is zero. If the diagram of the first principal stress was shown by Mr. Dedel for Fig. 18, it would have been approximately a rectangle. A crack near the heel would immediately reduce his rectangle of stresses to a triangle. Again, referring to Fig. 18, as far as can be determined from inspection, the diagram of the second principal stresses shows a straight-line variation. Such cannot be the case for even the simple triangular profile.

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[46] "Shrinkage and Cracks in Concrete Dams," by Fredrik Vogt, 1930.

Mr. Dedel develops an equation for the stresses at a point below the haunch by simply ignoring the existence of the haunch and the full thickness of the buttress and haunch. His Equations (110) and (113), if applied to the face, are identical in value to the writer's Equations (26) and (27). Mr. Dedel's Equation (112) is identical to that in the paper, preceding Equation (26). It would seem that Mr. Dedel's entire criticism of the paper is that no formulas were developed to care for approximate solutions during preliminary stages of design, but when he attempts to make an exact analysis, he concurs that "the exact method of analysis similar to the author's method must be used." A curve to assist in the selection of a proper base width of approximate trial solutions to eliminate tension is described later in this discussion. Mr. Nelidov has derived analytical equations for buttresses having an abrupt change in section, by which Mr. Birke's and Mr. Dedel's particular case can be solved for a monolithical homogeneous section.

Mr. Stich has raised the question of checking Equations (26) and (27) and has stated that these equations are only true for special cases. It can be shown that S_u and S_d approach S_o; and also that p_u and p_d approach p_o as Δ approaches zero. If the planes are taken as Δ distance apart, S_u and S_d approach equality and also p_u and p_d approach equality and, on Plane 0, are equal for all cases. Therefore, the equations as given are mathematically correct for all cases.

Mr. Cook takes exception to the basic assumption, namely, trapezoidal loading. This assumption is invariably made in the design of large dams, and will have to be used until something better, based on either a new theory or experimental data, is produced. He refers to a model test and states: "If the hypothesis of linear distribution of stress were true, the shearing stress would be distributed parabolically." Based on the trapezoidal law, the horizontal or vertical shear in a rectangular beam is distributed parabolically (the beam having both shear and moment at the section considered), but the maximum shear is an equation of a higher degree. An equation for maximum shear in an irregular section would be very complex. The shear is maximum on two planes, making 45° with the planes of principal stress, and is zero on the planes of principal stress. Shear deformation measurements, if not properly made, would give misleading results. The determination of maximum shear in a model by elastic deformations is not as simple as Mr. Cook infers. The photo-elastic method does show maximum shear, but as stated by Dr. Max Mark Frocht,[47] the solution of principal stresses is not simple. As Mr. Cook's shear hypothesis is in error, he cannot use it to prove that the trapezoidal law is in error.

The writer concurs with Mr. Cook that model tests may be an aid in the art of dam design, but thus far they cannot be substituted for analytical methods of determining stresses.

The writer cannot concur with Mr. Birke in the statement that all important dams recently built have been analyzed by the method for determining principal stresses, which are practical and theoretically correct.

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[47] *Transactions*, Am. Soc. Mech. Engrs., Vol. 53 (1931).

The part of the buttresses of the Rodriquez Dam, built in 1930, do not have any inclined joints, and, in fact, designs of the same dam made in 1928 show only vertical normal stresses. It has been reported that the amount of reinforcing steel was increased after construction started on the Lake Pleasant Dam,[48] Sutherland Dam, and Stony Gorge Dam.[49] Furthermore, a paper on the details of design and construction[50] for the Stony Gorge Dam makes no mention of principal stresses.

Mr. Birke makes the statement in his discussion that if properly arranged contraction joints are placed along the trajectories of the first principal stress, and if the second principal stress is either compression or zero, there will be no tendency for the joints to open and, as the shear is zero, there will be no tendency to slide. Mr. Birke assumes, therefore, that the structure is able to take the stresses necessary for monolithic action and inquires why it should not be so designed. Shearing movement along actual cracks in the aforementioned dams, and in others,[51] refute Mr. Birke's theories in this respect.

Cracks in the buttresses of the Lake Hodges Dam have been observed by many engineers for years. Micrometer readings across the cracks have been made for ten years, and show a steady yearly increase in crack openings. Some of the cracks now (1933) are open about 0.3 in. It is obvious, therefore, that the buttresses of this structure are not acting as a monolith, since tension or shear cannot be transmitted across the open joints. This one case furnishes sufficient evidence to prove that a large buttress should be designed in separate units.

Various theories[52] have been advanced as to the probable cause of failure of materials, among them (1) maximum stress; (2) maximum strain; and (3) maximum shear. No theory yet advanced appears to account for all the physical phenomenon observed.

The maximum stress theory or a maximum working stress is commonly used in building codes, but it has been shown that most materials, if restrained laterally, will carry in compression many times the ultimate load for the same materials when unrestrained. Such observations would lead to the adoption of the maximum strain theory. If the maximum strain theory is used, Poisson's ratio must be included in all calculations.

After principal stresses, as defined in the paper, have been obtained by any convenient method, true stresses may be computed from the formulas given in most textbooks, namely, if T is true stress, f_1 and f_2 are principal stresses, and μ is Poisson's ratio:

$$T_1 = f_1 - \mu f_2 \qquad (140)$$

and,

$$T_2 = f_2 - \mu f_1 \qquad (141)$$

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[48] *Western Construction News*, April 25, 1929, p. 199.

[49] *Loc. cit.*, August 10, 1928, p. 494.

[50] *Loc. cit.*, August 10, 1928, p. 490; and *Engineering News-Record*, July 11, 1929, p. 46.

[51] *Engineering News-Record*, January 31, 1924, p. 182.

[52] "Plasticity," Nadai, Engineering Society Monographs, p. 59.

To eliminate tension near the up-stream face due to the deformation, f_2 must be made equal to $\frac{f_1}{\mu}$, or,

$$f_2 = \frac{f_1}{\mu} \qquad (142)$$

Using this criterion, the ratio of base width to height becomes large, and it probably would be a more economical design to use steel reinforcing instead of widening the section. Steel also should be used to care for stresses due to temperature changes and shrinkage, but this subject is beyond the scope of the paper.

Mr. Birke makes reference to the Big Dalton Dam as an example of a "well established" method of locating joints along the direction of principal stresses. The full amount of steel crosses the so-called joint in this dam, and even one-third of the concrete is continuous. This so-called joint as shown in Fig. 23 is a misnomer, as it is only a line of weakness formed by inserting building paper from each face one-third the thickness of the buttress. Since this dam was built in 1929, and to date (1933), no appreciable quantity of water has been held in storage, no conclusions can be drawn as to the effectiveness of this untried joint in providing monolithic action without ill effects from the secondary stresses that are involved.

As a direct answer to Mr. Birke's question, an opening of a construction joint or a crack causes a redistribution of stresses; hence, no section larger than the distance between open joints or cracks should be analyzed as a monolith. The redistributed stress, in some cases, may be more than double the original stress.

The writer concurs with Mr. Stich and Mr. Birke that, in order to eliminate or decrease the effect of shrinkage, some joints parallel with the axis are necessary for massive concrete dams; but he cannot concur that a joint on the line of first principal stress is a "cure-all" for dam design. Hydrostatic pressure in the joint or shrinkage will alter the stress distribution, and the joint will no longer be a line of zero shear.

Mr. von Bergen has extended the formulas to indicate sections of minimum tension. A formula for sections having zero tension should be of value. Very simple tables or charts can be made to show the base width required for a gravity section straight in plan when no tension exists in the heel, and when various weights of concrete and various assumptions of uplift are used.[53] If it is desired to design a buttress of continuous triangular profile that has no tension, approximate limiting conditions of base width may be tabulated.

The buttress type of dam has more variables than the straight gravity dam, but the curves shown in Fig. 31 were derived by making the following assumptions: (1) The triangular section is a monolith; (2) the base is rectangular; (3) the weight of masonry is 150 lb per cu ft; (4) liquid in storage has variable densities; (5) no tension occurs in second principal stress at the up-stream face; and (6) Poisson's ratio is neglected. Assumption (4) eliminates the question of type of facing, but allows for the variable ratio of buttress spacing to buttress thickness.

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[53] *Western Construction News*, October 25, 1928, p. 662.

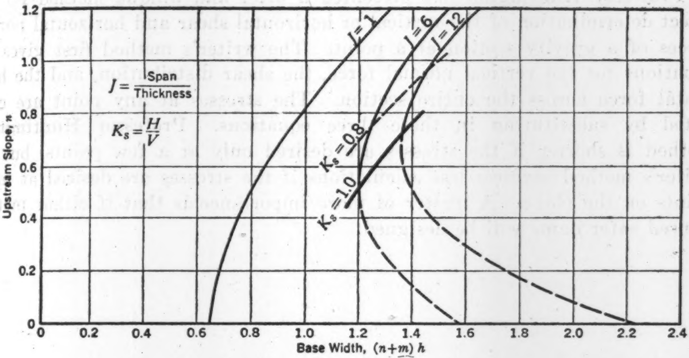

FIG. 31.—UP-STREAM SLOPE, BASE WIDTH, AND SLIDING FACTOR FOR STRAIGHT GRAVITY DAMS DESIGNED FOR ZERO STRESS NORMAL TO UP-STREAM FACE (NEGLECTING UPLIFT), WHEN FILLED WITH A LIQUID OF DENSITY OF *J*.

An example will illustrate: Given $n = 0.5$; buttress spacing, 60 ft; buttress thickness, 5 ft (or the equivalent of a liquid with a density of twelve times that of water acting on a gravity dam); and $J = \frac{60}{5} = 12$. It is found that $m = 0.89h$. With an up-stream slope of 0.5 and a down-stream slope of 0.89, the base width is $1.39h$.

It will be observed that the economic limit of the up-stream slope is 0 for a density of 1; 0.5, for a density of 6; and 0.75, for a density of 12. The batter of the up-stream slope is limited more by the sliding factor than by any other condition.

If this example were designed on the basis that T_2 shall equal zero (Poisson's ration $= \frac{1}{6}$), it is found that $m = 1.46h$. Such a requirement would increase the base width by 40% and indicates that the use of reinforcing steel would make a more economical design.

Mr. von Bergen has discussed steel reinforcement. This subject is of economic importance in the design of a buttress dam, but will not be discussed, as it is beyond the scope of the paper, which is concerned with the determination of principal stresses.

The writer finds nothing fundamentally wrong with the procedure of analysis for continuous and homogeneous triangular sections as used by Messrs. Birke and Dedel, but cannot agree with their results when applied to practical cases. Fig. 30, submitted by Mr. Davis as an example of design which he believes has the second principal stress in compression, illustrates this point. It is regretted that sufficient dimensions were not given to make a complete check of his computations. By computing the buttress thickness from the given principal stresses, it is found that the second principal stress is a tension of 63 lb per sq in., and the true tension, T_2, is 104 lb per sq in. To make f_2 equal to zero, the base must be widened 20 per cent.

Professor Huntington has presented a brief and unique method for the direct determination of the vertical or horizontal shear and horizontal normal forces of a gravity section at a point. The writer's method first gives the equations for the vertical normal force, the shear distribution, and the horizontal force across the entire section. The stresses at any point are computed by substituting in these three equations. Professor Huntington's method is shorter if the stresses are desired only at a few points, but the writer's method involves less calculations if the stresses are desired at many points on the plane. A matter of more importance is that if either method is used safer dams will be designed.

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STRESSES IN REINFORCED CONCRETE DUE TO VOLUME CHANGES

By C. P. Vetter,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Searcy B. Slack, Charles S. Whitney, M. Hirschthal, Walter Dreyer, Ralph E. Spaulding, A. H. Beyer, and A. G. Solakian, Lester L. Meyer, and C. P. Vetter.

Synopsis

The purpose of this paper is to present a rational analysis of the stresses occurring in a continuous reinforced concrete structure which, due to variations in moisture content and in temperature, is subject to linear expansions or contractions.

Formulas and diagrams are given whereby the required reinforcement may be determined for any combination of the various agencies which produce volume changes.

Although cracks in structures of this kind cannot be entirely avoided it is possible, through suitable reinforcement, to make the individual cracks exceedingly small. It is shown, however, that the ratio of steel to concrete required for this purpose is relatively high and, in many cases, it will be considered prohibitive.

It is hoped, therefore, that one of the results of this investigation will be the increased use of definite contraction joints in all but very thin structures.

Notation.—Algebraic terms used in this paper are defined as they first appear and, in addition, are arranged alphabetically in Appendix I, for further reference.

Shrinkage Stresses

By "shrinkage" and "swelling" is understood the change in volume of concrete due to drying out and water-soaking, respectively, as distinct from the contraction and expansion due to changes in temperature. The shrinkage alone without temperature variations will first be considered.

Let it be assumed that, at some time during the initial shrinkage, cracks have formed a distance, L in. apart. These cracks are caused by the tension

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Note.—Published in February 1932, *Proceedings*.

[1] San Francisco, Calif.

in the concrete having exceeded the ultimate tensile strength. The formation of the cracks will lower the concrete stresses temporarily, but as the shrinkage proceeds, these stresses will again grow until they reach the tensile strength of the concrete, and new cracks will form between the earlier ones.

At any stage of this development there is a certain relation between the stresses in the block confined between two cracks, the shrinkage, and the distance, L. The first step is to determine this relation and then to use the equations established in this way on the ultimate stage of the development when (after the cracks, L in. apart, have formed) the concrete has again just reached the ultimate strength, and new cracks are just about to form.

Considering a unit, L in. long, between cracks formed at an earlier stage, it is readily seen that the concrete will attempt to shrink between the cracks, but that it is partly held back by the reinforcement steel. This will cause tension in the concrete, compression in the steel, and bond stresses between the steel and the concrete. These bond stresses are confined to the vicinity of the cracks. Beginning at a crack and proceeding toward the center of the block the bond will gradually produce increasing compression in the steel and tension in the concrete until a point is reached where the shortening per unit length of the steel equals the net shortening per unit length of the concrete, the latter being made up of the shrinkage less the elongation due to tension. Beyond this point there is, then, no relative movement of concrete and steel and, therefore, no bond stresses and no further changes in concrete and steel stresses. This picture of the stress distribution was first suggested by Professor Emil Mörsch for the somewhat similar case of a relatively short reinforced concrete block subject to shrinkage.[2] It has recently (and after the writing of this paper) been confirmed by tests conducted in England.[3]

Furthermore, it is realized that, for a continuous wall, the total length of the steel bars must remain unchanged. The steel, therefore, must be in tension for part of its length near the cracks, in order to offset the compression near the center of the blocks, the total shortening due to compression being equal to the total elongation due to tension. This will cause a certain amount of slippage between the concrete and the steel near the cracks.

The question arises as to whether this slippage affects the distribution of bond stresses. European investigators[4] have found that even a small slip decreases the bond materially. American tests, however, show[5] that the bond stress is not materially decreased by small movements. For the purpose of simplifying the mathematics, it is assumed that the bond stress is constant. The following conditions then prevail (see Fig. 1):

(1) At a point near the center of the unit, the shortening per unit length of steel and of concrete must be equal, or:

$$\frac{f'_s}{E_s} = z - \frac{f'_c}{E_c} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

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[2] "Der Eisenbetonbau," Stuttgart, 1920, pp. 125–129; see, also, *Mitteilungen* über Forschungsarbeiten des Material-prüfungsanstalt, Stuttgart, Nos. 72–74.

[3] *Technical Paper No. 11*, "Studies in Reinforced Concrete II: Shrinkage," Dept. of Scientific and Industrial Research, Bldg. Research, H. M. Stationery Office, London, England; see, also, *Engineering* (Lond.), August 7, 1931, p. 181.

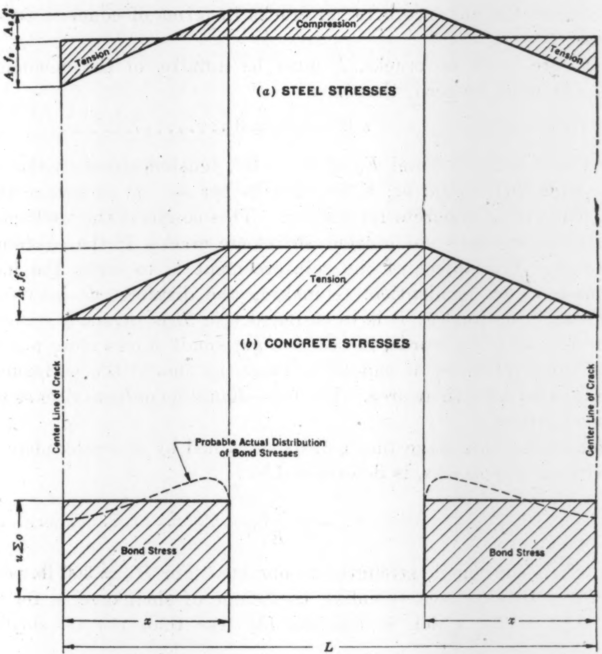
[4] *Zeitschrift* des Vereins Deutscher Ingenieure, 1911, p. 859.

[5] Concrete Engineers' Handbook, Hool and Johnson, New York, 1918, p. 267.

in which, respectively, f'_s and f'_c are compression stress in steel and tension stress in concrete; E_s and E_c are the moduli of elasticity of steel and concrete; and z is the coefficient of shrinkage of concrete. Let $n = \frac{E_s}{E_c}$, then Equation (1) is,

$$f'_s = z\,E_s - n\,f'_c \quad\text{.......................(2)}$$

(2) The total bond stress must equal the total tension in the concrete, and also the total change in steel stress, or if x is the distance along the bar


$A_s f'_s$
$A_s f_s$
Compression
Tension
Tension
$A_c f'_c$
Tension
Center Line of Crack
Center Line of Crack
Probable Actual Distribution of Bond Stresses
$u \Sigma o$
Bond Stress
Bond Stress
x
x
L

(a) STEEL STRESSES

(b) CONCRETE STRESSES

(c) BOND STRESSES

FIG. 1.—STRESS DISTRIBUTION BETWEEN CRACKS OF CONTINUOUS REINFORCED CONCRETE MEMBER SUBJECT TO SHRINKAGE

over which bond is effective, u = bond stress; Σo = circumference of the reinforcement bars; and A_c and A_s = concrete and steel areas, respectively:

$$u\,x\,\Sigma o = A_c\,f'_c = A_s\,(f_s + f'_s) \quad\text{................(3)}$$

(3) The total shortening of the steel must be equal to its total elongation, or:

$$\frac{1}{E_s} \times \frac{1}{2} \times L \times f'_s = \frac{1}{E_s} \times \frac{1}{2} \times x \times (f'_s + f_s)$$

and,

$$L = \frac{x\,(f'_s + f_s)}{f'_s} \quad\text{.........................(4)}$$

Substituting from Equations (2) and (3):

$$L = \frac{A_c\, f'_c}{u\, \Sigma o} \times \frac{A_c}{A_s} \times f'_c \times \frac{1}{z\, E_s - n\, f'_c}$$

Let $p = \frac{A_s}{A_c}$; and $q = \frac{\Sigma o}{A_s}$; then,

$$L = \frac{(f'_c)^2}{n\, p^2\, q\, u\, (z\, E_c - f'_c)} \quad \text{..................(5)}$$

which expresses the distance between cracks in terms of concrete tension and bond stress.

If there are to be no cracks, L must be infinite, or the denominator in Equation (5) must be zero; that is,

$$z\, E_c - f'_c = 0 \quad \text{..........................(6)}$$

With $z = 0.5 \times 10^{-3}$ and $E_c = 3 \times 10^6$, tension stress in the concrete, from Equation (6), would be, $f'_c = 1\,500$ lb. per sq. in., or several times the ultimate strength of concrete for tension. This confirms the well-known fact that reinforcement does not prevent shrinkage cracks, if the concrete is not free to move. The object of the reinforcement is to make the individual cracks small, which is identical to making the distance, L, between cracks small. From Equation (5) it is to be noted that L is inversely proportionate to p, q, and u; in other words, in order to get small cracks close together, the percentage of reinforcement should be great, as should the perimeter of the bars as compared with their area. The bars should be deformed so as to insure a high bond stress.

The maximum shrinkage that can be sustained by concrete, plain or reinforced, without any cracks, is determined by:

$$z = \frac{S'_c}{E_c} \quad \text{............................(7)}$$

in which, S'_c is the tensile strength of concrete. For $S'_c = 300$ lb. per sq. in., and $E_c = 3 \times 10^6$, the corresponding coefficient of shrinkage, z, for the concrete, is 0.10×10^{-3}, which is considerably less than normal shrinkage of concrete.

There is, however, another point which must be considered. The tensile stress in the steel, f_s, must not exceed the yield point, or the cracks will open.

From Equation (3):

$$f_s = \frac{f'_c}{p} - f'_s \quad \text{.........................(8)}$$

Substituting from Equation (2),

$$f_s = f'_c \left(\frac{1}{p} + n \right) - z\, E_s \quad \text{..............(9)}$$

For $f_s = S_s$ (the elastic limit of the steel), and $f'_c = S'_c$:

$$p = \frac{S'_c}{S_s + z\, E_s - n\, S'_c} \quad \text{...................(10)}$$

The ratio, p, is inversely proportional to S_s and z; therefore, reinforcement steel with a high elastic limit, such as cold-twisted bars, should be used. The highest percentage of steel will be required when z is small, that is, in the beginning of the shrinkage period. It has been demonstrated that, for shrinkages less than the value determined by Equation (7), no cracks would form. This value, therefore, should be used for determining p; thus, in Table 1, Equation (11a) gives $p = 0.60\%$ for $S'_c = 300$ lb. per sq. in., and for $S_s = 50\,000$ lb. per sq. in.

The distance between cracks when the concrete stress has just reached the tensile strength, but before the new cracks form, is determined by Equation (11e) (Table 1). Thus, for $p = 0.60\%$, $u = 300$ lb. per sq. in., $q = 8$ ($\frac{1}{2}$-in. round bars), $E_s = 30 \times 10^6$, and $n = 10$, Equation (11e) shows that L varies from infinity to $L = 87$ in., when z varies from 0.10×10^{-3} to 0.50×10^{-3}

These equations hold as long as $L \geqq 2\,x$. If the shrinkage should continue after the bond stress zone has reached the center of the unit, there would be no further changes in the stresses, but the cracks would increase in width due to continued slippage between concrete and steel.

The critical value of z may be determined by inserting values from Equation (3) and Equation (11e) (Table 1), in the equation, $L = 2\,x$, or,

$$\frac{(S'_c)^2}{n\,p^2 q\,u\,(z_1 E_c - S'_c)} = \frac{2\,A_c\,S'_c}{u\,\Sigma\,o} \quad \ldots\ldots\ldots\ldots\ldots (15)$$

Solving for z, Equation (15) takes the form of Equation (11d) in Table 1.

If p is determined by Equation (11a) (Table 1), the critical value of z becomes,

$$z_1 = \frac{S_s + 2\,n\,S'_c}{2\,E_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

As before, let $S_s = 50\,000$; $S'_c = 300$; $n = 10$; and $E_s = 30 \times 10^6$; then the critical value, z_1, is found equal to 0.98×10^{-3}. Greater shrinkage than this will not occur with Portland cement concrete.

TEMPERATURE STRESSES

The same reasoning which was used for the shrinkage stresses may now be applied to the temperature stresses (considered without accompanying shrinkage).

In this paper the term, "temperature drop," is used to denote the difference between the temperature at which the concrete has set and the temperature considered.

At first, before cracks appear, concrete and steel stresses are constant, that is,

$$\phi'_s = \epsilon\,t \times E_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

and,

$$\phi'_c = \epsilon\,t \times E_c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

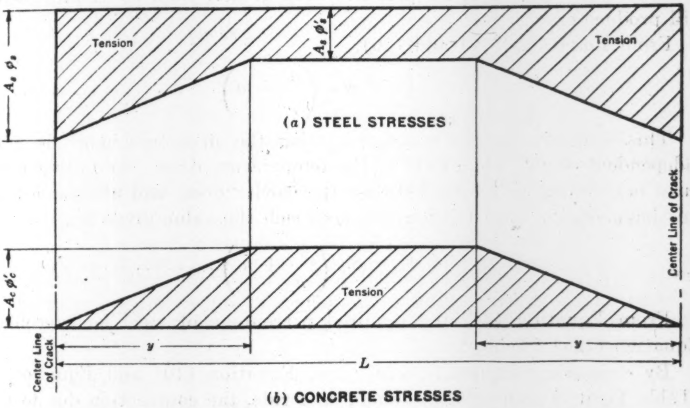
in which, the symbol, ϕ, represents a stress due to temperature drop as dis-

TABLE 1.—SPECIAL ARRANGEMENT OF IMPORTANT DESIGN FORMULAS

Item	Design condition	Shrinkage only, Equation (11)	Temperature drop only, Equation (12)	Indoor structures, shrinkage, and temperature drop, Equation (13)	Outdoor structures, swelling, and temperature drop, Equation (14)
(*a*)	Minimum reinforcement	$p = \frac{S'_c}{S_s}$	$p = \frac{S'_c}{S_s - n S'_c}$	$p = \frac{S'_c}{S_s - n S'_c}$	$p = \frac{S'_c}{S_s - w E_s - n S'_c}$
(*b*)	Maximum temperature drop for minimum reinforcement............................	....	$T = \frac{S_s + n S'_c}{2 \epsilon E_s}$	$T = \frac{S_s + n S'_c}{2 \epsilon E_s}$	$T = \frac{S_s + n S'_c + w E_s}{2 \epsilon E_s}$
(*c*)	Reinforcement for greater temperature drop....	....	$p = \frac{S'_c}{2 (S_s - T \epsilon E_s)}$	$p = \frac{S'_c}{2 (S_s - T \epsilon E_s)}$	$p = \frac{S'_c}{2 (S_s - T \epsilon E_s)}$
(*d*)	Critical volume change	$z_1 = \frac{S'_c}{E_c} \left(\frac{1}{2 pn} + 1 \right)$	$\epsilon t_1 = \frac{S'_c}{E_c} \left(\frac{1}{2 pn} + 1 \right)$	$\epsilon t_1 + z_1 = \frac{S'_c}{E_c} \left(\frac{1}{2 pn} + 1 \right)$	$\epsilon t_1 - w_1 = \frac{S'_c}{E_c} \left(\frac{1}{2 pn} + 1 \right)$
(*e*)	Distance between cracks..................	$L = \frac{(S'_c)^2}{n p^2 q u (z E_c - S'_c)}$	$L = \frac{(S'_c)^2}{n p^2 q u (\epsilon t E_c - S'_c)}$	$L = \frac{(S'_c)^2}{n p^2 q u [(\epsilon t + z) E_c - S'_c}$	$L = \frac{(S'_c)^2}{n p^2 q u [(\epsilon t - w) E_c - S'_c]}$
(*f*)	Minimum distance between cracks	$L_{\text{min.}} = \frac{2 S'_c}{p q u}$	$L_{\text{min.}} = \frac{2 S'_c}{p q u}$	$L_{\text{min.}} = \frac{2 S'_c}{p q u}$	$L_{\text{min.}} = \frac{2 S'_c}{p q u}$

tinct from f which represented a stress due to shrinkage (the sub-symbols having the same meaning as before); ϵ represents the unit contraction per degree temperature drop; and t, the temperature drop, in degrees.

When the concrete stress exceeds the ultimate strength, the first cracks will form. Between the cracks the concrete attempts to contract, but is held back partly by the steel. There will be bond stresses between concrete and steel near the cracks and none near the center, where the stresses will be constant. The stress distribution is shown on Fig. 2. The distribution of bond stress is the same as in Fig. 1(*c*), except that the length, x, should be represented by y.



(*b*) CONCRETE STRESSES

FIG. 2.—STRESS DISTRIBUTION BETWEEN CRACKS OF CONTINUOUS REINFORCED CONCRETE MEMBER SUBJECT TO TEMPERATURE DROP.

The following equations may be written:

$$\frac{\phi'_s}{E_s} - \epsilon t = \frac{\phi'_c}{E_c} - \epsilon t \quad \ldots\ldots\ldots\ldots (19)$$

which expresses the fact that near the middle of the unit there is no relative movement of concrete and steel;

$$y \times u \times \Sigma o = A_c \phi'_c = A_s (\phi_s - \phi'_s) \quad \ldots\ldots\ldots\ldots (20)$$

which expresses the fact that the total bond stress is equal to the total concrete stress and to the total increment in steel stress; and,

$$\tfrac{1}{2} L \epsilon t E_s = \tfrac{1}{2} y (\phi_s + \phi'_s) + (\tfrac{1}{2} L - y) \phi'_s \quad \ldots\ldots\ldots\ldots (21)$$

which expresses the fact that the shortening of the steel due to temperature drop is equal to the elongation due to tension.

Inserting appropriate values from Equations (19) and (20) in Equation (21):

$$L = \frac{(\phi'_c)^2}{n\, p^2 q\, u\, (E_c \epsilon t - \phi'_c)} \quad \ldots\ldots\ldots\ldots (22)$$

or, for $\phi'_c = S'_c$, Equation (22) takes the form of Equation (12*e*) in Table 1.

Comparing Equations (5) and (22), it is seen that the distance between cracks depends on the unit contraction of the concrete and that it is independent of the manner in which such contraction has been brought about, whether by shrinkage or by temperature drop.

In the same way as for shrinkage, there will be no cracks if $E_c \epsilon t \leqq S'_c$; or, if $\epsilon t \leqq \frac{S'_c}{E_c}$. With the numerical values used previously, and with $\epsilon = 0.67 \times 10^{-5}$, $t = 15°$ Fahr. A temperature drop greater than 15° Fahr., therefore, will produce cracks.

From Equations (19) and (20),

$$\phi_s = \phi'_c \left(\frac{1}{p} + n\right) \ldots\ldots\ldots\ldots (23)$$

This discloses the interesting fact that the steel tension at the crack is independent of the magnitude of the temperature drop. At falling temperatures new cracks will form between the earlier ones, and always just before this happens, the steel tension will approach the value given by,

$$\phi_s = S'_c \left(\frac{1}{p} + n\right) \ldots\ldots\ldots\ldots (24)$$

From Equation (24) the required percentage of steel is determined by Equation (12*a*) (Table 1).

By comparing Equation (12*a*) with Equation (10) and Equation (11*a*) (Table 1), it is seen that, although, as a rule, the contraction due to shrinkage exceeds greatly that due to temperature drop, the reinforcement required by the latter is always the greater. With the numerical values used in the preceding examples, $p = 0.64\%$ by Equation (12*a*).

As before, Equations (19) to (24) hold good as long as $L \geqq 2\,y$. If the temperature drop continues after the bond stress zone has reached the center of the unit, no further cracks can form and the steel stress will increase.

The critical temperature drop is determined by inserting values of y and L from Equations (20) and (12*e*) (Table 1); thus,

$$\frac{(S'_c)^2}{n\,p^2\,q\,u\,(E_c \epsilon t_1 - S'_c)} = 2\,\frac{A_c\,S'_c}{u\,\Sigma o} \ldots\ldots\ldots\ldots (25)$$

or,

$$t_1 = \frac{S'_c\left(\frac{1}{2\,p\,n} + 1\right)}{\epsilon\,E_c} \ldots\ldots\ldots\ldots (26)$$

If p is determined by Equation (12*a*) (Table 1),

$$t_1 = \frac{S_s + n\,S'_c}{2\,\epsilon\,E_c} \ldots\ldots\ldots\ldots (27)$$

For the same numerical values as before, $t_1 = 132°$ Fahr. For temperature drops to the value given by Equation (27), minimum reinforcement may be used, as given by Equation (12*a*) (Table 1). If a still greater temperature drop may be expected, the reinforcement must be increased.

If the temperature drops an additional increment of t_2 degrees below the critical temperature given by Equation (27), so that the total temperature drop is $T = t_1 + t_2$, then,

$$t_2 \epsilon E_s = S_s - \phi_s \quad \dots\dots\dots\dots (28)$$

Inserting ϕ_s from Equation (24),

$$t_2 \epsilon E_s = S_s - S'_c \left(\frac{1}{p} + n \right) \dots\dots\dots\dots (29)$$

or,

$$T = t_1 + t_2 = \frac{S_s - \dfrac{S'_c}{2p}}{\epsilon E_s} \quad \dots\dots\dots\dots (30)$$

Solving for p, Equation (30) takes the form of Equation (12*c*) (Table 1). This is the value that should be used for temperature drops greater than the critical as determined by Equation (26) or Equation (27).

COMBINED SHRINKAGE AND TEMPERATURE STRESSES

A reinforced concrete unit may be subject simultaneously to both shrinkage and temperature drop. The combined stresses may be distributed as shown on Fig. 3. It is there assumed that the temperature drop takes place first and is followed by shrinkage. The order might just as well have been reversed as the resultant total stresses would have been the same.

It is at once evident that the sum of Equations (2) and (19), and the sum of Equations (3) and (20) still hold good. Let, $F'_c = f'_c + \phi'_c$; and, $F_s = f_s + \phi_s$. Then, by adding Equations (3) and (20) and substituting from the sum of Equations (2) and (19):

$$F_s = F'_c \left(\frac{1}{p} + n \right) - z\,E_s \quad \dots\dots\dots\dots (31)$$

For $F'_c = S'_c$, Equation (31) is identical to Equation (9) and the steel stress for combined shrinkage and temperature drop is seen to be less than that for temperature drop alone, as given by Equation (24).

It seems paradoxical that the addition of shrinkage to an existing temperature drop should reduce the steel tension. It must be remembered, however, that Equation (24) expresses an equilibrium in which the concrete between the cracks is just on the point of breaking. If shrinkage is added, the unit will break into shorter units in which the concrete stress is again equal to the tension strength, but the steel tension corresponding to this stress distribution is less than for temperature drop alone.

By substituting S_s for F_s, in Equation (31), and solving for p, an equation identical to Equation (10) is derived. As before, the highest value of p will be required when z is small; but the case for combined stress differs from shrinkage stress in that the minimum value for z in the former case is zero, while the smallest value in the latter case was expressed by $z = \dfrac{S'_c}{E_c}$.

The value of p to be used for design, therefore, is determined by Equation (12a) (Table 1). In other words, for combined shrinkage and temperature drop, the reinforcement is determined by the case of zero shrinkage. The distance between cracks is found by expressing the condition that the total shortening of the steel is equal to the total elongation, or:

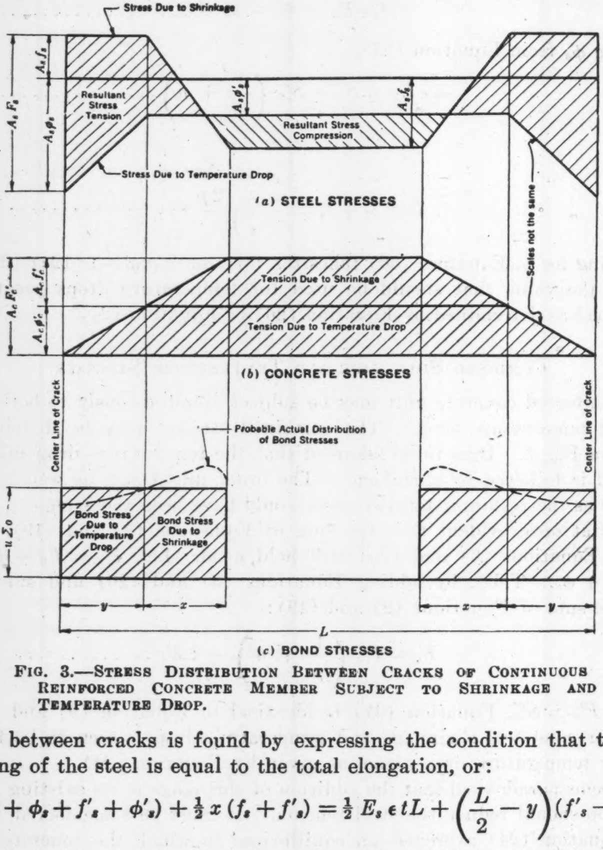

Stress Due to Shrinkage
$A_s f_s$
$A_s F_s$
$A_s \phi_s$
Resultant Stress Tension
$A_s \phi'_s$
$A_s f_s$
Resultant Stress Compression
Stress Due to Temperature Drop
(a) STEEL STRESSES
Scales not the same
$A_c F'_c$
$A_c f_c$
$A_c \phi_c$
Tension Due to Shrinkage
Tension Due to Temperature Drop
(b) CONCRETE STRESSES
Center Line of Crack
Center Line of Crack
Probable Actual Distribution of Bond Stresses
$u\,\Sigma o$
Bond Stress Due to Temperature Drop
Bond Stress Due to Shrinkage
y
x
x
y
L
(c) BOND STRESSES

FIG. 3.—STRESS DISTRIBUTION BETWEEN CRACKS OF CONTINUOUS REINFORCED CONCRETE MEMBER SUBJECT TO SHRINKAGE AND TEMPERATURE DROP.

$$\tfrac{1}{2}\, y\,(f_s + \phi_s + f'_s + \phi'_s) + \tfrac{1}{2}\, x\,(f_s + f'_s) = \tfrac{1}{2}\, E_s\, \epsilon\, t\, L + \left(\frac{L}{2} - y\right)(f'_s - \phi'_s)$$

By substituting x and y from Equations (3) and (20), and solving for L:

$$L = \frac{A_c\,(F'_c)^2}{u\,p\,\Sigma\,o\,(E_s\,\epsilon\,t + z\,E_s - n\,F'_c)} \quad \ldots\ldots\ldots\ldots (32)$$

or, by substituting S'_c for F'_c, Equation (32) takes the form of Equation (13e) (Table 1).

By comparing Equation (13e) with Equations (11e) and (12e) (Table 1), it is seen that the distance between cracks is shorter for combined shrinkage and temperature drop, than is the case when any of the shortening agencies act alone. This explains the fact that the steel tension is less for combined shrinkage and temperature drop than for temperature drop alone.

The critical volume change is found from the condition: $2\,(x + y) = L$; thus,

$$z + \epsilon t_1 = \left(\frac{1}{2\,p\,n} + 1\right)\frac{S'_c}{E_c} \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

which is similar to Equation (26) for temperature drop alone, and Equation (11*d*) (Table 1), for shrinkage alone.

If p is determined by Equation (12*a*) (Table 1):

$$z + \epsilon t_1 = \frac{S_s + n S'_c}{2\,E_s} \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

If the temperature drops an additional increment of t_2 degrees below the critical temperature given by Equation (13*e*) (Table 1), so that the total temperature drop is $T = t_1 + t_2$,

$$t_2\,\epsilon\,E_s = S_s - F_s = S_s - S'_c\left(\frac{1}{p} + n\right) + z\,E_s \ldots\ldots\ldots (35)$$

which, with Equation (33), reduces to the form, Equation (30).

If p in Equation (30) is determined by Equation (12*a*) (Table 1), then, for combined temperature drop and shrinkage:

$$T = \frac{S_s + n\,S'_c}{2\,\epsilon\,E_s} \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

For a temperature drop greater than that given by Equation (36), the reinforcement must be increased beyond the value given by Equation (12*a*) (Table 1). The value of p is found from Equation (12*c*) (Table 1), for combined stress due to temperature drop and shrinkage.

COMBINED SWELLING AND TEMPERATURE STRESSES

In this paper shrinkage is understood to mean a decrease in length of the concrete unit due to drying. Concrete, however, may also be subject to swelling or an increase in length if it is submerged in water and is water-soaked. If the temperature remains constant, the concrete will be in uniform compression throughout its length and although this may cause difficulties in some structures, the subject lies outside the scope of this paper.

If, however, the temperature drops sufficiently to create a tension in the concrete beyond the tensile strength, cracks will form and a stress distribution will take place similar to that described for shrinkage and temperature drop. A typical stress distribution is shown in Fig. 4.

If swelling is treated as a negative shrinkage ($z < 0$), formulas may be established which are identical to Equations (9), (10), and (31). However, maximum reinforcement is no longer determined by $z = 0$, which condition leads to Equation (12*a*) (Table 1), but by the minimum value of z (maximum swelling).

If the unit swelling is given a positive value, w, then p is as given by Equation (14*a*) (Table 1) and Equation (13*e*) (Table 1) takes the form of Equation (14e) (Table 1), indicating that cracks will form if $E_c\,(\epsilon\, t - w) > S'_c$, or $\epsilon\, t > \dfrac{S'_r}{E_c} + w$.

With $w = 0.1 \times 10^{-3}$, and other numerical values as before, $t > 30°$ Fahr. For temperature drops greater than that determined by the equation,

$$\epsilon\, t = \frac{S'_c}{E_c} + w \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

cracks will form even if the concrete has swelled.

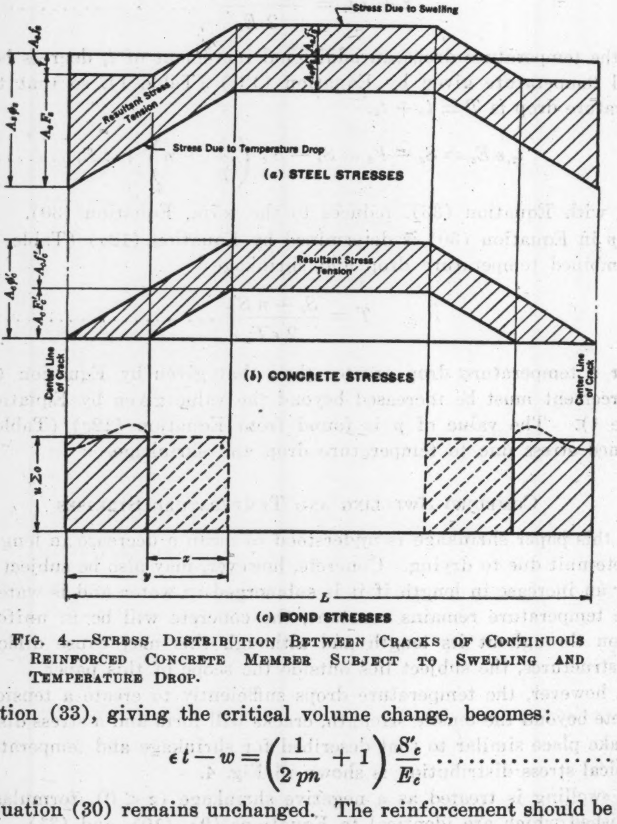

Stress Due to Swelling
$A_s f_s$
$A_s \phi_s$
$A_s F_s$
$A_s f_s$
Resultant Stress Tension
Stress Due to Temperature Drop
(a) STEEL STRESSES
$A_c \phi'_c$
$A_c f'_c$
$A_c F'_c$
Resultant Stress Tension
Center Line of Crack
(b) CONCRETE STRESSES
Center Line of Crack
$u\,\Sigma o$
x
y
L
(c) BOND STRESSES

FIG. 4.—STRESS DISTRIBUTION BETWEEN CRACKS OF CONTINUOUS REINFORCED CONCRETE MEMBER SUBJECT TO SWELLING AND TEMPERATURE DROP.

Equation (33), giving the critical volume change, becomes:

$$\epsilon\, t - w = \left(\frac{1}{2\,pn} + 1\right)\frac{S'_c}{E_c} \;\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

while Equation (30) remains unchanged. The reinforcement should be determined by Equation (14a) (Table 1), for temperature drops smaller than

$$T = \frac{S_s + \dfrac{1}{2p}\, S'_c}{\epsilon\, E_s}$$

, which may also be written as shown in Equation (14b) (Table 1).

For temperature drops greater than that given by Equation (14*b*) (Table 1), the reinforcement is determined by Equation (31).

The minimum distance between cracks for all cases discussed in this paper is given by Table 1, Item (*f*).

CONCLUSIONS

The formulas which have been developed are applicable to such structures as concrete flumes and canal linings, retaining walls, road pavements, floors in warehouses and large buildings, etc., in which contraction joints have not been provided. It has been shown that a ratio of reinforcement of from 0.60 to 0.75% is required to prevent unsightly cracks or water leaks. These values are far in excess of what has hitherto been deemed necessary, but it is believed that practice has shown these earlier values to be inadequate. The general subject of volume changes in concrete due to water content has not been treated in this paper. Reference is made to exhaustive tests by Professor S. H. White which have been published from time to time over a period of twenty years by the American Society for Testing Materials.[6]

It has been shown that the greatest ratio of reinforcement is required for concrete subject, simultaneously, to swelling and temperature drop. Although the swelling would tend to counteract the effect of the temperature contraction, a temperature drop of about 30° Fahr. will be sufficient to create cracks in the concrete, and the steel stresses at these cracks are higher than for any other combination of volume changes. It has been shown, furthermore, that shrinkage alone and combined shrinkage and temperature drop are less dangerous than temperature drop alone without shrinkage.

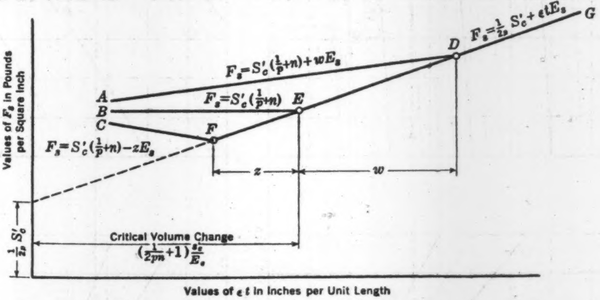


FIG. 5.—VARIATION OF STEEL STRESS WITH VOLUME CHANGE.

A volume change designated "critical" has been defined, which is dependent on the ratio of reinforcement and the strength characteristics of the concrete and the steel. For temperature drops in excess of the value corresponding to the critical volume change the temperature drop alone determines the reinforcement which is now independent of any volume change due to moisture.

The relationship between steel stresses and volume changes has been shown schematically in Fig. 5. It has there been assumed that the volume changes

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[6] As, for example, *Proceedings*, A. S. T. M., Vol. 28, Pt. II, p. 398.

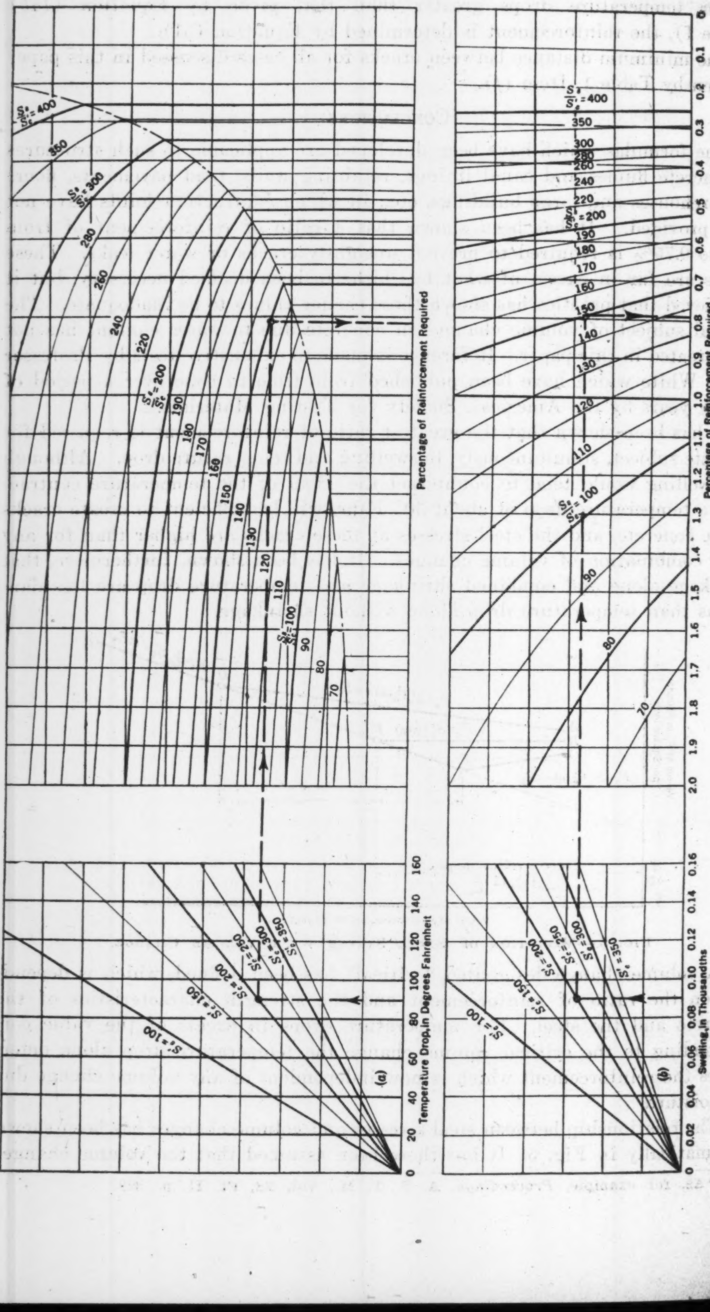

FIG. 6.—CURVES FOR USE IN DETERMINING PERCENTAGE OF REINFORCEMENT FOR ANY CONDITION OF VOLUME CHANGE.

due to moisture vary rectilinearly with the temperature drop. The diagram has no scale. Line $A\ D$ represents swelling and temperature drop; Line $B\ E$, temperature drop alone; and Line $C\ F$, shrinkage and temperature drop. Points D, E, and F represent the temperature drops prevailing at the time the total volume change equals the critical value. Beyond these points the steel stresses vary rectilinearly with the temperature drop. It is interesting to notice that in this region the steel stress is greater by a constant quantity, $\frac{S'_c}{2p}$, than it would be in a continuous steel rod without any concrete.

For design purposes Lines CFG should not be used because a temperature drop without accompanying shrinkage is always possible. Lines ADG should be used for outdoor structures subject to swelling, and Lines BEG, for all other structures. A summary of the design formulas is given in Table 1. A diagram giving the required amount of reinforcement for any condition of volume change is shown in Fig. 6. In these curves, $n = 10$; $\varepsilon = 0.67 \times 10^{-5}$; and $E_s = 3 \times 10^6$. Enter Fig. 6(a) for any given set of conditions and note the percentage obtained. If the structure is subject to swelling, check the first value by entering Fig. 6(b). Compare the values thus obtained and adopt the larger of the two.

APPENDIX I

NOTATION

The algebraic symbols introduced in this paper are defined as follows:

f = unit stress due to shrinkage: f_s, tension in steel; f'_s, compression in steel; f_c, compression in concrete; f'_c, tension in concrete.

n = ratio of moduli of elasticity, $\frac{E_s}{E_c}$.

Σo = sum of perimeters of bars.

p = steel area ratio, $\frac{A_s}{A_c}$.

q = ratio of perimeter of bars to area $\left(q = \frac{\Sigma o}{A_s} \text{ and } p \times q = \frac{\Sigma o}{A_c}\right)$

t = drop in temperature; t_1 = temperature drop corresponding to critical volume change; t_2 = additional drop.

u = unit bond stress between concrete and steel.

w = unit swelling caused by water-soaking.

x and y = distances over which effective bond occurs.

z = coefficient of shrinkage for concrete; z_1 = critical value.

A = area: A_s, of steel; A_c, of concrete.

E = modulus of elasticity: E_s, of steel; E_c, of concrete.

F = combined stress: $F'_c = f'_c + \phi'_c$; $F_s = f_s + \phi_s$.

L = distance between shrinkage cracks.

S = tensile strength: S'_c, of concrete; S_s, elastic limit of steel.

T = total temperature drop.

ϵ = coefficient of thermal expansion of both steel and concrete.

ϕ = stress due to a drop in temperature: ϕ_s, steel stress at a crack; ϕ'_s, steel stress at the center between cracks; ϕ'_c, concrete stress at the center between cracks.

DISCUSSION

Searcy B. Slack,[7] M. Am. Soc. C. E. (by letter).—The author has handled very clearly a mathematical analysis of the stresses that should occur in a continuous reinforced concrete structure due to variations in moisture, temperature, and shrinkage. The mathematical analysis, of course, begins with a series of assumptions, and the data on which these assumptions are based are not clearly set forth in the paper.

The assumption is made that the coefficient of expansion of concrete and the coefficient of expansion of steel due to temperature changes are the same. The same factor is used in reducing the formulas. Measurements made on these coefficients show that there is quite a wide variation in the thermal coefficient of concrete. This factor seems to vary between 0.000004 and 0.000006 per degree Fahrenheit. Most of the data indicate that 0.0000050 for ordinary structural concrete is approximately correct. The thermal coefficient of steel has generally been determined to be between 0.0000063 and 0.0000068, the value generally used being 0.0000065. It will thus be seen that there is quite a difference in the thermal coefficients of concrete and steel, and that this difference in the rate of expansion of the two materials is sufficient to cause a rather wide variation in the stresses, that may be expected at different temperatures.

Another factor that should be taken into account in analyzing stresses of this kind is the time at which the analysis is made. If the structure is fairly large, the heat caused by the setting of the concrete will not be dissipated for some time after the concrete has been placed. The initial heat in the concrete causes a rise in temperature in both the concrete and the steel. This occurs while the concrete is taking its final set and bond is developing between the concrete and the steel. When this heat due to chemical action of the cement is dissipated and the temperature falls, initial tension is caused in the steel reinforcement due to the fall in temperature. To illustrate: A usual case would be concrete with a thermal coefficient of 0.000005 per degree Fahrenheit and steel with a thermal coefficient of 0.0000065. The difference between the two thermal coefficients is 0.0000015. In a column 2 ft. square, the usual temperature rise would be from 70° to 120° Fahr. This would give a 50° fall in temperature following the set of the concrete; the reinforcement steel would be in initial tension amounting to 2 250 lb. per sq. in. This initial tension would gradually be relieved by the shrinkage of the concrete or the plastic flow, but several months would be required to relieve it. Temperature and stress curves for a case of this kind were presented[8] by the writer in 1929.

The author has presented a very good picture of the bond stress in steel. This picture should be extended or modified to include bond stress developed by the difference in thermal coefficients of the two materials.

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[7] Decatur, Ga.

[8] "The Behavior of a Reinforced Concrete Arch During Construction," *Proceedings*, Am. Soc. C. E., November, 1929, Papers and Discussions, p. 2279.

CHARLES S. WHITNEY,[9] M. AM. SOC. C. E. (by letter).—While the author has made a commendable effort to analyze the stresses in a continuous reinforced concrete structure due to volume changes, the writer believes that his basic assumptions are such that his formulas are of little practical value. The following should be considered:

1.—The author assumes the structure to be restrained at its ends so that "the total length of the steel bars must remain unchanged." He further assumes that no restraint exists between the ends, the concrete being free to crack at intervals where it will. He states: "The formulas * * * are applicable to such structures as concrete flumes and canal linings, retaining walls, road pavements, floors in warehouses and large buildings, etc. * * *." As a matter of fact no long structure exists, which is not subject to considerable restraint throughout its lengths from its supports. A continuous wall or pavement is restrained to such an extent by friction of the earth that it would not act as if held only at the ends. The floor of a concrete building is not fixed at the ends, but is restrained by its supports at intervals and cannot be put in the same class as a continuous pavement. In the latter case, the end movement which, of course, must exist, would have comparatively little effect, but the two are similar, inasmuch as they both are restrained to an indeterminate degree by their supports, which fact the author has neglected.

2.—The concrete has been considered as elastic with a definite value of the modulus of elasticity (assumed as 3 000 000 lb. per sq. in.), whereas concrete is to a considerable degree plastic and the effective value of the modulus of elasticity varies greatly with the time and rate of application of the load. It may be less than 1 000 000 for shrinkage and greater than 4 000 000 for temperature changes at later ages. Plastic flow greatly modifies the effects of shrinkage and temperature changes.[10]

3.—The coefficient of thermal expansion of concrete varies within a wide range and is not usually the same as that of steel, as the author assumes. The difference is so great that considerable stress is produced in the steel and concrete by a large change in temperature.

The problem is very much more complicated than the author's analysis indicates, so much so that it cannot be solved by the formulas presented. No general solution is possible for the variety of structures mentioned. The behavior of actual structures is probably the best indication and, even from them, general conclusions are made uncertain by differences in the properties of the materials, the special arrangement of the structures, and the climatic conditions.

M. HIRSCHTHAL,[11] M. AM. SOC. C. E. (by letter).—Great credit is due Mr. Vetter as a pioneer in the mathematical analysis of stresses due to volume and temperature changes in concrete—a field greatly in need of such analysis—and for the clear-cut handling of the mathematics and its application. The results obtained by him, however, are surprising, requiring almost prohibitive (economically) percentages of steel reinforcement to meet the condition of eliminating the major effects of volume and temperature changes occurring from the time the concrete is poured until it has been subjected to a cycle of exposure.

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[9] Cons. Engr., Milwaukee, Wis.

[10] "Plain and Reinforced Concrete Arches, a Progress Report on the Limitations of the Theory of Elasticity, and the Effect of Plastic Flow, Shrinkage, Temperature Variations and the Freyssinet Method of Adjustment", Report of Committee 312, Am. Concrete Inst., by Charles S. Whitney, Author-Chairman, *Journal*, Am. Concrete Inst., March, 1932.

[11] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

Such large results are undoubtedly due to the fact that the author has disregarded or overlooked some of the complications that enter into the problem. The principal factor that was overlooked is that resulting from the condition or fact that these volume changes, either in setting of concrete (or swelling), or in the exposure to rise and fall of temperature, occur not only in the longitudinal direction, *L*, but at right angles (transversely) in the same plane as well, and, in addition, at right angles to the plane just referred to, resulting in a condition differing materially from that treated by the author.

The shrinkage or temperature stresses at right angles to the direction, *L*, cause a reduction of the lateral deformation or strain resulting from the shrinkage or contraction due to temperature changes in the direction, *L*. The relationship between the lateral and longitudinal deformations is expressed by Poisson's ratio. This same ratio would have to be applied when considering the plane at right angles to the surface considered, thus further affecting the deformations respectively caused by the shrinkage stresses in each of the three directions. In other words, a cubical particle in the body of the concrete (a parallelopiped) would be subjected, by the shrinkage stresses, to deformations each respectively reduced by the stresses on the other planes so that the net result would be a considerable reduction of the value expressed by the symbol, z, in the author's analysis. In fact, the value of z would be found, in all likelihood, to lie between one-half and one-third the result derived and yield a percentage of reinforcement more nearly conforming to good practice for that purpose—about 0.25 per cent.

Another factor entering into this analysis is the restraint exerted upon the concrete body in question, preventing its freedom to move under stresses due to shrinkage or temperature changes. This restraint may take the form of vertical support (columns or piers) of horizontal members, such as decks, slabs or parapets, or of counterforts for retaining walls (in both cases these restraining factors may vary in their spacing or distance apart), or it may be the friction of the filling material behind non-counterforted walls, when movement occurs, which also restrains the free movement of the wall to changes due to temperature or to shrinkage in setting.

Still another factor entering into this problem, is that in connection with rise of temperature in concrete in the course of its manufacture due to the chemical interaction of its ingredients. This rise of temperature in concrete, in setting, affects the reinforcing steel so that when the concrete has set and is at the approximate temperature of the outer air, the reinforcing steel will have been subjected to that change of temperature resulting in deformations, including change of length, which will be contrary to the author's assumption that the length of the reinforcing steel in a wall of a certain length remains constant. This will affect the results obtained by the author in Equations (1) to (5).

Following this, the author assumes that "if there are to be no cracks, L must be infinite." This is only true for an infinite length of wall or other structure. For a given wall (without cracks), L is certainly definitely finite; it is the total length of wall, the length between expansion joints (if any), or the length between construction joints which are, in reality, partial expansion joints. A definite value for L will greatly alter the formula derived (Equation (6)) and, of course the subsequent equations derived therefrom.

After deriving the value of p (percentage of reinforcement) in Equation (10), the author remarks that it "is inversely proportional to S_s and z", which appear in the denominator, but z is a function of S'_c, which, in turn, is in the numerator of Equation (10), so that in reality the ratio is not an inverse one, but more or less complicated by this relationship.

Still another factor entering into the problem of stresses due to temperature changes (and for that matter those due to volume changes) is the probable presence of shearing stresses due to the fact that the exposed surfaces are subjected to greater changes of temperature than the laminæ farther removed from the surface, or in the interior of the mass, and the tendency to change their length varies, causing longitudinal shearing stresses which would have a further effect on the existence of pure tensile or compressive stresses assumed.

In assuming values for z, or coefficients of shrinkage, the question of curing must be considered. A value of 0.0005 for z assumes no attempt at curing and, even so, it is somewhat high except for higher strength concretes. Coefficients of shrinkage for concrete that has been protected against the evaporation of water for a period of ten days have been found to be as low as 0.00012 and with less care in this protection, between the foregoing figure and 0.00025, so that the value assumed by the author is 100% high and, therefore, will result in much higher stresses than would be true for cured concrete. While it is unfortunately true that little attempt is made to cure vertical members, such as columns, piers, walls, or abutments, the fact remains that such curing will result in the elimination of cracks due to that cause, and it is much cheaper to specify some form of curing—as, for example, playing a hose on the surface two or three times a day for 10 to 14 days—than the addition of reinforcement. On horizontal members, such as decks of bridges, there is no difficulty of maintaining curing conditions of one kind or another. As proof, the writer wishes to cite the fact, that on the approaches to the Lackawanna Railroad Bridge over the Hackensack River, consisting of girderless flat slabs on high columns, the contractor was persuaded to use as a curing agent the sand that was used later as the fine aggregate of the next section of viaduct.

Furthermore, the writer desires to call attention to the fact that the coefficient of shrinkage of concrete varies with the cement content as well as with the cement-water ratio. It is not in direct proportion, because the high-strength concretes up to 7 000 and 8 000 lb. per sq. in. (Freyssinet[12] writes seriously of the use of 15 000-lb concrete) behave differently in the matter of shrinkage than the ordinary concretes, and that behavior has not yet been definitely established. It will require careful research work to give more information on the subject of coefficients of shrinkage of higher strength concrete.

This criticism is not intended to detract from the value of the author's work because the writer appreciates the fact that it is one thing to initiate and develop a theoretical analysis and apply it to a practical use, and another to criticize the product thus presented.

While on the subject of shrinkage cracks, the writer wishes to take advantage of this opportunity to refute a statement frequently made, that no parapet or other surface of considerable length can be constructed of reinforced concrete without surface cracks developing at frequent intervals. In connection with improvements along the line of the Lackawanna Railroad since 1922, or before, a number of long flat-slab viaducts have been designed and built with expansion joints 250 to 300 ft. apart, both with fascia columns and with cantilever fascias. In only a few instances have any cracks appeared and these have been traceable to other causes, such as unequal settlement of foundations on compressible soils or on piles. These structures have stood the test of quite a few years of service and without an undue amount of temperature reinforcement.

The paper by Mr. Vetter should tend to emphasize the need of proper curing of concrete so as to avoid the rapid evaporation of the moisture content which is the main cause of the excessive volume changes that occur in uncured concrete.

WALTER DREYER,[13] M. AM. SOC. C. E. (by letter).—The writer is interested in Mr. Vetter's conclusion that from 0.60 to 0.75% of reinforcement is required in reinforced concrete in order to prevent unsightly cracks or water leaks. The Pacific Gas and Electric Company has constructed a number of reinforced concrete flumes and the prevention of unsightly cracks in these structures has always been one of the design problems. The earlier installations (1918 to 1920) followed the recommendation[14] of the Joint Committee on Concrete and Reinforced Concrete, which called for a transverse reinforcement of 0.3 per cent. These structures developed cracks at intervals, more or less regular, of 20 to 25 ft. These cracks are sufficient in size to permit water to seep through with consequent frost or ice action tending to increase the size of the cracks.

About 1926 preliminary studies were made on a hydro-electric project to be constructed on the Mokelumne River. These studies soon indicated that about 20 miles of reinforced concrete flume with a capacity of 550 cu. ft. per sec., would be constructed. About this time it also became necessary to construct a section of flume of nearly the same size and 1 600 ft. in length on another of the Company canals. It was decided to install varying quantities of longitudinal reinforcement in this flume in order to ascertain experimentally the amount of steel required to give satisfactory results. The information obtained from the experiment was to be used as a basis for the design of the longer Mokelumne flume.

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[12] *Civil Engineering*, February, 1932, p. 93.

[13] Asst. Chf., Div. of Civ. Eng., Pacific Gas & Elec. Co., San Francisco, Calif.

[14] *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 1140.

The flume is 10 ft. wide by 7 ft. deep, the side-walls and floor having an average thickness of 6 in. The 1 600-ft. length was divided into four equal sections, each with a different quantity of longitudinal steel. The first section has 0.27%; the second, 0.31%; and third, 0.47%; and the fourth, 0.65%; no expansion joints were provided. Before water was turned in, the shrinkage cracks were marked and stationed. During the first part of September, 1928, the section was inspected while carrying 500 sec-ft. of water and the comparative size of the cracks and the amount of leakage were noted.

At that time, the section having 0.25% longitudinal wall reinforcement showed cracks which developed leaks sufficient in size to cause water to drip from a shoulder left by the outside forms at the bottom of the wall. These main cracks were spaced in general from 15 to 25 ft. apart. In the section having 0.31% reinforcement, the main cracks were spaced in general from 15 to 35 ft. apart, the leaks were smaller, and, in many cases, the wall was only damp to the bottom. In the section having 0.47% wall reinforcement, the main cracks averaged about 25 ft. apart, and the wall in the majority of cases was only wet or damp. In the section having 0.65% wall reinforcement, the cracks were very small and were spaced 75 to 100 ft. apart.

Since the quantity of water diminished to a small flow before the advent of cold weather, no observation was made under winter conditions. On May 22, 1929, an observation was made with more than 500 sec-ft. in the canal, the water temperature being 41° Fahr. It was observed that in the section having 0.27% longitudinal reinforcement, the leaks in general had increased and one in particular (which appeared to be at the end of a day's pour) had reached the proportions of approximately 0.5 gal. per min. Several leaks showed efflorescence and, in one case, disintegration, which was probably due to ice and frost during the winter. An increase in leaks and damp spots was also observed in the section having 0.31% reinforcement, although to a lesser degree. In both the aforementioned sections the amount of leakage was considered undesirable for such structures as reinforced concrete flumes when severe winters prevail. The leakage in the first section could endanger certain types of soil foundations, where bench flumes are concerned.

The last two sections, having 0.47% and 0.65% reinforcement, showed satisfactory conditions, outside of an increase in number of sweat spots. The last section has few surface cracks for this type of construction.

As a result of this test it was concluded that at least 0.65% of longitudinal steel was required for satisfactory construction. This conclusion, arrived at entirely by experiment, is almost identical with that reached by Mr. Vetter as the result of a theoretical study.

The Mokelumne flume, as a result of the foregoing information, was designed and constructed with contraction joints at intervals of 20 ft. This construction, using about 0.2% longitudinal steel between the joints, which were made with a copper water-stop, proved to be less expensive than providing the quantity of longitudinal steel required to prevent cracking.

RALPH E. SPAULDING,[15] M. AM. SOC. C. E. (by letter).—Volume changes in reinforced concrete, particularly with reference to building structures, have interested the writer for more than twenty years. The only volume change in reinforced concrete building structures that is worthy of attention as disturbing the respective computed stresses in the concrete and steel is the change of shrinkage due to drying. The amount of such shrinkage is somewhat variable, depending upon many factors, but its average is about 0.00035. In many cases it is considerably greater. The shrinkage of the structure between expansion joints (or over the structure as a whole if there are no expansion joints), if there is no steel reinforcement, would be the amount of the shrinkage factor (0.00035) times the distance between expansion joints.

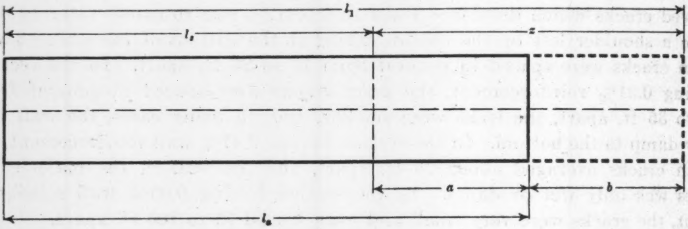

FIG. 7.

When steel reinforcement is introduced the shortening becomes a factor of the percentage of steel, the elasticity of the concrete, and the elasticity of the steel.

This is best illustrated, perhaps, by reference to Fig. 7. In addition to Mr. Vetter's notation, let

l_1 = original unit of length as cast = 1.0;
l_s = length the unit would assume if there were no steel to resist the shrinkage = $1 - z = 0.99965$;
l_a = length actually assumed by the reinforced unit when shrinkage is resisted by the steel;
a = induced stretching in the concrete;
b = induced shortening in the steel;
f_{cs} = unit stress in concrete due to the induced stretching; and,
f_{ss} = unit stress in steel due to induced shortening.

The actual unit shortening of the reinforced concrete, b, may be computed by:

$$b = \frac{\dfrac{z}{np}}{1 + \dfrac{1}{np}} \quad \text{..............................} (39)$$

and the unit stretching of the concrete by,

$$a = z - b \quad \text{..............................} (40)$$

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[15] Cons. Engr., Jacksonville, Fla.

The comparative values of f_{cs} and f_{ss} are not a direct factor of n, as Mr. Vetter seems to indicate by his Equation (1), but vary with p and the respective elasticities as well. For example, assume that $A_c = 200$, $A_s = 1$, and $n = 10$. From Equation (39), $b = 0.0003333$; and from Equation (40), $a = z - b = 0.00001667$. Then, $f_{cs} = a\,E_c = 50$ lb. per sq. in. tension, and $f_{ss} = b\,E_s = 10\,000$ lb. per sq. in. compression

To study its practical application in a structure, assume a structural column, 12 in. square, having eight $\frac{1}{2}$-in. square reinforcement bars. In that case, $A_c = 144$, $A_s = 2$, and $p = 0.01389$. Assume that the breaking strength of concrete $= 2\,500$ lb. per sq. in., $E_c = 2\,500\,000$, $n = 12$, and $z = 0.00035$. By Equations (39) and (40), $b = 0.0003$ and $a = 0.00005$, respectively; $f_{cs} = a\,E_c = 125$ lb. per sq. in. tension, and $f_{ss} = b\,E_s = 9\,000$ lb. per sq. in. compression. Therefore, at no load the column steel is pre-stressed to 9 000 lb. per sq. in. and the concrete is negatively stressed to 125 lb. per sq. in. Beyond this point the changes of stress in the concrete and steel, due to loadings, are directly proportionate to n.

The load on the column necessary to bring the concrete stress down to zero would be $f_{cs}\ (A_c + n\,A_s) = 125\ (144 + 12 \times 2) = 21\,000$ lb. total load, and, at this point, the stress in the steel would be, $f_{sz} + 125\ n = 9\,000 + (125 \times 12) = 10\,500$ lb. per sq. in.

If the total load on the column (according to design) is 100 000 lb., the stress in the concrete would become 470 lb. per sq. in. and the total final stress in the steel, 16 143 lb. per sq. in., whereas as computed by ordinary methods, without regard to shrinkage, the computed unit stresses would have been 595 and 7 143, respectively.

As another example, consider a round column having eight $\frac{3}{4}$-in., round, vertical reinforcement bars, and in which $A_c = 113$, $A_s = 3.536$, and $p = 0.0313$. Assume that $n = 10$, $z = 0.00035$, and the breaking strength of concrete is 3 000 lb. per sq. in. The column was originally designed for a total load of 178 000 lb., or with $f_c = 1\,200$ lb. per sq. in. and $f_s = 12\,000$ lb. per sq. in. (neglecting shrinkage in the concrete). The true stresses due to the inevitable shrinkage in the concrete are found by Equations (39) and (40) to be $b = 0.000266$ and $a = z - b = 0.000084$. As before, $f_{cz} = a\,E_c = 252$ lb. per sq. in. induced tension, and $f_{sz} = b\,E_s = 7\,980$ lb. per sq. in. induced compression.

Applying the computed load of 178 000 lb., this load would produce stresses (exclusive of the shrinkage stresses) of 1 200 lb. per sq. in. in the concrete and 12 000 lb. per sq. in. in the steel. Therefore, the true stresses which exist in the column as actually built are: $f_c = 1\,200 - f_{cz} = 1\,200 - 252 = 948$ lb. per sq. in., and $f_s = 12\,000 + f_{sz} = 12\,000 + 7\,980 = 19\,980$ lb., per sq. in., the actual stress in the concrete being only about 80% of the computed stress by ordinary formulas, and the actual stress in the vertical steel being about 165% of the computed stress.

It is evident then that the shrinkage in the concrete produces great changes in the computed stresses in both the concrete and the steel, reducing the concrete stresses and increasing the steel stresses. Furthermore, the shrinkage value of 0.00035 is by no means unusual, often it is much greater; certain it is that there is always some shrinkage of the concrete in a building structure, which produces approximately the foregoing results. Many designers are anxious to pre-stress the vertical steel in reinforced columns, when as a matter of fact the vertical steel is always pre-stressed, this being taken care of by the nature of the concrete itself.

This same shrinkage of concrete takes place, of course, in the beams and slabs of the structure, as well as in the columns, but its results are more complicated. Consider a typical floor-slab, as shown in Fig. 8, in connection with the following assumptions: 2 000-lb. concrete; $E_c = 2\,000\,000$; $n = 15$; w = load = 325 lb. per sq. ft.; l = span = 12 ft.; and W = total load per foot of width = 3 900 lb. The slab is continuous over many spans.

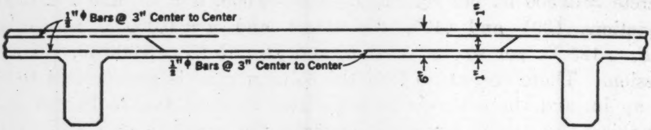
½" ϕ Bars @ 3" Center to Center
1"
½" ϕ Bars @ 3" Center to Center
6"
1"

FIG. 8.

With such a floor-slab (neglecting shrinkage of concrete), considering the reinforcement as shown, the true bending moments are somewhat less than $\frac{1}{24}\,Wl$ between supports and somewhat more than $-\frac{1}{12}\,Wl$ over the supports, being about $0.0365Wl$ and $-0.0895Wl$, respectively.[16] Still assuming no shrinkage in the concrete the computed maximum stresses in the concrete and steel at the total load of 325 lb. per sq. ft. would be about as follows: Between supports, $f_c = 350$, and $f_s = 6\,250$; over supports, $f_c = 675$, $f_s = 16\,000$, and $f'_s = 5\,300$.

Neglecting all loading for the time being, and computing the stresses due to shrinkage, taking the shrinkage factor as 0.00035, and using Equations (39) and (40), it is found that over the supports the concrete is stressed to 172 lb. per sq. in. tension and the steel (both upper and lower layers) is stressed to 7 900 lb. per sq. in. compression. Between the supports—since the steel is all in one face, and, therefore, eccentric with the center line of tension of the concrete as it shrinks—the concrete usually ruptures on the upper face, either in minute "hair"-cracks, or, if there has been a construction joint in the center of the span, by a definite single crack easily observed. If no rupture takes place the stresses in the concrete and steel, by Equations (39) and (40), will be approximately, for concrete, 98 lb. per sq. in. tension, and for steel, 9 000 lb. per sq. in. compression, which is the reverse of the condition for which it was designed. In either event the slab between supports is unable to resist any positive moment until it has deflected sufficiently to overcome the shrinkage in the upper face.

As load is applied to the slab it is resisted at first only by the cantilever action over the supports. At these points the moment couple of the steel in the upper part, and both the steel and the concrete in the lower part, carry the entire load until the slab deflects sufficiently to overcome the shrinkage in the compressive concrete, and until that takes place the bending moments are: Over the supports, $M = -\frac{1}{8}\ Wl$; and between the supports, $M = 0$. The moments continue, as stated, until the concrete computed for compression is brought into bearing. For all practical purposes this is accomplished when the induced tensile stress in the compression concrete over the supports (f_{cs}) is reduced to 0. Since this concrete (normally in compression) now has an induced tension of 172 lb. per sq. in., the load necessary to overcome that would be the amount of loading which would normally (not considering shrinkage) stress the concrete over the supports to 172 lb. per sq. in. at a bending moment of $-\frac{1}{8}\ Wl$, or computing by regular formulas, $W =$ about 680 lb. ($w =$ about 56 lb. per sq. ft.). At this time the added stress in the compressive steel over the supports is about 1 310 lb. per sq. in.

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[16] See curves for bending moments in concrete beams with variable moments of inertia, Concrete Engineers' Handbook, Hool and Johnson, N. Y., 1918, Article 54, pp. 339–341.

Beyond this point, and up to the total loading of $W = 325 \times 12 = 3\,900$ lb., or an added loading of $3\,900 - 680 = 3\,220$ lb., the additional stresses would be those computed by using the then true bending moments of $-0.0895\ Wl$ over supports and $0.0365\ Wl$ between supports. These additional unit stresses are approximately: Over the supports, $f_s = 13\ 200$; $f_c = 580$; and $f'_s = 4\ 370$; and between supports, $f_s = 5\ 150$; and $f_c = 290$.

At full loading the stresses in the tensile steel both over and between the supports are sufficiently great so that the concrete on these faces can no longer carry tension, and under full load any tensile stresses in the concrete in these faces can be eliminated as is regularly the practice in such calculations. Combining the stresses obtained the results are as indicated in Table 2.

TABLE 2.—FULL LOAD STRESSES IN CONCRETE AND STEEL, IN POUNDS PER SQUARE INCH

Stress symbol	Stress due to shrinkage of concrete	Stress due to enough load to bring concrete into compressive bearing	Stress due to remainder of load	Total final unit stress
Over supports:				
f_s	....	—3 900	—13 200	—17 100
f_c	—172	+172	+580	+580
f'_c	+7 900	+1 290	+4 370	+13 560
Between supports:				
f_s	....	....	—5 150	—5 150
f_c	....	....	+290	+290

Table 3 is a summary of all the cases as given, all of which are typical for any building structure.

It must be remembered that shrinkage always takes place in a reinforced concrete building structure, and produces the following results:

(*a*) It reduces the stresses in the concrete, usually from 15% to 25% less than the concrete stresses as predetermined by ordinary methods.

(*b*) It always acts to pre-stress the compressive steel, the actual stress being usually more than twice the stress as ordinarily computed; it greatly increases the efficiency of compressive reinforcement.

(*c*) It increases the negative moments over supports, and, of course, decreases the positive moments, causing much more of the load to be carried by cantilever action.

(*d*) It increases, somewhat, the stresses in the tensile steel over supports.

Taken as a whole the changes in stress brought about by this shrinkage of the concrete are not dangerous; on the contrary, although often annoying, they are rather advantageous. This is particularly true in the case of columns. When the shrinkage action causes stresses in the cantilever steel that are too high, those friends of ignorance—yield point and elastic recovery—step in and help to equalize the stresses. While this shrinkage is not a disadvantage in the ordinary or average case, nevertheless designing engineers should bear in mind that the shrinkage is inevitable, and there may be occasional cases where to overlook its existence would result in danger to the structure.

TABLE 3.—TYPICAL STRESSES IN CONCRETE AND STEEL FOR REINFORCED CONCRETE STRUCTURES

Member	Stresses as computed, neglecting shrinkage of concrete	Approximate true stresses because of shrinkage of concrete	Approximate variation increase or decrease, percentage
Column No. 1:			
f_s	+7 140	+16 140	+126
f_c	+595	+470	−21
Column No. 2:			
f_s	+12 000	+19 980	+66
f_c	+1 200	+950	−21
Floor-Slab:			
Over supports:			
f_s	−16 000	−17 100	+7
f_c	+675	+580	−14
f'_s	+5 300	+13 560	+156
Between supports:			
f_s	−6 250	−5 150	−17
f_c	+350	+290	−17

A. H. BEYER,[17] M. AM. SOC. C. E., and A. G. SOLAKIAN,[18] ASSOC M. AM. SOC. C. E. (by letter).—The writers have made an attempt to check by photo-elastic methods[19] the theoretical analysis made by Mr. Vetter of the stresses in reinforced concrete due to volume changes. They have confined themselves to the simplest case, namely, the shrinkage stresses developed in a simple reinforced concrete member.

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[17] Prof. of Civ. Eng., Columbia Univ., New York, N. Y.

[18] Lecturer, Mech. Eng., and Research Associate in Civ. Eng., Columbia Univ., New York, N. Y.

[19] "A Treatise on Photo-Elasticity," by Dr. E. G. Coker and Dr. L. N. G. Filon, Cambridge Univ. Press, Lond., 1931.

It was found that bakelite reinforced with aluminum rods has about the same relative physical properties as concrete reinforced with steel. The moduli of elasticity of aluminum and bakelite are, respectively, 675 000 lb. per sq. in., and 9 800 000 lb. per sq. in., giving a ratio of 14.5, compared with a ratio of 15 for concrete and steel. The coefficients of expansion at room temperature for bakelite and aluminum are about the same, namely, 0.000035 per 0° cent. Furthermore, the internal stresses developed in a specimen of bakelite reinforced with an aluminum rod during the processes of cooling and curing are qualitatively representative of the internal stresses developed in a reinforced concrete structure due to the shrinkage of the concrete.

The specimen analyzed consisted of a rectangular plate with a length three times its width and reinforced with an aluminum rod, 0.03 in. in diameter, all as shown in Fig. 9. It was made by pouring the molten material into a metal form, 1¼ in. deep, 2¼ in. wide, and 6 in. long, which contained at its geometrical axis an aluminum rod. The specimen was cooled in about two days in a special curing oven to room temperature. The two flat surfaces were then machined and polished so as to secure a specimen of a uniform thickness of 1⅛ in., with perfectly parallel faces.

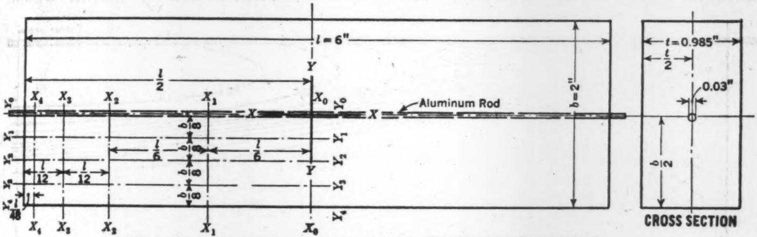

FIG. 9.—DETAILS OF REINFORCED BAKELITE SPECIMEN.

An internal stress distribution in this specimen is believed to be similar to the internal stress distribution developed by shrinkage in a reinforced concrete member unrestrained at its ends and reinforced at its center by one longitudinal rod.

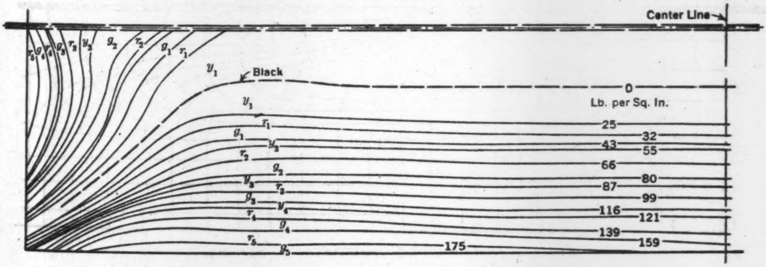

FIG. 10.—ISOCHROMATICS. LINES OF CONSTANT MAXIMUM SHEAR STRESS.

To determine the actual stress distribution, the specimen was placed in the path of circular polarized white light under which the color fringes were revealed. Fig. 10 shows the boundaries of the color fringes on one side of the middle transverse axis of the plate and for a thickness of specimen of 0.985 in. The colors—yellow, green, and red—are denoted by *y*, *g*, and *r*, respectively, and the subscript in each case denotes the order; thus, r_5 denotes fifth order red. In Fig. 10 each fringe line represents a contour along which the maximum shear intensity is constant. The evaluation of these stresses for each fringe line, technically known as isochromatic bands, was made by means of a test beam subjected to bending moments of known intensity.

The directions of the principal stresses were obtained from the isoclinic curves shown in Fig. 11. These curves were obtained by passing plane polarized white light through the specimen, using angular settings at intervals of 10° for the crossed Nicols,[20] always keeping the longitudinal axis of the specimen in a fixed horizontal position. For any given position of the Nicols, narrow black bands appeared, the so-called isoclinic bands, within which the direction of the principal stresses coincide with the axis of inclination of the Nicols.

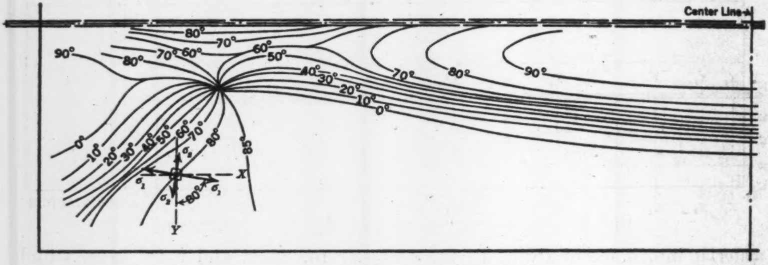

FIG. 11.—ISOCLINICS.

The principal stress trajectories thus obtained from these isoclinics by graphical methods are noted in Fig. 12 which shows at each point the direction of the two principal stresses, σ_1 and σ_2, but not their intensities. In order to obtain sharper and better defined isoclinic lines the thickness of the original specimen was reduced by machining to 0.328 in.

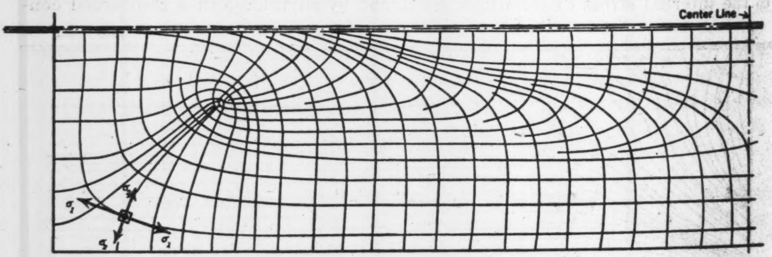

FIG. 12.—DIRECTIONS OF PRINCIPAL STRESSES.

Fig. 13 shows the isoclinic lines for a 20° inclination of the Nicols. Only the lines for the lower left quadrant are shown. The isoclinics for the upper

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[20] For a definition, see "A Treatise on Photo-Elasticity," p. 58.

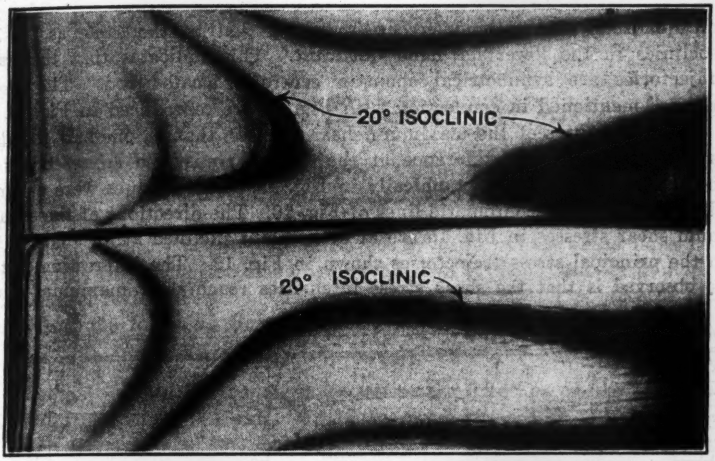
20° ISOCLINIC
20° ISOCLINIC

Fig. 13.—Twenty-Degree Isoclinics (Taken with Plane Polarized White Light).

Fig. 14.—Stress Fringes in a Test Specimen (Taken with Circular Polarized White Light).

left quadrant are the complementary values, that is, the same as for 70° isoclinics in the lower left-hand quadrant. This indicates that the stress trajectories are symmetrical about a centrally located rod. The "color fringes" mentioned in connection with Fig. 10 are reproduced in Fig. 14.

The intensities of the maximum shear stresses, that is, one-half the algebraic principal stress difference at the several transverse cross-sections of the specimen, are shown graphically in Fig. 15. These values were obtained directly from the isochromatic lines of Fig. 10. The directions of these maximum shear stresses in Fig. 15 are at every point inclined at an angle of 45° to the principal stress trajectories shown in Fig. 12. The interesting fact to be observed is that the shear stress intensities reach their maximum values where the reinforcement enters the specimen and, also, at the outer longitudinal edges of the specimen.

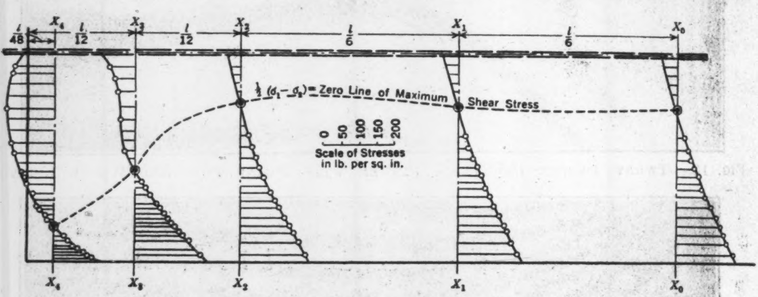

FIG. 15.—INTENSITIES OF MAXIMUM $\frac{\sigma_1-\sigma_2}{2}$ STRESSES.

The components of the principal stresses parallel and perpendicular to the axis of the reinforcing rod are computed from formulas based upon the assumption of a co-planor stress analysis. Although the actual stress distribution resulting from shrinkage is, strictly speaking, a three-dimensional one, it is believed that no large errors are introduced when the thickness of the specimen is relatively small compared to the width.

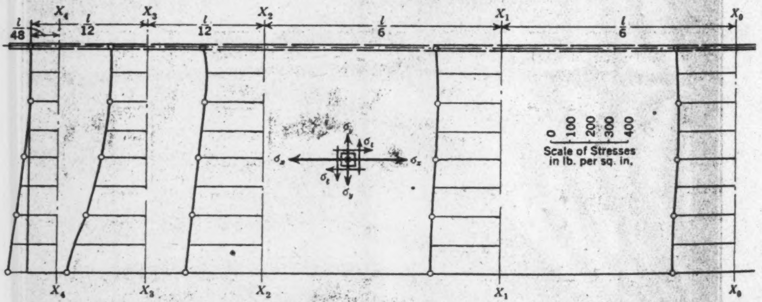

FIG. 16.—INTENSITIES OF LONGITUDINAL COMPONENTS OF MAXIMUM PRINCIPAL STRESSES.

Based upon a two-dimensional stress analysis, the Maxwell-Filon formula has been used to compute the stress intensity in the test specimen parallel to the axis of the rod, as follows:

$$\sigma_x = \sigma_{x0} - \int_{x0}^{x} \frac{\partial \tau}{\partial y}\, dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

in which, at any section, σ_{xo}, is a boundary stress parallel to the axis of the specimen. At this point, σ_{yo} is equal to 0, and $\partial\tau$ is the rate of change in the shear stress with respect to the y-axis. The shear stress, τ, is computed from the formula, $\tau = (\sigma_1 - \sigma_2) \sin\phi \cos\phi$. The values of σ_x have been evaluated at the various sections by graphical integration and its numerical values are shown in Fig. 16. The corresponding value for σ_y may be computed from the following formula:

$$\sigma_y = \sigma_x - (\sigma_1 - \sigma_2) \cos 2\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

In Equation (42) ϕ is the angle between the direction of the maximum principal stress and the X-axis.

The final results of the photo-elastic analysis that has been made are shown on Fig. 17. The upper curve (Fig. 17(*a*)), shows the average tensile stresses parallel to the axis of the specimen at the various sections in the specimen as a result of shrinkage. Note the short distance within which the tensile stresses, which amount to about 350 lb. per sq. in., become constant.

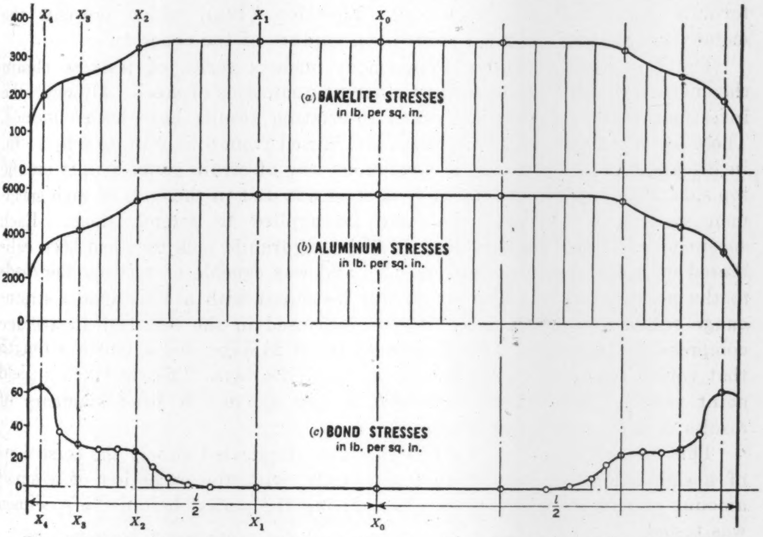

FIG. 17.—SHRINKAGE STRESSES DEVELOPED IN A REINFORCED BAKELITE SPECIMEN.

The middle curve (Fig. 17(*b*)), shows the corresponding compressive stresses in the aluminum rod. This is practically constant for two-thirds of the length of the rod and amounts to approximately 6 000 lb. per sq. in.

The bond stresses shown in the lower curve (Fig. 17(*c*)), reached the highest values at the ends, namely, about 65 lb. per sq. in., which gradually reduced to zero at about the quarter-points.

From the foregoing comments it is seen that the photo-elastic investigation made by the writers corroborated qualitatively the assumptions made by the author in computing by theoretical methods the maximum tensile stresses set up in the concrete, the compressive stresses in the steel, and the bond stresses between the steel and the concrete.

LESTER L. MEYER,[21] ASSOC. M. AM. SOC. C. E. (by letter).—The author's detailed analysis of this important subject is fundamental and complete. It is a subject that has been little discussed in this fashion in current engineering literature, although the method of design has been used by the Los Angeles Water Department for reinforcing concrete slabs on the face of earth dams. The design formulas used were those derived by Julian Hinds, M. Am. Soc. C. E., which are essentially the same as those given by the author, with the exception of the bond assumption which Mr. Hinds assumes to vary uniformly from a maximum at the crack to zero at the center of the block. This difference in the bond assumption, however, does not affect the only really useful formula for actual design, namely, Equation (12*a*), which prevents the metal from passing the yield point upon rupture of the concrete.

The Los Angeles Water Department made a series of tests to obtain the characteristics of slabs having different quantities of steel. Although the investigation is not complete, some interesting results have been obtained. These test slabs were 15 ft. in length and varied from 6 by 6 in. to 8 by 11 in. in cross-section. There was a continuous row of strain-gauge points on the top side, 10 in. apart. A loop of $\frac{3}{4}$-in. steel was cast in the ends of each specimen so that a tension load could be applied to stretch them. Each specimen was tested by stretching it with a hydraulic jack that had been calibrated on a Universal testing machine and was capable of reading the loads to the nearest 200 lb. The strain was measured with a Whittemore strain-gauge reading to 0.0001 in. The concrete used in the test had an average compressive strength of 2 800 lb. per sq. in. at 28 days, and a tensile strength that varied from 205 to 255 lb. per sq. in. at 28 days. The steel had a yield point varying from 45 000 to 58 000 lb. per sq. in. A brief summary of results is shown in Table 4.

This method of pulling the test specimen duplicated closely the conditions of a slab subject to temperature drop only, with the exception of a small amount of shrinkage that took place during the period before the specimen was tested.

As the slab was subjected to more and more tension, it expanded until at a unit expansion of from 0.00008 to 0.00011 the first crack occurred (this usually corresponded to a stress of about 200 lb. per sq. in. in the concrete). Application of additional load after the first crack had occurred, would never give a unit strain in an uncracked segment greater than this first strain just prior to the first crack. Additional load simply produced additional cracking and, hence, additional over-all expansion of the total length of the slab.

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[21] Asst. Civ. Engr., Bureau of Water-Works and Supply, Los Angeles, Calif. Mr. Meyer died on December 27, 1932.

Those familiar with concrete are aware that its tensile strength is much more erratic than its compressive strength and that carefully made concrete may vary widely in tensile strength in a single batch. For this reason the first crack usually occurs at the place of the weakest concrete in tension, and as the load increases cracks occur successively at places of increasing ultimate tensile strength. The uncracked segments remaining are a case of the survival of the strongest concrete. Table 4 shows that specimens having the greatest steel ratios developed the greatest ultimate tensile strength of the concrete. This variation in tensile strength makes it impossible to compare results of tests with the author's theories, except in a very general way.

TABLE 4.—Summary of Results of Tests on Concrete Test Slabs

Specimen No.	Steel ratio	Number of bars	Size of bars, in inches	Value of q	Age tested, in days	Maximum tensile strength developed in test, in kips	Maximum unit elongation of slab before reaching yield point
7b............	0.25	2	$\frac{1}{4}$	16.0	38	102	0.00010
6b............	0.35	2	$\frac{1}{4}$	16.0	39	205	0.00027
1b............	0.50	1	$\frac{1}{2}$	8.0	40	240	0.00065
2b............	0.75	2	$\frac{1}{2}$	8 0	41	290	0.00086
3b............	1.00	1	$\frac{5}{8}$	6.4	42	260	0.00105
4b............	1.25	2	$\frac{5}{8}$	6.4	46	360	0 00108
5b............	1.50	2	$\frac{5}{8}$	6.4	45	394	0.00114
1a............	0.50	1	$\frac{3}{4}$	5.3	53	202	0.00129
3a............	0.49	2	$\frac{1}{2}$	8.0	53	223	0 00102
5a............	0.50	4	$\frac{3}{8}$	10.7	53	227	0.00099
6a............	0.50	9*	$\frac{1}{4}$	16.0	53	241	0.00103

Specimen No.	Temperature drop corresponding to maximum elongation, in degrees	Temperature drop from Equation (27), in degrees	Average opening of cracks at maximum temperature drop, in inches	Average distance between cracks before yield point was reached, in inches	Average distance between cracks after yield point, in inches	$L_{\text{min.}}$ from Equation(12f) (Table 1), in inches†
7b............	15	110	0.0062	120	120	50
6b............	40	160	0 0122	70	70	36
1b............	96	137	0 0111	20	16	50
2b............	129	118	0.0208	22	15	33
3b............	157	84	0.0161	16	8	31
4b............	161	98	0.0210	22	9	25
5b............	171	95	0.0260	22	10	21
1a............	193	115	0 0310	25	20	75
3a............	150	127	0.0305	35	12	51
5a............	148	144	0.0248	30	10	37
6a............	153	138	0.0201	16	15	25

* Plain bars, all others deformed.
† Assuming unit tensile and bond strength equal.

Another indeterminate factor is the bond stress. D. A. Abrams, M. Am. Soc. C. E., found[22] that as the bar slips, the unit bond resistance increases and that in the case of deformed bars the bond stress may reach three or four times the tensile strength after considerable slip has taken place. For plain bars the maximum bond stress, which occurs approximately after a movement of 0.01 in., is more nearly equal to the tensile strength. These ideas, as well as the author's Equation (12*f*) (Table 1), are verified somewhat by comparing the results of the test with Equation (12*f*), as shown in Table 4, assuming the bond stress equal to the tensile strength. It is noticed that the plain bars used in the specimen, No. 6*a*, show the closest agreement with Equation (12*f*). The other specimens, in which deformed bars were used, indicate unit bond stresses from two to four times the tensile strength.

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[22] *Bulletin 71*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill.

Opening of Cracks.—At rupture, the cracks opened from 0.004 to 0.02 in., the average being 0.01 in. On removing the load the crack would close from 50 to 70% of the amount it had opened; but in no case would it close entirely regardless of the amount of steel. In two instances invisible cracks occurred before the load was applied. At point of failure of the next crack, these cracks had opened only a small amount compared with those rupturing during stress. Probably a considerable part of the opening of the cracks is due to bond slip caused by impact stress at the time of rupture.

The writer believes that the amount the cracks open, is an important consideration. Obviously, it was neglected by the author because of the impossibility of treating it by pure mathematics. However, further experimental work might bring out the factors governing the width of cracks. By observing the summary of the results of the test, it is seen that although some specimens could resist a temperature drop of 150° Fahr., or more, the cracks produced would have widths of 0.03 in., or more. In hydraulic structures this would permit considerable water leakage; it would be unsightly, and, in some cases, dangerous. The maximum width of cracks that should be tolerated in a structure depends, of course, on its use. For hydraulic structures, a crack of more than 0.005 in. in width probably will not seal itself by autogenous healing and swelling as is commonly the case for fine cracks in new concrete hydraulic structures. If cracks could be forced to occur at more frequent intervals by moulding indentations in the slab at predetermined intervals, the cracks might be made smaller. The effect of such indentation would be virtually to increase the steel ratio, which (according to the author's formula) should produce cracks at more frequent intervals.

The writer believes that it is apparent from the results of the tests that an equation for maximum temperature drop should be based on a limiting stress in the steel at the crack, and not on the critical penetration of bond stresses into the uncracked segment; however, the tests also show that when Equation (12*a*) (Table 1), is satisfied, there is more than ample provision for temperature requirements.

C. P. VETTER,[28] ASSOC. M. AM. SOC. C. E. (by letter).—The contributions to the discussion of this paper have been most gratifying. Many interesting points have been raised which were insufficiently or less clearly dealt with in the paper. This refers especially to the influence of Poisson's ratio and the effect of a difference in the coefficients of thermal expansion of concrete and steel.

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[28] San Francisco, Calif.

The writer wishes to thank those who have thus, by constructive criticism, helped to make the paper serve its purpose, that is, to throw some additional light on a subject that has been much neglected.

Mr. Slack points out that the thermal coefficients of expansions for concrete and steel are not actually equal, as assumed in the paper. This assumption of equal coefficients of expansion was introduced purposely in order to facilitate the general conception of the subject rather than to simplify the mathematics. Even with this simplification the subject is somewhat intricate and difficult to present clearly. Furthermore, the term, thermal coefficient of expansion (and contraction), when applied to concrete, is not the well-defined quantity Mr. Slack seems to indicate. It changes with the brand of cement, with the richness of the mix, and with the age of the specimen. It is not the same for increasing and decreasing temperatures, nor for wet and dry specimens. Carefully executed tests[34] of dry specimens, 90 days old, have given a mean value of 0.69×10^{-5} for increasing temperature and 0.56×10^{-5} for decreasing temperature, as against a value for steel of 0.67×10^{-5}.

A difference of 0.15×10^{-5} between the coefficients for steel and concrete, as suggested by Mr. Slack, seems high, but even so, the effect on the formulas developed in the paper is quite negligible. For example, Equation (12*a*), Table 1, which gives the ratio of reinforcement required for a temperature drop less than the critical, becomes:

$$p = \frac{S'_c}{S_s - n S'_c - E_s t\,(\epsilon_s - \epsilon_c)} \qquad \text{(43)}$$

in which, ϵ_s and ϵ_c denote the thermal coefficients for steel and concrete, respectively. Using the numerical values given by Mr. Slack: $\epsilon_s - \epsilon_c = 0.15 \times 10^{-5}$, $t = 50°$ Fahr., and, furthermore, when $S'_c = 300$ lb per sq in., $S_s = 50\,000$ lb per sq in., $E_s = 30.10^6$, and $n = 10$, Equation (12*a*) gives $p = 0.0064$. By Equation (43), $p = 0.0067$. The difference is not impressive bearing in mind that both S'_c and n may vary as much as 25% even in carefully conducted laboratory tests.

Mr. Slack properly draws attention to the considerable effect of a difference between the thermal coefficients in a reinforced concrete unit which is otherwise free to contract. This subject is outside the scope of the paper which deals with restrained units only. Due to the restraint, such units are subject to quite different forces. Mr. Slack's numerical calculations are somewhat approximate. The exact formula for the steel tension in an unrestrained unit is:

$$\phi'_s = \frac{E_s t\,(\epsilon_s - \epsilon_c)}{1 + n p} \qquad \text{(44)}$$

which, for $p = 0.01$, $t = 50°$ Fahr., $\epsilon_s - \epsilon_c = 0.15 \times 10^{-5}$, $n = 10$, and $E_s = 30 \times 10^6$, gives $\phi'_s = 2\,050$ lb per sq in., and for $p = 0.02$, $\phi'_s = 1\,870$ lb per sq in., as against Mr. Slack's 2 250 lb per sq in.

Mr. Whitney lists a number of factors that have not been taken into account, such as plastic flow and variations of the modulus of elasticity and of the coefficient of expansion of the concrete, and concludes:

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[34] *Deutsche Ausschuss für Eisenbeton, No. 23.*

> "The problem is very much more complicated than the author's analysis indicates, so much so that it can not be solved by the formulas presented. No general solution is possible for the variety of structures mentioned."

The writer fully realizes the importance of these factors, which have been disregarded in his analysis. However, it should not be overlooked that these same factors likewise enter into and materially effect every design of every reinforced concrete structure. There is no doubt but that the ordinary methods used in computing "stresses" in reinforced concrete fall far short of the conciseness that engineers are wont to demand when dealing with other materials; yet innumerable structures have been designed by the use of these short-cut methods, not because of ignorance of their shortcomings, but because knowledge of the factors enumerated by Mr. Whitney is as yet so vague that their inclusion in many problems seems a wasted effort, and because the elimination of these factors does not endanger the ordinary structure. When it comes to provisions for shrinkage and temperature variations, not even a pretense of theoretical consideration has been made and (what is far worse), the structures designed by the celebrated rule, "three-tenths of one per cent. for temperature," have failed utterly in this respect, as every large floor, every dam, every retaining wall, and every flume lining bear witness. The writer has attempted to open this question to discussion from a rational and scientific point of view and it is regrettable that Mr. Whitney did not extend his discussion to show how the factors mentioned by him could be taken into account for more rational design.

Mr. Hirschthal raises a number of interesting points, the discussion of which will help greatly to clarify the problem. He suggests that in a concrete unit restrained in two perpendicular directions the effect of Poisson's ratio will materially reduce the reinforcement required to control cracks in either of the two directions. Unfortunately, this is not so. Let it be assumed, as suggested by St. Venant, that failure of a material in tension takes place when the elongation in one direction has reached a critical value. Let it further be assumed that the unit considered is both infinitely long and infinitely wide, or that it is equally restrained and equally reinforced in two directions. Then the stresses in the two directions must also be equal at a given moment and the elongation of the concrete per unit length, e_c, for a given stress, f'_c will be expressed by:

$$e_c = \frac{1}{E_c} f'_c \, \frac{m-1}{m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

in which, m is Poisson's ratio. Equation (1) then becomes:

$$\frac{f'_s}{E_s} = z - \frac{f'c \, \frac{m-1}{m}}{E_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

and Equation (2):

$$f'_s = zE_s - n \, \frac{m-1}{m} f'_c \ldots\ldots\ldots\ldots\ldots\ldots\ldots (47)$$

Equation (3) and Equation (4) remain unchanged, and Equation (5) becomes:

$$L = \frac{(f'_c)^2}{n\, p^2\, q\, u\, \left(z\, E_c - \frac{m-1}{m} f'_c\right)} \quad \ldots\ldots\ldots\ldots\ldots(48)$$

In going from Equation (48) to the equivalent of Equation (11e), Table 1, it must be remembered that the stress at which fracture takes place is expressed by $S'_c \frac{m}{m-1}$, if S'_c is the breaking strength determined on the testing machine; therefore,

$$L = \frac{(S'_c)^2 \left(\frac{m}{m-1}\right)^2}{n\, p^2\, q\, u\, (zE_c - S'_c)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(49)$$

and Equation (9) becomes:

$$f_s = f'_c \left(\frac{1}{p} + n \frac{m-1}{m}\right) - z\, E_s$$

from which,

$$p = \frac{f'_c}{f_s + zE_s - n \frac{m-1}{m} f'_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots(50)$$

which, in turn, for $f_s = S_s$ and $f'_c = \frac{m}{m-1} S'_c$, becomes:

$$p = \frac{S'_c \frac{m}{m-1}}{f_s + z\, E_s - n\, S'_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(51)$$

which corresponds to Equation (10). Inserting in Equation (51) $f_s = S_s$ and $z = \frac{S'_c}{E_c}$, which is the value that in Equation (49) makes $L = \infty$, Equation (51) becomes:

$$p = \frac{S'_c \frac{m}{m-1}}{S_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(52)$$

which is equivalent to Equation (11a), Table 1.

The value of m for concrete is not well established, but it is probably about 4; therefore, in a reinforced concrete unit restrained in two directions, the reinforcement in each direction for shrinkage should be 33% greater than would be the case in a unit restrained in only one direction.

By similar reasoning, for temperature drop, Equation (12e), Table 1, takes the form:

$$L = \frac{(S'_c)^2 \left(\frac{m}{m-1}\right)^2}{n\, p^2\, q\, u\, (E_c\, \epsilon\, t - S'_c)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(53)$$

and Equation (12*a*), the form:

$$p = \frac{S'_c \dfrac{m}{m-1}}{S_s - n\,S'_c} \qquad \text{(54)}$$

By similar manipulation of Equations (25) to (30), inclusive, it will be seen that Equation (27) remains unchanged and Equation (30) becomes:

$$T = \frac{S_s - \dfrac{S'_c \dfrac{m}{m-1}}{2\,p}}{\epsilon\,E_s} \qquad \text{(55)}$$

and, therefore, Equation (12*c*), Table 1, becomes:

$$p = \frac{S'_c \dfrac{m}{m-1}}{2\,(S_s - T\,\epsilon\,E_s)} \qquad \text{(56)}$$

It appears to be a general rule that in a structure, subject to shrinkage and temperature drop, which is continuous or restrained in two directions, the reinforcement in each direction should be $\frac{m}{m-1}$ times the reinforcement required in structures that are restrained in only one direction. This ratio has the approximate value, 1.33.

Mr. Hirschthal further suggests that shrinkage and temperature stresses in a concrete unit may be influenced by the restraint exerted on the unit by columns, piers, counterforts, and by friction against the foundation. The writer cannot believe that such restraining influences can have any material effect on concrete structures such as those discussed in the paper. Before any cracks have formed, there is no movement of any points of the unit. In the case of shrinkage the shortening of any element equals the elongation due to tension, and the same is the case for temperature drops. When the tension reaches the breaking strength of the concrete, a crack will form and the concrete elements near the crack will move an infinitely small distance away from it. Aside from these small elastic movements, which change directions on either side of the crack, there is no movement of the unit as a whole. It is inconceivable that friction, for instance, can have any appreciable influence on movements of this kind; it would be analogous to say that a steel rod in a testing machine would show a measurably higher strength because it was surrounded by sand which exerted friction against the rod. However, the restraining influences mentioned by Mr. Hirschthal are of considerable importance in another respect in that they cause a concrete unit, which is not infinitely long or not absolutely restrained, to follow approximately the same laws as those established for very long units. In a road pavement provided with expansion joints, for example, the friction may cause a comparatively short unit between joints to act as a restrained unit, and if it is insufficiently reinforced it may crack between the expansion joints.

In his closing remarks, Mr. Hirschthal emphasizes the importance of curing concrete structures in order to minimize shrinkage; the writer fully concurs in this, but the statement that curing for 10 days will reduce shrinkage to about one-fourth, is at variance with the data obtained by other investigators. Fig. 18 gives results of a representative series of tests.[24] Based on these tests it would require curing under water for about 80 days to obtain the reduction in shrinkage that was obtained by Mr. Hirschthal in 10 days. The writer would hazard the guess that Mr. Hirschthal's specimens have not been under observation for a sufficient length of time.

Mr. Dreyer makes a most valuable contribution to the discussion of shrinkage and temperature cracks. The series of tests conducted by the Pacific Gas and Electric Company is the only known rational attack on that much cherished belief that 0.3% reinforcement will prevent cracks in concrete. Mr. Dreyer's concluding remarks: "As a result of this test it was concluded that at least 0.65% of longitudinal steel was required for satisfactory construction," are most gratifying, and the writer believes that had other organizations and individuals had the courage to face the issue in the same manner, considerable expense and the construction of many "eyesores" would have been avoided.

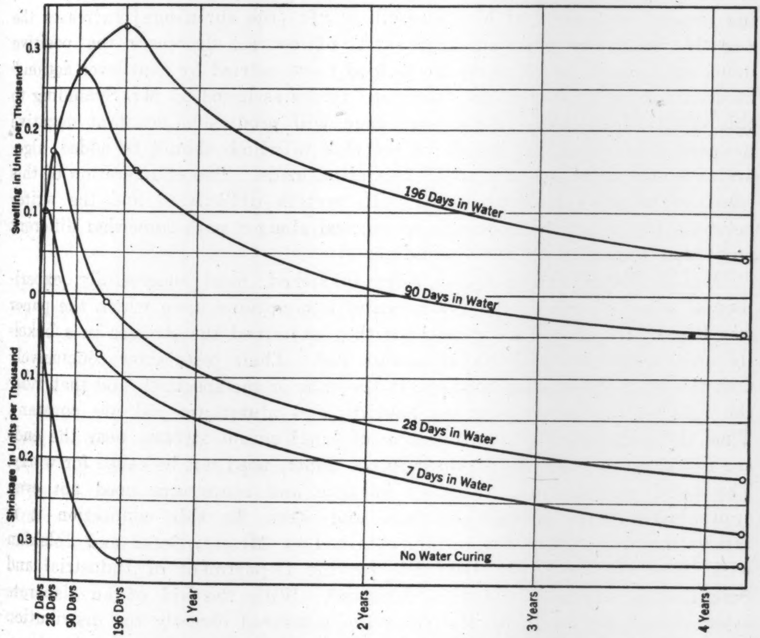

FIG. 18.—SHRINKAGE OF CEMENT MORTARS, 1 : 3, SUBJECTED TO WATER CURING FOR VARIOUS LENGTHS OF TIME.

Mr. Spaulding discusses in some detail the effect of shrinkage on an unrestrained reinforced concrete unit for the behavior of which he establishes Equation (39). Contrary to the opinion expressed by Mr. Spaulding, this equation can be obtained directly from the writer's Equation (9), by making $f_s = 0$, thus expressing that the unit is unrestrained. Equation (39) may also be written:

$$f'_c = \frac{z\,E_c}{\frac{1}{np} + 1} \qquad (57)$$

in which form it was given, about twelve years ago, by Professor Emil Mörsch.[25] Since Professor Mörsch's treatment is original, complete, and generally known, the writer saw no reason for including the treatment of unrestrained units in his paper.[26]

Mr. Spaulding goes on to investigate the effect of shrinkage on a reinforced concrete floor-slab. Although the writer fully agrees with the following general statement of Mr. Spaulding: "It [the shrinkage] increases the negative moments over supports, and, of course, decreases the positive moments, causing much more of the load to be carried by cantilever action," he does not fully agree to the numerical values assigned by Mr. Spaulding to this effect of shrinkage. The shrinkage will produce a constant negative moment throughout the length of the slab to which should be added, algebraically, the moments produced by external loads. The enumeration of this constant negative moment meets with certain difficulties, and the writer believes that it will be necessary to proceed along a path somewhat different from that suggested by Mr. Spaulding.

Messrs. Beyer and Solakian have furnished, most successfully, experimental proof of one of the fundamental assumptions upon which the paper was based. By photo-elastic methods they examined the stresses in a bakelite unit reinforced with an aluminum rod. Their tests show conclusively that the bond stresses are confined to the ends of the specimen and that, near the middle, the stresses in the bakelite and aluminum rod are constant. That the increments per unit length of longitudinal stresses near the ends are not constant, such as assumed in the paper, need not be cause for worry. The bond characteristics between bakelite and aluminum need not even approximate those between concrete and steel. In this connection it is interesting to compare the results obtained by Messrs. Beyer and Solakian with those of Mr. W. H. Glanville for the Department of Industrial and Scientific Research, in London, England. With the aid of an elaborate extensometer arrangement Mr. Glanville measured directly the distribution of the strain in a steel rod encased in a concrete unit subject to shrinkage. One of his diagrams is shown in Fig. 19. It is seen that the increment per unit length of strain in the steel rod is almost exactly constant near the end of the specimen and zero near the middle.

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[25] "Der Eisenbetonbau," Stuttgart, 1920, p. 127.

[26] For reference to Prof. Mörsch's work on this subject, see p. 1040.

More puzzling in the tests by Messrs. Beyer and Solakian is the distribution of the longitudinal stresses, σ_x, over the cross-section of the bakelite (see Fig. 16). It seems difficult to understand why, near the ends of the unit, the tension should be the greatest at a distance farthest from the reinforcement rod. In this experiment the aluminum rod is the stress-producing agency and it would be expected that the ends of the specimen would be slightly conical with the apex at the center of the rod. Since the strain is greater near the rod, the stress should likewise be greater at this point.

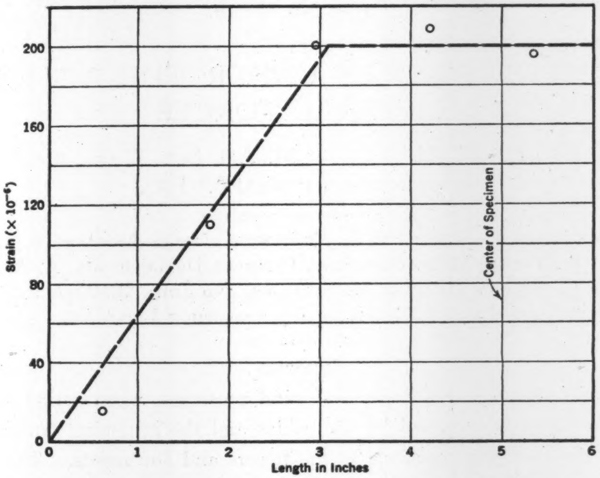

FIG. 19.—STRAIN IN A STEEL ROD ENCASED IN A CONCRETE UNIT SUBJECT TO SHRINKAGE (FROM TESTS BY W. H. GLANVILLE).

It would have made an interesting check on the stress distribution had the actual shortening of the aluminum rod been measured directly.

The writer learns with the greatest regret that Mr. Meyer, who had also contributed to this discussion, passed away after a short illness on December 27, 1932. Mr. Meyer's discussion was clear-cut and to the point. He contributed the results of tests conducted by the Los Angeles Water Department, from which he deducted some interesting conclusions. It is evident that in Mr. Meyer the profession has lost an able investigator.

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SUSPENSION BRIDGES UNDER THE ACTION OF LATERAL FORCES

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SYNOPSIS

Lateral forces, such as horizontal wind pressures, when acting on a suspension bridge are sustained by the cables and the stiffening trusses, which transmit the resulting reactions to the towers and abutments. The hangers which connect the trusses to the cables force the two loaded systems to work together so that the deformation on one system exerts its influence on the other.

As soon as the bridge swings out of its vertical plane the effect of the dead and live load comes into action and tends to reduce the horizontal deflections. Consequently, a redistribution of the acting wind load follows until equilibrium is established between all acting forces.

Compared with the suspended structure, the cables themselves offer a small exposed area to wind pressure, but part of the forces which act on the suspended structure are transmitted through the hangers to the cables. The magnitude of the transmitted forces depends on the respective stiffness of the two systems and the ratio of the wind pressure acting on them.

The stiffness of the cables is in direct, linear proportion to the pull in the cable and in inverse proportion to the square of the span length. The cable pull expresses the resisting effect of the gravity forces.

The stiffness of the truss is in direct, linear proportion to its horizontal moment of inertia and in inverse proportion to the fourth power of the span length.

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The two methods treated in this paper will lead to the determination of the stresses in the stiffening trusses caused by wind pressure. As the title suggests, the first method—"The Uniform Distribution Method"—assumes a uniform distribution of wind pressure on the cables and trusses. It is an approximate procedure and the derived results, which, by assumption, can be expressed in explicit form, have to be judged accordingly. The second—"The Elastic Distribution Method"—does not assume the simplification of the first method. The results are derived from balancing the deflections of the two systems, the cables and the trusses. Such distribution of wind pressure between cables and trusses may be found by trial computations which will result in a balanced position of the two combined systems.

THE UNIFORM DISTRIBUTION METHOD

This method applies only to a bridge the elevation of which is symmetrical about mid-span, and for the assumption that the wind exerts a pressure which is distributed uniformly over the entire span length. Both assumptions usually agree with practical cases.

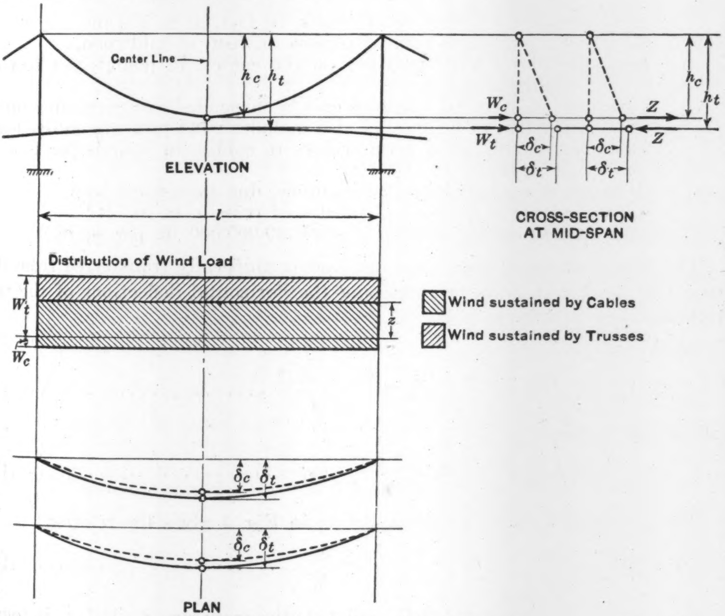
Center Line
h_c
h_t
ELEVATION
l
W_c
W_t
Z
Z
δ_c
δ_t
h_c
h_t
CROSS-SECTION AT MID-SPAN
Distribution of Wind Load
W_t
W_c
z
▧ Wind sustained by Cables
▨ Wind sustained by Trusses
δ_c
δ_t
δ_c
δ_t
PLAN

FIG. 1.—UNIFORM DISTRIBUTION METHOD.

To simplify the procedure further, the assumption is introduced that the wind pressure transferred from the trusses to the cables is also distributed uniformly over the entire length of the span. Consequently, the horizontal deformations of a cable will form a parabola and, since the elevation is a curve following the same law, all points of the deflected cable will remain in one plane. This plane rotates about a line connecting the two bearing points of the cable on the tower saddles. To combine the deflections of the two systems, the trusses and the cables, the assumption is made that at mid-span the deflected truss lies in the plane of the cable, as shown in cross-section in Fig. 1.

It can scarcely be expected that the actual behavior of the structure will conform to such simplifications. However, it affords a means of expressing in a simple form the effect of forces in two systems which depend on each other and yet behave so differently elastically. It will be shown that the method is of great usefulness, especially in cases where its results do not develop decided discrepancies with those of a more trenchant analysis, which is not based on such simplifying assumptions, but involves a much greater amount of work.

In the following analysis let:

l = length of span, in feet.
h_c = versine of cables, in feet.
h_t = versine of cables, in feet, plus length of hanger at mid-span, in feet.
δ_c = horizontal deflection of cables, in feet, at mid-span.
δ_t = horizontal deflection of trusses, in feet, at mid-span.
w_c = horizontal wind pressure on the cables, in pounds per foot of bridge.
w_t = horizontal wind pressure on the suspended structure, in pounds per foot of bridge; w_t also includes wind pressure on live load.
z = transferred wind from trusses to cables, in pounds per foot of bridge.
H = horizontal cable pull, in pounds, due to vertical load.
I = equivalent moment of inertia of trusses, in in.^{2} ft.^{2}
E = modulus of elasticity of steel, 29 000 000 lb. per sq. in.

The first assumption that the wind load is uniformly transferred from the trusses to the cables permits expressing the horizontal deflections of the two systems. For mid-span, it is:

In the cables,

$$\delta_c = \frac{(w_c + z)\, l^2}{8\, H} \quad \text{..........................} (1)$$

and in the trusses,

$$\delta_t = \frac{5\, l^4}{384\, E\, I}\, (w_t - z) \quad \text{........................} (2)$$

The assumption shown in cross-section in Fig. 1 gives the relation:

$$\frac{h_c}{h_t} = \frac{\delta_c}{\delta_t} \quad \text{..............................} (3)$$

Combining Equations (1), (2), and (3), the transferred wind, z, is found as follows:

$$z = \frac{h_c\, l^2\, H\, w_t - 9.6\, E\, I\, h_t\, w_c}{h_c\, l^2\, H + 9.6\, E\, I\, h_t} \quad \text{....................} (4)$$

In Equation (4) let $a = h_c\, l^2\, H$, and $b = 9.6\; h_t\, E\, I$; then,

$$z = \frac{a\, w_t - b\, w_c}{a + b} = \frac{w_t}{1 + \frac{b}{a}} - \frac{w_c}{1 + \frac{a}{b}} \ldots\ldots\ldots\ldots (5)$$

It will be noted that $\frac{b}{a}$ represents the ratio of the stiffness of truss to that of the loaded cable.

Having determined the wind load transferred from the trusses to the cables and having assumed beforehand that it is uniform, the computation of the stresses in the trusses caused by it becomes simple.

THE ELASTIC DISTRIBUTION METHOD

This method derives the distribution of the wind pressure between the cables and the trusses by balancing their simultaneous deformations. The procedure is similar to the one used in the first method, namely, to find an expression for the deflections of the two systems and a relation between them.

Let h_x = hanger length, in feet.
δ_{cx} = horizontal deflection of cables, in feet.
δ_{tx} = horizontal deflection of trusses, in feet.
$\Delta\delta_x$ = difference in horizontal deflections between cables and trusses, in feet.
p_{cx} = wind sustained by cables, in pounds per foot of bridge.
p_{tx} = wind sustained by trusses, in pounds per foot of bridge.
p_{sx} = dead load of suspended structure and live load, in pounds per foot of bridge.
I_x = moment of inertia, in in.^{2} ft.^{2} The index, x, indicates the value at section x ft. from the end of the span.

Horizontal Deflections of the Trusses.—The horizontal deflection of the trusses, δ_{tx}, is given in the form of the differential equation of the elastic line:

$$d^2\,\delta_{tx} = \frac{M_x}{E\, I_x}\, d\, x^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

which may be written,

$$E\, I_c \frac{d^2\,\delta_{tx}}{d\, x^2} = \frac{I_c}{I_x}\, M_x \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which, M_x is the bending moment in the trusses and I_c, any constant moment of inertia. The moment of inertia, I_c, is mostly so chosen that the term, $\frac{I_c}{I_x}$, becomes unity for as many sections as possible.

The algebraical integration of Equation (7) can only be solved for simple cases, that is, it must be possible to express the two terms, M_x and $\frac{I_c}{I_x}$, as functions of x. Such uniformity can scarcely be expected and it is best, therefore, to apply a well-known interpretation. It consists in recognizing that the right side of Equation (7), that is, the term, $\frac{I_c}{I_x}\, M_x$, where taken as a loading on the trusses results in a moment diagram which, divided by $E\,I_c$, will give the desired deflection curve. The procedure is plainly shown in Fig. 2 (*a*) to Fig. 2 (*e*), inclusive.

These curves also show the procedure to determine the horizontal deflection of the trusses under the action of any distribution of horizontal forces.

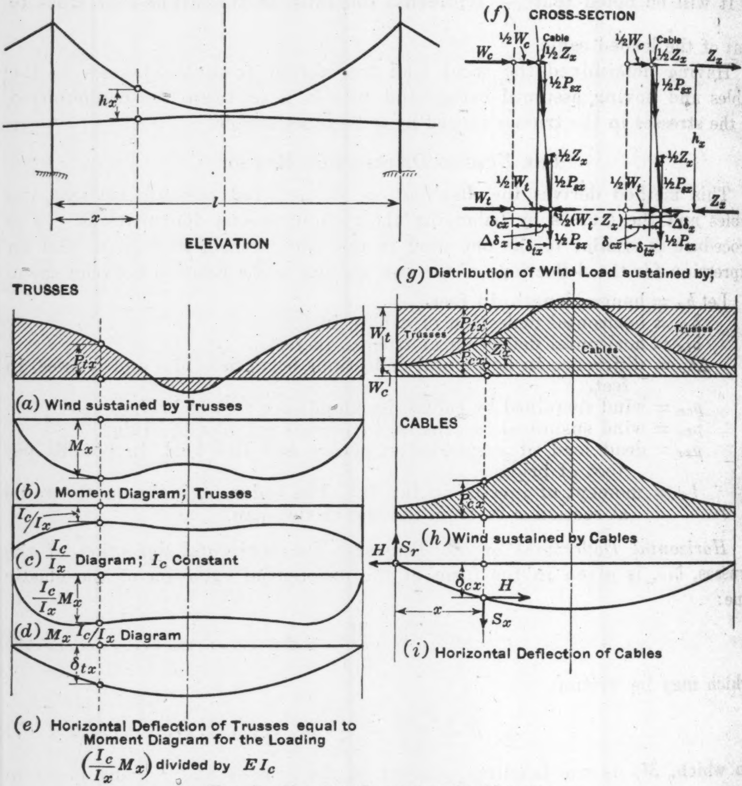
ELEVATION
h_x
x
l
(f) CROSS-SECTION
$\frac{1}{2}W_c$
Cable
W_c
$\frac{1}{2}Z_x$
Z_x
$\frac{1}{2}P_{sx}$
h_x
$\frac{1}{2}W_t$
W_t
$\frac{1}{2}(W_t - Z_x)$
δ_{cx}
$\Delta\delta_x$
δ_{tx}
(g) Distribution of Wind Load sustained by;
W_t
W_c
Trusses
Cables
P_{tx}
P_{cx}
Z_x
TRUSSES
P_{tx}
(a) Wind sustained by Trusses
M_x
(b) Moment Diagram; Trusses
I_c/I_x
(c) $\frac{I_c}{I_x}$ Diagram; I_c Constant
$\frac{I_c}{I_x}M_x$
(d) $M_x\ I_c/I_x$ Diagram
δ_{tx}
(e) Horizontal Deflection of Trusses equal to Moment Diagram for the Loading $\left(\frac{I_c}{I_x}M_x\right)$ divided by EI_c
CABLES
P_{cx}
(h) Wind sustained by Cables
H
S_r
δ_{cx}
H
x
S_x
(i) Horizontal Deflection of Cables

FIG. 2.—ELASTIC DISTRIBUTION METHOD.

The total wind load acting on the structure is plotted on a diagram consisting of w_c and w_t. At the towers, of course, the distribution of the wind load between cable and trusses is given. Along the span any distribution may be assumed. Such a probable distribution is indicated by the curve drawn on the load diagram. The wind load thus assumed, sustained by the trusses, is then plotted along the span on Fig. 2 (*a*). The moments for the load distribution shown in Fig. 2 (*a*) are then computed and plotted, as shown in Fig. 2 (*b*). The curve of the ratio of an assumed constant moment of inertia to the actual moment of inertia of the trusses is plotted along the span, as shown in Fig. 2 (*c*). The ordinates of Fig. 2 (*b*) and Fig. 2 (*c*) are then multiplied by each other and plotted as in Fig. 2 (*d*). The products are divided by $E\,I_c$ and plotted as in Fig. 2 (*e*). The resulting ordinates in Fig. 2 (*e*) represent the horizontal deflections of the trusses in accordance with the arbitrary distribution assumed at the beginning.

Of course, the diagrams in Fig. 2 (*b*) to Fig. 2 (*e*) are merely illustrative and, as a rule, are not plotted. The values of the respective ordinates are computed most conveniently in tabular form.

Horizontal Deflections of the Cables.—The remainder of the total wind load which was not assumed to be sustained by the trusses, must be sustained by the cables.

This loading is illustrated in Fig. 2 (*h*). To determine the horizontal curve which the cable will take under this assumed load, the following consideration will be of use. For a section distant x ft. from the end of the span, the moment equation for the cables reads (from Fig. 2 (*i*)):

$$H\,\delta_{cx} = S_r\,x - \int_0^x p_{cx}\,x\,dx \quad\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

The right side of this equation is the same as the moment, M_{ox}, in a simple beam of the span, l, and the loading, p_{cx}. The horizontal deflection of the cable, therefore, can be written:

$$\delta_{cx} = \frac{M_{ox}}{H} \quad\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Relation Between the Deflections of the Trusses and the Cables.—The relation between the horizontal deflections of the trusses and the cables must now be established, and the behavior of the two systems must be equated in order to arrive at the actual deflections.

To establish a relation between the deformations of the two systems, any cross-section may be considered as shown in Fig. 2. A moment equation about the centers of the cables will give the equation:

$$h_x\left[\frac{w_{tx}}{2} - \frac{1}{2}\,(\,w_{tx} - z_x\,)\right] = \Delta\,\delta_x\,\frac{p_{sx}}{2} \quad\ldots\ldots\ldots\ldots(10)$$

from which,

$$\Delta\,\delta_x = \frac{h_x}{p_{sx}}\,z_x \quad\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

in which, z_x denotes the intensity of the wind load transferred from the trusses to the cables. To arrive at the actual distribution, this transferred wind load, z_x, must be found by trial computations which will satisfy all three equations, (7), (9), and (11). The determination is made by the method of successive approximations.

The method can be applied for any loading condition, including live load, as well as wind pressure. It can also meet specifications which require the wind on the live load to be treated as a moving load in combination with the highest stresses caused by live load and temperature.

The solution may be made graphically or analytically. The analytical method, however, was found to be the more convenient if systematically treated; the results are also more accurate than those derived graphically.

Example of Wind Distribution in a 4 000-Foot Span.—To demonstrate the application of the Uniform Distribution Method and the Elastic Distribution Method, a numerical example of each will be given as applied to a suspension bridge with a span of 4 000 ft., for which a design has been made. It will be denoted as Proposed Design No. 2.

Example 1

Let $l = 4\,000$ ft.; h_c (normal) $= 500$ ft.; dead load $= 22\,000$ lb. per ft. of bridge; live load $= 6\,000$ lb. per ft. of bridge; $w_t = 1\,150$ lb. per ft.; and $w_c = 150$ lb. per ft.

The vertical deflection at mid-span for a 6 000-lb. live load, at the highest temperature (+ 40° Fahr. above normal) is 9.0 ft., downward. Therefore, $h_c = 500 + 9 = 509$ ft. and $h_t = 509 + 10 = 519$ ft. Furthermore, the horizontal cable pull equals H (for dead load and for live load at highest temperature) $= 110\,000\,000$ lb.; and I (constant) $= 1\,620\,000$ in.^{2} ft.^{2}

The wind load transferred from trusses to cables is determined by Equation (5); thus:

$$a = h_c\, l^2\, H = 509 \times 4\,000^2 \times 110\,000\,000 = 8.958 \times 10^{17}$$

$$b = 9.6\, h_t\, EI = 9.6 \times 29\,000\,000 \times 1\,620\,000 \times 519 = 2.340 \times 10^{17}$$

and,

$$z = \frac{8.958 \times 1\,150 - 2.340 \times 150}{8.959 + 2.340} = 880 \text{ lb.}$$

Therefore, the wind load sustained by the trusses equals 1 150 − 880 = 270 lb. per ft.; and the wind load sustained by the cables equals 880 + 150 = 1 030 lb. per ft.

By Equation (1) the horizontal deflection of the cable at mid-span equals:

$$\delta_c = \frac{1\,030 \times 4\,000^2}{8 \times 110\,000\,000} = 18.7 \text{ ft.; and } \delta_t = \frac{519}{509} \times \delta_c = 19.1 \text{ ft.}$$

The moment curve for the trusses is parabolic; for a loading of 270 lb., the mid-span moment is $M = \dfrac{270 \times 4\,000^2}{8} = 540\,000\,000$ ft-lb.; and the reactions of the wind forces, on one tower, are:

At the top,

$$R = 0.5\,(1\,030 \times 4\,000) = 2\,060\,000 \text{ lb.}$$

and at the elevation of the trusses,

$$R = 0.5\,(270 \times 4\,000) = 540\,000 \text{ lb.}$$

Example 2

In addition to the values in Example 1, let $E = 29\,000$ kips per in.^{2}; $\lambda = \dfrac{4\,000}{20} = 200$; $\delta_{tx} = \dfrac{\lambda^4}{E\,I_c} M_{mx} = 0.03406\, M_{mx}$; $\delta_{cx} = \dfrac{\lambda^2}{H} M_{ox} = 0.3636\, M_{ox}$; and p_{sx} = live load (6 kips) plus dead load of suspended structure (16 kips), or 22 kips per ft. of bridge (constant).

The application of the Elastic Distribution Method to this design and the analytical procedure followed, is shown in Table 1; it is illustrated diagrammatically in Fig. 3. A comparison of design characteristics is given in Table 2.

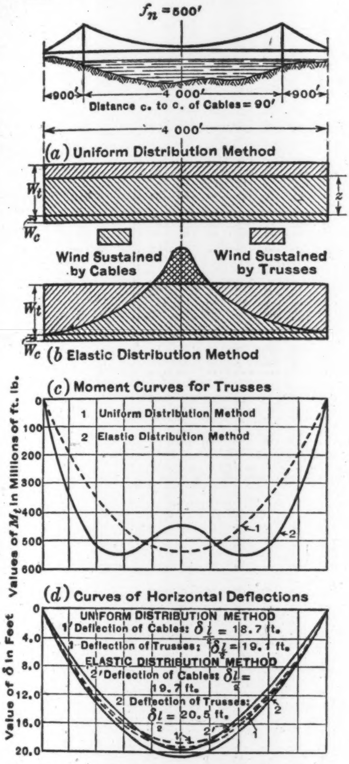

FIG. 3.—PROPOSED DESIGN NO. 2.

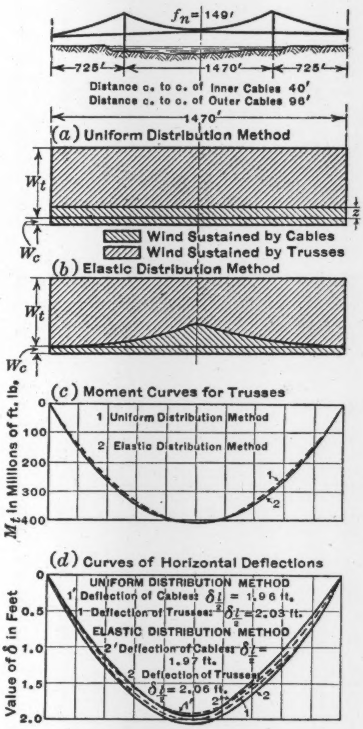

FIG. 4.—MANHATTAN BRIDGE.

The wind reactions for one tower are found to be: At the top,

$$R = 7.56 \times 200 + 0.15 \times 100 = 1\,527 \text{ kips}$$

and, at the elevation of the trusses,

$$R = 958 + 1.15 \times 100 = 1\,073 \text{ kips}$$

At first, the wind load, z_x, transferred from the trusses to the cables was assumed, and the computation was made on the basis of this guess of distribution. The assumption of z_x must be changed and the computation done over again until the differences in the horizontal deflections between trusses and cables, as determined by Equations (7) and (8), are practically equal to the differences computed by Equation (11). The span of 4 000 ft. was divided into twenty equal parts, each part having a length, λ = 200 ft. The trans-

TABLE 1.—ELASTIC DISTRIBUTION METHOD APPLIED TO PROPOSED DESIGN No. 2

x, in feet	p_{tx}, in kips per foot	S'_x, in kips	M'_x, in kip-feet	$\frac{I_c}{I_x}$	$\frac{I_c}{I_x} M'_x$, in kip-feet	S_{mx}, in kip-feet²	M_{mx}, in kip-feet³	δ_{tx}, in feet	p_{cx}, in kips per foot	S_{0x}, in kips	M_{0x}, in kip-feet	δ_{cx}, in feet	h_x, in feet	$\frac{h_x}{p_{sx}}$, in feet² per kip	z_x, in kips per foot	$\Delta\delta_x = \frac{h_x}{p_{sx}} z_x$, in feet	$\Delta\delta_x = \delta_{tx} - \delta_{cx}$ in feet	S_x, in kips	M_x, in 1 000 kip-ft.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
		4.790				106.58				7.560								958	
0.05 *l*	1.105		4.790	1.0	4.790		106.58	3.6	0.195		7.560	2.7	435.2	19.78	0.045	0.9	0.9		192
		3.685				101.78				7.365								737	
0.10 *l*	1.045		8.475	1.0	8.475		208.36	7.1	0.255		14.925	5.4	346.0	15.73	0.105	1.7	1.7		339
		2.640				93.31				7.110								528	
0.15 *l*	0.970		11.115	1.0	11.115		301.67	10.3	0.330		22.035	8.0	267.2	12.15	0.180	2.2	2.3		445
		1.670				82.19				6.780								334	
0.20 *l*	0.860		12.785	1.0	12.785		383.86	13.1	0.440		28.815	10.5	199.0	9.05	0.290	2.6	2.6		511
		0.810				69.41				6.340								162	
0.25 *l*	0.740		13.595	1.0	13.595		453.27	15.4	0.560		35.155	12.8	141.2	6.42	0.410	2.6	2.6		544
		0.070				55.81				5.780								14	
0.30 *l*	0.600		13.665	1.0	13.665		509.08	17.3	0.700		40.935	14.9	94.0	4.27	0.550	2.3	2.4		547
		—0.530				42.15				5.080								—106	
0.35 *l*	0.360		13.135	1.0	13.135		551.23	18.8	0.940		46.015	16.7	57.2	2.60	0.790	2.0	2.1		525
		—0.890				29.01				4.140								—178	
0.40 *l*	0.035		12.245	1.0	12.245		580.24	19.8	1.265		50.155	18.2	31.0	1.41	1.115	1.6	1.6		490
		—0.925				16.77				2.875								—185	
0.45 *l*	—0.500		11.320	1.0	11.320		597.01	20.3	1.800		53.030	19.3	15.2	0.69	1.650	1.1	1.0		453
		—0.425				5.45				1.075								—85	
0.50 *l*	—0.850		10.895	1.0	10.895		602.46	20.5	2.150		54.105	19.7	10.0	0.45	2.000	0.9	0.8		436

TABLE 2.—COMPARISON OF DESIGN CHARACTERISTICS

Item	Description	Unit	Proposed Design No. 2 (Fig. 3)	Manhattan Bridge (Fig. 4)	Delaware River Bridge (Fig. 5)	Detroit River Bridge (Fig. 6)	Proposed Design No. 1 (Fig. 7)	George Washington Bridge (Fig. 8)
1	Span length	Feet	4 000	1 470	1 750	1 840	2 640	3 500
2	Dead load of cables	Pounds*	6 000	4 280	4 200	1 600	4 000	13 000
3	Dead load of suspended structure	Pounds*	16 000	19 000	21 800	10 800	12 000	26 000
4	Total dead load	Pounds*	22 000	23 280	26 000	12 400	16 000	39 000
5	Live load	Pounds*	6 000	16 000	12 000	0	6 000	4 000
6	Dead load plus live load	Pounds*	28 000	39 280	38 000	12 400	22 000	43 000
7	Horizontal cable pull, H, due to dead and live load and temperature change	Thousand pounds	110 000	63 800	69 060	25 800	71 240	194 950
8	Horizontal Wind Pressure: w_t, on trusses	Pounds*	1 150	1 800	1 200	670	1 150	1 200
9	w_c, on cables	Pounds*	150	200	150	160	150	200
10	Transferred wind load from trusses to cables, z	Pounds*	880	264	306	255	618	1 060
	Lateral Deflections in Trusses at Mid-Span by Elastic Distribution:							
11	Cable not resisting	Feet	82.0	2.4	3.4	11.4	20.8	90
12	Cable resisting	Feet	20.5	2.06	2.6	7.2	10.1	11.4
13	Percentage reduction in lateral deflection because of cable action		75	12	23	37	51	87
14	Deflection in cables	Feet	19.7	1.97	2.52	6.9	9.7	10.3
	Deflection by Uniform Distribution:							
15	Trusses	Feet	19.1	2.03	2.58	7.0	9.6	10.3
16	Cables	Feet	18.7	1.96	2.52	6.8	9.4	9.9

* Pounds per foot of bridge.

ferred wind load, z_x, was assumed and thereby the loads sustained by the trusses and by the cables were fixed. Table 1 contains twenty columns which are explained as follows:

Column (1) = distance, x, in feet, from end of span.
Column (2) = p_{tx}, wind pressure sustained by trusses, in kips per foot of bridge.
Column (3) = S'_x, shear for loading, p_{tx}, in kips, reduced by λ.
Column (4) = M'_x moment for loading, p_{tx}, in kip-feet, reduced by λ^2.
Column (5) = $\frac{I_c}{I_x}$, ratio of moments of inertia for the various sections of the trusses; I_c is constant.
Column (6) = $\frac{I_c}{I_x} M_x$, in kip-feet, reduced by λ^2.
Column (7) = S_{mx}, shear, in kip-feet,[2] for loading, $\frac{I_c}{I_x} M'_x$, reduced by λ^3.
Column (8) = M_{mx}, moment in kip-feet[3], for loading, $\frac{I_c}{I_x} M'_x$, reduced by λ^4.
Column (9) = δ_{tx}, horizontal deflection of trusses, in feet, $= \frac{\lambda^4}{E I_c} M_{mx}$ = values in Column (8) multiplied by $\frac{\lambda^4}{E I_c}$.
Column (10) = p_{cx}, wind pressure sustained by cables, in kips per foot of bridge.
Column (11) = S_{ox}, shear for loading, p_{cx}, in kips, reduced by λ.
Column (12) = M_{ox}, moment for loading, p_{cx}, in kip-feet, reduced by λ^2.
Column (13) = δ_{cx}, horizontal deflection of cables, in feet $= \frac{\lambda^2}{H} M_{ox}$ = the values in Column (12) multiplied by $\frac{\lambda^2}{H}$.
Column (14) = h_x, hanger length, in feet.
Column (15) = $\frac{h_x}{p_{sx}}$, in feet[2] per kip; p_{sx} is sum of dead load of suspended structure and live load on it, in kips per foot of bridge.
Column (16) = z_x, transferred wind load from trusses to cables, in kips per foot of bridge.
Column (17) = $\Delta\delta_x = \frac{h_x}{p_{sx}} z_x$, difference in horizontal deflections between trusses and cables.
Column (18) = $\Delta\delta_x = (\delta_{tx} - \delta_{cx})$, difference in horizontal deflections between trusses and cables.
Column (19) = S_x, shear in trusses, in kips, equal to S'_x in Column (3) multiplied by λ.
Column (20) = M_x, moment in trusses, in kip-feet, equal to M'_x in Column (4) multiplied by λ^2.

The assumptions of z_x should be changed until the values of $\Delta\delta_x$ in Column (17) and Column (18) coincide.

COMPARATIVE RESULTS OF SIX BRIDGES

Six bridges of various span lengths and various loads have been investigated by both the Uniform Distribution Method and the Elastic Distribution Method. The two methods do not give decided differences in moments for the shorter span bridges. For the Manhattan Bridge (Fig. 4) and the Delaware River Bridge (Fig. 5), the Uniform Distribution Method gives slightly lower values, with the exception at mid-span, where both curves coincide. The difference, however, does not exceed 2.2% at quarter-span for the Manhattan Bridge, and 5.2% for the Delaware River Bridge, taking the Elastic Distribution Method at 100 per cent.

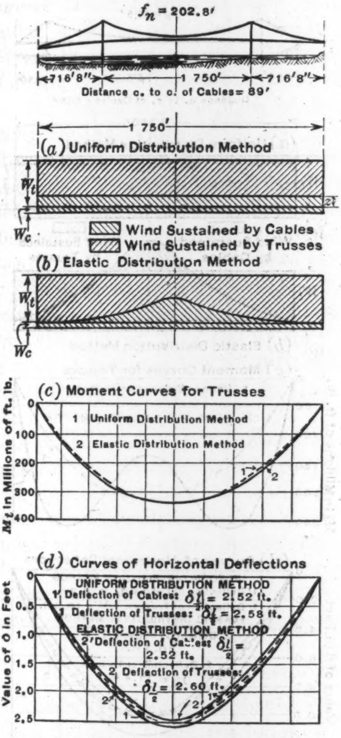

FIG. 5.—DELAWARE RIVER BRIDGE.

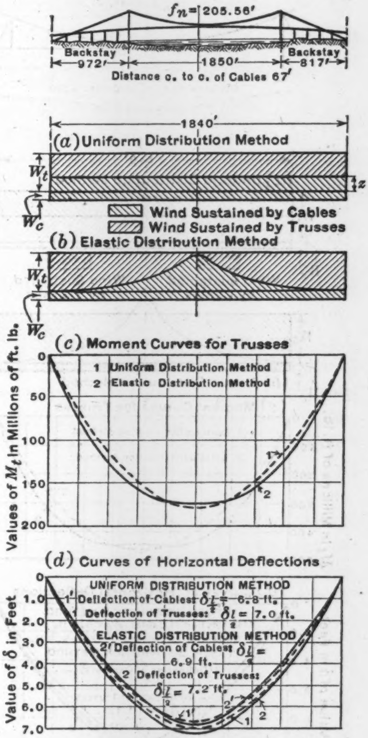

FIG. 6.—DETROIT RIVER BRIDGE.

With increasing spans the difference in the results of the two methods begins to grow. Toward the ends of the trusses the results of the Uniform Distribution Method are lower than those derived from the Elastic Distribution Method, whereas the opposite is true for the center portion. For the Detroit River Bridge (Fig. 6), the Uniform Distribution Method gives 8% less at quarter-span and 1.7% more at mid-span than the Elastic Distribution Method. The corresponding values for "Proposed Design No. 1." (Fig. 7), George Washington Bridge (Fig. 8), and "Proposed Design No. 2" (Fig. 3), are 13%, 37%, and 26% less at quarter-span, respectively, and 5%, 61%, and 24% more at mid-span.

It is of interest to know to what extent the cables help to relieve the stresses in the trusses. Introducing the moment value in an unrestrained truss as 100%, not suspended from the cables, the Elastic Distribution Method gives for the various bridges:

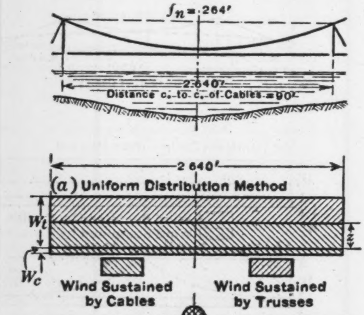

FIG. 7.—PROPOSED DESIGN NO. 1.

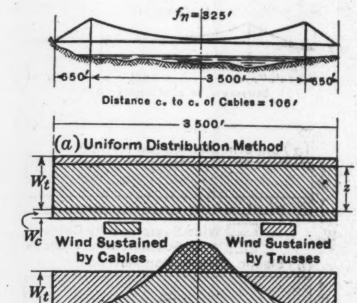

FIG. 8.—GEORGE WASHINGTON BRIDGE.

Bridge	Quarter-span (Percentage)	Mid-span (Percentage)
Manhattan	87	85
Delaware	79	74
Detroit	67	61
Design No. 1	53	44
George Washington	19	7.2
Design No. 2	32	13

Attention is drawn to the fact that in all cases the Elastic Distribution Method gives lower wind reactions at the tops of the towers. Table 3 gives a comparison of the wind reactions on the towers by the two methods.

From the study of the moment curves of the various bridges, it may be concluded that for spans up to 1 500 ft. of moderate live-load capacity and up to 1 750 ft. of large live-load capacity, the horizontal moments in the trusses due to horizontal wind loads are practically the same, computed by either method. It is also conclusive that, while as a rule the moments by the Elastic Distribution Method are somewhat larger, the much simpler Uniform Distribution Method may well be used.

The moments in bridges of longer spans begin to show more marked differences as evidenced in the moment diagrams and, in the cases of the George Washington Bridge (Fig. 8) and the Proposed Design No. 2 (Fig. 3), they attain totally different values for the two methods. It is evident that for bridges of spans exceeding those in Figs. 3 to 8, inclusive, the Elastic Distribution Method only should be used to obtain a more correct distribution of the wind load and more correct resulting stresses.

The tabulation of the wind reactions (Table 3) as computed by the two methods shows that the reduced wind reactions at the tower tops resulting from the application of the Elastic Distribution Method already have attained a value of 10% for the 1 850-ft. span of the Detroit River Bridge and have reached 20% for the 3 500-ft. span of the George Washington Bridge and the 4 000-ft. span of Design No. 2. Considering that with increasing length of span the height of the towers grows in proportion, the substantial importance of the Elastic Distribution Method for the design of the towers of long-span bridges becomes manifest. Not only is the lateral reaction at the tower top reduced, requiring less wind-bracing in the towers and resulting in less lateral deflection due to shear, but the usually greater tower height of the longer spans causes higher overturning moments and higher wind stresses in the tower legs above the stiffening truss supports, and these are much benefited by the reduced reactions of the Elastic Distribution Method.

LATERAL DEFLECTION OF TOWER TOPS

Towers in suspension bridges of great length are proportionately tall. Under full wind loads, the lateral displacement of the tops of the towers may reach sufficient magnitude to deserve attention in the computation of the wind stresses in the stiffening truss. In the case of the Golden Gate Bridge, with a span of 4 200 ft., these displacements are about 16 in. The lateral displacements of the stiffening truss, as well as its bending moment, will be increased thereby. In such cases a computation should be made to determine the additional stresses involved.

COMPARATIVE LATERAL STIFFNESS OF BRIDGES

The stability and lateral stiffness of a bridge depend on its width. General bridge specifications usually prescribe a minimum for the ratio of span to width. Even for spans of moderate lengths the lateral stiffness of the

TABLE 3.—COMPARISON OF WIND REACTIONS

Item	Description	UNIFORM DISTRIBUTION		ELASTIC DISTRIBUTION		Percentage difference	UNIFORM DISTRIBUTION		ELASTIC DISTRIBUTION		Percentage difference
		Kips	Percentage	Kips	Percentage		Kips	Percentage	Kips	Percentage	
		MANHATTAN BRIDGE					DELAWARE RIVER BRIDGE				
1	Top	341	23	281	19	4	399.0	34	305.35	26	8
2	Truss level	1 129	77	1 189	81	4	782.25	66	875.9	74	8
3	Total	1 470	100	1 470	100	....	1 181.25	100	1 181.25	100	....
4	Transferred	194	13.5	144	9.5	4	268	23	174.35	15	8
		DETROIT RIVER BRIDGE					PROPOSED DESIGN No. 1				
5	Top	382	50	304	40	10	1 014	59	746	43	16
6	Truss level	382	50	460	60	10	702	41	970	57	16
7	Total	764	100	764	100	....	1 716	100	1 716	100	....
8	Transferred	235	31	157	21	10	816	48	548	32	16
		GEORGE WASHINGTON BRIDGE					PROPOSED DESIGN No. 2				
9	Top	2 205	90	1 697	69	21	2 060	79	1 527	59	20
10	Truss level	245	10	753	31	21	540	21	1 073	41	20
11	Total	2 450	100	2 450	100	....	2 600	100	2 600	100	....
12	Transferred	1 855	76	1 347	55	21	1 760	68	1 227	48	20

trusses already begins to claim consideration in the determination of the width. For long-span bridges the required width between trusses raises a serious problem.

In suspension bridges the center of gravity is so low below the points of support that the suspended structure is always in stable equilibrium and the lateral stability of the towers only has to be considered. The study of the behavior of suspension bridges under the action of lateral loads has also disclosed that they are relatively stiffer than simple trusses of the same moments of inertia and that their relative stiffness increases with the length of the span.

Table 2, Items 11, 12, and 13, showing the lateral deflections at mid-span of the bridges studied in this paper, will serve to demonstrate the relative stiffness of simple to suspended trusses.

The effect of the cable in reducing the lateral deflection of the trusses is so great that in the longer spans the resulting deflection is one-half, one-quarter and one-eighth of those which would result without the action of the cables. It can be stated, in other words, that long-span suspension bridges are two to four times laterally stiffer than simple spans and arches of the same width and chord area. This characteristic of long-span suspension bridges is of the utmost importance in discussing the required width between trusses.

DISCUSSION

P. L. Pratley,[3] Esq. (by letter).—A notable and ingenious contribution to the modern and more precise application of scientific analysis to the design of suspension bridges is presented in this paper. The authors' adaptation of the Melan exact theory regarding the distribution of load between cable and stiffening truss to actual design has become established among engineers concerned with this type of structure and their demonstration regarding the contribution of the upper chords of a stiffening truss without top laterals in resisting horizontal bending moments, while perhaps not so well known, is of similar economic value.

The desirability of some analysis regarding the "restitution" accomplished by the cable when the lateral truss deflects horizontally under wind pressure has been in the writer's mind for some years and he, therefore, appreciates highly the investigations published in this paper.

The suggested elastic distribution method, while involving considerable computation by trial and error, is intrinsically simple, and with the examples presented in the paper at hand for guidance, should be readily applicable to those cases in practice that are likely to need it. Happily perhaps, from the point of view of the designer, only spans of considerable length are to be found in this category, and as the authors show even the Detroit span (1 850 ft.) is not of sufficient magnitude to suffer from any serious quantitative inaccuracy in proportioning, if the simpler system is used in the calculations. The actual shape of the curve for z is interesting to the writer because, in studying some suspension designs in his office, subsequent to the work on the Detroit Ambassador Bridge, he assumed (as an approximate interpretation of the rather obvious truth that the rate of participation by the cable varies along the span), a triangular distribution having a maximum at the center. In the actual case subjected to analysis at the time, the proportion at mid-span was 37.5% of W_t, taken by the cable. While no analysis was made at the time to substantiate this choice and while it is evident that the maximum central figure is a function of the geometrical and physical properties of the combined truss-and-cable system, it is gratifying to note that the actual variation, as demonstrated, is somewhat of this nature. The pertinent figures in the case were: $l = 1\,500$ ft.; $h_c = 170$ ft.; $h_t = 190$ ft. to the plane of the floor; $I = 114\,000$ in.^{2} ft.^{2}; $w_t = 400$ lb.; $w_c = 100$ lb.; and $H = 9\,000$ kips. Therefore, by the authors' simpler method, $a = 3.44 \times 10^{15}$; $b = 6.04 \times 10^{15}$; and $z = 81.5$, thus showing a "restitution" factor of 20.75 per cent. For this length of span, however, it is manifest from the paper that the simpler method furnishes a satisfactorily close approximation for the truss moments. The fact that z becomes negative in Figs. 3, 7, and 8, while not expected perhaps from a first consideration, is nevertheless quite capable of physical appreciation as one visualizes the trend of actual deflections. The writer is tempted to wonder whether the peak percentage at the center could be expressed approximately for the first trial computations as a function of I, H, and l, but not having the necessary figures for all the spans treated by the authors he has not been able to pursue this inquiry to any length.

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The importance of the variation in these long spans is well brought out in the graphical presentation of the moment curves, Figs. 7, 3, and 8, for the old and new Golden Gate designs and the George Washington Bridge. It is presumed that in Fig. 7 the 2 640-ft. span is treated as a pure suspension span comparable to the others, without any attempt being made to gauge the effect of the cantilever arm distortions which must occur under the assumed wind load. It might be of interest to inquire whether the truss lateral deflection is considered as taking place at the floor elevation, or elsewhere, and whether the floor system, lateral system, and truss web members are calculated as contributing to the lateral moment of inertia, in addition to the four chords?

The conclusions under "Comparative Results of Six Bridges" appear to be sound, and the economy resultant upon the use of the more exact method is decidedly worthy of notice both for its own sake and as being one further demonstration of the valuable effect of closer analysis for this type of indeterminate structure. The closing remarks of the authors on the effect of lateral stiffness are also of grave importance and serve to raise again in the writer's mind the question of "cradling" cables. One wonders whether this question too might be made the subject of some up-to-date study.

In concluding, the writer would again express his feeling that the paper is of tremendous value to those interested in this branch of bridge design, and he feels sure that the methods suggested will be accepted with gratitude by the profession and from them will develop a recognized improvement in practice and a still further economy in design.

CHARLES A. ELLIS,[4] M. AM. SOC. C. E. (by letter).—In this paper the authors have made a contribution to the theory of suspension bridges which is at once unique and valuable—unique from an academic viewpoint because it shows clearly how the cables, hangers, and stiffening trusses participate and co-operate in performing one of the duties for which the structure as a whole is designed; and valuable from a practical engineering standpoint because it gives a rational solution, in which no unwarranted liberties have been taken, to a hitherto perplexing problem. In reading the paper one must admire what amounts to genius displayed by the authors in the skilful manner whereby they have correlated and combined three simple fundamental principles most effectively in accomplishing their objective. These principles are the deflection of a simple beam, the funicular polygon, and the triangle of forces.

Since learning of the "Elastic Distribution Method" from the authors some years ago, the writer has had occasion to use it in several instances; and in doing so has discovered an algebraic solution which obviates the necessity of limiting the method to the "cut-and-try" solution described in the paper. Both the algebraic and the "cut-and-try" solutions give identical results, and the choice between them is a matter of personal preference. The algebraic solution of the authors' "elastic distribution method" will be illustrated by two examples. The first is a solution for maximum bending moments; the second, for maximum shears. In so far as possible the authors' nomenclature will be followed.

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[4] Evanston, Ill.

It may be well to emphasize a few points, as follows:

(1) A clear distinction should be kept in mind between the unit wind pressure acting on the trusses, which is a known constant quantity represented by w_t, and the unit wind pressures sustained by the trusses, which are unknown variable quantities represented by p_{tx} and for which the deflections, δ_{tx}, are to be determined as in a simple beam by the semi-graphical process of integration by summation. This process should commend itself to all engineers (professors and practitioners alike) because of its superiority over the classic and generally taught method of algebraic integration, which becomes very cumbersome if the moment of inertia varies or the load cannot be expressed as a continuous and simple function of one variable.

(2) A similar distinction should also be recognized between the unit wind pressure acting on the cables, which is a known constant quantity represented by w_c, and the unit wind pressures sustained by the cables, which are unknown variable quantities represented by p_{cx} and for which the deflections, δ_{cx}, are to be determined upon the conservative assumption that the cables resist the moment of the forces at any point, not by virtue of their stiffness *per se*, but rather by the product of the horizontal pull, H, and the deflection, δ_{cx}.

(3) The differences, $w_t - p_{tx} = z_x$, are the unit wind pressures transferred from the trusses to the cables by virtue of the inclination of the hangers, brought about by the difference in the deflections of the trusses and cables and represented by $\Delta\,\delta_x$.

(4) The truss deflections, δ_{tx}, are measured from the vertical plane through the longitudinal center line of the span at the truss elevation; while the cable deflections, δ_{cx}, are measured from the vertical plane through the tower tops. The horizontal distance, T, between these planes at either tower is the transverse deflection of the tower top when referred to the center line of the tower at the truss elevation. When the wind load is symmetrical, T is the same at each tower and the two planes are parallel; otherwise, T is a variable. The difference between the truss and cable deflections at any point, therefore, is,

$$\Delta\delta_x = \delta_{tx} - (\delta_{cx} + T)$$

or,

$$\delta_{tx} - \delta_{cx} = \Delta\delta_x + T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

(5) The hanger lengths, h_x, and the vertical loads, p_{sx}, sustained by the hangers are known; hence, from the triangle of forces,

$$h_x : \Delta\delta_x :: p_{sx} : z_x$$

or,

$$\Delta\delta_x = \frac{h_x}{p_{sx}}\, z_x$$

whence,

$$\delta_{tx} - \delta_{cx} = \frac{h_x}{p_{sx}}\, z_x + T \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The two examples which follow are taken from computations made by the writer in designing the Golden Gate suspension span. The reader will note that the general data given herewith differ slightly from those used by the authors in their Proposed Design No. 2 which is an earlier study of the same structure.

Example 1.—Maximum Bending Moments.—Let $l = 4\,200$ ft.; $h_c = 475$ ft.; dead load $= 15.3$ kips per ft. of bridge; live load $= 4$ kips per ft. of bridge; $p_{sx} = 19.3$ kips per ft. of bridge; $w_t = 1.1$ kips per ft.; $w_c = 0.2$ kips per ft.; $H =$ horizontal cable pull per bridge for live and dead loads $= 118\,310$ kips; $E = 29\,000$ kips per in.^{2}; and I (constant) $= 1\,479\,110$ in.^{2} ft.^{2} The span is divided into twenty equal parts, each division having the length, $\lambda = \frac{4\,200}{20} = 210$ ft. Then, $\frac{\lambda^4}{EI} = 0.0453397$; and, $\frac{\lambda^2}{H} = 0.37288$.

The expressions in Table 4 relate to the deflections of the trusses. The column numbers correspond to those used by the authors in their Table 1.

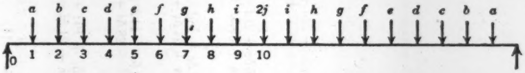

FIG. 9.

The load sustained by the trusses and for which the deflections are to be determined is of varying intensity. In Fig. 9, let *a*, *b*,...*i*, 2*j*, represent the intensity of this load at the various division points. The intensity, 2*j*, was used instead of *j*, in order to simplify the coefficients in the columns which follow. The intensity, *a*, at Point 1, may be taken as the average for the division length, λ, of which the point, 1, is the middle. Hence, the load over this division length may be replaced by the concentrated load, λa, acting at Point 1. Therefore, the quantities in Column (2), Table 4, and as shown in Fig. 9, when multiplied by λ, may be treated as concentrated loads at the division points. The expressions in Column (3), when multiplied by λ, represent the net reaction and shears between the division points; and those in Column (4), when multiplied by λ^2, represent the bending moments at the division points. From these data a moment diagram may be constructed, the area of which is now to be considered as a load on the structure. This load may be represented by a series of concentrated loads at the division points, each load being the product of λ and the moment ordinate for that point. The expressions in Column (4), when multiplied by λ^2, represent the moment ordinates; and when multiplied by λ^3, they represent the area-moment loads. For this loading the expressions in Column (7), when multiplied by λ^3, represent the net reaction and shears which, when divided by $E\ I$, represent the slope of the elastic curve. The expressions in Column (8), when multiplied by λ^4, represent the moments for said loading which, when divided by $E\ I$, represent the deflections, δ_{tx}, Columns corresponding to the authors' Column (5) and (6), Table 1, have been omitted since they serve only when I cannot be assumed constant.

Writing the expressions in Columns (3), (4), (7) and (8), Table 4, is little more than a procession of mechanical operations in addition and can be done very quickly; moreover, an independent check can be made by observing the

TABLE 4.—Expressions Relating to the Deflections of the Trusses (Example 1).

Division points (1)	p_{tx}, in kips per foot (2)	S'_x, in kips per foot (3)	M'_x, in kips per foot (4)	S_{mx}, in kips per foot (7)	M_{mx}, in kips per foot (8)
		$a+b+c+d+e+f+g+h+i+j$		$9.5a + 18b + 25.5c + 32d + 37.5e + 42f$ $+45.5g + 48h + 49.5i + 50j$	
1	a		$a + b + c + d + e + f + g + h + i + j$		$9.5a + 18b + 25.5c + 32d + 37.5e + 42f$ $+45.5g + 48h + 49.5i + 50j$
		$b+c+d+e+f+g+h+i+j$		$8.5a + 17b + 24.5c + 31d + 36.5e + 41f$ $+44.5g + 47h + 48.5i + 49j$	
2	b		$a + 2b + 2c + 2d + 2e + 2f + 2g + 2h + 2i + 2j$		$18a + 35b + 50c + 63d + 74e + 83f + 90g$ $+95h + 98i + 99j$
		$c+d+e+f+g+h+i+j$		$7.5a + 15b + 22.5c + 29d + 34.5e + 39f$ $+42.5g + 45h + 46.5i + 47j$	
3	c		$a + 2b + 3c + 3d + 3e + 3f + 3g + 3h + 3i + 3j$		$25.5a + 50b + 72.5c + 92d + 108.5e + 122f$ $+132.5g + 140h + 144.5i + 146j$
		$d+e+f+g+h+i+j$		$6.5a + 13b + 19.5c + 26d + 31.5e + 36f$ $+39.5g + 42h + 43.5i + 44j$	
4	d		$a + 2b + 3c + 4d + 4e + 4f + 4g + 4h + 4i + 4j$		$32a + 63b + 92c + 118d + 140e + 158f + 172g$ $+ 182h + 188i + 190j$
		$e+f+g+h+i+j$		$5.5a + 11b + 16.5c + 22d + 27.5e + 32f$ $+35.5g + 38h + 39.5i + 40j$	
5	e		$a + 2b + 3c + 4d + 5e + 5f + 5g + 5h + 5i + 5j$		$37.5a + 74b + 108.5c + 140d + 167.5e + 190f$ $+207.5g + 220h + 227.5i + 230j$
		$f+g+h+i+j$		$4.5a + 9b + 13.5c + 18d + 22.5e + 27f$ $+30.5g + 33h + 34.5i + 35j$	
6	f		$a + 2b + 3c + 4d + 5e + 6f + 6g + 6h + 6i + 6j$		$42a + 83b + 122c + 158d + 190e + 217f + 238g$ $+ 253h + 262i + 265j$
		$g+h+i+j$		$3.5a + 7b + 10.5c + 14d + 17.5e + 21f$ $+24.5g + 27h + 28.5i + 29j$	
7	g		$a + 2b + 3c + 4d + 5e + 6f + 7g + 7h + 7i + 7j$		$45.5a + 90b + 132.5c + 172d + 207.5e + 238f$ $+262.5g + 280h + 290.5i + 294j$
		$h+i+j$		$2.5a + 5b + 7.5c + 10d + 12.5e + 15f$ $+17.5g + 20h + 21.5i + 22j$	
8	h		$a + 2b + 3c + 4d + 5e + 6f + 7g + 8h + 8i + 8j$		$48a + 95b + 140c + 182d + 220e + 253f$ $+280g + 300h + 312i + 316j$
		$i+j$		$1.5a + 3b + 4.5c + 6d + 7.5e + 9f + 10.5g$ $+ 12h + 13.5i + 14j$	
9	i		$a + 2b + 3c + 4d + 5e + 6f + 7g + 8h + 9i + 9j$		$49.5a + 98b + 144.5c + 188d + 227.5e + 262f$ $+290.5g + 312h + 325.5i + 330j$
		j		$0.5a + b + 1.5c + 2d + 2.5e + 3f + 3.5g$ $+4h + 4.5i + 5j$	
10	$2j$		$a + 2b + 3c + 4d + 5e + 6f + 7g + 8h + 9i + 10j$		$50a + 99b + 146c + 190d + 230e + 265f$ $+294g + 316h + 330i + 335j$

following regularity in the coefficients of the terms in the expressions of Column (8). Beginning with the first term in the first row observe the same sequence in coefficients, whether reading to the right in the row or downward in the column; and the same is true when beginning with the second term in the second row, or the third term in the third row, etc. Hereafter, this phenomenon will be referred to as the "rule of coefficients".

TABLE 5.—EXPRESSIONS RELATING TO THE DEFLECTIONS OF THE CABLES (EXAMPLE 1)

Division points (1)	p_{cx}, in kips per foot (10)	S_{0x}, in kips per foot (11)	M_{0x}, in kips per foot (12)
		$12.35-a-b-c-d-e-f-g-h-i-j$	
1.......	$1.3-a$		$12.25-a-b-c-d-e-f-g-h-i-j$
		$11.05-b-c-d-e-f-g-h-i-j$	
2.......	$1.3-b$		$23.40-a-2b-2c-2d-2e-2f-2g-2h-2i-2j$
		$9.75-c-d-e-f-g-h-i-j$	
3.......	$1.3-c$		$33.15-a-2b-3c-3d-3e-3f-3g-3h-3i-3j$
....	....	....	....
		$0.65-j$	
10.......	$1.3-2j$		$65.00-a-2b-3c-4d-5e-6f-7g-8h-9i-10j$

The expressions in Column (8), Table 4, remain the same for any structure in which twenty divisions are used.

Corresponding expressions for the deflections of the cables are compiled as demonstrated in Table 5. The total unit wind pressure acting on the cables and trusses is 1.3 kips per ft., and the expressions in Column (10). Table 5, follow directly from Column (2), Table 4, which, when multiplied by λ, represent the concentrated loads sustained by the cable. The expressions in Column (11), Table 5, when multiplied by λ, represent the net reaction and shears; and those in Column (12), when multiplied by λ^2, represent the moments which, when divided by H, represent the cable deflections, δ_{cx}, when referred to a vertical plane through the tower tops.

The expressions in Column (12) remain the same for any structure in which twenty divisions are used, except the numerical term in each expression, which is proportional to the total unit wind pressure on the structure. Note, also, that beginning with a in the first row of Column (12), the rule of coefficients applies.

The unit of measure of all terms and expressions in Tables 4 and 5 is kips per foot, or $\frac{\text{kips}}{\text{feet}}$; hence, the following units of measure:

Shear:

$$\lambda S'_x = \text{feet} \times \frac{\text{kips}}{\text{feet}} = \text{kips}$$

Bending moment:

$$\lambda^2 M'_x = \text{feet}^2 \times \frac{\text{kips}}{\text{feet}} = \text{kip-feet}$$

Shear for area-moment loads:

$$\lambda^3 S_{mx} = \text{feet}^3 \times \frac{\text{kips}}{\text{feet}} = \text{kip-feet}^2$$

Slope of elastic curve:

$$\frac{\lambda^3}{EI} S_{mx} = \frac{\text{feet}^3}{\dfrac{\text{kips}}{\text{inches}^2} \times \text{inches}^2\ \text{feet}^2} \times \frac{\text{kips}}{\text{feet}} = \frac{\text{feet}}{\text{feet}} \text{ (an abstract number)}$$

Deflection:

$$\delta_{tx} = \frac{\lambda^4}{EI} M_{mx} = \frac{\text{feet}^4}{\dfrac{\text{kips}}{\text{inches}^2} \times \text{inches}^2\ \text{feet}^2} \times \frac{\text{kips}}{\text{feet}} = \text{feet}\ .$$

Shear or transverse component in cables:

$$\lambda S_{ox} = \text{feet} \times \frac{\text{kips}}{\text{feet}} = \text{kips}$$

Deflection:

$$\delta_{cx} = \frac{\lambda^2}{H} M_{ox} = \frac{\text{feet}^2}{\text{kips}} \times \frac{\text{kips}}{\text{feet}} = \text{feet}$$

The expressions relating to the hangers are found for the ten points, as demonstrated in Table 6. The hanger lengths are given in Column (14), from

TABLE 6.—EXPRESSIONS RELATING TO THE DEFLECTIONS OF THE HANGERS (EXAMPLE 1).

Division points (1)	h_x, in feet (14)	$\frac{h_x}{p_{sx}}$ in feet² per kip (15)	z_x, in kips per foot (16)	$\Delta\delta_x = \frac{h_x}{p_{sx}} z_x$, in feet (17)	$\Delta\delta_x + T = \Delta\delta_x + 1.142$, in feet (117)
1..................	408.81	21.182	1.1—a	23.300—21.182a	24.442—21.182a
2..................	324 66	16 822	1.1—b	18.504—16.822b	19.646—16.822b
3..................	250.41	12.975	1.1—c	14.273—12.975c	15.415—12.975c
....	......	......	......	....	....
10..................	7.86	0.407	1.1—2j	0.448— 0.814j	1.590— 0.814j

which follows Column (15), since $p_{sx} = 19.3$ kips is constant. The expressions in Column (16) follow directly from Column (2), Table 1, since $z_x = w_t - p_{cx}$. Column (117) has been added to include the tower-top deflections, $T = 1.142$ ft. Taking these into account, the fundamental equation may be written as follows:

$$\delta_{tx} - \delta_{cx} = \Delta\delta_x + T$$

or,

$$\frac{\lambda^4}{EI} M_{mx} - \frac{\lambda^2}{H} M_{ox} = \Delta\delta_x + T$$

whence,

$$0.0453397\ M_{mx} - 0.37288\ M_{ox} = \Delta\delta_x + 1.142 \ldots\ldots\ldots\ldots (14)$$

By substituting the expressions for M_{mx}, M_{ox}, and $\Delta\delta_x$ in Equation (14), ten independent simultaneous equations containing ten unknown quantities may be written and solved. These equations have been omitted in this discussion. If the equations are written consecutively, beginning with Equation 1 for Point 1, with each equation transposed so that the unknown terms appear in alphabetical order beginning with a, and all numerical terms combined in the second member, a check of the coefficients may be made by the rule of coefficients. The labor involved in the solution is greatly reduced if a computing machine is used. A few hints concerning the solution may not be out of place. Begin with a and proceed alphabetically in the elimination. Note that in each equation one unknown quantity has a comparatively large coefficient, a in Equation 1, b in Equation 2, etc. This fact makes possible the choosing of one equation for the subtrahend in each elimination so that few, if any, changes in signs will result.

Several methods of solution by approximation are available to one favoring this procedure. Perhaps the best of these is assigning values to five unknown quantities in five of the equations and solving for the five remaining unknowns; for example, near the end of the truss the variable unit load sustained by the trusses differs but slightly from the load, w_t, acting on them, due to the proximity of the truss support and the distance from the trusses to the cables; and this sustained unit load decreases from the end of the trusses toward their center. Suppose that the values assigned are, $a = w_t = 1.1$; $b = 1.0$; $c = 0.9$; $d = 0.8$; and, $e = 0.7$. These values are substituted for a. b, c, d, and e in Equations 6, 7, 8, 9, and 10, the equations in which the coefficients of a, b, c. d, and e are comparatively small; and the equations are solved for f, g, h, i, and j. The values thus found are substituted in Equations 1, 2, 3, 4, and 5, and the equations solved for a, b, c, d, and e. These latter values are now taken in place of those originally assigned and the process is repeated until sufficient agreement is reached.

The results of the solution are illustrated in Column (2) of Table 7, and the remaining columns computed therefrom for a check by noting the close agreement for each division in Columns (117) and (18). The solution for maximum bending moments was the objective and, except for the check, the computations need not have been carried beyond Column (4).

The length of trusses between supports is 4 148 ft.; consequently, in finding the reaction, shears, and bending moments for the trusses, it is probably more nearly accurate to consider the length of each division as, $\lambda = \frac{4\,148}{20} = 207.4$; or $\lambda^2 = 4\,302$.

The bending moments at the division points are the quantities in Column (4), Table 7, multiplied by λ^2. The bending moments at the panel points of the truss are obtained by drawing a moment diagram and scaling the ordinates at the panel points. The shear in the end division is $S = 4.4316\lambda = 919$ kips; to which is added the load on one-half the end division to obtain the truss reaction; thus,

$$R = 919 + (1.1 \times 103.7) = 1\,033 \text{ kips}$$

The angular deflection of the trusses at the end is,

$$\alpha = \frac{\lambda^3}{E\,I} S_{mx} = \frac{91.8705\,\lambda^3}{E\,I} = 0.01911$$

For the cable, the length of each division is, $\lambda = \frac{4\,200}{20} = 210$; and the cable reaction on the tower is $R = 7.9184\,\lambda + (0.2 \times 110) = 1\,685$ kips.

TABLE 7.—COMPUTATION OF BENDING MOMENTS (EXAMPLE 1)

Division points (1)	p_{tx}, in kips per foot (2)	S'_x, in kips per foot (3)	M'_x, in kips per foot (4)	S_{mx}, in kips per foot (7)	M_{mx}, in kips per foot (8)	$\delta_{tx} = \frac{\lambda^4}{EI} M_{mx}$, in feet (9)	$p_{cx} = 1.3 - p_{tx}$, in kips per foot (10)	S_{ox}, in kips per foot (11)	M_{ox}, in kips per foot (12)	$\delta_{cx} = \frac{\lambda^2}{H} M_{ox}$, in feet (13)	$\frac{h_x}{p_{sx}}$, in feet² per kip (15)	$z_x = 1.1 - p_{tx}$, in kips per foot (16)	$\Delta\delta_x + T = \frac{h_x}{p_{sx}} z_x + 1.142$, in feet (117)	$\delta_{tx} - \delta_{cx}$, in feet (18)
		4.4316		91.8705				7.9184						
1	1.0961		4.4316		91.8705	4.17	0.2039		7.9184	2.95	21.182	0.0039	1.22	1.22
		3.3355		87.4389				7.7145						
2	1.0297		7.7671		179.3094	8.13	0.2703		15.6329	5.83	16.822	0.0703	2.32	2.30
		2.3058		79.6718				7.4442						
3	0.9435		10.0729		258.9812	11.74	0.3565		23.0771	8.60	12.975	0.1565	3.17	3.14
......	....	....	....	....	....	....	....	....	....	....	....	....	....	....
		−0.4880		4.2664				1.1380						
10	−0.9760		8.5327		508.2434	23.04	2.2760		56.4673	21.05	0.407	2.0760	1.99	1.99

TABLE 8.—EXPRESSIONS RELATING TO THE DEFLECTIONS OF THE TRUSSES (EXAMPLE 2)

Division points (1)	p_{tx}, in kips per foot (2)	S'_x, in kips per foot (3)	M'_x, in kips per foot (4)	S_{mx}, in kips per foot (7)	M_{mx}, in kips per foot (8)
		$8h + 7g + 6f + 5e + 4d + 3c + 2b + a$		$48h + 59.5g + 64f + 62.5e + 56d + 45.5c + 32b + 16.5a$	
1	0		$8h + 7g + 6f + 5e + 4d + 3c + 2b + a$		$48h + 59.5g + 64f + 62.5e + 56d + 45.5c + 32b + 16.5a$
		$8h + 7g + 6f + 5e + 4d + 3c + 2b + a$		$40h + 52.5g + 58f + 57.5e + 52d + 42.5c + 30b + 15.5a$	
2	$10h$		$16h + 14g + 12f + 10e + 8d + 6c + 4b + 2a$		$88h + 112g + 122f + 120e + 108d + 88c + 62b + 32a$
		$-2h + 7g + 6f + 5e + 4d + 3c + 2b + a$		$24h + 38.5g + 46f + 47.5e + 44d + 36.5c + 26b + 13.5a$	
3	$10g$		$14h + 21g + 18f + 15e + 12d + 9c + 6b + 3a$		$112h + 150.5g + 168f + 167.5e + 152d + 124.5c + 88b + 45.5a$
		$-2h - 3g + 6f + 5e + 4d + 3c + 2b + a$		$10h + 17.5g + 28f + 32.5e + 32d + 27.5c + 20b + 10.5a$	
......	......	......	......	......	......
9	$10a$		$2h + 3g + 4f + 5e + 6d + 7c + 8b + 9a$		$32h + 45.5g + 56f + 62.5e + 64d + 59.5c + 48b + 28.5a$
		$-2h - 3g - 4f - 5e - 6d - 7c - 8b - 9a$		$-32h - 45.5g - 56f - 62.5e - 64d - 59.5c - 48b - 28.5a$	

Example 2.—Maximum Shears.—Ten divisions were considered ample for sufficient precision in determining the shears. The computations were first made for the loading shown in Fig. 10. The expressions for the trusses in Table 8 and for the cables in Table 9, are developed in the same manner as in Example 1. Note that, beginning with the coefficient of a in the expressions for Point 1, and reading to the left and downward, the expressions in Columns (8) and (12) may be checked by the rule of coefficients. The expressions for the reactions at the top and bottom of Columns (3), (7), and (11), are determined by moments.

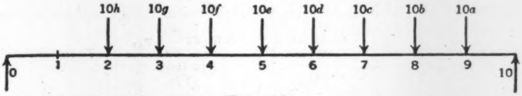

FIG. 10.

Because of the unsymmetrical loading, the deflections of the tower tops are unequal, and the compensating numerical term, T, which is constan; in all equations for Example 1, now becomes a variable as given in Table 10. The tower-top deflections are determined from the deflection in Example 1 by assuming that the deflection of each is proportioned to the total wind reaction from cables and trusses. This assumption is, of course, only approximately correct, but is used in lieu of a more exact method simply because the wind reactions thus found are statically determinate, as if the whole structure were a simple beam. Having thus established the tower deflections, the deflection, T, at each point, is found by interpolation. Full live load was assumed as in Example 1: $\lambda = \frac{4\,200}{10} = 420$; $\frac{\lambda^4}{E\,I} = 0.725435$; and, $\frac{\lambda^2}{H} = 1.49151$.

TABLE 9.—EXPRESSIONS RELATING TO THE DEFLECTIONS OF THE CABLES (EXAMPLE 2).

Division points (1)	p_{cx}, in kips per foot (10)	S_{0x}, in kips per foot (11)	M_{0x}, in kips per foot (12)
		$4.68-8h-7g-6f-5e-4d-3c-2b-a$	
1……	0		$4.68-8h-7g-6f-5e-4d-3c-2b-a$
		$4.68-8h-7g-6f-5e-4d-3c-3b-a$	
2……	$1.3-10h$		$9.36-16h-14g-12f-10e-8d-6c-4b-2a$
		$3.38+2h-7g-6f-5e-4d-3c-2b-a$	
3……	$1.3-10g$		$12.74-14h-21g-18f-15e-12d-9c-6b-3a$
		$2.08+2h+3g-6f-5e-4d-3c-2b-a$	
……	……	……	……
9……	$1.3-10a$		$5.72-2h-3g-4f-5e-6d-7c-8b-9a$
		$-5.72+2h+3g+4f+5e+6d+7c+8b+9a$	

Eight equations are required and only eight can be written because, while there are nine expressions each for M_{mx} and M_{ox}, there is no expression for $\Delta\delta_x$ for Point 1. The equations are written for Points 2 to 9, inclusive, and numbered accordingly. The equations are omitted in this discussion. If each equation is transposed so that the first member contains the unknown terms in reverse alphabetical order beginning with h, the coefficients may be checked by the rule of coefficients.

Much labor may be saved if the elimination begins with *a* and proceeds alphabetically until the value of *h* is found; for by so doing the solutions for the shears when Points 3 to 9, 4 to 9, and 5 to 9 are loaded, are much simplified. To illustrate, suppose that the equations are to be written when Points 3 to 9, inclusive, are loaded. In Table 8, Column (2), Point 2, make $p_{tx} = 10\,h = 0$, and the only change in all the expressions in this table is the elimination of all terms containing *h*. In Table 9, Column (10), Point 2, make $p_{cx} = 1.3 - 10\,h = 0$, and the only changes are in the numerical terms and the elimination of all terms containing *h*. The expressions for $\Delta\delta_x$ for Points 3 to 9, in Column (17), Table 10, remain unchanged; but the tower deflections, *T*, will be changed. The result is that the first member of Equations 3 to 9 will be precisely the same as in the previous solution, except that the first term in each equation containing *h* has disappeared. All the work, therefore, in the solution will be confined to the same manipulation of the numerical terms in the second member of the equations that was followed in the previous solution.

TABLE 10.—EXPRESSIONS RELATING TO THE DEFLECTIONS OF THE HANGERS (EXAMPLE 2).

Division points (1)	$\frac{h_x}{p_{sx}}$, in feet² per kip (15)	z_x, in kips per foot (16)	$\Delta\delta_x = \frac{h_x}{p_{sx}} z_x$, in feet (17)	*T*, in feet	$\Delta l_x + T$, in feet (117)
2....................	9.640	1.1—10*h*	10.604— 96.40*h*	1.007	11.611— 96.40*h*
3....................	4.511	1.1—10*g*	4.962— 45.11*g*	1.015	5 977— 45.11*g*
4....................	1.433	1.1—10*f*	1.576— 14.33*f*	1.020	2.596— 14.33*f*
....	......	....	....	....	....
9....................	16.822	1.1—10*a*	18.504—168.22*a*	1.067	19.571—168.22*a*

The values of S'_x, as determined for the various loadings, are given in Table 11.

TABLE 11.—COMPUTATION OF SHEARS (EXAMPLE 2).

Division points	POINTS LOADED			
	S'_x, in kips per foot (2)—(9)	S'_x, in kips per foot (3)—(9)	S'_x, in kips per foot (4)—(9)	S'_x, in kips per foot (5)—(9)
0				
	+1.18778	+0.71825	+0.42800	+0.26628
1				
	+1.18778	+0.71825	+0.42800	+0.26628
2				
	+0.31675	+0.71825	+0.42800	+0.26628
3				
	—0.27500	+0.03477	+0.42800	+0.26628
4				
	—0.33906	—0.17735	+0.09006	+0.26628
5				
	+0.42156	+0.43602	+0.54931	+0.73535
6				
	+0.43896	+0.46181	+0.49258	+0.59609
7				
	—0.08558	—0.07474	—0.05226	+0.01642
8				
	—0.91430	—0.90582	—0.88507	—0.82847
9				
	—1.93890	—1.92948	—1.90657	—1.85076
10				

In Division 1–2, or its counterpart, Division 8–9, the maximum is 1.18778
In Division 2–3, or its counterpart, Division 7–8, the maximum is 0.71825
In Division 3–4, or its counterpart, Division 6–7, the maximum is 0.59309
In Division 4–5, or its counterpart, Division 5–6, the maximum is 0.73535

In computing the shear, $\lambda S'_x$, the length of the division points is, $\lambda = \frac{4\,148}{10} = 414.8$. The shear in the end division, 0–1, was obtained from the loading in Example 1.

The authors' paper sheds much light on the question of adequate widths of suspension bridges. If the trusses in Example 1 of this discussion were treated as a simple span, receiving no assistance from the cables, the deflection at mid-span would be:

$$\frac{5 \times 1.1 \times 4\,200^4}{384 \times 29\,000 \times 1\,479\,110} = 104 \text{ ft.}$$

With the assistance of the cables the deflection was found to be 23 ft.; hence this suspension system has 4.5 times the lateral stiffness of a simple span of the same dimensions. This fact should be sufficient evidence that any method—either rational or empirical—which quite properly may be used in arriving at the width of a simple truss span, is entirely foreign to any discussion of the required width of a suspension bridge.

E. L. PAVLO,[5] ESQ. (by letter).—Owing to the great length of spans in suspension bridges, the elastic behavior and the induced stresses in this type of structure under the action of lateral forces are of considerable importance. As shown by the examples in the paper, the wind pressure on such spans amounts to more than 1 000 lb. per lin. ft. of bridge and the resulting stresses are of a magnitude sufficient to affect the determination of the width of structure. In addition, as a considerable part of the applied pressure is transferred from the trusses to the cables and thence to the saddles, the design of the towers is also affected. It may be stated also that the sag ratio, and therefore the general dimensions, of the structure are dependent upon the wind stresses in as much as the lateral deflections are proportional to the sag ratio for a given length of span. It will thus be seen that the wind stresses play an important part in the design of suspension bridges and their rigorous analysis is fully justified.

In this respect, in connection with the present tendency toward increased spans, this paper is of timely importance. The authors are to be commended for the clear and concise presentation of the problem of wind distribution and for the practical methods developed for its solution. The paper also contains much valuable information concerning several existing bridges. The rather condensed nature of the paper affords the writer an opportunity to elucidate some of the remaining aspects of the problem, as developed by him,[6] and to suggest three possible means of avoiding the procedure of successive approximations in the elastic distribution method described by the authors:

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[5] Asst. Engr., Robinson & Steinman, New York, N. Y.

[6] See thesis submitted to Massachusetts Institute of Technology in partial fulfillment of the requirements for the degree of Doctor of Science.

(1) By deriving and solving the differential equation of the deflection curve of the stiffening trusses;

(2) By writing the deflection equations and solving them simultaneously; and,

(3) By first assuming that the pressure transferred from the trusses to the cables varies as a parabola.

In addition to the notation introduced by the authors, the following will be used:

Let, H_a = horizonal component of cable stress before the advent of wind pressure, total for all cables.
H_w = additional horizontal component of cable stress due to wind pressure.
$H = H_a + H_w$.
$w = w_t + w_c$.
X = actual intensity of wind pressure sustained by trusses at any section.
r = intensity of wind pressure transferred from trusses to hangers at any section, called restitution.
Q = actual shear at any section of the truss.
M = actual moment at any section of the truss.
M_o = moment due w_c.
M_w = moment due to w.
M_c = moment due to $r + w_c$.
u = transverse tower deflection.

(1).—The Elastic Curve Method.—Referring to Fig. 11(*a*), the moment at any section, *A-A*, is,

$$M = M_w - H\,\delta_{cx} \quad \text{.......................} (15)$$

From Fig. 11(*c*), by taking moments around the cable, and noting that $\Delta\,\delta_x = \delta_x - \delta_{cx}$,

$$r\,(h_t - y)\,\cos\theta = p_{sx}\,(\delta_{tx} - \delta_{cx})$$

Substituting unity for $\cos\theta$ (that is, by taking the length of hanger equal to its vertical projection, since the difference is negligible), solving for δ_{cx} and substituting δ_{cx} in Equation (15);

$$M = M_w - H\left[\delta_{tx} - r\,\frac{(h_t - y)}{p_{sx}}\right] \quad \text{............} (16)$$

In Equation (16), in case of full loading,

$$M_w = w\,x\,\frac{l - x}{2} \quad \text{........................} (17)$$

By definition,

$$r = w_t - X \text{.........................} (18)$$

The load, *X*, can be expressed in terms of δ_{tx} as its fourth derivative multiplied by *E I;* thus,

$$X = EI\,\frac{d^4\,\delta_{tx}}{dx^4} \quad \text{.......................} (19)$$

Substituting Equation (19) in Equation (18),

$$r = w_t - EI \frac{d^4 \delta_{tx}}{dx^4} \qquad \text{(20)}$$

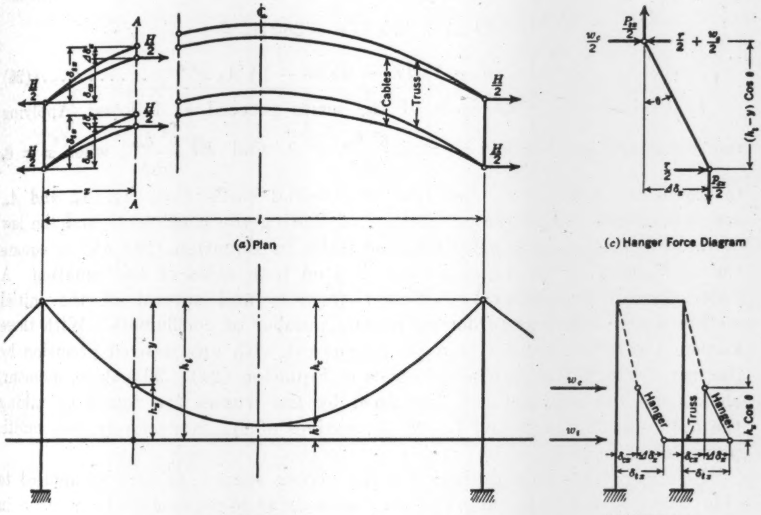
A
$\frac{H}{2}$
δ_{tx}
$\Delta\delta_x$
δ_{cx}
x
A
l
℄
Cables
Truss
(a) Plan
$\frac{P_{tx}}{2}$
$\frac{w_c}{2}$
$\frac{r}{2} + \frac{w_t}{2}$
θ
$(h_t - y)$ Cos θ
$\frac{r}{2}$
$\Delta\delta_x$
(c) Hanger Force Diagram
y
h_t
h_c
h_x
h
(b) Elevation
w_c
w_t
Hanger
Truss
Hanger
h_x Cos θ
δ_{cx}
$\Delta\delta_x$
δ_{tx}
(d) Section A-A

FIG. 11.—SUSPENSION BRIDGE UNDER THE ACTION OF LATERAL FORCES.

The equation of the cable curve referred to the origin at the top of the tower, is:

$$y = 4\, h_c\, x \frac{l - x}{l^2} \qquad \text{(21)}$$

Substituting M_w, r, and y from Equations (17), (20), and (21) in Equation (16),

$$M = w\, x \frac{l-x}{2} - H \left\{ \delta_{tx} - \frac{1}{p_{sx}} \left(w_t - EI \frac{d^4 \delta_{tx}}{dx^4} \right) \left[h_t - 4\, h_c\, x \frac{l-x}{l^2} \right] \right\} \qquad \text{(22)}$$

In the differential notation, $M = -EI \frac{d^2 \delta_{tx}}{dx^2}$; hence Equation (22) becomes,

$$-EI \frac{d^2 \delta_{tx}}{dx^2} = w\, x \frac{l-x}{2}$$
$$- H \left\{ \delta_{tx} - \frac{1}{p_{sx}} \left(w_t - EI \frac{d^4 \delta_{tx}}{dx^4} \right) \left[h_t - 4\, h_c\, x \frac{l-x}{l^2} \right] \right\} \qquad \text{(23)}$$

which is the differential equation of the elastic curve of stiffening trusses. This equation is not an integrable type, but can be solved by the use of the series:

$$\delta_{tx} = A_0 + A_1 x + A_2 x^2 + \ldots + A_n x^n \qquad \text{(24)}$$

$$\frac{d^2\,\delta_{tx}}{dx^2} = 2\,A_2 + 6\,A_3\,x + 12\,A_4\,x^2 + \ldots + n\,(n-1)\,A_n\,x^{n-2} \ldots\ldots (25)$$

and,

$$\frac{d^4\,\delta_{tx}}{dx^4} = 24\,A_4 + 120\,A_5\,x + 360\,A_6\,x^2 + \ldots$$
$$\ldots + n\,(n-1)\,(n-2)\,(n-3)\,A_n\,x^{n-4} \ldots\ldots\ldots\ldots\ldots (26)$$

To evaluate the coefficients of the series proceed as follows: Applying the boundary conditions, $\delta_{tx} = 0$, $\frac{d^2\,\delta_{tx}}{dx^2} = 0$, and $EI\,\frac{d^4\,\delta_{tx}}{dx^4}$, when $x = 0$, to Equations (24), (25), and (26), the initial coefficients, A_0, A_2, and A_4, are determined. The general method of finding the coefficients and the law of their formation is to substitute the series in Equation (23) and to equate the coefficients of the same powers of x on both sides of the equation. A knowledge of the general coefficient formula and several of the initial coefficients is sufficient to determine any number of coefficients. With these known, the deflection, δ_{tx}, can be determined with any desired precision by the use of a sufficient number of terms in Equation (24). The slope, moment, shear, and the load actually sustained by the trusses are found by taking the first, second, third, and fourth derivatives of δ_{tx}, respectively, and multiplying them by EI.

Equation (23) was derived for the center span. It can be applied to side spans as well; the only change necessary is to replace h_t by $x \tan \alpha$ in Equation (23), $\tan \alpha$ being the slope of the side cable chord.

From the theoretical viewpoint the method has the advantage that it is quite general and that all load functions can be expressed in terms of a single variable, x. In case of partial loading the application of the foregoing method becomes involved and in this case the following method will be found more advantageous to apply.

(2).—*Method of Deflections.*—Part of the wind pressure applied to the trusses is transferred in the form of unknown concentrated loads through the hangers to the cables. The structure is, therefore, indeterminate to the nth degree, n being the number of hangers. To obtain an exact solution it is necessary, therefore, to use n independent equations with n unknowns. In case of a symmetrically loaded structure the number of unknowns and equations will be $\frac{n}{2}$, since the unknowns at the panels equi-distant from the center will be equal. For practical purposes, however, it is unnecessary to obtain the distribution of wind pressure at every panel point. Owing to the regularity of the distribution curve it suffices to determine its ordinates at less frequent intervals. For this reason the span may be divided into a smaller number of panels thereby greatly reducing the number of the required equations.

In the present analysis the span is divided into ten equal panels and the applied pressure extends over the entire span. The indeterminacy of the structure is thus reduced to the fifth degree, necessitating the use of five equations. These equations will be written in the form, $\delta_{tx} = \delta_{cx} + \Delta\,\delta_x$ (see Fig. 11(*d*)).

Let X_1, X_2, X_3, X_4, and X_5, be the actual intensity of wind pressure sustained by the lateral system at Panel Points 1, 2, 3, 4, and 5, respectively. The truss and cable loading diagrams will then be as shown in Fig. 12(*a*) and Fig. 12(*b*). The loading diagram for the cable is shown for all cables combined.

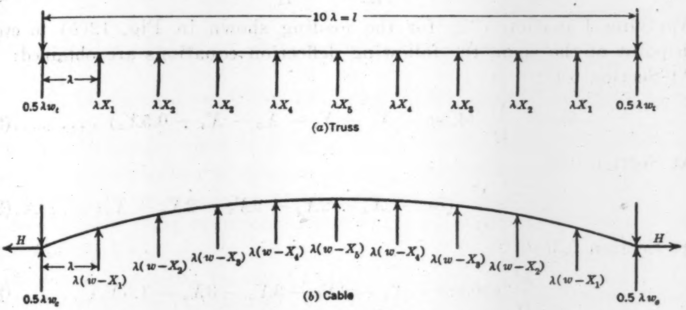

FIG. 12.—LOADING DIAGRAMS.

Deflection δ_{tx} will be found by means of the moment-area principle as demonstrated in Tables 12 and 13.

TABLE 12.—WIND LOADS, SHEARS, AND MOMENTS

Section	Load	Shear, *Q*	Moment, *M*
0		$\lambda(X_1 + X_2 + X_3 + X_4 + 0.5X_5)$	0
0.1*l*	λX_1	$\lambda(X_2 + X_3 + X_4 + 0.5X_5)$	$\lambda^2(X_1 + X_2 + X_3 + X_4 + 0.5X_5)$
0.2*l*	λX_2	$\lambda(X_3 + X_4 + 0.X5_5)$	$\lambda^2(X_1 + 2X_2 + 2X_3 + 2X_4 + X_5)$
0.3*l*	λX_3	$\lambda(X_4 + 0.5X_5)$	$\lambda^2(X_1 + 2X_2 + 3X_3 + 3X_4 + 1.5X_5)$
0.4*l*	λX_4	$\lambda(0.5X_5)$	$\lambda^2(X_1 + 2X_2 + 3X_3 + 4X_4 + 2X_5)$
0.5*l*	λX_5		$\lambda^2(X_1 + 2X_2 + 3X_3 + 4X_4 + 2.5X_5)$

TABLE 13.—MOMENT AREAS AND TRUSS DEFLECTIONS

Section	Moment area	Deflection, δ_{tx}	Equation No.
0	$\lambda^3(4.5X_1 + 8X_2 + 10.5X_3 + 12X_4 + 6.25X_5)$	0	
0.1*l*	$\lambda^3(3.5X_1 + 7X_2 + 9.5X_3 + 11X_4 + 5.75X_5)$	$\frac{\lambda^4}{EI}(4.5X_1 + 8X_2 + 10.5X_3 + 12X_4 + 6.25X_5)$	(27)
0.2*l*	$\lambda^3(2.5X_1 + 5X_2 + 7.5X_3 + 9X_4 + 4.75X_5)$	$\frac{\lambda^4}{EI}(8X_1 + 15X_2 + 20X_3 + 23X_4 + 12X_5)$	(28)
0.3*l*	$\lambda^3(1.5X_1 + 3X_2 + 4.5X_3 + 6X_4 + 3.25X_5)$	$\frac{\lambda^4}{EI}(10.5X_1 + 20X_2 + 27.5X_3 + 32X_4 + 16.75X_5)$	(29)
0.4*l*	$\lambda^3(0.5X_1 + X_2 + 1.5X_3 + 2X_4 + 1.25X_5)$	$\frac{\lambda^4}{EI}(12X_1 + 23X_2 + 32X_3 + 38X_4 + 20X_5)$	(30)
0.5*l*		$\frac{\lambda^4}{EI}(12.5X_1 + 24X_2 + 33.5X_3 + 40X_4 + 21.5X_5)$	(31)

Cable deflections at any point may be obtained by taking moments at the cable loads about the vertical axis passing through the point and dividing by H; thus:

$$\delta_{tc} = \frac{M_c}{H} = \frac{M_w - M}{H} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

Applying Equation (32) for the loading shown in Fig. 12(*b*) to every tenth point of the span, the following deflection equations are obtained:

At Section 0.1*l*:

$$\delta_{cx} = \frac{\lambda^2}{H}\,(4.5w - X_1 - X_2 - X_3 - X_4 - 0.5X_5) \quad \ldots\ldots\ldots (33)$$

At Section 0.2*l*:

$$\delta_{cx} = \frac{\lambda^2}{H}\,(8w - X_1 - 2X_2 - 2X_3 - 2X_4 - X_5) \quad \ldots\ldots\ldots (34)$$

At Section 0.3*l*:

$$\delta_{cx} = \frac{\lambda^2}{H}\,(10.5w - X_1 - 2X_2 - 3X_3 - 3X_4 - 1.5X_5) \quad \ldots\ldots\ldots (35)$$

At Section 0.4*l*:

$$\delta_{cx} = \frac{\lambda^2}{H}\,(12w - X_1 - 2X_2 - 3X_3 - 4X_4 - 2X_5) \quad \ldots\ldots\ldots (36)$$

At Section 0.5*l*:

$$\delta_{cx} = \frac{\lambda^2}{H}\,(12.5w - X_1 - 2X_2 - 3X_3 - 4X_4 - 2.5X_5) \quad \ldots\ldots\ldots (37)$$

Hanger deflections can be found by taking moments of hanger loads about the cable. From Fig. 11(*c*), and noting that $h_t - y = h_x$,

$$\Delta\,\delta_x = \frac{h_x}{p_{sx}}\,(w_t - X) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

Applying the relation, $\delta_{tx} = \delta_{cx} + \Delta\,\delta_x$, to every tenth point as far as the center of the span, using the deflections as given by Equations (27) to (31), (33) to (37), and (38), for the five points of the half span, the following five equations with five linear unknowns are obtained:

$$\frac{\lambda^4}{EI}\,(4.5X_1 + 8X_2 + 10.5X_3 + 12X_4 + 6.25X_5)$$
$$= \frac{\lambda^2}{H}\,(4.5w - X_1 - X_2 - X_3 - X_4 - 0.5\,X_5) + \frac{h_1}{p_s}\,(w_t - X_1) \quad \ldots\ldots (39)$$

$$\frac{\lambda^4}{EI}\,(8X_1 + 15X_2 + 20X_3 + 23X_4 + 12X_5)$$
$$= \frac{\lambda^2}{H}\,(8w - X_1 - 2X_2 - 2X_3 - 2X_4 - X_5) + \frac{h_2}{p_s}\,(w_t - X_2) \quad \ldots\ldots (40)$$

$$\frac{\lambda^4}{EI}\,(10.5X_1 + 20X_2 + 27.5X_3 + 32X_4 + 16.75X_5)$$
$$= \frac{\lambda^2}{H}\,(10.5w - X_1 - 2X_2 - 3X_3 - 3X_4 - 1.5X_5) + \frac{h_3}{p_s}\,(w_t - X_3) \quad \ldots (41)$$

$$\frac{\lambda^4}{EI}(12X_1 + 23X_2 + 32X_3 + 38X_4 + 20X_5)$$

$$= \frac{\lambda^2}{H}(12w - X_1 - 2X_2 - 3X_3 - 4X_4 - 2X_5) + \frac{h_4}{p_s}(w_t - X_4) \quad \ldots(42)$$

and,

$$\frac{\lambda^4}{EI}(12.5X_1 + 24X_2 + 33.5X_3 + 40X_4 + 21.5X_5)$$

$$= \frac{\lambda^2}{H}(12.5w - X_1 - 2X_2 - 3X_3 - 4X_4 - 2.5X_5) + \frac{h_5}{p_s}(w_t - X_5) \ldots(43)$$

The simultaneous solution of these equations furnishes the actual wind intensities sustained by the trusses at every tenth point of the span. With these known, shears, moments, and deflections can be readily evaluated. In the derivation of these equations, I represents either a constant or equivalent moment of inertia.

In case it is desired to use a variable I, the only difference will be in finding truss deflections. Instead of finding the deflections by taking moments of moment areas and dividing them by EI, it will be necessary to use the moment of the $\frac{M}{I}$ area and divide it by E. The method of deflection is also applicable to partial wind loadings and the side spans. A procedure analogous to the foregoing should be used in the derivation of equations. In both cases it is necessary to use a greater number of equations for the same accuracy on account of lack of symmetry in the loading and in the structure. Figs. 13, 14, and 15 show the results of applying the Method of Deflections to the

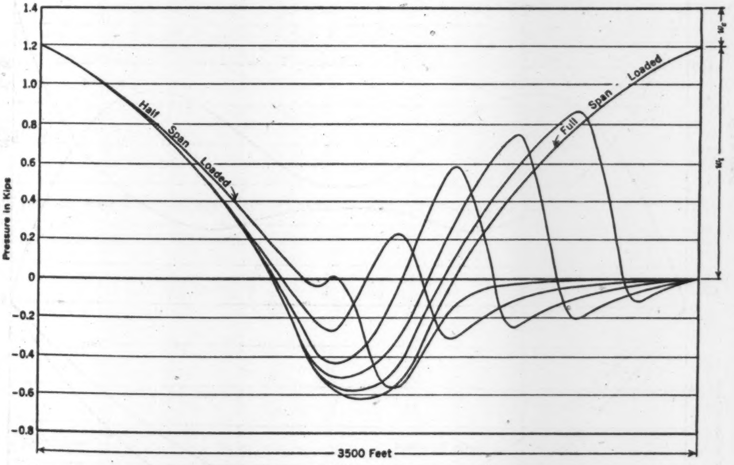

FIG. 13.—GEORGE WASHINGTON BRIDGE: DISTRIBUTION DIAGRAMS FOR FULL AND PARTIAL LOADINGS.

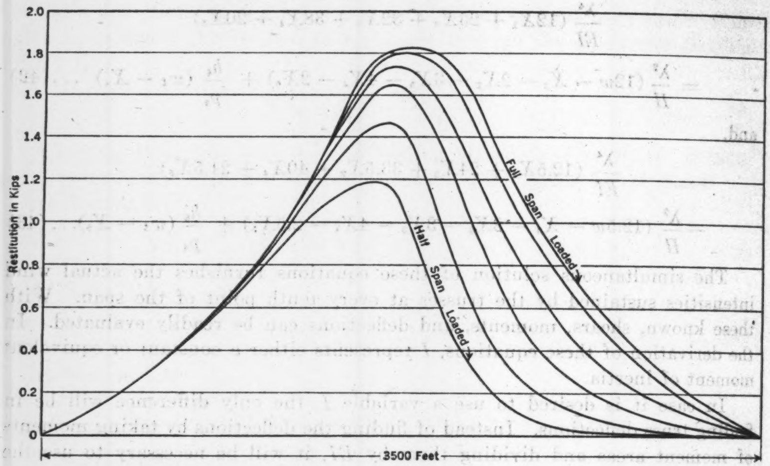

FIG. 14.—GEORGE WASHINGTON BRIDGE: RESTITUTION DIAGRAMS FOR PARTIAL WIND LOADINGS

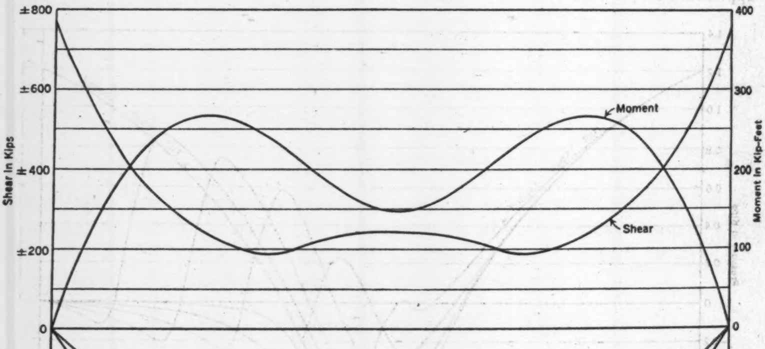

FIG. 15.—GEORGE WASHINGTON BRIDGE: MAXIMUM SHEAR, MOMENT, AND DEFLECTION DIAGRAMS.

George Washington Bridge under various conditions of loading. Ten equations were used in the analysis. The results for the full loading were identically checked by the elastic curve method described herein.

(*3*).—*Approximate Method.*—It can easily be perceived that there is no transfer of wind pressure from the stiffening trusses to the hangers at the ends of trusses since there is no deflection at these points. It is also obvious that the restitution, *r*, will be maximum at the center of the span in case of full wind pressure. For this reason the uniform wind distribution is not representative of the actual conditions. A closer approximation of the true nature of the distribution, as suggested by the results of the exact methods, may be had by assuming a parabolic variation of the restitution, *r*, as shown in Fig. 16.

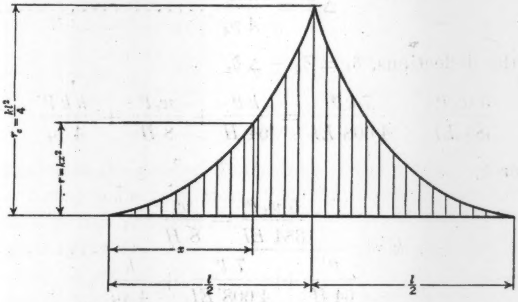

FIG. 16.—PARABOLIC RESTITUTION DIAGRAM.

On the basis of this assumption the restitution is then given by the equation:

$$r = kx^2 \quad \text{.......................} (44)$$

The pressure sustained by the trusses is expressed then, $w_t - r = w_t - kx^2$.

The shear at any section, x, is:

$$Q = w_t\left(\frac{l}{2} - x\right) - \left(\frac{kl^3}{24} - \frac{kx^3}{3}\right) \quad \text{.............} (45)$$

The moment at any section, x, is:

$$M = w_t\, x\left(\frac{l - x}{2}\right) - \left(\frac{kl^3x}{24} - \frac{kx^4}{12}\right) \text{............} (46)$$

The deflection at any section, x, is:

$$\delta_{tx} = \frac{1}{EI}\left[\frac{w_t\, x^3}{3}\left(\frac{x}{12} - \frac{l}{6}\right) + \frac{k\, l^3\, x^3}{144} - \frac{kx^6}{360} + \frac{w_t\, l^3 x}{24} - \frac{3\, k\, l^5 x}{640}\right] \text{..} (47)$$

and the truss deflection at the center of the span is:

$$\delta_t = \frac{5\, w_t\, l^4}{384\, EI} - \frac{7\, kl^6}{4\,608\, EI} \quad \text{..................} (48)$$

By isolating the cable and taking the moments about the center, $H\delta_c = \frac{k\,l^4}{48} - \frac{k\,l^4}{192} + \frac{w_c\,l^2}{8}$; whence,

$$\delta_c = \frac{k\,l^4}{64\,H} + \frac{w_c\,l^2}{8\,H} \qquad \text{(49)}$$

By Equation (38) $\Delta\,\delta_x = \frac{h_x}{p_{sx}}\,r$; whence,

$$\Delta\,\delta = \frac{hkl^2}{4\,p_s} \qquad \text{(50)}$$

Equating the deflections, $\delta_t = \delta_c + \Delta\,\delta$,

$$\frac{5\,w_t\,l^4}{384\,EI} - \frac{7\,k\,l^6}{4\,608\,EI} = \frac{k\,l^4}{64\,H} + \frac{w_c\,l^2}{8\,H} + \frac{h\,k\,l^2}{4\,p_s}$$

and solving for k,

$$k = \frac{\dfrac{5\,w_t\,l^2}{384\,EI} - \dfrac{w_c}{8\,H}}{\dfrac{l^2}{64\,H} + \dfrac{7\,l^4}{4\,608\,EI} + \dfrac{h}{4\,p_s}} \qquad \text{(51)}$$

With k known, the moments, shears, and deflections can be readily determined by the use of Equations (45) to (47).

The method gives very close results as compared with more exact methods of analysis.

Determination of H_w.—In all the previous analyses, H has been treated as a known quantity. It should be noted that it is also unknown since it involves the unknown H_w. In as much as H_w can be found only after the actual cable loads are obtained, it is necessary first to assume a certain H_w at the beginning of the analysis, to use $H = H_a + H_w$, and, after finding the actual cable loads, to check H_w. If the discrepancy between the assumed and the actually computed H_w is substantial, another trial H_w may be used, and the process continued until the desired agreement between the assumed and the computed H_w is reached. The additional horizontal component, H_w, does not usually amount to more than 2% of the total H. Its omission from the computations is on the safe side in so far as wind stresses are concerned.

Effect of Tower Deflection.—In case of appreciable transverse tower deflection the true deflection relation becomes $\delta_{tx} = \delta_{cx} + \Delta\,\delta_x + u$. Equations (39) to (43) should be modified accordingly by adding the constant, u, to the right-hand side of these equations. In Equations (23) and (51), this change should be introduced in the derivation.

It is hoped that the foregoing discussion will indicate some preferable variations in the analysis of wind-pressure distribution as presented by the authors.

Ralph A. Tudor,[7] **Jun. Am. Soc. C. E.** (by letter).—Tests made on a model of one of the suspension bridges proposed and studied for crossing the west channel of the Bay between San Francisco and Oakland, Calif., offered a rare opportunity to investigate the method developed by Messrs. Moisseiff and Lienhard to determine lateral deflections and stresses produced in the suspended structure by wind. Heretofore, this particular problem had not been answered satisfactorily, and it is gratifying that by analytical and mechanical methods a close check of results has been obtained.

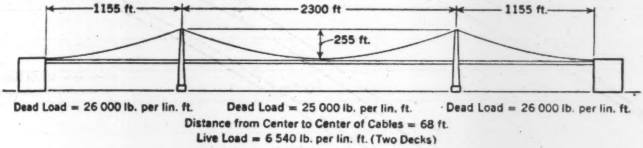
1155 ft.
2300 ft
1155 ft.
255 ft.
Dead Load = 26 000 lb. per lin. ft.
Dead Load = 25 000 lb. per lin. ft.
Dead Load = 26 000 lb. per lin. ft.
Distance from Center to Center of Cables = 68 ft.
Live Load = 6 540 lb. per lin. ft. (Two Decks)

Fig. 17.—Elevation of Bridge.

Fig. 17 illustrates the general dimensions of the prototype, which was reproduced with a linear scale reduction of 1 : 100. Since a complete description of the models and test procedure has been published elsewhere[8] these details are not repeated herewith.

Comparative values of truss reactions and deflections of cables and truss at mid-span were as listed in Table 14.

TABLE 14.—Comparison of Truss Reactions and Deflections

Item	Method	Truss Reaction at the Tower, in Kips		Cable Deflection at the Center Line of the Main Span, in Inches		Truss Deflection at the Center Line of the Main Span, in Inches	
		Full live load	No live load	Full live load	No live load	Full live load	No live load
1	Uniform distribution	657	461	70	50	73	52
2	Elastic distribution.	858	583	69*	49*	76*	54*
3	Model test........	989	659	67*	48*	74*	53*

* See, also, Figs. 18(*a*) and 18(*b*).

Fig. 18(*a*) clearly shows the extremely close agreement obtained in the deflection of all points along the truss and cable by the elastic distribution method and the model with no live load on the structure. Fig. 18(*b*) shows the comparative values with full live load on the bridge. Fig. 19 illustrates the load distribution curves as computed for the two cases.

Due to the fact that bending moments caused by wind were determined with partial live loads on the model it was difficult to make a direct comparison of values. However, such a comparison was made with live load covering approximately the middle-third of the main span and the following bending moments, in foot-kips, at the center of the span were found: Elastic distribution method, 3 300, and model tests, 3 020.

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[8] Univ. of California *Publication in Engineering*, Vol. 3, No. 2.

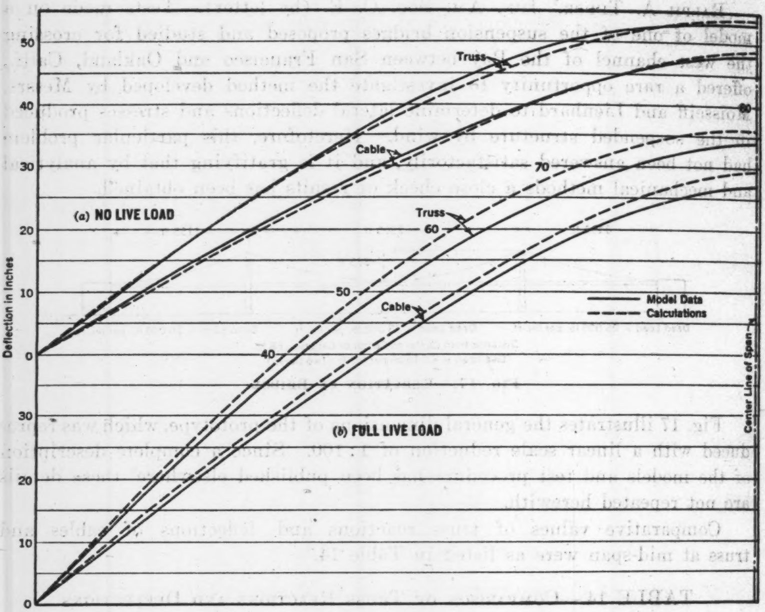

FIG. 18.—LATERAL DEFLECTION OF TRUSS AND CABLE.

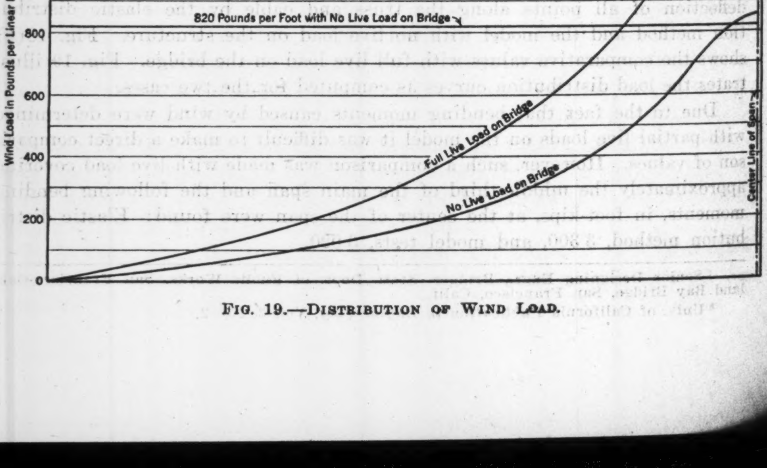

FIG. 19.—DISTRIBUTION OF WIND LOAD.

Conclusions.—As indicated by Messrs. Moisseiff and Lienhard, the uniform distribution method is simple of application, but the results obtained may be somewhat in error. It is sufficiently accurate, however, for determining maximum lateral deflections and for preliminary calculations of moments and shears.

The elastic distribution method is without appreciable error and provides the designer with a reliable means of obtaining bending moments, shears, and deflections of the stiffening truss of a suspension bridge.

The studies upon which this discussion is based clearly indicate the inherent lateral stability of a suspension bridge with adequate towers. Actual deflections are more greatly influenced by the dead load of the structure than by the stiffness of the trusses. The assumption of the following absurdity supports this statement: Consider the bridge discussed herein with all functions unaltered, except that the trusses are constructed so that they have zero moments of inertia about a vertical axis and are unrestrained laterally at the towers. Obviously, all wind must be carried by the cables to the towers. The deflection of the center of the main span truss is then found, by the elastic distribution method, to be only 11.1 ft. when the structure is fully loaded. This compares with a value of 6.2 ft. as found on the model and 6.3 ft. as found by calculation, with a truss of normal lateral stiffness.

F. H. CONSTANT,[9] M. AM. SOC. C. E. (by letter).—Much thought has been given to stresses in suspension bridges under the action of vertical loads, but those due to lateral forces have received scant attention. However, in narrow and moderately long spans, the latter may easily be considerably greater than those produced by the live load. This paper, in presenting a method, at once correct and simple, for the determination of the wind stresses in the lateral system of a suspension bridge, is very timely.

The practical usefulness of any trial-and-error method increases with the data acquired by repeated applications. With this in mind the writer would like to add another example, that of the Mt. Hope Bridge, in Rhode Island, to those already cited by the authors. In Fig. 20 are shown wind chord stresses for this bridge by each of the authors' two methods, for the case of wind acting uniformly over the entire main span.

The data used in the calculations are as follows: $l = 1\,192$; $h_c = 118.57$; $h_t = 124.57$; $w_c = 40$; $w_t = 360$; H (both cables, for combined dead load and live load over the entire center span, summer temperature) $= 9\,588\,000$; I (wind truss) $= 19\,600$; $E = 29\,000\,000$; P_{sx} (combined dead load and live load) $= 6\,800$; width of bridge (centers of stiffening trusses) $= 34$; and hanger length at center $= 6$.

By the uniform distribution method, $z = 241$ lb., leaving 119 lb. per lin. ft. as the uniform load to be taken by the truss. The corresponding chord stresses are shown by the parabola, Fig. 20.

In applying the elastic distribution method, the variation in I was taken into account, $\frac{I_c}{I_x}$ being 1.00 between the center and the quarter-point, and

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[9] Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

increasing to 1.30 at the end. Fig. 21 is the load distribution diagram (compare with the authors' examples). The synchronism between the cable and truss deflections at each of the ten sections chosen for the half-span, checked to within 0.10 ft. and generally was much closer. The corresponding chord stresses are shown by the upper curve in Fig. 20. For full wind loading over a symmetrical span the method is simple of application and accurate in results.

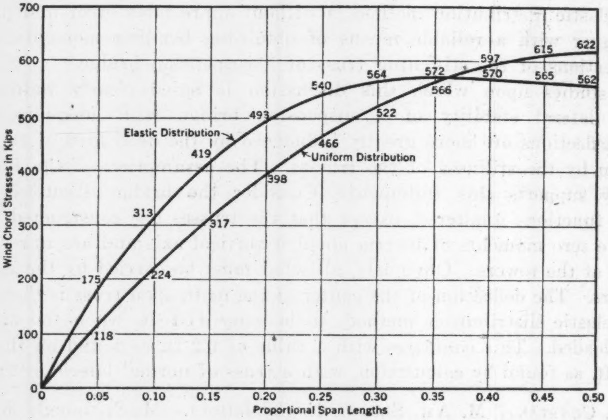

FIG. 20.—WIND CHORD STRESSES BY ELASTIC AND UNIFORM DISTRIBUTION METHODS.

The maximum live load chord stresses in the stiffening truss, however, do not arise when the span is fully loaded, but when the live load extends over only a part of the span. This not only changes the value of *H*, but as some of the lateral wind force must be due to the exposed surface of the live load, the actual wind loading on the span at the time the live load moments

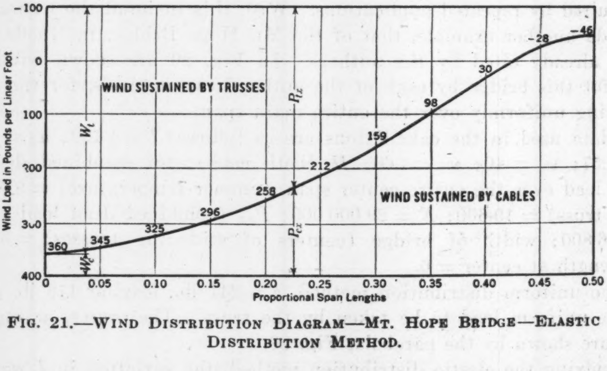

FIG. 21.—WIND DISTRIBUTION DIAGRAM—MT. HOPE BRIDGE—ELASTIC DISTRIBUTION METHOD.

are maximum, cannot be uniform over the entire span. In fact, this simple case is scarcely applicable at all, since every case of maximum live load moment is accompanied by a corresponding case of unsymmetrical wind loading; and, similarly, for the maximum diagonal stresses in the lateral system that occur under partial loading.

In the end spans, even if the loading is taken as uniform, the hanger lengths are not, and another case of dissymmetry arises. The authors' elastic distribution method is general enough to cover these cases, but it is important to point out the practical difficulties and the fact that the authors' simple loading case is itself an approximation.

Probably the most satisfactory solution of the wind problem in suspension bridges is obtained by the use of a model, such as was constructed at the University of California for the proposed San Francisco Bay Bridge. With such a model all combinations of both vertical and lateral loading conditions can be tried out in a short time. It would be of much interest in connection with this paper, if the results of those tests could be given and compared with those by the authors' methods.

CHARLES DERLETH, JR.,[10] M. AM. SOC. C. E. (by letter).—San Francisco Bay offers wide water barriers to easy traffic between the parts or boroughs of its metropolitan community. At San Francisco the barriers are much wider and deeper than at New York. For the past twenty-five years, in the writer's experience, costs of various bridge and tunnel projects have been bitterly debated and adequate truss dimensions disputed. There were those who clung to the belief that width of bridge and depth of truss must bear a direct ratio to length of span; and that weight and cost must bear a direct ratio to some function or power of the span, irrespective of whether the bridge is short or long, and without distinction in type. The contribution by Messrs. Moisseiff and Lienhard is of fundamental value in the solution of bridge building problems in San Francisco.

The Golden Gate requires a span of not less than 4 000 ft. The Consulting Board of Engineers for the Golden Gate Bridge and Highway District, after thorough examination, have fixed this span at 4 200 ft. The Engineering Board for the California Toll Bridge Authority, in preparing plans for the San Francisco-Oakland Bay Bridge *via* Yerba Buena Island, provisionally determined upon a double suspension bridge with central anchorage involving spans exceeding 2 300 ft. The only attractive alternative to this design would be the usual three-span bridge with a central opening of not less than 4 100 ft., and two long side spans. Foundation conditions require these dimensions.

It is evident then that for either the Golden Gate Bridge, or the trans-Bay projects, long spans are mandatory, and width of bridge is of the utmost importance. Messrs. Moisseiff and Lienhard have presented a notable and ingenious argument throwing light upon the capacity of long-span suspension bridges to withstand lateral wind forces.

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[10] Cons. Engr.; and Dean, Coll. of Eng., Univ. of California, Berkeley, Calif.

Great public projects in their pioneering days frequently meet with criticism and obstruction, due to conflict between proposed improvements and the existing order. The writer has no quarrel with those who object, enjoin, or criticize. In the long run a needed improvement receives impetus from such debating, which stimulates thorough study of the questions raised, and leads to discovery and invention in the science of design and to improvement in the art of construction. The paper had its inception in the early debates on the Golden Gate design. The authors were seeking a convincing argument to justify shallow stiffening trusses and slender wind-bracing for a 4 000-ft. span.

Testimony offered at San Francisco in opposition to the early Golden Gate Bridge designs argued that the designs presented were of inadequate width; that they did not have the necessary lateral stability. The proposed width between cables for the Golden Gate Bridge is 90 ft. Engineers of considerable accomplishment argued that, for a 4 000-ft. span, these widths should exceed 100 ft. and, perhaps, approach 200 ft. Necessarily, their estimates of cost greatly exceeded those of the criticized design. Messrs. Moisseiff and Lienhard's analysis completely answers these criticisms and gives to their paper a fundamental practical and public value, in addition to the merit which should be accorded them for an elegant analysis of a difficult problem.

They have clearly shown that the long-span suspension bridge is different in nature from simple trusses, arches, and cantilevers, and even from suspension spans of less length. One might speak of different species in a genus of suspension bridges. With long spans the cable becomes by far the stiffest member in the bridge system. It reduces the importance of the stiffening truss for vertical load distribution and likewise eliminates largely the necessity for heavy transverse wind framing and great width in the plane of the bridge floor. Sometimes, the writer likes to describe the theory of Messrs. Moisseiff and Lienhard as assigning to the floor system the nature of a kite in the wind, with the cable and suspenders acting as the restraining strings and tail. This may be a poetic rather than a mechanical figure of speech, but the analogy is clear. The authors have caused bridge engineers to consider long-span suspension structures from a newer point of view with respect to transverse resistance when compared to the usual treatment for wind forces in a simple bridge truss, an arch, or a cantilever.

Thus, the elastic distribution method is found to show that the cables relieve the stiffening trusses of wind load moments about 15% for an East River Bridge of 1 500-ft. span, while for the 3 500-ft. George Washington Bridge the relief is more than 80 per cent. For the Golden Gate Bridge, with two cables and a 90-ft. width of roadway, the relief is more than 50 per cent. These increments of relief are so substantial that, in long-span work, they silence all arguments for unnecessary floor widths, heavy wind framing, cable cradling, or other devices such as lateral stay cables where topography permits. They also silence claims for greatly increased costs, such as were advanced in the pioneer investigations at the Golden Gate.

The elastic distribution method adds additional arguments for the accuracy of cost analysis and the sufficiency of long-span designs, in that the authors show reduced wind reactions at the tower tops resulting from the application of the elastic distribution method; a reduction of 10% for the 1 850-ft. Detroit span, 20% for the George Washington Bridge, and about the same amount for the contemplated structure at the Golden Gate. This advantage becomes especially valuable for San Francisco Bay bridges when the possibilities of wind and other vibrations are considered. The elastic distribution method shows that the tall steel towers require less wind-bracing and receive less lateral deflection than heretofore assumed.

The application of these methods to the bridge from San Francisco to Oakland has justified narrower widths and double-deck floors; thus substantial savings are effected not only in the superstructure, but also in the foundations, whereas in earlier studies it was argued that for this enterprise long spans would demand one floor instead of two and widths exceeding 100 ft. instead of the present contemplated width of 66 ft. between cables and stiffening trusses.

It is gratifying to know that the models studied at the University of California[8] for proposed San Francisco bridges have checked closely the authors' theory.

A. A. EREMIN,[11] ASSOC. M. AM. SOC. C. E. (by letter).—In this paper the authors have shown some simplified methods of distributing lateral forces between the cables and the trusses.

In the uniform distribution method it was assumed that the transferred wind load from trusses to cables is distributed uniformly over the entire span length. However, the actual distribution is similar to that determined by the authors in the elastic distribution method (Fig. 2(*g*)).

A slight modification of the authors' Equation (5) is required to vary the assumption of wind-load distribution. It is also of interest to find an approximate expression for the effect of tower deflection on distribution of lateral forces. Fig. 22(*a*) shows the transferred wind load in the form of a parabola with the maximum transferred load, *z*, at mid-span. The cross-section in Fig. 22 shows deflections of the towers, the cables, and the trusses at mid-span.

Using the authors' notation, the horizontal deflection of the cables at mid-span for a load that varies as shown in Fig. 22(*a*), is:

$$\delta_c = \frac{l^2 (1.2\, w_c + z)}{9.6\, H} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (52)$$

and deflection of the trusses is:

$$\delta_t = \frac{l^4 (1.2\, w_t - z)}{92.5\, E I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

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From the cross-section, Fig. 22, the relation follows that:

$$\frac{h_t}{h_c} = \frac{\delta_t - \delta_a}{\delta_c} \quad \text{.........................} (54)$$

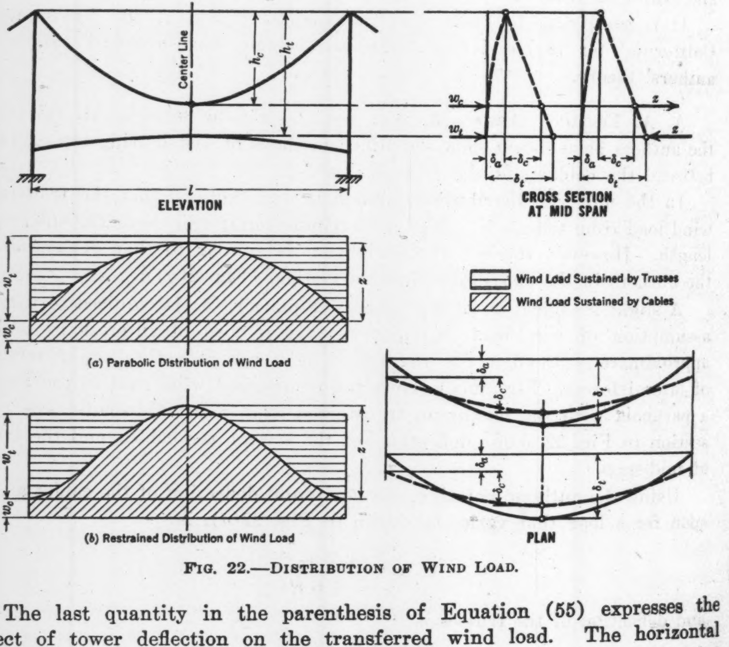
in which, δ_a is the deflection of the towers at the top. Combining Equations (52), (53), and (54), the transferred wind load, z, is:

$$z = \frac{11.5\, H\, h_c\, l^4\, w_t - 111.0\, E\, I\, h_t\, l^2\, w_c - 890.0\, E\, I\, H\, \delta_a\, h_c}{92.5\, E\, I\, h_t\, l^2 + 9.6\, H\, h_c\, l^4}$$

or,

$$z = K \left(\frac{w_t}{1 + \frac{b}{a}} - \frac{w_c}{1 + \frac{a}{b}} - \frac{\frac{c}{b}}{1 + \frac{a}{b}} \right) \quad \text{..............} (55)$$

in which, $K = 1.2$, and $c = 77.5\, E\, I\, H\, \delta_a\, h_c$. Values of the other factors are the same as those in Equation (5).


Center Line
h_c
h_t
w_c
w_t
z
z
δ_a
δ_c
δ_t
δ_a
δ_c
δ_t
l
ELEVATION
CROSS SECTION
AT MID SPAN
w_t
w_c
z
Wind Load Sustained by Trusses
Wind Load Sustained by Cables
δ_a
δ_c
δ_t
δ_a
δ_c
δ_t
w_t
w_c
z
PLAN

(a) Parabolic Distribution of Wind Load

(b) Restrained Distribution of Wind Load

FIG. 22.—DISTRIBUTION OF WIND LOAD.

The last quantity in the parenthesis of Equation (55) expresses the effect of tower deflection on the transferred wind load. The horizontal deflection of the towers at the top, δ_a, may be determined by the successive approximations.

In a long-span suspension bridge the ends of the stiffening trusses are restrained, due to the action of their own weight. Therefore, the distribution of the lateral forces will be similar to that shown in Fig. 22(*b*), in which the transferred wind load is in the form of a curve, with the maximum ordinate, z, at mid-span. An approximate algebraic expression of this curve may be assumed, as follows:

$$y = \frac{z\,(4\,x^2 - l^2)^2}{l^4} \qquad (56)$$

Thus, at mid-span, $x = 0$ and $y = z$.

The transferred wind load, z, Fig. 22(*b*), may be determined by Equation (55), in which case, $K = 1.35$ must be used. It is interesting to compare the effect of distributing wind load according to the parabola in Fig. 22(*a*), a curve, such as Fig. 22(*b*), and the distribution methods developed by the authors.

TABLE 15.—COMPARISON OF DISTRIBUTION METHODS

Distribution of wind load	Wind Reaction on One Tower, in Kips	
	Top	Truss level
Uniform *	2 060	540
Parabolic	1 700	900
Shown in Fig. 22 (*b*)	1 565	1 035
Elastic*	1 527	1 073

* Computed by the authors.

Table 15 shows wind-load reactions on the towers of a suspension bridge, with a span of 4 000 ft., which has been denoted by the authors as proposed Design No. 2. These reactions were computed with the assumption that the towers were not deflected. Referring to Design No. 2, the effect of a horizontal deflection of 1 ft. at the top of the towers, on the transferred wind load at mid-span, computed by the last quantity in the parenthesis of Equation (55), is approximately 14 lb.

From Table 15 it is evident that the reaction at the top of the tower computed with the assumption of wind-load distribution, as shown in Fig. 22(*b*), is only about 4% greater than the reaction determined by the elastic distribution method. However, computation by Equation (55) is much easier than that by the elastic distribution method.

ELMER C. OSGOOD,[12] JUN. AM. SOC. C. E. (by letter).—In textbooks, little can be found on the subject of stresses due to lateral forces in suspension bridges, although many texts deal quite thoroughly with stresses caused by vertical forces. Furthermore, the validity of some of the assumptions made in the few analyses that can be found is questionable. Consequently, the writer has made an investigation:[13] (1) To determine the importance of stresses due to lateral forces; (2) to find a method by which these stresses might be obtained with an exactness consistent with their importance; (3) to determine if, and for what cases, approximate methods could be used; and (4) to discover what assumptions are valid for a sufficiently exact general solution.

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[12] Greenfield, Mass.

[13] Presented as a thesis to Rensselaer Polytechnic Institute in partial fulfillment of the requirements for the degree of Doctor of Engineering in Civil Engineering.

Some of the conclusions were determined for the writer's benefit and are no doubt well known to those who have designed suspension bridges, and, therefore, will not be mentioned. Many of the others have been presented by the authors and, since they agree, need no further comment. Some of the results, which it is believed are not generally known, or which are not generally available, are presented herein.

The nature of the problem was investigated thoroughly, and a relation between the component parts of the suspended system was set up similar to that used by the authors in the elastic distribution method. The writer then investigated two approximate theories: (1) That presented[14] by D. B. Steinman, M. Am. Soc. C. E., and (2) that presented[15] by Johnson, Bryan, and Turneaure

Sufficient reason for accepting some of the assumptions made in the approximate theories could not be found; therefore, a more exact solution was sought. The attempts resulted in failure in so far as reaching the objective was concerned, but in each case the reasons why the method should fail, were found. Evidently, no entirely analytical exact solution has yet been devised, although efforts no doubt have been made to do so. It seems quite probable that more attempts will be made to this end in the future, because present methods are long and tedious. It would be well, then, that any logical attempts that have failed, be made generally known, since this would prevent not only possible duplication of useless effort when the reason for failure could be shown, but might also point the way to a different solution for some one else.

From the relation set up between the parts of the suspended system, using only those assumptions which the authors used in the elastic distribution method, the writer was able to derive the equation:

$$E\,I\,\frac{d^2\,\delta_{tx}}{dx^2} - H\,\delta_{tx} + \frac{H\left[h_t - \frac{4\,h_c\,x}{l^2}\,(l-x)\right]\left[w_t - E\,I\,\frac{d^4\,\delta_{tx}}{d\,x^4}\right]}{p_{sx} - u_x}$$
$$= (w_t + w_c)\,\frac{x^2}{2} + K_1\,x + K_2 \qquad (57)$$

The authors' notation has been used as far as possible. The factor, u_x, is the variable load per unit length of span sustained by the beam action of the vertical truss system at a distance, x, from one end; K_1 and K_2 are constants to be determined from the conditions of loading. The expression for u_x may be found in terms of x by any of the common formulas for vertical truss analysis. When no vertical trusses are used, $u_x = 0$.

Therefore, the equation can be made to contain, other than constants, only the independent variable, x, and its dependent variable, δ_{tx}. Although the writer did not know of, and could not find, any method of direct integration, mathematically a solution is possible by the use of an infinite series. An infinite power series,

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[14] "Suspension Bridges and Cantilevers", by D. B. Steinman, D. Van Nostrand Co., N. Y., 1913, p. 75.

[15] "Modern Framed Structures", by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, Pt. II, Tenth Edition, John Wiley & Sons, N. Y., 1929, p. 273.

$$\delta_{tx} = A + Bx + Cx^2 + Dx^3 + \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

was assumed. The necessary derivatives of this equation may be found and substituted in Equation (57). The sums of the coefficients of like powers of x may then be equated to zero and the values of the constants, A, B, C, etc., determined. A sufficient number of terms must be used in the series to insure that it be convergent.

By several trials in characteristic problems, it was found that, for the number of terms used in the series (13 were used in some cases), the equation when solved for large values of x did not even begin to converge. It might be argued that the equation could be used for finding values for the part of the span in which x is small, but it may be proved that, because one of the necessary conditions which must be introduced is that $\delta_{tx} = 0$ when $x = l$, the equation does not result in correct values for any part of the span unless it is convergent for all points. The only inference that could be drawn, therefore, was that the infinite power series is a form unsuited to express the deflection curve of the horizontal trusses.

It was found that another reason for the failure of this method was the necessity of expanding u_x into an infinite power series to express it in terms of x and constants. This series also required too large a number of terms to make it convergent.

An attempt was made to solve the equation by assuming an infinite trigonometric series similar to that used by Mr. S. Timoshenko,[18] in his analysis of vertical trusses. The equation was shown to be quickly convergent for the case that he considered, and it is reasonable to suppose that a similar one for horizontal deflection should be of like character. No trouble was encountered in a consideration of the internal work; but a treatment of the external work led to an expression which was so complicated (since it necessitated the use of the derivatives of different types of functions), that the method failed.

The writer had reached this stage, when, through the courtesy of G. M. Rapp, Assoc. M. Am. Soc. C. E., he learned of the elastic distribution method. He decided then to discontinue further investigation for a method that was entirely analytical.

Comparisons were made on the Delaware River Bridge, the Detroit River Bridge, and the George Washington Bridge, using the two approximate theories previously mentioned and the elastic distribution method. The uniform distribution method described by the authors was not known to the writer. Mr. Steinman's method, however, is somewhat similar, although, in the longer spans it results in somewhat greater discrepancies than the authors' approximate method. It was found, nevertheless, that, for the types of bridges for which the approximate methods investigated by the writer were obviously intended (that is, for those in which the span length, wind load, and truss stiffness were such as to make impossible large cable restraint), these theories gave sufficiently accurate results to be used. Mr. Steinman's method is very simple in its application, and has a tendency toward safe values, especially in the longer spans. The discrepancies in the results of the method of Johnson, Bryan, and Turneaure are of about the same order of magnitude, but tend in the opposite direction. The moment curves of neither method reverse near the center in long spans as does the true moment curve. For the George Washington Bridge, because of these characteristics, the Steinman method gave nearly true values for the stresses near the ends of the span; whereas, the other method resulted in values very close to the true ones at the center.

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[18] "The Stiffness of Suspension Bridges", by S. Timoshenko, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 377.

In the derivation of both the approximate methods which were investigated by the writer (as is also the case at mid-span in the authors' uniform distribution method), the assumption was made that the hangers, prolonged at any point of the span, would intersect the line joining the tower tops. To determine how true such an assumption was, the writer plotted the hanger directions at several points for each bridge. It was found that the variations in the direction of the hangers from the assumed direction were greater for those spans for which the discrepancies between the results of the approximate and the elastic distribution method were greater, and much in the same ratio. This indicated that the assumption made in the approximate methods has a major effect upon the accuracy of the method and is not valid for a solution covering all cases.

The writer investigated the effect of not considering the additional stress in the cable caused by lateral forces when computing the truss stresses induced by those forces. For the George Washington Bridge, under wind over the entire span, this additional stress was found to be only a small part of 1% of the total cable stress, and, therefore, was of negligible importance. Since in this bridge there is a large cable restraint on the truss, it is evident that, in others, which are laterally stiffer and shorter, the percentage is much less.

The importance of the stresses induced in the vertical trusses as a result of the cable and hanger inclination caused by the horizontal deflection, with consequent uplift, was also determined. For the George Washington Bridge under wind over the entire span, the vertical rise at the center caused by the horizontal displacement was found to be a small fraction of a foot. The stresses induced in the vertical trusses by such a deflection are of negligible importance. As in the preceding case, in bridges which are laterally stiffer and shorter, these stresses would be even less.

An investigation was made for the Delaware River Bridge and the Detroit River Bridge to determine how closely the position of loading for maximum live load and temperature stress at any point compared with that for maximum total stress (including, also, stress due to wind on truss and moving load). Using increments in the loaded length of one panel, it was found that for most points the positions for the two cases were the same, whereas for a few points the difference was only one panel length. The greatest difference in the total stresses for the two cases was found to be less than 0.2% of the maximum total stress.

The effect of temperature upon wind stresses was determined for the Delaware River Bridge. It was found that the maximum difference between the stresses at extreme and mean temperatures was only about 0.5% of the stress at mean temperature.

Wind stresses in the chords of the side spans of the Delaware River Bridge were investigated. These stresses were found to be only about 5% of those induced by vertical loading. Since in long suspension bridges the tendency is toward relatively short side spans, it is evident that in the side spans of these bridges, lateral stresses are of even less importance. Furthermore, since the cable causes very little restraint upon the side spans, simple beam methods are sufficiently exact to determine these stresses.

The writer is glad to find that the conclusions he had drawn regarding the span lengths for which approximate methods could be used are practically the same as those of the authors. In no way do any of his findings disagree with the authors' proposals. The purpose of his discussion is rather to illustrate some of the difficulties of obtaining a simple but exact solution, and to clear up a few points of doubt which might occur to the student, if not to the engineer familiar with suspension bridge design.

Charles M. Spofford,[17] M. Am. Soc. C. E., and John B. Wilbur,[18] Assoc. M. Am. Soc. C. E. (by letter).—This paper furnishes a long needed treatment, in English, of the subject of wind stresses in suspension bridges, and the authors are to be commended for presenting it to the Society.

The authors' elastic distribution method is essentially a method of trial. Like other trial methods, it involves considerable labor, and any process that will shorten the calculations is helpful. The writers consider that the following method of determining approximately the forces of restitution may be used to advantage in applying the elastic distribution method given by Messrs. Moisseiff and Lienhard, thus reducing the number of trials necessary.

This method is based upon the excellent treatment of the subject given by E. Pigeaud,[19] supplemented by certain charts and tables prepared by the writers and A. F. Ghiglione, Jun. Am. Soc. C. E.

Notation.—In addition to the notation adopted by the authors, let,

a = one-half the length of the central span, in feet,
ma = the distance from center of span to any point under consideration;
K = a constant, depending upon variation in transfer of wind stresses along the span;
k = a constant depending upon magnitude of the restitution force resulting from the slope of the hangers due to horizontal deflection of the truss;

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[18] Instr., Dept. of Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.

[19] *Le Genie Civil*, Vol. 87, July, 1925.

I = average value of moment of inertia of lateral truss (an approximate average value of I based upon the moment of inertia of chords alone may be used, or a more exact value may be computed by determining the center deflection of the truss under a full uniform load by one of the usual methods, equating it to the center deflection of a beam under uniform load and solving for I);

M_r = moment on truss due to restitution;

M_w = moment on truss due to wind load, assuming truss as end supported;

V_r = end shear on wind truss due to restitution;

V_w = end shear on wind truss due to wind load, assuming truss as end supported;

δ_{cw} = total lateral deflection of the cable at any section, due to wind, assuming no lateral force in hangers;

δ_{tw} = total lateral deflection of the truss at any section, due to wind, assuming no lateral force in hangers;

δ_{cr} = total lateral deflection of the cable at any section, due to the lateral forces in the hangers;

δ_{tr} = total lateral deflection of the truss at any section, due to the lateral forces in the hangers;

$G = \dfrac{k}{\Delta\,\delta}$, with $\Delta\delta$ measured at the quarter-point of the main span;

$$F_m = \frac{1 - m^2}{1 + K m^2};$$

$$F_{1(m)} = \int_0^m \frac{1 - m^2}{1 + K m^2}\, dm;$$

$$F_{2(m)} = \int_m^1 F_{1(m)}\, dm;$$

$$F_{3(m)} = \int_0^m F_{2(m)}\, dm; \text{ an}$$

$$F_{4(m)} = \int_m^1 F_{3(m)}\, dm.$$

From Equation (11), which is an exact expression:

$$z_x = \frac{p_{sx}\ \Delta\ \delta_x}{h_x} \quad \text{..........................(59)}$$

A study of the distribution of the wind forces to the various hangers of the actual bridge shows that the following equation suggested by Pigeaud may be used to determine z_x with a considerable degree of precision for the case of a uniform load over the entire span:

$$z_x = p_{sx}\, \frac{k\ (1 - m^2)}{1 + K m^2} = p_{sx}\, k\, F_{(m)} \quad \text{................(60)}$$

Equations (59) and (60) give,

$$K = \frac{1}{m^2}\left[\frac{k h_x\ (1 - m^2)}{\Delta\ \delta_x} - 1\right] \quad \text{..................(61)}$$

Let $G = \dfrac{k}{\Delta\delta_{(0.5)}}$ (in which, the subscript in the denominator indicates that $\Delta\delta$ is to be measured at the quarter-point of the span; that is, where $m = 0.5$), and plot a curve for G against span length. The result is shown in Fig. 23, which is based upon the solution of a number of important suspension bridges.

Substituting the value of G in Equation (61) and letting $m = 0.5$, gives the following equation for K:

$$K = 3\;Gh_{(0.5)} - 4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (62)$$

from which the value of K may be obtained by using the value of G from Fig. 23 and the value of h at the quarter-point of the span. For example, consider a bridge with a span of 1 750 ft., with a length, $h_{0.5}$, of 58 ft. at the quarter-point. From Fig. 23, $G = 0.0805$ and from Equation (62), $K = 10$. It is to be noted that, since Gh is a ratio, values of G are in units of $\dfrac{1}{\text{distance.}}$

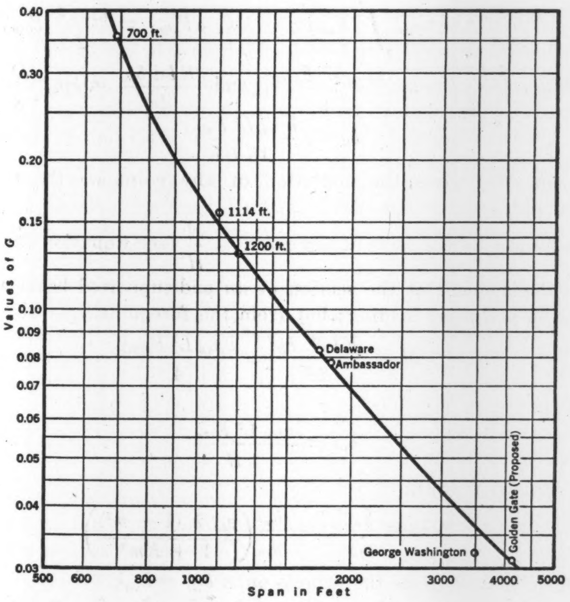

FIG. 23.—CURVE OF G FOR USE IN EQUATION (62).

With K known, the value of the constant, k, may readily be determined, as follows: It is evident that,

$$\delta_{cw(0)} + \delta_{cr(0)} + \Delta\,\delta_{(0)} + \delta_{tr(0)} = \delta_{tw(0)} \ldots\ldots\ldots\ldots\ldots (63)$$

It is also apparent that $\delta_{tw(0)} = -\frac{5}{384}\,\frac{w_t L^4}{E I}$, and $\delta_{cw(0)} = \frac{w_c L^2}{8 H}$, in which, H is the sum of horizontal components in both cables.

From Mechanics,

$$\frac{d^2 \delta_{tr}}{dx^2} = \frac{M}{E I} \quad \text{.......................}(64)$$

Furthermore, the first differential of the moment on a beam divided by EI equals the shear divided by EI, and the second differential of the moment divided by EI equals the load per foot divided by EI; hence,

$$\frac{d^2 M}{dx^2} = E I \frac{d^4 \delta_{tr}}{dx^4} = z_x = k\, p_{sx} \left(\frac{1 - m^2}{1 + km^2}\right) = kp_{sx}\, F_{(m)} \text{........}(65)$$

from which, the shear due to restitution is equal to:

$$\frac{E I\, d^3 \delta_{tr}}{dx^3} = kp_{sx} \int_0^x F_m\, dx = kp_{sx}\; aF_{1(m)} = \frac{kp_{sx} L}{2}\, F_{1(m)} \text{......}(66)$$

Therefore, $\frac{E I\, d^2 \delta_{tr}}{dx^2} = \frac{kp_{sx} L^2 F_{2(m)}}{4}$; and $\frac{E I\, d\, \delta_{tr}}{dx} = kp_{sx}\, \frac{L^3}{8}\, F_{3(m)}$; and,

$$\delta_{tr} = \frac{kp_{sx} L^4 F_{4(m)}}{16\, E I} \text{.......................}(67)$$

Equation (67) gives the deflection of any point on the truss due to restitution only.

The deflection of the cable at any point $= \frac{M}{H}$ (see Equation (9)), in which, M equals the moment at the section of an end-supported beam at the same point, due to a similar loading; but from the foregoing,

$$M = \frac{E I\, d^2\; \delta_{tr}}{dx^2} = \frac{kp_{sx} L^2 F_{2(m)}}{4} \text{.................}(68)$$

therefore,

$$\delta_{cr} = \frac{kp_{sx} L^2 F_{2(m)}}{4\, H}.$$

From Equation (59),

$$\Delta\, \delta_x = \frac{h_x}{p_{sx}}\; z_x = \frac{h_x}{p_{sx}} \left(\frac{p_{sx}\; k\; (1 - m^2)}{1 + Km^2}\right)$$

which, when $m = 0$, gives the expression, $\Delta\, \delta_{(0)} = h_{(0)} k$.

Substituting the foregoing values of δ_{tw}, δ_{cw}, δ_{tr}, δ_{cr}, and $\Delta\, \delta_0$ in Equation (63) and solving for k gives,

$$k = \frac{\frac{5}{384}\, \frac{w_t L^4}{E I} - \frac{w_c L^2}{8 H}}{\frac{p_{sx}\; L^4}{16\, E I}\, F_{4(0)} + \frac{p_{sx}\; L^2}{4\, H}\, F_{2(0)} + h_0} \text{.............}(69)$$

Values of $F_{2(0)}$ and $F_{4(0)}$ may be computed for various values of K and tabulated for repeated use. Thus, k may be readily determined for any given bridge. With K and k known, the value of z_x at any point of the truss may be easily obtained from Equation (60).

The values of z_x thus obtained may be used as trial values in the elastic distribution method and should reduce materially the number of trials necessary to give a satisfactory solution of the problem.

The application of this method to the determination of trial values of z_x for use in the elastic distribution method, as applied to the 4 000-ft. span solved by Messrs. Moisseiff and Lienhard, is as follows: From Fig. 23, $G = 0.0318$; whence, by Equation (62): $K = 3(0.0318)\ (141.2) - 4 = 9.47$; $\frac{5}{384}\,\frac{w_t L^4}{EI} = 81.595$; $\frac{w_c L^2}{8\,H} = 2.727$; $\frac{p_{sx}\ L^4}{16\ EI} = 7492.550$; and $\frac{p_{sx}\ L^2}{4\ H} = 800.000$. From the pre-arranged table, $F_{2(0)} = 0.261$; $F_{4(0)} = 0.100$; and hence, by Equation (69),

$$k = \frac{81.595 - 2.727}{7492.550\ (0.100) + 800\ (0.261) + 10.00} = 0.0815$$

Table 16 gives the trial values of z_x, as computed by the writers' method, as compared with the final values determined by the authors' elastic distribution method.

TABLE 16.—COMPARISON OF TRIAL VALUES OF z_x (IN POUNDS PER FOOT), AS COMPUTED BY THE WRITERS' METHOD, COMPARED WITH THE FINAL VALUES DETERMINED BY MESSRS. MOISSEIFF AND LIENHARD BY THE ELASTIC DISTRIBUTION METHOD

m	Trial values by writers' method	Final values by elastic distribution method	m	Trial values by writers' method	Final values by elastic distribution method
0.0............	1 793	2 000	0.6............	260	290
0.1............	1 623	1 650	0.7............	162	180
0.2............	1 248	1 115	0.8............	92	105
0.3............	881	790	0.9............	39	45
0.4............	599	550	1.0............	0	0
0.5............	400	410	....	....	....

The value of the truss moments due to lateral forces may be computed directly without the determination of z_x, by using curves of $F_{2(m)}$ plotted against m for various values of K.

From Equation (68) the moment in the truss due to restitution equals $M_r = \frac{k p_{sx} L^2}{4}\,F_{2(m)}$. The moment, M_w, on the truss, due to wind force, corresponds to the moment on an end-supported beam; hence, the actual moment carried by the truss equals $M_w - M_r$.

If the moments are found directly for the same 4 000-ft. span, the results shown in Table 17 are obtained by utilizing the aforementioned set of curves for $F_{2(m)}$. For purposes of comparison, the values obtained by the elastic distribution method are also inserted in this table. The results show the maximum discrepancy in the moment to occur where $m = 0.2$. The discrepancy at this point amounts to $\frac{18}{490} = 3.7\%$; hence, this method is sufficiently precise without further investigation to permit the making of a preliminary design, which can then be tested by the elastic distribution method, using the moment curve thus determined for the first trial.

TABLE 17.—COMPARISON OF RESULTANT MOMENTS BY TWO METHODS, IN THOUSANDS OF FOOT-KIPS.

m	$F_{2(m)}$	M_r	M_w	Writers' method (approximate)	Elastic distribution method
0.0	0.261	1 872	2 300	428	436
0.1	0.255	1 829	2 277	448	453
0.2	0.242	1 736	2 208	472	490
0.3	0.221	1 585	2 093	508	525
0.4	0.195	1 399	1 932	533	547
0.5	0.166	1 191	1 725	534	544
0.6	0.134	961	1 472	511	511
0.7	0.101	725	1 173	448	445
0.8	0.068	488	828	340	339
0.9	0.034	244	437	193	192
1.0	0	0	0	0	0

The checking of $\Delta\,\delta_x$ as computed from the various deflections at any point against the value computed from the assumed z_x by the relation, $\Delta\,\delta_x = \frac{h_x}{p_{sx}} z_x$, may be conveniently accomplished by the use of a set of curves giving values of $F_{4(m)}$, plotted against m for various values of K. This amounts to checking the value of K taken from Fig. 23.

The determination of the shearing forces may be accomplished as follows: By Equation (66), choosing $F_{1(1.0)}$ from predetermined tabular values, $V_r = 0.0815(22)\ (2\,000)\ (0.345) = 1\,239$ kips, and $V_w = 2\,000(1.150) = 2\,300$ kips. Therefore, the actual end shear equals $2\,300 - 1\,239 = 1\,061$ kips.

In the design of diagonals of the lateral truss, it is necessary to compute the maximum shears at points along the span due to partial wind loads. While the methods outlined by Messrs. Moisseiff and Lienhard are applicable to this condition of loading, the process of determining by trial the distribution of wind transfer is more difficult than for full wind load, since the curve of z_x is no longer symmetrical. An approximate method of obtaining these maximum moments as developed[20] by Mr. W. C. Bodycomb, Jr., may be summarized by the following three steps:

1.—Find the variation and magnitude of the force of restitution for:

(*a*) Wind load over entire span;

(*b*) Wind load from span center to right end.

2.—From the solutions under Step (1), determine the following quantities:

V_{LRC} = left end shear due to restitution with load extending from span center to right end.

V_{LRE} = left end shear due to force of restitution when span is fully loaded.

ΔR_c = total force of restitution from left end to the center, with load extending from span center to right end.

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[20] In a thesis submitted to Massachusetts Institute of Technology in partial fulfillment of the requirement for the degree of Master of Science.

3.—Insert these quantities in the empirical equation:

$$V_{\max} = \left[\frac{w_t L}{8} + \frac{3}{8} V_{LRB} + \Delta R_c\right] m^2 + \left[\frac{w_t L}{4} + V_{LRC} - \frac{11}{8} V_{LRB} - 2\,\Delta R_c\right] m$$

$$+ \left[\frac{w_t L}{8} - V_{LRC} + \Delta R_c\right] \text{ ....} (70)$$

Substitution of V_{LRC}, V_{LRB}, and ΔR_c in Equation (70) reduces it to the form,

$$V_{\max.} = A m^2 + B m + C \text{ ....} (71)$$

in which, A, B, and C are constants for a given bridge. The maximum shear for any value of m is easily computed from Equation (71). As is the case with most involved investigations of indeterminate structures, the time required for computations has made it impossible to compare exact solutions for a large number of bridges with the values given by this empirical curve. For such bridges as have been investigated, however, excellent results were obtained for spans up to 2 000 ft., and reasonably close values for greater spans. Equation (71) should be valuable for preliminary design.

A graphical solution for the various deflections is straightforward, and has been found particularly advantageous for partial wind loads. In connection with the graphical solution one must assume both the type of variation and the magnitude of the z_x curve. The following method will eliminate the necessity of assuming the magnitude correctly:

1.—By the methods of graphical statics construct:

(*a*) A curve of δ_{tw}.
(*b*) A curve of δ_{cw}.

2.—Assume a curve of z_x.

3.—On the basis of the assumed curve of z_x, construct curves for δ_{tr} and δ_{cr} (by the methods of graphical statics) and compute $\Delta\,\delta_x$ at points along the span, by the relation, $\Delta\,\delta_x = h_x \dfrac{z_x}{p_{sx}}$.

4.—At any convenient point (span center in case of full loading), determine the value of ϕ by,

$$\phi = \frac{\delta_{tw} - \delta_{cw}}{\delta_{tr} + \delta_{cr} + \Delta\,\delta_x} \text{ ....} (72)$$

All the deflections to be used in Equation (72) are measured at the point under consideration.

5.—Multiply all values of δ_{tr}, δ_{cr}, and $\Delta\,\delta_x$, from Step (3) by ϕ. At the point where ϕ is determined, Equation (63) will now hold. Whether or not it holds at other points depends on the type of variation of the z_x curve assumed. If Equation (63) holds at all points, the true values of z_x equal the assumed values multiplied by ϕ.

Factors not usually taken into consideration in computing wind transfer include tower movement perpendicular to span length, and increase in cable stress due to direct wind load. Either of these factors may be taken into account by a series of successive solutions, although the latter factor is easily shown to be so small as to be usually negligible. The question may be raised as to what portion, if any, of the live load should be considered as acting in computing restitution forces. Assuming the live load to extend over too great a length of roadway gives too large values for H and p_{sx}, the effect of which is to increase z_x and decrease the moments and shears on the lateral truss, which is on the unsafe side as far as the lateral truss stresses are concerned. Considerable uncertainty also exists regarding proper wind loads, time effect, inertia effect, and stiffening effect to the lateral system provided by the roadway.

GLENN B. WOODRUFF,[21] M. AM. SOC. C. E. (by letter).—Ever since Mr. Moisseiff, in designing the Manhattan Bridge, developed the mathematics of the deflection theory, engineers have been able to calculate accurately the stresses due to vertical loads in a suspension bridge. It is gratifying that Mr. Moisseiff, in collaboration with Mr. Lienhard, has furnished an equally accurate method of determining the stresses due to horizontal loads.

The mathematical processes are developed so clearly that they are subject to no criticism. In addition, the results have been confirmed by model tests. The writer will attempt only to emphasize some of the authors' conclusions.

In any bridge, except the suspension type, horizontal displacements of the structure may cause additive stresses of a serious nature. With a suspension bridge, the same degree of rigidity is not required as with other types. Furthermore, as the authors point out, the action of the cables adds greatly to the rigidity of the system.

Additional examples may further illustrate the point. The adopted design for the San Francisco-Oakland Bay Bridge has center spans of 2 310 ft. from center to center of towers, the stiffening truss span being 2 294 ft. The horizontal moment of inertia is 750 000 in.^{2} ft.^{2}

The lateral force on the unloaded structure has been assumed as 900 lb. per lin. ft. of the suspended structure plus 135 lb. per lin. ft. of cable. The distribution of the horizontal force between the trusses and the cables, by both theories, is shown by Fig. 24. The stresses by the two theories are, as follows:

	Uniform	Elastic
Moment at center of truss, in kip-feet....	329 000	340 000
Reaction at tower top, in kips............	614	451
Reaction at roadway, in kips............	573	736

The horizontal deflections of the system under different assumptions are, as follows:

	Deflections, in Feet	
	Cables	Truss
As designed	6.7	7.5
Assuming no relief from truss ($I_h = 0$)...	13.0	14.4
Assuming no relief from cables..........	....	14.0

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[21] Engr. of Design, San Francisco Bay Bridge Comm., San Francisco, Calif.

In one of the preliminary designs on this project a 3 800-ft. span was considered. It was desirable to use a truss spacing of not more than 72 ft. With a wind load of 1 050 lb. per lin. ft., the horizonal deflection was only 15.5 ft., or $\frac{1}{245}$ of the span.

It is believed that the foregoing with the other examples in the paper and discussions, is sufficient to demonstrate that the usual empirical rules for truss spacing have no application to suspension bridges. It is evident that the minimum truss spacing should vary as an inverse power of the span length, and quite possibly a spacing of 60 ft. would prove sufficient for the maximum possible span.

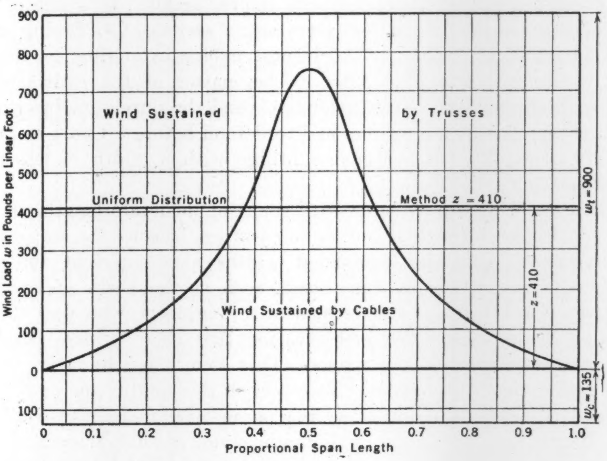

FIG. 24.

If there is any criticism to be made of the paper it might be that the authors do not recommend more strongly that the elastic distribution method be used for all final designs, the uniform distribution method being reserved for preliminary calculations only. As the authors point out, the difference in the moments calculated by the two methods is not great for spans of less than 1 500 ft. In the design of the lateral system, however, the uniform distribution method gives results erring on the unsafe side (see Table 3) by 13% for the Delaware River Bridge, 20% for the Detroit Bridge, and even more for the longer spans.

LEON S. MOISSEIFF,[32] M. AM. SOC. C. E., AND FREDERICK LIENHARD,[33] ESQ. (by letter).—The object in presenting this paper was to bring to the attention of engineers engaged in building long-span bridges the great lateral stability of suspension bridges, and to lay before them ready and tried methods by which they could compute the moments, shears, and deflections produced by lateral forces. The work presented is an outcome of the evolution of bridge building. With the growing demand for bridges of longer and longer spans the required width of bridge for a given long span became of decisive importance. The standard rules generally adopted by engineers for short and rigid bridges evidently do not apply to long spans; a ratio of one-twentieth of a span of 4 000 ft., according to these rules, demands a width of bridge manifestly uneconomical and impracticable. It is one of the curiosities of engineering that a few years ago a system of stiffening suspension bridges laterally by means of horizontal cables extending far beyond the trusses was proposed and patented. The matter of the satisfactory width of long-span suspension bridges extends beyond the purely engineering sphere. Some time ago one of the writers had to defend before a Court of Justice the feasibility of building a suspension bridge with a width of less than one-twentieth of the span.

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[32] Cons. Engr., New York, N. Y.

[33] Asst. Engr., The Port of New York Authority, New York, N. Y.

As long as bridges were built with spans of moderate lengths their lateral stability and behavior were of minor importance, because the requirements for carrying capacity usually demanded widths also sufficient for structural purposes. Longer spans in some cases, such as the Firth of Forth Bridge, were given special study and treatment. Engineers who deal with longer suspension bridges know from observation that these bridges develop much lateral stability under heavy winds. This knowledge however, was one of quality rather than of quantity. To establish this quality on a scientific basis a rational theory was required so that numerical quantities could be computed. Further, to verify the predicted behavior of such bridges, observations would have to be made on bridges under the action of lateral loads. As this is nearly impossible or, at any rate, impracticable, recourse must be made to observations on suitable models of bridges. This has been done by the engineers of the San Francisco-Oakland Bay Bridge.

At the time of the design of the Manhattan Bridge, with a width of 96 ft., the strengthening of the Williamsburg Bridge, with a width of 67 ft., and the design of the Delaware River Bridge, with a width of 89 ft., it was felt that these bridges were sufficiently wide to allow for the application of a simple and direct formula for the relief extended by the cables to the trusses. The formula used was that of the uniform distribution method as given in the paper. In proportioning the chords of the stiffening trusses of the Delaware River Bridge it became apparent that the wind stresses began to aspire to the control of the sectional area of the chords, and that a more profound and correct analysis of the wind effect was demanded than was offered by the uniform distribution method.

It should be explained here that in the relatively shorter spans of these bridges, chord sections of the stiffening trusses were determined by the effect of the live load and temperature, and that the wind stresses were not of sufficient magnitude to exceed the usual allowance made for wind by increasing the unit stresses. When, later, a study was made of the wind stresses on the George Washington Bridge, with a ratio of width to span of 33, and the Ambassador Bridge, at Detroit, Mich., with a ratio of 31, a more penetrating analysis was evolved, as given in the elastic distribution method.

The essence of this method is nothing more than the recognition of the phenomenon of the great resistance of the strained cables to distortion, on which phenomenon the deflection theory for vertical loads is based. From the standpoint of time, the uniform distribution method was not a simplification of the elastic distribution method, nor an intended approximation of it. The latter is a later and more thorough analysis of the behavior of a suspension bridge under the action of lateral forces. The elastic distribution method, as in the case of all progress of human knowledge, is a product of the necessities and demands of life. Bridges of longer span called for more accurate methods of analysis, structurally and economically, and these were brought forward in due time.

The manner of approach selected was that which appeared to the writers to be the most workable and most lucid. The choice of mathematical tools in many instances, is a purely subjective matter depending on the technical education and mental preference of the investigator. Many roads lead to Rome, and various mathematical approaches may bring the same results. It is important that Rome be reached. The numerical results of the investigation displayed characteristics of the behavior of suspension bridges, which were not known and not expected, but which with a clearer understanding should have been foreseen. The writers refer here to the characteristic projection of the wind-distribution curve in longer spans, showing a pull at the center of the truss in the opposite direction to the wind pressure.

The use of the moment-area principle appears to be a simple and pictorial way to present the bending moments in the trusses and to determine their magnitudes. The method is absolutely correct within its assumptions and can be applied to secure results as accurate as desired. Engineers are not likely to forget, however, that they deal with wind forces the magnitude, continuity, and extent of which are uncertain, and that the specified forces are likely to vary within wide limits. To apply hair-splitting accuracy here, would be quixotic.

Mr. Ellis has developed a process of applying the elastic distribution method in which all steps are algebraic and which leads to the solution of a number of simultaneous equations. He has established the equations for uniform as well as for partial loads; and he has also included the effect of the transverse deflection of the tower tops in these equations. The procedure is clear in its course and elegant in its construction. The "rule of coefficients" to which Mr. Ellis calls attention in establishing these equations will serve as an aid and a check of the work. By solving the simultaneous equations established for the given case of loading the "cut and try" procedure will be avoided. As stated by Mr. Ellis, both the algebraic and the writers' solutions give identical results.

In his study of the problem, Dr. Pavlo (as in the case of Mr. Ellis) has arrived at simultaneous equations. The discussions of Messrs. Ellis and Pavlo complement each other. The diagrams of distribution for partial wind loads on the George Washington Bridge are instructive. Dr. Pavlo also proposes an approximate method which is based on a convex parabolic restitution curve of the wind load, which approaches the forms of the curves shown in the paper. The proposed formula should give good results and can be readily applied.

Mr. Eremin's proposed approximation is an attempt in the wrong direction. His distribution curve is concave instead of convex as that of Dr. Pavlo and as are the plotted results of computations and tests. Mr. Eremin is mistaken in stating that "in a long-span suspension bridge the ends of the stiffening trusses are restrained, due to the action of their own weight." They are not restrained, unless especially designed in that way.

Mr. Pratley has made use of the assumption of a triangular distribution of the wind load with the peak at the center of the span. A formula based on this assumption can readily be deduced. It bears a marked resemblance to Equation (51) deduced by Dr. Pavlo for the convex parabolic assumption. The arithmetical application of either formula is equally simple, but Equation (51) will give the closer results. Mr. Pratley's question as to whether the floor system, lateral system, and truss web members are calculated as contributing to the lateral moment of inertia, in addition to the four chords, is answered in the negative. The effect of the lateral moment of inertia, of course, is not in linear proportion to the load transferred from the trusses to the cables and is not in linear proportion to the resulting moments and deflections. Its effect is rather small. In fact, the small effect of the width of the bridge is to some extent the basis of the paper. This is drastically illustrated by doubling the value of I in Example 1. The amount of wind load transferred from the trusses to the cables will change from 880 lb. to 702 lb. per lin. ft., a reduction of not more than 20 per cent.

Mr. Osgood also has chosen as his thesis the effect of lateral forces on suspension bridges. As others have done, he has sought a general analytical solution of the problem and has found it difficult of application. His findings of the relative value of the effect of the limiting assumptions made in the elastic distribution method are of interest and value.

The development and application by Professor Spofford and Mr. Wilbur of Pigeaud's paper on the action of wind on the trusses of a suspension bridge is another approach to the solution of the problem. It is scarcely the shortest. Long-span suspension bridges are not often built, and each case is of sufficient importance to demand the full attention of the designing engineers. Tables and curves prepared beforehand may all have their use but, to become thoroughly familiar with them, the engineer must dig his way through by himself. The discussion by Professor Spofford and Mr. Wilbur is, however, an important contribution to the subject.

Professor Constant's discussion has added the valuable diagrams which show the wind distribution and the comparative chord stresses for the Mt. Hope Bridge.

Mr. Woodruff's discussion gives the interesting comparative results of wind action as computed by the uniform and the elastic distribution methods for the newest of the long-span suspension bridges, the twin bridges over the West Bay at San Francisco, Calif.

Models were built in connection with the San Francisco-Oakland Bay Bridge at the University of California to a scale of 1:100. In addition to the effect of vertical loads, the effect of lateral loads was observed carefully. It is highly gratifying to find in Mr. Tudor's discussion an experimental verification both of the phenomenon of stability and the writers' method of its evaluation.

Dean Derleth clearly brings out the structural and economical value of the lateral stability of suspension bridges, which the paper attempts to demonstrate. He shows that the application of the theory set forth, has made it possible to limit the width of the 4 200-ft. span of the Golden Gate Bridge to 90 ft., and that of the San Francisco-Oakland Bay Twin Bridges of 2 310-ft. spans to 66 ft. The economies realized by the application of the writers' theory are considerable.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1850

CONSTRUCTION METHODS ON THE WELLAND SHIP CANAL[1]

By E. G. Cameron,[2] Esq.

Synopsis

The growth of the Welland Canal of 1829 to the Welland Ship Canal of 1931 represents a vast change in construction methods. Of the total cost of $130 000 000, about 60% was for earth and rock excavation and concrete. A description of the methods used in handling these three items of construction constitutes the body of the paper.

The letting of contracts, the division of the work into sections, the labor force and rates, and the difficulties and delays occasioned by the World War, are discussed preliminary to methods and costs.

In the study of costs it is to be borne in mind that they represent two periods of markedly different dollar values, namely, pre-war and post-war. The writer has provided extensive data showing the wide range of contract prices, but has confined his information to field costs, leaving the reader to use his own preference in adding overhead and rental charges.

Labor problems, wages paid, machine and plant outputs, together with descriptions of hydraulic dredging, embankment, and breakwater construction, rock drilling and removal methods, will be of interest to contracting engineers generally.

That section of the paper devoted to concrete and a description of the plant layouts, forms, and the methods of constructing reinforced concrete cribs afloat, and their various uses in structures, should be of particular interest to those who specialize in concrete.

The final section of the paper, devoted to the construction of a monolithic concrete siphon culvert, in which six tubes, each 20 ft. in diameter, carry

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[1] Presented at the Joint Meeting of the Construction and Waterways Divisions, Milwaukee, Wis., July 11, 1929.

[2] Prin. Asst. Engr., Welland Ship Canal, St. Catharines, Ont., Canada.

the waters of Chippawa Creek under the 30-ft. draft prism of the canal, treats of the construction and use of a large cellular steel sheet-pile cofferdam and the methods and difficulties experienced in carrying excavation and concrete operations in steel sheet-pile trenches sunk to a depth of 80 ft. in very soft clay.

HISTORY

Nature imposed an almost unsurmountable obstacle to the development of an inland waterway from the head of the Great Lakes to the Atlantic seaboard, by placing Lake Erie at an elevation of 525½ ft. above the next link in the chain—Lake Ontario.

From the days of Champlain (1600) until early in the Nineteenth Century, the limited but ever-growing commerce of the Great Lakes was portaged across the Niagara Peninsula from Queenstown, Ont., Canada, the limit of navigation in the Lower Niagara River, to Chippawa, Ont., where the Upper Niagara was again navigable through to Lake Erie.

On November 30, 1824, a private company largely financed by English and American capital, began the construction of a canal to provide a waterway with an 8-ft. draft between the two lakes. The route followed the winding valley of Twelve-Mile Creek from its outlet into Lake Ontario at Port Dalhousie, Ont., to its source in the face of the Niagara Escarpment in the vicinity of Thorold, Ont. (See Fig. 1). From this point, at the upper plateau level of the Peninsula, an entirely artificial channel was excavated due south to Port Robinson, Ont.—midway between the lakes. Chippawa Creek and the Upper Niagara River were used from Port Robinson to Lake Erie.

First Canal.—The construction of this first Welland Canal, using 40 wooden locks, each 110 ft. long by 22 ft. wide, was entirely effected by pick and shovel, supplemented at the lock structures by the broad-axe. The only means of transporting the excavation was by wheel-barrow. The rather meager record of this early undertaking registers the fact that, as a result of advertising a reward of £5 for the best mechanical contrivance for the removal of the excavation from the prism, a windlass was perfected by means of which the loaded wheel-barrows were hauled out of the prism and to the top of the adjacent spoil banks. Immigrant labor, very largely originating from Ireland, provided the working force on construction, and the rate of pay was 63 cents per day.

On November 30, 1829, just five years after the first sod was turned, two vessels—the *Ann and John,* of Toronto, Ont., and the *R. H. Broughton,* of Youngstown, N. Y.—passed through the completed canal from Lake Ontario to Lake Erie. In the first decisive engagement between Nature and Man, the latter had come off victorious.

Although the Erie Canal, across New York State to the Hudson River, had been previously completed, the combined accommodation of the two canals rapidly became inadequate to meet the increasing demands of shipping.

Second Canal.—In 1841 the Government of Upper Canada purchased this first canal from the Company, and immediately replaced it by a new Welland Canal of 9-ft. draft. This canal followed the same route; it had 27 locks of masonry construction, each 150 ft. long by 26.5 ft. wide. A few years later this second canal was deepened to 10-ft. draft and extended through to Lake Erie. The lock section of this canal is still intact and the splendid stonework and its remarkable state of preservation bear witness to the skill and integrity of the artisans of nearly a century ago.

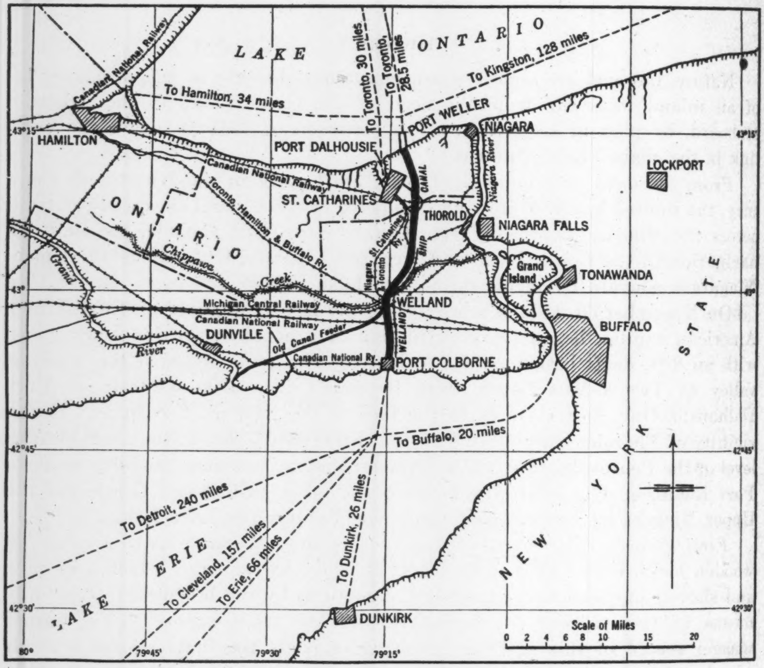

LAKE ONTARIO
To Toronto, 30 miles
To Toronto, 28.5 miles
To Kingston, 128 miles
Canadian National Railway
To Hamilton, 34 miles
PORT WELLER
NIAGARA
HAMILTON
PORT DALHOUSIE
43°15'
Canadian National Railway
LOCKPORT
CANAL
ST. CATHARINES
Toronto Hamilton & Buffalo Ry.
THOROLD
Niagara River
NIAGARA FALLS
ONTARIO
Niagara, St. Catharines & Toronto Ry.
SHIP
Chippawa
Creek
Grand Island
TONAWANDA
Grand
43°
Michigan Central Railway
WELLAND
Canadian National Railway
BUFFALO
DUNVILLE
Old Canal Feeder
River
Canadian National Ry.
WELLAND
PORT COLBORNE
To Buffalo, 20 miles
42°45'
To Detroit, 240 miles
To Dunkirk, 26 miles
NEW YORK STATE
LAKE ERIE
To Cleveland, 157 miles
To Erie, 66 miles
42°30'
DUNKIRK
Scale of Miles
0 2 4 6 8 10 15 20
80°
79°45'
79°30'
79°15'

FIG. 1.—MAP OF NIAGARA PENINSULA, SHOWING ROUTE OF WELLAND SHIP CANAL.

Third Canal.—The construction of the third Welland Canal (which was entirely replaced by the Welland Ship Canal in 1930), was inaugurated in 1870 by the Dominion of Canada. By 1887 a waterway, providing a 14-ft. draft, with 25 locks, each 270 ft. long by 45 ft. wide, was available for shipping. This was a much straighter and more direct route than that followed by the preceding canals across the lower plateau of the Peninsula to the foot of the Escarpment. On reaching the upper plateau level, much the same alignment as the previous canals was followed to Port Colborne, Ont.

As compared with present-day construction practices, comparatively primitive methods were utilized even on the third Welland Canal. The pick and shovel still held sway, assisted by teams and scrapers, while at the lock structures, derricks (chiefly hand-operated) were the only mechanical means then available for handling the masonry. A small steam shovel was introduced on the work toward the end of this construction period. However, as this type of equipment was then only in the development stage, the machine proved more of a novelty and an object of curiosity than an assistance in the actual execution of the work. Skilled artisans, thoroughly schooled in the intricacies of their trades, were paid about one-half the wage now paid the poorest class of unskilled labor.

This improved and enlarged waterway, together with a similar canal system along the St. Lawrence River from Lake Ontario to sea level at Montreal, Que., Canada, met the immediate demands of inland water commerce until the beginning of the Twentieth Century, but failed to keep pace with the ever-increasing size of cargo carriers being built on the Upper Lakes. Traffic congestion and cargo transshipment on Lake Erie, at the junction of 14-ft. navigation, with the greater draft accommodation of the Upper Lakes, was a logical result and demonstrated the necessity of a waterway across the Niagara Peninsula adequate for the accommodation of all Upper Lakes shipping.

Welland Ship Canal—The Fourth Canal.—With this object in view, investigations of various routes across the Niagara Peninsula were made, and the route along Ten-Mile Creek was finally adopted in 1913 for the construction of the Welland Ship Canal.

CONSTRUCTION CONTRACTS

Since the Confederation of the Canadian Provinces into the Dominion of Canada in 1867 the construction and operation of the Canadian canal systems have been administered by the Department of Railways and Canals. Its executive head is an elected Minister of the Federal Government, assisted by an appointed Chief Engineer, who directs all the engineering features of the Department's affairs. Construction of each of the canal systems is directed by an Engineer-in-Charge. Operating canals function under the direction of Superintending Engineers.

Method of Letting Contracts.—In initiating the construction of the Welland Ship Canal, the Department of Railways and Canals adhered to its standard practice of having the work done by unit price contracts.

The magnitude of the work and the construction difficulties involved made it necessary to adopt a procedure that would insure the work being placed in the hands of only thoroughly experienced and financially responsible contractors. To effect this, the entire work was divided into sections, each with an estimated cost considerably in excess of $1 000 000. In order to qualify, bidders were required to deposit security to the amount of 5% of their tenders. The contract for each division of the work was awarded to the bidder who, possessing the necessary credentials of experience and financial ability, submitted the lowest tender.

In the actual execution of the work the contractor was permitted to enlist the assistance of smaller competent contractors, by sub-letting portions of his division to parties who, in turn, could satisfactorily establish their ability to do the work allotted to them. In this case, however, the general contractor was free to make his own selection quite irrespective of whether it was based on the lowest sub-tender prices for the work.

That the foregoing procedure is an entirely satisfactory and effective method of handling the construction of such a large undertaking is best indicated by the fact that, while unit prices prevailing were low when the work was done, in no case during the entire period of construction did a general contractor or any of his sub-contractors fail to carry through his work in a manner satisfactory to the Department.

At the inception of construction in 1915 the work was divided, for the purpose of awarding contracts, into nine sections as shown in Fig. 2. Section No. 1 embraced the work at the north or Lake Ontario end and Section No. 9 incorporated the Port Colborne Harbor work at the south, or Lake Erie, end of the canal. Later, the number of sections was reduced to eight by combining the land and harbor portions of the south end of the canal at Port Colborne.

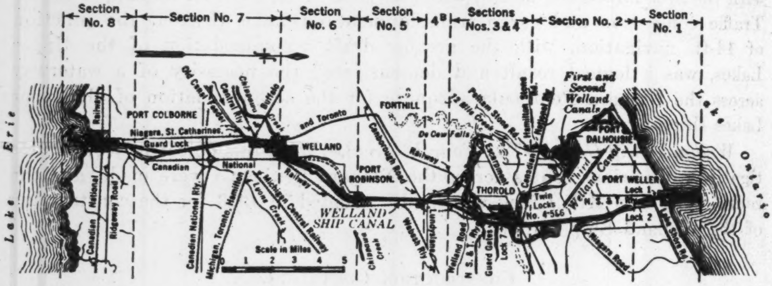

Section No. 8
Section No. 7
Section No. 6
Section No. 5
4B
Sections Nos. 3 & 4
Section No. 2
Section No. 1
Lake Erie
Lake Ontario
Railway
PORT COLBORNE
Niagara, St. Catharines,
Guard Lock
Canadian
National
Canadian
National
Ridgeway Road
Old Canal Feeder
Central Rly
Chippawa
Buffalo
Creek
and Toronto
WELLAND
National
Canadian National Rly.
Michigan, Toronto, Hamilton
Lyons Creek
Michigan Central Railway
Railway
Scale in Miles
0 1 2 3 4 5
PORT ROBINSON.
Camborough Road
WELLAND SHIP CANAL
Chippawa Creek
FONTHILL
Railway
Wabash Rly
Lundy's Lane
De Cew Falls
12 Mile Creek
Pelham Stone Rd.
Escarpment
Welland Road
N. S. & T. Rly
Guard Gates
Lock 7
THOROLD
Hamilton Stone Rd.
National Rly
Canadian
Twin Locks No. 4-5 & 6
First and Second Welland Canals
PORT DALHOUSIE
Third Welland Canal
PORT WELLER
Lock 1
N. S. & T. Rly
Lock 2
Niagara Road
Lake Shore Rd.

FIG. 2.—PRISM SECTIONS, WELLAND SHIP CANAL

Sections Nos. 1, 2, and 3 comprised the much heavier lock division of the canal from Lake Ontario to the summit, or Lake Erie, level. With a view to making the completion of all parts of the canal synchronize these sections were first let by contract and work was actually begun in the fall of 1913, together with Section No. 5—the northerly part of the summit level.

Delay Due to World War.—The outbreak of the World War in 1914 soon had its effect on construction activities on these sections of the work. By the spring of 1916 both labor and materials had become so scarce from diversion for war purposes, and the financial burden of the war had become so heavy on Canada, that the complete suspension of construction became necessary and the original contracts were cancelled.

The Department paid each contractor in full for all work done, and took over all plant at its cost on the work, less a percentage for depreciation equivalent to the percentage of the entire contract value actually done at the time of cancellation. Expenditures on the Welland Ship Canal up to the time all work was suspended approximated $16 000 000.

Post-War Activities.—Construction operations were completely suspended until the spring of 1919, when, as a means of furnishing employment to the many men returning again to civil life after the unsettling effects of the war, work was resumed on Sections Nos. 1, 2, 3, and 5, on a "cost-plus" basis, by the original contractors.

While this arrangement met the immediate object for which it was devised, it created other conditions that greatly retarded the progress of the work and so augmented the cost that the Department discontinued the percentage contracts in 1921 and thereafter adhered to the unit-price basis under which the remainder of the work was contracted. The partly completed Sections Nos. 1 and 2 were combined under one contract, as were also Sections Nos. 3 and 4. The contractors to whom the sections previously under construction were awarded, were required to take over the plant and equipment acquired by the Department with the cancellation of the pre-war contracts, at a valuation based on the prices paid at the time of its acquisition by the Department.

Before the war, general contracts were let for Sections Nos. 1, 2, 3, 4A, and 5, aggregating $12 017 000. After the war, contracts for the remainder of the canal and the gate-yard were let for a total of $62 932 000. The work embraced in these contracts does not include such major items of the Ship Canal requirements as bridges, lock-gates, valves, and operating equipment, machinery, and electrical apparatus. These, with a number of smaller features, were provided by entirely separate unit-price contracts.

Working plant and equipment with a value of $8 730 000 was placed on the various divisions of the work, including the equipment acquired on Sections Nos. 1, 2, 3, 4, and 5, as previously explained.

LABOR RATES

With modern and up-to-date equipment available, labor was still the governing factor of the construction operations on the Welland Ship Canal. In all public works constructed by the Dominion of Canada, the Federal Department of Labor exercises jurisdiction in the matter of fixing the minimum rates which must be paid the various classes employed.

At the inception of construction on any public work the Department of Labor investigates conditions prevailing in the immediate area of such work, and from the data thus obtained compiles a schedule of minimum wage rates which it requires shall be paid labor on the public work in question. This schedule of rates is furnished the parties tendering for their guidance in estimating costs, but is subject to revision by the Labor Department should conditions change during the period of construction.

The Department of Labor applied the foregoing procedure to this work, and the minimum rates of the principal classes of labor, their year by year fluctuations, and the application of overtime thereto, are given in Table 1. Only in a few exceptional cases did the contractors pay in excess of these minimum rates.

TABLE 1.—MINIMUM FAIR WAGE SCHEDULE IN EFFECT ON THE WELLAND SHIP CANAL

Class of labor	1913–16	1919–20	1921	1922	1923–24	1925–26	1927–30
Laborers	$0.20	$0.40	$0.375	$0.35	$0.40	$0.35	....
Expert laborers	0.20	0.45	0.45	0.45	0.50	0.50	....
Handymen	0.25	0.50	0.48	0.45	0.50	0.45	....
Concrete finishers	0.25	0.50	0.52	0.50	0.55	0.50	....
Carpenters	0.35	0.70	0.70	0.70	0.80	0.80	$0.85
Carpenter helpers	0.20	0.50	0.50	0.45	0.50	0.45	....
Locomotive runners	0.38	0.60	0.60	0.60	0.65	0.60	....
Firemen, locomotive crane	0.21	0.52	0.45	0.45	0.50	0.50	....
Brakemen	0.24	0.50	0.50	0.45	0.50	0.45	....
Conductors	0.36	0.60	0.60	0.60	0.65	0.60	....
Shovel runners	0.48	0.87	0.87	0.87	0.92	0.92	1.00
Shovel cranesmen	0.345	0.68	0.68	0.68	0.73	0.73	0.80
Shovel firemen	0.20	0.52	0.52	0.50	0.55	0.55	0.65
Dragline operators	0.35	0.56	0.56	0.50	0.55	0.45	....
Car repairers	0.20	0.55	0.56	0.50	0.55	0.45	....
Blacksmiths	0.30	0.70	0.70	0.60	0.65	0.60	....
Blacksmith helpers	0.20	0.50	0.50	0.45	0.50	0.45	....
Boiler-makers	0.35	0.75	0.75	0.60	0.65	0.60	....
Boiler-maker helpers	0.20	0.50	0.50	0.45	0.50	0.45	....
Machinists	0.30	0.70	0.72	0.65	0.70	0.65	0.70
Machinist helpers	0.20	0.50	0.50	0.45	0.50	0.45	....
Pipe fitters	0.25	0.60	0.70	0.70	0.75	0.65	....
Riggers	0.30	0.65	0.65	0.65	0.65	0.65	....
Rigger helpers	0.20	0.50	0.50	0.50	0.50	0.45	....
Drill runners	0.225	0.55	0.52	0.45	0.50	0.50	....
Powdermen	0.30	0.40	0.60	0.45	0.50	0.45	....
Hoist runners	0.35	0.65	0.70	0.70	0.75	0.70	0.75
Locomotive crane operators	0.35	0.80	0.75	0.60	0.60	0.75	0.80
Iron workers	0.30	0.85	0.80	0.80	0.80	0.80	....
Electricians	0.30	0.60	0.70	0.70	0.75	0.70	....

In cases of dispute between the contractor and his employees as to the fairness of rates being paid, such controversies were referred to the Federal Department of Labor which, after investigating the circumstances, would establish rulings governing the conditions in dispute, and both the contractor and his employees were obligated to abide by such prescribed regulations.

The normal working day was 10 hours throughout the period of construction, except during the years 1919–20, when an 8-hour day was in force for unskilled labor. The working day for skilled labor varied from 8 to 10 hours, depending on the class. In 1930, the 8-hour working day was adopted for all classes of labor. In all cases, time and one-half was allowed for overtime, with double time for Sundays and recognized labor holidays.

As the duration and extent of the construction operations each year were largely governed by weather conditions, the number of men engaged in the Ship Canal work varied from month to month. It reached a minimum during January or February each year when concrete and earth operations were largely suspended, while the maximum monthly average force during the post-war period of operations ranged from 1 462 in November, 1920, to 3 728 in August, 1926.

There was a marked increase in this force from 1924, because of construction on Sections Nos. 6, 7, and 8, which were put under contract during this period. The force reached a maximum in 1926, because the work on the lower sections, Nos. 1, 2, 3, and 4, was still in full swing with the extensive opening up of construction operations on the upper sections, Nos. 6, 7, and 8.

SUMMARY OF COSTS

The cost of excavation of earth and rock, and of the concrete of the various structures, constitutes approximately 58% of the total of $129 954 000 (see Table 2). The description of procedure which follows will be confined largely to these three features of the work.

TABLE 2.—DISTRIBUTION OF TOTAL COST, WELLAND SHIP CANAL

Description of items	Total cost	Pre-war cost	Expenditures to December 31, 1931	
			Dollars	Percentage
Sections Nos. 1 and 2	$18 725 000	$6 672 000	$18 675 000	99.7
Sections Nos. 3 and 4	26 189 000	4 666 000	25 125 000	95.9
Section No. 4A	75 000	75 000	75 000	100.0
Section No. 4B	738 000	0	655 000	88.6
Section No. 5	4 868 000	1 401 000	4 829 000	99.2
Section No. 6	10 557 000	0	9 800 000	92.8
Section No. 7	10 354 000	0	10 010 000	96.6
Section No. 8	12 136 000	255 000	11 379 000	93.8
Cement	11 482 000	1 006 000	11 410 000	99.4
Steel gates	5 773 000	0	5 538 000	96.0
Bridges	6 850 000	208 000	6 839 000	99.8
Lock metal work and machinery	4 961 000	160 000	4 500 000	90.7
Right of way	1 926 000	853 000	1 905 000	98.9
Transmission line and equipment	1 820 000	0	1 775 000	97.5
Power house and equipment	809 000	0	490 000	60.6
Construction railway	1 914 000	260 000	1 860 000	97.2
Gate-dock	450 000	0	450 000	100.0
Gate lifter	782 000	0	765 000	97.8
Buildings	543 000	0	500 000	92.1
Unwatering gates	302 000	0	302 000	100.0
Engineering	8 700 000	430 000	8 376 000	96.3
Totals	$129 954 000	$15 986 000	$125 258 000	96.4
Included in above are:				
Earth excavation	$25 232 000	$4 661 000	$24 455 000	96.9
Rock excavation	18 903 000	2 195 000	18 259 000	96.6
Concrete and cement	31 354 000	1 420 000	30 774 000	98.2

Earth Excavation

The earth removed in the construction of the Welland Ship Canal represents, approximately, 20% of the total cost of the entire undertaking, about 50 000 cu. yd. of excavation being involved.

Costs, Quantities, and Unit Prices.—Table 3 gives the range of contract unit prices for earth excavation with the quantities moved. It will be noted that the prices range from $0.1975 per cu. yd., in connection with the dredging on Section No. 1 during the pre-war period, to $4.86 per cu. yd. for tunnel excavation in connection with the gate-dock construction at Port Weller, Ont. The average is $0.4501 per cu. yd.

It is also to be noted that earth excavation prices prevailing during the pre-war period, from 1913 to 1916, were practically doubled on these same sections in the post-war period. In addition to the quantities in Table 3, handled on a unit-price basis, 2 639 000 cu. yd. were removed from Sections Nos. 1, 2, 3, 4, and 5, during the "cost plus" period from 1919 to 1922.

In the three cases (see Table 3) in which the unit price for excavation equals or exceeds $1.00 per cu. yd., this high price is due to some special operation being necessary in connection with the excavation work; as, for instance, in the case of the $1.50 price on Section No. 6, excavation was removed from within deep steel sheet-pile enclosures. The $4.86 price at the gate-dock was for earth removed in the construction of the emptying tunnel from this structure, by tunneling methods.

Plant.— Standard types of excavating equipment were generally used throughout the construction operations. The dragline took a very prominent part and proved particularly effective in the excavation of the prism sections where flat side slopes did not lend themselves to economical trimming by shovels. The much greater reach of the dragline, both horizontally and vertically, largely eliminated the extensive track-shifting operations that shovel operations would have necessitated. In many cases shovels excavated the center core and the draglines finished along the slopes, all loading into standard-gauge dump cars.

TABLE 3.—UNIT PRICES FOR EARTH EXCAVATION

Section Nos.	Period	Index	Quantities, in cubic yards	Unit price	Total cost
1	1913–16	Wet	1 570 000	$0.1975	$310 075
1	1913–16	Dry	1 685 000	0.34	585 537
2	1913–16	Prism	3 132 000	0.33	1 033 560
2	1913–16	Lock	2 010 000	0.40	804 000
2	1913–16	Breast walls	23 000	0.75	17 250
1 and 2	1922–30	Wet	1 167 000	0.84	991 950
1 and 2	1922–31	Dry	3 853 000	0.645	2 485 185
3	1913–16	General dry	2 254 000	0.35	788 900
3	1913–16	Trench	3 000	0.60	1 800
3	1913–16	Trench, deep	6 000	0.90	5 400
3 and 4	1921–31	General	5 200 000	0.60	3 120 000
4A	1913–16	General	79 000	0.35	27 650
4B	1926–30	Wet and dry	396 000	0.40	158 400
5	1913–16	Wet and dry	3 817 000	0.275	1 049 675
5	1921–31	Wet and dry	1 058 000	0.66	698 280
6	1926–31	Prism	9 399 000	0.375	3 616 265
6	1926–30	Siphon, open	631 000	0.63	397 530
6	1926–30	Siphon, trench	183 000	1.50	274 500
6	1926–30	Culvert, wet	1 066 000	0.45	479 700
7	1926–31	Wet and dry	10 213 000	0.42	4 281 060
8	1925–30	Dry	1 037 000	0.67	694 790
8	1925–30	Wet (harbor)	415 000	0.60	249 000
8	1925–30	Wet (lake)	20 000	1.00	20 000
Gate-dock	1928–30	Tunnel	4 000	4.86	19 440
Gate-dock	1928–30	General	180 000	0.40	72 000
Totals	....	....	49 281 000	0.4501	$22 181 947

The construction of the entirely artificial harbor at the north end of the Ship Canal at Port Weller, in Lake Ontario, necessitated the formation of two embankments, each extending 1½ miles out into the lake from the original shore line. These embankments were made with the surplus earth and rock excavation originating on the north end of the work between Thorold and Lake Ontario.

This material was removed from the prism at a number of different points, to its final destination in the lake, by several contracting organizations.

Construction Railway.—To unify this lake disposal, the Department of Railways and Canals provided a double-track, standard-gauge railway from the foot of the Escarpment at Thorold to Lake Ontario, a distance of 7½ miles, and managed it with a complete dispatching force. Sufficient sidings were provided from the railway to the canal prism to accommodate all excavation operations under way. Upon their arrival on the main line from any of the sidings, all contractors' trains operated under train orders, the block sys-

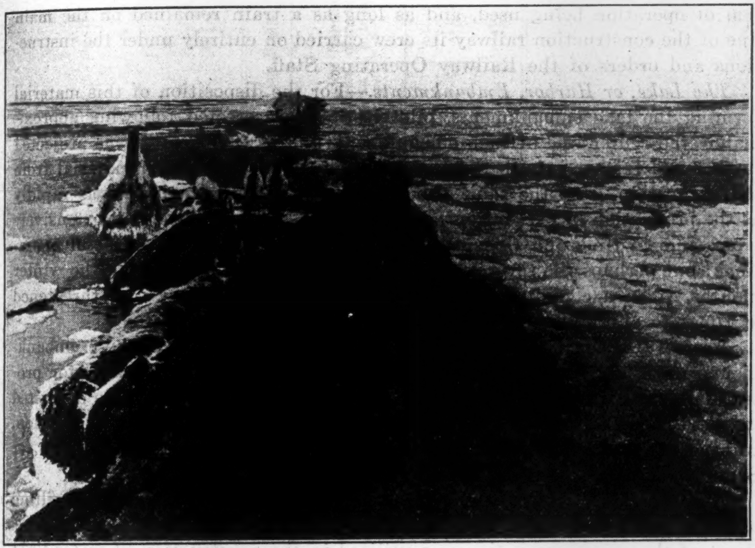


FIG. 3.—GAP BETWEEN WEST DUMP AND TRESTLE AT PORT WELLER HARBOR

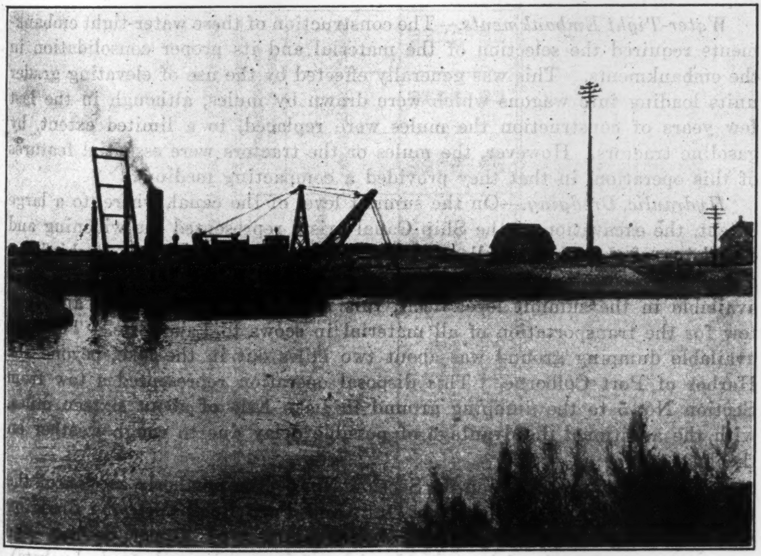


FIG. 4.—A 20-INCH HYDRAULIC DREDGE IN SPECIAL PUMPING BASIN

tem of operation being used, and as long as a train remained on the main line of the construction railway its crew carried on entirely under the instructions and orders of the Railway Operating Staff.

The Lake, or Harbor, Embankments.—For the disposition of this material forming the lake embankments, pile trestles were erected following approximately the center line of each embankment, and as each trestle was projected farther and farther into the lake it was immediately filled with material from the excavation. At the same time the trestle fills were widened as rapidly as the production of excavation permitted. These lake fills settled extensively from time to time and this introduced serious delays to the disposal operations; but perhaps the severest conditions were experienced during the winter months, when a combination of low temperature and heavy seas threatened at times to eliminate the embankments entirely. (See Fig. 3.)

From 1913, when the first of the excavation was placed in these embankments, until 1927, when they were completed with a facing of rock for protection, approximately 10 000 000 cu. yd. of excavated material were deposited in these construction units. All the operations involved in the excavation, transportation, and final disposal, were done under the unit prices given for Sections Nos. 1, 2, and 3. (See Table 3.)

A proportion of the necessary excavation was required for the back-filling of the structures and this was usually placed with comparatively short hauls. Some of the prism excavation was utilized in the construction of water-tight embankments along the prism where canal-water level would be above the original contour of the ground.

Water-Tight Embankments.—The construction of these water-tight embankments required the selection of the material and its proper consolidation in the embankments. This was generally effected by the use of elevating grader units loading into wagons which were drawn by mules, although in the last few years of construction the mules were replaced, to a limited extent, by gasoline tractors. However, the mules or the tractors were essential features of this operation, in that they provided a compacting medium.

Hydraulic Dredging.—On the summit level of the canal, where, to a large extent, the excavation of the Ship Canal prism represented the widening and deepening of the Third Welland Canal, excavation was carried on chiefly as a dredging operation; and as no disposal grounds for dredged material were available in the summit level itself, this introduced an expensive and long tow for the transportation of all material in scows to Lake Erie. The only available dumping ground was about two miles out in the lake, beyond the Harbor of Port Colborne. This disposal operation represented a tow from Section No. 5 to the dumping ground in Lake Erie of about sixteen miles, with the additional disadvantage of possible delay due to rough weather on the lake.

In handling this situation on Section No. 5 (the northerly section of the summit level, between Allanburg, Ont., and Port Robinson), the dredging contractor began operations with two standard type dipper dredges—the *Monarch* and the *Excelsior* (fitted with 55-ft. spuds and 5-yd. buckets). (See Table 4.) These dredges loaded into bottom-dump scows of all-steel

TABLE 4.—COMPARISON OF DIMENSIONS; DREDGES USED ON WELLAND SHIP CANAL.

Name of dredge	Type	Construction	DIMENSIONS OF HULL, IN FEET			Diameter of suction, in inches	Diameter of discharge, in inches	MAIN PUMP ENGINE OR MOTOR			MAIN PUMP	
			Length	Beam	Depth			Type	Horse-power	Normal full load speed, in revolutions per minute	Outside diameter, in feet	Impeller diameter, in inches
Monarch.............	Dipper.......	All-steel....	100	36	10	....	....	....	....	....	....	....
Excelsior.............	Dipper.......	All-steel....	100	36	10	....	....	....	....	....	....	....
Primrose.............	Hydraulic....	All-steel....	136	42	11	20	20	Triple expansion.....	800	200	....	....
Stewart.............	Hydraulic....	All-steel....	100	36	11	20	20	Direct-connected variable speed, electric motor.	1 500	500	8	57
Shuniah.............	Hydraulic....	All-steel....	125	40	10	22	20		1 500	500	8	57
Tornado.............	Hydraulic....	All-steel....	175	42	12	24	24	Triple expansion.....	1 800	182	10	60

construction, with capacities of 500 cu. yd. In this initial operation the excavation and loading operation cost 30½ cents per cu. yd., while the towing and disposal cost 40 cents per cu. yd., making a total cost for the dipper dredge excavation (with disposal in Lake Erie) of 70½ cents per cu. yd.

The long tow, combined with the lost time experienced at Port Colborne with the disposal operation in the lake being held up by rough weather, so reduced the output of the dredges that a more continuous method of disposal became necessary. To effect this the contractor placed a 20-in. hydraulic dredge, the *Primrose,* on the section (see Table 4).

However, as there was no territory available for use as a hydraulic disposal ground, throughout the length of the section, it was not possible for this dredge to excavate in the prism and discharge directly to the available disposal ground. Instead, the dredge was located in a basin excavated on one side of the canal prism entirely clear of the navigation channel, with the hydraulic disposal area immediately alongside. The output of the dipper dredges, excavating the prism, was brought by scow, and dumped in front of the hydraulic dredge, which rehandled it into the disposal area (see Fig. 4). By this arrangement the average tow was reduced from 16 to 2 miles, and the frequent delays due to rough weather on Lake Erie were entirely eliminated.

With the consequent vastly increased output obtained by adopting this method of disposal, and because of the marked reduction in the towing distance, the cost of transportation to the hydraulic dredge was 18 cents per cu. yd., as against 40 cents for disposal in Lake Erie. The additional rehandling cost for the dredge was 14 cents per cu. yd., making the total cost of this second method, 62½ cents per cu. yd. (exclusive of plant rental charge), using the hydraulic dredge, as against 70½ cents for lake disposal.

The foregoing methods of excavation were used on Section No. 5, both in the pre-war period and again in the post-war period, but the costs of these operations, as quoted, apply only to the post-war period of operation, when the unit price per cubic yard for earth excavation was 66 cents. Approximately 900 000 cu. yd. of earth were moved.

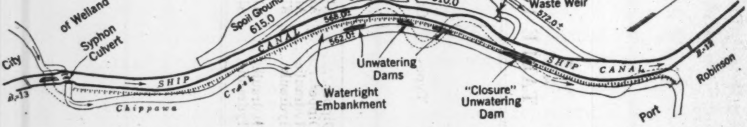


FIG. 5.—PLAN SHOWING RELOCATION OF CHIPPAWA CREEK, NEW PRISM LOCATION, SECTION NO. 6.

Operations in Section No. 6.—The construction of Section No. 6, extending from Port Robinson to Welland, Ont., was started in the fall of 1925, and while this section also forms a portion of the summit level, it represented the construction of an entirely new prism, involving as well the relocation of Chippawa Creek. These conditions are indicated in Fig. 5.

The diversion of Chippawa Creek from the area occupied by the Ship Canal prism to a location immediately alongside, and the excavation of the canal prism, required the moving of about 10 800 000 cu. yd. of earth. Furthermore, it necessitated the construction of a water-tight embankment practically throughout the entire extent of the section to separate the canal prism from the immediately adjacent Chippawa Creek. The water level in the creek was approximately 10 ft. lower than the water level in the Third Welland Canal.

The excavation of this prism was effected by the combined use of electrically operated hydraulic dredges and draglines. In addition, the extensive embankment work involved the use of several grading units which excavated material from the prism and transferred it in wagons, drawn by mules or gasoline tractors. The material was deposited in the embankment in thin layers, and the travel of the mules or tractors provided the required consolidating medium.

Two electric draglines fitted with 175-ft. booms and 5-yd. buckets (see Fig. 6) were utilized chiefly in excavating the slopes of the prism above water level. Their output was sidecast into the middle of the prism for final disposal by the hydraulic dredges. This procedure, although involving the rehandling of a large quantity of material, was found more economical than hauling the material by train to the final point of disposal.

The hydraulic excavation of the prism presented several interesting features. The two hydraulic dredges used—the *Stewart* and the *Shuniah* (see Table 4)—were operated entirely by electricity. The main pump, cutter, ladder, and spud hoists, as well as all general service pumps, were motor-driven.

The pontoon and land discharge lines were of 20-in. welded steel pipe. Besides the equipment described in Table 4, an independent motor of 150-h. p. capacity ran the cutter, while a 60-h. p. motor operated the ladder and spud hoist. All these motors worked on a frequency of 25 cycles and a pressure of 2 200 volts.

One of the first operations necessary on Section No. 6 was the excavation of the new channel for Chippawa Creek, in order to divert this stream from the Ship Canal prism. Accordingly, the two dredges, were brought up Chippawa Creek from the Niagara River. The hydraulic spoil areas were formed immediately adjacent to the prism by draglines casting up an earth embankment around the perimeter of each disposal area. A timber waste weir was introduced at some point in this embankment, of such construction that by the use of stop-logs the waste water was allowed to drain from the disposal area back to the creek, after the desired settlement had occurred.

In the excavation of the creek diversions the dredges pumped the material an average distance of about 1 500 ft. against an average reverse head of 50 ft. The tops of the disposal-area embankments were generally at a height of approximately 50 ft. above the water level in Chippawa Creek.

With the completion of the channel diversions of the creek, the portions of the original creek bed in the prism area were isolated from the new creek channel by earth and rock dams, and after these isolated areas were unwatered, the construction of the water-tight embankment separating the prism from the creek was carried on by the grading units as previously described (see Fig. 5). However, unsatisfactory progress with the construction of this dividing embankment caused the contractor to adopt special procedure in order to maintain the hydraulic dredges in operation.

It was impossible to connect the new prism of the Ship Canal to the prism of the Third Welland Canal at either the north or south ends, until the water-tight embankment between the new prism and Chippawa Creek was entirely completed to its full height. Therefore, the dredge, *Stewart,* was placed in the flooded portion of the new prism area. The unwatering dam was then completed and this latter operation isolated the dredge entirely, both from Chippawa Creek and the Third Canal.

To permit the construction of the water-tight embankment along this portion of the prism, inside the unwatering dam, the dredge excavated a small area in the prism, at her isolated position, to a depth 12 ft. below Ship Canal grade. Then, with the unwatering dam completed between the Ship Canal prism and Chippawa Creek, the dredge itself unwatered all this area, except the part excavated below prism grade, where it remained afloat and idle while the construction of the adjacent water-tight embankment, at this point, was continued.

When this embankment was built to 15 ft. above Ship Canal grade, the area of the prism occupied by the dredge was again flooded by pumping water from the creek over the partly completed embankment into the prism area. With this area flooded to a depth of 12 ft. above Ship Canal grade to give the dredge flotation, the excavation operation was resumed. This stage is very well indicated by Fig. 7.

At the beginning of this phase of the hydraulic operation, the dredge operated on a discharge line about 1 000 ft. long and against a reverse head of 60 ft.—the difference in level between the initial water level in the partly flooded prism and the top of the embankment around the hydraulic disposal area in use. As the construction of the adjacent water-tight embankment of the prism progressed, the contractor was permitted to raise the level in which the dredge operated, thus reducing the reverse head on his discharge.

In this operation, the waste water from the disposal area was returned immediately to the area of the prism in which the dredge operated. Any water lost or additional water required to raise the level, was supplied, as before described, by pumping from the adjacent Chippawa Creek.

In the summer of 1929 the dividing embankment between the new prism and the creek was completed to its full height, and in September of that year the *Stewart* cut through from the new Ship Canal prism to the Third Canal, at Port Robinson.

The *Shuniah* was not isolated in the Ship Canal prism like the *Stewart,* but was utilized, instead, in the excavation of that portion of the new prism at the south end of Section No. 6 which represents largely an operation of widening and deepening the Third Canal prism. (See Fig. 5.) In this operation, a hard gravelly clay was encountered, and as the prism excavation

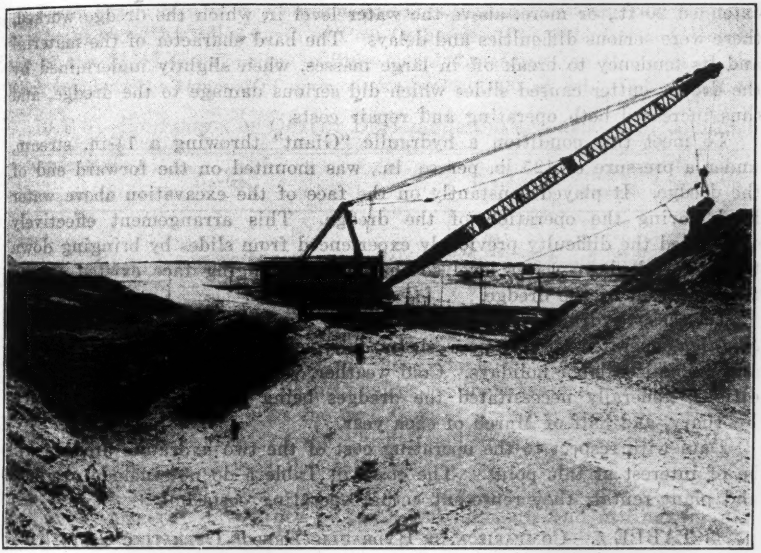


FIG. 6.—ELECTRIC DRAGLINE EXCAVATING THE SLOPES.

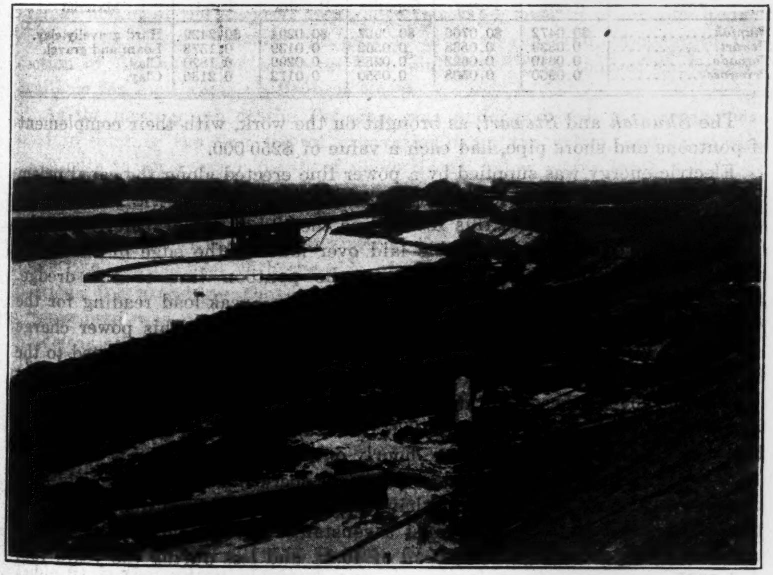


FIG. 7.—HYDRAULIC DREDGE IN FLOODED CANAL PRISM; RIVER BEYOND.

extended 20 ft., or more, above the water level in which the dredge worked, there were serious difficulties and delays. The hard character of the material and its tendency to break off in large masses, when slightly undermined by the dredge cutter caused slides which did serious damage to the dredge, and thus increased both operating and repair costs.

To meet this condition a hydraulic "Giant" throwing a $1\frac{1}{2}$-in. stream, under a pressure of 125 lb. per sq. in., was mounted on the forward end of the dredge. It played constantly on the face of the excavation above water level during the operation of the dredge. This arrangement effectively eliminated the difficulty previously experienced from slides by bringing down the material above water level so that no overhanging face existed at any time in front of the dredge.

In these hydraulic operations the two dredges, the *Stewart* and the *Shuniah,* operated three 8-hour shifts daily, closing down only on Sundays and recognized labor holidays. Cold weather, with its accompanying ice conditions, generally necessitated the dredges being laid up during January, February, and half of March of each year.

Data with respect to the operating cost of the two hydraulic dredges will be of interest at this point. The costs in Table 5 do not include overhead and plant rental; they represent actual operating costs only.

TABLE 5.—COMPARISON OF HYDRAULIC DREDGE OPERATING COSTS PER CUBIC YARD

Dredge	Wages	Power	Supplies	Repairs	Total	Material
Shuniah.............	$0.0472	$0.0706	$0.0957	$0.0294	$0.2429	Hard gravelly clay.
Stewart.............	0.0339	0.0538	0.0562	0.0139	0.1578	Loam and gravel.
Tornado.............	0.0640	0.0623	0.0258	0.0299	0.1820	Clay.
Primrose.............	0.0900	0.0508	0.0550	0.0172	0.2130	Clay.

The *Shuniah* and *Stewart,* as brought on the work, with their complement of pontoons and shore pipe, had each a value of $250 000.

Electric energy was supplied by a power line erected along the canal prism from one of the local distribution systems of the Hydro-Electric Power Commission of Ontario. From this feeder line the power was transferred to the dredge by a flexible armored cable laid over land to the edge of the water area and carried across the water on the pontoon line to the dredge. The monthly power charge was based on a 10-min., peak-load reading for the month and averaged $4 000 per month for each dredge. This power charge applied only during the months that the dredge was actually connected to the Commission's distribution system.

The output of these hydraulic dredges, and the actual number of hours worked during the three seasons, 1926, 1927, and 1928, are listed in Table 6. From this information the average hourly output of each dredge is deduced.

It will be noted that the dredge, *Shuniah,* operating in hard gravelly clay, obtained a higher hourly output than the *Stewart.* This is due to the fact that the latter had to pump against a constant reverse head of 60 ft., while the *Shuniah* operated against a head of 40 ft. and less during the years 1927 and 1928.

Earth Excavation in Section No. 7.—This section extended from Welland to a point about 1.5 miles north of Port Colborne. The widening and deepening of the Third Canal prism to Ship Canal dimensions on this part of the work involved the removal of approximately 9 800 000 cu. yd. This operation was accomplished very largely by the use of two steam-driven hydraulic dredges, the *Primrose* and *Tornado* (see Table 4), assisted by two draglines, with 80-ft. booms and using 2.5-yd. buckets. The latter excavated the material on the new west slope of the Ship Canal prism above water level and sidecast it into the dredging area. As the east slope of the Ship Canal prism corresponded with the east slope of the Third Canal no similar excavation, above water level, was necessary on that side.

TABLE 6.—COMPARISON OF OUTPUT DURING THE SEASONS 1926, 1927, AND 1928

Dredge	Output, in cubic yards	Hours operated	Output per hour, in cubic yards
Shuniah	2 036 565	11 369.5	179
Stewart	1 645 132	10 590.0	155
Tornado	3 866 382	9 734.0	397
Primrose	1 231 433	8 874.0	139

Hydraulic disposal areas placed as conveniently as possible to the west side of the prism were outlined by earth embankments and utilized for disposal in the same manner as described for Section No. 6. These areas were so located along the extent of the section that at no time did either of the hydraulic dredges operate on a pipe line more than 6 500 ft. in length. The average height of the disposal area above the water level in which the dredges operated was 25 ft., because the general level of the land adjacent to this section of the work was more uniform and lower than that of the ground available for spoil areas on Section No. 6.

As the main pump of the *Primrose* was under-powered, this dredge was used in digging the first lift of excavation from the prism, and it carried these cuts to an average depth of about 14 ft. below water level.

The *Tornado* followed the *Primrose* by carrying the excavation operations to the final Ship Canal grade, 26½ ft. below water level. In both cases the dredges operated with a floating pontoon line not longer than 1 000 ft. The shore line was parallel with the side of the cut and was equipped with connections for attaching the pontoon line at the required intervals. This arrangement was necessary in order to keep the slack part of the pontoon line clear at all times of that portion of the canal prism used by navigation. These dredges operated three 8-hour shifts per day, being shut down only for 24 hours on Sundays and labor holidays.

The operating costs of the dredges, *Tornado* and *Primrose,* exclusive of overhead and plant rental, were as shown in Table 5. The dredge, *Tornado,* delivered on the work, with its complement of pontoons and discharge line, was valued at $350 000, while the *Primrose,* similarly equipped, was valued at $225 000.

A matter of equal interest in connection with these two machines (see Table 6) is the output obtained, and the actual number of hours operated, during three seasons, from which is deduced the average output per hour. It will be noted that the hourly yardage for the *Tornado* was very much in excess of that maintained by the *Primrose.* This was due partly to the fact that the *Tornado* was a larger machine, using 24-in. equipment as against 20-in. equipment on the *Primrose.* However, a greater contributing cause was the lack of power on the main pump of the *Primrose* (see Table 4).

Rock Excavation

As in the case of earth excavation, the rock was generally removed by shovels loading into standard-gauge equipment in the dry sections, and dipper dredges loading into scows in the submarine portions of the work.

Contract Prices and Quantities.—At only three points in the extent of the Ship Canal was rock found in very large quantities although, as shown in Table 7, some rock excavation was included in every section of the work. A considerable range in unit prices also prevailed, and, again, it is to be noticed that rock excavation prices in the post-war period were more than double those prevailing in the contracts awarded prior to the war.

TABLE 7.—CONTRACT PRICES FOR ROCK EXCAVATION

Section	Period	Index	Quantities, in cubic yards	Unit price	Cost
No. 1	1913–16	Wet	86 000	$4.05	$348 300
No. 1	1913–16	Dry	57 000	1.05	59 850
No. 2	1913–16	General	19 000	1.40	26 600
Nos. 1 and 2	1922–31	Wet	57 000	10.00	570 000
Nos. 1 and 2	1922–30	Dry	90 000	2.75	247 500
No. 3	1913–16	General	1 295 000	1.10	1 424 500
Nos. 3 and 4	1922–31	General	1 832 000	1.56	2 857 920
No. 4A	1913–16	General	1 000	2.00	2 000
No. 4B	1926–31	Wet and dry	190 000	1.90	361 000
No. 5	1913–16	Wet and dry	29 000	3.45	100 050
No. 5	1922–31	Wet and dry	88 000	6.25	550 000
No. 6	1926–31	General	57 000	3.15	179 550
No. 7	1925–31	Wet and dry	1 548 000	1.90	2 941 200
No. 8	1924–31	Dry	2 107 000	1.50	3 160 500
No. 8	1924–31	Wet (harbor)	679 000	3.88	2 634 520
No. 8	1924–31	Wet (lake)	165 000	4.90	808 500
Total			8 300 000	$1.96	$16 271 990

In addition to the quantity covered by contracts, about 720 000 cu. yd. of rock were removed from Sections Nos. 2, 3, and 5, during the "cost plus" period, making the total rock excavation quantity 9 020 000 cu. yd. removed at the end of 1931.

The major rock excavation operations occurred in the construction of Sections Nos. 3, 4, 7, and 8, and some details of these three larger operations follow.

Rock Excavation in Sections Nos. 3 and 4.—The approach of the Ship Canal on an alignment normal to the Niagara Escarpment necessitated extensive rock cutting for the accommodation of the flight lock structure at this point. The average width of this cut was 300 ft. and the depth varied from 20 to 100 ft. In addition, south of the flight lock structure, Lock No. 7 was also located in rock cutting, as well as a portion of the prism of Section No. 4 south of Lock No. 7.

This situation necessitated, in all, the removal of about 3 000 000 cu. yd. of rock and a considerable portion of it was a good grade of limestone and sandstone. The contractor on Section No. 3 was required to select suitable grades of rock from the excavation and pass it through a crushing plant, thus reducing it to the sizes required for the concrete aggregate on the work. This crushed stone was provided free of cost to the contractors on Sections Nos. 1 and 2, while on Section No. 3 the contractor was allowed to use any surplus available for his own concrete purposes without compensation from the Department for the crushing.

Accordingly, the crushing plant was located on the lower plateau, a short distance north of the foot of the flight locks. All suitable rock from the excavation was sent there immediately, while the unsuitable shaly rock was utilized for local back-filling. Standard 60 to 80-ton railway shovels were used in this operation, loading into 6 and 12-cu. yd. side-dump cars. The 6-yd. cars were used most, because of their convenience in dumping at the primary crusher of the crushing plant.

It was desired to reduce the over-break to a minimum, in order not only to reduce the rock excavation quantity, but in the same way to keep down the quantity of concrete in the structures. Therefore, rock channelers were used in Section No. 3 in the pre-war period. The channel cut was put down on the neat alignment of the back of the lock-wall, in 6-ft. lifts. However, as the rock was extremely variable in its characteristics, with alternate strata of hard limestone, or sandstone and soft shale, the channeling operation proved extremely slow and expensive. In resuming work on this section after the war, close drilling was adopted as an alternative to the previous operation of channeling.

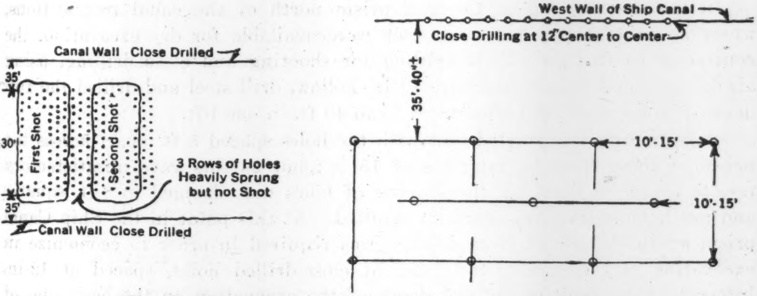


FIG. 8.—PLANS SHOWING METHOD OF CLOSE DRILLING.

In the close-drilling operation a row of vertical holes was drilled on the alignment desired for the limits of the rock cutting, these holes being drilled at intervals of about 12 in. to whatever depth the excavation was being carried (see Fig. 8). In blasting none of the blasting medium was placed in the close-drilled holes. When any part of the rock, so outlined by close drilling, was shot, the effect was to break it only to the line of the close-drilled holes, leaving a very uniform solid face exposed as illustrated subsequently under "Operations in Section No. 8."

Operations in Section No. 7.—Generally throughout the extent of the summit level, no rock was encountered above the grade of the canal prism from Allanburg south to Humberstone, Ont., at the south end of Section No. 7, but between this latter point and the harbor at Port Colborne practically all the necessary excavation for the canal and its structures was in rock.

To eliminate two very sharp bends of the Third Canal alignment as it approached Port Colborne, the Ship Canal location left the Third Canal entirely, and for the last two miles north of Port Colborne followed a perfectly straight line which involved an angular crossing of the Third Canal. (See Fig. 9.) This alignment placed about 3 000 ft. of the new prism at the south end of Section No. 7 entirely clear of the present Welland Canal north of the point of crossing. South of this point of crossing on Section No. 8 the remainder of the new prism, including the pit for the guard-lock, No. 8, for a distance of 8 000 ft., was located entirely clear of the Third Canal and so was made available for excavation "in the dry."

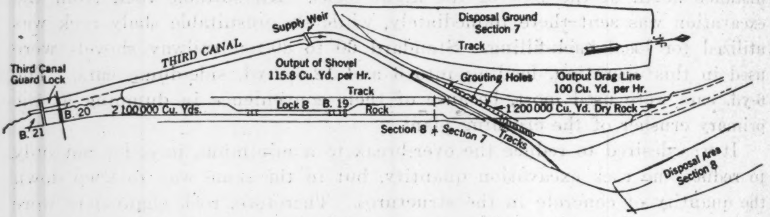


FIG. 9.—PLANS SHOWING TWO BENDS (REMOVED) IN OLD CANAL.

In the first section of the new prism north of the canal intersections, where about 1 200 000 cu. yd. of rock were available for dry excavation, the contractor carried on all his drilling for shooting and close drilling, using air-driven stand drills fitted with 1⅛-in. hollow, drill steel and drilled the full depth of the cut which varied from 35 to 40 ft. in one lift.

This drilling was started first with the holes spaced 5 ft. apart across the prism of these rows, at intervals of 15 ft.; but as this gave unsatisfactory results from the shooting, the spacing of holes was changed to 8-ft. squares and much more uniform break-up resulted. At this point in the Ship Canal prism again the use of close drilling was required in order to economize in excavation and concrete, and lines of close-drilled holes, spaced at 12-in. intervals, were sunk to the full depth of the excavation on the back line of the concrete walls facing this part of the prism.

No holes for shooting were drilled closer than the depth of the cutting from the line of this close drilling (35 to 40 ft. (see Fig. 8)); and in loading for shooting, no dynamite was placed in the close drilled holes, but by springing and shooting the holes in the center of the prism with heavy charges of 70% and 40% dynamite all the rock was well broken up. In no case did the effect of the shooting extend beyond the line of the close drilled holes on either side of the prism.

The most interesting features of this rock operation were the methods and equipment used for excavation and disposal. The contractor used a dragline with a 100-ft. boom and a 6-yd. bucket. Although it was rather unusual for this class of work, it proved effective, and, at the outset, had a distinct advantage over the usual shovel operation.

Much seepage occurred through the seams of stratified limestone at the south end of the area, immediately alongside of the Third Canal prism. Although the contractor placed two 12-in. pumps, together with the one 6-in. and one 8-in. centrifugal pumps in use, to unwater this small initial pit excavation, seepage considerably in excess of this pumping capacity was experienced. It was necessary, therefore, for him to excavate for some time in the water, and, in this respect, the dragline proved most effective.

To overcome this seepage a row of holes at 5-ft. intervals was drilled in the rock, parallel to the side of the Third Canal prism across the end of the dry rock cut. When the water in the partly excavated dry prism was at the same level as in the adjacent Third Canal prism, cement grout was pumped into the fissures of the rock by means of a standard grout machine. More than 100 cu. yd. of grout were injected, and when the unwatering of the cut was again attempted, two 10-in. electrically driven centrifugal pumps unwatered the cut completely in less than a day and thereafter a single pump operation kept the cut clear of water as the excavation proceeded.

The average haul from the point of loading to the disposal ground was about 7 500 ft. Two trains, each with one locomotive and three 30-yd. cars, were able to handle the full output of the dragline, making the round trip to and from the dump in 15 min. In September, 1925, this machine removed 80 500 cu. yd. of rock from the prism, and its average monthly output, representing an average loading rate of 100 cu. yd. per hour of actual working time, was 45 200 cu. yd. The dragline was working a continuous 24-hour shift when this maximum monthly output was obtained.

The unit operating costs were, as follows:

Item	Dragline excavator	Six-yard shovel
Wages	$0.0775	$0.1223
Power	0.0341	....
Supplies	0.0212	0.0644
Repairs	0.0070	0.1395
Total	0.1398	0.3262
Material	Rock	Rock; Section No. 8

The $1\frac{3}{4}$-in. drag cable was renewed every 10 days and the 1-in. hoist cable every 30 days.

The dragline was operated entirely by electricity, the main motor having a rating of 225 h.p., the hoist motor, 175 h.p., and the swinging motor, 75 h.p., all operating on 60-cycle, 2 200-volt energy.

The cost of rock excavation, in Section No. 7, in dollars per cubic yard, exclusive of overhead and plant rental charges, was as listed in Table 8.

The value of the plant and equipment installed for this operation was $300 000. A centrally located, 2 200-cu. ft. compressor, operated by electricity, delivered air to the drills at 100 lb. pressure. The average rate of drilling was 22 ft. per hour, based on actual drilling time. The average depth of hole was 38 ft. and holes were completed with one set-up of the drills.

Each cubic yard of rock removed required 8 in. of drilling, including close drilling. The dynamite required for each cubic yard was 1.5 lb. Shots were limited to about 12 000 cu. yd. of rock each, to minimize the movement of the dragline clear of the area being shot.

TABLE 8.—UNIT COSTS OF ROCK EXCAVATION PER CUBIC YARD

Operation	Section No. 7	Section No. 8
Drilling	$0.1592	$0.1290
Blasting	0.4724	0.2870
Excavating and loading	0.1398	0.3550
Transportation	0.1723	0.2460
Disposal	0.0863	0.1530
Field operating cost	$1.03	$1.1700

For all rock excavation on Section No. 7, both dry and wet, the Department paid the contractor at the unit price of $1.90 per cu. yd. About 378 000 cu. yd. of rock at the north and south ends of the dry work, at the intersections of the two canal prisms, required removal by dredging.

Operations in Section No. 8.—The dry rock excavation on Section No. 8, which included the removal of about 2 100 000 cu. yd., was in many respects similar to the operation previously described, but by comparison it had several interesting features.

The rock in this section was a stratified limestone formation typified by heavy beds interspaced with many seams of soft shale. The depth of the cutting averaged 35 ft., with the width of the excavation varying from a minimum of 90 ft. in the lock area to a minimum of 220 ft. in the prism above and below the lock.

On account of a heavy ledge of rock that occurred throughout the excavation approximately midway in the depth of the cutting, and in order to expedite the work by avoiding the use of extremely long steel, the drilling was done in two lifts of about equal depth. It was done with drills of the anvil block type equipped with independent air motors for rotating the drill steel. Drills were mounted on frames, each about 15 ft. high, on which they were raised and lowered by means of a small winch. These machines used hollow drill steel 1¼ in. in diameter, with starting bits 2⅞ in. across the wings, the bits of the longer drills being reduced by ⅛ in. with each increase in length.

At the outset the holes were spaced at the corners of 15-ft. squares. No holes were drilled for shooting within 35 ft. of the limit of the excavation line, where close drilled holes of 12-in. spacing were put down to the full depth of the cut (see Fig. 10). After these holes were shot the excavation revealed that the rock had broken very unsatisfactorily. Still retaining the 15-ft. spacing of holes an additional hole was drilled for about one-half the depth in the center of each 15-ft. square, with the idea that with this hole loaded midway of the depth of the cut the break up would be better (see

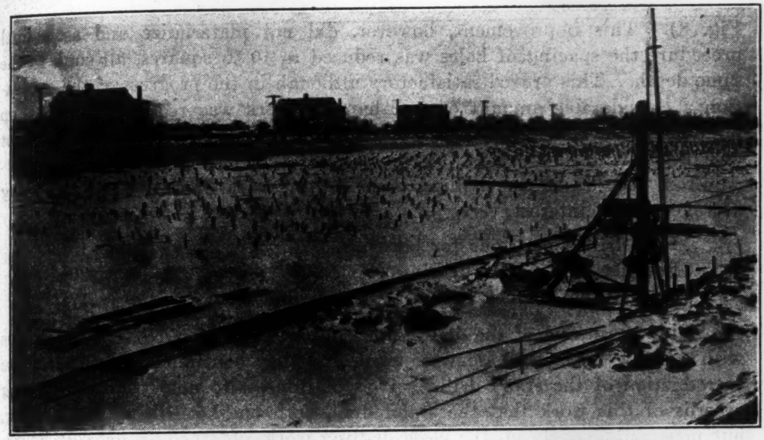


FIG. 10.—ROCK DRILLING IN LOWER ENTRANCE TO LOCK NO. 8.

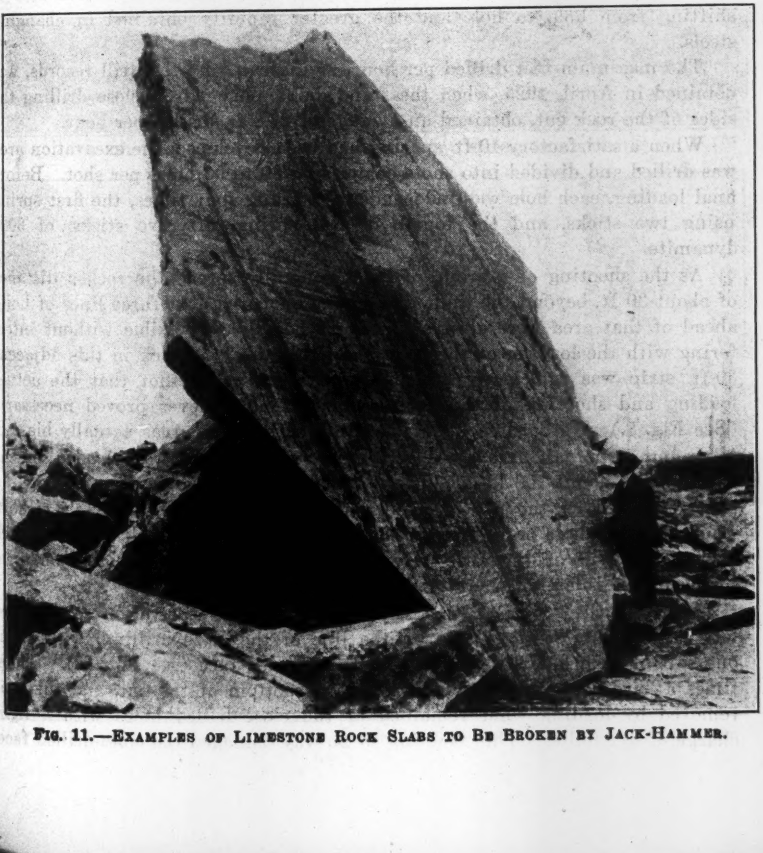


FIG. 11.—EXAMPLES OF LIMESTONE ROCK SLABS TO BE BROKEN BY JACK-HAMMER.

Fig. 8). This improvement, however, did not materialize and as a final procedure the spacing of holes was reduced to 10-ft. squares, all sunk to the same depth. This proved satisfactory although in the progress of the excavation a considerable amount of jack-hammer work was required to break up large pieces of the rock slabs not effectively broken by the original shot (see Fig. 11).

From records of the drilling done in this rock operation the following averages are of interest:

Feet drilled per hour for blasting....................	15.3
Feet per hour, close drilling	20.5
All drilling, in feet, including jack-hammers..........	17.6

At the beginning of the drilling operations in Section No. 8, tripod drills with a screw feed of 30 in. were used, and they continued in use after the introduction of the stand drills. It is of interest to note from the drilling records of this work that the stand drill, both on the drilling for blasting, but more particularly on the close drilling work, obtained an average hourly output almost 50% in excess of the tripod drill. This was due to ease of shifting from hole to hole and the greater rapidity obtained in changing steels.

The maximum feet drilled per hour, as indicated by the drill records, was obtained in April, 1925, when the stand drills, engaged on close drilling the sides of the rock cut, obtained an average of 27.8 ft. drilled per hour.

When a satisfactory 10-ft. spacing had been developed, the excavation area was drilled and divided into shots comprising 80 to 90 holes per shot. Before final loading, each hole was independently sprung four times, the first spring using two sticks, and the fourth or last spring, fifty-five sticks, of 60% dynamite.

As the shooting of a group of holes generally spread the rock a distance of about 30 ft. beyond the limit of the area shot, the next three lines of holes ahead of that area were always sprung as heavily as possible without interfering with the loading of the holes. In this way, the rock in this adjacent 30-ft. strip was sufficiently broken up with the main shot that the actual loading and shooting of these three rows of holes never proved necessary. (See Fig. 8.) No hard rock was encountered between areas actually blasted.

In preparing an area for blasting, 80 to 100 holes were loaded in the bottom with about 100 lb. of 40% dynamite. With two caps in each hole a shot was fired by the use of a current from an adjacent 220-volt circuit. Each of these shots move from 8 000 to 10 000 cu. yd. of rock. In this rock operation, 1.1 lb. of dynamite were used per cu. yd. of rock, in the combined springing and blasting operation.

Although the rock was of a variable nature, the excavation in all cases left a uniformly straight face on the line of the close drilling, with about one-half of each hole still showing in the exposed face (see Fig. 12). Sometimes a hard toe would be found along the bottom of the cut, and this was removed by loading what remained of the close-drilled holes with a light charge of dynamite. This shooting in no way damaged the close-drilled face.

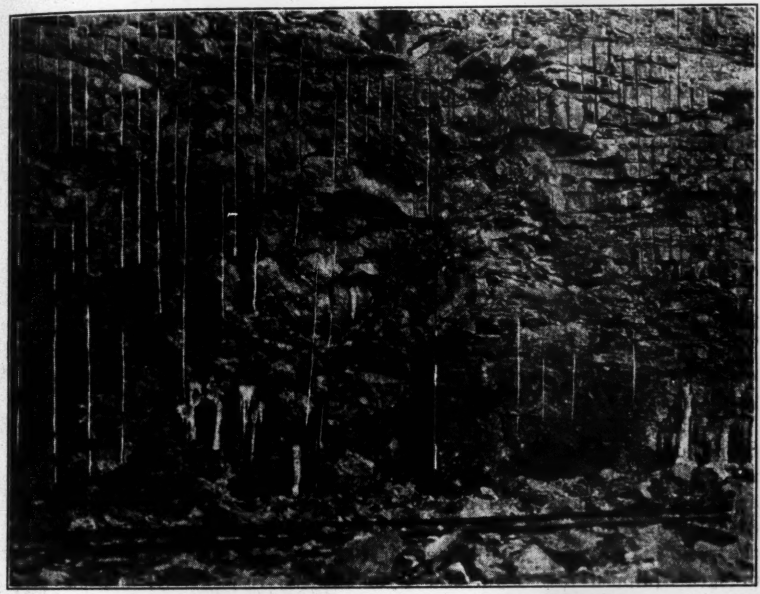


FIG. 12.—EVEN ROCK FACE SECURED BY CLOSE DRILLING.

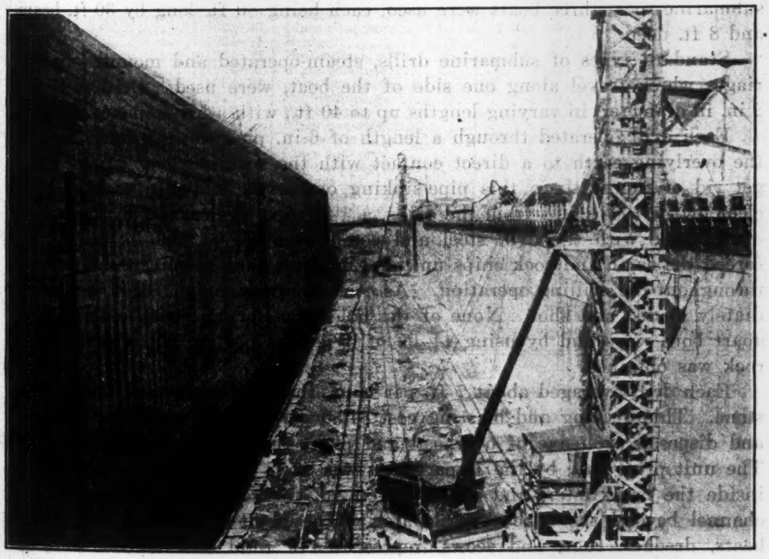


FIG. 13.—VIEW OF ARRANGEMENT OF CONCRETE PLANT FOR THE CONSTRUCTION OF LOCK NO. 1.

This rock was removed in two lifts, the first extending down approximately one-half the total depth of the cutting. Two standard railway shovels with bucket capacities of 3½ and 2½ yd., assisted by a small revolving shovel with a 1¼-yd. bucket, were used in the removal of this first lift. The small machine cleaned up behind the larger shovels in order to allow the drilling of the second lift to follow closely behind the first lift excavation.

For the excavation of the second lift to final grade, a revolving shovel, with a 6-yd. dipper and an 85-ft. boom, was used. It could load into cars 68 ft. above the level of its own tracks and could clear a 90-ft. width at grade. The machine was mounted on four standard-gauge trucks, one in each corner of the base, and was self-propelled.

The combined excavating and loading operations of the 6-yd., 3½-yd., 2½-yd., and 1¼-yd. shovels cost 35½ cents per cu. yd., as shown in Table 8. The total value of the plant and equipment on the rock work in Section No. 8 was $500 000. For the dry rock excavation on this section, the Department paid the contractor at the unit price of $1.50 per cu. yd.

In Port Colborne Harbor the work done in connection with the Welland Ship Canal consisted of approximately doubling the harbor width and deepening the entire harbor from an existing 20-ft. draft to an ultimate 30-ft. draft at extreme low-water level of Lake Erie, involving the removal of about 844 000 cu. yd. of rock, as well as about 300 000 cu. yd. of earth overlying the rock. This work was started in 1925 being carried on by two dipper dredges equipped with 6-yd. and 4-yd. buckets, respectively, and was completed in 1931, except the final cleaning up and sweeping. In drilling the submarine rock, three boats were used, each being 80 ft. long by 30-ft. beam, and 8 ft. deep.

Standard types of submarine drills, steam-operated and mounted in carriages which travel along one side of the boat, were used. Drill steel was 2 in. in diameter, in varying lengths up to 40 ft., with a bit diameter of 3¾ in.

Each drill operated through a length of 6-in. pipe worked down through the overlying earth to a direct contact with the rock surface. In order to get rid of the earth in this pipe-sinking operation, and also to keep holes clear of the rock chips from the drills, a ½-in pipe was carried to the bottom of the hole with the drill steel and compressed air at 100 lb. per sq. in. forced the earth and rock chips up the 6-in. pipe, thus keeping the hole clear throughout the drilling operation. As each hole was completed, it was immediately loaded and shot. None of the holes was sprung; with spacing 5 ft. apart both ways and by using 1¼ lb. of dynamite per cu. yd., a well broken rock was obtained.

Each drill averaged about 7 ft. per hour, the rock being very tough limestone. This drilling and blasting cost $1.50 per cu. yd., while the dredging and disposal, exclusive of overhead and plant rental, cost $0.75 per cu. yd. The unit price paid by the Department was $3.88 per cu. yd. in the harbor, inside the breakwater, and $4.90 per cu yd. for rock removed in the lake channel beyond the harbor breakwater. The plant in use, including drill boats, dredges, tugs, and scows, represented a total cost on the work of $650 000.

Concrete

A variety of uses was made of concrete in the construction of the Welland Ship Canal. Not only were the principal structures, such as the locks, regulating weirs, and waste weirs, of concrete construction throughout, but all bridge substructures, whether built "in the dry" or by submarine operations, dock walls, harbor and breakwater cribs, and the protection of the prism slopes, were entirely of concrete construction. This range of application naturally resulted in a corresponding range of unit prices being obtained in placing all the work under contract, as indicated in Table 9.

TABLE 9.—CONTRACT PRICES FOR CONCRETE

(All exclusive of cement)

Section Nos.	Period	Index	Quantities, in cubic yards	Unit price per cubic yard	Amount
1	1913–16	Mass concrete	168 000	$2.55	$428 400
1	1913–16	Harbor cribs	18 000	3.05	54 900
1	1913–16	Retaining walls	8 500	2.85	24 225
1	1913–16	Mass concrete	1 000	3.95	3 950
1	1913–16	Reinforced concrete	9 500	4.65	44 175
2	1913–16	Mass concrete	182 000	2.57	467 740
2	1913–16	Bridges	1 500	5.30	7 950
2	1913–16	Bridge walls	20 000	5.40	108 000
2	1913–16	Mass concrete	1 000	4.70	4 700
2	1913–16	Reinforced concrete	1 000	8.50	8 500
1 and 2	1922–31	Mass concrete	487 000	5.25	2 556 750
1 and 2	1922–30	Harbor cribs	29 500	7.00	206 500
1 and 2	1922–30	Bridges	52 500	12.00	630 000
1 and 2	1922–30	Protection	89 000	7.50	667 500
3	1913–16	Mass concrete	47 000	2.65	124 550
3	1913–16	Bridges	21 000	6.00	126 000
3 and 4	1921–30	Mass concrete	1 321 000	4.14	5 468 940
3 and 4	1921–31	Bridges	103 500	9.10	941 850
3 and 4	1921–30	Protection	35 500	7.87	279 385
4A	1913–16	Mass concrete	3 000	5.25	15 750
4B	1926–30	Mass concrete	4 000	10.00	40 000
4B	1926–30	Protection	2 200	7.50	16 500
5	1913–16	Protection	1 500	5.00	7 500
5	1922–30	Mass concrete	9 000	10.00	90 000
5	1922–30	Protection	1 000	11.00	11 000
6	1926–30	Mass concrete	11 500	8.00	92 000
6	1926–30	Siphon culvert	97 500	7.50	731 250
6	1926–30	Bridges	2 000	15.25	3 050
6	1926–30	Protection	26 500	7.50	198 750
7	1925–30	Mass concrete	112 000	8.00	896 000
7	1925–30	Protection	17 000	7.00	119 000
8	1924–30	Mass concrete	340 000	4.70	1 598 000
8	1924–30	Bridge cribs	37 000	15.00	555 000
8	1924–30	Bridges	10 000	11.00	110 000
8	1924–30	Road concrete	1 500	8.00	12 000
Gate-dock	1928–30	Mass concrete	38 000	5.95	226 100
Gate-dock	1928–30	Tunnel concrete	1 500	15.50	23 250
Totals			3 311 700	$5.10	$16 899 165

Besides the concrete placed under the various contracts, an additional 247 000 cu. yd. were placed during the cost plus period, making the total concrete placed in the Welland Ship Canal, 3 558 700 cu. yd. at the end of December, 1931.

The usual 100% or more increase in prices from pre-war to post-war periods is immediately apparent. The post-war prices varied from $4.14 for heavy mass concrete work, as in the flight-lock structure, to $15.50 for smaller quantities entering into the construction of the light section, heavily reinforced bridges, cribs, and tunnel construction. In all cases the cement was supplied by the Department and delivered in cars on the commercial railway sidings nearest the work. Contracts were made with the cement companies by the Department for each year's total cement requirements.

All concrete operations on the Welland Ship Canal were conducted in accordance with one standard of procedure. The cement was supplied in cloth bags of 87½ lb. net weight, to the contractors, being subjected, before acceptance and delivery, to the standard tests prescribed by the Canadian Engineering Standards Association.

The concrete aggregate—stone, sand, or graded gravel—was supplied by the contractor under the unit price per cubic yard for concrete, except in the case of Sections Nos. 1 and 2, where the stone was furnished by the Department to the contractor, at the Section No. 3 crushing plant.

Testing.—All operations of mixing and placing the concrete were closely supervised by a competent staff of inspectors allocated to the various concrete plants. In addition, a special department, equipped with a complete laboratory, was maintained to assist in the control of the concrete operations. It functioned as a very essential unit of the entire concreting organization. In addition to carrying on the routine cement sampling and testing, the men in this department took every day from each plant, a 6 by 12-in., or an 8 by 16-in. cylinder of the concrete as it was being poured, and from these, after curing for a period of 28 days, they determined the compressive strength of the concrete as produced.

These tests guided and confirmed the judgment of the Field Inspection Staff in proportioning the mixtures to meet the strength required in the various classes of concrete. In the heavy wall sections, using the proportions of 1:2.5:5, an average compressive strength of 2 000 lb. per sq. in. was obtained, while in lighter sections, in the reinforced concrete work, or where water-tightness was an essential feature, the proportions of 1:2:4 gave a compressive strength of 3 000 lb. per sq. in., or better.

Plant Layout.—In the design of his concrete plants, the contractor was required to provide such measuring devices as would permit the accurate determination of proportions for the mix.

A typical single-lock structure with its regulating and waste weirs represented a quantity of about 360 000 cu. yd. of concrete to be placed in a distance of only about 1 500 ft. This condition justified the general use of central mixing plants which system was adopted. Distribution was effected by traveling hoist towers, derricks, and chuting systems.

At Lock No. 1 the concrete from the central plant at the north end was transferred by rail to traveling hoist towers, by which it was elevated, and with short chutes, placed in the forms. (See Fig. 13.) At Lock No. 2 a similar central plant delivered the concrete into 4-yd. buckets which were transferred on cars to traveling derricks, in the lock chamber. The derricks, in turn, hoisted the buckets to place. At Lock No. 3 the procedure was similar to that at Lock No. 2, except that every effective use was made of a large 120-ft. boom drag line which as a crane handled the concrete to place.

At Lock No. 8 all concrete was placed by standard locomotive cranes with 60-ft. booms, the walls of the structure being only about 46 ft. high above chamber-floor level.

A much more extensive and elaborate concrete plant was used for the flight lock structure on Section No. 3, at Thorold. Twin Locks Nos. 4, 5, and 6 in flight, required the placing of 1 070 000 cu. yd. of concrete in the three continuous walls which constituted this structure, all confined to a distance of 3 035 ft., exclusive of entrance walls (see Fig. 14).

The twin lock chambers of standard dimensions are separated by a center wall, 60 ft. thick and 85.9 ft. high above the chamber floor level. The lower half of this wall is practically solid, and the upper half is divided into two gravity sections with a central separating space for rock-filling. The side walls of the chambers have a maximum base thickness of 47 ft.

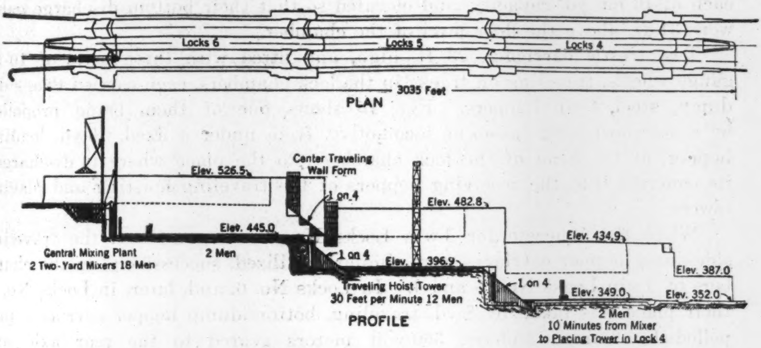

Locks 6
Locks 5
Locks 4
3035 Feet
PLAN
Center Traveling Wall Form
Elev. 526.5
1 on 4
Elev. 482.8
Elev. 445.0
Elev. 434.9
Central Mixing Plant
2 Two-Yard Mixers 18 Men
2 Men
1 on 4
Elev. 396.9
Elev. 387.0
Traveling Hoist Tower
30 Feet per Minute 12 Men
1 on 4
Elev. 349.0
Elev. 352.0
2 Men
PROFILE
10 Minutes from Mixer to Placing Tower in Lock 4

FIG. 14.—ARRANGEMENT OF CONCRETE PLANT FOR THE CONSTRUCTION OF FLIGHT LOCKS NOS. 4, 5, AND 6.

As each of these locks has the standard lock lift of 46.5 ft., the floor level of the upper entrance of Lock No. 6 is 139.5 ft. above the floor level of Lock No. 4. Construction was started at the upper or southerly end of Twin Locks No. 6, where the central mixing plant was located.

This mixing plant consisted of duplex bins of open-frame timber construction, each with a capacity, in separate compartments, for 350 cu. yd. of sand and 430 cu. yd. of stone, with supply tracks over the tops of the bins; 15-in. radial bin gates, spaced at 6-ft. intervals, discharged the aggregate on 30-in. belt conveyors, running longitudinally under each bin, which, in turn, fed a second belt inclined at 20 degrees. The inclined conveyor elevated the materials into the stone and sand bins of the mixing plant located vertically over the mixers, these bins having capacities of 73 cu. yd. for stone and 44 cu. yd. for sand. Batch hoppers or measuring bins, each with a capacity of 80 cu. ft. and provided with separate sand and stone compartments, were fed by vertical gates in the bottoms of the aggregate bins.

Two standard non-tilting mixers, each of 56 cu. ft. capacity, and separately driven by 50-h.p. motors, at a drum speed of 12 rev. per min., were located directly under the measuring bins. They discharged directly into

60-cu. ft. tower hoist buckets in steel hoisting towers, 134 ft. high, located immediately in front of the mixers. The connecting chutes between the towers and loading hoppers were lined with $\frac{1}{8}$-in. steel plate which required replacement after handling 50 000 cu. yd. of concrete.

As the mixing plant was designed to serve for the construction of the entire flight lock structure, provision was necessary for the distribution of the concrete throughout the 3 035-ft. extent of this structure, and it is in this connection that the unusual and interesting features were introduced.

The concrete from the mixer hoist towers was passed by chutes to loading hoppers in the upper end of the east and west chambers of Twin Locks No. 6, each of 16 cu. yd. capacity, and elevated so that their bottom discharge gates were 31 ft. above the floor level of the chamber.

Two trestle carriages, 23 ft. high, and fitted with three pairs of 10-ft. gauge wheels, traveling on tracks in the lock chambers, each carried two, side-dump, steel 4-yd. hoppers. Fig. 15 shows one of them being propelled by a standard-gauge gasoline locomotive, from under a fixed, 16-yd. loading hopper, at the head of the lock chambers, to the place where it discharged its concrete into the receiving hoppers of the traveling hoisting and placing towers.

When the concrete for Twin Locks No. 6 was completed the traveling side-dump hopper carriages were similarly utilized, successively, in the chambers of Twin Locks Nos. 5 and 4. In Locks No. 6, and, later, in Locks No. 5, their place was taken by 8-yd. traveling, bottom-dump hopper carriages propelled by 35-h.p., 3-phase, 550-volt motors geared to the rear axle and operated by one man.

The traveling, hoisting, and placing towers, one in each flight of the twin lock chambers, were of structural steel construction throughout. (See Fig. 16.)

The main tower was 30 ft. square and 150 ft. high. It was mounted on an I-beam base, 47 ft. square, with twelve cast-steel wheels, 36 in. in diameter, traveling on two lines of 100-lb. rails set at 33 ft. $9\frac{1}{2}$-in. gauge. On each side of the tower, adjacent to the lock walls, a bucket hoist tower extended the full height of the main tower, and accommodated a 2-yd. hoist bucket with its lower, fixed loading hopper and upper, movable receiving hopper. To each of the movable receiving hoppers a 60-ft. boom chute was attached which could, when necessary, carry a 50-ft. counterweight chute. The receiving hopper with its attached chutes was raised or lowered as a unit by a hoist on the tower base. These details are illustrated diagrammatically in Fig. 17. Fig. 18 shows the operating platform layout.

A 12-yd. hopper on the side of the hoisting and placing tower adjacent to the center wall of the lock chamber, received the concrete as delivered from the mixing plant by the traveling trestle car (Fig. 15). From this hopper it was elevated by the bucket hoist on the center-wall side of the main tower to an intermediate fixed receiving hopper and then it was transferred to another 12-yd. fixed hopper on the opposite side of the main tower by inclined chutes. This second hopper supplied the bucket hoist operating on the side of the placing tower adjacent to the side wall of the lock chamber.

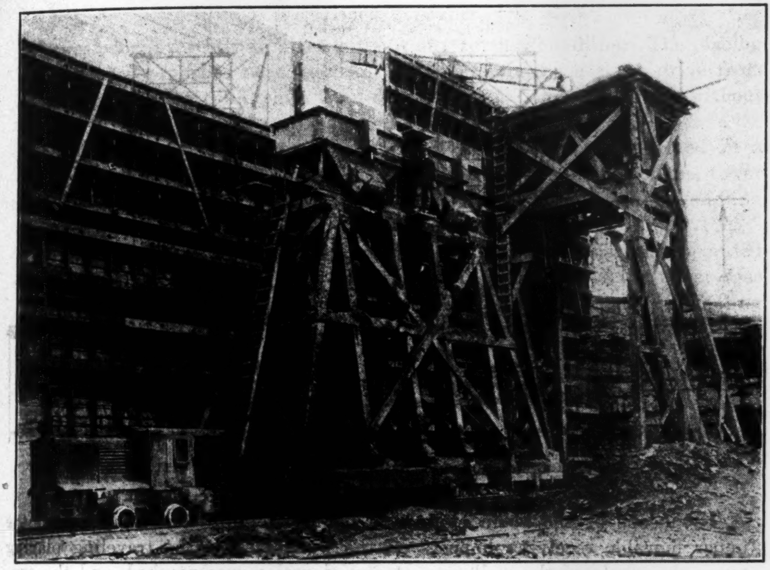


FIG. 15.—VIEW OF FIXED AND PORTABLE CONCRETE HOPPERS.

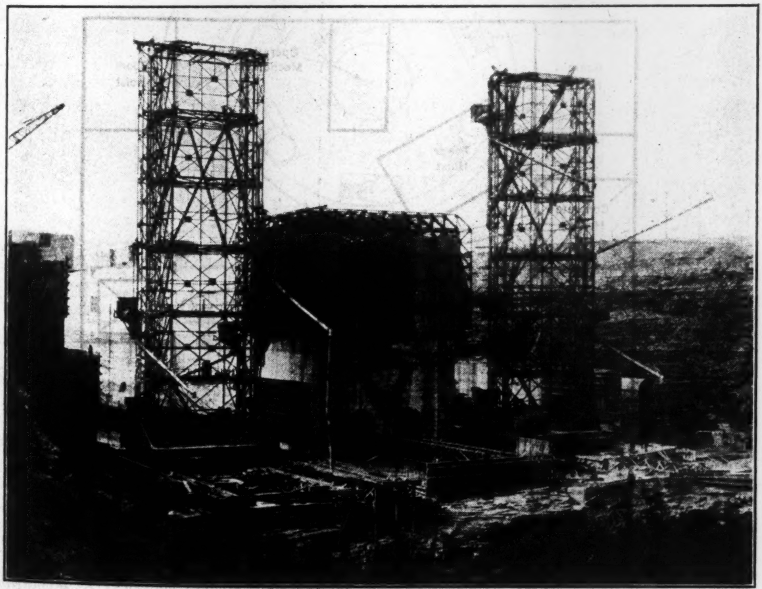


FIG. 16.—TRAVELING, HOISTING, AND PLACING TOWERS USED IN CONSTRUCTION OF FLIGHT LOCKS

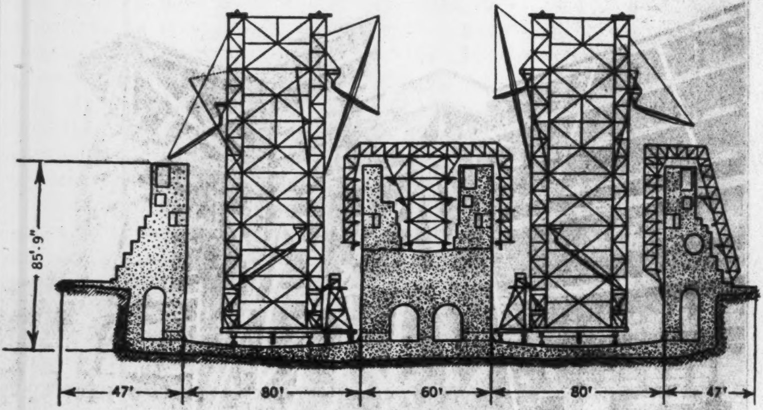

85' 9"
47'
80'
60'
80'
47'

FIG. 17.—OPERATION LAYOUT FOR RECEIVING HOPPER WITH ATTACHED CHUTES.

By means of the boom chute and counterweight chute attached to the upper movable receiving hoppers on the wall sides of the traveling placing tower the whole of the wall area at any elevation was reached, and the con-

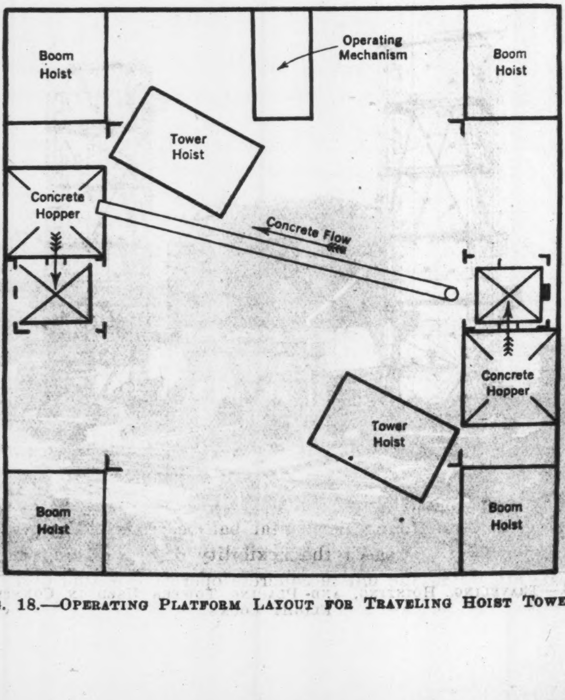

Boom Hoist
Operating Mechanism
Boom Hoist
Tower Hoist
Concrete Hopper
Concrete Flow
Concrete Hopper
Tower Hoist
Boom Hoist
Boom Hoist

FIG. 18.—OPERATING PLATFORM LAYOUT FOR TRAVELING HOIST TOWER.

crete was poured directly in place without any further handling. The placing tower was self-propelled at a speed of 30 ft. per min. by a chain drive from a 50-h.p. motor, and could shift from one placing position to another without delay.

On each corner of the main tower a 50-ft. timber boom operated in a vertical plane by means of a single-drum, electric hoist mounted on the tower base. These booms, with a lifting capacity of 2 tons, were used for handling chutes, forms, and other similar concrete operations.

All the equipment on each traveling placing tower was electrically operated and all machines and motors were located on the platform of the tower base (Fig. 18). This arrangement necessitated the use of 13 000 ft. of hoisting cable and the solution of a number of intricate reaving problems because of the limited space available for all the operating equipment.

The bucket hoists were fitted with a safety device mounted on the countershaft. This consisted of a series of hardened steel rollers working in tapered slots on spring mountings. Wedging action permitted the countershaft to rotate in one direction only, so that in case of a power interruption the safety device automatically locked the hoist machinery and prevented the load from dropping. This safety clutch is shown in Fig. 19.

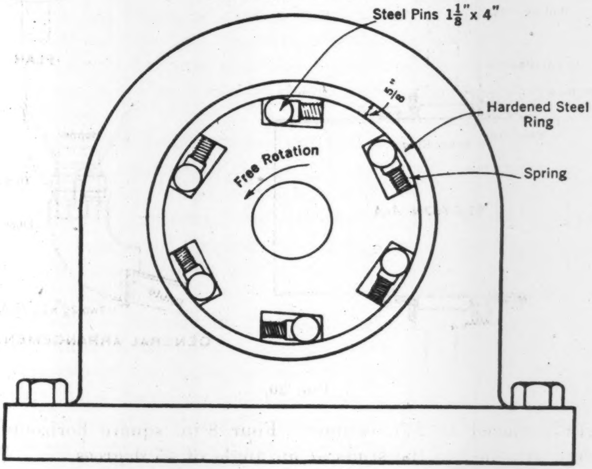
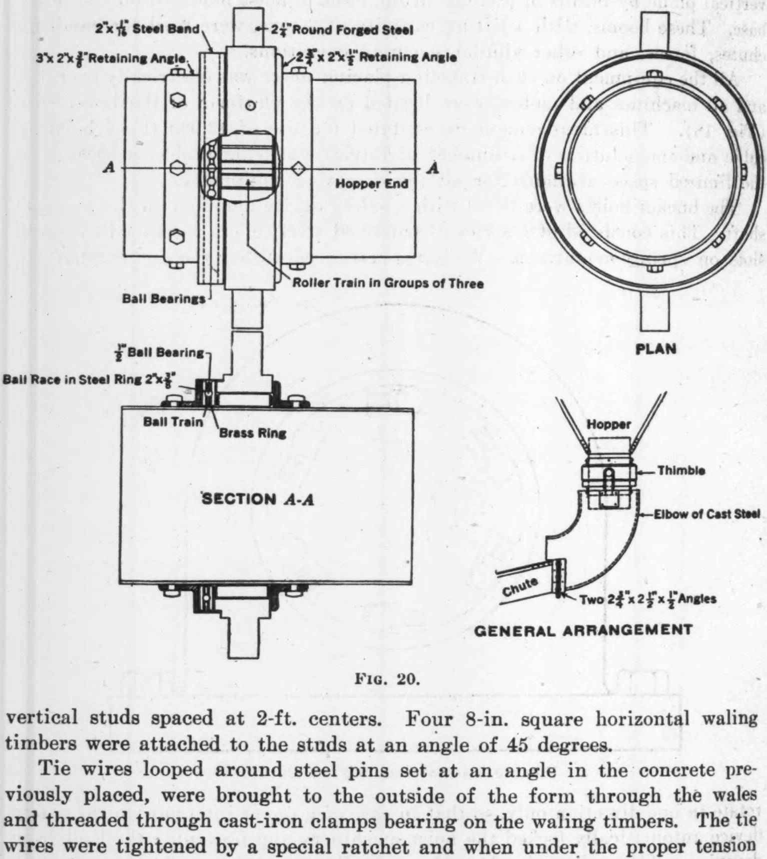


FIG. 19.—DETAIL OF SAFETY CLUTCH.

Another tower equipment detail of special interest was the cast-steel elbow by which the placing chutes were attached to the receiving hoppers of the tower hoists (see Fig. 20). It was fitted with a thimble of special design having a vertical roller and horizontal ball-bearings. The introduction of these elbows greatly increased the flexibility of the placing chutes. They functioned throughout the entire concrete operation at the flight locks without renewal.

Traveling Steel Form.—The lock walls were built in monoliths of standard 60-ft. lengths, variations from this length being made only where structural details required it.

Accordingly, the wooden forms were made up in panels, 20 ft. long by 12 ft. high, and consisted of $1\frac{3}{4}$-in. dressed fir lumber assembled on 2 by 6-in. vertical studs spaced at 2-ft. centers. Four 8-in. square horizontal waling timbers were attached to the studs at an angle of 45 degrees.



2"x 1/16" Steel Band
2 1/4" Round Forged Steel
3"x 2"x 3/8" Retaining Angle
2 3/4" x 2"x 1/2" Retaining Angle
A
A
Hopper End
Roller Train in Groups of Three
Ball Bearings
1/2" Ball Bearing
Ball Race in Steel Ring 2"x 3/8"
Ball Train
Brass Ring
SECTION A-A
PLAN
Hopper
Thimble
Elbow of Cast Steel
Chute
Two 2 3/4" x 2 1/2" x 1/2" Angles
GENERAL ARRANGEMENT

Fig. 20.

Tie wires looped around steel pins set at an angle in the concrete previously placed, were brought to the outside of the form through the wales and threaded through cast-iron clamps bearing on the waling timbers. The tie wires were tightened by a special ratchet and when under the proper tension were held by a set screw in the cast-iron clamps. The wooden forms, thus secured, held a 6-ft. lift of concrete—poured continuously—in a satisfactory manner and without movement. The introduction of a 2 by 3-in. finishing strip along the top of each lift of panel form permitted the concrete being placed to a height of 2 in. above the top of the panel. By removing this finishing strip before setting the next lift of forms, a 2-in. concrete height of the finished face was exposed and provided exact alignment for the forms above. This system of forming was used in the construction of the solid lower half of the center wall, the lower half of the west wall, and all of the east wall.

The upper half of the center wall, for 700 ft. of the lock-chamber length, consists of two separate gravity sections with a space of 24 ft. between their bases, to effect economy in concrete. It also contains the longitudinal mooring passage, the cable tunnel, and the drainage tunnel, all involving extensive form work. With a view to reducing form costs, a traveling steel form was used for this part of the work. Accordingly, a structural steel, **T**-shaped frame, 60 ft. long, was provided to carry the wooden forms outlining the two adjacent gravity walls as a unit (see Fig. 17).

This steel frame was made up of seven transverse truss sections spaced at 10-ft. centers. Each consisted of a central tower truss, 42 ft. high, an overhead section, 78 ft. long, with cantilever arms extending in a **T**-shape out beyond the wall faces. The 42-ft. vertical trusses suspended from either cantilever arm, were unified by an intermediate system of horizontal and diagonal bracing.

The wooden forms, all assembled and secured together as a unit, were suspended from this frame or gantry by turnbuckles and steamboat ratchets. The turnbuckles and ratchets, together with hardwood wedges inserted along a series of horizontal **I**-beams, provided a quick and ready means of aligning the forms for use, and they could be released quickly after the concrete of the walls was in place.

The suspended vertical trusses were tied to the central tower truss by 2½-in., through rods, spaced 10 ft. apart in two rows, 4 ft. and 18 ft. above the base of the frame. These vertical trusses were also secured at the ends of the gantry by four horizontal steel trusses which, at the same time, carried the timber-end bulkheads forming the ends of the 60-ft. monoliths. The entire traveling form assembly was carried on a series of wheels, running on four lines of 100-lb. rail.

The longitudinal tunnels in the upper gravity wall sections, were constructed by means of collapsible steel forms, suspended from the overhead gantry by a series of 1-in. bolts and held laterally by a similar series of horizontal bolts. This traveling, 60-ft., center-wall form unit, complete with wooden face forms and enclosed tunnel forms, weighed 300 tons and was moved horizontally from monolith to monolith by a heavy winch.

Another unit of similar construction was provided for the upper gravity section of the west wall of the west lock chambers. The support, however, was two 100-lb. rails, one carried on brackets fastened to the face of the lower part of the wall (see Fig. 21), and the other laid on the anchor slab in the rear of the gravity section proper. Lateral support between the front and back vertical trusses was provided by 2½-in through bolts and horizontal end trusses, as on the center wall form. The weight of the west wall form unit complete was 250 tons.

The traveling steel form units provided for the construction of the center and side walls did not prove to be as successful as anticipated. They were expensive to handle and set up, but the greatest expense was in transferring them from one lock to the next.

Moving Traveling Forms Between Locks.—On November 17, 1923, the traveling, center-wall form unit, weighing 300 tons, was transferred from Lock No. 6 to Lock No. 5. A timber skidway, supported at an inclination of 6 on 25, by timber bents spaced at 4-ft. centers, was constructed, extending from the unit form track level of the upper lock wall to below the similar track level of the lower lock wall. From the upper to the lower track level the vertical distance was 47 ft. 9 in. The two sets of inclined stringers were faced on their upper sides with a $\frac{1}{8}$-in. steel plate, coated with tallow, before the carrying cradle was erected.

The lowering operation was performed by two steam hoisting engines specially designed for this work and securely anchored to the center wall concrete at the lower end of the skidway. With a worm-gear reduction of 52 to 1 the engines (operating at 100 rev. per min.) gave a drum speed of just less than 2 rev. per min., and paid the cable off the drum at the rate of 12 ft. per min.

The $\frac{3}{4}$-in. plow-steel cable passed from each hoist through 18-in. single blocks secured to the concrete on the proper alignment in front of each hoist, and up the skidway below the cradle to two sets of 7-sheave, 18-in. blocks located above the form cradle.

The lower blocks were secured to the cradle with seven loops of 1-in. steel cable, wound continuously round the bridle timbers of the cradle and reaved through the head-pin of the blocks. The upper blocks were similarly held by 1-in. cables reaved through the head-pin and secured to the concrete at the top of the skidway. In order to keep the cables parallel to the inclination of the skidway, while paying out in the lowering operation, a structural steel bent fitted with a hardwood saddle was located on the concrete above the skidway with the anchor cables securing the upper sheave blocks passing over the saddle of this bent.

The becket lines from the two lowering cables were each passed through separate 3-in. pipes and heavy spiral springs, set in a concrete anchor block secured to the upper lock-wall concrete, and then around single sheave blocks, being clamped together between the sheaves. This arrangement equalized the load and synchronized the hoisting engines, while heavy cast-steel buffers clamped to the cables about 18 in. from where they passed through the 3-in. pipes and springs, provided an emergency means of automatically stopping the cradle if either lowering cable should fail.

With the lowering cable being paid off the hoist drum at the rate of 12 ft. per min., the center-wall form and its supporting cradle moved down the skidway at the rate of slightly less than 1 ft. per min. The lowering operation required 4 hours.

When lowering the center wall form unit the second time, from the Lock No. 5 to the Lock No. 4 level, on August 1, 1925, a shackle on one of the cables failed. The form unit was precipitated into the east chamber of Lock No. 4 and completely wrecked. This structure was not replaced, and the entire center wall of Twin Locks No. 4 was built with wood panel forms. The two hoisting and placing towers were successfully lowered to the floor

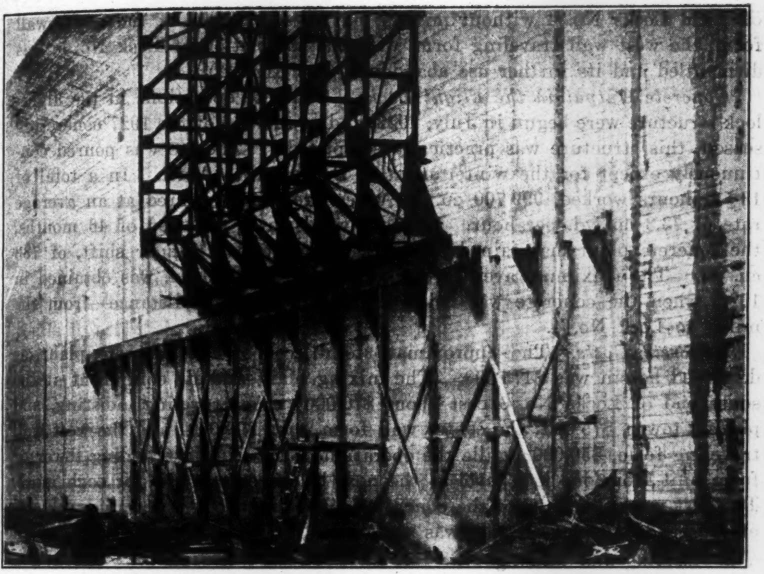


FIG. 21.—BRACKET SUPPORT FOR TRAVELING FORM UNIT.

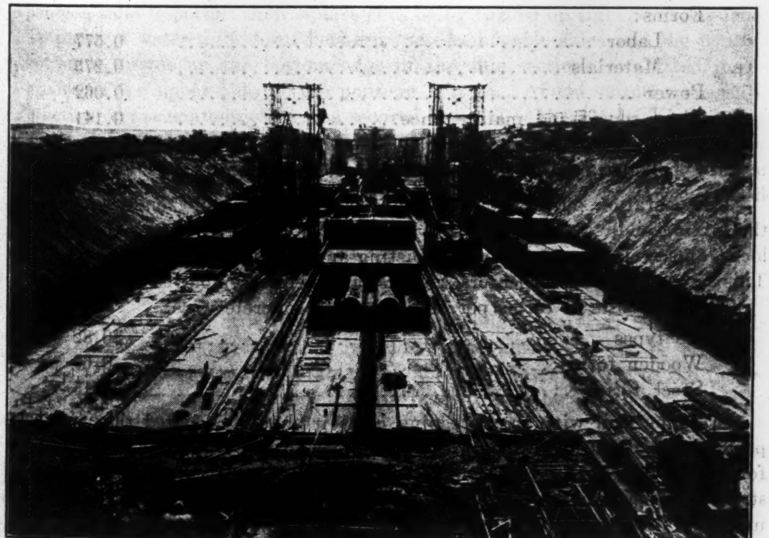


FIG. 22.—CONSTRUCTION VIEW IN LOCK NO. 4.

of Twin Locks No. 4 without mishap, but after the loss of the center-wall form, the west wall traveling form, with the completion of Lock No. 5, was dismantled and its further use abandoned (see Fig. 22).

Concrete Output in the Flight Locks.—Concrete operations in the flight-lock structure were begun in July, 1922, and at the end of the 1927 concreting season, this structure was practically completed. Concrete was poured continuously except for the winter months, December to April. In a total of 13 511 hours worked, 999 700 cu. yd. of concrete were poured at an average rate of 73.3 cu. yd. per hour. In other words, over a period of 46 months, the concrete plant maintained an average output, per 10-hour shift, of 733 cu. yd. The maximum average output per shift of 849 yd. was obtained in 1926, when the concrete was being moved the greatest distance—from the mixer to Lock No. 4.

Concrete Costs.—The approximate total cost of the concrete plant as described herein was $475 000. The mixing plant and its equipment at the south end of Locks No. 6 cost about $70 000; each traveling, hoisting, and placing tower, $75 000; the center wall form unit, $90 000; and the west wall traveling form, $50 000; while the operation of lowering a traveling tower or form unit from one lock level to the next below, cost $7 850 for each move. The cost per cubic yard of the concrete in the flight-lock structure, exclusive of overhead and plant rental, was as follows:

Mixing and Placing:	
Labor	$0.490
Materials	1.375
Forms:	
Labor	0.577
Materials	0.275
Power	0.062
Plant repair and maintenance	0.141
Total	$2.920

The operation of the concrete plant was carried on by eighteen men at the mixing end, six men operating the transfer hopper carriages through the locks, and twelve men on the two hoisting and placing towers. (See Fig. 14.) This is exclusive of carpenters, riggers, and helpers employed on forms, etc.

The unit costs of forms per cubic yard of concrete were:

All types	$0.82
Wooden forms	0.69
Steel forms	1.15

In connection with the foregoing costs it should be noted that, on that part of the flight-lock structure where steel forms were used, the area of form work per cubic yard of concrete was slightly more than twice that of the structure where wooden forms were used. Furthermore, the center wall form unit was used only for the placing of 220 000 cu. yd. of concrete in Locks Nos. 6 and 5, while had it not been destroyed by the accident, it would have accommodated an additional 110 000 cu. yd. of the center wall of Lock No. 4.

For the concrete placed in the flight lock structure the Department paid the contractor at his unit price of $4.14 per cu. yd. and supplied, as well, the cement which cost the Department between 1922 and 1928 an average price of $2.21 per bbl. There were 1⅓ bbl. of cement used per yard of concrete in this structure.

CONCRETE CRIBS

The use of reinforced concrete cribs instead of the usual type of timber cribs for under-water foundations in the Harbors of Port Weller and Port Colborne and in the summit level of the Welland Ship Canal presents some interesting construction features.

Cribs for Harbor Breakwater.—The artificial harbor at Port Weller, built by projecting two embankments out into Lake Ontario, required retaining walls to establish definitely, the outer entrance to the harbor as well as the inner docking space. These walls were founded 35 ft. below lake level on a clay bottom.

Reinforced concrete cribs, 109 ft. 8 in. long by 37 ft. 8 in. wide, and 34 ft. deep, of cellular construction, were provided for the portion of the retaining wall section up to lake level. Their construction afloat was effected in the following manner: A timber pontoon, 111 ft. 6½ in. long by 39 ft. 2½ in. wide, and 12 in. deep, of rigid mill-floor frame construction, was built. It was fitted with removable timber sides, 7 ft. high, which were secured to the pontoon bottom by means of vertical through bolts. The bolts were spaced at 6-ft. centers around the perimeter of the pontoon and were fastened into special castings set in the bottom. The tops projected sufficiently above the pontoon sides to permit their square ends being turned up tight with wrenches. To insure a water-tight joint between these removable sides and the pontoon bottom a tongue on the bottom edge of the side sections was fitted into a groove in the upper side of the pontoon bottom. At the corners the miter junction of the pontoon sides was secured by bolts held by special lug fittings attached to the sides. When the pontoon was assembled and with all the joints caulked, it represented a large water-tight box capable of carrying an interior load which could sink it in the water to a depth of 7 ft., while it still remained afloat.

The interior dimensions of the pontoon with the sides erected, corresponded to the over-all length and width of the concrete crib built therein and so for a height of 7 ft. it served as the external form work. For the remainder of the crib, panel forms of convenient size were utilized in the construction of the exterior and interior cross-walls.

By locating the concrete mixing plant immediately adjacent to the face of the dock where the crib was being constructed, and by equipping it with a tower hoist and a chute flexible enough to reach the full width of the crib, all points in the length of the crib were reached by the simple expedient of floating the whole structure past the concrete plant. Furthermore, by the use of two pontoons, the continuous construction of several concrete cribs could be carried on at one time.

By the time the weight of concrete submerged the carrying pontoon to within 6 in. of the top of the 7-ft. side walls, the concrete was about 12 ft.

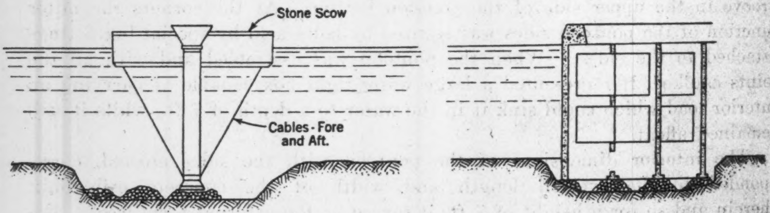
above the floor level, and the concrete crib itself with this height of wall was capable of remaining afloat without the assistance of the pontoon.

A rectangular opening was left in the bottom of each cell to permit filling when the crib was sunk to its final position. Each of these rectangular openings was closed by removable timber bottoms, while the crib was still under construction and supported by the pontoon. The bottoms were securely wedged down and held in place by knee-braces so as to be water-tight during the construction of the concrete crib and until finally it was sunk in its permanent position.

When the concrete was of sufficient height to permit the crib to remain afloat unassisted, the removable sides of the pontoon were released. By means of concrete blocks placed on the bottom of the pontoon below the timber bottoms, before these latter were put in position, the pontoon was sunk clear of the bottom of the concrete crib and taken away for use in starting the construction of another crib. From this point on, the construction of each crib was merely a matter of erecting forms and steelwork and pouring concrete until the full height of the crib was obtained.

The initial operation of constructing the cribs for Port Weller Harbor was done in the Harbor of Port Dalhousie, about 4.5 miles west of the point where the cribs were used. This was necessary because no protected area was available at first at the new harbor site. Accordingly, these first cribs, when fully completed, were towed by tug from Port Dalhousie to their site at Port Weller where they were sunk immediately on a previously prepared foundation.

Preparing Foundation for Harbor Cribs.—The foundation for the cribs was prepared by first excavating over the area to be occupied to a depth of 2 ft. below the bottom level of the crib. On this excavated foundation area windrows of crushed stone were placed by means of a pipe with a flared lower end. This was suspended from a scow with the lower end of the pipe carried at the elevation of the crib bottom. The windrows were placed to correspond with the longitudinal side and center walls of the crib itself, and by moving the scow along, during the process of placing the stone through the pipe, they were all leveled off at the required elevation (see Fig. 23). The completed crib was floated to a position over its final location and was securely held by a dredge while being sunk by filling with water.



LAYING STONE FOR CRIB SEATS

CRIB PLACED AND BACKFILLED

FIG. 23.—SKETCHES SHOWING METHODS OF PREPARING BOTTOM FOR BREAKWATER CRIBS.

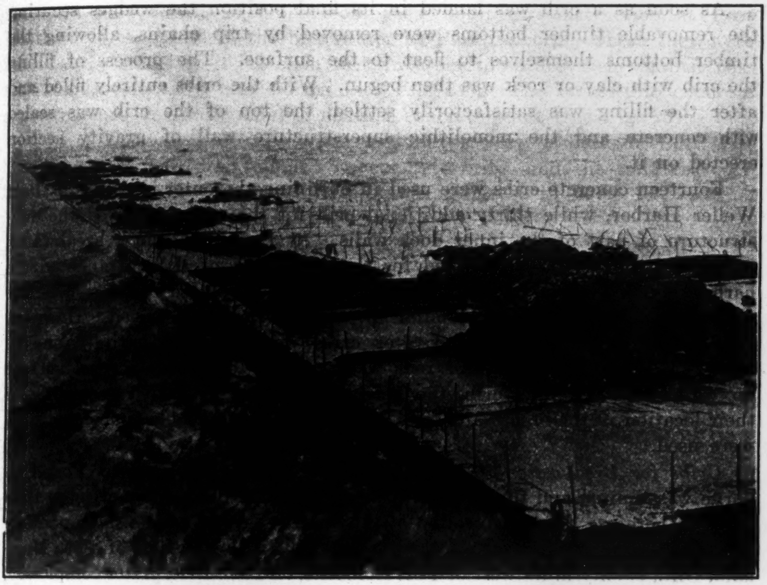


FIG. 24.—REINFORCED CONCRETE CRIBS IN PLACE.

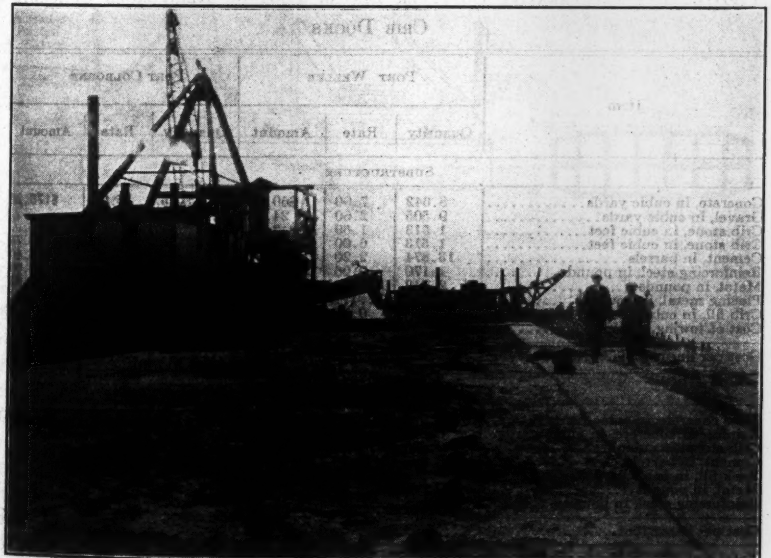


FIG. 25.—BREAKWATER AT PORT COLBORNE READY FOR SUPERSTRUCTURE.

As soon as a crib was landed in its final position the wedges securing the removable timber bottoms were removed by trip chains, allowing the timber bottoms themselves to float to the surface. The process of filling the crib with clay or rock was then begun. With the cribs entirely filled and after the filling was satisfactorily settled, the top of the crib was sealed with concrete and the monolithic superstructure wall of gravity section erected on it.

Fourteen concrete cribs were used in outlining the outer entrance at Port Weller Harbor, while thirty additional cribs of this type provided the substructure of part of the inner dock walls. At Port Colborne a breakwater, 2 000 ft. in length, was provided for the protection of the entrance to the harbor, using reinforced concrete cribs built and placed with a concrete superstructure, in a manner similar to that previously described. However, as at Port Colborne, the breakwater cribs were founded entirely on the rock and only sufficient of this rock was removed to provide a level surface on which to place each crib, the heights of the cribs varied according to their location in the breakwater, and cribs, without removable timber bottoms, were used.

Cribs for Breakwater at Port Colborne.—The harbor space at Port Colborne was limited and the maximum depth of water there was not more than 19 ft. Therefore, the construction of the cribs afloat, using the same pontoon procedure as previously described, was followed in the Harbor of Port Maitland, Ont., twenty miles west of Port Colborne, on Lake Erie. After completion,

TABLE 10.—COMPARATIVE COST PER LINEAR FOOT OF REINFORCED CONCRETE CRIB DOCKS

Item	Port Weller			Port Colborne		
	Quantity	Rate	Amount	Quantity	Rate	Amount
Substructure						
Concrete, in cubic yards	8.642	7.00	$60.49	11.75	15.00	$176.25
Gravel, in cubic yards	9.505	2.60	24.71	....	......	....
Crib stone, in cubic feet	1.513	1.50	2.27	....	......	....
Crib stone, in cubic feet	1.513	6.00	9.08	2.25	7.00	15.75
Cement, in barrels	13.874	2.20	30.52	18.0	2.30	41.40
Reinforcing steel, in pounds	1.170	0.06	70.20	15.30	0.04¾	72.60
Metal, in pounds	2.742	0.12	0.33	10.9	0.05	0.55
Placing metal, in pounds	....	......	....	10.9	0.02	0.22
Crib fill, in cubic yards	40.324	0.75	30.24	35.3	0.50	17.65
Cost of towing, placing	....	......	13.51	....	......	11.30
Cost per linear foot	....	......	$241.35	....	......	$335.72
Superstructure						
Mass concrete, in cubic yards	1.585	5.25	$8.32	4.7	4.70	$22.10
Reinforced concrete, in cubic yards	0.870	7.00	6.09	....	......	....
Crib stone, in cubic feet	2.000	1.50	3.00	....	......	....
Cement, in barrels	3.683	2.20	8.10	9.	2.30	20.70
Rock fill, in cubic yards	7.500	0.75	5.62	8.6	0.50	4.30
Metal, in pounds	0.060	0.35	0.02	....	......	....
Reinforcing steel, in pounds	30.0	0.06	3.60	....	......	....
Cost per linear foot	....	......	$34.75	....	......	$47.10
Total cost of dock per linear foot	....	......	$276.10	....	......	$382.82

the cribs were towed 20 miles from their point of construction at Port Maitland to their site in the breakwater at Port Colborne, where they were immediately sunk and filled with rock in order to secure them against possible displacement by lake storms (see Fig. 24). The monolithic concrete superstructure was built using a floating concrete plant (see Fig. 25).

Cost of Docking Facilities.—The cost of the Port Weller and Port Colborne docking facilities, founded on concrete cribs, and the unit cost per linear foot of the Port Colborne Breakwater, are given in Table 10.

In the Port Colborne Breakwater, these costs were:

Substructure per linear foot	$332.79
Superstructure per linear foot	109.70
Total	$442.49

The breakwater cost is higher than that of the docking facilities because of the larger cribs, the heavy concrete superstructure, and the heavy stone fill on the seaward side.

Cribs for Construction of Bridge Piers.—An even more interesting use was made of the reinforced concrete crib in the construction of the bridge substructures on the summit level of the canal between Thorold and Port Colborne.

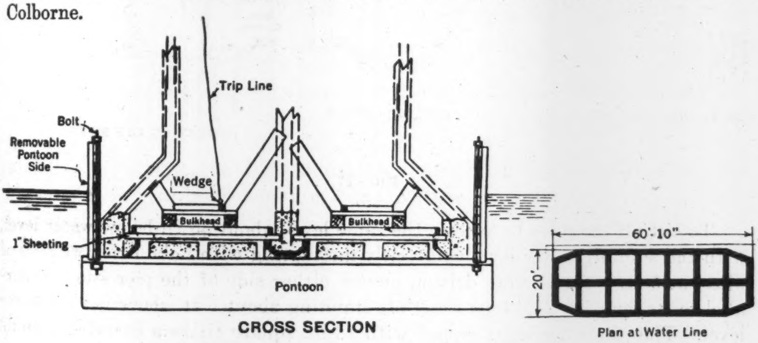
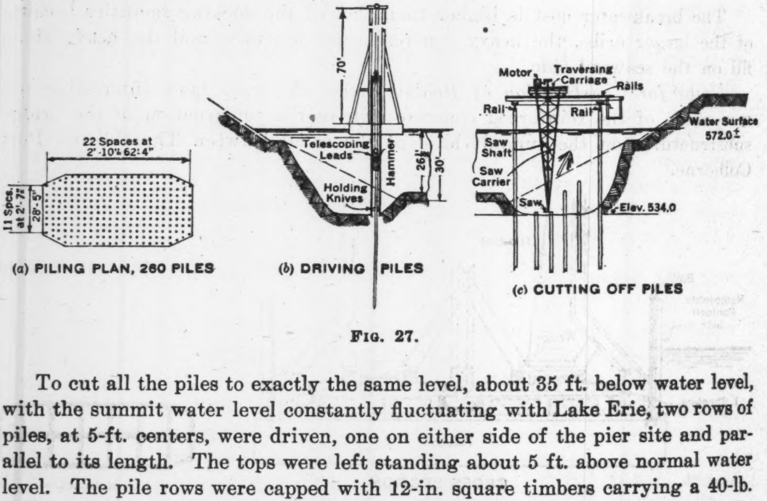


FIG. 26.—CRIB CONSTRUCTION ON PONTOON.

The summit level of the Third Canal was widened and deepened to the enlarged section of the Ship Canal, and, in consequence, all brige piers were built in the water, in order to avoid very difficult and expensive unwatering operations. Vertical lift bridges of 200 ft. clear span were provided for all railway and highway crossings of the summit level of the Ship Canal, and the main piers of these bridges are of rectangular section approximately 65 ft. long and 30 ft. wide, with an average height of 38 ft.

In order to eliminate the necessity of using deep coffer-dams an alternative method was adopted, by which cellular reinforced concrete caissons, with exterior dimensions the same as the bridge piers, were built, afloat. In building these cribs, a timber pontoon with removable sides of the exact base dimensions of the pier, was used, following closely the procedure for the construction of the harbor cribs. (See Fig. 26.) In this case, as at Port Weller, removable timber covers were provided in the bottom opening of each cell of the crib. After the crib was placed and the covers were removed, the concrete filling would occupy the space below and around the supporting foundation piles, bonding the pier to its foundation (see Fig. 28).

As rock at the bridge sites on the summit level was at considerable depth below Ship Canal grade, the piers were founded on timber bearing-piles driven to refusal. For this purpose a floating pile-driver, equipped with telescopic leads and a submersible steam hammer, was used. The pier area was first excavated to 5 ft., below prism grade by dredging. These piles which varied in length, according to soil conditions, from 30 to 60 ft., were spaced at 3-ft. centers, and 260 piles were used in each pier foundation. (See Fig. 27(*a*).)



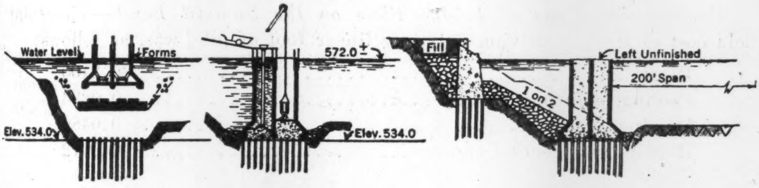
(*a*) PILING PLAN, 260 PILES (*b*) DRIVING PILES (*c*) CUTTING OFF PILES

FIG. 27.

To cut all the piles to exactly the same level, about 35 ft. below water level, with the summit water level constantly fluctuating with Lake Erie, two rows of piles, at 5-ft. centers, were driven, one on either side of the pier site and parallel to its length. The tops were left standing about 5 ft. above normal water level. The pile rows were capped with 12-in. square timbers carrying a 40-lb. rail. A timber carriage with double-flanged wheels ran on the rails of the pile bents. This carriage, in turn, was similarly fitted on top with a pair of rails on stringers located transversely to the length of the pier. Traveling on the upper track was a second carriage to which was attached the upper end of a structural steel frame of triangular section, 48 ft. long and hanging down into the water. Attached to the one vertical face of this frame, by a series of bearings, was a $3\frac{1}{2}$-in. steel shaft suspended by a thrust-bearing carried by the deck of the upper carriage. To the lower end of this shaft a 52-in. circular saw was mounted on a horizontal plane, and on the upper end, above the thrust-bearing, was attached a 23-in. crown pulley belt driven by a 75-h. p. electric motor on the upper carriage deck. The arrangement is shown diagrammatically in Fig. 27(*c*).

In operation, the submerged saw was started clear of the pile foundation area. The foundation piles were cut off at the required level by moving the lower and upper carriages lengthwise and crosswise as required. The average time taken to cut off all the piles in a pier foundation by this method was 40 hours.

A pier caisson was constructed on its pontoon until, with the walls built up about 12 ft., it had sufficient buoyancy to float by itself. The removable sides were then released, and the pontoon bottom dropped away from the crib with the weight of concrete blocks placed on the pontoon under the removable bottoms, before the caisson construction was started. Three of these operations are shown in Fig. 28.



(a) CRIB FLOATING, PONTOON REMOVED

(b) FILLING CRIB WITH CONCRETE, WET AND DRY

(c) PIER PLACEMENT COMPLETED

FIG. 28.

During the period of 2 to 3 hours that the caisson setting was under way, canal navigation was not allowed to pass the site. This permitted the use of setting tackle, anchored to either side of the prism, and also prevented the surge created by a passing vessel from disturbing the alignment of the caisson when landing on its foundation. All caissons built and placed by this procedure were landed within 3 in. laterally and longitudinally of their theoretical positions.

Immediately after a pier caisson was placed on its foundation piling and completely filled with water, coarse stone, from the excavation, was dumped by scow around the outside, to secure it further and to prevent the concrete filling from escaping. The removable timber bottoms were next released by removing the wedges and knee-braces with a trip line operated from above water level. (See Fig. 26.)

With the bottom bulkheads and sheeting removed and the caisson base outlined with stone filling, all cells of the caisson, as well as the space beneath and around the supporting piles, were filled to a height of 10 ft. above the caisson bottom, with concrete placed under water by means of a tremie or bottom-dump bucket (see Fig. 28(*b*)). This weight of concrete in the bottom of the caisson permitted the cells to be unwatered, one at a time, and the remainder of the concrete filling placed in the dry.

To allow for any possible settlement after the concrete had been poured, the top of the pier was left unfinished until immediately preceding the erection of the bridge superstructure. This precaution proved unnecessary, as no settlement was observed at any of the piers after the bottom concrete filling was in place. The weight superimposed on each of these bridge piers, by the steelwork of a vertical lift bridge, is 1 100 tons.

Six working days were required to assemble the pontoon with its removable sides and place the bottom forms and reinforcement steel ready for the first pour of concrete. Seventeen working days were required to pour the concrete of the caisson to its full height and to sink the completed caisson on its foundation.

Seventeen more working days were required after the caisson was placed on its foundation to remove the bottom bulkheads and sheeting, to seal the cell bottoms with under-water concrete, and then to unwater the caisson and pour the remainder of the concrete "in the dry."

Construction Costs of Bridge Piers on the Summit Level.—The total field cost of the pile-driving only, per linear foot of pile, was, as follows:

Piles	$0.3100
Peeling, pointing, rafting	0.0233
Driving	0.0480
Repairs and general expense	0.0372
Total field cost of pile-driving	$0.4185

The cost per pile for cutting off was:

Erection of cut-off outfit at pier site	$1.064
Operating cut-off outfit	2.159
Dismantling and removing forms	0.396
Pile cut-off outfit, first cost	2.353
Total cost of cutting off one pile	$5.972

The foundation piling varied in length from 30 to 60 ft., but the cost of "cutting off" bears no relation to the length of piles used. The entire value of the cut-off outfit is absorbed in the cost of cut-off given previously because it cannot be considered plant in the ordinary sense, its further use elsewhere being highly improbable.

The cost per cubic yard of concrete in the piers was, as follows:

Assembling and unwatering pontoon	$0.95
Forms	1.09
Gravel	1.98
Handling cement (supplied by Department)	0.09
Handling reinforcement steel	0.49
Mixing and placing	2.22
Total field cost	$6.82

One pontoon and one set of 1-in. panel forms, costing $11 000, served for the construction of ten piers.

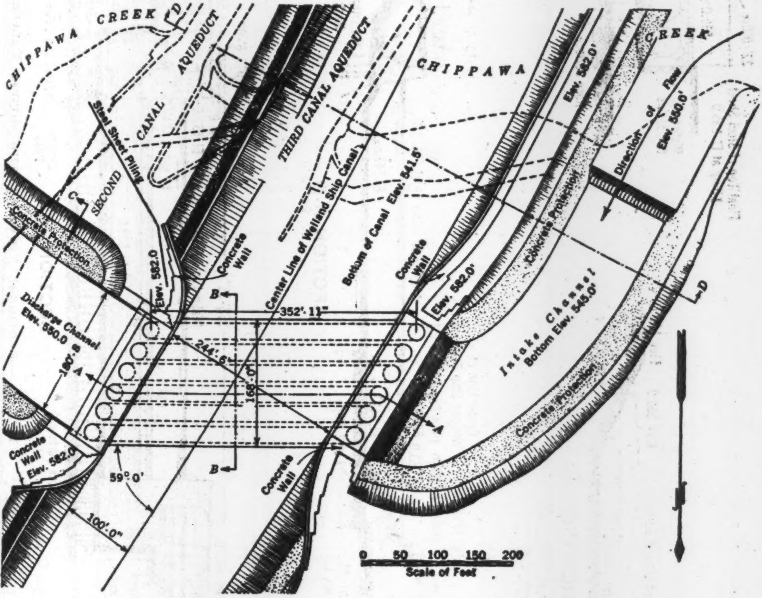
The contractor was paid at the following unit prices by the Department for the various items of work entering into the construction of the bridge piers:

Mass concrete per cubic yard	$8.00
Timber bearing piles per linear foot	1.25
Rock-fill around pier per cubic yard	1.25
Steel reinforcement per pound	0.05
Excavation (earth) per cubic yard	0.42

In addition, the Department paid the contractor a bulk sum of $40 000 for each bridge substructure, as completed, to cover the additional cost of excavation and all other operations involved in the substructure construction.

CHIPPAWA CREEK SIPHON CULVERT

A siphon culvert was provided on Section No. 6 at Welland, for the passage of Chippawa Creek under the Ship Canal prism. This presented one of the most interesting general problems solved in the construction of the Welland Ship Canal. Fig. 29 is the plan view of this crossing, showing over-all dimensions and general characteristics. Details essential to an understanding of the problem are shown in Fig. 30.



CHIPPAWA CREEK
D
AQUEDUCT
CANAL
SECOND
Steel Sheet Piling
THIRD CANAL AQUEDUCT
Center Line of Welland Ship Canal
Bottom of Canal Elev. 541.5'
CHIPPAWA
CREEK
Elev. 582.0'
Direction of Flow
Elev. 550.0'
C
Concrete Protection
Elev. 582.0
Concrete Wall
B
352'-11"
244'-6"
168'-0"
180'-8
Discharge Channel Elev. 550.0
A
Concrete Wall Elev. 582.0
59° 0'
100'-0"
Concrete Wall
Concrete Wall
Elev. 582.0'
Concrete Protection
Intake Channel Bottom Elev. 545.0'
Concrete Protection
D
A
0 50 100 150 200
Scale of Feet

FIG. 29.—PLAN OF SIPHON CULVERT IN SECTION NO. 6.

The structure extended for nearly one-half its length out into the prism of the Third Welland Canal. This end had to be enclosed by a cellular steel sheet-pile coffer-dam (see Fig. 33).

Steel Sheet-Piling.—Borings taken over the site before the final design was adopted, showed the rock surface at an average elevation of 483.5,

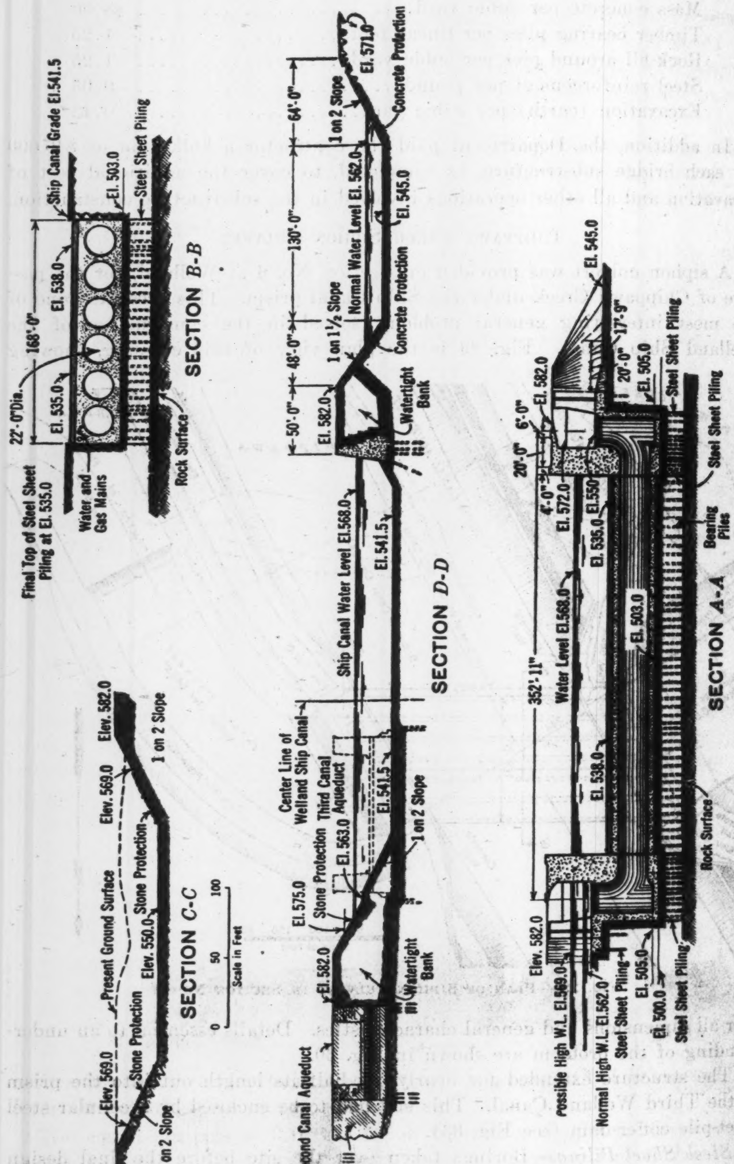

Final Top of Steel Sheet Piling at El. 535.0
22'-0" Dia.
El. 535.0
168'-0"
El. 538.0
Ship Canal Grade El. 541.5
El. 503.0
Steel Sheet Piling
Water and Gas Mains
Rock Surface
SECTION B-B
1 on 2 Slope
Elev. 569.0
Stone Protection
Present Ground Surface
Elev. 550.0
Stone Protection
Elev. 569.0
Elev. 582.0
1 on 2 Slope
SECTION C-C
0
50
100
Scale in Feet
Second Canal Aqueduct
Fill
Watertight Bank
El. 582.0
El. 575.0
Stone Protection
Center Line of Welland Ship Canal
Third Canal Aqueduct
El. 563.0
El. 541.5
1 on 2 Slope
Ship Canal Water Level El. 568.0
El. 541.5
SECTION D-D
50'-0"
48'-0"
130'-0"
64'-0"
El. 582.0
Watertight Bank
1 on 1½ Slope
Concrete Protection
Normal Water Level El. 562.0
El. 545.0
1 on 2 Slope
Concrete Protection
El. 571.0
Possible High W.L. El. 569.0
Normal High W.L. El. 562.0
Elev. 582.0
Steel Sheet Piling
El. 505.0
El. 500.0
Steel Sheet Piling
Rock Surface
El. 538.0
352'-11"
Water Level El. 568.0
El. 535.0
El. 503.0
El. 572.0
El. 550
4'-0"
20'-0"
6'-0"
El. 582.0
20'-0"
17'-9"
El. 505.0
El. 545.0
Steel Sheet Piling
Steel Sheet Piling
Bearing Piles
Steel Sheet Piling
SECTION A-A

FIG. 30.—SECTIONAL VIEWS OF THE SIPHON CULVERT.

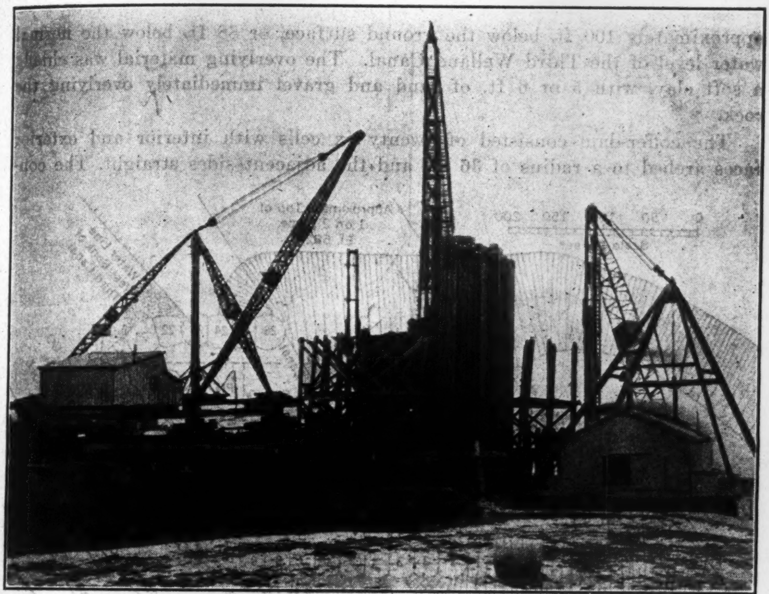


FIG. 31.—CELLULAR COFFER-DAM, ASSEMBLED AND READY FOR DRIVING.

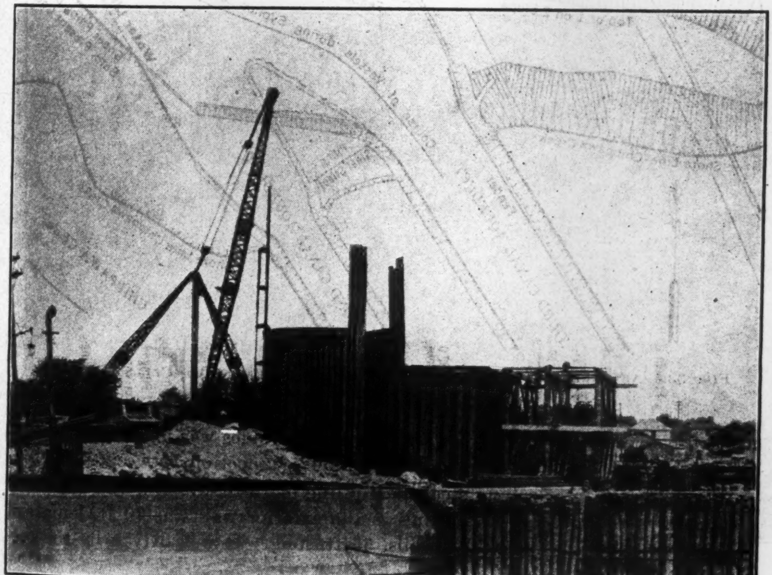


FIG. 32.—DRIVING SHEET-PILING AT CHIPPAWA CREEK SIPHON CULVERT.

approximately 100 ft. below the ground surface, or 88 ft. below the normal water level of the Third Welland Canal. The overlying material was chiefly a soft clay, with 5 or 6 ft. of sand and gravel immediately overlying the rock.

The coffer-dam consisted of twenty-six cells with interior and exterior faces arched to a radius of 36 ft., and the adjacent sides straight. The connection between the arched sides and the straight sides was made by the use of fabricated **Y**-piles, thus forming a continuous connection between all cells.

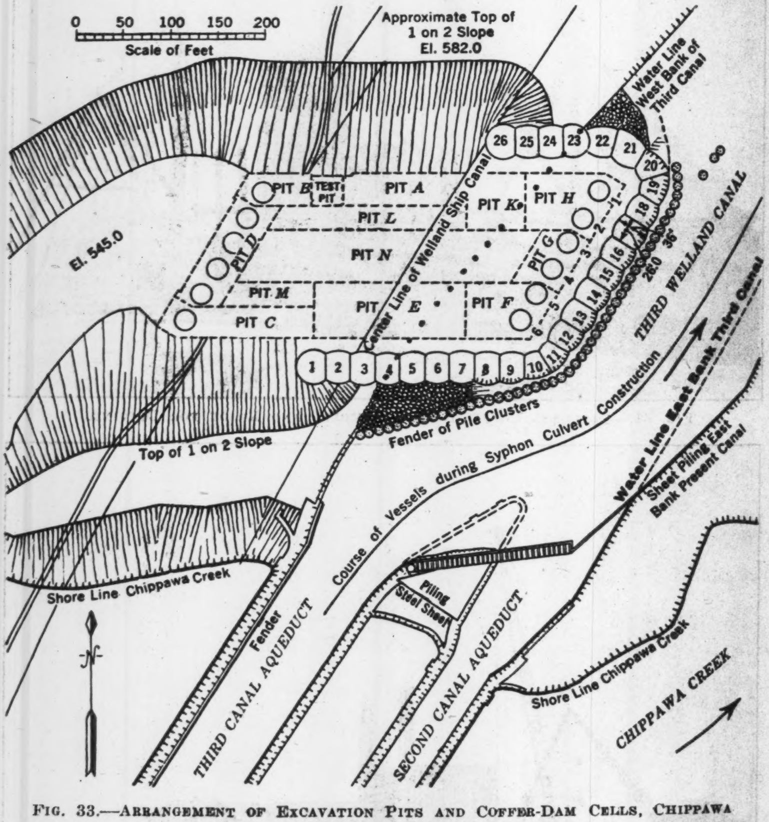

0 50 100 150 200
Scale of Feet
Approximate Top of 1 on 2 Slope El. 582.0
Water Line West Bank of Third Canal
26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
PIT B
TEST PIT
PIT A
PIT K
PIT H
PIT L
PIT D
PIT N
PIT G
PIT M
PIT E
PIT F
PIT C
1 2 3 4 5 6
Center Line of Welland Ship Canal
El. 545.0
26.0
36'
THIRD WELLAND CANAL
Water Line East Bank Third Canal
Fender of Pile Clusters
Course of Vessels during Syphon Culvert Construction
Top of 1 on 2 Slope
Sheet Piling East Bank Present Canal
Shore Line Chippawa Creek
Steel Sheet Piling
Fender
THIRD CANAL AQUEDUCT
SECOND CANAL AQUEDUCT
Shore Line Chippawa Creek
CHIPPAWA CREEK
N

FIG. 33.—ARRANGEMENT OF EXCAVATION PITS AND COFFER-DAM CELLS, CHIPPAWA CREEK SIPHON.

Straight web interlocking steel sheet-piling of 15-in. by $\frac{3}{8}$-in. and $12\frac{3}{4}$-in. by $\frac{3}{8}$-in. sections was used. The piling in the exterior wall was 70 ft. long, and that in the inner and partition walls, 55 ft. long. It was driven by the preassembly method.

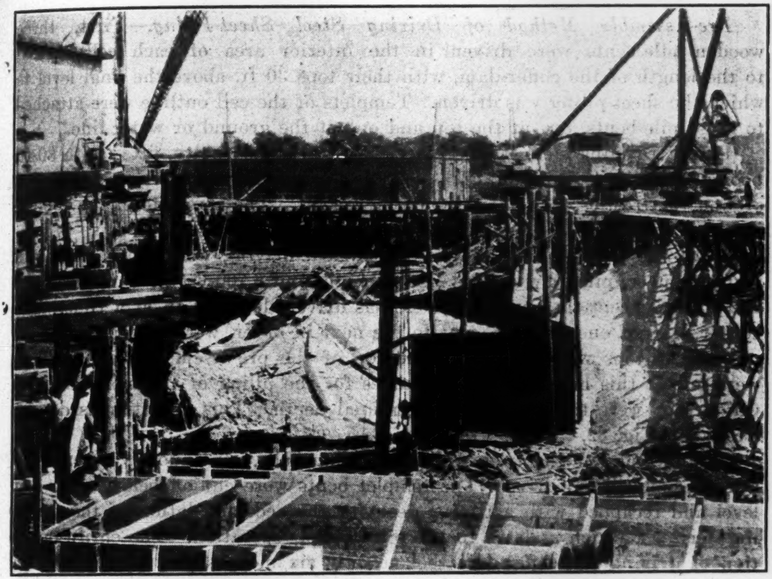


FIG. 34.—SETTLEMENT OF SHEET-PILES IN PIT *H*.

FIG. 35.—COMPLEX BRACING SYSTEM REQUIRED FOR PIT *N*.

Pre-Assembly Method of Driving Steel Sheet-Piling.—First, three wooden pile-bents were driven in the interior area of each cell normal to the length of the coffer-dam, with their tops 30 ft. above the final level to which the sheet-piling was driven. Templets of the cell outline were attached to these pile-bents, one at the top and one at the ground or water-line.

Using traveling steel stiff-leg derricks of 10-ton capacity, fitted with 80-ft. booms, the sheet-piling of each cell was assembled in a vertical position around the trestle templets as shown in Fig. 31. The complete circle, including the corner Y-piles, was interlocked as assembled.

With the piling of a cell thus assembled, driving was begun, with stiff frame, timber, pile-driving leads, 80 ft. high, and 6 700-lb. air hammers. All the piling, except the corner Y-piles, was driven down 10 ft. at a time. The corner piles of one cell, adjacent to the next in order, were driven down only until their tops were level with the top of the pre-assembly trestle. They were left at this height to act as guides for the piling of the next cell, with which they were then driven to their final depth. This operation is demonstrated in Fig. 32.

In this manner the twenty-six coffer-dam cells, continuously interlocked, were assembled and driven. The templet bents were cut off below coffer-dam level and recapped with a timber deck for the accommodation of the traveling derricks, which assisted in the pile-driving operation. The entire operation of driving the coffer-dam took exactly six months, and represented the handling of 1 700 000 lb. of steel sheet-piling.

Excavation Operations.—To reduce the depth of the trench excavation for the culvert foundation, the area inside the coffer-dam was dredged to the grade of the Ship Canal prism, Elevation 541.5, while the coffer-dam was under construction. When all the cells were completed, they were filled with hard clay from the dry excavation, but as the culvert site in the meantime had been dredged, this clay filling was dumped into the water, in the cells.

The cellular coffer-dam and its filling were completed on July 31, 1926, and the enclosed area was then unwatered. With the interior water level lowered 20 ft. below the exterior canal level, a corner Y-pile, along the east side failed, and the cells of this side of the coffer-dam tilted toward the pit. The maximum movement at the top of the coffer-dam was 4.5 ft.

This accident clearly indicated lack of lateral stiffness of the coffer-dam cells due to the clay filling being wet and not yet compacted when subjected to the water pressure. Therefore, the interior of the coffer-dam was immediately flooded to canal level and the necessary interior support was provided by depositing gravel against the inner side.

The interior level was then lowered (beginning October 6, 1926) at the rate of 3 ft. in 24 hours. This slow rate allowed the coffer-dam to be observed for further deflection, and it also allowed the water to drain from the supporting gravel. No further deflection occurred.

Test borings indicated the material to be penetrated below Ship Canal grade, in the interior of the coffer-dam, to be soft clay. The foundation construction therefore was carried on within steel sheet-pile enclosures, securely braced with timber as excavated.

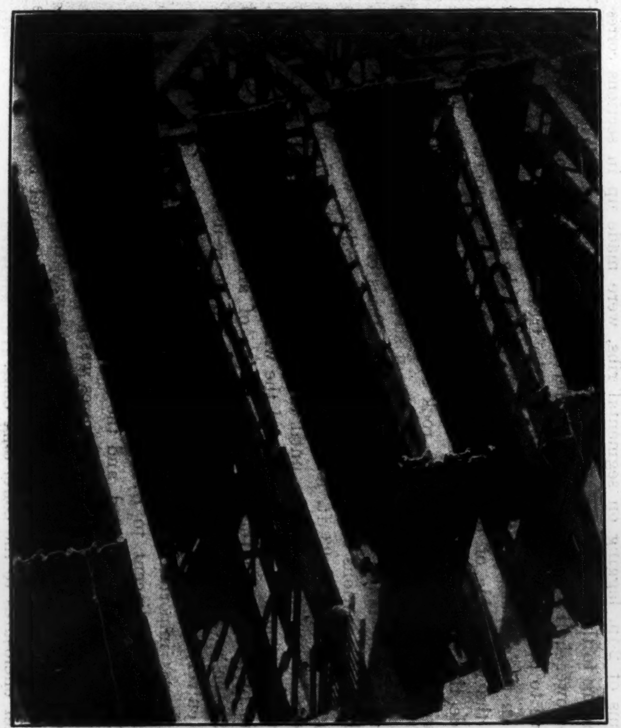


FIG. 36.—METHOD OF INTRODUCING THROUGH BRACING WITHOUT DISTURBING OTHER BRACING IN PLACE.

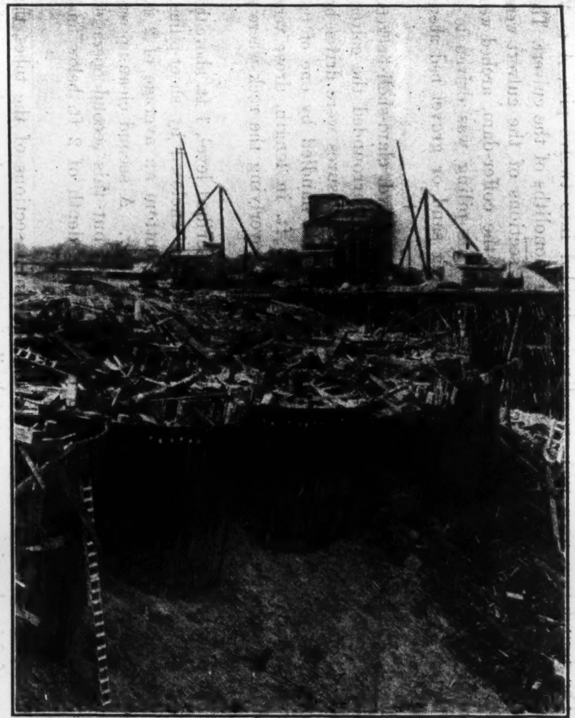


FIG. 37.—VIEW SHOWING COFFER-DAM CELLS IN PLACE WITH TIMBER BRACING REMOVED.

The foundation area was accordingly divided into twelve pits, *A* to *N* (see Fig. 33), each accommodating one or more monoliths of the culvert. The letters indicate the order in which the various sections of the culvert were built. In the west end of the area, enclosed by the coffer-dam, arched web sheet-piling was driven to outline each pit. This piling was driven to a contact with the rock or well into the overlying sand or gravel bed above the rock.

Earth was excavated by traveling derricks with 1½-yd. clam-shell buckets, working from a construction trestle that completely surrounded the culvert area. The foundation timber bearing piles in all enclosures were driven by a 4 000-lb., air-operated hammer in suspended leads handled by one of the traveling derricks. The piles, which averaged 20 ft. in length, drove very readily through the soft clay, down to the gravel overlying the rock where a good bearing was obtained.

The piles were then cut off by hand at the prescribed level, 1 ft. above the bottom of the excavation. Because the pits were enclosed by sheet-piling, the piles driven into the foundation raised the bottom an average of 2 ft. above the level to which it was taken out originally. A second clean-up was necessary, therefore, before placing the concrete; but this second operation was later eliminated by excavating each pit to a depth of 2 ft. below final grade before any bearing piles were driven.

In the enclosures incorporating the horizontal sections of the tubes, the forms of 2-in. lagging on segmental ribs, were made up in sections corresponding to the vertical distance between frames of bracing. When placed for the lower half of the circle, these forms were anchored to the concrete in place to prevent their being lifted by the fresh concrete.

In the east half of the culvert area, where the bank of gravel was placed on the inner side of the coffer-dam cells for their support, the gravel fill made the interior ground surface 70 ft. above the foundation level of the structure and 83 ft. above the rock surface.

This condition necessitated the use of steel sheet-piling 83 ft. long for the easterly enclosures. To overcome the difficulties of handling, pre-assembling, and driving sections of such extreme length, this piling was provided in two lengths, 60 ft. and 23 ft. long. Alternate 60 and 23-ft. lengths were first assembled and driven down until the upper sections could be placed on top of the previously driven piling and the whole was then driven to final depth. This procedure had the added advantage of allowing the piling reclaimed from the previously completed enclosures, to be re-used in the east end of the culvert area.

It was important to maintain the supporting power of the gravel bank, as the pits adjacent to it, along the east and south sides of the culvert area, were being excavated, and the concrete was being placed. As removed, the gravel from these outer enclosures was piled in similar form, as originally placed, along the sides of these enclosures away from the outer coffer-dam. Timber bracing transferred the support of this gravel bank through the outer enclosures to the coffer-dam cells.

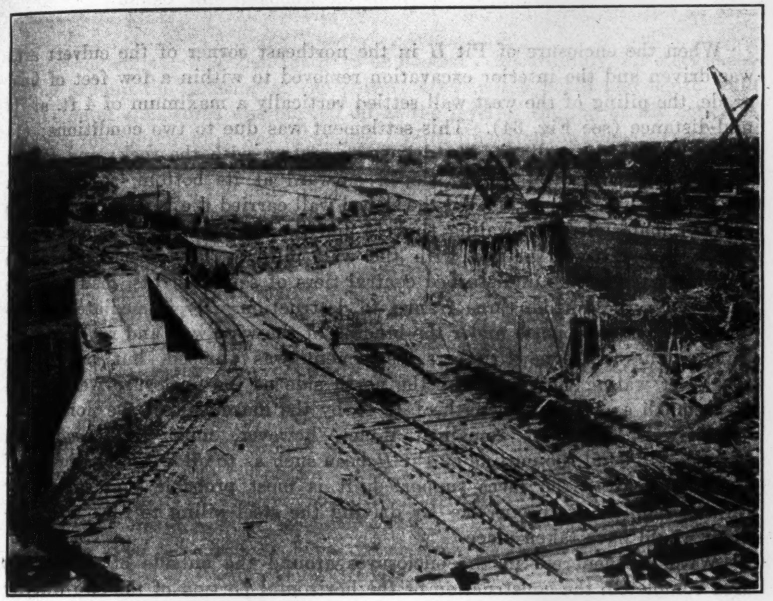


FIG. 38.—CHIPPAWA SIPHON CULVERT READY TO BE FLOODED, SEPTEMBER 10, 1928.

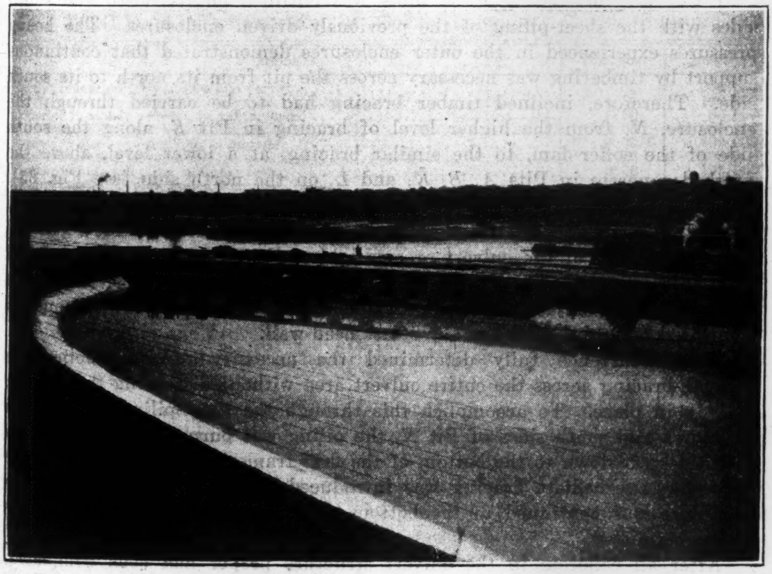


FIG. 39.—GENERAL VIEW OF CHIPPAWA SIPHON CULVERT LOOKING NORTHEAST. ENTRANCE CHANNEL IN FOREGROUND.

When the enclosure of Pit *H* in the northeast corner of the culvert area was driven and the interior excavation removed to within a few feet of final grade, the piling of the west wall settled vertically a maximum of 4 ft. at its mid-distance (see Fig. 34). This settlement was due to two conditions: (1) The piling of the wall was not driven to contact with the rock; and (2) in driving, this west wall piling sloped outward at its bottom, so that with the interior excavation removed the sloped wall carried the vertical load of the exterior material. The waling timbers and their supporting diagonal struts along the west wall settled with the wall piling. This concentrated the exterior loads on the undisturbed central tiers of square struts, loading some of them in the bottom three frames to destruction. By immediately introducing vertical supports under the bracing of the west wall and replacing the failed struts with new timbers, settlement was stopped, but not before the lower waling timbers along the north side of the enclosure were bowed in as much as 4 ft. in a length of 12 ft. by the movement of the north wall. These waling timbers remained intact, however, until reinforced with new struts. Had the outline of Pit *H* been such as to allow the entire use of struts normal to the faces supported, it is most probable that no timber failures would have occurred. By driving the steel piling to rock no settlement would have taken place.

With the concrete in the enclosures around the outside of the culvert area completed, the construction of the horizontal portion of the two interior tubes, 3 and 4, in Pit *N* (see Fig. 33), was the last operation.

Pit *N* accommodating these tubes was outlined on its north and south sides with the sheet-piling of the previously driven enclosures. The heavy pressures experienced in the outer enclosures demonstrated that continuous support by timbering was necessary across the pit from its north to its south side. Therefore, inclined timber bracing had to be carried through the enclosure, *N,* from the higher level of bracing in Pit *E,* along the south side of the coffer-dam, to the similar bracing, at a lower level, above the finished concrete in Pits *A, B, K,* and *L,* on the north side (see Fig. 35).

A very convincing demonstration of the severity of the external pressure exerted, was given when the completed concrete monolith in Pit *F* in the southeast corner of the culvert area weighing about 12 000 tons was moved as a unit approximately 2 ft. horizontally from its original position by the pressure of the adjacent portion of the outer coffer-dam. This monolith comprised one-third of the massive east head-wall.

This occurrence fully determined the necessity of introducing the through-bracing across the entire culvert area without disturbing the bracing already in place. To accomplish this through the sheet-piling in place on the north and south sides of Pit *N,* the piling was burned off opposite each inclined strut down to the bottom of the first frame level, and through these gaps the intermediate bracing was introduced, as shown in Fig. 36. This procedure was continued to the bottom of Pit *N,* as rapidly as the excavation permitted.

After all concrete in the culvert structure proper had been completed, the interior through-bracing was no longer required, because the prism area of the culvert was then ready to be flooded. Accordingly, all steel sheet-piling around the exterior of the culvert area in the north and south walls of the outer enclosures was burned off at the level of the top of the concrete, and the through-bracing was taken out. With this interior timbering entirely removed and the cells along the south side of the outer coffer-dam free of any interior support, the cells that had deflected when first unwatered gave no further sign of movement (see Fig. 37). No doubt, this was due to the fact that the clay fill in the cells had become dry and thoroughly consolidated.

The cells of the coffer-dam in the prism north and south of the culvert were then removed, the culvert area was flooded, and the new Ship Canal channel established over the siphon culvert. Fig. 38 is a view showing the siphon culvert ready to be flooded. On December 7, 1928, just two years and ten months after the construction of the cellular coffer-dam was started, the first boat passed over the new structure. Fig. 39 is a view of the completed siphon culvert.

The excavation of the intake and discharge channels of the culvert was completed in 1929 and Chippawa Creek was diverted through the new structure on September 5, 1929. The north and south wing-walls of the siphon culvert were completed on December 6, 1929. The Second and Third Canal aqueducts were then isolated from the creek in the canal prism and that of the Third Canal was removed by dredging in 1931.

For the construction of the Chippawa Creek siphon culvert, the contractor was paid at the contract unit prices listed in Table 11.

TABLE 11.—DISTRIBUTION OF COSTS, CHIPPAWA CREEK SIPHON

Items	Unit cost	Quantity
Earth excavation (open), in cubic yards	$0.63	300 000
Earth excavation (pit), in cubic yards	1.50	170 000
Concrete in siphon culvert, in cubic yards	7.50	84 000
Steel sheet-piling (driven), in pounds	0.05	15 000 000
Removing steel sheet-piling, in pounds	0.0075	6 489 000
Reinforcing steel, in pounds	0.05	808 000
Timber foundation piles, in linear feet	0.80	196 000
Timber trestle piles, in linear feet	0.85	246 000
British Columbia fir timber bracing, in thousand feet board measure	100.0	4 000
Hardwood bracing, in thousand feet board measure	150.00	999

In all cases where steel sheet-piling, timber bracing, etc., were removed by the contractor, these materials became his property, If they were in good usable condition, their re-use in the work was permitted, and the contractor was paid the prevailing contract prices. The total cost of the siphon culvert structure, including materials, such as cement, supplied by the Department, was $3 800 000.

While the contractor generally developed his own construction methods, the problem of building the siphon culvert was so involved and embraced so many operations that it was necessary for the Engineering Department of the Welland Ship Canal to cover the complete cycle of operations in detail in the specification. Practically the only detail of this operation left to the contractor's discretion was the selection of the plant.

ACKNOWLEDGMENTS

The writer is particularly indebted to A. J. Grant, Engineer in Charge of the Welland Ship Canal, for his review of this paper.

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STRESSES IN INCLINED ARCHES OF MULTIPLE-ARCH DAMS

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WITH DISCUSSION BY MESSRS. VICTOR H. COCHRANE, PAUL BAUMAN, LARS R. JORGENSEN, FRED A. NOETZLI, W. J. STICH, AND GEORGE E. GOODALL AND IVAN M. NELIDOV

SYNOPSIS

The paper here presented contains the derivation of analytical formulas for moments, thrusts, and shears in the inclined circular arches of multiple-arch dams. All the equations were derived by integration through application of the theorem of Castigliano, and they include the internal elastic work due to shear.

As they are all necessarily long and cumbersome, coefficients for obtaining moments, thrusts, and shears were computed, and curves representing these coefficients are presented for the convenience of the designing engineer. The water load on the arch may be divided into two parts, the first being the uniform water load due to the head measured from the free water surface to the arch crown, and the second, the variable water load which results from the inclination of the arch. In addition to the water load, the dead weight of the arch and the temperature stresses must be considered, and the moments, thrusts, and shears of all these influences must be combined algebraically to determine maximum stresses.

INTRODUCTION

In the past, the moments, thrusts, and shears due to variable water load and dead load have been determined by graphical methods or by the analytical method developed by Dr. N. Kelen.[3] This latter method provides

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NOTE.—Published in March, 1932, *Proceedings*.

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[3] "Die Staumauern", von Dr.-Ing. N. Kelen, 1926, p. 44.

curves from which the crown thrust may be obtained directly, but does not include curves or coefficients from which the abutment thrust, abutment shear, and bending moments may be directly obtained. Dr. Kelen's formulas omit the effect of the internal elastic work due to shear.

The late William Cain, M. Am. Soc. C. E., in his paper[4] entitled, "The Circular Arch Under Normal Loads," derived the necessary formulas for the moments, thrusts, and shears due to uniform water load and temperature changes. Professor Cain did not consider the problem of the non-uniform loads imposed on the inclined arches of multiple-arch dams, namely, the variable water load and the dead weight of the arch.

The derivations which follow are for the moments, thrusts, and shears due to the variable water load and the dead weight of the arch. All the formulas include the internal elastic work due to shear.

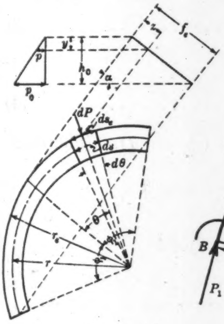

FIG. 1

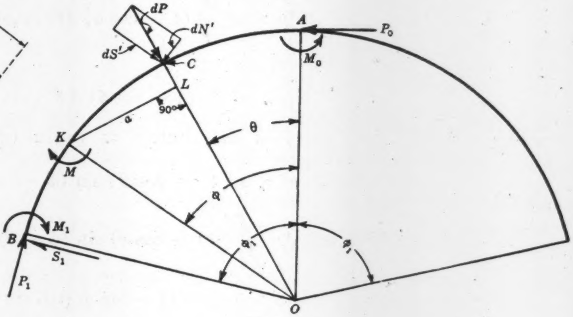

FIG. 2

MOMENTS, THRUSTS, AND SHEARS DUE TO VARIABLE WATER LOAD FOR ARCHES WITH FIXED ENDS

Fig. 1 represents a section of the arch taken normal to the generator of the arch barrel, and also the hydrostatic pressure diagram. The arch is considered as being fixed at the ends.

Let, $t =$ uniform radial thickness of arch, in feet;
$r =$ radius of center line of arch, in feet;
$r_e =$ radius of extrados, in feet;
$w =$ unit weight of water, in pounds per cubic foot;
$\theta =$ angle between the crown and any point;
$\phi_1 =$ one-half the central angle of the arch;
$\alpha =$ angle of inclination of arch generator with the horizontal;
$y =$ vertical distance of any point on the arch below the crown;
$h_o =$ vertical distance between the crown and the springing line of the arch;
$p_o =$ pressure due to the head, h_o; and,
$p =$ pressure due to the head, y.

In the derivation that follows, the only load acting on the arch is the water load which varies from zero at the crown to p_o at the springing line due to the inclination of the arch, α.

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[4] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 233.

From Fig. 1, $p_\circ = wh_\circ$; $p = p_\circ \dfrac{y}{h_0} = p_0 \dfrac{z}{f_e}$; $f_e = \dfrac{h_0}{\cos \alpha}$; and, $z = r_e - r_e \cos \theta = r_e (1 - \cos \theta)$. Therefore,

$$p = p_0 r_e \frac{(1 - \cos \theta)}{h_0} \cos \alpha = r_e (1 - \cos \theta)\, w \cos \alpha. \quad \ldots\ldots (1)$$

Since $w \cos \alpha = w'$ is a constant for any given inclination of the arch generator, Equation (1) may be written:

$$p = w' r_e (1 - \cos \theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From Fig. 2, $dp = p\, ds_e = w' r_e (1 - \cos \theta)\, ds_e$; and $\dfrac{ds_e}{ds} = \dfrac{r_e}{r}$, or $ds_e = \dfrac{r_e}{r}\, ds$. Therefore,

$$dp = w' \frac{r^2_e}{r} (1 - \cos \theta)\, ds \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

but $ds = rd\,\theta$, and, therefore,

$$dp = w' r^2_e (1 - \cos \theta)\, d\,\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Furthermore, $dM' = +\, a\, dP$; but $a = r \sin (\phi - \theta)$. Therefore,

$$dM' = +\, w' r_e^2 r (1 - \cos \theta) \sin (\phi - \theta)\, d\,\theta$$

and,

$$M' = w' r_e^2 r \int_0^\phi (1 - \cos \theta) \sin (\phi - \theta)\, d\,\theta$$

$$= w' r_e^2 r \Big[\sin^2 \phi + \cos \phi (\cos \phi - 1) - \sin \phi (0.5 \sin \phi \cos \phi + 0.5\, \phi) + 0.5 \cos \phi \sin^2 \phi \Big]$$

or,

$$M' = w' r_e^2 r (1 - \cos \phi - 0.5\, \phi \sin \phi) \quad \ldots\ldots\ldots\ldots\ldots (5)$$

Equation (5) gives the moment due to the external loads at the point, K. As each of the elementary loads, dP, is radial, it follows that the resultant of all these loads, P, between the crown and K must pass through the center of curvature of the arch. This resultant, $\Sigma\, P$, may be resolved into two components, the first being radial to K representing the shearing component of the external loads, and the second, tangential and representing the component of thrust, N'.

The force, N', is equal to M' divided by the radius, or,

$$N' = \frac{M'}{r} = w' r^2_e (1 - \cos \phi - 0.5\, \phi \sin \phi) \quad \ldots\ldots\ldots\ldots (6)$$

Taking moments about K of all the forces from the crown to K,

$$M = M_0 + w' r_e^2 r (1 - \cos \phi - 0.5\, \phi \sin \phi) - P_0 r (1 - \cos \phi) \quad \ldots\ldots (7)$$

$$P = P_0 \cos \phi + w' r^2_e (1 - \cos \phi - 0.5\, \phi \sin \phi) \quad \ldots\ldots\ldots (8)$$

At any point, K (Fig. 2), the shearing component of an elementary load, dP, is the component parallel to OK, thus,

$$dS' = dP \cos(\phi - \theta) \ldots\ldots\ldots\ldots (9)$$

Substituting dP from Equation (4), Equation (9) becomes,

$$dS' = w' r^2_e (1 - \cos\theta) \cos(\phi - \theta)\, d\theta \ldots\ldots\ldots\ldots (10)$$

The total shearing component of all external loads from A to K is,

$$S' = \int_0^{\phi} w' r^2_e (1 - \cos\theta) \cos(\phi - \theta)\, d\theta$$

or,

$$S' = \frac{w' r^2_e}{2} (\sin\phi - \phi\cos\phi) \ldots\ldots\ldots\ldots (11)$$

The total shear at K is equal to the sum of the components of all forces parallel to OK, between the crown and K, or,

$$S = \frac{w' r^2_e}{2} (\sin\phi - \phi\cos\phi) - P_o \sin\phi \ldots\ldots\ldots\ldots (12)$$

The total internal elastic work for the semi-arch due to M, P, and S, is,

$$L = \frac{r}{2\,EI} \int_0^{\phi_1} M^2\, d\phi + \frac{r}{2\,Et} \int_0^{\phi_1} P^2\, d\phi + \frac{2.88\, r}{2\,Et} \int_0^{\phi_1} S^2\, d\phi \ldots (13)$$

Since the rotation of the crown section is zero, $\frac{\partial L}{\partial M_o} = 0$; also, since there is no displacement of the crown section in the direction of P_o, $\frac{\partial L}{\partial P_o} = 0$.

Taking partial derivatives of Equations (7), (8), (12), and (13), $\frac{\partial M}{\partial M_o} = 1$; $\frac{\partial M}{\partial P_o} = -r(1 - \cos\phi)$; $\frac{\partial P}{\partial M_o} = 0$; $\frac{\partial P}{\partial P_o} = \cos\phi$; $\frac{\partial S}{\partial M_o} = 0$; $\frac{\partial S}{\partial P_o} = -\sin\phi$; and $\frac{\partial L}{\partial M_o} = \frac{r}{EI} \int_0^{\phi_1} \frac{M\,\partial M\, d\phi}{\partial M_o} = 0$. Therefore,

$$\int_0^{\phi_1} M\, d\phi = 0 \ldots\ldots\ldots\ldots (14)$$

By the aid of Equation (7) this reduces to,

$$\int_0^{\phi_1} [M_o + w' r^2_e r (1 - \cos\phi - \frac{1}{2}\phi\sin\phi) - P_o r (1 - \cos\phi)]\, d\phi = 0$$

or,

$$M_o \phi_1 + w' r^2_e r (\phi_1 - \frac{3}{2}\sin\phi_1 + \frac{1}{2}\phi_1\cos\phi_1) - P_o r (\phi_1 - \sin\phi_1) = 0 \ldots (15)$$

From $\frac{\partial L}{\partial P_o} = 0$,

$$\frac{r}{I} \int_0^{\phi_1} M \frac{\partial M}{\partial P_o}\, d\phi + \frac{r}{t} \int_0^{\phi_1} P \frac{\partial P}{\partial P_o}\, d\phi + \frac{2.88\, r}{t} \int_0^{\phi_1} S \frac{\partial S}{\partial P_o}\, d\phi = 0 \ldots (16)$$

Multiplying by $\frac{I}{r}$ and making $\frac{I}{t} = k^2$, this becomes,

$$- r \int_0^{\phi_1} M (1 - \cos \phi)\, d\phi + k^2 \int_0^{\phi_1} P \cos \phi\, d\phi$$

$$+ 2.88 k^2 \int_0^{\phi_1} S (- \sin \phi)\, d\phi = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

By Equation (14), $\int_0^{\phi_1} M d\phi = 0$, so the first term of Equation (17) becomes,

$$r \int_0^{\phi_1} M \cos \phi\, d\phi = r \int_0^{\phi_1} [M_o + w' r^2_e\, r \left(1 - \cos \phi - \frac{1}{2} \phi \sin \phi\right)$$

$$- P_o r (1 - \cos \phi)] \cos \phi\, d\phi \;\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

which reduces to the form,

$$r \int_0^{\phi_1} M \cos \phi\, d\phi = r \left[M_o \sin \phi_1 + w' r^2_e\, r \left(\sin \phi_1 - \frac{5}{8} \sin \phi_1 \cos \phi_1 \right.\right.$$

$$\left.\left. - \frac{3 \phi_1}{8} - \frac{\phi_1}{4} \sin^2 \phi_1 \right) - P_o r \left(\sin \phi_1 - \frac{1}{2} \sin \phi_1 \cos \phi_1 - \frac{\phi_1}{2} \right) \right] \ldots\ldots (19)$$

Likewise, the second term of Equation (17) becomes,

$$k^2 \int_0^{\phi} P \cos \phi\, d\phi = k^2 \int_0^{\phi_1} \left[P_o \cos \phi + w' r^2_e \left(1 - \cos \phi - \frac{\phi}{2} \sin \phi \right) \right] \cos \phi\, d\phi \,..(20)$$

or,

$$k^2 \int_0^{\phi_1} P \cos \phi\, d\phi = k^2 \left[P_o \left(\frac{1}{2} \sin \phi_1 \cos \phi_1 + \frac{\phi_1}{2} \right) \right.$$

$$\left. + w' r^2_c \left(\sin \phi_1 - \frac{5}{8} \sin \phi_1 \cos \phi_1 - \frac{3 \phi_1}{8} - \frac{\phi_1}{4} \sin^2 \phi_1 \right) \right] \ldots\ldots (21)$$

Finally, the third term of Equation (17) becomes,

$$- 2.88 k^2 \int_0^{\phi_1} S \sin \phi\, d\phi$$

$$= - 2.88 k^2 \int_0^{\phi_1} \left[\frac{w' r^2_e}{2} (\sin \phi - \phi \cos \phi) - P_o \sin \phi \right] \sin \phi\, d\phi \;\ldots (22)$$

or,

$$- 2.88 k^2 \int_0^{\phi_1} S \sin \phi\, d\phi = - 2.88 k^2 \left[\frac{w' r^2_e}{4} \left(- \frac{3}{4} \sin 2 \phi_1 + \phi_1 + \frac{\phi_1 \cos 2 \phi_1}{2} \right) \right.$$

$$\left. - P_o \left(\frac{\phi_1}{2} - \frac{\sin 2 \phi_1}{4} \right) \right] \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

After Equations (19), (21), and (23) have been substituted in Equation (17), M_o may be eliminated by multiplying through by ϕ_1, and subtracting the result from Equation (15), multiplied by $r \sin \phi_1$.

The next step is to replace $\sin \phi_1 \cos \phi_1$, by its equivalent, $\frac{1}{2} \sin 2 \phi_1$, and solve for P_o. The final result is:

$$P_0 = w' r^2_e \Bigg\{\left(2 \sin \phi_1 - \frac{3 \sin^2 \phi_1}{\phi} + \frac{\sin 2 \phi_1}{2}\right)$$

$$- \left(1 + \frac{k^2}{r^2}\right)\left(2 \sin \phi_1 - \frac{5}{8} \sin 2 \phi_1 - \phi_1 + \frac{\phi_1 \cos 2 \phi_1}{4}\right)$$

$$+ 5.76 \frac{k^2}{r_2}\left[\frac{1}{4}\left(- \frac{3}{4} \sin 2 \phi_1 + \phi_1 + \frac{\phi_1 \cos 2 \phi_1}{2}\right)\right]\Bigg\} \div \Bigg\{\left(1 + \frac{k^2}{r^2}\right)\left(\phi_1 + \frac{\sin 2 \phi_1}{2}\right)$$

$$- \frac{2 \sin^2 \phi_1}{\phi_1} + 2.88 \frac{k^2}{r^2}\left(\phi_1 - \frac{\sin 2 \phi_1}{2}\right)\Bigg\} \quad \ldots\ldots\ldots\ldots (24)$$

The denominator of Equation (24) checks Professor Cain's expression for D_8 divided by ϕ_1.[5]

Since w' is the unit weight of water multiplied by the cosine of the angle of inclination of the arch, and r_e and $2\phi_1$, for a given dam, are known, Equation (24) for the crown thrust may be written,

$$P_0 = w' r^2_e \, Z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which, Z embraces terms containing only $\frac{t}{r}$ and functions of the central angle, $2 \phi_1$. For the convenience of the designing engineer, numerous values of the coefficient, Z, have been calculated for all values of $\frac{t}{r}$ and $2 \phi_1$ likely to be met in the design of multiple-arch dams and, from these coefficients, curves have been plotted so that the crown thrust may readily be computed. The values of the coefficient, Z, are plotted in Fig. 3.

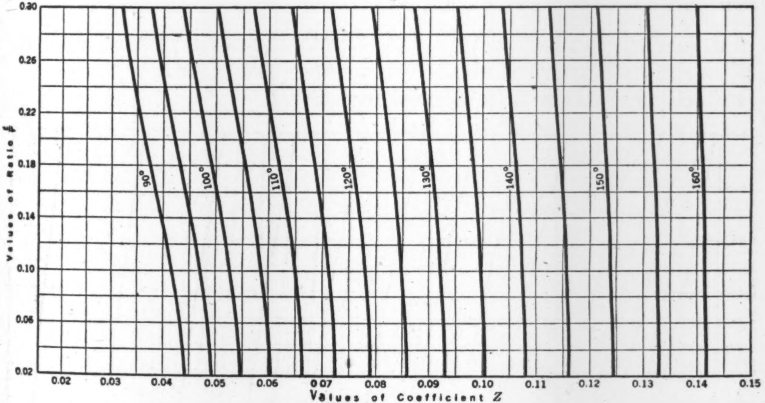

FIG. 3.—CROWN THRUST DUE TO VARIABLE WATER LOAD ON MULTIPLE-ARCH DAMS, $P_0 = w' r^2_e Z$

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[5] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), pp. 237 and 269.

If ϕ_1 is substituted for ϕ in Equation (8), the thrust at the abutment may be written in terms of Z, as follows:

$$P_1 = w' r^2_e \left[Z \cos \phi_1 + \left(1 - \cos \phi_1 - \frac{1}{2} \phi_1 \sin \phi_1 \right) \right] = w' r^2_e Z_1 \ldots (26)$$

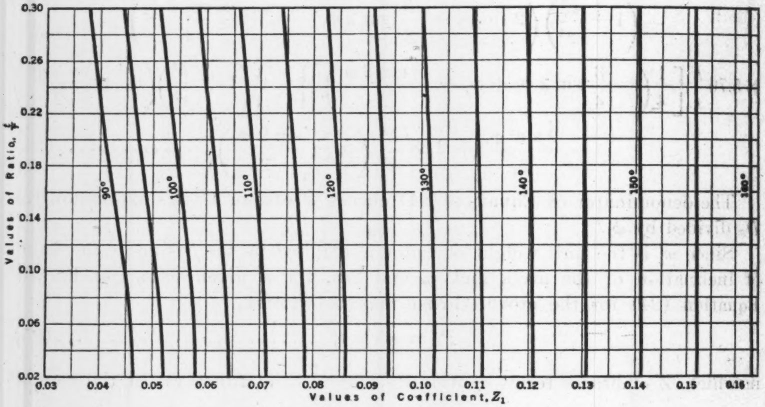

FIG. 4.—ABUTMENT THRUST DUE TO VARIABLE WATER LOAD ON MULTIPLE-ARCH DAMS, $P_1 = w' r^2_e Z_1$

in which, the value of Z_1 may be obtained from Fig. 4. Then, from Equation (15), the crown moment, M_O, is:

$$M_O = w' r^2_e r \left[\left(1 - \frac{\sin \phi_1}{\phi_1} \right) Z - \left(1 - \frac{3 \sin \phi_1}{2 \phi_1} + \frac{1}{2} \cos \phi_1 \right) \right] = w' r^2_e r Z_2 \,. (27)$$

in which, the value of Z_2 is obtained from Fig. 5.

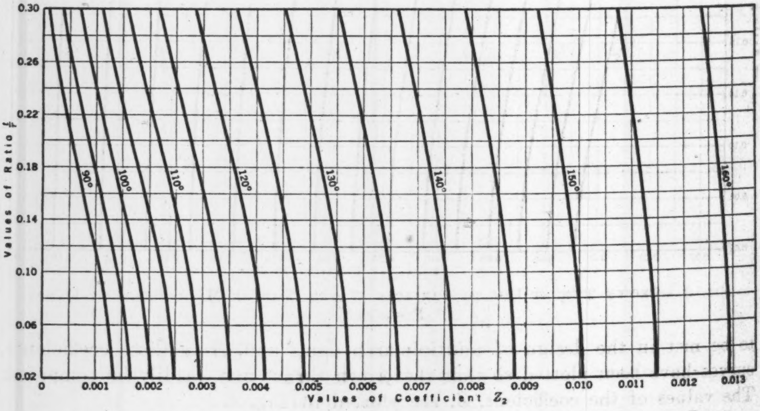

FIG. 5.—CROWN MOMENT DUE TO VARIABLE WATER LOAD ON MULTIPLE-ARCH DAMS, $M_O = w' r^2_e r Z_2$

The moment at the abutment is obtained by combining Equations (7) and (25), with the following result:

$$M_1 = -w'\, r_e^2\, r \left[Z \left(\frac{\sin \phi_1}{\phi_1} - \cos \phi_1 \right) + \frac{3}{2} \left(\cos \phi_1 + \frac{\phi_1 \sin \phi_1}{3} - \frac{\sin \phi_1}{\phi_1} \right) \right]$$
$$= w'\, r^2_e\, r\, Z_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which, Z_3 is obtained from Fig. 6.

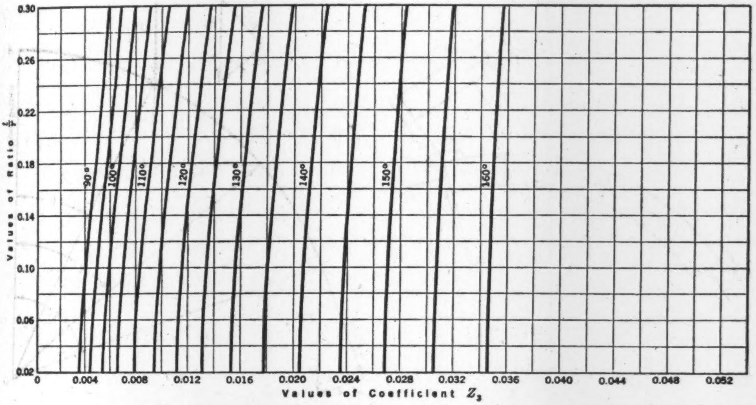

FIG. 6.—ABUTMENT MOMENT DUE TO VARIABLE WATER LOAD ON MULTIPLE-ARCH DAMS, $M_1 = w'\, r^2_e\, r\, Z_3$

From Equation (12), the shear at the abutment is equal to,

$$S_1 = w'\, r^2_e \left[\tfrac{1}{2} (\sin \phi_1 - \phi_1 \cos \phi_1) - Z \sin \phi_1 \right] = w'\, r^2_e\, Z_4 \ldots\ldots\ldots (29)$$

in which, Z_4 is obtained from Fig. 7.

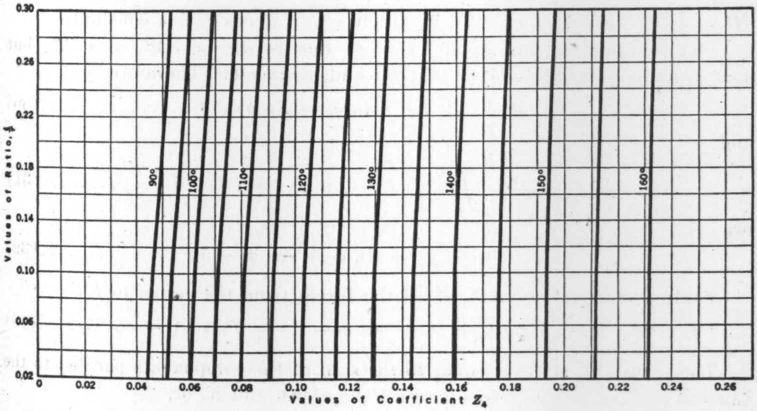

FIG. 7.—ABUTMENT SHEAR DUE TO VARIABLE WATER LOAD ON MULTIPLE-ARCH DAMS, $S_1 = w'\, r^2_e\, Z_4$

MOMENTS, THRUSTS, AND SHEARS DUE TO WEIGHT OF ARCH FOR ARCHES WITH FIXED ENDS

In determining the stresses due to the weight of the arch which is considered as fixed at the abutments, the only forces to be considered are the components of the weight of the elementary lengths, *ds,* normal to the generator of the arch barrel acting parallel to the plane of the buttresses (Fig. 8). The weight of an elementary length of arch, *ds* (for an arch slice of 1 ft. slant height) is $w_c\ t\ ds$, in which, w_c = unit weight of concrete, in pounds per cubic foot.

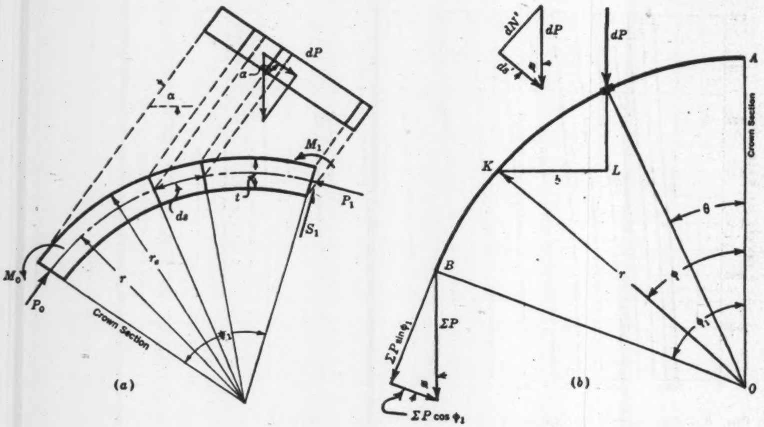

FIG. 8

The down-stream component of weight of the elementary length is, $dP = w_c\ t\ ds \cos \alpha = w'_c\ t\ ds$, in which, $w'_c = w \cos \alpha$, is a constant.

The bending moment at K (Fig. 8) due to dP is, $dM' = a\ dP$; but $a = r\ (\sin \phi - \sin \theta)$; $dP = w'_c t\ ds$; and, $ds = r d\theta$. Therefore,

$$dM' = w'_c\ t\ r^2\ (\sin \phi - \sin \theta)\ d\,\theta \ldots\ldots\ldots\ldots\ldots (30)$$

and,

$$M' = \int_0^{\phi} w'_c\ t\ r^2\ (\sin \phi - \sin \theta)\ d\,\theta \ldots\ldots\ldots\ldots (31)$$

or,

$$M' = w'_c\ t\ r^2\ (\phi \sin \phi + \cos \phi - 1) \ldots\ldots\ldots\ldots\ldots (32)$$

Taking moments about K of all the forces from the crown to K:

$$M = M_0 + w'_c\ t\ r^2\ (\phi \sin \phi + \cos \phi - 1) - P_0\ r\ (1 - \cos \phi) \ldots (33)$$

The thrust, *P,* at *K,* is equal to the sum of the components parallel to the tangent at *K* of all the forces between the crown and *K,* or,

$$P = P_0 \cos \phi + \Sigma\ P \sin \phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

Furthermore,

$$\Sigma P = \int_0^{\phi} w'_c\, t\, ds = \int_0^{\phi} w'_c\, t\, r\, d\theta = w'_c\, t\, r\, \phi \quad \ldots\ldots\ldots (35)$$

and hence Equation (34) becomes,

$$P = P_0 \cos \phi + w'_c\, t\, r\, \phi \sin \phi \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

The shear is equal to,

$$S = w'_c\ t r\, \phi \cos \phi - P_0 \sin \phi \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

Taking partial derivatives of Equations (33), (36), and (37): $\frac{\partial M}{\partial M_0} = 1$; $\frac{\partial M}{\partial P_0} = -r\,(1 - \cos \phi)$; $\frac{\partial P}{\partial M_0} = 0$; $\frac{\partial P}{\partial P_0} = \cos \phi$; $\frac{\partial S}{\partial M_0} = 0$; and $\frac{\partial S}{\partial P_0} = -\sin \phi$.

The internal elastic work, L, for the semi-arch, due to M, P, and S, is:

$$L = \frac{r}{2\,EI} \int_0^{\phi_1} M^2\, d\phi + \frac{r}{2\,Et} \int_0^{\phi_1} P^2\, d\phi + \frac{2.88\, r}{2\, Et} \int_0^{\phi_1} S^2\, d\phi \ldots (38)$$

Since the rotation of the crown section is zero, $\frac{\partial L}{\partial M_0} = 0$; and, also, since there is no translation of the crown section, $\frac{\partial L}{\partial P_0} = 0$:

$$\frac{\partial L}{\partial M_0} = \frac{r}{EI} \int_0^{\phi_1} M\, \frac{\partial M}{\partial M_0}\, d\phi = 0 \quad \ldots\ldots\ldots\ldots\ldots (39)$$

therefore,

$$\int_0^{\phi_1} M d\phi = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

By the aid of Equation (33), this becomes:

$$\int_0^{\phi_1} \Big[M_0 + w'_c\, t\, r^2\, (\phi \sin \phi + \cos \phi - 1) - P_0\, (r - r \cos \phi) \Big]\, d\phi = 0$$

$$\Big[M_0\, \phi + w'_c\, t\, r^2\, (\sin \phi - \phi \cos \phi + \sin \phi - \phi\,) - P_0 r\, (\phi - \sin \phi) \Big]_0^{\phi_1} = 0$$

or,

$$M_0 \phi_1 + w'_c\, t\, r^2\, (2 \sin \phi_1 - \phi_1 \cos \phi_1 - \phi_1) - P_0 r\, (\phi_1 - \sin \phi_1) = 0 \ldots (41)$$

From $\frac{\partial L}{\partial P_0} = 0$:

$$\frac{r}{I} \int_0^{\phi_1} M \frac{\partial M}{\partial P_0}\, d\phi + \frac{r}{t} \int_0^{\phi_1} P\, \frac{\partial P}{\partial P_0}\, d\phi + \frac{2.88\, r}{t} \int_0^{\phi_1} S\ \frac{\partial S}{\partial P_0}\, d\phi = 0\ . (42)$$

Multiplying by $\frac{I}{r}$ and making $\frac{I}{t} = k^2$, Equation (42) becomes:

$$-r \int_0^{\phi_1} M\, (1 - \cos \phi)\, d\phi + k^2 \int_0^{\phi_1} P \cos \phi\, d\phi$$

$$+ 2.88 k^2 \int_0^{\phi_1} S\, (-\sin \phi)\, d\phi = 0 \quad \ldots\ldots\ldots\ldots\ldots (43)$$

By Equation (40) the first term of Equation (43) becomes:

$$r\int_0^{\phi_1} M\cos\phi\,d\phi = r\int_0^{\phi_1}\Big[M_o + w'_c\,t\,r^2(\phi\sin\phi+\cos\phi-1) - P_o r\,(1-\cos\phi)\Big]\cos\phi\,d\phi \;\ldots\ldots\ldots\ldots\ldots\ldots(44)$$

which returns to the form:

$$r\int_0^{\phi_1} M\cos\phi\,d\phi = r\left[M_o\sin\phi_1 + w'_c\,t\,r^2\left(\frac{3}{8}\sin 2\phi_1 - \frac{\phi_1\cos 2\phi_1}{4} + \frac{\phi_1}{2} - \sin\phi_1\right) - P_o r\left(\sin\phi_1 - \frac{\sin 2\phi_1}{4} - \frac{\phi_1}{2}\right)\right]\ldots\ldots\ldots\ldots(45)$$

Likewise, the second term of Equation (43) becomes:

$$k^2\int_0^{\phi_1} P\cos\phi\,d\phi = k^2\int_0^{\phi_1}\Big[P_o\cos\phi + w'_c\,t\,r\,\phi\sin\phi\Big]\cos\phi\,d\phi\ldots(46)$$

which reduces to the form,

$$k^2\int_0^{\phi_1} P\cos\phi\,d\phi = k^2\left[P_o\left(\frac{1}{4}\sin 2\phi_1 + \frac{1}{2}\phi_1\right) + w'_c\frac{tr}{4}\left(\frac{1}{2}\sin 2\phi_1 - \phi_1\cos 2\phi_1\right)\right]\ldots\ldots\ldots\ldots(47)$$

Finally, the third term of Equation (43) becomes:

$$-2.88k^2\int_0^{\phi_1} S\sin\phi\,d\phi$$

$$= -2.88k^2\int_0^{\phi_1}\Big[w'_c\,t\,r\,\phi\cos\phi - P_o\sin\phi\Big]\sin\phi\,d\phi\;\ldots\ldots(48)$$

or,

$$-2.88k^2\int_0^{\phi_1} S\sin\phi\,d\phi = -2.88k^2\left[\frac{1}{4}w'_c\,t\,r\left(\frac{1}{2}\sin 2\phi_1 - \phi_1\cos 2\phi_1\right) - P_o\left(\frac{1}{2}\phi_1 - \frac{1}{4}\sin 2\phi_1\right)\right]\ldots\ldots\ldots\ldots\ldots\ldots(49)$$

After Equations (45), (47), and (49) have been substituted in Equation (43), M_o may be eliminated by multiplying through by ϕ_1, and subtracting the result from Equation (41) multiplied by $r\sin\phi_1$. The next step is to solve for P_o and simplify by multiplying both numerator and denominator by $\frac{2}{\phi_1\,r^2}$. The final result is:

$$P_o = w'_c\,tr\frac{\left(\frac{4\sin^2\phi_1}{\phi_1} - \frac{7}{4}\sin 2\phi_1 + \frac{\phi_1\cos 2\phi_1}{2} - \phi_1\right) + 1.88\frac{k^2}{r^2}\left(\frac{1}{4}\sin 2\phi_1 - \frac{\phi_1\cos 2\phi_1}{2}\right)}{\left(1+\frac{k^2}{r^2}\right)\left(\phi_1 + \frac{\sin 2\phi_1}{2}\right) - \frac{2\sin^2\phi_1}{\phi_1} + 2.88\frac{k^2}{r^2}\left(\phi_1 - \frac{\sin 2\phi_1}{2}\right)}.\quad(50)$$

Equation (50) may be written:

$$P_0 = w'_c\, t\, r\, Z' \text{ ....} (51)$$

By means of Equations (36) and (51), the formula for abutment thrust is:

$$P_1 = w'_c\, t\, r\, [Z' \cos \phi_1 + \phi_1 \sin \phi_1] = w'_c\, t\, r\, Z'_1 \text{ ....} (52)$$

Equation (41), which gives the moment at the crown, may be written:

$$M_0 = w'_c\, t\, r^2 \left[Z' \left(1 - \frac{\sin \phi_1}{\phi_1} \right) - \left(\frac{2 \sin \phi_1}{\phi_1} - \cos \phi_1 - 1 \right) \right] = w'_c\, t\, r^2\, Z'_2 \text{..} (53)$$

From Equations (33), (51), and (53), the bending moment at the abutment now becomes:

$$M_1 = - w'_c\, t\, r^2 \left[Z' \left(\frac{\sin \phi_1}{\phi_1} - \cos \phi_1 \right) + \left(\frac{2 \sin \phi_1}{\phi_1} - 2 \cos \phi_1 - \phi_1 \sin \phi_1 \right) \right]$$

$$= w'_c\, t\, r^2\, Z_3 \text{ ....} (54)$$

The shear at the abutment is, from Equation (37),

$$S_1 = w'_c\, t\, r\, (\phi_1 \cos \phi_1 - Z' \sin \phi_1) = w'_c\, t\, r\, Z'_4 \text{ ....} (55)$$

Moments, Thrusts, and Shears Due to Uniform Water Load for Arches with Fixed Ends

In the preceding part of this paper, equations were derived for the moments, thrusts, and shears due to the variable water load and the dead load of the arch itself. The formulas derived by the late Professor Cain, in his paper,[5] "The Circular Arch Under Normal Loads," have been used as a basis for curves similar to Figs. 3 to 7, inclusive, which serve as a convenient means of obtaining the stresses due to the uniform water load. As in the foregoing remarks, the formulas are those which include the effect of shear.

According to Professor Cain's Equation (10)[6], $(pr - P_0 = \dfrac{pr\, 2 \phi_1 \sin \phi_1\, k^2}{D_s\, r^2}$; but $pr = p_e r_e$ and $r_e = r + \dfrac{t}{2} = r \left(1 + \dfrac{t}{2r} \right)$; also $k^2 = \dfrac{t^2}{12}$. Therefore,

$$pr - P_0 = p_e r \left(1 + \frac{t}{2r} \right) \frac{2\, \phi_1 \sin \phi_1\, t^2}{12\, D_s\, r^2} = p_e r\, Q_0 \text{ ....} (56)$$

in which, $Q_0 = \left(1 + \dfrac{t}{2r} \right) \dfrac{2\, \phi_1 \sin \phi_1\, t^2}{12\, D_s\, r^2}$.

All the equations for moments and thrusts at the crown and abutments, as given by Professor Cain, may be expressed in terms of the coefficient, Q_0, as follows:

The thrust at the crown is:

$$P_0 = \left[\left(1 + \frac{t}{2r} \right) - Q_0 \right] p_e\, r \text{ ....} (57)$$

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[6] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 237.

in which, the coefficient of $p_e r$ is obtained from specially prepared curves similar to Figs. 3 to 7, inclusive.

The abutment thrust is:

$$P_1 = p_e r \left[\left(1 + \frac{t}{2\,r} \right) - Q_0 \cos \phi_1 \right] \dots\dots\dots\dots\dots (58)$$

The moment at the crown section is:

$$M_0 = -\, p_e r^2 \left(1 - \frac{\sin \phi_1}{\phi_1} \right) Q_0 \dots\dots\dots\dots\dots\dots (59)$$

The moment at the abutment is:

$$M_1 = p_e r^2 \left(\frac{\sin \phi_1}{\phi_1} - \cos \phi_1 \right) Q_0 \dots\dots\dots\dots\dots (60)$$

The shear at the abutment is:

$$S_1 = p_e r \; Q_0 \sin \phi_1 \dots\dots\dots\dots\dots\dots\dots\dots (61)$$

Temperature Stresses

The crown thrust for the arch with fixed ends as given by Professor Cain is $H = \epsilon\, \Delta\, tEI\; \frac{2\,\phi_1 \sin \phi_1}{r^2\, D_s}$, which may also be written:

$$H = \epsilon\, \Delta\, t\, Et\; \frac{2\,\phi_1 \sin \phi_1\, t^2}{12\, D_s\, r^2} \dots\dots\dots\dots\dots\dots\dots (62)$$

in which, $\frac{2\,\phi_1 \sin \phi_1\, t^2}{12\, D_s\, r^2}$ is obtained from specially prepared curves.

The abutment thrust is equal to:

$$H_1 = H \cos \phi_1 \dots\dots\dots\dots\dots\dots\dots\dots (63)$$

and the shear is:

$$S_1 = H \sin \phi_1 \dots\dots\dots\dots\dots\dots\dots\dots (64)$$

For a fall in temperature, the thrusts are negative and the shear is positive, whereas for a rise in temperature the signs are reversed.

The moment at the crown is:

$$M_0 = -\, Hr \left(1 - \frac{\sin \phi_1}{\phi_1} \right) \dots\dots\dots\dots\dots\dots (65)$$

and the moment at the abutment is:

$$M_1 = Hr \left(\frac{\sin\ \phi_1}{\phi_1} - \cos \phi_1 \right) \dots\dots\dots\dots\dots\dots (66)$$

For a fall in temperature the signs of the moments are as shown, but for a rise in temperature, the signs are reversed. For convenience in making numerical computations, values of $\left(1 - \frac{\sin \phi_1}{\phi_1} \right)$ and $\left(\frac{\sin \phi_1}{\phi_1} - \cos \phi_1 \right)$ are read from the specially prepared curves previously mentioned.

MOMENTS, THRUSTS, AND SHEARS DUE TO VARIABLE WATER LOAD FOR ARCHES WITH HINGED ENDS

The following derivation is similar to that for the arch with "fixed ends" up to the application of the partial derivatives of the "Work Equation" (13). As before, there is no translation of the crown section in the direction of P_0 due to P_0 and $\frac{\partial L}{\partial P_0} = 0$.

The general formula for moment (Equation (7)), becomes equal to zero at the abutment when ϕ_1 is substituted for ϕ; thus,

$$M_o = P_o r\,(1 - \cos\phi_1) - w'\,r^2_e\,r\,(1 - \cos\phi_1 - \frac{\phi_1}{2}\sin\phi_1) \ldots\ldots (67)$$

Substituting this value of M_o in the general moment equation (Equation (7)),

$$M = P_o r\,(\cos\phi - \cos\phi_1) + w'r^2_e\,r(\cos\phi_1 + \frac{\phi_1}{2}\sin\phi_1 - \cos\phi - \frac{\phi}{2}\sin\phi) \,.\,.\, (68)$$

Taking partial derivatives, as before, of the equations for moment, thrust, and shear, with respect to P_o: $\frac{\partial M}{\partial P_0} = r\,(\cos\phi - \cos\phi_1)$; $\frac{\partial P}{\partial P_0} = \cos\phi$; $\frac{\partial S}{\partial P_0} = -\sin\phi$; and,

$$\frac{\partial L}{\partial P_0} = r\int_0^{\phi_1} M\,(\cos\phi - \cos\phi_1)\,d\phi + k^2\int_0^{\phi_1} P\cos\phi\,d\phi$$

$$- 2.88k^2\int_0^{\phi_1} S\sin\phi\,d\phi = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots (69)$$

Integrating and solving for P_o:

$$P_o = w'r^2_e\Bigg[2\,\phi_1 - \frac{15\sin 2\,\phi_1}{8} + \frac{7\,\phi_1\cos 2\,\phi_1}{4}$$

$$+ \frac{\phi^2_1\sin 2\,\phi_1}{2} - \frac{k^2}{r^2}\left(2\sin\phi_1 - \phi_1 - \frac{5\sin 2\,\phi_1}{8} + \frac{\phi_1\cos 2\,\phi_1}{4}\right)$$

$$+ 2.88\,\frac{k^2}{r^2}\left(\frac{\phi_1}{2} - \frac{3\sin 2\,\phi_1}{8} + \frac{\phi_1\cos 2\,\phi_1}{4}\right)\Bigg] \div \Bigg[\phi_1\,(2 + \cos 2\,\phi_1) - \frac{3\sin 2\,\phi_1}{2}$$

$$+ \frac{k^2}{r^2}\left(\phi_1 + \frac{\sin 2\phi_1}{2}\right) + 2.88\,\frac{k^2}{r^2}\left(\phi_1 - \frac{\sin 2\,\phi_1}{2}\right)\Bigg] \ldots\ldots\ldots (70)$$

The abutment thrust (P_1), crown moment (M_o), and abutment shear (S_1), may be obtained from Equations (8), (67) and (12), respectively, by introducing the value of P_o obtained from Equation (70) and by replacing ϕ by ϕ_1 in Equations (8) and (12).

MOMENTS, THRUSTS, AND SHEARS, DUE TO WEIGHT OF ARCH FOR ARCHES WITH HINGED ENDS

Similar to the derivation of formulas for variable water load for arches with hinged ends, the general expression for moment is:

$$M = P_o r\,(\cos\phi - \cos\phi_1) + w'_c\,t r^2\,(\phi\sin\phi + \cos\phi - \phi_1\sin\phi_1 - \cos\phi_1)\;..(71)$$

The partial derivatives to be substituted in the work equation, $\left(\frac{\partial L}{\partial P_o} = 0\right)$, are: $\frac{\partial M}{\partial P_o} = r\,(\cos\phi - \cos\phi_1)$; $\frac{\partial P}{\partial P_o} = \cos\phi$; and $\frac{\partial S}{\partial P_o} = -\sin\phi$. The crown thrust is:

$$P_o = w'_c\,tr\,\frac{\frac{9}{4}\sin 2\phi_1 - \phi_1\left(\frac{5}{2}\cos 2\phi_1 + 2 + \phi_1\sin 2\phi_1\right) + 1.88\frac{k^2}{r^2}\left(\frac{\sin 2\phi_1}{4} - \frac{\phi_1\cos 2\phi_1}{2}\right)}{\phi_1(2 + \cos 2\phi_1) - \frac{3}{2}\sin 2\phi_1 + \frac{k^2}{r^2}\left(\phi_1 + \frac{\sin 2\phi_1}{2}\right) + 2.88\frac{k^2}{r^2}\left(\phi_1 - \frac{\sin 2\phi_1}{2}\right)}\;.(72)$$

The abutment thrust and shear and the crown moment are then obtained by the same method as for variable water load.

MOMENTS, THRUSTS, AND SHEARS FOR UNIFORM WATER LOAD AND TEMPERATURE FOR ARCHES WITH HINGED ENDS

Formulas for moments, thrusts, and shears, due to these influences are the same as those derived by Professor Cain,[7] except that the influence of shear is included; thus,

$$(pr - P_o) = \frac{(2\,pr\sin\phi_1)\,\frac{k^2}{r^2}}{B_s}\;\ldots\ldots\ldots\ldots\ldots\ldots(73)$$

and,

$$H = \frac{EI}{r^2 B_s}\,2\,\epsilon\,\Delta t\sin\phi_1\;\ldots\ldots\ldots\ldots\ldots\ldots(74)$$

in which,

$$B_s = \phi_1(2 + \cos 2\phi_1) - \frac{3}{2}\sin 2\phi_1 + \frac{k^2}{r^2}\left(\phi_1 + \frac{\sin 2\phi_1}{2}\right) + 2.88\frac{k^2}{r^2}\left(\phi_1 - \frac{\sin 2\phi_1}{2}\right)$$

CONCLUSIONS

In this paper it has been possible only to present a thorough description of the method of analysis proposed, with selected sections of the curves that have been prepared. These should serve as a basis of discussion. The original paper, with a complete set of the curves, has been placed in the Engineering Societies Library.[8]

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[7] "The Circular Arch Under Normal Loads", by William Cain, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), pp. 242 and 247.

[8] Address The Director, Engineering Societies Library, 33 West 39th Street, New York, N. Y., directly, for photostatic copies.

By reference to Table 1 (Appendix) certain definite conclusions may be drawn as to the importance of the various influences considered. It will be seen that the effect of the non-uniform loads (variable water load and weight of arch) is of increasing importance as the arch span is increased. This is especially true in the upper part of the dam where the bending moments due to the non-uniform loads are several times as large as those due to the uniform water load alone. In recent years the trend of design of multiple-arch dams has been toward an increase in arch span lengths. Span lengths of 40 ft. or more, are not uncommon in recent practice.

It will also be seen that reinforcing steel is necessary in both faces of the arch, as the sign of the summation of bending moments at a given section may be either positive or negative, and the arch should be reinforced for both conditions. In the past, some multiple-arch dams have been designed with reinforcing steel in one face of the arch only.

Maximum compressive stresses in the concrete will occur, in general, for full reservoir and changes in temperature. Maximum tensile stresses in the reinforcing steel may be expected when the reservoir is empty, accompanied also by changes in temperature.

As has been shown by the "Numerical Example" in the Appendix, the labor of determining the stresses in the inclined arches of multiple-arch dams by the formulas herein derived and by the aid of the diagrams proposed, is reduced to a minimum.

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APPENDIX

In order to demonstrate the use of the various curves and the effects of the various influences, the moments, thrusts, and shears for two typical multiple-arch dams of considerably different dimensions are given.

Numerical Example.—The first, which will be called Dam No. 1, has a clear spacing of buttresses of 40 ft. and a constant radius at the intrados of 24.5 ft. The increase in thickness of the arch rings is obtained by increasing the radius of the extrados from 26.5 ft. at the crest to 30.0 ft. at the base, with thicknesses of arch ranging from 2 to 5.5 ft. in a total height of dam of 168 ft. The central angles of the arches vary from 143 to 146 degrees. The calculations are for an arch ring of 1 ft. slant height, and the elevations given refer to the elevation of the inclined arch at its springing line. The generator of the arch barrel makes an angle of 47° with the horizontal. High-water surface is at Elevation 208. The weight of concrete is taken as 150 lb. per cu. ft., and the weight of water at 62.5 lb. per cu. ft. One set of calculations will be shown for Elevation 140 to illustrate the method, and the moments, thrusts, and shears for the entire arch will be tabulated.

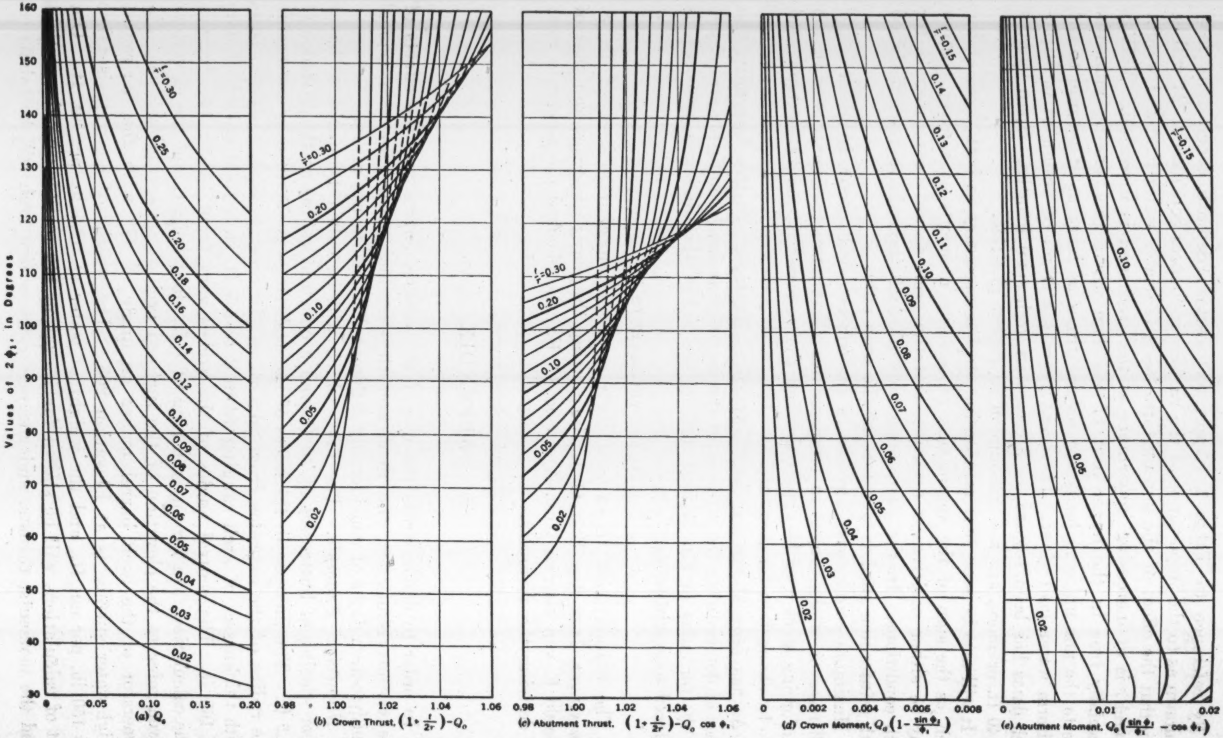

FIG. 9.—CROWN AND ABUTMENT THRUSTS AND MOMENTS IN ARCH DAMS FOR WHICH $\frac{t}{r}$ DOES NOT EXCEED 0.3.

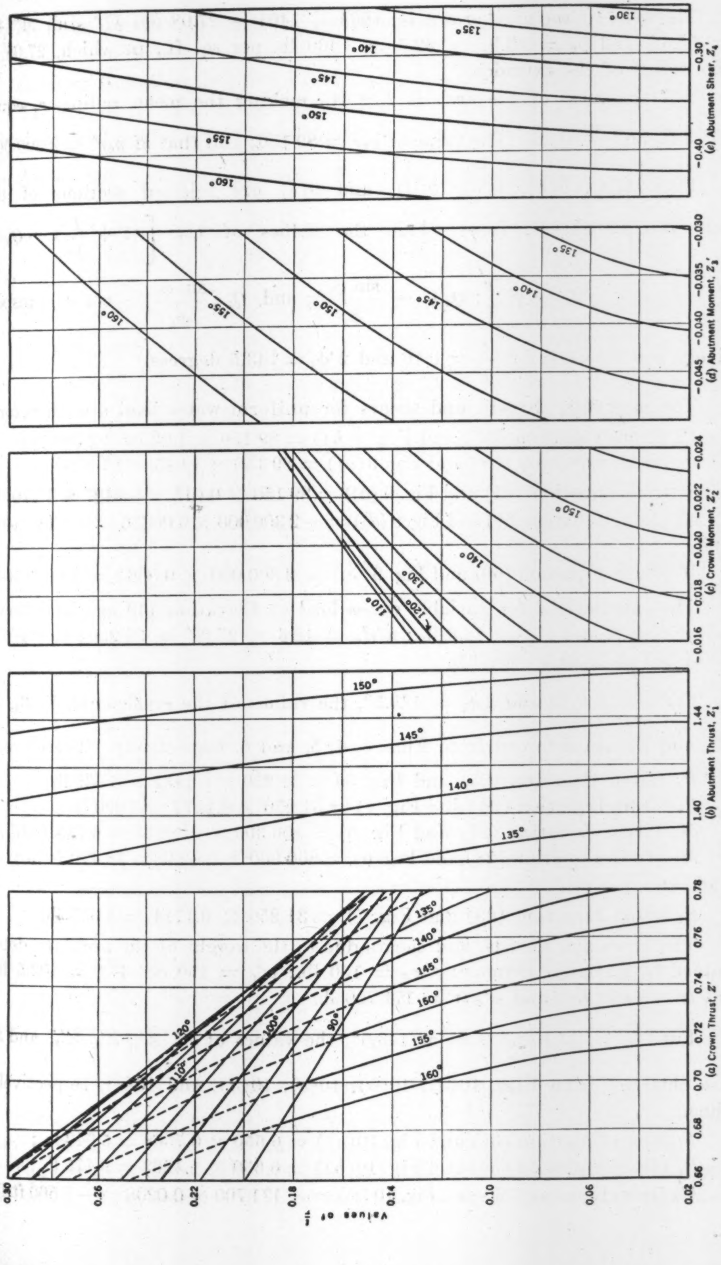

FIG. 10.—CROWN AND ABUTMENT THRUSTS AND MOMENTS DUE TO DEAD LOAD.

The head at the arch crown is: $(208 - 140) - 27.08 \cos 47° \sinh 71° 45'$ $= 55.32$, and $p_e = 55.32 \times 62.5 = 3\,460$ lb. per sq. ft., in which, 27.08 is the radius of the extrados.

The thickness of the arch is 2.58 ft., making the mean radius, r, equal 25.79 ft. and $\frac{t}{r}$, 0.10. The value of $p_e r = 89\,150$; and that of $p_e r^2 = 2\,300\,000$.

Figs. 9(a), 9(b), 9(c), 9(d), and 9(e) are selected sections of the computation curves, from which the values of Q_0; $\left[(1 + \frac{t}{2r}) - Q_0\right]$; $\left[(1 + \frac{t}{2r}) - Q_0 \cos \phi_1\right]$; $Q_0 (1 - \frac{\sin \phi_1}{\phi_1})$; and, $Q_0 (\frac{\sin \phi_1}{\phi_1} - \cos \phi_1)$, respectively, are obtained for $\frac{t}{r} = 0.10$ and $2\,\phi_1 = 143.5$ degrees.

The moments, thrusts, and shears for uniform water load are, therefore:

P_0 (from Equation (57) and Fig. 9(b)) $= 89\,150 \times 1.035 = 92\,300$ lb;

P_1 (from Equation (58) and Fig. 9(c)) $= 89\,150 \times 1.045 = 93\,200$ lb.;

S_1 (from Equation (61) and Fig. 9 (a)) $= 89\,150 \times 0.015 \times 0.9497 = 1\,270$ lb.;

M_0 (from Equation (59) and Fig. 9 (d)) $= -2\,300\,000 \times 0.00356 = -8\,185$ ft-lb.; and,

M_1 (from Equation (60) and Fig. 9 (e)) $= 2\,300\,000 \times 0.0065 = 14\,940$ ft-lb.

The calculations for variable water load at Elevation 140 are, as follows: $w' = 62.5 \times \cos 47° = 42.6$ lb.; $w' r_e^2 = 42.6 \times 27.08^2 = 31\,250$; and $w' r_e^2 r$ $= 31\,250 \times 25.79 = 806\,300$.

For $\frac{t}{r} = 0.10$ and $2\,\phi_1 = 143.5°$, the values of the coefficients, Z, Z_1, Z_2, Z_3, and Z_4, are obtained from Figs. 3, 4, 5, and 6, respectively. Therefore,

P_0 (from Equation (25) and Fig. 3) $= 31\,250 \times 0.1132 = 3\,535$ lb;

P_1 (from Equation (26) and Fig. 4) $= 31\,250 \times 0.1277 = 3\,990$ lb.;

M_0 (from Equation (27) and Fig. 5) $= 806\,300 \times 0.00833 = 6\,715$ ft-lb.;

M_1 (from Equation (28) and Fig. 6) $= 806\,300 \times 0.0229 = 18\,470$ ft-lb.; and,

S_1 (from Equation (29) and Fig. 7) $= 31\,250 \times 0.1714 = 5\,355$ lb.

The moments, thrusts, and shears due to the weight of the arch are determined in a similar manner: $w_c = 150$ lb.; $w'_c = 150 \cos 47° = 102.5$ lb.; $w'_c\, t r = 6\,650$ lb.; and $w'_c\, t r^2 = 171\,700$ lb.

For $\frac{t}{r} = 0.10$ and $2\,\phi_1 = 143.5°$, the values of Z', Z'_1, Z'_2, Z'_3, and Z'_4 are obtained from Figs. 10(a), 10(b), 10(c), 10(d), and 10(e), respectively; thus,

P_0 (from Equation (51) and Fig. 10(a)) $= 6\,650 \times 0.7568 = 5\,040$ lb.;

P_1 (from Equation (52) and Fig. 10(b)) $= 6\,650 \times 1.4263 = 9\,475$ lb.;

M_0 (from Equation (53) and Fig. 10 (c)) $= -171\,700 \times 0.0208 = -3\,560$ ft-lb;

M_1 (from Equation (54) and Fig. 10 (*d*)) $= -171\,700 \times 0.385 = -6\,605$ ft-lb.; and,

S_1 (from Equation (55) and Fig. 10(*e*)) $= -6\,650 \times 0.3283 = -2\,183$ lb.

The moments, thrusts, and shears due to a 20° drop in temperature are obtained, as follows: $\varepsilon = 0.0000055$; $\Delta\, t = -20°$; and $E = 4\,000\,000 \times 144 = 576\,000\,000$ lb. per sq. ft. For $\dfrac{t}{r} = 0.10$ and $2\,\phi_1 = 143.5°$, the values of $\dfrac{2\,\phi_1 \sin \phi_1\, t^2}{12\, D_s\, r^2}$, $\left(\dfrac{1 - \sin \phi_1}{\phi_1}\right)$, and $\left(\dfrac{\sin \phi_1}{\phi_1} - \cos \phi_1\right)$ are obtained from Fig. 11. Then,

$H_o = -0.0000055 \times 20 \times 576\,000\,000 \times 2.58 \times 0.0148 = -2\,420$ lb.;

$H_1 = H_o \cos \phi_1 = -757$ lb.;

$M_o = -H_o r \left(1 - \dfrac{\sin \phi_1}{\phi_1}\right) = -2\,420 \times 25.79 \times 0.242 = -15\,120$ ft-lb.;

$M_1 = H_o r \left(\dfrac{\sin \phi_1}{\phi_1} - \cos \phi_1\right) = 2\,420 \times 25.79 \times 0.446 = 27\,860$ ft-lb.; and,

$S_1 = H_o \sin \phi_1 = 2\,300$ lb.

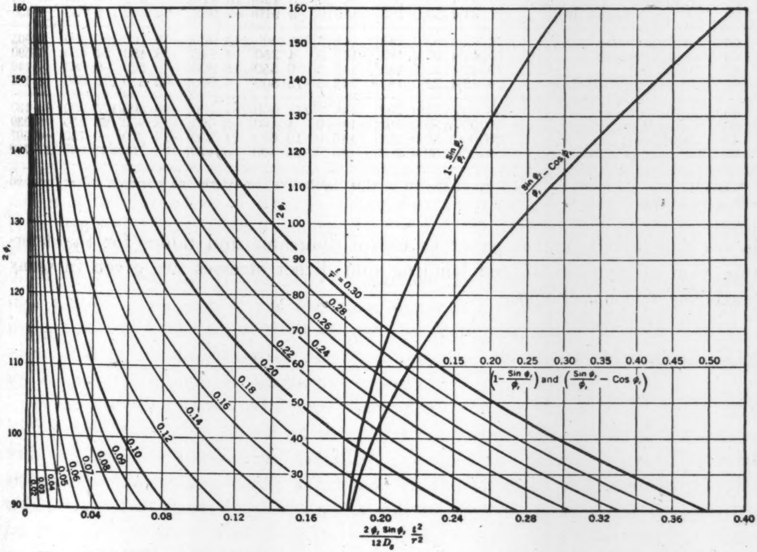

FIG. 11.—CURVES FOR CALCULATION OF TEMPERATURE STRESSES IN ARCH DAMS

For a 20° rise in temperature, the moments, thrusts, and shears are the same as those given for a 20° fall in temperature, but with the signs reversed.

The summary of moments, thrusts, and shears for the entire dam are given in Table 1. The determination of the unit stresses in the concrete and steel

TABLE 1.—SUMMARY OF MOMENTS, THRUSTS, AND SHEARS, DAM No. 1*

Elevation	Loading	r_e, in feet	t, in feet	$\frac{t}{r}$	$2\phi_1$, in degrees	P_O, in pounds	P_1, in pounds	M_O, in foot-pounds	M_1, in foot-pounds	S, in pounds
180	Uniform water...........	26.50	2.00	0.079	143.0	25 730	25 900	-1 402	2 613	228
180	Variable water..........	26.50	2.00	0.079	143.0	3 365	3 790	6 310	17 160	5 070
180	Dead load...............	26.50	2.00	0.079	143.0	4 000	7 465	-2 545	-5 400	-1 732
180	20° temperature drop.....	26.50	2.00	0.079	143.0	-1 165	-370	-7 120	13 170	1 100
160	Uniform water...........	26.50	2.00	0.079	143.0	58 600	59 000	-3 192	5 940	511
160	Variable water..........	26.50	2.00	0.079	143.0	3 365	3 790	6 310	17 160	5 070
160	Dead load...............	26.50	2.00	0.079	143.0	4 000	7 465	-2 545	-5 400	-1 732
160	20° temperature drop.....	26.50	2.00	0.079	143.0	-1 165	-370	-7 120	13 170	1 100
140	Uniform water...........	27.08	2.58	0.100	143.5	92 300	93 200	-8 185	14 940	1 270
140	Variable water..........	27.08	2.58	0.100	143.5	3 535	3 990	6 715	18 470	5 355
140	Dead load...............	27.08	2.58	0.100	143.5	5 040	9 475	-3 560	-6 605	-2 183
140	20° temperature drop.....	27.08	2.58	0.100	143.5	-2 420	-757	-15 120	27 860	2 300
120	Uniform water...........	27.67	3.17	0.122	144.0	127 100	129 200	-16 790	31 300	2 442
120	Variable water..........	27.67	3.17	0.122	144.0	3 720	4 200	7 150	19 900	5 660
120	Dead load...............	27.67	3.17	0.122	144.0	6 183	11 800	-4 900	-7 840	-2 690
180	20° temperature drop.....	27.67	3.17	0.122	144.0	-4 260	-1 315	-27 040	48 900	4 040
100	Uniform water...........	28.25	3.75	0.142	144.5	162 300	165 500	-29 200	53 400	4 150
100	Variable water..........	28.25	3.75	0.142	144.5	3 860	4 380	7 540	21 200	5 920
100	Dead load...............	28.25	3.75	0.142	144.5	7 320	14 130	-6 450	-8 850	-3 182
100	20° temperature drop.....	28.25	3.75	0.142	144.5	-6 765	-2 060	-43 700	80 400	6 435
80	Uniform water...........	28.83	4.33	0.163	145.0	199 000	204 000	-46 070	84 250	6 725
80	Variable water..........	28.83	4.33	0.163	145.0	4 065	4 670	7 990	22 750	6 260
80	Dead load...............	28.83	4.33	0.163	145.0	8 435	16 470	-8 325	-9 335	-3 660
80	20° temperature drop.....	28.83	4.33	0.163	145.0	-9 910	-2 980	-65 200	119 700	9 450
60	Uniform water...........	29.42	4.92	0.183	145.5	236 000	243 000	-68 100	125 300	9 905
60	Variable water..........	29.42	4.92	0.183	145.5	4 250	4 840	8 480	24 450	6 590
60	Dead load...............	29.42	4.92	0.183	145.5	9 550	18 900	-10 510	-9 683	-4 140
60	20° temperature drop.....	29.42	4.92	0.183	145.5	-13 800	-4 090	-93 350	170 000	13 180
40	Uniform water...........	30.00	5.50	0.202	146.0	274 000	284 000	-96 300	176 900	13 130
40	Variable water..........	30.00	5.50	0.202	146.0	4 440	5 065	8 950	26 150	6 930
40	Dead load...............	30.00	5.50	0.202	146.0	10 630	21 350	-12 970	-9 730	-4 607
40	20° temperature drop.....	30.00	5.50	0.202	146.0	-18 500	-5 410	-125 700	231 000	17 700

* The signs of all moments and thrusts shown in this table are considered as positive unless marked with a minus sign.

is not considered in this paper as useful diagrams and tables for the determination of these combined bending and direct stresses are given in many textbooks and handbooks.

DISCUSSION

VICTOR H. COCHRANE,[9] M. AM. SOC. C. E. (by letter).—The authors are to be commended for their clear and complete solution of the problem of inclined arches for dams. The writer appreciates the great amount of labor involved in the preparation of this paper, as he has had occasion to develop similar data for use in connection with the design of a large multiple-arch dam having a central angle of 160° and a buttress spacing of 70 ft. In this work he had the assistance of Donald Witten, M. Am. Soc. C. E.

The writer's formulas were derived for precisely the same conditions as those used by the authors, except that nothing was done in regard to the hinged arch, which seems to have little or no application in multiple-arch dams. In the development of formulas for the inclined arch the chief difficulty lies in the derivation of expressions for the horizontal thrust at the crown. All other formulas required are comparatively simple. The shears are always small, and, ordinarily, they do not need to be computed. The writer's formulas for the horizontal thrust were derived by a method that differs entirely from that used by the authors, and the two sets of formulas appear at first glance to be very different. However, a comparison of charts by both methods seems to show that they give the same results. For the uniform water load it is found that the expression derived by the late Professor Cain can be reduced to that derived by the writer.

At first, it was intended to prepare a set of curves similar to those presented by the authors, but this plan was not fully carried out. It was found that the data are better suited to tables than to diagrams, for the reason that nearly all the terms in the various formulas are independent of the mean radius or the thickness of the rib, being functions of the central angle alone. If the central angle is kept constant, or nearly so, the values of these terms are good for any section of the arch. Those terms involving the radius or thickness are few, as will be shown. Tables can readily cover a wider range than curves. The former are the more accurate. They are more easily prepared and are more convenient to use. A compact table such as Table 2, when used in connection with the formulas of Table 3, is substantially the equivalent of the authors' numerous curves.

If no curves are at hand and the central angle has an odd value not included in the table, it is quite easy to compute the required constants with the aid of a calculating machine, provided the formulas are simplified and the work is systematically arranged. In the example given herein the calculations are set forth in full, and it will be seen that the work involved is far from formidable. The computation of the required values takes but little more time than is needed for reading the curves.

Four load conditions and corresponding constants are expressed as follows:

Load 1.—Arch Weight:

$$K_1 = w_0\, t\, r \cos \alpha \qquad (75a)$$

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Load 2.—Uniform Water Load:

$$K_2 = w\,h\,r_e \qquad (75b)$$

Load 3.—Variable Water Load:

$$K_3 = w\,r_e^2 \cos\alpha \qquad (75c)$$

Load 4.—Temperature Change:

$$K_4 = \varepsilon\,\Delta_t\,E \qquad (75d)$$

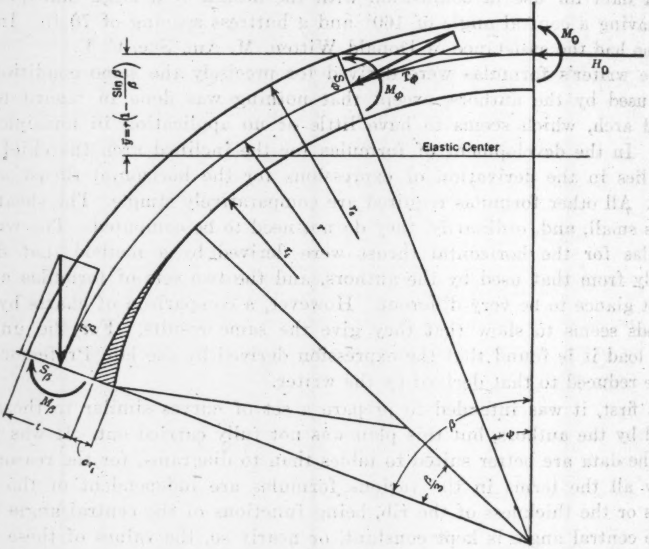

FIG. 12.

In Equation (75*d*), Δ_t is positive for a rise in temperature. Equations (75) are the same as the load constants given by the authors, except that r_e (Fig. 12) is used in Equation (75*b*) as well as in Equation (75*c*).

TABLE 2.—VALUES OF FUNCTIONS OF β

β	A	B	C	D	F	J	Q	S
60.......	3.24907	0.04782	0.89922	0.18298	0.04025	0.00378	0.15399	0.00951
65.......	3.68164	0.06941	1.04551	0.26419	0.05650	0.00645	0.17516	0.01298
70.......	4.13609	0.09760	1.18186	0.36929	0.07650	0.01055	0.19628	0.01729
75.......	4.60891	0.13346	1.30061	0.50180	0.10027	0.01660	0.21701	0.02254
80.......	5.09600	0.17807	1.39408	0.66498	0.12758	0.02524	0.23699	0.02885

β	$1-\frac{\sin\beta}{\beta}$	$\beta\sin\beta+\cos\beta-1$	$\sin^2\frac{\beta}{2}$	$\sin\frac{\beta}{2}\cos\frac{\beta}{2}$	$1-\frac{\beta\sin\beta}{2}-\cos\beta$	$\sin\beta-\beta\cos\beta$	$\frac{\sin\beta}{\beta}-\cos\beta$
60................	0.17301	0.40690	0.25000	0.43301	0.04655	0.34242	0.32699
65................	0.20111	0.45079	0.28869	0.45315	0.06330	0.42687	0.37627
70................	0.23085	0.49007	0.32899	0.46985	0.08396	0.52183	0.42713
75................	0.26209	0.52321	0.37059	0.48296	0.10899	0.62714	0.47909
80................	0.29468	0.54870	0.41318	0.49240	0.13883	0.74235	0.53167

The necessary formulas for thrusts, moments, and shears are arranged for convenience in Table 2, for the four load conditions. The constants for substitution in Table 2 are:

$$A = \beta\ (m + 1) - \frac{m - 1}{2} \sin 2\,\beta \ldots\ldots\ldots\ldots (76a)$$

$$B = \beta + \frac{\sin 2\,\beta}{2} - \frac{2 \sin^2 \beta}{\beta} \ldots\ldots\ldots\ldots\ldots\ldots (76b)$$

$$C = \frac{m - 1}{4} (\sin 2\,\beta - 2\,\beta \cos 2\,\beta) \ldots\ldots\ldots\ldots (76c)$$

$$D = \frac{m + 2}{2}\,\beta - \frac{3\,m - 5}{8} \sin 2\,\beta + \frac{m - 1}{4}\,\beta \cos 2\,\beta - 2 \sin \beta \ldots (76d)$$

$$F = \frac{4 \sin^2 \beta}{\beta} - \beta \sin^2 \beta - \frac{7}{4} \sin 2\,\beta - \frac{\beta}{2} \ldots\ldots\ldots\ldots (76e)$$

$$J = \frac{3}{4}\,\beta + \frac{9}{8} \sin 2\,\beta + \frac{\beta \sin^2 \beta}{2} - \frac{3 \sin^2 \beta}{\beta} \ldots\ldots\ldots (76f)$$

$$Q = \frac{2 \sin \beta}{\beta} - \cos \beta - 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (76g)$$

and,

$$S = 1 + \frac{\cos \beta}{2} - \frac{3}{2}\,\frac{\sin \beta}{\beta} \ldots\ldots\ldots\ldots\ldots\ldots (76h)$$

TABLE 3.—FORMULAS FOR THRUSTS, MOMENTS, AND SHEARS

Load	$H_0 =$	$M_0 =$	T_ϕ and S_ϕ (for springing section use β for ϕ)	$M_\phi =$
1	$K_1\,\frac{C + uF}{A + uB}$	$K_1 r\,Q - H_0 \bar{y}$	$T_\phi = K_1 \phi \sin \phi + H_0 \cos \phi$ $S_\phi = K_1 \phi \cos \phi - H_0 \sin \phi$	$M_0 + H_0 r\ (1 - \cos \phi)$ $- K_1 r\ (\phi \sin \phi + \cos \phi - 1)$
2	$K_2\,\frac{A - 2 \sin \beta + uB}{A + uB}$	$(K_2 - H_0)\,\bar{y}$	$T_\phi = 2\,K_2 \sin^2 \frac{\phi}{2} + H_0 \cos \phi$ $S_\phi = 2\,K_2 \sin \frac{\phi}{2} \cos \frac{\phi}{2} - H_0 \sin \phi$	$M_0 + H_0 r\ (1 - \cos \phi)$ $- 2\,K_2 r \sin^2 \frac{\phi}{2}$
3	$K_3\,\frac{D + uJ}{A + uB}$	$K_3 r S - H_0 \bar{y}$	$T_\phi = K_3 \left(1 - \frac{\phi \sin \phi}{2} - \cos \phi\right) + H_0 \cos \phi$ $S_\phi = \frac{K_3}{2} (\sin \phi - \phi \cos \phi) - H_0 \sin \phi$	$M_0 + H_0 r\ (1 - \cos \phi)$ $- K_3 r \left(1 - \frac{\phi \sin \phi}{2} - \cos \phi\right)$
4	$K_4 t\,\frac{2 \sin \beta}{A + uB}$	$- H_0 \bar{y}$	$T_\phi = H_0 \cos \phi$ $S_\phi = - H_0 \sin \phi$	$H_0 r \left(\frac{\sin \beta}{\beta} - \cos \phi\right)$

The factor, *m*, depends upon the ratio, $\frac{E}{G}$, and the shear distribution; the value, 2.88, may be used. If $m = 2.88$, the terms, *A*, *C*, and *D*, in Equations (76) reduce to the following:

$$A = 3.88\,\beta - 0.94 \sin 2\,\beta \ldots\ldots\ldots\ldots\ldots\ldots (77a)$$

$$C = 0.47\ (\sin 2\,\beta - 2\,\beta \cos 2\,\beta) \ldots\ldots\ldots\ldots\ldots (77b)$$

and,

$$D = 2.44\,\beta - 0.455 \sin 2\,\beta + 0.47\,\beta \cos 2\,\beta - 2 \sin \beta \ldots\ldots (77c)$$

In practice, the writer arranges Fig. 12, Equations (75) and (76), and Table 2 on a single sheet for ready reference. Having computed the constants, A, B, C, D, F, J, Q, and S for the central angle chosen, the formulas for H_0 and M_0 become very simple. It will be noted that all the expressions for H_0 in Table 3 have the same denominator, $A + uB$, in which, $u = 12\left(\frac{r}{t}\right)^2$. For example, if $\beta = 75°$; $\frac{t}{r} = 0.30$; and $u = 133.33$; from Table 2:

Dead load,

$$H_0 = \frac{14.67}{22.41} K_1 = 0.655\, K_1 \qquad \text{(78a)}$$

Uniform water load,

$$H_0 = \frac{22.41 - 1.93}{22.41} K_2 = 0.914\, K_2 \qquad \text{(78b)}$$

Variable water load,

$$H_0 = \frac{2.715}{22.41} K_3 = 0.1212\, K_3 \qquad \text{(78c)}$$

Fall in temperature,

$$H_0 = -K_4\, t\, \frac{1.93}{22.41} = -0.086\, K_4\, t \qquad \text{(78d)}$$

These values all agree with those shown in the authors' diagrams. Some of the coefficients for T_β, M_β, and S_β, are given in Table 2, which illustrates a part of a list of values of functions for half-central angles at 5° intervals. The table used in practice extends from $\beta = 35°$ to $\beta = 90°$, and other values may be found by interpolation if for any reason the central angle chosen is not included. It will be noted in Table 3 that, with the exception of the "load constants," the only terms that vary with the mean radius or thickness of the rib are $\overline{y}$ and u. In the expression for horizontal thrust due to the uniform water load, the terms, A, in the denominator, and, $A - 2 \sin \beta$, in the numerator, represent the effect of shear and arch-shortening. If these terms are dropped, $H_0 = K_2$, which is the so-called cylinder formula. If shear and direct stress are taken into account, the coefficient of K_2 must be less than unity. The expression, $H_0 - K_2$, is the force applied at the elastic center in combination with the cylinder thrust, K_2. In the expressions for H_0 for variable water load and dead load, the first terms of the numerator and denominator represent the effect of shear and direct stress, and the last terms show the influence of bending moments. When numerical substitutions are made, the relative influence of shear and direct stress, as compared with bending moments, is readily seen. The effect of the curvature of the arch was investigated and found to be negligible for values of $\frac{t}{r}$ up to 0.30, or greater.

In order to illustrate the use of these formulas, calculations are given for the authors' example, in which $\beta = 71°\ 45' = 1.25227$ radians. The trigonometric values required are, as follows: $\sin\beta = 0.94970$; $\cos\beta = 0.31316$; $\sin 2\beta = 0.59482$; $\cos 2\beta = -0.80386$; $\sin\frac{\beta}{2} = 0.58602$; $\cos\frac{\beta}{2} = 0.81030$; $\sin^2\beta = 0.90193$; $\sin^2\frac{\beta}{2} = 0.34342$; $\beta\sin\beta = 1.18928$; $\beta\cos\beta = 0.39216$; $\beta\sin^2\beta = 1.12946$; $\frac{\sin\beta}{\beta} = 0.75838$; $\frac{\sin^2\beta}{\beta} = 0.72023$; $\beta\cos 2\beta = -1.00665$; $\sin\beta\cos\beta = 0.29741$; $\sin\frac{\beta}{2}\cos\frac{\beta}{2} = 0.47485$; $\beta\sin\beta + \cos\beta - 1 = 0.50244$; $1 - \frac{\beta\sin\beta}{2} - \cos\beta = 0.09220$; $\sin\beta - \beta\cos\beta = 0.55754$; and, $\frac{\sin\beta}{\beta} - \cos\beta = 0.44522$.

The functions of β are (from Equations (76)):

$A = 3.88 \times 1.2523 - 0.94 \times 0.59482 = 4.300$;
$B = 1.2523 + 0.29741 - 1.44046 = 0.1093$;
$C = 0.47\,(0.59482 + 2.01330) = 1.2258$;
$D = 2.44 \times 1.2523 - 0.455 \times 0.59482 - 0.47 \times 1.00665 - 1.89940 = 0.41244$;
$F = 2.88092 - 1.12946 - 1.75 \times 0.59482 - 0.62613 = 0.08439$;
$J = 0.93920 + 1.125 \times 0.59482 + 0.56473 - 2.16069 = 0.01241$;
$Q = 1.51676 - 1.31316 = 0.20360$; and,
$S = 1.15658 - 1.13757 = 0.01901$.

For $r = 25.79$; $t = 2.58$; h (depth over crown) $= 55.32$; $u = 12 \times 10^2 = 1\,200$; and, $\bar{y} = 0.24162\, r = 6.231$; $K_1 = 150 \times 2.58 \times 25.79 \times 0.6820 = 6\,807$; $K_2 = 62.5 \times 55.32 \times 27.08 = 93\,630$; $K_3 = 42.6 \times 27.08^2 = 31\,240$; and, $K_4 = -576\,000\,000 \times 20 \times 0.0000055 = -63\,360$. Then: $K_1 r = 175\,550$; $K_2 r = 2\,414\,700$; and, $K_3 r = 805\,700$.

The terms required for computing the four values of H_o are: $A + uB = 4.30 + 1\,200 \times 0.1093 = 135.46$; $C + uF = 1.226 + 1\,200 \times 0.08439 = 102.50$; and, $D + uJ = 0.41 + 1\,200 \times 0.01241 = 15.30$.

The values of thrusts, moments, and shears (with the authors' corresponding values in parentheses) are:

Load 1.—Weight of Arch:

$H_0 = 0.757 \times 6\,807 = 5\,150$ (5 040); and $H_0 r = 132\,800$
$M_0 = 175\,550 \times 0.2036 - 5\,150 \times 6.231 = 3\,650$ ($-$ 3 560)
$T_\beta = 6\,807 \times 1.1893 + 5\,150 \times 0.3132 = 9\,710$ (9 475)
$S_\beta = 6\,807 \times 0.3922 - 5\,150 \times 0.9497 = -2\,220$ ($-$ 2 183)
$M_\beta = 3\,650 + 132\,800 \times 0.6868 - 175\,550 \times 0.5024 = 6\,660$ ($-$ 6 605)

Load 2.—Uniform Water Load:

$H_0 = 0.986 \times 93\,630 = 92\,320$ (92 300); and $H_0 r = 2\,380\,900$
$M_0 = (93\,630 - 92\,320)\,6.231 = 8\,160$ ($-$ 8 185)
$T_\beta = 93\,630 \times 0.6868 + 92\,320 \times 0.3132 = 93\,220$ (93 200)
$S_\beta = 93\,630 \times 0.9496 - 92\,320 \times 0.9497 = 1\,240$ (1 270)
$M_\beta = 8\,160 + 2\,380\,900 \times 0.6868 - 2\,414\,700 \times 0.6868$
$= -15\,050$ (+ 14 940)

Load 3.—**Variable Water Load:**

$$
\begin{aligned}
H_0 &= 0.1129 \times 31\,240 = 3\,530\ (3\,535);\ \text{and}\ H_0 r = 91\,040 \\
M_0 &= 805\,700 \times 0.01901 - 3\,530 \times 6.231 = -\,6\,680\ (+\,6\,715) \\
T_\beta &= 31\,240 \times 0.0922 + 3\,530 \times 0.3132 = 3\,990\ (3\,990) \\
S_\beta &= 15\,620 \times 0.5575 - 3\,530 \times 0.9497 = 5\,360\ (5\,355) \\
M_\beta &= -\,6\,680 + 91\,040 \times 0.6868 - 805\,700 \times 0.0922 \\
&= -\,18\,440\ (+\,18\,470)
\end{aligned}
$$

Load 4.—**Drop in Temperature:**

$$
\begin{aligned}
K_4\,t &= -\,63\,360 \times 2.58 = -163\,470 \\
H_0 &= 0.01401 \times -163\,470 = -\,2\,290\ (-\,2\,420) \\
M_0 &= +\,2\,290 \times 6.231 = +\,14\,270\ (-15\,120) \\
T_\beta &= -\,2\,290 \times 0.3132 = -\,717\ (-\,757) \\
S_\beta &= +\,2\,290 \times 0.9497 = +\,2\,175\ (+\,2\,300) \\
M_\beta &= -\,59\,060 \times 0.4452 = -\,26\,290\ \ (+\,27\,860)
\end{aligned}
$$

The authors call attention to the large bending moments for sections near the water surface. Unfavorable conditions may occur throughout the range of the draw-down, although elements some distance below the top of the dam derive support from adjacent elements above the water surface. At low water the arch in the vicinity of the water surface may actually be more severely stressed than it would be with the reservoir full.

Thickening of Arch at Haunches.—In the authors' example the temperature drop is 20 degrees. No mention is made of shrinkage, which acts like a fall in temperature. While the effect of shrinkage may be reduced by plastic deformation, it may be nevertheless much greater than that of the temperature change. Other factors tending to set up bending stresses in the arch are the water-soaking of the up-stream face and a difference in temperature between the two faces. All these conditions may be provided for by making a liberal allowance for temperature drop. In some recent bridge designs the equivalent temperature drop has been as much as 120 degrees. In order to determine the proper amount and location of the reinforcement, it is necessary to make an adequate allowance for temperature changes and shrinkage, with perhaps higher working stresses than are customary.

Owing to the large bending moments set up by changes in the length of the arch, the stresses will always be greatest at the crown and springing sections, and least at the plane of the elastic center, which is at the depth, $\bar{y}$, below the crown. The stress at the abutment section is much greater than that at the crown. For instance, consider the authors' example, and the same case with the temperature drop increased to 80° to include the effect of shrinkage. For the purpose of making a rough comparison of the stresses at the crown and abutment sections, it is assumed in Table 4 that the arch is capable of resisting tensile stresses. For reinforced sections, making the usual assumption that the concrete carries no tension, the inequality of stresses will be even greater. For the 80° drop the arch would have to be reinforced, and the maximum springing stress would be about double the maximum crown stress.

These considerations lead inevitably to the conclusion that a fixed arch barrel of uniform thickness from crown to springing is inefficient, and should not be used. For a given working stress the crown section may be made thinner than the abutment section. In other words, the arch should be thicker at the abutment than at the center. The thickening should begin about as far below the elastic center as the crown is above it. The desired result can perhaps best be obtained by making the intrados three-centered, as shown in Fig. 12. The point of compound curve is at an angular distance, $\frac{\beta}{3}$, from the springing. The radius of the end section is chosen so as to make the increase in thickness, $c\,r_t$, any desired amount. In order to simplify the form work, it is desirable to keep the intrados constant throughout the arch barrel. If this is done, $c\,r_t$ must also be constant. This gives a greater relative thickening for the thin arches near the top, which, fortunately, is what is needed. It will be noted that the arch axis shifts at the abutment a distance equal to one-half the thickening. The bending moment with reference to the new axis is changed by the amount, $\frac{c r_t}{2} T_\beta$.

TABLE 4.—COMPARISON OF STRESSES AT CROWN AND ABUTMENT SECTIONS

Load	CROWN SECTION				SPRINGING SECTION			
	Thrust, in pounds	Moment, in foot-pounds	Maximum	Minimum	Thrust, in pounds	Moment, in foot-pounds	Maximum	Minimum
1..............	+ 5 150	+ 3 650	......	......	+ 9 710	+ 6 660	......	....
2..............	+92 320	+ 8 160	......	......	+93 220	—15 050	......	....
3..............	+ 3 530	— 6 680	......	......	+ 3 990	—18 440	......	....
4..............	— 2 290	+14 270	......	......	— 720	—26 290	......	....
Total, 20°....	+98 710	+19 400	— 387*	— 144*	+106 200	—53 120	— 618*	+ 47†
Total, 80°....	+91 840	+62 210	— 635*	+ 143†	+104 040	—131 990	—1 106*	+ 546†

* Minus = compression. † Plus = tension.

In the case previously considered, assume that the springing section is thickened so that $c = 0.03$, and that the temperature drop is 80 degrees. Then, $c\,r_t = 0.03 \times 24.5 = 0.73$ ft.; and, $t = 2.58 + 0.73 = 3.31$ ft. Inasmuch as the arch is stiffer, the moments will be increased, but part of the increase will be offset by the shift of the axis. If the thrusts and moments for this case are assumed to be the same as for the unthickened arch, the approximate stresses for the thickened arch are found to be as follows: The direct stress is $+\frac{104\,040}{3.31} = +31\,430$; the bending stress is, $\pm 131\,990 \times \frac{6}{3.31^2} = \pm 72\,280$; the combined stress, in pounds per square inch, is — 720 or + 284; and, as computed for the unthickened arch, it is — 1 106 or + 546. The comparatively small thickening reduces springing stresses materially, and the reinforcement will no doubt be lessened.

The writer has investigated the effect of the proposed thickening on the thrusts and moments as computed for arches of uniform thickness. The method was to calculate the changes in crown deflections due to the increase in thickness, for various values of $\frac{t}{r}$, β, and c, using summations at 5° intervals. These values were plotted into curves for reference in design.

If the abutment section is thickened so as to make the crown and springing stresses approximately equal, the effect of the change on the stresses is not great, and may well be neglected. For example, if the arch previously considered is thickened in this manner, the maximum crown and springing stresses are, respectively, about 5% and 10% greater than those previously computed.

PAUL BAUMAN,[10] M. AM. SOC. C. E. (by letter).—In the "Introduction," the authors state that the analytical method developed by Dr. N. Kelen[8] "provides curves from which the crown thrust may be obtained directly, but does not include curves or coefficients from which the abutment thrust, abutment shear, and bending moments may be directly obtained." They state also that Dr. Kelen's formulas omit the effect of the internal elastic work due to shear. These statements are misleading because Dr. Kelen actually shows curves for the determination of the governing stresses; that is, at the abutment, both due to water load and dead load. No such "short-cut" is offered by the authors.

The introduction of the influence of shear is a refinement, but is more of academic than of practical value as far as circular, inclined arches of constant thickness are concerned. The allowable span of such arches is small as compared to the allowable span of pressure-line arches; that is, arches in which the center line coincides with the line of support in the statically determinate basic system.

To form an idea of the influence of shear, apply Dr. Kelen's formulas and diagrams to the authors' "Numerical Example" in the Appendix.

As $v = \frac{t}{l}$, the extreme fiber stresses at the abutment due to water load are:

$$f_w = \frac{P_1}{t} \pm \frac{6\,M_1}{t^2} = \frac{P_1}{v\,l} \pm \frac{6\,M_1}{v^2\,l^2}$$

or abbreviated,

$$f_w = [fw']\,w'l \quad \text{.......................} (79)$$

in which, l equals one-half the span.

Thus, for the stress at the extrados,

$$fw_e = [fw'_e]\,w'l \quad \text{......................} (80a)$$

and for the stress at the intrados,

$$fw_i = [fw'_i]\,w'l \quad \text{......................} (80b)$$

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[10] Chf. Designer, Quinton, Code & Hill-Leeds & Barnard, Engrs., Consolidated, Los Angeles, Calif.

From Dr. Kelen's diagrams, which hold for the metric system only, $fw'_e = -14.30$; and $fw'_i = +17.20$; and from the authors' numerical example: $w' = 42.6$ lb. per cu. ft. $= 0.632$ metric ton per cu. m.; and, $l = r \sin 71 - 45 = 25.79 \times 0.9497 = 24.50$ ft. $= 7.47$ m.

Consequently,

$$fw_e = -14.30 \times 0.682 \times 7.47 = -72.85 \text{ metric tons per sq. m.}$$
$$= -7.53 \text{ tons per sq. ft.}$$

and,

$$fw_i = +17.20 \times 0.632 \times 7.47 = +87.60 \text{ metric tons per sq. m.}$$
$$= +9.05 \text{ tons per sq. ft.}$$

Referring to Table 1 for Elevation 140 and solving Equations (80):

$$fw_e = \frac{P_1}{t} - \frac{6\,M_1}{t^2} = \frac{3\,990}{2.58} - \frac{6 \times 18\,470}{6.66} = -15\,083 \text{ lb. per sq. ft.}$$
$$= -7.542 \text{ tons per sq. ft.}$$

and,

$$fw_i = \frac{P_1}{t} + \frac{6\,M_1}{t^2} = +18\,177 \text{ lb. per sq. ft.} = +9.089 \text{ tons per sq. ft.}$$

Similarly, for dead load and Dr. Kelen's formulas:

$$fc_e = [fc'_e]\, w'_c\, l \qquad \text{(81}a\text{)}$$

and,

$$fc_i = [fc'_i]\, w'_c\, l \qquad \text{(81}b\text{)}$$

in which, $fc'_e = +3.85$; and $fc'_i = -0.90$.

From the authors' example, $w'_c = 102.50$ lb. per cu. ft. $= 1.637$ metric tons per cu. ft. Introducing these values in Equations (81),

$$fc_e = +3.85 \times 1.637 \times 7.47 = 47.08 \text{ metric tons per sq. ft.}$$
$$= +4.86 \text{ tons per sq. ft.}$$

and,

$$fc_i = -0.90 \times 1.637 \times 7.47 = -11.00 \text{ metric tons per sq. ft.}$$
$$= -1.135 \text{ tons per sq. ft.}$$

Based on Table 1 for Elevation 140:

$$fc_e = \frac{9\,475}{2.58} + \frac{6 \times 6\,605}{6.66} = +9\,618 \text{ lb. per sq. ft.} = +4.809 \text{ tons per sq. ft.}$$

and,

$$fc_i = \frac{9\,475}{2.58} - \frac{6 \times 6\,605}{6.66} = -2\,272 \text{ lb. per sq. ft.} = -1.136 \text{ tons per sq. ft.}$$

The foregoing shows that for practical purposes the two results are equivalent; that is, the work of deformation of shear in arches of short spans and large central angles, such as with ordinary multiple-arch dams, is small.

From an economic standpoint, the circular arch of constant thickness is not desirable for an inclined arch barrel, because the crown thickness is governed by the abutment thickness and thereby becomes over-dimensioned.

In all homogeneous structures it is desirable to keep the governing stresses uniform. The ideal condition naturally would be to have uniform stresses throughout such a structure, as suggested by Herman Schorer, Assoc. M. Am. Soc. C. E., for buttresses.[11] With arches, however, it is quite a different matter as, according to an old Chinese saying, "it [the arch] never sleeps." Arches, and particularly concrete arches under constant live load, are subject to continuous change in shape due to a change of temperature and relative humidity of the surrounding air.

Uniform strength, therefore, could only be temporarily reached in the case of a structure being designed as a pressure-line arch of uniform strength subject to compensating elastic thrusts due to live load, dead load, and to change in temperature and moisture, respectively. Use of this compensating feature has been made with long-span bridge arches in closing them at such a temperature that least stresses will result for a mean yearly temperature.

The difference in the stress distribution and the volume of an inclined pressure-line arch and a circular one may best be illustrated by an example. For a multiple-arch dam in Southern California, with a buttress spacing of 60 ft., a maximum height of 180 ft., more or less, and an inclination of the arch generator of 45°, the arches were designed as pressure-line arches (three-centered) and the elastic thrust, H_e, due to live load, dead load, and a change in temperature, was determined by the equation:

$$H_e = H_{eN} + H_{eT} = \frac{-\sum_0^{\phi_1} p \cos\phi \frac{\Delta s}{l} - \epsilon \Delta t l E}{\sum_0^{\phi_1} y^2 \Delta g + \sum_0^{\phi_1} \cos^2\phi \frac{\Delta s}{t} + 3 \sum_0^{\phi_1} \sin^2\phi \frac{\Delta s}{t}} \quad ..(82)$$

in which, for a 1-ft. arch slice, with the crown 169 ft. below the water surface:

Δg = elastic weight of element = $\frac{\Delta s}{I}$;
I = moment of inertia;
Δs = length of arch element = 3.355;
y = ordinates of center line of arch from x-axis through the elastic center;
ϵ = coefficient of thermal expansion = 0.000005;
Δt = change in temperature = $-5°$ Fahr.;
l = one-half span = 26.50 ft.;
E = modulus of elasticity = 2.50×10^6 lb. per sq. in.;
ϕ_1 = one-half central angle = 66° 30′.

The solution of Equation (82) is most conveniently arranged in tabular form and the detailed steps are not presented herein. For the stated conditions, H_e is found equal to $-$ 29 920 lb. The corresponding stresses at the crown are: $f_c = +418$ lb. per sq. in., and $f'_c = +167.40$ and -238.60; and the combined stresses are $f''_c = +585.40$ and $+179.40$ lb. per sq. in. Likewise, at the abutment (springing line), $f_s = +393$ lb. per sq. in.; and $f'_s = +321.0$ and -347.0; and the combined stresses are, $f''_s = +46.0$ and $+714.0$ lb. per sq. in.

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[11] "The Buttressed Dam of Uniform Strength," *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 666.

This three-centered arch may be approximated by a circular arch of the following dimensions: $r = 29.40$ ft.; $r^2 = 865$; $t = 5.98$ ft. (based on equal volume); $\frac{t}{r} = 0.2033$; $r_e = 32.39$ ft.; $r_e^2 = 1\,050$; $\phi_1 = 65° \; 00'$; and $\alpha = 45° \; 00'$.

The head at the crown of the arch is 169 ft. and: $p_e = 169 \times 62.50 = 10\,565$ lb. per sq. ft.; $p_e r = 310\,500$; $p_e r^2 = 9\,133\,000$; $Q_0 = 0.076$; $\left(1 + \frac{t}{2r}\right) - Q_0 = 1.018$; $\left(1 + \frac{t}{2r}\right) - Q \cos \phi_1 = 1.065$; $Q_0 \left(1 - \frac{\sin \phi_1}{\phi_1}\right) = 0.0147$; $Q_0 \left(\frac{\sin \phi_1}{\phi_1} - \cos \phi_1\right) = 0.0293$; $w' = 62.50 \times 0.707 = 44.20$ lb. per cu. ft.; $w' r^2_e = 46\,420$; and $w' r_e^2 \, r = 1\,365\,000$. Hence, $Z = 0.09$; $Z_1 = 0.102$; $Z_2 = 0.00515$; and, $Z_3 = 0.0165$.

Furthermore, $w'_o = 150 \times 0.707 = 106$ lb. per cu. ft.; $w'_c \, tr = 18\,650$; and $w'_c \, tr^2 = 548\,200$. Similarly, $Z' = 0.736$; $Z'_1 = 1.34$; $Z'_2 = 0.0274$; and, $Z'_3 = 0.012$.

Stresses at the crown are found to be as follows:

For Uniform Water Load:

$$P_0 = 1.018 \times 310\,500 = +\,316\,300 \text{ lb.};$$

and,

$$M_0 = -\,0.0147 \times 9\,133\,000 = -\,134\,250 \text{ ft-lb.}$$

For Variable Water Load:

$$P_0 = 0.09 \times 46\,420 = +\,4\,175 \text{ lb.}$$

and,

$$M_0 = +\,0.00515 \times 1\,365\,000 = +\,7\,030 \text{ ft-lb.}$$

For Dead Load:

$$P_0 = 0.736 \times 18\,650 = +\,13\,725 \text{ lb.}$$

and,

$$M_0 = -\,0.0274 \times 548\,200 = -\,15\,025 \text{ ft-lb.}$$

For Change in Temperature of − 5° Fahr.:

$$H = -\,0.000005 \times 5 \times 2.50 \times 10^6 \times 144 \times 5.98 \times 0.075 = -\,4\,040 \text{ lb.}$$

and,

$$M_0 = -\,4\,040 \times 29.40 \times 0.193 = -\,22\,930 \text{ ft-lb.}$$

The combined results are:

$$P_0 = +\,316\,300 + 4\,175 + 13\,725 - 4\,040 = +\,330\,160 \text{ lb.}$$

and,

$$M_0 = -\,134\,250 + 7\,030 - 15\,025 - 22\,930 = -\,165\,175 \text{ ft-lb.}$$

and the corresponding stresses are:

$$f''_e = + \frac{330\,160}{5.98 \times 144} \pm \frac{6 \times 165\,175}{5.98^2 \times 144} = \begin{cases} +\,576.0 \text{ lb. per sq. in. (extrados)} \\ +\,191.0 \text{ lb. per sq. in. (intrados)} \end{cases}$$

The stresses at the abutment are found to be, as follows:

For Uniform Water Load:

$$P_1 = +\ 1.065 \times 310\,500 = +\ 330\,700 \text{ lb.}$$

and,

$$M_1 = +\ 0.0293 \times 9\,133\,000 = +\ 267\,700 \text{ ft-lb.}$$

For Variable Water Load:

$$P_1 = +\ 0.102 \times 46\,420 = +\ 4\,737 \text{ lb.}$$

and,

$$M_1 = +\ 0.0165 \times 1\,365\,000 = +\ 22\,530 \text{ ft-lb.}$$

For Dead Load:

$$P_1 = +\ 1.34 \times 18\,650 = +\ 25\,000 \text{ lb.}$$

and,

$$M_1 = -\ 0.012 \times 548\,200 = -\ 6\,575 \text{ ft-lb.}$$

For Temperature Change of − 5° Fahr.:

$$H_1 = -\ 4\,040 \times 0.4226 = -\ 1\,710 \text{ lb.}$$

and,

$$M_1 = +\ 0.385 \times 117\,300 = +\ 45\,200 \text{ ft-lb.}$$

The combined results are:

$$P_1 = +\ 330\,700 \times 4\,737 \times 25\,000 - 1\,710 = +\ 358\,727 \text{ lb.}$$

and,

$$M_1 = +\ 267\,700 + 22\,530 - 6\,575 + 45\,200 = +\ 328\,855 \text{ ft-lb.}$$

and, the corresponding stresses are:

$$f''_s = + \frac{358\,727}{5.98 \times 144} \pm \frac{6 \times 328\,855}{5.98^2 \times 144} = \left\{ \begin{array}{l} +\ 34 \text{ lb. per sq. in. (extrados)} \\ +\ 800 \text{ lb. per sq. in. (intrados)} \end{array} \right.$$

Thus, with the circular arch, the maximum crown stress is 2% smaller than the maximum crown stress of the pressure-line arch of equal volume, but the maximum governing abutment stress of the circular arch is 12% greater than the corresponding stress of the pressure-line arch. This shows that geometrical shapes of structures which lend themselves beautifully to mathematical analysis are not necessarily economical.

For an inclination of the arch generator of between 45° and 50° and a central angle between 130° and 145°, which holds true for ordinary multiple-arch dams, the economic limit of buttress spacing for circular arches is 40 ft., more or less, and about 400 ft. for the pressure-line arch.

Lars R. Jorgensen,[12] M. Am. Soc. C. E. (by letter).—Perhaps the most complete analysis of the stresses in inclined arches of multiple-arch dams, to be found in the English language, is that presented in this paper. Taken together with recent papers on the investigation of stresses in buttresses and their distribution (Holmes,[13] Schorer,[11] Jakobsen,[14] and Noetzli[15]), the stress analysis of the complete multiple-arch dam is now in a satisfactory stage of completion.

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[12] Cons. Hydro-Elec. Engr., Berkeley, Calif.

[13] "Determination of Principal Stresses in Buttresses and Gravity Dams," by W. H. Holmes, Assoc. M. Am. Soc. C. E., see p. 971.

[14] "Stresses in Gravity Dams by Principle of Least Work," by B. F. Jakobsen, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 489.

[15] "Improved Type of Multiple-Arch Dam," by Fred A. Noetzli, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 342.

Not the least valuable contribution of the authors is the large amount of work put into the graphical representation of necessary coefficients. All the final formulas for moments, thrust, and shear would be very lengthy except for the coefficient, Z, which once for all has been calculated by the authors and the results put into the form of curves for all conditions most likely to occur in ordinary design and construction work. The curves are drawn to such scales that values of Z can be read from them with great enough accuracy to use for all cases. For even greater accuracy, photostat copies of the original curves, to a larger scale, are available, as stated in the paper.

Since all the final formulas consist of only two to three factors besides the coefficient, Z, a slide-rule can be used, whereas if Z could not be taken from curves, but had to be computed every time, a calculating machine would be absolutely necessary and the total time consumed in calculating the stresses in an arch of a multiple-arch dam would be days instead of hours.

The material is well presented. At the close of the paper, it is stated that the unit stresses in the concrete and steel are not considered in the paper since such information can be found in many handbooks. For the sake of completeness it would be helpful if the authors indicated in their closure the desirability of increasing the arch thickness near and at the abutments and gave an approximate solution for finding the new location of the neutral axis.

The authors recommend reinforcing steel in both faces. It is evident that steel is required in both faces at the abutments; but it is not so evident that reinforcing steel is required along the up-stream face at the crown. A rise in temperature with reservoir empty could be assumed compensated for by the shrinkage and permanent rib-shortening as far as tensile stresses in the up-stream face at the crown are concerned.

The writer has noticed at times, that wave action on a multiple-arch dam has the effect of creating a temporary higher water level at the abutments than at the crown, say, a maximum of 5 ft. The wave will climb the V perhaps at the time the valley reaches the crown. This would be of no importance with the water low in the reservoir, but with the water close to the crest, the phenomenon may call for steel in the up-stream face near the crest, if the arch is not provided with a stiffening rib or cornice.

FRED A. NOETZLI,[16] M. AM. SOC. C. E. (by letter).—Considerable painstaking labor was involved in developing the formulas, computing the coefficients, and platting the numerous diagrams given in this paper. The diagrams will be helpful in the design of multiple-arch dams. As mentioned by the authors, curves serving a similar purpose have been previously published by Dr. N. Kelen,[3] who did not, however, consider it necessary to include the effect of the internal elastic work due to shear. Diagrams by means of which the stresses from uniform water pressure can be obtained directly from the

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[16] Cons. Hydr. Engr., Los Angeles, Calif. Mr. Noetzli died May 24, 1933.

"cylinder" stresses, have also been published.[17] The corresponding thrusts and moments can be readily computed, if desired, from the equation,

$$f_c^{u,d} = \frac{N}{t} \pm \frac{6\,M}{t^2} \quad \dotfill (83)$$

in which, f_c^u and f_c^d are the stresses, in pounds per square foot, at the up-stream and at the down-stream faces, respectively.

In a number of the earliest multiple-arch dams the central angle of the arches is 120 degrees. The application of the elastic theory to such arches indicated that a larger center angle is desirable to keep the arch stresses within permissible limits. In the authors' example, the central angle of the arches is about 145 degrees. For long arch spans and large central angles the influence of the dead load and of the variable water-load becomes increasingly important.

The paper gives a complete and theoretically exact method of designing the arches of multiple-arch dams. Further improvements in the art are to be looked for rather from the point of view of construction than from that of design.

W. J. Stich,[18] Assoc. M. Am. Soc. C. E. (by letter).—This paper graphically presents data by which an exact solution of the stresses in the inclined uniformly thick circular arches of multiple-arch dams becomes a relatively simple matter.

The tremendous amount of labor involved in the preparation of a paper of this nature can perhaps only be fully realized by the authors themselves, and much commendation is due them for the convenient form in which the coefficients and formulas are presented. The writer can appreciate this fact, to some extent, because he undertook to check the first sixty odd formulas developed in connection with obtaining the coefficients for moment, thrust, and shear due to variable water load, weight of arch, temperature, and uniform water load for arches with fixed ends.

In order to determine the increase in tensile stress developed and, therefore, the additional reinforcement required, when the variable water load and weight component are included in the analysis of the arch stresses, a calculation was made by the writer for an arch ring practically at the crest of Dam No. 1 as given by the authors in the "Numerical Example" in the Appendix. The critical arch ring selected, will serve to indicate the degree of consideration merited by the non-uniform loads (weight component and variable water load) acting on the arch.

In conformity with the basic data given by the authors for Dam No. 1, the following physical data were selected by the writer for the arch ring at Elevation 195 (abutment): $r_e = 26.50$ ft.; $t = 2.00$ ft.; $\frac{t}{r} = 0.079$; $2\,\phi_1 = 143.0°$; and temperature drop = 20 degrees. The head at the arch crown is: (208—195) — 26.50 cos 47° × versin 71° 30′ = 0.66 ft., and $p_e = 0.66 \times 62.5 = 41.25$ lb. per sq. ft.

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[17] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 539.

[18] Civ. Engr., Los Angeles City Board of Education, Los Angeles, Calif.

By means of the authors' curves and using the foregoing physical data the summary of moments, thrusts, and shears in Table 5 was obtained for Elevation 195.

Analysis of Table 5 indicates that under the conditions assumed a critical tensile stress of 233 lb. per sq. in. is developed at the abutment extrados of the arch ring in question. By means of the standard formulas for the design of reinforced concrete arches subject to bending and direct stress, the longitudinal steel required to reinforce the arch ring adequately is then fixed as 1-in. square bars at 18-in. centers for both up-stream and down-stream faces.

TABLE 5.—COMPUTATION OF MOMENTS, THRUSTS, AND SHEARS FOR ELEVATION 195

Loading	r_e, in feet	t, in feet	$\frac{t}{r}$	$2\phi_1$, in degrees	P_0, in pounds	P_1, in pounds	M_0, in foot-pounds	M_1, in foot-pounds	S_1, in pounds
Uniform water........	26.50	2.00	0.079	143.0	1 086	1 090	—59	110	2.7
Variable water........	26.50	2.00	0.079	143.0	3 380	3 800	6 338	17 400	5 050
Dead load...........	26.50	2.00	0.079	143.0	3 982	7 450	—2 570	—5 330	—1 725
20° temperature drop.	26.50	2.00	0.079	143.0	—1 268	—402	—7 590	14 210	1 202

In order to investigate the influence of the non-uniform loads (variable water load and weight of arch) upon the critical stresses developed and upon the amount of reinforcement required, a second analysis of the arch ring at Elevation 195 was made by the writer. In this instance, curves based on the formulas developed by the late Professor Cain, in his paper entitled "The Circular Arch Under Normal Loads,"[4] were used to determine the crown and abutment stresses.

By means of these curves and with the further assumptions of a 20° temperature drop and a uniform water load on the arch corresponding to the average head between the crown and the abutment, a critical tensile stress of 109 lb. per sq. in. was developed at the abutment extrados for Elevation 195. Assuming in this case that all the tension is to be absorbed by the steel, $\frac{3}{4}$-in. round bars, at 14-in. centers, would be required in each face. This spacing agrees with the arbitrary minimum as utilized in present-day practice.

The results of this investigation would indicate that an increase in reinforcing steel of approximately 80% is required when the effect of the non-uniform loads is included in analyzing the arch ring stresses of multiple-arch dams. It should be noted, however, that this increase applies only to the upper arches and down to a level of approximately 100 ft. below the crest. At elevations lower than this, the bending moments induced by the variable water load and weight component become relatively small; and, furthermore, the weight component tends to equalize the load distribution on the arch.

Unfortunately, a more economical distribution of material and stresses is obtained in the design of multiple-arch dams when the normal arch is made thicker at the abutments than at the crown. This increase in thickness may be attained advantageously by making the intrados of the arch three-centered. In this case, the arch stresses developed at the abutments will be less critical than those indicated by applying coefficients such as those introduced by the authors, which are based on the analysis of a circular arch of constant thickness. A more exact method of analysis would dictate the use of the elastic theory wherein the non-uniformity of the load due to the slope of the arch barrel, as well as the variation in thickness of the arch, are given due consideration.

George E. Goodall[19] and Ivan M. Nelidov,[20] Assoc. Members, Am. Soc. C. E. (by letter).—One of the discussers, Mr. Cochrane, presents formulas for moments, thrusts, and shears due to water load, dead load, and temperature changes in the inclined arches of multiple-arch dams. Although these formulas were derived by a different method and appear to be entirely different from those of the writers, they give the same numerical results. By substituting the quantities given in his Equations (76) and (77) into the equations for the crown thrust, H_0, given in Table 3, and multiplying the numerator and denominator of the resulting expressions by $\frac{k^2}{r^2}$, the equations for the crown thrust may be shown to be identical to those of the writers. The close agreement of the numerical results is thus accounted for and affords a valuable check on the accuracy of the work.

Mr. Cochrane expresses doubt as to the applicability of the "two-hinged" arch to multiple-arch dams. The writers undertook the derivation of the "hinged" arch formulas in order to analyze two completed dams of this type. In the case of these dams, the ends of the arches were rounded, and every effort was made in construction to cause the arches to act as if they were hinged at the ends. It is interesting, in this connection, to note that a more beneficial stress distribution is obtained in these "hinged" multiple-arch dams than in similar structures having arches with fixed ends.

It has been stated by Mr. Cochrane that the shears are always small and, ordinarily, need not be computed. This will usually be true in the more recent designs, but in some of the older designs, shearing stresses of 90 lb per sq in. have been found. Unit shearing stresses of this magnitude would not be objectionable were it not for the fact that they were accompanied by tensile stresses at the same section. In the later designs of multiple-arch dams with their greater buttress spacing, larger central angles, and smaller ratio of thickness to mean radius, the shearing stresses are less important.

In the "Numerical Example" given by the writers, the temperature drop was chosen as 20° arbitrarily to demonstrate the method. It is well known that temperature changes greatly in excess of this may be expected. That the effect of shrinkage is of great importance must be admitted, but unfortunately the data on which to base any computations are very meager and until more is known of shrinkage quantitatively, it would seem to be wiser to take care of it by using a larger factor of safety, or by using a greater drop in temperature, as Mr. Cochrane suggests.

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[19] Oakland, Calif.

[20] Senior Engr. of Hydr. Structure Design, State Dept. of Public Works, Sacramento, Calif.

If it is desired to investigate the effect of a difference in temperature between the extrados and the intrados, the crown thrust due to such difference becomes:

$$P_0 = \left[\Delta\, x - \theta\, r_n \left(\frac{1 - \sin\phi_1}{\phi_1} \right) \right] \frac{Et}{r}\, \frac{2\, k^2}{D'_s\, r^2} \ldots\ldots\ldots\ldots (84)$$

in which, $\frac{2\, k^2}{D'_s\, r^2}$ is obtained by dividing the value of $\frac{2\,\phi_1 \sin\phi_1\, t^2}{12\, D_s\, r^2}$ (taken from Fig. 11) by $\sin\phi_1$. In Equation (84) the other terms are obtained from the following:

$$\Delta\, x = \frac{r}{t}\,(u_e\, r_e - u_i\, r_i)\,(\phi_1 - \sin\phi_1) + u\, r \sin\phi_1 \ldots\ldots\ldots (85)$$

and,

$$\theta = (u_e\, r_e - u_i\, r_i)\,\frac{\phi_1}{t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (86)$$

in which, u_e = unit deformation of extrados; u = unit deformation of center line; and u_i = unit deformation of intrados.

The thrust at any point is:

$$P = P_0 \cos\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (87)$$

the moment at any point is:

$$M = P_0\, r \left(\frac{\sin\phi_1}{\phi_1} - \cos\phi \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (88)$$

and the shear at any point is:

$$S = -\, P_0 \sin\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (89)$$

Mr. Cochrane, as well as Mr. Jorgensen, suggests the advisability of increasing the thickness of the arches at the abutments. This has been recognized in some of the more recent designs. The approximate method used by the writers in their own work is that suggested by Mr. Cochrane. If the amount of increase in abutment thickness is great when compared to the crown thickness, it would be better to make the more exact calculations by some method that considers the variable thickness of the arch ring, such as the formulas of von Müller-Breslau.[21]

Mr. Jorgensen questions the writers' statement that reinforcing steel is required in both faces of the arch. He admits the necessity of this when the abutment section is considered. In the arches of multiple-arch dams having moderately large central angles, the variable water load causes large positive moments at the crown. If the water surface is drawn down to the arch under consideration, it only requires a small rise in temperature to induce tensile stresses at the extrados of the crown. Because of the thin sections used in the arches, the rise of temperature in the concrete due to setting is quite small, and its subsequent dissipation would be equivalent to only a slight drop in temperature which would be insufficient to eliminate the tensile stresses.

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[21] *Transactions*, Am Soc. C. E., Vol. 93 (1929), pp. 1296–1304.

The tendency to an increase in the central angle of the arches in recent designs is pointed out by Mr. Noetzli. Any one who has analyzed even a few typical multiple-arch dams will agree with Mr. Noetzli as to the advisability of this course. With better methods of designing buttresses and the application of the elastic theory to the analysis of the arches already an accomplished fact, the writers can only concur in Mr. Noetzli's assertion that "further improvements in the art are to be looked for rather from the point of view of construction than from that of design."

Mr. Stich presents calculations which demonstrate that even if the elastic theory is applied to the determination of stresses due to uniform water load, the effect of the weight of arch and variable water load must be considered. He, like others, points to the advisability of increasing the thickness of the arches at the abutments.

The writers made the statement in the "Introduction" to their paper that Dr. Kelen did not present curves from which the bending moments, abutment thrust, and shear could be obtained directly. Mr. Bauman asserts that this statement is misleading and that Dr. Kelen presented curves from which the unit stresses at the abutment may be obtained directly. It is true that these stress curves are included in Dr. Kelen's book, but they are for plain concrete. Inasmuch as the arches of all multiple-arch dams on which the writers have been able to gather data are reinforced, it was considered that if the curves were to cover the widest field of usefulness they should express the moments, thrusts, and shears from which the unit stresses for any percentage of reinforcement could be computed.

As Mr. Bauman states, the circular arch is not the most advantageous for the inclined arches of multiple-arch dams. However, the writers were presented with the problem of determining the stresses in more than twenty multiple-arch dams already in existence and of these structures, all had circular arches, except two.

According to Mr. Bauman, "the introduction of the influence of shear is a refinement, but is more of academic than of practical value * * *." In support of this contention he introduces stresses for Elevation 140 of the writers' example computed both with and without the influence of the shearing forces. The arch at this elevation has a thickness of only 2.58 ft and $\frac{r}{t}$ is 0.100. For such a value of $\frac{t}{r}$, the influence of the shearing forces is small, but had he chosen the arch at Elevation 40 where the thickness was 5.50 ft and $\frac{t}{r}$ was equal to 0.202, he would have found the effect of the shearing forces to be considerably greater.

To demonstrate the effect of shear on this arch the stresses have been computed for the various systems of loads with the effect of shear included and also with that influence neglected. When the elastic work of shear is omitted, the formulas for moments, thrusts, and shears are the same as those given in the writers' paper except that all terms containing the factor, 2.88, are omitted. For the arch at Elevation 40, it has been found from these calculations that omitting the effect of shear causes an error in the stresses due to uniform water load of 21% at the extrados at the abutment. For variable water load the maximum error due to the neglect of the work done by shear is at the extrados at the crown and amounts to 22 per cent. In the analysis for dead load, the maximum error occurs at the intrados at the crown and is 10 per cent. For the combined stresses due to the total load, the error due to omitting the effect of shear from the calculations is just 20% at the extrados at the abutment. The writers would consider the inclusion of a factor that affects the final result by 20% as having practical as well as academic value.

The writers concur in Mr. Bauman's statement that uniform strength throughout the arch ring is desirable but, unfortunately, this desirable condition is not easy of accomplishment.

In conclusion, the writers wish to express their appreciation of the efforts of those who entered into the discussion of this paper.

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DEVELOPMENT OF HYDRO-ELECTRIC POWER AS AN AID TO IRRIGATION[1]

BY CHARLES C. CRAGIN,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. A. H. MARKWART AND R. V. MEIKLE

SYNOPSIS

Fundamentally, the economic feasibility of hydro-electric development in connection with an irrigation project presents the same problems for consideration and is governed by the same limiting factors as any other hydro-electric development, with the exception that certain additional restricting considerations are introduced in the case of the combined development.

This paper discusses these restrictions and propounds three fundamental principles to govern the engineer in judging the feasibility of combination. Eight basic essentials for a successful hydro-electric power system, and the fields most appropriate for the use of power in connection with an irrigation project are presented. Supporting illustrations are drawn from the Salt River Project in Arizona. Six conclusions are listed at the end of the paper.

In an irrigation project, the reclamation of large areas of land and the farmer's investment in land, buildings, and other improvements is dependent entirely on the availability of an assured water supply. The availability of an acre-foot of water, or the lack of it, may mean success to the farmer's agricultural operations on the one hand, or failure, with possible loss of his entire investment, on the other. The loss due to the lack of an acre-foot of water for irrigation may be anything from a few hundred dollars to several thousand dollars; whereas the loss of revenue to a power system would be relatively small.

It is conceivable that in a project which includes a power development in connection with irrigation storage a situation might arise such that in the face of an insistent demand for power and a concurrent need for money, there would be a strong temptation to waste stored water in order to realize immediate cash revenue. This is a practice which might be attended with serious agricultural losses due to resulting shortage of irrigation water at a critical time; and it might defeat the main purpose of the project. Such serious shortage of irrigation water might affect the power market even more than the loss of the same quantity of water for the production of power. The following fundamental principle therefore must be accepted, namely:

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[1] Presented at the meeting of the Irrigation Division, Sacramento, Calif., April 24, 1930.

[2] Gen. Mgr. and Chf. Engr., Salt River Val. Water Users' Assoc., Phoenix, Ariz.

A.—Power development in connection with an irrigation project is justified only when there is a little or no interference with the irrigation system.

This is an economic fact; and, undoubtedly, it is the basis of the laws concerning this subject in most of the Western States.

The term, "irrigation project," is ordinarily understood to mean an engineering development having for its purpose the irrigation of otherwise arid land for agricultural purposes. The property involved is usually owned by a large number of individuals whose main interests and training have to do with farming. A power project is ordinarily proposed and constructed as an investment of established capital, usually, in the more modern installations, for the purpose of re-selling the power produced to the general public for all purposes ranging from small domestic uses up to large industrial uses. The operating organization of a purely irrigation enterprise is rarely adapted to the production and marketing of such a commodity as electrical energy. This is a highly specialized industry and requires the services of a force of specialists entirely foreign to the activities encountered in an irrigation enterprise. To a public utility or a power company operating independently on a fairly large scale (in which case there would be ample assurance of an adequate stand-by service), a development which might reasonably be expected to show a profit of from 8 to 10% on the investment, would be considered a favorable undertaking. The same physical conditions and the same development made thereunder by an irrigation project might be a very doubtful investment. A much larger profit on the increment investment should be the minimum set for the same development by the irrigation project.

Most irrigation enterprises are developed to the maximum extent economically feasible at the moment, with or without provision for future expansion, depending on local conditions. Financing is frequently difficult and the expenditure required per acre of land benefited sometimes approaches the allowable limit. To saddle such an enterprise with the risk of having to develop power during water shortage, or other periods of stress, such as to become a burden to land already heavily taxed, might easily place the financial affairs of the entire irrigation enterprise in serious jeopardy and even cause many individuals to lose their entire investment.

The power business constantly requires large sums of money to meet the ordinary growth of a community and the increased uses of power, to keep step with invention and improvement in the electrical industry, and to meet the standards of better service set by the public utilities of the country. Mobility and ability to develop speed in financing are essential; but these are almost always absent in irrigation projects, practically all of which require votes of their members or stockholders for all substantial financing. One large power development in connection with irrigation requires a three to one majority for authorizing any expenditure greater than $100 000 in any year. Bond issues of this nature are always subject to legal contest by any portion of the minority. Known examples of six months' delay have been caused from litigation carried on by less than 1% of the stockholders. The Salt River Project in Arizona develops more than 40% of all power produced by irrigation projects. This project, on account of favorable physical market condition, has had a wide margin between gross and operating revenue. This has made possible a concession in rates in return for the advance in cash of more than $2 500 000 from its customers toward the necessary investment. In addition, other rate concessions have been made in return for minimum guaranties to purchase as much as $40 per kw-year for wholesale blocks of 7 000 kw. and up. More than $9 000 000 worth of bonds out of the total outstanding of $14 000 000 on the project were sold advantageously by pledging these guaranties, in addition to liens on the project.

Another item to be considered in connection with the necessity of a wide margin of profit is the effect on the credit of individual farms. The writer's experience leads him to believe that one of the most important causes of agriculture's trouble to-day is lack of proper financing of the farm. A reasonably profitable power development in connection with an irrigation development, which might affect disadvantageously the personal credit of an individual member of such project would be a most uncertain benefit. A relatively small curtailment of this individual's credit would more than wipe out the share of the reasonable profits due him from the power system as a member; hence, the following principle:

B.—A power development, therefore, made in connection with an irrigation development, may be considered advisable only when the margin of profit assured is greatly in excess of that required in an independent power development.

It does not follow, however, that a feasible power installation in connection with an irrigation project must remain undeveloped because the margin of profit is too small to justify the undertaking as a part of the irrigation system. In projects with only a reasonable margin of profit, the element of risk may often be shifted to other shoulders by leasing or selling the power privileges under appropriate contract which will assure freedom from interference with the water supply or operation of the irrigation system, and provide power for project needs under favorable terms. If the physical and hydrological conditions, taken in connection with the power demand, are not sufficiently favorable to make such a lease or sale readily possible to a responsible public utility, power company, or industry, it may be taken for granted that the development is one which the irrigation project should under no circumstances undertake. In the event, however, that the element of risk is substantially eliminated, and it appears advisable that the development be made by the irrigation project itself, it would still be good business for the output of such a power plant to be disposed of in one or more large blocks rather than for the irrigation project to engage in the general retail distribution of power. In other words:

C.—Only under the most unusual circumstances will an irrigation project be justified in engaging in the general power business.

This general statement, as made, is presented as a third fundamental factor of almost equal importance with the two principles before enunciated, differentiating between a hydro-electric development in connection with irrigation and a hydro-electric development made independently.

One advantage often possessed by a power system constructed in connection with an irrigation project, which is usually not possessed by a development made for power alone, is that irrigation developments of sufficient magnitude to contemplate the installation of a power system on any considerable scale are generally organized as irrigation districts under State laws or are Federal Reclamation projects. In either case, the project is exempt from taxation, and in addition enjoys whatever benefit there may be in freedom from control by State regulating bodies; but such freedom from taxation should not be considered wholly as profit. As far as such a project is concerned it is obvious that the profit from this tax-free circumstance is limited by the amount of the lost taxes which are made up by tax-payers other than the project members. The benefit from this would be the elimination of that amount of financial risk represented by the amount of the annual taxes thus saved, and not by the full amount of the taxes.

The basic essentials for a successful hydro-electric power system are too elementary and well-known to require elaboration. Briefly, these essentials are, as follows:

1.—Physical and hydrological conditions must be favorable.
2.—Market must be assured.
3.—Hydro-electric costs must compare favorably with steam costs.
4.—There should be freedom from competition.
5.—The market must be within economical transmission distance.
6.—The load must be adapted to the production capacity.
7.—Financing must be economically feasible.
8.—Design and construction of works must be of high character.

These and other factors apply to any power development. A hydro-electric development made in connection with an irrigation project, however, as before stated, would be considered feasible only to the extent that the irrigation system is not interfered with and in cases where the margin of profit is much larger than in an independent development. In view of the great number of irrigation storage dams existing and proposed in the United States and in other countries, it might be expected that the opportunities afforded for hydro-electric development in connection with such storage dams would be very numerous. The truth, however, falls short of this to a rather surprising degree. Of the twenty-eight irrigation projects of the United States Bureau of Reclamation operating on December 31, 1931, on which there are forty-two storage dams, hydro-electric power has been developed on only twelve. On three of these projects, the developments are of small capacity only—less than 200 kv-a.—so that there are actually only nine commercial developments. Of the remaining plants, the largest installations are the Salt River Project, 83 210 kv-a.; Boise, 11 875 kv-a.; Minidoka, 11 540 kv-a.; North Platte, 7 750 kv-a.; Riverton, 1 000 kv-a.; Shoshone, 2 000 kv-a.; Strawberry Valley, 1 000 kv-a.; and Yuma, 2 000 kv-a. The gross capacity of all plants is

122 960 kv-a., of which more than two-thirds is on the Salt River Project. The output for the year ending June 30, 1929, for the entire twelve projects was 345 300 000 kw-hr., of which 206 000 000 was produced by the Salt River Project, without the new Stewart Mountain development, finished in March, 1930. The foregoing figures are quoted from the report of the Commissioner of Reclamation for the fiscal year ending June 30, 1929.

Power plant capacity of installations other than the Salt River Project as of June 30, 1931, was 39 977 kv-a., which produced (including steam power output), 160 796 586 kw-hr., totaling gross sales of $773 436.27. The power plants of the Salt River Project, however, of this date, total 83 210 kv-a., and produced (including steam power), 294 464 301 kw-hr., or a total gross sales of $2 686 301.72.

Almost every condition that can arise in connection with a proposed hydro-electric development exists on the Salt River Project. The project operates eight hydro-electric plants with a combined generating capacity of 103 000 h.p., including the one at Stewart Mountain Dam. These plants are located at the Roosevelt, Horse Mesa, Mormon Flat, and Stewart Mountain Dams, on Salt River, and at drops on the canal system in the Salt River Valley. Horse Mesa, Mormon Flat, and Stewart Mountain Dams and plants have been built in the six years since 1924, the Stewart Mountain Dam having been completed in March, 1930. The Salt River Project is primarily an irrigation enterprise and serves water directly to 250 000 acres of project land, and to 90 000 acres of non-project land. By project land is meant that which is entitled to water from Roosevelt, Horse Mesa, Mormon Flat, and Stewart Mountain Reservoirs and signed up in the Salt River Valley Water Users' Association, the organization of 9 500 farmers who own the land and operate the project. The non-project land served, consists of various areas receiving pumped water from the project; these developments owe their existence largely to the availability of hydro-electric power from the Association's plants. Owing to the number and variety of power installations on the Salt River Project, and the fact that nearly all factors to be considered in any power development connected with irrigation will be found on this project, it will be used to illustrate points to be brought out in this paper.

USE OF POWER,
WITH ILLUSTRATIONS FROM THE SALT RIVER PROJECT, ARIZONA

The fields most appropriate for the use of hydro-electric power developed in connection with an irrigation project, are the supplying of power in large blocks: (1) For irrigation pumping; (2) for mining and other industrial uses; and (3) to public utilities distributing and serving power at retail for general domestic and kindred purposes. Ordinarily, the organization and functions of an irrigation project are not adapted to the distribution of power at retail for domestic consumption. This statement is meant to apply to conditions in general, but like any general statement it is subject to modification when applied to a particular problem. In this case, an exception might well be made as to the delivery of power for domestic and farm use to the land owners in an irrigation project that owns a hydro-electric power system capable of serving them without thereby entering into destructive competition with an existing utility.

Many isolated cases will also be found in which service cannot be obtained from other systems, but in which such service is feasible from the lines of an irrigation project. In any event, the service of electricity for retail consumption should be undertaken only in connection with an irrigation project in a new territory not already served from any other source. Where such new territory adjoins or overlaps territory served from other sources, competition and duplication of service lines and other installations should be avoided by agreement with other organizations serving power in that field.

IRRIGATION PUMPING — THE IDEAL LOAD

The ideal load for a hydro-electric system, as part of an irrigation project, would ordinarily be an irrigation pumping load. Preferably, as before stated, this should be on the irrigation project that includes the power system, or on adjacent lands where the conditions of time and quantity of use are identical or closely similar. The use of power for drainage pumping would come under the same head, but would have the additional advantage of allowable latitude in time of operating the drainage pumps, so as to make possible the use of off-peak power.

In the design of a hydro-electric plant appurtenant to an irrigation system, the known irrigation pumping and drainage loads that may be available to increase the load factor of the power system as a whole, may properly be considered in fixing the justifiable generating capacity to be installed. The fact that these loads are known will warrant the installation of generating capacity over and above the assured load for other uses by the amount of the pumping loads. The use of power for other project purposes of a permanent nature, such as the lighting and operation of project works, will often constitute a first demand on the project power system, but, ordinarily, the sum total of such uses will be a small percentage of the entire output of the system.

POWER FOR PUBLIC UTILITIES, MINES, AND LARGE INDUSTRIES

In the sale of power in large blocks to public utilities, mines, and industrial enterprises, the demand that can best be synchronized with the output of the system will be most desirable. When the customer owns a steam plant capable of supplying his needs or, at least, the deficiency in hydro-electric power, it is possible to co-ordinate the two sources of power so that the maximum load factor will be obtained from the hydro-electric system. Power for industrial or general domestic uses must be firm power and to the extent that there are daily and seasonal fluctuations in the uses of power which can not economically be met by the hydro-electric system, the difference must be made up by stand-by steam plants.

A power market is rarely created quickly—it is a growth. Such an existing market must necessarily have been supplied from some source (usually from steam), prior to the development of hydro-electric power designed to supply energy either in place of, or as a supplement to, the steam power.

Where this condition is found to exist, it may be taken advantage of in the hydro-electric development by designing the hydro-electric plant to care for the growth in the market above needs previously supplied from steam power and, in addition, to take the place of the steam power to a greater or less extent. The power rate must be sufficiently lower than the cost of generation by steam to warrant the substitution, this rate being at the same time sufficiently above the cost of hydro-electric generation to be profitable in the hydro-electric system. Where this is done in such a way as to obviate the necessity, which would otherwise have existed, of rebuilding, replacing, or modernizing an existing steam plant, the two may be co-ordinated to particular advantage, since, in this case, the steam plant may still be retained as an auxiliary stand-by while the hydro-electric system becomes the main source of supply.

An example of this exists on the Salt River Project. The financing of the Horse Mesa Dam and its 44 000-h. p. plant was greatly facilitated by long-term power contracts with the Inspiration Consolidated Copper Company. This Company had a 40 000-h. p. steam plant, its maximum requirements for power being approximately 30 000 h. p. The steam plant was approaching obsolescence and it was, therefore, good business for the Mining Company to contract for the entire output of the Horse Mesa Plant, retaining its steam plant as an auxiliary and depending on Horse Mesa for its main supply. This steam plant, under the contract, in turn, becomes an auxiliary to the hydro-electric system of the Salt River Project, and is available to make up any shortage in power required at times for other customers. The only investment the Association was obliged to make for this 40 000-h. p. stand-by service was approximately $350 000 to bring the boiler capacity of the Inspiration Steam Plant up to the capacity of the generators.

A local condition on the Salt River Project unfavorable to steam, but favorable to hydro-electric plants, is the cost of fuel oil, the freight rate on fuel being $1.13 to $1.16 per bbl., or more than twice the price of the oil in 1930. This situation has changed temporarily, in that fuel can be contracted for short terms now (1932) at 50 cents per bbl., plus freight of about 87 cents. With this excessive fuel cost, steam power can not be generated for less than approximately 7 mills per kw-hr. in a modern plant; whereas, in the older type of plant mentioned, the cost of generation was nearer 1 cent. The increment cost of the hydro-electric power is approximately 3½ mills. The contract price of 6½ mills received by the Association for power delivered to the Mining Company is, therefore, a favorable one both to the Company and to the Association. The minimum annual payments guaranteed by this contract are more than sufficient to take care of the interest and principal on the development bonds of the Horse Mesa Dam, and the stand-by service makes firm a large block of Association power which otherwise would have to be sold as dump power.

Irrigation and Drainage Pumping Loads

The existence of an underground water supply susceptible of pumping for irrigation at economical cost, as before stated, may justify the installation of generating facilities to furnish the power needed. The same is true of drainage pumping. Similarly, the availability of cheap power may warrant an irrigation pumping development which otherwise would not be feasible. The Salt River Project presents a notable example. In 1919, 23 000 acres of land were added to the 185 000 acres of the project, due to the development of pumps for utilizing the water from its large underground supply. The primary purpose of these pumps was drainage, but the installations were soon found to be an asset rather than a liability, due to the value of the water for irrigation, and the fact that operation for drainage could be effected largely by off-peak power. Beginning with 38 pumps in 1919, the installations by the Association were increased to a total of 206 at the end of 1930, the capacity ranging from a fraction of 1 cu. ft. per sec. to as much as 15 cu. ft. per sec. Twenty-six of these pumps, located at points where the water was of little value on Association lands, were sold to the Roosevelt Irrigation District, consisting of 40 000 acres adjacent to the Salt River Project on the west.

This District has installed 26 additional pumps within the Project and others within its own boundaries, and, in this manner, a dependable water supply has been developed. Power is furnished them from Salt River Project plants so that, not only is drainage provided for Project lands in the area where District pumps are operated, but a profitable outlet is provided for a substantial block of power. The total capacity of the 180 pumps operated by the Association is approximately 350 000 acre-ft. of water per irrigation season, if needed. During the abnormally dry years, 1920-31, this proved to be a valuable reserve to supplement the Association's irrigation supply from stored water and unregulated river flow. With high steam costs, this would not have been as economical as with with hydro-electric power and stand-by. The total project use of power for pumping in the year 1928–29 was 33 150 000 kw-hr., valued at $331 500.

The total water pumped by the Salt River Project was 259 000 acre-ft., of which only 26 000 acre-ft. was drainage water not susceptible of irrigation use, although a conservative estimate of the annual drainage pumping requirement of the project would be 200 000 acre-ft. The drainage benefits, therefore, may be considered as being almost entirely in addition to the value of the water for irrigation which, itself, is nearly $400 000. The Association also supplies power direct, for irrigation pumping on 20 000 acres of land west of Phoenix, and for pumping by the Roosevelt Water Conservation District. The latter is a 40 000-acre tract east of the Project, the water supply of which consists of the output from forty-seven wells and river water raised 50 ft., by pumps, from the main Southside Canal of the Salt River Project. Other irrigation enterprises supplied with power by the Project's hydro-electric system, include several electrical districts in the near-by Casa Grande Valley, and a number of isolated small irrigation developments. The irrigation pumping load of the Salt River Project, in addition to 33 150 000 kw-hr. used by its own pumps, amounted, in 1929, to 62 200 000 kw-hr. The aggregate pumping load on the Project's system for that year was, therefore, approximately 95 350 000 kw-hr., which equals the estimated combined output to be expected from the Mormon Flat Plant and the Stewart Mountain Plant.

RECLAMATION OF ADDITIONAL LAND

A direct irrigation benefit on the Salt River Project from its power development is the increase of 10 000 acres in the Project area, made possible by the construction of the Mormon Flat Dam, which, itself, was purely a power development. This dam, however, impounds the run-off from approximately 300 sq. miles of the area below the Roosevelt Dam, and this stored water was thus made available for irrigation purposes.

The entire cost of this particular dam is charged to the power system, and in this case the irrigation benefit is entirely incidental. This project was not feasible if charged to irrigation alone, but the power feature made the development profitable.

POWER FOR CONSTRUCTION

The Salt River Project furnishes an example of the use of power from its own hydro-electric system in the construction of both the irrigation and power features of the Project. The power used in the construction of the Roosevelt Dam, begun by the United States Reclamation Service in 1906 and completed in February, 1911, was furnished entirely from a temporary hydro-electric plant (later incorporated in the permanent plant). This was a 1 000-kw. installation, obtaining its head and water supply by means of a 200-cu. ft. per sec. power canal, 19 miles in length.

The power canal was designed to be a permanent feature of the hydroelectric system, being intended for use in times when the water level in the reservoir was low. In the later construction at Horse Mesa, Mormon Flat, and Stewart Mountain, all power was furnished from the Association system, being brought into the sites of the several dams over the same lines subsequently used to connect their respective outputs to the main transmission system.

RETAIL DELIVERY OF POWER FOR GENERAL USE

The Salt River Project also furnishes one of the instances illustrating conditions under which an irrigation enterprise may engage advantageously in the retail distribution and service of electric power for domestic and similar purposes.

There are 9 500 farms on the property embraced by the Project, a highly developed area, approximately 40 miles in length along its greatest dimension, and including 250 000 acres of irrigated farms and orchards. Practically one-half these farms are in close proximity to cities, towns, and other congested districts. In 1928–29, 700 miles of power lines were constructed in the valley of the Salt River Project, making possible the service of electricity to every farm. This development was made in connection with the construction of the Stewart Mountain Dam and power plant, a development which was financed after the making of a contract with a local public utility company, providing for the purchase of power from the plant and assuring minimum payments sufficient to take care of the indebtedness.

An important feature of the contract consisted of an agreement to avoid duplication of investment and a division of territory to be served from the lines of the respective parties, under the approval of the State regulating body and the Secretary of the Interior. The Company serves 15% of the Project and the Association serves the other 85 per cent. By this agreement, the Association avoids any competition whatever with the existing public utility company, and the Company agrees to use the Irrigation Project's power, insuring a profitable market for 50 years. This has been a very successful development, and although the work of electrification was only completed in June, 1929, more than 80% of all homes on the Project had been connected by April 1, 1930. The revenue from this service in less than a year was sufficient to cover all additional costs, including interest and depreciation. The rapid building up of load was largely accomplished by establishing a profitable merchandising department. A well-arranged salesroom and force of ambitious and capable salesmen was found to be practically an essential. By this means not only was the number of connections brought to a profitable point in a short time, but the character of the load was greatly improved by the sale of many ranges, refrigerators, and other applicances.

AID IN REGULATING THE RUNNING OF IRRIGATION WATER

An advantage gained from the power system on the Salt River Project, which may or may not find a parallel in other developments, is the closer regulation of the water discharged from the Roosevelt Reservoir, due to the construction of dams on the river nearer the point of diversion of the canal system. The Roosevelt Dam is 77 miles by road east of Phoenix, Ariz., approximately 45 miles above the Granite Reef (diversion) Dam at the head of the main canals serving the lands on both sides of the Salt River. It provides the main irrigation storage for the Salt River Project.

With the construction of the lowest dam at Stewart Mountain, the 45 miles have been reduced to 12 and practically true, demand-water service is available to the farmers. Practically any head of water, for any length of time, is delivered to the farm within 24 hours of the time for which it was ordered. This is 24 hours better than deliveries prior to the completion of the dam.

DEVELOPMENT OF RECREATIONAL AREAS

A benefit not usually considered in connection with a strictly utilitarian development is the outstanding value for recreational purposes of the Horse Mesa, Mormon Flat, and Stewart Mountain Lakes. In a country essentially arid in character, any considerable body of water affords the element of novelty to an extent which greatly increases its popularity. These three dams create a continuous chain of lakes, 35 miles long, along the Salt River in the heart of spectacular rugged mountains, with cliffs rising at times 2 000 ft. sheer from the water's edge. The lakes extend to the Roosevelt Dam which, itself, creates a lake 25 miles in length.

The distance from Salt River Valley and the winding nature of the road has always greatly limited the use of Roosevelt Lake for recreational purposes. The lower lakes, however, are readily accessible. Stewart Mountain Lake in particular is only 40 miles distant, a matter of an hour's drive over an excellent automobile highway from the City of Phoenix, with a population of 70 000. This recreational use, however, has its value from a strictly commercial standpoint, and it has been the incentive for the construction of high-class modern highways, for the advent of a large and growing winter population.

General Range of Benefits from Power

Using the Salt River Project as an illustration, it has been shown that benefits to be derived from hydro-electric power installations in connection with irrigation works may range all the way from the use of power for construction and other project purposes to the development of power as a commercial proposition independent of irrigation works, but providing incidental irrigation benefits in the form of better water service and permitting the reclamation of additional land. The conditions described, while peculiar to the Salt River Project, are not so unusual, but that they may be found singly or in combination in other kindred enterprises.

Development of Salt River Project Power System and Value in Money

The first power developed was purely for use in construction. With the completion of the permanent Roosevelt Plant, however, the capacity was made adequate to supply current in bulk to a local public utility company for general retail distribution. The original South Consolidated Power Plant and the Arizona Falls Power Plant were installed at drops on the main canals, with the Arizona Cross-Cut Power Plant (total, 10 000 h. p.), to meet the growing demand for power from the Project system. These developments were financed by direct assessments. The Chandler Power Plant, a 600-kw. installation made in 1919, was financed by the Magma Copper Company, under a contract whereby that Company agreed to take the entire output of the plant.

In 1920, the combined generating capacity of the Roosevelt Plant and the four valley plants was 20 000 h. p., and the investment of the Association in the power system was $4 500 000. The growth of the power demand in the meantime had been such that the Association was faced with the problem of installing additional generating capacity to take care of the market, or of leaving the field open for such installations by other competing interests. The Association's power business was undoubtedly profitable and its expansion, therefore, was simply a question of whether or not, as an irrigation enterprise, its officers were disposed to expand its power interests to any degree warranted by the market. An exhaustive investigation and study of all the conditions was completed and embraced in a comprehensive report in February, 1922.

The total yearly power consumption of the State of Arizona at that time was 500 000 000 kw-hr., of which 400 000 000 kw-hr. was within a radius of 100 miles of the Project plants, more than one-half these being within reach of existing transmission lines. Most of this load was generated by steam at costs approximating 1 cent per kw-hr. It was shown to be possible by the installation of 15-ft. gates in the Roosevelt Spillways, and by the construction of three dams on the Salt River below the Roosevelt Dam, to make available a net head of 729 ft. out of a total of 832 ft. between the top of the gates at Roosevelt and the crest of Granite Reef (diversion) Dam. The increased generating capacity found to be economically feasible was 85 000 h. p., which was susceptible of immediate absorption by the then existing market.

The sites along the entire Salt River had been reserved for the benefit of the Salt River Project by the Secretary of the Interior in 1903, but it was obvious that in the event that the Association failed to avail itself of those sites and to develop their potentialities it could not reasonably be expected that they would not eventually be awarded to some one able and willing to develop them. This was the logical source of power to meet the local demand, and since the Association was already in the power business on a fairly large scale, the protection of its existing $4 500 000 investment made the further expansion of this activity a necessity, in addition to its advisability from the standpoint of a profitable undertaking.

The construction period extended over seven years, from 1923 to March, 1930. The increased storage facilities provided by the three dams in connection with the 15-ft. gates at Roosevelt, amounted to 648 000 acre-ft., bringing the gross storage on Salt River to 2 015 000 acre-ft. Although this additional storage was created solely for power purposes, and although the hydrographic records of Salt River indicated that its development would not have been warranted by the irrigation benefits alone; yet it is nevertheless true that this additional water would be available in unusual periods of drouth, if needed, to increase the irrigation supply for the project lands.

The main benefit of this development, however, is to be measured in actual dollars and cents. Disregarding the loss which might have been sustained had the Association failed to make this investment, leaving it to be developed by competing interests, it is estimated that once the reservoirs have been filled the gross annual power income will exceed $3 500 000, with a net profit of nearly $1 500 000, including power used for Project purposes. This income is expected to pay all Project indebtedness, all operation and maintenance costs, and to leave a substantial surplus besides. In comparison with the agricultural part of this enterprise, the net anticipated annual power income approximates $7.00 for each acre of land, which is a cash crop available whether or not the land itself is cultivated. During the eleven years of drouth (1920–1931) and during the period of development, the net profit averaged more than $600 000 per year, or $2.62 per acre per year. These profits and estimated future profits are based on the additional costs and investment incident to the production of power, and do not include any part of the original investment in Roosevelt Dam or the canal system, except the actual power structure and equipment.

In the last analysis, the benefits accruing to any irrigation enterprise from power developed in connection therewith could probably be reduced to terms of dollars and cents, even as the benefits from the irrigation development itself. In the case of the Salt River Project, the cost of works installed for irrigation purposes, and which would have been necessary therefor, with or without the power, are charged entirely to irrigation, even if the power actually developed in connection with such works, such, for instance, as that at Roosevelt Dam, would have been entirely infeasible without the irrigation features of the installation. Similarly, the entire cost of works installed for power purposes, such as the Mormon Flat Dam, are charged to power even if an incidental irrigation benefit is derived, such as, in this case, the development of a water supply for 10 000 additional acres of land.

The actual investment on the Salt River Project for all Project works is approximately $29 000 000, although the replacement value would greatly exceed this amount. Considered as a power project alone, the revenues from this development, at 6%, would pay interest on an investment of $37 000 000. The actual amount of the investment charged to power, however, is $18 000 000.

OTHER POWER DEVELOPMENTS IN CONNECTION WITH IRRIGATION PROJECTS

The San Carlos Project, under the Coolidge Dam, in Arizona, opened in 1929, presents physical features somewhat similar to those on the Salt River Project, and it is not impossible that its development may eventually follow along the lines found successful on the older one. The Coolidge Dam is situated at the head of a box canyon on the Gila River, not dissimilar to that of the Salt River. The river has a fall of nearly 500 ft. in the canyon below the dam and several power sites exist to make this development feasible.

CONCLUSIONS

The foregoing comments furnish arguments in favor of the following conclusions:

1.—The power development must not interfere with the irrigation project.

2.—The margin of profit should be much greater than in an ordinary hydro-electric development.

3.—If such a margin of profit is not available, power rights should be sold to produce power at the cost of steam or competing power.

4.—Only in rare instances, and under favorable conditions, should power be retailed by promoters of an irrigation project.

5.—Great benefit accrues to most projects from availability of power at low-increment cost for drainage and supplemental irrigation pumping, or both.

6.—A wide divergence exists between projects, some showing more benefits than others, but, in general, the benefits have overwhelmingly exceeded the drawbacks.

DISCUSSION

A. H. Markwart,[3] M. Am. Soc. C. E. (by letter).—The writer was much impressed with Mr. Cragin's paper. The significant point gained from it is that irrigation power has had its golden opportunity in Arizona. Obviously, the basic factor in that situation is the high cost of fuel.

The value of irrigation power is, fundamentally, the value of the fuel saved in equivalent steam plants, since practically every irrigation power development requires a kilowatt of steam capacity for each kilowatt of irrigation hydro-electric power. This becomes necessary by reason of the general stipulation that irrigation draft shall govern the release of water. Thus, power demand, when not synchronized with irrigation draft, must be met by supplemental steam. To summarize, the cost of fuel alone, less transmission costs, sets the value of irrigation hydro-electric power, up to the point where all hydro-electric power on a load-carrying basis rather than on an irrigation draft basis, is cheaper than all steam-electric power at load centers.

If fuel is high in price, the irrigation power development will have the advantage because its investment is low, this being limited to the generating equipment, the penstock, and perhaps a short conduit. The dam, the reservoir, and other parts normally required for developing head in a power project, are here necessary to and chargeable to the irrigation project.

In general, irrigation power can be absorbed most readily in a regional system having a relatively high annual load factor, such as the systems in Central and Southern California. Irrigation and drainage pumping are, of course, ideal as irrigation power loads. A low load-factor system is not helpful in absorbing irrigation hydro-electric power.

In California at present (1930), irrigation power is suffering from the presence of enormous quantities of cheap fuel, both oil and natural gas. While fuel oil delivered in Arizona might cost $1 per bbl. for transportation only, in addition to the value of the oil, in California there are great quantities of natural gas available and going to waste, and which should be conserved as is any other natural resource. Under these circumstances, it will be somewhat more difficult for irrigation power to do as well in the future as it has in the past. While the falling waters of an irrigation project should be conserved, as a natural resource, the natural resources in gas, which unlike water have no possibility of being replenished, should likewise be protected.

R. V. Meikle,[4] M. Am. Soc. C. E. (by letter).—The benefits to be derived by the development of hydro-electric power in connection with irrigation storage have been clearly presented in this paper. In his analysis of the subject, the author's conclusions are generally applicable to the power development by irrigation districts on the Stanislaus, Tuolumne, and Merced Rivers in the San Joaquin Valley of California where the total installed capacity is 120 000 h. p. and the output is about 390 000 000 kw-hr. yearly. The power from these California developments is disposed of in a different manner for each project. One project receives storage water for irrigation free, in exchange for its power rights to the power company; another project sells the entire output wholesale under a 40-year contract; the third project combines distribution within its area and wholesale delivery to the power company under long-term contracts. In each instance, the development of hydro-electric power has been a substantial aid to irrigation.

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[3] Vice-Pres., Pacific Gas & Elec. Co., San Francisco, Calif.

[4] Chf. Engr., Turlock Irrig. Dist., Turlock, Calif.

The sale of power generated at these irrigation storage developments to the power company under a long-term contract is undoubtedly the safest and easiest method of disposing of the power output.

It is possible for an irrigation enterprise to engage in the general power business successfully, but this step should not be taken unless it is impossible to make a reasonable and fair agreement for the sale of the output, to the local power company. In an irrigation enterprise, the taxpayers should receive the first consideration in any matter that affects the financial structure of their project. The distribution of power by the irrigation project may lead to the paralleling of lines and destructive competition with the local power company. Reduction of power rates on the project resulting from competition or low rates made to please the voters, may not benefit the taxpayer to the extent that he would be benefited by the income that could be derived from the sale of the power to the power company without the risks involved in power distribution. In the California district where power is distributed at very low rates, the taxpayers represent only 60% of the power customers.

The question of future power distribution by irrigation districts is becoming of less importance in California, due to the inability of the districts to finance such expensive undertakings and to the fact that the power companies have been co-operating with the irrigation districts to the fullest extent possible in the handling of the power output from the district projects. Furthermore, lower domestic rates may be expected from the power companies. To increase the use of electricity, the rates must be low enough for the average family or customer to take advantage of the many uses of electric power.

The lowest domestic power rate under the irrigation district distribution, of the projects mentioned is, as follows: 15 kw-hr., $1.00; next 15 kw-hr., $0.05; next 50 kw-hr., $0.025; next 120 kw-hr., $0.02; and more than 200 kw-hr., $0.01. This rate, under the conditions of rural distribution, is possibly too low, but no money has been lost in supplying the 5 000 customers under the rate, and the growth of business, in kilowatt-hours, notwithstanding the economic depression, has continued at the rate of 4% yearly.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 1853

IMPROVEMENT OF HUDSON RIVER BY NARROWING THE NAVIGABLE FAIRWAY

By Lynne J. Bevan,[1] M. Am. Soc. C. E.

With Discussion by Messrs. William B. Moss, Lytle Brown, H. A. Marmer, and Lynne J. Bevan

Synopsis

This paper applies to the problem of pier extensions in the Hudson River west of the Island of Manhattan, the broad hydraulic principles of waterway design. It presents data indicating that successive encroachments on the navigable fairway by the extension of piers have set in action natural forces which have scoured the river bottom, actually increased the cross-section area at a substantial saving in dredging expense, and, contrary to generally accepted statements, have actually decreased tidal current velocities.

Narrowing a broad natural waterway and simultaneously increasing its depth tends to improve the hydraulic regimen of the waterway; it decreases the tendency to create cross-currents, increases the hydraulic radius, decreases the mean velocity necessary for a definite quantity of water, and tends to make velocity more uniform from the surface to the bottom.

Numerous illustrations of the general acceptance of this hydraulic principle are recorded in technical publications. An impressive illustration of its application occurs at the mouth of the Columbia River, on the Pacific Coast, where two jetties north and south, have concentrated the tidal flow, so that to-day (1932) a single 40-ft. channel, 1 mile wide, is easily maintained, whereas the old natural channels, usually less than 20 ft. deep, wandered aimlessly over a 7-mile front.

Owing to conflicting interests, real and imaginary, as between the States of New York and New Jersey, between trans-oceanic and local commerce, between economics and politics, and between various other elements, efforts to apply this principle to the Hudson River, by advancing pierhead lines from both the New York and the New Jersey shores, have always met with opposition.

Note.—Published in May, 1932, *Proceedings*.

[1] Cons. Engr., New York, N. Y.

In 1767, the shore lines of the Hudson opposite Lower Manhattan were approximately twice as far apart as are the pierhead lines in 1932. In 1857, a pierhead line on the Manhattan shore was established by the United States War Department, and since that date several lines farther from both the New York and New Jersey shores have been approved by that Department. These advances usually have been requested by trans-oceanic steamship interests that required longer piers at which to dock vessels of ever-increasing lengths. They have been opposed by towboat companies and other interests that have advocated the maintenance of the wide-water areas with which Nature has bountifully endowed New York Harbor. This opposition has averred that pierhead line advances have increased greatly the velocity of the tidal currents, and, hence, have handicapped navigation.

TABLE 1.—COMPARISON OF CROSS-SECTION AREAS OF THE HUDSON RIVER, IN SQUARE FEET, BETWEEN PIERHEAD LINES AS ESTABLISHED IN 1930 AND AS ESTABLISHED IN THE YEARS SHOWN

Description	1845	1855	1874	1882	1912	1930
Barclay Street Section:						
Between pierhead lines in year named.	....	....	154 398	139 260	142 940	141 900
Between 1930 pierheads; section in year named	128 554	129 585	143 782	132 369	142 940	141 900
Chelsea Piers (18th Street):						
Between pierhead lines in year named.	....	....	144 116	149 915	159 979	159 500
Between 1930 pierheads; section in year named	128 798	139 332	142 591	148 882	159 979	159 500
Typical Sections at 39th Street, 41st Street, and 52d Street:						
Between pierhead lines in year named.	....	....	125 728	123 446	120 695	123 200
Between 1930 pierheads; section in year named	....	....	114 428	112 746	120 695	123 200

The data correlated in this paper were assembled with the hope of developing the effect of pierhead line advances on the hydraulics of the river. Cross-section areas (in square feet) between pierhead lines at various points along the west shore of Manhattan are compared in Table 1. Their locations are shown in Fig. 1 and the Chelsea Sections, plotted in Fig. 2, are illustrative.

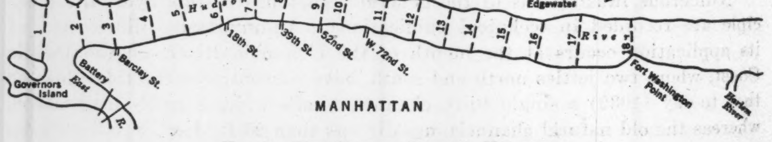

FIG 1.—KEY MAP TO SECTION NUMBERS

These cross-sections show that in the mid-Manhattan area natural forces have compensated for the advance of pierhead lines by scouring of the river bottom. For instance, in the Chelsea Sections, where the last pierhead line modification occurred in 1897, the cross-section area in 1874 was 144 116 sq. ft. between the pierhead lines of that date, and, in 1912, the cross-section area was 159 979 sq. ft. between the pierhead lines as then fixed, and this area was maintained until 1930.

The second series consists of cross-sections at eighteen, approximately equi-distant, locations between Governors Island and Fort Washington, for 1855, 1910, and 1930 (see Fig. 1). The sections for 1855 and 1910 were traced from a map dated November 28, 1922, entitled "Sections Showing Comparison of Channel Depths of 1855 and 1910" in the United States Engineer Office, First New York District, and the sections for 1930 were plotted from the 1930 edition of the United States Coast and Geodetic Survey Chart. The rough location of these sections is indicated in Fig. 1 and the defining limits in each case are, approximately, as given in Table 2.

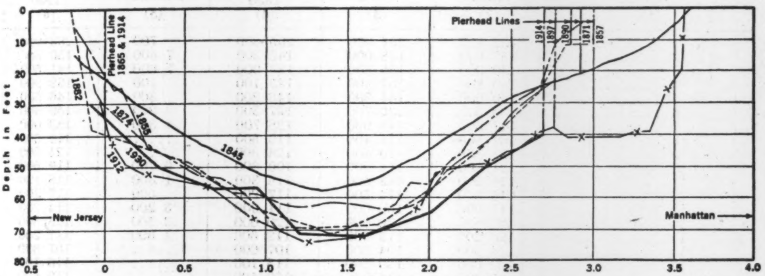

FIG. 2.—CROSS-SECTIONS OF CHANNEL OF THE HUDSON RIVER IN THE VICINITY OF THE CHELSEA PIERS (18TH STREET)

TABLE 2.—LOCATION OF SECTIONS SHOWN IN FIG. 1

Section No.	From New York City	To New Jersey	Section No.	From New York City	To New Jersey
1....	Governors Island...	Ellis Island	9....	West 52d Street....	West Shore Terminal, Weehawken
2....	Pier 1, the Battery..	Communipaw			
3....	Pier 14, Fulton Street	Pennsylvania Railroad Ferry, Jersey City	10....	West 61st Street...	10th Street, West New York
			11....	West 77th Street...	Guttenberg
4....	Desbrosses Street Ferry..........	Erie Ferry, Jersey City	12....	West 89th Street...	North Bergen
			13....	West 101st Street...	Shady Side, N. Bergen
5....	West 10th Street...	Delaware Lackawanna and Western Ferry, Hoboken	14....	West 113th Street..	New York Susquehanna and Western Railroad, Edgewater
6....	West 15th Street...	Castle Point, Hoboken			
			15....	West 125th Street..	Edgewater
7....	West 25th Street...	Weehawken Cave	16....	West 138th Street..	Fort Lee
8....	West 37th Street...	Delaware and Hudson Basin, Weehawken	17....	West 155th Street..	Fort Lee
			18....	Fort Washington Point...........	Fort Lee

Four typical sections are graphically shown in Figs. 3, 4, 5, and 6. The areas are listed in Tables 3 and 4. Column (6), in each case, shows the actual 1930 areas while Column (4) indicates the part of the total area for which natural forces are responsible. It is obtained by deducting the area dredged (Column (5)) from the total 1930 area.

The cross-section areas listed in Tables 3 and 4 show, with greater detail, that natural forces have compensated for the advances of pierhead lines. It is a simple corollary of increased cross-section areas, from correspondingly increased depth by scouring by natural forces, that substantial costs of dredging have been saved.

TABLE 3.—HUDSON RIVER CROSS-SECTION AREAS, IN SQUARE FEET, MEASURED TO PIERHEAD LINES AS OF THE DATES OF THE COLUMN HEADINGS

Section No. (1)	1855 (2)	1910 (3)	By Nature, 1930 (4)	Dredged (5)	Total, 1930 (6)
1	222 800	251 000	265 300	700	266 000
2	145 000	148 000	143 200	7 600	150 800
3	140 300	135 500	133 900	7 600	141 500
4	140 800	137 400	132 100	100	132 200
5	150 000	144 300	146 200	300	146 500
6	133 000	136 700	157 300	2 200	159 500
7	117 000	118 000	128 700	4 300	133 000
8	107 800	114 400	119 500	......	119 500
9	121 000	120 400	124 800	......	124 800
10	121 600	114 000	109 800	9 700	119 500
11	116 200	113 000	116 300	1 800	118 100
12	115 800	114 500	117 200	500	117 700
13	118 900	109 000	111 000	3 200	114 200
14	100 800	120 000	120 000	3 500	123 500
15	120 200	113 000	112 600	4 600	117 200
16	114 900	104 000	107 600	......	107 600
17	129 000	120 500	115 100	......	115 100
18	135 500	144 300	159 500	......	159 500
Mean	130 590	131 010	134 450	2 560	137 010

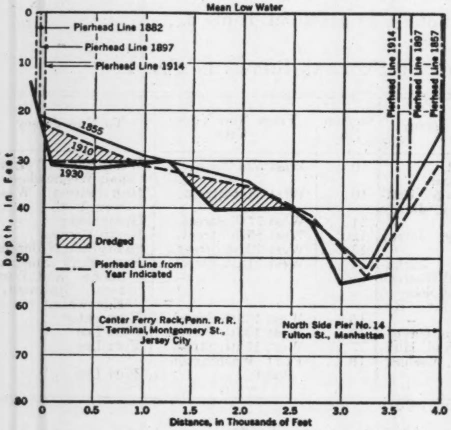

FIG. 3.—COMPARATIVE DEPTHS IN SECTION 3

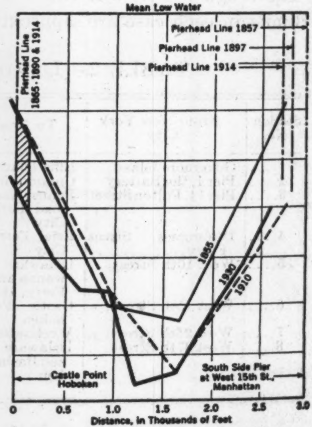

FIG. 4.—COMPARATIVE DEPTHS IN SECTION 6

A third series of six cross-sections was prepared in order to learn the effect, if any, of pierhead line advances upon the channel up stream from New York. These sections illustrate the differences in the channel bottom off

Haverstraw, N. Y., 36 miles north of the Battery, in 1916 and 1927 (see Fig. 7). As illustrative of this series, Figs. 8, 9, and 10 show typical profiles. The section areas were not planimeterd; but from the diagrams it is evident that Nature is maintaining these up-stream channels, and that no injury is resulting from pier extensions.

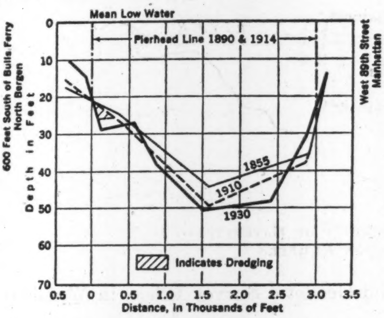

FIG. 5.—COMPARATIVE DEPTHS IN SECTION 12

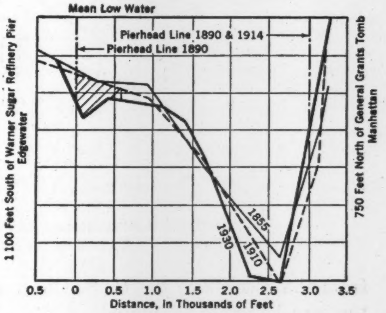

FIG. 6.—COMPARATIVE DEPTHS IN SECTION 15

These sections supplement the conclusion of the New York Harbor Line Board which, in 1911, compared cross-sections of the Hudson River at Yonkers, Fort Montgomery, and Poughkeepsie, as of 1955-61, with cross-sections at the same points as of 1903-05. After finding "practically no change," the Board concluded that it was fair to assume that no injurious effects on the river to the north of the city limits were found, that can be ascribed with certainty to the extension of harbor lines in New York.

TABLE 4.—HUDSON RIVER CROSS-SECTION AREAS, IN SQUARE FEET, MEASURED TO THE 1930 PIERHEAD LINES

Section No. (1)	1855 (2)	1910 (3)	By Nature, 1930 (4)	Dredged (5)	Total, 1930 (6)
1	223 000	251 000	265 300	700	266 000
2	140 800	143 500	143 200	7 600	150 800
3	122 400	126 200	133 900	7 600	141 500
4	118 100	128 400	132 100	100	132 200
5	127 300	135 000	146 200	300	146 500
6	128 300	146 200	157 300	2 200	159 500
7	104 300	118 100	128 700	4 300	133 000
8	91 500	114 400	119 500	.....	119 500
9	103 500	120 300	124 800	.....	124 800
10	106 900	113 900	109 800	9 700	119 500
11	97 100	113 000	116 300	1 800	118 100
12	104 000	114 400	117 200	500	117 700
13	102 000	109 000	111 000	3 200	114 200
14	93 600	120 000	120 000	3 500	123 500
15	104 900	113 200	112 600	4 600	117 200
16	93 600	103 900	107 600	.....	107 600
17	103 700	106 500	115 100	.....	115 100
18	133 600	144 300	159 500	.....	159 500
Mean	116 590	128 960	134 450	2 560	137 010

Tidal currents, during the period covered by these cross-sections (1855 to 1930), have remained substantially constant. This statement is based upon the common understanding that tidal currents exhibit changes in the strength of the current that correspond closely with the range exhibited by tides. As indicative of constancy in tidal range, the values for the tide at Yonkers, from the records of the U. S. Coast and Geodetic Survey Office, in Washington, are herewith tabulated:

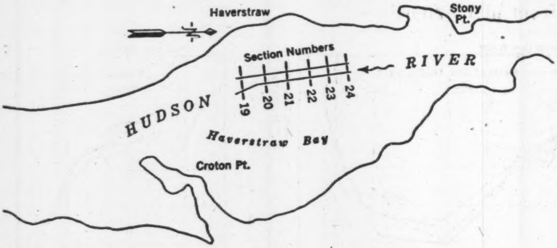

Fig. 7.—Hudson River in Vicinity of Haverstraw, N. Y., Key Map to Section Numbers

August 5 to 25, 1853......	3.94 ft...	Compared with Governors Island
July 16 to 27, 1855........	4.08 ft...	Compared with Governors Island
August 13 to November 14, 1898	3.77 ft...	Compared with Fort Hamilton

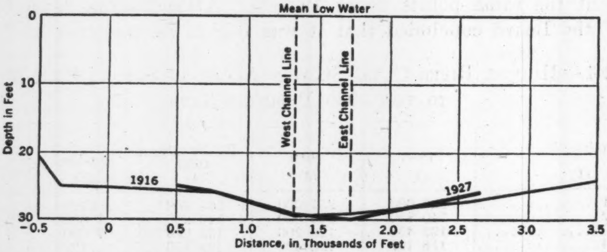

Fig. 8.—Comparative Depths in Section 20, Haverstraw, N. Y.

Table 5 shows the ranges derived from observations for three months (July to September) of each of the years indicated and corrected for longitude, or moon's node.

TABLE 5.—Tidal Range at Yonkers, N. Y.

Year	Range, in feet	Year	Range, in feet	Year	Range, in feet
1910................	3.40	1917................	3.47	1924................	3.42
1911................	3.33	1918................	3.35	1925................	3.52
1912................	3.19	1919................	3.52	1926................	3.52
1913................	3.33	1921................	3.48	1927................	3.48
1914................	3.32	1922................	3.57	1928................	3.61
1915................	3.40	1923................	3.42	1929................	3.57
1916................	3.35	....	....	....	....

The New York Harbor Line Board investigation of 1911, found also that there had been practically no change in the rate of progress of the tidal wave between Governors Island and Tivoli (16 miles below Hudson, N. Y.) since 1855, and no change in the tidal range.

The Metropolitan Sewerage Commission (of New York City), consisting of five eminent engineers made extensive studies of tides and tidal currents in New York Harbor, and submitted a voluminous report under date of April 30, 1910. The following is quoted from the report (page 181):

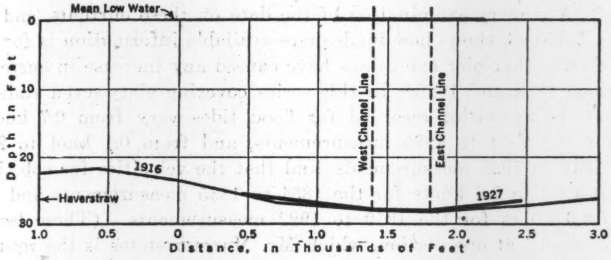

FIG. 9.—COMPARATIVE DEPTHS IN SECTION 22, HAVERSTRAW, N. Y.

> "The extension of piers into rivers, especially the Hudson River would reduce the amount of tide water passing up and down. However, neither the reclamation of shore areas nor the extension of piers into the Hudson seem, up to the present time, to have sensibly interfered with the tide. For instance, the mean range of tide at Dobbs Ferry, determined by observations in the years 1856, 1858, 1885, 1886, and 1900 has the values of 3.71, 3.69, 3.58, 3.60 and 3.66 feet, respectively, and these figures are apparently sufficiently close to cover yearly variations."

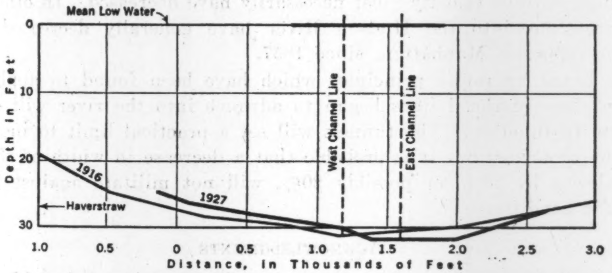

FIG. 10.—COMPARATIVE DEPTHS IN SECTION 24, HAVERSTRAW, N. Y.

The paradox between the first sentence of this quotation and the findings in the following sentences is doubtless due to the fact that the Hudson River bottom is of fine material, readily susceptible to scouring action of temporarily slightly increased velocities, and, once the scouring starts, it proceeds somewhat beyond the previous point of stabilization. The greater cross-section areas may be considered reasonably permanent because the narrowing of the channel has improved the uniformity of currents from the surface to the bed of the river.

To an engineer experienced in stream-flow measurement the tremendous difficulty in obtaining reliable tidal current-velocity measurements in the Hudson River are readily appreciated. However, such measurements opposite Manhattan have been compiled and briefly discussed by H. A. Marmer, M. Am. Soc. C. E., in a paper[3] entitled, "Tides and Currents in New York Harbor." A cursory examination of the data on these currents, and particularly his Table 63, shows how inadequate available information is for drawing the conclusion that pier extensions have caused any increase in current velocity between 1854 and 1922. In this series covering sixty-seven stations it is seen that the velocities recorded for flood tides vary from 0.7 knot to 1.9 knots for the 1854 to 1885 measurements, and from 0.1 knot to 2.1 knots for the 1919 to 1922 measurements, and that the velocities for ebb tides vary from 1.0 knot to 2.8 knots for the 1854 to 1885 measurements and from 0.9 knot to 2.6 knots for the 1919 to 1922 measurements. (These last figures omit the record at one station, which Mr. Marmer states is the result of the use of a tidal range correction factor which is undoubtedly too large.)

The fact too rarely realized that stream flow measurements require tremendously greater skill, time for observations, and experienced judgment than soundings, may explain the ready acceptance of a few early flow measurements as a basis for the assumption of increase in tidal current velocities in the Hudson, as was done in some of the early reports on this subject.

The conclusion of the data assembled is that, inasmuch as the cross-section areas of the Hudson River between the Battery and Fort Washington have increased substantially between 1855 and 1930, and the tidal flow as shown by the mean range of tide at observation stations a short distance above Manhattan has remained substantially constant during this period, the mean tidal current velocity must necessarily have decreased. In other words, pier extensions into the Hudson River have generally decreased current velocities opposite Manhattan, since 1857.

The broad hydraulic principles which have been found to apply in the Hudson since pierhead lines began to advance into the river will doubtless continue to function. The tunnels will set a practical limit to increases of depth by scouring; but it is probable that a decrease in width of the navigable fairway by 10%, or possibly 20%, will not militate against favorable hydraulic conditions.

ACKNOWLEDGMENTS

Acknowledgment for assistance in assembling material for this paper is gratefully made to Robert A. Lesler, and H. A. Marmer, Members, Am. Soc. C. E., and to the officers and civilian engineers in the U. S. Army Engineer Office, First District, New York. Most of the data in this paper were obtained from maps and records in that Office and from the U. S. Coast and Geodetic Survey Chart for 1930.

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[3] *Special Publication No. 111,* Serial No. 285, U. S. Coast and Geodetic Survey, p. 124 *et seq.*

DISCUSSION

William B. Moss,[3] Assoc. M. Am. Soc. C. E. (by letter).—Considered hydraulically, the author's opening statement is undoubtedly sound. The continued application of the narrowing process would probably benefit the hydraulic regimen; but that it would improve the Hudson River for navigation is questionable. Improvement, from the standpoint of the port engineer, must co-ordinate and, to an extent, subordinate hydraulic principles with, and to, the demands of safe navigation and harbor accommodation. A wide channel, easily maintained at the maximum usable depth, is generally much better than a narrow and deeper fairway.

The Lower Hudson River has a variety of important commercial servitudes. In its lower part it is practically a sub-port lined on both shores with berths for the largest merchant ships in deep sea, coastal, and river traffic. Its volume of cross-traffic and the congestion existing at times are too well known to need description. Above the bend at West 23d Street large areas are given over to the anchorage of vessels, including naval anchorages for capital ships, in the deep channel adjacent to the New York shore north of West 72d Street. Just below the river bed, are important tunnels, protected by comparatively thin layers of mud and clay from possible serious injury through the sinking of a vessel and/or an explosion of its cargo in close proximity to the shells of the tubes. Extra channel width is an important consideration as a safety factor in case of such a contingency. The river is the thoroughfare for vessels and flotilla tows bound to and from the distant up-river port of Albany and other river points. It passes on to these northern river areas a tidal movement of the progressive wave type, as well as the great tidal stream, with extensive hydraulic reactions.

Prior to 1890 harbor lines were established by State and city authorities; since that date they have been fixed by the Federal authorities. Owing to the conflict of interest mentioned by the author, it is probable that not all the subsequent modifications of these lines have had the sole purpose of improving the river for the benefit of navigation. The points fixing the present least width between pierhead lines on the New Jersey and New York shores at West 23d Street, Manhattan, were established by pier lines adopted by the Riparian Board of New Jersey in 1865 on the one shore, and by the City of New York in 1871, on the other.

In recommending Federal harbor lines in 1890, the first New York Harbor Line Board stated[4] its opinion that harbor lines should have been drawn tangent to Castle Point, N. J., since it projects so far beyond the general shore line. However, since such a modification would have interfered with vested rights, duly acquired under the State laws of New Jersey, the Board recommended the adoption of lines fixed by the laws of New York and New Jersey at that time. An exception was made in the pierhead line between the Battery and West 23d Street, New York City, which was extended outward to accommodate navigation. The maximum extension was about 155 ft. at Canal Street. According to this report the original width of the river at West 14th Street had been materially impaired by artificial encroachments on both banks; and this result was deplored because, originally, the rocky headland known as Castle Point had formed a marked gorge which should not have been made worse by artificial shore extensions. These statements indicate that this first Federal Harbor Line Board decided, against its inclination and judgment, to allow encroachments to continue, which had been legalized before the War Department received authority to regulate these matters.

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[4] Rept., U. S. Corps of Engrs., 1890, pp. 816 and 821.

Pre-war current observations and data appeared to support the opinion that the velocity of the river current had increased. New and comprehensive data were published[2] in 1925 by the U. S. Coast and Geodetic Survey in the pamphlet, entitled "Tides and Currents in New York Harbor," by H. A. Marmer, M. Am. Soc. C. E., which contains tabulations of tidal current observations from records of the Survey between 1854 and 1922. These data do not indicate any large percentage of change in the maximum tidal velocities of the Lower Hudson River during the period covered. Tables 63 and 64 of the pamphlet indicate observed ebb-current velocities at several stations and at the several periods, reading between 2.5 and 2.8 knots per hour and an observation in 1919 giving a velocity of 3.11 knots, all reduced to normal tide values. The latter observation, which is No. 58 in Tables 63 and 64, is discredited, as it is out of line with other observations in the vicinity made in 1922 (Nos. 57 and 59), although it resulted from two days' observations. Its location off Fort Washington Point, where practically no contraction in the width of the stream has occurred, eliminates it from consideration in discussion of the author's subject. It is mentioned only as it indicates the familiar and disconcerting manner in which an individual observation frequently breaks up the continuity of a chain of data, and raises an element of doubt. Observations in the tables numbered 2, 16, 19, and 55 of the pamphlet are also difficult to reconcile with others in their vicinities.

In Table 64,[5] the more recent observations Nos. 1, 8, 10, 13, 18, 24, 26, 29, 34, 42, 49, 57, and 63, include both pole and meter readings for approximately the same depths, which differ from 5 to 25% from each other. It seems to the writer, in view of these discrepancies, and the extreme difficulty in obtaining accurate current velocity measurements mentioned by the author (as well as the absence of information as to the depth of the earlier observations) that there is little to warrant a definite conclusion that the strength of the tidal currents has either decreased or increased noticeably since 1854. A conclusion that it has not increased nor decreased largely is believed to be justified by the observations.

Meter observations of the ebb current at strength were made by the U. S. Engineers on December 26 and 27, 1912, which showed maximum velocities of nearly 6 ft. per sec., or 3½ knots per hour. On these dates, lunar phase produced spring tides which were recorded as about 1½ ft. above normal.

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[5] *Special Publication No. 111,* U. S. Coast and Geodetic Survey, p. 132.

Reduced to normal tide these observed velocities would be about 2.7 knots, which agrees fairly well with the compiled maximum velocities of the Coast and Geodetic Survey pamphlet. These current observations were taken at intervals of 5 ft. in depth in the thread of the swiftest current at three stations off Barclay Street, Chelsea Piers, and 52d Street, Manhattan, respectively. They agreed with the Coast Survey determinations in showing a fairly regular decrease in current with depth at two stations, but, in the Chelsea Section, a nearly uniform velocity was observed in 1912 between the 10 and 50-ft. depths, the average rate of flow being from 2.1 to 2.2 knots, reduced to normal tidal range. The effect of the tidal currents on deep-draft vessels will be proportional to the mean velocity of the vertical section they displace. Normally, this velocity will be considerably greater than near-surface flows at flood strength. The velocity near the bottom will be the controlling factor in erosion by tidal current.

The paper by Mr. Bevan exhibits quite a collection of curves and tabular data which deal with changes in the cross-sectional areas of the lower river at intervals and which were originally compiled principally from U. S. Coast and Geodetic Survey charts. Close examination reveals inaccuracies in assumption and basic data, which weaken the conclusions apparently reached. Errors in assumption are: (1) That the effective cross-section is included between the pierhead lines; and (2) that the charts used show soundings of the date of publication. Errors in data include: (1) Using charts with insufficient soundings; (2) not considering dredging that increased a cross-section; and (3) excluding free areas shoreward of the pierhead line not occupied by structures at the given dates and sections. Critical study has been made of Sections 3, 6, 12, and 15, shown by the author's Figs. 3 to 6, inclusive, as well as Fig. 2. For the purpose the writer used Coast Survey charts of 1930 issue. To these were transferred soundings of 1855 taken from office tracings of two charts entitled: "U. S. Coast Survey, A. D. Bache, Superintendent, New York Bay and Harbor Surveys in 1855, Copied for the Commissioners for the Preservation and Protection of the Harbor of New York from Encroachments." They are Hydrographic Sheets Nos. 477 and 496 (scale 1: 10 000).

Table 1 and Fig. 2 of Mr. Bevan's paper show a fairly uniform deepening across channel at the Chelsea Pier Section between 1845 and 1855, the increase of section being 10 500 sq. ft. About one-half this amount was lost by the advance of the shore prior to 1855. At this early period a pier extended outshore to about the position of the present pierhead line and caused an additional obstruction beyond the pier line of 1855. Figs. 2 and 4 show a pierhead line of 1914 about 100 ft. channelward of the 1897 pierhead line, which is an error, as the 1897 line remains the established line. In this section shoaling is indicated along the New York shore between 1855 and 1874, followed by deepening in 1912. Between these dates the Chelsea Piers were built and the shoal area in front of them was dredged more than 40 ft. deep, and 997 000 cu. yd. of material were removed outside the pierhead line on a frontage of about 3 400 ft., with an average width of about 500 ft. This deepening would result in increases of cross-section of 6 000 and 9 000 sq. ft., the lower value probably applying to the place of the section.

In the sections shown, only a few widely separated soundings were obtained from the older maps. As a result the bottom contours are long straight lines with only three to four breaks across channel. If more frequent soundings are taken from the original hydrographic sheets and the entire clear portion of the river between existing structures is included, the results show considerably less increase of section between 1855 and 1930. It must be pointed out that many soundings on the Coast Survey chart of 1930 are quite old. Revised depths are given on the chart only wherever the U. S. Engineers have dredged or where they have made surveys in more recent years. Thus, on Chart No. 369.8, nearly all soundings northerly of West 158th Street date back to the survey of 1885-86, and many of those in the deeper water south of this point are of the same date. Since the soundings available and the harbor lines were the same at Fort Washington Point both in 1910 and 1930, the areas of cross-sections should be practically the same for both dates, but the author's Tables 3 and 4 show a difference of 15 200 sq. ft. as of these dates, or nearly 10 per cent. Even on Coast Survey Chart No. 369.4, issued in 1930, through the deeper sections, where untouched by dredging or not covered by a later survey, a few of the original soundings of 1885-86 are still retained. Most of the older soundings, however, date from 1899 to 1910. Thus, in the vicinity of Section 3 (Fig. 3), due to much dredging, out of fifty-three soundings only two refer back to the 1885 survey and seven to the 1899-1910 survey; but at Section 6 (Fig. 4), out of a total of forty-three, three are from 1885 and twenty-four from 1899-1910; while, at Section 9 (for which no diagram is shown), out of forty soundings, six date from 1885 and four from 1899-1910. The so-called 1930 data, therefore, refer to mixed dates and not to the year 1930.

Referring to the section numbers of Tables 3 and 4, the amount of advancement of the physical shore by structures or fill in various sections is approximately as follows:

Section	Shore advance, in feet	Section	Shore advance, in feet
2	600	10	1 500
3	800	11	1 000
4	1 500	12	700
5	1 600	13	600
6	270–320	14	500
7	1 500	15	500
8	1 200	16–18	0
9	1 100		

It will be seen that Table 3 shows a decrease, or a slight increase, in area in most of the sections where the largest contraction of stream has occurred, notably at Sections 3, 4, 5, 10, 11, and 12.

The writer has made comparison of the river sections from the Coast Survey charts of 1855 and 1930 at the author's Sections 3, 6, 12, and 15. Two tiers (three in Section 6) of reference squares have been drawn on the charts across the river at each section, and the approximate amount of deepening or shoaling in the 75-year period has been estimated. These data are shown by Table 6, the average change in depth, in feet, for each lettered square of each tier being given by figures. A preceding plus sign indicates deepening, and a minus sign, shoaling.

TABLE 6.—ESTIMATED DEEPENING OR SHOALING IN SECTIONS 3, 6, 12, AND 15

Tier	Description	Reference Grid Letters								
		A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
SECTION 3: FULTON STREET										
North..	Total change....	—10	+9	+2	+3	+7	+3	±0	+1	±0
	Dredging.......	.....	+9	+2	.....	+4.5	+1.5	.....	.....	.....
South..	Total change....	—5	+6	+1	+3	+7	+2	—1	+3	±0
	Dredging.......	.....	+6	+0.5	+1	+6	+1	.....	.....	.....
SECTION 6: CHELSEA SECTION										
North..	Total change....	+7	+5	+8	+8	+2	+5	.....	.....	.....
	Dredging.......	+2	.....	.....	.....	+2	+5	.....	.....	.....
Middle.	Total change....	+8	+4	+5	+4	+3	+5	.....	.....	.....
	Dredging.......	+2	.....	.....	.....	+3	+5	.....	.....	.....
South..	Total change....	+5	+3	+1	—1	+1	+6	.....	.....	.....
	Dredging.......	+3	.....	.....	.....	+1	+6	.....	.....	.....
SECTION 12: WEST 89TH STREET										
North..	Total change....	—10	—1	+3	+4	+4	+5	+2	—20	.....
	Dredging.......	.....	.....	+3	.....	.....	.....	.....	.....	.....
South..	Total change....	—10	+2	+5	+7	+4	+4	+2	—15	.....
	Dredging.......	.....	.....	+4	.....	.....	.....	.....	.....	.....
SECTION 15: WEST 138TH STREET										
North..	Total change....	+1	+4	+3	+1⅓	+7	+9	—3	.....	.....
	Dredging.......	+1	+1⅓	.....	.....	.....	.....	.....	.....	.....
South..	Total change....	—1	+7	+4	+4	+4	+4	—12	.....	.....
	Dredging.......	.....	+3	.....	.....	.....	.....	.....	.....	.....

An examination of Table 6 shows clearly a deepening of considerable areas. A striking difference is indicated between results in the north and south tiers at Section 6, and it is obvious that there is less increase in section across the lower tier than the upper. Thus it was found that by drawing the section line across the river about opposite 6th Street, Hoboken, N. J., the areas would be approximately 155 000 sq. ft. for the 1855 soundings, and 150 000 sq. ft. for the 1930 map, the latter including about 5% of dredging.

Taking into consideration the actual physical shores as limited by fill and structures at the dates indicated, the writer has computed the areas of the four cross-sections, in square feet, using for the 1930 sections the Coast Survey charts of the 1930 issue (see Table 7).

TABLE 7.—AREAS OF SECTIONS 3, 6, 12, AND 15, IN SQUARE FEET

Section	1855	Natural, 1930	Dredging	Total 1930
3..............................	163 000	149 000	7 500	156 500
6..............................	149 000	151 000	8 000	159 000
12..............................	131 000	117 500	500	118 000
15..............................	128 000	121 000	5 000	126 000

A recomputation of the 1855 cross-section at Fort Washington Point (Section 18), gives an area of about 145 000 sq. ft. The great depth at this point, the impossibility of accurate development of the bottom because of the small scale of the maps, and the infrequency of soundings accentuate the unreliability of the results.

That there has been deepening of parts of the channel is evident from the comparison of the earlier and latest soundings. That it is due to erosion of the bottom, which is known to be generally quite soft, is readily deduced. In general, the greatest increase in depth has been in the deeper areas, close to the thread of deepest water, which apparently has not shifted materially in the past seventy-five years. Some of this deepening may be attributed to the disturbance of the bottom by large vessels, maneuvering to and from docks along shore. This disturbance is very active when a vessel's powerful screws are reversed in checking speed, or when the vessel is started ahead or astern. Unquestionably a large increase by erosion beyond a depth of about 45 ft., involving large quantities of material, is not desirable from the standpoint of improvement, because the eroded material will be deposited in some other part of the channel or harbor, and will add to the cost of dredging in other places.

Although much larger sectional areas occur up stream and down stream from the Holland Tunnel, it is interesting to observe that near or along the line of the crossing (Canal Street), there is now (1932) an average depth of only 42 ft. and a section between pier ends of only 133 000 sq. ft. West of the middle the river has been dredged an average of about 3½ ft., which increased the section about 7 000 sq. ft. If this section is dredged from pierhead line to pierhead line to an average depth of 45 ft., the effective area will then be about 140 000 sq. ft. along the tunnel line. This would leave about 16 ft. of cover over the tunnel.

A study of the maps and surveys indicates that at times there has been slight shoaling on the New Jersey shore in the vicinity of the Delaware, Lackawanna and Western Railroad Ferry at Jersey City, N. J., and on the New York shore between 19th and 30th Streets. A recent development is the shoaling along the east shore between West 75th Street and West 152d Street, Manhattan, which is probably due largely to the fill made in extending Riverside Park. It may be probable that a tidal stream, particularly on the ebb, has the tendency to concentrate its energy along the line of deepest water, where higher velocities will occur, than it had before the deepening and concentration took place. Ambrose Channel, which was cut through the bar at the entrance to New York Harbor and deepened from 40 to 45 ft., now has tidal velocities only slightly less than those of the Lower Hudson River. However, they are considerably higher than were formerly observed in the adjacent shoaler channels that have been superseded by Ambrose Channel, although it is probable that the cross-section of the mouth of the Lower Bay is larger than at that former period.

In the absence of more reliable data and better interpretation of those at hand, it is believed that the conclusions reached and the implications suggested by Mr. Bevan are far from decisive. Even if the paper had demonstrated conclusively that the river's hydraulic regimen had been improved and its current velocities reduced, there is great uncertainty as to whether or not this hypothetical effect could be attributed solely to the narrowing caused by advancing the shore and pier lines. Other factors should be considered. Minor changes in the ranges and heights of tides produce marked current changes. Channel deepening and widening both in the Lower Hudson and in other parts of the harbor and its tributaries and entrances, and large changes in the volume of the tidal prism over the long period under consideration, probably have had an induced or direct effect upon the tidal reaction and currents. There is no question that the author's suggested method of "improvement" by narrowing solely to secure better hydraulic conditions, would injure rather than improve the Hudson River for navigation, even if it were not largely infeasible because of the existence of such barriers as the vehicular and railroad tunnels.

LYTLE BROWN,[8] M. AM. SOC. C. E. (by letter).—Data are presented in this paper, purporting to show that the successive encroachments of the piers into the fairway of the Hudson River in New York Harbor, have increased the cross-sectional area and decreased the tidal current velocities. Years ago apprehension was expressed by engineers of high standing that the contraction of the river by pier encroachment would produce an undesirable increase in tidal velocities. Experience has shown that the apprehension was not well founded. Whether such increase in cross-section as may have occurred, should be ascribed to the contraction or to other causes is an open question. Aside from the dredging done by the Federal Government, scour from ships' propellers may be a material factor. This waterway is extensively used by the largest vessels afloat. Their propellers are not far from the bottom when they pass through the channel, and the strong currents set up have a pronounced action in scouring the soft material in the river bottom.

The part of New York Harbor formed by the Hudson River is perhaps the most important artery of commerce in the world. It is well known that the commerce of the Port of New York far exceeds that of any other port of any nation, and the Hudson River Channel is the heart of the port. The number of vessel trips recorded in the Hudson River Channel exceeds 1 200 000 per annum. The commerce in a normal year, exclusive of car-ferry traffic and cargoes in transit, is about 40 000 000 tons, with a value of approximately $7 000 000 000. More than 100 000 000 persons are transported yearly over the channel. The cost of maintaining the channel averaged $125 000 per year during the five years, from 1927 to 1932. It is obvious that the exigencies of commerce completely overshadow any engineering problem of maintaining the depths in the harbor.

A comprehensive study of collisions in New York Harbor shows that during the five years, 1926 to 1930, inclusive, 118 collisions occurred in the Hudson River Channel, 6 vessels being sunk by collision. The average annual damages from collision in this section of the harbor exceed $300 000.

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[8] Maj.-Gen., U. S. Army; Chf. of Engrs., U. S. Army, Washington, D. C.

These figures show why it is that the War Department, in the administration of the laws for the protection and preservation of the navigable waters of the United States, is not inclined to view with favor the proposals advanced from time to time to further constrict the width of this great avenue of commerce.

H. A. MARMER,[7] M. AM. SOC. C. E. (by letter).—In this paper four theses are put forward by the author: (1) That successive encroachments on the navigable fairway of the Lower Hudson have set in action natural forces which have scoured the river bottom; (2) that this scouring has been of such magnitude as to more than counterbalance the restriction in width, having actually increased the cross-sectional area of the waterway; (3) that the range of the tide in the river has remained practically constant, in spite of the extension of the pierhead line into the channel of the river; and (4) that the velocity of the tidal currents has actually decreased.

Taking up each of these in turn, the author's comprehensive analysis of the changes of the cross-sectional areas in various sections of the Lower Hudson, as presented in Tables 3 and 4, clearly proves the first thesis. This analysis brings out the fact that an increase in the depth of the river has taken place coincident with the advance of the pierhead lines into the channel.

These tables at the same time show that the increase in depth has more than compensated for the restriction in width due to the extension of the pierhead line into the channel. As between 1855 and 1930, Table 3 shows that the average cross-sectional area has increased from 130 590 sq. ft. to 137 010 sq. ft., an increase of 5 per cent. Of this increase only 2 560 sq. ft., or 2%, was due to dredging. The second of the author's theses, namely, that the scouring has been of such magnitude as to have increased the average cross-sectional area of the Lower Hudson, is thus established. It is to be noted, however, that this increase in cross-sectional area is an average increase, and that for seven of the eighteen sections, the area now is less than in 1855, in spite of the dredging that has been done in these sections.

The third thesis, that relative to range of tide, presents a much more difficult question than that centering in the change of area of cross-section, because the latter can be determined readily from the charts showing the hydrographic features of the river at different dates. However, the range of the tide at any place varies from day to day, from month to month, and from year to year. Quite apart from the effects of wind, weather, and fresh-water run-off, the variations in range go through various cycles, with periods up to nineteen years. It follows, therefore, that two series of observations made at different times at the same place will give different ranges of tide, unless these observations in each case cover a period of nineteen years.

In connection with most engineering operations, it is impractical to observe tides over periods of nineteen years. Instead, observations are made over much shorter periods of time—from a month to a year—and the results derived from these observations are reduced to mean values.

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On the open coast, or in large coastal bodies of water, the range of the tide from a given series of observations may be reduced to a mean value either by comparison with simultaneous observations at some primary tide station, or by the use of a factor, depending on the longitude of the moon's node. Investigation proves that even from so short a series of observations as a month the value of the mean range can generally be determined correctly within 0.1 ft.

For tides in tidal rivers, however, the determination of the mean range is much more difficult than on the open coast or in large bays or sounds because, in addition to the fluctuations in range due to purely tidal causes, there are the fluctuations arising from the variations in fresh-water run-off. In streams that carry the drainage waters from large areas, the variations in range due to the fluctuations in fresh-water flow are frequently much greater than those due to purely tidal causes.

It is obvious, therefore, that the determination of the mean range of tide at any point in a tidal river from a short series of observations is far from a simple matter. Theoretically, it is possible to correct the range for the fluctuations in fresh-water flow; but practically this is not feasible for the simple reason that the data requisite for this are not at hand.

In the Hudson River, the fresh-water flow, as a rule, is greatest in the spring months and least in the summer and fall months. The true tidal range, therefore, can be determined more accurately from observations covering the summer and fall months, when the disturbing effects of the fresh-water flow are least. In Table 5 the author gives the mean range of the tide at Yonkers, N. Y., for a number of years, as determined from observations covering the three months of July, August, and September. This table illustrates clearly that in a tidal river the range of the tide derived from several months of observations and reduced to a mean value by correction for the longitude of the moon's node, will exhibit fluctuations of 0.1, or even, 0.2 ft., from year to year.

Such fluctuations in the values of the mean range are to be ascribed to difficulties involved in the observations themselves, and to fluctuations arising from the effects of wind, weather, and fresh-water run-off. It follows, therefore, that if from observations at any point in the Hudson River a difference in the mean range of as much as 0.1 or 0.2 ft. is found, as between two years, it cannot be concluded that this difference indicates a change in the range at that place. The author concludes that the data of Table 5 are indicative of the constancy of the range at Yonkers.

While the range for any one year in Table 5 cannot be considered as giving the mean range to better than 0.1 ft., the mean value for a group of three years is obviously a much closer approximation to the mean value than any one year. If from the data of Table 5 averages are formed for 3-year groups, the following values are derived:

1910–12..........	3.31 ft.	1921–23..........	3.49 ft.
1913–15..........	3.35 ft.	1924–26..........	3.49 ft.
1916–18..........	3.39 ft.	1927–29..........	3.55 ft.

It is a striking fact that each of the values for the 3-year groups shows an increase in range from 1910 to 1929, the difference between 1910–12 and 1927–29 being 0.24 ft. This is altogether too large to be ascribed to such fluctuations as characterize the means from year to year. The conclusion is inescapable that, on the basis of the data in Table 5, there has been an increase in range of tide at Yonkers between 1910 and 1929, of about 0.25 ft.

If the range of tide at Yonkers derived from the more recent observations, is compared with the earlier observations cited by the author in Table 5, it appears that there has been a decrease from about 4.0 ft. in the Fifties, and 3¾ ft. in 1898, to the present value of 3.55 ft. To evaluate, properly, the effects of the extension of the pierhead line on the range of the tide, it will be necessary to consider these changes in range in greater detail—the decrease to 1910 and the subsequent increase.

The fourth thesis in the paper, that relating to the current, offers greater difficulties than the thesis relating to the tide. As the author states, current measurements involve numerous difficulties. In addition, the current at any point changes from day to day. In part, this change is of a periodic nature, similar to that in tides, and, in part, it is of a non-periodic nature, due to fluctuations in wind, weather, and fresh-water run-off. A further complication arises from the fact that the current in any cross-section varies from point to point both in horizontal and vertical directions.

To determine whether or not there has been any change in the tidal current in a river, from short series of observations made at different times, is thus not a simple matter. To begin with, the observations must be made at the same point, in order that differences due to differences in position may not be introduced; and, in the second place, the results must be reduced to mean values. This is done most conveniently by taking the changes in the current to be proportional to the changes in range of the tide.

The current observations that have been made in the Hudson River, have been primarily for the purposes of the mariner, and to bring out the local peculiarities at different points. These observations, therefore, do not lend themselves readily to determining whether or not there has been a change in the velocity of the current consequent on the extension of the pierhead line into the channel.

The author concludes that the mean tidal current velocity must have decreased as a result of the pierhead extension into the Hudson, inasmuch as the cross-sectional areas have increased, while the range of the tide has remained substantially constant. This is a logical deduction in so far as the tidal current alone is concerned; but the Hudson River also serves as a channel for the drainage waters from a large territory. These drainage waters set constantly seaward, giving rise to a non-tidal current, which lessens the flood current and increases the ebb current.

Since the drainage waters are less dense than the sea water brought in by the tide they tend to remain on the surface. This is evidenced by the fact that, in the Hudson River, the velocity of the ebb current is greater than the flood near the surface, while below mid-depth the flood current is equal to or frequently even greater than the ebb. A decrease in the width of the channel, therefore, will decrease, in effect, the cross-sectional area of the channel for the drainage waters and give rise to an increased velocity for the ebb current.

In assembling and discussing the data bearing on this problem of the improvement of the Hudson River, the author has brought forward an important engineering problem which merits much more research than has been given it previously in this country.

Lynne J. Bevan,[8] M. Am. Soc. C. E. (by letter).—The writer appreciates the constructive criticism presented in the discussion. General Brown's suggestion that ships' propellers may be a material factor in scour is interesting. Intense agitation near the river bottom tends to disturb the stability of the bed and gives Nature a better chance to operate. If propeller agitation is a considerable factor in scour, is it not reasonable to expect the maximum scour along the line most used by the larger steamers, and in the areas agitated by their warping-in operations? From data in their offices, the Engineers of the U. S. War Department might be able to contribute something definite in this connection. From a casual inspection of the sections plotted, however, the scour appears to have occurred fully as much, if not to an even greater extent, along the New Jersey side as along the more intensely navigated Manhattan side.

General Brown's comments concerning the commerce raises questions outside the scope of the paper. It is suggested, however, that the 119 collisions between 1926 and 1930, inclusive, were in a large measure caused not by congestion—which is comparatively non-existent in the Hudson River—but either by poor visibility or by negligence in navigation. Who will argue that the 119 collisions are more than would have occurred if the width of the river at the Lackawanna and Erie Ferries had not been reduced since 1767 from 6 000 ft to 3 000 ft?

In connection with the discussion by Mr. Moss, the writer would like to explain that his original interest in this hydraulic problem as applied to the Hudson River was aroused in 1931 when he was retained in behalf of an applicant for pierhead extension and discovered that the Secretaries of War in 1913 and in 1920 had denied similar applications with memoranda stating that as a result of pier extensions, "the speed of the tide has already been increased over 60 per cent." Consequently, the intent in the paper under discussion was to bring consideration of the Hudson River problem out into the realm of sound and broad hydraulic theory. However, Mr. Moss' assertions of "inaccuracies and errors in assumptions and basic data" used, should not pass unchallenged.

There may be some inadvertent errors, but most of the alleged errors were deliberate as they were in the direction of conservatism in the main thesis. The first "error in assumption," namely, that the effective cross-section is included between the pierhead lines, was deliberately incorporated since this assumption is less favorable to the thesis and hence makes the argument less susceptible to criticism of details. This holds whether Mr. Moss' contention refers to a pier extending westerly from Manhattan in 1855 to the 1930 pierhead line, or simply to tidal water flowing behind later pierhead lines. In general, if the effective cross-section area is increased to admit the tidal flow back of the pierhead line, the areas in Table 4, Columns (4) and (6), are increased more than those in Columns (2) and (3), and thus the mean velocity of the tidal currents is still further reduced in comparison with the velocities of earlier years.

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[8] Cons. Engr., New York, N. Y.

As to the second "error in assumption," that is, "that the charts show soundings of the date of publication," the writer, of course, made no assumption that the soundings shown on the 1930 chart were made in 1930. He simply used the 1930 chart as the best information available concerning depths existing in 1930—as do navigators, the Army engineers, the U. S. Coast and Geodetic Survey, and other interested parties. Through the courtesy of the late Robert Lee Faris, M. Am. Soc. C. E., the writer was furnished, in May, 1931, with copies of the 1930 charts with the dates of the surveys indicated thereon. The part of the river from Section 4 to Section 9 shows 52.8% of the area surveyed subsequent to 1910. It is regrettable, of course, for the purpose of this discussion, that absolutely complete sounding data for each date considered are not available; but the absence of recent soundings in a substantial part of the river bed may be an indication, perhaps, that shoaling has not occurred sufficiently to interfere seriously with navigation.

Table 6 shows quite generally a deepening of the fairway. The only shoaling in excess of the nominal amount of 1.0 ft has occurred in grid areas largely behind the pierhead lines. In the case of the grid, 12-*A*, it is entirely behind the bulk-head line! The mean depth between the 1930 pierhead lines of the eighteen sections of Table 4 was 35.9 ft in 1855, and 42.2 ft by the 1930 charts. This is an increase of 17.6%, of which 15.3% is due to scouring by Nature and 2.3% to dredging.

As to Table 7, it has only to be pointed out that, while at the exact section used structures and fill did not reach out to the established pierhead lines, a short distance above and below the exact section, structures are shown substantially out to the pierhead line. For instance, the area of Section 3 in 1930 is presumably computed, in Table 7, to the end of the Pennsylvania Ferry racks on the New Jersey side which are more than 300 ft inside the pierhead line; but the flow-control pier at the north end of the ferry racks extends almost to the pierhead line. In 1855, the irregularity of structures was, doubtless, greater than it is to-day.

Mr. Moss correctly notes that in Figs. 2 and 4 the 1914 and 1897 pierhead lines coincide, and that this is legally in error. The 1914 line was shown 100 ft out from the 1897 line for the reason that Pier 54 below, and Piers 59 and 60 above, Section 6, had been built out 100 ft under temporary permit, and this outer line was used as the conservative one, and to avoid the criticism just applied to Table 7.

Mr. Marmer's discussion is greatly appreciated as it comes from many years of competent study of tides. Had the writer had the benefit of frequent consultation with Mr. Marmer during the preparation of the paper, he might have endeavored to apply corrections due to the considerable decrease in tidal range from 1885 to 1910 and the partial return since 1910. The change during the entire period from 1855 to 1930 shows 10% decrease in tidal range, and, hence, aproximately the same decrease in tidal current quantities, and thus, in general, adds a factor of safety to the conclusions in the writer's paper.

As to the effect of the run-off of the Hudson River drainage area of 13 400 sq miles, the writer wishes to make reference to the comprehensive paper on "Tidal Phenomena in New York Harbor," by H. de B. Parsons, M. Am. Soc. C. E. This work provides comparisons of land-water discharge volumes with tide-water volumes, as follows:[9]

> "The mean yearly discharge of land-water run-off past Battery Place for 6 lunar hours is 543 600 000 cu. ft. * * * The average ebb flow will be the mean given above plus one-half of this net discharge, and the average flood volume will be the mean less one-half of the net discharge, or:
>
> "Average volume of ebb current, 6 990 600 000 cu. ft.
> "Average volume of flood current, 5 903 400 000 cu. ft."

Thus, land-water drainage is only 7.8% of the ebb tide.

Since 1855 the width of the fairway between pierhead lines has been decreased roughly 12.3%, so that the percentage increase in the ebb-current velocity near the surface might be 0.96% under Mr. Marmer's suggestion of substantial separation of fresh and salt water. The effect of this increase, as it applies to deep-draft vessels, is largely offset by the corresponding decrease in current velocities at greater depths.

However, the very fact that average depths have been increased by Nature in compensation of the narrowing of the river is suggestive that an increase of ebb-current velocities has taken place near the river bed. This idea conforms with the general hydraulic experience recalled in the second paragraph of the paper, that narrowing a channel tends to make velocity more uniform from the surface to the bottom. If this is so, the probabilities are that surface velocities have actually decreased because, as was shown in the paper, the mean velocity has decreased.

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[9] *Transactions*, Am. Soc. C. E., LXXVI (1913), p. 2038.

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APPLICATION OF DURATION CURVES TO HYDRO-ELECTRIC STUDIES

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WITH DISCUSSION BY MESSRS. PAUL F. KRUSE, DONALD H. MATTERN, JAMES E. STEWART, DON JOHNSTONE, L. T. GUY, JOEL D. JUSTIN, C. MAXWELL STANLEY, F. KNAPP, ROBERT F. EWALD, AND G. H. HICKOX AND G. O. WESSENAUER

SYNOPSIS

This paper outlines an accurate method of determining, from duration curves, the reliable power rate, and plant and reservoir capacities for a combination of hydro-electric plants and reservoirs on streams with similar flow distribution, but not necessarily in the same basin.

The method can be utilized for these problems only by studying data for exactly one reservoir cycle. This procedure is fundamental, and may have been overlooked by other investigators; at least, no study has been found in which this factor has been emphasized.

The use of the duration-curve method for hydro-electric studies is explained in this paper by applying it to some simple hypothetical combinations of hydro-electric plants. Then, for an actual proposed hydro-electric combination, the results obtained by the use of the curves are compared with those obtained by analytical week-by-week studies (analyzed by days where necessary).

INTRODUCTION

Investigation of the power possibilities of Cheat River, by the West Virginia Power and Transmission Company, indicated the desirability of a rapid, fairly accurate method of determining the reliable power rate, and plant and reservoir capacities to be expected from various plant combinations.

Lake Lynn, a 50 000-kw., run-of-river, hydro-electric plant, is 3.5 miles up stream from the mouth of Cheat River, near Morgantown, W. Va., and

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eventually there may be about a dozen plants in operation in that river basin. The question of determining the order, capacity, and storage for succeeding plant installations gives rise to a series of problems that are difficult to solve by existing methods.

Previous studies indicated that the next hydro-electric construction after the Lake Lynn plant should be a storage development above two high-head plants on the Blackwater River. The storage reservoir should have not only sufficient capacity to equalize and control its tributary stream flow and two-plant output, but also enough to aid in maintaining a high uniform power rate when acting in combination with the Lake Lynn plant. The problems of determining the combined dependable output from the Lake Lynn and Blackwater plants, and also the proper installed capacity for the latter, are complicated by the fact that occasionally potential energy drawn from storage may not be developed at the Lake Lynn plant. The occasional failure of storage water to create energy at this plant is due to the fact that some of the water would reach that point when the plant would be working at its maximum efficient capacity. A study of flow and resulting energy, based on monthly data, fails to reveal the extent of this loss. Accordingly, several analytical studies were made on a weekly flow basis, with detailed study of each day when the Lake Lynn plant would be working at, or close to, its maximum efficient capacity. The analytical studies thus became essentially day-by-day studies and involved excessive work. Therefore, it was obvious that some time could be spent profitably in devising a method which would expedite the solution of the complicated problems involved in the still later developments.

The two well-known methods of solving hydro-electric problems are the hydrograph and the mass curve. Some investigators have used duration curves for solving certain phases of hydro-electric problems, but no evidence has been found to indicate that such curves have been used to the extent outlined herein. In studies of power development in Northern West Virginia, F. W. Scheidenhelm and R. M. Riegel, Members, Am. Soc. C. E., used the duration curve in a manner that approaches more closely the method outlined in this paper than any others which could be found in a search of engineering literature.

This paper indicates that when the duration-curve method is applied to data for a complete reservoir cycle, it gives quick, accurate determinations of maximum rate of output, total output, division of output, and plant and reservoir capacities. Furthermore, a comparison indicates nearly as high a degree of accuracy for the results of the duration-curve method as for the more tedious hydrograph method.

NOMENCLATURE

Terms which might be misunderstood are defined, as follows:

Flow Duration Curve.—A curve secured from an arrangement of daily stream flows in the order of their magnitudes. The ordinates represent flow, and the abscissa for any ordinate represents the percentage of time during which flows greater than that ordinate occur.

Output Duration Curve.—A curve secured from an arrangement of daily stream power capacities in the order of their magnitudes. The ordinates represent kilowatts, and the abscissa for any ordinate represents the percentage of time during which stream-power capacities greater than that ordinate occur.

Reservoir Replenishment, or Rate of Reservoir Filling.—The rate of inflow to the reservoir, minus the rate of draft. Replenishment occurs only when the inflow exceeds the draft, and hence it is always a positive quantity.

Reservoir Depletion, or Rate of Reservoir Emptying.—The rate of draft on the reservoir, minus the rate of inflow. Depletion occurs only when the draft exceeds the inflow, and hence it, too, is always a positive quantity.

Run-of-River Plant.—A hydro-electric plant that depends upon the daily flow of the river for its energy. It may have pondage, but not storage.

Storage Plant.—A hydro-electric plant that receives all its water directly from a large reservoir. The output of a storage plant is not affected by daily fluctuations in flow.

Partial Storage Plant.—A hydro-electric plant the tributary drainage area of which contains a storage reservoir with no plant intervening between it and the plant in question. Part of the flow from the drainage area above the plant is regulated by the storage reservoir. The remainder of the flow is unregulated.

Primary Energy.—Dependable energy from a plant or group of plants; that is, energy that can be relied upon for carrying a portion of the system load for which no other source of supply is regularly maintained.

Secondary Energy.—Energy other than primary, generated by the plants.

Waste Energy.—Energy lost in water that is by-passed necessarily, usually resulting from floods.

Controlled Energy.—Energy obtained from such flow as is regulated by a storage reservoir.

Uncontrolled Energy.—Energy obtained from unregulated flow.

Potential Energy.—Energy at a plant as limited by stream flow, plant efficiency, and average net head.

BASIC ASSUMPTIONS

The duration-curve method, as developed in this study, is based on the following assumptions:

(1) That the streams under consideration have the same distribution of flow; that is, flows that have the same duration-curve abscissa occur at the same time on each of the streams.

(2) That the water discharged from an upper plant is immediately available for use at the next down-stream plant.

(3) That the storage available at each reservoir is unlimited.

GENERAL DESCRIPTION

The first step in solving any hydro-electric problem by the duration curve is the selection of a period of flow for analysis, generally from a study of the mass curve. The selected period ordinarily will extend over several dry years, and will give the minimum reliable rate of controlled flow from storage. Its length, however, may vary from a few months to several years, depending upon the nature of the storage facilities.

It is axiomatic that the duration-curve method gives results for the period for which the curves are made and that the curves must be made up from all, but no more than, the data for the period chosen. In other words, the duration curve for determining dependable output and plant capacities must be prepared from flow records for a complete cycle, a resorvoir cycle being the elapsed time from full reservoir to full reservoir.

The duration curve for determining storage reservoir capacity must be prepared from flow records either for the period from the beginning of the cycle to the time of maximum draw-down, or for the period from the maximum draw-down to the end of the cycle.

The entire drainage area supplying water to the plants of a combined development is divided into a number of parts, each part being the area, tributary to a plant, below all up-stream dams. An output duration curve is then prepared for each of the partial areas. Each of these curves includes all the potential energy derived from flow from the area in question, at all the down-stream plants. For reasons which will appear later, the potential energy of each plant is shown separately on the duration-curve diagram.

After constructing output duration curves for each area, the curves representing uncontrolled energy are added, in order to make a composite curve. This is done by adding graphically the ordinates corresponding to the same abscissa of each of the separate uncontrolled energy curves. The curves of controlled energy are then added to the composite curve of uncontrolled energy in such a way as to give the maximum dependable energy output for the resulting combined uncontrolled and controlled energy curve. The plant capacities, as well as the maximum dependable energy output rate, are read directly from this final curve.

The maximum dependable energy output rate is applied to the storage-requirement duration curves in order to determine the demand on the stored water. The difference between the reservoir draft and inflow is the required storage capacity.

The duration-curve method may best be explained by considering its application to a few hypothetical cases.

THEORETICAL APPLICATIONS

Case 1.—Combination of a Run-of-River Plant and a Storage Plant on Adjacent Streams.—It is often desirable to install hydro-electric plants on two adjacent streams in such a manner that the two plants in combination will maintain a uniform dependable output. It may be that only one of the streams presents storage facilities, so that the problem of determining the dependable output becomes complicated. Accordingly, the method of handling the problems presented by two such plants is discussed herein.

It is assumed that in the river basin of Fig 1 there is a run-of-river plant, *B*, and a storage plant, *C*, and that any desired plant capacities can be installed at each of them. Flow duration curves are not used directly for determining output rate and plant capacity; hence, they need not be constructed. The output duration curves are constructed by collecting and tabulating the daily flows as if to make a flow duration curve. It will be found convenient to collect the daily flows in groups. The range of each group should not exceed 5% of its median value, in order to insure accurate results. If the daily flows are thus arranged, it is important that each group be represented by its lowest value and not by its mean, because any point on a duration curve indicates the time during which that particular flow is equalled or exceeded, and use of the mean will give duration curves with ordinates that are too large. These daily values (or groups of values) are then multiplied by a constant for each particular plant to convert them to power. This constant is the term, $\frac{He}{11.8}$, in the equation:

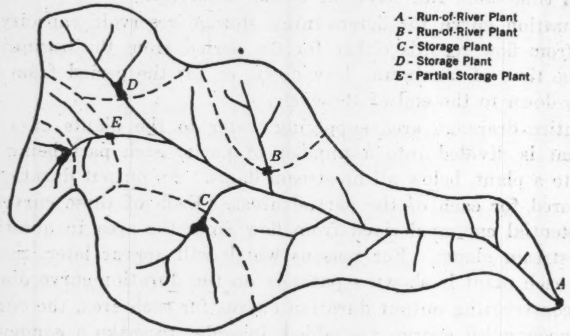

FIG. 1.—RELATIVE LOCATION OF PLANTS

$$KW = Q\,\frac{He}{11.8} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, KW is output, in kilowatts; Q is flow, in cubic feet per second; H, head on plant, in feet; and, e, plant efficiency. These power ordinates are plotted against the percentage of total time during which they are equalled or exceeded, thus forming the output duration curve.

In this example the area under the curve of Fig. 2(*a*) represents the total potential energy at Plant *C*. (The subdivided areas refer to Case 2, explained subsequently.) As drawn, the curve indicates the normal stream frequency, but as the flow from the area above Plant *C* can be controlled completely by a storage reservoir, it may be converted into energy at such times and at such rates as desired; hence it is not correct to state that Fig. 2(*a*) represents the output frequency of Plant *C*. This curve is useful only as an area; consequently, in practice, it is not necessary to construct it but merely to compute its area from the power values of the duration tabulation, which is more accurate than measuring the attenuated area of a duration curve with a planimeter.

The area under Curve *B* in Fig. 2(*b*) represents the total potential energy at Plant *B*, a run-of-river plant with uncontrolled flow that must be used as it occurs.

The determination of the primary energy obtainable from the combination of Plants *B* and *C* is made as follows: It is assumed that the demand for power is of such a nature that a uniform rate of output is required. This is represented on a duration curve by a horizontal line, indicating an equal rate of output at all times. The ordinates of Curve *B*, Fig. 2(*b*), cannot be changed since Plant *B* has no storage and the power must be developed when the water is available, or it will be lost. Since all the energy of Plant *C*, represented in Fig. 2(*a*), is controlled energy, it may be drawn upon in such amounts and at such times as desired. If it is drawn in such a manner as to furnish a uniform rate of energy output from the combination with Plant *B*, the duration curve of their combined energy output may be shown graphically as Line *DD′* in Fig. 2(*b*). The energy output of Plant *C* is represented by the shaded area, *AD′L*, which equals the area of Fig. 2(*a*), and represents the manner in which the output from Plant *C* is drawn. The total area under the line, *DD′*, is the total primary energy that can be developed at the two plants.

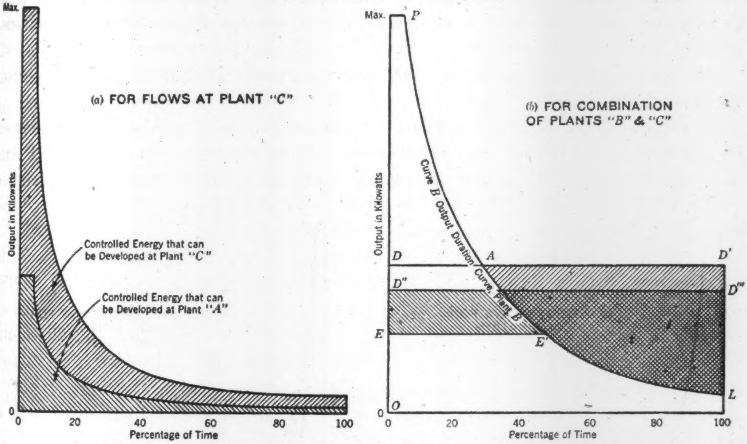

FIG. 2.—OUTPUT DURATION CURVES SHOWING PRIMARY ENERGY OBTAINABLE FROM PLANTS *B* AND *C*.

The installed capacities required for each of the two plants can be read from Fig. 2(*b*). The ordinate, *OD*, is the dependable output of the two plants, and it also represents the installed capacity required at Plant *B* to maintain the uniform output, *OD*; the ordinate, *LD′*, represents the similar required capacity at Plant *C*.

The area above *DA* and under the curve represents potential energy at Plant *B* that cannot be developed with a plant capacity limited to *OD*. Part of it may be realized as secondary energy by increasing the capacity of Plant *B*. The amount of such increase, if any, should be determined by a cost and value study.

The next step is the determination of the storage reservoir capacity controlling the flow to Plant *C*. This is accomplished by means of output duration curves for the flow occurring during the period extending from the time of full reservoir to the time of maximum draw-down. The month of maximum draw-down can easily be determined from a mass curve of reservoir inflow plotted by months. The day of maximum draw-down can then be determined by plotting the mass curve of reservoir inflow for the critical month.

Output duration curves for the flow at Plants *B* and *C* for this period (from full reservoir to point of maximum draw-down) are constructed as shown in Fig. 3. The dependable output of the two plants, as represented by the ordinate, *OD*, of Fig. 2(*b*), is applied to the duration curve for Plant *B* (Fig. 3(*a*)) to obtain the horizontal line, *DD′*. The area, *MD′N*, represents the total demand on the storage reservoir for that period. This demand is plotted in Fig. 3(*b*) with the inflow duration curve for Plant *C* (full reservoir to point of maximum draw-down). The curve, *PTQ*, is the duration curve of inflow to the reservoir in terms of potential energy. The curve, *M′TN′*, is the duration curve of energy demand at Plant *C*; the ordinates are those of the area, *MD′N*, in Fig. 3(*a*).

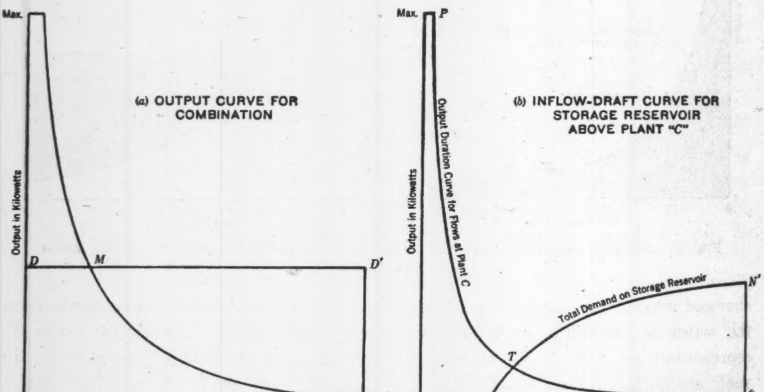

FIG. 3.—DURATION CURVES FOR COMPUTING STORAGE CAPACITY CONTROLLING FLOW OF PLANT *C*.

In Fig. 3(*b*), the area, *M′TQK*, is common to both the inflow and outflow areas and represents energy that is drawn from the reservoir as fast as it enters. The area, *TN′Q*, represents that demand on energy which must be provided by storage, and the area, *OPTM′*, that energy from inflow which must be stored until needed. The ordinates of the area, *TN′Q*, represent reservoir depletion and those of the area, *OPTM′*, reservoir replenishment.

Since the area, *M′TN′K*, represents the total demand on the storage reservoir and the area, *OPTQK*, the total inflow to the storage reservoir during the period from full reservoir to point of maximum draw-down, the difference between these areas must represent the net amount drawn from the storage reservoir during the period. Since the period extended to the point of maximum draw-down, the net amount drawn from the reservoir represents the maximum storage capacity needed in that period. Of course, the area is in terms of energy and must be transformed to second-foot-days, or acre-feet, by using the energy constant previously determined for Plant *C*.

In a similar manner, the draw-down at any point can be obtained from duration curves constructed for the flow during the period beginning with full reservoir and ending with the point studied. All the duration curves needed for a study can be made at the same time by distributing the data into the necessary periods when the flows are being collected into groups.

In the preceding discussion of plant capacities it was assumed that the desired plant capacity could be installed at *B*. In some cases the capacity of Plant *B* as determined by the ordinate, *OD* (Fig. 2(*b*)), may be larger than is needed. In other cases, Plant *B* may have been installed already and it is then desired to install the second plant on another stream with a view to regulating the output from both plants.

If Plant *B* has been installed, the method of attacking the problem is a modification of the foregoing. Let the capacity of Plant *B* be some quantity as *OE* shown in Fig. 2(*b*). Then, the area under the line, *EE′*, can be developed as primary energy at Plant *B*, while that above the line will be wasted.

The area representing controlled energy (Fig. 2(*a*)) now may be placed above the line, *EE′L* (Fig. 2(*b*)), as indicated by the shaded area, *D″D‴LE′E*. As before, the area under the line, *D″D‴*, represents the total primary energy that can be developed at the two plants. The capacity of Plant *C* may be read from Fig. 2(*b*) as the ordinate, *LD‴*. The necessary storage capacity at Plant *C* is determined in a manner similar to that previously described.

Thus far, it has been assumed that the reservoir capacity is unlimited. If the storage requirement as determined is too large to be developed economically, the procedure is to return to the mass curve, lower the rate line, and use the data for the shortened period in another study.

It may be noted here that the rate line used on a mass curve of flow, from the area above Plant *B*, say, is not the rate line for the combination of plants; but it may be used to determine the period of reservoir depletion and replenishment when the streams have the same distribution of flow, as stated in the first basic assumption.

Case 2.—Combination of a Run-of-River Plant and a Storage Plant on the Same Stream.—The treatment of the case in which the two plants are on the same stream is very similar to that in Case 1. Fig. 1 shows the respective locations of the plants, *A* being a run-of-river plant, and *C*, a storage plant with complete control of the flow above *C*. First, consider that unlimited plant capacity can be installed at both plants.

The outstanding feature of this set-up, differentiating it from the first set-up, is the fact that all water used for energy at Plant *C* is again available at Plant *A*, where it must either be utilized for energy or by-passed. Determination of the quantity by-passed is the problem demanding solution.

The curve, *PSL*, of Fig. 4(*a*) is the output duration curve for flows at Plant *A*, which, of course, does not include the flows from the area above Plant *C*. The output duration curve for flows from the area above Plant *C* as drawn in Fig. 2(*a*) differentiates between the amounts of potential energy at Plants *A* and *C*. For example, 1 cu. ft. per sec. from the area above Plant *C* is considered to be capable of generating two units of energy at Plant *C* and one unit of energy at Plant *A*.

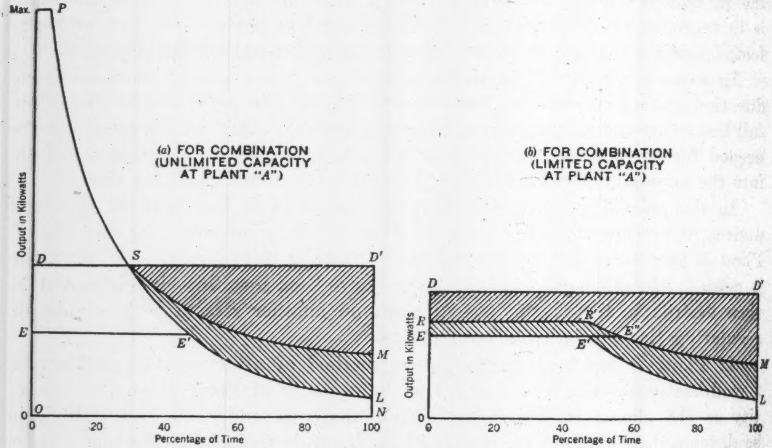

FIG. 4.—OUTPUT DURATION CURVES FOR CASE 2; RUN-OF-RIVER PLANT AND STORAGE PLANT ON SAME STREAM

The maximum uniform dependable output from the combination may be obtained as explained for the preceding case. In Fig. 4 (*a*), it will be noted that each ordinate of the area representing controlled energy is still so divided as to represent two units of energy developed at Plant *C* for each unit of energy developed at Plant *A*. The ordinate, *OD*, represents the uniform dependable output from the combination and also the capacity to be installed at Plant *A*. The ordinate, *MD'*, represents the capacity to be installed at Plant *C*. The area, *ODSMN*, represents the energy developed at Plant *A*, and the area, *SD'M*, the energy developed at Plant *C*.

When the capacity of Plant *A* is limited to *OE*, as in Fig. 4(*a*), the areas of Fig. 2(*a*) are applied as shown in Fig. 4(*b*). Since the capacity of Plant *A* is limited to *OE*, all potential energy (Area *EE''R'R*) above the line, *EE″*, that is shown as being available at that plant, cannot be developed and must be by-passed. The effect of this reduction in energy is to lower the rate line, *DD′* until this area is compensated.

In making this adjustment, it should be noted that at all points to the right of *E″*, the ratio of controlled energy developed at Plant *C* to controlled energy developed at Plant *A* must be the same after the adjustment as before. The installed capacity at Plant *C* is equal to the ordinate, *MD′*, of Fig. 4(*b*) after the adjustment has been made. The reservoir capacity required at Plant *C* may be determined in a manner similar to that described in Case 1.

Case 3.—Combination of a Run-of-River Plant and a Partial Storage Plant on the Same Stream.—Consider the case of a partial storage plant, *E*, on the same stream and above *A* in Fig. 1. Water from the storage reservoir may be carried to Plant *E* by a canal, tunnel, or other suitable means. Provision is also made at Plant *E* to utilize the energy of the flow from the area below the storage reservoir and above Plant *E*.

Let the curve, *PSL*, of Fig. 4(*a*), be the output duration curve for flows at Plant *A* (not including those from above Plant *E*). Fig. 5(*a*) is a composite output duration curve for flows at Plant *E*, distinguishing not only between flows from the area below the reservoir (uncontrolled) and flows at the reservoir dam site (controlled), but also between the potential energy of these flows at Plants *E* and *A*.

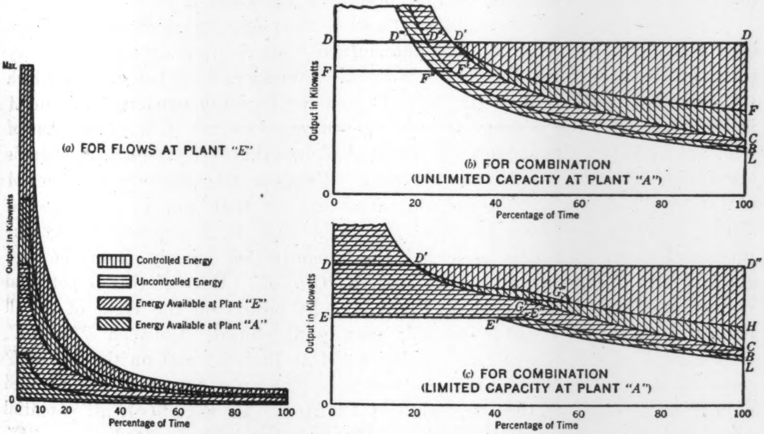

FIG. 5.—OUTPUT DURATION CURVES FOR CASE 3; RUN-OF-RIVER PLANT AND PARTIAL STORAGE PLANT ON SAME STREAM

Fig. 5(*b*) shows the re-arrangement of the duration curves when unlimited capacity is available at both plants. The uncontrolled energy ordinates of Figs. 4(*a*) and 5(*a*) are added graphically and represent the total energy which either must be utilized immediately, or lost. The area representing controlled energy is distributed as shown. The ordinate, *OD*, represents the dependable output from the combination of plants. The ordinate, *OD,* also represents the capacity of Plant *A*. The sum of the ordinates, *BC* and *FD,* represents the capacity of Plant *E*.

With regard to the production of primary energy: It may be seen by referring to Fig. 5(*b*) that, with the arrangement there shown, Plant *A* is forced to operate at the capacity, *OD,* during periods of high flow, while Plant *E* does not begin producing primary energy until the point, *D″*, is reached. It will be noticed that potential uncontrolled energy at Plant *E* in amounts greater than the ordinate, *FD,* is available at any point to the left of *D′*, indicating that the capacity of Plant *A* may be reduced to *OF* and that Plant *E* may be operated at a rate, *FD,* until *D′* is reached, at which point draft on storage begins. It should be noted in this connection that when the capacity of Plant *A* is reduced to *OF,* all the potential energy at that plant from the drainage area above Plant *E* lying above the line, *FF′* (Area *D‴D″F′F″*), cannot be developed and, therefore, is wasted.

The total energy developed at Plants *A* and *E* is represented by the area under *DD*. The division of energy between the plants is indicated by the legend. The storage requirement for the reservoir above Plant *E* is determined in a manner similar to that described for Case 1.

Thus far, in the discussion of Case 3, the capacity of both plants has been considered as unlimited and determined solely as that capacity necessary to maintain a uniform dependable output. Consider, now, the case in which the capacity of Plant *A* is limited. Unless the limiting capacity of Plant *A* is less than *OF,* the arrangement of the curves does not differ from that of Fig. 5(*b*). When the capacity of Plant *A* is less than *OF,* the arrangement is similar, but with a slight modification. Suppose the capacity of Plant *A* is *OE,* in Fig. 5(*c*). The uncontrolled energy that can be developed at Plant *A* from the area between Plants *A* and *E* is represented by the unshaded area. To this is added the uncontrolled energy from the area above Plant *E* represented by the area, *EDG′CLE′*. Part of this is potential energy at Plant *E* and part at Plant *A*. At points on the right of *E″* all potential energy at Plant *A* actually may be developed. Between *E″* and *E′*, only a part of this energy may be developed at Plant *A* and on the left of *E′* none of it may be developed. Accordingly, the only potential uncontrolled energy above *EE″* is that available at Plant *E*. In Fig. 5(*c*), all potential uncontrolled energy not available at Plant *A* has been omitted above *EE″*. The potential controlled energy at both plants (Fig. 5(*a*)) is then applied to Fig. 5(*c*) (Area *D′D″CG′*) to maintain a uniform dependable rate, shown by the line, *DD″*.

None of the potential controlled energy at Plant *A* can be developed at points to the left of *E″* in Fig. 5(*c*), and only a part of such energy for a short distance to the right of *E″*. The area, *G′D′G″*, representing this wasted energy must be discarded and the vacant space filled with energy available at Plant *E*. The wasted energy lowers the line, *DD″*, and reduces the uniform dependable output.

The storage requirement for the reservoir above Plant *E* may be determined in a manner similar to that outlined for Case 1.

Case 4.—Combination of a Run-of-River Plant and Two Storage Plants.—One more set-up will be discussed briefly to illustrate further possibilities. Consider the case of three plants—*A, C,* and *D*—arranged as illustrated in Fig. 1, with Plant *A,* a run-of-river plant, and Plants *C* and *D,* storage plants located above Plant *A,* but on different branches of the main stream. To facilitate comparison, assume that Plants *C* and *D* have identical possibilities for power development. Fig. 6(*a*) shows one distribution of the controlled energies from Plants *C* and *D*. In this diagram, the energy from Plant *C* is placed in the most favorable position; that is, the operating conditions for Plant *C* are better than those for Plant *D* in that the plant and storage reservoir capacities for Plant *C* are less than if the Plant *C* energy had been placed in the position occupied by the Plant *D* energy.

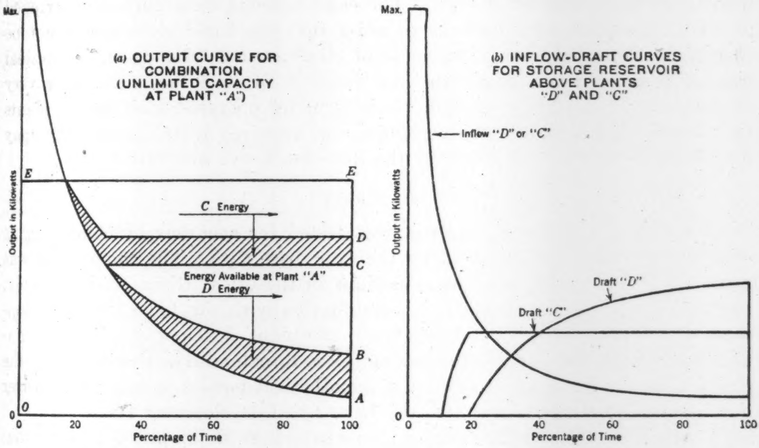

FIG. 6.—OUTPUT CURVES FOR CASE 4; RUN-OF-RIVER PLANT AND TWO STORAGE PLANTS

For example, the required installed capacity at Plant *C* is the ordinate, *DE,* while the necessary capacity at Plant *D* is the ordinate, *BC.* The advantage enjoyed by Plant *C* with regard to storage reservoir capacity requirements may be seen by referring to Fig. 6(*b*). While for the complete reservoir cycle the energy produced at Plant *D* equals that produced at Plant *C,* for the period from full reservoir to point of maximum draw-down, the energy demand on Plant *D* is greater than that on Plant *C* as shown by the areas under the respective draft curves of Fig. 6(*b*). Since the storage requirement is the difference between the areas under the draft and inflow curves, the storage requirement of Plant *C* will be the smaller.

It is possible to operate Plants *C* and *D* so that their plant and storage capacities are the same. This may be accomplished by distributing the controlled energies so that the outputs of the two plants are equal at all times. Since Plants *C* and *D* usually do not have similar characteristics, the advantage of one over the other may be measured by considering the value of the output weighed against the cost of development, as indicated by the required plant and reservoir capacities. If capacity at Plant *A* is limited, the procedure is the same, except that care must be taken to discard all potential energy at Plant *A* that cannot be developed when that plant is operating at its maximum capacity. Storage requirements are handled as in all other cases.

It is thought that sufficient examples have been given to illustrate the method of using duration curves. The cases selected were purposely stripped of all unnecessary complications in order that the basic ideas might not be obscured. Consider a single example of eliminated detail: In any practical case of a combination of plants, the question of load factor becomes very important. If the hydro-electric plant is being operated with steam plants and is used only to carry the peak load, the required installed capacity may be several times that indicated by the duration curve ordinate.

Accuracy

The duration curve method has been tested for accuracy by comparing it with the hydrograph method, using the same base data. The comparison was made for two proposed plant combinations in the Cheat River Basin, namely, (*a*) the Big Sandy development in combination with the Lake Lynn development (Case 2); and (*b*), a Blackwater two-plant development in combination with the Lake Lynn development (Case 3, extended). Furthermore, the comparison has been made (*c*) for a hypothetical partial storage Blackwater development in combination with the Lake Lynn development (Case 3).

The Big Sandy and Blackwater plants are planned to operate in combination with the Lake Lynn hydro-electric plant and large base load steam plants. Accordingly, in Studies (*a*) and (*b*) it was assumed that the hydro-electric plants would be operated so as to carry the peak of the system load. Therefore, plant and system load factors were considered. In Study (*c*), a load factor of 100% was assumed for both plants.

The detailed investigation involved studies of the reservoir cycles by the hydrograph method, using weekly flows. In addition, the daily flows were scanned, and all large flows were properly divided into primary, secondary, and waste energy, so that, for all practical purposes, the investigations were studies of daily flows.

The reservoir storage capacity was determined in Studies (*a*) and (*b*) by a modification of the method described in this paper, in which accuracy was sacrificed for speed. Study (*c*) was made to check the exact method of determining reservoir storage capacity and, hence, was stripped of unnecessary complications. In these studies, the desirable plant capacities were determined in a manner common to both the hydrograph and duration curve methods.

Table 1 records the percentage variation obtained by the two methods for the various quantities involved. In evaluating the "error," the hydrograph method was considered correct and the duration curve method at fault.

It may be seen that the error in the determination of the total primary energy was less than 0.5 per cent. For Studies (*a*) and (*b*), in which a modified method of storage determination was used, the storage capacity required was obtained within an error of 1.5 per cent. On the other hand, when the exact method was used, as in Study (*c*), the error was reduced to less than 0.5 per cent.

TABLE 1.—Comparison of Results

Item No.	Description*	Hydrograph method	Duration curve method	Error	Error (percentages)
	Study (*a*) — Big Sandy Development				
1	Lake Lynn uncontrolled Class *A* primary energy	503 400	491 000	—12 400	— 2.46
2	Lake Lynn controlled Class *A* primary energy	87 000	94 500	+ 7 500	+ 0.86
3	Big Sandy controlled Class *A* primary energy	760 900	772 400	+11 500	+ 1.51
4	Total Class *A* primary energy	1 351 300	1 357 900	+ 6 600	+ 0.49
5	Class *B* primary energy	89 000	89 700	+ 700	+ 0.79
6	Secondary energy	158 200	154 200	— 4 000	— 2.53
7	Waste energy	110 500	104 300	— 6 200	— 5.61
8	Plant use	5 100	4 900	— 200	— 3.92
9	Total energy	1 714 100	1 711 000	— 3 100	— 0.18
10	Reservoir capacity, in thousands of acre-feet	299	298	— 1	— 0.33
	Study (*b*) — Blackwater Development				
11	Lake Lynn uncontrolled Class *A* primary energy	528 900	511 800	—17 100	— 3.23
12	Lake Lynn controlled Class *A* primary energy	41 500	49 500	+ 8 000	+19.28
13	Blackwater uncontrolled Class *A* primary energy	294 700	299 900	+ 5 200	+ 1.76
14	Blackwater controlled Class *A* primary energy	560 100	564 400	+ 4 300	+ 0.77
15	Total Class *A* primary energy	1 425 200	1 425 600	+ 400	+ 0.03
16	Class *B* primary energy	94 400	94 200	— 200	— 0.21
17	Secondary energy	186 500	188 500	+ 2 000	+ 1.07
18	Waste energy	119 700	141 300	+21 600	+18.05
19	Plant use	5 800	4 600	— 1 200	—20.69
20	Total energy	1 831 600	1 854 200	+22 600	+ 1.23
21	Reservoir capacity, in thousands of acre-feet	132.7	131.0	— 1.7	— 1.28
22	Secondary and waste energy at Blackwater	14 300	8 600	— 5 700	—39.9
	Study (*c*) — Blackwater Development				
23	Lake Lynn uncontrolled primary energy	112 670	112 700	+ 30	+ 0.03
24	Lake Lynn controlled primary energy	8 208	8 111	— 97	— 1.18
25	Blackwater uncontrolled primary energy	34 707	33 460	— 1 247	— 3.59
26	Blackwater controlled primary energy	113 463	113 610	+ 147	+ 0.13
27	Total primary energy	269 048	267 881	— 1 167	— 0.43
28	Secondary energy	22 126	24 364	+ 2 238	+10.11
29	Waste energy	62 122	61 119	— 1 003	— 1.61
30	Total energy	353 457	353 364	— 93	— 0.03
31	Lake Lynn plant, capacity, in kilowatts	25 000	25 000	......	......
32	Blackwater plant, capacity, in kilowatts	28 000	28 400	— 200	— 0.70
33	Reservoir capacity, in thousands of acre-feet	110.7	110.9	+ 0.2	+ 0.18

* Unless otherwise noted all values, multiplied by 1 000, are in kilowatt-hours.

The error involved in the use of the duration curve method is dependent upon the degree of similarity in flow distribution at the various plants. In the case of Cheat River Basin, there is some dissimilarity in flow among tributaries and the main river. However, the error introduced by the use of the duration curve is not greater than the assumptions as to average head, efficiency, etc.

GENERAL DISCUSSION

Applicability.—The duration curve method was developed primarily to expedite preliminary investigations of power and storage projects for conditions in which the streams do not vary widely in their characteristics. Some of the tributaries of the Cheat River are more "flashy" than the main stream, but even so, their hydrographs are strikingly similar in point of flow distribution to those of the main stream.

It is thought that the method is particularly applicable to hydro-electric studies for streams with similar flow characteristics. It can be utilized for preliminary comparative studies of projects, for studies of developments involving two or more storage reservoirs, and for storage studies in which ample storage is available at a reasonable cost. A shorter cycle of reservoir depletion and replenishment must be used in cases where ample storage is not available at a reasonable cost.

In the theoretical discussion, it was assumed that the water discharged from an upper plant would be immediately available for use at the next down-stream plant. Such a condition could never exist, but, in practice, the manner of plant operation will eliminate much of the error caused by this assumption. Most plants are supplied from storage or diversion ponds which are able to store the inflow of one or more days. Water in quantities equivalent to that discharged at an up-stream plant can be drawn from the pond with the certainty that it will be refilled by the up-stream flow. In general, such operation will enable the duration curve method to be used for many hydro-electric studies.

Many factors have been entirely omitted from the discussion in order that the process might not be encumbered with too much detail. Among the more important are load factor (mentioned previously), seasonal variation of load, regulation of flow by diversion dams, variation of head due to reservoir draw-down, evaporation from the reservoir, etc.

The duration curve method should be used with caution, or not at all, where streams on widely separated water-sheds are being considered for obtaining a uniform power rate from two or more hydro-electric developments.

Advantages.—The method outlined has several advantages that recommend it for preliminary investigation. It is rapid and accurate, requiring little work for preparation, except the construction of flow duration curves for the plants considered. It eliminates much of the "cut-and-try" inherent in the week-by-week hydrograph studies wherein a rate of use must be assumed and a study completed with this rate before it is known whether the reservoir will fill. The duration curve method insures a full reservoir at the end of the period of study. The results, when finally obtained, are presented graphically, showing more readily and effectively than any tabulation, the distribution of load and the effects to be expected from a change in that distribution.

ACKNOWLEDGMENT

The writers are indebted to James E. Stewart, M. Am. Soc. C. E., for assistance in the preparation of this paper. Without his active interest and helpful suggestions, this study would not have been made.

DISCUSSION

Paul F. Kruse,[3] M. Am. Soc. C. E. (by letter).—This paper presents an interesting application of the graphic method of analyzing certain hydro-electric problems commonly known as the "duration-curve method." Instead of plotting the duration graphs in terms of water flow and time, the authors go a step farther and plot them in terms of power flow or power rate and time.

What appears to the writer as one of the chief advantages of this procedure over the older flow duration-curve method for studies, such as those depicted by the authors, relates to the factor of head. The power capacity or rate available at any water-power site is a function of the product of the quantity of water flowing and the head through which it falls. In the use of the ordinary water-flow duration curves in studies of the combined operation of two or more plants on different streams, or at different sites on the same stream, the head available at each site must be taken into account and separate determinations must be made to obtain the power capacities and the energy that can be secured from the combined development.

By translating stream flow into power rate at each site and plotting this against time, thus taking into account the factor of head before plotting the graphs (as has been done by the authors), such power-output duration curves for two or more sites may be directly applied one to another for the purposes cited of obtaining the combined energy output of the two or more plants under consideration and the capacities required in each plant for such combined operation. The ordinates of the output-duration curve being in units of power and the abscissas in units of time, the area under the curve represents energy, and the area of any one diagram may be transferred or applied to any other diagram of the same scale.

The term, "flow duration curves," in the authors' statements of "Advantages," is believed to have been used erroneously. As stated in the paper under Case 1, "flow duration curves are not used directly for determining output rate and plant capacity; hence, they need not be constructed." The authors' procedure is sufficiently novel and noteworthy to warrant the use of terminology consistent with the analysis which expresses or implies the advantage relating to the factor of head, and to this end it is suggested that it may well become known as the "output duration curve," as used elsewhere in the paper by the authors, or more specifically the "power rate-duration curve" method of analysis.

Donald H. Mattern,[4] Jun. Am. Soc. C. E. (by letter).—The authors are to be commended for their presentation of this new application of duration curves. Their examples ably demonstrate its great versatility under suitable conditions, that is, where the streams under consideration have similar flow characteristics. It is thought advisable to emphasize this point because, to use the method on widely separated streams, some of flashy and others of more regular characteristics, having different flow distributions, would introduce conditions that could not be handled satisfactorily. It is obvious that two streams which have their heavy flow periods occurring several months apart could have duration curves that were similar in shape, and yet proper utilization of the water could not be accomplished by the method suggested by the authors.

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[3] With Sanderson & Porter, New York, N. Y.

[4] Tyrone, Pa.

It would be interesting to learn why "output in kilowatts" was used as ordinates in all their work instead of "cubic feet per second." It seems that to use the basic factor would aid in keeping the fundamental object in mind, namely, utilization of the water flowing in the stream. Use of "cubic feet per second" would eliminate the reduced effective heads that result at high-water periods and, in the case of a plant at a storage reservoir, when depletion or filling is in progress. Changing unit efficiencies under different head and operating conditions would also then be eliminated, with a resulting increase of accuracy and usefulness of final results.

It is true that this method is used for preliminary purposes only, but the work necessary to secure these benefits entails no additional effort. The writer has used both types of ordinates in somewhat similar studies and with those dealing with plant operation, and he has come to the conclusion that use of "cubic feet per second" gives more accurate and understandable results.

JAMES E. STEWART,[5] M. AM. SOC. C. E. (by letter).—Graphical analysis is the preferable method of studying engineering problems when the results are of approximately the same accuracy as those obtained by more laborious computation methods, and the writer believes that this paper marks a notable advance in the application of graphical analysis to hydro-electric studies.

After the method outlined by Messrs. Hickox and Wessenauer has been thoroughly mastered, users of it should be able to obtain approximately the same results as by the use of the hydrograph (computation) method, with the great advantage of having such results in much more satisfactory form and accomplishing the work in about one-fourth the time necessary with the hydrograph method, aided by the fastest computing machines.

Checking time is reduced as much as the computing time, and where results are needed before complete checks on computations can be made, the results obtained by the duration-curve method, on the average, can be depended upon to be more nearly free of error than when carried out by the hydrograph method. This is due to the practically self-checking features of the duration-curve method.

It is believed that any hydro-electric engineer who thoroughly masters the duration-curve method will never return to the hydrograph method, except perhaps for an occasional "spot" check. The difficulty in bringing about the universal use of the duration-curve method for the type of hydro-electric problems outlined in the paper, does not lie in the method or in any particular difficulty in learning it. Rather the difficulty lies first, in human nature which is inherently slow to adopt anything new no matter how meritorious it may be; and, second, to the fact that unless the studies are extensive there will be no gain in total time due to the time used in learning the duration-curve method. Nevertheless, the writer believes that it is only a matter of time until the duration-curve method will displace the more laborious older methods wherever it is applicable.

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[5] Hydr. Engr., West Virginia Power & Transmission Co., Pittsburgh, Pa.

DON JOHNSTONE,[6] JUN. AM. SOC. C. E. (by letter).—That the authors have presented a method of hydro-electric investigation entailing less work than the old hydrograph method is scarcely to be questioned. The severe limitations imposed by two of the basic assumptions, however, reduce its usefulness considerably, and the writer believes that even in those cases to which it is applicable, a study by means of mass curves will yield the same results with less effort, as no re-arrangement of flows out of chronological order is involved.

The assumption that the streams under consideration have the same distribution of flow should be discussed further. The authors mention that the hydrographs of the tributaries of the Cheat River are "strikingly similar" to those of the main stream; yet no two streams have exactly similar hydrographs, and it is becoming increasingly important, with the development of systems involving plants located 100 miles or more apart, to be able to study, quickly and simply, hydro-electric possibilities on streams with widely varying hydrographs. Just how much variation in characteristics can take place before the method becomes inapplicable? It may be mentioned here that a system study by means of the mass curve makes allowance for any variation of flow distribution which may exist between the various plants, however great it may be.

The assumption which causes the greatest limitation of usefulness, however, is the third, requiring "that the storage available at each reservoir" be "unlimited." The writer's experience has been that it is a rare case in which sufficient storage is available to regulate completely the flow of the stream. Still more rare is the case in which it would be economical to use that amount of storage. In general, topographic features set definite limits on the amount of available storage, and a method which permits of starting from this ordinarily known point (as does the mass-curve method) has a far wider range of application than one which entails a basic assumption which is so often unwarranted.

Furthermore, it should be emphasized that, having assumed unlimited storage at a reservoir plant, it is necessary to make a "stab in the dark" in selecting the average effective head to be applied to its discharge. The selection may be so far from the actual value that a repetition of the study will be almost imperative; when the study is begun with a definite assumption concerning storage, however, the average head—and hence the power rate—can be arrived at very closely in the first trial, as will be shown later.

The method as presented by the authors may permit of some simplification. For example, in determining the storage reservoir capacity in Case 1, considerable unnecessary work has been done. As the total demand on storage (represented in Fig. 3 by the area, *MD'N*) is known, and the date of maximum draw-down is determined from a mass curve, there is no apparent reason, in the example given, for re-arranging flows in order of magnitude. All that is actually required is to add the inflows occurring between full reservoir and the point of maximum draw-down, and subtract their sum from the total demand on storage.

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[6] With U. S. Waterways Experiment Station, Vicksburg, Miss.

As it is apparent from the authors' paper that the applicability of mass-curve methods to system studies is not generally appreciated, and as the statement has been made that such methods are applicable to all the cases covered in their examples, the writer feels that it is not out of place in this discussion to describe them in sufficient detail to support his contention.

The common Rippl mass-curve serves for the study of a single plant. When system operation is involved, the "mass-curve of equivalent flow" must be introduced. Its construction is best explained by an example.

Assume a system of three hydro-electric plants located along a single stream. Let the actual (natural) flow at Plant No. 1 (in acre-feet per month) be designated Q_1, the simultaneous discharge at the second plant by Q_2, and at the lowest plant by Q_3. The power which could be produced by this natural flow is proportional to,

$$Q_1 (H_1 + H_2 + H_3) + (Q_2 - Q_1) (H_2 + H_3 + (Q_3 - Q_2) H_3$$

in which, H_1, H_2, and H_3 are the average operating heads at the various plants. Now, compute the hypothetical flow at the up-stream plant, which would be required to produce the same amount of power if no down-stream inflows occurred. This quantity, designated Q_e, is, therefore,

$$Q_1 + \frac{(Q_2 - Q_1)\ (H_2 + H_3)}{(H_1 + H_2 + H_3)} + \frac{(Q_3 - Q_2)\ H_3}{(H_1 + H_2 + H_3)}$$

In other words, for each month, Q_e is the flow at the up-stream plant, plus the incremental inflow at each plant down stream therefrom, multiplied by the ratio of the head through which that increment can act to the total head in the system. The mass curve of equivalent flow is plotted from the successive summation of Q_e.

As the curve has been computed in acre-feet, storages can be applied to it as to an ordinary mass curve, and there may be one reservoir in the system, or more than one. It is only necessary to remember that the equivalent storages must be used; that is, the storage capacity of each reservoir must be corrected by the same factors which were applied previously to the flows. Then, suppose a reservoir of 100 000 acre-ft. capacity to be located at the uppermost plant, and another of equal size at the second plant. If one-half the developable head of the stream occurs at the up-stream plant, then the water in the second reservoir is the equivalent of only 50 000 acre-ft. at the upper plant, and the total equivalent storage capacity of the system is 150 000 acre-ft.

The mean equivalent flow to which the stream can be regulated can be determined in the same manner as the mean flow from the ordinary mass curve. The total power (prime) is the product of this flow, the total average head of the system, and the proper factors for conversion to kilowatts.

The demand on equivalent storage in any month is obviously the difference between the mean equivalent flow to which the stream is to be regulated and the equivalent flow for the particular month. The storage requirement is then divided among the various reservoirs according to some fixed rule. In the example already introduced, suppose it has been determined that equal quantities should be drafted from the two reservoirs, and that in a certain month 10 000 acre-ft. of equivalent storage is necessary to the maintenance of prime. Then, if Q is the actual quantity drawn from each reservoir, $Q + \frac{1}{2}Q = 10\,000$, and Q is 6 667 acre-ft. (Note that 6 667 acre-ft. drawn from the lower reservoir has an "equivalent" value of 3 333 acre-ft., since it acts through only one-half the total head of the system.)

The next step in the study is to correct the natural flow at each plant by the draft or storage from all reservoirs up stream therefrom. The required installation at each plant is determined from the maximum regulated flow computed to occur at that point during a draw-down period. (The maximums at the various plants are not necessarily simultaneous.) Secondary output can also be calculated from the tables of regulated flow.

All the information required has thus been found by using a general method unhampered by the requirements of similarity of flow and unlimited storage, and without the necessity of going through the tedious work of constructing duration curves.

A claim for the authors' method is that when the work is completed a good graphical representation is obtained; but although it may be good from an engineering viewpoint, it is likely to be unintelligible to the layman. As the purpose of a study of this nature is primarily to interest persons other than engineers in a project, the study—not only the results, but the method as well—should be made easily comprehensible; and it should be recognized definitely that few curves are more abstruse and confusing than duration curves. There is a lay tendency (unconscious, perhaps, but perhaps, therefore, all the more firm), to look upon abscissas as representing time in chronological order—and it requires a specialized study to understand graphs which are not prepared in this manner. On the other hand, both the mass curve and the hydrograph have the characteristic abscissas which make them readily understandable to the layman; the mass-curve study is direct, requires no repetition of trials, and its results can be transferred to a hydrograph presentation if that is desired.

The effect of head variation at reservoir plants was omitted from the authors' treatise as a matter of detail. It would be interesting, however, to know how they propose to handle it. As far as the writer can see, the only way to bring head variation into account is to make an analytical study on at least a monthly mean flow basis. It becomes quite important, then, if an analytical study is out of the question, to make the proper assumption as regards average effective head to be applied to reservoir plants studied by duration curves or mass diagrams. Any error in this assumption introduces the same percentage of error in the calculation of prime rate.

The average head during the critical period (which is the head that must be used in setting the prime rate) is not as great as the average operating head for the entire period of record. The two common assumptions concerning the latter—(1) that average head equals gross head less one-third maximum draw-down, and (2) that average head equals gross head less draw-down corresponding to 50% useful storage capacity—are satisfactory for the purpose for which they were intended; that is, for determining the proper installation head for the turbines. When applied to duration-curve or mass-curve studies of the critical period they will indicate prime rates higher than could actually be maintained with the useful storage assumed.

Table 2 was compiled by the writer to aid in the selection of average head. As this head is a function both of the shape of the reservoir and of the distribution of flows during the critical period, nothing more than a general rule can be laid down, the actual selection in any particular case being made on the basis of similarity of conditions with those of some plant already studied analytically.

TABLE 2.—RATIO OF AVERAGE DRAW-DOWN DURING CRITICAL PERIOD TO MAXIMUM DRAW-DOWN

Reservoir No.	Gross head, in feet (1)	Maximum draft, in feet (2)	Average Q, in cubic feet per second (3)	Kilo-watts (4)	Average head, in feet (5)	Average draw-down (6)	Ratio (7)	Length of period, in months (8)
1........	193.5	51.1	6 487	72 000	163.6	29.9	0.585	9
2........	104.3	31.3	7 297	43 000	86.9	17.4	0.556	9
3........	109.2	29.7	3 028	18 700	91.0	18.2	0.613	9
4........	435.5	164.0	2 510	61 000	358.0	77.5	0.473	18
5........	99.0	21.0	3 087	18 900	90.3	8.7	0.414	16

The following will explain the entries in Table 2: Column (1) represents the static head at full reservoir; Column (2) is the draw-down at the end of the critical period; values in Column (3) were computed from the analytically determined, month-by-month discharge requirement to maintain the rate of Column (4); Column (4) is the rate, in kilowatts, maintained during the critical period; Column (5) is the head through which the average quantity (Column (3)) would have to pass to maintain the rate listed in Column (4) at 80% over-all plant efficiency; Column (6) is the difference of Column (5) from Column (1); Column (7) is the ratio of Column (6) to Column (2); and, Column (8) represents the time from the beginning of reservoir draw-down until the maximum draw-down occurs.

From these and similar observations the writer has concluded that the average head at a reservoir plant during the critical period should usually be taken as the gross head less 55 to 60% of the maximum draw-down, for purposes of mass-curve or hydrograph studies.

In Reservoirs Nos. 4 and 5, regulation extending over more than a year was involved. It happened that, in both cases, the demands during the first year were relatively small, the reservoir maintaining a fairly high level throughout the year, but failing to fill on account of low flows during the usual peak months. The average draw-down for the entire period is thus proportionately less, and the average head higher, than for the first three cases. Had the maximum demands come in the first year of draft the conditions would have been reversed.

L. T. Guy,[7] Assoc. M. Am. Soc. C. E. (by letter).—An interesting manner of using the duration curve, in solving the various problems pertaining to storage capacities, has been developed in this paper. As stated, this method has its particular application in the case of preliminary investigations, and the authors stress its advantages over "cut-and-try" hydrographic studies.

However, for the determination of storage capacities and the capacity of storage plants, the writer cannot see that the "duration curve" has any advantages over the "mass curve." On the contrary, the latter method appears to have some distinct advantages. As stated by the authors, their duration curve must be prepared from the flow records for a complete cycle, from full reservoir to full reservoir. The determination of this period necessitates the plotting of the mass curve and, having this curve, the duration curve is only an unnecessary complication.

A superior method of plotting mass curves, which was originally brought to the writer's notice by a Norwegian engineer, is shown in Fig. 7. This method has been found to have several advantages, and is thought to be not generally well known. Fig. 8 is plotted from the mass curve of a stream for which records are available over a period of thirty-nine years, including two or three periods of extremely dry flow. Curve 0 is for no failures in 39 years; Curve 1 is for one failure in 39 years; and Curve 4 is for four failures

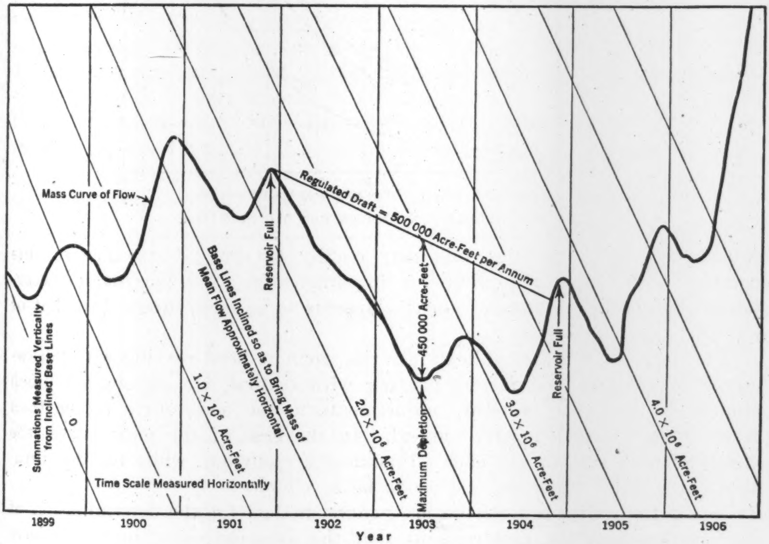

Fig. 7.—Mass Curve of Stream Covering Critical Period.

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[7] Asst. to Civ. Engr., State Electricity Comm. of Victoria, Melbourne, Victoria, Australia.

in 39 years. It will be noted that these curves indicate the storage capacity necessary to maintain any desired regulated flow.

It is clear that Fig. 8 can be used, not only for the stream to which these records refer, but also for other streams that have approximately the same flow characteristics, where records over long periods may be lacking. In the State of Victoria, Australia, many of the streams have remarkably similar flow characteristics, and may readily be grouped together in this manner, when making preliminary investigations. This means of ascertaining the necessary storage capacity appears to be more direct than that of plotting duration curves.

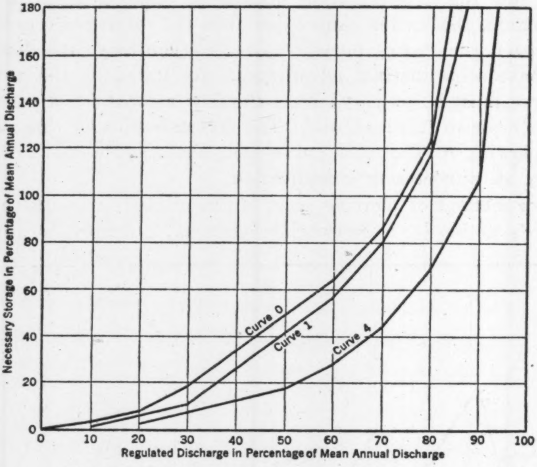

FIG. 8.—REGULATION CURVES FOR MITTA RIVER.

Furthermore, when complications arise, such as head varying with "drawdown" of storage, load factors varying with seasons, etc., it appears much simpler to make the necessary adjustments to the mass curve rather than resort to the duration-curve method. In the case of the mass curve, the effects of each step in any such adjustment are obvious, while for the duration curve, they would not appear to be so simple.

While the writer prefers the mass-curve method for the determination of storage problems, the fact remains that the duration curve may also have its place in this regard, and is undoubtedly essential when dealing with "run-of-stream" plants. In many cases, a combination of the two methods provides an excellent means of solving the problem.

JOEL D. JUSTIN,[8] M. AM. SOC. C. E. (by letter).—Over a wide range of problems involving load, steam and hydro-electric power, storage, stream flow, and various combinations of these factors, the duration curve is a timesaving and convenient tool, worthy of more general acceptance than it has received. The interesting paper by Messrs. Hickox and Wessenauer should serve to make a larger number of engineers familiar with the advantages of these curves. Although duration curves have been used by some engineers in connection with problems very similar to those outlined by the authors, it is probable that the precise manner of application which they present is quite novel.

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[8] Cons. Engr., Philadelphia, Pa.

Economic considerations are tied up so intimately at every step with studies of this kind that it is impracticable to eliminate "cut-and-try" methods entirely. Thus, after making the various assumptions required in such a study, computations are made, curves are drawn, and a "required" storage or installation is determined. The installation or storage determined by the methods outlined by the authors may not prove to be economically feasible. For this reason, it is well to keep constantly in mind the economic factors at each step in the computations.

In considering the question of storage for hydro-electric power, it is usually assumed that if relatively cheap storage is available, it is highly desirable to be able to turn out as much primary energy as possible. With modern interconnected power systems served by both steam and hydro-electric plants, sufficient storage to provide a large amount of primary energy might be an economic waste. Practically the entire function of storage in such systems is to provide enough primary energy to make the capacity of the hydro-electric plants "firm capacity"; that is, to make it capable of performing the same function as alternative steam capacity.

Fig. 9 shows a duration load curve for the peak December week of a typical power company, having a peak load of 100 000 kw. The load factor of the curve is approximately 53 per cent. To assume a simple case, there is only one hydro-electric plant in this system having a capacity of 30 000 kw., with a pond of sufficient size to provide weekly regulation, and there are also several steam plants. From this curve, it is found that by measuring down from the peak, 30 000 kw., the cross-hatched area containing 360 000 kw-hr. is cut off. If at the hydro-electric plant there are at least 360 000 kw-hr. available in a week of minimum December stream flow at the plant, then the

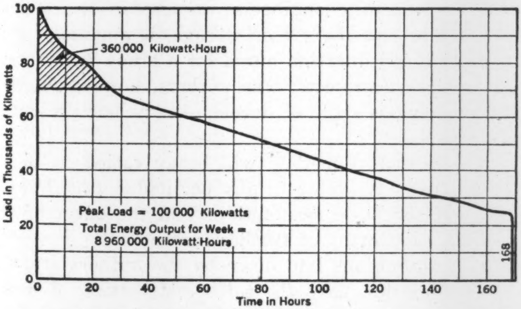

FIG. 9.—LOAD-DURATION CURVE, DECEMBER PEAK LOAD WEEK FOR A TYPICAL REGIONAL POWER COMPANY.

entire 30 000-kw. installation will be firm capacity. Although the capacity factor of the hydro-electric plant for the week will be only 7%, it has done exactly the same work that alternative steam capacity could do.

Additional primary energy in excess of the 360 000 kw-hr. required, would have had only the value of marginal steam-generated energy. However, additional storage might permit of the installation of more firm capacity later. The principal value of storage in a modern power system is the creation of firm hydro-electric capacity, and in making studies similar to those described by the authors this fact should be kept constantly in mind.

C. Maxwell Stanley,[9] Jun. Am. Soc. C. E. (by letter).—Any method that appears to present opportunities for reducing the time-consuming computations involved in hydrographic studies is of interest and is worthy of careful consideration. The computations necessary in studying combinations of proposed or existing hydro-electric plants in order to determine the most feasible and economic arrangement are indeed laborious, because of the numerous variables entering into the problem, and the several solutions which suggest themselves and which must be investigated carefully.

The authors present a method of analyzing such problems by the use of duration curves. There is no doubt but that their method is a time-saver as compared to more ordinary methods involving analysis by weeks or days. That the accuracy of their method is within allowable limits for such studies is proved by the data presented on comparative results of duration-curve and hydrograph methods; that is, it is proved at least for the particular conditions existing in their case. The duration-curve method presents the analysis in a compact graphical form which is visualized readily. The duration curve is not a new device to hydraulic engineers; its use in hydro-electric studies of a single plant, either run-of-river or storage, is generally accepted. The writer has used a combined duration curve for a study involving two run-of-river plants and a storage project. The authors, however, have carried the method beyond any previous stage of development; their presentation is both interesting and valuable.

The application of such a method will be as varied as the problems to which it will be applied. Its value in any particular case, however, depends upon the accuracy of the basic assumptions and the effect of the neglected factors. These items are worthy of further inspection.

The three basic assumptions laid down by the authors are, that the: (1) Streams under consideration have the same distribution of flow; (2) water discharged from the upper plant is immediately available at the next lower plant; and, (3) storage available at each reservoir is unlimited.

These assumptions are never true. The first appears to be the most vital to the practical application of the method, and, at the same time, the least likely of attainment. Obviously, the method cannot be applied satisfactorily without first ascertaining that the regime of the streams to be studied is sufficiently similar to avoid undue error. The failure to meet this first basic assumption will narrow the usefulness of the method. The second assumption should not generally limit the method. If the operation methods of the plants are co-ordinated in a proper manner and a reasonable amount of pondage is available, the variations can be cared for. If the plants are far apart the flow will tend to be dampened out naturally. If they are close together the assumption is practically a fact. The third assumption is not vital because, should the computed storage required exceed that available, it means simply that the problem must be re-worked on a basis of smaller outputs that will require less storage.

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[9] Engr. (Young & Stanley, Inc.), Muscatine, Iowa.

Therefore, in summing up the practical importance of the assumptions, it becomes evident that the first should be emphasized strongly and considered carefully. The streams under consideration must have similar distribution of flow.

What as to the factors omitted, some of which might easily be incorporated to widen the usefulness of the method? The authors list the following items: Load factor; seasonal variation of load; regulation of flow by diversion dams; variation in level due to reservoir draw-down; and evaporation from reservoir. To these might be added the following: Variation in head, due to back-water.

As stated in the paper the load factor is of extreme importance. In certain cases, the method presented may be altered slightly to permit an easier handling of the load factor. The authors use kilowatt units as ordinates. A simple conversion allows the ordinates to be plotted in terms of kilowatt-hours per day. Then, with either load factor or plant capacity known or assumed, the other is readily computed. Often, the load factor of the plants under consideration will be varied from time to time to meet changing conditions of load and stream flow. The plotting of ordinates in kilowatt-hours per day would be helpful in this respect. The growing tendency to develop hydro-electric plants as peak-load projects with relatively large capacities operating at low load factors places additional emphasis upon kilowatt-hours per day rather than merely kilowatt capacity.

Whether or not seasonal variations of load must be taken into consideration will depend upon the system load conditions. In many instances the omission of this factor will give misleading results. Where the plant will operate as a base-load plant, seasonal load variations will have little effect; but as the majority of the present-day installations are being developed as peak-load plants it would seem advisable, in order to widen the applicability of the duration-curve method, to provide some means of incorporating into the study the effect of seasonal load variations. In some instances the regime of the river studied may be sufficiently well established so that the high flows consistently fall in certain months and the low flows in others. Where this is the case a load graph, such as Fig. 10, may be plotted, replacing the horizontal load line of the authors' graphs and permitting other computations in accordance with their method.

In cases where the regime of the river is too variable to permit such a treatment and seasonal variation of load is an important item, other methods must be devised to allow the use of the duration-curve analysis.

Any effects of regulation of flow by diversion dams might be cared for by using the regulated flow in determining the values for the duration curves.

Variation of head due to reservoir draw-down will not be a factor in run-of-river plants. On storage projects, power computations using an average head should give sufficiently accurate results if the elevation-area curve for the reservoirs is a reasonably straight line through the changes of elevation required for draw-down. Such a method should certainly give more accurate results than could be obtained by omitting the loss of head due to draw-down.

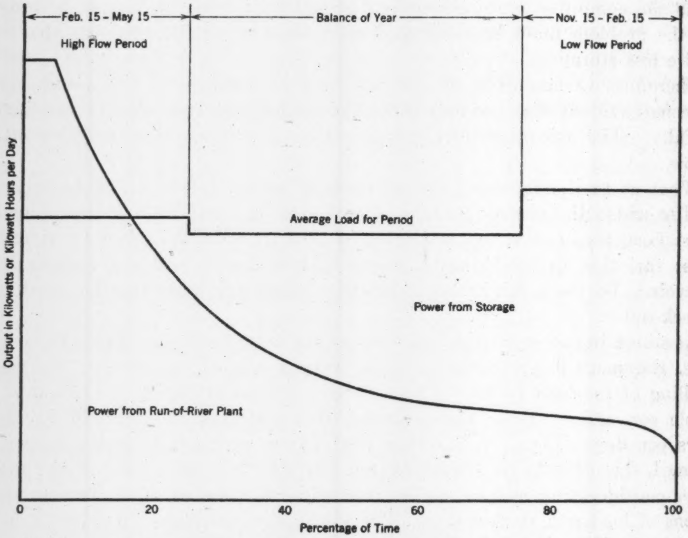

FIG. 10.—METHOD OF APPLYING SEASONAL VARIATION OF LOAD TO DURATION-CURVE METHOD.

A correction to the inflow of a storage reservoir can be made, which will allow approximately for evaporation. For instance, with a pond area of A acres and a total annual evaporation of d ft., an average reduction of flow of $q = \frac{A\ d\ 43\ 560}{8\ 760 \times 3\ 600}$ cu. ft. per sec. over the year will compensate for the evaporation. In some instances, the total correction should be spread over only a part of the year.

On run-of-river plants the output is usually affected by the back-water, which reduces the head. If the power computations for the duration curve are made, using the varying heads as determined from a head-discharge curve, all error from this source may be eliminated. Equation (1) would be replaced by:

$$K\ W = \frac{Q\,e\,h}{11.8} \quad \text{..........................} (2)$$

in which, h varies with Q. This alteration can be made with a minimum of time if the usual data showing the head-discharge relationship are available.

Many neglected or omitted factors could have been included without materially adding to the computations involved. The inclusion of these factors should extend the usefulness of the method where it is applicable.

F. KNAPP,[10] ESQ. (by letter).—The present tendency of engineering science leads always to the substitution of a number of isolated solutions by more general systematic methods, which do not perhaps correspond entirely to the individual properties of the case under consideration, but which by their power of general validity promise an abundance of new facts. Moreover, such a synthesis provides for a great economy of labor and time, thus fulfilling one of the engineer's obligations, namely, that of efficient and reliable service. Considering especially the mass of new hydro-electric literature, such a procedure becomes almost a necessity for the engineer who is endeavoring to keep up to date.

Any one can develop an unlimited number of variations of methods; but for daily use a general method of maximum simplicity without special exceptions combined with reasonable accuracy is required.

No doubt most young engineers bring from school an enormous quantity of scientific facts and a number of solutions of the most varied problems; but the coherence, the "Leitmotiv," is missing and the clear conception and synthesis come only after years of practical work, so that often simple fundamental questions are considered as difficult and complex.

Considering the authors' paper from the foregoing viewpoints, it seems to the writer that the assumptions made are restrictive to its general use. Large hydro-electric systems usually comprise a number of streams on widely separated water-sheds, thus contributing to the equalization of the total available natural stream flow. In any such study factors of capital importance, such as the effect of load factor and seasonal variation of load demand, as well as possibility of pondage, must be taken into account. The duration-curve method in such cases loses much of its simplicity. The authors assume a load demand of such a nature that a uniform rate of output is required. For large storage plants such an assumption is feasible, but the actual output of run-of-river plants depends so much on seasonal system load demand, load factor, installed capacity, pondage, and seasonal distribution of flow, that this assumption may lead to considerable error. Hence, the introduction of "use curves," "utilization factors," etc.

The importance of the problem of power output of several hydro-electric developments, supplying the same power market, demands a general unrestricted solution of easy applicability. Load-duration curves and hydrograph and mass curves are the chief "members" of the general solution, avoiding, however, the "cut-and-try" process inherent in the usual weekly or daily hydrograph studies.

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[10] Asst. Hydr. Engr., The São Paulo Tramway, Light & Power Co., Ltd., São Paulo, Brazil.

In 1914, Mr. C. T. Hutchinson proposed a power-energy curve,[11] which for the case under discussion has not obtained the proper consideration. According to Mr. N. E. Funk,[12]

> "The curve is constructed by plotting percentages of the total energy as abscissas against corresponding percentages of the total power or peak as ordinates. The percentage of the total energy corresponding to any percentage of the total power or peak is obtained by integrating the area of the load curve bounded by a horizontal line drawn at the desired power value through the contours of the load curve, the contours of the load curve below this horizontal line, the ordinates between the contours of the load curve and the base, and the base of the load."

This power energy curve is far more valuable than Mr. Funk's definition makes one suppose. Its value is derived from its quality as an integral curve of the load-duration curve. A thorough study of this integral curve reveals some very interesting and valuable features, among which is the possibility of reducing any load demand to a 100% load-factor demand. Mr. Hutchinson's integral curve, therefore, may be reduced to a single vertical line with sub-divisions in percentages of maximum yearly peak load. As the daily load demand varies with the seasons, a representative or equivalent daily load diagram may be chosen for each week or season. This load diagram may be reduced to 100% load factor as explained and the representative load demand lines arranged in chronological order. Thus, a two-dimensional total yearly load demand diagram is obtained, instead of a three dimensional "load demand mountain." Next adding the run-of-river plant hydrograph and reducing the ordinates to those of the corresponding 24-hour load demand, the diagram permits reading all necessary values directly or by means of mass curves. It shows, also, that deficiency and waste at the run-of-river plant may occur, so to speak, at the same time.

For example, the application of this procedure to the authors' Case 2 does not require Assumption (1) (see "Basic Assumptions"). Although a hydrograph is necessary, tedious day-by-day studies are avoided; Assumption (3) can also be avoided; while Assumption (2) requires a compensation pond at a convenient point between the two plants if the distance is considerable. The method proposed also avoids the authors' fundamental procedure of constructing output duration curves for exactly one complete reservoir cycle.

Thus far, the authors treated only the maximum possible output of several hydro-electric developments, supplying the same power market. The installed capacity of each station, however, must be determined by an economic study. The capacity of a run-of-river plant and, therefore, the cost of energy, is a function of the total system load demand. The greater the demand, the greater will be the capacity. However, the power company wishes not only to produce cheap power at Plant *A* or Plant *B*, but wishes to produce the total output as cheaply as possible. With expensive peak energy the most economical capacity of a run-of-river plant becomes far greater than that determined by consideration of the economic conditions of the run-of-river plant alone.

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[11] *Transactions*, Am. Inst. Elec. Engrs., Vol. XXXIII, p. 155.

[12] "Economics of Combined Hydro and Steam Power Systems," by N. E. Funk, *Transactions*, Second World Power Conference, Vol. XI, p. 281.

Robert F. Ewald,[13] M. Am. Soc. C. E. (by letter).—The uses of the duration curve have been presented in an admirable manner in this paper to determine the relative capacities of hydro-electric generating stations to be located on the same or on continuous water-sheds where natural flow varies in an identical manner.

If the storage available at economically feasible costs is large relative to the total annual run-off, the duration curve utilized in the manner proposed may be the only tool required for a reasonably accurate determination of the proper capacity. However, in most cases, in situations similar to those under consideration by the authors, the determination of the "reliable power rate" follows a study of the most economical size of storage reservoirs to be provided. That study necessitates consideration of the flow records for a period of several years and, for this purpose, the mass curve of stream flow in its various modified forms seems to be the most convenient tool. From this point, the procedure depends entirely on the set-up of the system to which the power is supplied.

In many cases, especially those in which large steam reserves are available, in order to utilize hydro-electrc energy to the greatest advantage, it is best to use variable power rates in the system, depending on reservoir and flow conditions. In order to develop the proper "rule curves" it is necessary to study the entire flow record repeatedly. For this purpose, the "energy draw-down diagram"[14] is a convenient tool. Such diagrams are readily constructed in a few hours and are of special value in studying systems utilizing large reservoirs, with relatively large draw-down, long conduits with widely varying losses, and turbines with widely varying efficiencies, because all these factors, together with leakage and evaporation loss, may be fully and easily evaluated in the preparation and use of the diagrams.

In the execution of such work it appears preferable to keep the final results in the form of graphs of power available and variation in reservoir levels, because these factors enable further study "by eye" with reference to further improvement in operation. The resulting power graphs form a series of power-duration curves from which an average duration curve may be prepared. This curve, however, has a much different significance from that discussed by the authors. To enable comprehensive understanding of the possibilities of a hydro-electric system, not only should the duration curve for reliable power rates be available for the period of minimum flow, as described by the authors, but (what seems to be more important), an average duration curve based upon application of the proposed rule curves should also be available.

The writer uses a form of mass curve that is of considerable interest—the curve of cumulative deviation from mean discharge. In its variations this curve swings about a horizontal line and, therefore, is very conveniently plotted on successive letter-sized sheets which may be easily attached to each other. The curve may be used not only in studying storage possibilities, but also systematic variations in the flow. This is an important matter, which will receive more consideration in the future.

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[13] Hydr. Engr., Aluminum Co. of America, Pittsburgh, Pa.

[14] "Power Rates in Hydro Development," by Robert F. Ewald, *Electrical World*, November 8, 1924, p. 993.

G. H. HICKOX[15] and G. O. WESSENAUER,[16] JUNIORS, AM. SOC. C. E. (by letter).—The writers have attempted to present, in as simple a fashion as possible, the basic ideas underlying their application of duration curves to hydro-electric studies. It was assumed that the paper would be of interest chiefly to those having had experience with similar problems. For this reason, analyses of the familiar mass curve and hydrograph methods were omitted, as well as the economic features inherent in all such problems.

Such factors as weekly variation of load, seasonal variation of load, variation of head due to reservoir draw-down, variations of turbine efficiency with head, evaporation from reservoir, pondage, regulation of flow by diversion dams, reduction of head due to back-water, plant load factor, system load factor, and other variables which occur in the practical application of any method of estimating power, and which, while important, were not essential in explaining the method devised, were purposely omitted. The same factors arise in connection with the use of the mass curve, and one who has had experience in dealing with them for use with such studies should have no difficulty in applying them to the duration curves.

A few of the discussers appear to think that the writers advocated the use of the duration curve for all the cases described. This does not necessarily follow. Their aim was to present the essential elements of an idea which is capable of endless variation. The illustrations selected were purposely simple ones, in order that the principles involved might be described without the fog of a multitude of extraneous factors. Any problem in power rates may be solved by the use of daily records (the hydrograph method), a great many problems by the mass curve, and a great many by the duration curve. Each has its own field of usefulness, and the choice of a method depends on the conditions of the problem to be solved. The mass curve is applicable to the simpler cases, and for these may be most desirable. For problems involving load factor and a number of plants the mass curve becomes less useful and the duration curve more useful. The mass-curve method falls down when the capacity at the run-of-river plant is limited, because it makes no provision for evaluating the energy lost when water must be by-passed at the lower plant due to the lack of capacity. The evaluation of this loss converts a mass-curve study into what is essentially a hydrograph study by days. Probably the most useful function of the mass curve is as an indicator of the critical period for reservoir operation. The duration-curve method as described fails only when the flows at the various sites are unduly dissimilar in point of distribution.

If the flow at two or more points under consideration is so dissimilar as to throw doubt on the results obtained by the duration curve, the effect of the dissimilarity can be evaluated by making a study of a simple set-up by the hydrograph method and comparing the results with those obtained by the duration curve. If the error is within permissible limits for the simple case, the duration curve method may be used safely for the more complicated cases.

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[15] Associate in Mech. Eng., Univ. of California, Berkeley, Calif.

[16] Asst. Engr., West Virginia Power & Transmission Co., Pittsburgh, Pa.

The second basic assumption, that flows at one plant are immediately available at the next down-stream plant, is not very important. In most cases, plants are close enough together and are provided with sufficient pondage so that water can be drawn from the lower ponds with positive assurance that it will be replaced within a short time by water liberated at the plants above.

The third basic assumption apparently was loosely stated. As used in the paper, "unlimited storage" is merely a relative term. Presumably, an investigation of a group of proposed developments would involve the determination of the primary energy rate, plant capacities, and reservoir capacities. Different values would be obtained by varying the length of the reservoir cycle considered. The results then would be analyzed from an economic and practical standpoint (including topographical limitations of storage). Therefore, if the period selected involves an impossible amount of storage capacity, the obvious procedure is to shorten the period. This point was brought out in the paper under the heading, "General Discussion", and was also emphasized in Mr. Stanley's excellent discussion. Mr. Johnstone's statement that the third basic assumption "causes the greatest limitation of usefulness," is thus seen to be incorrect.

Mr. Johnstone's belief that the mass curve is applicable to all problems which can be solved by the duration curve and that it will yield the same results with less effort is erroneous. The duration-curve method described, was developed for situations in which the mass curve was not readily applicable, namely, the combination of a proposed storage plant with an existing run-of-river plant with a fixed capacity. The combined plants were to operate with a load factor of 30 per cent. The capacity of the run-of-river plant was thus frequently exceeded when water was drawn from storage to produce power at the up-stream plant. The evaluation of power thus lost converts the mass-curve study into a daily hydrograph study requiring weeks of time, while the evaluation can be made with the duration curves in as many days.

Mr. Johnstone also questions the layman's understanding of the duration curve. It is not necessary to explain it if used only for preliminary studies. Final estimates should be made on the basis of hydrograph studies. However, the writers have found no more difficulty in explaining that an area means a definite quantity of power than they have in explaining the mass curve in which the reservoir is being depleted while the flow ordinate climbs steadily upward. Mr. Johnstone's comment on determination of storage capacity in Case 1 is well made.

The preceding discussion should be in part an answer to Mr. Mattern's query. It is necessary to reduce the flows at the various sites to a common denominator which is affected by head and efficiency. Since power output is the ultimate object of such studies, it was found convenient to use the kilowatt as the unit. Mr. Johnstone's equivalent flow, further modified to include efficiencies at each head, might be used, but this would still require ultimate conversion to kilowatts, and, in the meantime, would not give as clear a picture of the output from each plant.

Mr. Knapp seems to feel that the power-energy curve proposed by Mr. Hutchinson has a greater applicability than the duration-curve method. Mr. Hutchinson's power-energy curve is derived from a system load curve and is a convenient method of dealing with the load factor of parts of the system load. The power-energy curve is a useful tool in determining the effect of load factor, but it does not offer a complete analysis of the possibilities for development of a stream. Mr. Hutchinson used an integrated duration curve in his studies,[17] but his use was confined to a study of a run-of-river plant with a steam auxiliary plant.

Mr. Hutchinson's method avoids using data for one reservoir cycle by ignoring the effect of storage. Duration-curve studies have been commonly made on the basis of the average flow over the entire period of record, leading to fantastically high rates of power output. As far as the writers know, the only method of determining the reliable rate of output is by the use of data covering one reservoir cycle with a concurrent investigation of reservoir capacity to see if there is reserve enough to supply the deficiency. Estimates based on the lowest calendar year of record may also be grossly misleading in this respect.

Mr. Ewald mentions a modification of the mass curve which is cumulative about the mean discharge, and Mr. Guy calls attention to a similar curve. The writers agree that this form of mass curve is the most convenient one when using that method.

Mr. Kruse's suggestion that the term, "flow duration curves," as used in the paper under the heading, "Advantages," be changed to "output-duration curves," is well put. The paper states specifically that flow duration curves need not be constructed.

Mr. Justin, Mr. Knapp, and Mr. Ewald stress the importance of economic considerations. This factor can scarcely be over-emphasized. At the present cost of steam power development, a hydro-electric plant must be able to show a definite contribution to the primary energy requirement of the system in order to justify itself. No factor that will affect the amount or cost of output should be overlooked.

The writers believe that the duration-curve method permits a more exhaustive analysis of the possibilities of a group of developments at a given cost than is possible with any other known method without loss in accuracy. It is believed that engineers who familiarize themselves with this method will continue to use it.

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[17] "The Economical Capacity of a Combined Hydro-Electric and Steam Power Plant," by Cary T. Hutchinson, *Transactions*, Am. Inst. E. E., Vol. XXXIII, p. 155.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1855

THE COMPENSATED ARCH DAM

By A. V. KARPOV[1], M. Am. Soc. C. E.

With Discussion by Messrs. R. A. Sutherland, Lars R. Jorgensen, L. T. Evans, H. W. Sibert, Eugene Kalman, and A. V. Karpov

Synopsis

The paper is a theoretical analysis of an arch dam that is intended to simplify present-day design methods and bring to light the proper relationship between the profile of the dam site and the shape of the arch dam.

Conditions are investigated under which the dam can be relieved of unnecessary stresses. The compensated dam that is introduced, carries the stresses due to dead weight and water load, but it is relieved from stresses due to contraction or expansion of concrete and deflections of the foundations and abutments.

This investigation indicates a design that is believed to result in a safer and more economical structure.

I.—Methods of Designing Arch Dams

The theory of arch dam design has been given considerable attention in recent years. Quite a number of valuable papers have been published dealing with this complex problem. However, sufficient attention has not been paid to the shape of the dam, which, as in the case of most structures subjected to hydrostatic loads, is an important factor that influences the safe and economical design. A definite relationship exists between the shape of the dam site and the shapes of the vertical and horizontal cross-sections of the arch dam, which can be developed only if proper consideration is given to both the gravity and arch action and to the elastic properties of the ground on which the dam rests.

In general, all arch dam designs in the past can be divided into two classes: Those in which the gravity action is neglected, and those in which it is considered. The designs in which the gravity action is neglected are based on the assumption that the entire water load is carried by the hori-

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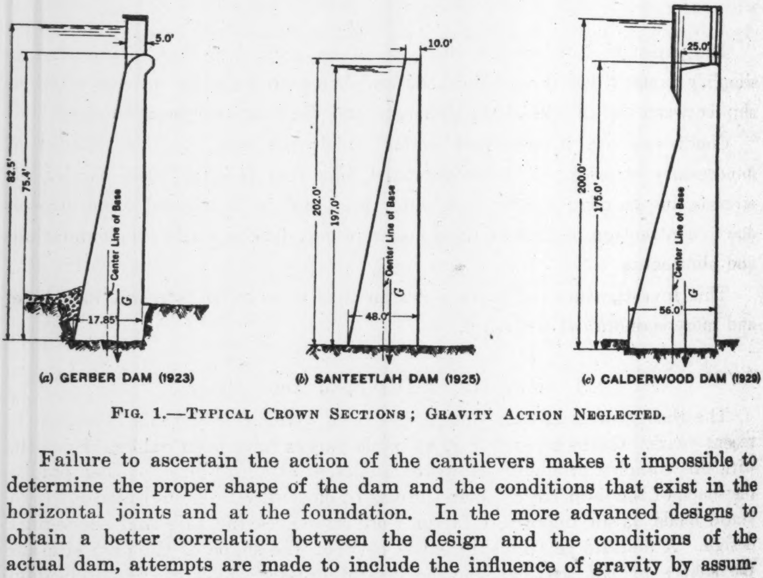
Note.—Published in April, 1932, *Proceedings*.

[1] Designing Engr., Hydr. Dept., Aluminum Co. of America, Pittsburgh, Pa.

zontal arches and the action of the vertical elements, usually called cantilevers, is neglected under the supposition that their influence is not very pronounced and merely serves to relieve the stresses in the horizontal elements.

Actually, in the lower parts of the dam, the action of the vertical elements changes the distribution of loading on the horizontal elements and may produce excessive moments that will result in heavy concentrations of stresses in some parts of the dam and uneconomically low stresses in other parts. In the higher parts, the influence of the vertical elements, or cantilevers, will be felt not only in the change of the distribution of loading, but also in the increase of the loading of the horizontal elements.

The statement is sometimes made that the influence of the vertical element may be neglected because it is counteracted by the effect of swelling when the dry up-stream face of the dam comes in contact with water. This contention seems to contradict the results of tests made on existing dams.



(a) GERBER DAM (1923) (b) SANTEETLAH DAM (1925) (c) CALDERWOOD DAM (1929)

FIG. 1.—TYPICAL CROWN SECTIONS; GRAVITY ACTION NEGLECTED.

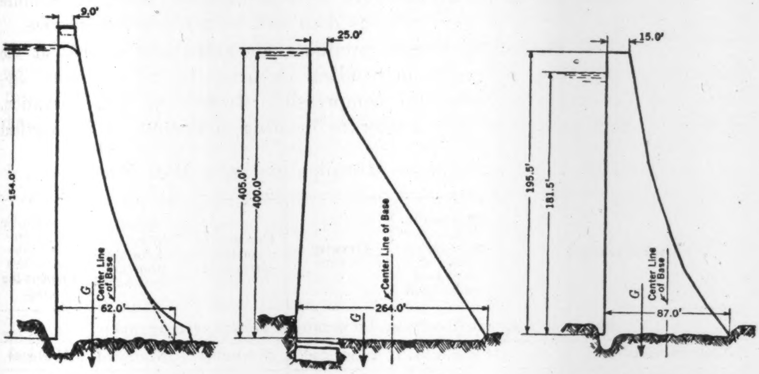
Failure to ascertain the action of the cantilevers makes it impossible to determine the proper shape of the dam and the conditions that exist in the horizontal joints and at the foundation. In the more advanced designs to obtain a better correlation between the design and the conditions of the actual dam, attempts are made to include the influence of gravity by assuming that only a part of the water load is resisted by the horizontal arches and the remainder by the vertical elements.

A difference that is most pronounced at the middle or crown can be noticed in the shapes of the vertical cross-sections. If the gravity action is neglected, the advantage of having a large central angle will be the governing factor, the larger angles showing more favorable computed stresses. The way to obtain such large angles, particularly at lower elevations, is to increase the thickness of the dam in the up-stream direction, as shown in Fig. 1, and it is customary at the same time to make the down-stream face nearly vertical.

If gravity action is considered, the central angle is not the major governing factor and vertical crown sections as shown in Fig. 2 will result, in which the up-stream face of the vertical crown element is made approximately vertical and the increased thickness is obtained by extending the lower parts of the dam in the down-stream direction.

The difference between these two types of sections is that, in Fig. 1, the gravity resultant of the section will intersect the base between its middle point and its down-stream face, and, in Fig. 2, the intersection point will be between the middle point of the base and its up-stream face.

Considering the dam under water load, this difference is of importance since in the first type the support that the arch receives from the gravity action is very slight. At the same time, the appreciable bending moments that are developed in the vertical element can be taken care of only by the opening of the horizontal joints, and by a considerable increase of the maximum stresses in horizontal joints, at the foundations, and at the upper horizontal arches.



(a) DEADWOOD DAM (1929) (b) OWYHEE DAM (1928) (c) GIBSON DAM (1926)

FIG. 2.—TYPICAL CROWN SECTIONS; GRAVITY ACTION CONSIDERED.

In Fig. 2 the arch not only gets all possible support from the gravity action of the vertical elements, but the bending moments in these elements are reduced, thus minimizing the opening of the horizontal joints and decreasing the stress in these joints, in the foundations, and in the upper horizontal arches. For these reasons, if for no other, the advantage of the second type (Fig. 2) is very great.

In methods used in the past the arch action of the horizontal elements of the dam was determined on the assumption of a continuous arch without joints. Actually, every commercial arch dam is built of a number of blocks with pronounced vertical, and less pronounced horizontal, joints. The design assumptions can be applied properly within the boundaries of a block, but not at the joints, where certain reservations are necessary. The most important of these is that only a limited amount of tension can be carried through the joints, and, consequently, if the estimated stresses show tension of any magnitude, a discrepancy will exist between the computed and actual stress distribution, which can be so large as to make the computed stresses useless.

In an arch dam the number of considerations that influence the behavior of the structure and its stresses will vary, depending on the distribution of moments and stresses. In a dam that has non-uniform stresses and moments in the horizontal arches, shear and twist will be introduced, and they must be considered. If the design could be made so that the stresses are kept uniform and the moments, as well as the complications due to shear and twist, are eliminated in the horizontal arches, then a scientifically safe design influenced only by a limited number of major factors can be developed, which will result in the most economical structure. If such a design is attempted, it will be found that, for each temperature change in the concrete, some modification will be necessary in the shape of the dam. Since that is obviously impossible, the best procedure is to design for some temperature corresponding to the mean to which the dam will be subjected in service.

The proposed method of design produces an arch dam which, at the assumed temperature, is free from bending moments in its horizontal sections and has uniform horizontal compression stresses at each elevation. If such a dam has reasonably moderate bending moments in its vertical sections, then in the vertical direction at each horizontal element there will be varying compression stresses.

TABLE 1.—Evolution of Methods of Arch Dam Design

Design method	Stresses in arches due to contraction of concrete under load	Gravity action	Continuity of the structure	Elastic deflections of foundations and abutments	Relationship of the profile of the site to this shape of the dam
All Load Assumed Taken by Independent Horizontal Arches					
Cylinder formula..............	Neglected	Neglected	Neglected	Neglected	Neglected
Application of theory of elasticity to arches. (Cain's formula, constant-angle dam, straight voussoir method, modified voussoir methods)....................	Considered	Neglected	Neglected	Neglected	Neglected
Load Distributed Between Horizontal Arches and Vertical Elements					
Original trial-load method (Bureau of Reclamation)..............	Considered	Considered	Neglected	Neglected	Neglected
Present trial-load method (Bureau of Reclamation)..............	Considered	Considered	Considered	Partly considered	Neglected
Compensated dam..............	Considered	Considered	Considered	Considered	Considered

The major factors that affect the behavior of such a dam and the magnitude of its stresses, at the assumed temperature, are the direct water pressure on the up-stream face, the compression in the horizontal arches, the varying vertical stresses, the horizontal shear in the vertical elements, the foundation conditions, and the conditions necessary to retain the continuity of the structure. Secondary factors that could be considered in such design are the influence of Poisson's ratio and possibly the swelling effect of the wetted concrete.

At temperatures other than the one for which the design is made bending moments will be introduced in the horizontal arches. These moments will make the horizontal stresses in the arches non-uniform and will introduce shear and twist, and the dam will be influenced by all the factors encountered in customary design. The important difference, however, is that in the customary design all these additional factors are present at any temperature and are of major importance. In the proposed design their influence on the arch will be of a secondary nature and they will occur only at temperatures differing from that assumed in the design. The interesting evolution of the different arch dam design methods is briefly summarized in Table 1.

II.—Outline of the Analysis

The simplification of the arch dam design by removing all non-essential factors is regarded as a most important step in introducing a scientific design method. The purpose of this analysis is to develop such a method by relieving the dam from bending moments in its horizontal arches. Arch dams so designed are herein designated "Compensated Arch Dams."

The procedure used for analyzing the conditions of an arch dam and for determining the main principles involved in the design is, as follows:

1.—The dam is divided into vertical elements defined by radial vertical planes, and horizontal elements defined by horizontal planes.

2.—The hydrostatic pressure applied to the dam is represented by radial forces acting at the up-stream face.

3.—A division of these forces is assumed so that they will exert varying loads on the vertical and horizontal dam elements.

4.—At first the horizontal arches are considered as a number of independent arches, each being in frictionless contact with the arch immediately above and below the one under consideration. Such an arch is free from the influence of gravity, and is restrained in the vertical direction, but has no restraint in the horizontal direction, except as provided by the abutments.

5.—In the investigation of the independent horizontal arches resultant forces and moments are taken with reference to the center line of the arch, and the proof is developed that such an arch cannot be relieved from bending moments if the assumptions are limited to changes in the shape. The problem is divided into two parts: First, to shape the horizontal arch so as to relieve it from bending moments due to external loadings; and, second, to provide some means of compensation that will relieve the properly shaped arch from moments due to rib-shortening, displacements of the abutments, and difference between the assumed temperature under service and the temperature at the time of closing the dam.

6.—The shape of an arch relieved from bending moments and the necessary compensation are determined, as well as the relationship between the radial, tangential, and torsional displacements of the loaded arch for any assumed deflections of the abutments.

7.—As in the case of the horizontal elements, the vertical elements are first considered as independent, as unrestrained from the adjoining elements, and as resting with sufficient friction at the foundation and horizontal joints to prevent slipping at any elevation. The difference in assumptions with regard to the horizontal arches and vertical elements relieves the horizontal arches from the influence of gravity and concentrates it on the vertical elements. The radial, tangential, and torsional displacements of the vertical element, due to the part of the water load taken care of by the element, are analyzed.

8.—The conditions imposed upon the independent horizontal arches and the vertical elements by the necessity of retaining the continuity of the structure are investigated. Continuity manifests itself in the form of a mutual restraint between the adjoining horizontal arches and the vertical elements, and requires the equalization of the radial, tangential, and torsional deflections of the independent horizontal arches and vertical elements. The conditions are formulated under which this requirement can be satisfied and by which the independent compensated horizontal arches can be transformed into elements that are restrained in the horizontal as well as in the vertical directions, at the same time remaining compensated and relieved from any bending moments. In this way the original assumption (independent frictionless horizontal arches and independent vertical elements) is eliminated and the structure is transformed into one that corresponds to the dam as actually built.

9.—Theoretical investigations are extended to cover unsymmetrical as well as symmetrical dam sites, and the influence of temperature changes, expansion, and contraction of concrete and elastic deformations of the foundations and abutments in a dam, are analyzed.

10.—A partly analytical and partly graphical design method is developed for the design of a compensated arch dam, which takes into full consideration the shape of the gorge at the dam site and in which the construction conditions are considered by insuring that the vertical joints of the loaded dam are under uniform compression.

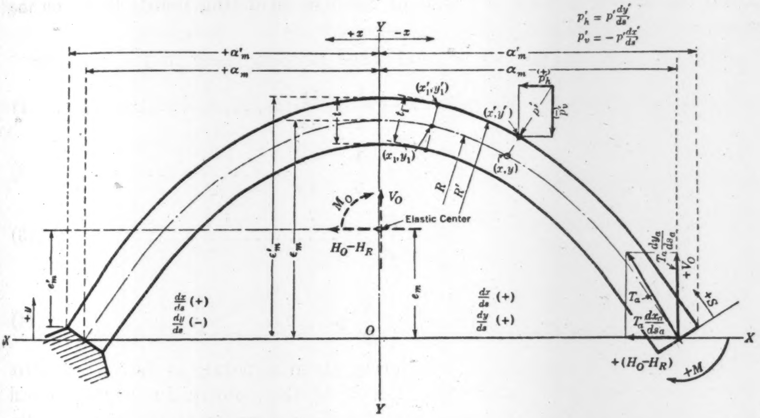
III.—General Equations of the Horizontal Arch Section Under Constant Temperature, Based on Static Conditions

The radial loads applied along a horizontal arch are not constant, but vary, depending on the topography of the dam site and the shapes of the vertical and horizontal elements of the dam. No general rule covering the manner in which the loads will vary, can be made; it can only be stated that the arch loads will be symmetrical for symmetrical canyons and unsymmetrical for unsymmetrical canyons and foundation conditions. The problem of finding the proper shape of the arch resolves itself into finding the equation of the center-line curve of the arch for which the changing radial loads applied at the up-stream face will produce zero bending moments at every point of the center-line curve. Such an arch is herein termed a "compensated arch."

Referring to Fig. 3, let a horizontal element of the dam (height = 1) be fixed at the left abutment, and let the abutment at the right be replaced by the reaction, T_a, in which the subscript, a, indicates the values of the different functions at the abutments. Since no moment is to appear at the center line of the arch, this reaction must be applied tangentially at the center line and is equal and opposite to the arch thrust.

If the elastic center method is applied, the right end of the arch (Fig. 3) is connected by a rigid bracket to the elastic center, and the reaction, T_a, is replaced by forces and moments applied at that center.

Let the forces and moments that are applied at the elastic center be divided into forces and moments due to the external loads and internal stresses. Assuming for the present a symmetrical arch and resolving these forces in their horizontal and vertical components, let,



$p'_h = p'\frac{dy'}{ds'}$
$p'_v = -p'\frac{dx'}{ds'}$
+x
Y
-x
$+\alpha'_m$
$-\alpha'_m$
$+\alpha_m$
$-\alpha_m$
(+)
p'_h
p'
p'_v
(x'_1, y'_1)
(x', y')
t
(x_1, y_1)
(x, y)
R
R'
M_O
V_O
Elastic Center
$H_O - H_R$
e'_m
e_m
$T_a\frac{dy_a}{ds_a}$
$+V_O$
T_a
$T_a\frac{dx_a}{ds_a}$
+S
$\frac{dx}{ds}$ (+)
$\frac{dy}{ds}$ (-)
$\frac{dx}{ds}$ (+)
$\frac{dy}{ds}$ (+)
+y
X
O
X
$+(H_O - H_R)$
+M
Y

FIG. 3

H_0 = the horizontal component due to external loads;
H_R = the horizontal component due to internal stresses;
V_0 = the vertical component due to external loads;
V_R = the vertical component due to internal stresses which is equal to zero for a symmetrical arch;
M_0 = the moment due to external loads; and,
M_R = the moment due to internal stresses which is equal to zero for a symmetrical arch.

Then, $(H_0 - H_R)$ and V_0 and M_0 are the total horizontal and vertical components and total moment, respectively, applied at the elastic center of a symmetrical arch due to all external loads and internal stresses. Let the Y-axis run through the elastic center and let,

x and y = the ordinates of the center line of the arch section;
t = the radial thickness of the arch section;
e_m = the distance of the elastic center from the X-axis;
$\pm\alpha_m$ = the limiting values of x;

ε_m = the limiting values of y;
T = the total thrust at the center line of the arch section;
s = the length of the center line of the arch section. The origin of s is assumed at the right end of the arch, and the direction of s is assumed positive from right to left; and,
R = the radius of curvature of the center line curve of the arch section.

Since, in general, the thickness of the section, t, is not constant, the up-stream face of the dam upon which the loads, p', are applied, must be treated as an independent variable with respect to the center line of the section. In order to conform to this fact, the values of x, y, s, a_m, e_m, ε_m, p, and R of the center line have corresponding values of x', y', s', a'_m, e'_m, ε'_m, p', and R' on the up-stream face of the dam, the corresponding points being on the same radii (Fig. 3).

Certain relationships can be established between these values, as follows:

$$x' = x - \frac{1}{2}\, t\, \frac{dy}{ds} \qquad \text{(1)}$$

$$y' = y + \frac{1}{2}\, t\, \frac{dx}{ds} \qquad \text{(2)}$$

$$ds' = \frac{R'}{R}\, ds \qquad \text{(3)}$$

and,

$$p' = \frac{R}{R'}\, p \qquad \text{(4)}$$

In taking the moments of the forces about a point, as (x_1, y_1), positive moments are considered clockwise. Let M_1 be the moment due to the external loads, p', and M_2, the moment due to the forces at the elastic center of the arch.

From the definition of a compensated arch:

$$M_1 + M_2 = 0 \qquad \text{(5)}$$

Keeping in mind the positive and negative values of x', y', s', and p', indicated in Fig. 3, the loads, p', may be resolved into their horizontal and vertical components:

$$p'_h = +\, p'\, \frac{dy'}{ds'} \qquad \text{(6)}$$

and,

$$p'_v = -\, p'\, \frac{dx'}{ds'} \qquad \text{(7)}$$

Furthermore, the reaction, T_a, may be resolved into its vertical and horizontal components. It is to be noticed that these components are, respectively: $T_a \dfrac{dy_a}{ds_a} = V_0$, and $T_a \dfrac{dx_a}{ds_a} = (H_0 - H_R)$.

The moment, m_1, due to the single force, p' applied at the point, $(x'; y')$, with reference to the point, $(x_1; y_1)$, is:

$$m_1 = -\, p'_v\,(x - x') + p_h\,(y - y')$$

and M_1, being the total moment at the point, $(x_1; y_1)$, of all the external forces, p', from the right abutment and up to the point, $(x_1; y_1)$, is expressed by $M_1 = \int_0^{s'_1} m_1\, ds'$.

Substituting Equation (6) and Equation (7):

$$M_1 = \int_{-a'_m}^{x'_1} p'\,(x_1 - x')\,dx' + \int_{o'}^{y'_1} p'\,(y_1 - y')\,dy' \ldots\ldots\ldots (8)$$

in which, o′ is the value of y' at the right abutment.

Taking the moment due to the forces at the elastic center:

$$M_2 = M_O + (H_O - H_R)\,(y_1 - e_m) - V_O\,x_1 \ldots\ldots\ldots\ldots (9)$$

and, when $y_1 = 0, x_1 = -\,a_m; M_2 = 0; M_O + (H_O - H_R)\,(-\,e_m) + V_O\,a_m = 0$; or,

$$M_O = (H_O - H_R)\,e_m - V_O\,a_m \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

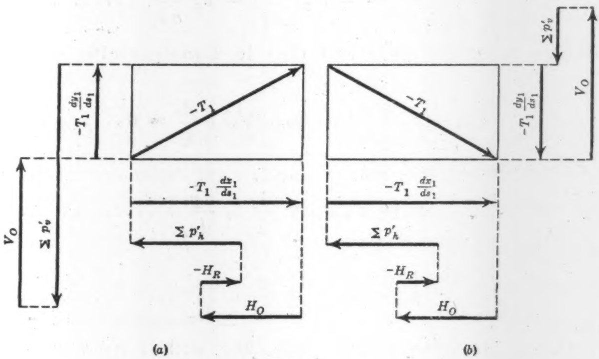


FIG. 4

Substituting Equation (10) in Equation (9), and collecting terms:

$$M_2 = (H_O - H_R)\,y_1 - V_O\,(a_m + x_1) \ldots\ldots\ldots\ldots\ldots (11)$$

From Fig. 4,

$$(H_O - H_R) + \Sigma\, p'_h - T_1\,\frac{d\,x_1}{d\,s_1} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

and,

$$V_O + \Sigma\, p'_v - T_1\,\frac{d\,y_1}{d\,s_1} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Since $p'_h = p'\,\frac{dy'}{d\,s'}$ and $\Sigma\, p'_h = \int_{o'}^{s'_1} p'_h\,ds'$, therefore, $\Sigma\, p'_h = + \int_{o'}^{y'_1} p'\,dy'$.

Likewise, $\Sigma\, p'_v = -\int_{-a'_m}^{x'_1} p'\,dx'$.

Substituting these values in Equations (12) and (13), and re-arranging:

$$\int_{o'}^{y'_1} p'\,dy' = T_1 \frac{dx_1}{ds_1} - (H_o - H_R) \quad \ldots\ldots\ldots\ldots (14)$$

and,

$$\int_{-a'm}^{x'_1} p'\,dx' = -\,T_1 \frac{dy_1}{ds_1} + V_o \ldots\ldots\ldots\ldots (15)$$

Differentiating Equation (5) with respect to s_1,

$$\frac{dM_1}{ds_1} + \frac{dM_2}{ds_1} = 0 \quad \ldots\ldots\ldots\ldots (16)$$

Differentiating Equation (8) with respect to s_1, substituting Equations (14) and (15), and re-arranging:

$$\frac{dM_1}{ds_1} = V_o \frac{dx_1}{ds_1} - (H_o - H_R)\frac{dy_1}{ds_1} + p'_1\,(x_1 - x'_1)\frac{dx'_1}{ds_1} + p'_1\,(y_1 - y'_1)\frac{dy'_1}{ds_1} \;\ldots (17)$$

Differentiating Equation (11) with respect to s_1,

$$\frac{dM_2}{ds_1} = (H_o - H_R)\frac{dy_1}{ds_1} - V_o \frac{dx_1}{ds_1} \quad \ldots\ldots\ldots\ldots (18)$$

Substituting Equations (17) and (18) in Equation (16) and cancelling terms:

$$(x_1 - x'_1)\frac{dx'_1}{ds_1} + (y_1 - y'_1)\frac{dy'_1}{ds_1} = 0 \ldots\ldots\ldots\ldots (19)$$

From Equations (1) and (2):

$$x_1 - x'_1 = +\,\frac{1}{2}\,t_1 \frac{dy_1}{ds_1} \ldots\ldots\ldots\ldots (20)$$

and,

$$y_1 - y'_1 = -\,\frac{1}{2}\,t_1 \frac{dx_1}{ds_1} \ldots\ldots\ldots\ldots (21)$$

Differentiating Equations (20) and (21) with respect to s_1 and re-arranging:

$$\frac{dx'_1}{ds_1} = \frac{dx_1}{ds_1} - \frac{1}{2}\frac{dt_1}{ds_1}\frac{dy_1}{ds_1} - \frac{1}{2}\,t_1\frac{d^2y_1}{ds^2_1} \quad \ldots\ldots\ldots\ldots (22)$$

and,

$$\frac{dy'_1}{ds_1} = \frac{dy_1}{ds_1} + \frac{1}{2}\frac{dt_1}{ds_1}\frac{dx_1}{ds_1} + \frac{1}{2}\,t_1\frac{d^2x_1}{ds^2_1} \quad \ldots\ldots\ldots\ldots (23)$$

From the calculus:

$$\frac{d^2y_1}{ds^2_1} = -\,\frac{1}{R_1}\frac{dx_1}{ds_1} \quad \ldots\ldots\ldots\ldots (24)$$

and,

$$\frac{d^2x_1}{ds^2_1} = +\,\frac{1}{R_1}\frac{dy_1}{ds_1} \quad \ldots\ldots\ldots\ldots (25)$$

Substituting Equation (24) in Equation (22) and re-arranging:

$$\frac{dx'_1}{ds_1} = \frac{dx_1}{ds_1}\left(1 + \frac{1}{2}\,\frac{t_1}{R_1}\right) - \frac{1}{2}\,\frac{dt_1}{ds_1}\,\frac{dy_1}{ds_1} \quad \ldots\ldots\ldots\ldots (26)$$

Likewise, from Equation (23),

$$\frac{dy'_1}{ds_1} = \frac{dy_1}{ds_1}\left(1 + \frac{1}{2}\,\frac{t_1}{R_1}\right) + \frac{1}{2}\,\frac{dt_1}{ds_1}\,\frac{dx_1}{ds_1} \quad \ldots\ldots\ldots\ldots (27)$$

Substituting Equations (20), (21), (26), and (27) in Equation (19), and simplifying,

$$\frac{dt_1}{ds_1} = 0 \quad \ldots\ldots\ldots\ldots (28)$$

which shows that the thickness of the arch section at a given elevation is constant.

Differentiating Equations (14) and (15) with respect to s_1:

$$p'_1\,\frac{dy'_1}{ds_1} = T_1\,\frac{d^2x_1}{ds^2_1} + \frac{dT_1}{ds_1}\,\frac{dx_1}{ds_1} \quad \ldots\ldots\ldots\ldots (29)$$

and,

$$p'_1\,\frac{dx'_1}{ds_1} = -\,T_1\,\frac{d^2y_1}{ds^2_1} - \frac{dT_1}{ds_1}\,\frac{dy_1}{ds_1} \quad \ldots\ldots\ldots\ldots (30)$$

Substituting Equation (22) in Equation (30), remembering Equation (28),

$$p'_1\left(\frac{dx_1}{ds_1} - \frac{1}{2}\,t\,\frac{d^2y_1}{ds^2_1}\right) = -\,T_1\,\frac{d^2y_1}{ds^2_1} - \frac{dT_1}{ds_1}\,\frac{dy_1}{ds_1} \quad \ldots\ldots (31)$$

Substituting Equation (24) in Equation (31) and re-arranging:

$$p'_1 = \frac{\dfrac{T_1}{R_1}\,\dfrac{dx_1}{ds_1} - \dfrac{dT_1}{ds_1}\,\dfrac{dy_1}{ds_1}}{\dfrac{dx_1}{ds_1}\left(1 + \dfrac{1}{2}\,\dfrac{t}{R_1}\right)} \quad \ldots\ldots\ldots\ldots (32)$$

Likewise, from Equation (29):

$$p'_1 = \frac{\dfrac{T_1}{R_1}\,\dfrac{dy_1}{ds_1} + \dfrac{dT_1}{ds_1}\,\dfrac{dx_1}{ds_1}}{\dfrac{dy_1}{ds_1}\left(1 + \dfrac{1}{2}\,\dfrac{t}{R_1}\right)} \quad \ldots\ldots\ldots\ldots (33)$$

Equating Equations (32) and (33) and simplifying:

$$\frac{dT_1}{ds_1} = 0 \quad \ldots\ldots\ldots\ldots (34)$$

which further shows that the thrust at a given elevation is constant.

Substituting Equation (34) in Equation (33) and re-arranging: $T = p'_1\ (R_1 + \frac{1}{2}\,t)$. Since t is constant, $R'_1 = R_1 + \frac{1}{2}\,t$ (see Fig. 3), and, therefore,

$$T = p'_1\,R'_1 \quad \ldots\ldots\ldots\ldots (35)$$

or, substituting Equation (4),

$$T = p_1\,R_1 \quad \ldots\ldots\ldots\ldots (36)$$

Summarizing, an independent, symmetrical, compensated, horizontal arch has the following properties:

(*a*) The thickness, t, is constant;

(*b*) The product of the up-stream radius, R', and the corresponding radial load, p', is equal to the product of the center-line radius, R, and the load on the center line, p, and is constant at any point of the arch;

(*c*) The thrust applied at the center line, T, is constant and equal to the product, $p\,R$ or $p'\,R'$; and,

(*d*) Since the thickness, t, is constant the elastic center is the center of gravity of the center line of the arch.

The same conclusions could be derived in a simpler way by making the assumption that the thickness, t, of the arch ring is constant. However, by making such an assumption no positive proof can be obtained that the arch ring with constant thickness is the only one that is relieved from bending moments, since the possibility would be open that an arch ring with varying thicknesses would be satisfactory.

By attacking the problem in this manner, the analysis proves positively that arch rings meeting the foregoing specifications are the only ones in which there are no bending moments.

IV.—Theory of Elasticity as Applied to an Independent Horizontal Compensated Arch at Constant and Varying Temperatures

The conditions developed in Section III must be met by every compensated arch, so that the static requirements are satisfied. In addition, the compensated arch must meet certain conditions to satisfy the requirements of the theory of elasticity. The only variable that has not been determined for a symmetrical arch is H_R, the horizontal component of the unbalanced part of the force applied at the elastic center. This variable does not depend on static considerations, but on the elastic properties of the material of the arch, on the coefficient of thermal expansion of this material, on the way in which the abutments are displaced under load, and on the difference in temperature of the arch material at the time when the arch is built and at any subsequent time under consideration.

Assume an arch under constant temperature conditions. Let the width equal unity and the cross-sectional area, t; then the relative deflection of two points in any desired direction is:

$$\Delta = \int_{s_1}^{s_2} \frac{1}{E}\,\frac{T}{t}\,n\,ds + \int_{s_1}^{s_2} \frac{M}{I}\,\frac{1}{E}\,m\,ds + \int_{s_1}^{s_2} \frac{V}{t}\,\frac{v}{E_s}\,C_s\,ds \ldots (37)$$

in which, T, M, and V are the total thrust, moment, and shear, respectively, at any point on the center line of the arch; n, m, and v are the thrust, moment, and shear, respectively, at any point on the center line of the arch, due to a unit load applied at the point, s_1, on the center line in the direction of the desired deflection; t is the thickness of the arch; E, the modulus of elasticity of the material in flexure; E_s, the modulus of elasticity of the material in shear; I, the moment of inertia of the cross-section about the center line; C_s, a constant depending upon the shape of the cross-section

(1.2 for rectangular sections); and s_1 and s_2 are the distances of the two points under consideration from a third point, as measured along the center line. The third point is assumed at the intersection of the center line of the arch and the right abutment (Fig. 5).

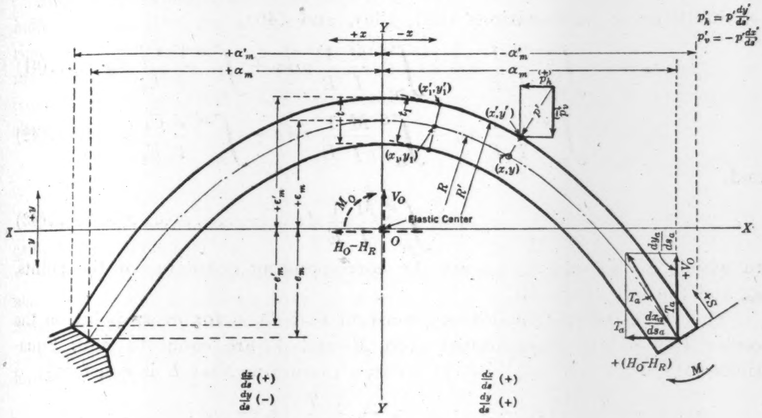

$p'_h = p'\frac{dy'}{ds'}$
$p'_v = -p'\frac{dx'}{ds'}$
+x
Y
−x
+α'm
−α'm
+αm
−αm
(+)
p'_h
p'_v
p'
(x'_1, y'_1)
(x', y')
t_1
t
(x_1, y_1)
(x, y)
R
R'
+e'm
+em
−e'm
−em
M_O
V_O
Elastic Center
X
X
O
$H_O - H_R$
+y
−y
$\frac{dy_a}{ds_a}$
$+V_O$
T_a
T_a
T_a
$\frac{dx_a}{ds_a}$
S
$+(H_O - H_R)$
M
$\frac{dx}{ds}$ (+)
$\frac{dy}{ds}$ (−)
Y
$\frac{dx}{ds}$ (+)
$\frac{dy}{ds}$ (+)

FIG. 5.

Making the elastic center and the elastic axes, the origin and axes of a co-ordinate system, the displacements of the point, s_1, are:

$$\Delta_x = \int_{s_1}^{S} \frac{T}{t}\,\frac{n_x}{E}\,ds + \int_{s_1}^{S} \frac{M}{I}\,\frac{m_x}{E}\,ds + \int_{s_1}^{S} \frac{V}{t}\,\frac{v_x}{E_s}\,C_s\,ds \quad \ldots\ldots(38)$$

$$\Delta_y = \int_{s_1}^{S} \frac{T}{t}\,\frac{n_y}{E}\,ds + \int_{s_1}^{S} \frac{M}{I}\,\frac{m_y}{E}\,ds + \int_{s_1}^{S} \frac{V}{t}\,\frac{v_y}{E_s}\,C_s\,ds \quad \ldots\ldots(39)$$

and,

$$\Delta_z = \int_{s_1}^{S} \frac{T}{t}\,\frac{n_z}{E}\,ds + \int_{s_1}^{S} \frac{M}{I}\,\frac{m_z}{E}\,ds + \int_{s_1}^{S} \frac{V}{t}\,\frac{v_z}{E_s}\,C_s\,ds \quad \ldots\ldots(40)$$

in which,

S = total length of the center line of the arch;
Δ_x = linear displacement of the point, s_1, in the direction of the X-axis;
Δ_y = linear displacement of the point, s_1, in the direction of the Y-axis;
Δ_z = angular displacement; and,
$\left.\begin{matrix} n_x, m_x, v_x \\ n_y, m_y, v_y \\ n_z, m_z, v_z \end{matrix}\right\}$ = the corresponding thrust, moment, and shear due to the unit load applied at the elastic center.

Assuming a symmetrical arch as shown in Fig. 5,

$$n_x = -\frac{dx}{ds}\,; \quad m_x = y\,; \quad v_x = +\frac{dy}{ds}$$

$$n_y = -\frac{dy}{ds}\ ;\ m_y = -x\ ;\ v_y = +\frac{dx}{ds}$$

$$n_z = 0\ ;\ m_z = 1\ ;\ v_z = 0$$

Substituting in Equations (38), (39), and (40),

$$\Delta_x = -\int_{x_1}^{x_2} \frac{T}{t}\,\frac{1}{E}\,dx + \int_{s_1}^{S} \frac{M}{I}\,\frac{1}{E}\,y\,ds + \int_{y_1}^{y_2} \frac{V}{t}\,\frac{C_s}{E_s}\,dy \quad \ldots\ldots(41)$$

$$\Delta_y = -\int_{y_1}^{y_2} \frac{T}{t}\,\frac{1}{E}\,dy - \int_{s_1}^{S} \frac{M}{I}\,\frac{1}{E}\,x\,ds + \int_{x_1}^{x_2} \frac{V}{t}\,\frac{C_s}{E_s}\,dx \quad \ldots\ldots(42)$$

and,

$$\Delta_z = \int_{s_1}^{S} \frac{M}{I}\,\frac{1}{E}\,ds \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(43)$$

in which, x_1, y_1, and x_2, y_2, are the corresponding ordinates of the points, s_1, and the left abutment.

The values of t, T, and I are constant and since for every point on the center line of the compensated arch M and V are equal to zero, Equations (41), (42), and (43) can be written (assuming that E is constant):

$$\Delta_x = -\frac{1}{E}\,\frac{T}{t}\,(x_2 - x_1)\ ;\ \Delta_y = -\frac{1}{E}\,\frac{T}{t}\,(y_2 - y_1)\ ;\ \text{and, } \Delta_z = 0$$

Following the previous assumption that the left abutment is fixed, the displacements, Δ_{xaR}, Δ_{yaR}, and Δ_{zaR}, of the right abutment can be obtained by substituting the lower and upper limits for the values of x, y, and s, corresponding to the left and right abutments, respectively.

For the symmetrical arch (Fig. 5):

$$x_1 = +\,a_m\ ;\ x_2 = -\,a_m$$

$$y_1 = -\,e_m\ ;\ y_2 = -\,e_m$$

Therefore,

$$\Delta_{xaR} = 2\,a_m\,\frac{1}{E}\,\frac{T}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(44)$$

$$\Delta_{yaR} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(45)$$

and,

$$\Delta_{zaR} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(46)$$

In order to "compensate" the arch Equation (44) requires that the right abutment must be moved horizontally in the positive direction or toward the left abutment for the distance, $2a_m\,\frac{1}{E}\,\frac{T}{t}$, which would be opposite to the horizontal movement that would take place as a result of the displacement of the abutment due to the arch thrust.

Temperature Effects.—Considering the effect of the change of temperature, a drop in temperature is analogous to the effect of rib-shortening and the term, $\frac{1}{E}\,\frac{T}{t}$, in Equation (44) can be substituted by $c_T\,T_T$, in which c_T is the coefficient of linear expansion of the material, and T_T is the temperature change, in degrees, being positive for a drop in temperature, and negative for a rise in temperature.

Making this substitution in Equation (44), $\Delta'_{xaR} = 2\,\alpha_m\, c_T\, T_T$, which indicates that, in order to compensate the arch for temperature changes, the right abutment (Fig. 5) is moved horizontally in the positive direction for the distance, $2\,\alpha_m\, c_T\, T_T$. The total horizonal movement that would be required in the positive direction, considering rib-shortening and temperature change, is:

$$\Delta_{xaR} = 2\,\alpha_m \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

If the abutments are considered as fixed, then $\Delta_{xaR} = 0$, and Equation (44) cannot be satisfied.

Consequently, a moment will appear in the arch; applying the theory of elasticity it can be shown that it is equal to a moment that is due to the force, H_R, applied at the elastic center. This moment will be $M_R = -\,H_R\, y$.

Neglecting V and substituting M_R for M in Equation (41), remembering Equation (47):

$$\Delta_{xaR} = 2\,\alpha_m \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) - \frac{H_R}{I}\,\frac{1}{E}\int_0^S y^2\, d\, s = 0 \ldots\ldots (48)$$

and,

$$H_R = 2\,\alpha_m \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) \frac{I\,E}{\displaystyle\int_0^S y^2\, d\, s} \ldots\ldots\ldots\ldots\ldots (49)$$

If the abutments are moved the value of H_R will be increased or decreased and, for the movement of the abutments equal to $2\,\alpha_m \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right)$, the force, H_R, will be equal to zero and the arch will be compensated.

If the displacements of the abutments under loading are considered and if the horizontal components of the displacements of the right and left abutments are D_{haR} and D_{haL}, respectively, then for the purpose of this analysis it can be assumed that the left abutment remains fixed, and the right abutment is displaced in the horizontal direction for the distance,

$$\Delta_{xaR} = -\,(D_{haL} - D_{haR})$$

Substituting this value in Equation (48),

$$2\,\alpha_m \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) - \frac{H_R}{I}\,\frac{1}{E}\int_0^s y^2\, d\, s = -\,(D_{haL} - D_{haR})$$

Whence,

$$H_R = \frac{I\,E}{\displaystyle\int_0^s y^2\, d\, s}\left[2\,\alpha_m \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) + (D_{haL} - D_{haR})\right]$$

The force, H_R, can be divided into three parts, $H_{RS} + H_{RT} + H_{RA}$, in which,

$$H_{RS} = 2\,\alpha_m \frac{T}{t} \frac{I}{\int_0^S y^2 d s} \quad \ldots\ldots\ldots\ldots\ldots\ldots (50)$$

$$H_{RT} = 2 \alpha_m \, c_T \, T_T \frac{I\,E}{\int_0^S y^2 d s} \quad \ldots\ldots\ldots\ldots\ldots\ldots (51)$$

and,

$$H_{RA} = (D_{haL} - D_{haR}) \frac{I\,E}{\int_0^S y^2 d s} \quad \ldots\ldots\ldots\ldots\ldots (52)$$

Equation (50) is the horizontal component of the force applied at the elastic center due to the elastic deformations of the material of the dam; Equation (51) is a similar force due to the changes of temperature of the material of the dam; and Equation (52) is a similar force due to the displacements of the abutments. The vertical components of all these forces are equal to zero for a symmetrical arch.

In order to compensate the arch, the right abutment is moved in the positive direction for the distance, $\Delta_{xaR} = 2\,\alpha_m \left(\frac{1}{E}\frac{T}{t} + c_T \, T_T\right) + (D_{haL} - D_{haR})$, which will make H_R, and, consequently, M_R, equal to zero.

Some other means can be provided that will make H_R equal to zero and, as far as the arch is concerned, will give the same result as a movement of the abutment. This cannot be done by simple changes in the shape of the arch, since the shape is determined from the static requirements.

The important conclusion to be derived from the elastic equations is that the only way to relieve the arch from bending moments is to shape it so as to conform with the conditions derived from statical considerations and to create conditions under which the force, H_R, due to rib-shortening, abutment deflections, and difference in temperature, is made equal to zero.

V.—Compensation of the Independent Horizontal Arch

One way to compensate the properly shaped arch for the moments, M_R, is to introduce stresses that are equal and opposite to those due to the moment, M_R. Mathematically, this is expressed by a force applied at the elastic center of the arch equal and opposite to H_R, neutralizing the action of H_R and compensating the arch in the same way as in the case of a movement of the abutment. It has been proposed to introduce such stresses by proper pressure grouting of the vertical construction joints, but this is open to certain objections.

A positive method of compensating the arch is to provide notches as illustrated in Fig. 6(*a*). The widths of the notches are extremely exaggerated to make the diagram clear. These notches can be designed so as to compensate the arch for the effect of rib-shortening, change in temperature from the time of closing the dam to the average temperature which the dam will attain in service, and for displacement of the abutments.

In an arch compensated in this manner, the deflections will differ from those in an uncompensated arch, and are due to two factors: First, the closing of the notches; and, second, the deformation due to axial compression.

Fig. 6(*b*) illustrates a section of the compensated arch adjacent to the right abutment. The notches in the down-stream face, as shown, will be

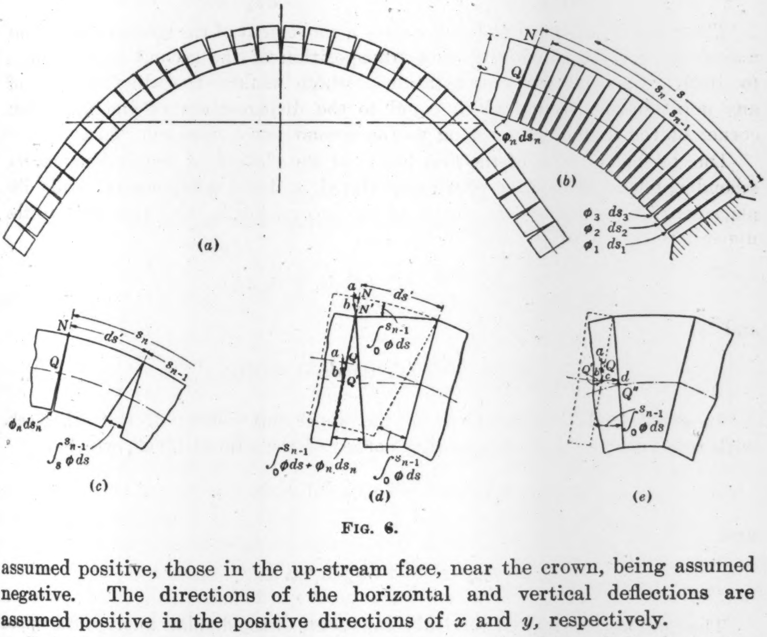


FIG. 6.

assumed positive, those in the up-stream face, near the crown, being assumed negative. The directions of the horizontal and vertical deflections are assumed positive in the positive directions of x and y, respectively.

Let ϕ be the angular measure of a notch per unit of length in the center of the arch. Then the angle of each notch will be $\phi\ ds$. The width of any notch at the face of the arch will be,

$$\Delta_b = t\ \phi\ ds \qquad (53)$$

Starting at the right abutment, let the angle of the first infinitely small notch be $\phi_1\ ds_1$ and that of the second, $\phi_2\ ds_2$ (Fig. 6(*b*)). As the first notch closes, the second one will open by the same angular amount, and, as long as the angles are small, the result will be $\phi_1\ ds_1 + \phi_2\ ds_2$. In turn, when the second notch is also closed the third notch, with a normal opening of $\phi_3\ ds_3$, will open by the amount the second one closed, or the resulting angle will be $\phi_1\ ds_1 + \phi_2\ ds_2 + \phi_3\ ds_3$.

Reducing the widths of the blocks until they are infinitely small, the resulting angle at s_{n-1}, for instance, after all the notches to the right of it have closed, will be $\int_0^{s_{n-1}} \phi \, ds$. When the first notch, with an opening, $\phi_1 \, ds_1$, closes, the block between it and the next notch turns at the same angle, rotating about the upper right corner. The horizontal and vertical components of the displacement of the upper left corner, due to this rotation, are:

$$\delta_{h1} = \frac{dy_1}{ds_1} \, ds'_1 \, (\phi_1 \, ds_1); \text{ and } \delta_{v1} = \frac{dx_1}{ds_1} \, ds'_1 \, (\phi_1 \, ds_1)$$

The rotation of the first block causes a movement of the second block. This movement may be considered as a transposition of the second block parallel to itself (that is, without rotation), which makes the displacement of any point in the second block equal to the displacement of the upper left corner of the first block, as long as the second notch does not close.

Due to the rotation of the first block, or the closing of the first notch, the second notch is opened, as previously stated, and the components of the displacement of the upper left corner of the second block, with reference to the upper left corner of the first block, are:

$$\delta_{h2} = \frac{dy_2}{ds_2} \, ds'_2 \, (\phi_1 \, ds_1 + \phi_2 \, ds_2)$$

and,

$$\delta_{v2} = \frac{dx_2}{ds_2} \, ds'_2 \, (\phi_1 \, ds_1 + \phi_2 \, ds_2)$$

In general, the displacement of the upper left corner of the nth block, with reference to the corresponding corner of the nth—1 block, will be:

$$\delta_{hn} = \frac{dy_n}{ds_n} \, ds'_n \, \Sigma_1^n \, \phi \, ds$$

and,

$$\delta_{vn} = \frac{dx_n}{ds_n} \, ds'_n \, \Sigma_1^n \, \phi \, ds$$

The total horizontal and vertical components of the displacement of the upper left corner of the nth block (designated as N in Fig. 6(b)), can be expressed as the sum of the horizontal and vertical displacements of the upper left corners of all the preceding blocks with reference to the corresponding corner of each preceding block, thus: $a' = \Sigma_1^n \, \delta_h$.

By decreasing the widths of the blocks until they are infinitely small, the following integral expressions for the displacement of the point from N to N' (Fig. 6(d)) may be written:

$$a = \int_0^{s_{n-1}} \frac{dy}{ds} \, ds' \int_0^{s} \phi \, ds; \text{ and, } b = \int_0^{s_{n-1}} \frac{dx}{ds} \, ds' \int_0^{s} \phi \, ds$$

which are also the displacements of the point from Q to Q' as indicated in Fig. 6(d).

After the notch between the nth $-$ 1 and nth blocks has closed, the resulting notch between the nth and nth $+$ 1 blocks will be: $\int_0^{s_n} \phi\, ds = \int_0^{s_{n-1}} \phi\, ds + \phi_n\, ds_n$, which may be considered equal to $\int_0^{s_{n-1}} \phi\, ds$ since both ϕ_n and ds_n are infinitely small. In order to close the arch, this notch must close independently of the preceding notches, and its closing displaces the point on the center line of the nth $+$ 1 block from Q' to Q'', as shown in Fig. 6(e). The horizontal and vertical components of this displacement are:

$$c = \frac{1}{2}\, t\, \frac{dx}{ds} \int_0^{s_{n-1}} \phi\, ds; \text{ and, } d = \frac{1}{2}\, t\, \frac{dy}{ds} \int_0^{s_{n-1}} \phi\, ds$$

After all the notches are closed, the center line of the arch is shortened by the axial compression and shortened or lengthened by the decrease or increase in temperature. The horizontal and vertical components of the shortening of the center line, neglecting the widths of the notches, are:

$$\delta_h = \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) \int_0^{s_{n-1}} \frac{dx}{ds}\, ds; \text{ and, } \delta_v = \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) \int_0^{s_{n-1}} \frac{dy}{ds}\, ds$$

The total horizontal and vertical components of the displacement of the point from Q to Q'' are:

$$D_h = + a - c - \delta_h \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

and,

$$D_v = - b - d - \delta_v \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

Due to the difference in the lengths of the down-stream face, in which the total width of the notch occurs, and the center line, in which only one-half the width of the notch appears, the uniform thrust over the section will cause a differential shortening between the down-stream face and the center line. This differential shortening may be conceived as a compressing together of the faces of the notch before it is closed, due to the thrust on the section, after which the notch is closed. This action of the notches under stress is a reversal of the sequence as compared with actual conditions in the arch.

Fig. 7 illustrates one-half a segment of the arch, of a length, ds, on the center line, with the open notch, $\phi\, ds$, shown on the right. The face of the notch, before and after the thrust is applied, is AB and $A'B'$, respectively. The angle between the radius, AO, and the face of the notch is $\frac{1}{2}\, \phi\, ds$.

Let the angle between the radii, AO and CO, be $\frac{1}{2}\, d\alpha$; the radius to the center line, R; and the radius to the down-stream face, R''. After the thrust is applied, AO will be moved to $A'O$ and the position of AB will be changed to $A'B'$. The angle between $A'O$ and $A'B'$ is designated as $\frac{1}{2}\, \beta\, ds$, which is the value of $\frac{1}{2}\, \phi\, ds$ after the faces of the notch are compressed together due to the action of the thrust, and which is to be substituted for $\frac{1}{2}\, \phi\, ds$ in the equations for the deflections of the arch. The angle between $A'O$ and AO will be $\left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right)\frac{1}{2}\, d\alpha$, considering change in length due to changes in temperature also.

Referring to Fig. 7:

$$DB + BE = DB' + B'E' + E'E \quad \dots\dots\dots\dots\dots\dots (56)$$

in which,

$$DB = \frac{1}{2}\, R''\, d\alpha - \frac{1}{2}\, t\, \phi\, ds$$

$$BE = \frac{1}{2}\, t\, \phi\, ds$$

$$DB' = \left(1 - \frac{1}{E}\frac{T}{t} - c_T\, T_T\right) DB = \frac{1}{2}\left(1 - \frac{1}{E}\frac{T}{t} - c_T\, T_T\right)(R''\, d\alpha - t\, \phi\, ds)$$

$$B'E' = \frac{1}{2}\, t\, \beta\, ds$$

and,

$$E'E = \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) \frac{1}{2}\; R''\, d\alpha$$

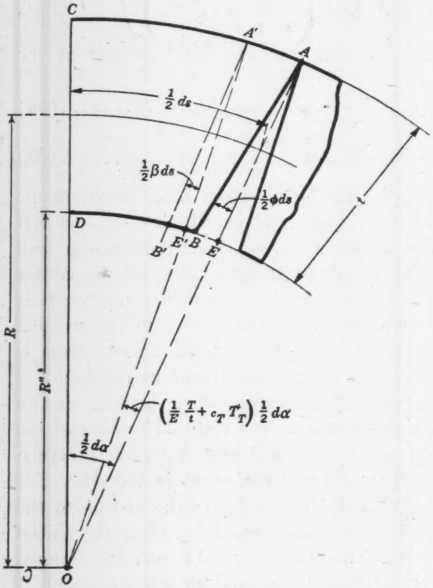

C
A'
A
$\frac{1}{2}\, ds$
t
$\frac{1}{2}\beta\, ds$
$\frac{1}{2}\phi\, ds$
D
E'
B
B'
E
R
R''
$\left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right)\frac{1}{2}\, d\alpha$
$\frac{1}{2}\, d\alpha$
O

FIG. 7.

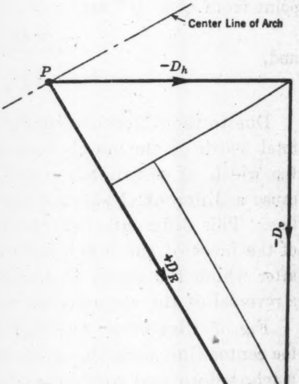

Center Line of Arch
P
$-D_h$
$-D_v$
$+D_R$
$-D_t$
P_1

FIG. 8.

In other words, Equation (56) equals,

$$R''\, d\alpha - t\, \phi\, ds + t\, \phi\, ds = R''\, d\alpha - t\, \phi\, ds$$

$$- \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right)\left(R''\, d\alpha - t\, \phi\, ds\right) + t\, \beta\, ds + \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) R''\, d\alpha$$

Whence,

$$\beta = \left(1 - \frac{1}{E}\,\frac{T}{t} - c_T\,T_T\right)\phi \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

An identical solution is arrived at by considering the notches in the upstream face.

If a displacement of the abutments occurs, an addition to the angle of the notch, $\phi\,ds$, must be made. The displacements of each abutment can be resolved into the vertical components, D_{vaL} and D_{vaR}, and the horizontal components, D_{haL} and D_{haR}, in which, the subscript, aL, refers to the left abutment, and aR to the right abutment.

Noting that $ds' = \frac{R'}{R}\,ds$ and adding the residual displacements of the abutment, the equations for the horizontal and vertical components of the deflections of the arch may be written from Equations (54) and (55), as follows:

$$D_h = \int_0^{y_{n-1}} \frac{R'}{R}\,dy \int_0^s \beta\,ds - \frac{1}{2}\,t\,\frac{dx}{ds}\int_0^s \beta\,ds - \left(\frac{1}{E}\,\frac{T}{t} + c_T\,T_T\right)\int_{-a_m}^{x_{n-1}} dx + D_{haR} \;\ldots\ldots\ldots\ldots (58)$$

and,

$$D_v = -\int_{-a_m}^{x_{n-1}} \frac{R'}{R}\,dx \int_0^s \beta\,ds - \frac{1}{2}\,t\,\frac{dy}{ds}\int_0^s \beta\,ds - \left(\frac{1}{E}\,\frac{T}{t} + c_T\,T_T\right)\int_0^{y_{n-1}} dy + D_{vaR} \;\ldots\ldots\ldots\ldots (59)$$

Differentiating Equation (58) with respect to s:

$$\frac{dD_h}{ds} = \frac{R'}{R}\,\frac{dy}{ds}\int_0^s \beta\,ds - \frac{1}{2}\,t\,\frac{d^2x}{ds^2}\int_0^s \beta\,ds - \frac{1}{2}\,t\,\frac{dx}{ds}\,\beta - \left(\frac{1}{E}\,\frac{T}{t} + c_T\,T_T\right)\frac{dx}{ds}$$

Multiplying by ds, substituting Equation (25), and collecting terms:

$$dD_h = dy\int_0^s \beta\,ds - \left(\frac{1}{2}\,t\,\beta + \frac{1}{E}\,\frac{T}{t} + c_T\,T_T\right)dx \;\ldots\ldots\ldots (60)$$

A similar solution of Equation (59) gives:

$$dD_v = -\,dx\int_0^s \beta\,ds - \left(\frac{1}{2}\,t\,\beta + \frac{1}{E}\,\frac{T}{t} + c_T\,T_T\right)dy \;\ldots\ldots (61)$$

Integrating Equations (60) and (61):

$$D_h = \int dy \int_0^s \beta\,ds - \int\left(\frac{1}{2}\,t\,\beta + \frac{1}{E}\,\frac{T}{t} + c_T\,T_T\right)dx + c_h \;\ldots (62)$$

and,

$$D_v = -\int dx \int \beta\,ds - \int\left(\frac{1}{2}\,t\,\beta + \frac{1}{E}\,\frac{T}{t} + c_T\,T_T\right)dy + c_v \;\ldots (63)$$

in which, c_h and c_v are integration constants.

When $s = 0$, the horizontal deflection is equal to the deflection of the right abutment. Referring to Equation (63):

$$D_{haR} - D_{haL} = \int_0^0 dy \int_0^0 \beta\, ds - \int_{-am}^{-am} \left(\frac{1}{2} t \beta + \frac{1}{E} \frac{T}{t} + c_T T_T \right) dx + c_h = c_h$$

Similarly, $D_{vaR} - D_{vaL} = c_v$. Substituting in Equations (62) and (63):

$$D_h = \int_0^y dy \int_0^s \beta\, ds - \int_{-am}^{x} \left(\frac{1}{2} t \beta + \frac{1}{E} \frac{T}{t} + c_T T_T \right) dx + D_{haR} - D_{haL}$$

and,

$$D_v = - \int_{-am}^{x} dx \int_0^s \beta\, ds - \int_0^y \left(\frac{1}{2} t \beta + \frac{1}{E} \frac{T}{t} + c_T T_T \right) dy + D_{vaR} - D_{vaL}$$

The radial deflection, D_R, and the tangential deflection, D_T, of a point on the center line of the arch may be found from the values of D_h and D_v.

Fig. 8 shows the deflection of a point, P, on the center line of the arch, to the position, P_1. Remembering that $\frac{dy}{ds}$ is negative and $\frac{dx}{ds}$ is positive on the left side, the following equations may be written:

$$D_R = - D_v \frac{dx}{ds} + D_h \frac{dy}{ds} \quad \ldots\ldots\ldots\ldots\ldots\ldots (64)$$

and,

$$D_T = + D_h \frac{dx}{ds} + D_v \frac{dy}{ds} \quad \ldots\ldots\ldots\ldots\ldots\ldots (65)$$

in which, D_R is positive in the down-stream direction, and D_T is positive in the positive direction of s.

Differentiating Equation (64) with respect to s,

$$\frac{d D_R}{ds} = - D_v \frac{d^2 x}{d s^2} + D_h \frac{d^2 y}{d s^2} - \frac{d D_v}{ds} \frac{dx}{ds} + \frac{d D_h}{ds} \frac{dy}{ds} \quad \ldots\ldots (66)$$

Substituting Equations (24) and (25) in Equation (66), and re-arranging:

$$\frac{d D_R}{ds} = - \frac{1}{R} \left(D_h \frac{dx}{ds} + D_v \frac{dy}{ds} \right) - \frac{d D_v}{ds} \frac{dx}{ds} + \frac{d D_h}{ds} \frac{dy}{ds}$$

Referring to Equation (65), substituting Equations (60) and (61), and simplifying:

$$\frac{d D_R}{ds} = - \frac{D_T}{R} + \int_0^s \beta\, ds \quad \ldots\ldots\ldots\ldots\ldots\ldots (67)$$

A similar solution from Equation (65) gives,

$$\frac{d D_T}{ds} = \frac{D_R}{R} - \frac{1}{2} t \beta - \frac{1}{E} \frac{T}{t} - c_T T_T \quad \ldots\ldots\ldots\ldots (68)$$

Substituting the limits of the right and left abutments in Equation (67),

$$\left(\frac{d D_R}{ds} \right)_{aR} = - \left(\frac{D_T}{R} \right)_{aR} + \int_0^0 \beta\, ds; \text{ and } \left(\frac{d D_R}{ds} \right)_{aL} = - \left(\frac{D_T}{R} \right)_{aL} + \int_0^S \beta\, ds$$

Since $\int_0^0 \beta\, ds = 0$ at the right abutment, $\left(\frac{dD_R}{ds}\right)_{aR} + \left(\frac{D_T}{R}\right)_{aR} = 0$. Likewise, $\left(\frac{dD_R}{ds}\right)_{aL} + \left(\frac{D_T}{R}\right)_{aL} = 0$ for a symmetrical arch, and, therefore,

$$\int_0^S \beta\, ds = 0 \quad \text{........................(69)}$$

Equation (68) can also be derived by equalizing the internal and external work. Neglecting the widths of the notches, the internal work done in the arch will be one-half the product of the thrust, T, times the deformation of the center line. The deformation of the center line is $\left(\frac{1}{E}\frac{T}{t} + c_T\, T_T\right)\int_0^S ds$.

The internal work done in the arch will be, $W_I = \frac{1}{2}\, T\left(\frac{1}{E}\frac{T}{t} + c_T\, T_T\right)\int_0^S ds$.

The external work due to the radial loads on the up-stream face is equal to one-half the sum of the products of the load at each point times the radial deflection at that point. From this must be deducted the external work done at the abutment. The external work due to the reaction, T, is one-half the product of the thrust times the tangential deflection at the abutment. This deflection is the tangential deflection on the center line of the arch plus the width of the notch on the center line at the abutment after all the notches toward the crown have been closed. This notch will be $\int_0^s \beta\, ds$, and the width on the center line will be $\frac{1}{2}\, t \int_0^s \beta\, ds$.

The equation for external work will be:

$$W_E = \frac{1}{2}\int_0^S p'\, ds'\, D_R - \frac{1}{2}\; T\int_0^S dD_T - \frac{1}{4}\; Tt\int_0^S \beta\, ds$$

Since $ds' = \frac{R'}{R}\, ds$ and $p'R' = T$:

$$W_E = \frac{1}{2}\, T\left[\int_0^S \frac{D_R}{R}\, ds - \int_0^S dD_T - \frac{1}{2}\, t\int_0^S \beta\, ds\right]$$

Equalizing the internal and external work:

$$\left(\frac{1}{E}\frac{T}{t} + c_T\, T_T\right)\int_0^S ds = \int_0^S \frac{D_R}{R}\, ds - \int_0^S dD_T - \frac{1}{2}\, t\int_0^S \beta\, ds$$

or,

$$\int_0^S dD_T = \int_0^S \frac{D_R}{R}\, ds - \frac{1}{2}\, t\int_0^S \beta\, ds - \left(\frac{1}{E}\frac{T}{t} + c_T\, T_T\right)\int_0^S ds \quad \text{..(70)}$$

Differentiating Equation (70) with respect to s, the result is a formula identical to Equation (68).

Dividing Equation (67) by Equation (68) and simplifying,

$$\frac{dD_R}{dD_T} = \frac{-D_T + R\int_0^s \beta\, ds}{D_R - R\left(\frac{1}{2}\, t\beta + \frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right)}$$

Further simplification gives:

$$D_R\, dD_R + D_T\, dD_T = R\, dD_T \int_0^s \beta\, ds + R\, dD_R \left(\frac{1}{2}\, t\,\beta + \frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) . \quad (71)$$

This differential equation gives the relationship between D_R and D_T.

A solution of Equation (71) is:

$$D_R = \left(\frac{1}{2}\, t\,\beta + \frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) R \quad \ldots\ldots\ldots\ldots\ldots (72)$$

and,

$$D_T = R \int_0^s \beta\, ds \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (73)$$

From Equation (72):

$$\beta = \frac{2}{t}\left[\frac{D_R}{R} - \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right)\right] \ldots\ldots\ldots\ldots\ldots (74)$$

therefore,

$$\int \beta\, ds = \frac{2}{t}\int \frac{D_R}{R}\, ds - \frac{2}{t}\left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right)\int ds \quad \ldots\ldots (75)$$

and,

$$\int_0^s \beta\, ds = \frac{2}{t}\int_0^s \frac{D_R}{R}\, ds - \frac{2}{t}\left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) s \quad \ldots\ldots\ldots (76)$$

From Equation (69):

$$\int_0^S \frac{D_R}{R}\, ds = \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) S \ldots\ldots\ldots\ldots\ldots\ldots (77)$$

Assuming a relationship between D_R and R, for instance,

$$D_R = \frac{\gamma}{R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (78)$$

in which, γ is a constant, and remembering that $R = \frac{T}{p}$, Equations (74) and (76) can be written:

$$\beta = \frac{2}{t}\left[\frac{\gamma}{T^2}\, p^2 - \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right)\right] \ldots\ldots\ldots\ldots\ldots (79)$$

$$\int_0^s \beta\, ds = \frac{2}{t}\,\frac{\gamma}{T^2}\int_0^s p^2\, ds - \frac{2}{t}\left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) s \ldots\ldots\ldots (80)$$

Remembering Equation (69):

$$\gamma = \left(\frac{1}{E}\,\frac{T}{t} + c_T\, T_T\right) \frac{T^2 S}{\int_0^S p^2\, ds} \quad \ldots\ldots\ldots\ldots\ldots (81)$$

Introducing Equation (81) in Equation (79):

$$\beta = \frac{2}{t}\left(\frac{1}{E}\,\frac{T}{t} + c_T\,T_T\right)\left(\frac{p^2 S}{\int_0^S p^2 ds} - 1\right) \text{..........} (82)$$

Finally, from Equations (72), (73), and (57):

$$D_R = \left(\frac{1}{E}\,\frac{T}{t} + c_T\,T_T\right)\frac{p\,T\,S}{\int_0^S p^2 ds} \text{..............} (83)$$

$$D_T = \frac{2}{t}\left(\frac{1}{E}\,\frac{T}{t} + c_T\,T_T\right)\frac{T}{p}\left\{\frac{\int_0^s p^2 ds}{\int_0^S p^2 ds}\,S - s\right\} \text{.....} (84)$$

and,

$$\phi = \frac{2}{t}\,\frac{\frac{1}{E}\,\frac{T}{t} + c_T\,T_T}{1 - \left(\frac{1}{E}\,\frac{T}{t} + c_T\,T_T\right)}\left\{\frac{p^2 S}{\int_0^S p^2\,ds} - 1\right\} \text{.......} (85)$$

Equation (53) gives the value of the width of the notch at the face of the dam for an infinitely small notch. For all practical purposes, the notches may be constructed at commensurate intervals, and their widths can be computed from the value of β for the mid-point of the length of the center line of the arch between the notches, which may be called s_N. This may be done as long as the notches are small enough to allow the arch to deflect within the elastic limit of the material.

Substituting s_N for ds in Equation (53) and remembering Equation (57):

$$\Delta_b = 2\,s_N\,\frac{\frac{1}{E}\,\frac{T}{t} + c_T\,T_T}{1 - \left(\frac{1}{E}\,\frac{T}{t} + c_T\,T_T\right)}\left\{\frac{p^2 S}{\int_0^S p^2\,ds} - 1\right\}$$

As far as the actual construction of the notches is concerned, one of the simplest methods is to place the concrete in alternate blocks and to provide the V-shaped joints, by means of elastic fillers placed between the adjoining blocks. During the time of loading the arch, the V-shaped joints will close and compress the fillers flat or squeeze them out. The difference between the present methods of design and the one proposed is that, at present, under zero load conditions the vertical joints have parallel faces and will be V-shaped after the load is applied. The proposed method will have V-shaped joints under zero load conditions and tightly compressed joints with parallel faces under full load.

The introduction of the V-shaped notches is a radical deviation from accepted design methods since the arbitrary assumption of a continuous horizontal arch is removed, and the arch is considered to be composed of a number of arch blocks in conformity with actual construction practice. The sizes of the V-shaped joints that are computed in accordance with the foregoing formulas represent the minimum necessary to compensate the arch. If a larger deflection of the horizontal arches is desired to improve the stress conditions of the vertical elements, that can be accomplished by increasing the size of the V-shaped joints.

Another way to compensate the horizontal arches without actually building the V-shaped joints is to take advantage of the contraction of concrete after it is placed. If the blocks are properly shaped and alternately placed, the contraction of concrete will result either in the actual forming of V-shaped joints, or in the introduction in the concrete of internal tension stresses the action of which will be the same as if V-shaped joints are provided.

VI.—Vertical Elements

In most of the present arch dam designs, the shapes of the vertical elements are chosen arbitrarily and proper attention is seldom given to the influence which these shapes have on the general behavior of the dam, and the conditions at the horizontal joints and foundations before and after the water load is applied. The following investigation of the action of the vertical elements is made under the assumption of straight-line stress distribution at the horizontal joints and foundations.

Each horizontal joint, as well as the foundation, carries a vertical load that is closely equal to the sum of the dead weight of the part of the dam resting on the joint and the vertical component of the total water load. The arch action has only a negligible effect on this load. The load resting on the horizontal joints and on the foundation produces certain stresses, the distribution of which is different before and after the water load is applied, and both cases should be considered in the design.

During construction the vertical elements are built up as independent piers and the stresses at the horizontal joints and at the foundations, as well as the deflections of the vertical elements, depend on the dead weight of the concrete only as long as no water load is applied. If the vertical cross-sections were made symmetrical with reference to the vertical center line, then the center of gravity of each section would always fall at the middle of the base and under the condition of no water load, the distribution of stresses at each joint would be uniform. The deflection would appear in the vertical direction only, the horizontal component of the deflection being equal to zero. If the shape of the vertical section is not symmetrical, then the center of gravity will be shifted, the stress distribution at the joints will be non-uniform, and, in addition to the vertical deflection, a horizontal deflection will be introduced.

As long as the gravity resultant falls within the middle third of the base at every point of the vertical section, no tension will occur; if the resultant is outside the middle third, tension will appear, and if the resultant is outside the base, the section will be unstable and may overturn during construction unless some means of support are provided.

Considering each vertical element as an independent pier even after the dam is built and closed, and after the part of the water load that is taken by the vertical element is applied, the distribution of stresses in the horizontal joints and at the foundation will depend upon the relative location of the resultant of the dead weight and water load forces.

The distribution of stresses can be specified in any desirable way, the limit being that different distributions will occur for conditions of water load and no water load, which would preclude a specification requiring a uniform distribution of stresses under both conditions. Among the possible specifications for stress distribution at the horizontal joints and at the foundation, the following may be of interest:

1.—No tension stresses are permissible in the design, with reservoir empty, and, for water load, the stress distribution must be uniform.

2.—No tension stresses for reservoir empty and, for water load, the compression at the up-stream face must be equal to, or greater than, the corresponding hydrostatic pressure in order to secure absolutely water-tight horizontal joints on the assumption that the horizontal construction joints would be water-tight under no-load conditions.

3.—No tension stresses are permissible under either water load conditions or reservoir empty.

4.—Reasonably low tension stresses are required under both conditions of loading.

The conventional designs of vertical dam sections cannot meet the first two specifications and generally the best that can be done with such designs is to have them meet the fourth specification. Specifications 1 and 2 require very special designs to make the sections safe under no-water load conditions. Specification 3 represents a reasonable compromise between the rigid requirements mentioned under Specifications 1 and 2 and the usual designs. In such sections a reversal of conditions under no-water load and under total water load must take place, the resultants in both cases intersecting the base within the middle third.

The study of the vertical element is most conveniently started at the crown of a symmetrical dam in which the combined gravity and arch action are most pronounced and the deflections are simplest. Considering such an element, it is necessary to realize the peculiarities of its behavior under water loading. At the lowest part of the dam, the total water pressure is resisted by the vertical elements with very small deflections. Disregarding the effect of Poisson's ratio and the deflections of the foundations, the arch element at the bottom of the dam has no load at all. At higher elevations the deflections of the vertical element increase and, consequently, a part of the water load is transferred into the horizontal arches. The part of the water load thus transferred increases until the elevation is reached at which the deflections of the vertical element are of such magnitude that the entire water load is carried by the horizontal arch element. At still higher elevations a reversal of conditions takes place. The vertical element does not take care of any water load; on the contrary, it increases the loadings of the horizontal arch elements. At these elevations the loading of the horizontal arch is greater than the corresponding water pressure and the vertical element is loaded in the up-stream direction.

In general, the action of each vertical element is such as to assume loadings that will modify the loading of the horizontal arch element, making it zero at the points of greatest hydrostatic pressure and increasing it toward the points of zero hydrostatic pressure.

Due to these loadings the vertical crown element of a symmetrical dam has certain horizontal and vertical deflections, that appear before and after the water load is applied, as follows:

(1) Vertical deflection due to the weight of the dam without water load;

(2) Horizontal radial deflection due to the weight of the dam without water load;

(3) Horizontal radial deflections due to the moments introduced by water load; and,

(4) Horizontal radial deflections due to the shear introduced by water load.

Deflection (1) can be neglected as far as the arch dam design is concerned, and only Deflections (2), (3), and (4), need be considered. The total deflection of the vertical element that enters into the design is the difference between the horizontal, radial deflections at water load and under no-load conditions.

The deflections of a vertical element anywhere between the crown and abutment, or the deflections of the crown element of an unsymmetrical dam, are of the same character under no-water load, but are much more complicated under water load.

The vertical crown element of a symmetrical dam is symmetrical and considering a part of such an element cut out by horizontal planes the resultants of the water pressure carried by the vertical element are applied in the same vertical plane. All other vertical elements of a dam are unsymmetrical and the resultants are applied at different planes. Fig. 9 illustrates the lower part of such an element. The radial load at each elevation is different in direction, as well as in magnitude, and, consequently, the loads are not in a vertical plane through the center of the base.

Due to the fact that all the forces are not parallel, the deflection of the element will consist of a radial deflection, tangential deflection, and torsion, as indicated by the dotted lines, which show the deflected position of the element, greatly exaggerated for clearness. The radial and tangential movements of the center of gravity, *G,* of the horizontal plane at the top of the element, to the position, *G′*, is shown, and the rotation of the axes through this point is also indicated. As a result, instead of the horizontal radial deflections of the symmetrical vertical element, deflections are introduced that are not radial. Considering the deflections of the centers of gravity of each horizontal section, the total can be resolved into radial, tangential, and torsional deflections, in a horizontal plane.

The proper procedure in determining the total deflection of a vertical section from the condition of no load to full water load would be to compute the horizontal deflections under water load due to the combined dead weight and water load and to deduct (with the proper sign) the deflections due to the dead weight only. If tension of any magnitude is developed in the horizontal joints of any of the vertical elements, then the deflections cannot be computed in the conventional way and the behavior of the horizontal joints must be considered.

In computing the deflections of a vertical element it is of importance to include the influence of the elastic deformations of the foundation rock on which the element rests, this deformation resulting in an increase of the horizontal deformations due to the moment and horizontal shear applied at the base. The deflection of the vertical dam elements is influenced considerably by the foundation deformation. This increased deflection becomes more and more important with increase in height of the dam.

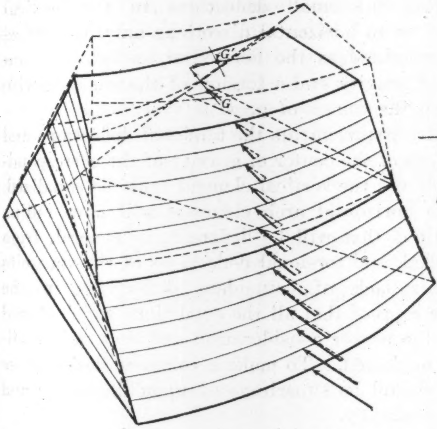


FIG. 9.

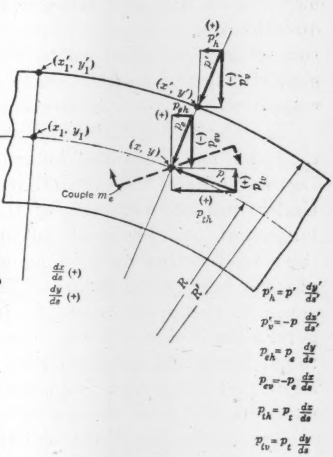


FIG. 10.

If the design does not take care of these conditions cracks in the concrete or the openings in the horizontal joints at one of the faces of the dam are inevitable.

VII.—CONTINUITY OF THE STRUCTURE

The investigation thus far has been based on the assumption of independent arches and independent vertical elements, so that the deflections of each point of a horizontal arch or vertical element were unrestrained. In order to transfer each of these independent arches and elements to an actual structure, a number of additional restrictions must be imposed upon the independent arches and elements, which will cause the dam to act as a unit, thereby retaining the continuity of the structure.

The dam can be considered as made up of a number of voussoirs that are formed by parallel horizontal planes and radial vertical planes. Each voussoir is a part of a horizontal arch, and, at the same time, it is a part of a vertical element.

Considering the horizontal arches and the vertical elements separately, the conception would be that the dam consists of a number of arch voussoirs and an equal number of voussoirs of the vertical element, that each voussoir of the horizontal arch is identical with a corresponding voussoir of a vertical element, and that these voussoirs coincide.

The continuity of the structure under load requires that the corresponding voussoirs that coincided before the load was applied, must coincide afterward. Due to the load application the horizontal arches and the vertical elements will deflect. Disregarding the small deflections in the vertical direction and considering deflections in horizontal directions only, the deflection of any voussoir can be represented as the horizontal, radial, and tangential deflections of its center of gravity and a torsion of the voussoir with respect to a vertical axis through the center of gravity.

In order to fulfill the continuity requirements the torsional deflections and the radial and tangential deflections of the center of gravity of the corresponding voussoirs of a horizontal arch and the vertical element must be equalized. Under load the voussoirs of the horizontal arch elements will have radial, tangential, and torsional deflections that will be different, in general, from the corresponding radial, tangential, and torsional deflections of the voussoirs of the vertical elements. This lack of coincident deflections in the voussoirs that should coincide, is a proof that all the conditions which should govern the design are not given the proper consideration and that some additional forces and moments were neglected. To make a complete design these additional forces and moments should be superimposed upon the forces and moments that were determined originally.

The continuity of the structure is a natural condition that will be assured irrespective of the design assumptions, and the additional conditions that must be considered can be found approximately by evaluating the additional forces and moments that are necessary to equalize the deflections of the coinciding voussoirs. These forces and moments must be applied simultaneously at the voussoirs of the horizontal arches and vertical elements, and they must be in equilibrium as far as the entire structure is concerned. They will represent forces and moment couples transmitted from the voussoirs of the horizontal arches to the voussoirs of the vertical elements and *vice versa.*

These moment couples and forces will disturb the compensation of the horizontal arches and a re-adjustment must be made to re-establish the conditions of compensation. Such re-adjustment can be made by trial, computing the moment couples and forces that are to be applied at the voussoirs of the vertical elements to effect the necessary additional displacements and torsions to bring them into the proper position. These moment couples and forces must be applied in opposite directions on the corresponding voussoirs of the horizontal arches, and as far as the arches are concerned they can be considered as external forces and moments.

These external forces and moments can be substituted by radial and tangential forces and moment couples that are applied at the centers of the arch voussoirs. The moment couples acting along the center line of the arch can be substituted by radial forces. If the moments are designated by m_e and the radial forces by p_e, then,

$$p_e = \frac{d\,m_e}{d\,s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (86)$$

The horizontal arch will finally be loaded by the radial forces, p', due to water load, and by the radial forces, p_e, and tangential forces, p_t, due to the action of the vertical element.

In order to determine the conditions necessary to retain the continuity of the structure, a procedure similar to that presented in Section III may be followed, introducing the radial forces, p_e, and the tangential forces, p_t, acting at the center line of the arch, together with the radial loads, p', acting on the up-stream face. These loads may be resolved, as follows, into their vertical and horizontal components (keeping in mind the positive and negative values of x, y, s, p', p_e, and p_t as indicated in Fig. 10):

$$p'_h = p'\,\frac{d\,y'}{d\,s'}\,;\; p_{eh} = p_e\,\frac{d\,y}{d\,s}\;;\; p_{th} = p_t\,\frac{d\,x}{d\,s}$$

$$p'_v = -\,p'\,\frac{d\,x'}{d\,s'}\,;\; p_{ev} = -\,p_e\,\frac{d\,x}{d\,s}\,;\; p_{tv} = p_t\,\frac{d\,y}{d\,s}$$

The new equations derived by using the foregoing loads on the arch are:

$$\frac{d\,t_1}{d\,s_1} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (87)$$

which is identical to Equation (28);

$$d\,T_1 = p_{t1}\,d\,s_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (88)$$

which corresponds with Equation (34); and,

$$T_1 = p'_1\,R'_1 + p_{e1}\,R_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (89)$$

which corresponds with Equation (36). Integrating Equation (88),

$$T_1 = \int_0^s p_t\,d\,s + T_{aR} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (90)$$

or $T_{aR} = T_1 - \int_0^s p_t\,d\,s$, in which, T_{aR} is the thrust at the right abutment.

Re-arranging Equation (89), and substituting Equation (4),

$$R_1 = \frac{T_1}{p_1 + p_{e1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (91)$$

Equation (90) shows that in order to have uniform thrust the tangential forces, p_t, applied at the center line of the arch must be made equal to zero and then from Equations (91) and (90), $R_1 = \frac{T_{aR}}{p_1 + p_{e1}}$, or, substituting Equation (86), $R_1 = \dfrac{T_{aR}}{p_1 + \dfrac{d\,m_{e1}}{d\,s_1}}$.

In a trial design if the radial deflections of the vertical elements are larger than the corresponding deflections of the arch blocks, the continuity of the structure may be satisfied by an increase in the size of the **V**-shaped vertical joints and a resulting increase in the deflections of the arch blocks. The vertical stresses on the down-stream face of the vertical element should be adjusted by this means in order to avoid or reduce tension and to reduce the avoidable overloading of the upper arches.

The steps by which a compensated dam is designed, taking into consideration the restrictions that the continuity of the structure impose upon the independent horizontal arches and independent vertical elements, may be summarized as follows:

1.—The radial deflections of the centers of gravity of the corresponding voussoirs of the independent horizontal arches and independent vertical elements are equalized by a properly assumed distribution of the water loads between the horizontal arches and the vertical elements.

2.—The tangential deflections of the centers of gravity of the corresponding voussoirs of the horizontal arches and vertical elements are equalized so as to eliminate the tangential force, p_t. This condition will govern the relative shapes of the dam at different horizontal elevations.

3.—The radii of the horizontal arches are varied so as to take care of the twisting moments, m_e, as well as the assumed part of the water load. The moments, m_e, are introduced in order to equalize the torsional deflections of the corresponding voussoirs.

4.—A trial design is made to determine the increase in the deflection of the horizontal arches that may be necessary to improve the conditions of the vertical elements.

Steps 1 and 3 must be followed in any case, but it is not necessary to follow Step 2 in order to have a compensated dam with zero bending moments at the horizontal arches. If this condition is not met, then a compensated dam will result with varying thrusts and stresses along the horizontal arches.

VIII.—Straight-Line Distribution of Stresses

In connection with these analyses it is important to visualize the errors due to the assumption of straight-line distribution of stress. This arbitrary assumption is commonly made in considering the vertical elements of a dam as well as the horizontal arches.

At the points where the vertical elements meet the foundations, or where the horizontal arches meet the abutments or the foundation rock, the distribution of stress is radically different from the assumed straight-line distribution. A considerable increase of stress toward the surface occurs with a corresponding decrease toward the middle of the section.

The only manner in which the stress can approach, at least approximately, the straight-line distribution is to avoid sudden changes from one section to another at the abutments, or from a section of a definite thickness to the indefinitely large foundations, by using fillets to transfer the stresses from the vertical elements or from the horizontal arches gradually to the foundations or abutments. Such details would seem necessary not only to have a better agreement between the assumed and actual conditions, but to avoid the high surface stresses at the juncture of the dam proper and the foundation rock or abutments.

IX.—INFLUENCES OF TEMPERATURE, EXPANSION, AND CONTRACTION OF CONCRETE, AND DISPLACEMENTS OF THE FOUNDATIONS AND ABUTMENTS

During construction, the temperature, due to chemical processes still going on in the concrete, will usually be much higher than the mean temperature that the dam will attain in service. Before the arch dam is closed, the changes in temperature of the separated piers or concrete blocks is of little concern in so far as they affect the design; but every time the horizontal arch is closed at any elevation, arch action is made possible at this elevation, and the changes in temperature will influence the stresses. From this point of view, the concrete temperature of a horizontal element at the time the last of the alternate blocks or piers is filled in, should be considered as the starting temperature to which all subsequent temperatures are to be referred. The temperatures at this time may be quite different in different blocks and at different points, but an average closing temperature at each elevation may be determined for the design. Due to the difference between the temperature of the dam in service as compard with the closing temperature, a certain re-adjustment of deflections and stresses must take place.

Considering a compensated arch dam with radii adjusted to the loadings, the change of temperature will change the deflections of the horizontal element. For a rise in temperature the horizontal element will deflect in the up-stream direction, and, consequently, the load transmitted to the horizontal arch element will increase, and the loading of the vertical element will decrease at the lower parts and increase at the higher parts of the dam. At the same time the horizontal element will become unbalanced (since the product of the new loading and of the up-stream radius of curvature becomes variable), and certain bending moments will be introduced in the horizontal arch due to external loadings.

The rise in temperature will increase the length of the horizontal arch and will introduce bending moments that oppose the moments due to rib-shortening.

Considering the horizontal arch under a rise in temperature the following counteracting influences will occur:

1.—The length of the arch will increase, which will increase the loading of the horizontal element; the increase of the loading, in turn, will increase the rib-shortening and will partly compensate the increase of the length due to rising temperature.

2.—The rising temperature will introduce bending moments that will be partly compensated by the bending moments due to the unbalancing of the product of the load times the up-stream radius.

Under falling temperature a reversal of these conditions will take place. Similar conditions will be encountered if volumetric changes occur in the concrete, as, for instance, in the swelling due to the so-called water-soaking effect.

In these and in similar cases it can be seen that the properly designed and compensated arch possesses a very remarkable self-adjusting feature, which results in a more uniform redistribution of stresses that are imposed upon the dam.

However, there is one important case for which the arch dam cannot be designed to improve conditions and must take all the "punishment," namely, when there is elastic deformation in the rock foundations. Such deformation has an important bearing on the stresses developed in the dam and, consequently, on the design.

In most structures these deformations have little effect on the design as long as they are proportional to the loadings, but in an arch dam their importance cannot be over-emphasized and the lack of reliable information on this subject is a considerable handicap. The assumption made so often, that the foundation rock deformations are negligible, does not conform with actual conditions, and results in inadequate designs.

This case stresses still more the importance of properly distributed V-shaped joints, relieving the arch from stresses that can be avoided.

X.—SYMMETRICAL AND UNSYMMETRICAL DAM SITES

In the application of this analysis to a symmetrical site a symmetrical arch dam will result; it will have a constant thickness at any given elevation and its radii will vary at different points at that elevation, being adjusted to correspond to the distribution of the loadings between the vertical and horizontal elements. The shape of the dam will relieve the horizontal arches from bending moments due to external loading, and the V-shaped vertical construction joints will relieve them from moments due to rib-shortening, difference in temperature, and displacements at the foundations and abutments. At the same time, the thrust and stress in the horizontal direction will be constant at each elevation.

By increasing the ratio of the radius of curvature to the span of the dam, the load sustained by the vertical element will increase until, for a ratio equal to infinity, all of it will be carried by the vertical element and the arch dam will be transformed into a straight gravity section. By decreasing this ratio the part of the loading carried by the horizontal arch elements will increase, the influence of gravity action will decrease, and the dam will approach a circular shape. The theoretical limit will be reached when all load is transmitted to the horizontal arch.

For an unsymmetrical gorge all the main characteristics of a compensated dam—constant thickness, constant product of horizontal arch loading and up-stream radius of curvature, constant thrust, and stress at each elevation—remain the same. Applying the same reasoning that was used for a symmetrical site to an unsymmetrical gorge, it will be found that an unsymmetrical arch dam will result, having the same properties as the symmetrical dam for a symmetrical canyon. In general, this analysis covers any kind of symmetrical and unsymmetrical dams within the limits of the gravity dam and the straight cylindrical segment. The same method of design can be applied to curved gravity dams.

As far as theory is concerned, no difference exists between an arch dam and a curved gravity dam, and both should be designed in the same way. When the radii of curvature of an arch dam are increased under the same maximum stress assumptions, the thickness of the vertical elements will increase and the stresses in the horizontal arches will decrease. Practically that would mean that the load at the sides of the gorge or at the abutments is decreased and the load at the bottom of the canyon or at the foundations is increased, until for a straight dam the sides of the canyon or the abutments are relieved from any load and the total load is taken care of by the foundations.

From a purely theoretical point of view, any desirable distribution of loads between the foundations and the sides or abutments is possible. For the straight gravity section the sides of the gorge are relieved from load, but the vertical section is heavy, and high stresses are developed in the foundations. By loading the sides of the gorge, the vertical section can be made lighter and the stresses can be reduced at the foundation.

For the gravity dam the stress assumptions of the vertical element at no-water load and at water load will govern the shape and size of the vertical cross-section, and practically only one design that meets these assumptions is possible. For a compensated arch dam a number of designs is possible which will meet the stress specification of the vertical element, but will differ as to the curvature of the horizontal elements and the size of the vertical cross-sections.

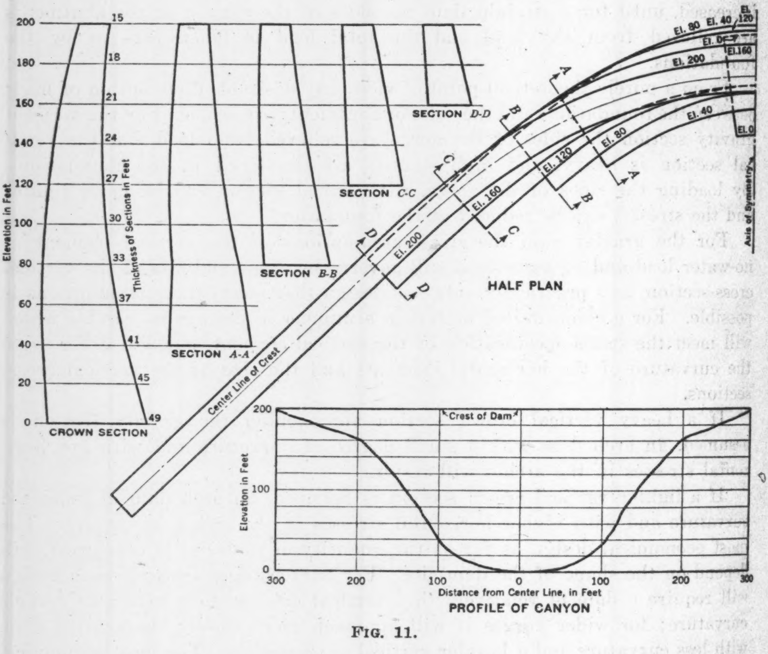
If a heavy vertical crown section approaching the gravity section is assumed, an arch dam with a small degree of curvature and with low horizontal stresses in the arches will result.

If a lighter vertical crown section is assumed, an arch dam of increased curvature and with higher horizontal stresses in the arches will result. The most economical design as far as the quantity of material is concerned, will depend on the shape of the dam site. For narrow, deep canyons such design will require a dam of relatively thin vertical cross-section with considerable curvature; for wider gorges it will approach more closely the gravity dam, with less curvature and a heavier vertical cross-section. The most economical design for each dam site can be determined by the comparison of a number of trial designs with differently assumed crown sections.

In choosing the type of dam for a given site, it is generally conceded that arch dams are economical only for narrow gorges, say, less than 1 000 ft. wide. Actually, the economy depends more on the height of the dam than on the width, and the limits of width that justify the use of arch dams increase rapidly with the increase of the height. For a dam more than 200 ft. high an arch dam probably would be more economical than a gravity dam for much wider gorges, provided the sides can withstand the loads safely without undue deflections.

XI.—Outlines of the Proposed Method of Design

An attempt to apply the results of this theoretical investigation to the design of an arch dam for a given dam site will show that, due to the large number of variables, it is rather difficult to solve the problem in the customary way. The solution is more feasible if the procedure is reversed, by making a number of assumptions concerning the cross-section of the dam, the loading distribution, and the arch deflection. The theoretical dam site that fits these assumptions should be found and a number of trials should be made under different assumptions until one of the theoretical sites will fit the actual dam site. The steps to be followed in these trials are as follows:



Elevation in Feet
200
180
160
140
120
100
80
60
40
20
0
15
18
21
24
27
30
33
37
41
45
49
Thickness of Sections in Feet
CROWN SECTION
SECTION A-A
SECTION B-B
SECTION C-C
SECTION D-D
Center Line of Crest
A
B
C
D
El. 80
El. 40
El. 120
El. 0
El. 160
El. 200
El. 40
El. 80
El. 120
El. 160
El. 200
El. 0
Axis of Symmetry
HALF PLAN
200
100
0
Elevation in Feet
Crest of Dam
300
200
100
0
100
200
300
Distance from Center Line, in Feet
PROFILE OF CANYON

FIG. 11.

1.—The vertical crown section and its loading curve are assumed and the deflection curve is drawn. The loading curve of the horizontal arch at the top of the dam and the length of the top arch must also be assumed.

2.—For the first attempt it is convenient to assume the radial deflections of the horizontal arches to be proportional to the loads sustained by these arches.

3.—Beginning at the top, determine the radius of curvature at the crown, which will give a radial deflection of the horizontal arch at the crown equal to the radial deflection of the corresponding point of the vertical crown section. By trial, determine the radius of curvature at the next lower horizontal arch which will have at each voussoir the same relative radial deflection with respect to the upper arch, as the vertical elements have with respect to the corresponding voussoirs. This procedure is continued for each successive lower elevation and the shape of the dam site is found that corresponds to the assumed set of conditions. Fig. 11 shows such a preliminary layout of an arch dam, with the shape of the resulting dam site.

4.—The tangential and torsional deflections of a few points in different horizontal arches are determined.

5.—The tangential and torsional deflections are computed at a number of elevations for a few vertical elements and these are compared with the corresponding deflections of the horizontal arches. The necessary adjustments are made in the shape of the dam and of the dam site that will equalize the tangential deflections, at the same time revising the radial deflections and the radii of curvature to take care of the additional moments introduced in the horizontal arches.

6.—This preliminary design is modified to take care of any secondary influences that the designer cares to include.

7.—After the final design is made, the sizes of the V-shaped joints are computed, taking into consideration the rib-shortening, abutment deflections, difference in temperatures, and the necessity (if any) of increasing the deflections of the horizontal arches.

ACKNOWLEDGMENTS

The writer has consulted freely the published works of various other writers. Acknowledgment is given to Mr. Philip B. Ramsey, who was instrumental in the preparation of this paper.

DISCUSSION

R. A. Sutherland,[2] Assoc. M. Soc. C. E. (by letter).—This paper is a valuable contribution to the knowledge of the subject of arch dams, and the author is to be congratulated on his candid presentation of a very unusual proposal for improving the loading conditions of such dams. No one who has given thought to the subject doubts that the design of arch dams can be and will be improved, but it is recognized that the number of unknown factors is so large that a perfect knowledge of load distribution will probably never be achieved. While model tests are undoubtedly helpful, the writer believes that they should be interpreted qualitatively rather than quantitatively; that is, as a guide toward a better understanding of load distribution rather than as a means of making exact deductions from the model as to the behavior of the full-sized dam.

The writer is glad that Mr. Karpov has shown the importance of considering the shape of the dam site, because this has a considerable effect, as far as can be known, in determining the relative distribution of load as between arch and beam at the different sections.

With regard to the proposal that V-shaped notches should be provided, the writer is of opinion that any removal of material from the arch must, in practice, reduce the share of load carried by it and increase that taken by beam action. Thus, if the design is based on a given distribution of load as between arch and beam, and the notches are then provided ostensibly to obtain uniform stress conditions in the arch, the result will be that the arch, in deflecting to close the notches (which must occur before it can take any appreciable share of load) will throw an undue load on the beams and falsify the assumptions on which the design was made. At the same time, twisting actions will be caused in the beam sections, which will probably cause secondary stresses just as pernicious as the mal-distribution of stress which it is aimed to avoid.

Apart from all this, the writer does not believe that it is possible to obtain uniform stress distribution in an arch by removing material therefrom in any manner. The gaps must first close before the arch takes load, and when they are closed, the arch is in just as bad case as a solid arch, with the further disadvantage of having its shear strength almost destroyed. Pre-stressing, as advocated by Fredrik Vogt, Assoc. M. Am. Soc. C. E., appears on theoretical grounds to be the only method by which uniform stresses in the loaded arch can be obtained. It may be urged that the closing of the gaps that are proposed actually results in pre-stressing the arch in the desired manner, but in practice the heavy concentrations of load, spalling of the concrete, etc., will destroy any nicely calculated effect of this kind.

Some approach to a better distribution of arch stresses can be obtained by varying the curvature to suit the loads, and the writer would look to developments in this direction for improvement in the practical design of arch dams.

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[2] Wellington, New Zealand.

LARS R. JORGENSEN,[3] M. AM. SOC. C. E. (by letter).—The author has proposed the design of an arch dam, in which the thrust at the abutments is to be applied tangent to the neutral axis of each arch ring of uniform thickness and in which single arch elements can be shifted automatically into the correct position under load to allow the thrust due to full load to be uniformly distributed over the arch sections along the arch ring. In such a case there would be no bending moment, which would be desirable.

Mr. Karpov has done considerable work on this design; he is working in a field in which there are still wide gaps and his contribution is valuable in helping to supply improvements.

In Part III, he states:

> "The problem of finding the proper shape of the arch resolves itself into finding the equation of the center-line curve of the arch for which the changing radial loads applied at the up-stream face will produce zero bending moments at every point of the center-line curve."

The greatest problem is to find the value of the changing radial loads on the arch at various points, as upon the accuracy of this depends the adequacy of the resulting design. As the author has stated, the cantilever carries most of the water load at the bottom, and at the top the arch generally has to carry a part of the cantilever load, in addition to all the water load.

It is well known that in order to compute the percentage of load taken by the various cantilevers several assumptions must be made, that are rather difficult to formulate correctly. Therefore, they are often omitted—especially the water-soaking effect and the difference in value of E between wet and dry concrete. Whether or not there will ever be actual vertical tension in the up-stream face of the cantilevers of an arch dam depends mainly upon the degree of water-soaking effect present in the section, but also to quite an extent on the difference in the amount of the total deformation of wet and dry concrete. The total deformation of dry concrete is approximately 1.6 times that of wet concrete. This fact will cause more load to be thrown on the arch than if, as is customary, E is assumed constant.

The water-soaking effect has thus far been omitted, when calculating the variable radial load on the arch slice, because less is known about it than any other factor. At its worst—that is, when the distribution varies from a maximum at the wet up-stream face to zero at the dry down-stream face—the arch may be forced to hold the cantilever back in addition to supporting all the load. At its best the water-soaking effect can be omitted. When designing an important structure the worst condition that can happen is of more interest than anything less than that. If the worst condition of loading is assumed, however, the working stresses can safely be chosen rather high.

The Diablo Dam on the Skagit River, in Washington, was designed to carry all the load on the arch without help from the cantilever. The section could have been made thinner (16 ft. at the crest and 98 ft. minimum at the bottom) had the cantilever action been taken into consideration and the possible water-soaking effect omitted, but it was assumed that the two actions would about equalize one another.

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[3] Cons. Hydro-Elec. Engr., Berkeley, Calif.

The crushing strength of the mass concrete in this dam at the age of one year was about 6 000 lb. per sq. in. It would appear to be safe to use maximum stresses, or at least maximum apparent stresses, of 1 000 lb. per sq. in. for such concrete in large dams, when assuming all the load as taken by the arch, since in such case the actual maximum stress should be less, whereas had the effect of the cantilever been included as supporting part of the load and had the water-soaking effect been omitted, the actual maximum stress certainly would be higher. Of course, material can always be saved by using higher stresses. If the cantilever is considered as taking a certain proportion of the load, using a constant *E,* omitting the water-soaking effect, and allowing 600 lb. per sq. in. for maximum compression, the actual stresses may well be about 1 000 lb. per sq. in. The water-soaking effect is always favorable to arch action and unfavorable toward forcing the cantilever to take load.

Concrete with a crushing strength of 6 000 lb. per sq. in., when one year old, will safely withstand a maximum calculated stress of 1 000 lb. per sq. in. To secure good concrete is much more important than to obtain mere low stress in a high dam since such concrete is also tight, and the life of a high concrete dam is in direct proportion to its tightness.

At one time the writer thought that bending moments in an arch dam could be eliminated just by changing the shape of the arch, making it rounder near the abutments, where the center line of pressure bends in a down-stream direction. This can not be done, however; it does require dividing the arch ring and shaping the voussoirs or blocks as shown in the author's Fig. 6, in order to allow the arch to adjust itself. More blocks will allow a finer adjustment than just a few blocks. Making the blocks 40 to 50 ft. long, as between ordinary contraction joints, would not accomplish the necessary adjustment and it becomes a pertinent practical question, therefore, how far to go with the division of the ring in order to avoid the bending.

The cost of constructing these blocks accurately and making them water-tight must at least balance the cost of taking care of the bending by means of added material at and near the abutments, where this added material incidentally takes care of the maximum shear also. The shearing strength of the concrete is an important item for holding the dam in place especially toward the bottom.

Considering the uncertainties in computing the variable radial loads on the arch elements and the many loose joints in the arch, the compensated arch should be dimensioned conservatively, because some bending may actually be thrown upon it.

Mr. Karpov, who is connected with a large corporation having first-class facilities for testing purposes, is in a favorable position for carrying on investigations of the greatest unknown influential factor in this connection, the general distribution of the water-soaking effect.

A study of the behavior of round or square horizontal concrete columns, subject to water pressure at one end and to the atmosphere at the other end, should yield valuable information about the distribution of the water-soaking effect throughout a body of concrete. It would be necessary to paint the sides of the specimens with a heavy coat of asphaltum to prevent evaporation and also to provide metal plugs at intervals for measuring purposes.

The writer believes that time and money spent on such an investigation would be worth while and suggests that the author go into this matter with the same determination as he has shown in his previous work. Before much faith can be placed on the positive existence of cantilever support, more information must be available on the distribution of water-soaking effect than now exists.

L. T. EVANS,[4] JUN. AM. SOC. C. E. (by letter).—Without a doubt, Mr. Karpov has made a valuable addition to engineering literature. For the most part his paper seems theoretically correct except for the statement that only arches of constant thickness can have zero moment at any point along the arch axis, neglecting the effect of rib-shortening. It is to be regretted that Mr. Karpov made this error because constant thickness is not necessary in order to have zero moment along the arch axis.

In checking over Mr. Karpov's work, equation by equation, there is no objection to be raised until the derivation of Equation (19) is reached. Every step is mathematically legitimate and no errors are involved. From the mathematician's standpoint all is well, but not from an engineer's standpoint. The engineer is interested in Equations (5), (8), and (11).

Consider a circle defined by the equation, $(x')^2_1 + (y')^2_1 = (R')^2$; from which, $\frac{dy'_1}{dx'_1} = -\frac{x'_1}{y'_1}$. Next, shift the origin so that $x'_1 = x_1 - x'$, and $y'_1 = y_1 - y'$. It follows that:

$$\frac{dy'}{dx'} = -\frac{x_1 - x'}{y_1 - y'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (92)$$

Equation (92) is at once recognized as a transposed form of Equation (19). Since Equation (19) is purely a statement of mathematics and can be derived by mathematical operations entirely free from all reference to an arch, it is obvious that it does not imply that the moment on the axis of an arch must equal zero. In other words, Mr. Karpov has lost all trace of his moment, shear, and thrust by the time he has reached Equation (19). As no moment, shear, or thrust terms are added in any operation between Equation (19) and Equation (28), it is impossible to see how any results obtained by any expression between these equations can be said to follow from the fact that the moment on the arch axis must equal zero.

Correct methods of attack have been devised, that eliminate the possibility of such errors. One method is to evaluate Equations (8) and (11) and set the sum equal to zero. Equation (8) can be expressed in such a manner that the thickness of the arch rib at any point appears as a variable. It will then be possible to study the effect of a variable thickness on the moments along the arch axis.

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[4] Long Beach, Calif.

If the origin of co-ordinates is taken at the "crown" of the arch on the axis of the arch, it is possible to set up an expression for the moment about the variable point, x, y, which is:

$$M_{xy} = \frac{p'}{2}\left[x^2 + \left(y + \frac{t_0}{2}\right)^2 - \frac{t_x^2}{4}\right] + M_0 + V_0 x - H_0 y \ldots\ldots (93)$$

In order to satisfy Equation (5):

$$\frac{p'}{2}\left[x^2 + \left(y + \frac{t_0}{2}\right)^2 - \frac{t^2_x}{4}\right] + V_0 x - H_0 y = 0 \ldots\ldots\ldots (94)$$

It is seen that Equation (94) can be satisfied by an infinite number of values of t_x, and as it is the equation of the neutral axis of the arch, it is obvious that there can be many shapes of arch ring that satisfy the conditions imposed by Equation (5). When the variable, t_x, is expressed in terms of x, y, and t_0, the equation of the neutral axis has a definite form.

Again, consider the problem in the light of the column analogy[5], as presented by Hardy Cross, M. Am. Soc. C. E. The change in moment will be given by the expression:

$$m_t = \frac{P}{A} + \frac{M'_x}{I'_x}\,x + \frac{M'_y}{I'_y}\,y \ldots\ldots\ldots\ldots\ldots\ldots (95)$$

and since it is assumed that there is to be no moment on the arch axis, the value of P, M'_x, and M'_y, must be equal to zero. As these factors are zero, the change in moment is also equal to zero and Equation (5) is satisfied independently of A, I'_x, and I'_y in Equation (95). As these terms are dependent upon the variation of the thickness, t_x, this is an undisputable proof that the condition of zero moment along the arch axis does not define the variation of the thickness of the rib.

It might also be mentioned that the moment at any point along the arch axis is equal to the product of the thrust at that point and the eccentricity. As long as the section of the rib is made symmetrical about the line of thrust, there is no moment, irrespective of the amount of section on each side of the pressure line.

This error detracts little from the value of Mr. Karpov's work; yet the writer feels that attention should be called to it for the benefit of engineering students who might use the paper as a reference.

H. W. SIBERT,[6] ESQ. (by letter).—In Part III of Mr. Karpov's paper, Equation (28) is incorrect. Hence, his conclusion that the thickness must be constant is erroneous. This failure to define p' has made it difficult to locate the source of his error. At the beginning of Part III, one gets the impression that p' is the radial load, a term which presupposes a circular arc; while immediately after Equation (4), p' is called the external load. In the discussion that follows, p' is considered to be the normal pressure of the water per unit area on the up-stream face at any point of the horizontal section of the dam shown in Fig. 3. Since p' is the pressure perpendicular to the up-stream face, it depends only upon the depth of the horizontal section below the surface of the water. Hence, p' is independent of x' and y'.

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[5] See *Bulletin No. 215*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

[6] Associate Prof. of Mathematics, Coll. of Eng. and Commerce, Univ. of Cincinnati, Cincinnati, Ohio.

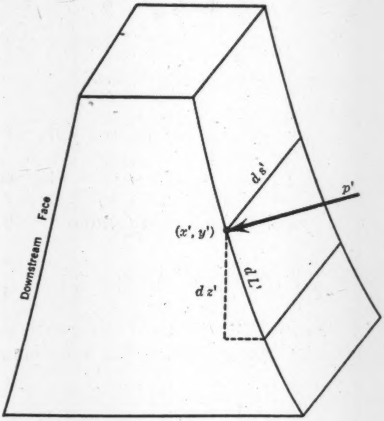

FIG. 12.

Consider an infinitesimally small area on the up-stream face having a length, dL', and a width, ds' (see Fig. 12). The normal force, dP', on this area is:

$$dP' = p' \, ds' \, dL' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (96)$$

If dz' is the vertical projection of dL', the horizontal component of this normal force is:

$$dP'_h = \frac{dz'}{dL'} dP' = p' \, ds' \, dz' \quad \ldots\ldots\ldots\ldots\ldots\ldots (97)$$

Thus, the horizontal component of the force per unit of height is:

$$dQ'_h = \frac{dP'_h}{dz'} = p' \, ds' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (98)$$

Furthermore, from Fig. 3 of the paper, the clockwise moment of dQ'_h about the point, (x_1, y_1), is:

$$dm' = (x_1 - x') \frac{dx'}{ds'} dQ'_h + (y_1 - y') \frac{dy'}{ds'} dQ'_h \ldots\ldots\ldots\ldots (99)$$

Hence, by use of Equation (98):

$$dm' = p' (x_1 - x') \, dx' + p' (y_1 - y') \, dy' \quad \ldots\ldots\ldots\ldots (100)$$

Finally, since $M_1 = \int \frac{dm'}{p}$, the identical value of M_1 given in Equation (8) of Mr. Karpov's paper is obtained.

Since not only p', but also x_1 and y_1 are independent of x' and y', the evaluation of the two integrals given in Equation (8) is an easy matter. Thus,

$$M_1 = \left[\frac{p'}{2}\,(x_1 - x')^2\right]_{-\alpha'_m}^{x'_1} - \left.\frac{p'}{2}\,(y_1 - y')^2\right]_{o'}^{y'_1} \quad \ldots\ldots\ldots(101)$$

or,

$$M_1 = \frac{p'}{2}\left[-(x_1 - x'_1)^2 + (x_1 + \alpha'_m)^2 - (y_1 - y'_1)^2 + (y_1 - o')^2\right].(102)$$

As the square of the hypotenuse equals the sum of the squares of the other two sides, $(x_1 - x'_1)^2 + (y_1 - y'_1)^2 = \frac{t^2_1}{4}$. The substitution of this result in Equation (102) gives the following value for M_1:

$$M_1 = \frac{p'}{2}\left[(x_1 + \alpha'_m)^2 + (y_1 - o')^2 - \frac{t^2_1}{4}\right]\ldots\ldots\ldots\ldots(103)$$

When this value of M_1 and the value of M_2 given in Equation (11) are substituted in Equation (5), the condition for zero moment at each section becomes,

$$\frac{p'}{2}\left[(x_1 + \alpha'_m)^2 + (y_1 - o')^2 - \frac{t^2_1}{4}\right] + (H_0 - H_R)\,y_1 - V_0\,(\alpha_m + x_1) = 0\,..(104)$$

As (x_1, y_1) is any point on the neutral axis, less confusion will result if this point is replaced by the variable point (x, y), and, at the same time, t_1 is replaced by t. Thus, the condition for zero moment at every section is,

$$\frac{p'}{2}\left[(x + \alpha'_m)^2 + (y_1 - o')^2 - \frac{t^2}{4}\right] + (H_0 - H_R)\,y - V_0\,(\alpha_m + x) = 0\,..(105)$$

For the particular case, in which $t = k$ (a constant), the neutral axis is the circular arc:

$$\frac{p'}{2}\left[(x + \alpha'_m)^2 + (y - o')^2 - \frac{k^2}{4}\right] + (H_0 - H_R)\,y - V_0\,(\alpha_m + x) = 0\,..(106)$$

Moreover, if $t = 2\,(y - o')$, the neutral axis is the parabolic arc:

$$\frac{p'}{2}\,(x + \alpha'_m)^2 + (H_0 - H_R)\,y - V_0\,(\alpha_m + x) = 0\ldots\ldots\ldots(107)$$

Furthermore, if $t = k + cx$ (k and c being constants and the numerical value of c being less than 2), the neutral axis is the elliptical arc:

$$\frac{p'}{2}\left[(x + \alpha'_m)^2 + (y - o')^2 - \frac{(k + cx)^2}{4}\right] + (H_0 - H_R)\,y - V_0\,(\alpha_m + x) = 0.(108)$$

Equation (105) shows that t can be any function of x and y. However, for each particular function of x and y assigned to t, the equation of the neutral axis is of a particular form, as has been demonstrated in the preceding paragraph.

EUGENE KALMAN,[7] M. AM. SOC. C. E. (by letter).—Original ideas and interesting results that are distinctly thought-provoking are presented by the author. The paper can be subdivided into three parts, dealing with: (1) The compensation of the arch; (2) the design of compensated dams; and (3) some general aspects of arch dam design.

The study of isolated arches, free from bending moments and shears, embraces a formidable problem, which many consider impossible of solution. The conception of notches as a structural expedient is intriguing. The author shows that an arch, if conveniently shaped, can be compensated by inserting a system of properly proportioned notches, provided they will close perfectly under the action of the load, thus developing uniform compressive stresses on the face of the notches. What he does not show, and what is scarcely susceptible of proof, is that a series of blocks strung between two fixed abutments will become adjusted, under the action of the load, exactly in the manner expected by the author, and not in some of the other innumerable ways that are possible, at least according to logic.

Before proceeding to the discussion of the author's results, a few remarks should be made as to the methods used in the paper. By and large, the mathematical machinery, including the selection of the symbols, is cumbersome, even to those thoroughly trained in mathematics. In addition, the author is unusually sparing of hints as to his aims. Many readers of more than average preparation will have asked themselves; "What is he driving at?". Heavy, formulistic mathematical developments are liked by neither engineers nor mathematicians.

Using five pages, Mr. Karpov proves that an arch fixed at one end and free at the other, will have bending moments and shearing stresses unless t is constant, and $T = p' R'$ is constant. This proof can be demonstrated much more simply. On the two faces of the arbitrary arch element in Fig. 13, moments and shears are excluded by assumption. Thus, the normal forces, T and $T + \frac{dT}{d\beta} d\beta$, as resultants of normal stresses uniformly distributed on the faces, will act at the centers of the faces. The force, $p' ds'$, acts in the center of the extrados element, ds', which may have an inclination, tan ω, with respect to the neutral axis and can be divided into components parallel and normal to the axis. The latter will meet the two lateral forces at their points of conjunction, so that this trio of forces will yield no bending moment with respect to that point, while the moment of the other component will be $p' ds' \sin \omega \frac{t}{2}$. This must be equal to zero. Hence, $\omega = 0$; that is, t is constant. Furthermore, the load acts in a direction normal to the neutral axis; and, evidently, $\frac{dT}{d\beta} = \frac{dT}{ds'}$. All this follows from the fact that $\omega = 0$. The condition of equilibrium of the three forces in the directions parallel and normal to the neutral axis yields $T + \frac{dT}{ds'} - T = \frac{dT}{ds'} = 0$; or T is a constant, and $T\, d\beta = p'\, ds' = p'\, R'\, d\beta$; or $p'\, R' = T$, respectively.

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[7] New York, N. Y.

Here the assumption is made that there is no shear on the faces of any section, an assumption which is not stated explicitly in the author's proof. However, it is tacitly introduced in Fig. 4, in which the horizontal and vertical components of load and end reactions are apportioned to cover the normal stress, T, only.

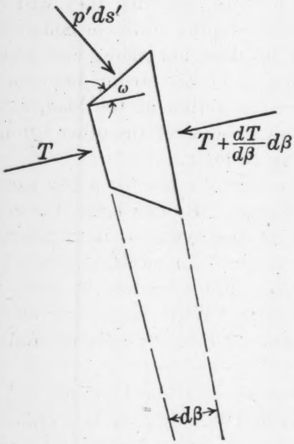

FIG. 13

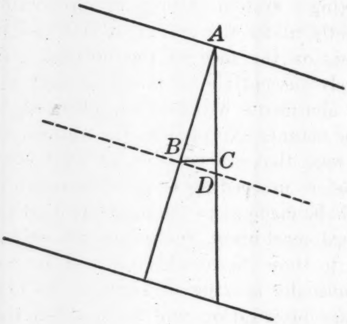

FIG. 14

In Mr. Karpov's exposition no use is made of his assumption, $M = M_1 + M_2 = 0$, as expressed by Equation (5); only the assumption, $\frac{dM}{ds'} = 0$, as expressed by Equation (16), is utilized, which means that in an arch fixed at one end and free at the other, the static condition, in which, t and T are constants, and $T = p'\, R'$, admits the presence of an invariable arbitrary moment, $\mathbf{M}$, along the entire arch. This is also seen in the writer's proof, since the relation,

$$p'\, ds' \sin \omega \, \frac{t}{2} + \mathbf{M} - \mathbf{M} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (109)$$

does not alter the results. Equation (109) expresses the absence of resultant bending moments if there are two moments, $\mathbf{M}$, that are equal and of opposite signs, acting on the two faces of the element. The presence of invariable moments is excluded only in a later stage of the investigation, when the notched arch span between two fixed abutments is analyzed.

The length of the author's demonstration as compared to the foregoing short derivation is due to the fact that instead of treating an arch element, he deals unnecessarily with the entire portion of the arch between an arbitrary section and the free end. Substantial "short-cuts" are possible even along that line of thought. For instance, that t is a constant can be inferred immediately from Equation (19), if the following two relations are considered:

$$\frac{d\left(\frac{t_1}{2}\right)^2}{ds_1} = \frac{d}{ds_1}\left[(x_1 - x_1')^2 + (y_1 - y_1')^2\right] = 2\,(x_1 - x_1')\left(\frac{dx_1}{ds_1} - \frac{dx_1'}{ds_1}\right)$$
$$+ 2\,(y_1 - y_1')\left(\frac{dy_1}{ds_1} - \frac{dy_1'}{ds_1}\right)$$

and,

$$(x_1 - s_1')\,\frac{dx_1}{ds_1} + (y_1 - y_1')\,\frac{dy_1}{ds_1} = 0 \;\ldots\ldots\ldots\ldots\;(110)$$

Equation (110) follows directly from Equations (1) and (2), or from elementary geometry in Fig. 14, which shows that,

$$(x_1 - x_1')\,\frac{dx_1}{ds_1} = \overline{BC}\,\frac{\overline{AC}}{\overline{AB}}\;;\text{ and }(y_1 - y_1')\,\frac{dy_1}{ds_1} = -\overline{AC}\,\frac{\overline{BC}}{\overline{AB}}$$

Similarly, the exposition of the theory of elasticity in Section IV is too mathematical and unnecessarily long. An arch, fixed at one end and free at the other, is of such a shape that, under the action of a well-defined load, only a uniformly distributed compression of constant magnitude develops on any of its sections. In such a case, however, it is self-evident that the individual displacement of one face of a given elementary block relative to the other face consists merely in a translation, without any rotation. Consequently, the deflection of the free—say, right—end section due to the deformation of the entire arch will also be pure translation. Its magnitude normal and parallel to the chord of the arch (depending on the individual displacement of the right face of an elementary block relative to its left face), is $\frac{dy}{ds}\,\frac{T}{Et}\,ds = \frac{T}{Et}\,dy$, and $\frac{dx}{ds}\,\frac{T}{Et}\,ds = \frac{T}{Et}\,dx$, respectively, the chord being the X-axis. Therefore, the total displacement of the free end section in the two directions, due to the total deformation of the arch, will be $\frac{T}{Et}\int dy$ and $\frac{T}{Et}\int dx$, or 0, and $\frac{TL}{Et}$, respectively, in which, L denotes the length of the chord. Furthermore, as is known universally, temperature changes and shrinkage have analogous effects on the deflection of the arch; therefore, the information in the latter part of Section IV, including Equations (47) to (52), is superfluous. Furthermore, it is certain to confuse many readers because of the unnecessary digression involved in the mention of a moment generated by temperature changes in the elastic center of the arch, a fact that has no bearing at all on the subsequent developments in the paper.

Substitution of that part through a single statement to the effect that the total translation of the free end section in the direction of the chord equals $2\alpha_m\left(\frac{T}{E\,t}+c_T\,T_T\right)+(D_{haL}-D_{haR})$, would add much to the clarity of this part of the paper.

The decrease in the opening of the notches, due to uniform compression and temperature changes, is evaluated in Fig. 7 and the supporting text. The writer doubts the necessity of "proving" Equation (57) by applying to it such complex mathematics. Underlying the author's reasoning, evidently, is the tacit assumption that, owing to the infinitesimal magnitude of the angle, $d\alpha$, the lines, $C\,A$ and $D\,B$ (Fig. 7), are straight lines. In fact, if they were circular segments, the line, $A\,B$, would become a curved line corresponding to $A'\,B'$ in Fig. 7, under the action of uniform compression. This would affect, seriously, the principal assumption of the paper, namely, that the joints of the closed notches will be straight lines as would be the case if uniform normal stresses and absence of shearing stresses are assumed. On the other hand, if the lines, $C\,A$ and $D\,B$, are treated as straight lines, Equation (57) follows immediately from the similarity of the diagrams.

Likewise, the method of deriving Equations (67) and (68) is unnecessarily cumbersome. These formulas are quite interesting and, from a theoretical point of view, valuable, but, unfortunately, few engineers will be tempted to fight their way through the skillful but wearisome deductions. Hence, the following graphical and simple derivation is presented.

Let the arch be considered fixed at the right end, as shown in Fig. 6(*b*), and free at the left end. First, consider the deflections of kinematic character; that is, the deflections due exclusively to the closing of the notches. In Fig. 15, Notches 1, 2, etc., close, one after another; the parts of the arch to the left of the hinges, 1, 2, etc., will undergo successive translations parallel to the left faces of the notches, 2, 3, etc. In this process of successive translations, Point i' of the extrados describes a certain path which is represented schematically in Fig. 16, by the broken line, $O\,I'$, composed of small chords, the last of them being derived by rotation around the center, h', in Fig. 15. Represented in Fig. 16 by $H'\,I'$, it is of the magnitude, $ds'\int_1^h \phi\,ds$, and has the direction normal to that of the chord, $h'\,i'$ (Fig. 15) immediately prior to the rotation around h'. The path of the hinge, j', may be represented by the same broken line as far as H', and, from that point, it will continue to I'' ($H'\,I''$ being parallel to the left face of the notch, i'), and then to the point, J', the chord, $I''\,J'$, being of the magnitude, $ds'\int_1^i \phi\,ds$, while its direction is normal to that of the chord, $i'\,j'$, immediately prior to the rotation around i'. During the rotation of the block, $h'\,i'$, around the hinge, h', the chord, $i'\,j'$, does not change its direction; thus, the chords, $H'\,I'$ and $I''\,J'$, have the directions of the radii of curvature of the arch in the points, i' and j', a fact which will be utilized later in this discussion.

The deflection at the point, i, of the neutral axis, can be represented by the same broken line, $O\,I'$, completed by a segment, $I'\,I$, representing the tangential displacement of Point i, due to the rotation of the block, $i'\,h'$, around h'. The direction of $I'\,I$ is normal to $H'\,I'$; its magnitude is $\frac{1}{2}\,t \int_1^h \phi\, ds$. By analogy, the deflection of the arch in the point, j, may be represented by the broken line, $O\,J'\,J$, the segment, $J'\,J$ being normal to $I''\,J'$, while its magnitude is $\frac{1}{2}\,t \int_1^i \phi\, ds$. It is easy to see that the magnitude of

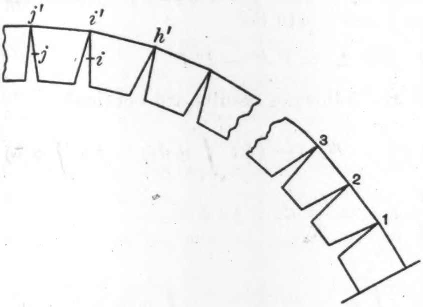

FIG. 15

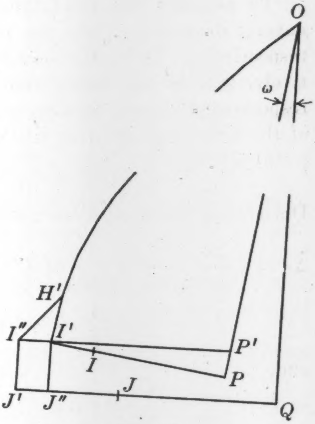

FIG. 16

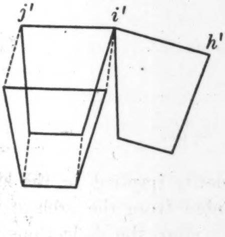

FIG. 17

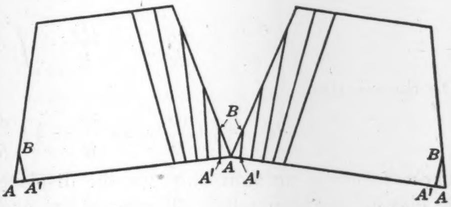

FIG. 18

$I''I'$ is negligible when compared to both $I'\,I$ and $I'\,H'$, so that the broken line, $H'\,I''\,J'$ can be substituted by $H'\,I'\,J''$, the segment, $I'\,J''$, being parallel to $I''\,J\,I$. This is seen from Fig. 17, which indicates that the tangential deflection of j' due to the rotation of i' around h' (that is, the segment, $I''\,I'$), is approximately $\frac{1}{2}\,\phi_i\,ds'_i \times H'\,I'$, or $\frac{1}{2}\,\phi_i\,ds'_i \times ds'_i \times \int_1^h \phi\,ds$, as compared with the magnitude of $H'\,I'$, or of $I'\,I$, which is $\frac{1}{2}\,t \int_1^h \phi\,ds$.

To evaluate the total tangential and radial deflections of a kinematic nature, draw lines from the point, O (Fig. 16) parallel to $H'\,I'$ and $I'\,J''$, respectively. Then, the segments, $O\,P$ and $P\,I$, and $O\,Q$ and $Q\,J$, respectively, will be the radial and tangential deflections at the points, i and j, respectively. Next, drawing a line parallel to $J''\,Q$ through I', the difference of the deflections relating to Points i and j, will be:

$$\Delta_T = Q\,J - P\,I, \text{ and } \Delta_R = I'\,J'' - P'\,P$$

Developing these relations further, the following results are obtained:

$$\Delta_T = (Q\,J'' - P\,I') - (J\,J'' - I\,I') = D_R\,\omega - (\tfrac{1}{2}\,t \int_1^i \phi\,ds) - \tfrac{1}{2}\,t \int_1^h \phi\,ds)$$

$$= \frac{D_R}{R}\,ds - \tfrac{1}{2}\,t \int_h^i \phi\,ds = \frac{D_R}{R}\,ds - \tfrac{1}{2}\,t\,\phi\,ds$$

and,

$$\Delta_R = I'\,J'' - P\,I'\omega = ds' \times \int_1^i \phi\,ds - (D_T + \tfrac{1}{2}\,t \int_1^h \phi\,ds)\,\omega$$

$$= ds' \int_1^i \phi\,ds - (D_T + \tfrac{1}{2}\,t \int_1^i \phi\,ds)\,\frac{d\,s}{R}$$

Since the magnitude of the angle, $P\,I'\,P$, is equal to that of $P\,O\,Q$, which is equal to the angle, ω, between the radii of curvatures in the points, i' and j', of the extrados (as explained in the foregoing), the derivatives of the tangential and radial deflections will be,

$$\frac{dD_T}{ds} = \frac{\Delta_T}{ds} = \frac{D_R}{R} - \tfrac{1}{2}\,t\,\phi \,\dots\dots\dots\dots\dots\dots (111)$$

and,

$$\frac{dD_R}{ds} = \frac{\Delta_R}{ds} = -\frac{D_T}{R} + \int_1^s \phi\,ds \,\dots\dots\dots\dots\dots\dots (112)$$

due to the relation,

$$ds' - \tfrac{1}{2}\,t\,\frac{ds}{R} = ds\,\frac{R'}{R} - \tfrac{1}{2}\,t\,\frac{ds}{R} = ds$$

These formulas account only for the displacements implied in the kinematic closing of the notches; all stresses are excluded from the arch at this stage of the investigation. Then, as a consecutive step, the deflections due to the effect of the uniform thrust, may be determined as follows: Suppose that each block is acted upon, successively, by the invariable thrust, immediately after the notch at the right of that block has been closed. This would mean a scheme of deflections due alternately to kinematic and mechanical processes. Then, for Points i and j, a broken line will result, which is somewhat different from that represented in Fig. 16. The ultimate position of i will be a certain point, I''', on the corresponding line, $I\,P$, at a distance, $ds_i\left(\frac{T}{E\,t} + c_T\,T_T\right)$, from I.

The compression of the block, $h'\,i'$, involves, of course, the immediate displacement of j in the same direction and in the same amount as $I\,I'''$, on the corresponding line, $J\,Q$. Subsequently, the notch, i', will close, a process which does not bring about any change in the relative positions as determined in studying the kinematic deflections. As a last step, Block $i'\,j'$ will be subjected to the action of the thrust, such action involving an additional displacement of j' along the line, $J\,Q$, of the amount, $ds_j\left(\frac{T}{E\,t} + c_T T_T\right)$. It is seen, therefore, that the thrust does not effect any change in Equation (112), for the derivative of the radial deflection, whereas the derivative of the tangential deflection (Equation (111)), will be decreased by the difference of the tangential displacements of the points, i and j, due to the thrust, which corresponds to the difference in the lengths of the axis. The proportion of these differences being $ds\left(\frac{T}{E\,t} + c_T\,T_T\right)$ to ds — that is, $\frac{T}{E\,t}\,c_T\,T_T$ — Equations (67) and (68) thus appear to be proved, after the obvious substitution of β for ϕ.

These remarks are merely intended to show that cumbersome, abstract, mathematical deductions can easily be replaced by shorter methods that are more attractive, because each step in the reasoning has a definite, clear bearing on the engineering problem treated; thus, they do not imply any criticism of the results.

The paper contains one error, however, and a few inconsistencies, to which attention must be called.

In order to obtain the solution of Equations (67) and (68)—that is, an expression for β in the function of the radial or tangential deflections, or *vice versa*—the author derived, and subsequently solved, Equation (71). This step is not warranted as it involves an erroneous solution of Equations (67) and (68). That system of simultaneous differential equations is quite at variance with the indeterminate differential equation, (71), which contains the derivatives of two independent unknown functions. The former system has a well-defined set, or pair, of solutions if β is the independent variable. Equation (71), on the contrary, admits of an infinite number of solutions. In fact, if any arbitrary function is selected for one of the two unknowns, D_T *or* D_R, Equation (71) admits of a definite solution for the other. Among all the erratic and absurd couples of functions that may be obtained in this manner for D_R and D_T, there will be only one couple satisfying Equations (67) and (68). From this, one may infer that it will not be easy, if at all possible, to produce this one pair of functions by considering Equation (71), instead of treating the total system, Equations (67) and (68). As a matter of fact, and most unfortunately, Equations (72) and

(73) claimed by the author to be the correct pair, are actually not the correct ones. They do satisfy Equation (71), but they do not satisfy the system of Equations (67) and (68). In fact, they do not satisfy either of those two differential equations, as a simple substitution shows. They cannot satisfy either of them because this would imply that their right members, and thus, also, their left members, should be equal to zero, which consequently implies that both D_T and D_R must be constants.

The correct pair of functions can be obtained by reducing the system of two unknowns to one differential equation with one unknown. For instance, by differentiation of Equation (67), the following relation will result:

$$\frac{d^2 D_R}{ds^2} = -\frac{1}{R}\frac{dD_T}{ds} + \frac{D_T}{R^2}\frac{dR}{ds} + \beta \text{ ....(113)}$$

Elimination of D_T and its derivatives from Equations (67), (68), and (113), will yield the differential equation:

$$\frac{d^2 D_R}{ds^2} + \frac{R'}{R}\frac{dD_R}{ds} + \frac{1}{R^2} D_R = \frac{1}{R}\left(\tfrac{1}{2}\, t\, \beta + R\,\beta + R' \int^s \beta\, ds + \frac{T}{E\,t} + c_T\, T_T\right) \text{..(114)}$$

From Equation (114), D_R may be determined, at least theoretically, if β is known. Naturally, it is hopeless to attempt the solution, because of the complexity of Equation (114), and for the reason that β, and especially R, will not be given as explicit functions of s. A graphical solution can be obtained, although the procedure will still be very wearisome, in fact, almost beyond human endurance. Conditions are simpler if D_R is given and β is to be determined, in which case Equation (114) will be used in the form:

$$\left(1 + \frac{t}{R}\right)\beta + \frac{dR}{ds}\int^s \beta\, ds = \frac{d^2 D_R}{ds^2} + \frac{1}{R}\frac{dR}{ds}\frac{dD_R}{ds} + \frac{1}{R^2} D_R$$
$$- \frac{1}{R}\left(\frac{T}{E\,t} + c_T\, T_T\right) \text{ ....(115)}$$

This is somewhat easier to treat, since it is of the first order in $\int^s \beta\, ds$, although analytically it cannot be solved, as R and D_R will not be known in explicit form.

Another inconsistency occurs in the derivation and enunciation of Equation (69), that is, $\int_0^s \beta\, ds = 0$, deduced from Equations (67) and (68), and claimed by the author to hold for symmetrical arches. As shown previously, Equations (67) and (68) hold for any kind of arches fixed at one end and free at the other end, either symmetrical or unsymmetrical, no matter what arbitrary assumptions are made as to position and magnitude of the notches. Hence, for such an arch, the algebraic sum of the notches need not, necessarily, be equal to zero; in other words, Equation (69) need not hold. If, however, the arch is to be compensated—that is, if the notches are to be arranged so that their closing will not result in a rotation of the free end section—Equation (69) holds true for that arch no matter whether it is symmetrical or unsymmetrical. Then it need not, and should not, be derived from Equations (67) and (68). It follows simply from the very definition of the compensated arch; in fact, it is the definition of the compensated arch.

If the conception of the compensated arch means that the free end section does not undergo any rotation, then Equation (69) must hold, since it is the expression for the amount of the rotation of the free end. If, furthermore, the term, "compensated arch," refers to an arch in which the kinematic closing of the notches on one side, and the thrust on the other side, cause the free end section to translate in the direction of the chord by equal amounts and in opposite senses, then it is obvious, that, in addition to Equation (69), the following relations must hold:

$$\int_0^S \beta\, Y\, ds + 2\, \alpha_m \left(\frac{T}{E\, t} + c_T\, T_T \right) + (D_{haL} - D_{haR}) = 0 \quad \ldots.(116)$$

and

$$\int_0^S \beta\, Z\, ds = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(117)$$

the latter referring to unsymmetrical arches. In Equations (116) and (117) Y and Z represent the distances of the notches—situated partly on the extrados and partly on the intrados—from two axes, that of Y being the chord of the arch. The foregoing additional criteria of the compensation are not mentioned in the paper. This is unfortunate, because they play the most important part in judging the possibilities of the method under discussion.

Still another point of the paper should be discussed, namely, the derivation of Equation (68) through equalizing the external and internal work in the arch. The author's reasoning holds, under certain conditions, for a part of the arch extending from the fixed end to an arbitrary section characterized by the arch length, s. Thus, Equation (70) holds if the upper limit of the integrals is changed from S to s, and then Equation (68) follows from Equation (70). This is not true for Equation (70) with S as the upper limit. The equality of the definite integrals of two functions between two given limits does not imply the identity of the functions. Another indication of the weakness of Equation (70) as it stands, is the presence of the definite integral, $\frac{1}{2}\, t \int_0^S \beta\, ds$, which, according to Equation (69), is equal to zero, and could be dropped without influencing the validity of Equation (70). In this case, Equation (68) would not evolve, even if the equality of the two definite integrals on both sides meant the indentity of the functions included under the integration signs.

There are other grounds for questioning the validity of Equation (70), which call for clarification of the manner in which the arch deforms. It must be borne in mind that a given load can cause the compensated arch between two fixed abutments to reach its final deformation in two different ways. One is an idealized procedure consisting of the following successive steps: (1) Temporary removal of the left abutment; (2) kinematic closing of the notches, without applying the load; (3) application of the given load on the extrados, and contemporary application of the thrust, T, on the free left-end section; and (4) since the arch is compensated, the effect of Steps (2) and (3) will be to keep the left-end section in its original position; hence, in Step (4), it is possible to replace the left-end abutment in its original position, in touch with the left-end section.

In contrast with this idealized process (which is well-defined, clear, handy for a quantitative analysis, and is used in the paper), the actual manner in which an arch deforms and is adjusted, between two fixed abutments, is unknown, and thus not susceptible even to a quantitative treatment. In the idealized process, the increase and the decrease in the chord length, affected by the closing of the notches and by the action of the load, respectively, have been allowed to take place in succession; but, actually, the gradual increase and decrease in the length of the chord must balance themselves at all times. This may be visualized as follows: At first, the effective arch will consist merely of a catenary passing through the connecting corners of the blocks. A certain initial compression will develop along this catenary, such that the original lengths, A-A, would become shortened to A'-A' (Fig. 18), and a centripetal displacement of the blocks would occur as they tended to fill in the space, B-B. Thus, the straight lines of contact, $A'\,B$, will become successively more and more inclined, as reproduced schematically in Fig. 18. Compression along the successively developing catenaries involves progressive decrease in the length of the chord; and adjustment of the blocks along contact lines of increasing inclination involves progressive decrease of curvature; that is, increase in the length of the chord. This increase and decrease must balance at all times during the process of deformation and adjustment.

The foregoing conception is necessarily a kind of fiction. No one knows whether it represents a picture even approximately true. Possibly, the actual process is not continuous, but discontinuous, involving occasional slips or abrupt changes in positions or stresses; possibly, the actual process occurs in different ways when it is repeated, by alternately releasing and loading the arch; possibly, the actual process is not even reversible (the gradual removal of the load may not imply the exact reversal of the process of adjustment during the gradual loading, either as to stresses, displacements, or time); possibly, the arch will not even return to its original position when released from the load, because the blocks may hinder each other in their effort to return to their original strainless condition; and, it is possible, above all, that under the action of the load, a final adjustment will take place which is quite different from the uniform distribution of compressive stress on the sections, as assumed in the paper. Everything may happen in a structure composed of loose blocks. Experimental studies would be very instructive from several points of view. The successive development of the compression surfaces may not be susceptible of optical measurements; but the strains could be estimated from the stresses, if a series of observations by the photo-elastic method would disclose the stress distributions in the successive phases of adjustment.

Under such circumstances, it seems to be unwarranted to evaluate the amount of work by using the formulas in this paper. The expression, $\frac{1}{2}\,P\,\delta$, for the work, is derived on the assumption that while the force increases uniformly from 0 to its end value, P, the displacement of its point of application in the direction of the force also increases uniformly from 0 to its end value, δ. This assumption is warranted for ordinary, continuous structures; but the principle of superposition involved is not warranted in the case of a structure made up of isolated units. The author does not, and cannot, present any facts to prove that the deflections increase uniformly with a uniform increase of the loads and stresses.

After having pointed out the possible errors, the next question concerns the practical aspects of the problems involved. For example, it will be necessary to form an idea as to the dimensions of the notches. In the following discussion, this magnitude will be evaluated, assuming that the arch is semicircular and that all notches are equi-distant and of equal magnitude. Such assumptions will yield characteristic results without computations that are too intricate.

The assumption of equi-distant notches of the same magnitude, ϕ, satisfies Equation (69). Calling ω the magnitude of the central angle which defies the distance of the notches, Equation (116) requires that:

$$\phi\left(R-\frac{t}{2}\right)\left\{\sin(n+1)\,\omega+\sin\,(n+2)\,\omega+\ldots+\sin 2\,n\,\omega\right\}$$
$$-\,\phi\left(R+\frac{t}{2}\right)\left\{\sin\omega+\sin 2\,\omega+\ldots+\sin n\,\omega\right\}=\tfrac{1}{2}\,\delta\;\ldots.(118)$$

in which, δ is the rib-shortening of the arch, and $4n$, the number of notches in the half circle. Equation (118) may also be written:

$$\phi\left(R-\frac{t}{2}\right)\sum_{i=1}^{n}\cos i\,\omega-\phi\left(R+\frac{t}{2}\right)\sum_{i=1}^{n}\sin i\,\omega=\tfrac{1}{2}\,\delta\;\ldots.(119)$$

From mathematics,

$$\sum\cos i\,\omega=\frac{\sin\dfrac{n\,\omega}{2}}{\sin\dfrac{\omega}{2}}\cos\frac{n+1}{2}\;\omega$$

and,

$$\sum\sin i\,\omega=\frac{\sin\dfrac{n\,\omega}{2}}{\sin\dfrac{\omega}{2}}\sin\frac{n+1}{2}\;\omega$$

If the number of the notches increases, as assumed in the paper, the values of $\frac{1}{2}\,n\,\omega$ and $\frac{1}{2}\,(n+1)\,\omega$ approach 22° 30′, and for all practical purposes:

$$\frac{\sqrt{2}}{4}\,\phi\left(R-\frac{t}{2}\right)-\frac{2-\sqrt{2}}{4}\,\phi\,\left(R+\frac{t}{2}\right)=\tfrac{1}{2}\,\delta\sin\tfrac{1}{2}\,\omega$$

or,

$$\phi\,(0.33\ldots R-0.5\,t)=\delta\sin\tfrac{1}{2}\,\omega$$

Substituting angles for the small sines, the proportion of ϕ and ω will be given approximately by the formula:

$$\frac{\phi}{\omega} = \frac{\delta}{\frac{2}{3} R - t} \quad \text{.......................} (120)$$

To apply this result to a practical case, let $R = 300$ ft; $t = 15$ ft; and $p = 15$ lb per sq in. Then, $\sigma = \frac{307.5}{15} 15 = 307.5$ lb per sq in. The rib-shortening due to the load alone is:

$$\delta = 2 R \frac{\sigma}{E} = 600 \text{ ft } \frac{307.5}{2\,000\,000} = 0.0922 \text{ ft}$$

Hence,

$$\frac{\phi}{\omega} = \frac{0.0922}{\frac{2}{3}\, 300 \text{ ft} - 15 \text{ ft}} = 0.0005$$

Assume, for instance, block lengths of 20 ft. The opening of the notches will then be:

$$\phi = 0.0005 \, \frac{20 \text{ ft}}{300 \text{ ft}} = \frac{1}{30\,000}$$

implying an angle of about 7″ of arc. Since the thickness of the arch is 15 ft, the foregoing value means that in the V-shaped void of a notch, having a height of 15 ft, the base will have a length of only $\frac{1}{160}$ in.

Such magnitudes, of course, are not practicable. It is not possible to cast two concrete walls with plane, vertical surfaces so that at 15 ft from their common origin, their distance should be $\frac{1}{160}$ in.

It is conceded that: (*a*) Equation (120), derived on the assumption of ever-decreasing block lengths, may require some revision, if block lengths of 20 ft are used; (*b*) the arches may be far from being circular; and (*c*) their dimensional characteristics may be different from those assumed in the foregoing computations. Consequently, the numerical results obtained herein are susceptible of changes; but their magnitude will not change substantially.

Next reverse the line of thought heretofore followed and investigate the consequences of assuming notches of reasonably large dimensions and of dropping, at the same time, the postulatum of equal and equi-distant notches throughout the arch.

The first thing to do is to establish a minimum practicable opening for the notches. Such a minimum, of course, is subject to differences of opinion, but it will be difficult if not impossible, to form cast-concrete plane surfaces, with angles smaller than about 2½ degrees. Hence, this arbitrary or empirical quantity will be accepted tentatively. In the foregoing example of the circular arch, such an angle implies a base length of about 4 in. in the V-shaped voids.

In order to study the arrangement of the notches further, Equation (116) may be written in the following form:

$$\Sigma \phi \times Y_1 - \Sigma \phi \times Y_2 = \frac{1}{2} \times 0.09 \text{ ft} \quad \text{.............} (121)$$

in which, the quantities, Y_1 and Y_2, are the distances, from the chord, of the hinges of the notches, situated on the down-stream and up-stream faces, respectively, as shown in Fig. 19. Each sum in Equation (121) may be replaced by a single product, composed of the concentration, or sum of the angles of the relative notches and an appropriate distance which may be termed their "middle distance." The summations of the up-stream and down-stream notches are equal, of course. If these sums are denoted by Ω, Equation (121) may be written:

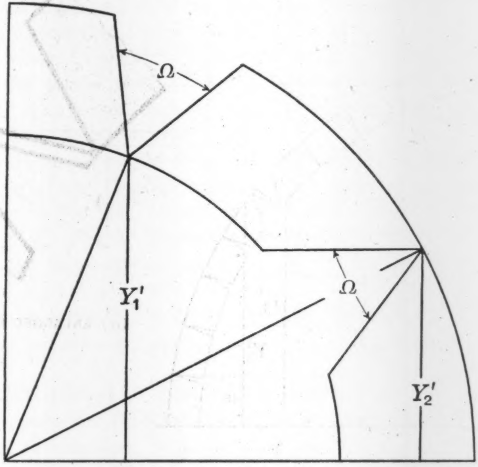

FIG. 19

$$Y'_1 - Y'_2 = \tfrac{1}{2} \times 0.09 : \Omega \quad\ldots\ldots\ldots\ldots\ldots\ldots (122)$$

It is assumed that no notch should be smaller than 2½°; therefore, the sums, Ω, will always be greater than $2.5 \times \frac{\pi}{360}$, and the difference of the two up-stream and down-stream "middle" co-ordinates, Y'_1 and Y'_2, will be always smaller than about 2 ft, if there is only one positive and one negative notch in the half arch; with the number of notches increasing, this difference decreases rapidly. In all cases, openings indicate that the two "middle" co-ordinates are practically of equal lengths. In other words, the radial vectors corresponding to the "concentrated" positive and negative notches are rather close to each other. Since at least one positive and one negative notch must be between the two radial vectors, and since these notches must balance all the other positive or negative notches outside the radii, it is clear that they will be quite large and will take almost all the space available between the radii.

This may be seen from Fig. 20, in which the arch is the same as the one treated previously. It will be assumed that notches with $2\frac{1}{2}°$ openings are situated at the distances, 45, 90, 135, and 170 ft from the chord, while Y'_1 and Y'_2 are assumed to be practically equal to 200 ft. A positive and a negative notch must be placed between the radial vectors corresponding to Y'_1 and Y'_2. Thus, the greatest distance of that negative notch from the chord cannot be more than about 205 ft. Noting the magnitude of that notch as x, and recalling the definition that the "middle distance" (that is, Y'_2), multiplied by the sum total of the negative notches is equal to the sum of the products of notches, by their distances from the chord, the following will result: $2\frac{1}{2}(45 + 90 + 135 + 170) + 205\,x = (4 \times 2\frac{1}{2} + x)\,200$; which gives $x = 72$ degrees.

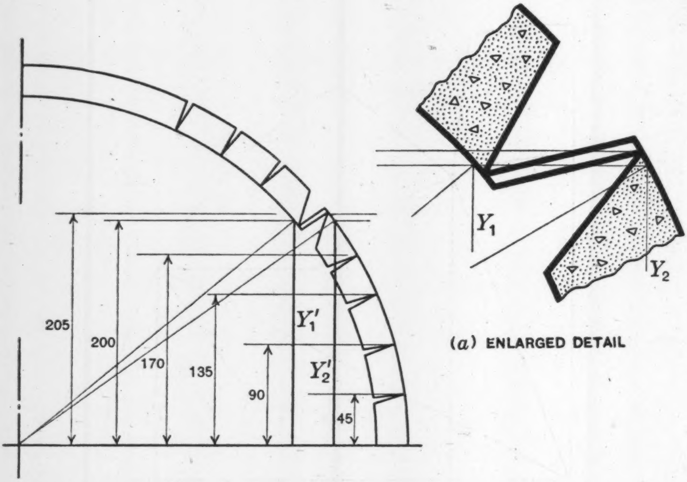

FIG. 20

Because of the fact that the sum of the positive notches must be equal to that of the negative notches, without regard to their distribution, it follows that the positive notch situated between the two radial vectors and balancing the other positive notches with respect to Y'_1, will be another angle of substantial magnitude. Thus, if it is at all possible to place both these wide notches between the two radial vectors, it is clear that there will be only a narrow transverse beam between the two notches, entirely isolated from the other regular-shaped blocks. An enlarged detail illustrating these conditions is reproduced in Fig. 20 (*a*), in which the transverse beam is shown not hatched (although it is obviously a part of the section), in order to avoid obscuring other details.

If the sizes of the notches are assumed to decrease gradually from the end section toward the region where the sign of notches changes, as suggested in the various illustrations in the paper, conditions are still more exaggerated. Suppose, for instance, that the four notches have the widths of 10, $7\frac{1}{2}$, 5, and $2\frac{1}{2}°$, respectively; then the foregoing relation will have the form: $10 \times 45 + 7\frac{1}{2} \times 90 + 5 \times 135 + 2\frac{1}{2} \times 170 + 205x = (2\frac{1}{2} + 5 + 7\frac{1}{2} + 10 + x)\,200$, or, $x = 555$ degrees.

Evidently, such values are of no use. They may be reduced, to a certain extent, by: (1) Increasing the available space between the two radial vectors, by increasing either the "middle distances" or the thickness of the arch; and (2) reducing the total opening of the notches either by decreasing their number, or the opening of each notch. As already noted, these provisions can be utilized only to a certain extent.

In fact, an increase of the values of Y'_1 and Y'_2 implies a reduction in the total opening of the positive notches; that is, those in the extrados. The maximum of space for the compensating notches between the radial vectors corresponding to Y'_1 and Y'_2 is realized when the former is the ordinate corresponding to the down-stream point of the symmetry section; but in this case there is, of course, no space for any positive notches to be compensated. At any rate, a superficial glance at the arch in Fig. 20 indicates that by increasing the "middle distances," conditions can not be improved very much.

They can be improved to some extent by increasing the thickness of the arch; but to increase the thickness of an arch just for the purpose of a better arrangement of the notches, is a waste of material. There are thin arches, such as the one shown in Fig. 20. The top arch in the author's model, in Fig. 11, is only slightly thicker than the arch now being discussed. Furthermore, a close study of geometrical conditions in thin and thick arches reveals that in reality not much is gained by thickening the arches because almost all the additional space between the two radial vectors must be utilized in order to accommodate the two balancing notches.

It would not be possible to construct notches with openings that are too small. On the other hand, to reduce the number of notches involves serious difficulties. Fig. 21 represents a central notch opened about 5 degrees. When the notches are closed, the faces of the central notch become vertical, while, for reasons of symmetry, the assumed thrust, T, will be horizontal. In other words, the thrust acts normal to the face of the notch, and not tangential to the neutral axis of the arch. This fact implies secondary stresses in addition to those depending on the usual stress régime in cylinders. To re-establish the latter régime, a perfect cylindrical segment may be secured by cutting off the notch, by means of a radius passing through its up-stream corner. The original thrust, T, acting in the neutral axis of the notch section must be resolved into the same thrust, T, acting at the center of the new section tangential to its neutral axis, and into a vectorial remainder, which represents the secondary stresses.

Utilizing the data given in Fig. 21, it is easy to verify the statement that the secondary stresses will originate from a bending moment of the magnitude, $\dfrac{1.9\text{ in.} \times 1.9\text{ in.}}{7.5\text{ ft}}\,T = 0.05T$, from a vertical force of the magnitude,

$T \sin \gamma = 0.0217T$, and from a horizontal force equal to the difference of T and of $T \cos \gamma$ (that is, $T - T \cos \gamma = 0.000235T$). All of them yield clockwise moments for any section, to the left of the notch.

Assume, now that the next notch is on the bisector of the left half arch, and that its horizontal and vertical distances from the points of application of the foregoing forces are 106 ft and 44 ft, respectively (Fig. 22). Then, the resulting bending moment will be:

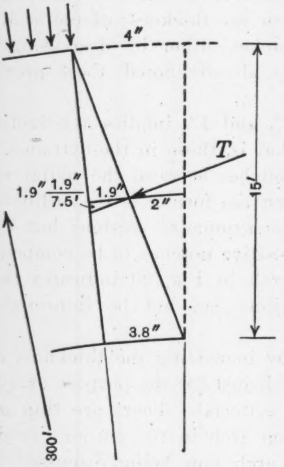

FIG. 21

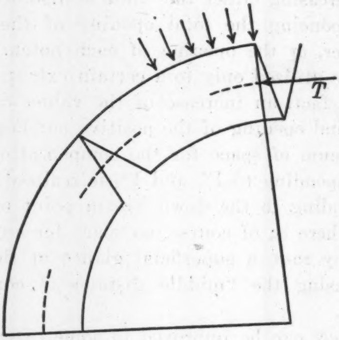

FIG. 22

$$M = 0.04T + 0.0217 \times 106 \times 12T + 0.000235 \times 44 \times 12T = 27.6T$$

This bending moment will cause secondary stresses, in the neighborhood of the bisector, in addition to the uniform compression between the two notches, created by the presence of T (acting tangentially on the radial section as corrected at the left side of the central notch) and p, the uniform water pressure.

The relative magnitude of cylinder and maximum secondary stresses is: $\frac{T}{t} : \frac{6 \times 27.6 \times T}{t^2} = 1 : \frac{165.6}{180} = 100 : \pm 92$. It is seen that, at least in this case, the extreme stresses in that particular section are $1.92 \frac{T}{t}$ and $0.08 \frac{T}{t}$, which are quite different from the assumed uniform stress distribution of the intensity of $\frac{T}{t}$.

Another trouble caused by blocks that are too long is the nullification of Equations (67) and (68), which have been derived on the assumption that the lengths of the blocks as well as the widths of the notches, are infinitesimal. Blocks of substantial lengths and notches of substantial widths change those formulas substantially. The problem presents two alternatives at this point. The notches must be appreciable, or they cannot be built. On the other hand, a reasonably large number of them must be built, or the assumption of a uniform compression in the arch is misleading, and the important formulas, Equations (67) and (68), lose their validity. Then, again, if there is a reasonably large number of notches having reasonably large widths, the region in which the notches pass from opening inward to opening outward (reserved for the two large compensating notches) may not be sufficient to accommodate both these large notches.

None of these difficulties is brought out by the author. Judging from the paper alone, it seems that there is practically no limitation to the arbitrary selection of the positions and widths of the notches; but, in reality, the procedure of selection is exacting because: (1) The total opening of the positive and negative notches must have the same value; and (2) since the difference of their static moments with respect to the chord is practically negligible, their lever arms then have practically equal lengths.

The author gives definite formulas for the widths of the notches, depending upon the radial deflections. This seems simple enough, but it is merely a shifting of the difficulties. The fact is that new troubles arise in selecting a compatible curve of deflections that are avoided in selecting a possible set of notches. Arbitrary deflection curves will not suffice.

If a certain deflection curve, D_R, is selected, it is necessary to determine whether it is compatible with the limitations set by Equations (69) and (116). To do so, the deflection curve selected must first be substituted in Equation (115) for D_R, and then Equation (115) must be solved. The function, β, thus obtained must be substituted into Equations (69) and (116). As a rule it will not satisfy these equations and another selection for D_R must be made, and substituted in Equation (115), in order to determine another function β, which must again be substituted in Equations (69) and (116), and so on, until the proper value is determined. The operations involved are rather forbidding. Conditions are made more difficult, if the tangential instead of the radial deflections are to be selected.

At this point, the author's selection of the radial deflection according to Equation (78) will be discussed. Substituting the value of β from Equation (79) in Equations (69) and (116), the following relations will result:

$$\int_0^S \frac{2}{t}\left(\frac{\gamma}{R^2} - C\right) ds = 0 \ldots\ldots\ldots\ldots\ldots (123)$$

and,

$$\int_0^S \frac{2}{t}\left(\frac{\gamma}{R^2} - C\right) Y\, ds = C\, L \ldots\ldots\ldots\ldots\ldots (124)$$

in which, L denotes the length of the chord, and C is the sign for $\dfrac{IT}{Et} - c_T T_T$.

In another form:

$$2\,\frac{\gamma}{t}\int_0^s \frac{1}{R^2}\,ds = 2\,\frac{C}{t}\int_0^s ds$$

and,

$$2\,\frac{\gamma}{t}\int_0^s \frac{1}{R^2}\,Y\,ds = 2\,\frac{C}{t}\int_0^s Y\,ds + C\,L$$

Dividing Equation (124) by Equation (123):

$$\int_0^s \frac{1}{R^2}\,Y\,ds : \int_0^s \frac{1}{R^2}\,ds = \int_0^s Y\,ds : \int_0^s ds + L\,t : 2\int_0^s ds \quad \text{...(125)}$$

Considering once more the top arch in Fig. 11 and making use of its dimensions, the second term of the right member of Equation (6) will have the value of about 0.4*t*.

The first term of the right-hand member represents the distance of the center of gravity of the arch from the chord. Now, if the second term of the right-hand member were equal to zero, or if it were negligible, the relation would express the fact that the distribution of the function, $\frac{1}{R^2}$, along the arch could not be uniform between the end section and the center section. A function, $\frac{1}{R^2}$, having constant values along the entire arch would be compatible with the formula; functions other than constant, if compatible with Equation (125), must increase and decrease from the end section to the center section. Arches of such characteristics are not, as a rule, used in dam engineering; radii of curvature usually decrease or increase uniformly, from end section to center section. A comparison of the value of that term with the distance of the center of gravity of the arch from the chord shows that Equation (125) admits only such uniform functions, $\frac{1}{R^2}$, for which the minimum value at the end section and the maximum value at the center section are nearly equal. Generally, this is not the case. The conditions of curvature involved are, at any rate, extremely different in the author's model arch (Fig. 11). For that arch, the left-hand member of Equation (125) (which represents the distance, from the chord, of the center of gravity of the hypothetical weights, $\frac{1}{R^2}$, situated on the arch) has a much larger value than the right-hand member, which represents the distance, from the chord, of the center of gravity of the arch itself, increased by the insignificant quantity, $0.4 \times t$. Thus, admitting only arches with radii of curvature ever increasing from center to end sections, Equation (125) would be impossible, and, hence, the author's choice of D_R according to Equation (78) appears to be erroneous. The writer modifies his statement with the words, "appears to be", because it is not feasible to segregate the influence of the assumption, Equation (78), from that of Equation (74) which is in either case erroneous as previously explained.

One further remark by the author should be corrected, namely, the statement near the end of Section V claiming that "the sizes of the V-shaped joints that are computed in accordance with the foregoing formulas represent the minimum necessary to compensate the arch." This statement is misleading. The formulas to which the author refers are manifestly those representing the proper solution of Equations (67) and (68), or Equations (82) or (85); but there is no evidence of the validity of the author's contention. The values of β as given by the foregoing formulas are correlated to a selected set of, say, radial deflections, but they do not represent the minimum notches necessary to compensate the arch. There are no smaller or larger notches compatible with that selected set of radial deflections; there is only one set of notches.

The writer candidly confesses that he was not able to grasp the author's ideas and his explanations pertaining to the process of compensating arch dams (Section XI and Section VII, following Equation (91)). It is regrettable that in a paper entitled "The Compensated Arch Dam," only half a page (Section VII), deals with the most difficult phase of this problem, and that only half a page (Section XI) is devoted to a problem that is the dream of dam engineers, namely, the reproduction of a dam having the most economic shape in a given canyon section. On the other hand, the author devotes about 36% of his paper to general aspects of arch dam design, which are common knowledge among those whom he may reasonably expect to understand his theory of the compensation of arch slices and to pass judgment upon its usefulness. This is a redactional inconsistency, which should have been avoided.

To pass a final judgment on the paper, the writer is pleased to give full credit to the author for the original conception and the searching treatment of the compensated arch. However, he could not help feeling disillusioned at the end of that treatment. The entire analysis is based on the assumption that the lengths of the blocks are infinitesimal. This implies that the widths of the notches are infinitesimal values of the second order. With commendable accuracy, the author thought it necessary to distinguish between ϕ and $\beta = \left(I - \frac{T}{Et} - c_T\, T_T\right) \phi = 0.99995\, \phi$, thus introducing the difference of $0.00005\, \phi$, which is an infinitesimal of the third order. After such painstaking analytical research of the widths of the notches, he concludes (near the end of Section V):

> "As far as the actual construction of the notches is concerned, one of the simplest methods is to place the concrete in alternate blocks and to provide the V-shaped joints, by means of elastic fillers placed between the adjoining blocks. During the time of loading the arch, the V-shaped joints will close and compress the fillers flat or squeeze them out."

However, the paper does not contain any information, nor does it attempt to analyze the following questions: How will the joint filler affect the action? How much will the filler material compress, or squeeze out? What will be the effect of such compression or squeezing on the notches and the arch?

Hence, the following alternatives: Either the author has, or has not, developed at least a rough quantitative analysis of the notches with fillers. If he has, he should have published that analysis of general validity instead of the one of particular validity now being discussed. Otherwise, he will have to handle the notches and fillers in the so-called "practical" way, in which case the searching theory of the ideal notches having no fillers looks very much like hair-splitting—up to third infinitesimals.

So much for isolated arches. As to the proposed method of compensating the entire dam, the writer ventures to present the following opinion: The method proposed in the paper is incomparably more complex than the trial-load method which, certainly, requires considerable labor. Recalling the great difficulties of selecting sets of deflections of isolated arches compatible with Equations (69) and (116), and appraising the formidable difficulties of such selections in arched dams, in which, in addition to the radial load, twisting moments and tangential shearing stresses must be dealt with, the labor required for the analysis of the dam as a whole seems to be prohibitive.

The writer objects to the complexity of the designing process only in a secondary way. His main objections to the type of dam proposed are based on the following:

(1) The uncertainty of its mechanical behavior;

(2) The impossibility of reconciling the practical problems of construction with the ideal assumptions of the paper; and

(3) The artificiality of the concept, implying the dissolution of a solid body of a structure into a great number of loose pieces. This involves an abandonment of the reserve strength that engineers properly associate with the idea of a monolithic mass structure, and there seems to be no adequate advantage in it.

A. V. KARPOV,[8] M. AM. SOC. C. E. (by letter).—These seems to be a rather large school of highly theoretical engineering thought that is based on the postulate that assumptions should be made to coincide with theoretical ideas, whether or not they are in accord with practical conditions.

The discussion by Professor Kalman is based on such a postulate. The last sentence of his discussion is all that is necessary to bring out the fundamental differences between him and the writer. The proper answer to this objection is that it has nothing to do with the subject under discussion. No matter if it suits or does not suit one's theoretical ideas, the cold fact is that no commercial arch dam was ever built as a monolithic structure; and it can never be built as a monolithic structure as long as engineers are limited to present materials and methods of construction. If there is any doubt on this point, it is only necessary to observe the behavior of the vertical construction joints at the up-stream and down-stream faces of an arch dam at different loadings. Such a dam can be built only as a number of adjoining loose blocks or piers, and the question at issue is how to make the best design under such limitations. The purpose of the paper was to give the theoretical answer to this question. The answer may be summarized as follows: Keeping the thickness of the horizontal arches uniform, and shaping them so as to suit the topography of the canyon, it is possible, theoretically, to build a dam without bending moments in the horizontal arches, if an infinitely large number of very small V-shaped vertical joints is properly distributed over the horizontal arches.

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[8] Designing Engr., Hydr. Dept., Aluminum Co. of America, Pittsburgh, Pa.

Professor Kalman agrees with that theoretical answer, but thinks that the methods used in the paper are too formal and cumbersome. He proves by a number of different methods, that t must be constant and that Equations (67) and (68) are correct, but he invariably prefers his own methods. Correctly, he points out that the solution of Equation (71) is wrong and, consequently, Equations (71) to (85), inclusive, must be corrected. His statement that $\int_0^s \beta\, ds = 0$, holds true only as long as the foundation deflections are such as not to cause rotation of the abutments. Although he already has proved Equation (68) by his own method, he questions Equation (70), stating (see following Equation (117)): "The equality of the definite integrals of two functions between two given limits does not imply the identity of the functions." That is a correct mathematical statement, the only objection being that if it is to apply to the derivation of Equation (70) it should be brought to a logical conclusion, as follows: "But if these two definite integrals are identical at each value of the limits, then the functions are identical."

The reasoning by which Professor Kalman compares the idealized and actual methods in which an arch deforms is a proper general statement, but has very little to do with a compensated arch dam. No matter what the intermediate conditions are, the stresses and deflections of an arch dam are defined exactly by the final conditions, provided the elastic limit is not overstepped and the deformation proceeds without slippage.

In an ordinary dam, considerable shearing forces are developed in the horizontal arches, that may result in two adjoining blocks slipping one with respect to the other. In a compensated dam, properly shaped, the development of shearing forces of any magnitude is practically excluded. Of course, the uniformity with which the hydraulic load is applied, is an additional factor that makes improbable any slippage in a compensated dam.

In his discussion of the practical aspects of compensation, Professor Kalman approaches the subject in a highly theoretical manner. Apparently, he thinks that there are definite limits to the size of the vertical V-shaped joints; but from a practical point of view, there are no difficulties with respect to their construction. An open joint was assumed in the original paper, in order to simplify the theoretical proof, but it was also stated that in the actual construction of the dam closed joints are proposed and elastic fillers are to be used. Fundamentally, it simply means that the dam is to be composed of two kinds of materials—one, the concrete of the main body, having a high modulus of elasticity, and the other, the V-shaped inserts, having a lower modulus of elasticity. By varying the thickness of the inserts and the modulus of elasticity of the insert material, any theoretically desirable thickness of joint may be constructed.

One method to accomplish this result would be to use sheets of compressible material of varying thickness in the vertical joints between adjacent blocks. For instance, if tar paper is used, the thickness of the joint filler can be changed by varying the number of sheets, and after the dam is loaded the residual filler will act as a part of the dam. Regarding compensation from this point of view, it would seem unnecessary to go into the involved speculations which are used by Professor Kalman to prove the impracticability of compensation owing to the impossibility of forming thin V-shaped joints.

The entire matter of compensation should be treated like many other engineering problems, on the basis of a reasonable compromise between theoretical and practical requirements. Theoretically, to have perfect compensation, an infinitely large number of V-shaped joints would be required. Practically, however, a limited number will reduce the bending moments so as to make them negligible, the moments being zero at the joints, with the possibility of having limited moments in the part of the horizontal arches between the joints.

Professor Kalman regrets that the original paper does not contain more about the actual design of a dam for a given canyon section. The writer believes that it is undesirable to include in a theoretical paper (which has a definite purpose and which is necessarily limited in length), an extended practical application of the problem. Such application should be the subject of a separate paper.

Mr. Evans and Professor Sibert do not agree that a uniform thickness of horizontal arches is a theoretically necessary condition. Mr. Evans is in error when he derives Equation (93); the moment, M_{xy}, at the point, x, y, is due to all the loads applied at the up-stream face of the arch and, as clearly shown by Equation (8), must be a summation of the moments due to each of the forces, p'.

Professor Sibert is in error when he derives Equation (101) by integration of Equation (100). This equation cannot be integrated in its general shape, because p' is a function of x' and y'.

The foregoing covers also the points raised by Mr. Sutherland.

Mr. Jorgensen raises an interesting question, relating to the "water-soaking effect" and the differences in behavior between wet and dry concrete. The writer believes that in an arch dam a number of important secondary influences must be considered, but nevertheless the prime factor is the elastic behavior of the dam and its foundations. A rational design is possible only if full consideration is given first to the prime factor—that is, the elastic behavior—and next, if corrections are made to cover the influence of secondary factors.

Mr. Jorgensen's reasoning leads to the conclusion that, since enough is not known about the water-soaking effect, the fundamental elastic behavior of the vertical elements of the dam, as well as the foundations, should be neglected. Such a conclusion can scarcely be defended from a theoretical point of view.

From a practical standpoint, the introduction of a phantom design by assuming that the dam acts as a number of independent horizontal arches, is a short-cut that results in a decidedly inferior structure.

The experimental study of the water-soaking effect is of considerable interest, and such study could be made by comparing the behavior of an actual arch dam and a model made of material that excludes the possibility of the water-soaking effect. It is expected that such a comparison would give at least some quantitative idea about the influence of this effect.

In conclusion, the theoretical status of the compensated arch dam is not changed. The discussion centers on the objections in the practical application. It is only natural that an attempt to introduce radical changes in accepted design methods will meet with considerable opposition, but if the theoretical foundation is proved, the first step is made in the practical application of the method proposed in the paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1856

STEEL COLUMN RESEARCH[1]

FINAL REPORT OF THE SPECIAL COMMITTEE

To the Board of Direction,
American Society of Civil Engineers:

The formation of the Special Committee on Steel Column Research was authorized in 1923. Its First Progress Report[2] consisted of a study of existing test data, classified and analyzed as far as appeared to be practicable. The Second Progress Report[3] gave the results of tests of eleven columns made with oblique loading, for the purpose of studying shearing stresses, tests of twelve columns under eccentric loading, and some tests on lacing-bars. This second report also contained a discussion of general matters pertaining to the strength of columns, including column formulas. Suggestions were also made as to further experimental work that seemed to be desirable. These suggestions included a study of effect of form, strength of lacing-bars, strength of outstanding flanges, effectiveness of batten-plates and diaphragms, and the conditions of a column in a structure as related to results of laboratory tests.

Since the second progress report, tests have been made on seventy-one columns, on a large number of lacing-bars, including flats, angles, and channels, and on eighteen short columns of **T**-section, for the purpose of determining the strength of outstanding flanges. No work has been done on columns with diaphragms, and no field work has been done on trusses. Inasmuch as the reliability of diaphragms can be safely predicated on the work already done, and as field work is at present quite out of the question on account of expense and time required, it appears desirable to terminate the work of the present Committee at this point.

The present report is divided into two parts: (I) A description and analysis of the tests made since the second progress report with some reference to the tests of the second report; and (II) general discussion and conclusions.

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[1] Presented at the Annual Meeting, New York, N. Y., January 18, 1933.

[2] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1485.

[3] *Loc. cit.*, Vol. 95 (1931), p. 1152.

PART I.—RESULTS OF TESTS

STRENGTH PROPERTIES OF THE MATERIAL

1.—Material conforming to the Standard Specifications of the American Society for Testing Materials for Standard Steel for Bridges (Serial Designation A7-29) was stipulated for all specimens.

The strength properties of the material were determined by the mill tests, laboratory tests in tension and compression on specimens cut from various parts of the section, and laboratory tests on short lengths of the full section. The average strength properties of the material used in all test pieces are given in Table 1. In most cases the variations in yield point in different parts of a given section were not as great as was found in some of the sections previously reported. The extreme variations in the average strength properties of the sections used are shown in Table 2. The range in proportional limit and in yield point was greater than the range in ultimate strength. Of the individual lengths of shape tested, 2% failed to meet the yield point, and 8% failed to meet the ultimate strength requirement of the specifications.

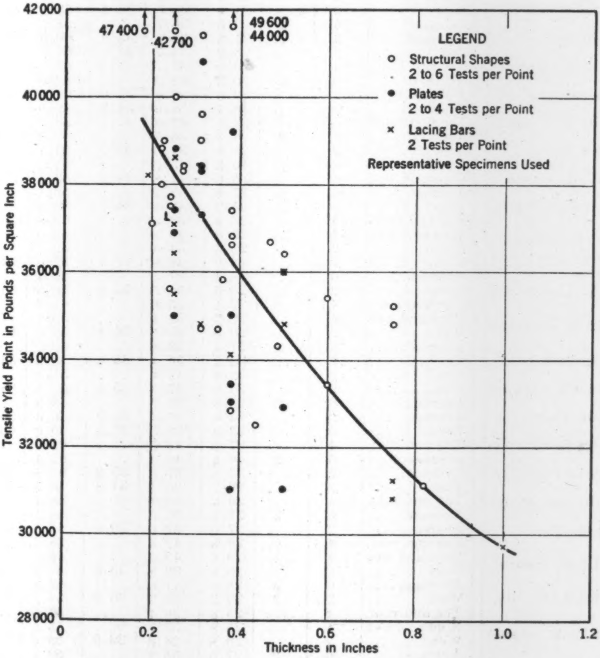

FIG. 1.—RELATION OF YIELD POINT TO THICKNESS OF SHAPE.

TABLE 1.—AVERAGE STRENGTH PROPERTIES OF MATERIAL IN TESTS OF SERIES (*D*) TO (*H*)

Column No.	Shape	Location of specimen	NUMBER OF SPECIMENS TESTED AT UNIVERSITY LABORATORY		TENSION, IN 1 000 POUNDS PER SQUARE INCH					COMPRESSION, IN 1 000 POUNDS PER SQUARE INCH	
						Yield Point		Ultimate Strength			
			Tension	Compression	Proportional limit*	University Laboratory*	Mill†	University Laboratory	Mill†	Proportional limit	Yield point
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
10, 11	**H**-column 8-in., 66-lb.	Toes	2	....	26.0	31.7	....	55.7	....	....	....
		Flanges	6	....	26.7	30.2	....	55.6	....	....	....
		Web	2	....	30.2	35.4	36.3	56.0	59.7	....	....
		Average	8	2	27.2	31.0	....	55.6	....	23.4	30.0
12, 13	10-in., 20-lb. channel (½-in. splice-plates)	Toe and root	4	....	32.6	35.4	....	61.8	....	....	....
		Root and web	4	....	33.4	36.3	....	61.8	....	....	....
		Web	2	....	32.8	36.6	34.7	62.4	57.5	....	....
		Average	6	2	32.6	36.0	....	62.0	....	25.6	33.9
	2¼ by ¼-in. lacing-bars	Average	....	....	....	....	39.0	....	60.4	....	....
14, 15	10-in., 15.3-lb. channel	Toe	2	....	30.9	36.8	....	59.3	....	....	....
		Toe and root	4	....	31.9	37.4	....	59.1	....	....	....
		Web	2	....	24.5	35.6	35.1	58.2	58.2	....	....
		Average	6	2	29.0	36.7	....	58.7	....	25.8	34.4
	12 by ¼-in. plate	Average	2	....	31.8	38.8	35.5	60.0	59.1	....	....
16, 17	5 by 3½ by 5/16-in. angles	Toe	2	....	28.2	34.7	35.9	61.0	59.9	....	....
		Toe and root	4	....	29.2	37.7	....	61.6	....	....	....
		Average	6	2	30.3	38.2	....	62.0	....	30.2	35.8
	10 by 5/16-in. plate	Average	2	....	32.0	38.4	35.0	64.5	58.1	....	....
18, 19	5 by 3½ by ½-in. angles	Toe	2	....	28.4	36.0	34.8	59.0	57.5	....	....
		Toe and root	4	....	27.6	35.8	....	58.9	....	....	....
		Average	6	2	28.8	36.5	....	59.2	....	26.6	33.6
	10 by ½-in. plate	Average	2	....	28.4	31.0	35.2	53.2	58.4	....	....
20, 21	8-in., 11.5-lb. channel	Toe and root	4	....	40.3	45.2	....	55.1	....	....	....
		Root and web	4	....	36.6	41.8	34.2	53.7	56.1	....	....
		Average	6	2	37.5	42.4	....	56.4	....	33.0	40.8
	8-in., 18.4-lb. I-beam	Web	2	2	35.9	38.4	34.7	58.0	57.7	30.0	37.8
22, 23, 42, 43	10-in., 20-lb. channel (½-in. splice-plates)	Toe and root	4	....	28.9	38.0	....	62.8	....	....	....
		Root and web	4	....	26.8	35.8	....	62.0	....	....	....
		Root	2	....	25.3	34.6	....	62.0	....	....	....
		Web	2	....	28.0	36.8	37.8	62.0	63.6	....	....
		Average	6	2	28.4	37.4	....	62.4	....	25.0	35.8
	2¼ by ¼-in. lacing-bars	Average	2	....	23.6	35.5	35.6	56.8	56.2	....	....
24, 25, 26, 27	10-in., 35-lb. channel	Toe and root	4	....	22.1	35.8	....	61.8	....	....	....
		Root and web	4	....	20.7	32.4	....	59.6	....	....	....
		Web	2	....	20.8	31.1	36.4	58.6	60.8	....	....
		Average	6	2	21.4	33.4	....	60.2	....	21.0	29.8
	2¼ by ¼-in. lacing-bars	Average	2	....	32.2	36.4	35.6	56.9	56.2	....	....
28, 29	10-in., 15.3-lb. channel	Toe	2	....	32.8	42.4	....	55.8	....	....	....
		Web	2	....	25.9	37.7	35.5	52.3	58.8	....	....
		Average	6	2	25.9	37.7	....	53.9	....	24.5	33.7‡
	12 by ¼-in. plate	Average	2	....	32.6	37.4	35.9	56.6	59.9	....	....
30, 31, 40, 41	5 by 3½ by 5/16-in. angles	Toe	2	....	32.3	39.0	36.4	59.9	60.7	....	....
		Toe and root	4	....	37.2	42.0	....	62.5	....	....	....
		Average	6	2	36.4	42.3	....	62.1	....	32.0	39.2‡
	10 by 5/16-in. plate	Average	2	....	32.8	37.3	35.9	61.8	59.8	....	....
32, 33	5 by 3½ by ½-in. angles	Toe	2	....	29.7	36.4	34.8	56.6	57.9	....	....
		Toe and root	4	....	26.4	35.1	....	58.1	....	....	....
		Average	6	2	27.4	36.5	....	58.1	....	24.0	36.2
	10 by ½-in. plate	Average	2	....	27.0	32.9	36.6	55.2	59.1	....	....
34, 35	8-in., 11.5-lb. channel	Toe and root	4	....	28.8	42.4	....	58.2	....	....	....
		Root and web	4	....	29.4	40.3	....	57.6	....	....	....
		Web	2	....	28.8	38.8	35.6	57.2	59.7	....	....
		Average	6	2	28.9	41.0	....	57.8	....	24.5	38.1‡
	8-in., 18.4-lb. I-beam	Web	2	2	32.2	38.3	36.9	58.2	61.1	24.5	38.0
36, 37, 38, 39	8-in., 18.75-lb. channel	Toe and root	4	....	25.6	36.1	....	57.0	....	....	....
		Root and web	4	....	25.8	33.7	....	55.5	....	....	....
		Web	2	....	29.8	34.3	36.9	55.4	62.0	....	....
		Average	6	2	27.6	35.2	....	56.2	....	28.0	33.5
	8-in., 20.5-lb. I-beam	Web	2	2	26.7	34.7	35.8	57.7	59.6	26.0	35.0
44, 45	8-in., 66-lb. **H**-column	Toe	2	....	31.5	32.7	....	57.6	....	....	....
		Flanges	6	....	27.5	30.9	....	56.6	....	....	....
		Web	2	....	30.3	33.4	37.4	56.8	60.3	....	....
		Average	8	2	28.0	31.3	....	56.6	....	24.5	31.2
46	8-in., 32-lb. **H**-column	Toe	2	....	34.8	39.6	....	60.4	....	....	....
		Web	1	....	41.8	41.8	43.7	63.9	64.3	....	....
		Average	7	1	35.2	38.6	....	61.7	....	26.0	40.9‡
47	8-in., 32-lb. **H**-column	Toe	2	....	34.8	41.8	....	61.1	....	....	....
		Web	1	....	38.3	40.4	42.2	59.2	60.9	....	....
		Average	7	2	32.4	38.0	....	58.9	....	25.6	37.6‡
80, 81	8-in., 32-lb. **H**-column	Toe	2	....	42.6	45.3	....	65.8	....	....	....
		Web	1	....	42.3	43.5	42.2	65.1	60.9	....	....
		Average	7	2	39.7	41.8	....	64.2	....	25.1	39.4‡

* Averages in this column are weighed. † Heat test data. ‡ Ultimate strength of full-sized prism.

TABLE 1.—(*Continued*)

Column No.	Shape	Location of specimen	Number of Specimens Tested at University Laboratory		Tension, in 1 000 Pounds per Square Inch					Compression, in 1 000 Pounds per Square Inch	
						Yield Point		Ultimate Strength			
(1)	(2)	(3)	Tension (4)	Compression (5)	Proportional limit* (6)	University Laboratory (7)*	Mill† (8)	University Laboratory (9)	Mill† (10)	Proportional limit (11)	Yield point (12)
82, 83	8-in., 32-lb. H-column.	Flanges	6	....	33.1	36.4	....	58.3	....	....	....
		Web	1	....	37.2	38.6	43.7	59.1	64.3	....	....
		Average	7	1	33.9	36.7	....	58.4	....	28.5	35.3‡
50, 51	6-in., 8.2-lb. channel	Toe	2	....	25.7	37.0	....	54.4	....	....	....
		Toe and root	4	....	24.2	37.4	....	53.6	....	....	....
		Web	2	....	34.7	37.1	33.8	54.5	56.2	....	....
		Average	6	2	27.7	37.3	....	53.9	....	29.5	35.8‡
	1½ by ¼-in. lacing-bar	Average	2	....	36.2	38.6	35.5	60.6	56.3	....	....
52, 53	6 by 4 by ⅜-in. angle	Toe (long)	2	....	27.2	37.4	36.1	58.0	60.2	....	....
		Toe and root	4	....	31.1	40.4	....	58.6	....	....	....
		Toe (short)	2	....	29.2	46.4	....	61.0	....	....	....
		Average	6	2	29.9	39.5	....	58.4	....	30.0	38.1‡
54, 55	3 by 2½ by ¼-in. angle	Toe (long)	2	....	37.9	40.0	35.7	61.9	57.3	....	....
		Toe and root	4	....	31.7	41.2	....	62.8	....	....	....
		Toe (short)	2	....	35.7	40.4	....	61.7	....	....	....
		Average	6	2	33.5	40.8	....	62.5	....	35.5	38.7‡
56, 57, 58, 59	4 by 4 by ⅜-in. angles	Toes	4	....	39.4	49.6	35.7	67.2	59.2	....	....
		Root	2	....	40.3	49.0	....	69.0	....	....	....
		Average	6	2	39.8	49.3	....	68.1	....	35.5	40.8‡
60, 61, 62	4 by 4 by ⅜-in. angles	Toes	10	....	36.6	44.0	35.7	62.7	59.2	....	....
		Average	16	....	36.7	43.6	....	63.1	....	35.7	42.4‡
	8 by ⅜-in. plate	Average	2	4	28.1	31.0	34.0	52.1	55.4	29.0	32.4
63, 64, 65	4 by 4 by ⅜-in. angles	Toes	10	....	36.6	44.0	35.7	62.7	59.2	....	....
		Average	16	....	36.7	43.6	....	63.1	....	35.7	42.4‡
	9½ by ⅜-in. plate	Average	2	4	29.1	33.0	34.0	55.0	55.4	30.0	32.0
66, 67, 68	4 by 4 by ⅜-in. angles	Toes	10	....	36.6	44.0	35.7	62.7	59.2	....	....
		Average	16	....	36.7	43.6	....	63.1	....	35.7	42.4‡
	11 by ⅜-in. plate	Average	2	4	32.6	33.4	34.2	57.3	56.3	31.6	33.7
69, 70, 71	4 by 4 by ⅜-in. angles	Toes	10	....	36.6	44.0	35.7	62.7	59.2	....	....
		Average	16	....	36.7	43.6	....	63.1	....	35.7	42.4‡
	13 by ⅜-in. plate	Average	2	4	34.0	35.0	35.1	62.2	58.2	29.8	32.0
72, 73, 74	4 by 4 by ⅜-in. angles	Toes	10	....	36.6	44.0	35.7	62.7	59.2	....	....
		Average	16	....	36.7	43.6	....	63.1	....	35.7	42.4‡
	15 by ⅜-in. plate	Average	4	8	37.4	39.2	34.2	66.1	56.2	36.8	38.7
75, 76, 77	4 by 4 by ½-in. angles	Toes	6	....	33.0	35.2	36.3	59.1	60.5	....	....
		Average	9	....	32.2	34.2	....	58.7	....	....	....
	15 by ⅜-in. plate	Average	4	8	37.4	39.2	34.2	66.1	56.2	36.8	38.7
B1	2¼ by 3/16-in. lacing-bar	Average	2	....	....	38.2	34.6	55.2	57.4	....	....
B1X			2	....	....	37.0	....	57.6	....	....	....
B2	2¼ by ¼-in. lacing-bar	Average	2	....	....	37.1	35.6	58.5	58.9	....	....
B2X			2	....	....	39.7	....	61.8	....	....	....
B3	2¼ by 5/16-in. lacing-bar	Average	2	....	....	34.8	35.6	56.4	58.9	....	....
B3X			2	....	....	37.6	....	60.0	....	....	....
B4, 5	2¼ by ⅜-in. lacing-bar	Average	2	....	....	34.1	35.9	55.6	61.2	....	....
B6–8, 31–36	8½ by 1-in. lacing-bar	Average	2	....	....	29.7	34.2	54.2	56.3	....	....
B9, 10, 20, 37–40	8 by ¾-in. lacing-bar	Average	2	....	....	30.8	34.6	55.3	57.0	....	....
B11-13 21–23	4 by ¾-in. lacing-bar	Average	2	....	....	31.2	35.0	55.4	59.7	....	....
B14-16 26, 27	6 by ⅝-in. lacing-bar	Average	2	....	....	36.0	36.6	54.2	61.1	....	....
B17-19 28–30	3 by ⅝-in. lacing-bar	Average	2	....	....	34.8	37.3	57.4	60.7	....	....
5 AD	1½ by 3/16-in. lacing-bar	Average	2	....	....	46.2	....	68.8	....	....	....
5 BD	1½ by ¼-in. lacing-bar	Average	2	....	....	41.4	....	62.6	....	....	....
5 CD	1½ by 5/16-in. lacing-bar	Average	2	....	....	41.6	....	62.2	....	....	....
5 AE	2½ by 3/16-in. lacing-bar	Average	2	....	....	41.0	....	54.2	....	....	....
5 BE	2½ by ¼-in. lacing-bar	Average	2	....	....	43.0	....	61.6	....	....	....
5 CE	2½ by 5/16-in. lacing-bar	Average	2	....	....	41.2	....	61.2	....	....	....
A1, 2	8 by 3½ by ¾-in. angle lacing-bar	Average	2	3	32.6	39.0	38.8	63.6	63.9	24.4	44.8‡ 32.6‡
A8, 9	5 by 4 by 7/16-in. angle lacing-bar	Average	3	3	34.2	38.3	35.8	56.6	58.2	23.8	33.5‡
A4–7	2 by 1½ by ¼-in. angle lacing-bar	Average	3	3	39.9	42.7	34.7	60.8	55.6	36.7	39.5‡
C4–6	7-in., 9.8-lb. channel lacing-bar	Average	3	3	34.3	38.5	33.8	57.7	55.2	30.3	36.7‡
C7–9	5-in., 11.5-lb. channel lacing-bar	Average	3	3	29.9	36.7	36.1	57.8	60.2	27.7	34.2‡
C1–3	3-in., 4.1-lb. channel lacing-bar	Average	3	3	37.2	47.4	34.1	62.7	55.5	30.7	39.5‡

* Averages in this column are weighed. † Heat test data. ‡ Ultimate strength of full-sized prism.

TABLE 2.—VARIATIONS IN STRENGTH PROPERTIES OF SPECIMENS IN TESTS OF SERIES (*D*) AND (*H*)

Property (1)	Kind of stress (2)	Highest average stress, in 1 000 lb. per sq. in. (3)	Shape having highest stress (4)	Lowest average stress, in 1,000 lb. per sq. in. (5)	Shape having lowest stress (6)	Ratio: Highest-lowest to specified minimum, in percentage (7)
Proportional limit..	Tension..	39.9	4 by 4 by ⅜-in. angle	21.4	10-in., 35-lb. channel..	62*
Yield point........	Tension..	49.3	4 by 4 by ⅜-in. angle	29.7	8½-in. by 1-in. lacing-bar......	65
Ultimate strength..	Tension..	68.8	1½ by 3/16-in. lacing-bar.............	52.1	8-in. by ⅜-in. plate..	30
Proportional limit..	Compression....	36.8	15 by ⅜-in. plate ...	21.0	10-in., 35-lb. channel..	53*
Yield point........	Compression....	42.4	4 by 4 by ⅜-in. angle	29.8	10-in., 35-lb. channel..	42*

* No minimum specified — based on 30 000 lb. per sq. in.

A summarized comparison of the mill and University Laboratory tests is given in Table 3. In this comparison, forty-six heats, six different shapes, forty sizes of shape in tension tests, and twenty-seven in compression tests, are represented. Weighted averages for a given shape were determined by weighting the specimens from different parts of a shape in proportion to the tributary area. Representative specimens were taken from the following locations: Plate, 3 in. from the edge; angle, from the toe; I-beam, from the middle of the web; channel, from the middle of the web; **H**-column, from the flange intermediate between toe and root; and lacing-bar, from the full section, or edge. It will be noted that the average ratios for yield point

TABLE 3.—COMPARISON OF TENSILE YIELD POINT AND ULTIMATE STRENGTH VALUES FROM MILL TESTS WITH RESULTS OF TESTS MADE AT THE UNIVERSITY LABORATORY; SERIES (*D*) TO (*H*)

	RATIO OF MILL YIELD POINT		RATIO OF MILL ULTIMATE		NUMBER OF		
	Laboratory Yield Point Based on		Laboratory Ultimate Based on				
(1)	Weighted averages for shapes (2)	Representative specimens (3)	Weighted averages for shapes (4)	Representative specimens (5)	Heats represented (6)	Shapes represented (7)	Sizes represented (8)
Average..................	0.98	0.99	1.01	1.02	46	6	40
Minimum................	0.72	0.72	0.85	0.85	46	6	40
Maximum................	1.20	1.17	1.13	1.13	46	6	40
Percentage of ratios within 5% of unity...........	37	43	56	52	......	......	....
Percentage of ratios within 10% of unity..........	67	72	93	89	......	......	....

and ultimate are nearly unity, but less than 50% of the yield point ratios and less than 60% of the ultimate strength ratios are within 5% of unity.

The tendency of the yield point to diminish as the thickness of the shape increases, is illustrated by the data in Fig. 1. The dispersion of the points indicates that other factors, such as differences in composition and rate of cooling, influenced the yield points of the specimens taken from the forty-six heats. It should be noted that the maximum thickness here represented is 1 in.

TABLE 4.—RELATIONSHIP BETWEEN YIELD POINTS IN TENSION AND COMPRESSION FOR SPECIMENS CUT FROM COLUMNS OF SERIES (*D*) AND (*E*)

Shape	NUMBER OF				AVERAGE YIELD POINT IN 1 000 POUNDS PER SQUARE INCH		Ratio: $\frac{T}{C}$
	Heat	Pieces sampled	Specimens Tested in				
			Tension	Compression	Tension T	Compression C	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
8-in., 66-lb. H-column	47497 18306	4	16	4	31.2	30.6	1.02
8-in., 32-lb. H-column	52E465 39F273	4	28	6	38.8	38.3	1.01
10-in., 35-lb. channel	161057	2	6	2	33.4	29.8	1.12
10-in., 20-lb. channel	19325 241001	4	12	4	36.7	34.8	1.05
10-in., 15.3-lb. channel	919708 25985 141048	4	12	4	37.2	34.0	1.09
8-in., 18.75-lb. channel	23998	2	6	2	35.2	33.5	1.05
8-in., 11.5-lb. channel	20089 181074	4	12	4	41.7	39.4	1.06
6-in., 8.2-lb. channel	221072	2	6	2	37.3	35.8	1.04
8-in., 20.5-lb. I-beam	131016	2	2	2	34.7	35.0	0.99
8-in., 18.4-lb. I-beam	19120 921046 171034	4	4	4	38.4	37.9	1.01
6 by 4 by ⅜-in. angles	221061	2	6	2	39.5	38.1	1.04
5 by 3½ by ½-in. angles	25224 15986	4	12	4	36.5	34.9	1.05
5 by 3½ by 5/16-in. angles	20342 191020	4	12	4	40.2	37.5	1.07
4 by 4 by ⅜-in. angles	921253	8	22	6	45.0	41.9	1.07
3 by 2½ by ¼-in. angles	....	2	6	2	40.8	38.7	1.05
15 by ⅜-in. plate	902557	4	4	3	39.2	38.7	1.01
13 by ⅜-in. plate	8583	2	2	1	35.0	32.0	1.09
11 by ⅜-in. plate	912125	2	2	2	33.4	33.7	0.99
9¼ by ⅜-in. plate	912142	2	2	2	33.0	32.0	1.03
8 by ⅜-in. plate	912142	2	2	2	31.0	32.4	0.96
8 by 3½ by ¾-in. angle lacing-bar	902193	3	2	3	39.0	32.6	1.20
5 by 4 by 7/16-in. angle lacing-bar	26827	3	3	3	38.3	33.5	1.14
2 by 1½ by ¼-in. angle lacing-bar	64420	3	3	3	42.7	39.5	1.08
7-in., 9.8-lb. channel lacing-bar	912161	3	3	3	38.5	36.7	1.05
5-in., 11.5-lb. channel lacing-bar	151028	3	3	3	36.7	34.2	1.07
3-in., 4.1-lb. channel lacing-bar	57385	3	3	3	47.4	39.5	1.20
Average ratio: $\frac{T}{C}$	....	......	.....	.....	......	......	1.06

Table 4 shows a comparison of yield points in tension and compression. On the average, the tension yield point is about 6% above the compression value, the maximum excess being 20 per cent. In the previous report[4] the average excess was 1.7% and the maximum, 5 per cent.

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[4] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1152.

Fig. 2 shows a number of typical stress-strain diagrams from compression tests of some of the sections used in the columns. Attention is called to the

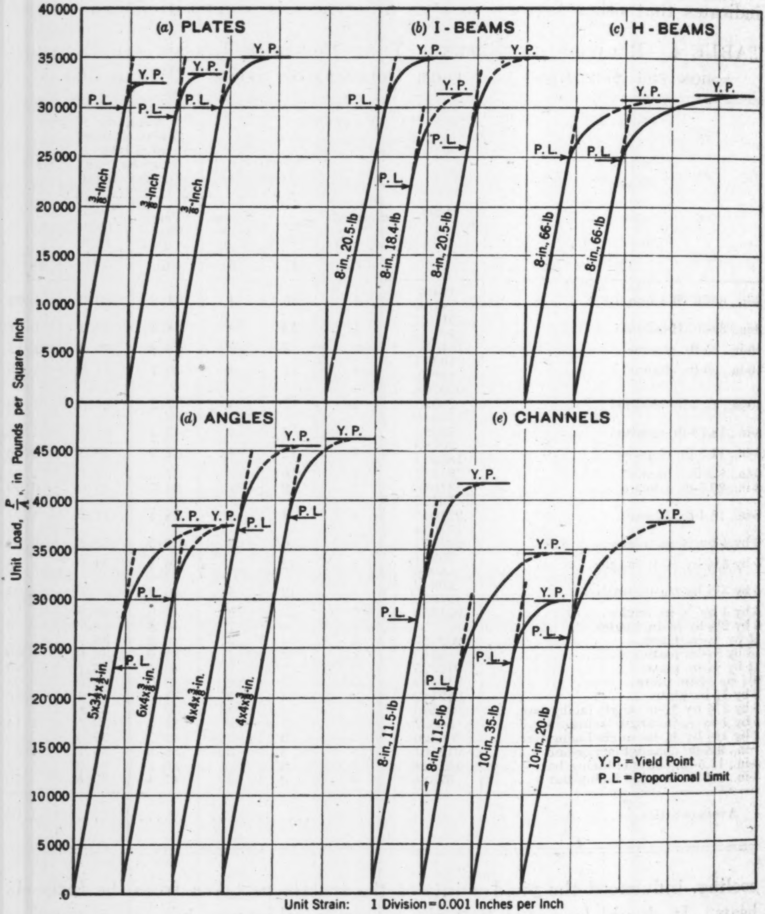

FIG. 2.—TYPICAL STRESS-STRAIN CURVES FOR SHORT PRISMS TESTED IN COMPRESSION (SPECIMENS TAKEN FROM MATERIAL FOR COLUMNS OF SERIES (*D*) AND (*G*).

wide differences among the curves, especially the values for proportional limit and yield point. The low proportional limit values of the channel sections appear to be characteristic.

TESTS OF COLUMNS

2.—The tests of columns are grouped in three series, following Series (*A*), (*B*), and (*C*) of the second progress report[5]; thus,

(*D*).—Forty-one columns of various forms tested with eccentric loads as in Series (*B*) of the 1929 report.

(*E*).—Six columns of three different forms tested with load applied with eccentricity diagonal to the principal axes.

(*F*).—Twenty-four tests of ten slender columns having values of $\frac{l}{r}$ from 102 to 204, tested with eccentric loads.

3.—Columns of Series (*D*).—The make-up, general dimensions, and properties of columns of Series (*D*) are given in Table 5. Columns Nos. 46, 47, 80, 81, 82, and 83 were cut from the 8-in., 32-lb **H**-columns of Series (*B*) of the second progress report.[6] In Columns Nos. 21, 35*R*, 34, 37, 38*R*, 39, 21, 35, 36, and 38, there were no batten-plates and the rivets were $\frac{5}{8}$ in. in diameter. The remainder of the columns were made with $\frac{3}{4}$-in. rivets. All columns of this series were tested with the roller end bearings used in former tests.[7] These bearings consisted of a nest of rollers bearing upon a curved surface having its center of curvature at the surface of the bearing plate.

As previously reported, these bearings for columns of the size tested are almost frictionless and closely approximate pivoted end bearings. Table 5 gives the direction of the roller axes in the tests (see Line *R-R* on each column). All these columns were tested with an eccentricity of bearing such as to make $\frac{ec}{r^2} = 0.5$, thus giving a stress on the extreme fiber at the beginning (before the deflection became significant) 50% greater than the average stress on one side and 50% less on the other. Five types of section were used and in each type, except the box-section, two thicknesses of shapes, slight and heavy. The values of $\frac{l}{r}$ ranged from about 30 to 100, with one value of 166. In conducting the tests, the columns were first centered by trial so that under a load producing a unit stress of about 15 000 lb per sq in., the deflection would be negligible. They were then shifted to produce the desired eccentricity, with respect to one principal axis.

In conducting the tests, deflections at the center, and strains at center, quarter-points, and ends were measured. Table 6 gives some of the results of strain measurements under loads of about two-thirds of the ultimate, and a comparison of stresses calculated from strains with stresses calculated from loads and deflections. The positions of the strain-gauge lines are indicated in Column (1), the top and bottom positions being $8\frac{1}{2}$ in. from the respective

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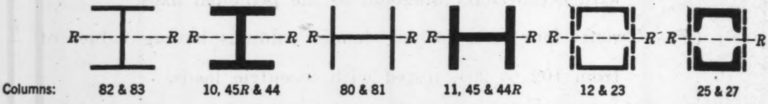
[5] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1153.

[6] *Loc. cit.*, p. 1185.

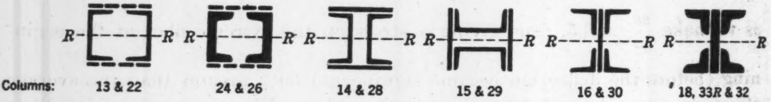
[7] *Loc. cit.*, p. 1161, Fig. 9.

TABLE 5.—DIMENSIONS AND PROPERTIES OF COLUMNS OF SERIES (*D*)

Column No.	Make-up	Length, in inches	Area, in Square Inches		Radius of Gyration, *r*, in Inches		Slenderness ratio, $\frac{l}{r}$	Distance, *c*, in inches	$\frac{c}{r^2}$
			Nominal	Actual	Nominal	Actual			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

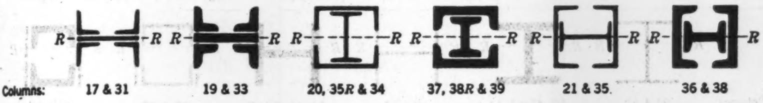


Column No.	Make-up	Length, in inches	Area Nominal	Area Actual	r Nominal	r Actual	$\frac{l}{r}$	*c*	$\frac{c}{r^2}$
82	8 in., 32-lb. H-column......	102.0	9.17	8.91	3.40	3.42	29.8	3.96	0.339
83	8-in., 32-lb. H-column......	204.1	9.17	8.90	3.40	3.42	59.8	3.96	0.339
10	8-in., 66-lb. H-column......	111.0	19.40	19.48	3.70	3.70	30.0	4.50	0.330
45*R*	8-in., 66-lb. H column......	211.0	19.40	19.29	3.70	3.66	57.6	4.46	0.333
44	8-in., 66-lb. H-column......	349.0	19.40	19.30	3.70	3.66	95.4	4.46	0.33?
80	8-in., 32-lb. H-column......	58.5	9.17	9.26	1.98	1.98	29.5	3.98	1.015
81	8-in., 32-lb H-column......	177.8	9.17	9.29	1.98	1.96	90.8	3.98	1.036
11	8-in., 66-lb. H-column......	63.2	19.40	19.45	2.11	2.11	30.0	4.20	0.943
45	8-in., 66-lb. H-column......	211.0	19.40	19.29	2.11	2.12	99.5	4.18	0.931
44*R*	8-in., 66-lb. H column......	349.0	19.40	19.30	2.11	2.10	166.2	4.16	0.946
12	10-in., 20-lb. channels....... 2¼ by ¼-in. lacing-bars...... ¼-in. batten-plates..........	164.0	11.72	11.45	5.44	5.40	30.4	5.93	0.203
23	10-in., 20-lb. channels....... 2¼ by ¼-in. lacing-bars...... ¼-in. batten-plates..........	349.2	11.72	11.52	5.44	5.50	63.6	6.05	0.200
25	10-in., 35-lb. channels....... 2¼ by ¼-in. lacing-bars...... ¼-in. batten-plates..........	159.2	20.54	20.30	5.36	5.52	28.8	6.15	0.202
27	10-in., 35-lb. channels....... 2¼ by ¼-in. lacing-bars...... ¼-in. batten-plates..........	349.3	20.54	20.13	5.36	5.38	65.0	6.03	0.208



Column No.	Make-up	Length, in inches	Area Nominal	Area Actual	r Nominal	r Actual	$\frac{l}{r}$	*c*	$\frac{c}{r^2}$
13	10-in., 20-lb. channels....... 2¼ by ¼-in. lacing-bars...... ¼ in. batten-plates..........	111.8	11.72	11.57	3.66	3.64	30.7	5.04	0.380
22	10-in., 20-lb channels....... 2¼ by ¼-in. lacing-bars...... ¼-in. batten-plates..........	349.1	11.72	11.42	3.66	3.65	95.8	5.04	0.379
24	10-in., 35-lb. channels....... 2¼ by ¼-in. lacing-bars...... ¼-in. batten-plates..........	102.2	20.54	20.25	3.34	3.33	30.7	5.03	0.454
26	10-in., 35-lb. channels....... 2¼ by ¼-in. lacing-bars...... ¼-in. batten-plates..........	335.1	20.54	20.10	3.34	3.34	100.4	5.04	0.450
14	10-in., 15.3-lb. channels..... 12 by ¼-in. plates..........	133.0	14.94	14.67	4.42	4.46	29.8	5.36	0.270
28	10-in., 15.3-lb. channels..... 12 by ¼-in. plates..........	349.0	14.94	15.31	4.42	4.39	79.5	5.36	0.278
15	10-in., 15.3-lb. channels..... 12 by ¼-in. plates..........	109.0	14.94	14.65	3.62	3.62	30.1	6.02	0.459
29	10-in., 15.3-lb. channels..... 12 by ¼-in. plates..........	349.0	14.94	14.70	3.62	3.58	97.5	5.98	0.467
16	5 by 3½ by $\frac{5}{16}$-in. angles..... 10 by $\frac{5}{16}$-in. plate..........	122.0	13.37	12.65	4.20	4.19	29.1	5.26	0.300
30	5 by 3½ by $\frac{5}{16}$-in. angles..... 10 by $\frac{5}{16}$-in. plate..........	346.0	13.37	13.69	4.20	4.22	82.1	5.24	0.294
18	5 by 3½ by ½-in. angles..... 10 by ½-in. plate............	124.0	21.00	21.09	4.16	4.16	29.8	5.23	0.302
33*R*	5 by 3½ by ½-in. angles..... 10 by ½-in. plate............	214.0	21.00	20.65	4.16	4.14	51.7	5.28	0.308
32	5 by 3½ by ½-in. angles...... 10 by ½-in. plate.............	346.0	21.00	20.73	4.16	4.13	83.8	5.29	0.310

TABLE 5.—(*Continued*)



Column No.	Make-up	Length, in inches	Area, in Square Inches: Nominal	Area, in Square Inches: Actual	Radius of Gyration, r, in Inches: Nominal	Radius of Gyration, r, in Inches: Actual	Slenderness ratio, $\frac{l}{r}$	Distance, c, in inches	$\frac{c}{r^2}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
17	5 by 3½ by $\frac{5}{16}$-in. angles..... 10 by $\frac{5}{16}$-in. plate...........	62.0	13.37	12.73	2.08	2.04	30.4	5.14	1.235
31	5 by 3½ by $\frac{5}{16}$-in. angles..... 10 by $\frac{5}{16}$-in. plate...........	206.0	13.37	13.64	2.08	2.07	99.5	5.16	1.203
19	5 by 3½ by ½-in. angles...... 10 by ½-in. plate............	64.0	21.00	20.89	2.16	2.15	29.8	5.22	1.130
33	5 by 3½ by ½-in. angles...... 10 by ½-in. plate............	214.0	21.00	20.65	2.16	2.19	97.8	5.33	1.108
20*	8-in., 11.5-lb. channels...... 8-in., 18.4-lb. I-beam........	105.6	12.06	11.95	3.51	3.60	29.4	4.35	0.336
35*R**	8-in., 11.5-lb. channels...... 8-in., 18.4-lb. I-beam........	239.0	12.06	11.98	3.51	3.58	66.8	4.43	0.346
34*	8-in., 11.5-lb. channels...... 8-in., 18.4-lb. I-beam........	348.1	12.06	12.16	3.51	3.57	97.5	4.38	0.345
37*	8-in., 18.75-lb. channels..... 8-in., 20.5-lb. I-beam........	112.0	16.95	16.71	3.71	3.73	30.0	4.52	0.325
38*R**	8-in., 18.75-lb. channels..... 8-in., 20.5 lb. I-beam........	232.0	16.95	16.47	3.71	3.73	62.2	4.52	0.325
39†	8-in., 18.75-lb. channels..... 8-in., 20.5-lb. I-beam........	346.1	16.95	16.47	3.71	3.72	93.2	4.51	0.326
21†	8-in., 11.5-lb. channels...... 8-in., 18.4-lb. I-beam........	71.5	12.06	11.95	2.38	2.38	30.0	4.09	0.722
35†	8-in., 11.5-lb. channels...... 8-in., 18 4-lb. I-beam........	239.0	12.06	11.98	2.38	2.38	100.4	4.08	0.718
36†	8-in., 18.75-lb. channels..... 8-in., 20.5-lb. I-beam........	70.0	16.95	16.50	2.32	2.31	30.3	4.02	0.754
38†	8-in., 18.75-lb. channels..... 8-in., 20.5-lb. I-beam........	232.0	16.95	16.47	2.32	2.32	100.0	4.02	0.747

* ⅝-in. rivets in these columns only; ¾-in. rivets in other columns. † No batten-plates in these colums.

ends. The stresses in Column (6) were computed by the formula,

$$S = \frac{P}{A}\left[1 + (e + D)\frac{c}{r^2}\right] \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

in which, $\frac{ec}{r^2} = 0.5$ and D denotes the deflection at the center of the respective gauges. The close agreement in results, especially at mid-point, indicates the accuracy of the work and the frictionless character of end bearings. Additional measurements were also made on certain thin columns to ascertain the lateral bending and twisting of the most stressed flange.

Table 7 gives results of these tests and a comparison between the calculated maximum fiber stresses and the properties of the material as determined by the tension specimens. As in Table 6, the calculated maximum fiber stress (Columns (8) and (9), Table 7) was determined from the measured deflection, D, by Equation (1). This formula assumes proportionality of stress to

TABLE 6.—COMPARISON OF MAXIMUM FIBER STRESSES IN COLUMNS OF SERIES (*D*), CALCULATED FROM STRAINS WITH STRESSES FROM LOADS AND DEFLECTIONS

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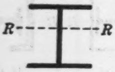
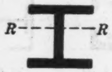
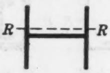


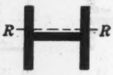
82 & 83



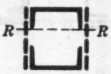
10, 45*R* & 44



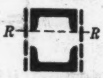
80 & 81



11, 45 & 44*R*



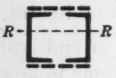
12 & 23



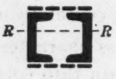
25 & 27

Column No. and Section (1)		Load, $\frac{P}{A}$, in pounds per square inch (2)	Unit Stresses in Outer Fibers on Concave Side of Column, in Pounds per Square Inch: Gauge *A* (3)	Gauge *B* (4)	Average, *A* + *B* (5)	Calculated from Equation (1) (6)	Percentage variation of gauge stress from average (7)	Ratio of stresses, $\frac{A + B}{2S}$ (8)
82	Top.........	14 730	21 800	23 200	22 500	22 100	3.1	1.01
	Middle......		24 000	21 600	22 800	22 600	5.3	1.02
	Bottom......		23 000	22 400	22 700	22 100	1.3	1.03
83	Top.........	14 890	24 900	20 100	22 500	22 200	10.7	1.01
	Middle......		25 900	23 200	24 550	24 300	5.5	1.01
	Bottom......		23 500	22 400	22 950	22 200	2.4	1.03
10	Top.........	14 960	24 100	20 400	22 250	22 600	8.3	0.98
	Middle......		23 300	21 400	22 350	22 900	4.2	0.98
	Bottom......		22 900	23 700	23 300	22 600	1.8	1.03
45*R*	Top.........	15 000	26 600	19 500	23 050	22 500	15.4	1.05
	Middle......		25 500	27 700	26 600	24 700	4.1	1.08
	Bottom......		22 700	24 600	23 650	22 500	4.0	1.05
44	Top.........	11 920	18 400	18 800	18 600	18 100	1.1	1.03
	Middle......		21 800	23 900	22 850	22 600	4.6	1.01
	Bottom......		19 700	18 100	18 900	18 100	4.2	1.04
80	Top.........	14 750	20 800	21 800	21 300	22 100	2.3	0.96
	Middle......		21 500	22 500	22 000	22 450	2.3	0.98
	Bottom......		20 400	21 400	20 900	22 100	2.4	0.95
81	Top.........	14 770	23 600	21 000	22 300	22 100	5.8	1.01
	Middle......		30 400	30 400	30 400	29 200	0.0	1.04
	Bottom......		22 200	22 600	22 400	22 100	0.9	1.01
11	Top.........	14 840	19 500	25 600	22 500	22 500	13.5	1.00
	Middle......		22 100	23 600	22 850	22 700	3.3	1.01
	Bottom......		24 300	22 100	23 200	22 500	4.7	1.03
45	Top.........	10 025	16 700	14 600	15 650	15 000	6.7	1.04
	Middle......		18 300	19 200	18 750	18 200	2.4	1.03
	Bottom......		14 400	15 800	15 100	15 000	4.6	1.01
44*R*	Top.........	5 900	11 400	4 650	8 025	8 860	42.1	0.91
	Middle......		14 100	13 200	13 650	13 400	3.3	1.02
	Bottom......		8 240	12 000	10 120	8 860	18.6	1.14
12	Top.........	14 890	24 700	18 600	21 650	22 500	14.1	0.96
	Middle......		24 600	21 700	23 150	22 900	6.3	1.01
	Bottom......		18 400	24 600	21 500	22 500	14.4	0.91
23	Top.........	14 790	18 300	23 900	21 100	22 200	13.3	0.95
	Middle......		23 900	24 900	24 400	24 600	2.0	0.99
	Bottom......		20 700	22 500	21 600	22 200	4.2	0.97
25	Top.........	14 900	22 900	22 100	22 500	22 350	1.8	1.01
	Middle......		24 000	24 000	24 000	22 800	0.0	1.05
	Bottom......		21 700	25 100	23 400	22 350	7.3	1.05
27	Top.........	14 850	23 400	24 400	23 750	22 400	1.5	1.06
	Middle......		25 800	24 700	25 250	25 300	2.2	1.00
	Bottom......		22 200	25 000	23 600	22 400	5.9	1.05

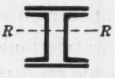
Columns:



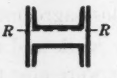
13 & 22



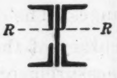
24 & 26



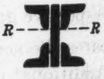
14 & 28



15 & 29



16 & 30

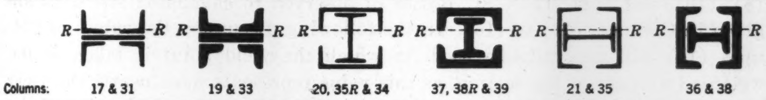


18, 33*R* & 32

Column No. and Section (1)		Load, $\frac{P}{A}$, in pounds per square inch (2)	Gauge *A* (3)	Gauge *B* (4)	Average, *A* + *B* (5)	Calculated from Equation (1) (6)	Percentage variation of gauge stress from average (7)	Ratio of stresses, $\frac{A + B}{2S}$ (8)
13	Top.........	14 870	20 200	23 600	21 900	22 400	7.8	0.98
	Middle......		23 500	22 700	23 100	22 800	1.7	1.01
	Bottom......		23 200	22 000	22 600	22 400	2.7	1.01
22	Top.........	11 830	16 800	17 200	17 000	17 700	1.2	0.96
	Middle......		21 900	21 700	21 800	22 200	0.5	0.98
	Bottom......		17 600	19 200	18 400	17 700	4.4	1.04
24	Top.........	14 850	19 200	25 600	22 400	22 300	14.3	1.00
	Middle......		22 500	24 100	23 300	22 750	3.4	1.02
	Bottom......		21 900	24 000	22 950	22 300	4.6	1.03
26	Top.........	11 870	22 400	12 700	17 550	17 800	27.6	0.99
	Middle......		23 300	23 000	23 150	23 100	0.6	1.00
	Bottom......		16 000	21 300	18 650	17 800	14.2	1.05
14	Top.........	15 100	26 000	20 700	23 350	22 800	11.4	1.02
	Middle......		23 400	23 100	23 250	23 100	0.6	1.01
	Bottom......		21 900	24 100	23 000	22 800	4.8	1.01

TABLE 6.—(*Continued*)

Column No. and Section (1)		Load, $\frac{P}{A}$, in pounds per square inch (2)	Unit Stresses in Outer Fibers on Concave Side of Column, in Pounds per Square Inch				Percentage variation of gauge stress from average (7)	Ratio of stresses, $\frac{A + B}{2S}$ (8)
			Gauge *A* (3)	Gauge *B* (4)	Average, *A* + *B* (5)	Calculated. from Equation (1) (6)		
28	Top.........	13 860	24 400	17 500	20 950	20 800	16.5	1.01
	Middle......		23 600	26 200	24 900	24 700	5.2	1.01
	Bottom......		21 700	22 800	22 250	20 800	2.5	1.07
15	Top.........	14 900	22 000	26 400	24 200	22 400	9.1	1.08
	Middle......		22 500	23 600	23 050	22 800	2.4	1.01
	Bottom......		25 200	21 400	23 300	22 400	8.2	1.04
29	Top.........	11 820	19 500	16 900	18 200	17 700	7.1	1.03
	Middle......		23 200	22 900	23 050	22 600	0.7	1.02
	Bottom......		20 200	17 800	19 000	17 700	6.3	1.07
16	Top.........	14 880	18 700	24 200	21 450	22 500	12.8	0.95
	Middle......		21 700	22 000	21 850	22 800	0.7	0.96
	Bottom......		23 300	19 000	21 150	22 500	10.2	0.94
30	Top.........	11 830	21 900	15 100	18 500	17 800	18.4	1.04
	Middle......		19 600	21 000	20 300	20 600	3.4	0.99
	Bottom......		19 500	18 100	18 800	17 800	3.7	1.06
18	Top.........	14 880	21 900	23 000	22 450	22 500	2.4	1.00
	Middle......		23 100	23 500	23 300	22 800	0.9	1.02
	Bottom......		26 400	19 700	23 050	22 500	14.5	1.02
33R	Top.........	14 990	23 100	17 800	20 450	23 100	13.0	0.89
	Middle......		23 300	25 600	24 450	24 700	4.7	0.99
	Bottom......		19 400	28 400	23 900	23 100	18.8	1.03
32	Top.........	13 860	26 300	13 700	20 000	20 800	31.5	0.96
	Middle......		22 600	27 000	24 800	25 100	8.9	0.99
	Bottom......		21 500	21 500	21 500	20 800	0.0	1.03
17	Top.........	14 880	18 400	25 900	22 150	22 500	16.9	0.99
	Middle......		22 900	22 500	22 700	22 700	0.9	1.00
	Bottom......		24 200	17 800	21 000	22 500	15.2	0.93
31	Top.........	11 860	18 800	16 200	17 500	17 600	7.4	0.99
	Middle......		21 500	23 300	22 400	22 200	4.0	1.01
	Bottom......		15 800	20 100	17 950	17 600	12.0	1.02
19	Top.........	14 900	21 100	23 000	22 050	22 600	4.3	0.98
	Middle......		21 700	23 800	22 750	22 800	4.6	1.00
	Bottom......		20 800	22 300	21 550	22 600	3.5	0.95
33	Top.........	11 870	17 700	17 300	17 500	17 650	1.1	0.99
	Middle......		21 800	22 100	21 950	21 700	0.7	1.01
	Bottom......		15 600	18 900	17 250	17 650	9.6	0.98
20	Top.........	14 870	21 800	20 900	21 350	22 300	2.1	0.96
	Middle......		23 200	22 500	22 850	22 600	1.5	1.01
	Bottom......		20 900	25 300	23 100	22 300	9.5	1.03
35*R*	Top.........	11 910	18 900	13 500	16 200	17 900	16.7	0.91
	Middle......		22 800	22 500	22 650	22 900	0.7	0.99
	Bottom......		15 900	18 000	16 950	17 900	6.2	0.95
34	Top.........	11 840	19 800	17 800	18 800	17 800	5.3	1.06
	Middle......		23 700	21 900	22 800	23 100	3.9	0.99
	Bottom......		18 200	21 300	19 750	17 800	7.8	1.11
37	Top.........	14 890	24 300	22 400	23 350	22 800	4.1	1.02
	Middle......		23 800	23 100	23 450	23 300	1.5	1.01
	Bottom......		17 600	29 800	23 700	22 800	25.7	1.04
38*R*	Top.........	13 880	21 200	22 900	22 050	20 800	3.9	1.06
	Middle......		22 800	22 800	22 800	22 900	0.0	1.00
	Bottom......		20 700	21 400	21 050	20 800	1.7	1.01
39	Top.........	13 820	20 800	22 800	21 800	20 800	4.6	1.05
	Middle......		27 400	27 200	27 300	27 500	0.4	0.99
	Bottom......		22 500	21 600	22 050	20 800	2.0	1.06
21	Top.........	14 870	17 800	22 900	20 350	22 500	12.5	0.90
	Middle......		22 200	22 600	22 400	22 700	0.9	0.99
	Bottom......		25 000	19 600	22 300	22 500	12.1	0.99
35	Top.........	15 000	19 900	27 000	23 450	22 500	15.1	1.04
	Middle......		25 500	26 400	25 950	25 300	1.7	1.03
	Bottom......		23 800	23 100	23 450	22 500	1.5	1.04
36	Top.........	14 880	19 800	21 400	20 600	22 300	3.9	1.04
	Middle......		23 300	22 300	22 800	22 800	2.2	1.00
	Bottom......		19 600	22 200	20 900	22 300	6.2	0.94
38	Top.........	11 830	19 600	21 600	20 600	17 750	4.9	1.16
	Middle......		21 900	22 900	22 400	22 700	2.2	0.99
	Bottom......		18 300	17 200	17 750	17 750	3.1	1.00



strain up to the maximum load, and, hence, where the calculated stress exceeds the yield point of the material the result is fictitious in a sense. It shows merely that the failure did not take place until some time after the yield point had been reached. Comparing the columns placed so that the maximum stress occurred on the edge of an outstanding flange with those where the maximum occurred over a broad face, as, for example, the **H**-columns, Nos. 80 to 44*R,* with Nos. 82 to 44, it will be seen that in the former group the calculated maximum fiber stress was considerably above the yield point, while in the latter group the difference was small. The same general results are shown in the other types of column.

Table 8 gives a comparison between the results of the tests and the calculated values of ultimate strength based on the theoretical formula for eccentric loaded columns (the "secant" formula) and various values of the yield point as indicated in the table. For example, p_1 in Column (6), Table 8, is computed from the formula,

$$p_1 = \frac{S_y}{1 + \frac{ec}{r^2} \sec \sqrt{\frac{p_1}{29\,400\,000}} \times \frac{l}{2r}} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

The same general formula is used in computing p'_1, p''_1 and p'''_1, in Columns (8), (10), and (12), Table 8. Ratios of observed to calculated strengths are given in Columns (13) to (16). In the following discussion the values of Column (14) will generally be used, in which the yield point is taken as the weighted average of the section, as this value represents most nearly the yield point of the material. Values from Column (13) from the yield point of the most stressed fibers, are used in some cases, but the difference between Columns (13) and (14) is small, the average values of the two columns differing by less than 1 per cent.

The application of the theoretical formula for eccentric loads has assumed proportionality of stress and strain up to the yield point and that the yield point determines the ultimate load. It is recognized that this is not strictly correct and that a more exact result could be obained for any particular case by allowing for curvature of the compressive stress-strain diagram of the material near the yield point after the methods of Engesser, Kármán, Westergaard, and others.[8] However, the great variations that exist in the stress-strain curves for different forms of section and for different parts of the same section, and the wide variation in the yield point itself (see Fig. 2), would appear to make such laborious methods of analysis of little practical value for general purposes. The yield point is a readily determined property and is recognized as of major importance in the fixing of working stresses. It seems desirable, therefore, to use this, if practicable, as a basis of study. The general effect of the errors involved account to a considerable extent for the effect of direction of bending as represented in Tables 7 and 8, and the relatively high ultimate strengths of the columns of Series (*E*).

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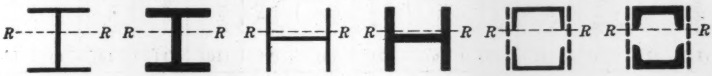
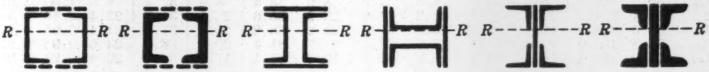
[8] **See paper by H. M. Westergaard, M. Am. Soc. C. E., on "Strength of Steel Columns," *Transactions*, Am. Soc. Mech. Engrs., Vol. 49–50, 1928.**

TABLE 7.—COMPARISON OF CALCULATED STRESSES WITH PROPORTIONAL LIMIT AND YIELD POINT OF MATERIAL FOR ECCENTRIC LOAD TESTS OF SERIES (*D*)



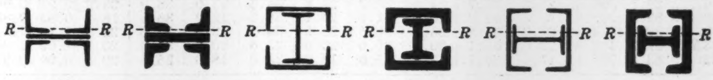
Columns: 82 & 83 10, 45*R* & 44 80 & 81 11, 45 & 44*R* 12 & 23 25 & 27

Column No. (1)	$\frac{l}{r}$ (2)	Applied Loads, $\frac{P}{A}$, in Pounds per Square Inch, at		Deflections, D, at Mid-Length, in Inches, at		Eccentricity, e, in inches (7)	Unit Fiber Stresses by Equation (1)		Average Tensile Properties of Metal, in Pounds per Square Inch	
		First scaling (3)	Maximum (4)	First scaling (5)	Maximum (6)		First scaling (8)	Maximum (9)	Proportional limit (10)	Yield point (11)
82.....	29.8	23 110	23 140	0.18	0.37	1.48	36 100	37 680	33 100	36 400
83.....	59.8	19 770	20 650	0.66	0.86	1.45	33 900	36 820	33 100	36 400
10.....	30.0	19 500	21 050	0.15	0.25	1.52	30 300	33 300	26 700	30 200
45*R*....	57.6	16 650	18 200	0.47	0.82	1.50	27 580	32 250	27 500	30 900
44.....	95.4	11 860	14 260	1.11	2.03	1.56	22 400	31 300	27 500	30 900
80.....	29.5	23 800	28 450	0.05	0.18	0.49	36 850	47 800	42 600	45 300
81.....	90.8	18 000	19 100	0.75	1.15	0.48	40 950	51 400	42 600	45 300
11.....	30.0	19 800	25 720	0.04	0.25	0.53	30 400	44 700	26 000	31 700
45.....	99.5	13 920	14 930	0.67	1.14	0.54	29 600	38 200	31 500	32 700
44*R*....	166.2	8 290	8 290	3.23	3.23	0.53	37 800	37 800	31 500	32 700
12.....	30.4	16 970	20 500	0.18	0.50	2.45	26 000	32 800	33 400	36 300
23.....	63.6	16 000	19 400	0.89	1.79	2.50	26 900	36 100	26 800	35 800
25.....	28.8	17 850	20 000	0.21	0.42	2.48	27 580	31 700	20 700	32 400
27.....	65.0	15 000	17 350	0.94	1.68	2.48	25 300	31 900	20 700	32 400



Columns: 13 & 22 24 & 26 14 & 28 15 & 29 16 & 30 18, 33*R* & 32

13.....	30.7	20 000	25 920	0.12	0.73	1.31	30 900	46 100	32 600	35 400
22.....	95.8	16 870	17 150	2.33	3.03	1.32	40 200	45 400	28 900	38 000
24.....	30.7	17 780	22 850	0.10	0.31	1.10	27 480	37 500	22 100	35 800
26.....	100.4	14 000	14 540	1.58	2.45	1.11	30 900	37 800	22 100	35 800
14.....	29.8	18 000	24 740	0.14	0.46	1.86	27 700	40 200	31 800	38 800
28.....	79.5	14 010	18 780	1.02	2.27	1.80	25 000	40 000	25 800	37 600
15.....	30.1	20 000	27 200	0.07	0.63	1.09	30 700	48 700	31 400	37 800
29.....	97.5	16 000	16 630	1.76	2.59	1.07	37 200	45 100	32 800	42 500
16.....	29.1	20 870	23 700	0.18	0.33	1.67	32 400	37 900	29 200	37 700
30.....	82.0	16 000	17 000	1.33	1.58	1.70	30 300	33 400	37 300	42 000
18.....	29.8	18 020	27 280	0.12	0.47	1.65	27 700	44 800	27 600	35 800
33*R*....	51.7	16 830	22 600	0.40	1.06	1.76	28 000	42 200	26 400	35 100
32.....	83.8	16 000	17 040	1.34	2.07	1.61	30 600	36 500	26 400	35 100

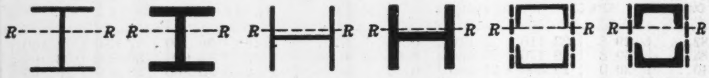


Columns: 17 & 31 19 & 33 20, 35*R* & 34 37, 38*R* & 39 21 & 35 36 & 38

17.....	30.4	21 000	28 950	0.04	0.21	0.40	32 500	50 900	28 200	34 700
31.....	99.5	14 000	18 290	0.45	1.12	0.41	28 500	51 900	32 300	39 000
19.....	29.8	21 630	28 500	0.06	0.85	0.44	33 900	70 200	28 400	36 000
33.....	97.8	16 000	19 640	0.57	1.03	0.44	33 900	51 600	29 700	36 500
20.....	29.4	26 000	26 000	0.44	0.76	1.45	42 500	45 300	36 600	41 800
35*R*....	66.8	22 230	22 230	1.50	1.50	1.44	44 900	44 900	29 400	40 300
34.....	97.5	17 000	17 000	3.35	3.35	1.45	45 200	45 200	29 400	40 300
37.....	30.0	22 600	22 600	0.47	0.47	1.64	38 100	38 100	25 800	33 700
38*R*....	62.2	19 880	19 880	1.14	1.14	1.54	37 200	37 200	25 800	33 700
39.....	93.2	15 890	15 890	2.80	2.80	1.54	38 400	38 400	25 800	33 700
21.....	30.0	20 000	30 000	0.07	1.06	0.69	31 000	68 000	40 300	45 200
35.....	100.4	16 060	16 540	1.25	1.59	0.70	38 600	43 700	28 800	42 400
36.....	30.3	19 000	26 200	0.06	0.34	0.66	29 300	45 950	25 500	36 100
38.....	100.0	14 810	15 420	0.97	1.81	0.67	32 950	44 050	25 500	36 100

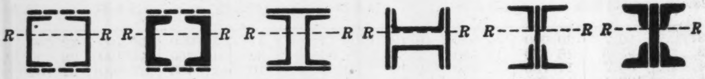
TABLE 8.—COMPARISON OF MAXIMUM UNIT LOADS WITH LOADS CALCULATED FROM YIELD POINT OF METAL FOR TESTS OF SERIES (*D*)

				UNIT LOADS, IN 1 000 POUNDS PER SQUARE INCH											
Column No. (1)	$\frac{l}{r}$ (2)	Maximum total load, in 1 000 lb. (3)	Maximum unit load, p, in 1 000 lb. per sq. in. (4)	Tensile yield point, S_y, of most stressed fibers (5)	Unit load, p_1, calculated from Equation (1) (6)	Tensile yield point, S'_y, of whole section (weighted) (7)	Unit load, p', calculated from S'_y (8)	Tensile yield point, S''_y, from representative specimens (9)	Unit load, p''_1, calculated from S''_y (10)	Tensile yield point, S'''_y, from mill test data (11)	Unit load, p'''_1, calculated from S'''_y (12)	Ratio, $\frac{p}{p_1}$, average, 1.02 (13)	Ratio, $\frac{p}{p'_1}$, average, 1.03 (14)	Ratio, $\frac{p}{p''_1}$, average, 1.04 (15)	Ratio, $\frac{p}{p'''_1}$, average, 1.03 (16)



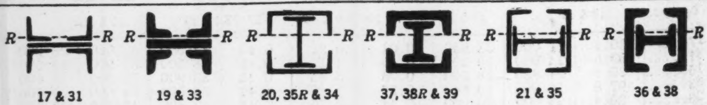
Columns: 82 & 83 10, 45*R* & 44 80 & 81 11, 45 & 44*R* 12 & 23 25 & 27

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
82....	29.8	206 2	23.1	36.4	23.5	36.7	23.6	38.6	24.8	43.7	28.0	0.99	0.98	0.93	0.83
83....	59.8	183.8	20.6	36.4	21.3	36.7	21.4	38.6	22.4	43.7	24.8	0.97	0.97	0.92	0.83
10....	30.0	410.7	21.0	30.2	19.6	31.0	20.1	35.4	22.8	36.3	23.4	1.07	1.05	0.92	0.90
45*R*..	57.6	351 3	18.2	30.9	18.5	31.3	18.8	33.4	19.9	37.4	21.9	0.98	0.97	0.92	0.83
44....	95.4	275.2	14.3	30.9	14.8	31.3	14.9	33.4	15.6	37.4	16.7	0.96	0.96	0.91	0.85
80....	29.5	263.4	28.4	45.3	29.1	41.8	26.9	43.5	28.0	42.2	27.2	0.98	1.06	1.02	1.05
81....	90.8	177.3	19.1	45.3	19.7	41.8	18.8	43.5	19.3	42.2	18.9	0.97	1.02	0.99	1.01
11....	30.0	500.1	25.7	31.7	20.6	31.0	20.1	35.4	22.9	36.3	23.4	1.25	1.28	1.12	1.10
45....	99.5	288.0	14.9	32.7	15.1	31.3	14.7	33.4	15.3	37.4	16.3	0.99	1.02	0.98	0.92
44*R*..	166.2	160.4	8.3	32.7	8.3	31.3	8.2	33.4	8.4	37.4	8.6	1.00	1.01	0.99	0.96
12....	30.4	234.7	20.5	36.3	23.3	35.9	23.2	36.6	23.6	34.7	22.4	0.88	0.88	0.87	0.92
23....	63.6	223.8	19.4	35.8	20.5	37.4	21.2	36.8	20.9	37.8	21.4	0.95	0.92	0.90	0.91
25....	28.8	406.0	20.0	32.4	21.0	33.4	21.7	31.1	20.2	36.4	23.6	0.95	0.92	0.99	0.85
27....	65.0	349.1	17.4	32.4	18.7	33.4	19.0	31.1	17.9	36.4	20.4	0.93	0.91	0.97	0.85



Columns: 13 & 22 24 & 26 14 & 28 15 & 29 16 & 30 18, 33*R* & 32

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
13....	30.7	300.0	25.9	35.4	22.8	35.9	23.2	36.6	23.6	34.7	22.4	1.14	1.12	1.10	1.16
22....	95.8	195.8	17.2	38.0	17.0	37.4	16.9	36.8	16.7	37.8	17.8	1.01	1.01	1.03	1.01
24....	30.7	462.9	22.8	35.8	23.1	33.4	21.6	31.1	20.2	36.4	23.5	0.99	1.06	1.13	0.97
26....	100.4	292.0	14.5	35.8	15.8	33.4	15.2	31.1	14.6	36.4	16.0	0.92	0.96	1.00	0.91
14....	29.8	363.0	24.7	38.8	24.9	36.7	23.7	36.9	23.8	35.5	22.9	0.99	1.04	1.04	1.08
28....	79.5	287.5	18.8	37.4	19.2	37.6	19.3	37.6	19.3	35.9	18.6	0.98	0.97	0.97	1.01
15....	30.1	398.7	27.2	37.8	24.4	36.7	23.7	36.9	23.8	35.3	22.8	1.11	1.15	1.14	1.19
29....	97.5	244.4	16.6	40.0	17.3	37.6	16.7	37.6	16.7	35.6	16.2	0.96	1.00	1.00	1.03
16....	29.1	300.0	23.7	37.7	24.4	38.2	24.7	35.6	23.0	36.9	23.2	0.97	0.96	1.03	1.02
30....	82.0	232.6	17.0	42.0	20.4	41.1	20.1	38.5	19.2	36.3	18.4	0.83	0.85	0.89	0.92
18....	29.8	574.9	27.3	35.8	23.2	36.5	23.6	34.8	22.5	34.8	22.5	1.18	1.16	1.21	1.21
33*R*..	51.7	466.8	22.6	35.1	20.7	35.6	20.9	35.6	20.9	35.3	20.8	1.09	1.08	1.08	1.09
32....	83.8	353.4	17.0	35.1	17.8	35.6	18.0	35.6	18.0	35.3	20.8	0.96	0.95	0.95	0.95



Columns: 17 & 31 19 & 33 20, 35*R* & 34 37, 38*R* & 39 21 & 35 36 & 38

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
17....	30.4	368.6	29.0	34.7	22.5	38.2	24.7	35.6	23.1	35.9	23.3	1.29	1.17	1.25	1.24
31....	99.5	249.3	18.3	39.0	16.9	41.1	17.3	38.5	16.7	36.4	16.2	1.08	1.06	1.09	1.13
19....	29.8	595.2	28.5	36.0	23.2	36.5	23.6	34.8	22.5	34.8	22.5	1.23	1.21	1.27	1.27
33....	97.8	247	19.6	36.5	16.5	35.6	16.2	35.6	16.2	34.8	16.0	1.19	1.21	1.21	1.22
20....	29.4	310.6	26.0	41.8	27.1	42.4	27.5	38.2	24.9	34.2	22.4	0.96	0.95	1.05	1.16
35*R*..	66.8	266.2	22.2	40.3	22.2	39.8	21.9	38.6	21.4	36.2	20.4	1.00	1.02	1.04	1.09
34....	97.5	206.9	17.0	40.3	17.3	39.8	17.2	38.6	16.9	36.2	16.3	0.98	0.99	1.01	1.04
37....	30.0	378.0	22.6	33.7	21.3	35.0	22.1	34.5	21.8	36.5	23.0	1.06	1.02	1.04	0.98
38*R*..	62.2	327.5	19.9	33.7	19.6	35.0	20.2	34.5	20.0	36.5	21.0	1.01	0.98	0.99	0.95
39....	93.2	261.8	15.9	33.7	16.1	35.0	16.5	34.5	16.4	36.5	17.0	0.99	0.96	0.97	0.94
21 ...	30.0	358.6	30.0	45.2	28.9	42.4	27.3	38.2	24.7	34.2	22.2	1.04	1.10	1.22	1.35
35....	100.4	231.0	16.5	42.4	17.3	39.8	16.7	38.6	16.5	36.2	15.9	0.95	0.99	1.00	1.04
36....	30.3	432.5	26.2	36.1	23.3	35.0	22.6	34.5	22.3	36.5	23.5	1.12	1.16	1.17	1.11
38....	100.0	254.0	15.4	36.1	15.9	35.0	15.6	34.5	15.5	36.5	16.0	0.97	0.99	1.00	0.96

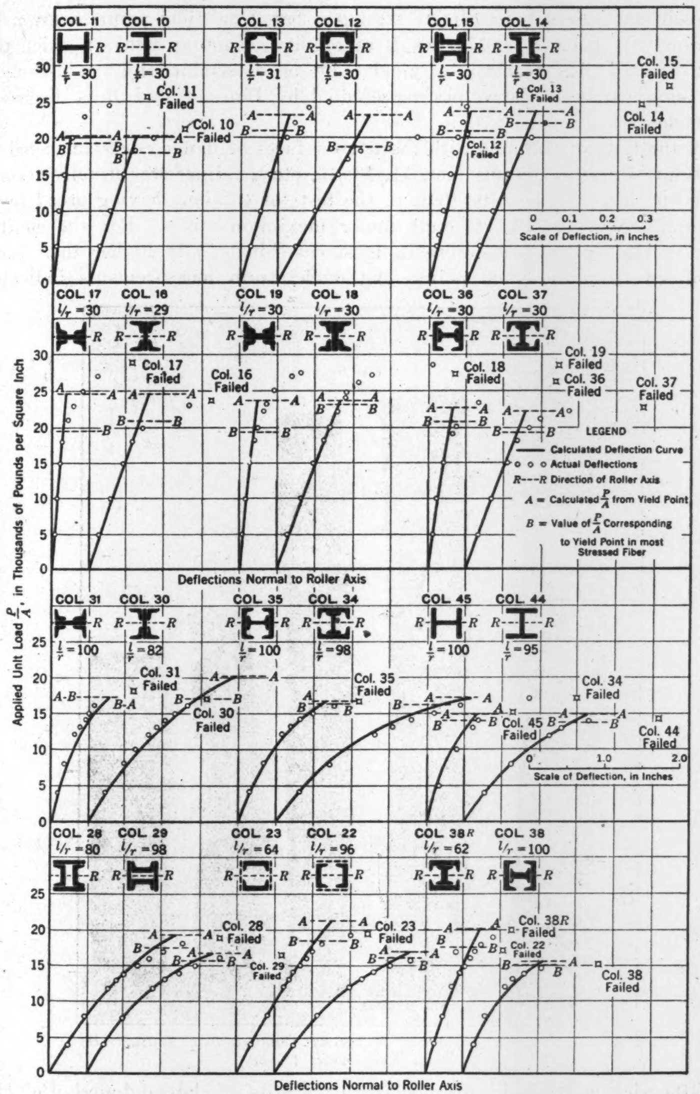


FIG. 3.—LOAD DEFLECTION CURVES FOR REPRESENTATIVE COLUMNS OF SERIES (*D*).

Fig. 3 shows the load-deflection relations for many of the tests. The curved lines are the calculated load-deflection curves, assuming elastic conditions and using values of *E* as determined by 20-in. strain-gauges on each column. The observed deflections are shown by the small circles. Lines *A* represent the calculated ultimate strengths based on yield point as given in Column (8), Table 8, and Lines *B* represent the actual loads at which the most stressed fiber reached its yield point as determined by strain-gauges. The significance of the values represented by Lines *A* and *B* is discussed subsequently.

In the tests of columns with the narrow faces or flange edges subjected to maximum stress (Columns Nos. 11, 13, 15, etc.), failure was usually accompanied by larger deflections than in the tests of columns having broad faces (Columns Nos. 10, 12, 14, etc.) under maximum stress. For the shorter columns the failure loads were in most cases materially higher than loads calculated from the yield point. Although strain measurements indicated that the yield point had been reached in the tests of plate-and-angle Columns Nos. 16 and 30 prior to failure, the failures occurred at loads somewhat less than the estimates based on yield point. These discrepancies may have been due to the effect of wind and waves in the thin flanges ($\frac{5}{16}$-in.) of these members and lateral bending of the flanges. The measurements at mid-

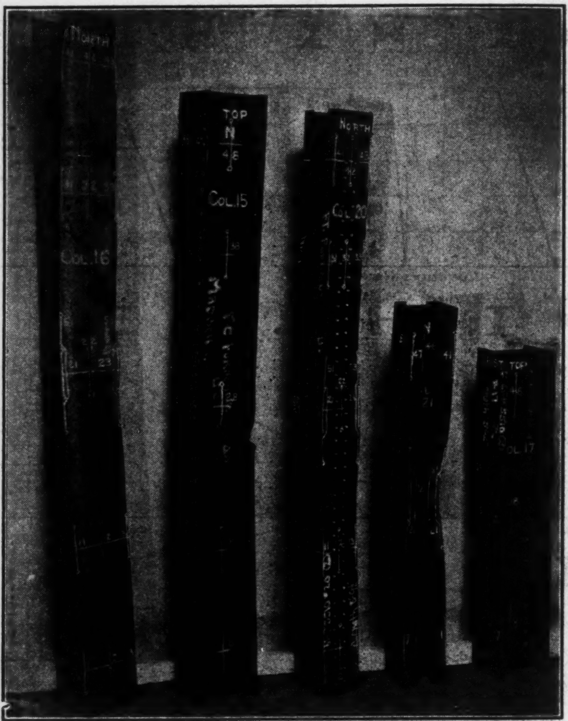


FIG. 4.—TYPICAL FAILURES OF SHORT COLUMNS OF SERIES (*D*) UNDER ECCENTRIC LOADS.

length showed a small amount of lateral bending, and there was perceptible crippling and twisting at the failure of both these columns (see Fig. 4). Column No. 18, with flanges $\frac{1}{2}$ in. thick, also exhibited twisting and buckling in the heavily stressed flange, but this phenomenon did not occur until the over-strain was very marked.

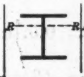
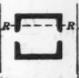
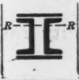
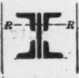
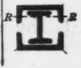
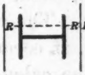
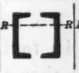
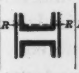
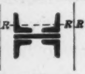
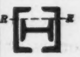
Typical failures of short box, and short channel, I-beam columns are also shown in Fig. 4. These results agree in general with those given on **H**-columns in the second progress report.

The inferences that may be drawn from the results given in Table 8 relate to three important elements of column strength: (*a*) Effect of direction of bending; (*b*) effect of form of column; and (*c*) relation between test results and strength values calculated from yield points. The first two topics are discussed herein; the last one will be considered in connection with results from Series (*F*).

3 (a).—Effect of Direction of Bending.—In Table 8 the differences due to direction of bending are much less than is indicated in Table 7; and where the value of $\frac{l}{r}$ is large, the differences are necessarily small, as the amount of deflection in such columns is so large that a considerable difference in yield point has a relatively small effect.

Considering the **H**-sections, the ratios for Columns Nos. 82 to 44 from Column (14), Table 8, average 0.99 and for Columns Nos. 80 to 44*R*, average 1.08, or about 9% better. Columns Nos. 12 to 27 average 0.91 and Columns Nos. 13 to 26 average 1.04, or 14% better. Columns Nos. 15 to 29 are about 7% better than Columns Nos. 14 to 28. The average for Columns Nos. 16 to 32 is 1.00 and that for Columns Nos. 17 to 33 is 1.16, or 16% better. Columns Nos. 20 to 39 average 0.99, and Columns Nos. 21 to 38 average 1.06, or 7% better. The average results are grouped according to form of section (plane of bending in a vertical direction) in Table 9.

TABLE 9.—COMPARISON OF RATIOS OF ACTUAL TO CALCULATED STRENGTH

	Description	Types:				
		A	*B*	*C*	*D*	*E*
	POSITION *A*					
Item	Types					
1	Approximate value of r, in inches........	3.5	5.4	4.4	4.2	3.6
2	Ratio of actual to calculated strengths....	0.99	0.91	1.005	1.00	0.99
	POSITION *B*					
Item	Types					
3	Approximate value of r, in inches........	2.0	3.5	3.6	2.0	2.4
4	Ratio of actual to calculated strengths....	1.08	1.04	1.075	1.16	1.06

In practice, Position *B* is likely to be the more significant as the value of *r* is the smaller in this direction; and it is of some importance to note that the actual strength is quite certain to exceed the calculated strength. Examining the individual values of Table 8, it is seen that in only three tests in Position *B* did the test result fall below the calculated, with a maximum deficiency of 4%, the others being 1 per cent.

3 (b).—Effect of Form of Column.—Referring to Table 9, it appears that there is little difference due to form when placed in Position *B*. In Position *A*, the strength is less in every case, but well up to the theoretical, except in the case of the laced channel form which is materially less. This deficiency in this form of column was indicated in the two tests in the second progress report, and has been mentioned elsewhere in this report. It is probably due in part to the lacing-bar connection being less efficient than plates and in part to the low proportional limit for channel sections (see Fig. 2). To overcome this weakness in Position *A*, the channels can be spread to insure bending in the other plane. In Position *B* this type appears also to be slightly weaker than the others, although the difference is small. Considering differences in values of *r*, Types *B* and *C* are the most economical of material, with Type *E* coming next, assuming the plane of bending as shown in Position *B*.

4.—Columns of Series (*E*).—These columns were tested by loading with an eccentricity diagonal to the principal axes. To permit bending in any direction, the columns were supported on well-greased spherical bearing blocks. Tests of these blocks gave an average friction coefficient of 1.7%, which would reduce the moment of an eccentric load by 1% for an eccentric ratio of 1.7 per cent. Since the resultant eccentricity was greater than this amount in every test of Series (*E*), it is believed that the frictional effects of the end bearings can be disregarded.

Table 10(*a*) gives dimensions and properties of these columns and the position of the applied loads with respect to each principal axis. For Axis 1-1, the eccentricity ratio, $\frac{ec}{r^2} = 0.5$, except in the case of Column No. 46, for which it is 1.0. For Axis 2-2, $\frac{ec}{r^2} = 1.00$ in each case.

Table 10(*b*) gives results of the tests and comparisons of fiber stresses calculated from deflections with yield point values. The ratios of the two values are given in Item No. 32. The very high ratios compared to the other series are to be noted. In fact, if eccentricity in one direction only be considered, it will be found that the calculated stresses, for this direction only, exceed the yield point values. This result appears to be explainable only on the ground that here the maximum fiber stress occurs at one corner of the section, which may be much overstressed without causing failure.

Typical deflection diagrams are shown in Fig. 5, with theoretical values added and the yield points marked.

In Table 10(*c*) fiber stresses calculated from deflections are compared with stresses calculated from strains and in Items Nos. (52), (55), (58), and

(61), there are also given the stresses calculated by the secant formula,

$$S = \frac{P}{A}\left(1 + \frac{e_1 c_1}{r^2_1} \sec \sqrt{\frac{P}{AE}}\ \frac{l}{2r_1} \pm \frac{e_2 c_2}{r^2_2} \sec \sqrt{\frac{P}{AE}}\ \frac{l}{2r_2}\right) \ldots\ldots(3)$$

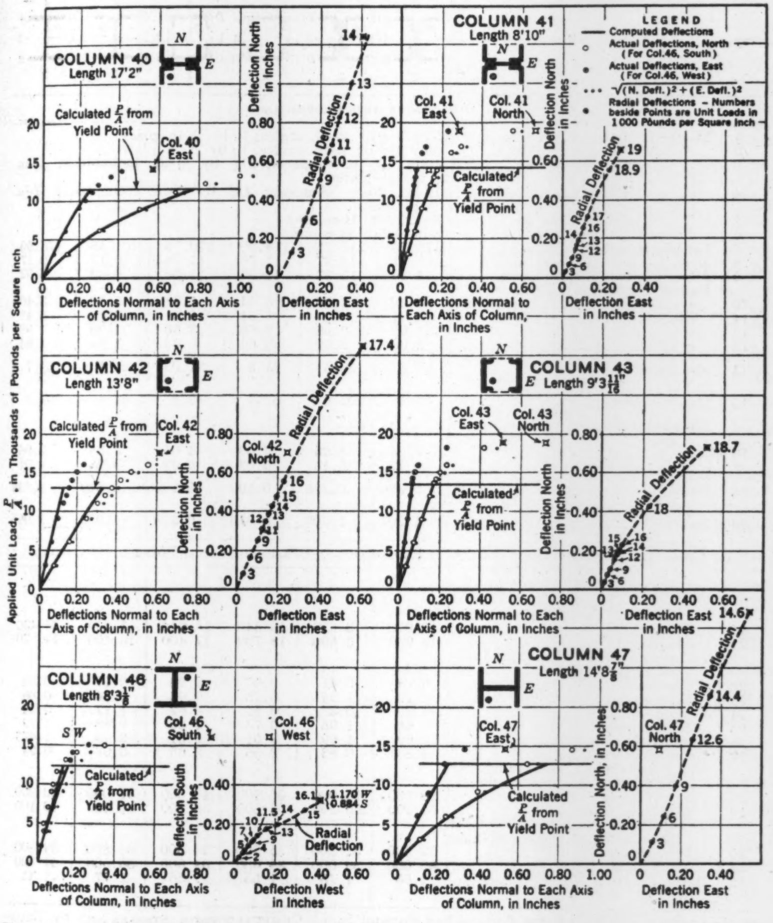


FIG. 5.—LOAD-DEFLECTION CURVES FOR COLUMNS OF SERIES (*E*), ECCENTRICALLY LOADED WITH RESPECT TO BOTH PRINCIPAL AXES.

The load considered is much less than the ultimate, the purpose of the comparisons being to check the method of calculation and the reliability of the apparatus. The agreement is seen to be very close. The effect of friction of bearings is so small that it is not in evidence in the results.

TABLE 10.—COMPARISON OF LOADS AND STRESSES; COLUMNS OF SERIES (*E*), ECCENTRICALLY LOADED WITH RESPECT TO BOTH PRINCIPAL AXES

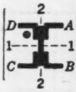
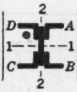
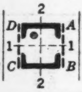
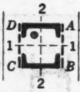
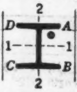
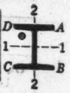
		COLUMN NOS.					
		41	40	43	42	46	47
Item	Description						
	(*a*) DIMENSIONS AND PROPERTIES						
1	Make-up: Column No. 41	5 by 3½ by 5/16-in. angles; 10 by 5/16-in. plate.					
2	Column No. 40	5 by 3½ by 5/16-in. angles; 10 by 5/16-in. plate.					
3	Column No. 43	10-in., 20-lb. channels; 2¼ by ¼-in. lacing-bars; ¼-in. batten-plates.					
4	Column No. 42	10-in., 20-lb. channels; 2¼ by ¼-in. lacing-bars; ¼-in. batten-plates.					
5	Column No. 46	8-in. 32-lb. H-column.					
6	Column No. 47	8-in., 32-lb. H-column.					
7	Length, in inches	112.75	212.75	118.50	170.75	101.48	176.90
8	Area, in square inches; nominal	13.37	13.37	11.76	11.76	9.17	9.17
9	Area, in square inches; actual	13.26	13.33	11.59	11.52	9.42	8.92
	Radius of gyration, in inches:						
10	r_1 nominal	4.20	4.20	5.44	5.44	3.40	3.40
11	r_1 actual	4.21	4.21	5.51	5.51	3.42	3.42
12	r_2 nominal	2.08	2.08	3.66	3.66	1.98	1.98
13	r_2 actual	2.06	2.07	3.66	3.65	1.94	1.94
14	Slenderness ratio, $\frac{l}{r_1}$	26.8	50.5	21.5	31.0	29.7	51.7
15	Slenderness ratio, $\frac{l}{r_2}$	54.8	102.6	32.4	46.8	52.4	91.2
	Distance to Most Stressed Fiber, in Inches:						
16	c_1	5.24	5.25	6.05	6.05	3.98	3.96
17	c_2	5.12	5.15	5.03	5.03	3.98	4.01
18	Ratio, $\frac{c_1}{r^2}$	0.296	0.296	0.199	0.199	0.340	0.339
19	Ratio, $\frac{c_2}{r^2_2}$	1.207	1.200	0.376	0.378	1.058	1.065
	(*b*) COMPUTED STRESSES COMPARED WITH PROPORTIONAL LIMIT AND YIELD POINT OF MATERIAL						
	Applied Loads, $\frac{P}{A}$, in Pounds per Square Inch, at:						
20	First scaling	13 970	13 620	13 640	16 640	11 500	14 430
21	Maximum	18 960	13 890	18 700	17 400	16 100	14 570
	Deflections at Mid-Length, in Inches, at:						
22	First scaling, D_1	0.08	0.41	0.07	0.28	0.11	0.33
23	First scaling, D_2	0.19	1.24	0.18	0.63	0.14	0.86
24	Maximum, D_1	0.29	0.57	0.51	0.62	1.17	0.54
25	Maximum, D_2	0.66	1.95	0.73	1.25	0.88	1.29
26	Eccentricities, in inches: e_1	1.69	1.69	2.51	2.51	2.93	1.475
27	Eccentricities, in inches: e_2	0.83	0.83	2.66	2.65	0.98	0.94
	Unit Fiber Stresses, r, in Pounds per Square Inch by Equation (3), at:						
28	First scaling	38 500	56 000	35 200	46 500	37 000	50 900
29	Maximum, S_m	64 200	69 500	53 800	53 900	70 200	59 100
	Tensile Properties of Most Stressed Fiber, in Pounds per Square Inch:						
30	Proportional limit	32 300	32 300	25 300	25 300	34 800	34 800
31	Yield point, S_y	39 000	39 000	34 600	34 600	39 600	41 800
32	Ratio, $\frac{S_m}{S_y}$ (Item No. 29 to Item No. 31)	1.65	1.78	1.55	1.56	1.77	1.41
	(*c*) MAXIMUM FIBER STRESS (IN POUNDS PER SQUARE INCH) COMPUTED FROM STRAINS AND BY EQUATION (3) COMPARED WITH MAXIMUM FIBER STRESSES COMPUTED FROM ACTUAL DEFLECTIONS (SEE ITEMS NOS. 14 AND 15 FOR SLENDERNESS RATIOS; ITEMS NOS. 18 AND 19 FOR $\frac{c}{r^2}$ RATIOS; AND ITEMS NOS. 26 AND 27 FOR ECCENTRICITIES).						
33	Load, $\frac{P}{A}$, in pound per square inch	5 880	5 900	8 830	8 830	9 530	8 770
	Unit Stresses, Upper Quarter Point, at:						
34	*A* strains	......	11 000	12 600	14 500	......	16 300
35	*A* deflections	......	10 200	13 400	13 700	......	15 600
36	*B* strains	......	–4 200	–3 600	–3 100	......	–7 400
37	*B* deflections	......	–4 600	–4 700	–5 200	......	–7 600

TABLE 10.—*Continued*

		COLUMN NOS.					
		41	40	43	42	46	47
	Unit Stresses, Upper Quarter Point, at:						
38	*C* strains	......	1 500	3 600	3 400	......	2 500
39	*C* deflections	......	1 600	4 200	3 900	......	2 000
40	*D* strains	......	15 400	22 700	20 500	......	23 800
41	*D* deflections	......	16 400	22 400	22 800	......	25 100
	Unit Stresses, Lower Quarter Point, at:						
42	*A* strains	......	10 400	13 200	12 000	......	14 400
43	*A* deflections	......	10 200	13 400	13 700	......	15 600
44	*B* strains	......	−5 300	−4 700	−3 100	......	−5 500
45	*B* deflections	......	−4 600	−4 700	−5 200	......	−7 600
46	*C* strains	......	900	3 500	4 500	......	3 500
47	*C* deflections	......	1 600	4 200	3 900	......	2 000
48	*D* strains	......	17 600	23 300	21 900	......	22 700
49	*D* deflections	......	16 400	22 400	22 800	......	25 100
	Unit Stresses, Mid-Length, at:						
50	*A* strains	8 940	10 500	13 000	13 600	30 700	14 800
51	*A* deflections	9 260	10 700	13 500	13 900	30 300	16 400
52	*A* Equation (3)	9 200	10 600	13 500	13 900	30 000	16 200
53	*B* strains	−2 840	−5 500	−5 000	−4 100	6 600	−7 400
54	*B* deflections	−3 500	−5 300	−4 800	−5 400	8 560	−8 600
55	*B* Equation (3)	−3 500	−5 100	−4 800	−5 400	8 300	−8 400
56	*C* strains	2 500	1 100	4 300	3 700	−5 500	3 000
57	*C* deflections	2 500	1 100	4 100	3 800	−11 200	1 200
58	*C* Equation (3)	2 500	1 200	4 100	3 800	−11 400	1 300
59	*D* strains	15 000	17 500	23 000	22 100	6 400	24 700
60	*D* deflections	15 300	17 100	22 500	23 100	10 500	26 200
61	*D* Equation (3)	15 200	16 900	22 500	23 000	10 800	25 900

(*d*) MAXIMUM UNIT LOADS, IN POUNDS PER SQUARE INCH, COMPARED WITH LOADS COMPUTED FROM YIELD POINT OF METAL

		41	40	43	42	46	47
62	**Maximum load, in pounds (for corresponding unit loads, see Item No. 21)**	251 500	185 000	216 800	200 400	151 800	130 000
63	**Most Stressed Fiber;:** **Tensile yield point, S_y**	39 000	39 000	34 600	34 600	39 600	41 800
64	**Unit load, p_1, computed from Item No. 63**	14 200	11 400	13 400	13 000	12 200	12 700
65	**Ratio, Item No. 21 to Item No. 64 (average = 1.30)**	1.34	1.22	1.40	1.34	1.32	1.15
	Entire Section (Weighted):						
66	**Tensile yield point, S'_y**	41 100	41 100	37 400	37 400	38 600	38 000
67	**Unit load, p'_1 computed from Item No. 66**	14 900	11 700	14 500	14 000	11 900	11 800
68	**Ratio, Item No. 21 to Item No. 67 (average = 1.26)**	1.27	1.19	1.29	1.24	1.35	1.24
	Representative Specimen:						
69	**Tensile yield point, S''_y**	38 500	38 500	36 800	36 800	41 800	40 400
70	**Unit load, p''_1, computed from Item No. 69**	14 100	11 200	14 300	13 800	12 800	12 400
71	**Ratio, Item No. 21 to Item No. 70 (average = 1.26)**	1.34	1.24	1.31	1.26	1.26	1.18
	Mill Test Data:						
72	**Tensile yield point, S'''_y**	36 300	36 300	37 800	37 800	43 700	42 200
73	**Unit load, p'''_r computed from Item No. 72**	13 300	10 800	14 600	14 100	13 400	12 800
74	**Ratio, Item No. 21 to Item No. 73 (average = 1.26)**	1.43	1.29	1.28	1.23	1.20	1.14
	Most Stressed Part:						
75	**Tensile yield point, S_y**	39 000	39 000	34 600	34 600	39 600	41 800
76	**Unit load, p_2 computed from Item No. 75, neglecting e_1**	17 200	12 700	16 600	16 000	17 300	14 500
77	**Ratio, Item No. 21 to Item No. 76 (average = 1.06)**	1.10	1.09	1.12	1.09	0.93	1.00

In Table 10 (*d*) the actual strength (Item No. 21) is compared with the strength calculated from the secant formula (Equation (3)), using various values for yield point. Individual results differ considerably, but the average excess of test strength over calculated values is about the same for all groups, approximately 26 per cent. This is much larger than the values for one-direction eccentricity, and indicates that the effect of eccentricity in both directions, which condition generally obtains in structures, may be discounted considerably in practice.

In Table 10(*d*), Items Nos. 75, 76 and 77, the maximum unit loads (Item No. 21) are compared with the calculated strengths based on yield point, considering only the eccentricity with respect to the Axis 2-2. These data show that a safe approximation of strength can be made by neglecting the lesser eccentricity ratio when it is not more than half the larger ratio. Excluding Column No. 46, for which the ratios were both unity, the test results exceed the calculated values by an average of 8 per cent.

5.—Columns of Series (*F*).—This series consisted of twenty-four tests of ten slender columns of values of $\frac{l}{r}$ from 102 to 204. The shapes were laced channels and double and single angles. Many of these shapes were axially loaded to the Euler load and then tested with values of $\frac{ec}{r^2}$ of 0.5. Columns Nos. 50 to 54 were tested with the roller bearings. The remainder of the columns of this series were loaded through the knife-edge bearings. Table 11(*a*) gives make-up, dimensions, position of bearing axis, and properties of these columns. For Columns Nos. 56 and 57, the subscripts, 3 and 4, in Table 11 refer to the minor and major principal axes, respectively.

Table 11(*b*) gives results of tests and comparisons of calculated stresses with yield point. In this case the variation in calculated stresses is large, due in part to variations in yield point of the metal composing the different columns, but due principally to differences in the breadth of flange carrying the maximum stress. This contention is sustained by the strength comparisons of Table 11(*c*). The ratios of actual to calculated strengths shown in Items Nos. 39, 40, and 41, of Table 11(*c*) are approximately unity, as would be expected in tests of such slender columns. The ratios for Columns Nos. 52 and 54, tested with their broad flanges receiving the maximum stress, are less than unity, whereas the ratios for nearly all the remaining columns tested with rollers parallel to a principal axis are unity, or slightly more. The low ratios for Column No. 59 are due most likely, to the marked twisting exhibited by this member during the test.

Load-deflection and load-twist relations for the eccentrically loaded slender columns of Series (*F*) are shown in Fig. 6. The direction of twist of the column viewed from the top is given by the curved arrow adjacent to the cross-section. For the columns having the bearing axes parallel to principal planes, the actual and theoretical deflections are in good agreement for maximum fiber stresses within the yield point of the metal. The twist noted in the tests of the double-angle columns was probably largely due to initial winds prevalent in these columns. Column No. 59 had a wind of about 2° in its length, which fact undoubtedly influenced the twist exhibited by it.

Columns Nos. 56 and 57 were tested with knife-edges parallel to one of the legs, thus producing an obliquity between the plane of the loading and the principal planes of these members. The calculations for stresses and loads for these two columns given in Tables 11(*b*) and 11(*c*) are based on the principal radii of gyration, as are also the deflection lines in Fig. 6. In making these calculations, the load line was considered to pass through each knife-edge at the intersection of the gravity axis which was normal to the knife-edge, and because of the partial restraint against bending in the plane of the knife-edges, the free length was taken at 0.7*l*. The deflection of this

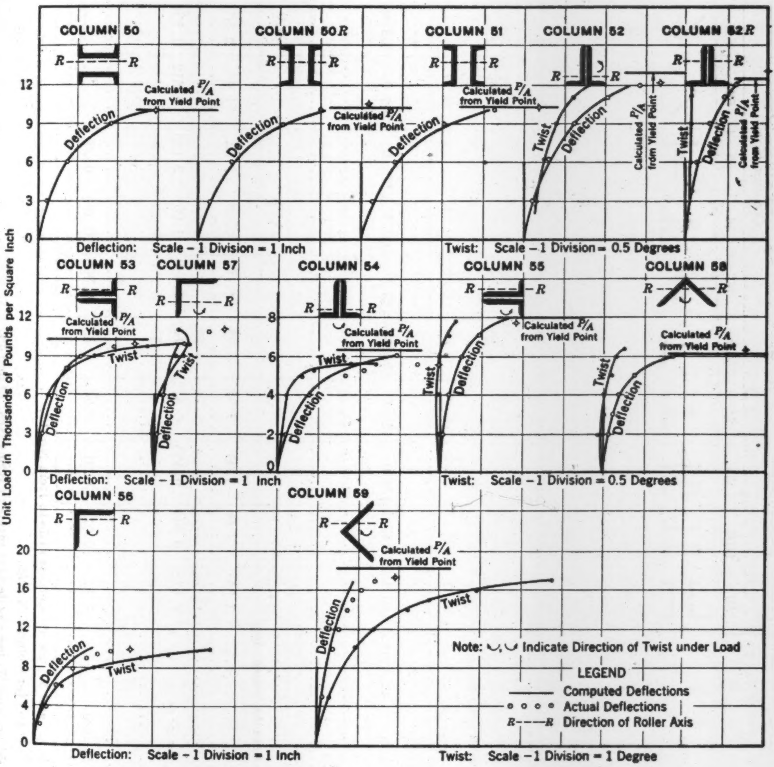
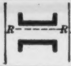
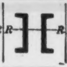
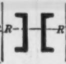
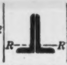
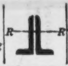
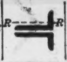
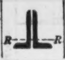
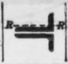
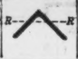
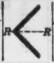
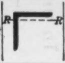
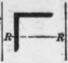
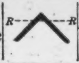
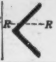


FIG. 6.—LOAD-DEFLECTION AND LOAD-TWIST CURVES FOR THE SLENDER COLUMNS OF SERIES (*F*), ECCENTRICALLY LOADED.

TABLE 11.—COMPARISON OF LOADS AND STRESSES, COLUMNS OF SERIES (*F*)

Item No.	Description	COLUMN NOS.													
		50	50*R*	51	52	52*R*	53	54	55	56*a*	56*b*	56*c*	57	58	59
															
	(*a*) DIMENSIONS AND PROPERTIES														
1	Make-up: Column Nos. 50, 50*R*, and 51	Two, 6-in., 8.2-lb. channels; 1½ by ¼-in. lacing bars.													
2	Column Nos. 52, 52*R*, and 53	Two, 6 by 4 by ⅜-in. angles; ⅜-in. fillers.													
3	Column Nos. 54 and 55...	Two, 3 by 2½ by ¼-in. angles; ¼-in. fillers.													
4	Columns Nos. 56 (*a*), 56 (*b*), 56 (*c*), 57, 58, and 59......	One, 4 by 4 by ⅜-in. angle.													
5	Length, *l*, in inches...............	349	349	349	244	244	244	190	193	161	161	161	161	161	161
	Area, in Square Inches:														
6	Nominal....................	4.78	4.78	4.78	7.22	7.22	7.22	2.62	2.62	2.86	2.86	2.86	2.86	2.86	2.86
7	Actual.......................	4.71	4.71	4.74	7.20	7.20	7.18	2.64	2.66	2.92	2.92	2.92	2.97	2.93	2.97
	Radius by Gyration, in Inches:														
8	r nominal..................	2.31	2.34	2.34	1.93	1.93	1.62	0.95	1.09	0.79	1.55	1.23	1.23	0.79	1.55
9	r actual.....................	2.32	2.37	2.36	1.90	1.90	1.59	0.94	1.10	0.79	1.57	1.24	1.25	0.79	1.57
10	r_3 nominal..................	......	......	......	......	......	......	......	......	......	......	0.79	0.79	......	......
11	r_3 actual.....................	......	......	......	......	......	......	......	......	......	......	0.79	0.80	......	......
	Slenderness Ratios:														
12	$\frac{l}{r}$...........................	150.4	147.3	147.9	128.5	128.5	153.5	202.1	175.5	204.0	102.5	130.0	129.0	204.0	102.5
13	$0.7 \frac{l}{r\ (\text{min})}$....................	......	......	......	......	......	......	......	......	......	......	*142.8	*140.7	......	......
14	$0.7 \frac{l}{r\ (\text{max})}$....................	......	......	......	......	......	......	......	......	......	......	§71.8	§71.8	......	......
15	$\frac{2\,l}{3r\ (\text{min})}$........................	......	......	......	......	......	......	......	......	......	136.0	......	......	......	......
	Distance to Most Remote Fiber, in Inches:														
16	c..............................	3.64	3.06	3.06	1.92	3.98	4.12	0.89	2.62	1.63	2.85	......	......	1.63	2.87
17	c_3.............................	......	......	......	......	......	......	......	......	......	......	1.63	1.20	......	......
18	c_4.............................	......	......	......	......	......	......	......	......	......	......	0	2.83	......	......
19	Ratio: $\frac{c}{r^2}$....................	0.678	0.544	0.550	0.532	1.102	1.630	1.007	2.165	2.612	1.156	......	......	2.612	1.163
20	$\frac{c_3}{r^2}$........................	......	......	......	......	......	......	......	......	......	......	2.61	1.88	......	......
21	$\frac{c_4}{r^2}$........................	......	......	......	......	......	......	......	......	......	......	0	1.15	......	......
22	Eccentricity, e, in Inches:.........	0.74	0.91	0.91	0.94	0.45	0.305	0.495	0.23	......	......	0.67	0.27	0.26†	0.43
23	$e_3 = e_4$..........................	......	......	......	......	......	......	......	......	......	......	0.47	0.19	......	......

* For Columns Nos. 56 (*c*) and 57, use slenderness ratio in Item No. 13.

† Eccentricity ratio, $\frac{ec}{r^2} = 0.68$ for Column No. 58.

§ For Columns Nos. 56 (*c*) and 57, use slenderness ratios in Items Nos. 13 and 14.

TABLE 11.—(*Continued*)

		Column Nos.													
		50	50*R*	51	52	52*R*	53	54	55	56*a*	56*b*	56*c*	57	58	59
	(*b*) Calculated Stresses Compared with Proportional Limit and Yield Point of Material, for Eccentric Load Tests (for Slenderness Ratios see Item No. 12*; for Eccentricities, see Item No. 22)														
24	Maximum applied load, $\frac{P}{A} = p$, in pounds per square inch	10 230	10 510	10 230	12 220	12 910	9 940	5 630	7 800	....	....	9 850	11 110	6 380	17 420
	Deflections, at Mid-Length, in Inches, at Maximum Loads:														
25	*D*	2.98	4.18	4.38	3.36	2.00	2.47	3.91	1.84	....	....	....	....	3.49	1.91
26	D_3	....	....	....	....	....	....	....	....	....	....	1.90	1.19	....	....
27	D_4	....	....	....	....	....	....	....	....	....	....	1.20	1.19	....	....
28	Unit fiber stresses by Equation (1), at maximum loads, in pounds per square inch	36 000	39 600	40 000	40 200	47 800	54 800	30 600	42 800	....	....	61 000	49 600	69 000	64 600
	Average Tensile Properties of Metal from Most Stressed Fibers, in Pounds per Square Inch:														
29	Proportional limit	25 700	24 200	24 200	31 100	27 200	29 200	31 700	35 700	....	....	39 800	39 400	40 300	39 400
30	Yield point,	37 000	37 400	37 400	40 400	37 400	46 400	41 200	40 400	....	....	49 300	49 600	49 000	49 600
	(*c*) Maximum Unit Loads Compared with Loads Calculated from Yield Point of Metal; Eccentric Load Tests (for Slenderness Ratios, see Item No. 12*; for Maximum Load, *p*, see Item No. 24; for Tensile Yield Point, S_y, of Most Stressed Fibers, see Item No. 30).														
31	Maximum total load, in thousands of pounds	48.2	49.5	48.5	88.0	93.0	71.4	14.9	20.8	....	....	28.7	33.0	18.7	51.8
32	Unit load, p_1, computed by Equation (2), from values of S_y, Item No 30	10.0	10.4	10.2	12.8	12.4	10.2	6.3	8.0	....	....	9.4‡	12.3‡	6.1	18.2
33	Tensile yield point, S'_y, of entire section (weighted)	37.3	37.3	37.3	39.5	39.5	39.5	40.8	40.8	....	....	49.3	49.3	49.3	49.3
34	Unit load, p'_1, computed by Equation (2), from values of S'_y, Item No. 32	10.0	10.3	10.2	12.6	12.7	9.8	6.3	8.0	....	....	9.4‡	12.3‡	6.1	18.1
35	Tensile yield point, S''_y, from representative sample	37.1	37.1	37.1	37.4	37.4	37.4	40.0	40.0	....	....	49.6	49.6	49.6	49.6
36	Unit load, p''_1, computed by Equation (2)from values of S''_y, Item No. 34	10.0	10.3	10.2	12.4	12.4	9.7	6.3	8.0	....	....	9.4‡	12.3‡	6.2	18.2
37	Tensile yield point, S'''_y from mill test data	33.8	33.8	33.8	36.1	36.1	36.1	35.7	35.7	....	....	35.7	35.7	35.7	35.7
38	Unit load, p'''_1 computed by Equation (2) from values of S'''_y, Item No 37	9.7	10.0	9.9	12.2	12.3	9.7	6.2	7.8	....	....	8.2‡	11.3‡	5.8	15.5
39	Ratio, $\frac{p}{p_1}$ Item No. 24 to Item No. 32	1.03	1.01	1.00	0.95	1.04	0.97	0.89	0.98	....	....	1.04	0.90	1.05	0.96
40	Ratio, $\frac{p}{p'_1}$ Item No. 24 to Item No. 34	1.02	1.02	1.00	0.97	1.02	1.01	0.89	0.98	....	....	1.04	0.90	1.05	0.96
41	Ratio, $\frac{p}{p''_1}$, Item No. 24 to Item No. 36.	1.02	1.02	1.00	0.98	1.04	1.02	0.89	0.98	....	....	1.04	0.90	1.03	0.96
42	Ratio, $\frac{p}{p'''_1}$, Item No. 24 to Item No. 38	1.05	1.05	1.03	1.00	1.05	1.02	0.90	1.00	....	....	1.20	0.98	1.10	1.12

* For Columns No. 56 (*c*) and 57, use slenderness ratio in Item No. 13.

‡ Computed by Equation (5).

TABLE 11.—(*Continued*)

		Column Nos.													
		50	50*R*	51	52	52*R*	53	54	55	56*a*	56*b*	56*c*	57	58	59
(*d*) Observed Euler Loads Compared with Computed Euler Loads when Columns Are Centrally Loaded (for Slenderness Ratios see Item No. 12*).															
43	Values of "Column E," in pounds per square inch x 10^6	30.5	....	30.3	29.1	....	20.0	30.3	29.3	30.1	30.1	30.1	30.1	30.1	....
	Euler Load, in Pounds per Square Inch:														
44	p, observed	13 750	....	14 520	18 580	....	12 160	7 500	10 150	6 910	17 540‖	12 890	14 130	6 660	....
45	p_1, computed from Item No. 43	13 300	....	13 700	17 400	....	12 200	7 330	9 390	7 140	16 050‖	14 570	15 010	7 140	....
46	p_2, computed from $E = 30 \times 10^6$	13 100	....	13 500	17 900	....	12 600	7 250	9 630	7 130	16 000‖	14 530	14 970	7 130	....
47	Ratio, $\frac{p}{p_1}$, Item No. 44 to Item No. 45	1.04	....	1.06	1.07	....	1.00	1.02	1.08	0.97	1.09	0.88	0.94	0.93	....
48	Ratio, $\frac{p}{p_2}$, Item No. 44 to Item No. 46	1.05	....	1.07	1.04	....	0.97	1.04	1.05	0.97	1.10	0.89	0.94	0.93	....

* For Columns Nos. 56 (*c*) and 57, use slenderness ratio in Item No. 13.

‖ For Column No. 56 (*b*) (column bent in plane of knife-edge).

free length was about 0.8 D. The maximum compressive fiber stresses for Columns Nos. 56 and 57 in Table 11(b) were then calculated from,

$$S = \frac{P}{A}\left(1 + (e_3 + 0.8\,D_3)\,\frac{c_3}{r^2{}_3} + (e_4 + 0.8\,D_4)\,\frac{c_4}{r^2{}_4}\right) \;\ldots\ldots(4)$$

using the values indicated in Tables 11(a) and 11(b). The unit loads in Table 11(c) were calculated by the formula,

$$p = \frac{S_y}{1 + \frac{e_3\,c_3}{r^2{}_3}\sec\sqrt{\frac{p}{E}}\left(\frac{0.70\,l}{2\,r_3}\right) + \left(\frac{e_4\,c_4}{r^2{}_4}\right)\sec\sqrt{\frac{p}{E}}\left(\frac{0.70\,l}{2\,r_4}\right)} \;\ldots\ldots(5)$$

The errors in the stresses, unit loads, and deflections, thus calculated, as the results indicate, are not large; hence the method affords a fair approximation for such calculations. Calculations of stresses and strengths based on the radius of gyration for the axis parallel to the knife-edge showed greater deviations from the observed values.

5(a).—Euler Loads for Columns of Series (F).—A comparison of the observed Euler loads for columns of Series (F) when centrally loaded with the calculated Euler loads appears in Table 11(d). The value of "Column" E (Item No. 43) was determined from 20-in. strain-gauge readings at mid-length except for Columns Nos. 56, 57, and 58, where the value of E from the corresponding tension specimens was used. Considering ratios for columns bending normal to the knife-edges, the ratios in Items Nos. 47 and 48 of Table 11(d) are between 0.93 and 1.08, and they average 1.02. The test of Column No. 56(b) with knife-edge in a plane of symmetry exemplified the behavior of a column free to bend as a pivoted end column in one principal plane and fixed in the other plane. This member was weakest in the plane of the knife-edges, and has been calculated as a pivoted end column of two-thirds the length. A fair estimate of the strengths of Columns No. 56(c) and 57 tested with the knife-edges parallel to one leg, has been made by considering the free length, 0.70, and using $r_{\text{min.}}$ in the Euler formula.

6.—The Range in Modulus of Elasticity in Columns of Series (D).—From readings on 20-in. strain-gauges taken at mid-length on thirty-six columns of Series (D), the average value of E was 29 400 000 lb per sq in., the same as the E for all series. Twenty-eight values (78%), were within ± 3% of the average; the maximum value was 30 900 000, and the minimum, 27 700 000, lb per sq in.

On thirty-one columns of Series (D) the modulus determinations from 8-in. strain-gauge readings at one or more positions on each column averaged 29 300 000 lb per sq in., with 77% within 3% of the mean value. On twelve of these columns the 8-in. gauges were read at the upper quarter, middle, and lower quarter points. The greatest variation in the values of E computed for each position for a single column was 9%, and the average variation was 4.3 per cent.

The foregoing data show that the variation in the value of E is small. Doubtless the real variation is much less than the data indicate, because of small inaccuracies in the strain-gauge measurements.

That the values of E for the opposite flanges of a given built-up column did not vary materially, is evidenced by the close agreement between eccentricity calulations and the position measurements for the center loading tests, the latter determined by trial (see Table 13). Further proof of the uni-

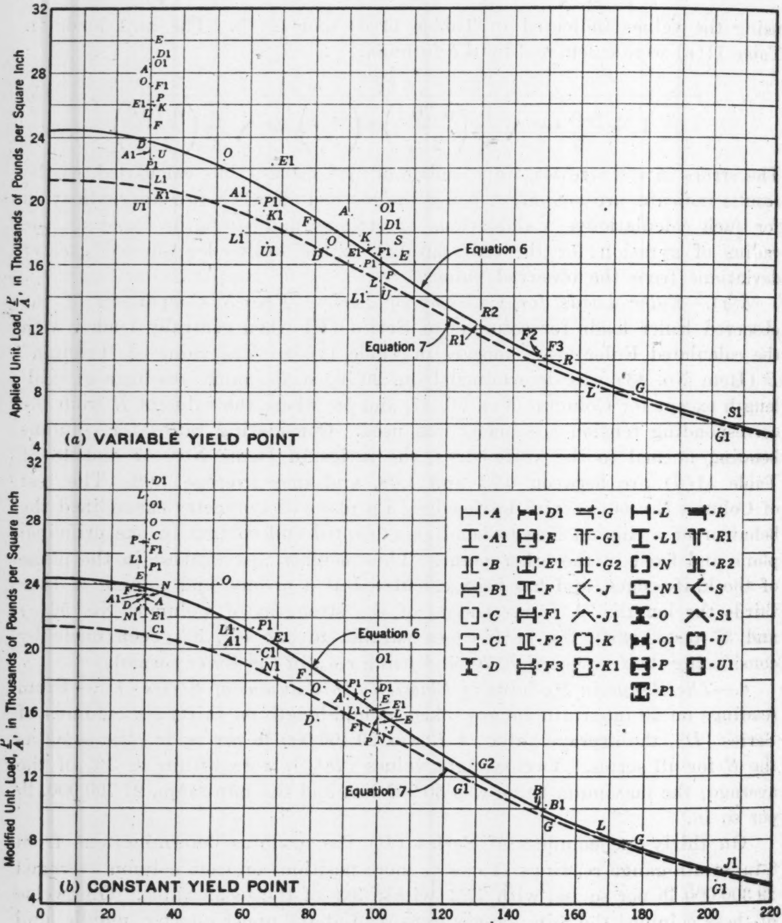


FIG. 7.—RELATIONSHIP OF MAXIMUM UNIT LOAD TO SLENDERNESS RATIO FOR COLUMNS TESTED UNDER ECCENTRIC LOADS $\left(\frac{ec}{Y^2}=0.5\right)$.

formity of E is given by the proximity of the calculated load-deflection curves to the observed data for the eccentric load tests of this and other series of tests.

7.—The Relationship of Strength and Slenderness Ratio.—The maximum unit loads for all columns tested with $\frac{ec}{r^2} = 0.5$ have been drawn in Fig. 7(*a*). The upper curve,

$$\frac{P}{A} = \frac{36\,600}{1 + 0.5 \sec \sqrt{\frac{P}{29\,400\,000\,A}}\left(\frac{l}{2\,r}\right)} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

represents the theoretical formula with the average yield point, 36 600 lb per sq in. from tensile specimens as the numerator, and average "Column" E in the denominator. The equation of the lower line,

$$\frac{P}{A} = \frac{32\,000}{1 + 0.5 \sec \sqrt{\frac{P}{29\,400\,000\,A}}\left(\frac{l}{2\,r}\right)} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

uses 32 000 for the yield point, a value, 1 000 lb per sq in. greater than the minimum weighted yield point of Table 8. It will be noted that the only results below this curve are three or four tests on channel columns and two tests on heavy **H**-sections the yield point of which was less than 32 000 lb per sq in.

In Fig. 7(*b*) the maximum unit loads for all columns have been modified to conform to a common yield point, 36 600 lb per sq in., and plotted against $\frac{l}{r}$. The modified maximum unit load, p', for each column was obtained from the assumed relation,

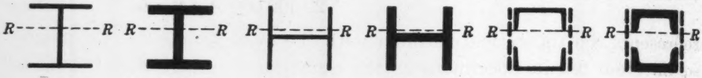
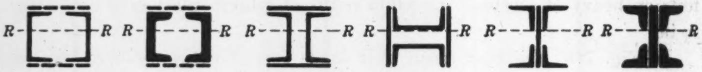
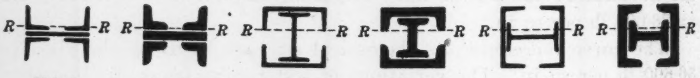
$$p' = \frac{p_2}{p_1}\, p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which, p is the maximum unit load; p_1, the unit load from the secant formula (Equation (3)) using the yield point of the most stressed fibers in the numerator; and p_2, the unit load from (Equation (3)) using yield point = 36 600 lb per sq in.

The curve represents the theoretical strength based on the yield point of 36 600 lb per sq in. The variations of test results from this line correspond to the values given in Table 8, Column (14). The most marked deviation from the curve occurs in the short column strengths at $\frac{l}{r} = 30$. Here, the strengths are, in most cases, appreciably above the curve based on yield-point stress. If the weighted average yield point had been used in calculating p_1 the test results would plot slightly higher, and variations from the theoretical curve would correspond to Column (14) of Table 8.

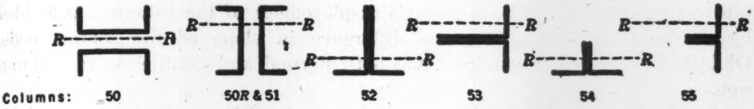
Had the more exact method of analysis been used the calculated ultimate loads for the shorter columns would have been greater, and would have corresponded more nearly to the test results, but in many cases the stresses in these short columns had greatly exceeded the yield point as shown in

TABLE 12.—ECCENTRICITIES AND CROOKEDNESS OF COLUMNS OF SERIES (*D*) AND (*F*). ECCENTRICITY IN ONE DIRECTION ONLY

Column No. (1)	Length, in inches (2)	Eccentricity of Center of Web Due to Rolling, in Inches		Crookedness, δ', at mid-section, in inches (5)	Center Loading Test Eccentricity as Calculated from Measurements, in Inches		
		Top (3)	Bottom (4)		Top (6)	Bottom (7)	Mid-section (8)
							
Columns:	82 & 83	10, 45*R* & 44	80 & 81	11, 45 & 44*R*	12 & 23	25 & 27	
10	111	−0.01	−0.01	0.00	+0.02	+0.02	+0.02
45*R*	211	+0.01	+0.01	−0.10	+0.11	+0.10	0.00
44	349	+0.01	0.00	−0.03	+0.04	+0.02	0.00
11	63.2	+0.11	+0.01	0.00	−0.05	0.00	−0.02
45	211	+0.03	+0.02	−0.04	+0.03	+0.06	0.00
44*R*	349	+0.05	−0.04	−0.04	+0.13	−0.01	+0.02
12	164	−0.01	+0.01	+0.02	0.00	+0.01	+0.02
23	349.2	0.00	−0.01	+0.10	+0.06	0.00	+0.13 +0.15*
25	159	+0.01	+0.01	−0.04	−0.03	−0.03	+0.01
27	349.3	−0.03	−0.01	+0.33	−0.24	−0.24	+0.09 +0.17*
							
Columns:	13 & 22	24 & 26	14 & 28	15 & 29	16 & 30	18, 33*R* & 32	
13	111.8	−0.01	+0.05	−0.04	0.00	0.00	−0.04
22	349.1	+0.02	+0.01	+0.08	−0.12	−0.09	−0.02
24	102	−0.01	0.00	−0.07	+0.07	+0.07	0.00
26	335	0.00	−0.01	−0.03	+0.06	+0.05	+0.02
14	133	−0.05	−0.01	+0.06	−0.05	−0.01	+0.03
28	349	−0.01	−0.01	+0.05	+0.05	0.00	+0.08 +0.07*
15	109	+0.02	+0.02	+0.02	−0.03	−0.06	−0.02
29	349	−0.02	−0.07	−0.09	+0.05	+0.03	−0.05
16	122	−0.01	−0.02	−0.02	0.00	+0.02	−0.01
30	346	+0.02	−0.03	+0.02	+0.04	0.00	+0.04
18	124	0.00	0.00	0.00	+0.01	−0.01	0.00
33*R*	214	−0.01	0.00	−0.03	+0.04	+0.08	+0.03
32	346	+0.01	+0.01	−0.10	+0.10	+0.10	0.00
							
Columns.	17 & 31	19 & 33	20, 35*R* & 34	37, 38*R* & 39	21 & 35	36 & 38	
17	62	0.00	+0.01	+0.01	+0.01	0.00	+0.02
31	206	0.00	0.00	0.00	+0.02	+0.02	+0.02
19	64	+0.01	−0.01	−0.02	−0.01	0.00	−0.02
33	214	0.00	0.00	−0.10	+0.10	+0.10	0.00
20	106	+0.02	0.00	−0.02	+0.04	+0.01	0.00
35*R*	239	0.00	−0.01	−0.11	+0.18	+0.20	+0.08 +0.08*
34	348	0.00	0.00	+0.28	−0.16	−0.18	+0.11 +0.10*
37	112	−0.02	+0.01	+0.04	−0.10	−0.09	−0.06
38*R*	232	+0.01	−0.01	+0.07	−0.09	−0.09	−0.02
39	346	−0.01	−0.01	+0.11	−0.15	−0.16	−0.04
21	72	+0.03	+0.02	0.00	+0.01	0.00	0.00
35	239	+0.03	+0.03	−0.01	+0.06	+0.08	+0.06 +0.01*
36	70	+0.01	+0.01	−0.02	−0.01	+0.01	−0.02
38	232	−0.02	−0.01	−0.02	+0.01	+0.01	−0.01

* Calculated from 20-in. strain-gauge measurements.

TABLE 12.—(*Continued*)



Column No. (1)	Length, in inches (2)	Eccentricity of Center of Web Due to Rolling, in Inches		Crookedness, e', at mid-section, in inches (5)	Center Loading Test Eccentricity as Calculated from Measurements, in Inches		
		Top (3)	Bottom (4)		Top (6)	Bottom (7)	Mid-section (8)
50	349	0.00	−0.01	+0.34	−0.38	−0.38	−0.04
50*R*	349	+0.02	0.00	+0.01	0.00	0.00	+0.01
51	349	0.00	0.00	+0.08	−0.10	−0.08	−0.01
52	244	0.00	0.00	−0.08	+0.12	+0.10	+0.03
53	244	−0.01	0.00	+0.05	+0.01	0.00	+0.06 +0.03*
54	190	0.00	0.00	−0.02	+0.02	+0.01	0.00
55	193	0.00	+0.01	+0.04	−0.06	−0.05	−0.02

* Calculated from 20-in. strain-gauge measurements.

Table 7, a point to be kept in mind in judging the significance of these high values. The differences would be small for the higher values of $\frac{l}{r}$.

8.—Effect of Time on Ultimate Strength of Columns.—In tests on steel columns under eccentric loads, there is more or less backing off of the load after the initial imposition of a high unit load due to inelastic action in some part of the column. This falling off of load is especially pronounced after imposition of the maximum load. To secure information concerning the proportion of maximum strength which can be considered as permanent, the strain was maintained on certain columns of Series (*D*) and (*F*) for varying intervals of time after the maximum load had been imposed. On four of these columns the strain was left on over night. At convenient intervals the scale-beam of the machine was balanced and the residual load determined.

The ratios in Column (8), of Table 12, show that 89%, or more, of the maximum load was held in these tests. Since the rate of drop off in load during the first minute or two is very rapid and the rate after 10 min is very small, and since these data were obtained after the maximum load had been imposed, it is probable that the strength of columns, such as those tested, under a steady load is at least 95% of the maximum testing-machine load.

9.—Crookedness and Obliquity Data.—The columns of Series (*D*), (*E*), and most of those of Series (*F*) were measured for crookedness and obliquity. All data refer to specimens as received, except the single-angle specimens which had initial bows of ¼ to ½ in. These were straightened prior to testing. The principal data are presented in Tables 13 and 14 and in Figs. 8 and 9.

Measurements were made on each column as it rested on the roller bearing in the testing machine just before testing. Crookedness measurements were made from vertical wires supported at the column ends a fixed distance from each corner. Readings to 0.01 in. were taken at quarter-points and middle on corners, mid-flange, and mid-web.

Data on rolling eccentricities were determined from measurements of the end cross-sections from which the locations of the gravity axes were computed.

Obliquity measurements were secured by leveling the base bearing block carefully with a transit bubble; then by rotating the lower bearing, the horizontal distance the top of the column had to be moved to level the column top was ascertained. The horizontal displacement of the column top divided by the column length gave the difference in slope of the column ends. Obliquity measurements were made both normal and parallel to the column web.

Comparison of the data on crookedness, e', with the rolling eccentricity data (see Columns (3), (4), and (5) of Table 13), shows that crookedness is much the more important. In the last three columns of Table 12 the eccentricities at the top, middle, and bottom sections have been calculated from position measurements and include allowance for rolling eccentricity and

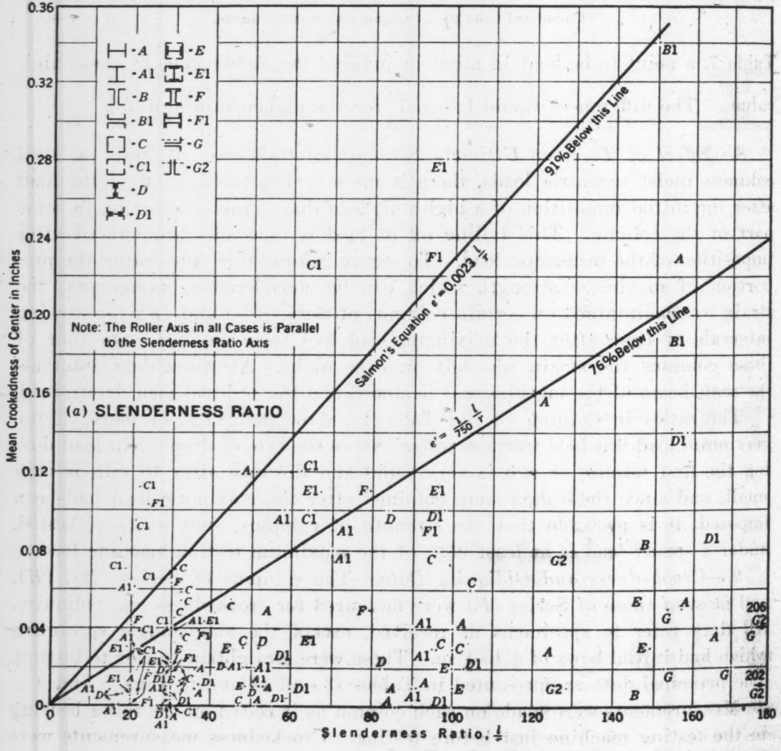
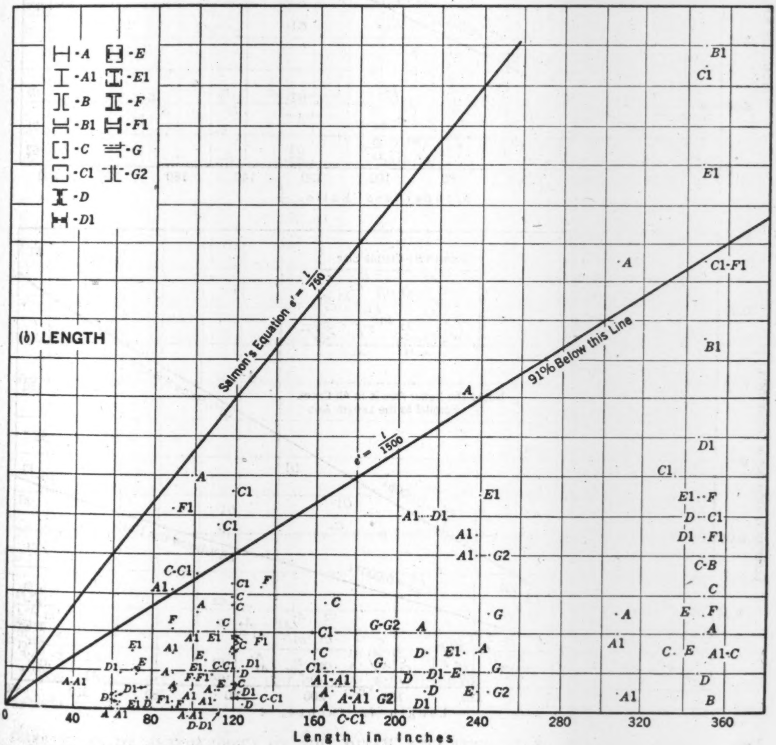


FIG. 8.—RELATIONSHIP OF CROOKEDNESS

crookedness. If both position measurements and the deflection and strain measurements, made in centering loadings, had been perfect, the results in Column (8) of Table 13 should approximate zero. For those specimens having the larger measured eccentricities at mid-section (Columns Nos. 23, 27, 28, 35*R*, 34, and 35), the eccentricities at mid-section were also calculated from the 20-in. strain-gauge readings taken during the centering loading and are placed immediately below the measured values in the table. These eccentricities from strain measurements check reasonably well the measured values. Considering Column No. 34, for example, the initial crookedness was 0.28 in. west. In the centering run the top was offset east 0.16 in. and the bottom 0.18 in.; hence, the middle was still 0.11 in. too far west, and the strain measurements checked this calculation to 0.01 in. The data in Columns (5), (6), (7), and (8) of Table 13 plainly show the important effect of crookedness on column eccentricity.

In Figs. 8 and 9 the crookedness, *e′*, has been plotted in various ways. Plotted against slenderness ratio (Fig. 8(*a*)), it is found that 91% of results



TO SLENDERNESS RATIO AND LENGTH

lie below the curve for Salmon's equation,

$$e' = 0.0023\,\frac{l}{r} = \frac{l}{435\,r} \dotfill (9)$$

and 76% lie below the curve,

$$e' = \frac{l}{750\,r} \dotfill (10)$$

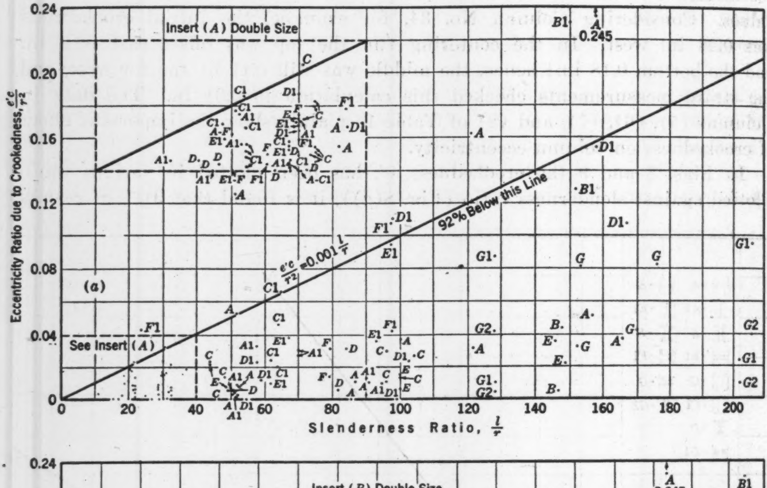
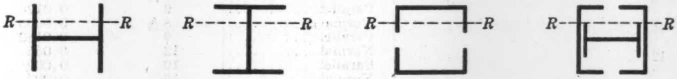
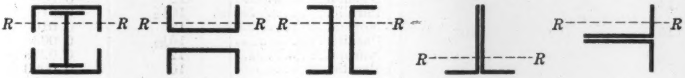


FIG. 9.—RELATIONSHIP OF ECCENTRICITY RATIO DUE TO CROOKEDNESS TO SLENDERNESS RATIO AND LENGTH.

TABLE 13.—EFFECT OF TIME ON THE ULTIMATE STRENGTH OF CERTAIN COLUMNS OF SERIES (*D*) AND (*F*)

Column No. (1)	Length in inches (2)	$\frac{l}{r}$ (3)	DURATION OF LOADING AFTER MAXIMUM: Hours (4)	Minutes (5	UNIT LOAD, IN POUNDS PER SQUARE INCH AT: Maximum, p (6)	End of period, p_c (7)	$\frac{p_c}{p}$ (8)
Columns:	 *H*6, 80 & 81; 45*R*; 25; 35*R*, 36 & 38						
H-6	39.6	20.0	0	17	28 850	27 000	0.94
80	58.5	29.5	0	10	28 450	26 200	0.92
81	177.8	90.8	1	10	19 100	17 000	0.89
45*R*	211.0	57.6	0	10	18 200	16 600	0.91
25	159.2	28.8	0	11	19 950	18 800	0.94
35*R*	239.0	100.4	15	15	19 280	17 180	0.89
36	70.0	30.3	0	10	26 200	24 500	0.94
38	232.0	100.0	0	16	15 450	14 000	0.91
Columns	 37; 50; 50*R* & 51; 52; 53						
37	112.0	30.0	1	6	22 600	20 500	0.91
50	349.0	150.4	14	45	10 200	9 500	0.93
50*R*	349.0	147.3	1	0	10 500	9 800	0.93
51	349.0	147.9	0	13	10 730	9 700	0.90
52	244.0	128.5	14	52	12 220	11 190	0.92
53	244.0	153.5	14	44	9 940	9 200	0.93

In Fig. 8(*b*), showing crookedness plotted against column length, all points lie below the line for Salmon's equation, $e' = \frac{l}{750}$, and 91% are below the line corresponding to $e' = \frac{l}{1\,500}$.

In Fig. 9(*a*) the eccentricity ratio due to crookedness, $\frac{e'\,c}{r^2}$, has been plotted against the slenderness ratio and 92% of the points lie below the line, $\frac{e'\,c}{r^2} = 0.001\,\frac{l}{r}$. In Fig. 9(*b*), the eccentricity ratio due to crookedness has been plotted against l; 97% of the points lie below the line, $\frac{e'\,c}{r^2} = 0.0003\,l$.

As is to be expected, none of the relations shown in Figs. 8 and 9 is very satisfactory, and in all cases most of the values are much below the few large ones. For purposes of theoretical application, as shown later, the relation shown in Fig. 9(*a*) is the most convenient and appears to be as con-

TABLE 14.—OBLIQUITIES IN TEST COLUMNS

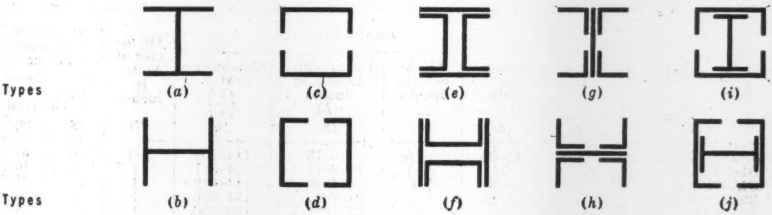
Column No. (1)	Length, in inches (2)	Plane of obliquity referred to web (3)	Nominal width of column in plane of obliquity, in inches (4)	OBLIQUITY: In the width of the column, in inches (5)	OBLIQUITY: Slope, in inches per inch (6)
10	111	Normal	$8\frac{5}{16}$	0.004	0.0005
		Parallel	9	0.026	0.0029
11	63	Normal	$8\frac{5}{16}$	0.000	0.0000
		Parallel	9	0.020	0.0022
44	349	Normal	$8\frac{5}{16}$	0.0125	0.0015
		Parallel	9	0.0126	0.0014
45	211	Normal	$8\frac{5}{16}$	0.0000	0.0000
		Parallel	9	0.0180	0.0020
12	164	Normal	12	0.055	0.0046
		Parallel	10	0.009	0.0009
13	112	Normal	12	0.003	0.0003
		Parallel	10	0.025	0.0025
22	349	Normal	12	0.0516	0.0043
		Parallel	10	0.0070	0.0007
23	349	Normal	12	0.0360	0.0030
		Parallel	10	0.0150	0.0015
24	102	Normal	12	0.047	0.0039
		Parallel	10	0.013	0.0013
25	159	Normal	12	0.014	0.0012
		Parallel	10	0.0010	0.0001
26	335	Normal	12	0.0252	0.0021
		Parallel	10	0.0040	0.0004
27	349	Normal	12	0.0276	0.0023
		Parallel	10	0.0050	0.0005
14	133	Normal	12	0.006	0.0005
		Parallel	10½	0.014	0.0013
15	109	Normal	12	0.010	0.0008
		Parallel	10½	0.003	0.0003
28	349	Normal	12	0.0228	0.0019
		Parallel	10	0.0090	0.0009
29	349	Normal	12	0.0228	0.0019
		Parallel	10	0.0410	0.0041
16	122	Normal	$10\frac{5}{16}$	0.005	0.0004
		Parallel	10½	0.009	0.0009
17	62	Normal	$10\frac{5}{16}$	0.003	0.0003
		Parallel	10½	0.000	0.0000
18	124	Normal	10½	0.019	0.0018
		Parallel	10½	0.005	0.0004
19	64	Normal	10½	0.026	0.0025
		Parallel	10½	0.007	0.0007
30	346	Normal	$10\frac{5}{16}$	0.0206	0.0020
		Parallel	10½	0.0220	0.0021
31	206	Normal	$10\frac{5}{16}$	0.0103	0.0010
		Parallel	10½	0.0220	0.0021
32	346	Normal	10½	0.0010	0.0001
		Parallel	10½	0.0095	0.0009
33	214	Normal	10½	0.0000	0.0000
		Parallel	10½	0.0179	0.0017
20	106	Normal	8	0.011	0.0014
		Parallel	8½	0.009	0.0011
21	72	Normal	8	0.002	0.0002
		Parallel	8½	0.007	0.0008
34	348	Normal	8	0.0032	0.0004
		Parallel	8½	0.0034	0.0004
35	239	Normal	8	0.0056	0.0007
		Parallel	8½	0.0272	0.0032
36	70	Normal	8	0.010	0.0012
		Parallel	9	0.035	0.0039
37	112	Normal	8	0.019	0.0024
		Parallel	9	0.058	0.0064
38	232	Normal	8	0.030	0.0038
		Parallel	9	0.009	0.0010
39	346	Normal	8	0.010	0.0013
		Parallel	9	0.021	0.0023
40	213	Normal	10¼	0.013	0.0013
		Parallel	10½	0.007	0.0007
50	349	Normal	7¼	0.046	0.0064
		Parallel	6	0.032	0.0054
51	349	Normal	7¼	0.023	0.0032
		Parallel	6	0.056	0.0094
52	244	Normal	8¼	0.053	0.0064
		Parallel	6	0.023	0.0039
53	244	Normal	8¼	0.027	0.0033
		Parallel	6	0.015	0.0025
54	190	Normal	5¼	0.004	0.0007
		Parallel	3	0.007	0.0024
55	193	Normal	5¼	0.051	0.0098
		Parallel	3	0.026	0.0085

sistent as those shown on the other diagrams. For $r = 0.8c$, the relation, $\frac{e' c}{r^2} = 0.001 \frac{l}{r}$, is equivalent to $e' = \frac{l}{1\,250}$.

The values of e' reported herein are, in general, somewhat greater than those given in the second progress report.

Table 15 shows the eccentricity ratios due to crookedness for the different types of cross-section. The box-sections and the plate and angle types exhibited the largest values, and rolled **H**-sections the smallest.

TABLE 15.—A COMPARISON OF AVERAGE ECCENTRICITY RATIOS DUE TO CROOKEDNESS OF COLUMNS OF VARIOUS TYPES OF CROSS-SECTION AND SLENDERNESS RATIO



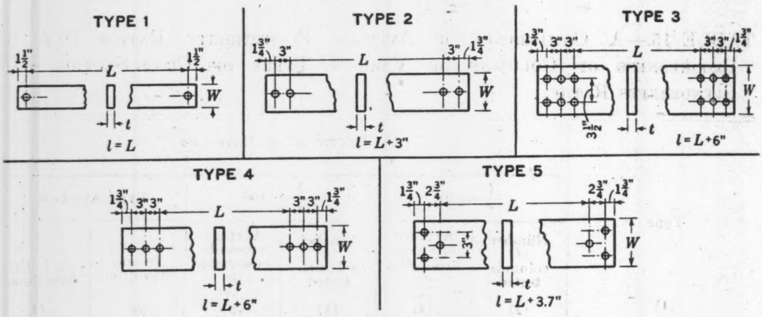
Type	ECCENTRICITY RATIO, $\frac{e'c}{r^2}$					
	$\frac{l}{r} = 30$		$\frac{l}{r} = 100$		TYPE AVERAGE	
	Number of columns tested	Average crookedness ratio, $\frac{e'c}{r^2}$	Number of columns tested	Average crookedness ratio, $\frac{e'c}{r^2}$	In one direction	In both directions
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(a)..........	5	0.010	1	0.010	0.010	0.017
(b)..........	3	0.021	3	0.029	0.025	
(c)..........	6	0.008	4	0.039	0.023	0.021
(d)..........	6	0.015	4	0.021	0.018	
(e)..........	2	0.010	2	0.023	0.016	0.030
(f)..........	2	0.014	2	0.075	0.044	
(g)..........	8	0.003	2	0.019	0.011	0.029
(h)..........	5	0.014	5	0.080	0.047	
(i)..........	4	0.008	4	0.045	0.026	0.021
(j)..........	4	0.015	4	0.020	0.017	
From equation, $\frac{e'c}{r^2} = 0.001\frac{l}{r}$	....	0.030	....	0.100	....	....

The obliquity measurements, Table 14, seem to bear no definite relation to length of column. They simply indicate the range in values which is likely to obtain in mill practice under the specifications. The average obliquity for these measurements was 0.0021 with 9% of the observations greater than 0.005.

Tests on Lacing-Bars

10.—Scope.—The tests herein reported consist of individual compression tests of 154 flat bars of various sizes, twenty-four tests in which the strength of the bars was determined by means of an obliquely loaded short column consisting of two channels connected by four single lacing-bars varying in size from $1\frac{1}{2}$ in. by $\frac{3}{16}$ in. to $2\frac{1}{2}$ in. by $\frac{5}{16}$ in., and nineteen obliquely loaded short columns connected by four double lacing-bars. The double lacing-bars were $2\frac{1}{4}$ in. wide; they varied from $\frac{3}{16}$ to $\frac{3}{8}$ in. in thickness and were either 11.3 or 15.56 in. long. They were of the same, or similar, stock as the bars tested singly.

TABLE 16.—Scope of Individual Tests of Flat Lacing-Bars



Holes $\frac{1}{64}$ in Larger than Diameter of Bolt

Group No.	Type No.	Range in: Width, in inches (w)	Range in: Thickness, in inches (t)	Range in: Unsupported length, in inches (L)	Range in: Slenderness ratio $\left(\frac{l}{r}\right)$	Diameter of bolts, in inches	Number of Specimens Tested: Per variable	Number of Specimens Tested: Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
5AD-6CE........	1	$1\frac{1}{2}$–$2\frac{1}{2}$	$\frac{3}{16}$–$\frac{5}{16}$	10.73	114–204	$\frac{5}{8}$–$\frac{7}{8}$	5	59
*B*1-*B*5..........	1	$2\frac{1}{4}$	$\frac{3}{16}$–$\frac{3}{8}$	11.3–15.5	104–317	$\frac{7}{8}$	3	16
*B*11-*B*19.........	2	3–6	$\frac{1}{2}$–$\frac{3}{4}$	14.4–43.4	114–219	1	3	31
*B*6-*B*10, *B*20......	3	8–$8\frac{1}{2}$	$\frac{3}{4}$–1	21.7–57.8	121–226	1	3	18
*B*21-*B*23.........	4	4	$\frac{3}{4}$	15.7–37.3	100–200	1	2	6
*B*24-*B*25.........	1–2	$2\frac{1}{4}$	$\frac{3}{8}$	16.2	150–178	1	2	4
*B*26-*B*27.........	5	6	$\frac{5}{8}$	14.4–21.7	125–176	1	2	4
*B*28-*B*30*........	2	3	$\frac{1}{2}$	14.4–28.8	119–219	1	2	6
*B*31-*B*40†........	3	8–$8\frac{1}{2}$	$\frac{3}{4}$–1	21.7–57.8	121–226	1	2	10
Total........	....	....	....	....	....	......	.....	154

* Bars bent 0.04 in. at mid-length prior to testing. † Bars tested in pairs (see Fig. 13).

In addition to the flat bars, tests were made on channel bars, fifty-three tested singly and four in pairs, and on angle bars, thirty tested singly and twelve in pairs.

The general scope of the individual tests on flat bars is given in Table 16, of the angle and channel bars in Table 17, of the column tests with single lacing-bars in Table 18, and of the column tests with double lacing-bars in

Table 19. In tests of single bars the distance between the centers of gravity of the end rivet groups has been used in calculating the slenderness ratio; the least radius of gyration has been used in all cases.

11.—Method of Testing.—Bars of Type 1 (single-bolt connections), except Groups *B*24 and *B*25, were tested as shown in Fig. 10, in a 200 000-lb Riehle 3-screw machine. This is the same arrangement as used in the few tests reported in 1929.[9] The remainder of the bars were tested in a 600 000-lb hydraulic machine. The ends of these bars were bolted to the flanges of a short piece of 116-lb, CB 105, **H**-column, as shown in Figs. 11 and 12. Bars of Groups *B*11, *B*12. *B*13, and part of Group *B*15, were tested with a 5-in. by 3-in. angle bolted to the **H**-column; for Groups *B*14, *B*16, *B*17, *B*19, and part of Groups *B*15 and *B*8, the angle was brazed, and for the remainder of the flat-bar tests it was welded as indicated in Fig. 11. The differences in the rigidities of these connections appeared to have little effect on the ultimate strength of the bars. Bars *B*31 to *B*40 were tested in pairs, using end attachments similar to those shown in Fig. 13. These ends were considerably more rigid than those used in the individual flat-bar tests.

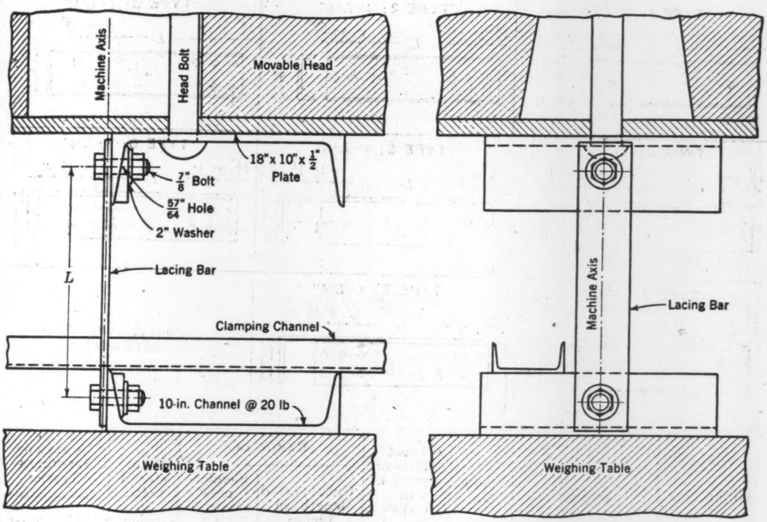

Machine Axis
Head Bolt
Movable Head
18" x 10" x 1/2" Plate
7/8" Bolt
57/64" Hole
2" Washer
L
Lacing Bar
Clamping Channel
10-in. Channel @ 20 lb
Weighing Table
Machine Axis
Lacing Bar
Weighing Table

FIG. 10.—METHOD OF TESTING SMALL LACING-BARS.

All angle and channel-bar tests were made with the flanges of the **H**-column held together by $\frac{3}{8}$-in. batten-plates, as shown in Fig. 13.

The properties of the bolt material used in fastening the bars are given in Table 20. All bolts and nuts were well oiled prior to fastening the bars. The nuts on the bolts fastening the flat bars of Type 1 and the angle bars with ⅝-in. bolts were tightened under a torque of 550 in-lb. The nuts on the 1¼-in. bolts were tightened with a torque of 3 000 in-lb. All other nuts received a torque of 1 500 in-lb. Following each test, attention was given to the condition of both bolts and holes, and whenever either became badly deformed, a new bolt was used, or a new hole was drilled, for the next test.

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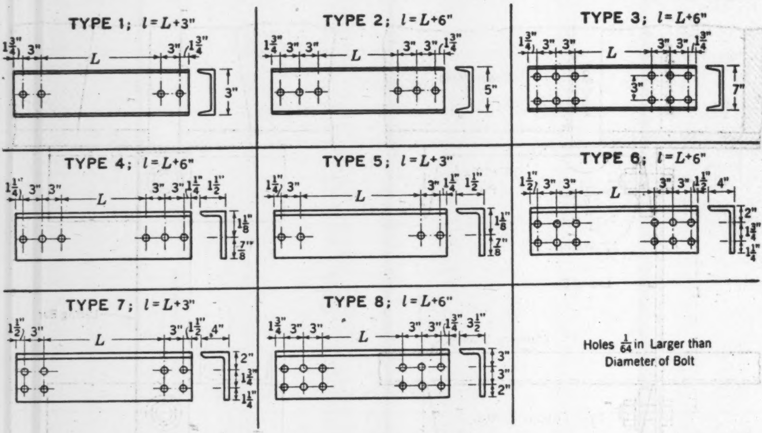
[9] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1179, Fig. 19.

TABLE 17.—Scope of Angle and Channel Lacing-Bar Tests


TYPE 1; l=L+3"
1¾" 3" L 3" 1¾"
3"
TYPE 2; l=L+6"
1¾" 3" 3" L 3" 3" 1¾"
5"
TYPE 3; l=L+6"
1¾" 3" 3" L 3" 3" 1¾"
3"
7"
TYPE 4; l=L+6"
1¼" 3" 3" L 3" 3" 1¼ 1½"
1⅛"
⅞"
TYPE 5; l=L+3"
1¼" 3" L 3" 1¼" 1½"
1⅛"
⅞"
TYPE 6; l=L+6"
1½" 3" 3" L 3" 3" 1½" 4"
2"
1¾"
1¼"
TYPE 7; l=L+3"
1½" 3" L 3" 1½" 4"
2"
1¾"
1¼"
TYPE 8; l=L+6"
1¾" 3" 3" L 3" 3" 1¾" 3½"
3"
3"
2"
Holes 1/64 in Larger than Diameter of Bolt

Group No.	Section	Type No.	Least radius of gyration in inches ($r_{\text{min.}}$)	Range in		Diameter of bolts, in inches	Number of Specimens Tested	
				Unsupported length, in inches (L)	Slenderness ratio $\left(\frac{l}{r}\right)$		Per variable	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
*C*1-*C*3, *C*1 *T*..	3-in., 4.1-lb. channel..........	1	0.41	20.5–82.0	57–206	1	5 & 2	17
*C*7-*C*9......	5-in., 11.5-lb. channel......	2	0.48	24.5–98.0	64–212	1	2–5	15
*C*9.........		2	0.48	49.0	116	1¼	2	2
*C*4-*C*6......	7-in., 9.8-lb. channel..........	3	0.58	29.2–118.0	61–216	1	3–6	23
*A*4-*A*7......	2 by 1½ by ¼-in. angle........	4	0.33	15.5–80.0	65–260	⅝	2–3	11
*A*4-*A*7......		5	0.33	21.5–86.0	74–269	⅝	2	8
*A*8-*A*9......	5 by 4 by $\frac{7}{16}$-in. angle........	6	0.85	53.5–113.0	70–140	⅝	4 & 5	9
*A*8-*A*9......		7	0.85	59.5–119.0	74–144	⅞	3 & 2	5
*A*1-*A*2......	8 by 3½ by ¾-in. angle........	8	0.77	88.0–123.0	122–168	1	4	8
*A*1.........		8	Individual bar test			......	1	1
Total....	....	.....	....	....	....	......	.....	99

Both strain and deflection measurements were made in the tests of the larger flat bars and in the tests of the angle and channel bars. These measurements were of considerable value during the tests in centering the load on the bars.

The alignment of nearly all bars was measured before and after bolting to the end attachments. The bow after fastening is recorded in Tables 18, 21, 22, and 23. In Column (2), Table 22, the designation, *I*½ *O*½, signifies that the inner bars were in compression over one-half the length of the column and that the outer bars were in compression over the remaining half-length of the column. The designation, $I + O$, signifies that the inner bars were in compression on one side of the column and the outer bars were in compression on the opposite side of the column.

The twenty-four column tests on flat bars, single lacing, were made as indicated in Fig. 14. In half the columns the bars carrying compression were placed inside the tension bars, and in the other half the compression bars were placed outside the tension bars. Analysis of the stress distribution shows that the average stress carried by each compression bar inclined as shown in Fig. 14 was 0.589 times the load, and this ratio was used in calculating the unit stresses in the bars. The stress on each tension bar was 0.455 times the load.

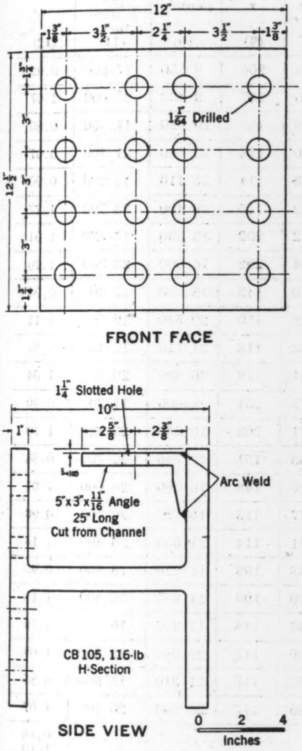

12"
1⅜"
3½"
2¼"
3½"
1⅜"
12½"
1¾"
3"
3"
3"
1¾"
1 1/64" Drilled
FRONT FACE
1¼" Slotted Hole
10"
1"
2⅝"
2⅜"
5"x3"x11/16" Angle
25" Long
Cut from Channel
Arc Weld
CB 105, 116-lb
H-Section
SIDE VIEW
0
2
4
Inches

FIG. 11.—DETAIL OF END ATTACHMENT FOR LACING-BARS

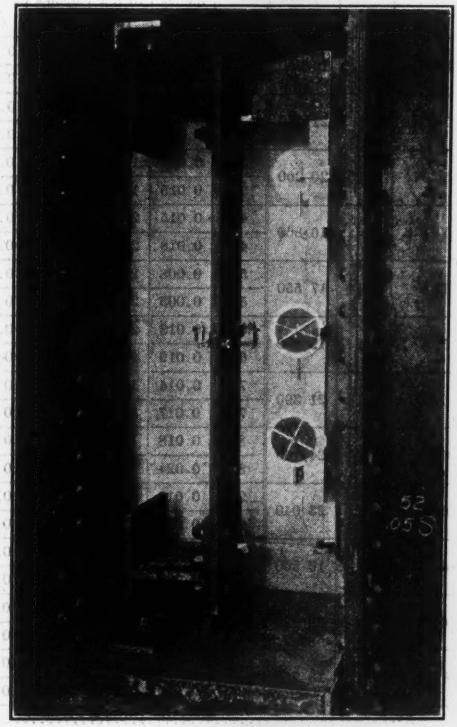


FIG. 12.—END ATTACHMENTS AND STRAIN-MEASURING DEVICES USED IN TESTING LARGE LACING-BARS, SERIES (H).

TABLE 18.—COMPARISON OF SINGLE BAR TESTS WITH COLUMN TESTS ON LACING-BARS

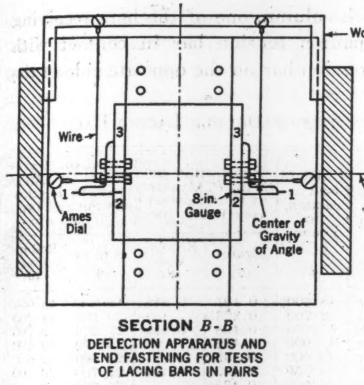
SINGLE BAR TESTS			SINGLE LACED "COLUMN" TESTS *								
					DIMENSIONS OF BAR THAT FAILED						
Group No.	Size of bolt, in inches	Average, $\frac{P}{A}$, of five tests, in pounds per square inch	Column No.†	Bow‡ over an 8-in. gauge length, in inches	Width, in inches	Thickness, in inches	Area of bar, A, in square inches	Slenderness ratio, $\frac{l}{r}$	Total load on column P, in pounds	Stress in compression bars, $0.589 \frac{P}{A}$, in pounds per square inch	Ratio, stress in column bar to stress in single bar
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
5AD	⅝	19 260	1 C	0.001	1.499	0.181	0.271	206	8 180	17 800	0.92
			1 T	0.011	1.498	0.180	0.270	207	9 430	20 600	1.07
5BD	⅝	21 760	2 C	0.006	1.502	0.245	0.368	152	10 930	17 500	0.80
			2 T	0.023	1.512	0.251	0.380	148	13 680	21 200	0.97
5CD	⅝	20 300	3 C	0.009	1.506	0.325	0.489	114	15 110	18 200	0.90
			3 T	0.015	1.497	0.322	0.482	115	19 400	23 700	1.17
5AE	⅝	16 860	4 C	0.015	2.510	0.184	0.462	202	13 390	17 100	1.01
			4 T	0.018	2.520	0.184	0.464	202	16 000	20 300	1.20
5BE	⅝	17 550	5 C	0.008	2.480	0.250	0.620	149	18 080	17 200	0.98
			5 T	0.005	2.490	0.248	0.618	150	20 540	19 600	1.11
5CE	⅞	19 600	6 C	0.013	2.490	0.314	0.783	118	21 710	16 400	0.84
			6 T	0.019	2.480	0.315	0.781	118	26 890	20 300	1.04
6AD	¾	21 390	7 C	0.014	1.500	0.182	0.273	204	8 000	17 200	0.80
			7 T	0.017	1.490	0.183	0.271	203	10 830	23 600	1.10
6BD	¾	24 160	8 C	0.018	1.470	0.247	0.363	151	12 745	20 700	0.86
			8 T	0.024	1.490	0.252	0.376	148	16 590	26 000	1.07
6CD	¾	23 040	9 C	0.015	1.480	0.322	0.477	115	16 725	20 700	0.90
			9 T	0.019	1.480	0.325	0.481	114	21 555	26 400	1.15
6AE	¾	16 300	10 C	0.021	2.500	0.187	0.468	199	12 220	15 400	0.95
			10 T	0.029	2.500	0.188	0.470	198	14 950	18 800	1.15
6BE	¾	20 820	11 C	0.011	2.480	0.252	0.625	148	17 350	16 400	0.79
			11 T	0.016	2.490	0.253	0.630	147	23 980	22 400	1.08
6CE	¾	18 750	12 C	0.017	2.490	0.318	0.792	117	21 310	15 900	0.85
			12 T	0.019	2.480	0.318	0.789	117	26 850	20 100	1.07
Average *C* ratio											0.88
Average *T* ratio											1.10
Maximum *C* ratio											1.01
Maximum *T* ratio											1.20

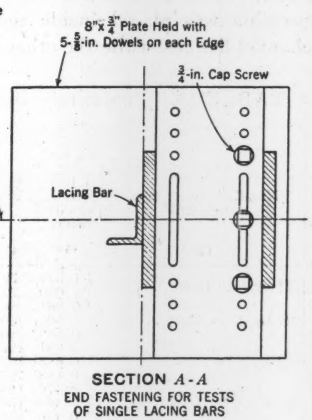
* See Fig. 14.

† *T* indicates that inner bars were in compression. *C* indicates that outer bars were in compression.

‡ All the bars were bowed away from the "column".

The set-up for the obliquely loaded columns with double lacing is shown in Fig. 15. The size of channels was 15-in., 33.9-lb. Each of the channels was stiffened longitudinally by 4 by 3½-in. by $\frac{3}{16}$-in. angles riveted to the back of the web, and the flanges of the channels were stiffened laterally by 1-in.


Wood Frame
Wire
3
3
1
2
8-in. Gauge
2
1
Ames Dial
Center of Gravity of Angle
Center Line
SECTION B-B
DEFLECTION APPARATUS AND END FASTENING FOR TESTS OF LACING BARS IN PAIRS


8"x¾" Plate Held with 5-⅝-in. Dowels on each Edge
¾-in. Cap Screw
Lacing Bar
SECTION A-A
END FASTENING FOR TESTS OF SINGLE LACING BARS

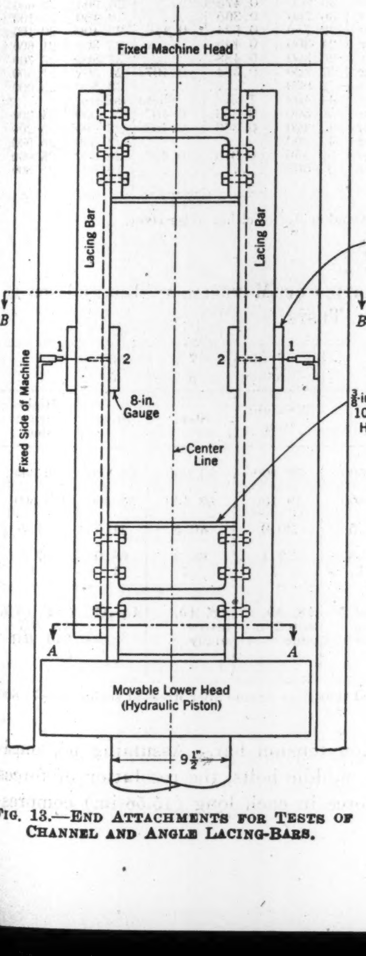

Fixed Machine Head
Lacing Bar
Lacing Bar
Position of Gauges for Strain Readings (8-in. Gauge)
B
B
1
2
2
1
Fixed Side of Machine
8-in. Gauge
⅜-in. Plate attached to 10-in.,116-lb. CB 105 H-Beam with 5-⅝-in. Dowels on each Edge
Center Line
A
A
Movable Lower Head (Hydraulic Piston)
9½"

FIG. 13.—END ATTACHMENTS FOR TESTS OF CHANNEL AND ANGLE LACING-BARS.

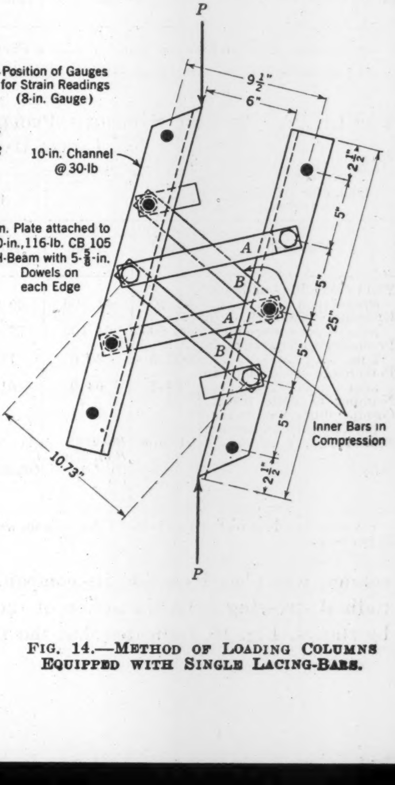

P
9½"
6"
10-in. Channel @ 30-lb
2½"
5"
A
B
5"
25"
A
B
5"
5"
Inner Bars in Compression
2½"
10.73"
P

FIG. 14.—METHOD OF LOADING COLUMNS EQUIPPED WITH SINGLE LACING-BARS.

long bolts shown at *B* in Fig. 15. In each column, one of the bars receiving compression was placed inside the companion tension bar in contact with the channel flanges, while the other compression bar on the opposite side of the column was placed outside its companion tension bar. Assuming no longitudinal stressing from the action of the middle bolts, the resolution of forces by statics, Fig. 16, indicates that the force in each long (15.56-in.) compres-

TABLE 19.—PROPERTIES AND STRENGTHS OF DOUBLE LACING-BARS

Column No. (1)	Nominal size of bars, in inches (2)	Area of failed bar, in square inches (3)	$\frac{l^*}{2r}$ (4)	Position of failed bar (5)	Ultimate load on column, in pounds (P) (6)	PROPORTION OF LOAD, P, CARRIED BY COMPRESSION BAR BASED ON:		AVERAGE UNIT STRESS IN FAILED BAR, IN POUNDS PER SQUARE INCH, BASED ON:	
						Strains (7)	Resolution (8)	Strains (9)	Resolution (10)
1	$2\frac{3}{4}$ by $\frac{3}{8}$ by 15.56	1.029	72	Outer	48 600	0.401	0.416	18 900	19 600
11		1.030	72	Outer	52 700	0.393	......	20 100	21 300
5	$2\frac{3}{4}$ by $\frac{5}{16}$ by 15.56	0.861	86	Inner	47 050	0.498	0.416	27 200	22 700
6		0.865	86	Outer	45 900	0.489	......	25 900	22 100
7		0.860	86	Outer	43 000	0.432	......	21 600	20 800
4	$2\frac{3}{4}$ by $\frac{1}{4}$ by 15.56........	0.691	107	Inner	43 350	0.442	0.416	27 700	26 100
8		0.688	107	Outer	39 000	0.475	......	26 900	23 600
9		0.693	107	Outer	36 750	0.365	......	19 400	22 100
2	$2\frac{3}{4}$ by $\frac{3}{16}$ by 15.56	0.470	158	Outer	22 750	0.524	0.416	25 400	20 100
3		0.468	158	Inner	24 300	0.446	......	23 300	21 600
10		0.470	158	Inner	26 800	0.488	......	27 800	23 700
18†	$2\frac{3}{4}$ by $\frac{3}{8}$ by 11.30	1.035	56	Inner	57 400	0.384	0.407	21 300	22 600
19†		1.021	56	Outer	62 600	......	......	23 500	25 000
16†	$2\frac{3}{4}$ by $\frac{5}{16}$ by 11.30	.....	63	Outer	44 400	Bolt	Shear	failure	
17†		0.849	64	Outer	50 500	0.353	0.407	21 000	24 200
14	$2\frac{3}{4}$ by $\frac{1}{4}$ by 11.30........	0.686	79	Outer	42 400	0.366	0.407	22 600	25 200
15		0.686	79	Inner	42 200	......	......	22 500	25 000
12	$2\frac{3}{4}$ by $\frac{3}{16}$ by 11.30	0.509	106	Outer	32 550	0.313	0.407	20 000	26 000
13		0.505	106	Inner	32 000	......	......	19 800	25 800

* Based on distance between center of middle rivet and center of either outer rivet, $\frac{l}{2}$.

† 1-in. bolts in these bars; ⅞-in. bolts in all others.

TABLE 20.—AVERAGE STRENGTH PROPERTIES OF MATERIAL IN BOLTS USED IN LACING-BAR TESTS

Diameter of bolt, in inches	$\frac{7}{8}$	1*	1	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{5}{8}$	1
Material	Steel	Steel	Steel	Wrought iron	Steel	Steel	High-carbon steel
Yield point, in pounds per square inch...........	41 400	41 500	39 600	37 100	33 900	39 800	70 300
Ultimate strength, in pounds per square inch.	60 900	64 100	77 800	49 100	52 400	55 900	119 500
Percentage elongation, in 2 in.................	33.5	30.0	18.5	30.0	40.5	31.5	24.0
Percentage reduction in area..............	61.7	64.0	61.8	52.1	66.3	68.0	57.2
Number of bolts tested.	2	2	1	1	1	1	1
Group numbers of lacing-bars, in which bolts were used...............	*B*1–*B*5	*B*6–*B*30, *B*35, *B*36, *C*1–*C*6, *C*9	*A*1–*A*2, *C*1–*C*3, *C*7, *C*8	*A*8, *A*9, tested in pairs	*A*8, *A*9, tested singly	*A*4–*A*7	*C*1*T*, *C*7*B*, *A*2, *B*37–*B*40, *B*31–*B*34

* A cross-bending test of two bolts of this size showed that the extreme fiber stress at scaling was 71 800 lb per sq in.

sion bar was 0.416 times the load, and the force in each long tension bar was 0.278 times the load (see Table 24). For the short 11.3-in. bars the corresponding ratios were 0.407 and 0.291, respectively. These ratios were used in computing the stresses in the bars by resolution. Stresses were also calculated by strain measurements. These measurements were made over 2-in. gauges with Olsen strain-gauges reading to 0.00003 in. on two gauges on the front and on the back of each bar (see Positions 3*A*, 3*X*, 4*A*, and 4*X*, Fig. 15). Strain readings were not practicable at the maximum loads carried, but stresses at the maximum were estimated by using the strains at loads near the maximum and multiplying these by the ratio of maximum load to load at which the gauge readings were taken. In deriving these values, the four readings on each bar were averaged and the average for the two compression bars taken as the final results. The results are given in Table 19.

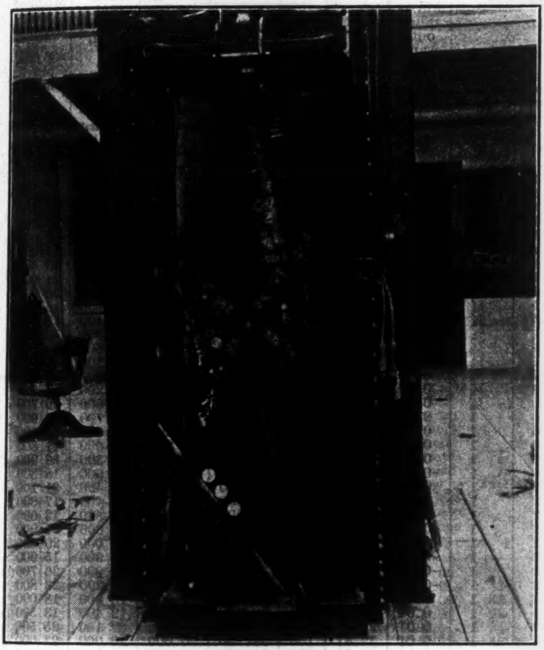


FIG. 15.—SET-UP FOR DOUBLE LACING-BAR TESTS.

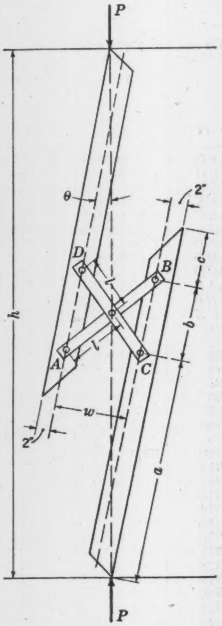


FIG. 16.—DIMENSIONS OF COLUMNS EQUIPPED WITH DOUBLE LACING-BARS.

RESULTS OF TESTS

12.—Tests on Flat Bars.—The results of the individual tests on flat bars are given in Table 21 and of the column tests on single lacing in Table 18. The strengths of the double lacing-bars calculated by resolution and from strains, are found in Table 19.

TABLE 21.—SUMMARY OF DATA ON INDIVIDUAL TESTS OF FLAT LACING-BARS. (EACH AVERAGE REPRESENTS FIVE BARS FOR SPECIMENS *5AD* TO *6CE*, THREE BARS FOR SPECIMENS *B*1 TO *B*20, AND TWO BARS FOR THE REMAINDER. BARS *B*31 TO *B*40 TESTED IN PAIRS)

Group No. (1)	Average dimensions, in inches (2)	Average area, in square inches (3)	Bolts		Maximum outward bow* at middle, in inches (6)	l, in inches (7)	$\frac{l}{r}$ (8)	Yield point of metal, in pounds per square inch (9)	Average unit load, $\frac{P}{A}$, in pounds per square inch (10)	Minimum unit load, $\frac{P}{A}$, in pounds per square inch (11)	Modified unit load for S_y =36 000, in pounds per square inch (12)
			Number (4)	Diameter, in inches (5)							
5AD	1.50 by 0.18	0.273	1	⅝	0.03	10.73	204	46 200	19 300	16 600	17 900
5BD	1.51 by 0.25	0.379	1	⅝	0.02	10.73	148	41 400	21 800	18 400	19 800
5CD	1.51 by 0.33	0.492	1	⅝	0.01	10.73	114	41 600	20 300	19 600	17 900
5AE	2.49 by 0.19	0.465	1	⅝	0.01	10.73	200	41 000	16 900	14 600	16 100
5BE	2.49 by 0.25	0.629	1	⅝	0.01	10.73	147	43 000	17 600	16 500	16 000
5CE	2.50 by 0.32	0.788	1	⅞	0.01	10.73	118	41 200	19 600	18 800	17 200
6AD	1.50 by 0.18	0.274	1	¾	0.02	10.73	204	46 200	21 400	19 600	19 800
6BD	1.51 by 0.25	0.378	1	¾	0.02	10.73	148	41 400	24 200	23 100	21 900
6CD	1.51 by 0.33	0.492	1	¾	0.00	10.73	114	41 600	23 000	21 500	20 200
6AE	2.50 by 0.19	0.464	1	¾	0.01	10.73	200	41 000	16 300	14 900	15 500
6BE	2 49 by 0.25	0.623	1	¾	0.01	10.73	148	43 000	20 800	18 400	18 700
6CE	2.49 by 0.32	0.787	1	¾	0.00	10.73	117	41 200	18 800	18 000	16 600
*B*1	2.76 by 0.17	0.470	1	⅞	−0.01	15.55	317	38 200	10 400	9 600	10 400
*B*2	2.74 by 0.26	0.702	1	⅞	.....	15.55	211	37 100	15 800	14 100	15 800
*B*3	2.77 by 0 31	0.864	1	⅞	0.00	15.55	173	34 800	17 100	16 600	17 400
*B*4	2.75 by 0.38	1.03	1	⅞	0.01	15.55	144	34 100	16 200	15 200	16 800
*B*5	2.76 by 0.38	1.03	1	⅞	0.00	11.30	104	34 100	18 600	17 900	19 700
*B*6	8.58 by 1.00	8.59	6	1	0.01	34.88	121	29 700	17 100	16 700	19 900
*B*7	8.62 by 1.00	8.60	6	1	0.04	49.31	172	29 700	14 600	14 400	16 100
*B*8	8.57 by 1.02	8.77	6	1	0.09	63.81	217	29 700	11 200	10 600	11 800
*B*9	7.99 by 0.74	5.95	6	1	0.01	27.69	129	30 800	18 600	18 300	20 900
*B*10	8.00 by 0.74	5.95	6	1	−0.01	38.50	179	30 800	16 900	15 900	18 300
*B*11	4.00 by 0.75	3.00	2	1	0.02	24.69	114	31 200	18 700	18 400	21 100
*B*12	4.01 by 0.75	3.01	2	1	0.01	35.50	164	31 200	15 700	15 400	17 000
*B*13	4.00 by 0.75	3.00	2	1	−0.03	46.38	215	31 200	13 000	11 500	13 600
*B*14	6.00 by 0 50	3.01	2	1	−0.01	17.38	121	36 000	21 900	21 600	21 900
*B*15	6.02 by 0.50	3.04	2	1	−0.01	24.63	169	36 000	20 000	18 000	20 000
*B*16	6.03 by 0.51	3.10	2	1	−0.01	31.81	215	36 000	15 900	14 900	15 900
*B*17	3.01 by 0.51	1.52	2	1	−0.01	17.38	120	34 800	26 100	24 900	26 700
*B*18	3.00 by 0.50	1.51	2	1	−0.01	24.62	170	34 800	23 400	22 200	23 800
*B*19	3.00 by 0.51	1.52	2	1	0.00	31.81	219	34 800	17 900	16 200	18 000
*B*20	7.99 by 0.76	6.04	6	1	0.04	49.38	226	30 800	13 000	12 400	13 500
*B*21	4.01 by 0.75	3.01	3	1	0.01	21.68	100	31 200	22 400	22 100	25 500
*B*22	4.00 by 0.75	3.00	3	1	0.01	32.47	150	31 200	20 000	20 000	22 000
*B*23	4.01 by 0.75	3.01	3	1	0.05	43.34	200	31 200	17 600	17 400	18 600
*B*24	2.75 by 0.38	1.04	2	1	−0.01	19.22	178	34 100	22 100	22 000	22 700
*B*25†	2.75 by 0.38	1.04	1	1	0.01	16.22	150	34 100	17 600	16 800	18 200
*B*26	5.96 by 0.50	2.98	3	1	0.01	18.06	125	36 000	23 500	23 200	23 500
*B*27	5.96 by 0.50	2.99	3	1	0.01	25.36	176	36 000	19 400	19 000	19 400
*B*28	3.01 by 0.50	1.52	2	1	0.04‡	17.37	119	34 800	24 100	......	24 700
*B*28	3.01 by 0.50	1.52	2	1	−0.04‡	17.37	119	34 800	31 400	......	32 200
*B*29	3.01 by 0.50	1.52	2	1	0.04‡	24.64	169	34 800	20 900	......	21 300
*B*29	3.01 by 0.50	1.52	2	1	−0.04‡	24.64	169	34 800	28 900	......	29 400
*B*30	3.01 by 0.51	1.52	2	1	0.04‡	31.80	219	34 800	18 800	......	18 900
*B*30	3.01 by 0.51	1.52	2	1	−0.04‡	31.80	219	34 800	23 800	......	24 000
*B*31–32	8.57 by 1.00	8.54	6	1	0.00	34.88	121	29 700	22 200	......	26 000
*B*33–34	8.60 by 1.00	8.59	6	1	0.00	49.50	171	29 700	20 600	......	22 800
*B*35–36	8.52 by 1.00	8.49	6	1	−0.04	63.75	221	29 700	16 700	......	17 600
*B*37–38	7.99 by 0 74	5.90	6	1	−0.03	27.62	129	30 800	24 400	......	27 500
*B*39–40	7.95 by 0.76	5.99	6	1	+0.01	49.00	224	30 800	16 700	......	17 400

* " Confined " bow.

† Retests.

‡ Bar bent as indicated prior to testing; " confined " bow same as " free " bow.

The material used in these tests varied considerably in yield point, as the data in Table 21 show. In order to study the effect of other variables to better advantage, the average observed ultimate strengths were modified, prior to plotting, to correspond approximately to a yield point of 36 000 lb per sq in. Coefficients for this purpose were determined by the "secant" column formula (Equation (3)), using $\frac{e\,c}{r^2} = 0.15$ and considering the length of the bar to be 0.55 of the length of a round end column. A considerable

TABLE 22.—DEFLECTIONS IN LACED-BAR COLUMNS DUE TO SHEAR

Column No. (1)	Bars carrying compression (2)	Span, in inches (3)	Length covered by lacing, L, in inches (4)	Shear from applied load, V, in pounds (5)	DEFLECTION, IN INCHES: Observed (elastic), D_0 (6)	DEFLECTION, IN INCHES: Due to bending D_b (7)	DEFLECTION, IN INCHES: Due to shear $D_s = D_0 - D_b$ (8)	$R = \frac{D_s}{L\,V} \times 10^{10}$ (9)
12....	$I\frac{1}{2}$ $O\frac{1}{2}$	162.5	139.5	4 945	0.1149	0.0882	0.0267	39
42....	$I\frac{1}{2}$ $O\frac{1}{2}$	163	140	5 000	0.1210	0.0867	0.0343	49
43....	$I+O$	110.7	87.7	5 075	0.0443	0.0283	0.0160	36
22....	$I\frac{1}{2}$ $O\frac{1}{2}$	343.6	324	1 000	0.1785	0.1640	0.0145	45
23....	$I\frac{1}{2}$ $O\frac{1}{2}$	348	325	500	0.0910	0.0841	0.0069	42
24....	$I\frac{1}{2}$ $O\frac{1}{2}$	101	78	5 000	0.0262	0.0121	0.0141	36
25....	$I+O$	158	135	5 000	0.0713	0.0460	0.0253	38
26....	$I+O$	329.6	310	1 000	0.0948	0.0852	0.0096	31
27....	$I+O$	348	328	1 000	0.1120	0.1006	0.0114	35

TABLE 23.—SUMMARY OF DATA ON INDIVIDUAL TESTS OF CHANNEL LACING-BARS

Group No. (1)	Number of tests (2)	SIZE: Depth, in inches 3	SIZE: Weight, in pounds per foot 4	Average area, in square inches 5	BOLTS: Number (6)	BOLTS: Diameter, in inches (7)	Maximum "confined" outward bow at middle, in inches (8)	L, in inches 9	Slenderness ratio, $\frac{l}{r}$, in inches 10	Yield point of metal, in pounds per square inch 11	UNIT LOAD, IN POUNDS PER SQUARE INCH: Average 12	UNIT LOAD, IN POUNDS PER SQUARE INCH: Minimum 13
C1T	2	3	4.1	1.25	2	1†	0.00	20.5	57	39 500	39 600†	38 500
C3	5	....	....	1.23	2	1	0.01	41.0	106	....	36 700	35 600
C2	5	....	....	1.24	2	1	0.00	61.5	155	....	28 400	27 600
C1	5	....	....	1.25	2	1	0.02	82.0	206	....	21 300	21 100
C7B	2	5	11.5	3.36	3	1†	0.01	24.5	64	34 200	33 400†	33 100
C9	3	....	....	3.35	3	1	0.00	49.0	116	....	30 300	30 200
C9	2	....	....	3.35	3	1‡	0.01	49.0	116	....	29 600	29 300
C8	3	....	....	3.35	3	1	0.02	73.5	164	....	24 100	23 800
C8	2	....	....	3.35	3	1‡	0.00	73.5	164	....	26 200‡	25 000
C7	5	....	....	3.37	3	1	0.06	98.0	212	....	18 100	17 400
C4T	6	7	9.8	2.90	6	1	0.04	29.2	61	36 700	34 900	32 600
C6	5	....	....	2.92	6	1	0.02	59.0	112	....	33 000	31 100
C5	5	....	....	2.92	6	1	0.06	88.5	163	....	25 600	24 000
C4	3	....	....	2.91	6	1	0.05	118.0	214	....	19 500	18 300
C4	2*	....	....	2.91	6	1	0.07	118.0	216	....	19 300	19 200

* Tested in pairs (see Fig. 13).

† Tested with high carbon-steel bolts.

‡ Tested with cold rolled steel bolts.

TABLE 24.—DIMENSIONS AND BAR LOADS (FIG. 16)

DIMENSIONS, IN INCHES: Bar length	*a*	*b*	*c*	*h*	*l*	*w*	θ	FORCE IN: *AB*	FORCE IN: *CD*
11.30	32	8	9	73	5.65	8.00	9° 27′	$+0.291P$	$-0.407P$
15.56	32	11	5.54	76.5	7.78	11.00	11° 19′	$+0.278P$	$-0.416P$

variation in assumed value of $\frac{e\,c}{r^2}$ would make little difference in this adjustment to a comparable basis. The modified values are given in Table 21.

Fig. 17 shows the results of the tests on the small flat bars with single bolt fastenings (with one exception). In this diagram the first group of bars listed is taken from the tests reported in 1929.[10] For the bars tested as double lacing, the half length is taken in computing $\frac{l}{r}$. Fig. 18 shows the results of tests on large bars with multiple bolt fastenings. In both groups the results are widely scattered, due very largely to differences in the rigidity of the end connection. In Fig. 17, all connections are made by single bolts with one exception, and for the larger bars the shearing stresses are high. The differences in the rigidity of the 10-in. and 15-in. channels to which the bars were bolted also doubtless affected the results somewhat.

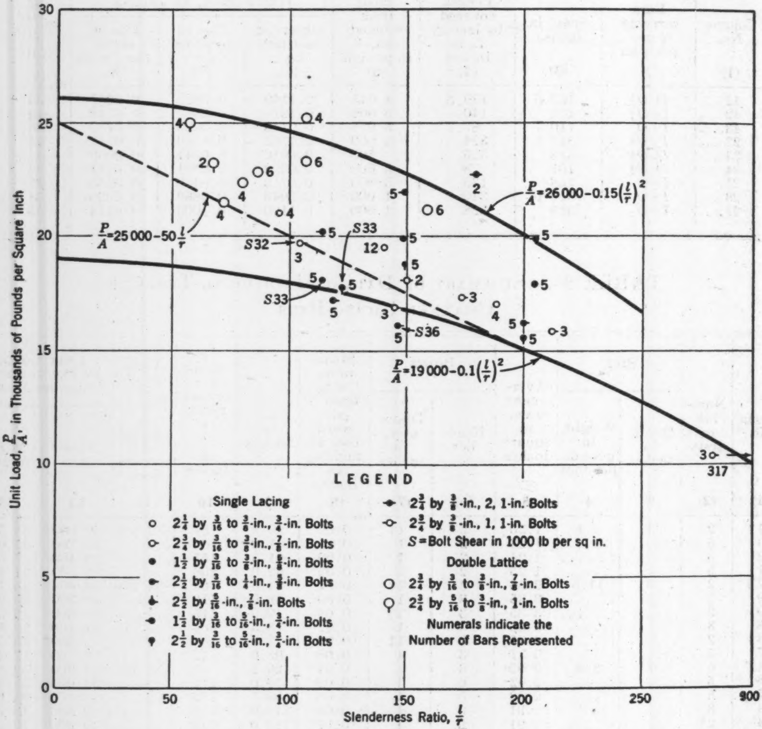


FIG. 17.—ULTIMATE STRENGTH-SLENDERNESS RATIO FOR SMALL RECTANGULAR LACING-BARS.

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[10] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1180.

Other factors which markedly affect strength of bars are tightness of bolts and crookedness of bar. In a few tests on 1½-in. by ¼-in. bars with ¾-in. bolts, increasing the torque applied to the nuts from 100 to 500 in-lb. caused an increase of 17% in bar strength. A comparison of data for the outward bowing bars of Groups *B*28, Table 21, with data for straight bars, Groups *B*17 to *B*19, shows that the 3-in. by ½-in. bars, with slenderness ratios of 120 to 170, were weakened about 10% by outward bows of 0.04 in. On the other hand, bars which bowed inward 0.04 in. were 20 to 30% stronger than straight bars.

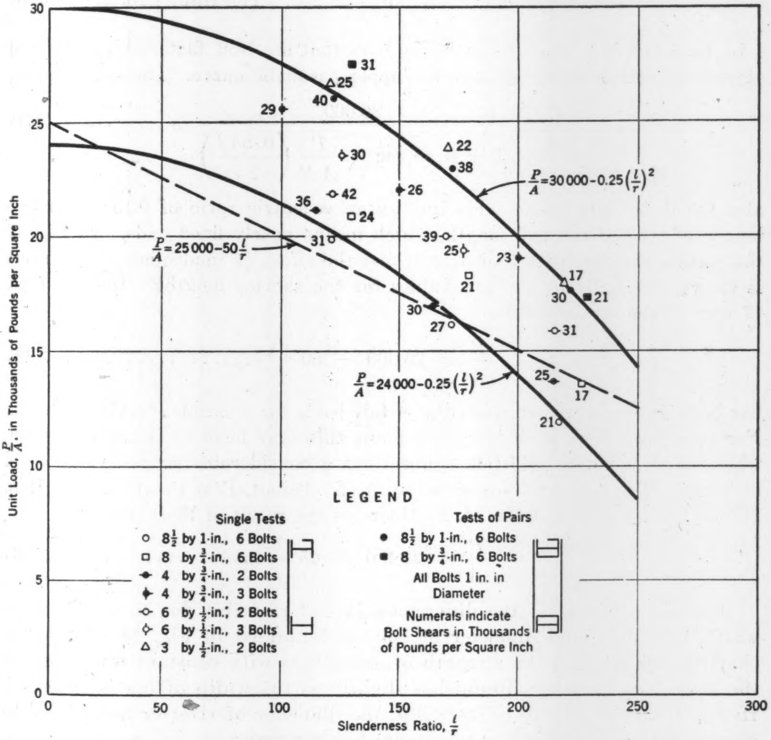

FIG. 18.—ULTIMATE STRENGTH-SLENDERNESS RATIO FOR LARGE RECTANGULAR LACING-BARS.

In the tests of Fig. 18, the **H**-sections forming the end supports are relatively rigid, and the multiple bolt connections used are more effective than the single bolts even if some of the shearing stresses are high. As a consequence, the values for the smaller ratios of $\frac{l}{r}$ are notably higher in Fig. 18

than in Fig. 17. Considering any one size of bar and connecting detail of the tests of Fig. 18, it will be found that the relation between strength and slenderness ratio is quite consistent and with a greater range covered would doubtless be found to follow the theoretical curve for a certain eccentricity and end restraint or free length of column. The two parabolas drawn in the diagram indicate approximately what may be expected as the limiting values of theoretical secant curves up to about $\frac{l}{r} = 200$. The results for the 3-in. by ½-in. bars and for the pairs of heavy bars that are best fastened, are in good agreement and lie fairly close to the upper parabolic curve. The secant curve,

$$\frac{P}{A} = \frac{36\,000}{1 + 0.15 \sec \sqrt{\frac{P}{A\,E}} \left(\frac{0.55\,l}{2\,r}\right)} \dots\dots\dots\dots\dots (11)$$

also fits these data well. This implies an eccentric ratio of 0.15 and a free length of 0.55 of the full length, which means nearly fixed ends. In Fig. 17 the results are much less consistent, and the effect of inadequate end support is shown especially by the low values for the shorter lengths. In both Figs. 17 and 18 the straight line,

$$\frac{P}{A} = 25\,000 - 50\,\frac{l}{r} \dots\dots\dots\dots\dots\dots\dots (12)$$

has been drawn and is suggested as a fair basis for a simple working formula. For very large bars with low yield point this may have to be reduced somewhat. For adequate multiple connections a considerably higher value could be taken. The formula[11] suggested by A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and Mr. H. F. Moore as the result of their tests is,

$$\frac{P}{A} = 21\,400 - 45\,\frac{l}{r} \dots\dots\dots\dots\dots\dots\dots (13)$$

Further evidence of the differences in end restraint due to variations in width, thickness, and diameter of bolt, is contained in Fig. 19. From this chart it appears that the strength of small bars with constant thickness and the same bolt diameter diminishes slightly as the width of bar is increased. Hence, if the bar width is increased, the diameter of rivet or bolt should be increased in order to secure maximum bar strength.

Table 18 and Fig. 19 give results of the tests on single bars by means of the short columns with single lacing and a comparison with individual ~~tests.~~ These results show, as did the data in previous reports, that the outer bars in compression are materially weaker than the inner bars in compression. This is due to the greater unsupported length of the outer bars. For these tests the outer bars in compression averaged 88% of the strength of bars tested singly, whereas the inner bars averaged 110% of the strength of the individual bars. The results of the tests on single bars thus represent a fair average of the results from the column tests.

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[11] *Transactions* Am. Soc. C. E., Vol. LXV (1909), p. 238, and Vol. 95 (1931), p. 1181.

The results of the column tests with double lacing are given in Table 19 and Fig. 17. It is interesting to note that even when the value of $\frac{l}{r}$ is based on the half-length of the bar the test results rate somewhat better than those for single bars. Furthermore, for the range in lengths used ($\frac{l}{2r}$ from 56 to 158), there was no material difference in strength. A flat value of 22 000 lb per sq in. would be a safe basis for calculating ultimate strength. This would correspond to an eccentric ratio for a short column of 0.64 for a yield point of 36 000 lb per sq in., or an eccentricity of connection of about one-tenth the thickness.

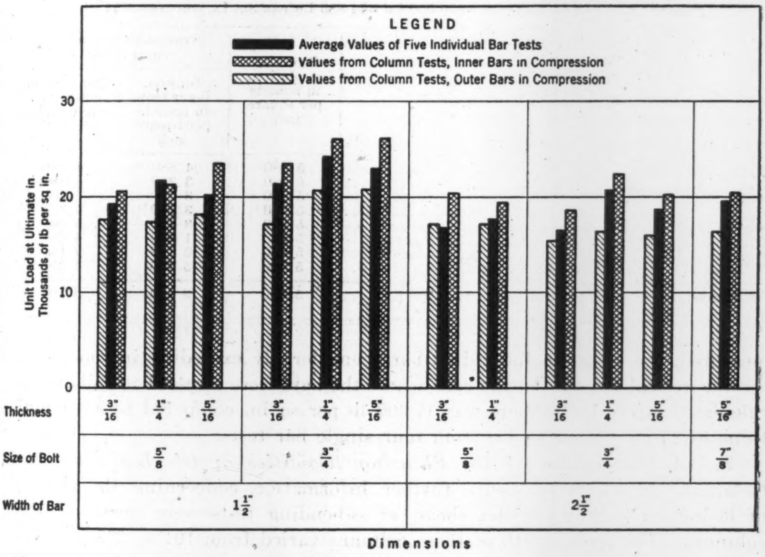

FIG. 19.—EFFECT OF SIZE OF BOLT AND WIDTH OF BAR ON THE COMPRESSIVE STRENGTH OF LACING-BARS.

13.—Results of Strain Measurements on Flat Lacing-Bars of Column No. 12.—During the tests of Column No. 12 (10-in. 20-lb channels, 2¼-in. by ¼-in. lacing) it was subjected to an oblique loading which produced transverse shear equal to 3% of the column load. Strains were measured on the narrow faces of the lacing-bars over 2-in. gauge lengths by means of an Olsen strain-gauge reading to 0.00003 in. Stresses were calculated by multiplying the strains by the modulus of elasticity. A comparison of the stresses calculated by strains and those obtained by resolution is given in Table 25. This comparison indicates that for the loading under consideration the average stress calculated from strains in the compression bars, which were all outer bars, was approximately 75% of the stresses calculated by resolution. In only one case was the stress-form-strain calculation slightly in excess of the stress determined from resolution.

The results of tests under oblique loading,[12] in which the shear was 5% of the direct load, shows a higher relative value of lacing-bar stress, as measured, the stress in individual bars considerably exceeding in some cases the theoretical stress. In the case where the bars were stressed to failure, the calculated stress in the bars was 17 600 lb per sq in. compared to a strength value of 17 200 lb per sq in. from four single bar tests.

TABLE 25.—UNIT STRESSES IN LACING-BARS OF COLUMN NO. 12 FOR 3 PER CENT. SHEAR LOADING

INCREMENT IN LOAD ON COLUMN = 170 500 LB = 14 880 LB PER SQ. IN.; SHEAR = 5 115 LB.

Panel No.	Stress by resolution, in pounds per square inch	Stresses Calculated from Strains: Tension, inner bars, in pounds per square inch	Stresses Calculated from Strains: Compression, outer bars, in pounds per square inch
2	5 080	4 880	1 395
3	5 080	3 260	3 950
4	5 080	3 020	3 260
5	5 080	3 260	5 350
24	5 080	3 720	2 560
25	5 080	1 860	4 880
26	5 080	3 720	4 180
27	5 080	3 490	4 650
Means	5 080	3 400	3 780

14.—A Comparison of the Shearing Rigidities of the Laced Channel Columns.—In order to secure further information concerning the rigidity of lacing-bar columns under shear, cross-bending tests were made on nine columns. The span length of these columns varied from 101 to 348 in. The shear loads and the principal data covering the tests are shown in Table 22. As stated previously in Column (2) of that table, the designation, $I\frac{1}{2}\ O\frac{1}{2}$, means that the inner bars were in compression over one-half the length of the column and that the outer bars were in compression over the remaining half length of the column. The designation, $I + O$, signifies that the inner bars were in compression on one side of the column and the outer bars were in compression on the opposite side of the column. In Column (6) of Table 22 are given the observed deflections; in Column (7), the calculated deflection due to bending; and in Column (8), the difference which is the deflection due to shear. In Column (9), the quantity,

$$R = \frac{D_s}{VL} \times 10^{10} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

indicates the relative rigidity of these columns with respect to shear, those with the larger values of R being less rigid than those having the smaller values. From these data it appears that the average value of R for the columns having the inner bars in compression on one side and the outer bars in compression on the other side was 35, whereas for the other columns it was 42.

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[12] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1177.

When the oblique loading lies in the median plane of the column parallel to the lacing, and the direction of the obliquity with respect to the column axis is known, the previous tests made by the Committee have shown that the greatest shearing resistance and rigidity will be obtained when all lacing-bars suffering compression are inner bars; but when the direction of the obliquity is unknown (which is the usual case before the designer), these latter tests indicate that the arrangement whereby the outer bars on one side of the column are under compression and the inner bars on the opposite side of the column are under compression is most effective in securing rigidity under shear.

15.—Tests on Channel Lacing-Bars.—The results of the tests on channel lacing-bars are summarized in Table 23. In Fig. 20(a), the average unit loads have been plotted without modification on account of yield point, and parabolic curves starting at the compressive yield points of the metal have been drawn through the groups of points.

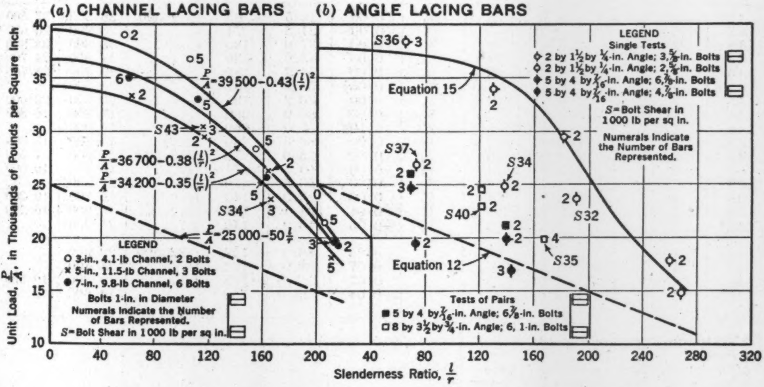

FIG. 20.—ULTIMATE STRENGTH-SLENDERNESS RATIO RELATIONSHIPS, CHANNEL LACING-BARS AND ANGLE LACING-BARS

The high results for the shorter lengths indicate very small eccentricity due to the rigidity of the multiple bolt connections and the relatively thin web metal. The results are much higher than those for flat bars; roughly, 50% greater than the values given by the straight line.

16.—Tests on Angle Lacing-Bars.—Table 26 contains the summary of results on angles. Average unit loads for each group unmodified for yield point have been plotted against slenderness ratio in Fig. 20(b). The light, 2 by 1½-in. by ¼-in. angles, well secured by three ⅝-in. bolts to the flanges of the heavy **H**-beams, typify the behavior of fixed end columns. These data agree well with the secant curve,

$$\frac{P}{A} = \frac{39\,500}{1 + 0.05 \sec \sqrt{\frac{P}{AE}} \left(\frac{0.5\,l}{2\,r} \right)} \quad \ldots\ldots\ldots\ldots (15)$$

for a fixed-end column with an eccentric ratio of 0.05. Two ⅝-in. bolts for the same angles failed to provide sufficient end restraint to develop the full strengths of the shorter members. Only one set of tests, the 5 by 4-in. angles, with four bolts, falls below the straight line, Equation (12).

TABLE 26.—SUMMARY OF DATA ON INDIVIDUAL TESTS OF ANGLE LACING-BARS

Group No. (1)	Number of tests (2)	Size of angle, in inches (3)	Average area, in square inches (4)	Bolts		Maximum "confined" outward bow at middle, in inches (7)	L, in inches (8)	Slenderness ratio, $\frac{l}{r}$, in inches (9)	Yield point of metal, in pounds per square inch (10)	Unit Load, in Pounds per Square Inch	
				Number (5)	Diameter, in inches (6)					Average (11)	Minimum (12)
*A*7	3	2 by 1½ by ¼	0.86	3	⅝	0.00	15.5	65	39 500	38 300	37 500
*A*7	2		0.85	2	⅝	0.00	21.5	74	....	26 800	26 600
*A*6	2		0.83	3	⅝	0.00	37.0	130	....	33 800	33 800
*A*6	2		0.83	2	⅝	0.00	43.0	139	....	24 800	24 700
*A*5	2		0.83	3	⅝	0.00	54.2	182	....	29 400	28 700
*A*5	1		0.83	3	⅝	0.05§	54.2	182	....	32 500§	....
*A*5	2		0.83	2	⅝	0.00	60.2	191	....	23 600	23 600
*A*4	2		0.83	3	⅝	0.00	80.0	260	....	17 800	16 800
*A*4	1		0.83	3	⅝	0.10§	80.0	260	....	16 000§	....
*A*4	2		0.83	2	⅝	0.00	86.0	269	....	14 800	13 300
*A*9	3	5 by 4 by $\frac{7}{16}$	3.69	6	⅞	0.00	53.5	70	33 500	24 600	24 500
*A*9	2		3.69	4	⅞	0.00	59.5	74	....	19 400	19 000
*A*9*	1†		3.69	6	⅞	0.00	53.5	70	....	25 800†	....
*A*8	2		3.72	4	⅞	0.00	113.0	140	....	19 800	19 200
*A*8	3		3.72	6	⅞	0.04	119.0	144	....	16 900	16 200
*A*8*	1†		3.72	6	⅞	−0.05	113.0	140	....	21 000†	....
*A*2	1*	8 by 3½ by ¾	8.24	6	1	0.05	88.0	122	32 600	22 900	....
*A*2	1*		8.21	6	1‡	0.07	88.0	122	....	24 400‡	....
*A*1	2*		8.20	6	1	0.07	123.0	168	....	19 800	19 500
*A*1	1		8.20	6	1	−0.04	123.0	168	....	14 600	....

* Tested in pairs (see Fig. 13).
† Retests
‡ Tested with high carbon-steel bolts.
§ Bars were bowed purposely as indicated.

Comparisons of the deflections of these small angle-bars of similar lengths show that, with the exception of the longest angles, those fastened with three bolts in a line at each end, deflected much less in the plane of attachment than did those fastened with two bolts. This was confirmed by the strain measurements which indicated that the stress in the most strained fiber (the tip of the fastened leg), was much less when the larger number of bolts was used. Furthermore, the strain evidence indicates that with the 3-bolt attachment the spreading of over-strain throughout the middle portion of the member occurred much less rapidly than with the 2-bolt attachment.

Observations during the test and the plots of the data in Fig. 20(*b*) show that the heavier angles attached by one leg only were not nearly so well restrained as the light angles. For the 5 by 4-in. by $\frac{7}{16}$-in. and the 8 by 3½-in. by ¾-in. angles, both fastened with six bolts at each end, the data indicate large end eccentricities. The reasons for the pronounced weakness of the 5 by 4-in. by $\frac{7}{16}$-in. angles tested with four bolts at each end are similar to those given for the 2 by $1\frac{1}{2}$-in by $\frac{1}{4}$-in. angles with 2-bolt attachment.

To ascertain the effect of deformation of the **H**-end attachments on the strength of the bars, tests of the heavier angles were made both singly and in pairs. The data show that the difference due to method of testing is small in the case of the 5 by 4-in. by $\frac{7}{16}$-in. angles. For the 8 by $3\frac{1}{2}$-in. by $\frac{3}{4}$-in. angles the **H**-ends were far too light to support them singly and were considerably deformed when the tests were made in pairs.

17.—The Effect of Stresses in Bolts on Lacing-Bar Strengths.—As may be ascertained from the data, there are numerous evidences of the existence of high bearing stresses (greater than 70 000 lb per sq in.) and shear stresses (greater than 35 000 lb per sq in.) in bolts at the failure of certain of the lacing-bars. Unless accompanied by large bending stresses such a condition does not appear to have caused material reduction in the strengths of the bars.

Although no rigorous calculation of bending stresses in bolts can be made, it is certain that such stresses of high magnitude existed in the thick bars and in the larger angles where the gravity axis was displaced considerably from the plane of attachment. It is believed that high bending stresses have a decided weakening effect on bar strength.

18.—Comparison of Shapes Tested as Lacing-Bars.—Of the shapes tested the channel appears to be the strongest for a given length of bar and cross-section area, and the flat bar the weakest. A channel section with somewhat wider flanges, like those used in ship-building, would doubtless be still better than those of the structural type.

19.—Conclusions Regarding Lacing-Bar Tests.—The strength of lacing-bars depends largely upon the end connections. For all single bolt connections of small flat bars and for the connections of the lesser strength of large bars, the ultimate strength is approximately given by Equation (12) for a yield point of 36 000 lb per sq in. By more adequate connections these values can be increased considerably, up to values approaching those of fixed-end columns.

The channel bar is a superior form for lacing-bars and, where adequate connections can be made, has a strength approximating that for fixed-end columns.

The angle bar has about the same strength as the flat bar for the larger sizes. Its advantage lies in its smaller value of $\frac{l}{r}$. The flat bar used in double lacing with connecting rivet has a strength equal to single flat bars of one-half the length. The strength of such bars is approximately equal to 60% of the yield point of the material.

The detailed results of the tests give much information to assist in the design of end connections to secure maximum efficiency.

The foregoing summary relates only to the compressive strength of bars. Convenience of detail is greatly in favor of the flat bar, and its strength in

double lacing is such as to make it well adapted to large columns. If shapes are used, the channel appears to be preferable to the angle.

TESTS OF OUTSTANDING PLATES OR T-SHAPED COLUMNS

20.—Method of Testing.—In order to secure further data concerning the limiting ratio of width of unsupported flange to thickness, eighteen T-shaped columns made up as shown in Table 27 were tested in the 600 000-lb hydraulic machine, using for the most part flat-end bearings. The principal variable was the outstanding width of the stem, or web-plate, often referred to herein as the outstanding width of flange. The properties of the material used in these tests are shown in Tables 1, 4, and 28. In testing with flat ends, each column was carefully centered in the machine, and the movable top head was supported at four points by 3½-in. cylinders which were wedged into place and a small initial load applied.

TABLE 27.—DIMENSIONS AND PROPERTIES OF WIDE-WEBBED T-SHAPED COLUMNS

Column No. (1)	Make-up, nominal dimensions (2)	Plate thickness, t, in inches (3)	WIDTH, IN INCHES: Total (4)	WIDTH, IN INCHES: Outstanding (5)	WIDTH, IN INCHES: Center of rivet, out, w. (6)	Area, in square inches (7)	Length, in inches† (8)	$\frac{w}{t}$‡ (9)
60*	Two 4 by 4 by ⅜-in. angles	0.376	8.48	4.46	5.98	8.86	75.0	15.9
61		0.378	8.37	4.29	5.87	8.96	75.0	15.5
62	One 8 by ⅜-in. plate	0.378	8.45	4.53	5.95	8.41	74.9	15.7
63	Two 4 by 4 by ⅜-in. angles	0.377	9.54	5.53	7.04	9.19	75.0	18.7
64†		0.378	9.56	5.53	7.06	9.15	75.0	18.7
65	One 9 by ⅜-in. plate	0.381	9.52	5.49	7.02	9.30	75.0	18.4
66		0.378	11.48	7.52	8.98	9.45	75.0	23.8
67	Two 4 by 4 by ⅜-in. angles	0.373	11.48	7.55	8.98	9.44	75.0	24.1
68		0.377	11.44	7.48	8.94	9.48	75.0	23.7
72*B**	One 11 by ⅜-in. plate	0.370	11.54	7.59	9.04	9.51	75.0	24.7
69	Two 4 by 4 by ⅜-in. angles	0.366	13.41	9.45	10.91	10.10	75.0	29.8
70		0.368	13.43	9.47	10.93	10.20	75.0	29.7
71	One 13 by ⅜-in. plate	0.370	13.42	9.47	10.92	10.15	75.0	29.5
72 and 72*A**	Two 4 by 4 by ⅜-in. angles	0.370	15.25	11.30	12.75	10.90	75.0	34.4
73		0.369	15.28	11.30	12.78	10.90	75.0	34.7
74	One 15 by ⅜-in. plate	0.380	15.37	11.40	12.87	11.15	75.0	33.9
75	Two 4 by 4 by ¾-in. angles	0.373	15.33	11.16	12.83	16.30	75.0	29.9
76		0.370	15.31	11.14	12.81	16.36	75.0	30.1
77	One 15 by ⅜-in. plate	0.370	15.34	11.17	12.84	16.44	74.9	30.2
76*A*	Two 4 by 4 by ¾-in. angles One 13½ by ⅜-in. plate.	0.370	13.74	9.57	11.24	15.75	75.0	25.9

* Tested with roller-bearings.

† Slenderness ratios based on minimum radii of gyration varied between 51 and 57.

‡ For columns with ⅜-in. angles (Column Nos. 60–74), w = width from rivet line to edge of plate; for columns with ¾-in. angles (Columns Nos. 75–76A), w = outstanding width as given in Column (5).

The initial crookedness of the stem and flange angles was then determined by deflectometer readings taken to 0.001 in. on horizontal lines (*A* to *X*, Fig. 21), or on alternate lines. These lines were spaced 3 in. apart. Reading points were spaced horizontally 2 to 4 in. apart, beginning at the free edge of the plate. The numbers in Fig. 22, for example, show points of measurement. Strain-gauge readings were also taken over 8-in. gauges centered 8 in. from the top and bottom and over 20-in. gauges at mid-length of the column. Figs. 21 and 23 show the location of the gauge lines. An initial load of 8 000 to 10 000-lb per sq in. was then applied, and the gauges read to determine how nearly the column was centered. If the column was not properly centered, the wedges on the top bearing were adjusted, and, when required, shims were inserted under the column ends. This procedure was repeated until the strain readings on the various gauges at mid-length were approximately equal. In so far as practicable, the strains in the end gauges were also equalized.

TABLE 28.—ULTIMATE STRENGTHS OF T-SHAPED COLUMNS

Column No. (1)	Thickness of angles, in inches (2)	$\frac{w}{t}$ (3)	Area, in square inches (4)	Maximum initial crookedness of plate within middle third of length, in inches (5)	PLATE PROPERTIES IN TENSION, IN POUNDS PER SQUARE INCH: Yield point (6)	PLATE PROPERTIES IN TENSION, IN POUNDS PER SQUARE INCH: Proportional limit (7)	ULTIMATE: Load, in pounds† (8)	ULTIMATE: Unit load, in pounds per square inch (9)
60*	¼ in.	15.9	8.86	0.05	31 000	28 100	266 000	30 000
61		15.5	8.96	0.02	......	......	351 000	39 200
62		15.7	8.41	0.02	......	......	294 000	34 950
						Ave.	322 500	37 070
63		18.7	9.19	0.05	33 000	=29 100	351 900	38 300
64*		18.7	9.15	0.06	......	......	274 500	30 000
65		18.4	9.30	0.05	......	......	335 800	36 100
						Ave.	344 000	37 200
66		23.8	9.45	0.04	33 400	=32 600	263 800	27 850
67		24.1	9.44	0.02	......	......	291 700	30 900
68		23.7	9.48	0.01	......	......	279 000	29 400
*72B**		24.7	9.51	0.02	39 200	37 400	301 100	31 700
						Ave.	278 200	29 380
69		29.8	10.10	0.03	35 000	=34 000	252 000	24 950
70		29.7	10.20	0.05	......	......	262 000	25 700
71		29.5	10.15	0.02	......	......	261 000	25 700
						Ave.	258 300	25 440
72		34.4	10.90	0.06	39 200	=37 400	....	......
*72A**		34.4	10.90	0.05	......	......	....	......
73		34.7	10.90	0.04	......	......	263 000	24 100
74		33.9	11.15	0.04	......	......	270 500	24 300
						Ave.	266 800	24 200
75	[illegible] in.	29.9	16.30	0.09	39 200	=37 400	470 500	28 900
76		30.1	16.36	0.12	......	......	....	......
77		30.2	16.44	0.02	......	......	481 000	29 300
						Ave.	475 800	29 100
76A		25.9	15.75	0.11	39 200	=37 400	488 200	31 000

* Tested with roller-bearings.

† In calculating average strength values, the columns tested with roller-bearings have been omitted.

After centering, a complete set of deflectometer and strain-gauge readings under an initial load of 5 000 lb was taken. Increments of load of 5 000 lb per sq in., or less, were then applied, and the strains, deflections, and twists read. The twists of the flange angles were read by pointers which moved relative to 0.01-in. scales clamped to the testing machine. Readings were taken at the top, bottom, middle, and quarter-points, as shown at the left of the column in the machine in Fig. 27. When the plate began to exhibit marked deflection, the increments of load were made smaller. After loads greater than 8 000 lb per sq, in. had been applied, they were released to their initial values to determine sets in deflections, twists, and strains. Deflection readings were usually taken on all vertical lines until lateral buckling was quite pronounced, and thereafter only on the outer and innermost lines.

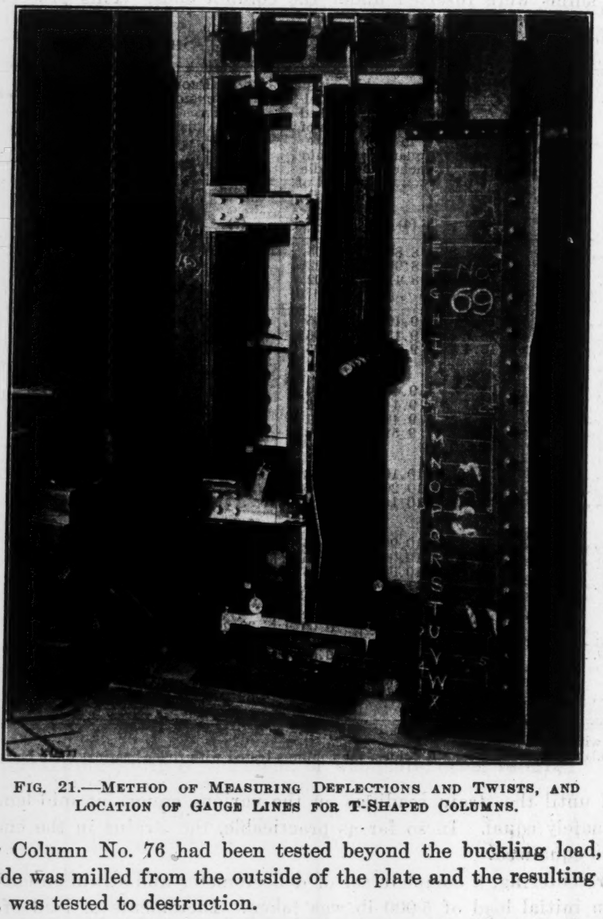

FIG. 21.—METHOD OF MEASURING DEFLECTIONS AND TWISTS, AND LOCATION OF GAUGE LINES FOR T-SHAPED COLUMNS.

After Column No. 76 had been tested beyond the buckling load, a strip $1\frac{1}{2}$ in. wide was milled from the outside of the plate and the resulting column, No. *76A*, was tested to destruction.

Columns Nos. 61, 65, 68, 69, 74, and 75 were tested with 3 by 2-in. by $\frac{1}{4}$-in. stiffener angles at top and bottom (see Fig. 21), but no stiffeners were used on the other columns. The effects of these stiffener angles appeared to be negligible.

In order to ascertain the effect of roller bearings in tests of such members, after the buckling load had been imposed on Column No. 72 under flat end bearings, it was numbered 72*A* and re-tested with the roller bearings until it buckled. Following this re-test, a strip, 3$\frac{3}{4}$-in. wide, was cut from the outside of the web, and it was again tested with roller bearings as Column No. 72*B*; Columns Nos. 60 and 64 were also tested with roller bearings. In all cases where roller bearings were used, the columns were placed so that the axes of the bearings were normal to the plate and with a slight eccentricity toward the free edge of the plate. Under these conditions, the ratio, $\frac{e\,c}{r^2}$, was less than 0.04.

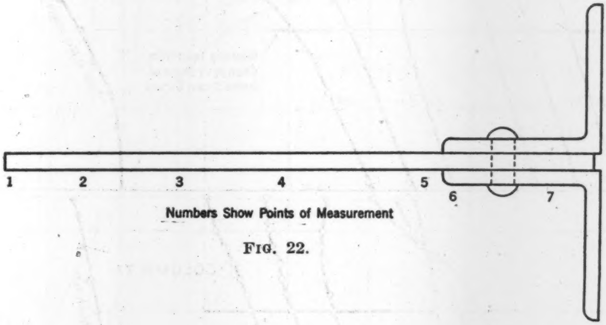

FIG. 22.

21.—Curves Plotted from Data.—From the test data were plotted: (*a*) Unit load-strain curves; (*b*) unit load-deflection curves; (*c*) unit load-set curves for the most deflected points of the outer edges; (*d*) curves showing the crookedness, deflection, and displacement (= deflection + crookedness) of the outer edge of plate at or near the buckling load; and (*e*) unit load-twist curves. Samples of these curves for Columns Nos. 66, 73, and 77 are shown in Fig. 23 and 24. Load-deflection curves for points of the outer edge having maximum deflections are shown for all columns in Fig. 25. In all cases these deflections are measured with respect to a line near the column axis and do not include the deflection of the column as a whole.

In addition to the foregoing data, elastic deflection curves, representing the lateral bending along given horizontal lines in the plate at, or near, the buckling load, were constructed as shown in Fig. 26.

22.—Discussion of Results of Tests.—During the progress of these tests, lateral deflection of the outstanding edge of the stems in the wider columns was very apparent. In the more crooked wide plates this began at low loads and there was no very pronounced "knee" in the load-deflection curve. Twisting was also apparent in all the wider columns having $\frac{3}{8}$-in. flange angles (Nos. 66 to 74). In most cases twisting began at relatively low loads and became very apparent in the vicinity of the buckling load. Most of the columns with the narrower plates and those having $\frac{3}{4}$-in. flange angles exhibited little twisting until the direct compressive stress was equal to the proportional limit.

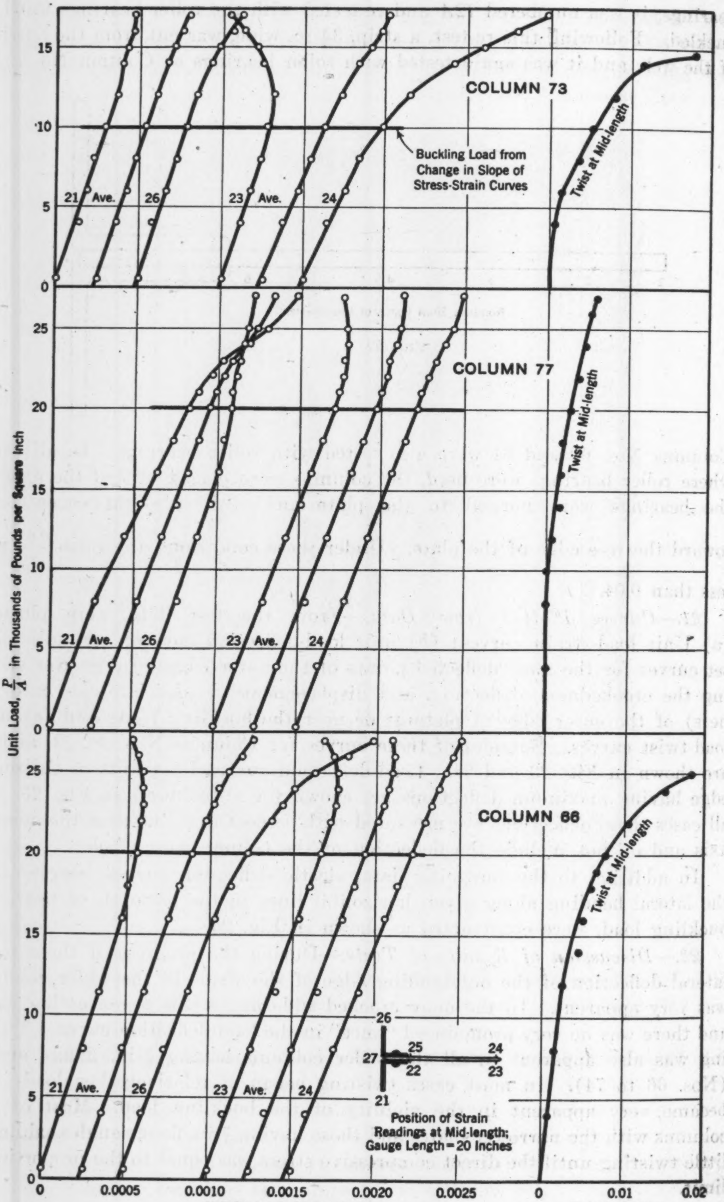

FIG. 23.—STRESS-STRAIN AND STRESS-TWIST CURVES FOR COLUMNS OF SERIES (G).

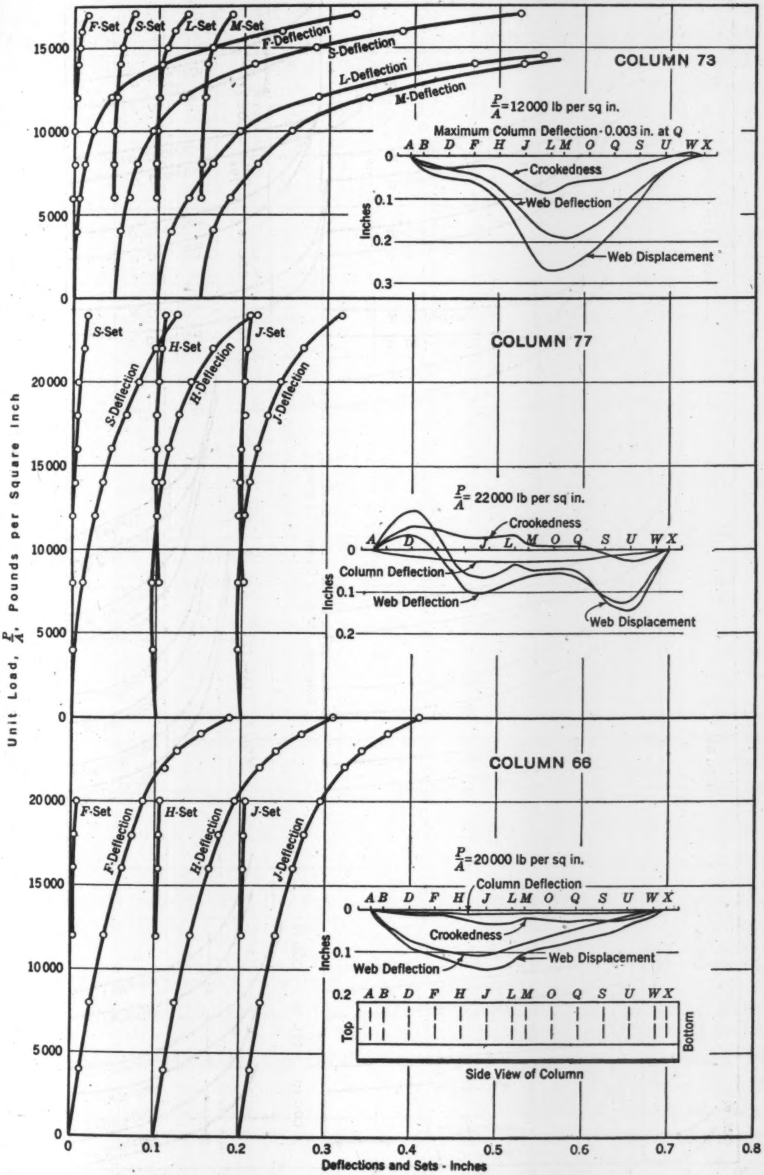

FIG. 24.—LOAD-DEFLECTION CURVES AND POSITION-DEFLECTION CURVES FOR SOME COLUMNS OF SERIES (G)

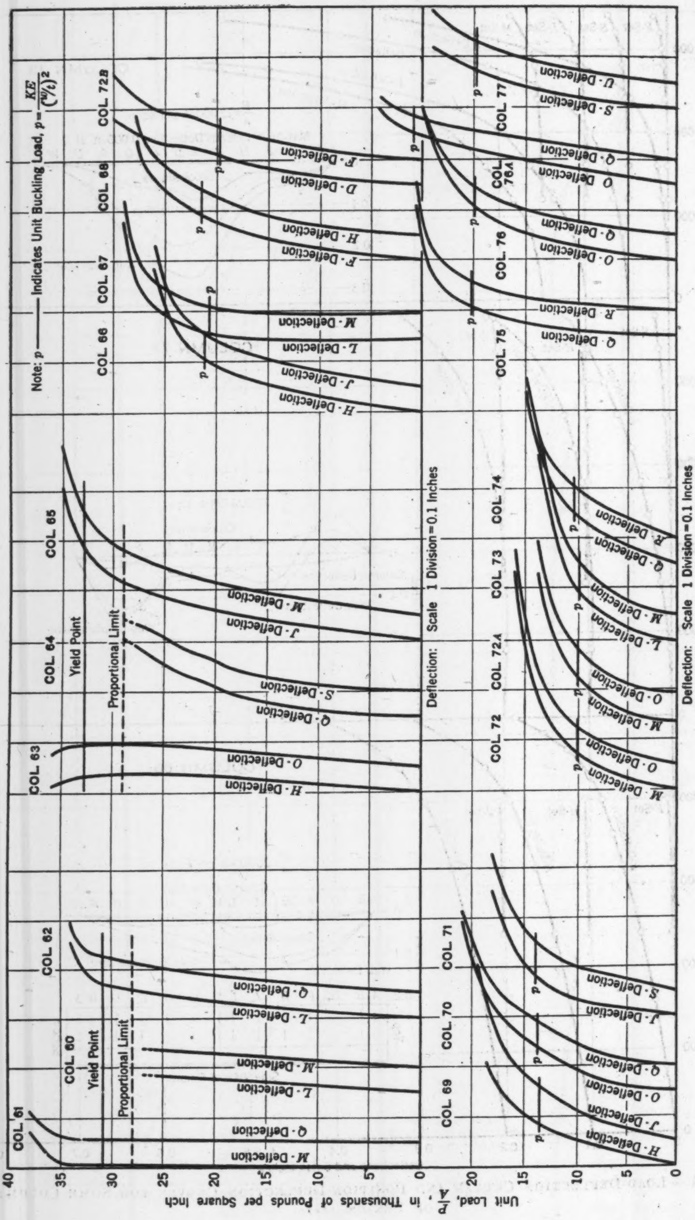

FIG. 25.—LOAD-DEFLECTION CURVES OF COLUMNS OF SERIES (G).

A study of the elastic-deflection data (see Fig. 26) for the columns having flange angles $\frac{3}{8}$ in. thick showed that these lines were nearly straight and that the lateral bending began well within the confines of the flange angles. On the other hand, as Fig. 26 shows, in the columns supported by $\frac{3}{4}$-in. flange angles, the elastic lines had appreciable curvature, and the deflection within the confines of the angles was small. Hence, in applying formulas to the results, the unsupported width, w, of the flange for Columns Nos. 60 to 74, with $\frac{3}{8}$-in. flange angles, has been taken from the free edge of the plate to the center line of rivets. For columns with $\frac{3}{4}$-in. flange angles the unsupported width has been taken as the distance from the free edge to the toe of the angles. Values of the ratio of unsupported width to thickness, $\frac{w}{t}$, calculated on this basis, are given in Table 27.

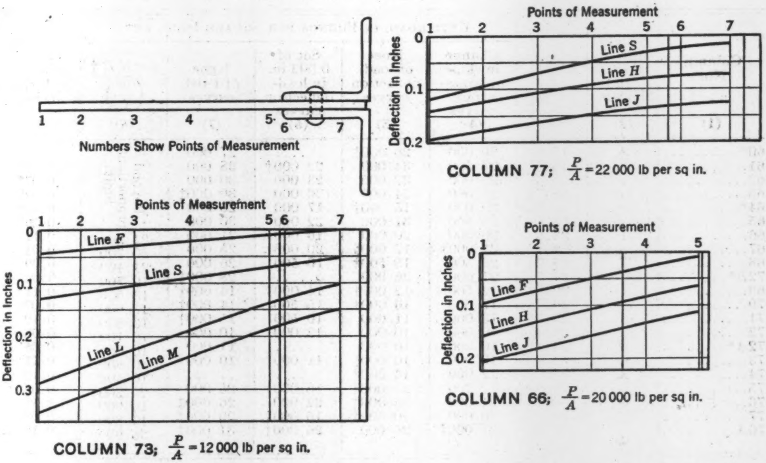

FIG. 26.—EXAMPLES OF ELASTIC DEFLECTIONS OF HORIZONTAL LINES IN THE PLATES OF COLUMNS NOS. 66, 73, AND 77.

Tables 28 and 29 show the principal results obtained from these tests. It should be noted that there was considerable crookedness in the free edges of the columns having the wider plates. Undoubtedly, these eccentricities weakened those members materially.

Unfortunately, the yield point of both the plates and the angles in these columns varied considerably (see Table 1). The average yield point for the angles was considerably higher than that for the plates. The steel in the columns having the wider plates, Nos. 72 to 77, apparently had a higher yield point and proportional limit than that in the narrow stemmed columns, Nos. 60 to 65. This difference undoubtedly affected the relative magnitude of the ultimate strengths of these two groups of columns. Several of the columns exhibited more than one half-wave in the free edge of the plate during the early part of the loading, but in all tests the plate edge eventually curved into a single half-wave as shown in Fig. 27. Failure was more sudden in the tests made with the roller bearings than with the flat bearings. For Columns Nos. 60 to 64, which were tested with roller bearings, the ultimate loads were materially lower than for companion columns tested with flat ends. This discrepancy is probably due to the larger inherent crookedness in the plates of both these columns.

TABLE 29.—BUCKLING UNIT LOADS FOR THE T-SHAPED COLUMNS DETERMINED BY DIFFERENT CRITERIA

Column No. (1)	Thickness of angles, in inches (2)	$\frac{w}{t}$ (3)	UNIT LOAD, IN POUNDS PER SQUARE INCH, AT: Change in slope of stress-strain curves (4)	Knee of load-deflection curves (5)	Set of 0.002 in. in load-deflection curves (6)	Knee of twist curves (7)	$p = \frac{KE}{\left(\frac{w}{t}\right)^2}$ † (8)	Deflection of free edge at load, p, in inches (9)
60*	⅜ in.	15.9	20 000	26 000‡	......	27 000	Beyond proportional limit	......
61		15.5	27 500	34 000	22 000†	38 000		0.01¶
62		15.7	26 000	32 000	23 000	30 000		0.07¶
63		18.7	26 000	34 000	20 000	30 000‡		0.04¶
64*		18.7	20 000	15 000†	17 000	20 000		0.13¶
65		18.4	25 500	31 000	22 000	30 000		0.16¶
66		23.8	20 000	20 000	16 000†	21 000	21 200	0.11
67		24.1	21 000†	17 000§	20 000†	25 000	20 700	0.01
68		23.7	20 500	18 500†	10 500	26 000	21 400	0.09
72*B**		24.7	21 000	26 000	......	28 000‡	19 700	0.06
69		29.8	12 500	12 000†	14 000†	14 000†	13 500	0.11
70		29.7	12 000	16 000†	15 500†	14 000†	13 600	0.09
71		29.5	13 000	11 000	10 500	15 000†	13 800	0.10
72		34.4	10 000	10 000	12 000	10 000	10 100	0.08
72*A**		34.4	10 000	10 000	......	11 000	10 000	0.08
73		34.7	10 000	10 000§	11 000†	10 000†	9 800	0.11
74	¼ in.	33.9	12 000	10 000†	......	.. ..	10 400	0.10
75		29.9	22 200	22 000†	16 000†	25 000	20 200	0.06
76		30.1	21 000	22 000†	22 000	26 000‡	19 900	0.11
77		30.2	20 000	20 000†	15 000†	29 000‡	19 800	0.08
76*A*		25.9	27 000†	26 000	26 000†	31 000‡	26 900	0.10

* Tested with roller-bearings.

† Indicates a poorly defined value.

‡ For Columns Nos. 66 to 74, $K = 0.4$; for the remainder, $K = 0.6$.

§ Edge of plate 0.08 in. out of plane of angle legs.

‖ Edge of plate 0.13 in. out of plane of angle legs.

¶ Taken at load of 30 000 lb per sq in.

In studying the behavior of these columns it becomes evident that two types or conditions of failure are involved, depending upon the width of the outstanding plate. For the smaller values of $\frac{w}{t}$ (15 to 20), the column strength is substantially equal to the yield point strength of the material, as is the case for other types of columns of the small value of $\frac{l}{r}$ of about 51 to 57 used herein. For wider plates the edge of the plate begins to buckle before failure, resulting in a curved stress-strain diagram, the point at which curvature begins depending upon the width of plate.

Considering the stability of the plate against buckling, there are available some useful theoretical studies. Professer H. W. March,[13] following the analysis of Professor S. Timoshenko, has derived the following formula for the elastic buckling unit load, p, for plates free on one edge, supported (not fixed) on the other, and loaded on the ends:

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[13] See Forest Service Publication: Report 382, National Advisory Committee for Aeronautics, by G. W. Trayer and H. W. March, 1931 (Supt. of Documents), Washington, D. C.

$$p = \frac{E}{\left(\frac{w}{t}\right)^2}\left[\frac{\pi^2}{12\,(1-m^2)}\left(\frac{w}{c}\right)^2 + \frac{1}{2\,(1+m)}\right] \ldots\ldots\ldots\ldots (16)$$

In the expression, m = Poisson's ratio, c is the length of half-wave in the free edge of plate, and w, the unsupported width of plate. If $m = 0.25$ this expression becomes,

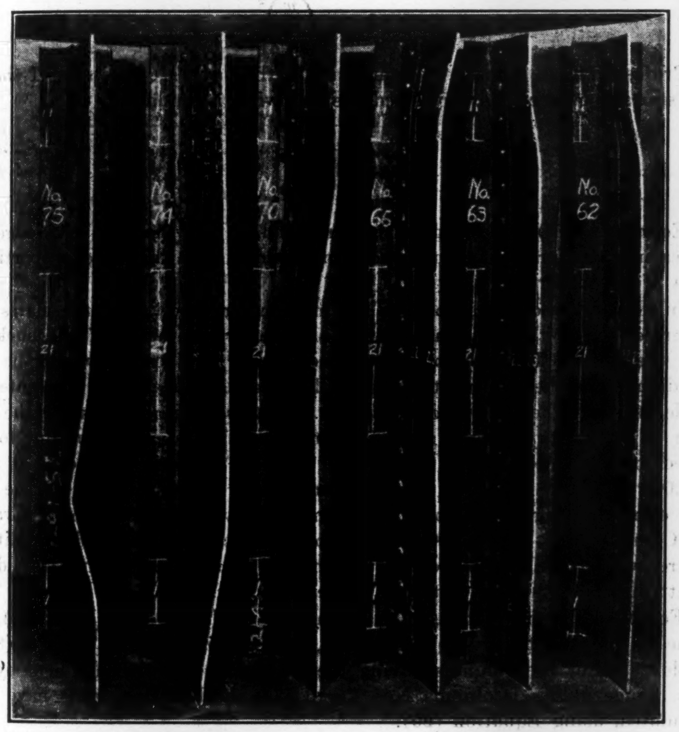

FIG. 27.—TYPICAL FAILURES OF T-SHAPED COLUMNS.

$$p = \frac{E}{\left(\frac{w}{t}\right)^2}\left[0.88\left(\frac{w}{c}\right)^2 + 0.4\right] \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

When the member is long the ratio, $\frac{w}{c}$, is very small and its square may be neglected; hence,

$$p = \frac{0.4\,E}{\left(\frac{w}{t}\right)^2} \qquad (18)$$

When the member is long and one edge is perfectly fixed, the theoretical expression for the buckling load is,

$$p = 1.16\,\frac{E}{\left(\frac{w}{t}\right)^2} \qquad (19)$$

Professor R. J. Roark[14] in connection with a series of tests on **T**-shaped and star-shaped sections, developed the following formula:

$$p = \frac{0.6\,E}{\left(\frac{w}{t}\right)^2} \qquad (20)$$

Equations (17) to (20) give the unit load, *p*, at which the free edge becomes unstable providing that such unit load is less than the proportional limit of the material.

In these tests, owing in part to the inherent crookedness of the plates and imperfections at the ends, no entirely satisfactory criterion of the buckling load was found. In order to secure a measure of unit load at which the outstanding flange became unstable the following data have been presented in Table 29: (*a*) The unit loads at which the stress-strain curves for the free edge of the plate changed slope; (*b*) the unit loads corresponding to the "knee" of the load-deflection curves; (*c*) the unit loads corresponding to a set of 0.002 in. in the deflection of the outer edge of the plate; and (*d*) the unit load corresponding to the knee of the unit load-twist curves. The low values for Columns Nos. 60 and 64 are due to initial crookedness and inability to secure even stress distribution on the ends of these members and should not be attributed to the use of the roller bearings.

For the columns with ⅜-in. flange angles (Columns Nos. 60 to 74), the value of *p*, calculated by Equation (18), has been inserted in Column (8) of Table 29. For the columns having the plate edge more firmly held by the ¾-in. flange angles (Columns Nos. 75 to 77), the buckling loads have been calculated, using Equation (20).

Column (9) of Table 29 shows the maximum observed lateral deflection of the free edge of the plate at the calculated buckling load, *p*.

In order to visualize better the relation of these buckling loads to deflections of the free edge, the values of the buckling loads of Column (8), Table 29, are shown by the heavy horizontal lines on the load-deflection curves of Fig. 25. Fig. 28 shows Equations (18) and (20) plotted against the observed unit loads. The observed buckling unit loads in Fig. 28 were obtained in most cases from the change in slope of the stress-strain curves, but in the flat end tests on the narrow columns Nos. 61 to 66, the load at the knee of the load-deflection curves was taken. The agreement between these observed unit loads and the equations is, on the whole, satisfactory.

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[14] *Engineering Record*, Vol. 74, p. 592.

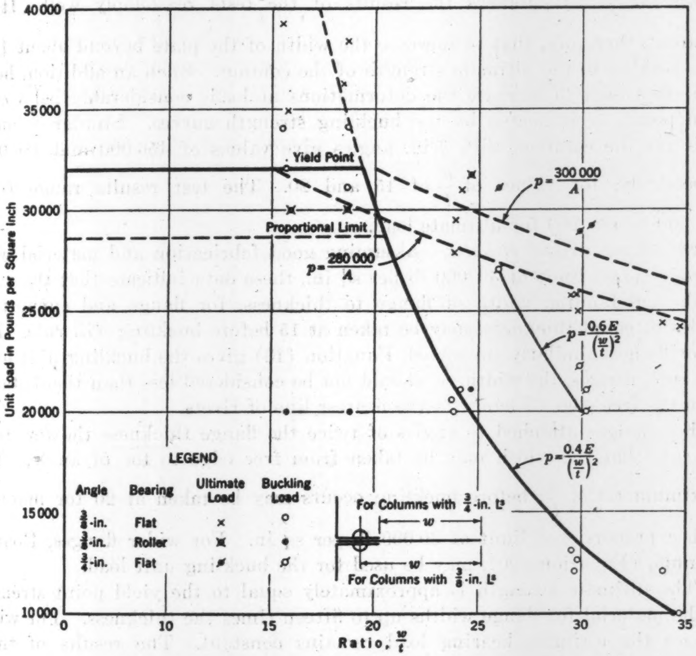


FIG. 28.—RELATIONSHIPS BETWEEN BUCKLING LOAD AND WIDTH THICKNESS RATIO OF OUTSTANDING FLANGE FOR COLUMNS OF SERIES (G).

Considering the ultimate loads carried by the several columns, it is to be noted from the results given in Table 28 that for values of $\frac{w}{t}$ greater than 18.7, and for the $\frac{3}{8}$-in. angles, the total ultimate load remains nearly constant and that, as a result, the ultimate load per square inch decreased considerably, as the ratio, $\frac{w}{t}$, increased. This condition is brought out in Fig. 28 by the two curves, $p = \frac{280\,000}{A}$ and $p = \frac{300\,000}{A}$. The first numerator, 280 000, is taken as the yield point load for a column with $\frac{w}{t} = 15$, $\frac{3}{8}$-in. angles, and the second, 300 000, for a column with $\frac{w}{t} = 20$.

These curves, therefore, represent the ultimate loads per square inch supported by the columns on the assumption that the additional width beyond 15 t or 20 t had no effect on the total load carried. The first curve drawn for $\frac{w}{t} = 15$ follows the results of the tests reasonably well. It is apparent, therefore, that to increase the width of the plate beyond about 15 t adds nothing to the ultimate strength of the column. Such an addition, however, does tend to increase the deformations at loads considerably below the yield point, as indicated by the buckling strength curves. Similar calculations for the columns with $\frac{3}{4}$-in. angles give values of 456 000 and 480 000, respectively, for values of $\frac{w}{t}$ of 15 and 20. The test results range from 470 500 to 488 200 for ultimate load.

23.—Summary of Results.—Assuming good fabrication and material with a proportional limit of 30 000 lb per sq in., these data indicate that the ratio of the outstanding width of flange to thickness, for flange and supporting angles of equal thickness, may be taken at 15 before buckling will rule. For wider flanges similarly supported, Equation (18) gives the buckling unit load. For such flanges, the width, w, should not be considered less than the distance from the free edge of flange to the nearest line of rivets.

For flanges attached to angles of twice the flange thickness the few tests indicate that the width may be taken from free edge to toe of angle. The maximum ratio, $\frac{w}{t}$, before buckling occurs may be taken at 20 for material with a proportional limit of 30 000 lb per sq in. For wider flanges, Roark's formula, (Equation (20), may be used for the buckling unit load.

The ultimate strength is approximately equal to the yield point strength of the material for flange widths up to fifteen times the thickness. For wider flanges the ultimate bearing load remains constant. The results of these tests may be safely applied to built-up I-sections having thin webs. They would indicate that the safe ratio of unsupported width to thickness may be taken at 30.

PART II.—THE STRENGTH OF COLUMNS: GENERAL DISCUSSION AND CONCLUSIONS

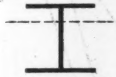
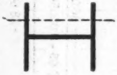
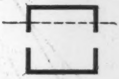
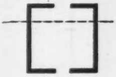
In the discussion of Part II it is assumed that the column is designed so as to act as an integral unit, namely, that the connecting details are adequate to secure integral action. The discussion will be divided into two parts: (1) Relation of test results to theoretical values; and (2) development of a rational column formula.

RELATION OF TEST RESULTS TO THEORETICAL VALUES.

24.—In Fig. 11 the results of the recent tests have been plotted on the basis of the average yield point of 36 600 lb per sq in., and from this curve and from Tables 8 and 11(c), the variations between test values and calculated values are indicated. The ratios range from a minimum of 85% to a maximum of 128 per cent. The higher values were obtained when the direction of bending was such as to permit a large over-strain on the edges of outstanding flanges. The effect of form is not great; the only case deserving notice is the laced channel column which shows a rather marked deficiency when the bending is in a plane at right angles to the webs. These tests were made with an eccentric ratio of 0.5.

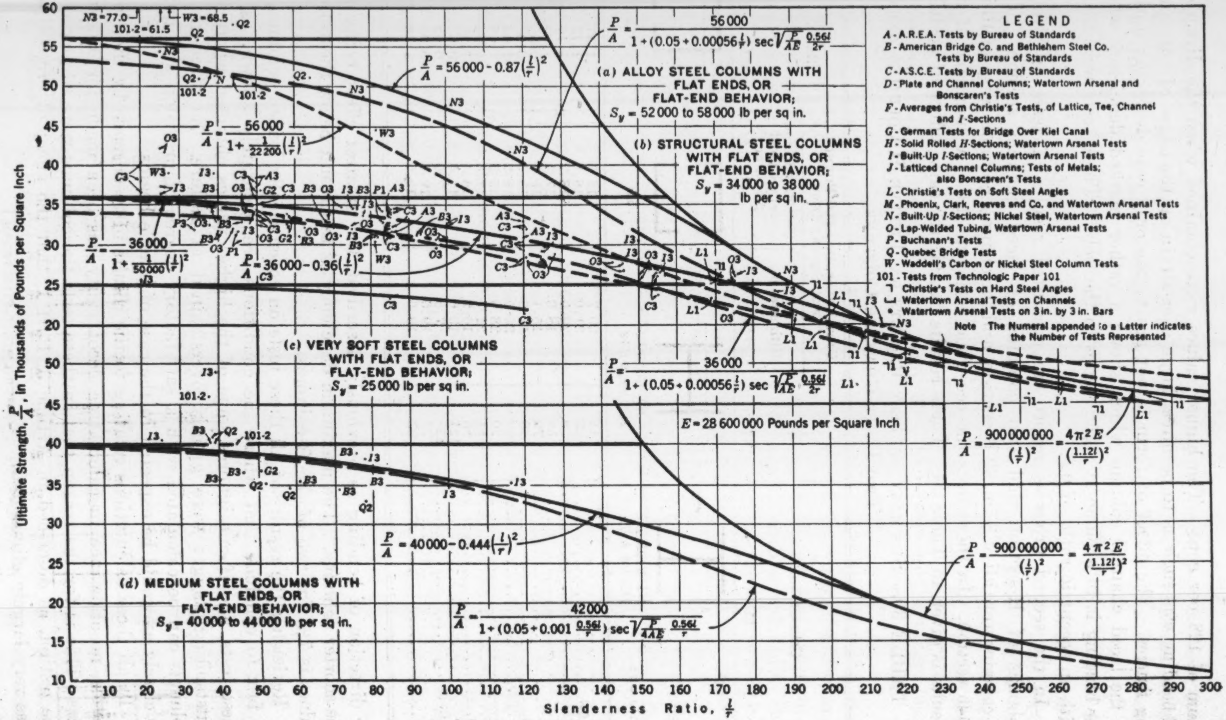
In the second progress report are presented results of tests on twelve columns of **H**-section and two columns of laced channel section. The **H**-columns were all of 8-in., 32-lb section, and the channel columns were of 10-in. 20-lb section. In these tests the eccentric ratio was 1.0. A summary of the results is given in Table 30. These results exhibit the same effect of position or direction of bending as previously described, the greatest effect being for the shorter columns. The maximum deficiency of test result below theoretical values is 7% for two of the **H**-columns.

TABLE 30.—SUMMARY OF TEST RESULTS ON TWELVE **H**-COLUMNS AND TWO LACED-CHANNEL COLUMNS

Column No. (1)	$\frac{l}{r}$ (2)	Unit Loads, in 1 000 Pounds per Square Inch: Ultimate, p (3)	Unit Loads, in 1 000 Pounds per Square Inch: Calculated from average yield point, p_1 (4)	Ratio, $\frac{p}{p_1}$ (5)
Columns:  *H*1, *H*10*B*, *H*2 *H*3, *H*4, & *H*5		 *H*1, *H*6, *H*7 *H*8, *H*9, & *H*10	 5*B*	 8*A*
*H*1	11	20.7	18.6	1.11
*H*10*B*	12	21.0	18.8	1.12
*H*2	29	20.0	18.2	1.10
*H*3	50	18.0	17.1	1.05
*H*4	70	15.1	15.8	0.96
*H*5	90	12.6	13.5	0.93
*H*1	20	26.9	18.2	1.48
*H*6	20	28.9	19.3	1.50
*H*7	51	21.7	17.6	1.24
*H*8	86	16.2	14.4	1.12
*H*9	121	10.6	11.4	0.93
*H*10	158	7.8	7.9	0.99
5*B*	22	19.0	19.8	0.95
8*A*	33	18.0	15.6	1.18

Reviewing the results of all these tests on eccentrically loaded columns, it is fair to conclude that where the conditions are accurately known, as in these tests, and if one ignores some of the higher values obtained where the outstanding flanges were greatly overstrained at failure, the strength of columns can be calculated with a satisfactory degree of precision and quite as closely as would be expected in the case of simple beams.

In all these calculations relating to the results of tests made by the Committee, no allowance in calculating eccentricity has been made for crookedness This is for the reason that in these tests the columns were so placed in the testing, as described in Article 3, Part I, that the effect of crookedness was very largely eliminated.


Ultimate Strength, $\frac{P}{A}$, in Thousands of Pounds per Square Inch
Slenderness Ratio, $\frac{l}{r}$
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230 240 250 260 270 280 290 300
60 55 50 45 40 35 30 25 20
50 45 40 35 30 25 20 15 10
N3 = 77.0
101-2 = 61.5
W3 = 68.5
$\frac{P}{A} = \frac{56\,000}{1 + (0.05 + 0.00056\frac{l}{r})\sec\sqrt{\frac{P}{AE}}\frac{0.56l}{2r}}$
$\frac{P}{A} = 56\,000 - 0.87\left(\frac{l}{r}\right)^2$
$\frac{P}{A} = \frac{56\,000}{1 + \frac{1}{22\,200}\left(\frac{l}{r}\right)^2}$
(a) ALLOY STEEL COLUMNS WITH FLAT ENDS, OR FLAT-END BEHAVIOR; S_y = 52 000 to 58 000 lb per sq in.
(b) STRUCTURAL STEEL COLUMNS WITH FLAT ENDS, OR FLAT-END BEHAVIOR; S_y = 34 000 to 38 000 lb per sq in.
$\frac{P}{A} = \frac{36\,000}{1 + \frac{1}{50\,000}\left(\frac{l}{r}\right)^2}$
$\frac{P}{A} = 36\,000 - 0.36\left(\frac{l}{r}\right)^2$
$\frac{P}{A} = \frac{36\,000}{1 + (0.05 + 0.00056\frac{l}{r})\sec\sqrt{\frac{P}{AE}}\frac{0.56l}{2r}}$
(c) VERY SOFT STEEL COLUMNS WITH FLAT ENDS, OR FLAT-END BEHAVIOR; S_y = 25 000 lb per sq in.
E = 28 600 000 Pounds per Square Inch
$\frac{P}{A} = \frac{900\,000\,000}{\left(\frac{l}{r}\right)^2} = \frac{4\pi^2 E}{\left(\frac{1.12l}{r}\right)^2}$
(d) MEDIUM STEEL COLUMNS WITH FLAT ENDS, OR FLAT-END BEHAVIOR; S_y = 40 000 to 44 000 lb per sq in.
$\frac{P}{A} = 40000 - 0.444\left(\frac{l}{r}\right)^2$
$\frac{P}{A} = \frac{42000}{1 + (0.05 + 0.001\frac{0.56l}{r})\sec\sqrt{\frac{P}{4AE}}\frac{0.56l}{r}}$
$\frac{P}{A} = \frac{900\,000\,000}{\left(\frac{l}{r}\right)^2} = \frac{4\pi^2 E}{\left(\frac{1.12l}{r}\right)^2}$
LEGEND
A - A.R.E.A. Tests by Bureau of Standards
B - American Bridge Co. and Bethlehem Steel Co. Tests by Bureau of Standards
C - A.S.C.E. Tests by Bureau of Standards
D - Plate and Channel Columns; Watertown Arsenal and Bonscaren's Tests
F - Averages from Christie's Tests, of Lattice, Tee, Channel and I-Sections
G - German Tests for Bridge Over Kiel Canal
H - Solid Rolled H-Sections; Watertown Arsenal Tests
I - Built-Up I-Sections; Watertown Arsenal Tests
J - Latticed Channel Columns; Tests of Metals; also Bonscaren's Tests
L - Christie's Tests on Soft Steel Angles
M - Phoenix, Clark, Reeves and Co. and Watertown Arsenal Tests
N - Built-Up I-Sections; Nickel Steel, Watertown Arsenal Tests
O - Lap-Welded Tubing, Watertown Arsenal Tests
P - Buchanan's Tests
Q - Quebec Bridge Tests
W - Waddell's Carbon or Nickel Steel Column Tests
101 - Tests from Technologic Paper 101
˥ Christie's Tests on Hard Steel Angles
⊔ Watertown Arsenal Tests on Channels
• Watertown Arsenal Tests on 3 in. by 3 in. Bars
Note The Numeral appended to a Letter indicates the Number of Tests Represented

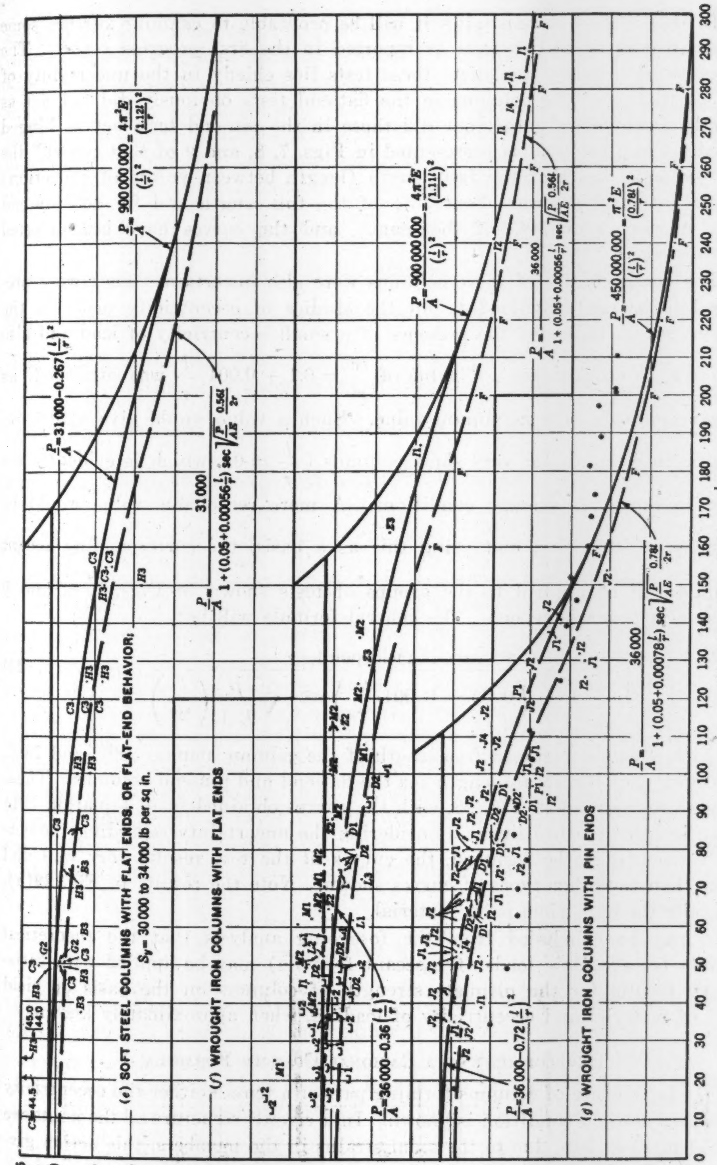


FIG. 29.—RELATION OF ULTIMATE STRENGTH OF STEEL COLUMNS TO SLENDERNESS RATIO.

In the light of these studies it will be profitable to examine further some of the results of earlier tests as reported in the first progress report. The difficulty of properly analyzing these tests lies chiefly in the uncertainty of end conditions. The columns in the flat-end tests obviously did not act as strictly fixed-end columns; nor did those in the pin-end tests act as hinged ended. From the studies represented in Figs. 7, 8, and 9 of that report,[15] the best estimate was that the free length (length between points of inflection) of the flat-end tests was about 56% of the full length and for the pin-end tests it was about 78% of the length, and the curves have been plotted accordingly.

The eccentricities of these columns were also uncertain. They were presumed to be centrally loaded, but the studies of eccentricity made in the present report indicated the presence of a small eccentricity of load and also some due to crookedness. A value of $\frac{ec}{r^2} = 0.1 + 0.001 \frac{l}{r}$ was suggested[16] as a near approach to a maximum value. Such a value would give a 10% reduction in strength for very short columns ($\frac{l}{r} = 0$), which is evidently too large to represent average conditions. A more reasonable value would be $\frac{ec}{r^2} = 0.05 + 0.001 \frac{l}{r}$, and using this as a basis, the corresponding secant formula will be applied to the groups of tests shown in Figs. 7, 8, and 9 of the first progress report. The general formula will be:

$$\frac{P}{A} = \frac{\text{Yield point}}{1 + \left(0.05 + 0.001 \frac{l'}{r}\right) \sec \sqrt{\frac{P}{EA}} \left(\frac{l'}{2r}\right)} \quad \text{..........} (21)$$

in which, l' is the assumed free length of the column, namely 56% and 78%, respectively, of the actual length for the flat-end and pin-end columns. These data are reproduced in Fig. 29, with the curves obtained from Equation (21) shown in heavy dashed lines. Considering the uncertainty regarding the correct value of l' to be assumed, the curves fit the test results very well and better than the other types of curves shown. Note the results in Fig. 29(*a*), especially the high yield point material.

It may be concluded from the foregoing analysis that the theoretical formula for eccentric loads (the secant formula) may be applied with satisfactory results for the ultimate strength of columns on the basis of yield point of material and eccentricity of loading, when approximately known.

DEVELOPMENT OF A RATIONAL COLUMN FORMULA

25.—In the case of columns forming part of a truss neither the eccentricity of loading nor end condition is known. In a riveted structure all the joints are twisted more or less, due to the axial strains in the members, this action giving rise to the so-called secondary stresses. These stresses vary greatly, being in some cases as high as 50% or more of the primary or axial unit stresses. Such high stresses occur rarely, except in the case of relatively deep members subjected to reversed curvature. For members bent in single curvature, such as many top-chord members, a value of 20 to 25% would be a reasonable maximum. In general, the more slender the member, the smaller the secondary stress. To translate secondary stress into equivalent eccentricity of load it is to be noted that the quantity, $\frac{ec}{r^2}$, or eccentric ratio, corresponds to the ratio of secondary to primary stress; that is, a load having such an eccentricity as to make $\frac{ec}{r^2} = 0.25$, will produce bending (secondary) stress at the ends of 25% of the axial stress.

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[15] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), pp. 1527–1529.

[16] *Loc. cit.*, p. 1544.

As a fair estimate of eccentricity of load for columns of all lengths, it is proposed to use $\frac{ec}{r^2} = 0.25$. This will include a reasonable value for crookedness also. If crookedness is represented by $\frac{ec}{r^2} = 0.001\,\frac{l}{r}$, then for $\frac{l}{r} = 100$, this factor would be 0.1, leaving for end eccentricity, $\frac{ec}{r^2} = 0.15$, which would be a fair value for the secondary stresses in a member of this slenderness.

In regard to the free length of column to be assumed, it is to be noted that in structures such as herein considered, the columns, although loaded eccentrically, are not free to turn at the ends; the angular movement at these points is controlled by the deformation of the structure as a whole. It was shown[17] in the second progress report, that the theoretical strength of such columns was nearly equal to that of an eccentrically loaded hinged-end column of half the length, with an eccentricity corresponding to the secondary stresses assumed at the ends. However, instead of assuming the half length, a free length of two-thirds or three-fourths might be taken as giving conservative values. Taking the larger value of three-fourths, for example, a rational formula is derived for ultimate strength, as follows,

$$\frac{P}{A} = \frac{\text{Yield point}}{1 + 0.25 \sec \sqrt{\frac{P}{EA}}\left(\frac{0.75\, l}{2\, r}\right)} \qquad \ldots\ldots\ldots\ldots (22)$$

It is believed that Equation (22) will give reasonable and conservative results for the ultimate strength of the ordinary compression member in a riveted structure. Its construction is logical, and in special cases the eccentric ratio of 0.25 and the assumed free column length may be modified to fit the conditions. Where the secondary stresses are computed, the actual ratio may be used (approximately the average of the two ends) plus about $0.001\,\frac{l}{r}$ for crookedness, instead of the constant, 0.25. This process may be followed to advantage in checking over the design of the various compression members where the secondary stresses have been computed.

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[17] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1220.

For the purpose of further study, Fig. 30 has been prepared, in which all the tests of the Committee have been reduced to an eccentric ratio of 0.25 instead of the value of 0.5 and 1.0 used in the tests. This reduction has been made by assuming that the deviation from the theoretical values, as given in Table 8, would be about one-half as great for an eccentric ratio of 0.25 as for the 0.5 ratio used, and about one-fourth as great as for the eccentric ratio of 1.0. These values may not be very exact, but they are based on the assumption that a test with no eccentricity would show failure at the average yield point of the material. For the shorter lengths, test results would somewhat exceed the yield point.

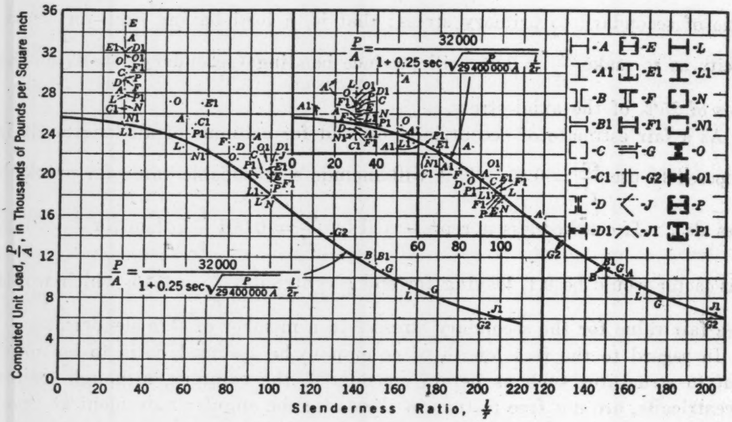

FIG. 30.—RELATIONSHIP BETWEEN COMPUTED MAXIMUM LOADS FOR $\frac{ec}{r^2}=0.25$, AND SLENDERNESS RATIO: (*a*) NO ALLOWANCE MADE FOR VARIABILITY OF YIELD POINT; (*b*) UNIT LOADS REDUCED TO CONSTANT YIELD POINT, $S_y = 32\,000$ POUNDS PER SQUARE INCH.

Fig. 30(*a*) shows the results of the tests so modified, using the actual yield points of the individual specimens; Fig. 30(*b*) shows the results reduced to a yield point of 32 000 lb per sq in. In both cases the secant curve corresponding to a yield point of 32 000 lb per sq in. and an eccentric ratio of 0.25 has been drawn. The deviations from the curve of the test results in Fig. 30(*a*) indicate what may be expected from columns of the material actually furnished; Fig. 30(*b*) shows deviations that may be expected due to differences in form and direction of bending, but with uniform yield point. In all cases, the direction of bending relative to the section shown on Fig. 30 is in a vertical plane.

It is evident from Fig. 30(*b*) that the ultimate strengths of columns loaded under known conditions can be closely calculated from the yield point strength of the material. From Fig. 30(*a*) it is also apparent that a yield point strength of 32 000 may well be used for columns of the grade of material used in these tests. The deficiency in strength below the theoretical is a maximum of 3½ per cent.

It should be recalled at this point that the maximum thickness of material used in these columns was about 0.9 in.

26.—A Useful Limit Point as a Measure of Column Strength.—In the foregoing discussion it has been shown that the yield point strength of the material is a satisfactory basis for calculating the ultimate strength of a column. It is true, however, that practically all the columns having a slenderness ratio of 100 or less showed some permanent deformation at loads somewhat less than the calculated ultimate values; and, in general, the shorter the column the greater is this difference. Expressed in another way, the deflections of the columns at loads approaching the yield point were greater than those calculated, and indicated some permanent set. This general behavior is well known and was taken into account by the earlier Special Committee of the Society on Steel Columns and Struts by defining[18] a "Useful Limit Point" (U. L. P.), and using this point, rather than the ultimate strength, as a basis for a suggested column formula. This U. L. P. was defined as the point on the stress-strain curve of the column under central loading at which the slope of the tangent was one-half that of the straight portion of the diagram. In the tests conducted by the present Committee, using eccentric loads, a U. L. P. could not be determined in the same manner, but strain-gauge measurements were made in all cases on the most stressed fibers, and the loads noted at which these fibers reached their proportional limits and also their yield points. The detailed results are given in Table 31, together with the ratios of the loads to the calculated ultimate. Another criterion of behavior is given by the load deflection curves as shown in Fig. 3. A deviation of the measured deflections from the calculated elastic values indicates inelastic deflection. In Table 31, the loads at which the actual deflections were 15% greater than the theoretical elastic deflections, are given in Column (6), and Column (9) gives the ratios of these loads to the calculated ultimate. These values average about the same as those of Column (8), although a few are considerably lower. Both series of values show again the relatively poor results from the channel sections. They also indicate weakness of the very thin sections used in Columns Nos. 16, 17, and 30. Some of the sections made up of channel and **I**-beams also showed up rather low by this method of comparison. The best relative results are from the **H**-sections and those made up of plates and channels.

As compared to yield point values the proportional limits are not well defined and are widely scattered. The yield points are well defined and in view of the small excess deflections at these corresponding loads and the large margin between these loads and the ultimate loads for the shorter columns, it would appear reasonable to take the values of Column (5), Table 31, as fairly representing a U. L. P. for the column. These values are plotted in

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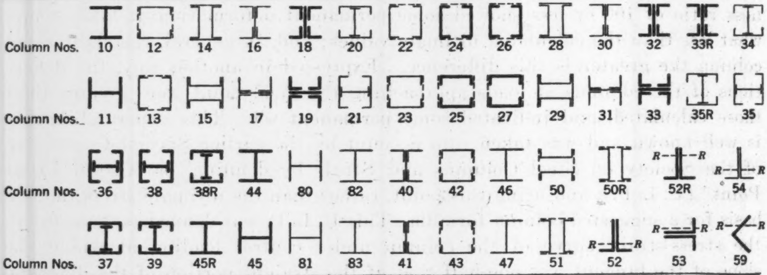
[18] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 1618.

TABLE 31.—COMPARISON OF UNIT LOADS CALCULATED BY SECANT FORMULA WITH UNIT LOADS AT WHICH PROPORTIONAL LIMIT OR YIELD POINT, IN MOST STRESSED FIBERS, OR 15 PER CENT. EXCESS DEFLECTION WAS OBSERVED.


Column Nos. 10 12 14 16 18 20 22 24 26 28 30 32 33R 34
Column Nos. 11 13 15 17 19 21 23 25 27 29 31 33 35R 35
Column Nos. 36 38 38R 44 80 82 40 42 46 50 50R 52R 54
R R R R
Column Nos. 37 39 45R 45 81 83 41 43 47 51 52 53 59
R R R R R R

		UNIT LOADS, IN 1 000 POUNDS PER SQUARE INCH						
Column No. (1)	$\frac{l}{r}$ (2)	Unit load calculated from secant formula,* p'_1 (3)	Unit load at which proportional limit was observed in most stressed fiber, p_L (4)	Unit load at which yield point was observed in most stressed fiber, p_y (5)	Unit load at which excess deflection was 15% more than calculated elastic deflection, p_e (6)	Ratio, $\frac{p_L}{p'_1}$ (7)	Ratio, $\frac{p_y}{p'_1}$ (8)	Ratio, $\frac{p_e}{p'_1}$ (9)
10.....	30	20.1	16.9	17.9	19.5	0.84	0.89	0.97
11.....	30	20.1	16.7	19.2	20.2	0.83	0.96	1.01
12.....	30	23.2	18.4	18.8	18.7	0.79	0.81	0.81
13.....	31	23.2	20.0	21.0	22.5	0.86	0.91	0.97
14.....	30	23.7	19.8	22.0	21.0	0.84	0.93	0.89
15.....	30	23.7	19.0	20.9	25.5	0.80	0.88	1.08
16.....	29	24.7	19.2	20.9	20.0	0.78	0.85	0.81
17.....	30	24.7	17.4	19.6	20.0	0.70	0.79	0.81
18.....	30	23.6	18.7	23.1	25.4	0.79	0.98	1.08
19.....	30	23.6	17.0	19.3	19.0	0.72	0.82	0.80
20.....	29	27.5	20.6	21.5	21.0	0.75	0.78	0.76
21.....	30	27.3	22.0	22.7	21.2	0.81	0.83	0.78
22.....	96	16.9	14.3	16.2	16.5	0.85	0.96	0.98
23.....	64	21.2	15.8	18.3	18.3	0.75	0.86	0.86
24.....	31	21.6	14.1	19.0	16.9	0.65	0.88	0.78
25.....	29	21.7	12.7	17.5	13.0	0.59	0.81	0.60
26.....	100	15.2	11.7	14.0	13.6	0.77	0.92	0.89
27.....	65	19.0	12.8	16.4	15.5	0.67	0.86	0.82
28.....	80	19.3	16.4	17.5	17.5	0.85	0.91	0.91
29.....	98	16.7	14.8	15.7	15.7	0.89	0.94	0.94
30.....	82	20.1	16.7	17.0	....‡	0.83	0.85	...
31.....	100	17.3	15.8	17.3†	17.9	0.91	1.00	1.03
32.....	84	18.0	14.3	16.5	17.0	0.79	0.92	0.94
33.....	98	16.2	14.3	16.2	19.6	0.88	1.00	1.21
33*R*...	52	20.9	15.2	18.4	20.0	0.73	0.88	0.96
34.....	98	17.2	14.2	16.2	16.0	0.83	0.94	0.93
35.....	100	16.7	14.0	16.3	15.9	0.84	0.98	0.95
35*R*...	67	21.9	15.8	18.8	20.7	0.72	0.86	0.94
36.....	30	22.6	16.5	20.7	18.0	0.73	0.92	0.80
37.....	30	22.1	16.2	19.3	19.5	0.73	0.87	0.88
38.....	100	15.6	13.1	15.1	15.1	0.84	0.97	0.97
38*R*...	62	20.2	15.3	17.7	17.3	0.76	0.88	0.86
39.....	93	16.5	12.8	14.9	15.2	0.78	0.90	0.92
44.....	95	14.9	13.0	13.8	14.2	0.87	0.93	0.95
45.....	100	14.7	13.8	14.0	14.2	0.94	0.95	0.97
45*R*...	58	18.8	14.6	15.3	16.3	0.78	0.81	0.87
80.....	30	26.9	24.5†	25.2†	24.8	0.91	0.94	0.92
81.....	91	18.8	17.4†	17.9†	17.7	0.93	0.95	0.94
82.....	30	23.6	21.2†	....	23.0	0.90	....	0.97
83.....	60	21.4	18.5	19.3	20.0	0.86	0.90	0.93
40.....	103	11.7	10.1	11.5	13.3	0.86	0.98	1.14

* Values in Column (3) were taken from Table 8, Column (9); Table 11(*c*), Column (9); and Table 10(*d*). Columns Nos. 40 to 43, 46, and 47 were tested with oblique eccentricity, and the values reported take account of actual eccentricities.

† Estimated.

‡ Column failed before 15% excess deflection was observed.

TABLE No. 31.—(*Continued*)

Column No.	$\frac{l}{r}$	UNIT LOADS, IN 1 000 POUNDS PER SQUARE INCH				Ratio, $\frac{p_L}{p'_1}$	Ratio, $\frac{p_y}{p'_1}$	Ratio, $\frac{p_e}{p'_1}$
		Unit load calculated from secant formula,* p'_1	Unit load at which proportional limit was observed in most stressed fiber, p_L	Unit load at which yield point was observed in most stressed fiber, p_y	Unit load at which excess deflection was 15% more than calculated elastic deflection, p_e			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
41.....	55	14.9	11.7	13.6	15.8	0.79	0.91	1.06
42.....	47	14.0	10.4	13.1	14.4	0.74	0.94	1.03
43.....	32	14.5	10.0	13.0	16.0	0.69	0.90	1.10
46.....	52	11.9	11.0	12.0	14.3	0.92	1.01	1.20
47.....	91	11.8	11.8	13.2	....‡	1.00	1.12	....
50.....	150	10.0	8.9	10.2	....‡	0.89	1.02	....
50*R*...	147	10.3	9.2	10.5†	10.5	0.89	1.02	1.02
51.....	148	10.2	9.0	10.2	....‡	0.88	1.00	....
52.....	128	12.6	11.5	12.1	12.2	0.91	0.96	0.97
52*R*...	128	12.7	10.8	12.4	12.8	0.85	0.98	1.01
53.....	154	9.8	8.9	9.6	9.7	0.91	0.98	0.99
54.....	202	6.3	5.6	....	4.7	0.89	....	0.75
59.....	102	18.1	16.5	....	12.0	0.91	....	0.66

* Values in Column (3) were taken from Table 8, Column (9); Table 11 (*c*), Column (9) and Table 10 (*d*) Columns Nos. 40 to 43 46, and 47 were tested with oblique eccentricity, and the values reported take account of actual eccentricities.

† Estimated.

‡ Column failed before 15% excess deflection was observed.

Fig. 31. The results are considerably less than those in Fig. 30(*a*) corresponding to the ratios of Column (7) of Table 31. Secant curves are shown for the average yield point of the material, 36 600, and the arbitrary values of 32 000 and 30 000. The effect of direction of bending is obvious and also the fact that the longer columns are less affected by over-strain of extreme fibers than the shorter ones. A value of 29 000 for the numerator of a secant formula would produce a curve having about the same relative position in this group of test values as the curve drawn for 32 000 in Fig. 30(*a*). This is a 10% reduction of the numerator constant, or a 10% reduction in load, for very short columns. For the long columns ($\frac{l}{r} = 100$, for example), the reduction is about 7% and for $\frac{l}{r} = 150$, only 3 per cent.

Whatever the basis of estimate may be, ultimate strength, or strength at over-strain, the most logical method of treating the question is to select a suitable numerator for the secant formula. Ultimate strengths based on yield point are more consistent than the values shown in Fig. 31, and with the selection of a proper value for the basic unit stress, or factor of safety, the results will be quite as satisfactory as those obtained by using the proportional limit or by a detailed consideration of a U. L. P. Starting with any given basic working stress for short columns and using the same factor of safety for all lengths, the use of a formula based on a value of 32 000 will give slightly lower working stresses for long columns than if the lower value of 29 000 or 30 000 is used for the numerator of the secant formula. Considering the margin of strength of the short column beyond its yield point, this relationship appears to be reasonable.

A general formula for working stresses, therefore, would be,

$$p = \frac{\dfrac{S_y}{m}}{1 + a \sec \sqrt{\dfrac{pm}{E}}\left(\dfrac{l'}{2r}\right)} \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

in which, p = working stress; S_y = yield point of material; m = factor of safety; a = eccentric ratio assumed; and l' = assumed free length of column.

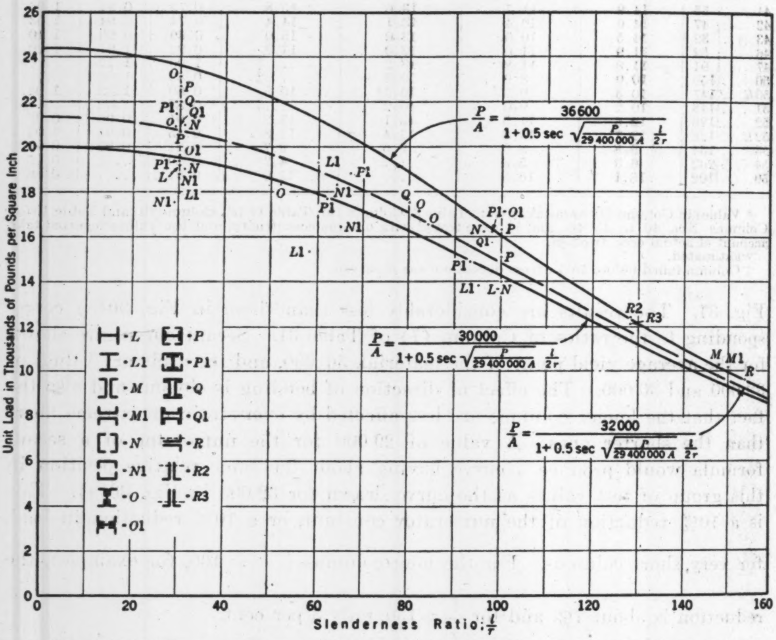


FIG. 31.—RELATIONSHIP BETWEEN UNIT LOAD CORRESPONDING TO YIELD POINT OF MATERIAL IN MOST STRESSED FIBERS AND SLENDERNESS RATIO, OF COLUMNS ECCENTRICALLY LOADED; FROM STRAIN-GAUGE MEASUREMENTS.

It is suggested, therefore, that the yield point may well be taken as the basis for ultimate strength calculations, and that working values may be obtained by applying a suitable factor of safety to the ultimate values calculated.

27.—The Parabola as a Working Formula.—An objection to the secant formula commonly raised is that it is somewhat complex, as values of $\frac{P}{A}$ for given values of $\frac{l}{r}$ can be determined only by successive approximations. It is to be noted, however, that values of $\frac{l}{r}$ for given values of $\frac{P}{A}$ can be computed directly so that curves and tables can readily be prepared. If a simpler formula is desired, it will be found that a parabolic curve may be drawn, which will closely approximate the secant curve up to a value of $\frac{l}{r}$ of 120 to 160, depending on the grade of steel and the free length assumed. The equation of this parabola will be, for ultimate strength,

$$p = \frac{P}{A} = \frac{S_y}{1 + a} - K\left(\frac{l}{r}\right)^2 \text{ ....} (24)$$

The constant, K, is well determined by assuming the parabola to cross the secant curve at $\frac{l'}{r} = 100$.

For $S_y = 32\,000$, $a = 0.25$, and $l' = \frac{3}{4}l$, the parabola will be:

$$p = 25\,600 - 0.425\left(\frac{l}{r}\right)^2 \text{ ....} (25)$$

Fig. 32 shows the relation between this curve and the corresponding secant curve. The maximum deviation is less than 2% up to a value of $\frac{l}{r} = 160$. For higher steels the curves separate more rapidly beyond the point of intersection.

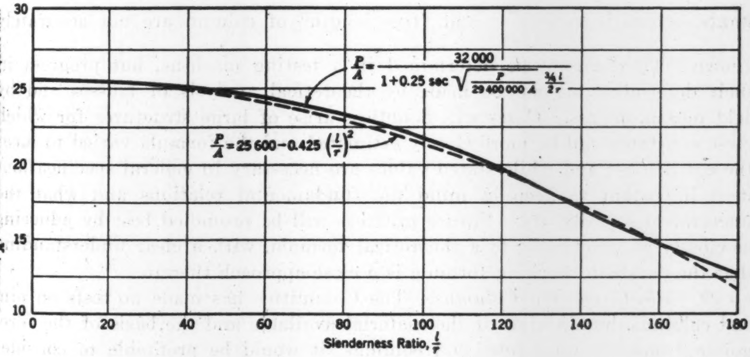


FIG. 32.—COMPARISON OF SECANT AND PARABOLIC FORMULAS FOR $\frac{ec}{r^2} = 0.25$.

For structural steel used for bridges under present specifications and where the working tensile unit stress is 18 000 lb per sq in., the Committee suggests the following working formula for compression members with riveted ends:

$$p = 15\,000 - 0.25\left(\frac{l}{r}\right)^2 \text{ ....} (26)$$

This formula gives a uniform factor of safety of 1.7 with respect to the ultimate values of Equation (25). Due allowance should be made for

material thicker than 1 in., unless the minimum yield point of such material exceeds 30 000 lb per sq in.

In judging the merit of the suggested factor of 1.7 it should be noted that an allowance of a 25% bending stress (for short lengths) has already been made. Assuming the same condition for the tension member the actual fiber stress would be 18 000 × 1.25 = 22 500, and a factor of 1.7 would imply an ultimate value of 38 250 lb per sq in., which is considerably beyond the average yield point.

For higher steels similar formulas can be developed, using a conservative value of yield point. Where columns of unusual length need to be considered, the secant formula itself should be used.

28.—Straight-Line Formulas.—It is obvious from an examination of the test results that a straight-line formula with a horizontal "stopper" at the left can be written that will be satisfactory up to a value of $\frac{l}{r}$ of 100 or more. Straight-line formulas are numerous, but they are not adapted to columns of unusual length, nor are they readily modified to fit a variety of steels without recourse to more fundamental relations. The Committee prefers, therefore to suggest the theoretical formula as a basis, with working parabolic formulas derived in a definite manner. The secant formula is correct in principle and lends itself readily to variations in conditions. The basic constants, eccentric ratio, $\frac{ec}{r^2}$, and "free length" of column are not accurately known. They cannot be determined in a testing machine, but progress in their determination can be made by theoretical analysis of trusses and by field measurements. Cases will doubtless arise of large structures for which these constants can be more closely estimated and the formula varied to meet the conditions; and while stated values are necessary in general specifications it is important to keep in mind the fundamental relations and what the uncertainties really are. Future progress will be promoted best by adhering as closely as practicable to a theoretical formula, with a clear understanding that the parabolic working formula is a close approach thereto.

29.—Pin-Connected Columns.—The Committee has made no tests on pin-end columns, but in view of the material available and the basis of the foregoing discussion on riveted-end columns, it would be profitable to consider the factors involved in the problem of the pin-end column.

It is well known that in a test, a pin-end member, if well centered, will behave as a flat-end member up to a point approaching the ultimate strength, and this was taken into account in grouping the results of tests in the first progress report previously mentioned.[19] It is also to be noted in Fig. 9[20] of the first progress report that where the pin-end member was notably weaker than a flat-end member, the corresponding free length of column was found to be about 78% of the total length. Whether a pin-end member actually turns on the pin depends on the frictional resistance and the turning moment developed by the deformations of the structure as a whole (represented by the secondary stresses) and by the bending of the column under its load.

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[19] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), pp. 1527–1528, Figs. 7 and 8.

[20] *Loc. cit.*, p. 1529.

It will be useful to estimate the bending moment or equivalent eccentricity in a pin-end column which is likely to exist at the time the column begins to turn on the pin. From data recently submitted to the Committee by O. E. Hovey, M. Am. Soc. C. E., the pin friction may be taken at about 27% and the ratio of radius of gyration to pin diameter at about 75% for vertical posts. For end posts it will be more than unity. Assuming values of 27% and 75%, respectively, and assuming further a radius of gyration equation to 0.4 of the depth, the bending moment at the point of turning is $M = \frac{0.27\ P\ D}{2} = 0.18\ Pr$, and the bending fiber stress is $f = \frac{Mc}{A\,r^2} = \frac{P}{A} \times 0.18\,\frac{c}{r}$. Taking $c = 1.25\ r$, approximately, $f = 0.225\,\frac{P}{A}$; that is, the bending stress is the same as would result in a hinged-end column with an eccentric ratio, $\frac{ec}{r^2}$, of 0.225. For smaller pins, as in the end post, the corresponding value of $\frac{ec}{r^2}$ would be less. For increased friction it would be more. This eccentric ratio of 0.225 is about what was assumed in developing the formula for riveted ends, and indicates that if the secondary stresses at the end are not in excess of 0.225, the member will probably act as a riveted end member. On the other hand, if the frictional resistance is less than that assumed, the member will turn on the pins, but the eccentricity will be limited to an amount corresponding to the pin friction. If a pin-end member were strictly a hinged-end member without end friction, there would be no eccentricity of load, except that due to crookedness and eccentricity of pin-hole. This would be very much less than the 25% assumed in deriving the column formula. For such a member, a theoretically correct column formula would follow a secant curve for a hinged-end member, but would cut the Y-axis at a point considerably higher than the equation for riveted-end columns with an eccentricity of 0.25. However, this is somewhat speculative, as a pin-end column is never free to turn at the ends.

Considering what happens as the load on a column approaches the ultimate, the analysis of the top chord section given in the second progress report shows that the riveted column is held to certain deformations at the ends and behaves at all points substantially as a fixed-end column with certain definite angular changes at the ends. In the case of a pin-end column approaching its ultimate load, such restraint could not be assumed, and the failing point would almost certainly be less than the riveted-end column.

Considering all these factors, it would appear reasonable to use the same theoretical formulas for pin-end columns as for columns with riveted ends, with the free length increased to, say, 85% of the full length. This is largely a matter of judgment, but it seems clear that 100% need not be taken, as this assumes frictionless ends, which would imply very slight eccentricity.

The parabolic formula corresponding to Equation (26) for riveted ends would then be,

$$p = 15\,000 - \frac{1}{3}\left(\frac{l}{r}\right)^2 \dots\dots\dots\dots\dots\dots (27)$$

30.—Strength of Columns Subject to Reversed Curvature.—Where the joint deformations are such as to bend a member into a reversed curve, as is frequently the case, the maximum stresses will be at the ends and will be equal to the direct stress plus the secondary stress. The secondary stresses are likely to be greater in such members, especially if relatively short, than where the member is bent in single curvature. On the other hand, since the point of inflection is near the center of the member, the effect of crookedness will be very small, and need scarcely be considered. If the tensile working stress is 18 000, for example, then a basic working stress of 15 000 lb per sq in. for columns would allow an excess of 20% of secondary stress in the short column over that in the tension members, with the same factor of safety measured by the yield point. This appears to be reasonable, and, therefore, it may be concluded that the same basic stress may well be used for both types of columns.

In the case of the column bent in reversed curvature, little or no consideration need be given to length of column, for the length ratios used in practice, as the maximum stress will be at the end. A flat value of 15 000, therefore, might be used for such columns, if sufficient material is added to the section to provide for any excessive bending stresses. For very long members, the effect of crookedness and buckling will be to increase, somewhat, the bending stresses at the ends over the values calculated as secondary stresses, but here, again, the values of the secondary stresses are likely to be small.

In ordinary practice the same formula will be used for all columns, but from the foregoing discussion it will be seen that a calculation of secondary stresses would enable the design to be adjusted somewhat to fit the requirements, thus giving a structure of more uniform strength.

31.—Effect of Weight of Member.—For long compression members placed in a horizontal or approximately horizontal position, the effect of the weight of the member needs to be considered. This effect may be divided for convenience into two parts: (*a*) The direct effect of the bending moments due to the weight of the member; and (*b*) the effect of the deflection of the member in adding to the eccentricity of the direct forces. The magnitude of both these effects depends greatly upon the manner of erection. If such members are supported only at the ends and are fully riveted in that position, both the center bending moment and deflection are those for a simply supported beam. The stresses due to the bending moment are closely given by the expression, $f = \frac{1}{25} c \left(\frac{l}{r}\right)^2$. The center deflection will be $\Delta = \frac{l^2}{240\,E}\left(\frac{l}{r}\right)^2$. This deflection will about as great as the assumed eccentricity due to crookedness for a value of $c\left(\frac{l}{r}\right)^3 = 7\ 300\ 000$. For example, when $c = 10$, $\frac{l}{r} = 90$.

If, on the other hand, the erection procedure is such that the member supports its load as a fixed-end beam the direct bending effect will be one-third of the foregoing, or $f = \frac{1}{75} c \left(\frac{l}{r}\right)^2$, and the deflection one-fifth. For $c = 10$ and $\frac{l}{r} = 100$ (rather large values), the former will be about 1 300 lb per sq in., and the latter about 28% of that assumed for crookedness, which is a negligible amount. Under the usual methods of design and erection the first condition will scarcely prevail, but the second condition will not be fully realized, except where care is taken to eliminate dead load deformation stresses, as is now done in the case of large and important structures. It may be concluded, therefore, that the effect of weight on eccentricity may be neglected, but that it should receive such consideration in respect to the direct bending stresses produced as the conditions of design and erection may require.

32.—Buckling of Columns in a Direction Perpendicular to the Plane of the Truss.—The preceding discussion of the behavior of columns in a truss has concerned itself primarily with the important case of a bridge or similar truss in which the principal loads act in the plane of the truss and the deformations in that plane are of chief significance. It is necessary to consider also deformations and possible buckling in a lateral direction. Considering the top chord members of the ordinary bridge truss, the secondary or deformation stresses in a lateral direction will be small so that eccentricity of load in that plane will be small. On the other hand, these members are not so firmly held at the ends in that plane, so that it is conceivable that one segment might bend outwardly and the adjoining segments inwardly. This greater freedom of action would in effect give a greater free length of column in a horizontal, than in a vertical, plane.

Taking both factors into consideration, it would appear that if a free length of 75% of the full length is assumed for vertical bending, with an eccentric ratio of 0.25, the resulting formula can safely be applied to lateral bending as well. In other words, the least value of r should be used in determining the slenderness ratio. Ordinarily, this will be the value taken about a horizontal axis. Whatever eccentricity there may be in a lateral direction will add, theoretically, somewhat to the maximum fiber stress, but the results of the tests of Series (E) and the considerations there noted (Article 6), indicate that this effect may be neglected. To keep lateral bending or eccentricity of load to a minimum, the lateral system should consist of double diagonals and transverse struts.

ACKNOWLEDGMENTS

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Columns for the tests of Series (*D*) and (*E*) were donated by the American Bridge Company. Some material for small lacing-bar tests was donated by the Wisconsin Bridge and Iron Works. Material for the tests of Series (*F*), (*G*), and (*H*), and for the larger lacing-bars, was given by the Illinois Steel Company, and fabricated gratuitously by the McClintic-Marshall Construction Company.

SUMMARY AND CONCLUSIONS

The more important results and conclusions reached by the Committee from its studies are here re-stated:

1.—Columns tested with hinged ends and eccentric loads give results both as to deflection and distribution of stress that agree very closely with the theoretical analysis within the limits of elasticity of the material.

2.—The ultimate strength of such columns agrees fairly well with the theoretical strength based upon the yield point of the material and assuming elastic conditions up to point of failure. For the longer columns this agreement is very close; for the shorter lengths, the actual strength is likely to be considerably greater than the theoretical, due to the approximate nature of the calculations. For the same reason, the direction of bending with respect to the form of section has some influence on the strength.

3.—Of the various sections tested the one consisting of two channels connected by lacing showed a distinct weakness when tested with the plane of bending perpendicular to webs of channels. When bent in the other direction, its rating was nearly the same as that of the other forms.

4.—The compression member of a truss acts, in general, as a column eccentrically loaded and more or less restrained at the ends. The amount of eccentricity and restraint are factors that affect the strength of the compression member greatly. They are of much greater influence than variations in form of section or of details of design under ordinary practice. They cannot be determined in a laboratory, but by the analysis and measurements of the deformations of trusses under load.

5.—A rational column formula can best be constructed on the basis of the theoretical formula for eccentrically loaded columns, using values of eccentricity and free length of column derived from truss analysis. The yield point strength of the material may be used in a formula for ultimate strength. The formula representing ultimate strength will then be,

$$\frac{P}{A} = \frac{\text{Yield point}}{1 + a \sec \sqrt{\frac{P}{A\,E}}\left(\frac{l'}{2r}\right)} \quad \text{.................(28)}$$

in which, a = eccentric ratio to be used, and l' = free length of column.

6.—For columns bent in single curvature, an eccentric ratio of $\frac{ec}{r^2} = 0.25$, to include both crookedness and eccentric application of load, and a free length of three-fourths the full length for riveted connected members are suggested as reasonable values for ordinary trusses.

7.—For the material used in these tests a value of 32 000 lb per sq in. is a reasonably conservative value for yield point. A few results lie somewhat below the curve based on this formula. The resulting formula for ultimate strength of a column in a truss is given by Equation (22), substituting a yield point stress of 32 000 lb per sq in.

8.—For a working formula the parabola is a good substitute for the theoretical formula up to values of $\frac{l}{r}$ of 160 for structural steel. Equation (25) represents this parabola. It deviates from the theoretical curve by less than 2% up to $\frac{l}{r} = 160$.

9.—As a working formula for bridge truss columns corresponding to a basic tensile unit stress of 18 000 lb per sq in., the Committee suggests the parabolic formula, Equation (26).

10.—For pin-end columns, the suggested formula is given by Equation (27).

11.—If it is desired to consider a U. L. P. instead of the ultimate strength as a basis, the numerator of the theoretical formula should be reduced accordingly—about 10 to 12% according to the estimates made herein. The results will not be greatly different from those obtained from the ultimate strength formula, providing the same basic stress of 15 000 lb per sq in. for short columns is used.

12.—The foregoing numerical formulas are based on a material for which the yield point is not less than 30 000 lb per sq in. The maximum thickness of material tested was 0.9 in. For thicker material, the constants may have to be modified or special pains taken to secure the yield point desired.

13.—For long horizontal members, the direct effect of weight may have to be considered.

14.—For large structures, the calculation of secondary stresses will enable the values for the eccentric ratio to be estimated more closely and the columns designed accordingly.

15.—Usually the effect of eccentricity in a plane at right angles to the plane of the principal truss may be neglected.

16.—Compression members bent into reversed curvature, due to loads applied with opposite eccentricities at the two ends, will have their maximum stress at the ends of the members. The allowable unit stress requires little reduction for length, but the eccentricities in this case (as shown by secondary stress calculation) are likely to be greater than the value of 0.25 assumed in the formula.

17.—The batten-plate column is suitable only where the eccentricity perpendicular to the webs is known to be slight.

18.—The strength of lacing-bars depends largely upon the end connections. For the ordinary connections and for a yield point of 36 000 lb per sq in., the minimum strength is given approximately by the straight-line formula, Equation (12). By more adequate connections the strength will approach that of fixed-end columns.

19.—Double lacing with connecting rivets has a strength measured by the half length of bar.

20.—For bars made of shapes, the channel bar is a superior form.

21.—Outstanding flanges in which the unsupported width does not exceed fifteen times the thickness will have a strength under centrally applied loads equal to the yield point of the material.

In the second progress report further work was suggested on columns with diaphragms and on field measurements. While no tests were made on columns with diaphragms, the tests on the various types indicate that the column with a diaphragm should be given full rating and is superior to one in which lacing is used. Field measurements would require an organization quite impracticable to secure at the present time.

The Committee, therefore, desires this to be its final report and asks that it be discharged.

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January 18, 1933. Special Committee on Steel Column Research.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1857

A CENTURY OF CIVIL ENGINEERING IN THE UNITED STATES

ADDRESS AT THE ANNUAL CONVENTION, CHICAGO, ILLINOIS, JUNE 27, 1933

BY ALONZO J. HAMMOND,[1] PRESIDENT, AM. SOC. C. E.

"When the mariner has been tossed for many days in thick weather, and on an unknown sea, he naturally avails himself of the first pause in the storm, the earliest glance of the sun, to take his latitude and ascertain how far the elements have driven him from his true course."

These immortal words of Daniel Webster, uttered 103 years ago, as a part of an oration, one of the most eloquent, forceful, and momentous in the annals of our history, established firmly the doctrine of government as the independent offspring of the popular will.

For the purpose of drawing a more striking picture of the advance in Civil Engineering in the past century, let us for a moment divert our attention to that of Finance, a subject we all like to discuss and of which we all know little. As Mark Twain said of the weather, "We all talk about it but no one does anything about it."

Let us draw the veil and see the banking picture of a century ago and compare it to the present situation, having in mind our recent "bank holiday."

The Bank of the United States had been first created in 1791, on plans suggested by Alexander Hamilton and opposed by Thomas Jefferson. Mr. Jefferson's party allowed the charter of the Bank to lapse in 1811, when the day of their power came, professing themselves as opposed to it on principle; but, in 1816, we find that the exigencies of finance had been powerful solvents of their scruples—they had changed their minds and had given the Bank another charter, running to 1836.

General Andrew Jackson came in, in 1828, "to simplify and purify the workings of the government * * *, to promote its economy and efficiency, and to maintain the rights of the people and of the states in its administration, * * *", so that from the outset he looked upon the Bank as an enemy of constitutional and democratic government.

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[1] Cons. Engr., Chicago, Ill.

The Bank was the depository of the Government, so to cripple it, he found it necessary to change and dismiss two Secretaries of the Treasury and, finally, found a pliant tool in Roger B. Taney, who issued the necessary order in 1833—a century ago—that all revenues should be deposited in State banks and that the deposits should gradually be withdrawn from the Bank of the United States.

There was immediate distress in the money market and a sudden flutter of credit, and only the sound condition of business at the moment prevented a crisis and disaster.

The Government deposits went to State banks, which became banks of issue, so that paper money seemed to pour suddenly from every town and hamlet, and speculation became a riot. The bubble of credit glistened resplendent with all the hues of a round globe, until General Jackson pricked it in 1836 by demanding that land agents of the Government accept nothing but gold and silver in payment for public lands. The whole fabric of credit was shattered and was followed by the depression of 1837—one of the worst in history of the United States.

What have we learned in finance in one hundred years?

The bankers' panic of 1907 caused a demand for new laws, resulting in the Federal Reserve Act of 1913.

That the Federal Reserve Banking System, as operated, has proved miserably inadequate to protect our banking institutions and credit facilities, is of so recent an occurrence as to require little or no comment. In 1928 and 1929 when speculation ran riot and the wealth of the world was pouring into Wall Street to cover the turnover of millions of shares of securities every day, rediscount paper of country banks furnishing money for legitimate business was called, in order that the insatiable maw of speculation might be appeased. As in 1837, we were led into disaster and a resulting worse depression than a century ago.

I have purposely presented this picture of similarity of conditions in finance prevailing a century ago with those of to-day, in order that we may give some serious thought to the future of our financial structure. Our idols have crashed, our financial wizards have faded; but have we in the making a structure that is as safe, sound, and solid as Gibraltar?

Another reason that I have had, is the making of a comparison of the state of the Art and Science of Civil Engineering of a century ago with that of to-day. I have used the word, Art, as prior to Science advisedly, as among the earlier of our structures, outside the benefit of previous practical experience, the designer depended largely on his judgment which made of his product an art rather than a scientific production.

What was the state of the Art of Civil Engineering in 1833?

What has been the advancement of Civil Engineering during the century now being celebrated, at the Century of Progress Exposition in Chicago, Ill.?

What have been the influences at work of benefit to our civilization? To make our lives more livable, happier, and more secure?

As expressed in my opening quotation let us pause, look around, and take stock of what the Civil Engineer has accomplished (and we shall endeavor to do this with pride, but also without laudatory phrasing).

Civil Engineering was defined by Thomas Tredgold in 1828 as the "Art of directing the great sources of Power in Nature for the use and convenience of Man."

Engineering was formerly divided into Military Engineering, which is the art of designing and constructing offensive and defensive works, and Civil Engineering, in a broad sense, as relating to other kinds of public works, machinery, etc.; so that, in modern usage, Civil Engineering is the art of planning, laying out, constructing public and private works, such as railroads, highways, canals, river improvements, and irrigation, aqueducts, water-works, sewers and sewage disposal plants, bridges and metal structures, lighthouses, docks, embankments, breakwaters, tunnels, subways, etc. In a comprehensive sense, Civil Engineering includes Architecture as a Mechanical Art, in distinction from Architecture as a Fine Art.

For the purpose of this discussion, I have limited myself to a study of the field of Civil Engineering as practiced in the United States; therefore, we must draw a picture of our conditions a century ago, when the United States consisted of twenty-six States and the Louisiana Purchase which reached westward to the boundaries of the Red River and the Rocky Mountains; the area was 60% of our present area and the population approximately 10 per cent.

In 1800, a man lived essentially the same kind of a life—as far as external conditions were concerned—as his prototype of 6 000 years before. He went as far as his own legs or his horses' legs could carry him. He dug his ditch, he mowed his hay, with the power of his two arms, or the help of his wife. In that early day the housewife wielded the axe—for stovewood, or for logs for the fireplace wherein the iron pot was hung over the fire.

To-day, the housewife has the electric oven—thermometer attachment and solenoid brake with time clock to cut off the "juice" at the end of the proper cooking period.

Let me quote from the Inaugural Address[2] of the late J. James R. Croes, Past-President, Am. Soc. C. E., in 1901, for a description of conditions existing a century prior to that time, and which may have been repeated in 1833:

> "One hundred years ago, the man of enterprise who resided 50 miles from a large city and wished to consult an engineer regarding a project for a new canal, arose before daylight, struck a spark from his flint and steel, which falling on a scrap of tinder, was blown by him into a flame and from that a tallow dip was lighted. In the same primitive manner the wood fire was kindled on the kitchen hearth and his breakfast was cooked in a pot and kettle suspended from the iron crane in the fireplace. Entering the cumbrous stage coach, hung on leather springs, which passed his door, he was driven over muddy roads, crossing the narrow streams on wooden trestle bridges and navigable rivers on a ferry boat, the paddle wheels of which were turned by

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[2] *Transactions*, Am. Soc. C. E., Vol. XLV (June, 1901), p. 599.

a mule on a treadmill. At last he was landed in the city where he walked through dirty streets paved with cobble stones until he reached his destination, a plain three-story brick building founded on sand, with a damp cellar and a cesspool in the back yard. Entering a dark hall, he climbed a wooden staircase and was ushered into a neat room, rag-carpeted, warmed by a wood fire on the open hearth and lighted by a sperm oil lamp with one wick, for it was dark by this time. No wonder that before proceeding to business he was glad to take a good stiff noggin of New England rum."

As the river and lake furnished the early means of transportation, it was logical that the canals should form the connecting links for such water transport.

The Erie Canal, begun in 1817, was finished in 1825, a distance of about 364 miles.

The passenger boats were 12 to 15 ft wide, from 40 to 80 ft long, and would draw (light draft) 12 in. of water.

The sleeping accommodations were very interesting. At 8:00 P. M., all passengers were turned out of the cabin, and the captain and crew suspended two rows of cots or hammocks from the ceiling, arranged in three tiers, one above the other. At 9:00 P. M., all were ordered below and each passenger was assigned to his bed. It must be taken possession of at once under penalty of being obliged to sleep on the floor, should the number of passengers exceed the number of beds—a circumstance of common occurrence.

At 5:00 A. M., every one was routed out, the beds were removed, and toilettes made by washing in a tin pan at the stern of the boat, filled by a long dipper from the canal. Towel, brush, and comb for general use were hung at the cabin door, their use being optional.

The Welland Canal was finished in 1829 with capacity for a 125-ton vessel.

The year 1825 was memorable in Ohio as, on May 19, General Lafayette crossed the Ohio River and stepped on the Ohio shore at Cincinnati in the midst of a patriotic demonstration unparalleled in the history of the State. On July 4, 1825, at Licking Summit, the dirt was turned for the Ohio Canal, and the work was completed in 1833, by pick and shovel.

The cost of the first boat on the Ohio Canal was $2 123.24, which included six horses for towing; 5 gal of tar, at 50 cents per gal; $1.00 for hemp for caulking; and 2 gal of whiskey, at 24 cents per gal, for the hands.

Freight from the coast to Western points was reduced from $125 to $25 per ton.

Thus was established a complete waterway from New York, N. Y., to New Orleans, La., as follows:

150 miles from New York to Albany, N. Y., *via* the Hudson River;
360 miles from Albany to Buffalo, N. Y., *via* the Erie Canal;
210 miles from Buffalo to Cleveland, Ohio, *via* Lake Erie;
309 miles from Cleveland to Portsmouth, Ohio, *via* the Ohio Canal; and,
1 620 miles from Portsmouth to New Orleans *via* the Ohio and Mississippi Rivers.

To 1831 there were 348 vessels built and in use on the Mississippi River, and Captain Basil Hall describes the life at the Port of New Orleans, as follows:

> "At every hour—I had almost said at every minute of the day—the magnificent steamboats which convey passengers from New Orleans into the heart of the western country, fire off their signal guns and dash away at a rate which makes one giddy even to think of."

About this time—1830—the first train (the De Witt Clinton) ran between Albany and Schenectady, N. Y., and there was only about 30 miles of railroad in the United States.

In 1830 and 1831, five miles of railroad were built from New Orleans to Lake Ponchartrain. This was the first railroad in America to use iron T-rails. Henry Clay, at this time at the zenith of his popularity, had the honor of the first ride over this railroad, as he was on a visit to New Orleans at the time of its completion.

It is said that as early as 1829 Peter Cooper experimented with a little engine of his own on the Baltimore and Ohio Railway and during a trial trip, his engine out-ran a gray horse attached to another car.

Prior to 1833 highways were not very numerous, and most of those laid out were in a neglected and wretched condition. About 1832, Andrew Jackson vetoed a bill providing a National subsidy for a road in Kentucky, as the section was all in one State.

An English engineer, David Stevenson, traveling in this country in 1832, writes that the highway was quite unfit for any vehicle but an American stage coach and any pilot but an American driver.

The corduroy road had a surface of trees cut 12 ft long, laid close together transversely, over which the coach advanced by leaps and starts.

Stevenson tells of a trip from Pittsburgh to Erie, Pa.—128 miles—which took 46 hours to make by coach, a rate of 2¾ miles per hour, with time out for breakfast, lunch, and tea—once upset and several times mired.

In 1849, all engines, water-wheels, prime movers, etc., amounted to about 10 000 000 hp in the United Stats; in 1930, there was well over 1 000 000 000 hp, a multiplication factor of a century. This may be visualized by taking 1 hp as equivalent to 10 men in work-producing ability. On this basis, in 1849, each man, woman, and child in the United States had the equivalent of 5 human slaves each, continuously available for service; in 1930, there was an equivalent of 100 slaves for each man, woman, and child.

In Boston, Mass., fecal matters were excluded by law from drains until 1833, and even then only liquid wastes were permitted. It is said that the first plans for sewerage in the United States were made in 1857 by the late Julius Walker Adams, Past-President, Am. Soc. C. E.

In 1830, there were forty-four water-works plants in the United States. Until 1832, New York City obtained its water from wells that became so badly polluted that, about 1832, approximately 600 hogsheads of water were brought in daily, at $1.25 per hogshead.

The first public water supply in Chicago, in 1831, was a public well.

At Philadelphia, Pa., water from the Schuylkill River was raised by water power into four large reservoirs and passed through a gravel filter. Double-acting force pumps, with a 6-ft stroke, were worked by cranks attached to axles of paddle-wheels which were 16 ft in diameter and 15 ft wide.

Bridge engineering prior to 1833, and for some years later, was an art—there being no scientific computation of stresses in bridge frames prior to the publication, in 1847, of the treatise, "A Work on Bridge Building," by the late Squire Whipple, Hon. M. Am. Soc. C. E., followed, in 1851, by General Herman Haupt's book "The General Theory of Bridge Construction." Before this period, bridge members were proportioned by the judgment of the builder.

That remarkable results were obtained is seen by some of the trusses invented in 1820 by Ithiel Town, and built of common plank up to spans of 220 ft.

A bridge over the Susquehanna River, at Columbia, Pa., was begun in 1832, finished in 1834, and was the most extensive arch bridge in the world at the time, as it had twenty-nine arches of 200 ft each, 30 ft wide, each arch rib consisting of two pieces, 7 by 14 in.—9 in. apart.

The early dams were largely of timber, and were rock-filled. Usually, trees were laid parallel with the stream on the river bed, with the butt down stream, and a mattress of logs and rocks was developed, which accumulated sediment and became fairly tight.

I have deliberately pictured in some detail the conditions of a century ago relative to Civil Engineering projects, so as to set up a comparison of the rather primitive developments of the period, with the progress of to-day wherein the Civil Engineer has not only been the agent for making life more livable, but has rendered such great economies by scientific planning as to make possible works of the greatest magnitude.

What, therefore, has the Civil Engineer accomplished in the past century, which has been outstanding in bringing our civilization to a higher plane of living, and which has added materially to the health, comfort, and well-being of our people?

Compare for a moment the Erie Canal with its 4-ft depth of water and its capacity for a small towboat with that of the Panama Canal with locks having 41-ft depths on the sill and capable of taking the 60 000-ton floating palaces. The canal was our earliest means of bulk transportation, connecting the rivers and lakes and, from 1815 to 1850, we constructed about 4 500 miles at a cost of about $214 000 000.

The building of locks, such as those at Sault Ste Marie, Mich., which pass about 90 000 000 tons of freight per year—the largest inland water traffic in the world; the New Welland Canal, which opens a channel to the Great Lakes for ocean-going vessels; the development of slack-water navigation on the Ohio River and its tributaries; the construction of new waterways, such as the Chicago-Illinois River-Lakes to the Gulf; the protection from floods, such as the Miami Conservancy District and the Lower Mississippi River levees and diversion channels; the building of harbors for both lake and ocean service; the construction of wharfs and docks, such as the great ore docks at Duluth, Minn., and coal docks at Lake Erie ports; the straightening of great river channels, such as is now in process of change in the Mississippi River below Vicksburg, Miss., which is working out new hydraulic problems, directed by tests made at the Vicksburg Hydraulic Testing Laboratory—these are all of the field of Civil Engineering and are vital to the development of water-borne commerce, and to the protection of the lives and the property of our citizens.

From the corduroy road, the cobble-stones of New York, and the dirt surface of the Wilderness Pike, there is a vast advance to the concrete highway, with its long-radius curves and banked turns, where the motor car may run 100 miles per hr with safety—just a century, so to speak. The smooth surface of the asphalt, wood block, and brick pavements, with a prospect in the near future of rubber which will more completely obliterate noise in our large cities, makes the cobble-stone a bad dream.

All these surfaces and constructions, as well as the various types of lower-cost pavements, are products of the Civil Engineer's skill; his function is to solve the economies of the street and highway problem, from the lightest to the heaviest demands.

In 1830, we had approximately 30 miles of railroads, with practically no rolling stock. It is a vast jump to 240 000 miles of railway, an annual income account of $5 000 000 000 to $6 000 000 000, and a book value of about $23 000 000 000; the growth of the century is staggering.

The man who has pioneered in the location and construction of these railroads is symbolized by a portrait hanging in the hall of the Headquarters of the Cleveland Engineering Society. It depicts the early locating engineer with his equipment, to which is added a rifle, a revolver, and a knife, the latter three having been very essential in crossing the Western prairies and mountains.

From the days of 1830 when sewage ran down the gutters, and hogsheads of water were peddled in the streets of New York, we jumped to the Lawrence (Mass.) Experimental Station of the Eighties, the work which moulded public opinion to the need for the treatment and purification of the cities' sewage.

The marvelous reduction in the death rate and the improvement in health of the average citizen is in large measure due to the practical work of the Sanitary Engineer.

The tap which turns loose a flood of pure drinking water free from all germs, tastes, and odors, also reveals the work of the Engineer in conserving Man in his pursuit of health and happiness.

The great pressure tunnel recently completed on the Island of Manhattan is again witness of the valuable work of the Engineer in bringing the waters from the distant mountains to the dweller in New York.

From the primitive dam of logs in the Tippecanoe River, we have come to the stage of the mighty Boulder Dam, with its 730-ft head and enormous power possibilities; and to the irrigation dams with their distribution of water which makes the desert blossom like the rose.

The early type of the wooden covered bridge has appealed to some of our editors as a facetious bit of engineering—a place for lovers to hide away, for hold-ups, etc.

That early bridge was built as a matter of engineering judgment and is in vast contrast to the 3 500-ft span of the George Washington Bridge, crossing the Hudson River, at New York, and other large bridges in prospect. These not only have scientific engineering analysis made of all stresses, and tests of all materials, but, in some cases, a model on a miniature scale is constructed and tested to check the calculations.

The growth of our cities vertically and horizontally is accomplished by the skill of the Engineer in his design of the steel frame of the skyscraper, and by burrowing his tunnels under the streets or by over-head construction of railways.

With due modesty, but with the utmost pride of accomplishment, the Civil Engineer looks back over the century with its epoch-making strides in various fields of transportation, of health and comfort of living, of protection of lives and property, of additions to the capital values of the Nation, with the thought that here is a life of usefulness, of upbuilding, of creating, of doing worthwhile things honestly, and economically, not because of the fee that comes with the job, but because his training of exactitude, of honesty of purpose, of direct dealing is inborn.

His greatest achievement is that of having his engineering work judged by his fellow men as of the greatest good to the greatest number.

It is meet, therefore, on this occasion—during the display of the Century of Progress in celebration of the birth of a great city—that we also shall have put on record some of the achievements of a profession which has been active, beneficent, and forceful in the making of that great city—Chicago.

MEMOIRS OF DECEASED MEMBERS

JOHN RIPLEY FREEMAN, Past-President and Hon. M. Am. Soc. C. E.[1]

Died October 6, 1932

John Ripley Freeman was born on July 27, 1855, in West Bridgton, Me., where his early life was spent on his father's farm and where he began his education at the country schools. Later, he attended the public schools of Portland, Me., and Lawrence, Mass. He entered the Massachusetts Institute of Technology, in Boston, Mass., in 1872, and became a student in the Department of Civil Engineering, graduating in June, 1876, with the degree of Bachelor of Science.

After graduation, Mr. Freeman went to work for the Essex Company, a water power company at Lawrence, Mass., on the Merrimack River, which had previously employed him during his vacations at the Institute. He soon became Principal Assistant Engineer to the late Hiram F. Mills, Hon. M. Am. Soc. C. E., the Chief Engineer of the Essex Company, one of the foremost hydraulic and sanitary engineers of his time and a member of that eminent New England school of hydraulicians represented by Charles S. Storrow, Hon. M. Am. Soc. C. E., Uriah Boyden, James B. Francis, Past-President and Hon. M. Am. Soc. C. E., and Joseph P. Davis, M. Am. Soc. C. E. Mr. Mills at that time was engaged on extensive hydraulic experiments, the results of which have only in part been made public, and he was also carrying on a large consulting practice devoted mostly to problems of water power, foundations, and factory construction. In all this work Mr. Freeman was an active assistant and there he laid the foundations for his remarkable career.

After ten years of this apprenticeship at Lawrence, Mr. Freeman resigned his position there in 1886 to become Engineer and Special Inspector for the Associated Factory Mutual Fire Insurance Companies, of Boston, Mass. He re-organized the corps of inspectors employed by those companies, conducted experiments looking to the improvement and standardization of fire prevention apparatus, and conducted scientific researches into the causes of fires. During this period he presented to the Society his papers entitled "Experiments Relating to the Hydraulics of Fire Streams"[2] and "The Nozzle as an Accurate Water Meter.[3] For the first of these papers he received the Norman (Gold) Medal of the Society in 1890, and for the second the Norman Medal of 1891. While in Boston Mr. Freeman arranged to give one-half his time to a consulting practice in water power, municipal water supply, and factory construction. There, he also began his long career in the public service, to which he gave so much of his life. He was a member of the Water Board of Winchester, Mass., where he made his home, and in 1895 and 1896 he was Engineer Member of the Metropolitan Water Board of Massachusetts, which was then engaged in preparing plans for the development of a large additional water supply from the Nashua River for the Boston Metropolitan District.

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[1] Memoir prepared by Walter E. Spear, M. Am. Soc. C. E.

[2] *Transactions*, Am. Soc. C. E., Vol. XXI (1889), p. 303.

[3] *Loc. cit.*, Vol. XXIV (1891), p. 492.

In 1896, Mr. Freeman left Boston to become President and Treasurer of the Massachusetts Mutual Fire Insurance Company and its Associated Companies, at Providence, R. I., a position which he held at the time of his death. Although best known to the profession as a Consulting Engineer in many fields, he gave fully of his time after he went to Providence to the insurance business and developed those natural talents as an executive and business man which gave him pre-eminence in the factory insurance field. His success was marked by a constant improvement in fire prevention methods and a steady reduction in insurance costs. His companies, which wrote about $65 000 000 of insurance in 1896, had about $3 000 000 000 on their books in 1932.

Mr. Freeman approached the problems of building construction and of safe-guarding life and preventing fire losses as an engineer and an insurance executive. He wrote extensively on fire protection matters and in 1905 issued a publication "The Safeguarding of Life in Theatres," which represented a comprehensive study of theatre fires, their causes, and means of prevention. In 1915, he presented to the International Engineering Congress at San Francisco, Calif., a paper on "The Fire Protection of Cities."

For many years Mr. Freeman was alive to the importance of adequate design and good construction in preventing the loss of life and property from earthquakes. He was a member of the Seismological Society of America and collected at Providence an unusual library on seismology. He devoted much effort to stimulating engineers, geologists, and seismologists to the importance of obtaining adequate information on the magnitude and character of earth movements in the vicinity of major seismic disturbances. He visited many earthquake regions in all parts of the world to study at first hand the causes of failures in buildings and other structures, and in 1932 published a book entitled "Earthquake Damage and Earthquake Insurance," which may be considered a textbook on the subject of earthquake-resisting design.

In an age of specialization, Mr. Freeman did not confine his work to those branches of engineering within which the civil engineer to-day is supposed to practice. He was at one time, for example, a civilian member of an Army Board on disappearing gun carriages. He was, however, pre-eminently an hydraulic engineer and his field was primarily in water power, river control, water supply, and allied problems of a sanitary and hydraulic character. Except for short periods on his larger undertakings he never employed a large organization in his consulting practice, preferring to give to each task a large measure of his personal attention. Mr. Freeman believed that a change of work afforded a complete rest. Engineering was his recreation, and he practiced it with an enthusiasm and thoroughness that made him early a prominent figure in the engineering world.

Mr. Freeman never lost his interest in the hydraulic and construction problems associated with the development of water power with which his early work in New England was identified. He wrote extensively on water power subjects and made many investigations and reports on water power projects in the United States, Canada, and Mexico. He organized in 1905 a staff of engineers for the preliminary surveys and investigations for the Feather River Development of the Great Western Power Company, in California, and subsequently advised on the Big Bend and Caribou Developments. In 1907 and 1908, he was Senior Consulting Engineer to the New York State Water Power Supply Commission, which was charged among other duties with that of conserving the water resources of the State. Among other storage projects which he studied was a reservoir on the Sacandaga River above Conklingville, N. Y., in the Adirondack Region, which has since been constructed. He was called in from time to time to advise upon the construction of the Massena Power Development on the St. Lawrence River, for the St. Lawrence River Power Company, and its successor at Massena, N. Y., the Aluminum Company of America; and afterward gave advice on ice difficulties and on back-water and other operating problems. He also advised the Aluminum Company of America on its power developments on several Southern rivers and on the design and construction of new aluminum smelting works at Niagara Falls, N. Y. He made investigations on water power development for the Canadian Government in Alberta, Manitoba, and British Columbia; designed high masonry dams for the Mexican Northern Power Company, and for the Pacific Gas and Electric Company at Lake Spaulding, California; designed and supervised the construction of a high masonry dam on the Missouri River, at Holter, Mont.; and prepared plans for many power projects which have not been executed, notably for the development of the Great Falls of the Potomac River, for a large hydro-electric project in the Lachine Rapids, on the St. Lawrence River, near Montreal, Que., Canada, and for a subterranean development for the Ontario Power Company, at Niagara Falls.

Not the least of Mr. Freeman's contributions in the broad field of Hydraulic Engineering were those on problems of river control and navigation. Among his writings on these matters was a paper on "Flood Control of the River Po in Italy"[4] which was presented at the meeting of the Society on June 6, 1928, and for which he received the J. James R. Croes Medal on January 21, 1931. In 1903, he was made Chief Engineer of the Charles River Dam Commission and prepared an exhaustive report on the project for the conversion of the lower estuary of the Charles River into a fresh-water lake, which has since been constructed. In 1904, he reported to the Massachusetts Metropolitan Park Commission on the improvement of the Mystic River and the drainage of the Fresh Pond marshes. In 1905, 1908, and 1915, Mr. Freeman was appointed by the President of the United States a member of Engineering Boards to report on a sea-level *versus* a lock canal and on problems of dam and lock foundations, and earth slides which blocked the Isthmus of Panama several times. From 1917 to 1920, he acted as Consulting Engineer to the Chinese Government on the improvement of the Grand Canal and the prevention of disastrous floods on the Yellow River and the Hwai River, organized a staff of engineers to investigate these problems, and went to China himself in 1919. Between 1924 and 1926, Mr. Freeman was a member of the Engineering Board of Review of the Sanitary District of Chicago and prepared a program for the regulation of the Great Lakes. His report on these matters included exhaustive studies of winter evaporation from the Great Lakes and of minor earth movements or tilting affecting the problem of lake levels.

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[4] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 161.

Mr. Freeman was greatly interested in the application of the hydraulic research laboratory to the study of the river and harbor problems. He made himself familiar with such laboratories in Europe, and presented to the Society a paper on the "The Need of a National Hydraulic Laboratory for the Solution of River Problems."[5] In order to stimulate the interest of American engineers in the laboratory approach to hydraulic problems he had translated and published in 1929 a series of monographs by leading European hydraulicians, entitled "Hydraulic Laboratory Practice." As a further means of educating American engineers to the importance of the hydraulic laboratory and to provide trained men to operate such laboratories when they were built, Mr. Freeman in 1923 gave $25 000 each to the American Society of Civil Engineers, the American Society of Mechanical Engineers, and the Boston Society of Civil Engineers, with which to provide for traveling scholarships in hydraulics open to young engineers and junior professors. The first scholarship was awarded by the Society in 1927 and since then, one or two men have studied each year in Europe. Mr. Freeman also conceived the idea of a National Hydraulic Laboratory in Washington, D. C., and was active in furthering the passage of the necessary legislation at Washington and in advising on its construction.

In the water supply field, Mr. Freeman had a long record of distinguished service, and his many published water supply reports made him perhaps best known to engineers in that field. In 1899, he was engaged by the Comptroller of The City of New York to investigate new sources of water supply for that city, and after eight months of intensive work he submitted an exhaustive report on all its possible water supply sources. In this report, he presented valuable experiments on the flow over the crests of model dams, a recomputation of the yield of the Croton System, and an estimate of the future water consumption of New York City, which has been well confirmed by subsequent events.

In 1903, he was a member of the Commission on Additional Water Supply of New York City, the so-called Burr-Hering-Freeman Commission, which investigated the yield and quality of all available sources of supply for the city and made a report which became the basis of the subsequent work of the Board of Water Supply. Mr. Freeman, with his insurance training, recognized the importance of ample water pressures in large cities and was largely responsible for the adoption of the Commission's recommendation for delivering the new supply in the City at a considerably higher level than the existing distributing reservoirs at Croton, a recommendation that was followed in the construction of the Catskill Works. In 1905, he was appointed Consulting Engineer to the Board of Water Supply of New York City, the body created in that year to construct the Catskill System. He was active in the organization of the Engineering Corps of the Board of Water Supply and in the planning and construction of the Catskill Works. He continued as a Consultant to that Board until his death.

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[5] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1033.

In 1906, Mr. Freeman was one of a commission of three engineers engaged to report on the Los Angeles Aqueduct, a project to bring the waters of Owens River 240 miles to the City of Los Angeles, Calif. In that work he recommended a location for the Owens River Aqueduct which made possible the development of power not previously considered feasible.

He was engaged in 1909 in the capacity of Consulting Engineer on the problem of new sources of water supply for the City of Baltimore, Md.; from 1909 to 1912, he was Consulting Engineer to the City of San Francisco, Calif., and planned the Hetch-Hetchy System, in which he made the development of electric power an important feature. Among other cities which had sought his advice on water supply matters were Nashua, N. H., Denver, Colo., Seattle, Wash., San Diego, Calif., and the City of Mexico, Mexico.

In addition to his country-wide public service, as a Consulting Hydraulic Engineer, Mr. Freeman was active during the World War as a member of the National Advisory Committee on Aeronautics, and reported at that time on the Hog Island Shipyard. He was also a member of the Visiting Committee of the Bureau of Standards, Washington, D. C. In Providence, which became his home after 1896, he identified himself with many local activities. In 1911, he made a study of city planning for the east side of Providence, including new highways, parkways, and parks. He also carried on as a private venture a large real estate development of higher character in the vicinity of his home. He served for ten years as a Director of the Rhode Island Hospital Trust Company and of the National Bank of Commerce, in Providence; in 1904, he was a member of the Rhode Island Metropolitan Park Commission; and, during the war, served as President of the Providence Gas Company.

Mr. Freeman was keenly interested in technical education and spoke and wrote on such educational matters. For forty years he was a member of the Corporation of the Massachusetts Institute of Technology, Boston and Cambridge, Mass. He was once offered the Presidency of the Massachusetts Institute of Technology and twice was offered the chair of Civil Engineering at Harvard University, but had to decline in each case, feeling that he was better suited to professional work.

On April 21, 1931, Mr. Freeman was given a testimonial dinner, sponsored by the Providence Engineering Society, at which engineers, scientists, educators, industrialists, and other friends paid him a remarkable tribute for his many varied accomplishments. He was an Honorary Member of the American Society of Mechanical Engineers, the Boston Society of Civil Engineers, the Providence Engineering Society, the New England Water

Works Association, the Badische Technische Hochschule, Karlsruhe, Germany, and Mitgleid des Wissenschaftlichen Beirats des Forschungsinstituts, in Munich and Walchensee, Bavaria, and a Past-President of the first two of these societies. He held honorary degrees from Brown University, Tufts College, the University of Pennsylvania, Yale University, and the Sächsische Technische Hochschule, Dresden, Germany.

At its meeting on January 17, 1933, the following Resolutions were unanimously adopted by the Board of Direction of the Society:

"*Whereas:* It has pleased Almighty God to remove from our midst our late Honorary Member and Past-President, John Ripley Freeman.

"*And Whereas:* We realize that in his death, the World has lost a most useful citizen; the Engineering Profession an able leader; and the American Society of Civil Engineers a valued and loyal member.

"*Now, Therefore, Be It Resolved:* That the Board of Direction of the American Society of Civil Engineers hereby records its profound sorrow at the passing of its respected and revered friend and supporter.

"*Be It Further Resolved:* That the sincere sympathy of the Board of Direction be conveyed to his bereaved family; and

"*Be It Further Resolved:* That a page be set aside in the Minutes of the Board upon which shall be inserted this Resolution, a copy of which shall also be transmitted to his family."

In 1888, Mr. Freeman was married to Elizabeth Farwell Clark who, with one daughter and four sons, survives him.

Mr. Freeman was elected a Junior of the American Society of Civil Engineers on June 7, 1882; a Member on April 3, 1889; and an Honorary Member on September 29, 1930. He served as Director from 1896 to 1898; as Vice-President in 1902 and 1903; and as President in 1922.

GEORGE FILLMORE SWAIN, Past-President and Hon. M. Am. Soc. C. E.[1]

DIED JULY 1, 1931

George Fillmore Swain was born on March 2, 1857, in San Francisco, Calif. His ancestors came from New England. He was eighth in direct descent from Richard Swain who came to America in 1635 and settled first in Hampton, N. H. A few years later, Richard Swain moved to Nantucket, Mass., and was one of nine who in 1659 purchased the English rights to the entire island from Thomas Mayhew who, however, retained a twentieth interest. The Indian rights were also promptly acquired. Nantucket, in its prime, was the leader in whale fishery, and this not only bred sturdiness of character, but also brought prosperity with its many advantages. George Swain could trace his ancestry among many of the best known families of Nantucket, the names of Coffin, Macy, Starbuck, and Bunker being prominent among others.

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[1] Memoir prepared by C. Frank Allen and Albert Haertlein, Members, Am. Soc. C. E.

His father, Robert Bunker Swain, was a native of Nantucket who moved to California in 1855, where he was engaged in the shipping and commission business. He became one of the leading merchants of San Francisco, served as President of the Chamber of Commerce, and also of the Mercantile Library Association. He was appointed Superintendent of the Branch Mint during President Lincoln's administration, and this work brought into play an unusual facility in the use of mathematics; the son's success in this direction had some root both by inheritance and example. The father's business position and his tastes led him to entertain many notable people, among them Camella Urso, distinguished violinist, Dr. Bellows, the leading Unitarian clergyman, and Mark Twain, whose stories from "Innocents Abroad" quite convulsed young Swain, who was allowed to listen from a near-by room. The elder Swain was an intimate of Starr King, well known in New England, and a power in holding California loyal in the Civil War. It is interesting to note that Bret Harte was his Secretary at the Mint.

George Swain relates that he was shy, timid, and without much physical courage or aggressiveness, and that his father to cure this had him take lessons in boxing and later in fencing. Quick in movement and perception, he became for a little chap expert with foils. When a school bully tried to pick on him he got his father's consent to fight him, used his boxing science, and was troubled no further. He also had to ride horseback, with horses not always too gentle, and thus learned to manage an unruly horse without fear and was never unseated. His father showed rare wisdom in these matters. Further, in preparation for college, he sent him to a Military School, conducted under strict discipline, which included guard mounting, inspection, and other military requirements. Young Swain became a good shot at target practice. He served as Adjutant as well as Captain of a company and was the Valedictorian of his Class. He rated his military training as one of the best things that ever happened to him. All these furnished a background and were elements which contributed to make George Swain the man he was; he had an unusual start, both by inheritance and environment, from his father's side.

His mother, Clara Fillmore, born in Lynn, Mass., was a woman of refined literary tastes. She died while George Swain was still young. His grandfather was the Rev. Daniel Fillmore, a Methodist minister, who was twice assigned to a pastorate in Nantucket. His grandmother, Susan French Clark, born in Plymouth Vt., also was of fine literary tastes. A small collection of her poems was printed; one of these, "The House I Live in," written by her on the occasion of her eighty-fifth birthday, had real merit. She lived to be nearly ninety-eight years old. At one time, she had kept a private school in Boston, Mass. His uncle, Dr. Charles Wesley Fillmore, was graduated from the Harvard Medical School in 1856 and studied further in Europe—in Paris, Berlin, Vienna, and elsewhere—specializing in eye and ear; on his return he became a surgeon in the Army and, later, engaged in practice in Providence, R. I. While in Europe Dr. Fillmore studied the violin successfully under some of the best masters, and this as well as his studies and life abroad furnished elements which afterward contributed to Professor Swain's success.

At the age of fifteen George Swain had decided to become an engineer, and not long after his father's death in 1872, he came East to his grandmother and his uncle, Dr. Fillmore, in Providence. He finally decided to enter the Massachusetts Institute of Technology at Boston, largely on account of its proximity to Providence. In some doubt as to adequate preparation in mathematics for the entrance examinations, he secured as a tutor a senior in Brown University, Benjamin Ide Wheeler, who later became President of the University of California. A letter from Dr. Fillmore was sent to Edward Everett Hale, who secured young Swain's boarding place in Boston. He was fortunate in the kind of people who became interested in him.

Not quite sixteen when he entered the Massachusetts Institute of Technology, he had a fine record of scholarship and was the ranking student of his class. He served as Adjutant of the Military Battalion for which his earlier training well fitted him.

Among the studies pursued, the course in logic taught by Professor George H. Howison appealed to him, and he took this course as an extra subject. The taste for study along this line and the reading done in this direction he had always valued highly as an important part of his training. He was an omnivorous reader. Bryant's poetry specially appealed to him, as it had also to his mother, and Emerson did much for his philosophy. He developed a fondness for Shakespeare, and this, in connection with his logic, certainly was conducive not only to clear thought, but also to habits of clear expression, which a training in mathematics and science sometimes fails to secure.

His teacher in Civil Engineering was Professor John B. Henck, a fine mathematician, a strict and effective teacher who demanded and received good work from his pupils and did more than any other member of the early faculty to fix the high standards of this institution. His influence on Professor Swain's later teaching must have been very great.

In 1877, young Swain received the degree of Bachelor of Science in the Course of Civil and Topographical Engineering. Not satisfied with this, he spent three years abroad, primarily for the study of civil engineering, in which he did fine work, but with the added advantage to him of acquiring a breadth of view and experience in life, a wide acquaintance with men of various training, and an opportunity to travel, which together formed a most valuable feature of his life abroad. He thus secured something which cannot be acquired from travel alone. Among the acquaintances he formed while abroad the late Dr. Arthur T. Hadley is worthy of mention. He acquired also an almost perfect knowledge of German, and in his travels often registered from Berlin rather than from America, to his material advantage.

He studied at the Royal Polytechnicum, in Berlin, under such able masters as Winkler, Goering, and Hagen, specializing in bridges and other structures, railroads, and hydraulics, subjects valuable quite as much for training as for understanding of the work covered. With superior natural and demonstrated ability, combined with an excellent foundation secured at the Massachusetts Institute of Technology, he was in a position to benefit well-nigh to the limit from the opportunities offered at the Royal Polytechnicum. At the close, he had substantially mastered the subjects of structural, hydraulic, and railroad engineering, and reached this country probably more highly trained in these subjects than any of his contemporaries.

Advanced degrees were then restricted to those who were to enter governmental service, so that he received no dgrees; but the substance he secured. He did, however, bring away flattering recommendations from Winkler and Goering. While in Germany he visited many engineering works with his professors. He attributed much of his later success to his years abroad, and not solely to his training in engineering or his mastery of German. He took advantage of his opportunity to hear music by great masters. He played the piano with skill and taste, and on his return to America spent many evenings playing such music as Beethoven sonatas with his uncle, the violinist. It is probable that his uncle, who had been trained abroad, was responsible for young Swain's going to Germany to study.

On his return in 1880, he was appointed Expert Special Agent of the Tenth Census of the United States, investigating water power in connection with manufacturing interests. He presented reports on the Middle Atlantic and on the South Atlantic water-sheds, and, later, on the water powers of New England.

In 1881, he was appointed Instructor in Civil Engineering at the Massachusetts Institute of Technology. Gen. Francis A. Walker who had been Superintendent of the Census, was in the same year appointed President of the Institute. George Swain had served under him while engaged on the Census, and the natural outcome was General Walker's early recognition of his abilities by his promotions to Assistant Professor in 1883 and to Associate Professor in 1886. He spent one summer with the Essex Company, at Lawrence, Mass., and another at Lowell, Mass., in the office of Locks and Canals under the late James B. Francis, Past-President and Hon. M. Am. Soc. C. E.

Early in 1887, occurred the widely known Bussey Bridge disaster. This furnished the first opportunity for Professor Swain to distinguished himself in the engineering field. At the investigation which followed, the quality of his training and his ability to analyze the causes of the accident so impressed the Railroad Commission of Massachusetts that he was appointed the first Expert Engineer of the Commission under a law newly passed, an office for which he was well fitted—one which he filled continuously and satisfactorily for many years, and which yielded him a rare experience. He examined and became responsible for more than 2 000 railroad and electric railway bridges—a tremendous burden. He steadily and unostentatiously brought them into condition, securing the confidence of railroad officials, and thereafter no accident happened for which he could be held responsible in the slightest degree. He served in this position for twenty-seven years.

Although Professor Swain had been told that he was too young to become the head of the Department of the Institute then vacant, the prestige of his railroad appointment led to his selection as Hayward Professor of Civil Engineering and head of the Department at the age of thirty (an early age for conservative Boston); however, his success more than justified his appointment. A remarkable development followed. At the time when he became Acting Head of the Department in 1887, he was the only full Professor; there were three Instructors and one Assistant. In 1909, there were twenty members of the Department, of whom ten were members of the Faculty. In 1887, there was no Department library worthy of the name; in 1909, it had become probably the best working library of Civil Engineering in the country, and it was due to his enthusiasm and judgment that this was accomplished.

While at the Massachusetts Institute of Technology he served for a time as Secretary of the Society of Arts connected with it, and for two years was Secretary of the Technology Alumni Association.

In 1909, Professor Swain was offered and accepted the Gordon McKay Professorship of Civil Engineering at what was then and for a time afterward the Graduate School of Applied Science, Harvard University. An important consideration in his acceptance was his expressed belief that this Graduate School marked an advance in the status of Engineering, putting it on the same basis as Law and Medicine, the schools of which for some time had been on a graduate basis at Harvard. He held the Gordon McKay Professorship at Harvard until his death, becoming Emeritus in 1929 when impaired health prevented his active participation in the class-room.

As a teacher, Professor Swain's standards were high. He demanded from his students not only diligence and attention but, beyond that, insisted upon clear thinking on their part. His work was very thorough, of a high grade, and enforced by a steady drill in the principles of the subject. Mere acquirement of an understanding of the subject in hand was secondary to clear thinking along the line of work. He was a drill-master *par excellence* in a subject which especially lent itself to such treatment. Perhaps no other man has contributed so much, so consistently, and for so long a period, to the development of correct methods of thought and high standards of scholarship in the field of Engineering Education.

He taught many subjects at the Massachusetts Institute of Technology in the early days, the important course in Hydraulics being for several years in his direct charge. Later, he confined his attention almost exclusively to structures, which he taught not only to undergraduate students, but also to graduate students. Fate, in part through the Railroad Commission, seems to have decreed that both his teaching and his engineering activity should be largely in the field of structural work.

In 1893, men engaged in teaching engineering in the United States and Canada organized the Society for the Promotion of Engineering Education. Professor Swain was an early President of this Society, in which he took for many years an active part. His attitude toward the work of teaching was well defined in his Presidential Address on "The Profession of Engineering Teaching," in which he called attention to the necessity that professors of engineering should cultivate the teaching side, however attractive the engineering side might be. In his Annual Address[2] as President of the American Society of Civil Engineers, he said:

"Our teachers are generally, I think, chosen upon an incorrect principle; they are appointed by reason of what they know; it seems to me they should be selected for what they are—for their ability to teach, and their power of enforcing scientific discipline."

This attitude toward teaching, shared by his colleagues in the Department, meant much to the success of his administration. His reputation among the teachers of the country placed him in the first rank. This is attested by the fact that the Society for the Promotion of Engineering Education in 1928 made him its choice as the first recipient of the Lamme Medal, awarded yearly for "accomplishment in technical teaching or actual advancement in the art of technical training." This medal, Professor Swain valued most highly.

As an engineer and as an official in connection with engineering work, he had a large and successful experience. Aside from his duties with the Railroad Commission, he served as Consulting or Designing Engineer for a number of bridges, movable as well as fixed, an unusual case being the reconstruction of the old chain suspension bridge at Newburyport, Mass., as a stiffened cable suspension bridge with a hinge at the center and with reinforced concrete towers.

In 1894, the Boston Transit Commission was organized to construct the Boston subways. Professor Swain was one of the first Commissioners appointed. He served on it for twenty-four years and was its Chairman for five years. The work was without precedent in this country, was well carried out, and within the original estimates. The Tremont Street Subway was the first under-street subway in the United States. The tunnel under Boston Harbor to East Boston was a part of the work of this Commission, as was also the Charlestown Bridge. The expenditure by the Commission during Professor Swain's years of service was in excess of $30 000 000.

He served upon approximately twenty commissions in Massachusetts to fix the method of eliminating grade crossings of highways with steam railroads, those of Worcester, Taunton, Newton, and Waltham, being among the most important. He was a member of the Commission to revise the Building Laws of Boston. He was also frequently called upon as an expert in Court cases, not only in Massachusetts, but elsewhere.

Professor Swain served as expert for the State Commission on the valuation of the assets and liabilities of the New York, New Haven and Hartford Railroad Company. This involved appraisals of properties and securities held by the railroads, a physical valuation, a consideration of economic problems, and a critical examination of books. He was in charge of the Royal Commission on Railways and Transportation of Canada, involving 13 000 miles of Canadian railways. He directed similar valuations of other railroads, both large and small, among which were the Chicago Elevated Railways as well as the New York Central Railroad. Altogether, these valuations ran up to many hundreds of millions of dollars.

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[2] "Some Tendencies and Problems of the Present Day and the Relation of the Engineer Thereto": Address at the Annual Convention, Ottawa, Ont., Canada, June 18, 1913, by George Fillmore Swain, President, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXVI (1913), p. 1112.

In 1908, he was appointed by President Theodore Roosevelt a member of the Inland Waterways Commission and also of the Conservation Commission. This followed the hearings of a Congressional Committee upon the proposed Appalachian Forest Reserve, at which Professor Swain took an important part.

In 1918, he was one of the representatives of the American Society of Civil Engineers, as part of a delegation of nine from the four Founder Engineering Societies of the United States, to confer with French Engineers regarding the adoption of a program for the rehabilitation of France. He served as Secretary of the Delegation as well as Chairman of two of the Sub-Committees during a stay in France of about one month. He was also a member of the Franco-American Engineering Commission organized in 1919.

For a number of years he was a member of the Board of Judges to select names for the Hall of Fame, at New York University, and this University, in 1906, conferred upon him the honorary degree of Doctor of Laws.

During the celebration of the Fiftieth Anniversary of the founding of the University of California in 1918, he delivered the Hitchcock Lectures. There was then conferred on him the honorary degree of Doctor of Laws by President Benjamin Ide Wheeler, his former tutor in Providence who must have had great satisfaction in conferring the degree upon his former pupil. Professor Swain was much gratified at receiving this honor from the State University of his native State.

In 1914, he delivered the Charles S. Lyman Lecture, at Yale University. He choose for his title "The Conservation of Water by Storage." This was published in book form by the Yale University Press.

There were early publications on hydraulics and on structures for the use of his classes; they were highly regarded, but were not given general circulation. The *Journal* of the Franklin Institute for 1883 contains articles by him on "Mohr's Graphical Theory of Earth Pressures" and on the "Application of Virtual Velocities to the Determination of the Deflection and Stresses of Frames"; in 1887 he contributed a paper to the Society "On the Calculation of Stresses in Bridges for the Actual Concentrated Loads,"[3] a feature of which was the use of "influence lines," and in March, 1919, a paper, "On a New Principle in the Theory of Structures."[4] The three last-mentioned papers have had a very material effect upon present practice in structural computations and investigations. He was active in contributing papers to various societies, and many official reports were made by him, sometimes individually, sometimes as a member of a board.

Late in life, he brought to the point of publication three volumes of a treatise upon which he had worked intermittently for thirty years. The first volume on "Strength of Materials" was published in 1924, as was also the second volume, "Fundamental Properties of Materials." The third volume, "Stresses, Graphical Statics and Masonry," was published in 1927. His sudden illness delayed and perhaps prevented the completion of the remaining volumes, which, in manuscript, are nearly finished. As might be expected, these books were of monumental character, and of great value to the profession. His book on "How to Study" (1917), has had a wide and continuing circulation, being required as a part of the Engineering Course in a number of colleges. A somewhat later book, "The Young Man and Civil Engineering," was one of a series touching various occupations or professions, and was well received.

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[3] *Transactions*, Am. Soc. C. E., Vol. XVII (1887), p. 21.

[4] *Loc cit.*, Vol. LXXXIII (1919-20), p. 622.

Professor Swain was a member of many engineering or allied societies, including: Boston Society of Civil Engineers, of which he was President and Honorary Member; New England Water Works Association; American Society of Mechanical Engineers; American Institute of Consulting Engineers; American Society for Testing Materials; Canadian Institute of Engineers; Institution of Civil Engineers of Great Britain; Society of Engineers of Hanover, Germany; American Railway Engineering Association; New England Railroad Club; National Academy of Arts and Sciences; American Association for the Advancement of Science, of which he had been a Vice-President; American Academy of Arts and Sciences; American Forestry Association; and Society for the Promotion of Engineering Education, of which he was Senior Past-President.

Perhaps the highest honor to which a civil engineer may aspire is the Presidency of the Society, and to this post Professor Swain was elected in 1913, the first Professor of Engineering to be so honored. This qualification was doubtless a factor in his selection, although this was justified either by his standing as an engineer or because of faithful and efficient work within the Society.

He had long been of the opinion that activity in the Society should precede rather than follow election. By this standard, he qualified well. He had contributed papers to the Society, had served on its important committees, had been an influential member of its Board of Direction, and a Vice-President. Additional distinctions came from his elevation to Honorary Membership in 1929. At this ceremony, which occurred at the Annual Meeting, in New York, N. Y., on January 15, 1930, he was introduced by Hardy Cross, M. Am. Soc. C. E., with the following remarks:

"Bred of stock which made New England famous, educated in that school of great engineers, the Massachusetts Institute of Technology, he brought to his life work a firm foundation of training and of character. I need not itemize his long career of service to his profession, to his Alma Mater, to Harvard University, to the City of Boston, to the Commonwealth of Massachusetts, and to our nation. His record is well known to you. You know especially of his services to this Society, through its publications, on its committees, and in administrative work. As an author he has given us a model of clear, precise, and forceful style. As a scholar he has been honest and accurate in detail and broad in vision. As an engineer he has shown discrimination in the choice of appropriate tools of thought and resourcefulness in application to engineering work. As a teacher he has been preeminent in his ability to inspire men and to train them.

"This man, this teacher, is no mere academic pedagogue. He was a man who never permitted mazes of mathematics and mazes of statistics to befog the vision of the men who studied under him. He was a prophet, a priest of clear individual thought and aggressive individual judgment."

Professor Swain had been a pronounced believer in the advantage to engineers of associating more largely with business and professional men not engineers. His Presidential Address to the Boston Society of Civil Engineers dealt with this subject; and, consistently, he had been a member in Boston of the Union Club, a general social organization; the St. Botolph Club, where literary or similar qualification is necessary; the Boston Art Club, for which he could qualify as a musician; the Commercial Club, whose name indicates its business character; and the Boston City Club, very cosmopolitan in character. He was also a member of the Harvard Club, the University Club, and the Unitarian Club, a denominational dining club that meets monthly.

Neither was he averse to recreation. He enjoyed whist and billiards, at both of which he played a good game. He was also, as has been stated, fond of music, and played the piano with skill and taste. It was his habit to do things well.

Professor Swain had a fine library, not only of scientific and professional books, but also along varied lines, including economics as well as general literature which appealed to him. His library contained many books in German and French, and he read freely in both languages, both as literature and as scientific treatises.

He was thrice married; to Katherine Kendrick Wheeler, who died in 1901; to Mary Hayden Lord, who died in 1914; and to Mary Augusta Rand, who survives him, as do also his daughters, Barbara and Clara, and a stepdaughter, Alice Rand.

Professor Swain's success may be attributed in part to inherited ability and fine environment at home and at school, both here and abroad; a training extended beyond that secured by most of his fellows; an orderly habit in his work and economy of effort, avoiding waste; a faculty for clear thought and clear expression; a remarkable capacity for hard, rapid, and concentrated work, together with an appreciation of "short-cuts" and effective methods. He possessed the valuable faculty of taking advantage of his opportunities, the lack of which talent has spelled failure to many otherwise able men.

It is not easy to record the sense of personal loss felt by his many friends who held him in high esteem. Many who visited him in his last sickness will rejoice that the honors so much appreciated by him, the Lamme Award and the Honorary Membership in the Society, happily came to him as a meed of cheer as the sun was slowly setting upon an active, well-spent, and useful life. He died July 1, 1931, at his summer home in New Hampshire.

Professor Swain was elected an Affiliate of the American Society of Civil Engineers on September 5, 1883; a Member on March 2, 1892; and an Honorary Member on October 7, 1929. He was elected and served as a Director from 1901 to 1903; Vice-President, in 1908 and 1909; and President, in 1913.

WILLIAM BARCLAY PARSONS, Hon. M. Am. Soc. C. E.[1]

DIED MAY 9, 1932

William Barclay Parsons was born in New York, N. Y., on April 15, 1859, the son of William Barclay and Eliza Glass (Livingston) Parsons. In 1871, he went to school at Torquay, Devonshire, England, and during the following four years studied under private tutors while traveling in France, Germany, and Italy. Returning to the United States in 1875, he entered Columbia College, from which he was graduated in 1879 with the degree of Bachelor of Arts. He continued his studies at the Columbia School of Mines until 1882, when he received the degree of Civil Engineer.

After receiving his degree, William Barclay Parsons joined the Engineering Staff of the Erie Railroad Company and, by 1885, had reached the rank of Division Engineer. Resigning this position, he commenced practice as a Consulting Engineer in New York City in association with his brother, H. de Berkeley Parsons, M. Am. Soc. C. E. He maintained this office for twenty years. Between 1885 and 1891, he acted as a Consultant on projects which banking houses were called upon to finance or which they had to take over to complete. During this time, he also served as Chief Engineer for the water-works of several communities, including Vicksburg and Natchez, Miss., and as Chief Engineer or Consultant on railway lines in the West and in Jamaica, British West Indies.

In 1891, the New York State Legislature authorized a commission to take measures looking toward the construction of the long proposed subways for New York City. The late William E. Worthen, Past-President and Hon. M. Am. Soc. C. E., was appointed Chief Engineer, and William Barclay Parsons, Deputy Chief Engineer. Although only 32 years old, his name was well known as a railway engineer on account of his books "Turnouts" and "Track" which he wrote during his engagement with the Erie Railroad Company. He prepared the general plans for a subway under Broadway, but when the Commission tried to sell the franchise at public auction only a nominal bid of $100 was received, and the plan was dropped.

In 1894, a new Rapid Transit Act was passed by the New York State Legislature authorizing another rapid transit commission to plan a subway and to offer its construction contract for public bidding. William Barclay Parsons was made Chief Engineer of the new Commission. After early progress in the studies of the design had been made, and the route laid down, the project lagged, due to legal difficulties and the outbreak of the Spanish-American War.

With the outbreak of the war, William Barclay Parsons entered the service as Captain in one of the Volunteer Engineer Regiments. He was soon promoted to be Chief of Engineers of the New York National Guard with the rank of Brigadier-General, and was placed in command of the Regiment at Peekskill, N. Y., to teach the principles of army engineering. This appointment lasted only for the summer of the war, and just as he was preparing to return to New York City, members of a syndicate headed by the late Senator Calvin Brice asked him to make surveys and plans for the construction of railway lines in China.

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[1] Memoir prepared by Messrs. Eugene Klapp and Henry M. Brinckerhoff, and W. J. Douglas, John P. Hogan, and E. E. Halmos, Members, Am. Soc. C. E.

During part of 1898 and 1899, he conducted the survey of 730 miles of railway line from Hankow to Canton and of 270 miles of reconnaissance on other lines, largely through regions previously never penetrated by foreigners. Subsequently, a company was organized to build the railroad, of which he was elected President, and served as such for several years. His experiences during that period are recorded in his book entitled "An American Engineer in China."

Late in 1899, legal difficulties in the way of the subway construction in New York City having been eliminated, William Barclay Parsons returned there to resume work on this project. On February 21, 1900, the contract was signed with Mr. John B. McDonald, who had the financial support of the late Mr. August Belmont, and on March 24, 1900, work had begun in front of City Hall at a point now marked by a bronze tablet in the City Hall Subway Station. William Barclay Parsons continued as Chief Engineer of the Rapid Transit Commission until operation was started in 1904.

William Barclay Parsons' most outstanding accomplishment will always be his service as Chief Engineer of the first Subway System in New York City. In this, he created a standard which, notwithstanding the advance in the science of engineering, both structural and mechanical, is still being followed and adhered to as the best that the human mind has been able to develop. The characteristic feature of his plans was the construction of shallow subways wherever the topography was favorable, passengers being able to reach the street level by means of short flights of stairs.

In 1904, he was appointed a member of the Isthmian Canal Commission and, early in 1905, he went to Panama as a member of the Committee of Engineers which reported in favor of the construction of a sea-level canal. This report subsequently was adopted by the majority of the International Board of Consulting Engineers consisting of eight American and five foreign engineers—all the foreign and three American members (including William Barclay Parsons) voting for it, while the minority of five American engineers favored the lock type of canal, which latter was approved by President Roosevelt in February, 1906.

In 1905, he organized the firm of William Barclay Parsons, Consulting Engineers, with Mr. Eugene Klapp as a partner, and immediately undertook the engineering and construction of the Steinway Tunnel under the East River from Manhattan to Queens at 42d Street, New York City.

The Steinway franchise had almost expired by limitation when the late Mr. August Belmont bought it after being advised by William Barclay Parsons that it would be possible to complete the tunnel in the allotted time, or near enough to it, so that a short extension certainly could be had. To build the tunnel in the short period available, he conceived the idea of building an artificial island in the East River near the south end of Blackwell's Island, where he sunk a shaft and drove two shields east and west, at the same time working from the New York and Long Island shores. In this manner, he had four headings under way at once with the result that the tunnel was finished and almost completely lined when the franchise expired, and there was no difficulty in extending the time to permit completion of the work. The success of the undertaking was due to the untiring energy which William Barclay Parsons put behind the work, in which he was ably assisted by the late St. John Clarke, M. Am. Soc. C. E.

In 1904, William Barclay Parsons was appointed to the Advisory Board of Engineers for the Royal Commission on London Traffic. He thus had, as an American citizen, the unique experience of serving on a Royal Commission. He served on the Board with the late Sir Benjamin Baker, Hon. M. Am. Soc. C. E., and the late Sir John Wolfe Wolfe-Barry, two of the best known and most distinguished English engineers. His work with the Royal Commission was highly praised by his fellow members, and he was made a Director of the District Railway of London. In 1908, he read a paper before the Institution of Civil Engineers, describing the design and construction of the New York Subways, for which he was awarded the Telford Gold Medal which he considered one of his most valuable possessions.

In 1909, William Barclay Parsons' firm changed its name to Barclay Parsons and Klapp, and its membership was increased by the inclusion of Mr. Henry M. Brinckerhoff as a partner. A fourth partner was added to the firm in the person of Walter J. Douglas, M. Am. Soc. C. E., and, in 1920, the name of the firm was changed to Parsons, Klapp, Brinckerhoff & Douglas. Subsequently, in 1926, John P. Hogan, M. Am. Soc. C. E., was admitted to partnership.

William Barclay Parsons became active in the design and construction of hydro-electric plants when the generation of electric energy by water power was in its infancy in the eastern part of the United States. From 1900 to 1902 he was associated with Mr. H. de B. Parsons in the design of the Spier Falls Plant, on the Hudson River, which, at that time, was a pioneer development; the dam designed by and constructed under the supervision of the Parsons brothers is still one of the classic examples of gravity dam structures.

In 1905, his firm acted as Consulting Engineers for the hydro-electric development on the Susquehanna River at McCall Ferry, Pa., and, in 1906 and 1907, it supervised the construction of the hydro-electric plant at Colliers, N. Y., on the same river. Between 1908 and 1911, the firm designed the hydro-electric plant at Ephratah, N. Y., on Garoga Creek, which, for a long time, was the highest-head water-power plant in the East, utilizing a fall of 300 ft. In this development there were introduced such modern devices as a surge tank and a pressure-relief valve. This plant effected an almost complete control of the water-shed through the construction of Peck's Lake Dam and the Garoga Dam, both impounding large volumes of storage.

From 1911 to 1914, the firm acted as Consulting Engineers to the Niagara, Lockport, and Ontario Power Company, designing and supervising the construction of that Company's hydro-electric plant near Altmar, N. Y. William Barclay Parsons took special interest in the further developments of Niagara Falls, and had plans prepared for the utilization of the head lost in the Rapids years before the Ontario Power Commission conceived its project for the Canadian side, which later resulted in the construction of the Chippewa-Queenston Development.

In the same period, William Barclay Parsons was active in various engineering works in Cuba; his firm designed and supervised the construction of the Almandares Bridge, a multiple arch of considerable magnitude, and of the docks of the Port of Havana Docks Company, the first all reinforced concrete docks on this hemisphere. In 1914, the firm also made a report and plans for the reclamation of Zapata Swamp, near Cienfuegos, Cuba, the project covering an area of 500 000 acres. He also studied and made plans for a great harbor at Progreso, Yucatan; this project, however, did not materialize because of the Mexican Revolution.

In 1905, William Barclay Parsons was chosen Chief Engineer of the Cape Cod Canal and spent nine years in designing and constructing this waterway which recently became the property of the Federal Government. With this project he materialized the dream of several centuries to permit an all land-locked waterway between Boston, Mass., and New York City. For the description of the design and construction of this work in his paper, "The Cape Cod Canal,"[2] he was awarded the Norman Medal by the American Society of Civil Engineers.

Much of the period from 1904 to 1917 was devoted by William Barclay Parsons to studies and reports on urban and interurban transit for San Francisco, Calif., Detroit, Mich., Cambridge, Mass., Baltimore, Md., Chicago, Ill., Philadelphia, Pa., Toronto, Ont., Canada, and other cities. In 1916 and 1917, he acted as Chairman of the Chicago Transit Commission reporting on the feasibility of subways in that city. Upon the request of the editors of the Encyclopedia Britannica, he contributed an article in the Eleventh Edition of the Encyclopedia on "Rapid Transit."

In 1915, William Barclay Parsons was appointed Chairman of the Joint Commission of Engineering Societies to urge the organization of an engineering reserve in the Army and, in 1916, his efforts were successful when the National Defense Act was passed creating the Officers Reserve Corps. He was commissioned a Major and helped to organize the regiment which became, when the United States entered the World War, the 11th Engineers.

Soon after Congress declared war, the Government appointed a Military Railways Commission to study the measures to be taken for the transportation of large bodies of soldiers and huge quantities of war material from the United States to France and to the battle-fronts. The members of this Commission were Majors William Barclay Parsons and W. J. Wilgus, Members, Am. Soc. C. E., Capt. A. B. Barber, U. S. Corps of Engineers, and

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[2] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1.

Messrs. W. A. Garrett and F. de St. Phalle. Sailing orders were received on May 10, 1917, and thus this Commission included the first American military men, except hospital units, to be sent overseas.

The Commission inspected the British and French battle-fronts, possible ports of entry for American troops, and main lines of railways leading to the fighting line. After thorough study of all facts and conditions, it reported to the War Department on what construction must be undertaken immediately if victory were to be won, and what materials, machinery, equipment, and man power must be supplied from the United States to carry out the plans for the transportation of troops.

Upon the completion of the Commission's report, Major Parsons was re-assigned to his regiment, the 11th Engineers, which was then in England, as its Lieutenant-Colonel. From that time until the spring of 1918, he served as second in command, and from May, 1918, until the regiment was disbanded in May, 1919, he served as its commanding officer, except for a brief period in the fall of 1918. On October 9, 1918, he was promoted to the rank of Colonel. The regiment crossed to France on August 7, 1917, and on September 30, reported the first American casualties. The 11th Engineers first served for about six months with the British Third Army and fought with distinction near Cambrai and Peronne. On November 30, 1917, they were far up at the front, repairing railroad lines taken from the enemy, when the Germans counter-attacked, and the Engineers had to fight with picks and shovels until they could reach their base where their arms were. In the spring of 1918, the regiment was in Arras and Bethune during the crisis of the great German offensive. The regiment was attached to the First American Army shortly after its formation and participated in the Meuse, St. Mihiel, and Argonne offensives.

Following the war, Colonel Parsons was transferred to the Engineer Reserve Corps, and, in 1922, was appointed Deputy Chief Engineer with the rank of Brigadier-General.

For his work as Chief of the Military Railways Commission and his valor on the battle-front, General Parsons received the Victory Medal with five clasps, the Distinguished Service Medal, and a citation for conspicuous service from the United States; the Distinguished Service Order from Great Britain; the Order of the Crown from Belgium; and the Legion of Honor from France.

After the war, General Parsons returned to New York City and continued as the Senior Member of the firm bearing his name. He took great interest in the firm's activities in designing and constructing hydro-electric plants on the Hudson River and was actively engaged in the planning of a huge hydro-electric plant on the Colorado River, at Diamond Creek, in Arizona, which contemplated a dam of 500-ft height and 1 000 000-hp. capacity.

In 1920, he was chosen Consulting Engineer for the New York Water Power Investigation, organized by several large public utility companies operating in New York State, the object of which was to determine the available and economically promising power sites throughout the State. This work lasted until 1923, and resulted in the collection of most valuable information and data which form the basis of the further development of water power on the streams of New York State.

Besides the activities already mentioned, General Parsons was chosen by the City of New York to represent the City's interests in its controversy with the Federal Government on the question of extending the pierhead lines into the North River.

In the post-war period, General Parsons devoted a great deal of his time to the writing of his book, "American Engineers in France," a review of the activities of the Engineer Corps during the war. Subsequently, he wrote "Robert Fulton and the Submarine," and collected a tremendous volume of data for his contemplated four-volume work to be entitled, "Engineers and Engineering of the Renaissance." At the time of his death, this last work was almost completed.

All his life General Parsons was very active in lines outside the Engineering Profession. He had been one of the Trustees of Columbia University since 1897 and was elected Chairman in 1917, serving in this capacity until his death. He was Chairman of the Joint Administrative Board of the Medical Center, which directed the building of the Presbyterian Hospital, where he died. He drove the final rivet on the ten-story framework on May 24, 1926.

General Parsons accepted the invitation of the Engineering Laboratories of Cambridge University (England), in 1929, and gave two lectures, one on subways and the other on skyscrapers, before very large and enthusiastic audiences. He was a member of the Institution of Civil Engineers of Great Britian, a Fellow of the American Academy of Arts and Sciences, Honorary Member of the Institution of Architects, and a member of Societé des Ingenieurs Civils de France. As a Trustee of Carnegie Institution, of Washington, D. C., he took special interest in the archæological explorations made by the Institution in Yucatan. He was a Trustee of the New York Public Library, and a Vestryman and Warden of Trinity Church, New York City. He was also Vestryman of All Saints Church, Navesink, N. J., Chairman of the Middletown Township Public Library Trustees, and Trustee of the Monmouth County Historical Association. He was a member of the University and Century Clubs, in New York City, of the Metropolitan Club. in Washington, D. C., of the Tuxedo, Downtown, Army and Navy, St. Anthony, Church, and Rumson Country Clubs, and of the Colonial Lords of Manors.

General Parsons was made Doctor of Laws by St. Johns College, Annapolis, Md., in 1909; Doctor of Science by Princeton University, Princeton, N. J., in 1920; Doctor of Engineering by Stevens Institute, Hoboken, N. J., in 1920; and Doctor of Science, by Trinity College, Hartford, Conn., in 1921.

He was a thorough believer in, and practitioner of, the classic definition of engineering, namely, that it is the science of controlling the forces of Nature to the use of Man. He had a thorough education in the mathematics of physics and of engineering; but little, if any, sympathy with problems which were purely academic and had no immediate bearing on the practical. In consequence, his whole career was engaged in a class of work in which the convenience or safety of mankind was enhanced by constructions which, while accomplishing both of these great objects, were at the same time financially feasible. He had a far-seeing imagination which led him to champion projects like the first New York Subway and the Cape Cod Canal, both of which had been under consideration for many years, but had not materialized because of supposed engineering or financial difficulties. Such difficulties never daunted General Parsons when he could see the ultimate advantages to be obtained.

He was entirely courageous, both morally and physically, and when he was sure of his facts and the object to be obtained was worth the risk, he took the risk with splendid courage. Such a man was necessarily more than a purely technical engineer. He had to have a rounded personality, a knowledge of finance, a clear appreciation of the relation between cost and return. He had to have the ability to express his opinions and ideas in convincing language, thereby to sway men's minds to his own enthusiasm and courage.

As an executive, General Parsons was strict, but always just toward both contractors and members of his own staff. Many millions of dollars were spent upon construction under his direction, and his rulings were accepted by clients and contractors with complete confidence in his fairness. He had the courage to go to his client and insist upon the payment of just and reasonable claims. He was a fine organizer, successful in his selection of subordinates, and a man who compelled the most loyal affection and respect of every one who worked with or under him. This was finely exemplified in the relations existing between him and the officers and enlisted personnel of his regiment in the World War, and, earlier in his career, between him and the great organization he built up for the Rapid Transit Commission.

His commanding presence greatly added to his influence, for his was a face that no one ever forgot after once seeing. With all the success and all the publicity to which he was accustomed, he was essentially modest as to his own worth.

General Parsons was known in New York City and throughout the United States by large numbers of people, and he was almost equally well known abroad in London, Paris, Rome, and elsewhere. He traveled a great deal for business, study, and leisure, and was a great reader; he had an extremely keen sense of observation and a wonderful memory. These accomplishments made him much sought for socially as he was a most interesting talker and conversationalist. He had a fine sense of humor and, at the same time, a deep feeling for the serious side of life. He was untiring in doing favors for his friends and in helping the men over whom he held command during the war. His memory will always be cherished as that of an exceptionally gifted and wonderfully kind man in the hearts of those who had the privilege of knowing him.

He was married to Anna DeWitt Reed, of New York City, on May 20, 1884. She survives, with two children, Mrs. Rudolph Weld, of Boston, Mass., and Dr. William Barclay Parsons, Jr., of New York City.

General Parsons was elected a Junior of the American Society of Civil Engineers on June 7, 1882, a Member on November 2, 1887, and an Honorary Member on November 16, 1925. He served as a Director of the Society from 1896 to 1898.

FREDERICK WILHELM ALBERT, M. Am. Soc. C. E.[1]

DIED AUGUST 23, 1932

Frederick Wilhelm Albert, the son of Allen Diehl and Sarah Ann (Faber) Albert, was born at Williamsport, Pa., on January 4, 1877.

He received his early education in the schools of Washington, D. C., and, in 1897, entered the employ of Carl E. Pelz, M. Am. Soc. C. E., of Washington, as Chainman and Rodman.

At the outbreak of the Spanish-American War in 1898, Mr. Albert volunteered for service in the United States Army, and as Third Sergeant, U. S. Volunteer Engineers, assisted in building camps, roads, wharves, refrigeration plants, and other military construction.

At the conclusion of the war he accepted a position as Copyist and Detail Draftsman for the Cahill Writing Machine Company, Washington, D. C., and remained with that Company until 1901. After a few months as Assistant Draftsman on minor design, detail, and assembling work in the Washington Office of the American Ordnance Company, he began a sixteen-year period of service with the Washington Water Department, during which time he held the positions of Assistant Draftsman, Chief Draftsman, Assistant Engineer, Engineer, and Assistant Superintendent.

During this time Mr. Albert pursued studies at the Corcoran Scientific School of Columbian (which later became George Washington) University, from which he received the Muth Prize for excellence in mechanical drawing, the degree of Bachelor of Science in Mechanical Engineering in 1905, and the degree of Mechanical Engineer in 1906.

In September, 1917, Mr. Albert received his commission as Captain, Corps of Engineers, U. S. Army, for service in the World War. In December, 1917, he was promoted to the rank of Major and placed in command of a Replacement Detachment forming at Camp Devens, Massachusetts, for the 14th Engineers. Later, he became second in command of the 33d Engineers at the same camp. In March, 1918, he was given command of the 516th Engineers at Camp Gordon, Georgia, and organized, equipped, and disciplined this organization for overseas service. In May, 1918, he was returned to Camp Devens in command of the 33d Engineers with the rank of Lieutenant-Colonel and assisted in completing its preparations for going overseas. In July, 1918, he arrived in France and became Assistant to the Section Engineer, Base Section No. 5, American Expeditionary Forces, where he had complete charge of all sewer, water, and mechanical installations. In this capacity, he constructed the Pontanazen Rest Camp, including buildings, roadways, walks, electric plants, drainage, water supply, latrines, delousing stations, and baths. This camp was of such a size as to accommodate 80 000 men and was located three miles from Brest. For this work, Colonel Albert was cited in general orders, A. E. F., and given a certificate "Distinguished Service." He was appointed Officer of Mérité Agricole by the French Government.

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[1] Memoir prepared by Dr. Duncan MacRae, Asst. Technical Director, Chemical Welfare Service, Edgewood Arsenal, Edgewood, Md.

In 1919, he was assigned to the Office of the Chief of Engineers, at Washington, and served with the Joint Commission on the Reclassification of Salaries of Federal and District Employees. During 1921 and 1922, he was engaged in special engineering investigations for Farish and Company, of New York, N. Y., and, in 1923, became Assistant Comptroller of that Company.

From 1924 to 1929, Colonel Albert was Engineer in Charge of the Water Department of Knoxville, Tenn., and, in this capacity, was responsible for the conduct of the entire Department and the rehabilitation of the River Pumping Station, Filtration Plant, and Distributing System. He also designed and supervised the construction of a new Supply and Distributing System.

In 1929, he was appointed Assistant Chief of the Engineering Division of the Edgewood Arsenal of the Chemical Warfare Service, United States Army. He came to this work in splendid physical condition and endeared himself to his associates by his unfailing courtesy and happy disposition. In 1931, he was afflicted with an incurable disease which slowly undermined his health. In the face of this affliction he displayed a remarkable courage, continuing to attend his work daily until a few days before his death, although suffering considerable pain. Colonel Albert died in Baltimore, Md., on August 23, 1932, and was buried with military honors in Arlington Cemetery.

On October 10, 1916, he was married to Helen Margaret Barker, of Burlington, Vt. He is survived by his widow and his son, Brooke.

He held a commission as Colonel in the Engineer Reserve, commanding the 306th Engineers. He was a member of the Theta Delta Chi Fraternity, and edited its publication "Shield of Theta Delta Chi;" a Director and, for ten years, the Secretary of the Washington Society of Engineers; and a member of the Society of American Military Engineers, and of the Cosmos Club.

Colonel Albert was elected an Associate Member of the American Society of Civil Engineers on March 4, 1914, and a Member on January 14, 1919.

TRUMAN HEMINWAY ALDRICH, M. Am. Soc. C. E.[1]

Died April 28, 1932

Truman Heminway Aldrich was born on October 17, 1848, at Palmyra, N. Y. He was the son of William Farrington Aldrich and Louisa Maria (Klapp) Aldrich, and a great-grandson of Samuel Augustus Barker, Aide on

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[1] Memoir prepared by William Bullard Allen, M. Am. Soc. C. E.

General Lafayette's Staff during the Revolutionary War. He was a kinsman of Thomas Bailey Aldrich, the author, and the Hon. Nelson W. Aldrich, late United States Senator from Rhode Island, and was a brother of the late William F. Aldrich, former Congressman from Alabama. For three hundred years, the Aldriches and their connections, many of them Quakers in religion, had been teachers, lawyers, bankers, and authors, and had filled many noted rôles in New England life.

Educated in the public schools of Palmyra, West Chester, Pa., Military Institute, and Rensselaer Polytechnic Institute, at Troy, N. Y., Truman Heminway Aldrich received from the latter institution on graduation in 1869 the degree of Mining Engineer.

After his graduation, Mr. Aldrich served six months as Assistant Engineer on the Montclair Railroad, in 1869; six months as Assistant Engineer on the Newark and Paterson Branch of the Erie Railroad, in 1870; and nine months as Assistant Engineer for the Essex County Road Board of Newark, N. J.

In 1871, he entered the banking business at Selma, Ala., from which he retired in 1873, when he began the operation, under lease, of coal mines at Montevallo, Ala., which he purchased soon afterward. After a successful coal-mining operation, Mr. Aldrich leased the property to his partner, Col. Cornelius Cadle, and to his younger brother, Mr. W. F. Aldrich, subsequently selling his interest to the latter. He then proceeded to explore the coal seams of the Warrior Field of Alabama with a view to the development of coal that admitted of profitable commercial handling in coke ovens, the Jefferson seam of coal being opened by him at this time. Mr. Aldrich's investigations led to the formation of the Pratt Coal and Coke Company. He was one of its organizers and was elected, on July 18, 1878, Superintendent of the Company's operations. The mine operated by the Company was notable as being the first mine opened in the Pratt seam of coal which is now extensively mined and which is recognized as producing the highest grade of coal in Alabama for blast-furnace-coke purposes. Mr. Aldrich resigned as Superintendent and as a Director of the Pratt Coal and Coke Company, on November 13, 1880.

After his resignation he assembled several thousand acres of coal land at Henry Ellen, Ala., which he soon disposed of and upon which mines were opened. He then purchased and developed at Dudley, Ala., what is now known as the Brookwood Property of the Sloss-Sheffield Steel and Iron Company. In 1883, he organized the Cahaba Coal Mining Company which began the development of a large acreage of coal land in the Cahaba Field of Alabama, Mr. Aldrich being elected President of the Company. He also formed the Excelsior Coal Company, in 1889, which with him as President operated two mines at Gurnee, Ala., in the Cahaba Basin, and the Export Coal Company which had the distinction of being the first organization to introduce Alabama coal to ports in Central America, South America, the West Indies, and Mexico. Success continued to follow Mr. Aldrich in his latest ventures and, on December 19, 1892, the Tennessee Coal, Iron, and Railroad Company (subsequently, a subsidiary of the United States Steel Corporation), purchased the properties of the Cahaba Coal Mining Company and the Excelsior Coal Company, which at that time included 7 active coal mines, 467 coke ovens, 16 miles of standard-gauge railroad track, and about 44 000 acres of land. On April 4, 1893, Mr. Aldrich was elected a Director and Second Vice-President of the Purchasing Company, being also designated as General Manager.

Mr. Aldrich severed his connection with the Tennessee Coal, Iron, and Railroad Company in 1894, and in the same year was nominated by the Republicans and endorsed by the Populists for Representative in Congress from the Birmingham, Ala., District. The Democratic nominee, Oscar W. Underwood, received the certificate of election which Mr. Aldrich contested. He was seated, on June 9, 1896, and served for the remainder of the term which ended March 3, 1897.

After his essay into politics, Mr. Aldrich returned to the coal business, organizing the Cahaba Southern Mining Company which opened and operated mines at Hargrove, Ala., which were sold in 1903. He opened and operated, in 1898, the Brilliant Coal Mines which he turned over to the management of his son. He then entered the service of the Sloss-Sheffield Steel and Iron Company as Assistant to the President and Manager of Mines. In this capacity, he frequently acted as President and on August 3, 1900, was elected as Acting President, which position he resigned in 1901.

Mr. Aldrich then gathered together an acreage of coal land on which were operated the Virginia Mines which were later sold to the Alabama Steel and Wire Company. Then, with his son, Mr. T. H. Aldrich, Jr., he organized the Hillabee Gold Mining Company which operated in Tallapoosa County, Alabama. In 1905, together with Mr. Peter B. Thomas, he repurchased the Montevallo Mines which he had operated in the early Seventies, and became President of the Montevallo Coal Company which was operating the mines.

In December, 1911, Mr. Aldrich was appointed by President Taft, as Postmaster at Birmingham, and served in that capacity until December, 1915, during which time he discontinued his mining activities.

On July 3, 1918, during the World War, he was appointed Regional Adviser to the War Industries Board of the United States. The Birmingham Zone under the auspices of the Regional Adviser comprised the States of Alabama, Tennessee, Mississippi, and parts of Florida and Louisiana. Mr Aldrich, as a "dollar-a-year man," gave his entire time to the duties of the position until relieved by instruction of Chairman Baruch, on November 23, 1918, after the signing of the Armistice. In addition to the routine duties of his office, there was organized at his suggestion the Alabama Manufacturers and Operators Association which continued to function after the war as a permanent organization for the promotion and protection of Alabama manufacturers and operators. He then opened an office in Birmingham as a Consulting Mining Engineer, being at the same time interested in gold mining in Tallapoosa County, Alabama, where he owned a large tract of land.

Mr. Aldrich had a more intimate knowledge of the mineral deposits in the Birmingham District than perhaps any other man and his was a large contribution to the development of the coal and iron industry. His most important work in this connection probably was his early investigations which disclosed the value of Alabama coal for coking purposes and resulted in the manufacture of successful blast-furnace coke therefrom.

He was an interesting conversationalist and delighted in relating the early history of the district in the development of which he played so major a part throughout half a century. In early boyhood he became interested in the science of geology, and this interest activated him throughout his career. For a number of years he was associated with the late Dr. Eugene Allen Smith, State Geologist of Alabama, and through him became interested in tertiary fossils. In 1930, Mr. Aldrich became connected with the Alabama State Geological Survey as Paleontologist. He was the author of numerous papers on research, fossil molusca, and Alabama geological work. In recognition of his services to the State and the University of Alabama in the field of geological research, the University conferred upon him the honorary degree of Doctor of Science. Dr. Walter B. Jones, who succeeded Dr. Smith as State Geologist, has paid Mr. Aldrich the following tribute:

> "So far as I know, he has had more to do with the industrial development of Alabama than any man of our generation. He started underground coal mining in Alabama in 1875 and was actively engaged in that industry for half a century. He was also engaged in gold mining in Alabama and was for several years manager of the old Tennessee Company.
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> "As great as were his accomplishments in the industrial development of Alabama, it is my opinion his scientific researches were even greater. Somehow he found time to assemble 20 000 of the 30 000 known species of marine shells. This collection he gave to the Alabama Museum. He is considered as one of the four most eminent tertiary paleontologists of all time, along with T. A. Conrad, Isaac Lea, and Sir Charles Lyell.
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> "He has described hundreds of new species of fossils from Alabama, and certainly no one has contributed so much to the solution of the complex geology of our coastal plain as Dr. Aldrich.
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> "He was easy of approach and admired and loved by all who knew him, and he was generous, perhaps to a fault. We have lost our best friend."

Mr. Aldrich was twice married, his first wife being Anna Morrison, of Newark, N. J. By this union, there were three children who, with four grandchildren, and two great-grandchildren, survive him, his children being Mrs. Georgia (Aldrich) Herron, of Cincinnati, Ohio, Truman Herbert Aldrich, of Birmingham, Ala., and Mrs. Marie (Aldrich) Cravner, of Biltmore, N. C. After the death of his first wife, he married Mrs. Helen Lee Harris, of Birmingham, who also survives him, together with four step-daughters and six step-grandchildren.

He was the first member from Alabama of the American Institute of Mining and Metallurgical Engineers; he was also a member of the Geological Society of America, the Southwestern Geological Society, the Malacological Society of London, England, and Paris, France, and of the Manhattan Club, of New York, N. Y.

Mr. Aldrich was elected a Member of the American Society of Civil Engineers on May 4, 1881.

HENRY CLAYTON ALLEN, M. Am. Soc. C. E.[1]

DIED AUGUST 5, 1932

Henry Clayton Allen, the son of Albert and Elvira Elizabeth (Stanford) Allen, was born on September 10, 1864, at Newark, N. J. After attending the common and High Schools of Newark, Mr. Allen moved to Syracuse, N. Y., with his parents, and was graduated from the Syracuse High School in 1882.

During the succeeding four years he was employed with the late H. Wadsworth Clarke, M. Am. Soc. C. E., of Syracuse, on surveys to establish the boundary lines between the State of New York and the States of New Jersey and Pennsylvania. On the completion of these surveys in the spring of 1886, Mr. Allen accepted an appointment as Assistant City Engineer of Syracuse.

In October, 1887, he was engaged as Leveler with the New Croton Aqueduct Commission, of New York, N. Y. In recognition of meritorious work performed, his promotion to Assistant Engineer followed in 1889, with assignment to special duty in supervising the correction of defective work.

Mr. Allen returned to Syracuse in 1890 to become Deputy City Engineer. He was appointed City Engineer in October, 1890, and held this position until February, 1896, during which time he designed and directed the construction of sewers, pavements, and other municipal work.

From February to June, 1896, he was employed by the Department of State Engineer of New York, in making surveys of the Oswego Canal. On the completion of this work, Mr. Allen engaged in private practice, designing and supervising the construction of the water supply for Marathon, N. Y., pavements in Cortland, Norwich, and Oxford, N. Y., and of a system of sewers for Dodgeville, Seneca Falls, and East Syracuse, N. Y. He also prepared plans for the electric railway from Syracuse to Baldwinsville, N. Y. In 1897, under contract with the State of New York, he undertook the reconstruction of a section of the old Erie Canal near Rochester and continued with the work until this improvement was suspended by the State.

In April, 1900, Mr. Allen was appointed Resident Engineer of the Middle Division of New York State Canals, in charge of public work, including highways, locks, dams, bridge construction, and canal maintenance and repair. His promotion to Special Deputy State Engineer in May, 1904, placed him in responsible charge of all the new Barge Canal work of the State. Following the election of a new State Engineer, Mr. Allen resigned from the service and was again appointed City Engineer of Syracuse on January 1, 1907, serving until December 31, 1913, when he relinquished this office to become Chief Engineer of the Syracuse Grade Crossing Commission and devoted all his time to the problems of grade elimination.

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[1] Memoir prepared by Glenn D. Holmes, M. Am. Soc. C. E.

Mr. Allen was appointed City Engineer of Syracuse for the third time on January 1, 1916, and occupied this post until 1921 when he opened an office to engage in private practice as a Consulting Engineer.

From June, 1907, to December, 1913, and from January to July, 1930, Mr. Allen served as a member of the Syracuse Intercepting Sewer Board. He resigned from this Board to become a member of the Grade Crossing Commission, serving gratuitously, and continuing as such until his death. He had given much of his time as Consultant to the Committee on Unemployment and Work Relief, in Syracuse.

In 1918, Syracuse University honored Mr. Allen by conferring upon him the degree of Master of Science in Engineering. Mr. Allen was a Charter Member and Past-President of the Technology Club of Syracuse, and, in May, 1932, was elected an Honorary Member in recognition of his outstanding accomplishments as an Engineer and his long and valued services to the community.

Nearly all his active life was devoted to the interests of his City and State. His advice and counsel on matters outside his own particular field of endeavor were continually sought, and he gave freely of his time and efforts in meeting these demands.

The fact that many knew him during his years of service with the city as "Straight Line Allen" was a tribute to his uncompromising honesty of policy as well as to the directness of his methods as an engineer. It was axiomatic that whatever work he planned and supervised, was well done. Serving under six Mayors, Mr. Allen had much to do with the steady expansion and development of Syracuse during the era of its most rapid growth.

With exceptional foresight regarding future expansions of the city he mapped its annexations, planned its sewers, sidewalks, and pavements, built its school houses, laid out its parks, and directed an extraordinary amount of its municipal activity. He was the trusted adviser of more than one Mayor in many matters not strictly concerned with engineering and few knew more of the machinery of government than he.

Possibly Mr. Allen's lack of a college training intensified his studious habits not only in professional subjects but also in other lines. He acquired a reading knowledge of technical French; he was a student of the history of the ancient inhabitants of America; he loved Nature studies, and was specially interested in his collection of local ferns.

On December 27, 1887, Mr. Allen was married to Anna Eloise Bayette, of Syracuse. He is survived by his widow, three children, Mrs. Eloise W. Holmes, Mrs. Marion S. Bushnell, and Henry B. Allen, and four grandchildren.

He was a member of the Archæological Society of Syracuse, the Chamber of Commerce, the Rotary Club, the University Club, and the Bellevue Country Club. He was a communicant and officer of Trinity Protestant Episcopal Church.

Mr. Allen was elected an Associate Member of the American Society of Civil Engineers on July 1, 1891, and a Member on January 3, 1905.

CHRISTIAN ANDERSEN, M. Am. Soc. C. E.[1]

DIED NOVEMBER 18, 1932

Christian Andersen, the son of Peter M. and Marie Andersen, was born on December 28, 1860, on a small island called Möen, in the Town of Stege, near Copenhagen, Denmark, where he received his early schooling. He was educated at the Royal Danish Military Academy, at Copenhagen, from which he was graduated in 1881, having received special instruction in Mathematics and the Sciences. Subsequently, he served one year in the Royal Danish Army as Second Lieutenant.

Upon the completion of his military service, Mr. Andersen came to the United States to live with his brother, H. J. Andersen, an Architect, in Spring Valley, Minn. His interest in Mathematics and the Sciences induced him to enter the University of Minnesota from which he was graduated with the degree of Bachelor of Science in the Civil Engineering Course in 1888. From 1884 to 1888, during his vacation periods, he was employed in his brother's office and on miscellaneous land surveying and railroad work.

In 1886, Mr. Andersen undertook his first engineering work, as Division Engineer for the Minneapolis, Lyndale, and Minnetonka Railway Company, having charge of the location and construction of ten miles of railroad. In 1887, before graduating, he also occupied the position of Resident Engineer of the Northern Pacific Railway Company on the Moure-Edgle Extension.

On the completion of his course at the University of Minnesota, he moved West and finally located in Oregon, where he became particularly interested in railway engineering. In 1888, he was Topographer under the direction of W. P. Watson, Locating Engineer of the Washington and Idaho Railroad, and, in 1889, he was promoted to the position of Locating Engineer. His associates on this work state that he showed exceptional ability in railroad location and construction.

In 1890, Mr. Andersen moved to Astoria, Ore., where a partnership was formed with Messrs. Henry Thielson and Charles F. Lester, and an office was opened for general engineering practice in the Northwest. In addition to performing the municipal engineering for the City of Astoria during 1890 and 1891, Mr. Andersen designed and constructed a sewer system for Ellensburg, Wash., a great many wharves and docks in Astoria, and did some land surveying under contract.

In 1891, he moved to Portland, Ore., where he was engaged as Assistant City Engineer, and his duties as such embraced all branches of engineering work. Several of the employees of the City Engineer's Office at this time recall Mr. Andersen and his work during that period and speak very highly of his ability as an engineer. He resigned this position in 1895 to engage again in private practice.

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[1] Memoir prepared by O. Laurgaard, and D. C. Henny, Members, Am. Soc. C. E.

During the period from 1895 to 1899, he was in private practice with offices in Portland, and his work covered miscellaneous projects of all kinds, including municipal engineering, land surveying, subdivision work, contract surveying, and light-house construction for the Federal Government. He also built a large salmon cannery in Alaska during the season of 1898.

From 1899 to 1902 he was engaged on difficult location work for The Dalles Southern Railroad Company and built light-houses for the United States Government in Alaska.

After having successfully passed a Civil Service examination for Irrigation Engineer in 1903, Mr. Andersen was appointed as Engineer of the United States Reclamation Service, and began his work in the State of Washington in May, 1903. During 1903 and 1904, he was in charge of preliminary surveys for the Palouse Project and the Big Bend Project (later known as the Columbia Basin Project). Subsequently, Mr. Andersen was placed in charge of the preparation of design and estimates for the Big Bend, Palouse, and Okanogan Projects, with Spokane, Wash., as headquarters.

In the spring of 1905, he was in charge of field investigation surveys, hydrographic investigations, and the preparation of preliminary plans, specifications, and estimates for the Okanogan Project, with headquarters at Conconully, Wash. On the completion of the field work he was assigned to office work, at the headquarters of the Reclamation Service, at Portland.

From November, 1905, until the summer of 1910, Mr. Andersen was in charge of the construction of the Okanogan Project. The work involved the construction of the Conconully Dam and Reservoir, the Diversion Dam and main canals, and a complete distribution system. On the completion of this project, Mr. Andersen was assigned to the Yakina Valley, Washington, on general investigations. During a part of 1910, Mr. and Mrs. Andersen made a trip abroad, visiting his former home and relatives.

In 1911, he resigned from the Reclamation Service to enter private work at Spokane. After spending approximately one year there, he moved to his home which he had continually maintained in Portland.

From 1912 until the World War, Mr. Andersen devoted most of his time to study and research. In particular, he made extensive research into the field of light and optics, and invented and received patents for many improvements to the motion-picture machine. For a part of this time he was also engaged in the practice of Christian Science, of which he had been a devotee for a number of years.

When the World War broke out, Mr. Andersen applied for and received a commission as Captain in the Ordnance Department, U. S. Army, under date of August 1, 1918, and was assigned to work in Washington, D. C.

When he was mustered out of the service on March 25, 1919, he was transferred to the Federal Bureau of Roads in the Department of Agriculture as Senior Highway Engineer and stationed at Portland. He supervised Federal highway work in Oregon and Washington until December 31, 1930, when he was retired on account of age limit. During his service with the Bureau of Roads, he supervised all highways in the State of Washington receiving Federal aid and, in addition, during the last four years, he received the title of Materials Engineer in charge of the testing and co-ordination of all materials used in highway construction.

He was known to all his co-workers in the Federal Bureau of Roads as a man of good morals and high principles, ruggedly honest, with a quiet disposition, and always willing to assist wherever and whenever it was possible. He was also known to his co-workers as a brilliant mathematician with a thorough knowledge of the exact sciences.

For the last two years of his life, Mr. Andersen continued to conduct his research and investigation work. He persisted until the end which came rather unexpectedly while he was quietly resting. He was held in high regard by all his associates and friends to whom his sudden passing came as a severe blow.

For many years he had been an active member of the Sixth Church of Christ, Scientist, in Portland. During 1931 and 1932 he was a Director of this church and a member of the Building Committee engaged in the construction of a beautiful new edifice. Mr. Andersen had been a highly respected member of Washington Masonic Lodge No. 46 in Portland since May 17, 1895. He was also an active member of the American Legion, and joined this organization at the first opportunity. He was very patriotic and always took a keen interest in those affairs and matters that pertained to the Nation's welfare. He was a member of the Northwest Society of Highway Engineers, having become a member in July, 1917, at the first meeting of that Society after its organization.

Mr. Andersen was a man of large physique, well-proportioned, and of striking appearance. He was as a general rule very quiet and unassuming, and never sought publicity for himself or the limelight for his actions. On account of being reserved and sparing in his words, he hid his sympathetic nature under an apparently somewhat gruff exterior. He had a large heart and was always ready to help those who required his services. He was meticulous in all his arrangements and undertakings, and any one having dealings with him would soon discover that he was ruggedly honest. Countless friends sincerely mourn his passing.

He was married on July 2, 1890, to Gertrude Cook, daughter of Capt. Horatio and Kate Daley Cook, in Portland. Mrs. Andersen survives him.

Mr. Andersen was elected a Member of the American Society of Civil Engineers on January 2, 1901.

HERBERT LEANDER AULLS, M. Am. Soc. C. E.[1]

DIED APRIL 25, 1931

Herbert Leander Aulls was born in Washtenaw County, Michigan, on June 14, 1852, and was the son of James Taylor and Eliza (Randall) Aulls.

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[1] Memoir prepared by Herbert S. Crocker. Past-President, Am. Soc. C. E.

Mr. Aulls was a lineal descendant of William Aulls, a younger son of an Irish nobleman, who came to this country and settled in Pennsylvania in pre-Revolutionary days. William Aulls later removed to Pleasant Valley, New York, where in the spring of 1793, he made the first clearing and built the first house in the Valley.

The boyhood days of Herbert Leander Aulls were spent in Michigan. In 1871, he entered the University of Michigan, where he took a two years' special course in engineering. Because of a severe illness, he was unable to complete the courses necessary for graduation.

Upon leaving college, Mr. Aulls removed to Central California, and, from 1874 to 1876, was employed as Chainman, Rodman, Levelman, and Transitman on irrigation work. From 1877 to 1880, inclusive, he was engaged in general engineering work, principally drafting, alternating his time between the States of Michigan, California, and Colorado. All this time he was acquiring engineering experience which served him well when he began to assume responsible positions in the years that followed.

On March 14, 1881, Mr. Aulls assumed a permanent residence in Denver, Colo., and this city he considered his home during the remainder of his life. In April of this year he was appointed Division Engineer of the Denver, South Park and Pacific Railroad Company, on its Breckenridge Division, and was engaged in making surveys which resulted in the completion of the construction of the narrow-gauge line from Breckenridge to Leadville, Colo., in 1883. As a matter of history, it may be here stated that the Denver, South Park and Pacific Railroad Company later became merged with other short lines to form, on April 1, 1890, the Union Pacific, Denver and Gulf Railway Company. On November 19, 1898, this latter road was sold under foreclosure to a purchasing committee, which on December 29, 1898, conveyed the property to the Colorado and Southern Railway Company.

In the latter part of 1881, Mr. Aulls became Chief Assistant Engineer of the Denver and New Orleans Railroad, this road being another of the constituent lines of which the Colorado and Southern Railway was formed. His duties included charge of the drafting-room and office work in connection with the preliminary and location surveys of what is now the main line of the Colorado and Southern Railway between Pueblo, Colo., and the Canadian River in Northwestern Texas, as well as construction on four divisions of the road between Denver and Pueblo. This engagement continued until the latter part of 1882, when the road between Denver and Pueblo was put in operation, after which the Engineering Department was suspended.

After the November, 1882, election, Mr. Aulls was appointed Assistant City Engineer of Denver, in which capacity he performed largely routine duties of the office. In October, 1883, the city granted him thirty days' leave of absence in order that he might complete surveys in the Elk Mountains north of Gunnison, Colo., under "Orders of Survey" from the United States Surveyor General. Exposure while engaged upon this and some underground work in mines caused his illness and close confinement until April, 1884.

In the meantime, he had resigned his position with the City of Denver in December, 1883, and acting in accordance with medical advice, decided to leave Colorado temporarily. Accordingly, in July, 1884, he entered the employ of the firm of Raymond and Campbell, Engineers and Contractors, at Council Bluffs, Iowa, which firm was engaged in the construction of highway bridges. His duties with this firm consisted of general office work.

In August, 1885, Mr. Aulls entered the service of the Union Pacific Railroad Company, in Omaha, Nebr., as Chief Draftsman. His work included inspections and the supervision of erection of bridges. This position he retained until 1886. Leaving Omaha on April 26, 1887, he returned to Denver and formed a partnership with the late Harvey C. Lowrie, M. Am. Soc. C. E., under the firm name of Lowrie and Aulls. This firm was engaged in general engineering work, mostly land surveying and reporting upon irrigation projects. A few years later, this partnership was dissolved, and on July 29, 1890, the firm of Martin, Aulls, and Follett engaged in similar work. The activities of the late W. W. Follett, M. Am. Soc. C. E., of this firm, in work for the United States Geological Survey led to various engagements, among which was the location of pipe lines in the State of California in the winter of 1891-92. Some time during this period, Mr. Follett severed his relations with the firm, and the work was carried on by Messrs. Martin and Aulls.

A recital of the detailed activities of these engineers would throw light upon the difficulties and inadequate remuneration of engineering practice in sparsely settled country, engaged largely in pioneering of irrigation projects.

Records show that in the early Nineties, Mr. Aulls turned his attention to bridge construction, his firm having been appointed Engineers charged with the preparation of plans and construction of a proposed viaduct across the railroad yards on the line of 14th Street, in Denver. This project failed of completion, due to lack of funds of the Development District which had promoted it. However, in 1894, in partnership with Messrs. M. J. Patterson and W. R. Hand—under the firm name of Aulls, Hand, and Patterson—Mr. Aulls was largely responsible for the design and construction of the 38th Street Bridge across the South Platte River, in Denver. On the completion of this work, the firm was retained to design and superintend the construction of a bridge over Cherry Creek, at Broadway, also in Denver. The design of this bridge involved some unique features requiring unusual ingenuity, the 125-ft steel plate girders being perhaps the longest straight girder spans designed to that time. The records in the office of the City Clerk of Pueblo, show that Mr. Aulls was retained by that City, in addition to his structural practice, as Consulting Engineer during the years 1899 to 1901, in the design of the first sewerage system of that city. However, the construction of this system was delayed and was executed by others a few years later.

In 1899, Mr. Aulls went to Alamagordo, N. Mex., where for a short time he was employed in the office of the Chief Engineer of the El Paso and Northeastern Railroad Company, a line then under construction. Following this, he was employed in a similar capacity by the Oregon Short Line Railroad Company in Salt Lake City, Utah, and, in 1901, he returned to Denver where, after a brief engagement as Assistant Engineer in the City Department of Public Works, he returned to railroad practice and became Office Engineer of the Denver, Northwestern and Pacific Railroad Company, with which Company he remained until 1904.

He went to Mexico in 1905 as Resident Engineer for the Rio Grande, Sierra Madre and Pacific Railway Company, with J. P. Hallihan, M. Am. Soc. C. E., Chief Engineer. He was stationed at Nueva Casas Grandes, Chihuahua, for a year and, later, the offices were transferred to Juarez, and Mr. Aulls established a residence in El Paso, Tex. During 1907, he was Acting Chief Engineer for the road.

During 1908 and 1909, he did some work with the International Boundary Commission, with the late Mr. W. W. Follett, Consulting Engineer of the American Section.

In 1911, Mr. Aulls went to South America with a New York Company as Locating Engineer for the Ferrocarril Huancayo a' Ayacucho, in Peru. He remained for a year, but the effect of the high altitude and manner of living in the interior of Peru compelled him to return to the United States, and he never entirely recovered from the effects of this expedition.

From 1914 to 1917, Mr. Aulls was employed by the writer as Office Engineer, during the construction of the Colfax-Larimer and 23d Streets Viaducts, in Denver.

In 1918, he entered the service of the Colorado and Southern Railway Company and became Bridge Engineer for that road, which position he held until the time of his retirement in February, 1927, at the age of 75. After his retirement, Mr. Aulls spent much of his time on amateur color photography, in which he had always been interested, and which had been a pleasant pastime most of his life. He died in Denver on April 24, 1931, after a brief illness.

A man of high principle, sound judgment, and fine character, Mr. Aulls had those qualities which endeared him to those with whom he came in contact in a personal way. Although modest and retiring, he never hesitated to stand up for what he believed to be the right course of action. He had a judicial mind, and in controversy could be relied upon to weigh fairly the opinions of his opponents. He was loyal to his associates and possessed of a kindly and serene disposition. He left this world without enemies and with a most enviable record in his field of operations. Mr. Aulls was not a member of any church organization, although he was in sympathy with all broad religious ideas and trends. He attended Plymouth Congregational Church, in Denver.

He was married on June 5, 1877, to Alice A. Grace, of St. Clair, Mich., and is survived by his widow and a daughter, Ina T. Aulls, of Denver.

Mr. Aulls was elected a Member of the American Society of Civil Engineers on July 1, 1885.

FRANCIS WHEELWRIGHT BELKNAP, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 22, 1932

Francis Wheelwright Belknap, the son of Charles and Fanny (Wheelwright) Belknap, was born in Baltimore, Md., on January 26, 1874. He was descended on the paternal side from Samuel Belknap who came from England during the early part of the Eighteenth Century and settled in East Windsor, Conn., and on the maternal side from John Wheelwright who migrated to New England, in 1636, as one of the settlers of the Massachusetts Bay Colony.

Mr. Belknap's preliminary education was received in a private school in Baltimore and the High School of Newport, R. I. His college training was obtained at the Massachusetts Institute of Technology, at Cambridge, Mass., from which he was granduated in 1895 with the degree of Bachelor of Science.

Shortly after his graduation, Mr. Belknap started his business career as Assistant Engineer with the Massachusetts Highway Commission at Boston, Mass., which position he held for two years.

From 1897 to 1898 he was one of the Engineers engaged in the Nicaraguan Canal Survey. In 1898 and 1899, he was employed as Draftsman with the Dock Department of the City of New York, and, from 1900 to 1904, he was Assistant Engineer with the same Department, during the latter two years of which he had responsible charge of the Chelsea Section improvement.

From 1904 until his death on September 22, 1932, Mr. Belknap was Engineer, General Manager, and President of the Phoenix Construction Company, of New York. During this period he directed and had charge of two hundred contracts, which included the substructures of the Penham Creek Viaduct and the Erie Railroad Bridge, at Hackensack, N. J.; West Shore Pier 5, at Weehawken, N. J.; and the Ferry Terminal, as well as foundations for power houses, bulkhead, dock and sea walls, and intake and dischage tunnels, at Stapleton, Staten Island, N. Y. Mr. Belknap also designed bulkhead walls, foundations, piers, etc., such as the Travers Island Bulkhead, the Albany, N. Y., Dock Wall for the Hudson Navigation Company, the Company's Dock, at Flushing, N. Y., and the Coney Island Boardwalk. At the outbreak of the World War he took his entire organization to France to build docks for the American Expeditionary Forces.

He strove always to produce excellent workmanship and many times refused to bid on work when he knew the plans proposed were inadequate to the needs of the client.

Mr. Belknap was a member of the Military Order of the Loyal Legion of the United States, and his clubs included the Oakland Golf Club, New York Yacht Club, and the University Club, of New York. His pastimes were golf and sailing. He was a member of the New York Thirty-Foot Association, through his ownership of the *Alera* and the *Amorita*.

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[1] Memoir prepared from information supplied by Charles Belknap, Esq., Everett, Mass.

He was married on July 29, 1899, to Millicent Rebecca Grant, the daughter of Donald Fraser Grant, of Boston, Mass., and Newport, R. I., who survives him, together with their five children, Millicent F., Francis P., Charles, Elizabeth M., and Donald F. G. Belknap.

Mr. Belknap was elected an Associate Member of the American Society of Civil Engineers on February 1, 1905, and a Member on June 24, 1914.

LOUIS DAVID BLAUVELT, M. Am. Soc. C. E.[1]

Died October 26, 1930

Louis David Blauvelt was born in Tappen, N. Y., on April 8, 1867. He came of sturdy Dutch ancestry and distinguished Revolutionary stock. His early education was obtained at Rutgers Preparatory School and at Rutgers University, New Brunswick, N. J. At the latter institution he was a member of the Class of 1886, but left college in 1885 without completing his course. The Honorary Degree of Civil Engineer was conferred on him by the University on June 13, 1916. He was a member of Delta Phi Fraternity.

Soon after leaving college, Mr. Blauvelt decided to take up the practice of his profession in the West. Accordingly, he removed to Colorado, and was engaged in railway engineering there, as well as in the States of New Mexico and Washington. From 1889 to 1892, he held minor positions; but in the latter year, he became Chief Engineer of the Pecos Irrigation and Improvement Company at Eddy, N. Mex., in which capacity he had charge of the construction of dams, canals, and frame and concrete structures, together with highways auxiliary to the project. This project included the Lake Avallon Dam, at Seven Rivers, N. Mex. The cost of this work was approximately $3 000 000. This engagement continued until 1894.

From December, 1894 to February, 1896, Mr. Blauvelt was Assistant to the Signal Engineer of the Chicago and Northern Pacific Railway Company, at Chicago, Ill. He then returned to the State of his adoption, and after a brief service with the Midland Terminal Railway Company, at Cripple Creek, Colo., engaged in mining until May, 1899. During this period, he was leasing on his own account, and was Superintendent of the "C. O. D.," Jefferson, and Hoosier Mines. Upon abandoning mining work, he spent about three years as Locating and Division Engineer in charge of 30 miles of heavy mountain construction, for the Colorado Springs and Cripple Creek District Railway Company. On the completion of this work, he spent several months as Locating Engineer of the San Pedro, Los Angeles, and Salt Lake Railway.

In May, 1902, Mr. Blauvelt accepted a similar position with the Denver, Northwestern and Pacific Railway Company (Moffat Road, and more recently known as the Denver and Salt Lake Railway Company). His connection with this Company continued without interruption until 1908, and the various positions which he occupied may be briefly summarized as follows: August, 1902 to April, 1904, Resident Engineer in charge of locating parties, and Division Engineer in charge of construction; May, 1904, to April, 1905, Office Engineer in Chief Engineer's Office, and Engineer, Maintenance of Way, in charge of bridges, buildings, and track; and 1905 to 1908, inclusive, Assistant Chief Engineer, serving under the Chief Engineer, the late Horace A. Sumner, M. Am. Soc. C. E. This railway construction extended from Denver, northwesterly to Steamboat Springs, Colo., a distance of 191 miles. It included 65 miles of difficult and even spectacular location across the Continental Divide of the Rocky Mountains, and involved about 44 tunnels having an aggregate length of 17 000 lin. ft. It is known as one of the most scenic roads ever constructed, and has attracted tourists from all over the world. To have been intimately and responsibly connected with its construction would be considered a high honor by any railway engineer.

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[1] Memoir prepared by Herbert S. Crocker, Past-President, Am. Soc. C. E., and Oliver T. Reedy, M. Am. Soc. C. E.

Upon the completion of this work Mr. Blauvelt was appointed, in 1910, General Manager and Chief Engineer of the San Luis Southern Railway, with headquarters at Blanca, Colo. In this capacity, he constructed 32 miles of railroad and laid out town sites. The approximate cost of this work was $600 000.

In 1911, the Denver, Northwestern and Pacific Railway Company having decided to extend its lines from Steamboat Springs to Craig, Colo., a distance of 41 miles, Mr. Blauvelt was chosen as Chief Engineer. His duties were those usually pertaining to this position, involving complete supervision of the plans and field construction. In 1914, upon the death of the General Superintendent, he took over the duties of that office and continued the occupancy of the dual position until 1918, when the construction work was completed. In the meantime, the Railroad Company had been re-organized and had become the Denver and Salt Lake Railway Company.

Mr. Blauvelt then turned his attention to the hydraulic field, and, for about five months, or until May, 1918, was Chief Engineer of the Maxwell Irrigation Company in New Mexico, in charge of the construction of dams. This position he relinquished in order to enter military service in connection with the World War.

With the commission of Major, Quartermaster Corps, dated May 17, 1918, he entered the service on May 21, 1918, and, on June 1, was assigned to duty as Assistant to the Construction Quartermaster, Edgewood Arsenal, Edgewood, Md., where he was placed in charge of the railways and water supply of that project. In July of the same year, he was appointed Assistant Constructing Officer, Edgewood Plant, upon the taking over of the construction work of the chemical and shell-filling plants from the Ordnance Department, by the Construction Division. On September 13, 1918, he was transferred as Constructing Officer to the Western Cartridge Company on its plant extension at East Alton, Ill. This plant was to produce 3 000 000 small arms cartridges daily. Work on this project was suspended December 5, 1918, and in March, 1919, Major Blauvelt was transferred to Fort Bliss, Texas, where he was engaged in construction work on cantonments. There still remained many routine duties incident to shutting down the various projects, and Major Blauvelt remained in service until August 2, 1919, when he was honorably discharged at Fort D. A. Russell, Wyoming.

After his discharge Major Blauvelt returned to Denver, where he opened an office under the firm name of Burrowes and Blauvelt, and was engaged as a Consultant upon various matters of a local character. During this period of private practice, Major Blauvelt was appointed Consulting Engineer for the Colorado Railroad Commission, which body was charged with the study of Transcontinental Divide tunnels in Colorado. Later, when a Moffat Tunnel Improvement District was formed and the building of that tunnel was a certainty, the Tunnel Commission appointed Major Blauvelt as its Chief Engineer; but this position he resigned to accept an appointment by the Governor to the important office of State Highway Engineer of Colorado, on May 7, 1921. This appointment did not prevent the continuance of his active interest in the affairs of the Moffat Tunnel, as he was retained by the Tunnel Commission as a member of the Board of Consulting Engineers, which, in general, approved all plans and details of construction.

From the date of his appointment until his death, after a protracted illness, Major Blauvelt retained the position of State Highway Engineer. The duties of this office are too well known to be described here. Major Blauvelt proved to be a highly capable executive and an engineer of vision. He combined the qualities of firmness and diplomacy, and won to himself the loyalty of the members of his extensive organization. His was the duty of conceiving and planning the extensive system of highways which have been a comfort to the citizens of Colorado, and an attraction for those living outside the borders of the State. He leaves as a monument to his professional ability a permanent record of highways and bridges well designed and of high-class construction.

Major Blauvelt became a member of the American Association of State Highway Officials in 1921. He was placed on many important committees of the Association, and for several years was a member of the Executive Committee. In 1924, he was elected Secretary and, in 1925, became Vice-President. The following year he was elected President of the Association and, in his honor, the National Convention was held in Denver.

As a man, Major Blauvelt was of a retiring disposition, slow to make friends, but likable and sincere. As an engineer and executive, he was loyal, straightforward, and competent. He will always be remembered in Colorado as an important factor in the upbuilding of the State. He was a just and honorable man, of unfailing courtesy and kindness.

He was married on March 10, 1895, to Helen Donahue who, with one daughter, Mrs. Howard E. Parks, of Denver, and one sister, Mrs. A. O. Miller, of Montclair, N. J., survives him.

Major Blauvelt was elected a Member of the American Society of Civil Engineers on May 6, 1908.

WILLARD LEWIS BOWKER, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 18, 1932

Willard Lewis Bowker was born in Walpole, Mass., on October 2, 1867. He attended the Walpole Public Schools, Chauncy Hall Preparatory School, and the Massachusetts Institute of Technology for two years with the Class of 1890.

For two years, 1889 and 1890, Mr. Bowker was with the City Engineer's Office in Bangor, Me., surveying and drafting on street and sewer construction. From 1891 to 1900 and again from 1902 until 1912, Mr. Bowker was with an organization engaged in the engineering of pulp, paper, and power plants. D. H. and A. B. Tower, A. B. Tower, A. B. Tower and Company, Tower and Wallace, and Joseph H. Wallace and Company kept Mr. Bowker employed for twenty years, during which period he was in charge of some of the most important pulp and paper-mill developments handled by this organization, with headquarters at Holyoke, Mass., and New York, N. Y. To describe his work would be to tell the history of paper-mill progress from 1890 to 1912. Included in this work were the preparation of plans and specifications, preliminary surveys, field supervision of construction of dams, reservoirs, canals, power plants, water-works, paper and pulp mills, including complete mechanical equipment, with appraisals and preparation of reports of existing or prospective plants.

For a two-year interval, 1901-02, Mr. Bowker was with the American Writing Paper Company, Holyoke, Mass., as Engineer on repairs, alterations, and new construction relating to the thirty mills then controlled by that Company. For a portion of that time, Mr. Bowker acted as Secretary for the Appraisal Committee on the formation of the Box Board Combination in charge of all field work on about fifty mills then being considered.

His professional work further included responsible supervision for construction of plants for the following organizations: Hudson River Water Power and Paper Company, Mechanicville, N. Y.; Glenn Manufacturing Company, Berlin, N. H.; Racquette River Pulp and Paper Company, Potsdam, N. Y.; Kimberly and Clark Company, Neenah, Wis.; various companies at Holyoke, Mass.; Duncan Company, Mechanicville, N. Y.; Bangor Water-Works, Bangor, Me.; and B. D. Rising Paper Company, Housatonic, Mass.

Due to protracted illness from 1912 to 1915, Mr. Bowker returned from New York City to his boyhood home in Walpole, Mass., devoting himself principally to farming activities which kept him in the open air, although engaging in occasional engineering work.

In November, 1915, on complete recovery, he took up the work of fire protection engineering with the Inspection Department of the Associated Factory Mutual Fire Insurance Companies, having headquarters at Boston,

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[1] Memoir prepared by Joseph H. Wallace, M. Am. Soc. C. E., and O. W. Stewart, Esq., Boston, Mass.

Mass., continuing as Fire Protection Engineer and Inspector to the end of his professional career. To this work in the fire insurance field, he brought a wealth of experience in construction and industrial lines which made his judgment of real value to his employers. During the seventeen years in which he represented the Factory Mutual Companies, he made inspections and reported upon some of the largest and most important manufacturing properties in the United States. He was not only a hard and conscientious worker, but was ever helpful, and especially genial, to all with whom he was associated.

Mr. Bowker was known as "Old Reliability" by those associated with him in business and was loved by all who came in contact with him.

In 1900, he was married to Ruby Bellingham, of Wausaukee, Wis. He is survived by his widow and four sons.

Mr. Bowker was elected a Member of the American Society of Civil Engineers on June 3, 1903.

CON MORRISON BUCK, M. Am. Soc. C. E.[1]

Died January 26, 1933

Con Morrison Buck, the son of Charles Addison and Emily (Morrison) Buck, was born in Oskaloosa, Kans., on November 10, 1873. He attended the Oskaloosa High School and, later, the Kansas State Agricultural College, at Manhattan, Kans., where he obtained his degree of Bachelor of Science in 1896. The Master of Science degree followed in 1898 and the Civil Engineer degree in 1916. He was a member of the Phi Kappa Phi and Sigma Tau Honorary Fraternities.

More than twenty-eight years of Mr. Buck's engineering career was with the Atchison, Topeka and Santa Fé Railway Company. He also was employed by the Missouri Pacific Railroad Company, and was in charge of a change-of-line project for the Kansas City Southern Railway Company. He served the City of Manhattan, Kans., as City Engineer, and also conducted a private engineering practice in that city. During 1912 and the first half of 1913, Mr. Buck was Construction Engineer in charge of location and construction of about twenty-five miles of standard gauge railroad for the Central Coal and Coke Company at Neame and Carson, La.

His services with the Atchison, Topeka and Santa Fé Railway Company were not continuous, but during his early connections he held various positions from Chainman to Office Engineer in the Chief Engineer's Office at Topeka, Kans. In August, 1913, he assumed charge of the construction of a reservoir and delivery works for the Santa Fé Railway Company, at Guthrie, Okla. The following year he was made Division Engineer of the Eastern and Kansas City Divisions, with headquarters at Topeka. In 1927, he became Locating Engineer for the Santa Fé Eastern Lines, reporting to the Chief Engineer at Topeka. Due to ill-health, it was necessary for Mr. Buck to retire from active service on January 1, 1932.

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[1] Memoir prepared by a Committee of the Kansas State Section, consisting of G. J. Bell, A. B. Griggs, and H. M. Swope, Members, Am. Soc. C. E.

Mr. Buck's engineering ability was held in high regard by his associates and his influence over the young men under his supervision was very helpful to them in the early years of their experience, as he was always ready to advise upon proper methods of handling new problems that came to them.

He was a representative of The Kansas Engineering Society on American Engineering Council; a Director and Past-President of The Kansas Engineering Society, a Past-President of the Kansas State Section of the American Society of Civil Engineers, and a member of the Topeka Engineers Club.

Mr. Buck had been in failing health for several years prior to his death, which occurred at his home in Topeka, following a cerebral hemorrhage.

He was married on June 20, 1899, to Winifred Anna Houghton, of Manhattan, Kans. He is survived by his widow and their two children, Alfred Houghton and Nadine Emily. He interested himself in local affairs and was affiliated with the Protestant Episcopal Church and the Masonic bodies.

Mr. Buck was elected an Associate Member of the American Society of Civil Engineers on December 6, 1905, and a Member on March 2, 1915. He was elected a Director of the Society in 1927, but due to his ill-health, was forced to resign before attending a meeting in that capacity.

HARRY BURGESS, M. Am. Soc. C. E.[1]

Died March 18, 1933

Harry Burgess, the son of James and Elizabeth (Foster) Burgess, was born at Starkville, Miss., on February 22, 1872.

He attended the Agricultural and Mechanical College of Mississippi through the Junior Year. In 1891, he entered the United States Military Academy at West Point, N. Y., from which he was graduated in 1895, second in his class. At the Military Academy, Cadet Burgess demonstrated two qualities conclusively. He proved that he was an excellent student of decidedly scientific bent of mind, and a man of the soundest character, entitled thereby to the respect of every one.

On graduation from the Military Academy, he was assigned to the Corps of Engineers of the United States Army. In that branch of the Service he passed through every grade from Second Lieutenant to Colonel, inclusive, and without exception performed outstanding service in each grade. The dates of his promotion to the various grades in the Army were as follows: First Lieutenant, July 5, 1898; Captain, January 23, 1904; Major, September 21, 1909; Lieutenant-Colonel, November 27, 1916; Colonel, July 1, 1920; and Brigadier-General, June 1, 1932.

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[1] Memoir prepared by Maj.-Gen. Lytle Brown, Chf. of Engrs., U. S. A., M. Am. Soc. C. E.

Some of the important works on which Lieutenant Burgess was employed soon after his graduation were, as follows: In Savannah, Ga., he was Assistant to the District Engineer, entrusted, among other things, with the development of Savannah Harbor. This work has resulted in making at Savannah one of the great harbors of the United States, suitable for entry by the modern ships of the world. He was Assistant also in the development of Mobile, Ala., Harbor. In 1898, he attended the Engineer School of Application at Willets Point, N. Y., and during the Spanish-American War, he was engaged in the submarine mine defense of the harbors at Galveston, Tex., and New Orleans, La.

In 1900, Lieutenant Burgess was assigned as Instructor of Engineering at the United States Military Academy, but due to military operations in the Philippine Islands, he was sent there after only a few months as Instructor. He served with the Engineer Battalion in the operations of the Philippine Insurrection. In 1902 and 1903, he was Adjutant of the 1st Battalion of Engineers at Jefferson Barracks, Missouri, and Fort Leavenworth, Kansas.

From 1903 to the entry of the United States into the World War, he was engaged as District Engineer in active charge of work at Louisville, Ky., New Orleans, La., Nashville, Tenn., and Detroit, Mich., and, in this period, served on various Engineer Boards in the examination of irrigation projects. His work during this time embraced the improvement of the Ohio River, the navigation works on the Great Lakes, the flood-protection works on the Lower Mississippi, and the general navigation works on the Tennessee River, as well as the planning of the power development at Muscle Shoals, Ala.

In 1917, after the entry of the United States in the World War, Colonel Burgess organized the 16th Engineer Railway Regiment at Detroit, commanded it, and took it to France. In France, he was Corps Engineer and Commander of the 305th Engineers. He returned to the United States from France in 1919 in command of the 16th Engineer Regiment.

From 1919 to 1924, Colonel Burgess was District Engineer at Rock Island, Ill., in charge of work on the Upper Mississippi, member of the Mississippi River Commission, and Commandant of the United States Engineer School, at Fort Humphreys, Virginia.

In 1924, he became Engineer of Maintenance of the Panama Canal, and, in 1928, Governor of the Panama Canal Zone. He served continually in the tropics at Panama as Engineer of Maintenance and Governor for eight years. Such service will tax the strength and health of the most enduring man. In 1932, when his tour of duty as Governor of the Canal Zone was finished, as a recognition of exceptional service, he was made a Brigadier-General of the Line of the Army. This reward came much later than it was due, and was to be but briefly enjoyed. He died on March 19, 1933, at the Army Hospital at Hot Springs, Ark.

General Burgess is survived by his widow, Mary Lillington McKoy whom he married at New Orleans, in 1912.

He was an honor and an ornament to the American Society of Civil Engineers and to the Corps of Engineers of the United States Army. He was a thorough engineer, as well as a soldier, but outstanding above these professional qualifications was his high honor and thorough loyalty to his friends, his family, his superiors, and to his country. There was nothing about General Burgess akin to self-seeking, but always a most thorough dedication of himself to his duty. His life ended before the usual time allotted by the Creator, but not before the accomplishment of great good through years of unswerving devotion to all tasks as they came to hand, whether they were great or small.

He was a member of the American Society of Military Engineers, and his clubs included the Army and Navy of Manila, Philippine Islands, and Washington, D. C.; Union and Century of Panama, as well as several Army Service Clubs.

General Burgess was elected a Member of the American Society of Civil Engineers on October 5, 1909.

HARRY EZRA CAMERON, M. Am. Soc. C. E.[1]

Died March 13, 1933

Harry Ezra Cameron was born at Carrollton, Ohio, on December 10, 1879. He was the son of Cyrus Cameron and Hannah (Denny) Cameron. When he was six years old, his mother died, and when he was twelve, he lost his father. He then went to Lisbon, Ohio, to make his home with his uncle and aunt, Mr. and Mrs. Abraham Moore.

He received his education in the public schools of Lisbon, and was graduated in 1901, from the Engineering College of Ohio State University, Columbus, Ohio, with the degree of Civil Engineer.

Mr. Cameron began the practice of his profession immediately afterward with the Columbus Bridge Company. In 1902, he started with the McClintic-Marshall Construction Company, at Rankin, Pa., and worked in its Drafting-Room until the end of 1904.

On January 1, 1905, he became associated with the American Bridge Company and continued this connection until the time of his death. He was employed in the Detail Drafting-Room at the Ambridge Plant until 1910, most of the time as Squad Foreman, in charge of the preparation of shop drawings for a number of large bridges.

From 1910 until his death, Mr. Cameron was in the Designing Office of the American Bridge Company, at Pittsburgh, Pa., in charge of the design of many important bridges, such as the Kentucky and Indiana Terminal Railroad Bridge, over the Ohio River, at Louisville, Ky., the Ohio Connecting Railroad Bridge, over the Ohio River, at Pittsburgh; a cantilever bridge for the LaBelle Iron Works, at Steubenville, Ohio; the 565-ft ore-handling bridge, for the Pittsburgh and Conneaut Dock Company, at Conneaut Harbor, Ohio; the 650-ft coal-handling bridge for the Carnegie Steel Company, at

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[1] Memoir prepared by F. S. Merrill and V. D. Beard, Members, Am. Soc. C. E.

Coal Valley, Pa., and the Pittsburgh and West Virginia Railroad Bridge, over the Monongahela River, at Pittsburgh.

Mr. Cameron was in charge of the engineering work in connection with planning and supervising shop and erection procedure on these bridges designed by him and on numerous other bridges, such as the Chesapeake and Ohio Railway Continuous Bridge, over the Ohio River, at Cincinnati, Ohio; highway cantilever bridges, over the Ohio River, at Ambridge and Rochester, Pa.; and the Lake Union Bridge, at Seattle, Wash.; also, the eye-bar suspension bridges, over the Allegheny River, at 6th, 7th, and 9th Streets, Pittsburgh, and the parallel wire suspension bridge, over the Monongahela River, at 10th Street, Pittsburgh.

He was also in charge of the preparation of a series of designs for railroad turn-tables and for draw-span machinery. He made numerous inspections of old bridges and was in charge of reinforcing them to bring them up to modern practice. He made an independent investigation of the stability during erection of such structures as the Hell Gate and Kill van Kull Arch Bridges, at New York, N. Y.

All this experience was very broadening, and Mr. Cameron developed into an engineer whose opinions were widely respected and much sought after by all who came in contact with him. He had a pleasing personality and was much liked by all his associates and made many firm friends wherever he went.

He was a member of, and a consistent attendant at, the Dormont, Pa., Presbyterian Church. He was also a member of Dormont Lodge, F. and A. M., St. Clair Chapter No. 305, R. A. M., and the Almas Masonic Club.

Mr. Cameron was an ardent sportsman, and, at the time of his death, was President of the Clary Rod and Gun Club, and also a member of the Castle Shannon Golf Club. He was a member of the Engineers Society of Western Pennsylvania.

He was married on October 18, 1906, to Elizabeth Kinsvatter, at East Liverpool, Ohio, and is survived by his widow and two children, Dorothy Marie and Harry Robert.

Mr. Cameron was elected a Member of the American Society of Civil Engineers on October 15, 1923.

FRANK TAYLOR CHAMBERS, M. Am. Soc. C. E.[1]

Died November 10, 1932

Frank Taylor Chambers, the son of Henry and Annie C. (Weisiger) Chambers, was born in Louisville, Ky., on July 1, 1870. He was graduated from the Louisville High School in 1888, and from Rensselaer Polytechnic Institute, Troy, N. Y., in 1892, with the degree of Civil Engineer.

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[1] Memoir prepared by Capt. G. S. Burrell, C. E. C., U. S. N., M. Am. Soc. C. E.

After a short time in electrical switchboard drafting, Mr. Chambers was employed (1892 to 1895) by the Snead and Company Iron Works, Louisville, in drafting and estimating, and three months on the erection of observation towers in Chickamauga National Park. He was then engaged for seven months with Henry Aiken, Consulting Mechanical Engineer, in charge of structural steel designing and inspection. This was followed by a few months with the Pittsburgh Bridge Company, and from October, 1896, to July, 1897, he was in the Office of the Supervising Architect, United States Treasury Department, designing structural steelwork for various Federal buildings.

Having attained the highest standing in a competitive examination, Mr. Chambers was commissioned, on July 23, 1897, a Civil Engineer in the United States Navy, with the rank of Lieutenant (Junior Grade). During the next twenty years, he performed tours of duty at the following Naval Stations: Tutuila, Samoa; Tiburon, Calif.; and Great Lakes, Illinois; and at the following Navy Yards: New York, N. Y., Philadelphia, Pa., Norfolk, Va., Charleston, S. C., and Mare Island, California. He also was on duty in the Bureau of Yards and Docks, United States Navy Department, as follows, 1898-1901, 1903-04, 1904-06, 1907-09, 1911-12, and 1916-17. These assignments included a great variety of design and construction work, embracing coaling plants, dry docks, piers and quay walls, power plants, and various types of buildings, power distribution systems, etc.

In the meantime, he had been promoted successively, in due course, to the various ranks in the Civil Engineer Corps, as follows: 1901, Lieutenant; 1906, Lieutenant Commander; 1911, Commander; and, 1914, Captain (equivalent to the rank of Colonel in the Army).

In 1906-07, Lt.-Comdr. Chambers was appointed by the President a member of the Chesapeake and Delaware Canal Commission which investigated and recommended the location and details for a 35-ft draft ship canal between Baltimore, Md., and the mouth of the Delaware River. In 1906, he was a member of the National Advisory Board on Fuel and Structural Materials. In 1910, he served as a member of the Joint Army and Navy Board on Water Approaches to the Navy Yard, at Mare Island.

In 1917, Captain Chambers acted as Chairman of a Board of Naval Officers to select a site for a fuel-oil base for the Navy on Chesapeake Bay.

During the World War, he served as Officer-in-Charge of the construction of the Naval Operating Base, at Hampton Roads, Norfolk, Va., until November, 1917, when he was detailed to duty with the United States Shipping Board. He served at first in charge of dry-dock and shipyard planning and, later, as Chief Engineer and Member of the Port Facilities Commission. In the summer of 1918, Captain Chambers was sent abroad to study port facilities to develop means for the more expeditious movement of American ships in British and French ports. His Shipping Board duties continued until 1926. Concurrently, from 1919 to 1926, he served as Consulting Engineer on Port Facilities to the Board of Engineers for Rivers and Harbors, United States War Department, and advised on port development on all coasts of the United States and also on inland and river terminals. From

1917 to 1926, he served also as a member of the Commission on Navy Yards and Naval Stations, U. S. Navy Department.

For his war-time services in the Navy and with the Shipping Board, Captain Chambers received a special letter of commendation from the Secretary of the Navy.

From April, 1926, to April, 1929, he served as Public Works Officer of the Navy Yard, Pearl Harbor, Hawaii, and of the Fourteenth Naval District, where a large volume of new construction was in progress. In April, 1927, Captain Chambers was promoted to the rank of Rear-Admiral in the Corps of Civil Engineers. From 1929 to 1930, he served as Public Works Officer of the Third Naval District, with headquarters in New York, N. Y.

In October, 1930, Admiral Chambers was placed in charge of the Oil Office in the Navy Department, as Director of Naval Petroleum and Oil Shale Reserves, which duty he continued until his death which occurred on November 10, 1932, at his home in Washington, D. C. He was buried in the National Cemetery at Arlington, Va., with the full Naval honors of his rank.

Admiral Chambers was a member of the Permanent International Association of Navigation Congresses, and a member of the Permanent Commission. As an Official Delegate of the United States, he attended the XIIIth Congress in London, England, in 1923, and the XVth Congress in Venice, Italy, in 1931. He represented the American Section at the Business Meeting in Antwerp, Belgium, in 1924. He was also a member of the American Association of Port Authorities, the Advisory Committee of the Federal Oil Conservation Board, the Sigma Xi Fraternity, and the National Geographic Society.

He was a member of the following clubs: Army and Navy, Chevy Chase, and Cosmos, of Washington, D. C.; and of the Rittenhouse and University, of Philadelphia, Pa. He had been a member of Calvary Protestant Episcopal Church, Louisville, since his boyhood.

Admiral Chambers was the author of "English Port Facilities" (U. S. Shipping Board); "Water Terminals and Transfer Facilities" (War Department); and various magazine articles. He had delivered numerous addresses on the subject of port facilities and terminal developments.

He was married in Washington, D. C., on February 14, 1920, to Florence Newell Pease, who survives him. He is also survived by three brothers, John, Henry, and Henning Chambers, of Louisville, Ky.

His naval career of thirty-five years was replete with distinguished service in widely diversified fields of endeavor. He was possessed of a keen analytical mind and of great earnestness and zeal for the Navy and for his profession. In his death, the Navy has lost a highly capable officer and the Society a distinguished member.

Admiral Chambers was elected an Associate Member of the American Society of Civil Engineers on April 6, 1898, and a Member on March 3, 1903.

ROBERT MORRIS CLAYTON, M. Am. Soc. C. E.[1]

DIED JULY 4, 1931

Robert Morris Clayton, the son of Ephraim and Nancy (McElroy) Clayton, was born at Marion, N. C., on March 31, 1845. His grandfather was Lambert Clayton, born in 1755, one of the first settlers in Western North Carolina and a soldier during the Revolution.

Mr. Clayton was raised, and educated until he was sixteen years old at private schools, in Asheville, N. C. He enlisted in April, 1861, in the Confederate Army, in the first company recruited in Asheville (which later was attached to the First North Carolina Regiment, under Col. D. H. Hill), and fought in the first infantry battle, at Bethel Church, Va. His regiment being disbanded, he enlisted again in the spring of 1862, in the 60th North Carolina Regiment, at Warm Springs, N. C., under Col. J. A. McDowell, attached to the Western Army commanded by Gen. Braxton Bragg, with headquarters at Murfreesboro, Tenn. He was with this regiment until the close of the war and surrendered at Greensboro, N. C., with Gen. Joseph E. Johnson, in April, 1865. His regiment was in practically all the battles of the Western Army, at Murfreesboro, Jackson, Miss., Chickamauga, Ga., Missionary Ridge, Tenn., etc., as well as Atlanta and Jonesboro, Ga. He was made First Lieutenant of his Company when he was 18 years old, and commanded the Company in all the battles. He escaped without any serious wounds. After the war, he returned to Asheville, and was associated with his four brothers in farm work.

At the end of 1866, Captain Clayton commenced his work in engineering. In 1867, he was Rodman for the Cincinnati, Cumberland Gap and Charleston Railroad Company; in 1868, Assistant Engineer with the Western North Carolina Railroad Company; in 1869 and 1870, Assistant Engineer in Charge of Location, Western Division, Western North Carolina Railroad; from 1871 to 1874, Resident Engineer, Atlanta and Charlotte Air Line Railroad Company; in 1874, Engineer in Charge of Location, for the Spartanburg and Asheville Railroad Company; and from 1875 to 1878, Superintendent of Construction on the Augusta Canal.

Captain Clayton began his activities as a Municipal Engineer in 1879, when he was elected City Engineer of Atlanta, Ga. At that time Atlanta had a population of about 60 000 and an area of 9½ sq. miles. He served two terms as City Engineer. When he was defeated for a third term, he engaged in the private practice of engineering for two years. He was re-elected in 1885, and served as City Engineer and, later, as Chief of Construction of the City of Atlanta until the end of 1914, when he retired voluntarily. At that time, the city had a population of 170 000 and an area of 26½ sq. miles.

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[1] Memoir prepared by C. E. Kauffmann and William A. Hansell, Members, Am. Soc. C. E.

When Captain Clayton began his service, the municipal improvements of streets, sewers, water supply, etc., were very inadequate. Roadways were mostly unpaved, with the exception of a few blocks around the center of the city, which were paved with Belgian blocks, laid on a sand bed, and, also, a few sidewalks paved with flagstones. At once he advocated the laying of smooth, up-to-date pavements, selecting granite cubes and vitrified brick for that purpose. Through his foresight, he was instrumental in adjusting grades and widening streets, which resulted in great benefit to the future growth and development of Atlanta.

Captain Clayton introduced and completed the adjustment and enlargement of the sewerage system, as well as the construction of an entirely new water-works plant. After lengthy and tedious negotiations, he succeeded in securing the late Rudolph Hering, M. Am. Soc. C. E., as Consultant, and as the result of mutual study of the problems, these improvements were executed under a bond issue; thus, to a large extent, the sanitary sewers were separated from the combination sewers and three Imhoff tank sewage disposal plants were built (1910-12). The largest had a capacity of 8 000 000 gal., and, at that time, was the first large Imhoff tank plant in the United States.

The water supply of Atlanta was taken from the South River and from an Artesian well in the center of the city. Considering the various sources of water supply from the northern section of Georgia, Captain Clayton recommended the use of water from the Chattahoochee River. Again, he was placed in charge of preparing the plans for the present water-works system, requiring a pumping station at the river and one at the city with a head of 260 ft. above the river station; all buildings and equipment are of the most modern design and type, and while new equipment has been installed since that time, on account of the growth of the city, several of the pumps and the Hyatt Filter Plant, said to be the first filter plant of this type in America, are still (1932) in operation.

As ever, he had foresight, in the selection of the type of bridges, for, in 1901, when he built the Forsyth Street Bridge, across the Railroad Yard, the most modern forms of construction, with vitrified brick pavement, were applied, with the result that later bridges, about fifteen in all, were constructed with solid floors.

As his advice was sought by most of the various City Departments, when difficult problems were involved, his counsel and judgment were accepted. During 1904, the City installed, under his advice, a garbage destructor plant of 125-ton capacity. By 1913, this plant was outgrown, and the City Officials, under the advice of Captain Clayton, built a new plant of 250-ton capacity, which was constructed, in its entirety, under his direct supervision. His activities were so numerous and manifold in all branches of the municipality that it is impossible to include all his achievements.

Captain Clayton was a Past-President of the American Water Works Association. He was a Past Commander of the United Confederate Veterans' Association. He was also Vice-President of the American Savings Bank and President of the Hibernia Savings Building and Loan Association, of Atlanta. In politics, he was a Democrat.

He was a Presbyterian and an Elder in the Central Presbyterian Church, of Atlanta.

His greatest recreation was fishing and, in his later years, he went every winter to Florida, to enjoy this sport. When his physical condition became such that he knew his end was not far off, Captain Clayton came back to Atlanta from Florida because he wanted to spend his last days where he had so many friends and where he had lived so many happy and useful years.

His ability and wisdom as an Engineer were extraordinary, and these, combined with his extensive general information, make him of inestimable value to the City of Atlanta. He was one of the most active, admired, and beloved officials, and a man of unquestioned honesty, uprightness, and integrity.

He was married on June 2, 1875, to Felixina Hansell, the daughter of Gen. Andrew J. Hansell, of Roswell, Ga.

Captain Clayton was elected a Member of the American Society of Civil Engineers on March 4, 1896.

BRANCH HARRIS COLBY, M. Am. Soc. C. E.[1]

DIED JANUARY 3, 1933

Branch Harris Colby was born on July 20, 1854, at Cherry Valley, Ohio, the son of Lewis and Celestia (Rice) Colby. He was graduated from the University of Michigan in 1877 with the degree of Bachelor of Science (later, called Bachelor of Science in Civil Engineering), and took a post-graduate course in Mining Engineering the following year.

His first work was as Assistant Engineer on the primary triangulation of the Great Lakes and on primary base-line measurements. Then, for almost five years, Mr. Colby was United States Assistant Engineer with the Mississippi River Commission, employed in the field and office on triangulation and topographic and hydrographic work.

In 1884, he engaged in private practice, devoting his attention principally to the design and construction of electric light central stations, which he followed until 1889, except for about eighteen months in 1886 and 1887, when he made the survey for the Portage Lake Ship Canal in Michigan, and for Sandusky (Ohio) Harbor; he also had charge of the dredging in that harbor. Late in 1888, he returned for a short time to the Mississippi River Commission as United States Assistant Engineer on the topographic survey of the Mississippi River.

On May 1, 1889, Mr. Colby was appointed by the late Robert E. McMath, M. Am. Soc. C. E., then Sewer Commissioner of the City of St. Louis, Mo., to take charge of a topographic survey for the purpose of perfecting plans for the drainage of the city. He devised a metal protractor for plotting by polar co-ordinates, especially designed and constructed for this survey. He also designed a slide-rule for computing the difference of elevation between two points when distance and vertical angle are known, to be used in conjunction with Ockerson's tables. These tables published by the United States Government, were prepared by the late John A. Ockerson, Past-President, Am. Soc. C. E., when he was engaged on the survey of the Great Lakes. Mr. Colby stated in a paper[2] read before the Engineers' Club of St. Louis, in November, 1892, that the slide-rule was about three times as rapid as the tables and almost as accurate.

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[1] Memoir prepared by Edward E. Wall, M. Am. Soc. C. E.

He continued in charge of this work until April, 1895, when he was appointed Sewer Commissioner for a four-year term. While in this position, there was designed and built under his supervision and direction, several large trunk sewers, the largest of which was the Tower Grove Storm Sewer, of horse-shoe shape, 12½ ft in height by 15 ft in width. An inspection of the older sewers of the city showed repairs to be badly needed in many places, so that he inaugurated a system of repairing them from the inside, without tearing up the pavements and excavating from the surface. In many cases, not only was the bottom renewed or the arch restored, but the entire structure was rebuilt. He devised a slide-rule for determining the proper size of sewers adequate to carry the run-off for various slopes and characters of drainage areas.

As Sewer Commissioner, Mr. Colby was a member of the Board of Public Improvements of St. Louis, the duties of which were to pass on all questions and propositions pertaining to public work in the City. In conjunction with two other members of the Board, he prepared plans and specifications for lighting the city by electricity and for placing all wires underground. The ideas proposed in these plans were ahead of the times, and the electric companies refused to bid on them until modified. Much of his time while Sewer Commissioner was devoted to the work of the Board, and his influence on its policies and decisions was clearly and conclusively evident.

At the close of his term, he was engaged by the American Car and Foundry Company, as Civil and Consulting Engineer for about two years, after which he resumed his private practice in which he continued until his retirement in 1920, except for the time he spent with the Shipping Board after the entrance of the Federal Government into the World War.

Mr. Colby was appointed Engineer on the building of steel ships at Hog Island, Pennsylvania, in 1917. Here, he learned ship construction in detail, and was sent to Chicago, Ill., to supervise the manufacture of parts in the plants there. In July, 1918, he was ordered to San Francisco, Calif., to take charge of the construction of the Government Island Shipyard, where it was proposed to build concrete ships.

He completed the work there in about one-half the time allotted by the Shipping Board and within the estimated cost. He was recalled to Philadelphia, Pa., and made a member of the Board to investigate and report upon the condition and operation of shipyards on the Atlantic Coast. He was then sent to Jacksonville, Fla., as Resident Engineer in charge of the construction of a dry dock, marine railway, and repair plant. This was considered an unpleasant and difficult job, because of the negligence and mismanagement that had characterized the inception and progress of the work up to the time Mr. Colby took hold of it. He soon brought matters to rights and was busily engaged there until the close of the war.

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[2] "Recent Survey of St. Louis: Its Method and Results," *Journal,* Assoc. of Eng. Societies, January, 1893, Vol. 12, No. 1.

The last work upon which he was engaged as an Engineer was in 1920 on a sewerage system at Muscle Shoals, Ala. While there he suffered an almost fatal attack of angina pectoris, after which he was obliged to give up all hope of resuming active practice. Throughout his entire career, Mr. Colby was extremely particular to have everything done by himself, or by his subordinates, as correct as human ability could make it.

He had the faculty of writing good English and the knack of good delivery in speaking. After delivering an address in St. Louis on "How Maps are Made," he received twenty-one invitations to repeat it before different organizations.

In 1881, when Mr. Colby was United States Assistant Engineer engaged in a topographic survey of the Mississippi River, wild ducks and geese were abundant, and there were always hunters with every engineering party. Mr. Colby was an expert shot with the rifle. A favorite sport with him was to wait until he got two ducks in line and shoot their heads off with one bullet.

He was an excellent chess player and well-known member of the St. Louis Chess Club. Before the days of bridge, he was a lover of the old-fashioned English game of whist, and invented a method of playing duplicate whist in which the hands played by one couple on any deal, would be turned over to their opponents the next deal. This infallibly showed up the poorest players and, therefore, enjoyed only a brief period of popularity.

After his enforced retirement in 1920, Mr. Colby was more or less closely confined to his home at Normal, Ill., sometimes unable to go out for days or weeks at a time. Occasionally, he would go to Bloomington, Ill., to see his friends and acquaintances and once or twice he traveled to St. Louis. During the last twelve years of his life, he was several times able to go as far as New York City, by exercising great precautions. He greatly enjoyed a visit to Mackinac Island and the journey there and back home on two occasions during those years of invalidism. His heart disease brought about a complication of other troubles, finally resulting in his death on January 3, 1933.

"B. H. Colby," as he always signed his name, was in the active practice of Civil Engineering for more than forty years, 1877 to September, 1920, when he was forced to retire on account of ill health. During those years his work covered a wide range of engineering experience—triangulation and topographic surveys, establishment of electric light control stations, direction of dredging work, city surveying, designing and constructing sewers, consulting engineer for manufacturing plants, building a shipyard, a dry dock, and a marine railway, not to speak of countless minor activities forced upon him directly or indirectly in the pursuit of these major duties.

Consequently, he was a man of broad vision, not too exacting in his expectations and requirements of those associated with him, and working under him, but always clear in his mind how much latitude should be allowed in every case. This made him universally respected and esteemed alike by his men, his employers, and his acquaintances.

Mr. Colby was President of the Engineers' Club of St. Louis in 1899 and, at one time, Vice-President of the American Society for Municipal Improvements. He was well known among the Scottish Rite Masons in St. Louis, having served as Senior Warden in the Lodge of Perfection for seven years (1912-1918). He and his family belonged to the Unitarian Church and he was one of the Trustees of Unity Church for years.

He was married on June 28, 1883, to Minnie Bary, of Detroit, Mich. They had two daughters who survive him, Vine (Mrs. Charles O. McCasland), of Pasadena, Calif., and Dorothy (Mrs. Victor H. Lawn), of Radburn, Fair Lawn, N. J.

Mr. Colby was elected a Member of the American Society of Civil Engineers on June 5, 1895.

WINFIELD WALKER CONARD, M. Am. Soc. C. E.[1]

Died September 21, 1932

Winfield Walker Conard was born on November 11, 1876, at Port Kennedy, near Valley Forge, Pa. He was the son of Edward Bright Conard and Ella Virginia (Walker) Conard.

Mr. Conard attended the public schools of Upper Merion Township and the Friends' Central School, at Philadelphia, Pa., from which he was graduated in 1893. In the fall of that year he matriculated at the University of Pennsylvania and completed the Civil Engineering Course, graduating in 1897, with the degree of Bachelor of Science.

After his graduation, Mr. Conard began as Draftsman with the Pencoyd Iron Works, Wissahickon, Pa. In 1903, he wrote a thesis on the "Three-Hinged Arch," and for this work he received, the same year, the degree of Civil Engineer from the University of Pennsylvania.

In 1900, when the American Bridge Company was formed, taking over the Pencoyd Plant, of the A. and P. Roberts Company, Mr. Conard continued at Pencoyd, in the capacity of Designer and Estimator of highway and railway bridges of all types, until May, 1904. He then entered the employ of the Phœnix Iron Company, at Phœnixville, Pa., where he continued his efficient work in the design of structural steel for buildings. He returned to the American Bridge Company in May, 1905, and from that time until his death was engaged with that Company in estimating, designing, and the sale of structural steel.

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[1] Memoir prepared by Victor G. Thomassen, M. Am. Soc. C. E.

Mr. Conard practiced his profession effectively, with a deep sense of loyalty and conscientious endeavor. He was widely read in literature pertaining to the sciences, and was a member of the American Association for the Advancement of Science. He developed and held patents on a system of reinforced concrete, and his interest in motoring led to his development of a patent which he held on a pulverized coal-burning automobile.

He won the respect and friendship of all with whom he came in contact by his unselfish devotion and sterling honestry. Unassuming, and with a lovable nature, he dedicated his life to his profession and to the service of those around him.

Mr. Conard was a member of the Society of Friends, and a strong and active leader in that organization.

He was married on April 8, 1903, to Mary Kemble Walker, of New Centreville, Pa. Mrs. Conard died in 1927. He is survived by a sister, Mrs. Lila C. C. Walker, and a niece, E. Virginia Walker, of Norristown, Pa.

Mr. Conard was elected a Member of the American Society of Civil Engineers on June 6, 1921.

ALGER ADAMS CONGER, M. Am. Soc. C. E.[1]

DIED MAY 26, 1931

Alger Adams Conger was born at Gouverneur, N. Y., on June 5, 1875. He was the son of Gerrit S. and Martha (Church) Conger. His early years were spent in the vicinity of his birthplace where on the St. Lawrence River and Black Lake he acquired a healthy and life long interest in aquatic sports, especially deck canoe sailing. Mr. Conger obtained his early education at the Grammar and High Schools at Gouverneur and was graduated from the Civil Engineering Course, at Cornell University, Ithaca, N. Y., in 1897.

After a short time spent in the office of his uncle, Daniel W. Church, III, in Chicago, Ill., he accepted his first engagement which was with the United States Board of Engineers on Deep Waterways and extended from 1897 to 1899. The personnel of this Board, the late Alfred Noble, Past-President, Am. Soc. C. E., and Brig.-Gen. Charles W. Raymond, U. S. A., and George Y. Wisner, Members, Am. Soc. C. E., was a guaranty that the training obtained under its guidance would be of the highest order. Mr. Conger was first engaged on topographic and hydrographic surveys and, later, on the design and estimates that formed the basis for the report by the Board on the proposed ship canal from the Great Lakes to the Atlantic Ocean.

Mr. Conger continued on similar work from 1899 to 1900 with the Isthmian Canal Commission, first, on surveys at Panama, and, later, at Washington, D. C., on preparation of plans and estimates. From 1900 to 1901, he was engaged with the New York State Barge Canal Commission, at Albany, N. Y., on preliminary plans and estimates for the Mohawk Division of the Barge Canal. During these years Mr. Conger was able to gain an insight into a wide variety of engineering problems as well as to form lasting associations with many able engineers.

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[1] Memoir prepared by James H. Brace, M. Am. Soc. C. E., and C. R. Bliss, Assoc. M. Am. Soc. C. E.

From 1901 to 1903, while employed by the Michigan-Lake Superior Power Company, he first entered his real work—that of hydro-electric development—on the canal and power house then being built at Sault Ste. Marie, Mich. This work was done in part by contract and in part by the Company's own forces. Mr. Conger's duties involved both those of a Superintendent of Construction and a Resident Engineer, and he displayed tact and energy as well as engineering skill under trying circumstances. This experience gave him a familiarity with the difficulties encountered in the field and a sympathy with the men who were trying to carry out the work, that benefited him through all his later years.

From 1903 to 1906, he was again employed on the New York State Barge Canal, at Albany, preparing plans and contracts for the actual construction of the canalized river division.

In March, 1906, he joined the organization of J. G. White, Incorporated, of New York, N. Y., as Assistant Hydraulic Engineer and, while with that Company, was engaged on the design of the following: Plant at West Buxton, Me.; Comerio Development, in Porto Rico; Ocmulgee Development, in Georgia; La Crosse Development, in Wisconsin; and the San Joaquin Development, in California. He also went to Venezuela and prepared plans and estimates for the Mamo Hydro-Electric Development. While with this organization Mr. Conger was in active charge of the design of the hydro-electric development at Vernon, Vt., on the Connecticut River. This plant, which was the first generating plant of, and, later, formed the nucleus of what is now the New England Power Association, was the beginning of a work and interest which was to continue during the remainder of his career.

When the Power Construction Company was formed in 1911 as the Engineering and Construction Subsidiary of the New England Power Company, Mr. Conger was made Hydraulic Engineer for the former Company and continued this association until he was made Hydraulic Engineer for the companies affiliated with the Association in 1924. He continued in this latter capacity until the time of his death, which occurred, after an illness of about a year, at Wellesley, Mass., on May 26, 1931.

In the earlier period, there were constructed under his engineering direction: The Deerfield No. 2; Deerfield No. 3; Deerfield No. 4; Somerset; Deerfield No. 5; and Searsburg Developments, in Massachusetts and Vermont, as well as many works of less importance carried out by the Company.

The hydro-electric developments in Massachusetts and Vermont, at Harriman and Sherman. on the Deerfield River, and at Bellows Falls, 15-Mile Falls, and McIndoes Falls, on the Connecticut River, installed during the latter period, just as surely exemplify the basic soundness of Mr. Conger's viewpoint as a Designer. It had been his constant endeavor to inculcate his ideas and ideals in the younger men who came under his direction, and, furthermore, he was always in constant touch with the designs as they progressed and active and vigorous in consultation regarding them. During this period, he also was in charge of investigations of future power possibilities.

Mr. Conger had the advantage of an ancestry that represents the best in American life. His father was a competent county lawyer, a probate Judge, and, in his later years, a lawyer whose practice was best described as a "friend of the widow and orphan." Through his mother, a charming woman with strongly developed charitable and business instincts, he was descended from an unusually long line of pioneers devoted to engineering pursuits. His great-great-grandfather, Jonathan Church, was a mill builder and interested himself in bringing a water supply into Lansingburg, N. Y. His great-grandfather, born in Brattleboro, Vt., spent his life in developing water powers and building mills in Vermont, and, later, in the vicinity of Waddington, N. Y. It is interesting to note that he also built a machine to bore out logs to be used as pipes in conveying water from Jamaica Pond to Boston, Mass. His grandfather, Daniel Church, took up the work, but rather as a mill and power owner and operator. Finally, his uncle, Daniel W. Church, III, also became a well-known civil engineer in the Middle West.

Always interested in the history and activities of the early pioneers, Mr. Conger found great pleasure in carrying on his later work in the localities where his ancestors had lived and wrought. From his grandfather he had inherited an interest in a tangled skein of mill and water-power privileges on the Oswegatchie River, in New York State. As required by the development of the art at that time, Mr. Church had divided the power into many parcels and disposed of these in different ways for use in a large number of small manufacturing plants. With changing conditions these latter were gradually closed, and the power is now mostly unused. It would again be serviceable under modern practice if it could be re-assembled into one project. It was characteristic of Mr. Conger that, in his last years, he added this project to his life-long hobby of interest in the history of Gouverneur and the genealogical histories of his own family and of those of many of his friends with whom he came in close contact. In this study he spent, with great interest and without hope of personal gain, much of his spare time in an attempt to trace out these old rights.

Naturally, then, Mr. Conger had by this inheritance and association a mind trained to weigh his problems and make his decisions with common sense, justice, and humanity. His associates valued his counsel, and he was a trusted adviser and valued friend to all who knew him. He succeeded to an unusual degree in passing on to others his sound, conservative viewpoint and thoroughness in searching out the truth. Those of a younger generation who were so fortunate as to work with him, find this training among their most valued assets.

Alger Conger, the engineer, will be remembered by his associates as one who never departed from the highest ideals of his profession. Alger Conger, the man, will be remembered by all who knew him as one in whom kindness, tact, and sympathy could always be counted on; as a good sportsman, and an unfailing friend. He did the work that life gave him to do with charity, fidelity, and honor—what more is possible?

Mr. Conger was a member of Gouverneur Lodge No. 217, F. and A. M. He was also a Member of the Boston Society of Civil Engineers, the Cornell Club of New England, American Canoe Association, Wellesley Country Club, and the Maugus Club, of Wellesley.

He was married in 1902 to Kathleen Noble, of Gouverneur, N. Y. She was also descended from the pioneers of the region, and they had known each other from childhood. She greatly aided him in many practical ways. Mrs. Conger and two sons, Robert Noble Conger and Gerrit Church Conger, survive him.

Mr. Conger was elected an Associate Member of the American Society of Civil Engineers on October 1, 1902, and a Member on March 12, 1918.

HERBERT CHAPIN DAGGETT, M. Am. Soc. C. E.[1]

DIED MARCH 17, 1932

Herbert Chapin Daggett was born to Thomas and Hannah (Chapin) Daggett on June 6, 1867, at Foxcraft, Me. He was graduated from the Massachusetts Institute of Technology in 1891, receiving the degree of Bachelor of Science, after having successfully pursued the course in Civil Engineering.

After his graduation, Mr. Daggett took a position as Assistant Engineer in the office of the Proprietors of the Locks and Canals on the Merrimack River, at Lowell, Mass. His duties in this position gave him a broad foundation for his life work in the development of water power. He became familiar with the details of a great variety of water-wheels and their settings, and various methods of transmission of the power developed. In these early days there were no direct-connected electric generators. He also received thorough training in water measurement and the flow of water through open and closed conduits.

In 1898 and 1899, he practiced as a Consulting Engineer, making independent reports and investigations. Among others were surveys and plans of water-power projects at Marble Falls, Tex., and Huntsville, Ala.

In 1899, Mr. Daggett became Chief Engineer of the Swain Turbine Manufacturing Company, at Lowell. His duties consisted in designing and superintending the manufacture of hydraulic turbines and in acting as Sales Engineer. In this capacity, he was largely responsible for the design of the turbine settings.

From 1901 to 1903 he was Sales Engineer in the Hydraulic Turbine Division of the Holyoke Machine Company, at Worcester, Mass. His duties were similar to those with the Swain Company, with perhaps less attention to detail and more to general design and development. Mr. Daggett's work with these two companies served to broaden his experience and intensify his knowledge of hydraulic turbine developments. At the same time, he was accumulating a great fund of information concerning the developed and undeveloped water-power sites in New England and adjacent territory.

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[1] Memoir prepared by George A. Jessop, Assoc. M. Am. Soc. C. E.

From 1903 to the time of his death he had charge of the New England Office of the S. Morgan Smith Company, manufacturers of hydraulic turbines and allied equipment, as Engineer and Sales Manager. In this position he widened his acquaintance with engineers and builders of hydraulic power plants. He saw the rapid introduction of electrical equipment and the change from the driving of machinery directly, or through various types of drives, by the water-wheel, to the direct connection of generators and the driving of machinery and equipment by motors.

He had a thorough knowledge of the various types of turbines and the particular conditions they were best suited to fulfill. Few, if any, important developments were made in New England without Mr. Daggett's advice. Because of his exact knowledge of the flow of the various rivers and streams, he was able to be of reliable assistance in determining the possibilities of undeveloped water power. His sound engineering knowledge was greatly valued, also, in improving old power developments. Perhaps his most noteworthy accomplishments were his broad knowledge of the water-power situation in New England and his wide acquaintanceship with hydraulic engineers. Always intent upon securing the very best results, his advice and co-operation were widely sought.

Mr. Daggett was a man of high ideals in his professional and personal life. He had the marked respect of all with whom he came in contact.

He was a Life Member of the Winthrop (Mass.) Yacht Club. His chief recreation for many years was associated with the activities of this organization. He was also a member of the Vermont Electrical Association and the Boston Society of Civil Engineers.

He was married on October 4, 1892, to Lenore Elizabeth Briggs, of Urbana, Ill. Mrs. Daggett died on June 7, 1925. He is survived by a daughter, Mrs. Helene E. (Daggett) Sweeney, a son, Harold C. Daggett, a sister, Mrs. Elizabeth Sturtevant, a brother, Sumner T. Daggett, and two grand-daughters, Ernestine and Frances Sweeney.

Mr. Daggett was elected a Member of the American Society of Civil Engineers on July 10, 1907.

LOUIS CHRISTIAN DATZ, M. Am. Soc. C. E.[1]

DIED MAY 20, 1932

Louis Christian Datz was born in New Orleans, La., on April 27, 1880, a son of Henry and Charlotte (Caezar) Datz. After his elementary education, he entered Tulane University, in New Orleans, from which he was graduated in 1901 with the degree of Bachelor of Engineering. He was a member of the Phi Kappa Sigma Fraternity.

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[1] Memoir prepared by R. J. Lockwood, M. Am. Soc. C. E.

After graduation Mr. Datz's first work was with J. O. Wright, a Construction Engineer of New Orleans, as Engineer in charge of designing and superintending the construction of dredges, derricks, and levee-building and cane-loading machinery. He remained in this position until February, 1903, when he became Assistant Engineer for the New Orleans Railway and Light Company.

From July, 1910, to January, 1911, there was a brief interlude in Mr. Datz's public utility work during which time he was Secretary and Graduate Manager of Athletics at Tulane University. In 1911, he joined the firm of Ford, Bacon, and Davis as Assistant Engineer and, a year later, when the American Cities Company was organized and the New Orleans property became a part of this group of Southern holdings, Mr. Datz returned to the New Orleans Railway and Light Company as Engineer of Maintenance of Way. He remained with this Company until February, 1914, serving successively as Engineer of Roadway and Superintendent of Track and Overhead.

In 1914, when the United Gas and Electric Corporation became interested in the American Cities Company and the United Gas and Electric Engineering Corporation established an office in the South, Mr. Datz was made Engineer of Southern Properties, with engineering supervision of the properties in New Orleans, Houston, Tex., Birmingham, Ala., Memphis and Knoxville, Tenn., and Little Rock, Ark. He remained with the American Cities Company until April, 1922, having served as Chief Engineer from 1919. From 1922 to 1924, he was Chief Engineer of the Memphis Power and Light Company.

Mr. Datz's varied experience in public utility work fitted him particularly well for the position he took in 1925 when he became Advisory Engineer for the Re-Organization Committee of the United Railways Company of St. Louis, Mo. In December, 1927, the St. Louis Public Service Company obtained control of the street railway system in St. Louis, and Mr. Datz was made Chief Engineer, which position he held at the time of his death.

Notwithstanding the many duties and responsibilities imposed upon him by his work, Mr. Datz found time to take an active part in various technical and social organizations. He was a member of the Louisiana Engineering Society (President in 1915), Engineers Club of St. Louis, American Electric Railway Engineering Association (President in 1922-23), American Society of Mechanical Engineers, American Society of Municipal Engineers, St. Louis Electrical Board of Trade, St. Louis Chamber of Commerce, and the St. Louis County Chamber of Commerce. He was also a member of the Rotary Club of St. Louis, the Louisiana Lodge of the Masonic Order, the Missouri Athletic Association, and the Westborough Country Club. He was a member of the Westminster Presbyterian Church, of which he was Chairman of the Finance Committee.

On June 27, 1905, Mr. Datz was married to May C. Eblen, of New Orleans. He is survived by his widow; a son, Warren E.; two daughters, Elise Charlotte and Rosemary; a brother, Henry E. Datz; and a sister, Mrs. Louise Gelder.

His keen sense of humor and his sympathetic nature won for Mr. Datz a host of friends. Always considerate of others, he would lend a helping hand wherever needed. He was a devoted husband, a loving father, a staunch friend, a Christian gentleman.

Mr. Datz was elected an Associate Member of the American Society of Civil Engineers on June 30, 1911, and a Member on March 2, 1915.

ARTHUR LINCOLN DAVIS, M. Am. Soc. C. E.[1]

DIED OCTOBER 19, 1932

Arthur Lincoln Davis, the son of Henry L. and Susan Davis, was born on August 17, 1866, in San Francisco, Calif. His father was prominently connected with the early development of the city, especially the street railway system, of which he became the head.

The son attended the public schools of San Francisco and, after graduation from High School, spent three years as a member of various surveying parties on railway location in California and adjoining States. This work gave him a fund of practical knowledge and experience that was of value to him later in his technical education and business career. In the fall of 1885, he entered the Massachusetts Institute of Technology, at Boston, Mass., from which he was graduated in 1889 with the degree of Bachelor of Science in Civil Engineering. He spent the following year at the Institute taking special work in Mechanical Engineering.

On account of the influence of his family, Mr. Davis might have returned to California and obtained a lucrative position, but he preferred to rely on his own ability for advancement. He therefore accepted a position as a Detail Draftsman with the Vermont Construction Company, of St. Albans, Vt. He was soon advanced to designing and estimating and, later, was placed in charge of that work for the Company. In 1892, he accepted the position of Bridge Engineer with the Central Vermont Railway Company, in charge of the design, construction, and maintenance of bridges and buildings. In 1893, he returned to the Vermont Construction Company as Chief Engineer and Manager, in which position he had charge of the designing, estimating, selling, fabricating, and erecting of all work taken by the Company. In 1897, Mr. Davis accepted a position with the Rutland Railroad Company as Chief Engineer in Charge of Maintenance of Track and Structures. In 1898, he went to the Berlin Iron Bridge Company, of East Berlin, Conn., as Assistant to the General Contracting Manager, in which position he had general charge of contracting.

Upon the merger in May, 1900, of the Berlin Iron Bridge Company with the American Bridge Company, Mr. Davis was made Assistant to the Vice-President in Charge of Structural Contracting. In 1903, he was appointed Assistant to the President of the American Bridge Company of New York, in charge of all contracting. In 1911, he was made Eastern Division Contracting Manager, and, in 1927, General Contracting Manager. In 1931, he was elected Vice-President and General Contracting Manager of the Company. From 1900 to the time of his death, Mr. Davis had negotiated or had had a major part in the negotiation of many of the contracts for the large and important steel structures of the United States.

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[1] Memoir prepared by J. E. Wadsworth, M. Am. Soc. C. E., and R. Fleming, Esq., New York, N. Y.

Because of his absolute honesty and fairness he had the confidence not only of his own Company, but of the customers to whom he sold steel. He had accumulated during his varied and active career a fund of general information that made him an excellent conversationalist and a delightful companion.

Any commentary on the life of Mr. Davis would not be complete that did not emphasize his fine personality and lovable character. As one of his associates expressed it: "For some men we have admiration for their ability, but for him we have not only admiration for his ability, but affection for himself." Another of his friends and associates said: "He was an honorable and upstanding citizen, one who at all times gave fully and freely of the best that was in him to his family, his fellow men, and his employer."

The General Managers of Sales Association of the United States Steel Corporation passed the following resolution in regard to Mr. Davis:

> "Members of the General Managers of Sales Association of the United States Steel Corporation have learned with profound sorrow and deep regret of the death on October 19, 1932, of Arthur Lincoln Davis, Vice-President of the American Bridge Company.
>
> "Mr. Davis had a long and distinguished career in his chosen profession, marked by his outstanding executive, administrative, and sales qualifications, which contributed greatly to the eminent position occupied by his Company in the industry.
>
> "Mr. Davis was a man of outstanding personality and unusual strength of character, tempered with a sympathetic understanding, courage, and loyalty which won for him the highest esteem and admiration of a wide circle of devoted friends. We wish to record our sense of the great loss to the American Bridge Company, to the United States Steel Corporation, and to this Association in the death of Mr. Davis, and to offer our deepest sympathy to the members of his family."

Mr. Davis was a member and a former Trustee of the Community Church of Mount Vernon, N. Y. He was a former Director of the Huguenot Trust Company of New Rochelle, N. Y., a member of the Engineers Club, the Railroad Club, and the Transportation Club, of New York City; and a former member of the Siwanoy Country Club, of Bronxville, N. Y., and of the New Rochelle Yacht Club.

He is survived by his widow, Kate (Macomber) Davis, and three daughters, Mrs. Laura D. Freeman, Mrs. Dorothy D. Abriel, and Mrs. Marion Davis Doyle, also by a half-brother, Laurens G. Davis, and a half-sister, Mrs. Hillman Lueddeman.

Mr. Davis was elected a Junior of the American Society of Civil Engineers on April 4, 1893, an Associate Member on October 7, 1896, and a Member on March 6, 1906.

PAUL DIDIER, M. Am. Soc. C. E.[1]

DIED MAY 19, 1928

Paul Didier was born at Sept-Fontaines, France, on June 12, 1850. He obtained his education at l'Ecole Industrielle, Athenee, Conservatoire et Politechnique, Paris, France, receiving the degree of Bachelor of Science, in August, 1869, and that of Construction Engineer, in August, 1872, after he had returned to school following his service in the Franco-Prussian War of 1870.

At the opening of that war Mr. Didier left school and was inducted into service with the Engineer Corps attached to the 143d Regiment of The Army of the East under Marshal Bazaine. Suffering the common fate of most of that Army, Mr. Didier was held a prisoner of war for many months at Metz and Mayence, in Germany. Finally escaping, he made his way back to Paris to find it in the grip of the disturbed political conditions which occurred in the closing months of the war, and he was unable to complete his education until after the restoration of peace.

Mr. Didier began his engineering career in September, 1872, with the Chemin de Fer des Nord-Est, at Lille, France, as a Designing Engineer in the Projecting Department, preparing plans of locations, bridges, and buildings. In March, 1874, he became "Chef de Section" on surveys, location, and construction of new lines from Hennin-Letard to Don (19 miles), and from Tubize to Brame-l'Alleud (27 miles). He continued in this work until the latter part of 1880, when he visited Constantinople, Turkey, with a view of accepting a position as an Instructor in the University there. Finding the conditions undesirable, he returned to France, and shortly thereafter came to the United States to join a brother who had preceded him to this country some years before.

Mr. Didier arrived in the United States in January, 1881, and entered the service of the Baltimore and Ohio Railroad Company, on February 2 of that year, as Chief of Field Party, engaged in the location of various lines in Western Pennsylvania and West Virginia. Desiring to become more familiar with American methods of bridge construction, Mr. Didier took a position as Draftsman with the Keystone Bridge Company from September, 1882, to March, 1884. He then returned to the service of the Baltimore and Ohio Railroad Company as Assistant Engineer, Maintenance of Way, of the Pittsburgh Division.

In August, 1886, he was appointed Chief Engineer of the Pittsburgh and Western Railroad Company, a line in which the Baltimore and Ohio was then interested and which came under the control of the latter Company in 1891. In addition to other improvements of a lesser nature which were carried out under Mr. Didier's supervision while in this position, the main line of the Pittsburgh and Western Railroad from Etna to New Castle Junction, Pa., was double-tracked and practically rebuilt. This work, involving about 50 miles, was begun in 1890, and required several years for its completion.

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[1] Memoir prepared by H. A. Lane, M. Am. Soc. C. E.

In 1902, the operations of the Pittsburgh and Western Railroad Company were combined with those of the Baltimore and Ohio Railroad Company, and in June, 1903, Mr. Didier was named District Engineer of the latter Company in charge of construction in the Pittsburgh District. He was promoted to the position of Principal Assistant Engineer in April, 1909, and continued as such until his death. Numerous important improvements were carried out under Mr. Didier's supervision during this period, but mention will be made of only the more important ones in order to indicate the varied character of the projects which he executed successfully.

He had charge of the construction of the Ribold Cut-Off, from Eidenau to Ribold, 8 miles in length, and of the Chewton Cut-Off, from Rockpoint to West Pittsburgh, also about 8 miles in length, both of which were constructed in 1901 and 1902 in connection with the arrangements which had been made for the use by the Buffalo, Rochester and Pittsburgh Railway Company of certain lines of the Baltimore and Ohio Railroad.

In 1903, work was started under Mr. Didier's supervision on the reconstruction of the Baltimore and Ohio's Line from Hazelton, W. Va., to Girard, Ohio, the construction of a new line from Girard to Ravenna, Ohio, and the rebuilding of the old line from Ravenna to Cuyahoga Falls, Ohio, a total distance of 54 miles. This work required a period of three years for completion and involved, in a district already congested with railroad lines, many intricate problems which were solved successfully. During this same period, another important project, which was carried out in 1905, was the standardization of the Northern Division running from Butler, to Mount Jewett, Pa. This line which was about 90 miles in length, had a gauge of 3 ft., and, in addition to providing standard gauge, the work also involved the revision of curvature in a number of places. Under Mr. Didier's supervision, the work was planned and prepared so carefully that the actual change from narrow gauge to standard gauge over the entire line was accomplished in one day.

In the period, 1907 to 1909, one of the most important improvements under Mr. Didier's supervision was the construction of part of the several lines now forming the Quemahoning Branch of the Baltimore and Ohio Railroad. This project opened an important new coal-producing territory and involved the construction of more than nine miles of railroad through mountainous country. Among the more important items carried out under Mr. Didier's supervision during 1910 and 1911 were the yards at Somerset, Pa., and track elevation at Columbus, Ohio.

Perhaps the most important construction project which Mr. Didier supervised for the Baltimore and Ohio Railroad Company was the double-tracking of the main line crossing the Allegheny Mountains at Sand Patch, Pa., and the construction of a new tunnel at that point. This work was started in 1911 and was completed in 1912. It involved the construction of additional track facilities from Foley to Keystone, Pa., a distance of about 12 miles, and a new double-track tunnel at the summit, with a length of about 4 400 ft. It was completed successfully and opened for service on Christmas Eve, 1912.

In 1913, Mr. Didier was among those who performed almost herculean labors in getting the lines of the Baltimore and Ohio Railroad Company in the Middle West re-opened after the disastrous floods in March of that year. Mr. Didier's task involved the reconstruction of tracks and bridges in the vicinity of Zanesville and Columbus, Ohio, which had been damaged seriously by the Muskingum and Scioto River floods. Among other accomplishments in this work, a temporary bridge, 400 ft. long, was constructed under his supervision over the Muskingum River, at Zanesville, in ten days against the handicap of high waters and a swift current. This work was completed in April, 1913, and by July of that year, a new permanent structure had been designed, constructed, and was opened for traffic.

In 1914, and in subsequent years, until he retired from active charge of construction work in 1918, Mr. Didier supervised other important projects, including the building of the Crozier's Run Branch, the reconstruction of the bridge over the Monongahela River, at Glenwood Junction, Pa., the construction of third and fourth tracks from Wheeling Junction, W. Va., to McKeesport, Pa., and the grade-crossing elimination at Liberty Avenue, Pittsburgh, including the approach to the proposed new Allegheny River Bridge.

In his personal characteristics Mr. Didier was of a genial disposition and had numerous friends both in and out of his chosen profession, who hold his memory in affectionate regard. One of his outstanding characteristics was the high esteem and pride which he held for his profession. Having chosen the field of Civil Engineering, he held the theory that he should confine his activities to that branch of scientific knowledge, and it is known to his intimates that on several occasions he refused opportunities that would have been greatly to his financial advantage, because he felt that they trespassed upon other fields of engineering. As an engineer, Mr. Didier combined the rare balance of the theoretical with the practical. He demanded the best both in design and construction of engineering works, but with this attitude he added a sense of judgment that produced the successful results that were uniformly characteristic of the projects under his supervision.

Mr. Didier is survived by his widow, Victorine, two sons, Paul and Victor, and a sister, Madam Catherine Verstagen, of Charleroi, Belgium.

He was a Charter Member of the American Railway Engineering and Maintenance Association and a member of the Franklin Institute. He was selected as a Delegate to the International Railway Congress in 1900 and attended the meetings held in Paris in that year. He was also a member of the Engineers Society of Pennsylvania, Engineers Society of Western Pennsylvania, and the American Association for the Advancement of Science, and served on the Pittsburgh Flood Commission and the City Planning Commission of Pittsburgh.

He was the American representative of the National Federation of Combatants, an organization of French and Belgians pledged to a humane alliance between the World War sufferers of the two countries. In January, 1922, the

King of Belgium appointed Mr. Didier as Belgian Consul in Western Pennsylvania and, later, bestowed upon him the rank of Chevalier of the Order of the Crown.

Mr. Didier was a Thirty-second Degree Mason, a member of the Blue Lodge, a Knight Templar, and a Noble of the Mystic Shrine. He also held membership in the Pittsburgh Chamber of Commerce, Pittsburgh Athletic Association, and Luther Burbank Society, and was a Charter Member of Allegheny Lodge No. 339, Benevolent and Protective Order of Elks.

Mr. Didier was elected a Member of the American Society of Civil Engineers on May 6, 1889. He became a Life Member in January, 1921.

BOYD EHLE, M. Am. Soc. C. E.[1]

DIED JUNE 5, 1932

Boyd Ehle, the son of Peter and Anna V. (Dockstader) Ehle, was born at his family home at Snells Bush, N. Y., on April 18, 1866. He was of Revolutionary Mohawk stock, the sixth in line from the pioneer missionary, Dominie John Jacob Ehle. Dominie Ehle was born in 1690, and passed his examinations in 1722. He was ordained in the ministry by the Bishop of London and took up his work among the Mohawk Indians, erecting the first mission in 1727 and, later, the stone house known as the Ehle home, near Nelliston, N. Y., which is still treasured as a relic.

Boyd Ehle was educated at the Nelliston District School and at the old Clinton Liberal Institute, at Fort Plain, N. Y.; he then entered Cornell University, at Ithaca, N. Y., from which he was graduated with the degree of Civil Engineer in 1886.

Immediately following his graduation, Mr. Ehle was employed on the location of the Fort Worth and Denver Railroad, in Colorado, as Chief Draftsman and Assistant Engineer. He then went to the old Nicaraguan Canal Construction Company as Chief Draftsman and, later, as Chief of Party; this employment ended in 1893. Subsequently, he located and built the Fort Plain and Richfield Springs Railroad, in New York State, near his home. Then followed the design and construction of the Fort Plain Water-Works. From 1895 to 1897, he was Assistant Engineer on the construction of the New York State Barge Canal, working on the new Erie Canal, and from 1897 to May, 1899, he was again on the Isthmus for the Nicaragua Canal Commission as Chief of Party, studying locations.

From October, 1899, to May, 1901, Mr. Ehle was Principal Assistant Engineer in charge of investigations under the Isthmian Canal Commission, on the Darien Surveys; he was put on this work by the late George S. Morison, Past-President, Am. Soc. C. E., who regarded him as one of the very ablest of field engineers. From June, 1901, to December, 1902, under the general direction of Mr. Morison, he studied the topography and hydraulics for a hydro-electric plant on the Susquehanna River, at McCall Ferry, Pa. Mr. Ehle then became Consulting Engineer on the water supply and sewage plant for Summit, N. J. Following that, until January, 1904, he was Resident Engineer, studying the railway crossing problems of the Michigan Central and Grand Trunk Railways, in Detroit, Mich., also under the direction of Mr. Morison.

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[1] Memoir prepared by Messrs. Cary T. Hutchinson and R. Garrabrant, New York, N. Y.

In 1905, he returned to Susquehanna River work, and until 1908 was Resident Engineer of the McCall Ferry Power Company, which constructed the hydro-electric plant of the Pennsylvania Water and Power Company, at Holtwood, Pa. After the banking failure of this Company in 1907, Mr. Ehle was put, by J. G. White and Company, in charge of the survey of a tract of 150 000 acres of undeveloped land in Southern Idaho, to be used for irrigation purposes. Upon the completion of this work he became Construction Superintendent of, and finished, the Tri-State Irrigation District, in Western Nebraska.

After he left J. G. White and Company in 1910, Mr. Ehle was sent to Haiti to investigate a hydro-electric power plant; later, came the water supply of Victoria, B. C., Canada, followed by the construction of an oil pipe line from Yarhola, Tex., to St. Louis, Mo., under the direction of the San Francisco Office of Sanderson and Porter, Consulting Engineers.

From 1918, when this work was completed, and for the next three or four years, he was employed by the Susquehanna Power Company in studies of the Susquehanna River for the development of the hydro-electric power at Conowingo, Md., subsequently built by Stone and Webster Corporation, which made extensive use of the locations and topography made by Mr. Ehle.

Boyd Ehle spent many years of his life, at several different periods, in the development of the water power of the Lower Susquehanna River, and probably his outstanding contribution to engineering progress in his time was the pioneer work he did in making preliminary studies and surveys which eventually led to the three great hydro-electric developments at Holtwood (formerly known as McCall Ferry), Conowingo, and Safe Harbor, Pa.

Hydraulic power development of the Susquehanna River attracted the attention of engineers and capitalists prior to 1900, but it was not until the turn of the century, when Mr. Ehle began an investigation under the direction of the late Mr. Morison and Mr. Cary T. Hutchinson, that the plans began to take tangible and practical form. This was the beginning of a long series of successful and unsuccessful attempts to develop the several sites. The developments that have finally resulted and the final designs have been the work of several separate organizations, but to Boyd Ehle must be given the credit for laying the foundation for the design of all three projects. He was instrumental in the final choice of the dam site for the Conowingo development. Although he was not directly connected with the recent development of the Safe Harbor site, it is an interesting fact and a tribute to the painstaking thoroughness and accuracy of his work, that important features of the hydraulic design and in particular the computation of back-water profiles were based on hydrographic work that Mr. Ehle had done nearly thirty years before. His early surveys in this reach of the river were also of the greatest value.

In 1919–1920 he was sent by Sanderson and Porter to South America for the Anaconda Copper Company, in connection with water supply at Portrerillos, Chile, and on a hydro-electric development. From 1922, he made trips to Colombia on the location of the railway from Bogota to Barranquilla; to Cerro de Pasco, Peru, on the design of a hydro-electric plant; and to Cuba on railway locations. These were Mr. Ehle's principal commissions; since September, 1929, he had been living on his farm near Little Falls, N. Y.

On June 7, 1896, Mr. Ehle was married to Grace Cook, who, with two sons, Louis C. and Ralph V., survives him.

Viewed in retrospect, the career of Boyd Ehle is amazingly colorful. His work took him not only to almost every section of the United States, but also over a large part of Central America and South America. He is said, for example, to be the only man who took active part in every one of the surveys leading to the final location of the Panama Canal. Peru, Chile, Haiti, Cuba, and Colombia are among the countries that have felt his influence and benefited from his ability. As a natural result of these extensive travels, Mr. Ehle was an extremely interesting talker, the effect of his anecdotes, told with rich detail of local color and seasoned with rare humor, was intensified by the matter-of-fact attitude that he held toward the discomforts and hazards of life under difficult, and often dangerous, conditions. Unfriendly natives, strange diseases, and the ever-hostile flora and fauna of the tropical jungles were to him minor annoyances. It was part of his philosophy that the man who goes resolutely and purposefully about his business, deviating neither to curry favor nor to cause unnecessary annoyance, would win his objective in due course—and this belief stood the test of experience.

Like most strong characters, Mr. Ehle was intense in his likes and dislikes. Those who were honored with his friendship found it sincere and unselfish. He sought no favors, but was always quick to perform them. On the other hand, incompetence, dishonesty, or insincerity were sure to arouse his active aversion. His appraisals of those with whom he came in contact, once arrived at, were seldom changed and were never doubtful, for he was as outspoken in criticism as he was in praise. Independence was his boast; attempts to curry favor, the one thing he could never forgive. Unusually free from petty prejudice, he numbered his friends among all creeds, nationalities, and conditions of life.

His associates soon learned that Boyd Ehle had a remarkable gift in his pioneering instinct, making little of very difficult situations; and that he had great ability in organizing expeditions into wild and unsettled territory. Once in the field, his subordinates were inspired by his quiet but firm leadership and gave to their leader the best they had in them.

He was an outstanding example of the engineer as "an ingenious man", to quote an old definition. To him, the solution of any technical problem, no matter how involved, was a relatively simple matter — get the facts, consider them fully and carefully, make a decision, act.

Mr. Ehle never bowed to authority; although the most noted hydraulic engineer in the United States said of the McCall Ferry Dam, when it was proposed:

> "I do not believe that a masonry structure 2 830 ft. long and some 50 ft. high with 19.3 ft. of water passing over it, is advisable, no matter how built, nor is to be considered, unless supposed to be built of huge, and in dam construction unprecedented, monoliths, for which special appliances would have to be built to handle and transport. Not wishing to call such a structure an impossibility, I have roughly, estimated for it, but regard my estimate as probably too low."

Mr. Ehle did not waver in his certainty that the construction not only was feasible, but could be built without special difficulty with ordinary plant and materials, and he was right.

Mr. Ehle was elected an Associate Member of the American Society of Civil Engineers on January 3, 1894, and a Member on February 1, 1910.

ARTHUR JOHN ELA, M. Am. Soc. C. E.[1]

DIED MARCH 21, 1932

Arthur John Ela was born on a farm in Norwich, Vt., on December 14, 1883, the only child of Albert B. and Amy (Coit) Ela. His training in the atmosphere of this New England home grounded in him at an early age those cardinal virtues of strict honesty and the stern sense of duty that governed him in his manhood.

His mother's death when he was twelve years old added to his responsibilities and deprived him of much of the care-free life that is the privilege of youth. It was the realization of this loss that caused Mr. Ela to make many sacrifices that his own sons might enjoy what he was denied. After the death of his mother, the father and son moved to Lebanon, N. H., where young Arthur completed his public school education. Having by this time decided that engineering was to be his life work, he allowed no obstacle to interfere with the attainment of this ambition, and entered Dartmouth College, at Hanover, N. H., from which he was graduated with the degree of Bachelor of Science in 1905.

In order to complete the formal education that he had mapped out for himself, it was necessary to secure funds by his own efforts. Just before his graduation, Mr. Ela had taken the competitive Civil Service Examination for the United States Coast and Geodetic Survey, and had passed with such high standing that he was one of the few to receive an appointment. Two years' service in this branch of governmental work made it financially possible for Mr. Ela to spend a year at the Thayer School of Civil Engineering, at Dartmouth College, from which he received the degree of Civil Engineer in 1908.

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[1] Memoir prepared by Asa C. Baldwin, Assoc. M. Am. Soc. C. E.

Equipped now with the necessary technical foundation, Mr. Ela was ready to enter seriously upon his engineering career. His previous experience with the Coast and Geodetic Survey had been most interesting, and so he re-entered that service and for many years devoted his time and energy to its work. His duties carried him to the outlying parts of Continental United States, to Puerto Rico, and to Alaska. Most of this time he was engaged in the responsible work of coast pilot revision and hydrographic surveys.

How well he executed every assignment given him can best be determined by the fact that the United States Lighthouse Service asked for his transfer to assist in its re-organization and for three years Mr. Ela was with this Bureau on construction and inspection work in Oregon, Washington, and Alaska.

Returning again to the U. S. Coast and Geodetic Survey in 1914, Mr. Ela was applying his talent to peace time work for the Survey, when the World War involved the United States. To Mr. Ela his duty was clear, and he immediately offered his services and was commissioned in October, 1917, Lieutenant, United States Naval Reserve Force. His knowledge of navigation peculiarly fitted him for his first assignment, the inspection of navigational instruments at the place of manufacture. However, in this work he was not entirely satisfied as he wished to take a more active part in the prosecution of the war. It was, therefore, a great satisfaction to him to be made Navigating Officer of the transport, U. S. S. *Zeelandia*.

The close of the World War marked a turning point in the professional life of Mr. Ela. For fifteen years he had been in Government Service which, for the most part, had taken him on the high seas and into remote sections away from civilization. In May, 1912, he had been married in Seattle, Wash., to Mary Bowlby, of Crete, Nebr. Two boys were born to them, Albert Byron in 1914, and Arthur John four years later. With only himself to consider, such hardships and inconveniences as his Government work had entailed made no difference, but he now felt it his duty to engage in work that would make it possible to keep his family together. Therefore, he accepted a position with the Citizens Light, Power, and Water Company of Ketchikan, Alaska, and, until 1926, was Engineer for this Company in charge of construction of the city water and power system.

In 1926, he entered private practice in Ketchikan and, from 1928 to 1930, engaged in what was probably the most important work of his career—the field investigation for power development in Southeastern Alaska, in connection with the establishment of pulp and paper mills by the Cameron-Chandler Interests. From 1930, until his death, Mr. Ela was associated with the United States Army Engineers in Seattle, in their study for Congress of the Columbia River Basin Project, acting as Editor-in-Chief of their Final Report.

The bare recital of facts regarding Mr. Ela's life as an Engineer covers only a small part of his many activities. Although he was naturally a student and found his recreation throughout his life in books, he also took his part in the social and civic life of his community. In college, he was a member of the Sigma Alpha Epsilon Fraternity; he was Past Master of the Ketchikan Masonic Lodge No. 159, F. and A. M.; a member of the Rotary Club, the Benevolent and Protective Order of Elks, and the American Legion of Ketchikan; and the Scottish Rite Masons, of Seattle. He organized the first Boy Scout Troop in Ketchikan, and gave much of his time to this splendid work.

For a number of years before his death, Mr. Ela was in poor health, and he must have known for some time that the end was near. In spite of this he continued his work with a will and determination to carry on. To his friends he was always cheerful and never complained of his ill health. His death is an irreparable loss to his family. Their consolation rests in the memory and heritage of a name that will ever be respected by Mr. Ela's friends and associates.

Mr. Ela was elected a Member of the American Society of Civil Engineers on August 18, 1930.

ROBERT LEE FARIS, M. Am. Soc. C. E.[1]

DIED OCTOBER 5, 1932

Robert Lee Faris was born at Caruthersville, Mo., on January 13, 1868, the son of James White and Willie Ann (Stovall) Faris. His early education was received in the public schools. In 1886, he entered the University of Missouri from which he was graduated as a Civil Engineer in 1890.

After serving for a year with the Corps of Engineers, United States Army, on the survey of the Missouri River, Mr. Faris entered the United States Coast and Geodetic Survey as a Recorder on May 12, 1891. On May 4, 1893, he was appointed Temporary Aide and shortly after was given a permanent position in the field force. He rose to the position of Assistant Director of the Survey in 1915, in which position he served until his death.

Mr. Faris' field experience in the Coast and Geodetic Survey was varied and interesting. From 1893 to 1895, he was a member of triangulation parties engaged in extending the arc of transcontinental triangulation across the Rocky Mountains. This work involved the occupation of peaks 12 000 to 14 000 ft. in height, in remote and almost inaccessible locations, and observations over lines having a maximum length of 183 miles. In 1896, he made field magnetic observations, and assisted in the determination of the difference of the astronomical longitude between Albany, N. Y., and Montreal, Que., Canada.

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[1] Memoir prepared by R. S. Patton, M. Am. Soc. C. E.

From 1898 to 1900 he was assigned to parties engaged in the survey of the Yukon Delta, Bering Sea, Alaska, and served at various times on the Coast and Geodetic Survey ships, *Taku, Yukon,* and *Patterson.* This work was of material value to the shipping which had increased greatly as a result of the stampede to the new gold fields of Alaska. During the season of 1901, he was Executive Officer of the Survey ship, *McArthur,* which was engaged on hydrographic and topographic surveys in the vicinity of Sanak Island and along the Alaska Peninsula.

From 1902 to 1906, Captain Faris served as Commanding Officer of the Survey ships, *Blake* and *Explorer,* making surveys along the Atlantic and Gulf Coasts and in Puerto Rico. During this period he successfully demonstrated the practicability of making magnetic observations at sea for dip, total intensity, and variation. As the result of this pioneer work, the other ships of the service were equipped with magnetic instruments, and a large volume of data in this hitherto little explored field was accumulated. The success attained by the Coast and Geodetic Survey in making magnetic observations at sea, led to the magnetic survey of the oceans by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, D. C., and its attendant benefits to the shipping of the world.

When Dr. Louis A. Bauer, Inspector of Magnetic Work of the Coast and Geodetic Survey, was chosen to become the head of the Department of Terrestrial Magnetism of the Carnegie Institution, it was only logical that Captain Faris should be his successor in the Survey's magnetic work. In September, 1906, he was appointed Inspector of Magnetic Work and Chief of the Division of Terrestrial Magnetism. Under his direction, notable progress was made in the magnetic survey of the United States, and marked improvements in instruments and methods were accomplished. Captain Faris took an active interest in problems of magnetic research, and was the author of several publications on the subject, all of which showed careful thought and an intimate knowledge of that field.

During the period from about 1907 to 1910, Captain Faris was a member of the Naval Battalion of the District of Columbia. He rose to the rank of Lieutenant Commander, and served as Executive Officer of the U. S. S. *Ozark.*

In November, 1914, he was appointed Assistant Inspector of Hydrography and Topography, and in March, 1915, he was made Assistant Director of the Coast and Geodetic Survey.

During the World War, while the Director was absent on military duty, Captain Faris, as Acting Director, ably guided the Survey through that trying period during which many of the experienced officers were on duty with the armed forces of the Nation.

In 1919, Captain Faris was appointed a member of the Mississippi River Commission, and he continued in that capacity until his death.

His broad technical knowledge, together with his sound practical judgment, made Captain Faris a valuable man in the Coast and Geodetic Survey, and his loss will be felt keenly. He had a fine sense of public duty and was kindly, sympathetic, and just to those under him.

Captain Faris was a Fellow of the American Association for the Advancement of Science; a member of the Committee on Navigation and Nautical Instruments of the National Research Council; a member of the Federal Board of Surveys and Maps; the Washington Academy of Sciences; Philosophical Society of Washington (President in 1921); Washington Society of Engineers (President in 1921); American Geophysical Union; and the International Society of Navigational Congress.

In 1897, he was married to Carrie Elizabeth Hellen, who survives him. He is also survived by two sons and three daughters: Robert Lee Faris, Jr., Helen Mills (Faris) Myers, Carolyn (Faris) Wheeler, Elizabeth R. (Faris) Short, and Charles William Faris.

Captain Faris was elected a Member of the American Society of Civil Engineers on March 14, 1916.

JOHN MOYER FARLEY, M. Am. Soc. C. E.[1]

DIED AUGUST 26, 1932

John Moyer Farley, the son of Worthington S. and Helen (Moyer) Farley, was born at Fort Plain, N. Y., on March 30, 1863.

After receiving his preliminary education at Fort Plain Seminary and Rutgers Preparatory School, Mr. Farley entered Rutgers University, at New Brunswick, N. J., from which he was graduated in 1886, with the degree of Bachelor of Science. In 1889, he received the degrees of Master of Science and Civil Engineer from his Alma Mater.

Soon after his graduation, Mr. Farley designed extensions to the sewerage system of New Brunswick. From March to June, 1887, he was Topographer for the Pennsylvania and New England Improvement Company, on railroad location from Delaware Water Gap, Pa., to Poughkeepsie (N. Y.) Bridge, and from July to September, 1887, Engineer in charge of the preliminary surveys and relocation of the Fort Plain and Richfield Springs Railroad. From October, 1887, to January, 1888, as Assistant to J. A. Kirby, of Port Chester, N. Y., and to J. H. Carpenter, of White Plains, N. Y., Mr. Farley had charge of real estate and park improvement, as well as the construction of bridges, etc. He also made the original surveys for the sewerage system of White Plains.

After serving as Assistant Engineer in charge of grading, storm-water drains, etc., for Capitol Park, as well as other municipal work, at Albany, N. Y., in 1888, and in charge of the construction of the substructure for the steel and iron highway bridge over the Mohawk River at Fort Plain, Mr. Farley, in 1889 and 1890, was engaged as Assistant Engineer on general canal work in the State of New York; Engineer for the Highway Commissioners at Glens Falls and South Glens Falls, N. Y., in charge of the construction of a steel bridge over the Hudson River at that point; Principal Assistant to the late John Bogart, M. Am. Soc. C. E., State Engineer; and Engineer for the Department of Health, New York State, in the planning of several sewerage systems. In 1890 and 1891, he was especially detailed by the State Engineer Department to make flow measurements for streams feeding into the Genesee Valley, as well as measurements of run-off on the Seneca and Cayuga Lakes Water-Shed. During this time he was also employed as Expert by the Board of Trade of Syracuse, N. Y., to examine and report on the condition of the pavements of that city.

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[1] Memoir prepared from information on file at the Headquarters of the Society.

From 1892 to 1900, as a member of the firm of Carpenter and Farley, he planned and superintendended the construction of the extensions to the sewerage system of White Plains, as well as the construction of highway bridges, and was engaged in all classes of general municipal work for that community. He was also employed as Expert by the Farmers Loan and Trust Company on water-shed and water supply investigations in connection with the suit of the Village of White Plains to acquire the water plant. He served as Engineer for the Board of Water Commissioners for White Plains from its organization in 1886 until 1911, and, as such, had charge of the design and construction of all improvements to the water supply system, including dams, reservoirs, filter plant, etc.

In 1901, Mr. Farley was appointed Engineer for the Board of Town Officials, of White Plains, in charge of the highways; he also served as Consulting Engineer on the four-span steel County Bridge over the Mohawk River between the Canajoharie and Palatine Bridges, which was 486 ft long, and cost approximately $65 000. In 1902, he planned and installed storm-water drains for a part of the Village of Mount Kisco, N. Y., and completed plans for road improvement work for that village, amounting to approximately $100 000. He also served as Consulting Engineer for the New Castle Water Company on the construction, design, and installation of a water plant for Pleasantville, N. Y., which included reservoirs, dams, elevated tank, etc. From 1903 to 1905, as Chief Engineer, Mr. Farley had charge of all surveys for the Westchester Traction Company and the Danbury and Harlem Traction Company, in Westchester County, New York, and in Connecticut; and made surveys and located lines for a steam railroad extension across the eastern and central parts of the State of New York for the Ballston Terminal Railroad Company (now the Eastern New York Railroad Company), including extensive improvements to the existing lines.

From 1904 to 1906, he was in partnership with Edward Wegmann, M. Am. Soc. C. E., under the firm name of Wegmann and Farley, engaged in general consulting work in connection with dams and reservoirs. As this partnership was entirely outside of his regular work, Mr. Farley, in 1905 and 1906, had charge of an extension to the sewerage system at Ballston Spa, N. Y. He also did consulting work on improvements to the water plant, and had charge of road and other municipal work in that village. He also designed sewerage and water supply plants for various private estates. From 1907 to 1908, Mr. Farley was employed by the Board of Water Commissioners to report on a proposed source of water supply for Batavia, N. Y. He served as Engineer for the Board of Town Officials on the improvement of the principal highways of Harrison, N. Y.

In 1908, he designed a system of sewers for Mount Kisco, which was completed in 1913; designed and superintended the construction of a sewage disposal plant and pumping station for Mount Vernon, N. Y. (completed in 1910); and made plans for sewerage systems for Harrison and Mamaroneck, N. Y. During this time, as Chief Engineer of the Award Company, Mr. Farley handled some of the most important of the water power and water supply cases against New York City in the Ashokan Reservoir condemnation proceedings, and as Chief Engineer of Shanley-Morrissey, Incorporated, he had charge of the construction of 60 miles of State road improvement. He was also employed by the Union Bag and Paper Company to make a report on its proposed hydro-electric installation of a 10 000-hp plant at Hudson Falls, N. Y.

In 1909, Mr. Farley investigated and reported on various construction propositions, including McNally's Aqueduct, estimated at $4 500 000, for the Bunting-Ball Company. He also appeared in Court proceedings as Expert Witness for Mount Vernon, Mount Kisco, Mamaroneck, and Ballston Spa, N. Y.; planned and superintended the construction of road improvements and macadamizing in Harrison; prepared a report and outlined plans for a water supply for Scarsdale, N. Y.; designed plans for a sewage disposal plant for Lynn, Mass.; and reported on proposed railroad improvement and elimination of grade crossing at White Plains. In 1910, he designed the intercepting filter wells and stand-pipe, of which latter he superintended the construction, for the Board of Water Commissioners at White Plains; and, as Engineer for the Scarsdale Estates, designed and superintended general improvements for the water supply, sewerage system, sewage disposal plant, road work, etc., and also made plans for a private water supply and sewage disposal plant in that village. He also prepared plans for a water supply system, as well as a sewerage system and sewage disposal plant for the Town of Harrison, and prepared plans and superintended the construction of the improvement of various town roads of that locality; prepared plans for various municipal improvements for the Village of Mamaroneck; made a report on the pollution of the Bronx River; prepared a report for a proposed water supply for the Keystone Coal and Coke Company in Western Pennsylvania; and was Consulting Engineer on the reconstruction of the West Penn Electric Railway.

In 1911, under the firm name of Farley and Braunworth, Mr. Farley did municipal improvement work for the Scarsdale Estates, the New Castle Water Company, of New Castle, N. Y., and Chappaqua, N. Y., and served as Village Engineer for Mamaroneck on considerable municipal improvement work; constructed a disposal plant to take care of trade wastes and domestic sewage for the Agasote Millboard Company, of Trenton, N. J.; and made surveys and sub-divisions for the Lawrenceville Realty Company, of Trenton, as well as laying out roads, etc. In 1912, he was engaged in municipal work for the Village of Mamaroneck, made plans, specifications, and estimates of the brick pavement for the Boston Post Road, and trunk sewers for other highways; made surveys and sub-divisions for several realty companies; and was employed on railroad construction for the New York Central and Hudson River Railroad Company.

From 1913 until 1921, Mr. Farley served as Consulting Engineer on improvements to the plant and dam of the New Castle (N. Y.) Water Company; made surveys and sub-divisions and engaged in municipal improvement work for the Gedney Farm Company, Scarsdale Estates, White Plains, Long Beach, Great Neck, Nelson Estates, at Peekskill, Nassau Haven, Philipse Manor, New Hyde Park, Bryn Mawr Park, at Yonkers, all in New York State. He also did considerable road improvement work on a number of private estates, among which was Ophir Farm, near White Plains, owned by the late Mrs. Whitelaw Reid, and surveys for numerous real estate developments, title companies, and property transfers. From October 1, 1917, to January 1, 1918, Mr. Farley made progress reports at Camp Merritt, Tenafly, N. J., for the United States Government, and from July 1 to December 31, 1918, he was Superintendent of Materials, Warehouses, and Magazines for the Woodbury, N. J., Bag Loading Plant, in charge of all materials used in the production of propellent charges for mobile artillery, for the American Expeditionary Forces in Europe. From 1921 until his death on August 26, 1932, Mr. Farley was engaged in general consulting engineering work, on surveys, highways, dams, reservoir construction, etc.

His wide professional knowledge in applied science and engineering was a most important contribution toward the betterment of life in the communities in which he worked.

The following tribute appeared on the editorial page of the *Daily Reporter*, of White Plains, N. Y., on August 29, 1932:

"White Plains has lost another of its older citizens to whom the present generation owes much for the fact that this is such a good community in which to live.

"John M. Farley, civil engineer, was identified with the progress of the old village and town, and of the city, for nearly half a century. He drew plans for the original sewer system; he constructed roads and bridges and waterworks. He was active in many movements tending for the betterment of conditions, generally.

"His fame as an engineer extended far beyond the confines of his city home. To list all the projects of which he had charge, or on which he was a consultant, would be to fill columns of space. The State of New York called him into its service many times, as did various municipalities. In White Plains he was busy almost constantly. As an engineer, and as a consultant, he was held in high regard by public bodies and by people laying out home plots or large developments.

"Mr. Farley was interested intensely in the welfare of animals. He was, for many years, in the work of the Society for the Prevention of Cruelty to Animals, and at the time of his death, was honorary president of the Central Westchester Humane Society.

"Too often do the people of today forget the fact that those who preceded them in the upbuilding of a community, had much more to do with that upbuilding than they themselves have had. It took real faith, and courage, too, to look into the White Plains of half a century back and envision for it a great future. Now it is easy, thanks to men such as Mr. Farley, who never lost their enthusiasm, and who gave a great deal of their time and of their energy, with their reward, most times, only their personal satisfaction at accomplishment.

"Those who knew him will remember him as a patient, kindly man, a hard worker, and an enthusiast in the effort he put to those things which interested him most."

He was a member of the Delta Phi and Phi Beta Fraternities, the Milton Point Casino, of Rye, N. Y., and the City Club and University Club, of White Plains. Mr. Farley attended the Presbyterian Church.

On October 22, 1889, he was married to Ella Shumway, of Fort Plain, N. Y., who survives him, together with their four children, Worthington, Walter, Dorothy, and Jack.

Mr. Farley was elected a Junior of the American Society of Civil Engineers on December 5, 1888; an Associate Member on July 1, 1891; and a Member on September 1, 1897.

WALTER HAMER GAHAGAN, M. Am. Soc. C. E.[1]

DIED DECEMBER 18, 1930

Walter Hamer Gahagan was born at Troy, Ohio, on February 14, 1864, the son of William Henry Harrison and Hannah (Smith) Gahagan. He was a descendant of William Gahagan who, with eighteen others, founded Dayton, Ohio, in April, 1796,

Walter Hamer Gahagan was educated at the Ohio State Technical School, and the Massachusetts Institute of Technology from which he was graduated in 1887. From the date of his graduation until 1899, he gained the experience and knowledge of construction necessary to fit him for his life's work as a Contracting Engineer.

In time he became Engineer in charge of the construction of many bridges of the Western railroads and, finally, of the foundation of the west pier of the Williamsburg Bridge Caisson, New York, N. Y., In 1899, Mr. Gahagan established a contracting business of his own, his first contracts being railroad construction. Out of the original business firm grew several large corporations: The Gahagan Dredging Company, W. H. Gahagan Realty Company Incorporated, the W. H. Gahagan, Incorporated, and the Gahagan Construction Corporation, all of which he founded, financed, and personally directed as President.

Among the sixty major contracts carried out by the Gahagan Companies were: One contract for the Peruvian Government, costing $1 000 000; twelve contracts for the Delaware, Lackawanna and Western Railroad Company amounting to $5 223 106; four contracts for the Lehigh Valley Railroad Company, amounting to $4 906 000; seven contracts for the United States Government, amounting to $4 276 977; five contracts for the Long Island Railroad Company for $5 488 000; and three contracts in and about the City of

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[1] Memoir prepared by Frederick L. Cranford, M. Am. Soc. C. E.

New York, costing $3 311 000, as well as seven contracts, including the Wanaque Dam, the Chesapeake and Delaware Canal, and improvements on the Hudson River, from Coxsackie to Hudson, N. Y., amounting to $8 874 446.

The magnitude of these operations required equipment worth $3 500 000 and involved excavation of earth (101 653 953 cu yd), excavation of rock (2 523 014 cu yd), and concrete (378 433 cu yd).

Mr. Gahagan was also a Director of the Manufacturers Trust Company and the North Side Holding Company, and was interested financially in many other enterprises. He was a member of the Associated General Contractors of America, the General Contractors Association of New York (President in 1925), and the Masonic Fraternity (Thirty-second Degree).

The world at large will remember Mr. Gahagan as an Engineer and a Contractor. A small group will remember him also as a loyal friend and a worthy competitor, with a knowledge of men, materials, and equipment which on several occasions developed new and successful methods in the great work of construction. Those who worked with him and for him, however, will remember him, too, as a lovable human being, endowed with the gift of leadership.

He was married on September 30, 1896, to Lillian Rose, the daughter of James Mussen, of Lodi, Wis. They had five children, William Corthell and Frederick Mussen, twins, Helen Mary Gahagan, Lillian, wife of Robert S. Walker, and Walter Hamer Gahagan, Jr.

Mr. Gahagan was elected a Junior of the American Society of Civil Engineers on September 5, 1888; an Associate Member on July 1, 1891; and a Member on April 3, 1901.

EMERSON WARREN GRANT, M. Am. Soc. C. E.[1]

DIED MAY 6, 1932

Emerson Warren Grant was born at Conway, Mich., on June 4, 1849. He was the son of Elisha Wilcox Grant, of Great Barrington, Mass., and Amelia Jane (Hitchcock) Grant, of South Dansville, N. Y. After his preparatory education, he attended Cornell University, at Ithaca, N. Y., in 1870 and 1871; and was graduated with the degree of Civil Engineer from the University of Michigan, Ann Arbor, Mich., in 1875.

Mr. Grant was employed on land surveying in Livingston County, Michigan, from 1875 to 1878, and on surveys of the Mississippi and Missouri Rivers in 1879 and 1880.

In March, 1880, he entered the service of the Atchison, Topeka, and Santa Fé Railway Company as Rodman, at Nickerson, Kans., and remained with that Company as Transitman, Assistant Engineer, and Resident Engineer, until November, 1888, and afterward, at intervals, to March, 1889, on the right of way for the extension of the line from Kansas City, Mo., to Chicago, Ill.

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[1] Memoir prepared by C. D. Purdon, Gilbert J. Bell, and Charles A. Morse, Members, Am. Soc. C. E.

From this time until August, 1890, Mr. Grant was employed on the extension of the Fort Worth and Rio Grande Railway, west from Granbury, Hood County, Tex., after which he was sent (apparently in the interest of the Atchison, Topeka, and Santa Fé) to California, where he was in charge of one of the locating parties between Bakersfield and San Francisco. When these parties were recalled, he had intermittent employment in the Land Department from December, 1890, to July, 1891.

From July, 1891, to February, 1892, he was in the service of the Argentine Smelting and Refining Company, on mine or topographical surveys, or on office work, at El Paso, Tex., Sierra Mojada, Mexico, and Argentine, Kans.

On February 8, 1892, Mr. Grant again entered the service of the Atchison, Topeka, and Santa Fé Railway Company as Assistant Engineer. Later, he served as Resident Engineer of the New Mexico Division; and afterward as Assistant Engineer in the General Office in Topeka, Kans., where he remained, engaged in general work, rate cases, and valuation until he was retired, on a pension, in June, 1925. He afterward lived quietly in Topeka until his death, on May 6, 1932.

Mr. Grant was married, on October 14, 1886, to Jennie E. Bradbury, of Topeka, and is survived by two daughters, Mrs. Harold (Grant) Loucks, of Arkansas City, Kans., and Mary A. Grant, of Topeka.

He was a member of the Kansas Engineering Society, and the Topeka Engineers Club.

Mr. Grant was a lovable man, and much esteemed by his fellow workers, and, indeed, by all who knew him. He was, however, of such a modest and retiring disposition, that he did not have a large circle of acquaintances; but all who knew him were his friends.

He had done a large amount of valuable work for the Atchison, Topeka and Santa Fé Railway Company, but never had much to say about it, and had seemingly no desire to advertise himself. During a long and fairly active life, his object seemed to be to get the work done—and well done—and seek no credit for it. He was never a talkative man, and never was known to make disparaging remarks about others.

Mr. Grant was elected a Member of the American Society of Civil Engineers on December 4, 1889.

EVERETT ADDISON HADLEY, M. Am. Soc. C. E.[1]

Died November 11, 1932

Everett Addison Hadley, the son of Frank Milan and Susan Lillian (Eastman) Hadley, was born at Lowell, Mass., on November 19, 1879. He received his early education in that city, having been graduated from the Lowell High School in 1897.

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[1] Memoir prepared by a Committee appointed by the St. Louis Section, consisting of W. J. Burton, *Chairman*, and E. O. Sweetser, Members, Am. Soc. C. E., and W. F. Cummings, Asst. Chf. Engr., B. & M. R. R., Boston, Mass.

Upon graduation, Mr. Hadley entered the employ of Smith and Brooks, Civil Engineers, as an Apprentice. In September, 1900, he undertook his first railroad work as Draftsman for the Boston and Maine Railroad Company. His energy and ambition for construction work resulted in his transfer, in June, 1901, to the Construction Department.

This work brought Mr. Hadley into contact with the surveys, plans, and estimates for double-tracking of parts of the former Fitchburg Railroad between Rotterdam, Johnsville, and Troy, N. Y., a distance of fifty-two miles. Shortly afterward, a program of grade-crossing elimination was begun, and Mr. Hadley was placed in charge of the surveys, design, and estimating of all this class of work on the Boston and Maine Railroad. This responsibility continued during the remainder of his engagement with that Company. Among the projects for which Mr. Hadley developed plans were the grade separations at Waltham, Mass., including the main line and the Watertown Branch; at Somerville, Mass., including Webster and Somerville Avenues and Medford, Dane, and Park Streets; at Fitchburg, Mass., including Rollstone Street and also the engine-house facilities; at Belmont, Mass., including the elevation of both the Fitchburg and the Massachusetts Central Railroads and the stone arch under-pass and the station; and, at Williamstown, Mass., including also a new freight yard and station. Mr. Hadley's enthusiasm and pride in good work were characteristic of the man.

In 1910, Mr. Hadley resigned his position with the Boston and Maine Railroad Company, in order to accept an offer from the Missouri Pacific Railroad Company to become Engineer of Design, and he then moved to St. Louis, Mo. The new work involved not only the actual design, but studies of the economics involved in the vast number of projects which together make up an improvement program on a large railroad.

Mr. Hadley's ability, not only for the detail work connected with design, but for securing the co-operation of his associates, soon led to his promotion to Principal Assistant Engineer, and, in 1915, to Chief Engineer. About the time of Mr. Hadley's going to the Missouri Pacific, the Company was beginning the process of evolution from the pioneer type of railroad, and during his term as Chief Engineer and under his direction more than $150 000 000 was spent in betterments to existing lines. These improvements included grade revisions, relocation of terminals, strengthening of bridges and track, automatic signals, double-tracking, and the enlargement of terminal and station facilities necessitated by increasing traffic and the operation of heavier power.

Among the important matters falling under Mr. Hadley's jurisdiction was the determination and adoption of standards for all detail parts of the railroad, including roadway, track, bridges, signals, water and fuel stations, shops, engine-houses, stations, and other buildings. Among the larger projects constructed under Mr. Hadley's direction may be mentioned the second-track work between St. Louis and Jefferson City, Mo., involving 110 miles of heavy work, with a considerable part on new alignment; the relocation and reconstruction work between Kansas City, Mo., and Hoisington, Kans., involving 70 miles of new alignment and the reduction of ruling gradients from a maximum of 1.5 to 0.7%; the construction of the heavy double-track lift bridge over the Arkansas River at Little Rock, Ark.; the construction of five large grain elevators at St. Louis and Kansas City, and at Omaha, Nebr., with a combined capacity of about 14 000 000 bushels; and the construction of the large Union Stations at Little Rock and Texarkana, Ark., and of the 22-story, Missouri Pacific Office Building, in St. Louis; as well as the Missouri Pacific Hospitals, at Little Rock and at St. Louis.

Not only did the larger projects, such as those mentioned, require a great amount of detail supervision and design, but the tens of thousands of smaller ones each required studies to determine justification and most economic design. That Mr. Hadley's efforts along these lines were in the right direction is perhaps best illustrated by the fact that, in 1929, the traffic (ton-miles) had doubled over what it had been when he became Chief Engineer, but this double volume of business was handled with only one-fourth more train-miles and, at the same time, at practically double the speed.

During Federal Control, Mr. Hadley was detached from the Missouri Pacific Railroad Company to be Engineering Assistant to the Regional Director of the Southwestern Region. This afforded him an opportunity to become acquainted with practically all the railroad officials of the Southwest, and few men had as wide a circle of friends and few enjoyed the confidence which every one had in Mr. Hadley. A contractor once remarked that Mr. Hadley was so fair that he was perfectly willing to have the latter represent both sides in a controversy with the Missouri Pacific. A railroad officer who had participated in many discussions in which Mr. Hadley represented the Missouri Pacific, writes:

> "To observe Everett Hadley in action was a rare privilege. His candor was inoffensive, his criticism constructive, his reasoning clear, his argument persuasive. He had the faculty of avoiding technical discussion with non-technical people—a rare talent for an engineer. He never permitted himself to be lost in a maze of non-essentials; he invariably stuck to the main theme."

While Chief Engineer of the Missouri Pacific Railroad Company, Mr. Hadley found time for many activities connected with memberships in engineering and other societies. In addition to his membership in the Society, he was a member of the American Railway Engineering Association, and, at the time of his death, was a Director of that Association, as well as Chairman of its Rivers and Harbors Committee and a member of its Rail and Standardization Committees. He was also Chairman of the Automatic Train Control Committee of the American Railway Association. Other memberships included the Engineers', Traffic, Missouri Athletic, and Bellerive Country Clubs, of St. Louis. He was a Director of the Railroad Young Men's Christian Association and of the Missouri Pacific Hospital Association. Soon after moving to St. Louis, Mr. Hadley became affiliated with the Pilgrim Congregational Church, serving a term as a member of the Board of Trustees. He was also a member of Tuscan Lodge, A. F. and A. M., Missouri Consistory No. 1, M. R. S., and Moolah Temple, A. A. O. N. M. S.

In 1902, Mr. Hadley was married to Lilla May Sturtevant. His domestic life was of singular clarity and felicity. In addition to Mrs. Hadley, he is survived by one son, Carleton Sturtevant, and also by his mother, Mrs. Frank Milan Hadley, and a sister Mrs. Harrison E. Byam, of Lowell, Mass., and by a brother, Walter E. Hadley, of Maplewood, N. J.

Following the funeral services at Pilgrim Congregational Church, at St. Louis, conducted by the Rev. Dr. J. T. Stocking, who had been Mr. Hadley's Pastor, the body was taken to Lowell, Mass. After a simple service at Lowell, conducted by the Rev. John H. Sargent, of Highland Congregational Church, Mr. Hadley was buried in Edson Cemetery.

Few men have the ability of making friends developed to the same extent as did Mr. Hadley. His genial and friendly disposition was outstanding both among his railroad associates and his wide circle of other friends.

> "He was a Christian gentleman in the truest sense of the word, and in his passing the Missouri Pacific has lost an executive of unusual ability; his employes have lost an officer who always had their interests at heart; and the other officers of the company have lost an associate that they could ill afford to spare, and a friend who was always ready with helpful suggestions and practical assistance."

Mr. Hadley was elected a Member of the American Society of Civil Engineers on October 11, 1920.

FERDINAND FINNEY HARRINGTON, M. Am. Soc. C. E.[1]

DIED FEBRUARY 8, 1932.

Ferdinand Finney Harrington, the son of Eugene C. and Harriet (Edwards) Harrington, was born in St. Louis, Mo., on November 18, 1869. His early education was received in the public schools of St. Louis. He then entered Washington University, at St. Louis, from which he was graduated in 1893 with the degree of Bachelor of Science in Civil Engineering. In 1894, he had conferred upon him the degree of Civil Engineer, *Magna cum Laude.*

While a student at Washington University, and during his summer vacations, Mr. Harrington was Assistant to the Engineer, Maintenance-of-Way Department, Wabash Railroad Company; he spent other summer vacations with the Missouri Pacific Railway Company, the Missouri River Commission, and in the Testing Laboratory of Washington University.

From 1894 to 1895, Mr. Harrington was Assistant to the late J. B. Johnson, M. Am. Soc. C. E., on important work undertaken for the United States Government in reference to the physical investigations of timber.

From April, 1895, to August, 1898, he was Engineer for the Board of Public Improvements, St. Louis, and, in addition to his other duties, had charge of the Testing Laboratory of that city where he made extensive original investigations.

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[1] Memoir prepared by S. H. Harrison, M. Am. Soc. C. E.

About August, 1898, there began the active building of Portland cement plants throughout the United States, at which time Mr. Harrington became associated with Booth, Garrett and Blair, Engineers and Chemists, of Philadelphia, Pa. He was Chief Inspector in charge of the inspection of structural materials at mills and shops, as well as Assistant Engineer on the design of Portland cement plants; this position he held until January, 1900, when he became identified with companies that afforded him opportunity to gather further information and experience in the field of mechanical engineering—specializing particularly in elevating and conveying machinery, power transmission, etc.

In 1900, he became associated with the Osborn Engineering Company, of Cleveland, Ohio, which at that time was engaged extensively in the new field of Portland cement plant engineering.

From August, 1900, to August, 1901, Mr. Harrington was Assistant Engineer of the Michigan Lake Superior Power Company, Sault Ste Marie, Mich., in charge of the construction of a power house and forebay for the development of a 40 000-h. p. hydro-electric plant.

He left the Power Company in 1901 to accept a position with the New York Central and Hudson River Railroad Company. While with the Railroad Company, he had responsible charge of the design of bridges, viaducts, draw-spans, transfer bridges, power houses, and other structural work. He also had charge of several field parties making surveys for the reconstruction of bridges; and, finally, was appointed Assistant to the Chief Engineer on general railway work.

In July, 1905, Mr. Harrington became identified with the Virginian Railway Company, of Norfolk, Va., as Bridge Engineer and Engineer of Structures. Later, he was appointed Chief Engineer of the Virginian Terminal Railway, a subsidiary of the Virginian Railway.

Mr. Harrington was an indefatigable worker, and a close student of technical matters. In July, 1913, he received a rating of 96% from the United States Civil Service Commission, in a competitive examination for the position of Senior Structural Engineer. This was the highest percentage obtained that year in the entire United States for the position in question. Also, in 1917, he passed a competitive examination held by the United States Civil Service Commission for the position of Designing Engineer, Bureau of Yards and Docks, Navy Department.

From November, 1918, to January, 1919, Mr. Harrington served as Office Engineer, Division of Capital Expenditures, United States Railroad Administration, at Washington, D. C. From January, 1919, to July, 1919, he was Engineer of Structures, on the Cunard Terminals, with the Todd-Robertson-Todd Engineering Corporation, of New York, N. Y. From July, 1919, and to the time of his accidental death on February 8, 1932, in Richmond, Va., Mr. Harrington was again identified with the Virginian Railway Company, and the Virginian Terminal Railway Company.

He was the author of numerous papers and articles on engineering subjects, such as those mentioned in the following list: "Experiments on Vitrified Paving Brick": Paper read before the Engineers' Club of St. Louis, June, 1896; "A New Testing Machine and the Cross-Breaking Test of Vitrified Paving Brick": Paper read before the Engineers' Club of St. Louis, January, 1897; "Underground Conduits in St. Louis": Paper read before the Engineers' Club of St. Louis, June, 1897; "The Sewalls Point Coal Pier of the Virginian Railway": Paper read before the Appalachian Engineering Association, Washington, D. C., November, 1909; "Ventilation of Allegheny Summit Tunnel, Virginian Railway";[2] "Coaling and Sanding Station on the Virginian Railway at Elmore, W. Va.";[3] and "Coal Piers on the Atlantic Coast, and the Virginian Railway Coal Pier No. 2, at Sewalls Point, Virginia": Paper read before the Engineer's Club of Hampton Roads, March, 1923.[4]

Mr. Harrington took a lively interest in several Engineering Societies, and in spite of his many activities, and a busy professional career, he took a keen interest in religious work, and in the welfare of young men. He was thorough, painstaking, and obliging, with a wonderful capacity for detailed work.

He was a member of Ruth Masonic Lodge, of Norfolk, Va., a member of Hampton Roads Engineers' Club, of which he had served as President from 1924 to 1925. He was, also, a member of the American Railway Engineering Association, and the American Association for the Advancement of Science.

Mr. Harrington was a man of high ideals, of a modest and unassuming disposition, and was held in affectionate esteem by his friends and associates. He took an active interest in Church affairs, serving as a Vestryman in the Protestant Episcopal Church of the Good Shepherd, at Norfolk, and also as Superintendent of the Sunday School. He was a kind and considerate husband, a loving father, and a man whose chief concern in life was to serve his fellow men.

On April 10, 1901, he was married to Anne V. Baldwin, daughter of the late William James Baldwin, M. Am. Soc. C. E. Mr. Harrington is survived by his widow, a son, Ferdinand Baldwin Harrington, and a sister, Mrs. Harriet L. Pretlow.

Mr. Harrington was elected an Associate Member of the American Society of Civil Engineers on May 3, 1899, and a Member on November 10, 1908.

RUSSELL THEODORE HARTMAN, M. Am. Soc. C. E.[1]

DIED MARCH 16, 1932

Russell Theodore Hartman, the son of August and Sophronia (Miller) Hartman, was born in Eddyville, Iowa, on September 13, 1872. He attended the public schools in that town and was graduated from the Eddyville High School at the age of sixteen.

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[2] *Engineering Record*, September 19, 1914, and *General Electric Review*, December, 1914.

[3] *Engineering Record*, January 2, 1915.

[4] *Dock and Harbor Authority*, London, England, October, 1923.

[1] Memoir prepared by Frank R. Hubbard, M. Am. Soc. C. E.

He inherited an inclination toward engineering and the mechanical arts and doubtless was also influenced in the choice of his life work by his early training. He worked as a boy with his father who was a Superintendent of Bridges and Buildings on the Chicago, Rock Island, and Pacific Railroad.

Mr. Hartman was the fourth son in a family of six boys and two girls. He was a student from his youth, and although he was modern in his thinking, he could readily have been classed with the old school of engineers in that he did not depend on books alone for his education. He learned from what he did and from what he saw others doing all about him at a time when new ideas and new methods were developing in his chosen field more rapidly than they were being presented in textbooks.

In the fall of 1891, he entered the State University of Iowa, at Iowa City, Iowa, and was graduated from that institution in June, 1896, with the degree of Bachelor of Science in Civil Engineering. While he received some support from home during his college course, on account of the large family and their moderate income, young Russell found it necessary to support himself in part while in the University and was out of college one whole year during his course to provide the necessary funds to continue his studies.

Immediately following his graduation Mr. Hartman was retained for two years by the University as an Instructor in Civil Engineering. During this time he continued his studies and in June, 1898, received his degree of Master of Science. The University, in 1906, gave him the degree of Civil Engineer.

On leaving the University in June, 1898, Mr. Hartman took a position as Draftsman with Stupp Brothers Bridge and Iron Works in St. Louis, Mo., detailing highway bridges. After about one year in St. Louis, he went with the Gillette-Herzog Manufacturing Company, of Minneapolis, Minn., as Draftsman, and after two years in that position entered the Branch Office of the American Bridge Company, at Salt Lake City, Utah, where he was engaged in designing, estimating, and contracting.

Mr. Hartman remained in Salt Lake City less than a year, when he moved to Des Moines, Iowa, taking up his duties as Chief Draftsman for the Des Moines Bridge and Iron Works (later, the Pittsburgh Des Moines Steel Company). He was with this firm in Des Moines for eleven years, rising to the position of Chief Engineer. It was in this latter position that he did his best work as a Structural Engineer; he assumed much of the responsibility for the design, fabrication, and erection of the many large contracts executed by this Company, and also assisted with the sales.

In 1912, Mr. Hartman organized the R. T. Hartman Company, Consulting Engineers and Manufacturers' Representatives, in Des Moines. About a year later, he sold his share in that firm and bought an interest in the Iowa Steel and Iron Works, of Cedar Rapids, Iowa. He then moved with his family to Cedar Rapids, and was appointed General Manager of the Iowa Steel and Iron Works, which position he held until 1920, when on account of ill-health he sold his interest in that firm and retired from regular business, except for such consulting work as his health would permit.

Before leaving Des Moines Mr. Hartman had a severe attack of rheumatism which weakened his heart and from which he never fully recovered. For more than twenty years he was seriously handicapped by ill health which came upon him in early manhood and checked a career in which he had already shown more than ordinary ability and promise of achievement as an Engineer.

Mr. Hartman was a member of the Methodist Episcopal Church, and the Engineers Club, in Cedar Rapids. He was a Life Member of the National Geographic Society, and while a resident of Des Moines, an active member of the Des Moines Engineers Club. He was always a devoted and faithful husband and father, interested and active in those affairs which build higher ideals and better lives. He gave even beyond the limit of his strength to his profession, his family, and his community.

He died at his home in Cedar Rapids, Iowa, on March 16, 1932. He was borne to his last resting place in the family lot in Glendale Cemetery, very near the site of the family home in Des Moines where three of his children were born and where their early lives were spent.

On January 11, 1899, he was married to Esther Spinden, of Iowa City, Iowa, the climax of a romance which began during his student days. They established their first home in Minneapolis. To this union were born four sons, three of whom, together with their mother, survive him. The three sons (Theodore, an Architect, in Chicago, Ill., Warren, a Civil Engineer, formerly with the Iowa State Highway Commission, and Edward, an Electrical Engineer, following the career of their father, received their education at the State University of Iowa. The fourth son, Richard, passed away at the age of six years.

Mr. Hartman was elected an Associate Member of the American Society of Civil Engineers on May 4, 1909, and a Member on November 1, 1910.

HARRISON WASHBURN HAYWARD, M. Am. Soc. C. E.[1]

DIED OCTOBER 18, 1932

Harrison Washburn Hayward was born at Dorchester, Mass., on January 1, 1873. He was the son of Edward S. and Emma (Washburn) Hayward.

After his preliminary education, he was graduated in 1896 from the Chemical Engineering Department of the Massachusetts Institute of Technology. After serving for two years on the Staff of that Department, he joined the Mechanical Engineering Staff on which, in 1908, he became an Assistant Professor in Applied Mechanics. In 1912, he was made an Associate Professor; and, in 1920, he became a full Professor in Materials of Engineering.

For a great many years Professor Hayward had taught Theoretical and Applied Mechanics and, since 1916, he had been in entire charge of the Testing Materials Laboratory at the Institute. During the World War he rendered valuable service to the Government in connection with materials, and immediately following the war he accepted a commission as Major in the Reserves.

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[1] Memoir prepared by Edward F. Miller, M. Am. Soc. C. E. Prof. Miller died June 12, 1933.

Professor Hayward completely revised "Materials of Construction, Their Manufacture, and Properties," written originally by A. P. Mills. The Fourth Revision of this book edited by Professor Hayward only recently came off the press. He was also the author of a book entitled "Notes on Calculus," used by the students in the Lowell Institute School, in which he had been a teacher since it was founded in October, 1903. This school is conducted at the Massachusetts Institute of Technology two or three evenings a week for the period from October to June, and is run for the benefit of factory superintendents, draftsmen, and men in similar occupations who desire the advantages of an engineering education, but being obliged to work during the day, cannot attend college. He took special pleasure in helping men of this type, who were anxious to better themselves, and was one of the best loved Professors on the Staff.

As a teacher, Professor Hayward was clear in his presentation of a subject, concise, and forceful. He was an instructor whom the students both respected and admired. He had been retained as an Expert and Consultant on Materials by a number of large industrial firms. He also held the position of Assistant Director of the Division of Industrial Co-Operation and Research, this Division having been organized at the time the Institute moved to its new site in Cambridge, to handle problems in engineering research brought in by outside companies.

Professor Hayward was a member of the American Society of Mechanical Engineers. He was also a member of the American Society for Testing Materials, the American Concrete Institute, the American Society for Steel Testing, the Society for the Promotion of Engineering Education, and the Boston Society of Civil Engineers. He also belonged to the Theta Xi Fraternity.

He died on October 18, 1932, following a heart attack which came on him very suddenly. He was married on November 4, 1902, to Mabel E. Holmes who survives him. There were no children.

Professor Hayward was elected an Affiliate of the American Society of Civil Engineers on June 6, 1906, and a Member on May 5, 1915.

ARTHUR RASCO HIRST, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 17, 1932

Arthur Rasco Hirst was born in Elmont, N. Y., on March 31, 1881, the youngest of five sons of John Hirst and Mary (Cornell) Hirst. His forefathers came to America from England about 1812. His maternal forefathers also came from England, and settled in New York State before the American Revolution.

At the early age of seven years, Mr. Hirst moved with his parents from their Long Island home to Cambridge, Md., where a farm had been purchased. Here he spent his youth, learning at first hand the nearness of Nature to life—a knowledge that paid him well in after life, enabling him always to appreciate the viewpoint of the other side of a problem. Here, in Cambridge, he attended Grade and High Schools, being always attentive to his studies and finishing as Valedictorian of his Class. After finishing High School, Mr. Hirst spent one year on the farm, after which he undertook his professional studies of Civil Engineering, a calling which he always admired and to which he gave his best efforts.

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[1] **Memoir prepared by H. J. Kuelling, M. Am. Soc. C. E.**

In 1900, he enrolled in the Junior Class of the Maryland Agricultural College and was graduated in 1902, with the degree of Bachelor of Science. Again, he was the highest in his class and was elected the Valedictorian. While Mr. Hirst was a Class A student, one must not think of him only as a student for he was exceedingly active in the physical affairs of school and college, playing on the college baseball team. In after life he always was active physically and was a real student of baseball and football statistics. His opinions on these matters were about as much respected by his co-workers as his professional decisions.

After finishing his college work in 1902, Mr. Hirst became associated with the Engineering Corps of the Pennsylvania Railway Company. He spent one year with this Company, after which he entered his chosen field of highway work with the Maryland Highway Commission. He felt that highway engineering was in its infancy, and offered a real field for engineering endeavors, an opinion that was amply verified in succeeding years.

Mr. Hirst remained with the Maryland Highway Commission as Engineer of Surveys and Plans until May, 1907, when he went West to join the Illinois Highway Commission as Engineer in Charge of Construction. This was done at the urgent request of A. N. Johnson, M. Am. Soc. C. E., an old friend, who was in charge of the Illinois work.

In August, 1907, those in charge of highway work in Wisconsin were seeking some one with proper experience to take charge of their work. After a study of the few available men, Mr. Hirst was chosen and moved to Wisconsin to undertake what proved to be his real life work, the planning and construction of a system of highways for that State. This was before there was even a highway commission in the State. He acted as Engineer in Charge of Surveys and Construction for the Wisconsin Geological and Natural History Survey, which had been assigned the work of highway study. This work continued until 1911, when the first State Highway Commission was formed by law, and Mr. Hirst was made its first State Highway Engineer, a position he held continuously until 1924.

During this decade of highway progress, he took a leading part in engineering, legal, and educational struggles in his own State and also was active in numerous surrounding States. He was honored as few highway engineers have been honored. He not only served through all the offices of the Mississippi Valley Highway Association, but was also elected President of the National Association of State Highway Officials.

A forceful and positive speaker and writer, his presence on a program always met with approval. He was never a straddler on any question—always hewing to the line, letting the chips fall where they might. This positiveness of character, while it made enemies for him, richly endeared him to his friends and fellow engineers. Always they knew that when "A. R." was handling their problem, it would be done efficiently and effectively. The present status of the Highway Engineering Profession owes much to such men as Mr. Hirst, who with others pioneered this new profession from a small beginning to the important position it now holds in American life.

After severing his connection with the Wisconsin Highway Commission in 1924, Mr. Hirst spent several years in the East as Chief Engineer for the American Vibrolithic Corporation. He then returned to Wisconsin, where he spent his remaining years engaged in consulting work. Much of this was for the United States Bureau of Public Roads.

He was married, in 1904, to Daisy Babylon, of Baltimore, Md., who, with two daughters, Louise Dallas and Anne Wood, survives him.

Mr. Hirst was an active member of the Protestant Episcopal Church, prominent in the affairs of his Parish and a member of the Vestry of Grace Church, Madison, Wis., for a number of years. It was from this church that he was carried to his last resting place in Forest Hill Cemetery, on the hill overlooking his favorite Lake Mendota.

In 1922, his Alma Mater extended him the courtesy of a degree of Doctor of Engineering, as a token of his marked success in his chosen field.

Mr. Hirst was elected a Member of the American Society of Civil Engineers on October 12, 1925.

CHARLES WARREN HUNT, M. Am. Soc. C. E.[1]

DIED JULY 23, 1932

Charles Warren Hunt was born in New York, N. Y., on May 19, 1858, the son of Charles Havens and Anna de Peyster (Livingston) Hunt. His father was an eminent lawyer in New York City; author of several books, including a biography of his wife's cousin, Chancellor Livingston; a member of the Century Club; and the son and grandson of physicians of Welsh ancestry, the Hunts having come to New York in Colonial days.

Through his mother, Mr. Hunt was the direct descendant of Robert Livingston, first Lord of the Manor of Livingston, and of Peter Van Brugh Livingston, Provincial Governor of New York (1775), as well as of Abraham de Peyster and Johannes de Peyster, early Knickerbocker Mayors of New Amsterdam. This side of his ancestry included eminent Huguenot, Scotch, and Dutch families, such as the Beekmans, de Peysters, Schuylers, and Van Dams, and others who founded New Amsterdam (later New York City).

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[1] Memoir prepared by George H. Pegram, Past-President and Hon. M. Am. Soc. C. E., *Chairman,* Lincoln Bush and Charles D. Marx, Past-Presidents, and Hon. Members, Am. Soc. C. E., George S. Davison, Past-President, Am. Soc. C. E., and Frederick C. Noble, M. Am. Soc. C. E. The Committee has made large use of a solicited letter from Mrs. Hunt, and the article in August, 1932, *Civil Engineering,* prepared by the Headquarters Staff of the Society.

With his brother, Livingston Hunt, he attended private schools in New York City and, later, Dr. Ledoux's Boarding School, in Cornwall-on-Hudson, N. Y. From there, at the age of fourteen, he entered New York University, from which he was graduated at eighteen with the degree of Bachelor of Science in Civil Engineering, the youngest member and the Valedictorian of the Class of 1876. In 1907, his Alma Mater conferred on him the Honorary Degree of Doctor of Laws.

Immediately after his graduation from college, Mr. Hunt entered upon engineering practice, getting experience in several lines—Government service on river and harbor work, municipal employ in the Brooklyn Parks and New York Dock Department, water-works engineering with the New Rochelle, N. Y., Water-Works, and elevated railroad construction of the Suburban Rapid Transit Company, later absorbed by the Manhattan Railway Company. For a number of years he was connected with the engineering offices of the late Charles B. Brush, M. Am. Soc. C. E., and J. James R. Croes, Past-President, Am. Soc. C. E., both nationally known members of the Society. He thus early became familiar with the organization and its ideals.

This path led to his appointment in 1892, as Assistant Secretary and Librarian of the Society under the late Francis Collingwood, M. Am. Soc. C. E., then Secretary. When Mr. Collingwood resigned in 1895, Mr. Hunt was promoted to the Secretaryship. Thus, at the age of thirty-seven, he became the executive head of the oldest National Engineering Society in America. He held this position until 1920, when he resigned on account of failing health, and became Secretary Emeritus.

The membership increased from 1 900 to 9 400 during the twenty-five years of his term as Secretary, which might be called the formative period of the Society. A master hand was needed, and, fortunately, it was supplied in Dr. Hunt. He did more to develop and sustain the practices, ideals, and traditions of the American Society of Civil Engineers than any other single influence. It was his lifework and he made it a great achievement.

No member who came into contact with Society Headquarters during his administration could fail to realize its high efficiency. Dr. Hunt was a stickler for form, and yet he inoculated every member of the staff with a vigorous sense of pride in the Society. No better evidence can be adduced than the continued enthusiastic service of that same staff.

It was not in the size of the Society only that Dr. Hunt's efforts were reflected. Its resources and influence among engineers also mounted. When he began his work the organization had headquarters in a building on West Twenty-third Street, which although small was ample at that time. Before long, these offices seemed crowded, and more commodious quarters were needed to meet the requirements of the growing organization.

Foreseeing the northward growth of the city, Dr. Hunt advised the purchase of the property on West 57th Street. This acquisition was made, and a Society House was built and occupied with fitting ceremony on November 24, 1897. Later, it was expanded by the purchase of an additional lot on which the building was extended. This was completed in 1906 and first used for the

Annual Meeting in January, 1906. This house sufficed for twenty years, or until 1917, when the Society joined with the other Founder Societies in its present quarters.

All these advances were made during Dr. Hunt's incumbency. How well his judgment has been vindicated and his planning borne fruit has been emphasized in recent years, for the 57th Street property has been yielding a handsome return on the Society's investment.

Among the means emphasized by Dr. Hunt for increasing interest in Society activities was the holding of technical meetings. Semi-monthly gatherings at the Society House were the rule, except during the summer. These meetings are still well remembered, especially by the older members. He also believed strongly in Society meetings held elsewhere, particularly the Annual Convention. Dr. Hunt was fond of recalling some of the most picturesque of these, such as the International Engineering Congress sponsored by the Society in connection with its Convention held at the St. Louis, Mo., Exposition in 1904; the celebrated Convention held in the City of Mexico, in 1907; and the International Engineering Congress at the Panama-Pacific Exposition in San Francisco, Calif., in 1915.

In all these meetings and conventions, the technical phases of engineering held the most prominent place. With the rapid expansion of the Society, however, not only in point of members, but in geographical distribution, it became increasingly difficult to serve a sufficient number through the meetings alone. There remained then the medium of Society publications and, accordingly, these were enlarged in size and scope under Dr. Hunt's direction. They became the largest single phase of Society endeavor. Both *Proceedings* and *Transactions* were edited under the direction of Dr. Hunt, and gained an enviable place in engineering literature so that through them the Society became known and respected throughout the world.

This work was peculiarly the outgrowth of Dr. Hunt's definite policy. A number of the papers he himself edited. In so far as the Annual Conventions were concerned, he was largely responsible for the programs. As an outgrowth of the St. Louis Meeting, in which Dr. Hunt took an especial pride, the Society was privileged to publish a noteworthy set of papers, included in six volumes of *Transactions*. This general phase of his work as Secretary is attested by an imposing array of engineering literature—Volumes XXVI to LXXXIII, inclusive, of *Transactions*—published under his direction.

He visualized the Society library as a living and useful tool. It had to be up to date, efficient, and always available. Originally, he was opposed to the thought of moving into the present Headquarters, but, later, he felt that one of the strongest incentives to do so was the opportunity of merging facilities to create the most important technical library in America.

His interest and pride in the Society is further attested by his writings on its history. In 1897, his "Historical Sketch of the American Society of Civil Engineers: 1852-1897" was published and sold in book form, and the proceeds were applied to the 57th Street Property Fund. At the Annual Convention, in Washington, D. C., on May 20, 1902, he delivered an address outlining briefly "The First Fifty Years of the American Society of Civil

Engineers: 1852-1902."[2] In 1918, he brought these historic records up to date in his paper on "The Activities of the American Society of Civil Engineers During the Past Twenty-Five Years."[3]

Replying to an inquiry from the Committee charged with collecting material for this memoir, Mrs. Hunt wrote:

> "It has been both interesting and gratifying to me to read over since his death, many, many letters from engineers in the United States and many other countries, written to him in appreciation of his help, encouragement and co-operation in their various undertakings and investigations while he was Secretary. I have also found many letters of regret written to him when he resigned.
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> 'Dr. Hunt was very fond of, and good at, playing all kinds of games. He was a good figure skater, an ardent base-ball fan, golfer, and motorist. For years he was a member of St. Andrews Golf Club, and during the past ten years he and I motored across the Continent three times, and he drove all the way himself. For many years he used to enjoy the camping and fishing jaunts which he and Mr. Alfred Noble took to Lake Superior.
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> "He had an unusually fine ear for music, a fine tenor voice, and played the piano by ear. You know what a wonderful memory he had and how he could fit a joke or anecdote to any occasion. He was a wonderful friend. When we were married in 1883, he was so thin and delicate looking that few thought he would live to any age, but had he lived until September, 1933, we would have celebrated our fiftieth anniversary."

Dr. Hunt was particularly fitted by heredity, education, experience, and training for his life work. He had a fine presence, a keen intellect, and a sterling character. His judgment was unusually sound and he supported it with pointed, forceful argument.

He could draft the most polished letter, or, if occasion demanded it, the most withering criticism. As a raconteur he was at his best. His comments were pithy, his appreciation of the foibles of humanity was keen, and his wit illuminated every narrative. Furthermore, he was possessed of a spirit of sportsmanship and camaraderie.

In all associations of men the ideals of fellowship and the promotion of of friendship are of the greatest value, and Dr. Hunt performed a great service in stressing play as a feature of Conventions and Annual Meetings of the Society to these ends.

A reformer, such as Dr. Hunt, must be somewhat of a dictator. It was unfortunate that what were at times thought to be unnecessarily dictatorial actions, developed vigorous antagonism. With his conscientious desire for the greatest good of the Society this was inevitable, and, in some cases, might have been considered inexcusable, had he not found fast friends in those who felt him to be motivated solely by the interest and prestige of the Society.

Another of those who were asked to contribute to this memoir, Onward Bates, Past-President and Hon. M. Am. Soc. C. E., wrote:

> "I hope justice will be done to Dr. Hunt's memory, for I believe no man has contributed more to our great Society than he has. He may have been thought annoying at times, but just think how much annoyance he was sub-

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[2] *Transactions*, Am. Soc. C. E., Vol. XLVIII (August, 1902), p. 220.

[3] *Loc. cit.*, Vol. LXXXII (1918), p. 1577.

jected to! Part of his job as Secretary was to entertain all the visitors to New York and to see that they had a good time and got all the Society could give them. I remember the annoyance I gave him in the years when I was a member of the Board of Direction, and now wonder how he could stand it. I recall a particular occasion when I was annoyed at him (my fault as well as his), but I did not let the sun go down on my wrath because I knew him so well. At this time I am glad to record myself both as his friend and also his admirer."

Dr. Hunt was very tenacious of his opinions. Of these opinions probably the most criticized was opposition to the Society moving from its own home in West 57th Street and joining with the other three Founder Societies in the Engineering Societies Building on West 39th Street; but this attitude was not unnatural in view of the fact that the American Society of Civil Engineers was originally formed to include all branches of Engineering and none of the other Societies was organized until years after it had become established.

It was assumed that the standards of membership in newer societies would be lowered on account of their desire for increase. As one is known by the company he keeps, Dr. Hunt felt that the prestige of the Society might be injured in the union of all societies in one house. At the later date at which this union took place, the standards of the other societies had risen.

Dr. Hunt was memorialized by a resolution drafted and presented by Vice-President Arthur S. Tuttle and adopted by the Board of Direction of the Society, on October 3, 1932, which, after a general review of his history, concludes as follows:

> "His services to the Society may, therefore, be considered as extending over a period of forty years, or one-half of the Society's life. From the beginning of this connection it was recognized that Dr. Hunt possessed an unusually forceful character, an excellent technical background, broad learning, a genius for finance, a keen sense of the obligation of membership and an unswerving devotion to the interests of the Society. It was recognized by those who knew him that the interests of the Society, as he saw them, were placed above every consideration of personal advantage or friendship, and that at all times and in all places he was ready to defend the Society against any proposal, which, in his judgment, threatened a loss of dignity or prestige.
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> "It was these qualities in its chief executive officer which were responsible for raising the standard of the American Society of Civil Engineers to its present high plane and for making its badge of membership a recognized evidence of qualification as a practicing engineer.
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> "The Board of Direction places this minute in its records as a tribute to Charles Warren Hunt and his long and conspicuously valuable service in the upbuilding of the Society."

In 1922, Dr. Hunt was made an Honorary Member of the Koninklijk Instituut van Ingenieurs in Holland. He was a member of the Engineers Club of New York for forty years. He also belonged to the Delta Phi Fraternity and was a member of the order of the Colonial Lords of Manors of America. He was on the first Committee of One Hundred to suggest eminent people for recognition in the Hall of Fame of New York University and was privileged to unveil the bust of Robert Fulton, with whom and with whose work, Dr. Hunt's ancestors were prominently identified.

He is survived by his widow, Mrs. Mary (Riggs) Hunt, a daughter, Alice Riggs Hunt, who resides with her mother in Hawthorne, N. Y., and by a son, Charles Warren Hunt, Jr., of San Francisco, Calif.

Dr. Hunt was elected a Member of the American Society of Civil Engineers on January 2, 1884.

BENJAMIN OLIVER JOHNSON, M. Am. Soc. C. E.[1]

Died June 27, 1932

Benjamin Oliver Johnson was born on May 25, 1878, in Winchester, Mass., the son of Oliver and Hannah Johnson. He prepared for college at the Worcester, Mass., High School, from which he was graduated in 1896. In the fall of that year he entered the Worcester Polytechnic Institute, where he completed a four-year course and received the degree of Bachelor of Science in Civil Engineering in the Class of 1900. He specialized in Sanitary and Water Supply Engineering.

Immediately after his graduation, on July 16, 1900, Mr. Johnson entered the service of the Northern Pacific Railway Company. His first railroad work was that of a track laborer. In December, 1900, he entered the Engineering Department as a Rodman at Dickinson, N. Dak., and after serving as a Levelman at Miles City, Mont., he was appointed Resident Engineer on maintenance of way with headquarters at Glendive, Mont. On November 18, 1902, he was made Roadmaster at Forsyth, Mont.

Mr. Johnson then left the service of the Northern Pacific Railway Company and, on July 22, 1904, was appointed Roadmaster on the Atchison, Topeka and Santa Fé Railroad, at Chillicothe, Mo. The following year, he returned to the Northern Pacific Railway Company and on April 10, 1905, was made Trainmaster at Dickinson, N. Dak., and, later, Work Train Trainmaster on an extensive grade and line revision at Missoula, Mont. On July 19, 1908, he was appointed Division Superintendent at Glendive. On August 10, 1911, he was transferred to Fargo, N. Dak., and on June 4, 1912, to Livingston, Mont. While in the latter position he also served as Manager of the Gilmore and Pittsburgh Railroad, a subsidiary of the Northern Pacific. His service was again broken by a leave of absence to enter service in the World War with the Russian Railway Corps. This leave was terminated December 20, 1920, when Mr. Johnson resigned from the Northern Pacific Railway Company to continue service in Russia beyond the war period.

On May 1, 1923, Mr. Johnson again entered the service of the Northern Pacific Railway Company as Assistant to the Operating Vice-President. On

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[1] Memoir prepared by W. L. Darling and M. F. Clements, Members, Am. Soc. C. E.

July 16, 1927, he was made Assistant to the President, where he served two years and was again transferred to the Vice-President's Office. On December 1, 1931, he was placed on the retired list, with a pension allowance, on account of ill health. He went to Phoenix, Ariz., for a short period, but returned to St. Paul, Minn., early in June and died in the Northern Pacific Hospital at St. Paul on June 27, 1932.

In October, 1917, Mr. Johnson enlisted as a Major in the Russian Railway Corps and, later, was promoted to the ranks of Lieutenant Colonel and Colonel, and, in the absence of the President, served as President *pro tem* of the Interallied Technical Board, which acted as advisory to the operating organization of the Trans-Siberian Railway. He received an honorable discharge from the Army in January, 1923. He was decorated with the French Legion of Honor, Chinese Order of Merit, Japanese Order of the Rising Sun, and Cjecho Slavals War Cross, being the only American to receive the latter decoration.

Mr. Johnson's service as a railroad engineer and operating officer was highly satisfactory. He handled many problems of railroad operation and maintenance successfully. He had one ambition about which all other interests centered—success in his chosen line of work—and to accomplish that end he recognized the necessity of a broad outlook on life. He had a pleasing personality and made many friends. He was a good conversationalist and enthusiastic in regard to the work in which he found interest. He was a born diplomat and always worked for harmony. He was a man of initiative, quick judgment, and had the ability to handle and direct men.

While in Russia, Mr. Johnson mastered a working knowledge of the Russian language and became intensely interested in the history, manners, and customs of the Russian people. He never lost an opportunity to impress upon others his viewpoint of the substantial qualities of this interesting nation. He was a good story teller and his unusual experiences in Russia were very entertaining. He was a lover of sports, especially tennis and golf.

On February 5, 1903, Mr. Johnson was married to Isabelle M. Hastings, at Worcester, Mass., who, with two daughters, Mrs. Jerome Locke, of New York, N. Y., and Mrs. George Roundtree, Jr., of Wilmington, N. C., survives him. He is also survived by his father, Mr. Oliver Johnson, and two sisters, Mrs. W. E. Stewart and Mrs. Lawrence McConnville, of Los Angeles, Calif.

He was a member of the Board of Trustees of Unity Church in St. Paul, and President of the Minnesota Unitarian Conference in 1931. He was a member of the Masonic Fraternity and Shrine, the American Legion, and a life member of the Rotary Club of Livingston, Mont. In St. Paul, he was a member of the Informal Club, the Interprofessional Men's Institute, Foreign Policy Association, Minnesota Club, St. Paul Athletic Club, and Midland Hills Golf Club. He was also a member of the American Railway Engineering Association and the Minnesota Federation of Architectural and Engineering Societies.

Mr. Johnson was elected a Member of the American Society of Civil Engineers on July 9, 1923.

WARREN BYRON KEIM, M. Am. Soc. C. E.[1]

DIED JUNE 2, 1932

Warren Byron Keim was born in Reading, Pa., on September 6, 1872, the son of Henry and Amanda Elizabeth (Bast) Keim. He was graduated from the Reading High School in 1889, and at once began an engineering career by engaging in municipal engineering, railroad surveying, water supply, and hydro-electric work. He completed his professional training by graduation in 1895 from Lehigh University, at Bethlehem, Pa., with the degree of Civil Engineer.

Early in his career Mr. Keim devoted two years, 1897-99, as Instructor in the Civil Engineering Department of Lehigh University. His thirty-seven years of sincere service to engineering was in association with William H. Dechant, Civil Engineer, Reading, Pa.; New Jersey Steel and Iron Company, Trenton, N. J.; Nathaniel Roberts, Consulting Engineer, Jersey City, N. J.; Clinton and Russel, Architects, and Milliken Brothers, New York, N. Y.; Phœnix Bridge Company, Phœnixville, Pa.; The Pennsylvania Steel Company, Steelton, Pa.; Bethlehem Steel Bridge Corporation, and The McClintic-Marshall Corporation, Bethlehem, Pa.

Mr. Keim's devotion to his profession, his comprehension of engineering details in the field of steel bridges and buildings and the erection thereof, and his constant accuracy in plan and computation, provided a continuous catalog of intensely interesting employment, completed projects, and contacts with noted engineers. Of the more important and enduring structures to which he contributed of his conscientious skill as Draftsman, Checker, Squad Chief, Designer, or Assistant Engineer may be mentioned (in chronological order) the following: Park Row Office Building (33 stories), and Jewelers Building, New York, N. Y.; Commonwealth Building, Allentown, Pa.; Susquehanna River Bridge, Pittston, Pa.; Bridge Shops for Pennsylvania Steel Company, Steelton, Pa.; Queensboro Bridge, New York, N. Y.; Canarsie Elevated, Brooklyn, N. Y.; Baltimore and Ohio Monongahela River Bridge, Fairmont, W. Va.; Central Railroad of New Jersey Ferry House, at 23d Street, New York, N. Y.; Open-Hearth Buildings for Pennsylvania Steel Company, Steelton, Pa.; Raleigh and Pamlico Sound Railroad Bridges, Raleigh, N. C.; Ditmars Avenue Station, for Public Service Commission, Astoria, Long Island; Harahan Bridge over the Mississippi, Memphis, Tenn.; Ohio River Bridge for Pennsylvania Lines West of Pittsburgh, Louisville, Ky.; Dirigible Hangar, for United States Navy Department, Lakehurst, N. J.; Coaldale Breaker, Lansford, Pa.; Hill-to-Hill Bridge, Bethlehem, Pa.; Citizens Bank Building, Baltimore, Md.; Delaware River Bridge Towers, Philadelphia, Pa., to Camden, N. J.; Packard Building, Philadelphia, Pa.; Central Railroad of New Jersey, Newark Bay Lift-Bridges, Elizabeth, N. J.; Peace Bridge, over Niagara River, Buffalo, N. Y., to Canada; Goethals Bridge, over Kill van

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[1] Memoir prepared by Meier George Hilpert, M. Am. Soc. C. E.

Kull, Elizabeth, N. J., to Staten Island, New York; and George Washington Bridge Towers, Fort Lee, N. J., to Fort Washington, New York, N. Y.

In private life Mr. Keim exemplified the true Christian and Churchman and dedicated his tithe, his every Sabbath, and his conscientious and earnest service to the Methodist Episcopal Church and Church School wherever he resided. His counsel was much sought and his churchly preferments many. His recent associations were with Wesley Methodist Episcopal Church, at Bethlehem, Pa., until April, 1931, as President of the Board of Trustees, Superintendent of the Intermediate-Senior Department, and member of the Building Committee for the Church School Building dedicated in 1932; and at the time of his death with Grace Methodist Episcopal Church, at Harrisburg, Pa., as member of the Board of Stewards, Assistant Secretary of the Official Church Board, and member of the Membership Committee.

At the time of his death, Mr. Keim held a Life Membership in Harrisburg Lodge No. 629, Free and Accepted Masons, in which he was initiated on May 23, 1905. He served as Worshipful Master during 1911. No Craftsman of this ancient Fraternity could have held its principles in higher reverence, or enjoyed greater respect from those who knew him.

Mr. Keim rendered valuable community service through his basic advice, especially on engineering matters, and through his membership in Civic Associations, Planning Commissions, and Parent-Teacher Associations, as a resident from 1910 to 1923, at Camp Hill, Pa.; and from 1923 to 1931, at Bethlehem, Pa. He always gave freely of his time and experience by contributing to the Engineering Profession proof-readings, briefs, and chapters to engineering texts.

Mr. Keim was a member of the Tau Beta Pi Fraternity, the American Association for the Advancement of Science, and the American Association of Engineers. He was a registered Professional Engineer in the Commonwealth of Pennsylvania.

On June 30, 1910, he was married to Bertha Guiles Young, of Harrisburg, who, with their son, David Young Keim, survives him.

Mr. Keim was elected an Associate Member of the American Society of Civil Engineers on January 4, 1905, and a Member on April 26, 1921.

JEREMIAH JOSEPH KENNEDY, M. Am. Soc. C. E.[1]

Died February 15, 1932

Jeremiah Joseph Kennedy was born in Philadelphia, Pa., on October 22, 1864. As a child, he was quite delicate and was unable to attend school after the age of ten years. This condition, however, did not prevent him from studying at home and, being ambitious, he embraced every oportunity to learn. He was particularly fond of mathematics and borrowed from the Apprentices' Library, in Philadelphia, all the books contained therein relating to this subject. Later in life, he was instrumental in helping poor boys further their education, through this same Library.

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[1] Memoir prepared by Robert H. Hammer, Esq., Brooklyn, N. Y.

Mr. Kennedy had a wide and varied career in the Engineering Profession. In January, 1882, he started as a Rodman with the Norfolk and Western Railroad Company. After a year he became Principal Assistant to Russell Thayer, M. Am. Soc. C. E., and was engaged in miscellaneous engineering activities. In 1884 and 1885, as Engineer for the Grubb Iron Works, Blue Ridge, Va., Mr. Kennedy constructed railway extensions, operated mines, and planned and constructed improvements for the old Lynchburg, Va., Iron Company.

Then followed varied engineering work with the Henry Disston and Sons Steel and Saw Works, Morris Tasker and Company, etc. From 1892 to 1894, he was engaged in designing and constructing gas-works, gas-holders, retorts, and accessories, and, in this period, he owned and operated the National Iron Works, at Linwood, Pa.

In May, 1894, Mr. Kennedy became Chief Engineer of White, Crosby, and Company, of Baltimore, Md., which Company later became the J. G. White and Company, of New York, N. Y. He planned, constructed, and operated, among the many engineering projects of this Company under his supervision, the Atlantic Highlands, Red Bank and Long Branch Electric Railway.

In February, 1901, Mr. Kennedy severed his connection with J. G. White and Company, to engage in private practice as a Consulting Engineer, in which he was active until his death.

He designed and constructed for the Whitney Reduction Company, in Rowan and Stanly Counties, North Carolina, a masonry dam, 38 ft high, together with a 4½-mile hydraulic canal to develop 20 000 hp. He designed and constructed hydro-electric developments for the Schuylkill Valley Illuminating Company, at Phœnixville, Pa.; for the Lynchburg Traction and Light Company, at Lynchburg, Va.; and for the American Gas Company, at Winooski and Essex Junction, Vt. He also planned and constructed improvements in the power plants of the Detroit and Lake Orion Railway, at Rochester, Mich., and for the Cortland County Traction Company, at Cortland and Homer, N. Y. For the Baldt Steel Company, he designed and constructed the buildings, furnaces, and equipment for its steel plant at New Castle, Del.

In this period from 1901 to 1932, Mr. Kennedy was also engaged in consultations, investigations, and re-organizations of many engineering projects in various parts of the country.

From 1904 to 1907, he was in charge of all operations and extensions to the underground pneumatic mail-tube systems in New York, Chicago, Ill., St. Louis, Mo., and Boston, Mass.

In 1908, Mr. Kennedy became financially interested in the Biograph Company, the pioneer company, with the Edison Company, in the motion picture industry. With the late Thomas A. Edison, he acquired and controlled the patents relating to the business, and, for many years, both Mr. Kennedy and Mr. Edison guided the motion picture industry through its early difficulties and developments.

Mr. Kennedy's ability as an organizer, his intimate and practical knowledge of engineering, his personal integrity, and his industrious and studious habits attracted men of national standing to him, and made him successful in the Engineering Profession.

Closely in touch with all engineering activities and technical advancements, Mr. Kennedy was a member of the American Society of Mechanical Engineers, the American Institute of Electrical Engineers, the American Institute of Mining and Metallurgical Engineers, the New York Electrical Society, the American Society for Testing Materials, the American Association for the Advancement of Science, the Illuminating Engineering Society, and the Franklin Institute. He had been a resident of New York City since 1896, and a member of the Lotos Club, the Engineers Club, and the Bankers Club of America.

Mr. Kennedy was elected a Member of the American Society of Civil Engineers, on June 4, 1902.

MORRIS KNOWLES, M. Am. Soc. C. E.[1]

DIED NOVEMBER 8, 1932

Morris Knowles, the son of Charles E. and Ellen B. (Richardson) Knowles, was born in Lawrence, Mass., on October 13, 1869. As a young man, he was a neighbor of the late Hiram F. Mills, Hon. M. Am. Soc. C. E., who at that time was conducting the historic studies at the Lawrence Experiment Station, in the field of water and sewage treatment. In part, at least, as a result of his contacts with Mr. Mills, Mr. Knowles elected to study engineering at the Massachusetts Institute of Technology, from which he was graduated in 1891, with the degree of Bachelor of Science.

From his early acquaintance and friendship with Mr. Mills and from his up-bringing in the atmosphere of sanitary engineering that prevailed in Lawrence at that time, Mr. Knowles continued in the practice of his profession under men who then or later became outstanding leaders in that field. During his student days at the Massachusetts Institute of Technology, he served an apprenticeship under the late Richard A. Hale, M. Am. Soc. C. E., then Principal Assistant Engineer, and, later, Chief Engineer, of the Essex Water Power Company, of Lawrence. During his vacations in 1890 and 1891, he served as Surveyor in the Plan Department of the Associated Factory Mutual Insurance Companies.

Upon graduation Mr. Knowles became Assistant Engineer for the East Jersey Water Company, under the late Clemens Herschel, Past-President and Hon. M. Am. Soc. C. E., Chief Engineer, and J. Waldo Smith, Hon. M. Am. Soc. C. E., Principal Assistant Engineer. From September, 1893, to May, 1897, he was Assistant Engineer with the Massachusetts State Board of Health, under the late Frederick P. Stearns, Past-President, Am. Soc. C. E.

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[1] Memoir prepared by George S. Davison, Past-President, Am. Soc. C. E., and Nathan B. Jacobs, M. Am. Soc. C. E.

Mr. Knowles served as Personal Assistant to Mr. Stearns in the latter's investigation of a water supply for Boston, Mass., and vicinity, leading to the formation of the Massachusetts Metropolitan Water Board, and the development of the Wachusett Reservoir and Aqueduct to supply Boston and twenty-eight surrounding cities and towns, with water.

At the same time, Mr. Knowles was a member of the Water Board of Lawrence, his home town. This marked the beginning of an intimate contact with its municipal water supply, which he always maintained, returning there many years later to build additions to the slow sand filter plant and to report on new sources of supply.

In 1897, Mr. Knowles became Resident Engineer of the Filtration Commission of Pittsburgh, Pa., and established a laboratory and experimental filters looking to the improvement of the city water supply. Following the completion of the report of the Pittsburgh Filtration Commission, and prior to the actual construction of the filters, Mr. Knowles went to Philadelphia, Pa., in 1899, to perform a similar service for that city in experimental work on filters and an investigation of a proposed new gravity water supply. In 1900, Mr. Knowles became Assistant Engineer under the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., and assisted the latter in the preparation of his classic report on additional water supply for the City of New York. From June, 1900, to July, 1901, he was Assistant Engineer in charge of the Spring Garden and Torresdale Testing Stations in connection with the improvement and filtration of the water supply of Philadelphia.

In August, 1901, Mr. Knowles returned to Pittsburgh to become Chief Engineer of the Pittsburgh Bureau of Filtration and to design and supervise the construction of the slow sand filters for the city's water supply on the Allegheny River, at Aspinwall, Pa.

Upon the completion of this filtration plant, Mr. Knowles devoted himself actively to the consulting engineering practice which he had formed in partnership with the late L. E. Chapin, M. Am. Soc. C. E., in Pittsburgh, a few years before. This continued, first as a partnership with Mr. Chapin, and, later, as a corporation under his own name until his death on November 8, 1932.

In private practice, Mr. Knowles designed an impounding dam for the development of an industrial water supply for the Tennessee Coal, Iron, and Railroad Company, on Village Creek in the Birmingham, Ala., District. He was a member of the Engineering Committee of the Flood Commission of Pittsburgh, which compiled an exhaustive report upon flood protection and prevention. Following the completion of the report in 1911, Mr. Knowles continued as a member of the Executive Committee of the Flood Commission. He was a member of the Board of Advisory Engineers of the Miami Conservancy District, in 1914, and Chief Engineer of the Essex Border Utilities Commission, Ontario, Canada, from 1916 to 1921.

During the World War, Mr. Knowles was Supervising Engineer in charge of the construction of Camp Meade and Camp McClellan, and, in 1918 and 1919, he was Chief Engineer of the Housing Division of the United States Shipping Board. He was a member of the Engineering Board of Review of the Chicago Sanitary District in 1924, and served this Board as Chairman of its Committee on Construction Program.

Mr. Knowles early sensed the impending participation of the Civil Engineer in the field of city planning and zoning, and, as a consequence of his pioneering activities in this direction, he became a member of the Advisory Committee on Zoning appointed by Herbert Hoover, Secretary of Commerce. Mr. Knowles served as Chairman of the City Planning Commission of Pittsburgh from 1922 to 1929, and as Chairman of the Zoning Board of Appeals of Pittsburgh from 1923 to 1927.

He designed and supervised the construction of sewage disposal and harbor improvements for Lynn, Mass., costing $2 500 000. He was Consulting Engineer to the Attorney General of Connecticut, in conducting comprehensive investigations in support of the latter's unsuccessful suit to restrain Massachusetts from the diversion of interstate waters in the extension of the Boston Metropolitan District water supply.

Mr. Knowles was Director of the Department of Sanitary Engineering, University of Pittsburgh, from 1911 to 1920, and, in 1929, was awarded the honorary degree of Doctor of Engineering by the University. He was Vice-Chairman of the Executive Committee of the Commission appointed by the Governor of Pennsylvania to study municipal consolidation in counties of the second class in Pennsylvania, and drew up a proposed charter for a federated city to amalgamate more than one hundred minor political subdivisions in Allegheny County into a Greater Pittsburgh. In 1931, Mr. Knowles was Chairman of the Committee on Utilities in President Hoover's Conference on Home Building and Home Ownership. He was a term member of the Corporation of the Massachusetts Institute of Technology from 1924 to 1929, and a Director of the Pittsburgh Chamber of Commerce from 1922 to 1930. He was a former Director of the National Conference on City Planning, and a member of the Institution of Civil Engineers (Great Britain), the Verein Deutscher Ingenieure (Germany), the Engineering Institute of Canada, the American Society of Mechanical Engineers, the Engineer's Society of Western Pennsylvania (President in 1923), the Franklin Institute, American Water Works Association, the New England Water Works Association, the Pennsylvania Water Works Association, the American Public Health Association, the American Academy of Political and Social Science, the National Institute of Social Science, the Boston Society of Civil Engineers, the National Municipal League, and the International Housing and Town Planning Association. He was also a member of the Sons of the American Revolution and of the Order of Founders and Patriots of America.

He was a member of the Duquesne Club and the University Club, Pittsburgh; the City Club and Engineers' Club, New York, N. Y.; the Cosmos Club, Washington, D. C.; the City Club, Boston, Mass.; and the University Club, Philadelphia. He was also a member of the Long Vue Country Club of Pittsburgh, and the Merrimac Valley Club of Lawrence, Mass. He was the author of "Industrial Housing" (1920), and many professional papers and contributions to the technical and scientific press.

Throughout his life Mr. Knowles was notable as one of those citizens who not only have the ability but also the willingness to be useful in civic affairs, or in any movement of an altruistic character. Seldom did he decline to enter wholeheartedly and actively into such a project, provided only he was made aware of its civic value.

It was not strange, therefore, that he was called on repeatedly to serve on various commissions and boards. While some of these were professional assignments, many were simply honorary in character and, therefore, a tribute to his great ability. Typical of these was his work as one of the Society's Representatives on an Advistory Committee passing on the ideal development of the City of Washington, D. C.; and on another committee also associated with Government work, having to do with advising on the Mississippi River flood-control projects, proposed within the Congress as a sequence of the 1927 flood.

During his busy life he found time to contribute his professional views to the public in various articles and papers. Many of these were presented to the Society and even during his latter years he was generous in this particular. He also gave freely of his time on organization procedure and endeavors. This was evidenced not only by his faithful service on the Publications Committee of the Society while he was a Director, but also by the frequent occasions on which he was consulted by many civic units on matters within his own technical purview.

Speaking on his feet, Mr. Knowles was at his best. His ideas were logical and the appropriate words came easily. His manner itself was no less convincing than his thoughts. In Board meetings, or in public assemblies, he had admirable presence and the happy faculty of transmitting his ideas pleasantly yet most effectively.

Perhaps it was due to his studied efforts as well as to his personal inclination to exert himself in worthy causes that made and kept for Mr. Knowles a wide circle of friends. In addition, his remarkable sense of diagnosis and his innate diplomacy made him marked as a leader among engineers.

He is survived by his widow, Mina (McDavitt) Knowles, one son, Morris, Jr., of Pittsburgh, and a daughter, Mrs. H. N. Schreuder, of New York City.

Mr. Knowles was elected a Junior of the American Society of Civil Engineers on October 4, 1892; an Associate Member on January 1, 1896; and a Member on April 1, 1903. He served as a Director of the Society from 1928 to 1930.

THOMAS MONAGHAN LAVELLE, M. Am. Soc. C. E.[1]

DIED MARCH 6, 1932

Thomas Monaghan Lavelle, the son of Anthony and Mary (Monaghan) Lavelle was born on July 10, 1866, at Uniontown, W. Va. He attended the public and Normal Schools of West Virginia until about 1890, when he entered the University of West Virginia, from which he was graduated in June, 1895, with the degree of Bachelor of Science in Civil Engineering. In the interim, he spent one year in teaching.

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[1] Memoir prepared from information on file at the Headquarters of the Society.

After leaving the University, Mr. Lavelle received his first position, from August to November, 1895, as Leveler for the Washington, Alexander and Mt. Vernon Railway Company. This work was on the construction of an extension of line from Alexandria, Va., to the end of Aqueduct Bridge opposite Georgetown, D. C. He wrote the descriptions for right-of-way deeds for all properties.

From November, 1895, to July, 1901, he was with the Pencoyd Iron Works, at Pencoyd, Pa., as follows: From November, 1895, to February 1896, as Assistant Locating Engineer; from February 1896, to the summer of 1899, as Locating Engineer; from the summer of 1899, to July, 1901, as Superintendent of Yards and Buildings; and from February, 1896, to July, 1901, he was in charge of construction, except mechanical installations, for which he had charge of foundations only. During this time open-hearth furnaces, gas producers, boiler, power, and cupola plants, and buildings for these installations were constructed by the Pencoyd Company. There was also various miscellaneous work: Track systems were revised and extended and almost entirely relocated and rebuilt; a railroad bridge was built across the Schuylkill River, to the East Side Plant which included a large machine shop and foundry plant with heavy retaining walls, intake wells from the river, track systems, etc.

From July, 1901, to July, 1908, Mr. Lavelle was employed by the American Bridge Company, as Engineer of Yards and Buildings. In this position, he had general charge of improvements at the Trenton, N. J., and Brooklyn, N. Y., Plants, including boiler and power plants, intake wells for water supply, and all work for complete operation. Mr. Lavelle had full charge of the Ambridge, Pa., Plant of the Company. He formulated methods of construction and designed and built track systems, including classification yards, water supply, drainage, flood protection (on Ohio River) in short, all construction incidental to the completion of a large industrial plant. He also directed the compilation of a large property book of all properties of the American Bridge Company, at Ambridge. The maps were white prints with black lines and were water-colored, different colors being used for the various tracts or parcels of land, and all the deeds were typed. This work required searches of records at the various Courthouses of the County, in regard to the land leased.

In 1909, Mr. Lavelle went to the Belmont Iron Works, at Philadelphia, Pa., in charge of miscellaneous construction work. In the same year he went to Pottsville, Pa., as Chief Engineer of the Eastern Steel Company. In this capacity he had charge of the construction of open-hearth furnaces, with the necessary buildings, power plants, etc., including low-pressure steam turbine installation, utilizing exhaust from rolling mill engines. A condenser plant and a cooling water system were constructed under his supervision, the water supply was studied and extended, the yards were revised, and the track systems rebuilt, etc.

In 1923, he became Supervising Engineer of Construction for The Ballinger Company, Architects and Engineers, of Philadelphia. In this position, he had charge of the construction of "The Board of Temperance Building", opposite the Capitol, in Washington, D. C. He had charge, also, of the construction of a large reinforced concrete warehouse and office building for the Sanitary Grocery Company, in Washington, as well as a twenty-two story apartment building at 22d and Walnut Streets, in Philadelphia. Afterward, Mr. Lavelle had charge of two extensions to the plant of the Atwater Kent Manufacturing Company, in Philadelphia. These extensions covered about five acres. His last work was in charge of the foundations for the Sibley Memorial Hospital, in Washington, D. C. Owing to poor health, he was forced to retire from active work in 1926.

Mr. Lavelle had one ideal always in mind—perfection—and, therefore, his employers always had the greatest confidence in his ability and honesty. Due to this ideal, he gained the respect of all those for whom he worked, as well as those with whom he was associated in his work.

On his retirement, in 1926, he went to Glendale, Calif., where he resided until his death. He is survived by his widow, Bertha S. Lavelle, and two sons, Kenneth and Julian, as well as two sisters, Julia and Anna Lavelle.

Mr. Lavelle was elected an Associate Member of the American Society of Civil Engineers on September 7, 1904, and a Member on June 30, 1911.

SIDNEY FRANCIS LEWIS, M. Am. Soc. C. E.[1]

Died February 8, 1933

Sidney Francis Lewis was born in New Orleans, La., on January 28, 1854. He was the son of Dr. John Hampden Lewis, a noted physician and surgeon prior to and following the war between the States, and Annie E. G. (De Britton) Lewis.

Sidney Francis Lewis received his early education in the public schools of his native city, was a graduate of the Boys' High School, and received his degree as Civil Engineer at the University of Virginia, Charlottesville, Va., in the Class of 1874.

After leaving college he began the practice of his profession and, in 1877, he entered the employ of the Board of State Engineers as a Draftsman and Assistant Engineer, during the administration of Governor Francis T. Nicholls. He was appointed to membership on the Board by Governor Louis A. Wiltz, in 1880, and served continuously until 1908, when he was made Chief Engineer for the New Orleans Levee Board.

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[1] Memoir prepared by a Committee of the Louisiana Section, consisting of A. F. Theard and John T. Eastwood, Members, Am. Soc. C. E., and Charles M. Kerr, Assoc. M. Am. Soc. C. E.

Regarded as one of the foremost civil engineers in the South, Mr. Lewis was chosen to be Chief Engineer of the New Orleans Levee District, in September, 1908, on the recommendation of Governor Jared Y. Sanders. He served in that capacity until September, 1916.

In 1920, he returned to his work on the Board of State Engineers under Governor John M. Parker. Subsequent Governors who appointed him to the Board were the Hon. Samuel D. McEnery, the Hon. Murphy J. Foster, the Hon. William Wright Heard, the Hon. Newton C. Blanchard, and the Hon. Henry L. Fuqua. The last Chief Executive under whom he acted on a public project was Governor O. H. Simpson, who filled the unexpired term of Governor Fuqua.

During his later years, Colonel Lewis had been employed by the Sewerage and Water Board of New Orleans, as Assistant Engineer.

Colonel Lewis was active in the State Militia for twelve years and retired as Colonel of Engineers on the Staff of Governor McEnery. He was also on the staffs of Governors Wiltz and Nicholls.

As a participant in the Young Men's Democratic Association movement in 1880, Colonel Lewis became interested in politics, which interest he retained throughout his life. At one time he was prominently associated with the Choctaw Club, and served as a President of that organization. He was a member of the Sons of the American Revolution, and formerly took part in the affairs of various Carnival groups, the Alumni of the Boys' High School, the Young Men's Gymnastic Club (now the New Orleans Athletic Club), and the Benevolent and Protective Order of Elks.

In 1898, he was one of the Charter Members of the Louisiana Engineering Society and became its first President.

He was twice married, his first wife being the former Clara Davis, who died in 1899. He is survived by his second wife, the former Evelina Nicaud, and nine children, as follows: J. Hampden and Sidney F. Lewis, Jr., Mrs. Lillian Lewis, Mrs. Clara (Lewis) Landry, Mrs. Robert Farrell, Mrs. L. E. Hogan, Mrs. William T. Cates, Mrs. Enid (Lewis) Pierce, and Mrs. Gordon Clay.

Colonel Lewis had many excellent business and professional qualities. Those who knew him always retained a high regard for his clear-sighted analysis of conditions, his high standard of fair dealing, his integrity, and his tactful treatment of all questions submitted to him. He was an Engineer of unquestioned ability—being active and energetic—and he endeared himself to all who knew him.

He loved the South and its people, and worked for them all his life. As a public official, he incurred the good will of his associates and commanded the respect of the public. Through his death his friends and the Engineering Profession have suffered a great loss.

Colonel Lewis was elected a Member of the American Society of Civil Engineers on May 4, 1881. At the time of his death he was the oldest member of the Louisiana Section of the Society.

JOHN WILLIAM LINK, M. Am. Soc. C. E.[1]

Died April 14, 1933

John William Link, the son of Preston J. and Fannie P. (Greenlee) Link, was born in Richmond, Va., on May 28, 1866. As a boy he attended the public schools of Richmond, and was graduated from the High School in 1884. Subsequently, he studied Drafting and Engineering at the Virginia Mechanics' Institute Night School of Technology.

Mr. Link first began work, in 1884, as a Clerk in the employ of the Vulcan Iron Works of Richmond, which position he held for three years. Then, from 1887 to 1890, he served as an Assistant Engineer during the construction of the City Hall at Richmond. The next year, 1891, found him as an Assistant to the late D. C. Humphreys, M. Am. Soc. C. E., on town layouts and municipal engineering in Basic City and Lexington, Va.; and from 1893 to 1896, he served, successively, as Draftsman, Computer, and Surveyor with the Missouri River Commission, with headquarters at St. Louis, Mo. For the next three years—from 1897 to 1900—Mr. Link was employed as an Office Assistant in the Designing Department of the Metropolitan Water Board, Boston, Mass., during which time the Wachusett Aqueduct was under construction, and studies were being made for the Wachusett Dam.

It was in 1901, however, that Mr. Link first began work on hydro-electric developments, in which field, with the exception of two years, he spent the remainder of his life. From 1901 to 1903, he was with the Niagara Falls Power Company, on hydro-electric design. In 1904, he was Principal Assistant Engineer to the late W. A. Brackenridge, M. Am. Soc. C. E., Consulting Engineer, Niagara Falls, N. Y., and was engaged on water power investigations and the preparation of reports, plans, and specifications, included among which were those for the plant of the Great Northern Development Company, at Duluth, Minn.; also, he was engaged on miscellaneous work for the New York State Barge Canal.

From 1905 to 1907, Mr. Link was in Columbus, Ohio, as Assistant Engineer to John H. Gregory, M. Am. Soc. C. E., in charge of the design of various water supply and sewage disposal structures included in the improved public works of that city. In 1908, he returned to the field of hydro-electric developments and for two years was Assistant Engineer to J. W. Rickey, M. Am. Soc. C. E., at Massena, N. Y., engaged on studies for a proposed water power development at Long Sault Rapids, on the St. Lawrence River, and in charge of remodeling the Massena Hydro-Electric Plant of the Aluminum Company of America.

In 1910, Mr. Link became associated with H. M. Byllesby and Company, in Chicago, Ill., as Chief Hydraulic Engineer, with which Company he remained until his death. It was during his twenty-three years of service with this organization and affiliated companies that his engineering talents found their fullest expression. Most notable of his achievements are the Rapidan, Coon Rapids, and Chippewa Falls Developments of the Northern States Power Company; the El Dorado Project of the Western States Gas and Electric Company; the Prospect No. 2 and Prospect No. 3 Developments of the California-Oregon Power Company; and the Albany and Big Fork Power Developments and the Albany Filtration Plant of the Mountain States Power Company. In addition, he directed investigations and examinations of numerous water-power projects totaling many thousands of horse-power capacity.

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[1] Memoir prepared by Stephen Wehner, Hydr. Engr., Byllesby Eng. & Management Corp., Chicago, Ill., assisted by John H. Gregory, M. Am. Soc. C. E., and by Brig.-Gen. Charles Keller, U. S. A. (*Retired*), M. Am. Soc. C. E.

Mr. Link was quiet and unassuming in manner, and somewhat reserved, except with his closest friends. In his uncompromising integrity and fine devotion to the interests of those he served, he was an outstanding example of what an Engineer should be who desires to conform to the highest standards of his profession.

Mr. Link was a member of the Boston Society of Civil Engineers and of the Western Society of Engineers.

In 1915, he was married to Edna M. Ramsay, of La Salle, Ill., who, with one daughter, Ruth, survives him.

Mr. Link was elected a Member of the American Society of Civil Engineers on November 12, 1913.

HERBERT PRESCOTT LINNELL, M. Am. Soc. C. E.[1]

DIED FEBRUARY 25, 1932

Herbert Prescott Linnell, the son of Benjamin F. and Sybel B. (Wheeler) Linnell, was born in Worcester, Mass., on January 8, 1874. He obtained his early education in the public schools of that city and, after graduating from the Worcester High School, entered the Worcester Polytechnic Institute, from which he was graduated in the Class of 1894, with the degree of Bachelor of Science in Civil Engineering.

Mr. Linnell's first work after leaving college was in North Adams, Mass., where he made surveys and plans for the development of properties, the design and layout of sewers, etc.

In 1896, he became Assistant City Engineer of North Adams, which position he held until 1899, when he bought out the private practice of Mr. F. S. Smith, a Civil Engineer in that city. From then until 1904, Mr. Linnell enlarged his engineering practice and acted as Consulting Engineer for most of the towns in Northern Berkshire County, as well as for most of the large corporations in that section.

In 1904, Mr. Linnell was appointed Assistant Resident Engineer at Colon, by the Panama Canal Commission, and spent two years, first, as Assistant, then as Resident, Engineer in the Canal Zone. His work at this time covered the design and construction of water supply systems, reservoirs, pumping stations, pipe lines, sewers, pavements, and general municipal requirements for the City of Colon. His Roosevelt Medal, which was given to all engineers who worked on the Isthmus for two years or more, was No. 123, and his family, which at that time consisted of his wife and two small sons, was the first American Engineer's family to live in Cristobal, where he was the first American School Commissioner.

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[1] Memoir prepared from information on file at the Headquarters of the Society.

Leaving the Isthmus in 1906, Mr. Linnell took the position of Office Engineer with the Atlantic, Gulf, and Pacific Company of New York, N. Y. For this Company he carried on investigations and made estimates and reports on various contracts in the United States, including many miles of the New York State Barge Canal, harbors and coaling stations for the United States Navy, and other projects in Brazil and Cuba.

In 1908, he went to the Philippine Islands as Vice-President and Chief Engineer of the Atlantic, Gulf, and Pacific Company of Manila, and for ten years his work included most of the principal contracts in the Islands. During this time, Mr. Linnell designed and built such works as docks at Manila, Cebu, and other places, water supply and sewers at Manila and Cebu—a $7 000 000 job—piers and quay walls, highway and railroad bridges, deep foundations, dry docks, and many of the engineering projects that were required for the development of the Islands. Much of this work was for the various branches of the United States Government, such as the Army, Navy, Quartermaster Department, and various Bureaus of Public Works. His work in the Philippine Islands totaled more than 600 contracts, costing several millions of dollars, and embraced almost every phase of engineering practice.

In 1918, Mr. Linnell returned to the United States and accepted a position as District Plant Engineer in charge of the Fifth District, with the United States Shipping Board, Emergency Fleet Corporation. In this capacity he had the supervision of sixteen shipyard plants from Pensacola, Fla., to New Orleans, La.

After the World War, in 1919, Mr. Linnell severed his connections with the Emergency Fleet Corporation and moved to Springfield, Mass., where for several years he was engaged in private practice as a Consulting Engineer. During this period, he attempted to enter the manufacturing field and, as President of the Federal Corporation of Westfield, Mass., undertook the manufacture of automobile accessories. However, he found that his interest and activity could best be satisfied only in engineering and, therefore, in 1926, he formed a connection with The Foundation Company of New York. From that time until his death, he carried on investigations for The Foundation Company. His work took him to many places in the United States, as well as to Mexico, Guatemala, Colombia, and Venezuela.

It was on an investigation project in Bogota, Colombia, that his heart first was affected and, because of the high altitude of Bogota, he was forced to return to the United States to recuperate. While in the States, he was retained as an expert on the Chicago Water Diversion controversy which was heard by Justice Charles E. Hughes. However, his work in Colombia was not finished and, in 1930, he went back to South America, returning to the United States in the fall of 1931.

His constitution, which was strong and robust enough to withstand the attacks of malaria and jungle fever in Panama, and many tropical and dangerous illnesses met with in his far wanderings, finally succumbed to a heart attack in New York City, on February 25, 1932, while he was on a business trip. Death was instantaneous. Mr. Linnell was borne to his last resting place in Hope Cemetery, Worcester, on February 29, 1932.

In 1897, Mr. Linnell was married to Josie A. Harrington, of Worcester, Mass. From that time until her death in the Philippine Islands in 1911, she shared his wanderings and fortunes. He is survived by two sons and a daughter: Herbert H. Linnell, of Port Arthur, Tex., Philip W. Linnell, of South Manchester, Conn., and Gladys J. Linnell, of Springfield, Mass. There are also two grandsons and a grand-daughter.

Mr. Linnell was a Thirty-second Degree Mason, Past Master of Greylock Lodge, North Adams, Mass., and a member of the Shrine. He was also a member of the Benevolent and Protective Order of Elks; and belonged to the Engineering Society of Western Massachusetts, and The Philippine Club.

He was familiar with the world and many of its far places. He was known by many, loved by many, and his numerous friends ranged from the lowly Filipino laborers on a construction job in the Far East, to the bank presidents, financiers, and men of affairs in the Western world. His life was one of achievement and the advancement of the Engineering Profession. His best epitaph might be "He was an Engineer."

Mr. Linnell was elected an Associate Member of the American Society of Civil Engineers on May 2, 1906, and a Member on February 4, 1914.

HENRY ROBERTSON LORDLY, M. Am. Soc. C. E.[1]

DIED OCTOBER 4, 1932

Henry Robertson Lordly was born at Rothesay, N. B., Canada, on December 12, 1868, the son of Albert James Lordly, Manufacturer, and Director of the Mechanics' Institute of St. John, N. B., and Martha Louise (Brennan) Lordly, daughter of Maj. Charles Brennan, of Fredericton, N. B. His paternal grandfather came over from England and was a Surgeon on one of the English steamers that carried the Colonial Loyalists to Canada; here, he met his future wife. After their marriage they lived at Chester, Nova Scotia, where Dr. Lordly practiced his profession until his death. Henry Robertson Lordly was prepared for college at the St. John Grammar School and at E. Mannings School, at St. John, N. B. Entering Cornell University, at Ithaca, N. Y., in the fall of 1889, he was duly graduated in 1893 with the degree of Civil Engineer.

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[1] Memoir prepared by Professor Clark S. Northup, Cornell Univ., Ithaca, N. Y.

Mr. Lordly began his professional career in the office of the Bridge Engineer of the Canadian Pacific Railway Company, in Montreal, Que., Canada. From 1895 to 1898, he was General Manager of the Maritime Auer Light Company. In 1897 and 1898, he was also Engineer in Charge of Construction, and Manager, of the Charlottetown, Prince Edward Island, Light, Heat, and Power Company. In 1899 and 1900 he was employed in consulting engineering work in Montreal, Toronto, and Hamilton, Ont.

In September, 1901, Mr. Lordly was made Engineer in Charge of new construction on the Lachine Canal, working under the general supervision of the Department of Railways and Canals of Canada. In the next two years he constructed Locks Nos. 1 and 2, Basin No. 1, underpinned the walls of Basin No. 2, constructed the slope walls of the upper reach, and finished the power house. The year 1903-04 he spent in work on Black's Bridge, in Montreal. In 1905, he built the Lachine Wharf, of reinforced concrete, rebuilt freight sheds, constructed subaqueous walls, paved the wharves, etc. The years from 1906 to 1908 were spent in underpinning the side basins, and the year 1909 was devoted to subaqueous wall construction, the building of waste weirs and a dry dock, etc. In 1909 and 1910, he built the St. Henri Basin, and worked on the bascule bridge, at Côte St. Paul, Que. In the following year, he constructed the Lachine Dam. In 1911-12, he was further engaged on the bascule bridge at Côte St. Paul (154-ft span and abutments). The next year he rebuilt and constructed waste weirs carried on granite paving. In 1913, he was engaged in widening the Lachine Canal, and the following year he was busy with power-house foundations and wasteways. The total value of the work, over the construction of which Mr. Lordly had entire charge, was more than $2 500 000.

Then came the World War. From his early years Mr. Lordly had been interested in military affairs. In 1885, he joined the 62d Fusileers, at St. John, N. B., as a bugler. In 1898, he was gazetted a Lieutenant in the Charlottetown Engineering Company, Canadian Militia, and won the Governor-General's Medal for rifle shooting. In 1905, he was gazetted Captain and, later, was promoted to be Major in the Corps of Guides, Canadian Militia. In 1907, he passed the promotion examination for officers in the Regular Army for the rank of Lieutenant-Colonel, and became President of the Montreal Military Institute. In the following year he passed the promotion test for Regular Army field work, produced a paper on "Methods of Measurements for Military Reconnaissance," and won the Medal of the Lieutenant-Governor of Quebec for the Special Aggregates competition in rifle shooting. On April 1, 1911, Colonel Lordly was commissioned Lieutenant-Colonel. In the same year he received the Long Service Medal for combined service as a private and as an officer.

At the outbreak of the World War he was in command of the Corps of Guides, in Montreal. He at once organized a detachment from the First Corps for the Canadian First Division. On March 15, 1916, he was authorized to organize the Fifth Overseas Pioneer Battalion, of which he was made Commander. Leaving with his command on November 24, 1916, he arrived at Bramshott Camp, in England, on December 7.

An experienced engineer, Colonel Lordly was destined not to remain long in a purely military office. On June 26, 1917, he was transferred to the Imperial Service for special engineering work, first in connection with the channels of war traffic and, later, in connection with the Big Guns Department as Advising or Special Emergency Engineer to push forward urgent work. In both these capacities he traveled in England, Wales, and Scotland in the next eighteen months approximately 51 000 miles. In June, 1918, he was one of the Colonial officers decorated for long service. He returned to Canada in April, 1919.

The following year, 1919-20, he spent at Cornell University as Fellow in Civil Engineering, being engaged on experiments with piles in clay. For this work he received the degree of Master of Civil Engineering. During the next two years he did consulting work in Montreal, and from January 23, 1923, until his death, he was a member of the Technical Staff of the Montreal City Hall.

On March 10, 1890 Colonel Lordly was made a Student Member of the Canadian Society of Civil Engineers. He became an Associate Member on May 11, 1897, a Member on May 11, 1905, and a Life Member on May 19, 1931. In 1919 he became a Fellow of the Society of Engineers of London, England. He was also a member (Chairman, 1912–1920), of the Executive Council of the Province of Quebec Rifle Association, as well as of the Zodiac Society of Cornell University, and the Sigma Xi and Phi Kappa Phi Fraternities.

In the spring of 1932, he finished a treatise on "The History of Bricks." In September, 1932, he won the prize offered by *The Montreal Daily Herald* for the best study of "How to Secure Cheaper Bread, Milk, and Ice for the People of Montreal."

Colonel Lordly was the author of the following papers: "The Transition Curve,"[2] read before the Canadian Society of Civil Engineers, on November 11, 1892; "The Present and the Future Prospects of Gas for Lighting, Heating, and Power Purposes"[3]; "Anti-Friction Bearings"[4] (this paper won for its author the Fuertes Medal of Cornell University); "Subaqueous Concrete Work,"[5] read before the Canadian Society of Civil Engineers on March 17, 1910; "Experiences in Subaqueous Concrete Work"[6]; "Lateral and Vertical Pressure Effect of Piles in Clay,"[7] read before the Society of Engineers, London, England, on October 4, 1920; "Waterproofing of Concrete Structures,"[8] read before the Society of Engineers, London, March 4, 1929. (For this paper the author received the Nursey Memorial Premium of the Society.)

On August 26, 1896, at Halifax, N. S., Colonel Lordly was married to Edith Agnes Lordly, a first cousin, daughter of Edward J. and Mary A. (Major) Lordly. Three daughters were born to them: Ruth Leigh (now

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[2] *Transactions*, Canadian Soc. C. E., Vol. VI, pp. 171–181.

[3] *Transactions*, Assoc. of Civ. Engrs., Cornell Univ., Vol. VI, 1898, pp. 80–90.

[4] *Loc. cit.*, Vol. X, 1902, pp. 120–136.

[5] *Transactions*, Canadian Soc. C. E., Vol. XXIV, pp. 122–150.

[6] *The Cornell Civil Engineer*, Vol. XXVIII, March, 1920, pp. 254–263.

[7] *Transactions*, Soc. of Engrs. (London), 1920, pp. 171–195; abridged in *The Cornell Civil Engineer*, Vol. XXIX, May, 1921, pp. 143–149, and p. ix.

[8] *Transactions*, Soc. of Engrs. (London), 1929, pp. 55-93.

Mrs. William Tullis Birmingham, of Mesa, Ariz.), Helen Warburton, and Edith Mary. Mrs. Lordly and their three daughters survive him, together with four brothers, Sterling, Charles, Walter, and Arthur, and a sister, Mrs. G. Fred Fisher, all of St. John, N. B.

Henry Lordly did resolutely whatever he undertook to do. An expert marksman, he guaranteed to hit the bull's-eye in every undertaking. He worked manfully to secure better engineering practice and a higher standard of intelligence and equipment for engineers. Endowed naturally with an abundance of common sense, he was accustomed to survey a project or a theory from many points of view, and to try to see it in its just proportions. To engineering theory and practice he made a substantial and enduring contribution; but he was always more than the engineer; he was interested in the art of living and in human welfare in its larger relations; and he attained a sound and satisfactory philosophy of life.

Colonel Lordly was elected a Member of the American Society of Civil Engineers on October 11, 1920.

JOHN ROBERT CLARKE MACREDIE, M. Am. Soc. C. E.[1]

DIED DECEMBER 5, 1932

John Robert Clarke Macredie was born of Scotch parentage at St. John, N. B., Canada, on June 13, 1880. As a youth his love of adventure took him to many out-of-the-way places both by land and sea. At the age of sixteen, during the school interval, this fondness for exploring induced him to join a railway location party as a packer. As such, he was required to pack supplies to the camp on his back. Strong for his years, he held his own quite ably with the older men and by the time his outing was over had made up his mind to study civil engineering and make it his future vocation.

Entering the University of New Brunswick in 1897 as a Science Student, Mr. Macredie still kept in touch with his field work, spending each vacation with railway and land parties, so that when he was graduated in the spring of 1901, he had a valuable grounding in the practical work of his chosen profession. Besides his degree of Bachelor of Science, he also received that of Bachelor of Engineering.

Immediately on leaving the University, Mr. Macredie was offered a position with the Albert Manufacturing Company, of Hillsborough, N. B., Canada, as Engineer in charge of its railway surveys, and did splendid work for that firm until November of the following year, when an opportunity to enter the service of the Canadian Pacific Railway Company, as Transitman, presented itself. Although this new post offered less pay, the possibilities were great for a young man equipped with Mr. Macredie's ambition and training, and he lost no time in making the change.

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[1] Memoir prepared by D. A. Livingstone, Esq., Vancouver, B. C., Canada.

During the eighteen months as Transitman his good work attracted the notice of his superiors, and when the important grade reduction work between Winnipeg, Man., and Fort William, Ont., Canada, was commenced, Mr. Macredie was sent to that undertaking with the rank of Resident Engineer, in full charge of the Kenora Section. The completion of grade betterment saw the beginning of the double-tracking and Mr. Macredie was retained on the Kenora Section in charge of that heavy work until it was finally completed in 1909.

With the rapid expansion of railways across the prairies, the Western Lines of the Canadian Pacific Railway Company claimed Mr. Macredie's services, and with a promotion to Assistant Engineer, he was sent to take charge of the building of the well-known mile-long Lethbridge Viaduct. Upon its completion, several other similar and important works were entrusted to his care. He was given charge of location surveys in difficult country, and supervised the construction of many pieces of heavy railway work, not the least of which was his assistance in blasting through the Selkirk Range of the famous Connaught Tunnel. When the Edmonton, Dunvegan, and British Columbia Railway reached the Peace River crossing in 1916, expert opinion was required, and Mr. Macredie's services were loaned to superintend the first and only bridge ever built over that mighty Arctic stream.

During his eight years with Western Lines he again won the attention of his Chief and when, in 1917, the post of District Engineer at Moose Jaw, Sask., Canada, became vacant, it was given to Mr. Macredie. In this position he was responsible for all engineering in the huge Saskatchewan District. He held this position at the time of his death.

Mr. Macredie's early love of adventure never deserted him. Divorced from his former outdoor activity, he sought recourse in books of travel and followed closely the exploits of early explorers in the Canadian West. Of La Verendrye and Kelsey he knew well. Samuel Hearn he followed step by step on that awful trip across the barren lands to the Coppermine River. With Sir Alexander Mackenzie on his dash across half a continent, he toiled up rapid, through canyon, and down perilous mountain stream, ever eager to see what lay around the next bend. With David Thompson he ventured on his historical voyage down the Columbia River that all but won for Canada the States of Oregon and Washington. A close student of their charts, he would often point out localities that figured in the narratives.

As a raconteur he was brilliant and witty. His prolific reading of the best authors, and his wonderfully retentive memory, gave him a marvelous hold over his listeners. His great sense of humor, his repartee, and his fund of stories made him a much sought-after and welcome guest. His home was ever open to a wide circle of friends among whom were many distinguished Canadian men and women.

Mr. Macredie was a member of the Anglican Church. In 1906, he was married to Alice Rhea Walker, of Kenora, Ont., Canada. He is survived by his widow, his daughter, Alice, and his sister, Helen Macredie, of Fredericton, N. B.

Keeping in close touch with all engineering activities, he was a Member of the Engineering Institute of Canada, the American Railway Association, and the Saskatchewan Professional Engineering Association. He served on the Council of the latter, and did much to further its aims in the engineering field.

Mr. Macredie was elected a Member of the American Society of Civil Engineers on December 12, 1926.

CARL ARTHUR McCLAIN, M. Am. Soc. C. E.[1]

DIED AUGUST 5, 1932

Carl Arthur McClain was born at Emerson, Iowa, on May 16, 1878. He was of Scotch and English ancestry, although his parents, Arthur and Caroline (Murray) McClain, and his grandparents, were American born.

Mr. McClain spent his early life on a farm near Emerson. He attended school at McClain District School on his father's farm, and was graduated from the Emerson High School. He then attended Tabor College, at Tabor, Iowa, and afterward taught school, farmed, and worked as Assistant to the County Surveyor.

In 1901, at the age of 23, Mr. McClain went West. He first went to Idaho where he farmed and managed a grain elevator, then to Oregon where he entered the University in March, 1902. He was graduated from the University of Oregon, in 1906, with the degree of Bachelor of Science and, in 1912, he received the degree of Civil Engineer. In 1915-16, he was a Graduate Student at Cornell University, at Ithaca, N. Y., and there received the degree of Master of Civil Engineering in 1916. He was a member of Phi Beta Kappa.

Mr. McClain's education was financed largely through his own efforts, and during the fourteen-year period after removing to Oregon, he worked as Recorder for the United States Coast and Geodetic Survey, and as Foreman on sewer construction. He was also engaged in the private practice of engineering at Eugene, Ore., and in teaching at the University of Oregon (1911 to 1914) as Assistant Professor of Civil Engineering. During the period of private practice, his work included designs for hydro-electric power, water supply, sewers, and hydraulic features of logging operations. In 1914 and 1915, he was Engineer for the Eugene Water Board, in charge of design and construction of water supply works.

Following his graduation from Cornell University Mr. McClain was an Instructor in the College of Civil Engineering for one year and, upon leaving Cornell, in 1917, he became Construction Engineer, Division of Water, at Cleveland, Ohio, for one year. In this position, he was actively connected with the construction of the Division Avenue Filter Plant.

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[1] Memoir prepared by R. E. Koon, M. Am. Soc. C. E.

In 1918, Mr. McClain was made General Superintendent-Secretary of the Water Board at Eugene, Ore., a non-partisan board having jurisdiction over both water and power utilities. This position was equivalent to that of Chief Engineer, but with broad administrative duties in addition thereto, and it was here that he found a service well suited to the expression of his outstanding ability as an executive. He occupied the position until his death. This may be said literally since he made an active inspection of the new filtration plant only a few hours before he died.

During the fourteen years that Mr. McClain guided the destiny of the municipally owned water and power utilities of Eugene, the properties were greatly expanded, from a physical value of $850 000 in 1918, to $6 000 000 in 1932. The major construction activities during this period included three distributing reservoirs; improvements and additions to the filter plant; a change in source of water supply requiring a 30-in. gravity pipe line with diversion and operating structures; a new filter and pumping plant with a daily capacity of 12 000 000 gal; improvement of the power canal, and the addition of a new unit to an existing hydro-power plant; the Leaburg Hydro-Electric Plant of 20 000 hp and an auxiliary steam power plant of 8 000 hp. The distributing facilities of both utilities were correspondingly extended to keep pace with the demands of a population which had more than doubled during these years.

To all this work, Mr. McClain gave devoted attention, even after ill health demanded more freedom from exacting service. This constant devotion to his work, his outstanding ability as an engineer and executive, and withall, his warm-hearted personality and splendid character, endeared him greatly to his associates, and he received the respect and admiration of all those with whom he had personal contact.

In 1908, he was married to Grace Parker, and one child, Dora, was born of this union. Grace (Parker) McClain died in 1923. In 1926, Mr. McClain was married to Mrs. Anna M. McClain, a playmate of his early childhood and the widow of a deceased elder brother. Mrs. McClain, his mother, a daughter, two brothers, and a sister survive him.

Early in life Mr. McClain became affiliated with the Presbyterian Church of which he was always an active member. He took much interest in civic affairs; was a member of the singing organization known as the Eugene Gleemen; and was active in service with the Young Men's Christian Association, and civic clubs. He had no hobby other than devotion to his work and to the moral, social, and material advancement of his home city.

While his duties and success as an executive required that he have a clear understanding of the problems of an electric utility, Mr. McClain was by training and personal inclination a water-works engineer. He was a member of the American Water Works Association, and always active in the meetings and councils of the Pacific Northwest Section.

Mr. McClain was elected an Associate Member of the American Society of Civil Engineers on September 12, 1916, and a Member on January 16, 1928.

ROLLO GLENROY MANNING, M. Am. Soc. C. E.[1]

Died March 10, 1933

Rollo Glenroy Manning, the son of Anthony and Sarah Manning, was born on January 28, 1868, in Cass County, Michigan. He was graduated from the High School of Elkhart, Ind., in 1886, and from the University of Michigan, Ann Arbor, Mich., in 1890, with the degree of Bachelor of Science in Civil Engineering. In 1902, he was given the degree of Civil Engineer.

After graduation, Mr. Manning immediately entered the Engineering Office of the then American Bridge Company of Chicago, Ill. From this office he went shortly afterward to the Edgemoor Bridge Company, of Wilmington, Del., and, in 1893, from there to the Toledo Bridge Company of Toledo, Ohio.

When the Toledo Bridge Company became the Toledo Plant of the American Bridge Company, in 1900, Mr. Manning was made Engineer of this plant, at which time he designed and developed a line of drafting-room furniture, which has since become standard.

In 1906, he was transferred to the Ambridge Plant of the American Bridge Company, as Chief Engineer, which position he filled until January, 1928, when he resigned to become active Vice-President of The Ambridge Savings and Trust Company, of which he had been a Director since 1909. He was in active charge of the Trust Department of this institution until July, 1931, when he was suddenly prostrated by an illness from which he never fully recovered.

Mr. Manning had unusual executive ability which, coupled with sterling honesty and a sense of civic duty, made him a leader in the communities in which he lived, as witnessed by the following positions filled by him at various times: Member of the Ambridge Water Commission; President of the Ambridge Welfare Federation; Trustee of the Laughlin Memorial Library; Director of the Ambridge Board of Trade; Director and Treasurer of the Ambridge Building and Loan Association; member of the Community and Country Clubs; Trustee of the Ambridge Methodist Episcopal Church, and Superintendent of its Sunday School. Certainly an enviable record.

He was married in 1895 to Clara E. Smith, of Elkhart, Ind., and is survived by his widow, two daughters, Dr. Helen M. Speelman, of Pittsburgh, Pa., and Sue K. Manning, of Chicago; one son, Professor Wayne E. Manning, Smith College, Northampton, Mass.; three grandchildren, two brothers, and one sister.

Mr. Manning was elected an Associate Member of the American Society of Civil Engineers on October 2, 1901, and a Member on October 30, 1906.

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[1] Memoir prepared by A. H. Smith, Assoc. M. Am. Soc. C. E.

CYRIL ERNEST DAVIS MARSHALL, M. Am. Soc. C. E.[1]

DIED FEBRUARY 16, 1933

Cyril Ernest Davis Marshall, the son of Ernest Neilson Marshall and Ella Maria (Davis) Marshall, was born at Kingston, Jamaica, British West Indies, on April 26, 1880. He received his early education in Jamaica and, later, in England, where he finished his engineering course at Dulwich College in 1897.

Mr. Marshall then returned to Jamaica and was employed by the Public Works Department for two years, when he took charge of an irrigation project at Vere for the firm of Purdon and Cox, of Kingston.

Early in 1903 he went to New York, N. Y., where he secured a position as Recorder and Instrumentman, on surveys for the Ashokan Reservoir for the New York City Water Supply. In November of the same year he became Assistant Engineer to Harold A. Parsons, City Engineer, of Stamford, Conn.

In 1904, Mr. Marshall went to Panama with a group of engineers and soon after was appointed Assistant Engineer in the Municipal Department of the Panama Canal, where he located, designed, and constructed the Rio Grande and Comacho Dams which supplied with water the City of Panama and the Culebra Cut Districts. He also built roads, sewers, drains, etc., for the middle third of the Canal Zone municipalities.

Returning to New York City, he was employed by the late Charles Wellford Leavitt, Jr., M. Am. Soc. C. E., as Assistant Engineer. As such, he was engaged on the engineering work of real estate developments in Garden City, Port Washington, Jamaica Estates, and Long Beach, N. Y.; he also designed water and sewerage systems for Garden City. He was in charge of the design and construction of the campus development at Lehigh University, Bethlehem, Pa., and of Forbes Field Baseball Park, at Pittsburgh, Pa., the first of the modern baseball stadia.

In April, 1910, Mr. Marshall engaged in private practice in Hempstead, N. Y., as Civil and Consulting Engineer, designing and constructing the sewerage system and sewage disposal plant at Hempstead, also real estate developments involving bulkheads, canal dredging, and reclamation work, roads, sidewalks, drains, and water and sewerage systems.

In 1917, he went to Joplin, Mo., where he was General Manager of the mines of the United Zinc Smelting Corporation, and, in 1920, he became associated with the Marland Oil Company, at Ponca City, Okla., as Efficiency Engineer.

The following year, he returned to Garden City, N. Y., to live, and resumed the practice of Civil Engineering at Hempstead. He became Consulting Engineer for the Villages of Floral Park and Cedarhurst, N. Y., and did considerable engineering for the County of Nassau. By 1926 Mr. Marshall had become Village Engineer for the Villages of Hempstead, Woodsburgh, Valley Stream, Rockville Centre, and, in succeeding years, for Patchogue and Jamaica Square, all on Long Island, New York.

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[1] Memoir prepared by Leslie S. Marshall, M. Am. Soc. C. E.

Mr. Marshall was widely known as an expert in Municipal Engineering. He designed and supervised the construction of the municipal power plant for the Village of Rockville Centre, which utilizes Diesel engines for the production of electricity for light and power. This plant is one of the largest of its kind and has been a model of efficiency and economy of operation. He was considered an authority on incineration of garbage, having built seven incinerators in Nassau and Suffolk Counties, New York.

He conceived the establishment of numerous districts for furnishing water to residents of Long Island, having thus built fourteen water-supply systems. At the time of his death, he was extending the water-supply system to the newly acquired sections of Hempstead, as well as carrying out the design for remodeling the water plant and installing new pumps, building aeration basins for the removal of carbon dioxide which is prevalent in Long Island water obtained from deep wells, etc. He also designed five sewerage systems for Long Island municipalities.

Mr. Marshall had a keen engineering mind and was outstanding as a business executive. His wide knowledge of the law affecting municipalities and engineering contracts made him invaluable to villages, special districts, and commissions that retained him. His personality, together with his readiness to help others, made a friend of every one with whom he came in contact. A very convincing speaker, his vast experience enabled him to guide communities in the development of public improvements with great success.

Outside of business, Mr. Marshall took great interest in young people, being one of those who organized the Young Women's Christian Association in Hempstead. Before his death, he had also taken part in making plans for a Young Men's Christian Association there. He was President of the Lions Club of Hempstead for two years and a Director of the Hempstead Association of Commerce, which he helped to organize. He also was made a Director of the Hempstead Bank, Central Nassau, Incorporated, The Franklin Shops, Incorporated, and the Rockville Centre Mortgage Company. He was a Charter Member of the Board of The Nassau County Chapter of the New York State Society of Professional Engineers, and a Thirty-second Degree Mason. An Episcopalian, Mr. Marshall was a member of the Cathedral of the Incarnation of Garden City.

He was fond of sports, and, in his youth, had taken part in International Cricket Matches. To the last, he was an exceptional tennis player.

Mr. Marshall was recovering from a attack of influenza, when he was stricken with coronary embolism and died suddenly at his home in Garden City on February 16, 1933. His death was a shock to an unusually wide circle of friends who admired, loved, and respected him for his outstanding personality, his unquestioned integrity, his cordiality, and his magnanimity of spirit.

While in Stamford, Conn., he had married Anne Elizabeth Krokstedt, daughter of Oscar Krokstedt, on May 23, 1905, who survives him, as well as his three sons, Ernest Davis Marshall, lawyer, Cyril Wilmot Marshall, and Richard Nosworthy Marshall; two brothers, Leslie S. Marshall, M. Am. Soc. C. E., and Wilmot N. Marshall, of Hempstead, N. Y.; and two sisters, Mrs. George Randall, of Los Angeles, Calif., and Mrs. Frederick Tremlett, of England.

Mr. Marshall was elected an Associate Member of the American Society of Civil Engineers on March 6, 1907, and a Member on April 17, 1918.

ARTHUR POMEROY MELTON, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 28, 1932

Arthur Pomeroy Melton was born at Nebo, Ill., on July 12, 1875, the second child and oldest son of Churchill Curren Melton (a native of Carroll County, Virginia) and Martha Ann (Wilkinson) Melton. His father was a schoolmaster, in addition to his vocation of farming. The son attended the country schools in Pike County, Illinois, until he was fourteen years old. At the age of seventeen he obtained employment with the Starke Nursery Company, at Louisiana, Mo., where he worked for six years. It was probably during this time that young Melton decided to study Civil Engineering. He first entered a business college at Dixon, Ill., and, later, Iowa State College at Ames, Iowa, from which he was graduated in 1901. His road was not an easy one, and during vacations it was necessary for him to work as a street car conductor in Chicago, Ill., St. Louis, Mo., and Memphis, Tenn., in order to meet his college expenses.

In June, 1901, Mr. Melton entered the employ of the Illinois Steel Company, at South Chicago, Ill., and was sent as a Draftsman to Iron Mountain, Wis. After a year's work, he was given the position of Instrumentman and Assistant Engineer in charge of the layout of heavy foundations at Buffington, Ind., for the new cement plant which was being built there by the Universal Portland Cement Company, later a subsidiary of the United States Steel Corporation. Thus he was employed for nearly two years when in March, 1904, he accepted the position of Factory Superintendent with the Miracle Pressed Stone Company, of Minneapolis, Minn. While with this Company he patented a machine for making concrete blocks, which was used extensively.

A warm friendship between Mr. Melton and Ralph Rowley, Chief Engineer of the Illinois Steel Company, at South Chicago, had continued during the time that Mr. Melton was in Minneapolis. In December, 1905, word came from Mr. Rowley that a new steel plant and city were to be built on the southern shores of Lake Michigan in Indiana, by the United States Steel Corporation, and Mr. Rowley invited Mr. Melton to return and assist in that huge task.

Accordingly, on March 12, 1906, Mr. Melton came to the site of what is now Gary, Ind., and in company with Mr. Rowley and Mr. Thomas C. Cutler (now Chief Engineer of the Missouri State Highway Commission) started the laying out and construction of the city with which Mr. Melton was to be identified for the remainder of his life. He served as Townsite Engineer until August, 1906, when the first Town Board was organized and he became Town Engineer, serving in that capacity until January 1, 1912. He also carried on a private engineering practice during that time. Later, when this consulting work became too heavy for one person, he formed the partnership of Melton and Cutler, but continued his public life—as City Comptroller of Gary—until October, 1912, when he was appointed City Engineer, which office he held until January, 1914.

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[1] Memoir prepared by William J. Titus and Charles Brossman, Members, Am. Soc. C. E.

That period from 1906 to 1914 marked the pioneer stage of the new City of Gary, and during that time Mr. Melton had been in charge of the design and construction of city improvements aggregating $3 000 000. From 1914 until America's entry into the World War in 1917, he devoted his time to consulting engineering and construction work. That activity included many extensive surveys, and he became an accepted authority on the Calumet Water-Shed as well as an oft-consulted expert on the inland waterway possibilities of the surrounding industrial region. During that same time, Mr. Melton planned the Town of Iroquois Falls, Ont., Canada, a paper-mill town situated about 400 miles north of Toronto, Ont., That work included plans for paving, sewage disposal, water supply, lighting, playgrounds, parks, etc. He also completed a similar project in Mark Townsite near Indiana Harbor, Ind.

Upon America's entry into the World War, Mr. Melton was among the first to leave from the Calumet Region, having been commissioned a Captain in the Engineers' Officers Reserve Corps, on February 23, 1917. He was called into active service on May 17 of that year and, abandoning his business, he went to Fort Leavenworth, Kansas, where his engineering training and executive ability were soon recognized.

He embarked for France on August 5, 1917, in company with Paul Hansen, M. Am. Soc. C. E. They and many others arrived at Liverpool, England, on August 17, and continued immediately to Brest, France, where Captain Melton received his first assignment. His work as a Military Engineer provides an interesting chapter of his life and constitutes an enviable record of achievement. His greatest task during the war was at Brest, the principal debarkation point for the American troops, where he had charge of the construction and operation of a modern water supply and sewage disposal system which served 150 000 troops and provided facilities for watering all ships in port. That was probably the most extensive water supply system developed by the American Expeditionary Forces during the war, and included a dam in the river, reservoirs, pumping stations, and an extensive distribution system reaching to various camps, barracks, and hospitals in the area.

Other such systems were established under his personal direction at the following towns in France: St. Nazaire, Kerhoun de Rochambeau, Savanay, Bazeilles, Angers, Rimaucourt, Dijon, Perigueux, Beau Desert, Mesves, Mars, Blois, Clos St. Victor, Allery, and Baumes. He also prepared studies and recommendations for camps and hospitals at the following points: Chatillon-sur-Seine, Nantes, Grand Blattereau, Chateauroux, Limoges, Contrexeville.

Mirabeau, and Vichy. While in France Captain Melton was attached to the Staff of the late General Mason N. Patrick, U. S. A., who later became Chief of Staff of the Army.

Captain Melton returned from France in December, 1918, and again entered private practice as a Consulting Engineer, serving in that capacity for the Town of Hobart, Ind. He also completed the work started before the war at Mark Townsite, near Indiana Harbor.

Enactment of the National Defense Act by Congress in 1920 found Captain Melton devoting all his spare time to the organization of an Engineer Regiment in Lake County, Indiana, as a part of the Indiana National Guard. He was very enthusiastic on the subject of National preparedness and had earned the commission of Major of Engineers in the Officers' Reserve Corps, which commission was dated April 25, 1919.

The organization of the regiment was completed and Federally recognized as the 113th Engineer Regiment on January 23, 1922, and Major Melton was promoted to be a Colonel and placed in command. On June 15, 1922, he was also commissioned as Colonel in the Indiana National Guard. He proudly held both these commissions until his death and continued in active charge of his Regiment with its full quota of officers and men at nearly all times. His military work became his chief hobby and so intense was his interest that, in 1924, he abandoned his engineering work for a time to attend the Army War College, in Washington, D. C.

In 1924, he launched a $250 000 armory building project in Gary, which was completed in June of that year. Throughout the post-war years and outside his activity in the National Guard and the Officers' Reserve Corps, he found time to aid and promote the American Legion in Gary, becoming the first Commander of Gary Memorial Post.

His civilian work during these years included the laying out, in 1923, of the town of Dune Acres, Ind., the planning and the supervision of construction of the large drainage project legally known as the Burns Ditch, which project extended through Lake and Porter Counties, in Indiana, and consisted of draining many of the huge marshes of the Calumet Region by a channel cut through the huge sand dunes to Lake Michigan, reclaiming many miles of waste land. Having been so closely linked with the development of the Calumet Region for more than a quarter of a century, Colonel Melton's ability was further recognized in June, 1929, when he was appointed by the Governor as a member of the State Highway Commission of Indiana. In that capacity he encouraged highway development throughout the State. He continued as a member of that Commission until his death.

For fifteen years Colonel Melton was Vice-President of the Gary Commercial Club and Chamber of Commerce, and Chairman of that organization's Committee on Inland Waterway Development. He was a member of the American Association of Military Engineers and a Charter Member of Gary Lodge of the Masonic Fraternity. He was an Honorary Member of the Gary Rotary Club and a Vice-President of the Chicago Regional Planning Association. He also served as Chairman of the Mississippi Valley Waterways Commission. He was a member of the Protestant Episcopal Church, serving in the capacity of Junior Warden of Christ Church in Gary, from its foundation until 1920. He was a member and for some years a Trustee of the Indiana Engineering Society. He was a Registered Professional Engineer in the States of Indiana and Michigan, and served for four years on the Examining Board for Professional Engineers in the former State.

On September 2, 1906, Colonel Melton was married to Edith B. Quilling, of Menominee, Wis., who, with his two sisters, Mrs. Clara Harris and Mrs. Bertha Hurd, of Los Angeles, Calif., and five brothers: Judge Harve L. Melton, of McAlester, Okla.; William G. Melton and Wilke R. Melton, of Denton, Mont.; Fred Melton, of Nebo, Ill,; and Louis Melton of Hull, Ill., survives him.

A biographer attempting to write a record of the life of Colonel Melton might write a volume, and in the end find he had omitted mentioning many important events, meritorious services, and unselfish endeavors. The best that can be accomplished, in view of the breadth and richness of Colonel Melton's life, is to bring out the high lights of a record of service to the community in which he lived, to the State, to the Nation, and to mankind, such as few men can leave behind them.

Colonel Melton was elected an Associate Member of the American Society of Civil Engineers on September 5, 1911, and a Member on June 2, 1920.

LEE HAUN MILLER, M. Am. Soc. C. E.[1]

Died April 9, 1933

Lee Haun Miller was born in Aylmer, Ont., Canada, on January 25, 1877, the son of Asa Miller and Mary (Haun) Miller. His father was engaged in farming and for many years participated actively in Canadian politics. Mr. Miller's ancestry on his father's side was Scotch, and his maternal ancestors date back to the time of Oliver Cromwell. They left England following the restoration of King Charles the Second. Subsequently, in 1783, all his ancestors who had located in New York State moved to Canada.

Mr. Miller was educated in the public schools of Aylmer and the Collegiate Institute, and was graduated from the School of Engineering of the University of Toronto, Toronto, Ont., in 1900. Before and during his academic training he spent two years in foundry and machine-shop practice.

After his graduation in 1900 Mr. Miller was employed for a year as Structural Steel Draftsman by Jones and Laughlin Steel Company, of Pittsburgh, Pa., and in a similar capacity for another year by the Brown Hoisting Machinery Company, of Cleveland, Ohio. From June, 1901, to December, 1904, he was employed by the Wellman-Seaver-Morgan Company, of Cleveland, as Structural Steel Detailer and afterward as Designer.

In December, 1904, Mr. Miller was engaged by the Interstate Engineering Company, of Cleveland, as Estimator and Designer on coal and ore-handling equipment, railroad turntables, and other similar structures. He was employed as Estimator and Designer for the McMyler Manufacturing Company, of Cleveland, from January, 1906, to May, 1908, principally on coal and ore-handling equipment, locomotive and traveling cranes, etc. In May, 1908, Mr. Miller became Structural Sales Agent for the Bethlehem Steel Company at Cleveland, which position he held until May, 1922. From 1916 to 1922 he was also Sales Engineer for the Bethlehem Steel Bridge Company. He became a naturalized citizen of the United States in 1910.

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[1] Memoir prepared by F. H. Frankland, M. Am. Soc. C. E.

In 1922, Mr. Miller began the work of organizing the American Institute of Steel Construction in the capacity of Managing Director and was actively associated with the development of the Institute until his death. In 1923, the Standard Specifications of the Institute were completed and Executive Offices were established in New York, N. Y., at which time Mr. Miller was appointed Chief Engineer with offices at Cleveland.

Mr. Miller was a man of outstanding personality and strength of character and, during the formative years of the Institute's history, his influence was invaluable in the development of activities that covered many new and unprecedented fields of research and promotion of the use of structural steel.

He was a member of the American Society for Testing Materials, the Cleveland Athletic Club, the Acacia Country Club, and was a Thirty-second Degree Mason, a Knight Templar, and a member of the Mystic Shrine. His hobbies were the study of history and sculpture.

He is survived by his widow, Annie (Graham) Miller, and two children, Mrs. Anabel Lee Hulme and Paul Graham Miller.

Mr. Miller was elected an Associate Member of the American Society of Civil Engineers on January 31, 1911, and a Member on October 15, 1923.

WILLIAM SELBY MITCHELL, M. Am. Soc. C. E.[1]

Died January 27, 1932

> "I would the great world grew like thee,
> Who grewest not alone in power
> And knowledge, but from hour to hour
> In reverence and charity."

The death of William Selby Mitchell has taken from the Engineering Profession one of its most capable and best-loved members, distinguished among his friends, acquaintances, and associates not only for his extensive knowledge of river hydraulics, which had been his study for more than half a century, but as well for his rare judgment of men and things, for his innate sense of justice, for his charity toward his fellows, and for the human touch that characterized his every action and tempered his thought and speech.

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[1] Memoir prepared by Edward E. Wall, M. Am. Soc. C. E., from data supplied for the most part by William M. Penniman, M. Am. Soc. C. E., and E. N. Parker, Upper Mississippi Valley Div., U. S. Engr. Dept. St. Louis, Mo.

Even as a boy he won the confidence of his playmates to such an extent that by common consent many disputes were left to him for decision and few appeals were made from his verdict; and so throughout his life, his opinion was often sought on many subjects, by his friends and associates, and always his word carried great weight toward the settlement of the mooted question.

Many tributes were paid to Mr. Mitchell, the man, in the letters written by those who knew him well, on hearing of his unexpected death, and through them all there ran the same expression of deep regard for the man and of great respect for his character and his professional attainments.

In the early Fifties the parents of William Selby Mitchell came from Wheeling, W. Va., to St. Louis, Mo., where their son was born on October 11, 1857. His early education was received in the public schools of St. Louis and at Clark's Academy, a private institution.

From 1874 to 1877, he studied engineering at the University of Virginia, at Charlottesville, Va., but did not graduate, as he was obliged to go to work one year before finishing the course. Possibly these years at college accentuated those traits of his character which led one of his friends, Major John C. Gotwals, U. S. Corps of Engineers, M. Am. Soc. C. E., to write of him: "He exhibited that sense of honor typical of the Old Dominion, accompanying it with a keen intelligence and exquisite sense of humor."

Mr. Mitchell's first engineering work was in East St. Louis, Ill., laying switch tracks in the railroad yards. Following this, he entered the Government service on August 14, 1878, as Rodman, being promoted, successively, to Instrumentman, Recorder, and through other grades in the United States Engineer Office at St. Louis, until 1881, when he was made Assistant Engineer. He remained in this position for twenty-three years, when, in 1904, he was appointed Principal Assistant Engineer of the St. Louis Office, in which capacity he served until the United States entered the World War in 1917.

During these years Mr. Mitchell had charge of all surveys and the collection of physical and hydrographic data in the St. Louis District; of the design of wood and steel river vessels, such as barges, towboats, and dredges, and their power plants; of river improvement works and all forms of construction used for the improvement of navigation in the District, embracing dikes, dams, jetties, revetments, etc.

He directed the investigation and surveys and made the estimates of the cost of a $3 000 000 drainage system with levee embankments for the protection of the American Bottoms, in Illinois. This report on local flood control, made about thirty years ago,[2] formed the basis of the East St. Louis Levee Project as it was afterward built. For a considerable period he had charge of the Mississippi River snagboats and the snagging operations between the mouth of the Missouri River and New Orleans, La., and of the dredging work in the St. Louis District.

He designed for Mississippi River service a fleet of four all-steel towboats of unusual size and power, and nineteen of the largest steel barges now in use on navigable rivers, at a total cost of $3 600 000. For the Missouri Southern Railway Company, he designed and built the large transfer boat, *Kellogg,* and acted as Consulting Engineer on the design and construction of the incline at Mudd Landing, Ill. He also served as Engineer and member of the United States Board on Experimental Towboats, and was a member of other engineering boards. His thorough knowledge and expert proficiency in matters pertaining to inland waterways were fully recognized and generally acknowledged by members of the Engineering Profession.

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[2] House Doc. 772, 59th Cong., 1st Session.

Mr. Mitchell was the author of several important engineering reports to the War Department pertaining to the Mississippi River, among which may be mentioned "Measurement of Stream Flow by Rod-Floats and Meters," and, "Design and Construction of Towboats and Barges."[3] In 1909, he prepared[4] a "Memorandum on a Project for Maintaining by Hydraulic Dredging Alone a Navigable Channel 14 Feet in Depth by 500 Feet in Width in the Mississippi River between St. Louis, Mo., and the Mouth of the Ohio River."

Those especially cognizant of the great value of Mr. Mitchell's judgment and knowledge were the officers of the Engineer Corps, who at various times were assigned to take charge of the U. S. Engineer Office at St. Louis, one of whom, Col. Wildur Willing, wrote of him:

> "He had few equals and no superior in solving the intricate problems of regulation and dredging presented by this mighty and unruly stream [the middle Mississippi River]. Perhaps his greatest service to the government he served so long was in the design of hydraulic dredges and other floating plant, for which he had unusual talent. * * * Mitchell won and held the highest respect from those with whom he served."

Another distinguished officer, Col. C. McD. Townsend, U. S. A. (*Retired*), M. Am. Soc. C. E., wrote:

> "He was not only a very able engineer but also most conscientious in the performance of his duties. He could be depended upon not only in cases of emergency but also in the daily routine, which is such a necessary adjunct in successfully running an office."

Major Gotwals, who has already been quoted, also wrote:

> "His physical work on the Mississippi River was noteworthy and permanent. Still more noteworthy in my opinion was his influence upon his fellow-workers, particularly the officers of the Corps of Engineers, to whom he essentially acted as a counselor and a friend. So training the character of many men who have taken an influential part in the life of the United States, I believe Mr. Mitchell's greatest work has been that of intellectual guidance and character training of the engineers and army officers with whom he has labored."

During the World War years (1917-1919), Mr. Mitchell was appointed District Engineer in charge of the U. S. Engineer Office at St. Louis, going back to his position as Principal Assistant Engineer from 1919 to 1922. On January 5, 1922, he was transferred to the office of the Division Engineer, Western Division, at St. Louis. Here, he reviewed and studied for the Division Engineer, all projects, plans, details of construction, reports, and papers passing through that office until, on October 10, 1929, it was consolidated with the Central Division and the addition of the First Chicago District to form the Upper Mississippi Valley Division. In this office, because of his ability, extensive knowledge, and ripened judgment on river and harbor affairs, his talents were utilized in a consulting capacity until October 31, 1931, when he was retired from the Government service under the provisions of the Federal Retirement Law.

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[3] Annual Repts., Chf. of Engrs., U. S. Army, 1901 and 1922, respectively.

[4] House Doc. 50, 61st Cong., 1st Session.

On August 14, 1928, after exactly fifty years of continuous service in the Engineer Department at Large, War Department, his associates in the St. Louis Engineer District gave an informal reception in his honor at the Engineers' Club in St. Louis, which was largely attended by engineers in and about that city. On November 2, 1931, a dinner was given in his honor, by his friends in the Office of the Division Engineer, the District Office of the Corps of Engineers at St. Louis, the United States Lighthouse Service Office, and the United States Steamboat Inspection Service.

Mr. Mitchell joined the Engineers' Club of St. Louis in 1884 and was made an Honorary Member in 1930. He was always actively interested in the affairs of local engineering organizations, and, in 1921, was President of the St. Louis Section of the Society.

Mr. Mitchell's death occurred on January 27, 1932, not quite three months after his retirement. He never married, but devoted the best part of his life to his two maiden aunts, who lived with him and constituted his family. Both of them lived until a short time prior to his death.

This attempt to portray this unforgettable man would be wretchedly incomplete unless an effort was made to put before the reader his standards of life and conduct, his kindness and understanding, his genial personality, and his discerning wit.

Possibly no better way could be found to do this than to quote from some of the many letters of his friends, written after his decease. A life-long friend, Mr. S. J. Waterman, wrote:

> "His never-failing good humor and consideration made him a welcome member of any group, and a rich store of anecdotes, suggested by the conversation (never introduced obtrusively), aptly illustrated his viewpoint or clinched an argument; this was so characteristic that a witty friend charged him with being 'in his anecdotage.' His wit was never caustic but ever kindly and mellow with the happy quality of making its object join heartily in the laugh at his expense. * * * In summing up these memories, one feels they portray the qualities of a gentleman, loyal to his ideals, inspiring to his associates in his standards, devoted to family and friends, genial and lovable, whose memory will long be cherished by those fortunate enough to have known him."

Colonel Willing, who was associated with Mr. Mitchell officially as well as socially, writes:

> "Others will no doubt sum up his splendid professional achievements. I shall try to pay some small tribute to the lovable human comrade. * * * With a wonderful sense of humor, fond of hearing and telling a good story, nothing vulgar ever escaped his lips. * * * Mitchell did not allow himself

to fall into the rut of professional study. He was a man of broad interests, well read in literature, philosophy, and history, and could throw a flood of light upon almost any question under discussion. He was a lover of good music and of the other fine arts."

This from the letter of another old friend, Col. George W. Goode, U. S. Army (*Retired*):

"I would love to express in words my affection and admiration for our dear friend, Will Mitchell, whom I have known from boyhood; the recital, if adequately portrayed, would read like a poem. I believe God puts men like him on earth to stabilize the world. * * * His serenity, his normal saneness, his sense of humor, his genuine good fellowship with all the world make you ashamed of yourself but proud that you can count yourself among his friends. * * * To the man or woman who came in intimate contact with him, the world seemed better and life more worth-while; and that I believe was his mission. May God bless him."

Mr. Mitchell's retirement in 1931 ended one of the most creditable and extended careers of any civilian in the Government service. To only a few men is given the opportunity to work continuously in any one service for more than fifty-three years, and to possess the personality and ability to advance steadily year after year in the esteem and regard of every one associated with him, from the lowest to the highest. His name will long remain in the memories of those who knew him well, as one who was an able engineer and an accomplished gentleman.

Mr. Mitchell was elected a Member of the American Society of Civil Engineers on June 1, 1909.

WESLEY CHARNOCK MOCK, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 15, 1931

Wesley Charnock Mock was born at Petaluma, Calif., on December 9, 1879, the son of Charles Wesley and Amy (Charnock) Mock. His father was the son of John Lewis Mock, of North Carolina, one of the early pioneers in California, who moved there by covered wagon from Missouri in 1853 and settled in the Petaluma Valley, where he planted the first orchard to be grown in that Valley. His mother came to the United States from England in 1873, and after spending several years in Boston, Mass., moved with her family to San Francisco, Calif.

In 1881, Mr. Mock's parents moved from California to Tacoma, Wash., where his father for a number of years held the position of Secretary of the School Board, and where Mr. Mock was educated in the public schools, specializing, in post-graduate work, in Mechanical Drawing.

He joined the Engineering Department of the Northern Pacific Railway Company in 1900 and remained with that Company for about nine years, first on survey work, then as Assistant Engineer on Maintenance of Way, in which position he had charge of ballasting, the relaying of tracks, and the construction of bridge foundations, and buildings. Later, he served as Field Draftsman and Transitman, and from April, 1904, to May, 1906, as Chief Draftsman in the Division Engineer's Office at Livingston, Mont. In June, 1906, Mr. Mock was appointed Chief Draftsman in the office of the Principal Assistant Engineer at Spokane, Wash., where he continued until April, 1909, his work including the design of timber bridges and buildings, concrete bridge foundations, and the making of cost estimates for construction work. This latter work was in connection with the location and construction of the Spokane, Portland, and Seattle Railway, which line was built jointly by the Northern Pacific and Great Northern Railway Companies.

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[1] Memoir prepared by Alfred Holmead, Esq., Houston, Tex.

In May, 1909, Mr. Mock joined the Engineering Department of the Spokane, Portland, and Seattle Railway Company and was appointed Resident Engineer on Construction and Maintenance of Way, in which position he served until 1914. The construction and maintenance of this railway presented a number of difficult engineering problems, and it was a matter of wide general interest at the time it was built.

For a time Mr. Mock was engaged in the private practice of surveying and drafting at Tacoma, during 1914 and 1915; and also within this period he was with the Commission of Public Docks at Portland, Ore., in connection with the erection of an extension to the Municipal Terminal. Later, he served for a short time with the Engineering Department of the Chicago, Milwaukee, and St. Paul Railway Company; and, in July, 1916, he joined the Oregon-Washington Railroad and Navigation Company, a part of the Union Pacific System, as Assistant Engineer on Federal Valuation.

The extensive inventory and valuation work of the Interstate Commerce Commission required much assistance and co-operation from the railway companies. In this work, Mr. Mock served with great diligence and understanding. His duties required extensive negotiations with the engineering forces of the Federal Commission, intricate cost analyses, and service on various railroad committees connected with the valuation work of the carriers. In August, 1923, he was appointed Valuation Engineer of the Oregon-Washington Railroad and Navigation Company. He resigned in 1926 and joined the Valuation Department of the Southern Pacific Company, at San Francisco, with which Company he was serving at the time of his death.

Mr. Mock was a member of several Committees of Railway Valuation Engineers and was frequently identified with the work of the Western Group of the Railway Presidents' Conference Committee on Federal Valuation at Chicago, Ill. He had exceptional ability as an engineer, was a keen student, and one whose work inspired confidence. His very practical knowledge of railway construction and maintenance was always manifest, and he had the faculty of realizing quickly the essential points of a question or problem. His associates invariably benefited by the soundness of his observations and judgment. In a multitude of details he would hold accurately to the end to be attained. In discussion, he was desirous always that the best thought might be evidenced and prevail. Mr. Mock was by nature observant, and was widely informed. He was fond of the outdoors and of photography, and was much interested also in the technical phases of radio.

He was married in August, 1904, to Adelia Lou Swain, at Tacoma, Wash. They had one child, a son, Wesley Charnock Mock, Jr., an Engineer with the Bureau of Standards, at Washington, D. C. He is survived by his widow and son, his mother, a brother, Ernest Thornton Mock, and two sisters.

Mr. Mock was elected a Member of the American Society of Civil Engineers on December 15, 1924.

FREDERICK HAYNES NEWELL, M. Am. Soc. C. E.[1]

DIED JULY 5, 1932

Frederick Haynes Newell was born in Bradford, Pa., on March 5, 1862, the son of Augustus William and Annie Maria (Haynes) Newell. His early education was obtained in the schools of Newton, Mass., and he was graduated from the Newton High School in June, 1881. Four years later, he received the degree of Bachelor of Science in Mining Engineering from the Massachusetts Institute of Technology.

During his period of schooling Mr. Newell served as Field Assistant, Levelman, and Transitman on various land and railroad surveys in the oil regions of Northwestern Pennsylvania.

After three years of field work in Colorado and other States, he was appointed on October 2, 1888, as Assistant Hydraulic Engineer of the United States Geological Survey, being the first Aid designated under Maj. John W. Powell to investigate the extent to which the arid regions of the United States might be reclaimed by irrigation. He advanced steadily in this work, expanding its scope, and was successively made Hydrographer and Chief of the Hydrographic Branch.

At this early period, Mr. Newell initiated and was in responsible charge of river flow surveys, and estimates of cost and capacity of reservoirs, investigations of ground-waters, and other allied work.

This preliminary work was directly succeeded by his co-operation with the late Senator Francis G. Newlands, George H. Maxwell, President of the National Irrigation Association, and others in the preparation and public presentation of various Congressional bills, one of which by the personal efforts of President Roosevelt became the Reclamation Act when signed by the latter on June 17, 1902. Immediately after that event, Mr. Newell was appointed Chief Engineer, under Charles D. Walcott, then Director of the United States Geological Survey.

During the next few years the organization of the Reclamation Service was developed, and plans were outlined for extensive work in each of the Western Arid States, work being started in most of them. In 1907, when Mr. Walcott was made Secretary of the Smithsonian Institution, the Reclamation Service was organized as a separate Bureau of the Department of the Interior with Mr. Newell as Director and the late Arthur P. Davis, Past-President, Am. Soc. C. E., as Chief Engineer. Construction was pushed rapidly until twenty-six projects, including reservoirs, canals, and related works were completed in whole, or in part, notably the Roosevelt, Shoshone, Arrowrock, Gunnison Tunnel, and others, involving an investment of more than $100 000 000, in one hundred dams, of which ten form reservoirs of National importance, also 25 miles of tunnels, 13 000 miles of irrigating canals and ditches, with regulation works, bridges, steam and hydro-electric power plants, transmission lines, pumps, and other devices for supplying water to about 20 000 farms. Special efforts were made to secure the greatest practicable economy and efficiency in the execution of the work and to supply the needs and desires of the settlers under them.

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[1] Memoir prepared by Allen B. McDaniel, M. Am. Soc. C. E.

In spite of many difficulties offered by Nature and man, progress was made and, year by year, the work was expanded, new and additional men being tried out and advanced as results warranted. The Reclamation Service became a training school for young men, and attained an international reputation for efficiency, *esprit de corps,* and constructive results. While Director of the Reclamation Service, Mr. Newell made several special surveys and investigations, notably a first-hand study of, and report on, the water resources of the Hawaiian Islands.

The breadth of his interests and activities is indicated in his work as one of the founders and the first Secretary of the National Geographic Society and also of the American Forestry Association. The early aim of both these organizations was to arouse public interest in the proper conservation and utilization of the natural resources of the United States.

The late President Roosevelt wrote of Mr. Newell and his services during this period of his life, as follows:

> "For fourteen years I have followed at first hand the work of Mr. Frederick H. Newell. I speak from personal knowledge when I say that he was one of the most loyal, disinterested and efficient public servants the United States had had throughout that period. I first came in touch with him when I was Governor, when I drew on him for aid and advice in formulating the proper conservation policy for the State of New York. During the years that I was President he was one of my right-hand men. It is too often the case in the United States that the men who are most prominent, who attract most attention, are inefficient or even vicious public servants, whereas the men who do the best work (I think, rather better than that done by the public servants of any other nation), pass almost unnoticed and without any adequate reward. Mr. Newell belongs to that small group. He is a public servant of whom it is the bald and literal truth to say, that by his services he has made all good American citizens his debtors."

In July, 1914, as the result of the re-organization of the Reclamation Service, Mr. Newell was made Consulting Engineer. About this time, he received the degree of Doctor of Engineering from the Case School of Applied Science, and it is significant that, in the later years of his life, he was universally addressed and referred to as Dr. Newell, a token of the respect and esteem in which he was held.

During the spring of 1915, Mr. Newell made a lecture tour of the schools and universities of the country, and was invited to become the Head of the Department of Civil Engineering of the University of Illinois. He succeeded the late Professor Ira O. Baker and re-organized and developed the civil engineering work over a period of five years. He also established an engineering library in the College of Engineering. During the participation of this country in the World War, he was instrumental in securing a course in Aviation at the University and in the establishment of a flying and training field near Rantoul, Ill.

Mr. Newell early realized the need of wider acquaintance and greater co-operation among engineers, and with this in mind he called a conference in Chicago, Ill. This meeting led to subsequent conferences, in which about 100 societies were represented. The final outcome was the American Association of Engineers, an all-inclusive organization modeled somewhat on the Washington Society of Engineers (of which he was one of the founders and second President), but embracing the whole country, and dedicated to welfare work and to encouraging engineers to become broader men and better citizens. Mr. Newell became President in 1919 when the membership was about 4 000 and under his active leadership the number of members increased rapidly to 16 000, and, later, reached a peak of about 23 000. Under his wise guidance the Association established branches or chapters in every important center of population; employment methods were devised by which thousands of engineers were aided; standards of employment, compensation, and of ethics were set forth; laws for the registration of engineers successfully secured in many States, and, in other ways, the status of the Engineer was improved. The technical societies have been stimulated to adopt and work out policies and programs of welfare, civic, and other non-professional activities as the result of the pioneer work of the Association and its "father," Frederick Haynes Newell.

In 1924, Mr. Newell co-operated with his friend, Allen B. McDaniel, M. Am. Soc. C. E., in the organization of The Research Service, a group of consultants in various fields of engineering and construction for the purpose of making researches and rendering service to States, municipalities, private concerns, and individuals. To the time of his sudden death, Mr. Newell was President of, and, at all times, an active participator in the work of, this unique agency. Among the more important projects which he largely handled or in which he collaborated were the following: Engineer of the Water Resources and Power Board of Pennsylvania; survey of the public works of Puerto Rico; survey and report on the water resources of Virginia; the control, conservation, and utilization of the flood waters of the Mississippi Basin; the construction of the Bahai Temple, at Wilmette, Ill.; and rural electrification and agriculture in Virginia.

Mr. Newell's last work was the compilation and editing of the book, "Planning and Building the City of Washington." This work was the result of the co-ordinated contributions of seventeen leaders of civic life in the Nation's Capital, and is the contribution of the Washington Society of Engineers to the Washington Bi-Centennial Celebration. In this work, Mr. Newell selfsacrificingly spent months of ceaseless effort and produced in a 250-page handbook a vast amount of interesting and valuable data concerning the City of Washington and its public works. This last work will endure as a memorial to his vision, love for his adopted community, devotion to his profession, and indefatigable energy.

He was also the author of "Oil Well Drilling" (1888), "Hydrography of the Arid Regions" (1891), "Agriculture by Irrigation" (1894), "The Public Lands of the United States" (1895), "Irrigation in the United States" (1902), "Hawaii, Its Natural Resources" (1909), "Principles of Irrigation Engineering" (1913), etc.

While engaged in some details of the work of the Local Committee on Unemployment and Public Works of the Washington Society of Engineers and American Engineering Council, he ended his services to humanity, his community, and his profession.

Mr. Newell was first of all a man; strong for the right, upholder of the highest ideals, worker for the common weal, lover of humanity, and engineer of outstanding ability and achievements. He was of high integrity, exemplary habits, and simple life. Devoted to his family, true to his friends, and always seeking an opportunity for service, his religion was embodied in "living the life."

On the reverse side of the Cullum Gold Medal awarded to Mr. Newell by the American Geographical Society in 1918, is this inscription.

> "He carried water from a mountain wilderness to turn the waste places of the desert into homes for freedom."

Mr. Newell was a member of the American Society of Mechanical Engineers; Washington Academy of Science, of which he was Vice-President in 1907; Western Society of Engineers, and Illinois Society of Engineers. He had also served as a Member of the United States Land Commissions, United States Inland Waterways Commission, National Advisory Board for Fuels and Structural Materials, and the Illinois State Board of Examiners of Structural Engineers. He belonged to the Cosmos Club of Washington, D. C.

He was married on April 3, 1900, to Effie Josephine Mackintosh, of Milton, Mass., who, with a son John Mackintosh, and two daughters, Josephine and Constance, survives him.

Mr. Newell was elected a Member of the American Society of Civil Engineers on December 5, 1900.

WILLIAM NELSON PAGE, M. Am. Soc. C. E.[1]

DIED MARCH 7, 1932

William Nelson Page, the son of Edwin Randolph and Olivia (Alexander) Page, was born in Campbell County, Virginia, on January 6, 1854. His ancestors on both sides of the family were born for nine generations in

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[1] Memoir prepared from information on file at the Headquarters of the Society.

England and Virginia. Mr. Page received his early education under private tutors, which was followed by an academic course almost entirely devoted to the study of mathematics.

At the age of seventeen, he began the practice of his profession as one of an engineering party engaged on the location and construction of the Chesapeake and Ohio Railroad through the New River Canyon, in Virginia, by which the New River Coal Field secured its outlet to the Atlantic seaboard. In 1873, he was engaged in the construction of shops at Huntington, W. Va., and, as Chief Engineer, constructed the Gauley Kanawha Railroad. During 1874, Mr. Page worked in the field with Professor David Thomas Ansted, an English geologist and scientist of eminent reputation, and, in 1876, he was in charge of an engineering party locating a double-track railroad from the Ohio River to Hampton Roads, Va.

From 1878 to 1885, Mr. Page was General Manager of the Hawk's Nest Coal Company, Hawksnest, W. Va., and built and operated for five years the Victoria Iron Furnace, at Goshen, Va., and its railroad connections. Having an inspired faith in the possibilities of the New River and Kanawha Coal Fields in West Virginia, he returned to that line of activity and, until 1889, he developed the large mining plant of the Mt. Carbon Coal Company, Limited, at Powellton, W. Va., and, as Chief Engineer, built the Powellton and Pocahontas Railroad, a branch line of the Chesapeake and Ohio Railroad, for transportation purposes.

In 1889, associated with New York financiers and bankers, Mr. Page organized and developed the Gauley Mountain Coal Company, at Ansted, W. Va., of which he was President and General Manager for twenty-eight years. Other companies of which he was President were the Loup Creek Colliery Company, which he organized in 1902, and for which he erected 500 coke ovens; the Mill Creek Colliery Company; the Deepwater Railway Company; and the Tidewater Railway Company.

Probably the greatest of Mr. Page's engineering achievements was the building of what is now the Virginian Railroad, an enterprise which was financed by the late Mr. Henry H. Rogers and which is not only a model of railroad construction, but crosses the Allegheny Mountains at the lowest gradient of any trans-Allegheny line.

In addition to his many general business activities, Mr. Page was also Consulting Engineer of the Amalgamated Copper Company; a Director of the Republic Iron and Steel Company; President of The Ansted National Bank; and Financial Agent for the Virginia Iron and Steel Company. He had also invented and erected a belt conveyor for transferring coal from standard railway cars into river barges, which was used on the Mississippi, Ohio, and Kanawha Rivers.

On account of impaired health, Mr. Page retired from all active business in 1917. Athough he had been confined to his home for many months previous by poor health, the immediate cause of his death, on March 7, 1932, was pneumonia, which because of his weakened physical condition was quickly fatal. On February 9, 1932, less than a month prior to his death, he and Mrs. Page had celebrated the Fiftieth Anniversary of their marriage.

Aside from his business career, Mr. Page also took an active part in civic and military duties, serving as Mayor of the City of Ansted for ten years and as Line and Field Officer in the West Virginia National Guard for twenty years. He was a devout member of the Protestant Episcopal Church, and a member of the Masonic Fraternity.

The technical societies in which he held membership were the American Institute of Mining and Metallurgical Engineers (of which he was Past Vice-President); Washington Society of Engineers; and the Geological Society of Washington, D. C. He was also a member of the National Geographic Society. As Chief of International Group of Awards No. 16 (Mining and Metallurgy), Mr. Page was awarded the Congressional Medal at the St. Louis, Mo., Exposition (Louisiana Purchase Centennial). Among his social clubs were the Cosmos Club, of Washington, the Westmoreland Club, of Richmond, Va., and the Engineers and Reform Clubs, of New York, N. Y.

On February 9, 1882, Mr. Page was married to Emma Gilham, of Richmond, Va., who died on April 14, 1933. He is survived by one son, Edwin Randolph Page, two daughters, Mrs. James Jones Marshall and Mary Josephine Page, and several grand-children.

He was distinguished both as a Civil and a Mining Engineer, and, as stated, was a pioneer and notable promoter in the development of the coal-mining industry of Southern West Virginia. He was an ideal husband and father, a faithful and generous friend, being all that that term implies, and a lover of his fellow men.

Mr. Page was elected a Member of the American Society of Civil Engineers on May 3, 1905.

SAMUEL HENRY PITCHER, M. Am. Soc. C. E.[1]

Died March 15, 1932

Samuel Henry Pitcher was born at Barbados, in the British West Indies, on November 19, 1863, and spent his early childhood on his father's sugar plantation. It was there he received his early education from a tutor, later entering Harrison College. In 1881, he began his technical education at McGill University, in Montreal, Que., Canada.

Like many who enter the romantic profession of Civil Engineering, Mr. Pitcher's work took him into the big open spaces, and to far-flung frontiers, notably the Hudson Bay country. Acquiring practical experience in various capacities, his work covered a broad field in water power development, plant construction, and bridge engineering.

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[1] Memoir prepared by A. V. Gillingham, Esq., Philadelphia, Pa.

In 1896, Mr. Pitcher went to Worcester, Mass., as Assistant City Engineer, serving in that capacity for twelve busy years. He then established the firm of Samuel H. Pitcher Company for the practice of his profession. He designed and supervised the construction of the Lake Quinsigamond Bridge which stands to-day as a symbol of architectural beauty and a monument to his genius.

He was one of the founder members of the Worcester Society of Civil Engineers and served as its first President. He was also a member of the Planning Board of the City of Worcester, of the American Water Works Association, and the Boston Society of Civil Engineers.

"Sam" Pitcher, as he was known to hundreds of his fellow citizens in Worcester, was very human. His apparent reserve and reticence were due probably to his innate modesty and shyness; but he was a man of very positive character. He had a deep faith and an abiding confidence in his fellowmen. Those who were fortunate enough to bridge the chasm between acquaintanceship and tried friendship found in him warmth, a depth of understanding, and enthusiasm for life and for his profession. They found him tactful, helpful, and an ardent disciple of the creed that it is more blessed to give than to receive. The opinion was frequently voiced that it was doubtful whether he had a single enemy. In his innate desire to be of constructive service, it was inevitable that he should become one of the early members of the Kiwanis Club.

In his social life, Mr. Pitcher was retiring and devoted much time to painting in water colors. With a natural flair for color, coupled with an unerring sense of proportion, he made rapid strides in his favorite avocation. As a hunter and fisherman, his enthusiasm knew no bounds; his endurance and hardihood as well as his skill were the envy of those who shared in these outdoor adventures.

Samuel Pitcher was widely and deeply read in both the classics and contemporary literature. He was particularly well posted on current events and took a deep interest in the political destiny of his adopted country. His religion did not so much embrace ritual and form as it did the broad practice of practical Christianity.

The last few years of his busy life brought to him sickness and suffering, which he faced with an equanimity and fortitude that won the admiration of his close associates. Under this stress, without the blare of bugle or the fanfare of drums, minus the panoply of war, Mr. Pitcher was a soldier, waging an unceasing battle against the ravages of disease, which eventually resulted in his death on March 15, 1932.

On October 21, 1891, he was married to Jennie Van Wagenan Terwilliger who survives him, together with a daughter, Marion Elizabeth (Pitcher) Gillingham, and two grand-daughters.

Mr. Pitcher was elected a Member of the American Society of Civil Engineers on March 2, 1909.

GEORGE ADDISON POSEY, M. Am. Soc. C. E.[1]

DIED AUGUST 3, 1932

George Addison Posey was born in Hermosillo, Sonora, Mexico, on September 28, 1883. He was the son of Dr. George Dixon Posey and Mary Edith (Stickle) Posey. His mother was the daughter of George Stickle, who arrived in California on August 20, 1849, and who became the principal citizen of the famous mining community of Angels Camp. He was the banker, postmaster, express agent, and leading merchant of the town. On his father's side, George Posey was descended from an aunt of George Washington, and he had several ancestors of Virginia stock who fought in the American Revolution.

George Addison Posey's early education was had in the schools of Berkeley, Calif. In August, 1902, he entered the University of California, in the College of Civil Engineering, and was graduated in May, 1906, with the degree of Bachelor of Science. He was a member of Tau Beta Pi Honorary Fraternity.

For several months after his graduation, Mr. Posey was employed as a Draftsman in the Maintenance-of-Way Department of the Southern Pacific Railroad Company. In October, 1906, he was appointed an Instructor in the College of Civil Engineering of the University of California. In July, 1907, be became Chief of Party, for the Mt. Whitney Power Company, on the location of reservoir sites and power plants, and, later, Resident Engineer of the Tule River Hydro-Electric Plant of that Company.

In June, 1909, Mr. Posey was employed by Haviland and Tibbetts, Civil Engineers, of San Francisco and Oakland, Calif., and from May, 1910, to October, 1921, was a member of the firm. During this period he was connected with the design of a wide variety of projects, including sewerage systems, concrete bridges and culverts, roads, streets, and tunnels. Some of the more important projects in this period were the location and construction of the Sacramento-Woodland Railroad (18 miles, electric); a report on harbor improvements for Richmond, Calif., followed by the design and construction of the Richmond Harbor Project, including the Richmond Tunnel and the Richmond Municipal Wharf; the Knights Landing Ridge Drainage District, the largest drainage project in California; sewerage systems for Pleasanton, Winters, Oroville, Lompoc, Etna, Stockton, and Santa Maria, Calif.; plans for the Richmond Municipal Water District; and water systems for Pleasanton, Corcoran, Santa Maria, and Lompoc.

In April, 1913, Mr. Posey was appointed Chief Deputy County Surveyor of Alameda County, California, which office he held until October, 1921, when on the death of his superior, he became County Surveyor. This office Mr. Posey held until his death, and it is evidence of the regard in which he was held in his community that he was elected and re-elected without appreciable opposition.

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[1] Memoir prepared by Harold Farnsworth Gray, M. Am. Soc. C. E.

Until the adoption of the new charter of Alameda County in January, 1927, Mr. Posey, in addition to his duties as County Surveyor, carried on an independent engineering practice in California, including streets, sewerage and water systems for sub-divisions in Alameda and Los Angeles Counties, and sewerage systems for the Niles, Irvington, and Union Sanitary Districts, in Alameda County.

Mr. Posey's work as County Surveyor was outstanding. Under the County Charter, all road and bridge work, both construction and maintenance, was concentrated in his office, instead of being divided, as is the usual practice, among the five supervisorial districts headed by a non-technical County Supervisor. By this arrangement, a definite, consistent policy of improvement and maintenance could be carried out, under competent engineering supervision. As a result, the roads in Alameda County were steadily improved, and low-cost pavements suited to traffic requirements were constructed and maintained adequately and economically. To-day, the County has a road system of acknowledged excellence.

Mr. Posey's greatest achievement, because of its size, the originality of its design, and its success in use, was the main traffic artery between Oakland and Alameda, passing under the Estuary, and appropriately named in honor of its designer, "The George A. Posey Tube." This structure, 3 545 ft. long, passes under an arm of San Francisco Bay at a maximum depth of roadway of 67.7 ft. below mean low water. At the time of its construction (it was completed on October 27, 1928), its cross-sectional area (37 ft. in diameter) was the largest of any subaqueous vehicular traffic tube in the world. The unique feature of its design and construction was the pre-casting of the tube in sections, in a dry dock. After casting and removal of forms, the sections were bulkheaded at each end, towed seven miles across San Francisco Bay, and sunk into position in a previously excavated trench. The section joints were then sealed outside with tremie concrete, and after the back-fill had been placed, the bulkheads were removed, the section was unwatered, and the joint completed from the interior.[3]

The Posey Tube is notable not only for its method of construction, but also for the perfection and economy of its ventilation and mechanical equipment. It was characteristic of the man that the most thorough study must be made of every detail of a project and that everything must be checked and re-checked until there was certainty that all was right. George Posey never permitted "guesses" in his practice. No man in public service was ever more conscientious in his endeavor to make certain the correctness of his design and construction.

Aside from the high order of his professional ability, and the fine and lovable quality of his personality, two characteristics of Mr. Posey stood out distinctly. One was his rigid honesty, not merely material or moral, but intellectual; the other was the extreme sensitiveness of his nature. The misery and misfortune of his fellow-man affected him keenly. Neither could he endure with indifference or composure the ignorant or biased criticism which is the frequent lot of the public official—it hurt him to the quick.

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[3] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 547.

Illustrative of the regard in which he was held, by high and low alike in his community, the East Bay Engineers' Club adopted the following memorial to him:

> "Conscientious, able, and just; scrupulously honest in thought, word, and deed; admired and respected by those who worked with him and for him; honored by the Engineering Profession for his great technical and administrative ability; loved by his friends, to whom he was loyal; his works will long survive to testify to the thoroughness, competence and integrity of his design.
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> "In him the people of Alameda County have lost a faithful public servant, and the East Bay Engineers' Club a valued and beloved member. To his bereaved family we extend our sincere sympathy in their great loss. May a valid pride in his distinguished career in some degree assuage their anguish."

Mr. Posey was a member of many organizations, among them being the Loyal Order of Moose, Woodmen of the World, Knights of the Round Table, Benevolent and Protective Order of Elks, and Masons; he was an officer in the Shrine. He was also a member of the Athens Athletic Club, in Oakland, the Society of California Pioneers, and the Orinda Country Club.

On September 16, 1914, he was married to Ethel Harriet Kesselring, who, with three children, Dorothy Carol, Barbara Jeanne, and George Addison, Junior, survives him.

Mr. Posey was elected an Associate Member of the American Society of Civil Engineers on February 6, 1912, and a Member on March 16, 1925.

SAMUEL MOREAU PURDY, M. Am. Soc. C. E.[1]

Died January 2, 1931

Samuel Moreau Purdy, the son of Victor Moreau and Julia Penelope (Larche) Purdy, was born at Lake Providence, La., on October 23, 1874. He was of Colonial stock, the Purdy family having originally settled in Rye, N. Y. His father served with distinction with the Confederate Army during the Civil War.

Young Purdy attended the public schools of his native State, and was graduated from the Louisiana State University in 1894. Following an urge to become an engineer, he continued his education with technical studies at Cornell University, Ithaca, N. Y., from which he was graduated with the degree of Civil Engineer in 1896. He was an excellent student, standing high in his classes, and was a member of the Kappa Sigma Fraternity. The influence of this well rounded education, supplemented by later reading and study, was apparent throughout his career in the efficient manner in which he approached and solved his problems, and in his diversified interests and broad outlook upon life.

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[1] Memoir prepared by Laurence B. Manley, M. Am. Soc. C. E.

After graduation Mr. Purdy's experience was similar to that of many other budding engineers. He began work as a Rodman with the United States Engineer Office, at New Orleans, La., and was engaged for a few months the following year as Rodman and Recorder for the United States Deep Waterways Commission, in New York, N. Y. During the summer of 1899 he obtained a position as Instrumentman on the Chicago and Northwestern Railroad, and made preliminary railway surveys for a branch line at Sanborn, Minn. From September, 1899, to August, 1900, Mr. Purdy was Assistant Engineer for the Niagara Falls Hydraulic Power and Manufacturing Company, Niagara Falls, N. Y., where he designed several features of an extension to Power House No. 2 below the cliff, including a steel penstock, and supervised their construction, the total cost of the work with which he was connected being about $1 000 000. In August, 1900, he was engaged as an Expert Computer assisting the late Emil Kuichling, M. Am. Soc. C. E., in hydraulic calculations connected with a report on the water supply for the New York State Barge Canal then in its inception. He also assisted Mr. Kuichling in his general practice on other details. Following this, was a short engagement with J. F. Whitman, Consulting Engineer, of Buffalo, N. Y., in connection with the appraisal of the water-works system of Jamestown, N. Y.

In January, 1901, Mr. Purdy obtained a position with the Bureau of Filtration of Philadelphia, Pa., as Assistant Engineer on the Lower Roxborough Plant, on which construction had been started. Here, he performed the duties of Field Assistant, directing the work of two field parties, giving lines and grades for this work, and making quantity surveys of the work done. On the completion of the Lower Roxborough Plant, he was transferred to the construction of the Torresdale Conduit, a pressure tunnel for conveying water from the Torresdale Filter Plant to the Ladner's Point Pumping Station. His duties in this connection were those of Field Engineer in charge of about one-half the length of the tunnel, including the establishment of lines and grades and the inspection of the construction. The tunnel was $10\frac{1}{2}$ ft in diameter, about $2\frac{3}{4}$ miles long, and about 100 ft underground. During this period, in his spare time, and in collaboration with his friend Glenn D. Holmes, M. Am. Soc. C. E., Mr. Purdy designed a sewage disposal plant for Batavia, N. Y. The plans were approved by the New York State Board of Health, but the plant was never built.

Following the engagement with the Bureau of Filtration of Philadelphia, Mr. Purdy entered into what was to constitute a large part of his life work —the construction of subways for underground transit. In July, 1903, he was engaged by the Philadelphia Rapid Transit Company as Assistant Engineer on the construction of the Market Street Subway and Elevated Railroad, in Philadelphia. He was first in charge of the field work of elevated construction in Delaware County, and on the completion of this work, was placed in charge of subway construction in Market Street at 15th Street, and the subway loop around the City Hall. Later, he assumed charge of the subway construction in Market Street east of City Hall to Eighth Street. He remained on this work under the supervision of Frank R. Fisher, Resident Engineer, until its completion in 1909.

This work was followed by two short engagements with contractors, first as Superintendent for Chiles and Witmer, of Reading, Pa., on the construction of a reinforced concrete water conduit along Maiden Creek, and, afterward, with the Milliard and Lupton Company, as the Company's Engineer in studying the methods of construction in a large contract for the New York State Barge Canal south of Rochester, N. Y.

While on this latter work Mr. Purdy was offered the position of Chief Engineer with the E. E. Smith Contracting Company, of New York, which had contracts for two sections of the Fourth Avenue Subway in Brooklyn, N. Y., amounting to a little more than $5 000 000. This work was followed by the construction of the three-track elevated line from Long Island City to Corona, N. Y., comprising the construction of the well-known reinforced concrete elevated structure and an extension of about four miles of steel elevated structure. The construction of the concrete part of the work comprising sixty-five arches dome-like in shape—that is, arched in two directions—involved unusual problems, the satisfactory solution of which was a matter of which Mr. Purdy was justly proud. The work with the E. E. Smith Contracting Company also included construction contracts for a section of four-track subway in Broadway—from Union Square to 26th Street—in Manhattan. This construction, which was directly under Mr. Purdy's charge, was particularly difficult on account of the congested nature of the street, both above and below ground, and the presence of two 36-in. water mains which were required to be maintained and supported for the length of the contract, as well as two 48-in. water mains which crossed the trench on a long diagonal at Fifth Avenue.

On January 16, 1918, Mr. Purdy resigned his position with the E. E. Smith Contracting Company to accept a position as Construction Supervisor for the J. G. White Engineering Corporation. He was immediately assigned to the general charge as Works Manager of the construction of a large steam-generating plant and appurtenant features at Muscle Shoals, Ala., a part of the large nitrate plant constructed by the Government at that place. The total cost of this work was about $14 000 000. Mr. Purdy also supervised other construction work for the J. G. White Engineering Corporation, including a $3 000 000 nitrate plant, near Syracuse, N. Y.; the additions to a chemical plant at Cleveland, Ohio, involving an expense of $1 250 000; the repairs and alterations to a chemical plant at Marcus Hook, Del.; and other jobs of lesser magnitude at Los Angeles, Calif., Passaic, N. J., and East St. Louis, Ill. After a brief engagement with Cross and Brown, Real Estate Agents, in New York, as General Superintendent in charge of the operation and maintenance of about thirty-five buildings of various types, Mr. Purdy returned to the J. G. White Engineering Corporation on May 1, 1922, in his former capacity. He was placed in charge of the construction of a steam power plant of 25 000-kw capacity at Pine Grove, Pa., costing about $3 500 000, including the construction of the steel power-transmission line to Pottsville, Pa. During this time he also acted as Business Administrator for the J. G. White Engineering Corporation in the construction of an oil storage plant for the United States Navy at Pearl Harbor, Hawaiian Islands. This work was administered from the New York Office of the Corporation.

On September 22, 1924, Mr. Purdy resigned his position with the J. G. White Engineering Corporation and was employed by the Department of City Transit of the City of Philadelphia, first as Assistant to the Chief Engineer, studying and reporting on various problems connected with the plans and construction for the Broad Street Subway, and afterward as Construction Engineer, which position he held until his death. In March, 1926, he was placed in charge of the construction of the four-track Broad Street Subway and its appurtenant features, including, in addition to the main subway construction, the station finish, track-laying, temporary and permanent paving contracts, the construction of the sub-station buildings, terminal yard buildings and equipment, and other miscellaneous work required for the completion and equipment of the subway, except the construction of cars and the contracts connected with the electrical equipment. Following the completion of the 6½ miles of subway construction in Broad Street, other contracts were entered into, the construction of which were under Mr. Purdy's supervision at the time of his death. These included the beginning of the work of straightening the tracks of the Philadelphia Rapid Transit Company beneath City Hall, the construction of a two-track branch of the Broad Street Subway in Ridge Avenue and Eighth Street, including the support of the Market Street Subway during the new construction below it, which was substantially completed during his life, and the beginning of the construction of a four-track subway and bridge in Market Street, at the Schuylkill River. The construction work for the Department of City Transit, either completed or begun under his supervision, involved the expenditure of about $100 000 000.

Mr. Purdy was well grounded in the fundamentals of his chosen profession and delighted to work out original solutions of these problems rather than slavishly to follow precedent. He had a keen analytical mind, combined with sound common sense, and an abundance of physical energy, which permitted him to attack his problems vigorously and to carry them through completely and thoroughly. He could make quick decisions, but was not given to snap judgments. By force of circumstances his professional career was concerned with heavy engineering construction in which he was an acknowledged expert, but his tastes ran to research and excursions into the realms of pure science. He had many and diversified interests and was well informed on current topics, including politics, finance, literature, music, and the fine arts. He was a ready talker, positive in his opinions, and with the background of his extensive experience and wide reading coupled with a retentive memory, a charming companion. During the later years of his life he pursued the hobby of the design and construction of radio receiving sets, of which he made a number for his friends.

Mr. Purdy was an example of the best type of engineer in the public service. Both in his capacity as a Contractor's Representative and as a City Administrator he was guided by the highest motives, and no suspicion of dishonest or under-handed procedure ever rested upon him. As an administrator of city work his example inspired and fortified his subordinates. If the necessity for it arose, he could wage a stiff fight for what he considered to be the right, and was no mean antagonist, but he was always a gentleman, dignified and courteous in his bearing, with thoughtful consideration at all times for others. He held the esteem and affection of his friends and associates, and the respect of all with whom he came in contact.

Mr. Purdy was a strongly religious man and was active in the Baptist Church. During the period from 1909 to 1924, when he lived in Brooklyn, he was active on, and became Vice-President of, the Board of Trustees of the Greenwood Baptist Church. At the time of his death, he was a member of the Trustees of the Chestnut Street Baptist Church, in Philadelphia.

On June 19, 1901, he was married to Fannie Minerva Lord, of Batavia, N. Y., who, with one son, and two grand-children, survives him.

He was a Registered Professional Engineer in the Commonwealth of Pennsylvania, and a member of the Engineers' Club of Philadelphia.

Mr. Purdy was elected a Member of the American Society of Civil Engineers on May 3, 1910.

FREDERICK GEORGE RAY, M. Am. Soc. C. E.[1]

DIED MARCH 21, 1932

Frederick George Ray, the son of James W. and Charlotte A. (Wicker) Ray, was born at Whitewater, Wis., on September 18, 1871. He received his early education at Parkersburg, Iowa, and at Cornell College, Mount Vernon, Iowa. In 1892, he was graduated from the University of Iowa, Iowa City, with the degree of Bachelor of Science in Civil Engineering.

Following graduation, Mr. Ray's first work was that of Contracting Agent for a bridge builder, at Manchester, Iowa, during 1891 and 1892. He then served as Recorder and Surveyor with the Mississippi River Commission from 1892 to 1900.

In May, 1901, he was transferred to the United States Lake Survey Office, at Detroit, Mich., and for nine years he was in charge of a steamer making surveys of: Apostle Islands, in Lake Superior (1901-02); Green Bay (1903-06); Straits of Mackinac (1907); and in the south end of Lake Michigan, off Chicago, Ill. (1908–09). While engaged in this work, he contributed largely to the betterment of methods used in hydrographic surveying.

In August, 1909, Mr. Ray was called from the field to become Principal Assistant Engineer in the Lake Survey Office in Detroit. During the World

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[1] Memoir prepared by a Committee consisting of Charles Y. Dixon, George H. Fenkell, Clarence W. Hubbell, Francis C. Shenehon, and Frank H. Stephenson, Members, Am. Soc. C. E.

War, from October, 1917, to January, 1920, he served as District Engineer in charge of the Office.

In December, 1920, feeling the need of a well-earned rest, he retired from the service; but in January, 1922, he returned to his former duties, to supervise the large amount of hydraulic study which became necessary because of changes and improvements in the connecting rivers of the Great Lakes. His long experience and recognized ability along these lines, made his services of particular value.

Mr. Ray first became interested in river hydraulics in 1908, when he was called upon to make a report on the effect upon lake levels caused by the construction of the Livingstone Channel in the Lower Detroit River, and to recommend means for the continued maintenance of such lake levels. He gave much thought and consideration to the derivation of natural discharge formulas for the St. Clair and Detroit Rivers, made extensive studies in regard to the effect upon lake levels of changes in the outlets, and in regard to possibilities of lake regulation and compensation. At the time of his death, he was engaged in revising the plan of compensating lake levels recommended by the St. Lawrence Waterway Board, and in devising means for maintenance of lake levels following the proposed deepening in Livingstone Channel, in the Detroit River.

He was a member of the Commission organized by the Chief of Engineers, United States Army, in February, 1931, to study formulas for flow in open channels, and to recommend the best formula for this purpose.

His very efficient work in the United States Lake Survey was made possible by the loyalty of his subordinates, who were inspired by his genial and kindly personality. In his consideration of problems, while giving due weight to the opinions of others, Mr. Ray was ever ready to express independently, yet courteously, his own views, with the purpose of arriving at correct solutions. As an engineer and as a man, he was held in high esteem.

Mr. Ray was a member of Corinthian Lodge No. 241, F. and A. M., at Detroit, Mich., the Detroit Athletic Club, and the Birmingham, Mich., Golf Club.

He was married on December 22, 1905, to Martha Lawton, of Ridgeway, Mich. There are two daughters, Ella G., and Charlotte.

Mr. Ray was elected a Member of the American Society of Civil Engineers on April 6, 1909.

FREDERICK JARRETT REINKE, M. Am. Soc. C. E.[1]

Died August 30, 1932

Frederick Jarrett Reinke, the son of Otto and Martha (Garner) Reinke, was born in Cassville, Wis., on July 30, 1867. He attended the High School of Cassville.

Mr. Reinke secured his first experience in Civil Engineering work in 1886 as a Rodman on the location and construction of the Minnesota and Northwestern Railroad, and, later, he was connected with construction work on the La Crosse, Black River Falls and Eastern Railroad and the Winona and Southwestern Railway, all in Minnesota.

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[1] Memoir prepared by Baxter L. Brown, M. Am. Soc. C. E.

In December, 1891, Mr. Reinke was appointed Resident Engineer on the construction of the Burlington, Cedar Rapids and Northern Railway, and served in this capacity until January, 1893.

Owing to the panic of 1893, and the consequent material reduction in engineering work, he was forced to seek other employment until August, 1898, when he was employed as Inspector with the United States Engineer Corps at Rock Island, Ill., on the Upper Mississippi River Improvement. In May, 1901, he became Resident Engineer with the St. Louis Valley Railway Company. The Missouri-Pacific System purchased the St. Louis Valley Railway in April, 1903, and Mr. Reinke was placed in charge of the construction of several large trestles on its White River Extension, remaining on this work until its completion in July, 1904. The remainder of the year was devoted to the location of a proposed electric railway in Indiana. During 1905, he was again in the employ of the United States Engineer Corps, at Rock Island.

The excellent record made by Mr. Reinke in connection with railroad construction resulted in his return to St. Louis, Mo., in October, 1905, as Assistant Chief Engineer of the St. Louis, Troy and Eastern Railroad, the East St. Louis, Columbia and Waterloo Railway, and, later, as Superintendent of the St. Louis Sand and Material Company, with which he was connected during the remainder of his life.

Mr. Reinke was held in the highest esteem by all who knew him for his honesty, loyalty, and strict attention to work placed in his charge. In fact, he sacrificed the pleasure and advantage of a large acquaintance among his fellow-workers in the Engineering Profession on account of his devotion to duty.

On October 12, 1892, he was married to Margaret Bell, at Osage, Iowa, who survives him. He is also survived by a son, Robert Charles, and a daughter, Thelma Marie.

Mr. Reinke was elected a Member of the American Society of Civil Engineers on July 2, 1913.

CARL JONAS RHODIN, M. Am. Soc. C. E.[1]

DIED MAY 3, 1932

Carl Jonas Rhodin, the son of Carl Joseph Theodore and Caroline Gustava Nathalia (Trahn) Rhodin, came of a family whose ancestry had been prominently identified with Swedish cultural history since the Reformation. He was born at Kongelf, Bohuslan, Sweden, on May 11, 1877. His elementary education in the Swedish preparatory schools was supplemented by special studies in foreign languages, of which he became proficient in five. He attended the Royal Technical University, of Stockholm, from which he was graduated in April, 1900, with the degree of Civil Engineer.

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[1] Memoir prepared by James T. Ryan, Assoc. M. Am. Soc. C. E.

His professional work began immediately afterward in the Department of Public Works of the City of Goteborg, as Assistant Engineer assigned to the design of harbor improvements. In January, 1901, he entered the service of the Royal Department of Roads and Waterways of Sweden, as Assistant Engineer of the Upper North District, with headquarters at Umea, which position he held until April, 1902.

Mr. Rhodin emigrated to America in May, 1902, and, after a brief experience as a Structural Draftsman with the firm of Snare and Triest, of New York, N. Y., was employed by C. Macdonald and Company, of New York, as Assistant Engineer and assigned to the construction of piers and foundations for a railway bridge across the Mississippi River, at Thebes, Ill. Upon the completion of this work in October, 1904, he moved to California and, in 1908, became an American citizen by naturalization.

His subsequent professional career was that of a general consultant and adviser, rather than that of a specialist in detail, and extended beyond the conventional technical field into the adjoining realms of finance, economics, and policy. His first work (1904 to 1906) was with the San Francisco Bridge Company, largely on plans and designs of bridges, foundations, wharves, etc., for contract bidding. During the great building era that followed the San Francisco fire, Mr. Rhodin contributed materially to the solution of the problems of design in earthquake-resistant structures. The development of water power, the rapid advance of industrial and rural electrification, and the numerous irrigation and water supply projects that were promoted in California during the ensuing period, afforded him an extensive range of employment and usefulness.

During 1909 and 1910, he was employed by the Southern Pacific Company to investigate the water-power resources required for an extensive electrification of its mountain lines in California and Oregon. From 1911 to 1916, in the capacity of Civil Engineer, he was attached to the San Francisco Offices of J. G. White and Company, Incorporated, of New York, which Company was employed as Consultants by the Pacific Gas and Electric Company, Mt. Whitney Power and Electric Company, Spring Valley Water Company, Oakland and Antioch Railway Company, Northern California Power Company, and numerous smaller utility and railway enterprises. In this position Mr. Rhodin's services were extremely varied and extensive, and included the supervision of design and construction of water supply, electric power, railway and pipe-line projects, studies and reports on the economic problems involved in utility service and industrial development, and the valuation of utility properties.

In May, 1916, Mr. Rhodin opened an office as Consulting Engineer in San Francisco and continued in active practice until his untimely death closed his career on May 3, 1932. During this period he served as Consulting Engineer for the Pacific Gas and Electric Company and the Great Western Power Company, as well as Valuation Engineer for the Great Western Company of California. His work in this connection consisted largely of appraisals, valuations, and reports for presentation before the California Railroad Commission, the United States District Courts, the United States Income Tax Department, as well as financial and engineering reports for presentation before the Commissioner of Corporations of the State of California. During this time the Pacific Gas and Electric Company and the Great Western Power Company were merged, and Mr. Rhodin served the merged Company as a Consultant until his death.

In addition, he became interested in civic affairs and took an active leadership in the study of some of the problems of urban development and municipal government. In 1924, he was made Chairman of the City Planning Section of the Commonwealth Club of California, on motion of Chief Justice William H. Waste, and, from 1924 to 1927, he was Executive Secretary of the Regional Plan Association for the San Francisco Bay Counties.

Mr. Rhodin was an enthusiastic yachtsman and took an active interest in the annual yachting events on the Pacific Coast, serving on executive committees in numerous regattas and frequently as judge. He was a recognized authority on handicapping and the author of a notable monograph on that subject.

He was a member of the Commonwealth, Engineers, Yacht, and Commercial Clubs, of San Francisco, of the Mt. Diablo and Claremont Country Clubs, and of the American Society of Swedish Engineers.

His widow, Grace Margaret (Cook) Rhodin, to whom he was married on April 22, 1908, and a son, Carl William, born in 1916, survive him.

When a man of character and ability emigrates from the land of his birth and achieves recognition as a leader in the professional and civic activities of his new National home, he brings to it a vital contribution to its material well-being as well as to its cultural development. The career of Carl Jonas Rhodin is an inspiring instance of this, and deserves the appreciative recognition of his fellow citizens as well as of his professional associates.

Mr. Rhodin was elected a Member of the American Society of Civil Engineers on December 3, 1912.

JOHN AUGUSTUS ROEBLING, M. Am. Soc. C. E.[1]

Died July 22, 1869

John Augustus Roebling was born on June 6, 1806, in Muhlhausen, Germany, which is located in Thuringen, at that time one of the poorest German States. Life was bleak and frugal and it was chiefly through the thrift and self-denial of his mother, that her youngest son, John, was enabled to secure a thorough education. At an early age, he entered Doctor Unger's Pedagogium, in Erfurt, where he won the admiring friendship of this well-known scholar. Later, he was enabled to enroll in the Royal Polytechnic School, at Berlin, Germany, where he studied architecture and engineering with Stuler and Rabe; bridge construction with Dietleyn; hydraulics with Eytelwein; languages in the regular course; and philosophy under the great Hegel, who openly avowed that John Roebling was his favorite pupil. He was graduated in 1826 with the degree of Civil Engineer.

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[1] Memoir compiled from data submitted by the John A. Roebling's Sons Company, Trenton, N. J.

Notwithstanding this distinctive training, John Roebling migrated to America in 1831 with the avowed intention of becoming a farmer. Upon his arrival in the United States, Mr. Roebling selected a tract of land in Western Pennsylvania in a little town named Germania (later known as Saxonburg), twenty-five miles from Pittsburgh. At that time this was considered the frontier region of the Far West.

Farming proved tedious to his progressive mind and fate seemed to ordain that Mr. Roebling's engineering skill should not remain fettered to a Pennsylvania plow handle.

The system of canals and portages, which afterward evolved and merged into the Pennsylvania Railroad, was then building haulways and canals past the hamlet of Saxonburg and fatefully under the eyes of the young German engineer. He abandoned the plow and plunged into the problems of construction where he belonged. He obtained the position of Assistant Engineer on the slack-water navigation of the Beaver River, a tributary of the Ohio River, where he located a feeder for the Pennsylvania State Canal, and for three years he was occupied in surveying and locating three lines of railways across the Alleghany Mountains, from Harrisburg to Pittsburgh, Pa. The road was ultimately built by the Pennsylvania Central Railway Company and not by the State.

His attention was directed to the tedious hauling of the canal-boats of the Portage Canal across the Alleghany Mountains. This was accomplished by building the boats in sections. When the base of the mountains was reached, these sections were loaded on cars and hauled up an incline with a manila rope windlass. By a like process they were lowered down the opposite side of the mountain. These clumsy manila ropes were several inches in diameter, expensive and short-lived. About this time, John Roebling read an article in a German paper that a strong rope of wires twisted together, had been made in Freiburg, in Saxony—where wire drawing had its birth.

John Roebling opined that if a wire rope could be made flexible enough to be wound on a windlass, it ought to cost but little more than the hemp cable and would possess greater tensile strength with one-fourth the diameter. Moreover, it would outlast a dozen ropes woven from the vegetable fiber. No one in America had ever made a wire rope, or even seen one, but the idea was at least worth a trial. So he built a rope walk on his farm at Saxonburg, purchased a quantity of wire suitable for his purpose, instructed his friends and neighbors in the art of rope twisting, and actually fabricated a wire rope which surprised his most buoyant expectations. He did not stop here. The wire rope led to the wire cable, later to be used in connection with canals.

A canal, which is really an artificial river, must sometimes cross a natural river—must somehow be made to flow above it. In this case, the canal becomes a wooden aqueduct. In those days the building of such an aqueduct was a large and hazardous undertaking for frequently the ice in the river would gorge and crush the piers and abutments, permitting the canal itself to join the procession and float off to sea, thus destroying the aqueduct.

John A. Roebling's inventive mind evolved an idea. While yet a student at the University, on a vacation hike through Northern Bavaria, he saw for the first time a bridge suspended by chains. He studied the structure, sketched it, and made it the subject of a thesis. He now recalled his youthful essay and conceived the idea of a cross-river aqueduct suspended from wire cables. This would eliminate piers. He submitted his designs to the engineers of a Canal Company that was planning to cross the Alleghany River at Pittsburgh. He frankly admitted that what he proposed to do was without precedent, but he insisted that his figures were correct and the advantages to be gained justified some risk.

His scheme appealed to the American spirit of shrewd adventure and daring enterprise, and he was ordered to proceed with the work, knowing that the outcome would either place him in the forefront of American Engineers, or ruin him forever. The undertaking was a success and led directly to the building of several of these suspended aqueducts which are still in use. A suspended aqueduct is really a suspension bridge carrying an enormous load. John A. Roebling recognized this fact and the whole world knows and has profited by the result of his ideas.

The crude factory at Saxonburg and the poor shipping facilities proved the undesirability of that location and, in 1848, John A. Roebling moved his factory and family to Trenton, N. J., a journey which at that time required several days. He was the architect of every building in his new plant, as well as the inventor and the designer of every piece of machinery that was installed in the factory. It was not until many years later that he could be persuaded to employ an assistant engineer or draftsman.

How and where John A. Roebling found time to do all he did—to attend scientific conventions; write voluminously for scientific journals; practice the flute and piano; study metaphysics; invent tools and machinery; make his own drawings for the Patent Office; design bridges and superintend their construction—bewilders the imagination.

During this period, 1851, he began the erection, at Niagara Falls, of a suspension bridge, with a double roadway, for railway and vehicular use. A truss system had been developed previously by the late Squire Whipple, Hon. M. Am. Soc. C. E.

For four years the work on this bridge continued without interruption, during the coldest Canadian winters, and it was completed in March, 1855. This suspension bridge with a clear span of 800 ft was supported by four wire cables, each 10 in. in diameter, and had two decks, connected by struts and diagonal tension rods, so that the superstructure formed a continuous hollow girder, stiff enough to support a rolling load, the weight being entirely supported by the cables. Due to the novelty of its design, most eminent engineers regarded this bridge as foredoomed to failure, but Mr. Roebling fought his detractors, undiscouraged, and with its complete success demonstrated by long use, the number of suspension bridges greatly increased. Through his effort, the use of parallel wire cables in place of chain cables has become all but universal.

In 1856, the foundation of the towers for the Ohio River Bridge at Cincinnati, Ohio, was built. This bridge has a clear span of 1 057 ft and proved to be still more remarkable than that over the Niagara River. After the completion of this bridge, due to the outbreak of the Civil War, there was a short cessation of similar enterprises.

The crowning achievement of his life was the conception of the Brooklyn Bridge, spanning the East River, between Manhattan and Brooklyn. This is indeed a marvel of daring skill, the work of a master mind—and a monument to its designer. Sadly enough John A. Roebling lost his life at the very inception of the actual construction work. He was making a survey near the Fulton Ferry slip on the Brooklyn side of the river when the abrupt entry of a ferry-boat crushed his foot between the piling and the rack on one of the slips. This accident occurred on June 28, 1869. Tetanus developed and he died in Brooklyn, on July 22, 1869. John A. Roebling's oldest son, Col. Washington A. Roebling, had been so closely associated with his father in the preparation of the design that he was capable of continuing the work with slight interruption.

It has been said that John A. Roebling worked too hard, pouring his strength and determination into every minute of the day. In this connection the following incident is related: During the Civil War, General Fremont sent for Mr. Roebling and kept him waiting in the anteroom. Mr. Roebling took a card and wrote upon it: "Sir, you are keeping me waiting. John Roebling has not the leisure to wait upon any man." His rule was to postpone a conference if the gentleman with whom he had an appointment happened to be five minutes late in keeping it. It was, of course, this extreme application of every instant that enabled Mr. Roebling to do the work of ten men. He worked too hard to have time to be happy.

At a meeting of the Board of Directors of the New York Bridge Company, held July 24, 1869, at the office of the Brooklyn Gas Company, to take into consideration the death of the Engineer of the Brooklyn Bridge, the following resolution was adopted[2]:

"*Resolved, First*—This Board receives the death of John A Roebling, Esq., with the deepest concern and sorrow. Connected with him officially for a considerable period, we have learned to appreciate his unsurpassed merits as an engineer and adviser in our work, and to admire his eminent genius and virtues as a man. In all his qualities which exalt human nature he deserved and had won our unqualified confidence and esteem.

"Regarding his death as a great loss, we bow with humble submission to the inscrutable ways of an All-Wise Providence in taking him from us in the midst of his usefulness and labors.

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[2] *Proceedings*, New York and Brooklyn Bridge (1867-1884), p. 326.

"*Second*—We sincerely condole with his wife and family in the dispensation now dealt to them, which deprives them of a husband, father and friend, and tender them our sincere sympathy and consolation in their affliction.

"*Third*—As a tribute of respect to Mr. Roebling's memory the members of this Board will attend his funeral at Trenton to-morrow."

He was married in 1836 to Johanna Herting, the oldest daughter of Ernst Herting, who had come to the United States from Muhlhausen in 1834. They had nine children of whom Col. Washington A. Roebling was the oldest.

Mr Roebling was elected a Member of the American Society of Civil Engineers on December 2, 1868.

ALBERT OWEN ROWSE, M. Am. Soc. C. E.[1]

DIED AUGUST 16, 1932

Albert Owen Rowse was born at Davenport, Iowa, on December 1, 1865. His parents, John Blake and Angeline (Owen) Rowse, were early settlers, having driven from Kentucky to Davenport in a covered wagon. His grandfather, Edward Rowse, was born at Newton Abbot, Devonshire, England, and his grandmother, Mercy Blake, was born at Bath, Me., of Puritan stock. Mr. Rowse attended the public schools of Davenport and was graduated from the High School of the same city. Later, he attended Griswold College, in Davenport, going to Grinnell, Iowa, when this school became Grinnell College.

His first employment in engineering work was on preliminary and location surveys, in 1884, for the Burlington, Cedar Rapids, and Northern Railway in Iowa, Minnesota, and Dakota, as Chainman and Auxiliary Transitman. For four years (1885 to 1888) he was on surveys for and construction of the Chicago, Rock Island, and Pacific Railroad, as Rodman, Levelman, Transitman, and Division Engineer.

In September, 1888, Mr. Rowse started work under the United States District Engineer, Corps of Engineers, War Department, on surveys of the Rock Island Rapids of the Mississippi River as Transitman, and as Inspector on Construction (by contract) of dikes and revetments in the vicinity of Muscatine, Iowa, and Keithsburg, Ill. This was the beginning of a long and successful career in the Government service, which was continuous, except during 1891 and a part of 1892, when he was engaged on surveys for a logging railroad in Northern Michigan; a large park near Lake Geneva, Wisconsin; and construction work on the Chicago, Rock Island, and Pacific Railroad, in Indian Territory.

From August, 1892, to November, 1894, he was Resident Engineer in charge of the construction of a section of the Illinois and Mississippi (Hennepin) Canal around the Lower Rapids of Rock River at Milan, Ill. This work consisted of three miles of canal trunk, two locks, one pontoon bridge, one railroad swing bridge, and one highway swing bridge.

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[1] Memoir prepared by Fred W. Honens, M. Am. Soc. C. E., assisted by G. C. Haydon and J. W. Woermann, Members, Am. Soc. C. E.

From 1894 to 1897, Mr. Rowse was in charge under the late L. L. Wheeler, M. Am. Soc. C. E., Assistant United States Engineer, of the field work for making topographical and right-of-way surveys for the Feeder and for the Western Section of the Illinois and Mississippi Canal, and in writing the descriptions of lands required for right-of-way and flowage purposes.

From 1897 to 1907, he was in charge of construction work on certain parts of the Eastern Section and the Feeder of the same canal under Assistant Engineers J. C. Long and L. L. Wheeler, respectively. During this time Mr. Rowse was engaged, on the Eastern Section, on Miles 17, 18, and 19, of the canal prism, and Locks 19, 20, and 21; and on the Feeder, on the canal trunk of Miles 1 to 8; the superstructure of one railroad bridge; seventeen highway bridges; an aqueduct with reinforced concrete lining and large automatic emergency gates; and three arch culverts, part by contract and part by hired labor. He was engaged, also, on the excavation of a rock cut of 80 000 cu yd on Mile 1, the material being removed by a narrow-gauge railroad and used for rip-rapping the banks of Miles 1 to 23, inclusive. From October, 1907, to February, 1910, he was in charge of maintenance and various works for completing and operating the canal.

During 1910 and 1911, Mr. Rowse made a survey of the Des Moines River from Des Moines, Iowa, to the mouth. This work consisted of a topographical survey, the preparation of designs and estimates for the canalization of 300 miles of the river, and investigations concerning the feasibility of hydro-electric installations, under the direction of the late Col. C. S. Riché, U. S. Corps of Engineers, F. Am. Soc. C. E., in charge of the Lake Survey Office, Detroit, Mich.

From December, 1911, to December 31, 1930, the date of his retirement because of disability, Mr. Rowse was employed on the improvement of the Missouri River and its branches, under the United States District Engineer, at Kansas City, Mo. He had charge of all contract and hired labor work on the Gasconade Section of the Missouri River, which consisted of the construction of revetments, dikes, and other protective works. He also had charge of the Gasconade Boat Yard where all major repairs and the greater part of the minor repairs were made to the numerous steamboats, snagboats, quarter-boats, barges, pile-drivers, and other floating plant in the Kansas City District, and where most of the new floating plant is built and equipped. This work reached its climax in 1928, when 1 500 men were employed and more than $2 000 000 was expended on work in the Gasconade Section and Boat Yard.

From September, 1914, to August, 1916, he had charge of all the work on that part of the Missouri River from Kansas City, Mo., to Fort Benton, Mont. This called for frequent inspections of the river, surveys, estimates, and plans of needed work, with reports and recommendations to the District Officer.

Mr. Rowse was a quiet unassuming man, diplomatic and considerate in his dealings with others, and in those who came to know him, he inspired confidence and trust. He was loved and respected for these qualities by the employees who worked under him, and was highly appreciated by the officers in charge and his associates in the work. Because of his unswerving integrity and upright dealings with his fellow men, it came to be known that his actions were above reproach and his decisions just.

A paragraph in the order of Capt. Theodore Wyman, Jr., U. S. Corps of Engineers, announcing his retirement on account of disability, shows the esteem in which Mr. Rowse was held:

> "Mr. Rowse's record is a splendid and remarkable one of long, faithful and distinguished service to the United States. Throughout his service, his work has been characterized by the highest quality, and the display of leadership, engineering skill, and devotion to duty of exceptional order. He has always been known as a man who could be relied upon to competently, energetically, and expeditiously complete any tasks assigned him. He exemplifies the finest of the members of the Engineer Department. The Department of this District suffers an irreparable loss in the retirement of Mr. Rowse. His record of service and his accomplishments should be an inspiration to the rest of us who will carry on where he was forced to leave off." And Captain Wyman added, "The hearty best wishes of the district and an ardent hope for the early recovery of his health will accompany Mr. Rowse in his retirement. It is to be hoped that we all may have 'well done' written in as large letters at the end of our record as is the case with Mr. Rowse."

On November 1, 1887, he was united in marriage to Fannie E. Eckert, at Meade Center, Kans. To this union were born two sons, George Albert and John Blake. The latter died at Sterling, Ill., in 1905. Their home changed a number of times due to the various positions held by Mr. Rowse, but he remained at Sterling for eighteen years, at Gasconade, Mo., for twenty years, and at Rock Island, Ill., after his retirement. He passed away in the latter city. The direct cause of his death was the result of cerebral hemorrhages, the first of which occurred shortly before his retirement. His body was placed in the family lot at Sterling.

He is survived by his widow, his son, George Albert, and a grand-daughter, Mrs. Betty Rowse Cavanaugh.

He was a member of the Presbyterian Church and during his residence in Sterling for a number of years was one of its honored officers. He was a member of Rock River Lodge, A. F. and A. M., at Sterling.

Mr. Rowse was elected a Member of the American Society of Civil Engineers on March 4, 1908.

COLEMAN SELLERS, M. Am. Soc. C. E.[1]

DIED DECEMBER 28, 1907

Coleman Sellers, whose paternal ancestor came to Pennsylvania in 1682. was born in Philadelphia, Pa., on January 28, 1827, the son of Coleman Sellers and Sophonisba (Peale) Sellers, about a month before the death of his maternal grandfather, Charles Willson Peale, the distinguished portrait painter of the Revolution and subsequently founder of the first Museum of

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[1] Memoir prepared by S. Howard Rippey, M. Am. Soc. C. E.

Natural History in America. His paternal grandfather, Nathan Sellers, an officer in the Philadelphia Militia in the Revolution, was recalled from the field by a special resolution of Congress to devote his mechanical skill to the manufacture of moulds required by the paper makers. His son, Coleman Sellers, Sr., further developed the manufacture of paper-making appliances, besides engine building, and his firm when called upon by the Columbia Railroad to build locomotives in 1834 first suggested iron frames, outside cylinders, and other important features in the design, which have survived as fundamental in the later development of the American locomotive.

With these antecedents and under the influence of the characteristics which marked both lines of his descent, Coleman Sellers developed the versatility and broad scope of his interests and activities by which, through his individual genius and energy, he contributed to the advancement of Applied Science during the Nineteenth Century.

His early education was secured in private schools, including that of John Haslem, in the Franklin Institute Building, and at Bolmar's Academy, West Chester, Pa., where his interest in natural philosophy led him to attempt to make instruments described in the lectures on the subject, such as an improved hygrometer which he contrived at this time, in his sixteenth year. After leaving school he was encouraged to learn farming and during the short time he was so engaged, when 17 years old, he invented a hay rake which anticipated the horse rake which, later, was re-invented by others and came into general use.

His mechanical propensities conspicuous in his school days were displayed when at the age of 19, while employed in surveying for a Kentucky railroad, he made a Locke hand-level for his use. Later, Mr. Sellers was employed as a Draftsman by his older brothers, who owned the Globe Rolling Mills, in Cincinnati, Ohio. Before reaching his majority he was made Superintendent and General Manager of the plant and in this connection assisted his elder brother, George Escol Sellers, in building the "hill-climbing" locomotives of his invention for the Panama Railroad. This experience resulted in his engagement, in 1851, by Niles and Company, Locomotive Builders of Cincinnati, as Foreman and, later, as Superintendent.

His design of special machine tools for locomotive building while with Niles and Company resulted in 1856 in his being induced by his kinsman, William Sellers, to return to Philadelphia and take charge of the Drafting Room of William Sellers and Company. This position widened the scope of his activities, and his improvements in the design of machine tools and the invention of a system of power transmission and other appliances resulted in his being admitted as a partner of the firm and Chief Engineer of the Works in 1873.

Dr. Sellers continued his active interest with the firm to its advantage until 1886, when ill health forced him to retire and, subsequently, he was induced to undertake a general professional practice. For more than thirty years his ever-expanding activities had covered a wide range of scientific investigation, primarily devoted to making the products of his firm preëminent for efficiency and high standard of performance, as recognized, not only in the United States, but abroad, and in so far as his inventions have advanced the "state of the art," he held a distinguished place in his special field of engineering.

While attaining this success in his chosen work, Dr. Sellers' abounding interest in applied science and the arts led him incidentally into various phases of research and invention. His work in the early development of photography and its application to his engineering practice was outstanding, and one of his important inventions covered the use of glycerine to keep plates wet indefinitely, which was of great utility and value in photography before the advent of dry plates. He was a pioneer in photography by artificial light, and, in 1873, delivered a course of lectures at the Franklin Institute in which this subject was reviewed. He was the first to discover and suggest the use of absorbent cotton in surgery, as now so universally applied, and he also invented the "kinematoscope," the forerunner of modern motion picture apparatus. His philanthropic activities during the Civil War were numerous and, later, his interest in the Society for the Prevention of Cruelty to Animals led him to call attention to the need for specially trained veterinarians. It was upon his initiative that the Veterinary Department of the University of Pennsylvania was founded, through the liberality of his friend and associate in the Society, Mr. J. B. Lippincott. His sympathies thus extended to the well being of animals, were characteristic of his ever generous contribution of service in educational and public affairs in which his experience and resources could be applied advantageously. He was a wise and sympathetic counsellor to his junior associates, always ready to assist in solving their specific problems, and in guiding and advancing them by his advice and influence into successful professional careers.

Upon retiring from active participation in the business of William Sellers and Company, Incorporated, Dr. Sellers retained his financial interest, and was ably represented by his son, the late Coleman Sellers, Jr., who, in due time, became President following the death of William Sellers.

Thus relieved of his former duties, Dr. Sellers was free when his health permitted to respond to the many demands made upon him for professional advice in his varied field of engineering. He also accepted the chair of Engineering Practice in the Stevens Institute of Technology, at Hoboken, N. J., and it was during this period that his recognized pre-eminence in his field of engineering led to his selection for the work by which perhaps he became best known to the public and which in its results marked the epoch-making advent of what has been termed the "Hydro-Electric Age."

Through the years prior to 1886 there had been some small local utilization of part of the head of water available at Niagara Falls. In 1875, a flour mill operating under 25 ft. of the 210-ft. head available, first developed power from the hydraulic canal for which ground had been broken in 1853. Other local industries, using varying small portions of the total head, were built to use the water supplied by this canal during the following years, but although there was practically unlimited water available, Niagara Falls was by 1886 only a village without means for any large-scale development or utilization of any substantial amount of the potential power of the Falls.

In 1886, initial steps were taken by some local manufacturers to secure the necessary legislation and financial support for a project to develop a larger percentage of the hydraulic power then going to waste. The only hope of utilizing such power if developed at that time was to attract to a village of less than 5 000 population a large number of manufacturers who would build individual plants to secure the cheap water power proposed to be created at mill sites. Although important legislation was secured and public interest aroused, the promoters of the undertaking were unable to devise any practical basis for creating such a demand for power in the form contemplated as to justify the expensive construction proposed. Continuing efforts to carry out the large development were made during a period of about three years, but all attempts to finance the project proved futile until, in August, 1889, a half-interest in the enterprise was offered to Winslow, Lanier and Company, New York bankers, whereupon the late Edward Dean Adams, F. Am. Soc. C. E., a partner in that firm, undertook to investigate the merit of the proposal. As a result of the technical guidance which he sought and his own great executive ability and financial resourcefulness, there was soon launched the "project" which as described was "destined to develop into the first successful application of Niagara power on a scale commensurate with the magnitude of the Falls and with the demands of modern industry." The Niagara Falls Power Company and its acting agent, The Cataract Construction Company, in executing this monumental work, availed themselves of the best talent in every related field of engineering, science, and practical experience. Dr. Sellers was called upon to consider the reports of the several engineering authorities consulted and upon the result of his investigations and report as to the feasiblity of the project, final action was taken and the dominant and responsible rôle was assigned to him by the Directors of the enterprise to its successful realization.

When the United States Government, as a World War measure, brought about the consolidation of the competing hydraulic power developments at Niagara Falls, the retiring Directors of The Niagara Falls Power Company prevailed upon Mr. Adams to prepare a history of the project "because of his great sentiment for and long association with the enterprise." This comprehensive work entitled "Niagara Power" was dedicated

"To
Coleman Sellers
1827–1907
Chief Engineer
The Niagara Falls Power Company
This record is dedicated
by the Author
as a tribute of
Honor and Affection"

This dedication, as well as the credit given by Mr. Adams throughout the record to Dr. Sellers' services in bringing about the successful results there attained, and upon which modern hydro-electric power development has been so largely based, is an ample recognition of the engineering skill and judgment of the Chief Engineer, by the man who, in an executive and financial capacity, was more than any other individual responsible for guiding the novel undertaking to practical commercial success.

Mr. Adams records that:

> "After careful inquiries for an engineer of broad experience in mechanical and electrical problems * * * Coleman Sellers of Philadelphia, was engaged about October 1, 1889, to investigate the conditions at Niagara Falls, to advise as to the development of power, and to consider the relative merits of the systems available for local and distant use, the extent of their commercial application, their economies as compared with steam, and their practical application to the project for the development of Niagara power * * *. Dr. Sellers made three formal reports during that period to Jan. 1, 1890, from which date his services were permanently established as consulting engineer of The Cataract Construction Company."

As the result of the studies, certain decisions were reached as to the basic features of the hydraulic tunnel plan, funds were subscribed in January, 1890, and the "Chief Engineer, Dr. Sellers, proceeded with the preparations for construction on the preliminary plan proposed, and organized an engineering staff for his assistance." Mr. Adams, President of The Cataract Construction Company, meanwhile proceeded to Europe in quest of information as to the state of the science of power development and the art of its transmission, especially in France and Switzerland, where limitations of fuel had led to intensive applications of water powers for industrial purposes. By May, 1890, Mr. Adams had visited many plants, had conferred with many European designers and operators of hydraulic stations, and had reached the conclusion that practice there was ahead of that in the United States (where abundant fuel resources and transportation facilities had naturally directed the greatest power development along the lines of highly efficient steam plants). On May 11, 1890, he therefore proposed to his Company the invitation of "American, English, French, and Swiss houses to submit competitive preliminary schemes to commission composed of Sellers, Edison, and English, French and Swiss engineers, one each." He also requested Dr. Sellers to join him in London at once, which Dr. Sellers did, and after reviewing the situation they together called on Sir William Thomson (Lord Kelvin) on June 2, when Sir William accepted a suggestion previously made to him that he serve as Chairman of an International Niagara Commission. The other members were promptly selected and the Commission was organized as follows: Sir William Thomson, *President,* Dr. Coleman Sellers, Philadelphia, Pa., Professor E. Mascart, Paris, France, Colonel Theodore Turrettini, Geneva, Switzerland, and the late William Cawthorne Unwin, Hon, M. Am. Soc. C. E., *Secretary.*

The first meeting of the Commission was held on June 21, 1890, and without delay it proceeded with the Company's program of offering a series of prizes and premiums to a number of selected representative designers in the several countries for the best practicable schemes for the development, transmission, and distribution of the power proposed to be created at Niagara Falls. The generous compensation offered the competing engineers and manufacturers ensured securing carefully studied proposals for the several features of the development, and the plans submitted covered a variety of methods and systems which represented the most advanced actual accomplishments to date in the entire world, backed by the scientific knowledge essential to visualizing an undertaking entirely beyond precedent. Dr. Sellers spent about nine months in Europe in the interests of the Niagara Project, during which time he inspected many developments, conferred with many engineers, and studied many novel proposals. In his activities on the Commission and in his reports and recommendations to his Company, he was able, therefore, to make decisions based not only upon his own experienced judgment and inventive genius, but was supported by a most comprehensive knowledge of the best that had been done in the world along related lines and enjoyed the advantage of full discussions with some of the world's best thinkers of the many novel problems involved in the Niagara enterprise.

No combined project for the development and distribution of power was submitted to the Commission which in its judgment could be recommended for adoption without considerable modification. On March 6, 1891, Dr. Sellers wrote to The Cataract Construction Company:

> "What has been accomplished by means of the International Commission may be expressed in a few words. It has shown us what we can reasonably hope to do; it has shown us in a degree how to do what is wanted and where we can expect to get what we require to do the work. It has brought the scheme before the world with a prestige that cannot be measured by dollars; it has enlisted the interest of the whole scientific world; it has made this company command the confidence of the world and won for its management respect, as wise, far-seeing, cautious business men, and not followers of any one or more visionary schemer or inventor."

In effect, the Commission assembled for the Company's consideration data covering the state of the art of power development and transmission, but at best this could only be a start in working out the problems influencing the design and construction of such a complete power project as was contemplated at Niagara, and a tremendous amount of pioneer engineering still had to be done by the Company's own organization.

Dr. Sellers had been appointed Consulting Engineer of The Cataract Construction Company (subsequently, Chief Engineer) and Chief Engineer of The Niagara Falls Power Company from January 1, 1890, when he immediately planned a staff to deal with the various phases of the preliminary work; he also organized the "Niagara Advisory Board of Engineers," including the late John Bogart and George B. Burbank, Members, Am. Soc. C. E., Clemens Herschel, Past-President and Hon. M. Am. Soc. C. E., Albert H. Porter, Assoc. M. Am. Soc. C. E., and Colonel Th. Turrettini, with Dr. Sellers as Chairman, which functioned until the end of 1892. (Dr. Sellers became President of the Niagara Falls Power Company in 1893, continuing also as Chief Engineer). When the Board of Engineers was abolished full supervision of the various engineering details rested with Dr. Sellers, requiring his approval of all construction plans, thus placing upon him full responsibility for deciding all engineering questions involved, and being in constant touch with every phase of the engineering work, he was able to co-ordinate all the elements entering into the creation of such a novel plant so as to ensure its ultimate operation as a harmonious "unit" of power.

It is unnecessary to repeat here an account of the Niagara Development except in so far as it may be required to indicate the magnitude of the task which faced the engineers. The requirements for the hydraulic works and machinery were beyond precedent in many respects, but assuming those problems to be capable of satisfactory solution, there remained the fundamental question concerning the best means of ensuring the efficient and commercially successful utilization of the power thus to be created. The plan of the original promoters for continuing the old type of industrial community based upon each manufacturer having his own turbine wheels was soon rejected in favor of a central turbine plant with large units of high efficiency operating under the greatest head available, but the question of how to transmit and apply the mechanical power created at the turbine shafts was much more difficult of solution because of the limited progress which had been made to that time along such lines. Serious consideration had to be given to the alternative methods of transmitting and utilizing power which were then being used successfully in different countries and some of which were presented to the International Commission in great detail and earnestly advocated by their proponents. Among the systems proposed and considered were mechanical transmission to near-by plants by shafting, gears, and belts; telodynamic or wire-rope transmission; high-pressure water transmission; compressed-air transmission and pneumatic utilization; and direct-current, and various alternating-current, electric systems. It serves to illustrate how significant to the whole modern power program were the decisions then reached when it is realized what opposition had to be overcome before agreement could be reached upon the adoption of alternating-current generation, transmission, and distribution, and what apparatus had to be developed for each part of such a system. Mr. Adams has stated that in 1889,

> "The thorough examination and resulting opinion, as expressed by so eminent an electrician as Mr. Edison, were convincing that the use of electricity had not then been sufficiently developed for its economical and profitable distribution in long-distance transmission for power purposes."

Of the many proposals received by the Niagara Commission in 1890, "only two * * * advocated alternating current, and Prof. Forbes alone, proposed polyphase currents, and his recommendations were based on theory and expectation rather than accomplishments." As late as 1893, Lord Kelvin proposed using direct-current generators in series. "Opposition to the use of alternating current was led in England by Lord Kelvin and in the United States by Thomas A. Edison."

In 1890, the late George Westinghouse, M. Am. Soc. C. E., while a proponent of alternating-current transmission for electrical currents, believed steam power could be generated more cheaply in Buffalo, N. Y., than power could be transmitted from Niagara. He expressed the further belief that pneumatic transmission to Buffalo would probably be adopted, thinking it might be found cheaper than electrical transmission and having certain practical advantages in application to existing conditions. Lord Kelvin cabled on May 1, 1893: "Trust you avoid gigantic mistake of adopting Alternating Current."

It is interesting to record that on August 23, 1897, after a visit to Niagara to see the development in operation, Lord Kelvin, then President of the British Institution of Electrical Engineers, wrote:

> "I was greatly pleased with all the engineering, both hydraulic and electric, which I saw in the power-house, and in the distribution of the power to the factories in the neighborhood and to the trolley lines and most particularly to Buffalo for the distant section of the line between Buffalo and Niagara. The results thoroughly justified Dr. Coleman Sellers in his adoption of the alternate-current system at a time when the promise of good results from it on a large scale was only beginning to come from the tentative work of a much smaller character."

In 1893, however, it was in the face of the opposition cited, and that of most manufacturers of electrical apparatus, to alternating current that a decision had to be reached, and Mr. Adams records that,

> "* * * the three technical advisers of the company finally united in recommending the alternating current system. Prof. Forbes for some years had consistently recommended it; Prof. Henry A. Rowland when first retained in 1889, advocated the continuous current system, but after thorough study of the matter and practical demonstrations, united with Prof. Forbes in its recommendation and Dr. Sellers recommended it for its economy and utility for the company's purposes."

On the basis of the recommendation made, the Directors of the Company on May 6, 1893, officially adopted polyphase alternating current, notwithstanding the limited extent to which apparatus for generating, transmitting, and using it had been developed, and this marked the beginning of the great power program which has so profoundly influenced all phases of modern life.

The original alternating-current generator submitted to the manufacturers was based on Dr. Sellers' design, in so far as its peculiar mechanical features were concerned, and for which he was granted a patent.

In a tribute to Dr. Sellers written for "Niagara Power," L. B. Stillwell, M. Am. Soc. C. E., stated:

> "The extent to which the judgment and painstaking investigation of Dr. Sellers influenced electrical development in America along lines now well established has not been generally recognized. As a member of the International Niagara Commission, he opposed successfully a resolution moved by Lord Kelvin himself which aimed to exclude from further consideration all systems of electrical power transmission other than direct-current systems. Dr. Sellers wisely took the ground that the Commission's knowledge of the possibilities of the alternating current at that time was not sufficient to justify action which would close the doors to that system. Fortunately his opinion prevailed. Few at this time would question the soundness of his judgment."

After the alternating-current system had been adopted, Mr. Stillwell adds,

"The electrical experts of the Cataract Construction Company failed to agree in regard to various essential features, and in exercising his power of supervision and approval of all plans, Dr. Sellers accepted heavy responsibility, not only in his own special field of mechanical engineering, but in the hydraulic and electrical fields as well.

"From 1889 to 1893, many important questions of engineering practice, in respect to which there is now practical unanimity of opinion, were earnestly debated. In those years, the Doctor, sound in his judgment, conciliatory toward others, untiring in his effort to secure for his company what would stand the test of time, was the adviser whose counsel chiefly guided the company in regard to all technical questions. * * *

"The far-reaching influence of the first plant of the Niagara Falls Power Company upon the development of the art of transmitting power by electricity has been widely recognized and instances might be multiplied in which his foresight avoided or skill surmounted difficulties which but for him would have handicapped seriously the remarkable engineering and industrial development to which that plant contributed so much of value."

While some one had referred to "the harnessing of Niagara" as the apogee of the remarkable career of Dr. Sellers, he was by no means satisfied, upon the demonstration of the complete success of the undertaking, to rest upon his laurels. Although retaining his interest in the Niagara Project during the successive extensions of the plants on the American and Canadian sides of the river, Dr. Sellers experienced increasing demands for his counsel in connection with the great number of power projects in this country and in other countries, which were promoted as a result of the conspicuous success of the Niagara undertaking. With his son, Horace Wells Sellers, and S. Howard Rippey, M. Am. Soc. C. E., both of whom had been associated with him for years in his general consulting practice, including the Niagara work, he formed the firm of Sellers and Rippey, Consulting Engineers, Philadelphia, in which, as Senior Partner, he continued to the time of his last illness to contribute of his wealth of engineering knowledge to the successful development of other power projects in many parts of the country, as well as to other technical problems presented to his firm for solution.

The following brief summary will indicate something of the range of activities covered by Dr. Sellers during his long and useful life: Patented the kinematoscope—the first application of photography to motion pictures (1861); recommended the use of absorbent cotton for surgical work (1861); Judge of Appeals, U. S. Centennial Exhibition (1876); Order of St. Olaf conferred on him by King of Sweden (1877); Professor of Mechanics, Franklin Institute (1881); Delegate of Franklin Institute to Tercentennial, University of Edinburgh (1884); member of Seybert Commission, University of Pennsylvania (1884); Professor of Engineering Practice, Stevens Institute of Technology (1888); Doctor of Engineering, Stevens Institute of Technology; Doctor of Science, University of Pennsylvania; President, The Franklin Institute, American Society of Mechanical Engineers, Photographic Society of Philadelphia, Pennsylvania Society for the Prevention of Cruelty to Animals, Pennsylvania Museum and School of Industrial Art, and the Niagara Falls Power Company; Vice-President, The American Philosophical Society; Chief Engineer, William Sellers and Company, The Niagara Falls Power Company, and The Cataract Construction Company; Chief Mechanical Engineer, Canadian Niagara Power Company; Charter Member, American

Society of Mechanical Engineers; Member, Society of Naval Architects and Marine Engineers, Institution of Civil Engineers (Great Britain), Institution of Mechanical Engineers (Great Britain), Pennsylvania Historical Society, Pennsylvania Academy of The Fine Arts, and University Club of Philadelphia; Corresponding Member, Society of Arts, Geneva, Switzerland; and Senior Partner, Sellers and Rippey, Consulting Engineers, Philadelphia.

He had secured about thirty patents upon his inventions while with William Sellers and Company, supplemented by subsequent patents in other fields of applied arts.

Dr. Sellers was married in 1851 to Cornelia Wells, daughter of Horace Wells, a prominent citizen and type founder of Cincinnati.

He died at his home in Philadelphia, on December 28, 1907, being survived by his widow, two sons, Coleman Sellers, Jr., and Horace Wells Sellers, and his daughter, Mrs. Sabin W. Colton, Jr.

Dr. Sellers was elected a Member of the American Society of Civil Engineers on May 3, 1876.

FRANK WOODWARD SKINNER, M. Am. Soc. C. E.[1]

DIED DECEMBER 24, 1932

Frank Woodward Skinner, the son of Horace and Harriet (Emery) Skinner, was born in Brownsville, N. Y., on June 6, 1858. He was graduated as a Civil Engineer from Cornell University, Ithaca, N. Y., in 1879.

Mr. Skinner's professional career started immediately after his graduation, as, respectively, Workman, Draftsman, and Assistant Engineer with the Pittsburgh Bridge Company on special shop and field duty. After a year or more of such general experience, he was promoted to be Assistant Engineer in charge of the reconstruction of the railroad suspension bridge across the Niagara River. On this work Mr. Skinner used steel rivets, for the first time on record in bridge building. Once, while walking on a 20-in. girder, 240 ft above the rapids, and carrying a white-hot rivet in a pair of tongs, he slipped, but recovering his balance, helped the gang to hammer the rivet in place, indicating that, like many other engineers of prominence, he was a man of iron nerve.

After finishing this reconstruction work he became Assistant Engineer of Erection for both the Pittsburgh Bridge Company and its associate organization, the famous erecting firm, Baird Brothers. His service with these firms was so satisfactory that in 1881 the Delaware Bridge Company secured him for Assistant Engineer on the erection of the Jumping Point Bridge, after the completion of which he became Bridge Engineer to Cooper, Hewitt and Company (New Jersey Steel and Iron Company), in charge of its fabricating shops and special rolling-mill operations. Mr. Skinner remained with that Company until 1884, when he was called to Montreal, Que., Canada, by the

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[1] Memoir prepared by J. A. L. Waddell and Eugene E. Halmos, Members, Am. Soc. C. E.

Dominion Bridge Company to be its Principal Assistant Engineer, mainly on erection, with which Company he spent two years, leaving to become Bridge Engineer of the St. Paul and Northern Pacific Railway Company.

In 1887, he was appointed Assistant Engineer and Engineer of Inspection on the Washington Arch Bridge, New York, N. Y., then the longest arch-ribbed bridge in the world, under the famous engineer, the late Theodore Cooper, M. Am. Soc. C. E.

After the completion of his work on that structure, Mr. Skinner made a radical change in his occupation, becoming, first, Associate Editor, and afterward, Senior Associate Editor of *Engineering Record,* at the same time practicing as a Consulting Engineer. Simultaneously, he served as Professor of Field Engineering in the College of Civil Engineering of Cornell University from 1894 to 1904.

Under his able management, the style and character of *Engineering Record* steadily improved both as a newspaper and as a scientific periodical, until its curve of progress crossed that of its only rival *Engineering News,* which had been for many years the semi-official organ of the Engineering Profession in America. The proprietors of the latter, in 1914, secured control of the stock of the former, and combined the two periodicals into the present *Engineering News-Record.*

After working for a time with the new organization, Mr. Skinner left in order to found and edit *Contracting* and to practice as a Consulting Engineer and Technical Author. During his spare time he wrote several standard engineering books, mainly on bridge work. These were well received by the profession, and they proved of special usefulness to the young practitioners in that line of activity.

After five years spent in developing *Contracting,* he sold his interest therein and became associated with its successor, *Public Works,* for which he wrote during the next two years. After this he devoted his entire time to his consulting practice, which included publicity of great constructions, developing, recording, emphasizing, presenting, and publishing vital, interesting, and important features of cement mills, dams, steel buildings, bridges, foundations, highways, tunnels, industrial and manufacturing plants, steam and hydro-electric power plants, and construction operations of all kinds.

In connection with this important and widely varied field, Mr. Skinner had the good fortune (and intense personal satisfaction) to be thrown in intimate contact with a large number of prominent Americans—engineers, contractors, promoters, bankers, technical authors, educators, and others of that ilk. It was his pleasure of late years to talk with his numerous friends, especially engineers of the younger generation, concerning these great men of the past and his share in their activities; and always everything he said was of unusual interest to his hearers.

He was always happiest when he could be of assistance to young engineers; never too busy to spend some time with a discouraged young engineer, advising, encouraging, and, whenever possible, aiding, if only with letters of introduction to prospective employers; he claimed that the best and biggest things in his life came through his friends, and it was his duty, as well as his pleasure, to pass on to other young men the kindnesses shown to him when he was a "young cub."

Mr. Skinner was a born orator, and had a splendid flow of language, his thoughts coming with great rapidity, his speech being phenomenally fluent, and his command of the English language uniquely marked. As an after-dinner speaker he was unsurpassed; for, when called upon, without any notice whatever, to speak upon any subject with which he was at all conversant, he would rise to his feet and charm all his hearers by the beauty of his eloquence and by his supreme power of oratory.

As a lecturer he was wonderfully successful, as was often demonstrated by his hearers, who, after listening to him for an hour and a half, would ply him with questions for an additional half hour, or even more, which questions invariably he was able to answer satisfactorily, because his fund of knowledge, especially concerning technical matters, was extraordinarily extensive. Mr. Skinner delivered construction lectures singly, in groups, and in regular courses at Maine and Wisconsin Universities, and at Yale, Harvard, Cornell, Brown, Princeton, McGill, Columbia, Massachusetts Institute of Technology, Rensselaer, Case School of Applied Science, Lewis Institute of Chicago, Pittsburgh Institute, and other well-known schools where engineering is taught.

His systematic habits and his deep interest in structural engineering led him to collect and collate a vast amount of information, particularly of an illustrative character. These files resembled more nearly a library than an office file. His photographs and lantern slides covered the long historical development of steel bridges of almost every conceivable type. It is doubtful whether another such collection exists. He himself estimated it as including more than two thousand lantern slides.

Owing to his vast experience in the study of field construction in so many and such varied lines, both as a practicing engineer and as an investigator for his newspapers, he was conceded, by many persons who ought to know, to be the best posted man in America on engineering construction in general.

In debate, Mr. Skinner was strong and forceful. About 1928 Sir Edwin Lutyens, a noted British architect, said that the Americans were not good builders and that, owing to faulty construction methods, the life of American skyscrapers was limited to forty years. Mr. Skinner made an exhaustive research for the American Institute of Steel Construction in order to refute this statement. Acting as spokesman for his Committee, he said that Sir Edwin's charges were evidently based on ignorance, for American skyscrapers are an unprecedented achievement of structural perfection, with very complete scientific and mechanical development, insuring safety, durability, and economic efficiency to a high degree.

Mr. Skinner's fame was not merely national—it was international; his writings appeared in the technical press of Europe, Asia, and Australasia. He was a regular contributor to *Engineering,* the leading technical paper of England, and his writings therein were highly appreciated by its readers. In its January 13, 1933, issue there appeared an editorial notice of his decease, evidently from the pen of his friend, Maj. P. J. Cowan, with whom he had corresponded for many years. That editorial says:

> "His connection with this Journal dates back many years, and has at one time and another been responsible for some of the most complete serial descriptions which it has been our privilege to publish. * * * .
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> "He was a man of very wide acquaintance in his country, and one with whom it was a pleasure to have dealings. He was courteous and considerate to a degree, and ever willing to give advice or assistance. His admiration for this Journal and the pride he felt on the publication of his articles in our columns were of the most ingenuous kind, and to any one who realized the frankness of his character, were of an obviously honest nature, and the result of seeing his work well made use of. In his passing we have lost a valued personal friend, and our readers a contributor who has done them great service."

Among Mr. Skinner's practical achievements in engineering there might be mentioned as most prominent the following: The designing of the St. John, N. B., Canada, Cantilever Bridge and its erecting plant; the direction of both shop and mill inspection on many large early steel bridges; the founding and developing in *Engineering Record* of a Department of Bridge Engineering, Structural Engineering, and Municipal Engineering; and the first sustained Departments of Civil Engineering construction, operations, methods, and plans ever published in any engineering magazine; the designing, building, and operation of the first power reamer for structural work; the creation of a university course of Construction Engineering; the invention and patenting of many features of steel sheet-piles, foundations, substructures, and concrete and reinforced concrete buildings and bridges; the invention of a universal fluid-pressure and volume-meter and of a fabricated pressure- and tension-joint for structural steel; and the invention of military portable steel houses, huts, and trench sheeting. During the early part of the World War, Mr. Skinner spent several months in England, Paris, and Belgium, in an endeavor to introduce these portable steel houses for troops, meeting with some success in Belgium.

His principal writings were: "Arch Bridges," "Specifications and Standards," "The Woolworth Building," "The Quebec Bridge," and "The New Quebec Bridge." He contributed widely to *Century Magazine, Harper's Weekly, McClure's Magazine,* and *Collier's Weekly.* He wrote about two hundred entirely different articles (all of which were published), and delivered sixty lectures, on the George Washington Bridge.

In connection with the American Association of Engineers, in 1931 and 1932, Mr. Skinner served as one of a committee of three on planning and editing an extensive work for prospective or student engineers entitled "Vocational Guidance in Engineering Lines." He lived long enough to see the entire manuscript completed, but was not permitted to see it published.

Beginning in 1887 he was a frequent contributor to Society publications on a wide variety of topics. Although most of his discussions were in the structural field, they were by no means confined to bridges. Dams, foundations, pipe lines, and wood conservation also were included, bearing evidence of the extent of his interest and deep technical knowledge.

Among the last of his writings was a paper on "Development of Erection of Girder Bridges" printed in the June, 1932, issue of *Civil Engineering*. In this publication, Mr. Skinner evinced a great interest. He had planned a series of papers on bridge erection of which, however, only this one was issued. At the time of his death he was working on the next and was looking forward to the prospect of further publication with characteristic youthful enthusiasm.

He was a Life Member of the Society and an Honorary Member of the Connecticut Society of Civil Engineers; a member and Past-President of the Brooklyn Engineers Club; and a member of the Cornell Society of Civil Engineers, the Railroad Bridge Engineers' Association, the American Association of Engineers, and other technical organizations.

Mr. Skinner was gifted with an unusually striking and pleasing personality that made him extremely popular with his numerous acquaintances and dearly loved by all who had the high privilege of knowing him intimately. He was a true and loyal friend; and he ever considered that nothing could be of too much trouble to do in the service of those for whom he truly cared. Unselfishness was one of his most distinctly marked characteristics. He certainly "did his bit" in many ways for the Engineering Profession, both at home and abroad, and he will be long remembered as one of its most useful devotees.

In 1881, he was married to Rachael Sumner, who survives him. Their only son, Alanson B., a noted ethnologist, author, and lecturer, was killed in an automobile accident in 1925 while in North Dakota on official business.

Mr. Skinner was elected a Member of the American Society of Civil Engineers on September 1, 1886.

FRANCIS PITT SMITH, M. Am. Soc. C. E.[1]

Died April 19, 1932

Francis Pitt Smith was born in New York, N. Y., on September 9, 1867, the son of Charles W. Smith and Elizabeth I. (Pitt) Smith. His father's family came from England in the early history of this country. His mother was a direct descendant of William Pitt, Earl of Chatham.

Francis Pitt Smith's early education was received at the Lyons Collegiate Institute, in New York City. He then entered the School of Mines, Columbia University, and, in 1888, was graduated with the degree of Bachelor of Philosophy from the course of Analytical and Applied Chemistry. From that time until 1897 he held several positions. While employed by the Eidoloscope Company, of New York, he designed and built machines for taking motion pictures and exhibiting them, originating principles that afterward proved of the greatest value toward perfecting the motion-picture machines of to-day. He also devised processes for the printing and developing of motion-picture films.

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[1] Memoir prepared by A. W. Dow, Esq., New York, N. Y.

From 1897 to 1902, Mr. Smith served as Chemist and General Superintendent of the Alcatraz Company, of New York, with refineries at various places in California. His duties consisted of the supervision of the manufacture and refining of both natural and oil asphalts and the manufacture of asphalt cement and paving mixtures. Practically all his work in California for this Company was of a pioneer nature, as the manufacture of such products was new to this district. The success of the Alcatraz Company and its subsidiaries was largely due to his skill and precise care. One of Mr. Smith's strong characteristics, which was constantly evident, was the precision which he brought to bear in all his professional undertakings. He planned and built refineries for the Alcatraz Company at Alcatraz Landing, Sisquoc, Carpinteria, and McKittrick, Calif. He also planned and built the oil refinery of the American Oil and Asphalt Company, at Los Angeles, Calif., which at that time was the largest plant on the Pacific Coast. At this plant some of the first asphalt from California oil was manufactured. He also designed and built paving plants for the Alcatraz Company at Los Angeles, San Francisco, Calif., Seattle, Wash., and Glasgow, Scotland.

He personally supervised the manufacture of pavements at many of these plants and, in Glasgow, laid the first sheet asphalt pavement that successfully withstood the severe climate of that city. In 1898, Mr. Smith laid the first California oil asphalt pavement in the Eastern United States, on Lafayette Street, in South Bend, Ind. This pavement lasted more than twenty-five years and was evidence that this class of asphalt could be used successfully.

At the formation of the Asphalt Company of America, commonly known in those days as the Asphalt Trust, which absorbed the Alcatraz Company, Mr. Smith resigned and took a position as Chemist and General Manager of the British Paving Company, Limited, of London, England, and laid a number of sheet asphalt pavements in London, Glasgow, and other cities in Great Britain.

From 1903 to 1906, he was Chemist and Asphalt Expert of the Union Oil Company of California; and from 1906 to 1907, Consulting Chemist and Asphalt Expert for the A. L. Barber Asphalt Company and its subsidiaries. In 1907, he entered into partnership with Mr. A. W. Dow, forming the firm of Dow and Smith, and was associated with this firm until his death.

In 1909, the firm of Dow and Smith was retained by Sir Wheaton Pearson for the Anglo-Mexican Petroleum Company, of London, with refineries at Minatitlan, Mexico. It was at Minatitlan that Mr. Smith worked out the process by which the first satisfactory paving asphalt from Mexican crude oil was manufactured.

In 1911, a number of experienced asphalt men in this country combined and formed the United States Asphalt Refining Company, with refineries at Baltimore, Md. Through Mr. Smith's influence the Anglo-Mexican Petroleum Company contracted with this new firm to supply the oil and allow it to manufacture asphalt under the supervision of Mr. Smith. This was really the beginning of the manufacture of Mexican asphalt in the United States and marks an epoch in the industry. The firm of Dow and Smith was retained by the Anglo-Mexican Petroleum Company under the direction of Lord Cowdray (Sir Wheaton Pearson), for twelve years, during which time Mr. Smith supervised the laying of many pavements in Europe for paving companies controlled by Lord Cowdray, and all asphalts manufactured at the Minatitlan refineries and by the United States Asphalt Company refineries at Baltimore, were passed upon by his firm before marketing. In connection with work for the firm, Mr. Smith also designed and erected several asphalt refineries, asphalt paving plants, and oil-distributing plants, and supervised the laying of bituminous pavements of various kinds in practically all the large cities in the United States and Canada, and for many of the States.

In 1914, Mr. Smith was appointed Delegate from the State of New York and the City of Baltimore, to the Third International Road Congress in London. In 1914, he was appointed Delegate for the State of New York to the American Mining Congress. He was a non-resident Lecturer on Highway Engineering at Columbia University from 1912 to 1916, and at the University of Michigan, Ann Arbor, Mich., from 1919 to 1926.

Mr. Smith took a most active part in the work of the technical societies to which he belonged. In the American Society for Municipal Improvements (later, the American Society for Municipal Engineers), he was Chairman of the Committee on Asphalt Paving in 1910, and from 1913 to 1918 and in 1920 and 1921. He was the first President of the Association of Asphalt Paving Technologists, organized in 1926. In the American Society for Testing Materials, he was Chairman of Committee D-4 on Roads and Paving Materials in 1923 and 1924, and Chairman of Committee D-8, Bituminous Waterproofing and Roofing Materials, in 1920 and 1921. Among the many technical societies of which Mr. Smith was a member were the American Institute of Chemical Engineers; American Society for Municipal Engineers, American Society for Testing Materials; American Chemical Society; Society of Chemical Industry; American Road Builders' Association; Association of Asphalt Paving Technologists; Permanent International Association of Road Congresses; Canadian Good Roads Association; and American Wood Preservers Association.

He was Associate Editor of the American Highway Engineers Pocketbook, and wrote numerous papers and articles for various technical journals on the subject of the manufacture and testing of asphalts and road oils and on the construction of bituminous pavements. His excellent work in writing and systematizing specifications for these materials has done much toward the advancement of this industry.

Mr. Smith was taken sick in the spring of 1931 and after a lingering illness which confined him to his home at Brevoort Farm, Mamaroneck, N. Y., he died on April 19, 1932.

He was married on October 14, 1890, to Alice Towle, who, with two sons, Henry Brevoort and Francis Pitt, Jr., survives him.

One of his most remarkable traits was his ability to analyze and tabulate most neatly and thoroughly all work in which he was engaged. He was a man of most precise habits, not only in his profession, but in his every-day life. His quiet friendly manner and his sincerity of purpose in everything he undertook, won him many friends.

Mr. Smith was elected a Member of the American Society of Civil Engineers on November 30, 1909.

THEODORE NELSON SPENCER, M. Am. Soc. C. E.[1]

DIED JULY 29, 1932

Theodore Nelson Spencer was born in Philadelphia, Pa., on July 4, 1866, the son of Theodore Nelson, and Adelaide (Horn), Spencer. He attended first a private school and then a Public High School, in Philadelphia. He was graduated from the University of Pennsylvania, with the degree of Bachelor of Science in 1887 and received that of Civil Engineer in 1888.

From 1888 to 1890 Mr. Spencer was with the Lake Erie and Western Railroad Company, in charge of a level party, and, later, of a transit party. From 1890 to 1891, he served the Norfolk and Western Railroad Company as Transitman, and, in 1891, he was in charge of a field party on a survey of the Town of Chester, S. C., for a land company.

From 1891 to 1893, Mr. Spencer was with the Pennsylvania Railroad Company on the Lines West of Pittsburgh, Pa., and was the first man in charge of double-track work for the World's Fair at Chicago, Ill., from New Galilee to State Line, Ohio. Later, he accepted a position as Resident Engineer on double-track construction on the Pittsburgh, Fort Wayne, and Chicago Railroad. In the summer of 1893, he took charge of a survey for a land company in Ohio.

The years of practical field experience obtained in the varied work mentioned undoubtedly grounded him in those habits of accuracy, thoroughness, and conscientiousness, for which his work was distinguished.

Mr. Spencer later moved to Philadelphia, and, from May, 1894, to July, 1913, he was with the Department of Public Works of that city, serving first as Transitman. Later, he was given charge of the Middle Section of the Pennsylvania Avenue Subway and Tunnel, a project of grade-crossing elimination entered into jointly by the City and the Philadelphia and Reading Railroad Company.

Mr. Spencer was then transferred to the Bureau of Water as Third Assistant Engineer, in charge of surveys and plans for the Torresdale Filtration Conduit. Subsequently, he became Second Assistant Engineer in charge of a field party making the preliminary surveys for the filtration plant and, later, was appointed First Assistant Engineer in charge of the Torresdale Conduit; he then was given charge of the construction of the Torresdale Intake, including two gate-houses in the Delaware River, and a coaling wharf. Following this, Mr. Spencer was appointed Assistant Engineer in charge of the completion of the preliminary filters, pumping stations, coal pocket, track scales, etc., for the Torresdale Filter Plant.

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[1] Memoir prepared by Stanley R. Smith, Assoc. M. Am. Soc. C. E.

He was then made Assistant Engineer in charge of the Pennypack Creek Sewage Disposal Plant, for the Bureau of Surveys, of Philadelphia, including the construction of a pumping station at Holmesburg Junction. On the completion of this work, he accepted a position as Supervising Engineer on the construction of a temporary convention hall.

In July, 1912, Mr. Spencer began work in the office of the Transit Commissioner. The Transit Commission was a body appointed to study the transit requirements of the City of Philadelphia and to outline a program for the transit lines. In July, 1913, the Department of City Transit was created by Act of Legislature, and Mr. Spencer was transferred to this Department, in which he held the title of Assistant Engineer in the Field during his period of service. He resigned from service with the City on January 31, 1932. In this last position, Mr. Spencer was in charge of the field surveys on which the design work of the Department was based. This work included surveys for the Frankford Elevated, the Broad Street Subway, with its various branches, the Frankford and Bustleton Surface Line, the extension west of the existing Market Street Subway, subway changes on the City Hall Section, and numerous other projects.

He was married in 1892 to Lillie Kindig, of Philadelphia. Mrs. Spencer, a son, and two daughters survive him.

Mr. Spencer was elected an Associate Member of the American Society of Civil Engineers on September 5, 1900, and a Member on March 4, 1914.

WILLIAM AIKEN STARRETT, M. Am. Soc. C. E.[1]

DIED MARCH 25, 1932

William Aiken Starrett was born at Lawrence, Kans., on June 14, 1877, a son of the Rev. William A. and Helen (Ekin) Starrett. He attended the Chicago, Ill., Preparatory Schools and spent two years at the University of Michigan as a member of the Class of 1897, leaving in 1895. He received his degree of Bachelor of Science in Civil Engineering in 1917, as of the Class of 1897, and, in 1931, the University conferred upon him the honorary degree of Doctor of Engineering.

After leaving the University, Mr. Starrett worked for a time for a large wholesale grocery firm in Chicago. In 1900, with his older brother, Theodore, he went to New York, N. Y., and entered the employ of the Thompson Starrett Company, of which his brothers, Theodore and Ralph, were organizers. He spent about three years with that Company, as Timekeeper, Field Engineer, and Building Superintendent. After several years as Building Superintendent on various projects in New York, he took charge of the erection of the Union Station, in Washington, D. C., and on its completion he was made General Superintendent and Vice-President of his Company. In 1913, Mr. Starrett left the Thompson Starrett Company to become a member of the firm of Starrett and Van Vleck, Architects, a connection which continued until 1918.

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[1] Memoir prepared by Albert R. Palmer, Esq., New York, N. Y.

In 1916, he spent some time in training at the Plattsburg, N. Y., Camp and with the Corps of Engineers. When the United States entered the World War, he was called to Washington to take charge of emergency construction work for the Council of National Defense, and was appointed Chairman of the Emergency Construction Division of the War Industries Board, serving throughout the war. This Division had charge of the building of all cantonments, hospitals, and army bases in the United States. He was commissioned a Captain in the Engineers Reserve Corps of the National Army in May, 1917, and a Major the following month. He was promoted to the rank of Colonel in the Quartermaster Corps in March, 1918, and, when discharged a year later, retained that rank in the Reserve Corps.

After the war, Colonel Starrett was made Vice-President of the George A. Fuller Company, an office which he retained until 1921. During this period, he was connected with the erection of several large buildings in Tokyo, Japan. The steel-frame buildings which he built then, withstood the earthquake of 1927.

In 1922, with his brothers, Paul and Ralph, and Mr. Andrew J. Eken, he founded Starrett Brothers, Incorporated, the firm later becoming Starrett Brothers and Eken, Incorporated, with Mr. Paul Starrett as President and the other members as Vice-Presidents. The Starrett Corporation, of New York, was organized in 1929 as the holding company of the Starrett realty and building enterprises, in all of which Colonel Starrett was an Officer or a Director, or both. As its President, he was active in the work of the Corporation up to the illness leading to his death. He was Vice-President and Director of Starrett Brothers and Eken, Incorporated, the building affiliate of the varied interests, which erected the Empire State Building and other large buildings of note, including the 70-story Forty Wall Street Building and the 19-story Starrett-Lehigh Terminal Building, in New York City, the Carew Tower, in Cincinnati, Ohio, the tallest building in the Ohio Valley, and the 40-story Ramsey Tower, the tallest structure in Oklahoma City, Okla. Colonel Starrett was also President of the Starrett Investing Corporation, the investing and realty affiliate; the Forty Wall Street Corporation; and the Wall and Hanover Street Realty Corporation; and a Director of several contingent interests. His outstanding achievements as a builder of skyscrapers made him an authority on that subject.

Colonel Starrett was called into consultation by President Hoover in August, 1931, in connection with a nation-wide program for enlivening the building trades. He spent the next several months in intensive work as a member of the President's Emergency Committee for Employment and as a

Director of the Gibson Emergency Committee in New York City. He was also engaged in public service in Madison, N. J., as its Mayor, on the Republican ticket, from 1920 to 1922. He declined renomination on account of increased building activities at that time.

He was the author of "Skyscrapers and the Men Who Build Them," published in 1928, as well as a frequent contributor to magazines. He had also made several addresses over National broadcasting chains in regard to the utilization in home building of experience secured in the erection of skyscrapers.

Colonel Starrett was a member of the American Society of Mechanical Engineers, the Society of American Military Engineers, and the Engineers' Club, in New York City. He was a former member of the American Institute of Architects.

He died at his home in Madison, N. J., on March 25, 1932, after an illness of three months. Surviving him are his widow, formerly Eloise Gedney, of East Orange, N. J., to whom he was married in 1900, a son, David, and a daughter, Helen Ruth, as well as his brother, Paul, and two sisters.

Colonel Starrett was elected an Affiliate of the American Society of Civil Engineers on October 7, 1914, and a Member on March 14, 1916.

VERNON CHRISTOPHER SUCKOW, M. Am. Soc. C. E.[1]

DIED FEBRUARY 12, 1932

Vernon Christopher Suckow, the son of Louis C. and Anna Suckow, was born at Janesville, Wis., on May 5, 1875. The family later moved to Milwaukee, Wis., where the son attended preparatory schools. He could be classed among the self-educated as he assumed his first duties in the engineering field immediately upon his completion of the Science Course at the East Division High School of Milwaukee.

Mr. Suckow started his professional career under the direction of Oscar Sonne, Consulting Engineer, of Milwaukee, during 1893, and was constantly engaged in engineering work throughout his life. Mr. Suckow felt the lack of technical training very keenly and, as a result, spent his evenings during the time he was with Mr. Sonne and for several succeeding years in careful study of the theory of his chosen profession. This habit of study remained with him, and he was always recognized as a well-informed man on general subjects as well as on the theory and practice of structural engineering.

During the latter part of 1894 he joined the staff of The Wisconsin Bridge and Iron Works, of Milwaukee, with which he remained for one year. For the next two years he was with The Milwaukee Bridge Company, also of Milwaukee. This early work was in connection with the design and cost analysis of structural steel bridges of all types.

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[1] Memoir prepared by W. R. Engstrom, Assoc. M. Am. Soc. C. E.

In 1897, Mr. Suckow entered the employ of The Edward P. Allis Company, as a Structural Designer and, during the two years that he was with this Company, he was in charge of some interesting work. During the next two years he was associated with Ralph Modjeski, M. Am. Soc. C. E., in charge of inspection. From 1901 to 1903, he was with the Mead-Morrison Company, of Chicago, Ill., as a Structural Designer. This engagement was followed by four years with Ritter and Mott, Consulting Engineers, also of Chicago.

In 1907, he responded to the urge to go West and opened an office as a Consulting Engineer at Seattle, Wash. He succeeded in developing a fine practice in the structural engineering field and, during the 25-year period preceding his death, was responsible for the structural design of many large commercial and monumental buildings. Mr. Suckow also maintained an office in Vancouver, B. C., Canada, for several years and had charge of the structural design of many of the fine buildings of that beautiful city. Included in this work was a group of buildings for the University of British Columbia.

Among the many projects with which he was connected in Seattle are several fine buildings at the University of Washington. The delightful Gothic design of this group presented some interesting structural problems. He was associated with the architectural firm of Bebb and Gould, of Seattle, on this work.

Mr. Suckow was retained by the Sound Construction Company, of Seattle, in connection with the construction of the State Capitol Buildings at Olympia, Wash. He also had charge of the structural design of the Civic Auditorium, at Portland, Ore., and, at a later date, was associated with the architectural firm of Schack, Young, and Meyers, in connection with a similar project at Seattle.

A list of building projects for which he originated and supervised the structural design would be truly imposing. It would include hospitals, banks, office buildings, hotels, and apartments, and numerous other commercial as well as monumental buildings. His record for work carried out in a careful, conscientious manner is quite remarkable considering the fact that all projects had his intimate attention and supervision.

Mr. Suckow served with the United States Army during the World War. He entered the service at the Presidio, San Francisco, Calif., on May 21, 1917, as Captain of Engineers, and was assigned as Engineer Depot Officer, at Camp Lewis, Washington. When he received his discharge from the service, on February 14, 1920, at the Army Supply Base, Norfolk, Va., he held the rank of Captain of Engineers, Commanding Officer, Motor Transport Corps of Port of Embarkation. He had no foreign service. He held a commission as Captain in the Engineers Reserves until October 17, 1923, when he was promoted to the rank of Major, which commission he held until his death. He was a member of the Society of American Military Engineers.

He was married in June, 1898, to Ruby Johnson, of Milwaukee, who is now living in Seattle. There were two children, a daughter, Hazel Laverne, who passed away at the age of nine, and a son, Herbert, who has his residence in Chicago. Mr. Suckow was married again in 1925 to Florence Medley, of Seattle, who passed away two years before he did.

Mr. Suckow took an active part in the affairs of the Society. He was a keen student of technical papers as published by the Society and enjoyed nothing more than a frank impartial discussion of the points of theory involved. He was a particularly progressive thinker and always intensely interested in new methods of design. During the later years of his life he did considerable research work in connection with composite column design. Unfortunately, this work was not completed.

He was a member of the American Concrete Institute, and although his early training was confined largely to structural steel designing, he soon branched out into the field of concrete engineering and, with his customary persistence, developed into a concrete designer of considerable reputation.

For many years he was a familiar figure around the Engineers Club of Seattle and, at the time of his death, was President of the Club. He was also a member of the Arctic Club of Seattle.

Mr. Suckow will long be remembered by all who knew him not only for his fine engineering ability, but for his sterling character. He was blessed with an extremely happy disposition and a spirit of good fellowship which won him a host of friends. He practiced engineering with a fine regard for the ethics of the profession and lived according to the same high standards. A young man throughout his life, with engineering his hobby as well as his profession, he will be remembered by his many friends with esteem, respect, and affection.

Mr. Suckow was elected a Member of the American Society of Civil Engineers on November 15, 1926.

HENRY ALLEN SYMONDS, M. Am. Soc. C. E.[1]

Died February 17, 1931

Henry Allen Symonds, the son of William Franklin and Abbie M. (Washburn) Symonds, was born at Hancock, N. H., on September 23, 1871. His ancestors were among the first settlers of the town, moving there from Groton, Mass., in 1779.

Mr. Symonds' earlier education was obtained in the public schools of Hancock. After graduation from the High School, he attended the University of New Hampshire, at Durham, N. H., for three years, Tufts College, at Tufts College, Mass., for one year, and then entered the Thayer School of Civil Engineering of Dartmouth College, at Hanover, N. H., from which he was graduated in 1894 with the degree of Civil Engineer.

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[1] Memoir prepared from information on file at the Headquarters of the Society.

In 1892, Mr. Symonds was employed as a Transitman for the City of Boston, Mass., and, in 1893, he served as Draftsman with the Boston Bridge Works; for five months of the same year, he was Chief of Party, City Engineer's Office, at Somerville, Mass.

Following his graduation from college, he held the following positions: 1894, Inspector of dredging with the Massachusetts Harbor and Land Commission; 1895 to 1899, Assistant Engineer with Messrs. Percy M. Blake and E. Worthington, Civil Engineers, engaged in general engineering practice; 1899 to 1904, Contracting Engineer for Booth and Company, Incorporated, of Boston, on the building of the first section of the Kittery and Portsmouth Electric Railroad, on the construction of the Springfield and Suffield Electric Railroad, the Sebago Basin for the Portland Water-Works, Portland, Me., and sewers, etc., at South Gardner, Mass.; 1904 to 1905 miscellaneous engineering and construction, including an engagement as Division Superintendent on a section of a 42-in. concrete pipe line at Cambridge, Mass., Superintendent of Construction of sewer filters at Whitinsville, Mass., and surveys for the location of the Uxbridge and Blackstone Electric Railroad; 1905 to 1907, Superintendent of the Water-Works and Engineer for Athol, Mass., on general rehabilitation work; 1907 to 1908, member of the firm of Blake and Symonds, engaged in water-supply engineering, including the design and construction of water-works at Westford and Wareham, Mass., and a new supply, pumping station, plant, etc., for the water-works at Provincetown, Mass.; 1908 to 1909, Superintendent of Construction for the J. G. White Company, of New York, N. Y., on a 42-in. lock-bar steel pipe line, 12 miles long, for Springfield, Mass.; 1909 to 1916, Contracting Engineer associated with Mr. H. Hanscom, of Boston, mostly on water supply, including the design and construction of water-works for Norton, Barnstable, Mattapoisett, Acton, Shrewsbury, and Groveland, Mass., Greenville, Me., Haverhill, N. H., and other places.

From 1916 until his death, Mr. Symonds was in private practice as Consulting and Constructing Engineer for water-works and sewage disposal plants, including the design and construction of the Warren and Lynnfield (Mass.), Water-Works; the construction of filters for the water supply systems of Stockbridge, Mansfield, Braintree, Shrewsbury, and Amesbury, Mass.; and filters, a new pumping plant, pipe line, etc., for Putnam, Conn., Water-Works, as well as numerous other problems in water supply, drainage, etc. During this period, he was also President and Director of the Norton, Mass., Water Company, the plant of which was sold to the town in 1924, and Manager of the Grafton Water Company, of Grafton, Mass. He also constructed the water supply systems of Chatham and Hyannis, Mass., known as the Barnstable Water Company, of which he was a Director and also Manager until his death.

Mr. Symonds was a member of the New England Water Works Association, having served as Editor of its *Journal* from 1917 to 1922, the Boston Society of Civil Engineers, the Thayer Society of Civil Engineers, and the Waban Neighborhood Club, of Waban, Mass.

He died suddenly, on February 17, 1931, of angina pectoris, while at a conference on a water supply system for the Town of Norwell, Mass. He was buried in Hancock.

He was married on June 3, 1896, to Eliza P. Tuttle, at Belmont, Mass., who survives him, with their children, Allen M., Marion E., Edmund A., and Robert T. Symonds.

Mr. Symonds was elected a Member of the American Society of Civil Engineers on October 1, 1926.

WILLIAM WOLCOTT TEFFT, M. Am. Soc. C. E.[1]

Died June 24, 1932

William Wolcott Tefft was born at Milbank, S. Dak., on February 28, 1882. He began his technical career in 1902 as a Rodman on electric railway construction in Michigan. From this, he went to dredging inspection for the Federal Government on the Detroit River. Later, he became Chief of Party on water-power surveys on the Muskegon River, in Michigan, and then Resident Engineer in the construction of a hydro-electric power plant at Superior, Mich.

In 1907, Mr. Tefft was graduated from the Engineering College at the University of Michigan as Bachelor of Science in Civil Engineering and, in 1930, was awarded the degree of Civil Engineer from the same institution.

Before 1911, he had supervision of field surveys for eleven dams, aggregating 78 000 hp, and was Resident Engineer during the construction of several hydro-electric developments. In 1911, Mr. Tefft became a member of the Fargo Engineering Company, of Jackson, Mich., retaining the connection until 1919. During this period he had responsible charge of the design of many plants, invented the Tefft conduit spillway which was utilized in the construction of several important dams, and conducted an investigation on water-power possibilities of the rivers of the Southern Peninsula of Michigan for the late Mr. W. A. Foote, founder of the Commonwealth Power Corporation. During the World War Mr. Tefft made a survey for the Federal Government to determine water-power possibilities in Michigan and Wisconsin, as well as a survey of the possibility of substituting water power for steam power in Michigan.

In 1919, he became associated with Hodenpyl, Hardy, and Company of New York, N. Y., and was made Chief Engineer of the Commonwealth Power Corporation and the Consumers Power Company. He was made Vice-President of the former in 1922, and, from 1924 to 1928, served as Chief Engineer of the Central Illinois Light Company, Southern Indiana Gas and

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[1] Memoir prepared by F. N. Menefee, Horace S. Hunt, and Henry L. Fruend, Members, Am. Soc. C. E.

Electric Company, Ohio Edison Company, Illinois Power Company, Northern Ohio Power and Light Company, and the Tennessee Electric Power Company. He then rejoined the Fargo Engineering Company as Vice-President, with which he remained until his death on June 24, 1932.

Mr. Tefft had an important part in the design and construction of several engineering projects of unique and remarkable character. One of these is the Alcona Dam on the Au Sable River, in Michigan. This dam of 11 000 hp is built on a quicksand foundation, and is regarded as the first successful project of its kind ever engineered under such difficulties. Another one of these outstanding projects was the Ox Bow Dam, on the Muskegon River, in Michigan, the highest dam in the world on an earth foundation with a spillway through it. Mr. Tefft was responsible for the construction of about 350 miles of 140 000-volt transmission lines in Michigan, Illinois, Tennessee, and Ohio, and of seven steam plants with 345 000 hp. He assisted his father-in-law, Mr. W. A. Foote, in the development of the first electric power undertaking in the world to transmit as high as 130 000 volts at long range. He had engineering supervision of 3 000 miles of transmission lines, and of forty-three hydro-electric, and twenty-five steam electric, plants.

He was a member of Sigma Xi, the American Society of Mechanical Engineers, the American Institute of Electrical Engineers, the American Society for the Advancement of Science, the Society of American Military Engineers, and the National Electric Light Association. He was a Director of the Michigan Engineering Society and member of the American Committee of the World Power Conference.

Mr. Tefft showed a strong and consistent interest in the activities of the Methodist Episcopal Church. He was President of the Board of Trustees of Albion College; President of the Methodist Foundation of Michigan; and President of the Board of Trustees of the First Methodist Episcopal Church, of Jackson. He was President of the Jackson Council of the Boy Scouts; a Director of the Young Men's Christian Association, and Chairman of the Finance Committee of the Michigan George Washington Bi-Centennial Commission.

Mr. Tefft was a Director of the Union and Peoples National Bank; President of the General Power Corporation; and belonged to the Jackson Rotary, City, Gun, and Country Clubs:

Briefly, this is the adult history of Mr. Tefft, an engineer of whom his home newspaper said editorially:

> "In the passing of William W. Tefft, Jackson has lost a valuable citizen. Mr. Tefft was vitally concerned with the welfare of this community. He supported those movements which contributed to that end, he was not afraid to oppose forces which he considered detrimental to the best interests of the people of Jackson.
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> "This man was a positive factor for good. He was willing to devote his resources and his time to the support of good causes. Yet he abstained rigorously from making a show of his virtues. Men of this type are no more plentiful to-day than they have been in any day or age. And a loss from their ranks is certain to be profoundly felt."

The universality of Mr. Tefft's activities is partly portrayed in these memoirs, his modesty is rightly mentioned in the editorial, but the effectiveness and the breadth of his life's work are further attested by the attendance at his funeral of Michigan's Governor, the Attorney General, and the President of Albion College, with many other prominent citizens from throughout the State.

His immediate survivors are his widow, Mrs. Ethel (Foote) Tefft, to whom he was married on October 8, 1910, a son, Robert, and a brother, Leo C. Tefft.

Mr. Tefft was elected a Junior of the American Society of Civil Engineers on March 31, 1908; an Associate Member on January 2, 1912; and a Member on October 10, 1916.

ARTHUR WEBSTER THOMPSON, M. Am. Soc. C. E.[1]

DIED NOVEMBER 9, 1930

Arthur Webster Thompson, the son of Seldon E. and Laverne B. (Webster) Thompson, was born on May 8, 1875, at Erie, Pa. Through his mother he was descended from the famous family which gave John Webster, one of Connecticut's early Governors, to the Colonies, and Noah Webster, author of the dictionary, to America. His father and his paternal grandfather were both veterans of the Civil War.

He attended public school and Allegheny College, Meadville, Pa., from which he was graduated in 1897 with the degree of Civil Engineer. He worked during summer vacations as Apprentice and, later, as Rodman on location work with the Pittsburgh, Bessemer and Lake Erie Railroad Company. His first real job was as Draftsman for the firm of Wilkins and Davison, in Pittsburgh, Pa.

He was subsequently employed as Instrumentman for the Pittsburgh and Lake Erie Railroad Company; Chief of Party on a survey, with the Baltimore and Ohio Railroad Company; Assistant Engineer for the Baltimore and Ohio Railroad Company, at Pittsburgh, in 1900; Division Engineer at Cumberland, Md., in 1901 and 1902, and at Pittsburgh in 1903; then back to Cumberland as Superintendent in 1903 to 1904; and Superintendent at Wheeling, W. Va., in 1904 to 1907.

In 1907, Mr. Thompson became Chief Engineer, Maintenance of Way, for the Baltimore and Ohio System, at Baltimore, Md. Three years later (1910) he was Chief Engineer of the System, and after eight months, he was named General Manager. In 1912, he became First Vice-President in charge of operations. Four years later (1916), he was given charge of traffic and commercial development; and, in 1918, engineering and operations were placed under his control.

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[1] Memoir prepared by Dwight P. Robinson, Esq., Philadelphia, Pa.

On June 1, 1918, he was appointed Federal Manager, by the United States Railroad Administration, of the Baltimore and Ohio Railroad, Western Maryland Railway, Cumberland and Pennsylvania Railroad, Cumberland Valley Railroad, Coal and Coke Railroad, Wheeling Terminal Railroad, and Staten Island Lines, with headquarters at Baltimore.

Some of the most difficult engineering feats in the history of railroading were accomplished by the Baltimore and Ohio System while Mr. Thompson directed its engineering forces. It fell to him to build the famous Magnolia Cut-Off, which cost $6 000 000 for less than six miles of road. For a paper which he wrote describing the Magnolia Cut-Off he was awarded a Gold Medal at the Annual Dinner of the Engineering Society of Western Pennsylvania, on February 8, 1915. He also worked out many other problems of improvements in grade and curvature in the mountainous region traversed by the road.

On February 1, 1919, Mr. Thompson was elected to the Presidency of the Philadelphia Company and its subsidiaries, of Pittsburgh, Pa. Under his leadership there was tremendous expansion of power production and distribution facilities in Pittsburgh. The great plant of the Duquesne Light Company on the Allegheny River was built, and successive additions made it one of the largest in the United States. The entire electrical generation and distribution system was re-organized through the construction of new sub-stations and additions to the "power-ring" of Pittsburgh.

The Pittsburgh Electric Railway System which had been in the hands of receivers, was rehabilitated financially and in its services so that, in 1924, it was released by the Receiver. In 1925, at the Annual Convention of the American Electric Railway Association, the Pittsburgh Electric Railway Company was awarded the Charles A. Coffin Prize, among thirteen competitors. President Shanahan, of the American Electric Railway Association, said:

> "But a few years ago the company to which the award is now made was faced with a series of difficult situations that have been overcome with success. The contour of the territory served presents many natural obstacles. In addition, it is necessary to give service over a wide area among numerous scattered communities.
>
> "Your Committee is led to believe that if the methods and practices there employed, as set forth in the presentation submitted, were applied to other communities, confidence in the industry would soon become so firmly established that there would be no serious difficulty in obtaining the flow of capital necessary for extensions and betterments. With such additional capital, wisely administered, the industry will be on the high road to improvement.
>
> "In view of the success with which it has surmounted the natural difficulties presented in the communities served, this Committee takes great pleasure in awarding the Charles A. Coffin gold medal for 1925 to the Pittsburgh Railways."

Mr. Thompson occupied the position of President of the Philadelphia Company until September 1, 1926, when he resigned to become President of the United Gas Improvement Company. During his direction of this Company a number of significant mergers and connections were accomplished with other public utilities. That with the Philadelphia Electric Company was accomplished in 1927, and close relations were also established with the Public Service Corporation of New Jersey. He was also responsible for the merger of four large engineering and construction companies—Dwight P. Robinson and Company, Incorporated, of New York, N. Y.; Public Service Production Corporation, of Newark, N. J.; Day and Zimmermann Engineering and Construction Company, of Philadelphia, Pa., and United Gas Improvement Contracting Company, of Philadelphia—into one of the largest construction organizations in the United States—United Engineers and Constructors Incorporated. This Company has undertaken engineering and construction service in all fields in many parts of the United States and several foreign countries. Mr. Thompson retired from the United Gas Improvement Company, on January 23, 1929, because of ill health.

Some of his other business interests were, as follows: President of the Board of Directors of Allegheny College; Chairman of the Board of Directors of the Philadelphia Electric Company, Philadelphia Gas Works Company, United Engineers and Constructors Incorporated, and McKinney Manufacturing Company; Director of the American Gas Company, Consolidated Gas, Electric Light and Power Company of Baltimore, Mohawk Hudson Power Corporation, Pittsburgh Equitable Meter Company, Public Service Corporation of New Jersey and its subsidiaries, Welsbach and Company, Pennsylvania Railroad Company, Union Trust Company of Pittsburgh, Eastern Rolling Mills Company of Baltimore, Missouri-Kansas-Texas Railroad Company, The Bellefield Company, Pittsburgh Hotels Company, Chamber of Commerce of Pittsburgh, Pittsburgh Athletic Association, Boy Scouts of America, Pittsburgh Skin and Cancer Foundation, Associated Charities of Pittsburgh, Girard Trust Company, Merchants National Bank, at Meadville, Pa.; and Trustee of the American Surety Company, of New York, N. Y.

He was a member of the following technical and other associations and clubs: American Electric Railway Association, National Electric Light Association, Engineers Society of Western Pennsylvania, Economic Club of New York, Franklin Institute, American Railway Guild, American Academy of Political and Social Science, American Civic Association, Pennsylvania State Chamber of Commerce, United States Chamber of Commerce, Philadelphia Chamber of Commerce, Public Charities Association of Philadelphia; Allegheny College Alumni Association, Alumni Association of St. John's College, Delta Tau Delta, Kappa Phi Kappa, Phi Beta Kappa (Swarthmore Chapter); Engineers Club, Rittenhouse Club, Philadelphia Cricket Club, Sunnybrook Golf Club, Wissahickon Farms, Incorporated (Philadelphia); Duquesne Club, Pittsburgh Club, Pittsburgh Skating Club, Allegheny Country Club (Pittsburgh); Ligonier Country Club, Rolling Rock Club, Rolling Rock Rod and Gun Club, Rolling Rock Hunt Club (Pennsylvania); Recess Club (New York); Maryland Club (Baltimore); Forest Haven Club (Alexandria, Va.); and Mastigouche Fish and Game Club (Montreal, Que., Canada).

On June 14, 1921, the honorary degree of Doctor of Laws was conferred upon Mr. Thompson by St. John's College, Annapolis, Md. At the time of the conferring of this degree, the following remarks were made:

"The honorary degree of Doctor of Laws conferred by St. John's College upon Arthur Webster Thompson was in recognition of his outstanding service and unusual activity in connection with the Maryland Council of Defense.

"Mr. Thompson was appointed a member of this group on June 27, 1917, by Governor Emerson C. Harrington.

"In meeting the emergency created by the World War he carried out the duties and responsibility of his office of Federal Manager of: Baltimore and Ohio Railroad, Western Maryland Railroad, Cumberland and Pennsylvania Railroad, Cumberland Valley Railroad, Coal and Coke Railroad, Wheeling Terminal Railroad, Staten Island Lines, by devoting his efforts tirelessly and unselfishly in the defense of the State, the safety of its people, the protection of property, and to aid the Government of the United States in the successful prosecution of the war."

The University of Pittsburgh conferred upon Mr. Thompson the honorary degree of Doctor of Laws, on June 15, 1921, with the following words:

"The honorary degree of Doctor of Laws was conferred upon Arthur Webster Thompson by the University of Pittsburgh on June 15, 1921, in recognition of his active interest in civic work in Pittsburgh while President of the Philadelphia Company.

"He was a Director of the Chamber of Commerce, Chairman of the Special Housing Committee of the Chamber, and President of the Chamber's Housing Corporation. He was also active as Chairman of the Chamber of Commerce Committee which went to Washington, D. C., and was successful in inducing the War Department to grant sufficient funds to complete the Emsworth Dam."

The honorary degree of Doctor of Science was conferred upon him by the University of Pennsylvania, on June 15, 1927. Dr. Josiah H. Penniman, Provost of the University of Pennsylvania, in conferring this degree, said:

"Arthur Webster Thompson, Son of Allegheny College and honored with degrees from the University of Pittsburgh and St. John's College.

"You have made for yourself a notable career as an Engineer, and you are recognized by the profession as one of its ablest members. You have by successive stages risen to positions of increasing responsibility until you now occupy the office of President of one of the great corporations.

"That your interests are in no way narrow or confined to any one field is evidenced by the learned societies of which you are a member and especially by the fact that you are (Chairman) of the Board of Trustees of the College from which you received your earlier education. Devoted to the interests of college and university education you have been and are a force in the world of constructive administrative thought.

"In recognition of these facts we have invited you to be present that you may receive from the University of Pennsylvania an honorary degree appropriate to your life work.

"By virtue of the authority committed to me by the mandamus of the Trustees I hereby confer upon you the degree of Doctor of Science."

Mr. Thompson's life is a significant example of the manner in which young men rise by industry and ability from routine occupation to positions of great responsibility in connection with the management of large enterprises. Perhaps the most significant testimonial as to his qualities is that contributed by

Mr. Daniel Willard, of the Baltimore and Ohio Railroad Company, who knew as well as any other living man the value of Mr. Thompson's services:

"My first meeting with Mr. Thompson was shortly after I became President of the Baltimore and Ohio Railroad in January, 1910. At that time he had the title of Chief Engineer Maintenance of Way and he had been in the Baltimore and Ohio Service some ten or twelve years. At the time of our first meeting he must have been about thirty-five years old. He was healthy, vigorous, mentally alert, and personally attractive. I found him very keenly interested in his work and full of enthusiasm.

"Shortly after I was made President it became necessary to appoint a new Chief Engineer for the Company. While I had known Mr. Thompson personally but a short time, he had impressed me so favorably that I did not hesitate to appoint him to the position above mentioned. The Company had authorized the expenditure of a large sum of money for improvements and extensions of the property, and the expenditure of this money came very largely under the direction of the Chief Engineer. Many of the problems confronting him required careful thought and the application of a high quality of engineering skill. He handled all such matters in a way that was eminently satisfactory.

"Later on in the same year Mr. Thompson became General Manager of the Baltimore and Ohio System and was obliged on that account to give up the office of Chief Engineer. While the duties of General Manager were in a sense new to him, nevertheless he took up the work with enthusiasm and in a short time gave evidence of such skill as an administrative officer that he was still further advanced to the position of Vice-President in charge of Operation and Construction.

"On July 1, 1916, Mr. Thompson was transferred from the Operating to the Traffic Department with the same title, that of Vice-President, and he continued in charge of the Traffic and Commercial Development Departments until June 1, 1918, when he was appointed Federal Manager, the railroads at that time being under Federal Control. For several months previous to Federal Control his jurisdiction embraced the Operating as well as the Traffic Department.

"Shortly after the end of the war he resigned all of his connections with the Baltimore and Ohio Railroad to become President of the Philadelphia Company.

"My relations with Mr. Thompson in a business way were very close and our personal relations became and remained intimate and to me most agreeable. Because his service with the Baltimore and Ohio Company had been so satisfactory and of such a high order, we were naturally sorry when he left but having in mind the uncertainty of the future of the railroads at that particular time, together with the opportunities which seemed to be connected with the new position which he was offered, he undoubtedly acted wisely in leaving the railroad service when he did, although the change meant a very serious loss to the Baltimore and Ohio organization.

"Mr. Thompson was a man of unusual ability, with a most attractive personality, and with seemingly inexhaustible energy and enthusiasm. He had already before his death achieved an uncommon measure of success, although he had scarcely reached the prime of life. If he had lived, there is no doubt that the qualities which I have mentioned would have carried him much further."

He is survived by his widow, Marion Dinwiddie (Gordon) Thompson, and two sons, Gordon and Rolland Dinwiddie.

Mr. Thompson was elected an Associate Member of the American Society of Civil Engineers on June 4, 1902, and a Member on April 5, 1910.

WILLIAM LOVE THOMPSON, M. Am. Soc. C. E.[1]

DIED FEBRUARY 23, 1932

William Love Thompson, the son of John Knox and Elizabeth (Love) Thompson, was born at Chester, Miss., on September 6, 1875.

His early education was obtained in the public schools of Mississippi. Although this was interrupted by an attack of inflammatory rheumatism, he entered the Preparatory Department of the Mississippi State College (then the Mississippi Agricultural and Mechanical College) at the age of seventeen, so equipping himself that he was graduated in 1897 with the degree of Bachelor of Science.

From July, 1897, until January, 1898, Mr. Thompson was employed on farm surveys, and surface and tile drainage. From February, 1898, until June of the same year, he was Maintenance Inspector for the Mississippi Levee Board, at Greenville, Miss. Obtaining a leave of absence, he enlisted as a Private, in Company A, 2d Mississippi Volunteer Infantry, during the war with Spain. He was discharged on December 12, 1898, and in January, 1899, returned to his work with the Levee Board. From September, 1899, to June, 1902, he served this Board as Construction Inspector, and from this latter date until April, 1905, as Assistant Engineer.

In April, 1905, Mr. Thompson resigned his position in order to accept an appointment as Instrumentman in the Culebra Division of the Panama Canal. He was promoted to Junior Assistant Engineer in September, 1907, and, subsequently, to Senior Assistant Engineer in April, 1911. Until November, 1912, he was in charge of Hydraulic Plant Operation with the Pacific Division, and from that date until March, 1913, he was in charge of lock construction, at Pedro Miguel. When the dirt slides in Culebra Cut were critically hindering the work, Mr. Thompson was placed in charge of the design of layout, installation, and operation of the Gold Hill sluicing project, reporting directly to the Chief Engineer, the late George W. Goethals, Hon. M. Am. Soc. C. E.

In July, 1913, Mr. Thompson resigned to accept the position of Chief Engineer of the Mississippi Levee Board, at Greenville, Miss. During his eight years in this office, he gave himself unstintedly to the development of his district, lowering construction costs, and raising the standards of safety. He gave valuable aid in the development of tower excavators for levee construction, and was the author of a pamphlet entitled "Backwater in the Yazoo Basin," the result of original back-water studies, which was published by the Mississippi Levee Board in 1918. He was many times before the Rivers and Harbors Committee of Congress, attesting to the need for flood-control appropriations.

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[1] Memoir prepared by Messrs. W. L. Thompson, Jr., Springfield, Ill., and R. L. Thompson, Dallas, Tex.

From 1922 to 1925, Mr. Thompson was engaged in consulting practice at Greenville, during which time he served as Engineer of three highway projects, three drainage projects, a flood-control project, a river and rail transfer project, and general water-power investigations.

On account of his extensive experience with earth-moving machinery and flood control, Mr. Thompson was offered the position of Superintending Engineer for the Indo-Burma Petroleum Company, on its sand flat reclamation and guide-wall project at Lanywa, in Upper Burma. Here, he was engaged for three years in the most interesting work of his career, in addition to which he made many and fast friends among the British engineers who were also employed by Steel Brothers and Company, Limited, of London, England, Managing Agents for the Indo-Burma Petroleum Company. In October, 1928, he returned to America and took a well-earned rest after his three years in the Tropic East.

In January, 1930, Mr. Thompson entered the employ of the Beauharnois Construction Company, of Beauharnois, Que., Canada, as Engineer in charge of dredging operations on its canal project between Lakes St. Louis and St. Francis, on the St. Lawrence River. He remained at headquarters in Valleyfield, Que., until August, 1930, when he resigned to accept a return engagement with the Indo-Burma Petroleum Company as Engineer for an extension of the guide-wall project, and an additional hydraulic fill to obviate high-water platforms for oil-well rigs. He remained on this work until his death on February 23, 1932, which was caused by a gastric hemorrhage, induced by an attack of dysentery in August, 1931.

Mr. Thompson was married in 1901 to Mabel Richardson, at Bolivar, Miss., by whom he is survived. Three children also survive him: William Love, Jr., Robert L., and Margaret Thompson.

Mr. Thompson was a Life Member of the International Association of Navigation Congresses, and belonged to several social clubs in Burma.

A character such as that of this man could not be confined within the borders of one nation. Born and reared in the country, he was one of the most cosmopolitan and broad-minded personalities. A substantial, high-minded citizen himself, he considered that the morals of others, as long as they did not conflict with his professional ideals, should have no bearing on his business relations with them, and, as a result, he succeeded in a marked way in keeping personalities out of his professional decisions. He always flayed the issue, not the person behind the issue. Because of his professional standards, it must be said that he made enemies; he was uncompromising (some said autocratic) in his criticism of shoddy or uneconomical workmanship whenever he found any in his province.

He was, then, beloved as a human being and respected, to say the least, as an engineer. One can readily see that this combination would find a home on any spot on the globe. American, English, South American, Scotch, Hindu, and Burmanese swallowed his professional medicine with a wry face at times, but never failed to respond to the generous humanity of the man himself.

In addition to a loving family, many warm friends all over the world revere the memory of Mr. Thompson as a generous human being and a conscientious engineer.

Mr. Thompson was elected a Member of the American Society of Civil Engineers on June 6, 1911.

MAXIMILIAN ERNST ROBERT TOLTZ, M. Am. Soc. C. E.[1]

DIED JANUARY 11, 1932

Maximilian Ernst Robert Toltz was born at Koeslin, Germany, on September 2, 1857, the son of Herrmann and Malvina (Beilfuss) Toltz. He attended the Royal Gymnasium of his native town, entering the Royal Bau Akademie, Berlin, Germany, in 1875 and graduating from that college as Civil Engineer in 1878. Coming to the United States in 1881 he became a naturalized citizen in 1892.

Mr. Toltz began his engineering career as a Draftsman in the Engineering Department of the St. Paul, Minneapolis and Manitoba Railway Company (an antecedent of the Great Northern Railway Company) in July, 1881, going to the Chicago, Burlington and Northern Railway Company (acquired later by the Chicago, Burlington and Quincy Railway Company) as Draftsman and Assistant Engineer in September, 1886. In July of the next year, he entered the service of the City of St. Paul, Minn., as Sewer Engineer, under the late L. W. Rundlett, M. Am. Soc. C. E., City Engineer. Mr. Toltz left the city service in June, 1890, to become Principal Assistant Engineer of the Minneapolis Western Railway Company (a subsidiary of the Great Northern Railway). In August, 1891, he was made Assistant Engineer of the Great Northern Railway Company; Bridge Engineer, in 1894; and Mechanical Engineer in charge of motive power, in December 1899, holding this latter position until February 1, 1903.

During the time he was with the Great Northern Railway Company, he built the Dale Street Shops, in St. Paul, and shops at Havre, and Great Falls, Mont., and at Spokane, Wash.

Mr. Toltz was credited with being a pioneer in the design and erection of grain elevators of all-steel construction, the first being built at Buffalo, N. Y., and another at West Superior, Wis., in 1899. The latter was said to be the second largest and fastest working grain elevator in the world, having a record of receiving more than 100 cars of wheat in 24 hours.

From 1904 to 1907, he was Consulting Mechanical Engineer for the Canadian Pacific Railway Company, and, at the same time, Vice-President and General Manager of the Manistee and Grand Rapids Railroads. He designed the Angus Shops at Montreal, Que., Canada, said to be the largest on the Continent; also, the Winnipeg Shops.

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[1] Memoir prepared by George H. Herrold, M. Am. Soc. C. E.

Going abroad in 1906 for the late James J. Hill, F. Am. Soc. C. E., President of the Great Northern Railway Company, Mr. Toltz studied electric furnaces for smelting iron ores; the making of coke from peat for iron puddling furnaces; improvements in steam locomotives; and the electrification of steam railroads. Upon his return from Europe, he equipped several locomotives of the Great Northern, Northern Pacific, and Burlington Railroads with superheaters which effected a saving in coal of from 23 to 26 per cent. These were said to have been the first locomotives in the United States to be equipped in this manner; at present, it is estimated that there are, in the country, about 65 000 locomotives designed for superheat, with a resulting reduction of coal consumption amounting to about 25 000 tons per year. It was also through his recommendation that the Great Northern Railway Company electrified the Cascade Tunnel.

Establishing himself as a Consulting Engineer at St. Paul, Minn., in 1907, Mr. Toltz made reports on the electrification of the Butte, Anaconda and Pacific, the Northern Pacific, and the Chicago, Milwaukee and St. Paul Railroads. He pioneered a new construction of permanent, fireproof ore docks used on the Great Northern, Northern Pacific, and "Soo" Railroads, of which Dock No. 4 of the Great Northern, at Superior, Wis., is credited with the Lakes' record for quick loading, the 10 000-ton S. S. *Wolverine* having been loaded there in 28½ min. During this time he also served as Consulting Engineer on shops for the Erie Railroad Company, and for a terminal grain elevator built at Baltimore, Md., for the Maryland and Western Railroad Company.

In 1911, Mr. Toltz organized the Toltz Engineering Company for general engineering practice. Among the buildings constructed by the firm may be mentioned the new Union Depot at St. Paul, and the Railroad Building housing the offices of the Great Northern Railway Company and the Northern Pacific Railroad Company, as well as the First National Bank of St. Paul.

He was active in the Minnesota National Guard. He was commissioned Captain in the Company of Engineers, 1st Artillery, on November 30, 1900, and honorably discharged on August 10, 1903. He was commissioned First Lieutenant in the Engineer Corps, on July 1, 1907, promoted to the rank of Captain on March 12, 1909, and to the grade of Major, on May 2, 1913. He was placed on the retired list on November 28, 1917. During the World War, Major Toltz served as a civilian engineer and contractor on hospital construction and other projects at Camp Dodge, Iowa, Fort McHenry, Maryland, Roland Park, Baltimore, Md., Denver, Colo., and Camp Normoyle, Texas.

Returning to civilian life he organized the firm of Toltz, King, and Day, Incorporated, of which he became President. The firm entered the general architectural and engineering field, devoting itself to the larger engineering and architectural projects in the Northwest. Among the undertakings to the credit of Mr. Toltz are the $1 000 000 Court House at St. Cloud, Minn., court houses at Valley City, N. Dak., Redfield, S. Dak., Aitkin, Minn., and Minot, N. Dak. Many of the larger manufacturing plants of the "Twin Cities" bear the impress of his ingenuity and ability. Among these projects may be mentioned the Island Station of the St. Paul Gas Light Company (now known as the Northern States Power Company), and municipal power plants at Willmar, Moorhead, and Austin, Minn. Mr. Toltz was the guiding spirit in many other activities of the firm. Notable work was accomplished in the valuation of the street railway system at Des Moines, Iowa, where the appraisals made were accepted as the basis for a settlement out of Court. Mr. Toltz was also one of a committee of three St. Paul engineers appointed by that City to establish depreciation of the street railway system, and the findings of the Committee were accepted by both the City and the Railroad and Warehouse Commission.

At the time he was engaged in the study of fuel and steam economies in Europe, in 1928, Mr. Toltz investigated the public works departments of about seven or eight European countries, with a view to facilitating his work in connection with American Engineering Council.

Mr. Toltz became a member of the Assembly of American Engineering Council on January 1, 1923, and served for the 2-year term ending January 1, 1925, as a representative of the American Society of Mechanical Engineers. He was re-appointed by that Society on January 1, 1929, returning to the Assembly for the 2-year term ending January 1, 1931. From that time until January, 1932, he filled out the unexpired term of Mr. Charles Piez, representative of the American Society of Mechanical Engineers.

He was Past Vice-President of the American Society of Mechanical Engineers; a member of the Power Test-Code Committee, and of the Fuel Section; a member of the American Institute of Electrical Engineers; Society of American Military Engineers; Verein Deutscher Ingenieure; Honorary Member of Pi Tau Sigma Fraternity; Past-President of the Engineers' Society of St. Paul, and the Minnesota Federation of Architectural and Engineering Societies; Director of the American Interprofessional Institute; Director of the St. Paul Community Chest; and a member of the Minnesota Club, St. Paul Athletic Club, University Club, and Somerset Country Club.

He represented the American Society of Mechanical Engineers at the 100th Anniversary of the Institution of Civil Engineers, in London, England, in 1928, and was a United States Delegate to the World Engineering Congress at Tokyo, Japan, in 1929.

Mr. Toltz was the holder of more than thirty patents issued on locomotive developments, ore docks, grain elevators, acetylene gas lighting for railroad cars, steel dump cars, and other industrial improvements. He was the author of "Superheated Steam and Its Economies;" "Steam Locomotives *versus* Electric Locomotives;" "Peat as Fuel;" "Peat-Coke;" "Electrification of Steam Railroads;" "Low Temperature Distillation of Low Grade Fuels;" "Lignite;" "Natural Gas for Fuel in Ceramic Plants of North Dakota;" etc.

He gave freely of his time and ability to the advancement of the Engineering Profession. He was interested in the younger members, as instanced by the establishment of three Student Funds of $15 000 each, for electrical, civil, and mechanical engineering students. He was mercurial in his habits of thought, versatile, and optimistic. He was loved by his friends and associates for his responsiveness of character, which was sympathetic and continuous. He took a great interest in civic and National affairs. He was active on the streets of St. Paul until a few days before his death.

Mr. Toltz was married on May 17, 1883, to Amelia Kramer, of a prominent St. Paul family, who died in September, 1918. There were two children, both of whom are deceased. Three grand-children survive him. Later, Mr. Toltz married L. Elizabeth Argue, who also survives him.

Mr. Toltz was elected an Associate Member of the American Society of Civil Engineers, on September 7, 1892, and a Member in May 13, 1929.

OTTO VON GELDERN, M. Am. Soc. C. E.[1]

Died February 17, 1932

Otto von Geldern, the son of Gustav von Geldern, was born in Berlin, Germany, on September 6, 1852. He was brought to the United States as a child, spending his youth at Sonoma, Calif. He was educated in part in this country and in part at Gymnasia and Technical Schools in Germany.

His first professional experience was with his father who was an Engineer and Architect. In 1872, Mr. von Geldern returned to California, from Germany, and served as an Assistant Engineer on the construction of the Mare Island Dry Dock. Many years later he gave an interesting account of this work before the Technical Society of the Pacific Coast.[2] He was consulted on the raising of the earth dam of the Chabot Water-Works, at Vallejo, Calif., and had other various small jobs pending his appointment as Hydrographic Assistant in the United States Coast and Geodetic Survey in 1876. This Government work kept him active on the Pacific Coast, mainly on harbor surveys. In 1880, he was made Assistant Engineer on the Cascade Locks, Columbia River, and thereafter remained in Oregon for some years, with the United States Engineers, principally in charge of field surveys and investigations of proposed river and harbor works.

In 1886, his activities, still with the United States Engineers, were transferred to California and terminated in 1889 when he established himself in private practice at San Francisco.

Because of his experience on the harbors of the Pacific Coast, Mr. von Geldern had become thoroughly familiar with the ebb and flow of the tides and his knowledge of this subject and of the problems connected with a delimitation of shore areas, made him an authority on tide-marsh and ocean-front ownerships. Of valuation matters, too, he made a specialty, his advice being frequently called for in proceedings before rate-fixing authorities and in the Courts.

Mr. von Geldern was exceptionally well educated and it was but natural, therefore, that upon the retirement of its first Secretary, Mr. George Specht, he should have been selected on January 24, 1890, by the Technical Society of the Pacific Coast to serve as its Secretary. Of this Society, he became the heart and soul, continuing in contact with its members long after its further activity had been rendered unnecessary by the establishment of Local Sections of the Founder Societies.

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[1] Memoir prepared by C. E. Grunsky, Past-President, Am. Soc. C. E., and L. M. Cox and E. F. Haas, Members, Am. Soc. C. E.

[2] "Notes on the Dry Dock and Coffer-Dam at Navy Yard, Mare Island, Calif.," *Transactions*, Technical Soc. of the Pacific Coast, Vol. 7 (January-March, 1890), pp. 9–28.

He had great command of language, delighting in producing occasional short poems and highly meritorious accounts of early California life and customs. It would have been an easy transition for him from engineer to poet and philosopher; but he found his principal pleasure and relaxation in solving intricate mathematical problems.

There are few in the profession who had established broader fields of contact, particularly with members of the Engineering Profession, than Mr. von Geldern. It was to him that reference was made when specific information about a fellow engineer was desired, and it was to him that the engineer when in need of advice would appeal. Such appeals, moreover, were never in vain.

Of the hydrographic work done by Mr. von Geldern—surveys of the Columbia River Bar; of the entrance to San Diego Harbor, and of the harbor itself; of San Pedro or Los Angeles Harbor; and the special surveys of Arch Rock, Shag Rock, Anita Rock, and Blossom Rock, in San Francisco Bay, are to be noted. He served for a time as Consulting Engineer to the Anti-Débris Association of California, assisting this organization in the assembling of data in protest against a continuation of mining by the hydraulic process. He was an Adviser to Sutter County, California, on flood-control problems and also Consulting Engineer to the City of Oakland on matters pertaining to the determination of tidal planes as waterfront boundaries. His advice was sought in connection with salt-marsh reclamation at Humboldt Bay, on Napa River, and at various points about San Francisco Bay. He planned head-works, and supervised their construction, for the Modesto Irrigation District; and he was a Consultant in the matter of the selection of a naval base and dry dock site on the northern shore of San Francisco Bay, in Marin County. He served the Government, too, in the matter of locating coast defense armament, about San Francisco Bay. In this connection, Mr. von Geldern invented a new and practical method of adjusting a modern coast defense gun, an account of which was published[3] at the Artillery School, Fortress Monroe, Virginia, upon command of Maj.-Gen. Arthur MacArthur, U. S. A., in January, 1905. This method included, in the complete adjustment of large guns and mortars, the determination of errors in azimuth, elevation, and verticality, as well as the errors of the telescopic gun-sight.

In professional circles Mr. von Geldern will be remembered as an able engineer and a member of that fine old school of practitioners who hold the ethics of their calling above gain. He was known and respected, however, far beyond the boundaries of professional fields. In his youth, he knew personally many of the men whose names made California history during the period of transition from Mexican to American control. Among others, he knew General Vallejo—as a youth knows older men—and was intimate with his family. More than any other, perhaps, he is responsible for clearing the General's record of the charge of opportunism made by inaccurate or prejudiced historians. In the course of the studies undertaken in the interests of General Vallejo's history, Mr. von Geldern developed into one of the best authorities on the history of the so-called Sonoma District of Mexican California, the Kearney "filibuster," and the frequently exaggerated episode that men refer to as the "Bear Flag Republic."

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[3] *Artillery Notes*, U. S. Coast Artillery School, 1905.

Mr. von Geldern's life was a full one—and not restricted to the attainment of his own selfish ambitions. He was an asset to his State and an example of the type of citizen on whom the country must depend for the maintenance of its best traditions.

Long a member of the California Academy of Sciences, Mr. von Geldern had served as a Vice-President and as a member of its Council since 1915; he was also a Past-President of the Astronomical Society of the Pacific. He was a member of various other scientific organizations and served for many years as a Trustee of the Mechanics' Institute of San Francisco, being its President at the time of his death. He was also a Regent of the University of California.

He was married on July 10, 1884, to Mathilda Schildt, by whom he is survived. He is also survived by his three sons: Dr. Charles E., Edward B., and Dr. Hans S. von Geldern, and by seven grand-children.

Mr. von Geldern was elected a Member of the American Society of Civil Engineers on May 4, 1892.

PHILIP RIDSDALE WARREN, M. Am. Soc. C. E.[1]

Died March 6, 1933

Philip Ridsdale Warren, the son of the Rev. W. Warren, was born in London, England, on September 7, 1874. An engineering course at University College, London, from 1891 to 1893, was followed by apprenticeships with Moreland and Sons, Mechanical Engineers, London, 1893 to 1894; with Sir John Wolfe Barry, Civil Engineer, London, 1894 to 1900, and Henry Marc Brunel. During the latter part of Mr. Warren's connection with Sir John Wolfe Barry, he was Assistant Engineer on the construction of dock works at Barry, South Wales, including railways, bridges, etc., pontoon landing stage for steamers, railway tunnel, and general terminal work. From 1899 to 1900, as Engineer in Charge, he designed and constructed the masonry and concrete graving dock at Barry, which was 600 ft long and cost $1 500 000.

In 1900, Mr. Warren accepted the position of Engineer with Coode, Son and Matthews, of London, and as such he had charge of surveys, designs, and preparation of working drawings for a railway (30 miles) in Malacca, Straits Settlements. He also had charge of the construction of harbor and dock works in Malay Peninsula at Penang, Singapore, Malacca, and Kuantan. From 1907 until 1910, as Chief Engineer for the same Company, Mr. Warren supervised the construction of the dock works at Singapore, including a wet dock of 34 acres and a deep-sea wharf a mile in length, as well as a dry dock, together with coffer-dams and all equipment, caissons, pumping plant, etc., at a cost of $7 500 000.

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[1] Memoir compiled from information on file at the Headquarters of the Society.

From 1910 to 1914, Mr. Warren was Chief Engineer for Norton Griffiths and Company, Contractors, of Canada, for which Company he designed dry docks at Quebec, Que., Sydney, N. S., and St. John, N. B., Canada. He also had direct charge of the construction of harbor works and of a dry dock and shipbuilding yard, at St. John, which included quay walls, a breakwater, etc., the cost of which was $13 000 000.

With the outbreak of the World War in 1914, Mr. Warren went with the Royal Engineers to France, where he constructed the Channel ferry terminus at Cherbourg, and railway depots at Oissel, Yvetot, Lery, and Havre. In 1919, he was made Director General of Transport in France, with the rank of Lieutenant-Colonel, and also had charge of the reconstruction of the Belgian Ports of Ostend and Zeebrugge. From 1920 to 1921, under the terms of the Peace Treaty, Colonel Warren had charge of the demolition of the fortifications and naval harbor of Helgoland, Germany.

From 1922 to 1923, as Head of a mission for Sir Alexander Gibb and Partners, Colonel Warren went to Brazil to prepare the engineering details of a plan for the development of iron mines, including 350 miles of railway and the building of a port at Santa Cruz. From 1924 to 1925, he had charge of a hydro-electric project in Newfoundland for the development of 100 000 hp, and the construction of a 400-ton paper mill; during 1926 and 1927, Colonel Warren was in charge of the construction of water-works in China. In 1928, he went on an engineering mission to Greece in connection with reclamation and development projects.

Of all Colonel Warren's activities, his most successful and that which perhaps gave him the greatest pleasure was when he served as a representative of Sir Alexander Gibb and Partners, on the National Council of Ways of Communications in the Republic of Colombia, from 1929 to 1933. In this position he acted as Technical Adviser to the Colombian Government on all hydraulic engineering matters. He succeeded in winning the confidence equally of colleagues, officials, and members of the Government, to a remarkable degree, which testified to his tact and fairness, as foreign advisers are rarely welcome anywhere, and it was often necessary for Colonel Warren to offer criticism of the work of officials and departments, and to make decisions in matters where local political feelings were deeply involved. It is certain that his work during this period contributed in the highest degree to the prestige of his country and countrymen in Colombia. When he was forced to leave the country early in February, 1933, on account of illness, the Minister of Public Works referred to him as irreplaceable.

He was married on April 28, 1903, to Evelyn A. Bevan, eldest daughter of the late F. A. Bevan, Esq., Chairman of Barclay's Bank, London, and of Trent Park, Herts, England, who survives him.

Colonel Warren was a member of the Institution of Civil Engineers, of England, and of the Engineering Institute of Canada. For his war and post-war services, he received the Order of the British Empire (Military) and the Belgian Order of the Crown. He was a member of the Sports Club, St. James Square, London.

Perhaps the outstanding trait in Colonel Warren's character was his undimmed and unquenchable belief in human nature. He was almost constitutionally unable to think ill of any one. It meant that at times his confidence was betrayed, and he was left bearing burden, or blame, which in no way belonged to him. He took that risk, however, and as a reward he gained the friendship and confidence of almost all with whom he came in touch, and found good where others did not suspect it to exist.

Colonel Warren was elected a Member of the American Society of Civil Engineers on July 2, 1913.

GARDNER STEWART WILLIAMS, M. Am. Soc. C. E.[1]

Died December 12, 1931

Gardner Stewart Williams, the son of Stewart Beach and Juliet Merritt (Ripley) Williams, was born in Saginaw, Mich., on October 22, 1866. His pioneer progenitors settled in Deerfield, Mass., about 1640 and his grandfather, Gardner Davenport Williams, came to Michigan in 1814. His maternal grandfather, Thomas Cornell Ripley, was graduated from Rensselaer Polytechnic Institute, at Troy, N. Y., in 1828, was elected a Representative to Congress as a Whig in 1846, and was a friend of Daniel Webster. Mr. Williams' cousin, Gardner Fuller Williams, was a well-known mining engineer with Cecil Rhodes in Africa.

Mr. Williams was graduated from the Saginaw High School in 1884, and from the University of Michigan, Ann Arbor, Mich., in 1889. Ten years later he received from his Alma Mater the degree of Civil Engineer, a distinction at Michigan corresponding to a doctorate.

From 1893 to 1898 he was Civil Engineer to the Board of Water Commissioners of Detroit, Mich.; from 1898 to 1904, Professor of Experimental Hydraulics, and in charge of the Hydraulic Laboratory, at Cornell University, at Ithaca, N. Y.; from 1904 to 1911, Professor of Civil, Hydraulic, and Sanitary Engineering at the University of Michigan and head of the Department of Civil Engineering; and from 1911 until his death, a private Consultant, with headquarters at Ann Arbor.

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[1] Memoir prepared by a Committee of the Detroit Section, consisting of Louis E. Ayres, Mortimer E. Cooley, George H. Fenkell, and Clarence W. Hubbell, Members, Am. Soc. C. E.

Particularly in his early days, Mr. Williams devoted much attention to research and experiment. While connected with the Board of Water Commissioners of Detroit, he investigated the serious typhoid epidemic of 1892 and succeeded in tracing the infection to dredging operations at Port Huron, Mich., a distance by water of 60 miles. He directed a survey of water currents and temperatures in the Detroit River, leading to the location of a new intake. He performed experiments on the use of water by lawn sprinklers and studied waste in the consumption of water. Also, in collaboration with Clarence W. Hubbell, M. Am. Soc. C. E., and George H. Fenkell, M. Am. Soc. C. E., he made an investigation of the flow of water in pipes, with particular reference to the effect of curvature on loss of head. A paper[2] on this work was awarded the Norman Medal of the American Society of Civil Engineers in 1902. While at Cornell, Mr. Williams made a study of the flow of water over dams for the International Waterways Commission, of which he was a member from 1903 to 1905.

He spent thirteen years as a University professor, six at Cornell and seven at Michigan. He was thorough, conscientious, and fair. He accomplished considerable toward raising the standards of engineering education and of professional pride. He visioned the future engineer as a leader, both in creative work and in the economic life of the nation. With him responsibility did not cease with the successful development of a project, but continued on and embraced the effects of that development on society. A man of strong convictions, of unswerving purposes, he argued forcefully; and, if unsuccessful, he smiled his tolerance and stood ready for the next issue. He was a delightful antagonist and a most comfortable colleague and friend.

As a Consulting Engineer, Mr. Williams' efforts were devoted mainly to the fields of water-works and water power; and he served as an expert witness in many important suits involving hydraulics. In 1902, at Ithaca, he designed and built one of the earlier mechanical water filtration plants in the United States and also the first dome dam. The latter structure, known as the Six-Mile Creek Dam, possesses unique features, which remained unused elsewhere prior to the construction of the Coolidge Dam across the Gila River near San Carlos, Ariz. About 1905 Mr. Williams introduced, at Sault Ste. Marie, Mich., the vertical water-wheel in an open-flume scroll setting, directly connected to an electric generator; and during the following ten years he developed in a number of water-power plants an effective stream-line draft-tube. In the building of earthen embankments for dams and reservoirs, he abandoned the standard core-wall construction and substituted underdrained banks of selected materials. He was an advocate of the use of the arch in dam construction, and the builder of several multiple-arch dams, the first at Sturgis, Mich., in 1910, and the last on the Ural River, in Russia, in 1930. This latter structure contains 101 arches and has a length of approximately 3 250 ft.

As early as 1903 Mr. Williams appeared before the United States Supreme Court as an expert in the Drainage Canal Case of the State of Missouri *vs.*

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[2] "Experiments at Detroit, Mich., on the Effect of Curvature upon the Flow of Water in Pipes," by Gardner S. Williams, Clarence W. Hubbell, and George H. Fenkell, Members Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. XLVII (1902), p. 1.

the State of Illinois and the Sanitary District of Chicago; from 1907 to 1914, he was an expert in the United States Circuit Court of Illinois in the case of the United States *vs.* the Sanitary District of Chicago; and early in 1927 he appeared as an expert in this same case in the hearings held before the Hon. Charles Evans Hughes, Special Master for the Supreme Court. His Court experience over a period of twenty-five years covered about thirty important cases involving the science and art of hydraulics.

For many years Mr. Williams was an active worker in the Society and a frequent contributor to its publications. He was a Charter Member of the American Institute of Consulting Engineers; and, from 1921 to the time of his death, he was a representative of the Detroit Engineering Society on American Engineering Council and had been Vice-President of the Council since 1923. In this relationship he served upon several of the most important committees of the Council. He was an active worker for a National Department of Public Works and, in 1931, became Chairman of the Council's Committees on Government Reorganization and Flood Control, a member of the Committee on Water Resources and Control, and of the Committee on Corps of Engineers.

Mr. Williams was an early advocate of the development of the St. Lawrence Waterway and of the water power connected therewith. He was recognized as an expert on the hydraulics of the Great Lakes and favored regulation to control lake levels. During the last years of his life he devoted much time to the hydraulics of the Mississippi River and, as a means of controlling floods and aiding navigation, proposed the straightening and deepening of the channel, as an alternative to levee construction.

Mr. Williams was commissioned a Major in the Engineer Officers Reserve Corps, United States Army, in 1917. During the World War he served on the Committee on Industrial Preparedness under the Council of National Defense, and, in 1917, acted as Chief Engineer on the construction of Camp Beauregard, Alexandria, La.

He was the author of the section on "Hydraulics" in American Civil Engineers Handbook, and, jointly with the late Allen Hazen, M. Am. Soc. C. E., of "Hydraulic Tables." His writings embrace a long list of professional papers and discussions.

In 1893, he was married to Jessie Benton Wright, who survives him, as does also a daughter, Harriet Ripley (Williams) Waite, and a son, William Wright Williams.

Mr. Williams' life was devoted to work, and his days were intensive and long. He gave unsparingly of his remarkable energy in the lines of activity to which he directed his attention. He made important additions to the literature of hydraulics, and contributed largely to engineering society organizations, his efforts there being to broaden their scope and crystallize their purposes. Through American Engineering Council he rendered valuable and much needed public service. He was a successful teacher and a definite force in shaping engineering education so as to attain higher standards in scholarship and to broaden it to meet the needs of the future.

Throughout his entire career may be discerned a singleness of unselfish purpose, the object of which was to improve the quality of engineering service, and to establish engineering definitely in the mind of the public as one of the learned professions.

Mr. Williams was elected an Associate Member of the American Society of Civil Engineers on October 2, 1895, and a Member, on December 6, 1899. He was a Director of the Society from 1908 to 1910, and a Vice-President in 1914 and 1915.

GEORGE MONROE WISNER, M. Am. Soc. C. E.[1]

Died August 26, 1932

George Monroe Wisner was born on February 9, 1870, in Detroit, Mich. He was the son of the late George Y. Wisner, M. Am. Soc. C. E., a hydraulic engineer of international reputation, and Carrie (Palmer) Wisner. The genealogy of the family shows that the original American Wisner came to these shores in Colonial times from Switzerland. Educated in the public schools of Detroit, George Monroe Wisner entered the University of Michigan, at Ann Arbor, Mich., and was graduated in June, 1892, with the degree of Bachelor of Science in Civil Engineering.

His engineering career began at the age of twenty-two, when he accepted a position with The Sanitary District of Chicago, Ill. Fifteen years later, Mr. Wisner became its Chief Engineer. He passed through the various grades: As Instrumentman, having charge of the construction of four rock sections between Lemont and Lockport, Ill.; as Assistant Engineer (1895-1897), having charge of cement testing, but employed chiefly on hydraulic and design work; (1897-1901) in charge of the Chicago River Improvement, consisting of the dredging and docking of the Chicago River and the construction of a "by-pass" between Adams and Van Buren Streets, along the west side of the river; and as Assistant Chief Engineer (1901-1907), he directed all construction work, including the building of docks, coffer-dams, bridges, both substructures and superstructures, and the water power development.

During his engineering life of about forty years, Mr. Wisner was connected throughout with the work of the Sanitary District, first with construction and, later, with operation and maintenance, and the various problems related thereto.

During his earlier years of service with the District he was identified with many important projects, including the construction of the Main Channel; the construction of the Des Plaines River Improvement through Joliet, Ill.; the widening of the Chicago River; and the construction of the water power development, at Lockport. Appointed Assistant Chief Engineer in July, 1901, Mr. Wisner held this position until his election as Chief Engineer in August, 1907. Under his direction as Chief Engineer, the North Shore Channel and the Calumet-Sag Channel, adjuncts to the Main Channel, were planned and built. The design and construction of the Morton Grove Sewage Treatment Works, the 52d Avenue Outlet Sewer, the West 39th Street Conduit, the Stock Yards Intercepting Sewer, the North Shore Intercepting Sewer, the Summit Outfall Sewer, the Evanston Intercepting Sewers and Pumping Station, the Des Plaines Intercepting Sewer, and the Calumet Intercepting Sewer and Pumping Station were all under his direction during his term of office.

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[1] Memoir prepared by a Committee of the Illinois Section, consisting of Langdon Pearse, Edward J. Kelly, and Robert I. Randolph, Members, Am. Soc. C. E.

When Chicago was experiencing its rapid growth both in population and industry, between 1900 and 1910, Mr. Wisner foresaw the necessity of sewage treatment to supplement the existing dilution system. He convinced the Board of Trustees of the need of a thorough study of the sanitary program and, early in 1909, secured an appropriation for a testing station for domestic sewage at the 39th Street Pumping Station, at which the then known methods of sewage treatment were tried. He also began the organization of a Sanitary Engineering Staff, equipped with a laboratory adequate to handle the biological, bacterial, and chemical problems, as well as coal, oil, and other routine analyses. Following this, he directed an intensive study of the Packingtown and Stockyards wastes of Chicago, the tannery wastes, and the corn products wastes, whereby for the first time the magnitude of the industrial waste problem in the Sanitary District was revealed.

He developed a keen interest in sewage treatment as then practiced in the United States and abroad. In 1910, he visited England, France, and Germany to inspect sewage works and to make a first-hand study of foreign practice. In 1911, he made a report to the Board of Trustees on sewage disposal works for the Sanitary District, laying the foundation for later developments. Following these and other investigations, the Des Plaines River Sewage Treatment Works and the Calumet Sewage Treatment Works were designed and constructed, the first units of the sewage treatment system. At the same time, Mr. Wisner was following the negotiations and litigation relating to the diversion from Lake Michigan, and was one of the principal witnesses in the case of the United States *vs.* The Sanitary District and, later, in the hearings before the Rivers and Harbors Committee of Congress, and became an expert in advising the lawyers in such matters.

He resigned as Chief Engineer of the Sanitary District in June, 1920, to enter private consulting practice, but was retained by the District as Consulting Engineer, a relationship which continued until his death. As a consultant, he proved to be a valuable adviser during the development of the sewage treatment program and the lake-level litigation (Wisconsin *et al vs.* Illinois *et al*), and was also instrumental in making great savings for the Sanitary District.

Mr. Wisner's private consulting work comprised harbor works in Labrador, dock work and shore protection in Florida, and drainage and road work in Cook County, Illinois. He served one year as Chairman of the Advisory

Subway Engineering Commission of the City of Chicago. For several years prior to his death he was deeply interested in the transportation problem of the City of Chicago.

His loyalty to his chosen associates brought to him many enduring friendships. His ability to judge men quickly and strip matters presented for his decision to the essentials for sound decisions made him an efficient executive. Mr. Wisner possessed tact and excellent business ability, with engineering talent. His outstanding characteristic was his dynamic personality. His indomitable spirit was best evidenced by his cheerful courage during the last six years of his life in meeting the handicap of total blindness. In spite of this, he kept up his interest in engineering works and increased his mental powers.

He was a member of the American Society of Mechanical Engineers and the Western Society of Engineers. His club affiliations included the Beta Theta Pi, Chicago Athletic Association, Chicago Engineers, University, South Shore Country, and Beverly Country Clubs. He took a great interest in athletic events, playing football when attending the University of Michigan. Throughout his working life, he kept up his skill in swimming. He also enjoyed yachting and travel.

In 1902, Mr. Wisner was married to Edith K. Young, a daughter of the late Caryl Young, who was one of the outstanding real estate operators in Chicago during the Nineties. They had no children.

He died suddenly on August 26, 1932, at his home, in Chicago, after a short illness. Besides the widow and a younger brother, Ralph, he left a host of friends to mourn his passing.

Mr. Wisner was elected a Member of the American Society of Civil Engineers on February 4, 1903.

HERBERT WALDO YORK, M. Am. Soc. C. E.[1]

Died November 21, 1932

Herbert Waldo York was born at West Roxbury, Mass., on February 18, 1864. He was the son of John and Julia A. (Hinckley) York. His father came to the United States from England in 1850, and was employed in the office of the Commissioner of Customs, United States Treasury Department, Washington, D. C., until his death. As a boy, Herbert attended the public schools and was graduated from the High School, at Washington.

From November, 1882, to November, 1885, Mr. York served as Assistant to the late Charles E. Emery, M. Am. Soc. C. E., who was then Consulting Engineer for the United States Revenue Marine (now United States Coast Guard Service). As such, his duties involved work on plans and specifications for new machinery and alterations and repairs to old equipment.

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[1] Memoir prepared by Lt.-Comdr. John Barnard, U. S. N. R. (*Retired*), New York, N. Y.

It was urged upon him at this time that he should acquire the sea service pre-requisite to an examination for a sea-going Marine Engineer, and it was arranged that he should go out in the Engineer's Department of that famous old United States Revenue Cutter, *Bear,* then being sent from the Atlantic Coast to the Pacific for the beginning of her internationally known services in Behring Sea and the Arctic. These services have only lately ended, after nearly fifty years of relief to distressed or icebound whalers, carrying medical assistance and succor to the scattered habitations along the shores, holding Court, and policing the Alaskan Coast and enforcing the international sealing laws.

Upon this ship's arrival in San Francisco, Calif., in the spring of 1886, Mr. York's sea service was still inadequate for promotion, and he continued on the ship for her first summer's patrol in the Arctic. Upon the *Bear's* return to San Francisco, Mr. York successfully passed the examinations for, and was commissioned as, a Second Assistant Engineer in the United States Revenue Marine.

From this service he resigned in December, 1886, to return East where he entered the employ of the New York Steam Company, of New York, N. Y., as Assistant Engineer in charge of building service connections, repairs to street mains, tests, etc., for one year, until November, 1887.

He then engaged in private consulting work until July, 1889, when he became Assistant Superintendent of the Phœnix Construction Company, on construction of electrical subways for The Consolidated Telegraph and Electric Subway Company. Mr. York was in charge of all construction south of Twenty-third Street, New York City, and, to January, 1890, had built about 110 miles of ducts. He then again engaged in private work until May, 1890, when he was employed for about six months as Superintendent on the construction of new shops for The Yale and Towne Manufacturing Company, in Stamford, Conn.

In November, 1890, Mr. York joined The United Electric Light and Power Company from which were run the first alternating-current circuits in New York City. This Company bought the old Brush Electric Illuminating Company and with it The United States Illuminating Company was merged. He joined this Company as Assistant Engineer and, in May, 1892, was made Chief Engineer in charge of operation and maintenance of all stations and of the design, construction, and operation of a new station on East Twenty-eighth Street and the East River, New York City.

In these days alternating-current generating plants have replaced the earlier and simpler direct-current stations, and much is now commonplace which then presented problems. Although to-day these problems are largely forgotten, they were serious in the older days and had to be solved if success was to result. Accordingly, it is interesting to hear from the first Electrical Engineer of that Twenty-eighth Street Station, of some of the devices originating with Herbert York for sychronizing those reciprocating engines. The first of these devices consisted of little whistles blowing at a given point of the stroke so that as the engineer brought up to speed an engine about to be cut in, the operator at the switchboard would hear its whistle gradually come into unison with the others. This method Mr. York later replaced by a control on the valve gear which automatically accomplished synchronization. On this, he obtained a patent which was acquired by The Westinghouse Company.

It was about this time that he became interested in Engineering Society work. He joined the Society and its sister organization, the American Society of Mechanical Engineers, and to both of them he remained loyal for the remainder of his life. Shortly after his election as a Junior of the Society, Mr. York submitted a paper on "The Twenty-Eighth Street Central Station of the United Electric Light and Power Company," which was accepted for publication.[3] As a result, in 1896, he was honored in receiving the Society's Collingwood Prize for Juniors, his being the second award of this highly esteemed distinction which has since included many notable engineers as its recipients.

He formed another association during his services with The United Electric Light and Power Company. He joined, on November 5, 1900, the First Battalion of the Naval Militia of New York State as Lieutenant (Junior Engineer), serving as Assistant Engineer of that Battalion, and transferring on November 15, 1904, to the duties of Gunnery Officer. In these positions he continued until September 30, 1911, when he was promoted to the rank of Lieutenant Commander, and served as Ordnance Officer on the Staff of the Commanding Officer of the New York Naval Militia.

In the meantime, Mr. York, on April 1, 1903, became Chief Engineer of the United Lead Company, holding that position until May 1, 1905, when he accepted the position of Engineering Manager for The J. G. White Engineering Corporation, which had undertaken the function of Consulting Engineers for the American Smelting and Refining Company, coupled with an understanding that Mr. York personally should handle its work. The attempt, however, proved too taxing and on June 30, 1906, he transferred to the American Smelting and Refining Company as Consulting Engineer, in which capacity he continued until his retirement on April 1, 1930, after twenty-five years of the widely varying requirements of that Company spread the length and breadth of the Western Hemisphere. It was these duties which prevented his participation in the World War.

His joining the Naval Militia was a natural outcome of his early training, and he became an integral part of the organization. So, when in 1917, practically all his associates were among the first to go into active service, he, too, eagerly sought duty and was keenly disappointed when the Navy Department deemed him too important a factor in munitions production. After the war, Mr. York continued his active efforts in the State Naval Service until he was retired at his own request on June 15, 1927, and was transferred to the State Reserve List with the rank of Commander. He also held at his death the rank of Lieutenant Commander on the Honorary Retired List of the United States Naval Reserve.

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[3] *Transactions*, Am. Soc. C. E., Vol. XXXV (1896), p. 429.

One of Commander York's outstanding characteristics was the ease with which he accomplished an astounding amount of work. He might well be compared with a ship of perfect lines making speed, blow high blow low, with the least possible disturbance in his progress and, apparently, with the minimum of propulsive effort. Allied with this and, possibly an important factor in this quietness of effort, was a sense of humor always in evidence. He had a peculiarly personal type of smile which almost never entirely disappeared and one might also say that he smiled in his speech; for his comments on almost any topic were illuminated with little quirks of expression that made them that much more succinct. He had, too, an extremely analytical mind which often reduced hard facts to their simplest terms. He never seemed to strive for friendships, he was always his own natural self to whom people gravitated, and this same sane disposition was not one to arouse the enmities of any. To know him was to feel spontaneous admiration.

Commander York was married on June 19, 1930, to Mrs. Jane Dolen-Lenz, who survives him.

He was a member of the Franklin Institute, and the American Institute of Mining and Metallurgical Engineers. His clubs included The Engineers' Club, of New York, the New York Yacht Club, and The Bankers Club, of New York.

Commander York was elected a Junior of the American Society of Civil Engineers on May 2, 1888, and a Member on June 3, 1896.

MORRIS ALEXANDER ZOOK, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 3, 1932

Morris Alexander Zook was born in Philadelphia, Pa., on March 2, 1859, the son of Henry M. and Sarah (Moore) Zook. His early education was obtained in the schools of Chester, Pa. He attended the Polytechnic College of the State of Pennsylvania (later, the University of Pennsylvania), and was graduated in 1879, with the degree of Civil Engineer.

During a period of fifty-three years, Mr. Zook was actively engaged in the construction, maintenance, operation, and valuation of railroads in different parts of the United States, Canada, and Cuba. He began work as Chainman on the Engineer Corps and steadily advanced through all the grades of engineering to the position of President. At the time of his death he was President of the Montana, Wyoming and Southern Railroad Company, with offices in New York, N. Y.; he was also a Consulting Engineer specializing in the valuation of railroads. His various activities are outlined briefly in the following summary:

Shenandoah Valley and Norfolk and Western Railway Companies, 1880 to 1883; Resident Engineer and Division Engineer in charge of heavy mountain construction and location; Resident Engineer from Riverton to Luray, Va.; Division Engineer in charge from Buchanan to Roanoke, Va.; and Locating Engineer, New River and Cripple Creek Extensions of the Norfolk and Western Railroad and its Terminal, at Norfolk, Va.

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[1] Memoir prepared by Edwin F. Wendt, M. Am. Soc. C. E.

Pennsylvania Lines West of Pittsburgh, 1883 to 1889: Assistant Engineer and Engineer of Maintenance, in charge of the Indianapolis and Vincennes Railroad, Indianapolis Union Railway, and the Chicago Division.

Chicago, Indianapolis and Louisville Railway Company, 1890 and 1891: One year as Engineer, Maintenance of Way, and one year as Superintendent.

The Southern Railway Company, Queen and Crescent Route, 1892 to 1897: Engineer, Maintenance of Way, and General Roadmaster in charge of the Alabama Great Southern Railway from Chattanooga, Tenn., to Meridian, Miss.

Louisville, Evansville and St. Louis Railway Company, 1898 to 1901: Three years as Engineer, Maintenance of Way, and Chief Engineer.

Central Railroad Company of New Jersey, 1902 and 1903: Two years as Special Engineer, in charge of important emergency work, and the reconstruction of the Lehigh and Susquehanna Division, due to floods in the Lehigh River.

Peabody Coal Company, of Chicago, Ill., 1904 and 1905: One year as Chief Engineer, one as Consulting Engineer, and one year as Chief Engineer of Construction and Manager of Railway Operation. During this period Mr. Zook located important railway lines in Southern Illinois for acquiring a large acreage of coal lands, constructed the Chicago and Illinois Midland Railway, and organized and operated the line for the first six months.

J. G. White and Company (of New York, N. Y.) Engineering Corporation, 1906 One year as Special Engineer, making expert examinations, reports, bids, etc., in various parts of the United States and Canada. Mr. Zook also went to Cuba in charge of engineering forces, where he located the Havana Central Railroad from Havana to Cienfuegos.

New York, New Haven and Hartford Railroad Company, 1907 and 1908: In company with H. W. Edwards, M. Am. Soc. C. E., he made a complete physical valuation of this property for John F. Stevens, Past-President and Hon. M. Am. Soc. C. E., who was then Vice-President of that railroad.

Consulting Engineer, 1909: Mr. Zook investigated coal properties in Montana and Pennsylvania.

Chicago Great Western Railroad Company, 1910 and 1911: Engineer of Construction in charge of the reconstruction and rehabilitation of this property. Later, Mr. Zook was in charge of joint terminals and facilities, and of transportation problems.

Associated with J. W. Kendrick, M. Am. Soc. C. E., 1912: During this time he was engaged on expert examination and reports of important railway systems, and made a complete physical valuation of the Quebec-Ontario Lines of the Canadian Pacific Railway Company, and other properties.

Grand Trunk Railway Company, 1913: He made a physical valuation of the joint lines of the Grand Trunk-Wabash Railways, in Canada, between

Buffalo, N. Y., Niagara Falls, Ont., and Detroit, Mich. He also made examinations with preliminary report and estimate of a new independent line of railroad from Norfolk, Va., to Washington, D. C.

Federal Valuation of Railroads, 1914 to 1921: Mr. Zook was Resident Engineer of the Bureau of Valuation, of the Interstate Commerce Commission, from January 2, 1914, to September 30, 1921, a period of seven years and nine months. He was General Assistant to the Hon. C. A. Prouty, Director of Valuation, and was actively in charge of the Cost Section and supervised the work required by Valuation Order No. 3. After the death of Judge Prouty, he left the Government Service on October 1, 1921.

Montana, Wyoming and Southern Railroad Company, 1922 to 1932: He was Vice-President of the Montana, Wyoming and Southern Railroad Company from March to September, 1922, and President, from September, 1922, to the time of his death on September 3, 1932. This is an important coal-carrying railroad in Montana.

Mr. Zook was a Consulting Engineer of wide experience, and during the past ten years he was engaged at different times with the Washington, Idaho and Montana Railroad, the Chicago and Illinois Midland Railroad, the Lake Champlain and Moriah Railroad, the Rescoe, Snyder and Pacific Railroad, and the Cumberland and Manchester Railroad Companies.

He was pre-eminent in his chosen field of railroad engineering, particularly in the economics of location, construction, and operation. He had a large experience in the valuation of railroads and public utilities, and was a pioneer in this field, having made important valuations before the Federal Valuation of Railroads was undertaken by the Government.

Mr. Zook was a member of the American Railway Engineering Association, Washington (D. C.) Society of Engineers, and the Chamber of Commerce of the State of New York. He was also a Licensed Engineer in the State of New York. He belonged to the Cosmos Club, of Washington, D. C., and the Columbia Club, of Indianapolis, Ind.

He was married on June 12, 1888, to Justine Schenck who, with a son, Henry M., survives him.

He was an earnest Christian, being at the time of his death a member of the First Baptist Church, of Plainfield, N. J. He was honest, gracious, and frank, loved by all who knew him for his sterling qualities of character, and respected by his co-workers for his outstanding ability and fairness. He was an enthusiastic Engineer and advocated the high standards of honor, diligent work, and efficiency for which the Engineering Profession stands. In every community in which he lived Mr. Zook was regarded as a most useful and highly respected citizen, and he was held in high esteem by his professional associates. His fine character, interesting personality, and love of his profession made for him a nation-wide circle of friends.

Mr. Zook was elected a Member of the American Society of Civil Engineers on October 4, 1899.

ARMAND JOHN CALLERI, Assoc. M. Am. Soc. C. E.[1]

Died June 3, 1931

Armand John Calleri, the son of Michele and Celestina Calleri, was born in Loomis, Calif., on September 15, 1900.

As a boy he attended the public schools at Oakland, Calif., the Oakland High School, and St. Mary's High School of that city. Mr. Calleri received his technical education at St. Mary's College, Oakland, from which he was graduated in May, 1922, with the degree of Bachelor of Science, *Maxima cum Laude*, in Civil Engineering. During the latter part of his college course, from August, 1921, to May, 1922, he was Instructor at St. Mary's High School in Elementary Chemistry, Mechanical Drawing, and Descriptive Geometry.

In June, 1922, Mr. Calleri entered the employ of Clyde Fuller Smith, Consulting Sanitary Engineer, as Draftsman, Structural Designer, and Assistant Engineer. As such, he was engaged on investigations, designs, etc., of activated sludge and Imhoff sewage disposal plants for the Towns of Lodi, Tracy, and Ojai, Calif.

From August, 1923, to July, 1924, as Junior Partner of the firm of Smith and Calleri, Consulting Sanitary Engineers, of Berkeley, Calif., Mr. Calleri made studies and structural designs for an Imhoff sewage disposal plant for Hollister, Calif., the construction of which he supervised. This was followed by studies and field investigations for a report on the sewage disposal of Santa Rosa, Calif. He also made studies and structural designs, and prepared the contract forms and specifications, for an activated sludge sewage disposal plant for Santa Rosa.

From July, 1924, to January, 1929, he was with the East Bay Water Company, of Oakland, as follows: From July, 1924, to May, 1925, as Draftsman and Designer engaged on studies and design for the Upper San Leandro Filtration Plant (12 000 000 gal daily capacity); from May, 1925 to May, 1926, as Resident Engineer on the construction of the aeration basin, coagulation tanks, sedimentation basins, filters, wash-water tank, filtered water reservoir, buildings, etc., of the same project, the cost of which was $750 000; and from May, 1926, to January 1, 1927, as Resident Engineer on the Upper San Leandro Project, including the completion of the filtration plant and the Upper San Leandro Dam and Spillway.

From August 15, to December 1, 1926, Mr. Calleri also served as Assistant Engineer in charge of field investigations of the Sacramento River as a source of supply for the Port Costa Water Company, which included the preparation of a preliminary report with plans and estimates of cost.

From January, 1927, to January, 1928, he was with the East Bay Water Company as Assistant Engineer on design, detailing costs, and final report, on the Upper San Leandro Project, and from January, 1928, to January, 1929, as Assistant Designing Engineer, part of this time being spent as

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[1] Memoir compiled from information on file at the Headquarters of the Society.

Designing Engineer with George W. Hawley, M. Am. Soc. C. E., Consulting Engineer, on hydraulic and structural designs for dam diversion and supply tunnel, control works, and spillway for the Webber Creek development of the El Dorado Irrigation District.

From January 1, 1929, to the time of his death on June 3, 1931, Mr. Calleri served the East Bay Municipal Utility District as Designing Engineer on the structural designs and the preparation of estimates and plans for the alteration and enlargement of San Pablo and Upper San Leandro Filtration Plants, and the re-design of Middle Bar Dam. He also served part of this time again as Designing Engineer for Mr. Hawley on the design and specifications for a 2 000 000-gal water filtration plant for Placerville, Calif.

Only thirty years of age when he passed away, Mr. Calleri was an engineer who possessed those qualities of judgment and leadership which pointed to a brilliant professional future. He was of unfailing good nature, and was possessed of a scholarly mind and sound judgment. All those with whom he came in contact respected his opinions, and he was always able to preserve his poise even under the most trying circumstances.

He was married on July 23, 1927, to Alice C. Gomer, of Oakland, who, with his parents and two sisters, Flora and Edna Calleri, survives him.

Mr. Calleri was elected an Associate Member of the American Society of Civil Engineers on March 5, 1928. He was also a member of the San Francisco Section of the Society.

FRANCIS FEHER, Assoc. M. Am. Soc. C. E.[1]

DIED APRIL 30, 1932

Francis Feher, the son of Peter and Hermina Feher, was born in Leva, Hungary, on December 19, 1890. He was very young when his father died, and his mother was not able to send him through High School. He worked for a while in the machine shop of the State Railroad as an Apprentice, trying in every possible way to go back to school. In this he succeeded. Ultimately, Mr. Feher received an appointment as a School Teacher, but he never taught, as his ambitions were not yet fulfilled.

In 1910, he entered the Royal Hungarian Technical University, and studied there until the outbreak of the World War. From October, 1915, until October, 1918, he served in the Austrian-Hungarian Army as Commander of the 6th Engineer Division. He was seriously wounded during the fighting in Italy, and this injury eventually caused his death. After the Armistice, in 1919, he received the degree of Civil Engineer from the Royal Technical University.

From 1919 to August, 1921, Mr. Feher was engaged with an architectural and engineering firm, in Budapest. Owing to economic conditions in Hungary, he was obliged to emigrate. In 1921, he went to The Netherlands and until 1922 he was with the Delta Concrete Company in The Hague. From May, 1922, to November, 1923, he was employed as a Structural Engineer in Rotterdam, Holland. In December, 1923, he came to the United States and settled in Cleveland, Ohio, where he lived until his death.

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[1] Memoir prepared by Ernest A. Weiss, Assoc. M. Am. Soc. C. E.

Mr. Feher was with the T. H. Brooks Company, as Draftsman, until April, 1924. From April, 1924, to 1926, he was employed by W. S. Ferguson Company as Structural Engineer, and from 1926 to May, 1928, he was with the Engineering Division of the City of Cleveland as Engineer of Structural Design.

In May, 1928, Mr. Feher went to the Cleveland Union Terminals where he was engaged until March, 1930, as a Designer. He was also employed as Designer, from April, 1930, to February, 1931, with the Austin Company, and from that date until December, 1931, he was with the Arthur McKee Company.

In this short time Mr. Feher had acquired a good reputation in the engineering circles of Cleveland, and he was highly esteemed by his associates and by his many friends. His sudden death was a severe blow to those who had come in contact with him.

He was married on January 24, 1917, to Irma Major, who, with his sister and his mother, survives him.

Mr. Feher was elected an Associate Member of the American Society of Civil Engineers on March 5, 1928.

DAVID PRYDE GILMORE, Assoc. M. Am. Soc. C. E.[1]

Died January 13, 1933

David Pryde Gilmore was born in Arnot, Pa., on May 6, 1885. He was the son of James and Agnes (Pryde) Gilmore, both of whom were of Scotch descent. He received his grade and High School education in Blossburg, Pa., where he was graduated in 1903. In the fall of that year he enrolled at the Pennsylvania State College, State College, Pa., from which he received his degree of Bachelor of Science in Civil Engineering in 1907.

In the fall of 1907, Mr. Gilmore was appointed Instructor in Civil Engineering at Pennsylvania State College, continuing in this capacity until September, 1909. At this time, he received an appointment as Instructor in Drawing and Descriptive Geometry at the State University of Iowa, Ames, Iowa. Here, he later served, successively, in the capacities of Assistant Professor of Mechanics and Hydraulics, and Associate Professor of Civil Engineering. He was granted the degree of Master of Science in 1913.

During the summer periods, Professor Gilmore enriched his experience by associating himself with the Chicago, Milwaukee and St. Paul Railway Company as Assistant Engineer on valuation and as a Detailer and Designer of viaduct piers and structural steel bridgework; with the Federal Bridge

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[1] Memoir prepared by Clair C. Johnston, Director of Civ. Eng., Coll. of Eng., Univ. of Detroit, Detroit, Mich.

Company, of Waukesha, Wis., as a Detailer and Designer; with the Government survey of the Miami Conservancy District on flood control; and, during the World War, with the United States Forest Products Laboratory, at Madison, Wis., investigating the physical properties of wood for airplane construction.

From September, 1924, to October, 1925, he served as Engineer in Charge of Tests for street paving and sewer materials for the City of Detroit, Mich. In October, 1925, he was appointed Professor and Head of the Civil Engineering Department, University of Detroit, serving in this position until his death. In this capacity he revised the curriculum and improved all the Civil Engineering Courses. His constructive leadership, quiet dignity, and genial personality endeared him to all his associates and to the student body.

The Rev. John P. Morrissey, of Loyola University, and former Regent of the Engineering College at the University of Detroit, has written of him:

> "Professor Gilmore was an outstanding member of his Profession, well versed in the practices and art of Civil Engineering. His greatest achievement was conducting the young men of his classes through the mazes of engineering problems, making clear in his concise manner the most abstruse principles of engineering theory. He was most exact in his demands on students and insisted on precise methods in the solution of problems. Such was his character and personality that he could lead his class through the most arduous work and make them like it. As with other men of this profession, Professor Gilmore's work was done in a quiet, unostentatious way, and the good he did will persist long after he has left the scene of his endeavor."

Professor Gilmore suffered a physical break-down in January, 1930. He recovered sufficiently to resume his work in September of the same year, and seemed to be improving gradually until he suffered a cerebral hemorrhage which caused his death.

In 1910, Professor Gilmore was married to Edna Katherine Kilbourne, culminating a romance that started in their Grammar School days. He is survived by his widow, and their only child, Roger.

He was a member of the Society for the Promotion of Engineering Education and the Detroit Engineering Society.

Professor Gilmore was elected an Associate Member of the American Society of Civil Engineers on June 4, 1928.

JOHN ALEXANDER McLEAN, Assoc. M. Am. Soc. C. E.[1]

DIED JULY 30, 1932

John Alexander McLean was born on March 29, 1881, at Fisher, Minn. He was the son of Alexander and Martha (Morrow) McLean, and was taken by his parents in infancy to Grafton, N. Dak., which has since been the family home.

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[1] Memoir prepared by Dr. George A. McFarland, President, State Teachers College, Minot, N. Dak.

Mr. McLean attended the public schools of Grafton through the High School, from which he was graduated in June, 1900. In the fall of 1901, he entered the University of North Dakota, matriculating in the Engineering College. After four years of study and training, he was graduated with the degree of Mining Engineer, in June, 1905.

Immediately following the completion of his engineering course, Mr. McLean was placed in charge of a field survey party on drainage work in his home county of Walsh by the County Engineer, Mr. H. H. Mott.

The son of a family of pioneers, he inherited the spirit of adventure and soon sought work in the newer West. In 1906, he left the home of his boyhood and was employed in and about the City of Seattle, Wash., in various capacities on sub-division work. His objective had been Alaska, for during his student days it had been engaging the attention of the world by its Yukon gold rush. In 1907, he secured a position as Chief of Party on "Claim Location Surveys" in the Behring Lake Coal Fields, Alaska, under the direction of the Engineer, Mr. M. A. Green.

Returning to the States in 1908, Mr. McLean was employed as Field Engineer for the F. C. Kelsey Consulting Engineers, of Portland, Ore., on surveys, plans, and estimates for municipal improvements in various cities in the State of Oregon. From 1908 to 1910, the Portland Water Supply System was being extended and reconstructed, and Mr. McLean found employment in various phases of this work as Transitman on pipe-line surveys, reservoir location, and, in the City offices, on the development of plans and estimates.

At the beginning of 1910, he received an appointment as Assistant Engineer with the United States Reclamation Service. His first responsibility under this appointment was as Chief of a survey party on distribution and location on the Sunnyside Project in Washington. The following year he was transferred to the Klamath Falls Project in Oregon, where he did engineering work on the canal extension of that project. Mr. McLean's efficiency and leadership attracted the attention of the authorities of Klamath Falls, and he was induced to resign from the Reclamation Service and to accept the position of City Engineer in that growing city. Under his administration—1912 to 1915— he had a wide and varied experience, growing out of the conditions surrounding a community rapidly changing from town to city requirements. Miles of paving were laid, sewerage and water-works had to be constructed, and many other civic improvements were inaugurated and completed under his direction.

In 1916, the R. C. Storrie Construction Company, of San Francisco, Calif., was seeking an engineer for an important development near the City of Las Vegas, N. Mex. The attention of the Company was directed to Mr. McLean, and he accepted the offer to take charge of this irrigation project as Chief Engineer. The objective was the reclamation of a large area of dry lands in that section. The work involved the construction of a 500 000-cu yd earth-fill dam; 25 miles of canal (capacity, 140 cu ft per sec), and a half-mile water-level rock tunnel—all of which was planned as subsidiary to the irrigation project.

After two years' work on this Las Vegas Project, Mr. McLean felt that he should offer his service to the United States, and he resigned his position with the R. C. Storrie Company and volunteered for service in the World War. He entered as Captain of Engineers in the United States Army and was attached to the Headquarters Construction (new camp), at Camp A. A. Humphreys, Virginia. He was assigned to work involving plans for the future development of the water and sewerage systems of this camp. After several months in this capacity, the war having terminated, he was honorably discharged and returned to his home at Grafton, in 1919.

The City of Crookston, Minn., in the county where he was born, learning of his availability, offered Mr. McLean the position as City Engineer. As such he had charge of all the city service systems, their upkeep and development; the city water and sewerage systems were extended and developed; much paving was laid; and bath houses and swimming pools were constructed. During the twelve years of his incumbency he held the confidence of the successive Mayors and Aldermen and the people of the City of Crookston to an eminent degree, serving through various local administrations. His judgment was sought on all city problems. He came toward the end of his twelve years' work to be practically a City Manager. In April, 1932, he resigned, in preparation for the acceptance of a position with the Minnesota State Highway Commission, as District Supervising Engineer in the development of the highway system of that State. The Crookston *Daily Times*, commenting on his resignation, made the following statement:

> "Summing up the above we find that through the initiative of a City Engineer and his willingness to fight for the interests of a city, Mr. McLean, through major savings, has saved the tax-payers of this city during the past few years the sum of $105 114.00."

This is a fine testimony to the alertness of Mr. McLean—not only as an engineer, but as a business man who could meet and convince county and State boards, railroad and utilities managers, and secure for the benefit of the people the rights to which they were entitled from these corporations.

Mr. McLean's life was tragically terminated on July 30, 1932. He was shot down on the walk within a block of his home, when returning to lunch, by a truckman whom he had discharged for insubordination from the employ of the City about two years before. He had no chance for his life, because he was shot in the back and died instantly. The assassin stepped out of his car and took his own life with the same instrument. Mr. McLean's body was sent to the home of his mother, at Grafton, where the funeral ceremonies were conducted by the church of his boyhood and the Masonic Lodge. The final ritual of the Lodge was read at the grave by the Hon. Walter L. Stockwell, Grand Secretary of Masonic Bodies of North Dakota, and Mr. McLean was laid to final rest by the side of his father.

Mr. McLean was a member of the Presbyterian Church, the Masonic Lodge, and the American Legion.

He was married on June 18, 1910, to Elsie W. McFarland, of Valley City, N. Dak., and is survived by his widow, his mother, three brothers, and three sisters. To an unusual degree he held the love and confidence of this large family and a much larger circle of other relatives and friends.

John A. McLean was a strong character, intense, frank, direct, absolutely incorruptible, loyal to every relation in life. He respected his profession—did his work in the light of the highest principles of ethics. He meant that every act of his should be worthy of himself. He made and held friends by his dependability and sincere efforts to serve the interests by which he was employed. Contractors and construction engineers with whom he did business give unanimous testimony to the fact that no temptation could swerve him from the strict line of duty. He was respected by all classes of people, and his tragic death was the first evidence that he had a single enemy in the community he had served so valiantly for twelve years.

Mr. McLean was elected an Associate Member of the American Society of Civil Engineers on April 3, 1922.

LESTER LOUIS MEYER, Assoc. M. Am. Soc. C. E.[1]

DIED DECEMBER 27, 1932

Lester Louis Meyer, the son of Louis N. and Rena (Bamberger) Meyer, was born in Los Angeles, Calif., on August 24, 1902. He was educated in the public schools of Los Angeles and, later, attended and was graduated from the University of California at Berkeley, Calif., in December, 1925, receiving the degree of Bachelor of Science in the College of Civil Engineering.

In February, 1926, Mr. Meyer began his engineering career as a Draftsman for the Department of Water and Power, City of Los Angeles. Because of his diligent work and remarkable understanding of hydraulic theory and practice, he was quickly promoted to the grade of Junior Civil Engineer in July, 1927. He remained in this position until September, 1931, at which time he was promoted to the position of Assistant Civil Engineer. His work included hydraulic studies of various routes for the Colorado River Aqueduct reports, paper locations, field examinations, water consumption studies, preliminary locations, and estimates of costs.

Mr. Meyer was Assistant to the Field Engineer in the inspection and tests of tunnels, siphons, and conduits of the existing Los Angeles Aqueduct, that were made to find a method of increasing its capacity, and planned a large part of the construction operations which were carried on in connection with this project. More recently he had become extremely interested in the economic design and construction of various types of hydraulic structures being built by the Water Department. He was fast becoming an expert on earth-fill dams, having carried on considerable original research work on this subject.

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[1] Memoir prepared by R. R. Proctor and S. B. S. Nelson, Assoc. Members, Am. Soc C. E.

He had made extensive trips in the interest of engineering. These trips were taken during his vacation periods, and the investigations were made solely because of his intense interest in engineering problems. He had written several papers and discussions on various engineering subjects, a few of which were published[2] in various engineering periodicals.

As an engineer, Mr. Meyer's success seemed assured. He was only thirty years old at the time of his death, but he had laid an unusual foundation for his work and had accomplished that which would do credit to a much older man. A man of likable personality, he was quiet and reserved, and loved by all who knew him.

He took an active interest in local affairs of the Society, and was a Registered Civil Engineer in the State of California. He was a Mason and a member of the local bodies of the Shrine, Scottish Rite, and Blue Lodge.

He was not married and is survived by his mother and father, a brother, Frederick, and a sister, Mrs. Schweitzer.

Mr. Meyer was elected a Junior of the American Society of Civil Engineers on October 1, 1926, and an Associate Member on January 25, 1932.

THOMAS HARTMAN OLDS, Assoc. M. Am. Soc. C. E.[1]

DIED FEBRUARY 10, 1932

Thomas Hartman Olds was born in Pontiac, Ill., on November 25, 1883. Several years of his early boyhood were spent in Mexico, where his father, Augustus G. Olds, a Mechanical Engineer, was employed.

Mr. Olds was graduated from the Agricultural and Mechanical College of Texas in 1902, with the degree of Bachelor of Science in Civil Engineering. After his graduation he returned to Mexico and, from June, 1902, to January, 1904, was with the Mexican Central Railroad Company, successively, as Draftsman, Instrumentman, Topographer, and Assistant Engineer, on railroad construction and maintenance of way. From January to March, 1904, he was an Engineer for the San Francisco del Oro Mines, in Parral, Mexico. He then entered the employ of the Mexican Light and Power Company, at Necaxa, Puebla, Mexico, remaining with that Company until August, 1905. During this period he was engaged on right-of-way surveys for a transmission line and was Resident Engineer, under Mr. Walter Diem, in charge of the construction of Tunnel No. 1 (1 100 m. long) and the Tezcapa (earth) Dam.

In September, 1905, Mr. Olds returned to the United States and entered Cornell University, from which he received the degree of Civil Engineer in June, 1907. He then again returned to Mexico, and from October, 1907, to October, 1908, was with the National Railways of Mexico as Transitman and

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[2] "Stresses in Reinforced Concrete Due to Volume Changes," (Discussion), see p. 1070; and "Percolation Slope in Dams Measured by New Device," *Engineering News-Record*, January 26, 1933.

[1] Memoir prepared by the late Lyman E. Bishop, M. Am. Soc. C. E., and William B. Freeman, M. Am. Soc. C. E.

Location Engineer. From October, 1908, until July, 1910, he was again employed by the Mexican Light and Power Company, at Necaxa. During this two-year period he was Assistant Engineer on the location of tunnels and, later, Resident Engineer, in charge of the construction of Los Reyes Dam and Dike, Coquila Canal, and Laguna Power Plants. From July to December, 1910, he was Assistant to the Engineer, Maintenance of Way, National Railways of Mexico, on about 7 000 miles of main lines, which were under this office. He then left Mexico because of the increasing revolutionary activities.

From December, 1910, to June, 1914, Mr. Olds was with the São Paulo Electric Company, at São Paulo, Brazil. The first six months of this period was occupied with preliminary surveys, estimates, and reports on a hydro-electric development of 50 000 h. p. In June, 1911, he was designated Assistant Chief Engineer, and the following three years were spent on the construction of the plant at a cost of approximately $5 000 000.

In June, 1914, Mr. Olds left Brazil and joined his father in Denver, Colo., with whom, until November, 1916, he was associated in the business of Machinery Manufacturers' Agents, designing, estimating, and constructing feed and grain mills.

Although past the draft age, when the United States entered the World War, Mr. Olds expressed his desire to serve his country by voluntary enlistment, but his application was held up until the Armistice by technicalities concerned with his physical examination.

During the two years, 1917 and 1918, Mr. Olds engaged in private practice in Denver as a Civil Engineer. He conducted surveys and investigations of three drainage projects, involving intensive underground water studies and determinations, and acted as Resident Engineer during their construction; also, from April to November, 1917, he was Resident Engineer and Superintendent of Construction under Charles W. Comstock, M. Am. Soc. C. E., Chief Engineer, on Costilla Dam, near Jarosa, N. Mex., an earth dam about 125 ft. in maximum height.

In 1919 Mr. Olds became associated with Fisher and Lowrie, Consulting Geologists, of Denver, and remained with them for more than ten years. For several years, he was a member of the firm. This firm was the Consulting Geologists for Midwest Oil Company, Midwest Refining Company, Merritt Oil Corporation, Elk Basin Petroleum Company, etc., and practically all the operating oil companies in Wyoming, as well as several in Oklahoma and Texas. The firm is generally credited with the discovery of the famous Salt Creek Oil Field, in Wyoming, and did the principal pioneering and development work in this field. Mr. Olds as an associate and member of the firm had an active part in the oil development field. He specialized particularly in appraisal work, income tax valuations, and probable oil recoveries. Millions of dollars in oil property valuations prepared by him were accepted as the basis for sales of property and for income tax settlements. He took a prominent and active part in the preparation of the Manual for the Oil and Gas Industry, which was published about 1918 by the United States Internal

Revenue Bureau of the Treasury Department. This Manual became the basis of oil property valuations for income tax purposes.

In 1929, Mr. Olds left the firm of Fisher, Lowrie and Olds, and joined the Midwest Oil Company, Saltmount Oil Company, and associated companies, as their Valuation Engineer. This position he held at the time of his death. He was active in his work until December, 1931, when he was confined to his bed by heart trouble. He failed steadily and passed away on February 10, 1932, at his home, in Denver.

He was married on January 4, 1922, to Gertrude Bunger. He is survived by his widow, a son, Robert, 4 years old, and a brother, Frederick H. Olds, of Colorado Springs, Colo.

Mr. Olds was a member of the American Association of Petroleum Geologists, and of the Colorado Society of Engineers. He was always active in the work of the technical societies, and for several years he represented the Rocky Mountain Association of Petroleum Geologists as a delegate to Colorado Engineering Council.

He was of a quiet and retiring disposition. He was always sincere, painstaking, and thorough, and an indefatigable worker. Because of his interest in his work he often spent many nights in studies to produce a finished product that would completely satisfy. He made few close friends, but these were always worth-while and staunch, who appreciated his high ideals and sterling qualities. He gained and held the respect and appreciation of all those for whom he worked, as well as those with whom his work associated him.

Mr. Olds was elected an Associate Member of the American Society of Civil Engineers on May 6, 1914. He was also a Member of the Colorado Section of the Society and served as its President in 1922-1923.

ALBERT HENRY ROSS, Assoc. M. Am. Soc. C. E.[1]

DIED JULY 6, 1932

Albert Henry Ross, the son of the Rev. Henry Ross and Mary (Held) Ross, was born at Quincy, Ill., on May 13, 1887. He was reared in a parsonage home and attended the public schools wherever his father was stationed as minister. He was graduated from the Decatur, Ill., High School, in 1905.

After a period of two years, during which time Mr. Ross was employed by the Decatur Bridge Company, he entered Millikin University, at Decatur, where he took his first two years of college work. After another period of several years with the Decatur Bridge Company, he entered the Engineering School of Washington University, at St. Louis, Mo., from which he was graduated in June, 1913, with the degree of Bachelor of Science in Civil Engineering.

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[1] Memoir compiled from information on file at the Headquarters of the Society.

Mr. Ross' first work after his graduation was with the Stupp Brothers Bridge and Iron Company, at St. Louis, where he was engaged in various capacities until his health failed in March, 1918. The breakdown was the probable result of overwork during the World War period, when so many structural steel companies were working at high pressure for the Government. He returned to the home of his parents in Decatur, in May, 1918.

After a rest of several months Mr. Ross became an Instructor in Surveying for the Students' Army Training Corps, at Millikin University, until the camp was closed, at the end of the war. His next work was as Building Inspector on the construction of grain elevators in various places near Decatur, which work was intended to keep him out of doors. He was fighting tuberculosis, and after another rest of several months, he took a position under the direction of the Decatur and Macon County Medical Association and the Visiting Nurses Association. This was in the nature of research work to show the need in Macon County for a Tuberculosis Sanatorium. As a result of this survey the Sanatorium was built and opened in 1921.

In September, 1919, Mr. Ross returned to his work with the Decatur Bridge Company (now the Mississippi Valley Structural Steel Company), but was again forced to give it up in November, 1924. This time he went into the Sanatorium for which he had made the tentative plans and survey. After another rest cure, Mr. Ross resumed his work in the fall of 1925, and continued until July 6, 1929, when he was forced to give it up once more and again entered the Sanatorium at Decatur, where he waged a faithful, persistent battle for almost three years. On May 25, 1932, he entered St. Luke's Hospital, in Chicago, Ill., for thoracoplasty operations, with the hope of collapsing his right lung. He had two stages of this rib operation and a very good collapse resulted. However, an embolism developed on July 1, and he passed away on July 6, 1932. His long persistent struggle for health was characteristic of his engineering training and instinct.

During his employment with the Decatur Bridge Company and, later, with the Mississippi Valley Structural Steel Company, Mr. Ross developed an excellent record as a Draftsman and an Engineer. Starting as a Draftsman, his work was such as to mark him for rapid advancement, and this progress continued to the time when his health became a serious handicap. He came to be regarded as one of the principal engineers of the Company and, without doubt, he would have been given still larger responsibility had his health been such as to permit it.

Equipped with the substantial foundation of a good engineering education, his ability and temperament were such as to enable him to make the most of his opportunities. Quiet, capable, studious, interested in his work, he was liked by all his associates and his passing leaves a void not easily filled.

Mr. Ross was a member of the Collimation Club, Washington University, and of the Engineers Club of St. Louis. He was a Mason and a member of the First Methodist Episcopal Church, at Decatur, Ill.

On September 1, 1915, he was married to Florence E. Steinmetz, at St. Louis. He is survived by his widow, his daughter, Ruth Helen; his brother, Edward W. Ross, of Pleasantville, N. Y.; his sisters, Ada E. Ross, of Chicago, Dr. Flora E. Ross, of Decatur, Mrs. H. D. Lemkau, of Peoria, Ill., and Mrs. R. W. Sleeter, of Medford, Ore.

Mr. Ross was elected an Associate Member of the American Society of Civil Engineers on October 1, 1928.

WILLIAM FRANCIS SHEEHAN, Assoc. M. Am. Soc. C. E.[1]

Died June 14, 1932

William Francis Sheehan was born in Mechanicville, N. Y., on June 19, 1892, the son of Josephine A., and the late James F. Sheehan.

Mr. Sheehan prepared himself for the Engineering Profession by study at Rensselaer Polytechnic Institute, at Troy, N. Y., Rutgers University, New Brunswick, N. J., and by courses in Analytical and Bacteriological Chemistry at the University of Buffalo, Buffalo, N. Y.

He began his engineering career in April, 1911, with the New York State Highway Department, where he served in grades up to and including that of Junior Assistant Engineer. In June, 1917, Mr. Sheehan entered the Engineering Department of the New York Central Railroad Company, in charge of a field party on location. This engagement was followed in February, 1918, by a connection with the Curtiss Aeroplane Company, in the capacity of Plant Engineer.

In September, 1918, Mr. Sheehan went with the American Brass Company as Field Engineer in charge of plant layout, and supervised the construction of mill buildings, railroad sidings, etc. In August, 1919, The Beaver Board Companies engaged him as Plant Engineer in connection with the development of new plant sites, and sources of supply. This work took Mr. Sheehan from Hudson Bay, in Canada, to Baton Rouge, La. This engagement was followed in 1921 by a period of private practice after which, in 1923, he became Resident Engineer in charge of the Williamsville Office of George C. Diehl, C. E., Incorporated. The Williamsville Office was opened to serve as the headquarters for the engineering projects incident to the development of the Town of Amherst and the Village of Williamsville, suburbs of the City of Buffalo, which were rapidly becoming residential centers of the better type.

As Resident Engineer for this Corporation Mr. Sheehan had under his direct charge the planning, design, and construction of pavements, sanitary sewers, and sewage treatment plants, storm sewers, and water projects, aggregating approximately $6 000 000. He was in charge of the design and construction of the sanitary sewerage system of Williamsville, which project involved the planning of an inverted siphon about 1 500 ft. in length.

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[1] Memoir prepared by Wallace V. R. Fretts, Esq., Williamsville, N. Y.

Shortly after the completion of this work, Mr. Sheehan planned and supervised a project to relieve a flood condition in Williamsville, occasioned by the annual overflow of Ellicott Creek.

On January 1, 1930, he left this work and with Messrs. Wallace V. R. Fretts and B. D. Tallamy, both of whom had been associated with Mr. Sheehan, formed the firm of Sheehan, Fretts, and Tallamy, Consulting Engineers. Mr. Sheehan continued as Town Engineer of the Town of Amherst until January 1, 1932, and as Village Engineer of Williamsville until May, 1932. In 1930 and 1931, the firm executed several projects, among which was the dam for Reservoir No. 3 of the Hornell, N. Y., Water Supply System, consisting of an earth-fill dam, about 90 ft. high and 400 ft. long.

Mr. Sheehan's death was the result of injuries sustained in an automobile accident which occurred on May 13, 1932, at Greigsville, N. Y. Interment was made in the family plot in Hudson View Cemetery, at Mechanicville.

He was married on Thanksgiving Day, 1920, to Ethel C. Ouchie, of Williamsville. He is survived by his widow; his mother, Mrs. Josephine Sheehan, of Mechanicville; two brothers, James M., of Mamaroneck, N. Y., and John H., of Mechanicville; and a sister, Mrs. Milburn D. Smith, of Fort Plain, N. Y.

He was a member of Amherst Lodge No. 981, F. and A. M., the Rotary Club of Williamsville, the Society of American Military Engineers, the New York State Society of Professional Engineers, the American Water Works Association, the Sewage Works Association, and the J. E. W. S., a social organization of Buffalo.

Mr. Sheehan was elected an Associate Member of the American Society of Civil Engineers on March 16, 1925.

JOSEPH EDWARD TEMPEST, Assoc. M. Am. Soc. C. E.[1]

Died September 12, 1932

Joseph Edward Tempest, the son of Hezekiah R. and Ann (Sugden) Tempest, was born in Keighley, Yorkshire, England, on February 28, 1872. He attended the public schools until he entered the Keighley Institute of Technology in 1888, which he attended until 1892, when he entered the South Kensington School of Art and Science, in London, England. He completed his studies in 1895 and for two years thereafter taught art in the Keighley Institute.

In 1897, Mr. Tempest went to Canada where he started the practice of his profession with the Hudson Bay and Western Railway Company, advancing in three years from Draftsman to Resident Engineer on Location. In 1901,

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[1] Memoir prepared by S. A. Hart, Assoc. M. Am. Soc. C. E.

Mr. Tempest became Assistant Engineer with the Grand Trunk Pacific Railway Company and, until 1907, as Locating Engineer and Resident Engineer, he remained with this Company during the most interesting period of its development, leaving it only to enter into contract work on its construction and in lumbering in Northern Canada.

In 1912, Mr. Tempest went to California and was employed by the City of Sacramento as a Draftsman, Office Engineer, and Assistant Engineer, until 1918, when he transferred to the service of the State of California and was engaged in reclamation, flood control, and irrigation work.

In 1921, Mr. Tempest's previous experience with municipal work stood him in good stead as he was appointed Assistant City Engineer of Sacramento, in which position he served until 1925. During this period the development of public projects in the city was intensive, including a modern filtration plant, garbage incinerator, modern wharves, sub-map and extensive street improvements, in all of which Mr. Tempest took an active and important part.

In 1925, he again answered the lure of contracting and from this time until shortly before his death was actively and successfully engaged in general contracting.

In 1907, Mr. Tempest was married, in Ontario, Canada, to Esther Cameron, who, with a daughter, Ann Sugden Tempest, and a son, Donald Cameron Tempest, survives him.

No record of Joseph Edward Tempest's life is complete without reference to the fact that besides being an Engineer he was a true artist. During all his life in the north woods of Canada and in California his spare time was spent in painting, and he left with his family and friends many water colors which prove by their excellence that he possessed a rare talent for art and which gave an insight into the beauty of a nature known only to his family and to his most intimate friends.

Mr. Tempest was elected an Associate Member of the American Society of Civil Engineers on March 16, 1925.

GEORGE EDWARD WELLS, Assoc. M. Am. Soc. C. E.[1]

DIED MAY 26, 1932

George Edward Wells, the son of Henry W. and Delphine Wells, was born at LeRaysville, Pa., on May 31, 1879. He received his primary education in the LeRaysville Public Schools.

When the family moved to Naugatuck, Conn., in 1893, Mr. Henry W. Wells, being a Contractor, interested his son in the building trade at which he worked from 1895 to 1904.

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[1] Memoir prepared by William A. Bartlett, Assoc. M. Am. Soc. C. E.

From 1904 to 1906, George Edward Wells was engaged with the Naugatuck Chemical Company, first as Draftsman and, later, as Superintendent of Construction. During this time, he applied himself, not only to the task of building construction and equipment installation, but to a study of commercial chemistry and the peculiar problems which made maintenance difficult. This study and training proved of great value in his subsequent work. During these years and for some time afterward, Mr. Wells took correspondence courses in Engineering.

In April, 1906, he entered the employ of The American Brass Company, later controlled by the Anaconda Copper Company. Here, he started as Draftsman on mill-building design, working untiringly to improve his opportunity. Reward for his industry came in February, 1907, when he was promoted to the position of Division Engineer in Charge of Buildings. This included not only the design and construction of new structures, but repairs and alterations to existing buildings. He reported directly to the Chief Engineer and had surveillance over all plants at that time owned and operated by The American Brass Company, in Waterbury, Torrington, and Ansonia, Conn., as well as in Kenosha, Wis.

Later, the building work increased, when during the World War additional plants were acquired in Buffalo and Hastings-on-Hudson, N. Y. All the plans for remodeling these plants were ably executed by Mr. Wells and the men working in his Building Department.

Long hours at the office and the great task of rushing building changes for increased production and the support of the soldiers in France was fought as hard by this courageous man as by any who went overseas. This extreme effort had its effect in undermining his health, so that in recent years his bodily resistance was greatly reduced.

While driving alone on the evening of May 26, 1932, and in rounding a street corner in Waterbury, the wheels of Mr. Wells' car skidded on the trolley tracks; the car was overturned and caught fire. Death was instantaneous, due probably to shock, and all efforts to rescue him were of no avail.

The esteem of his associates was well expressed in final tributes, and all who knew Mr. Wells remember him as a faithful friend, a hard worker, and a capable Engineer.

Mr. Wells was married to Katherine Shea, in 1926, and is survived by his widow, a son, George Edward Wells, Jr., by a former marriage, and two sisters, Dr. Mary E. Wells, of Vassar College, Poughkeepsie, N. Y., and Mrs. H. D. Gibbud, of Naugatuck, Conn.

He was a veteran member of Shepherd Lodge No. 78, Free and Accepted Masons, and also of Allerton Chapter No. 39, Royal Arch Masons, acting as Tyler of both Lodges for years, as well as a member of Waterbury Council No. 21, Royal and Select Master. He was Past Sachem of Natatuc Tribe No. 25, Improved Order of Red Men, and a member of Centennial Lodge No. 100, Independent Order of Odd Fellows, both of Naugatuck.

Mr. Wells was elected an Associate Member of the American Society of Civil Engineers on February 6, 1912.

PAUL REVERE WILLIAMSON, Assoc. M. Am. Soc. C. E.[1]

DIED JUNE 27, 1932

Paul Revere Williamson was born on August 8, 1888, at Charlestown, Mass. He was the son of the late Charles Henry and Mary Alice Williamson. The first Williamson family, originally from England, was among the pioneer settlers of Duxbury, Mass. At an early date the family moved to Marshfield, Mass., where its members became the first successful farmers of the community, in which they are still prominent.

Educated in the public schools of Marshfield, Paul Revere Williamson entered the Massachusetts Institute of Technology with the Class of 1912, and was graduated with the degree of Bachelor of Science in Civil Engineering.

His first work after graduation was with the American Bridge Company, as a Structural Draftsman. In the next few years, while he was gaining his experience, he held various positions, always in his chosen line—structural engineering—with the Chesapeake and Ohio Railroad Company, the Interstate Commerce Commission, the American Ice Company, and others.

In 1924, he moved to Hartford, Conn., where for six years he was with the firm of Buck and Sheldon, Incorporated, as Structural Engineer and Chief Draftsman. With the experience thus accumulated, he opened his own office in 1930 as Consulting Structural Engineer. His most important engagement was on the Federal Building and Post Office, in Hartford, a $1 500 000 project. On this building Mr. Williamson handled the complete structural design, including all reinforced concrete, steelwork, and caissons. This project was in the process of construction at the time of his death.

In 1920, Mr. Williamson was married to Mary B. DeLaplane, of York, Pa. They had no children.

He died suddenly of a heart attack on June 27, 1932, at his home in Hartford. Besides his widow, he leaves his mother; a sister, Mrs. E. A. Keene; and a brother, Henry Williamson.

Mr. Williamson was elected an Associate Member of the American Society of Civil Engineers on January 15, 1923.

FRANK HERMAN WINTERER, Assoc. M. Am. Soc. C. E.[2]

DIED SEPTEMBER 15, 1932

Frank Herman Winterer, the son of Frank J. and Margaret L. (Cuddy) Winterer, was born at Philadelphia, Pa., on September 26, 1898. He received his early education at the Roman Catholic High School, in that city, and after graduation, had a position with the Western Union Telegraph Company.

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[1] Memoir prepared from information supplied by Mrs. Williamson and on file at the Headquarters of the Society.

[2] Memoir prepared by Walter E. Witte, M. Am. Soc. C. E., and Charles A. Belz, Esq., Audubon, Pa.

After a year of general office work, Mr. Winterer took a position with the Link Belt Company, in which, in addition to the general improvement it brought with it, he had daily contact with technical men, and with engineering work that had always held a fascination for him. He promptly began a night course of study in Structural Engineering at the Drexel Institute, in Philadelphia, to supplement the work he was doing at the office, and to provide a solid groundwork of theory on which to build his structure of experience.

In 1921, he accepted a position with the Box Crane and Hoist Corporation, of Philadelphia, where he spent two years as Structural Draftsman on electric traveling cranes, runways, supports, and in laying out monorail hoist systems.

In 1923, during his fourth year at Drexel Institute, Mr. Winterer became interested in reinforced concrete and concrete structures, and to round out his engineering education, he obtained a position as Draftsman with the Concrete Steel Company, of Philadelphia, where he spent two years of excellent training in detailing and designing the variety of concrete structures that come to the Engineering Department of a Bar Company.

In 1925, he acquired a position with the Department of City Transit of Philadelphia. In this service, Mr. Winterer spent more than six years as a Structural Draftsman engaged on the design of all the various types of concrete structures incidental to subway construction. Never of a robust constitution, he suffered a heart attack which proved fatal on September 15, 1932.

Mr. Winterer was married on October 26, 1927, to Mary P. Carr, of Philadelphia, who with a son, Francis X. Winterer, survives him.

Although not having the advantage of a university education, Mr. Winterer had an intense interest in his chosen work, and a love for his profession that drove him to great efforts in his pursuit of the knowledge which many with the more formal education have casually thrust upon them. Given better health and a longer career, he undoubtedly would have gone far in his chosen field. He had a friendly approach, the rare gift of conversation, a high sense of ethics, and a loyalty in his friendships that made him generally liked and respected.

His early death is a great loss to his friends and also to the profession that may count as its greatest wealth, its young men whose professional instincts prove stronger than the dictates of business expediency.

Mr. Winterer was elected an Associate Member of the American Society of Civil Engineers on June 9, 1930.

FRANK DENNETT YOST, Assoc. M. Am. Soc. C. E.[1]

Died August 7, 1931

Frank Dennett Yost was born in Philadelphia, Pa., on April 15, 1884, the son of Mr. and Mrs. Penfield P. Yost. He was educated in the Asa Packer School and the Central Manual Training School, in Philadelphia.

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[1] Memoir prepared by H. W. Corp. M. Am. Soc. C. E.

In 1906, he enlisted in the United States Army and served in the Corps of Engineers, particularly on topographic surveys of the Island of Luzon, Philippine Islands.

In 1910, Mr. Yost passed the Government Civil Service examination for Civil Engineers and, at the request of the Philippine Government, was discharged from the Army to serve on land survey work for the newly organized Bureau of Lands which was undertaking the survey of all Government-owned lands in the Philippine Islands. On June 22, 1910, he was detailed to Cebu to act as Assistant with the party engaged in sub-division and identification work of the Talisay-Minglanilla Estate. On December 13, 1911, he was assigned as Assistant in the execution of the Santa Barbara Cadastral Project, after which he was engaged as Assistant on various cadastral projects, such as Bago, Bacolod, and Saravia.

On March 5, 1915, Mr. Yost was detailed as Assistant on the Pura Cadastral Project, and afterward on the La Paz Project. Later, in the early part of 1916, he was assigned as Assistant on the Conception Cadastral Project, and in May, 1916, he was designated Chief of Survey Party No. 5, stationed at Aringay, La Union. He served in this capacity until December 11, 1917, when he was appointed Supervising Surveyor. As such, he had under his jurisdiction the survey of the Aringay, Santo Tomas, and Rosario Cadastral Projects.

In the early part of 1918, Mr. Yost returned to the United States with the intention of enlisting in the Army to take part in World War activities. However, finding himself physically unfit for military service, he transferred to the Federal Forest Service as Surveyor, in which position he remained until the latter part of 1919. During his connection with the Forest Service, he was engaged in triangulation work in Albuquerque, N. Mex.

On January 19, 1920, he was reinstated in the Bureau of Lands, as Supervising Surveyor, with station at Apalit, Pampanga Province, Philippine Islands. By order of the Secretary of Agriculture and Natural Resources, under date of October 25, 1921, Mr. Yost was assigned with the Survey and Investigation Board detailed to Nueva Ecija, the function of which was the disposition of various problems affecting public lands in that Province. He remained with the Board until September 25, 1922, when he was detailed as Inspecting Surveyor of Northern Luzon, which then comprised the Provinces of Cagayan, Isabela, and Nueva Vizcaya, with headquarters at Echague, Isabela.

With the creation of the Inspection Division in 1923, Mr. Yost's designation was changed to Division Inspector. He was in charge of the Northeastern Luzon Inspection District until his death on August 7, 1931. This District comprises the Provinces of Batanes and Cagayan and the Sub-Province of Apayao, Mountain Province (Land District No. 1); the Province of Nueva Vizcaya and Sub-Province of Ifugao, Mountain Province (Land District No. 4); and the Province of Isabela (Land District No. 5).

Mr. Yost rendered valuable service with the Rubber Investigating Party sent out in 1923 by the United States Government to conduct investigations in Mindanao as to rubber possibilities in the Philippines. His service in gathering evidence to sustain the Director of Lands on important registration cases, involving vast tracts of lands in the Northern Provinces, was also valuable.

He died of heart failure on August 7, 1931, at the Hospital Español de Santiago, San Pedro Makati, Rizal, Philippine Islands, at the age of 48. He was a bachelor.

Mr. Yost was elected an Associate Member of the American Society of Civil Engineers on April 16, 1918.

JAMES McCORMICK BROCKWAY, Jun. Am. Soc. C. E.[1]

DIED SEPTEMBER 14, 1932

James McCormick Brockway was born on June 4, 1902, in Chicago, Ill. While he was still a lad, the family moved to Montana where his father, James A. Brockway, was engaged in the construction of irrigation systems. With this early environment it was natural that the boy took to irrigation engineering.

His preparatory schooling and the first courses of his college education were pursued in Montana. Later, the family moved to Oregon, and Mr. Brockway went to the University of California, at Berkeley, Calif. Financial necessity caused his years at college to be taken intermittently with years of work, always in some connection with irrigation engineering. Before graduation from the University of California, he had attained standing as Chief of Party and Instrumentman with the United States Bureau of Reclamation, on the Klamath Project, in Oregon. During his years at the University of California, his spare time and summers were occupied as Assistant to the writer, in computations connected with hydraulic problems and evaporation studies.

Immediately after finishing college in 1927, Mr. Brockway entered the service of the State of California as Assistant Hydraulic Engineer, in the Division of Water Rights, with the Sacramento-San Joaquin Water Supervisor. In this position he was engaged in making measurements and collecting records of the stream flow, diversions, return flow, and use of water along the Sacramento River, a very important investigation, upon which much of the future policy in regard to the Sacramento River must be based.

In the summer of 1931, L. M. Lawson, M. Am. Soc. C. E., International Boundary Commissioner and also International Water Commissioner, asked several engineers in California to recommend a man who had had extensive experience in the measurement of diversions from a large river, under varied and complex conditions, for similar service along the Rio Grande, between Texas and Mexico. The fact that James Brockway was unanimously and independently recommended by those familiar with the problems involved, showed he was the obvious man. These recommendations were accepted, and he left the service of the State of California to enter that of the International Water Commission in hydrographic work bearing on the division of waters for various uses between the United States and Mexico.

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[1] Memoir prepared by Fred C. Scobey, M. Am. Soc. C. E.

It was in the measurement of the unprecedented flood of the Rio Grande early in September, 1932, that he lost his life. Mr. Brockway and his associate, Mr. Fred Thomas, were making current meter measurements of the flood, opposite the Town of Hidalgo, from a small boat with an outboard motor. The boat struck some obstruction under a bridge, capsized, and sank immediately. Both men started for shore, which Mr. Thomas reached, about 200 ft. up stream from where Mr. Brockway was rapidly tiring. Mr. Thomas ran down along the edge of the water and went back into the river to help Mr. Brockway. By this time both were very tired and they sank together. Mr. Thomas succeeded in ridding himself of his trousers and shoes and rose to the surface where he was reached and dragged out by some Mexican workmen. He tried to get these workmen to go into the river and rescue Mr. Brockway, but he was not successful. The body was recovered after a long search and taken to Atlantic, Iowa, for interment.

Mr. Brockway was married in April, 1932, to Margaret Finkbine, of Atlantic, Iowa. His mother and two sisters, Florence and Dorothy, live in Portland, Ore. A brother, Stuart Brockway, is a Surveyor with the California Division of Highways. Another brother, Lyman, lives at Santa Monica, Calif.

Those who worked with James Brockway regarded him as much more than the rapid efficient worker that he was. He had a quiet, generous nature that endeared him to his associates. He was resourceful in devising methods of accomplishing difficult and complex water measurements. He would have gone a long way, as an hydraulic engineer, had he lived a few more years.

Mr. Brockway was elected a Junior of the American Society of Civil Engineers on March 11, 1929.

CHARLES WILLIAM IRONS, Jun. Am. Soc. C. E.[1]

DIED MARCH 5, 1932

Charles William Irons, the eldest son of Claud and Lena (Wright) Irons, was born in Linesville, Pa., on February 19, 1909. He was a worthy descendant of the pioneer Quaker founder of Linesville, Amos W. Line, after whom the town was named. He attended the local High School and, until he was graduated, worked in his spare hours in the Linesville Hatchery which is owned by his family.

He then entered Ohio Northern University, at Ada, Ohio, from which he was graduated in 1931, with honors and high distinction. He also received the University Prize which was given for the highest grades received in Mathematics in the four years. Professor J. T. Fairchild, Professor of Astronomy and Mathematics at the University, and Mr. Irons' instructor during his University course, has written the following:

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[1] Memoir prepared by the Rev. E. B. Dwyer, Linesville, Pa.

> "* * * Mr. Irons, having a special fondness for Mathematics, devoted his leisure hours to the broad field and hidden beauties of the Higher Mathematics, delving deep into the mine of original investigation. * * * His many friends are called to mourn the untimely loss of one of the brightest intellects that the Ohio Northern University ever produced. * * *."

During his last two years at the University, Mr. Irons was employed by the State Highway Department of Pennsylvania as Draftsman, at Clearfield, Pa., and as Assistant Inspector, at Franklin, Pa., under S. W. Jackson, M. Am. Soc. C. E., Division Engineer of Division No. 6, who contributes the following:

> "* * * Charles worked under my direction one summer as Assistant Draftsman, and, later, he was employed as Assistant Inspector after I left the Clearfield district.
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> "He was methodical and turned in neat, accurate drawings in the drafting-room. He was not the aggressive, boisterous type, but was rather reserved. He won the respect of his associates by his conscientious effort and thoroughness."

Mr. Irons was married on September 20, 1931, to Loine Ash, a student at Ohio Northern University. They began housekeeping on the home place, and Mr. Irons was employed in his father's hatchery. While engaged in the operations about the large incubators, he accidently received injuries from which he died on March 5, 1932. He is survived by his widow, by his father and mother, and by five brothers.

He was a member of Nu Theta Kappa Fraternity, and of the Baptist Church, at Linesville.

Mr. Irons was elected a Junior of the American Society of Civil Engineers on November 23, 1931.

JOHN EGON SKAFTE, Jun. Am. Soc. C. E.[1]

DIED NOVEMBER 10, 1932

John Egon Skafte, the son of George and Johanne Skafte, was born in Branderslev, Denmark, on March 14, 1905. He arrived in the United States on December 18, 1920, and after attending the University of California, at Los Angeles, Calif., for two years, he was graduated in 1928 from the California Institute of Technology, Pasadena, Calif., with the degree of Bachelor of Science. He continued his studies at this institution while acting as Assistant Instructor, and, in 1929, he received the degree of Master of Science in Civil Engineering.

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[1] Memoir prepared by W. J. Manetta, Assoc. M. Am. Soc. C. E.

Upon leaving the California Institute of Technology, Mr. Skafte was employed in the office of the Los Angeles County Surveyor. On September 1, 1930, he became a Designing Engineer for the Los Angeles County Flood Control District, and continued in this capacity until his death. During 1932, he was responsible for the design of many of the unusual and original structural features of the two large rock-fill dams on the San Gabriel River, near Azusa, Calif. His keen analytical mind, his unbiased scientific attitude toward all problems, and his sound training in the fundamentals of mathematics and the engineering sciences particularly qualified Mr. Skafte for this important work. His conscientiousness, painstaking thoroughness, and loyalty and devotion to duty were an inspiration to his fellow workers and associates.

Scientific pursuits did not occupy all of Mr Skafte's interests. He was very appreciative of the arts, being especially fond of classical music. Those fortunate close friends whom he entertained in his home will always remember the delightful evenings spent as his guest, enjoying the rendition of musical gems from his well chosen library. Mr. Skafte was a most genial host. His simplicity of manner and modest unassuming personality endeared him to all with whom he came in contact

He is survived by his parents, a brother, Walter, and a sister, Mrs. Magda (Skafte) Bush.

Mr. Skafte was elected a Junior of the American Society of Civil Engineers on October 14, 1929.

PHILIP HENRY WARD, Jun. Am. Soc. C. E.[1]

DIED DECEMBER 18, 1932

Philip Henry Ward was born at Cranford, N. J., on May 4, 1905, the only son of Philip H. and Janet H. Ward. While he was a child, the family moved to Bridgeport, Conn., and, later, up the Housatonic Valley. Mr. Ward received his elementary school training in the public schools of Bridgeport and at the Sanford School, at Reading Ridge, Conn.

The sight of the shipping in New York Harbor, or on Long Island Sound near Bridgeport, stirred in the boy the love and ardor for the sea that were his by ancestral right. His youth and young manhood were set to the rhythm of the worship of the sea, its heroes, and its ships. Nevertheless, he entered Rensselaer Polytechnic Institute, at Troy, N. Y., in the fall of 1923. His father had been an architect. Philip Henry Ward enrolled as a student in Civil Engineering, but when he received his appointment as Midshipman to the Naval Academy, at Annapolis, Md., in June, 1924, he gladly dropped the technical course at Rensselaer to enter upon his duties at Annapolis. After two years at the Naval Academy he was forced to resign because of ill-health; a broken arm, and failing eyesight and hearing, made it very certain he could not return.

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[1] Memoir prepared by Deane C. Ellsworth, Esq., Wilmington, Del.

With the dream of the sea still upon him, Mr. Ward turned again to Civil Engineering, planning to specialize in Naval Architecture and Marine Engineering. He spent a year and one-half at Rensselaer Polytechnic Institute, but ill-health still dogged him, and he transferred to the University of Southern California at Los Angeles, Calif., from which he was graduated in January, 1931, with the degree of Bachelor of Science in Civil Engineering.

He was already stricken with the malady that was to prove fatal, and before he had had a chance to practice his profession, Mr. Ward almost completely lost his eyesight as the result of a tumor of the brain which, in great measure, destroyed the optic nerve. He was operated upon for this, early in May, 1931, at the Hospital of the Neurological Institute in New York City, with but indifferent success.

During the succeeding year, Mr. Ward's physical health seemed gradually to improve, but the return of his sight was disappointingly slow. In spite of this handicap, however, his interest in Civil and Naval Engineering affairs did not flag. He was the author of several letters and comments in *Civil Engineering,* in the metropolitan newspapers, and elsewhere. With the help of his mother, he continued his studies in the mathematical and theoretical side of his profession. It seemed certain that, granted the full return of his health and sight, he would go far in his chosen work. He impressed all his associates by the rigorous nature of his analysis of any problem, and by the almost fanatical intensity with which he pursued his investigations and enthusiams.

With his mother, Mr. Ward returned to Lake Worth, Fla., in the autumn of 1932, hoping to find there, in the warmth and kindliness of the seashore, the final cure and restoration of his sight. Always an excellent swimmer, he seemed to benefit much from this exercise. He died from a cerebral hemorrhage while swimming in the sea, on the afternoon of December 18, 1932. He is survived by his mother, Mrs. Janet H. Ward.

Mr. Ward was elected a Junior of the American Society of Civil Engineers on May 25, 1931. He was also a member of the Society of American Military Engineers.

ERNEST ROBINSON ACKERMAN, Affiliate, Am. Soc. C. E.[1]

DIED OCTOBER 18, 1931

Ernest Robinson Ackerman, the son of James Hervey and Ellen (Morgan) Ackerman, was born in New York, N. Y., on June 17, 1863. When quite a young boy his parents moved to Plainfield, N. J., where his father soon became a leading citizen, holding at different times the offices of President of the Common Council and City Judge. Mr. Ackerman was educated in the schools of Plainfield and was graduated from the High School in 1880.

For many years, Mr. Ackerman was head of the Lawrence Cement Company which was established in 1832. He was one of the incorporators of the Lawrence Portland Cement Company, of Pennsylvania, of which he was President until 1925, when he retired to devote his time to other duties.

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[1] Memoir compiled from information on file at the Headquarters of the Society.

Mr. Ackerman served as a member of the Plainfield Common Council in 1891 and 1892; he was Republican Presidential Elector in 1896, and Secretary of the New Jersey Republican Presidential Electors in 1897. He served as a member of the New Jersey State Senate from 1905 to 1911, of which body he was President in 1911. During the latter year, while Governor Woodrow Wilson was absent from the State on several occasions, Mr. Ackerman served as Acting Governor. The passage of the first Civil Service law in the State of New Jersey was almost entirely due to his efforts, and he introduced and pushed to its final passage the first Employers' Liability Bill in that State. Mr. Ackerman was also a Delegate to the Republican National Conventions in 1908 and 1916, and in 1918 he was elected to Congress from the State of New Jersey, having been re-elected for his seventh term at the time of his death. During this period he had served as a member of the House Committee on Appropriations, and as a Member of the National Migratory Bird Commission. During the World War he acted as Federal Food Administrator for Union County, New Jersey.

Mr. Ackerman's many political activities were rivaled by his great interest in civic affairs. He was a member of the United States Chamber of Commerce, the New Jersey Chamber of Commerce, and the Chamber of Commerce of the Plainfields. He was a Trustee of Rutgers College, New Brunswick, N. J., from 1916 to 1920 and served as a member of the New Jersey Geological Survey and the New Jersey State Board of Education in 1928 and 1929. He was an Honorary Governor of Muhlenberg Hospital, at Plainfield, and a Director of the Young Men's Christian Association. He served also as a Director of the Central Railroad of New Jersey. He was a Fellow of the American Geographical Society, a Life Member of the New Jersey Historical Society, and a member of the Union League, Engineers', and Bankers' Clubs, and of the Merchants' Association, of New York, N. Y.

Mr. Ackerman was also a member of the American Philatelic Society and as the owner of a valuable stamp collection, he exchanged specimens with King George of England and came in contact with Emperor William of Germany, the Sultans of Turkey and Jehore, and the Maharajah of Benares, India. In his hunt for stamps, he traveled through many countries, having crossed the Atlantic Ocean more than fifty times and making two trips around the world.

Mr. Ackerman died on October 18, 1931, after being confined to his home by illness for about a week, although he had been in failing health for six months prior to his death. His funeral, which was held at the Presbyterian Church, in Plainfield, was one of the largest ever attended in that place, especially by men, many of whom were Congressional and State officials.

His character was of the highest, and his fellow-Congressman, the Hon. Charles A. Eaton, said of him:

"He was a white-souled, good man, in the noblest sense of the word. In an age drifting aimlessly amidst the wreckage of ancient ideals and moral standards, Ernest Ackerman remained steadfastly loyal to those principles of life and conduct which must forever abide as the immovable foundation of personal and national character."

In a telegram received at the time of Mr. Ackerman's death, President Hoover stated: "He has given a long and distinguished career to his country; an invaluable public servant."

He was married on February 11, 1892, at Cumberland, Md., to Mora L. Weber, who, together with one brother, Marion S. Ackerman, of Plainfield, and two sisters, Mrs. Robert Rushmore, also of Plainfield, and Mrs. Lydia (Ackerman) Murphy, of Washington, D. C., survives him.

Mr. Ackerman was elected an Affiliate of the American Society of Civil Engineers on May 1, 1900.

JOHN JOSEPH ALBRIGHT, F. Am. Soc. C. E.[1]

DIED AUGUST 20, 1931

John Joseph Albright, the son of Joseph J. and Elizabeth (Sellers) Albright, was born at Buchanan, Va., on January 18, 1848. In his early childhood, his parents moved to Scranton, Pa., where he attended the public schools. His later education was received at Williston Academy, at Easthampton, Mass., and at Rensselaer Polytechnic Institute, at Troy, N. Y., from which he was graduated in 1868 with the degree of Mechanical Engineer.

After his graduation Mr. Albright went into the coal business at Lewistown, Pa., and, later, conducted a similar business at Washington, D. C., but, in 1883, on account of the growth and development of his Western coal trade, he moved to Buffalo, N. Y., where he made his home.

Water-power generation of electricity and its distribution through the State of New York was also one of the most important of Mr. Albright's Buffalo enterprises. About 1896, he became interested in water power on the Madison River, in Montana, and on the Hudson River, at Mechanicville, N. Y.

In line with these ventures in 1901 he became associated with the Ontario Power Company, of Niagara Falls, Ont., Canada. Once firmly financed, plans for the Company grew until construction of its plant began in 1903. With a keen eye to the future, Mr. Albright and his associates, in 1904, negotiated and consummated a contract between the Ontario Power Company and the Niagara, Lockport, and Ontario Power Company for the purchase of Ontario power for the distribution of electricity throughout New York State. This contract, which is still in effect and will be until 1950, was signed while the plant of the Ontario Power Company was still under construction, and the negotiations on the part of the Niagara, Lockport, and Ontario Power Company were carried out by the Westinghouse Electric and Manufacturing Company, the controlling interest holder.

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[1] Memoir prepared by Frederick K. Wing, M. Am. Soc. C. E.

In 1905, Mr. Albright and his associates at the request of the Westinghouse Electric and Manufacturing Company took a substantial interest in the Niagara, Lockport, and Ontario Power Company, in order to enlarge their hydro-electric power scope. The Niagara, Lockport, and Ontario Power Company had been incorporated in 1894 at the behest of a business men's association of Lockport, N. Y. The charter granted by the Legislature enabled the Company to draw an unlimited water supply from the Niagara River, for which at that time there was no governmental restriction. As conceived by the organizers, a canal would be constructed from the river to a point near the City of Lockport, and there the water was to be discharged into the gorge of Eighteen-Mile Creek, at which point hydro-electric works could be constructed.

After Mr. Albright and his associates had awarded a contract for the construction of the canal, and the work had been started, Congress passed the Burton Bill, over the strong protests of the business interests of the Niagara frontier. The Burton Act limited the diversion of water from the river to the companies then actually tapping the Niagara. In the meantime, the Albright interests had completed the construction of a hydro-electric works on the Canadian side of Niagara Falls. The Burton Act, however, upset these well-laid plans at two different and vital points, preventing the construction of the canal and limiting the importation of power by the Niagara Company to 60 000 instead of 180 000 h. p., as provided in the contract with the Ontario Company. This legislation, together with the treaty with Canada that superseded it, struck a staggering blow to this ambitious enterprise, but Mr. Albright never lost confidence in the integrity and soundness of the plan.

A steam plant was built at Lyons, N. Y., the Salmon River, in Northern New York, was harnessed, power from the Oswego River was developed, and the transmission line, from the Canadian boundary to Syracuse, N. Y.—then the longest in the world—was completed, despite the many and formidable difficulties that arose from transmitting electric power over such long distances. By July 1, 1906, the Company's construction work had grown to such an extent that it was delivering electric power as far east as Syracuse, and, in 1913, Buffalo capital became the controlling factor in the Company when Mr. Albright's interests bought out the Westinghouse Company's block of stock. Coincident with this move came the rise of the Ontario Power Company to a position where it was recognized as the largest single generating plant on either side of the Niagara River.

In 1917, the Canadian Government, desiring to control its water power resources, negotiated with Mr. Albright for the purchase of the Ontario Power Company at a handsome figure. Mr. Albright, however, still retained his financial interest in the Niagara, Lockport, and Ontario Power Company, although he soon retired from active business. His son, Langdon Albright, is now the Vice-President of the Company.

Shortly before the Pan-American Exposition which was held in Buffalo, in 1901, Mr. Albright, in association with the late John G. Milburn, began negotiations for the purchase of about 3 000 acres of land now occupied by the Lackawanna Steel Company. The bringing of this great Steel Company to Buffalo, with all its attendant industrial interests, was another achievement of which Mr. Albright was very proud.

One of Mr. Albright's specialties was the rescue of apparently foundering companies, which he would re-organize until their financial status was beyond reproach. One of these was Plumb, Burdick, and Barnard, Bolt Manufacturers, which Company, with the aid of the late Edmund Hayes, M. Am. Soc. C. E., he re-organized and re-named the Buffalo Bolt Company, and it is now a going concern. In 1910, Mr. Albright took charge of the refinancing of the Locomobile Company, of Bridgeport, Conn., maker of fine automobiles, and succeeded in pulling it through a financial crisis. Railroad operations were also of great interest to him and he collaborated in several deals with Messrs. O. P. and M. J. Van Sweringen, of Cleveland, Ohio.

Mr. Albright's many business accomplishments were on a par with his numerous philanthropical interests. Two cities, Buffalo and Scranton, Pa., are especially indebted to him for generous gifts. Realizing that there was no adequate outlet for the work of local artists and no proper place for exhibiting recognized masterpieces of painting and sculpture, Mr. Albright presented to the City of Buffalo the Art Gallery in Delaware Park, which bears his name and which was built at a cost of $1 000 000. The City of Scranton owes its thanks to him for its library, which was erected on the site of his father's old homestead. Mr. Albright was also a benefactor of Smith College, at Northampton, Mass., which named a building after him, and of the University of Buffalo. He gave a Community House to Buffalo, which was among the first of the kind in the United States, and the establishment of the Nichols and Franklin Schools in that city was almost entirely due to his efforts.

General recognition of his abilities, of his public benefactions, and of his open-handed liberality with welfare projects, came two years before his death on February 22, 1929, when the University of Buffalo bestowed upon him the Chancellor's Medal, the highest public honor which the Institution has within its power to confer, for his contribution to the art and education of the City of Buffalo. The medal was conferred with the following citation:

> "John Joseph Albright, dauntless promoter of Buffalo's industrial development, generous contributor to institutions of welfare and of learning, donor of the Albright Art Gallery, exemplar of Buffalo's highest standards of civic responsibility and of unpretentious social conduct, who has created for his city's fame more than one lasting monument, and who not only through these foundations, but also through a life of simple greatness has dignified Buffalo in the eyes of the world."

In addition to being an admirer, patron, and collector of art, Mr. Albright was a lover of Nature. Many of his vacations were spent on his estate in the heart of the Adirondack Mountains, or on Jekyl Island, off the coast of Georgia. At these retreats, far from business, he found his recreation in the out-of-door life of which he was so fond. Mr. Albright's personal friends loved his keen sense of humor and as a story teller and practical joker he had few rivals.

On December 4, 1872, Mr. Albright was married to Harriet Langdon, of Washington, D. C., who died in 1895. Two years later, he was married to Susan Fuller, of Lancaster, Mass., who died on June 19, 1928. He is survived by two sons, Raymond and Langdon Albright, and one daughter, Mrs. Evan Hollister, who were born of the first marriage, and two sons, Fuller and John J. Albright, Jr., and three daughters, Mrs. Robert L. Reed, Mrs. Lester P. Faneuf, and Mrs. Laurance L. Hurd, children of his second marriage.

He was a member of the Buffalo, University, and Country Clubs, of Buffalo, of the Delta Phi Fraternity, of New York, N. Y., and of the Jekyl Island Club, of Brunswick, Ga.

Mr. Albright was elected a Fellow of the American Society of Civil Engineers on April 20, 1886.

On December 1, 1875, Mr. [illegible] was married to [illegible] Washington, D. C., who died in [illegible] Sarah Butler, of [illegible], Mass., who died [illegible] vived by two sons, [illegible] Edith [illegible], who were born of the first marriage, [illegible] and [illegible] of [illegible], and [illegible] Mrs. [illegible] marriage.

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